Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4.5-Secant/236-4.5.2.3

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**286**]. This is test number [236].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (286)	0.00 (0)
Mathematica	98.60 (282)	1.40 (4)
Maple	92.31 (264)	7.69 (22)
Fricas	83.22 (238)	16.78 (48)
Giac	78.67 (225)	21.33 (61)
Mupad	66.78 (191)	33.22 (95)
Maxima	58.04 (166)	41.96 (120)
Reduce	48.95 (140)	51.05 (146)
Sympy	0.35 (1)	99.65 (285)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

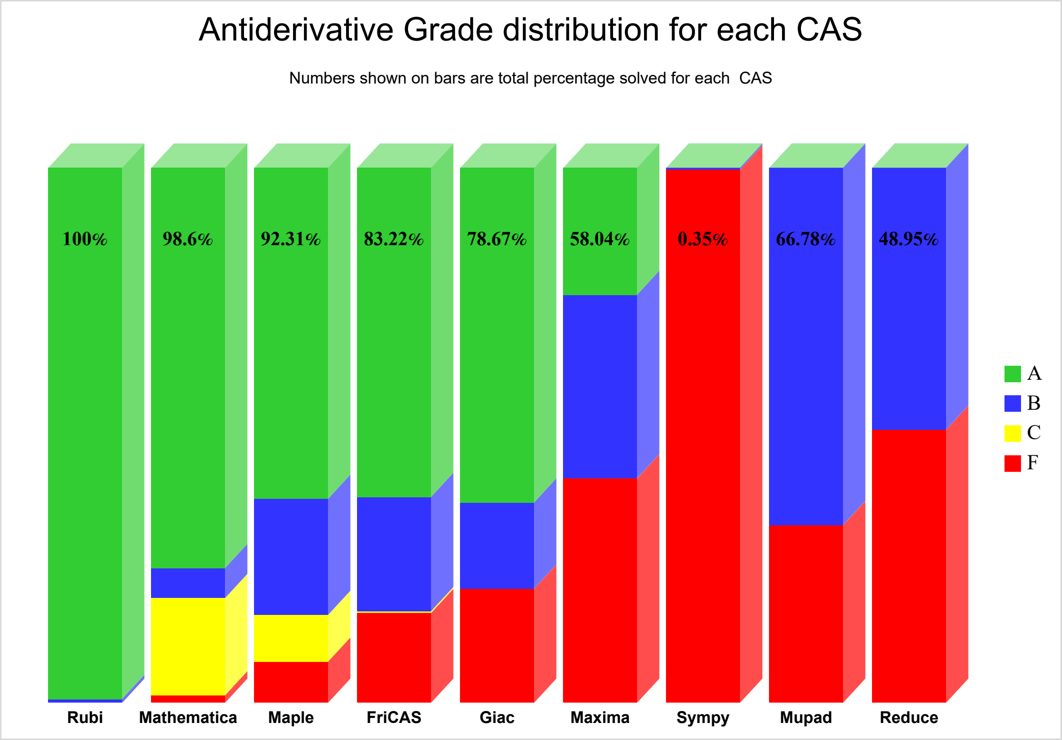
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

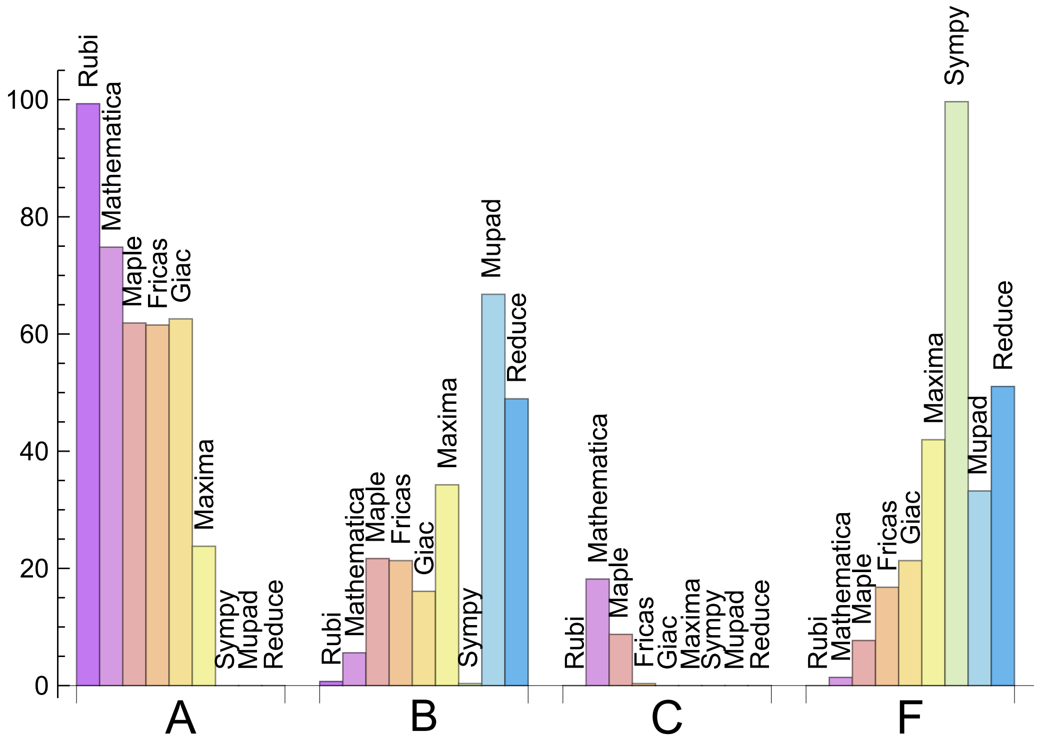
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.301	0.699	0.000	0.000
Mathematica	74.825	5.594	18.182	1.399
Giac	62.587	16.084	0.000	21.329
Maple	61.888	21.678	8.741	7.692
Fricas	61.538	21.329	0.350	16.783
Maxima	23.776	34.266	0.000	41.958
Mupad	0.000	66.783	0.000	33.217
Reduce	0.000	48.951	0.000	51.049
Sympy	0.000	0.350	0.000	99.650

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Maple	22	100.00	0.00	0.00
Fricas	48	68.75	31.25	0.00
Giac	61	78.69	0.00	21.31
Mupad	95	0.00	100.00	0.00
Maxima	120	58.33	6.67	35.00
Reduce	146	100.00	0.00	0.00
Sympy	285	84.91	15.09	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Reduce	0.19
Maxima	0.33
Giac	0.35
Rubi	0.60
Sympy	0.90
Mathematica	2.36
Maple	2.54
Fricas	2.97
Mupad	13.40

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	51.00	3.00	51.00	3.00
Rubi	141.19	1.08	120.50	1.00
Giac	183.32	1.34	121.00	1.18
Maple	229.03	1.80	158.00	1.30
Fricas	361.48	2.61	176.50	1.72
Maxima	453.47	4.44	219.00	2.21
Mathematica	463.93	2.38	84.00	0.95
Reduce	582.33	3.12	218.00	2.18
Mupad	803.47	4.71	158.00	1.46

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

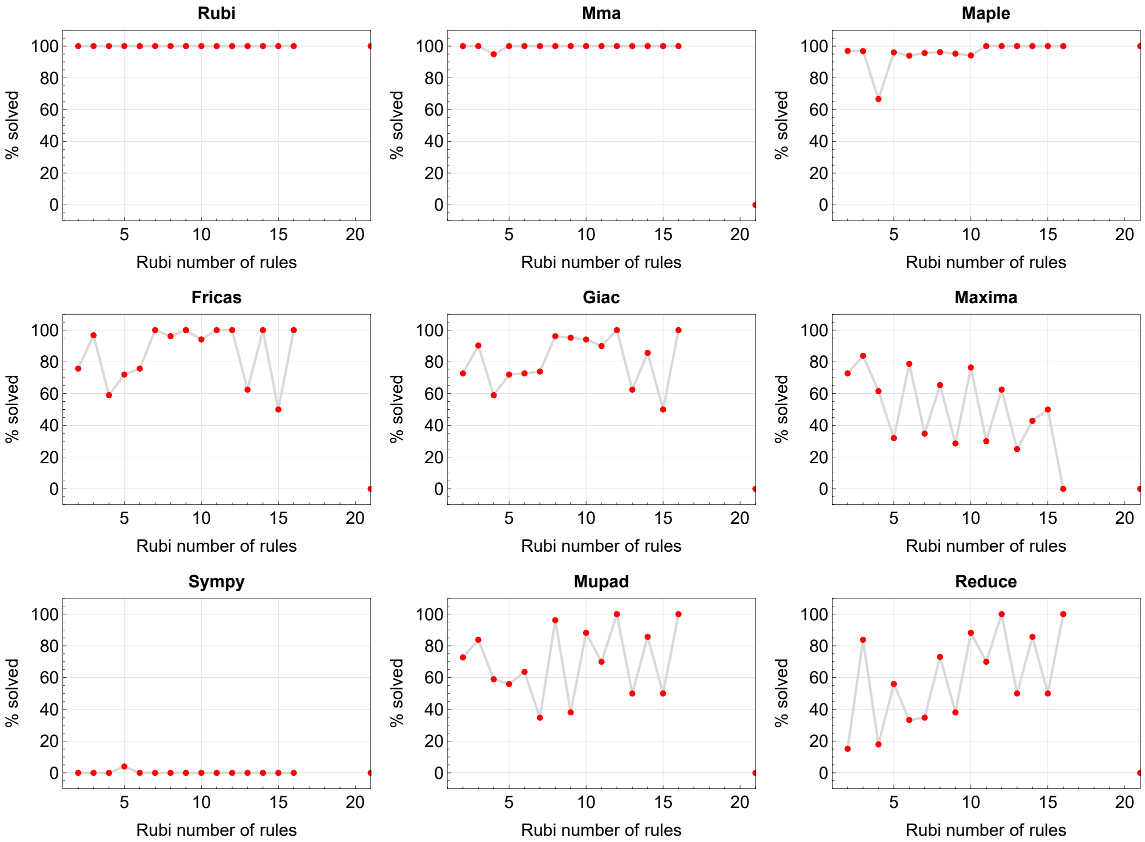


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

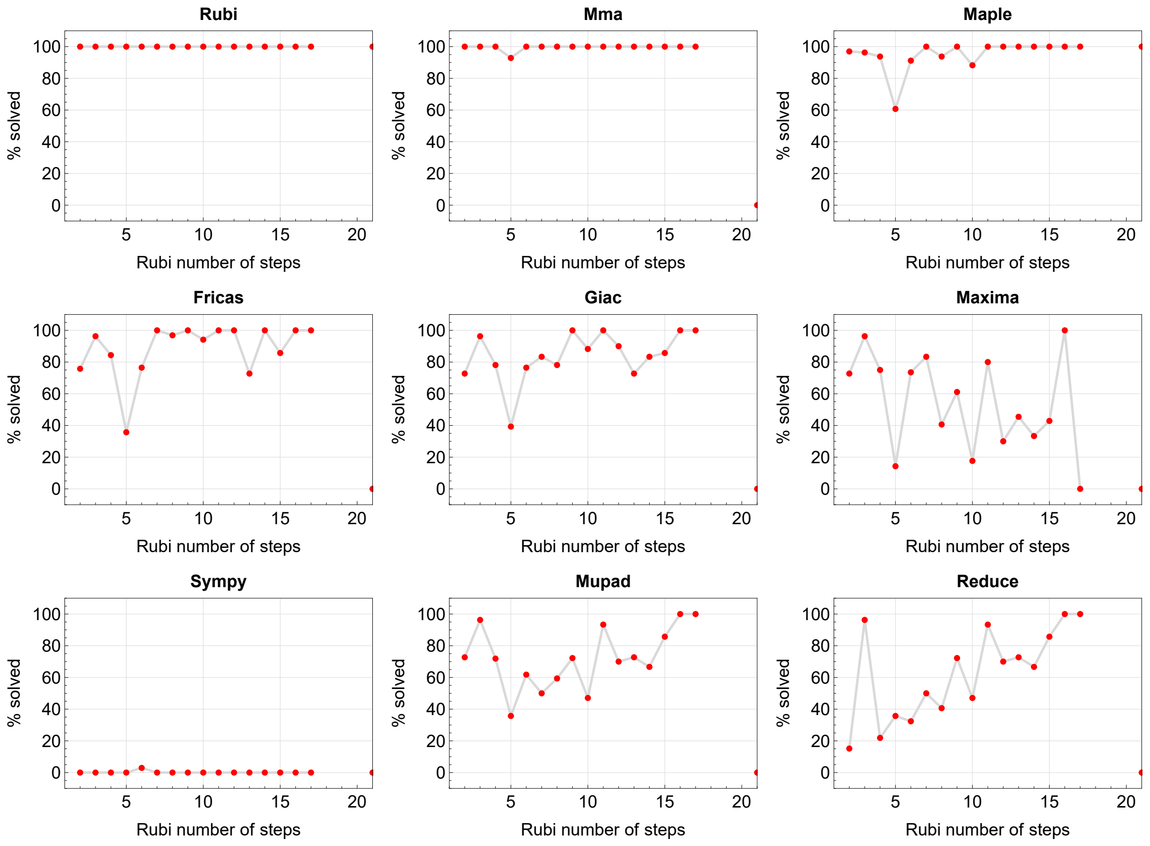


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

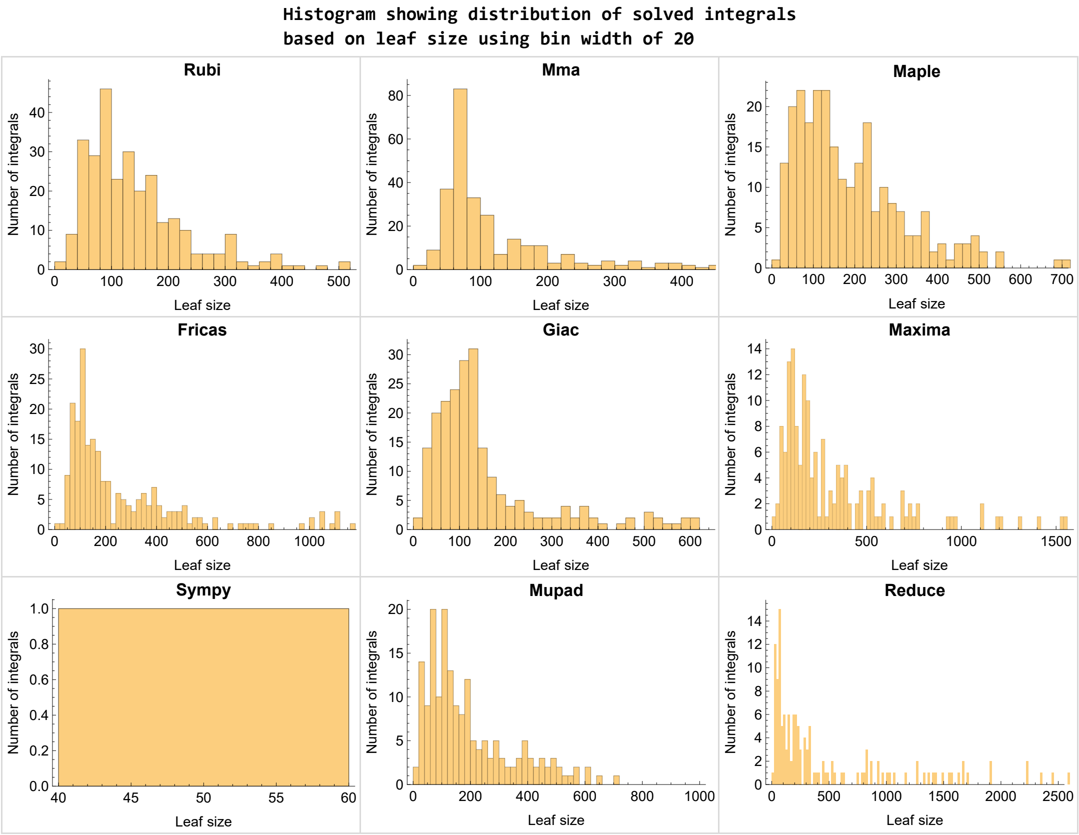


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

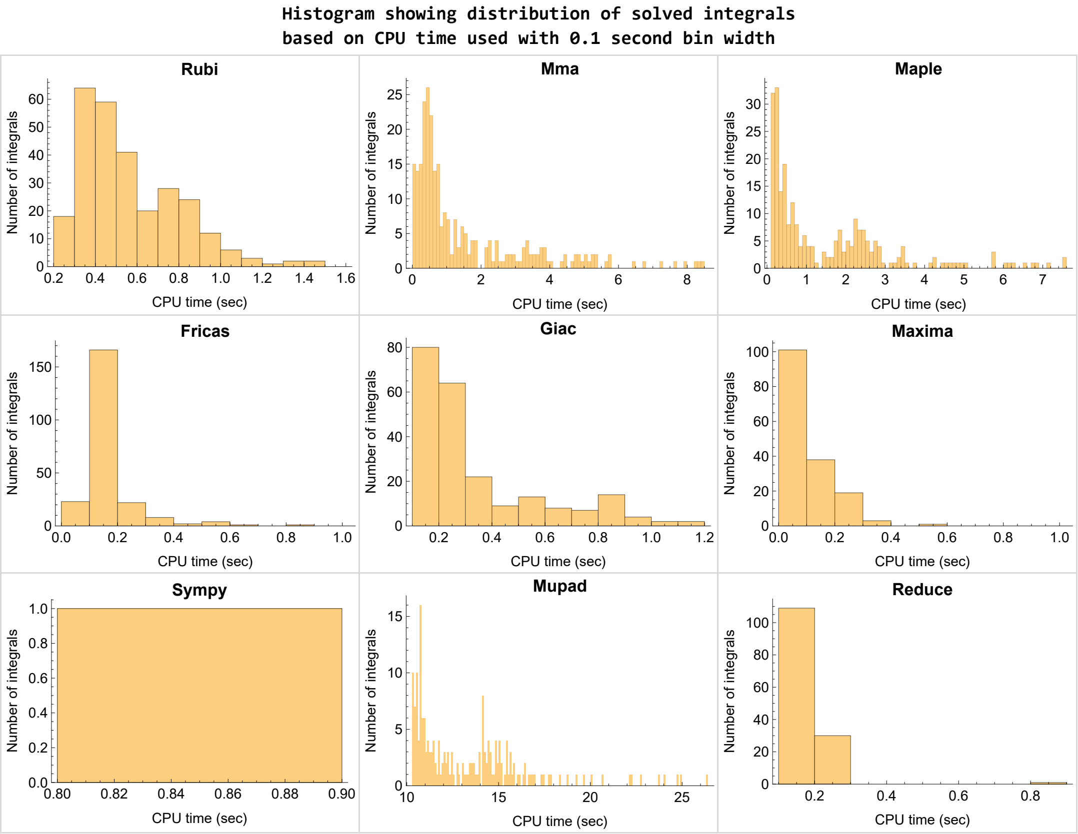


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

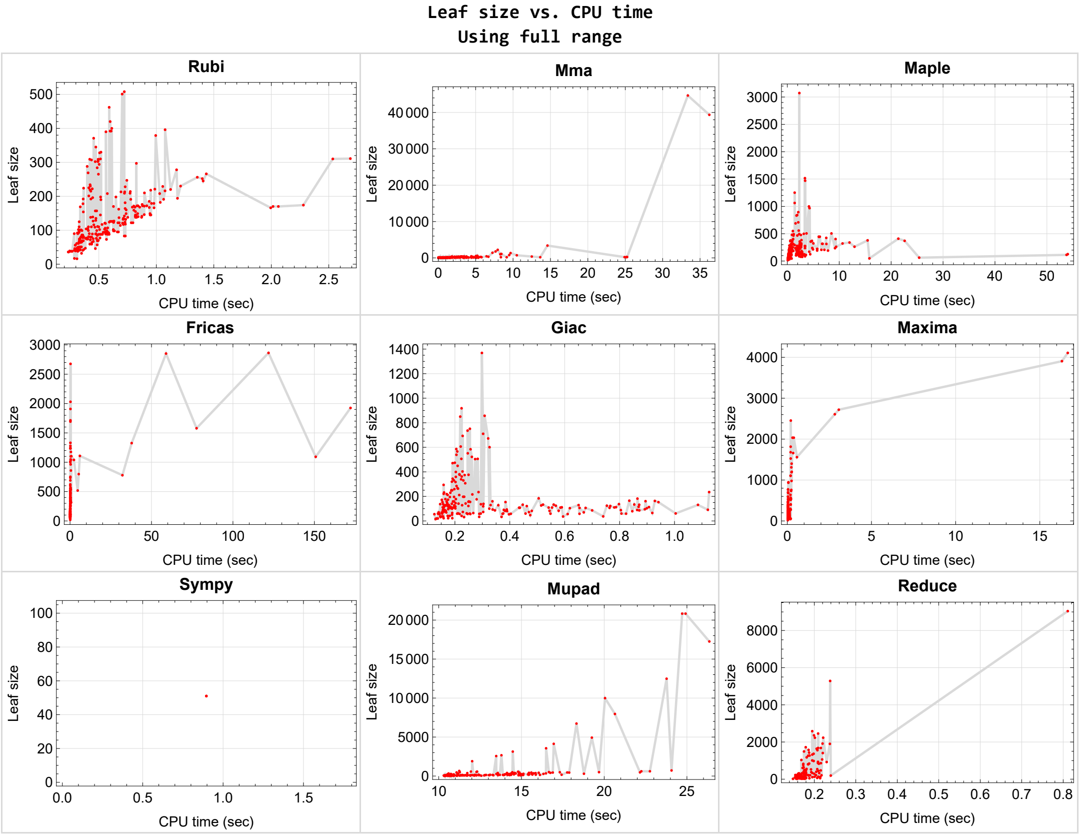


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{158, 174, 176, 181, 197, 198, 205, 206, 207, 208, 216, 223, 224, 232, 233, 265, 269, 275, 277, 281\}$

Maple $\{76,78,92,98,99,105,106,128,145,184,234,235,237,238,239,240,241,242,243,270\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

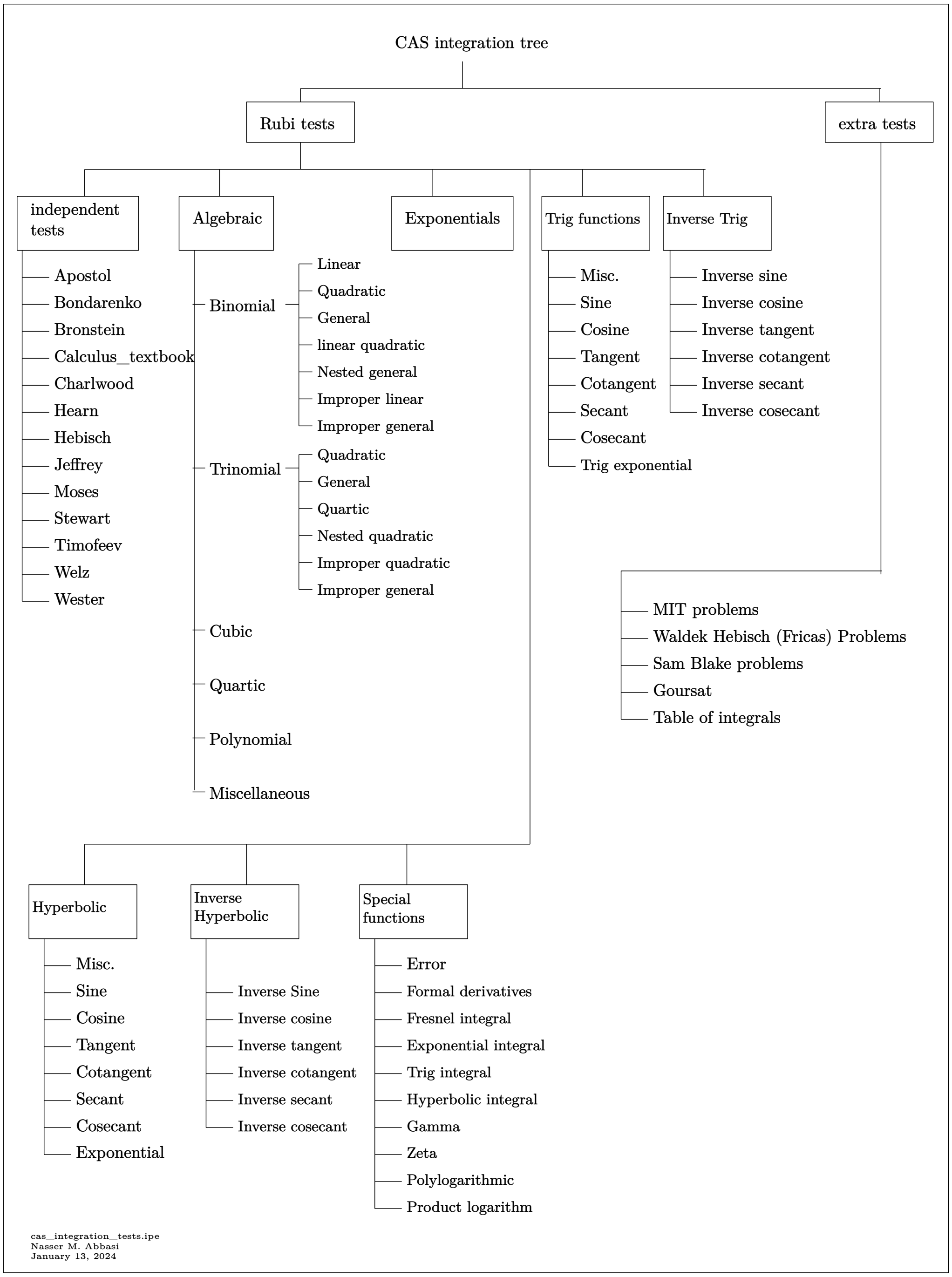
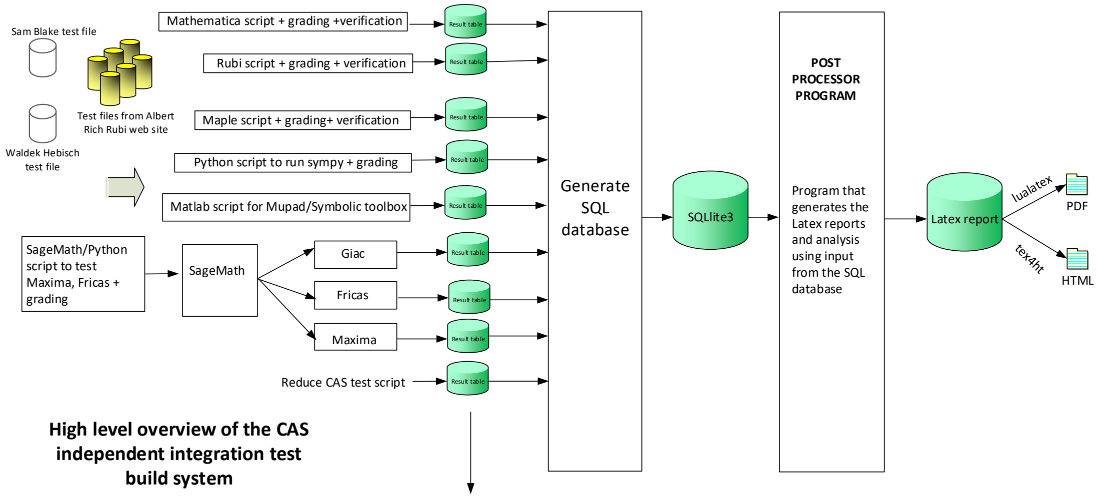


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286 }

B grade { 197, 212 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 175, 177, 178, 179, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 200, 201, 202, 203, 204, 209, 210, 211, 217, 218, 221, 226, 227, 229, 230, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 259, 260, 261, 262, 263, 264, 266, 267, 268, 271, 275, 276, 279, 281, 282, 283, 286 }

B grade { 3, 171, 174, 180, 181, 212, 213, 219, 220, 225, 228, 252, 253, 258, 273, 285 }

C grade { 15, 27, 28, 34, 35, 36, 42, 43, 44, 52, 53, 54, 70, 76, 78, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 158, 176, 183, 197, 198, 199, 205, 206, 207, 208, 214, 215, 216, 222, 223, 224, 231, 232, 233, 265, 269, 270, 272, 277, 280, 284 }

F normal fail { 154, 155, 274, 278 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 22, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 74, 75, 79, 80, 81, 82, 83, 86, 87, 88, 89, 93, 96, 103, 109, 113, 114, 115, 120, 121, 123, 124, 125, 129, 131, 132, 135, 163, 164, 168, 169, 171, 172, 173, 178, 180, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 269, 273, 275, 276, 279, 282, 285, 286 }

B grade { 66, 69, 70, 71, 72, 73, 76, 77, 78, 84, 85, 90, 91, 92, 94, 95, 98, 99, 100, 101, 102, 105, 106, 107, 108, 110, 111, 112, 116, 122, 126, 127, 128, 136, 137, 138, 139, 141, 142, 144, 145, 146, 147, 148, 149, 150, 170, 179, 182, 184, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 252, 281 }

C grade { 10, 12, 21, 23, 24, 25, 117, 118, 119, 130, 133, 134, 140, 143, 183, 267, 270, 271, 272, 274, 277, 278, 280, 283, 284 }

F normal fail { 68, 97, 104, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 174, 175, 176, 177 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 113, 114, 115, 120, 121, 123, 124, 125, 132, 138, 148, 149, 150, 156, 157, 158, 162, 163, 164, 168, 169, 171, 172, 173, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 193, 194, 195, 196, 197, 202, 203, 204, 205, 210, 211, 213, 214, 217, 218, 220, 221, 225, 226, 227, 228, 229, 230, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 262, 286 }

B grade { 17, 30, 56, 69, 74, 82, 107, 108, 111, 112, 116, 119, 122, 126, 130, 131, 136, 137, 141, 142, 143, 144, 146, 147, 170, 183, 184, 190, 191, 192, 198, 199, 200, 201, 206, 207, 208, 209, 212, 215, 216, 219, 222, 223, 224, 231, 232, 233, 234, 238, 250, 251, 252, 253, 254, 257, 259, 260, 261, 263, 285 }

C grade { 277 }

F normal fail { 110, 117, 118, 127, 128, 129, 133, 134, 135, 139, 140, 145, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 266, 267, 268, 273, 275, 276 }

F(-1) timedout fail { 258, 264, 265, 269, 270, 271, 272, 274, 278, 279, 280, 281, 282, 283, 284 }

F(-2) exception fail { }

Maxima

A grade { 2, 3, 4, 7, 8, 9, 14, 26, 38, 39, 40, 41, 47, 48, 49, 50, 51, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 93, 94, 95, 100, 101, 109, 110, 116, 118, 126, 129, 135, 136, 140, 141, 145, 147, 156, 157, 162, 163, 164, 168, 173, 183, 185, 186, 187, 188, 195, 196, 203, 221, 229, 230, 244, 245, 246, 247, 285, 286 }

B grade { 1, 5, 6, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 89, 96, 102, 103, 107, 108, 111, 112, 113, 114, 115, 117, 119, 120, 121, 122, 123, 124, 125, 127, 128, 130, 131, 132, 133, 134, 137, 138, 139, 142, 143, 146, 148, 150, 158, 169, 170, 171, 172, 179, 180, 181, 182, 193, 194, 202, 204, 210, 211, 212, 213, 217, 218, 219, 220, 225, 226, 227, 228 }

C grade { }

F normal fail { 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 82, 83, 84, 90, 91, 92, 97, 98, 99, 104, 105, 106, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 178, 184, 234, 235, 236, 237, 238, 240, 241, 242, 243, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

F(-1) timedout fail { 71, 72, 77, 78, 79, 80, 81, 85 }

F(-2) exception fail { 144, 149, 189, 190, 191, 192, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 214, 215, 216, 222, 223, 224, 231, 232, 233, 239, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

Giac

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 111, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 168, 169, 170, 171, 172, 173, 178, 180, 190, 195, 196, 199, 203, 204, 208, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 225, 226, 227, 229, 230, 248, 249, 255, 257, 261, 262, 263, 285, 286 }

B grade { 3, 14, 107, 108, 112, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 197, 198, 200, 201, 202, 205, 206, 207, 209, 217, 222, 223, 224, 228, 231, 232, 233, 238, 244, 245, 246, 247, 250, 251, 252, 253, 254, 256, 258, 259, 260 }

C grade { }

F normal fail { 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 174, 175, 179, 181, 182, 234, 235, 236, 237, 239, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

F(-1) timedout fail { }

F(-2) exception fail { 110, 117, 118, 127, 128, 129, 176, 177, 183, 240, 241, 242, 243 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 107, 108, 109, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 141, 146, 147, 157, 162, 163, 164, 168, 169, 170, 171, 172, 173, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 285, 286 }

C grade { }

F normal fail { }

F(-1) timedout fail { 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 110, 117, 118, 127, 128, 129, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { 170 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 102, 103, 104, 105, 108, 109, 110, 111, 112, 116, 117, 118, 119, 134, 135, 136, 137, 138, 140, 141, 142, 143, 146, 147, 148, 151, 152, 153, 154, 155, 158, 159, 160, 162, 163, 164, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 285, 286 }

F(-1) timedout fail { 64, 65, 71, 79, 80, 86, 93, 94, 100, 101, 106, 107, 113, 114, 115, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 139, 144, 145, 149, 150, 156, 157, 161, 165, 180, 182, 243, 278, 284 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 168, 169, 170, 171, 172, 173, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 285, 286 }

C grade { }

F normal fail { 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	104	154	159	215	131	0	145	242	176
N.S.	1	0.99	1.47	1.51	2.05	1.25	0.00	1.38	2.30	1.68
time (sec)	N/A	0.390	3.893	0.740	0.038	0.086	0.000	0.202	0.166	15.866

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	144	111	133	117	0	128	182	146
N.S.	1	1.00	1.67	1.29	1.55	1.36	0.00	1.49	2.12	1.70
time (sec)	N/A	0.352	0.940	0.348	0.033	0.085	0.000	0.169	0.177	14.148

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	56	132	98	108	103	0	111	150	114
N.S.	1	0.92	2.16	1.61	1.77	1.69	0.00	1.82	2.46	1.87
time (sec)	N/A	0.296	0.472	0.259	0.032	0.090	0.000	0.159	0.177	13.028

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	58	68	67	0	55	95	77
N.S.	1	1.00	1.00	1.53	1.79	1.76	0.00	1.45	2.50	2.03
time (sec)	N/A	0.317	0.019	0.171	0.027	0.083	0.000	0.125	0.156	11.451

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	77	47	101	66	0	60	66	31
N.S.	1	1.00	1.83	1.12	2.40	1.57	0.00	1.43	1.57	0.74
time (sec)	N/A	0.294	0.129	0.118	0.034	0.084	0.000	0.138	0.167	10.710

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	23	21	97	51	0	20	20	20
N.S.	1	1.00	0.64	0.58	2.69	1.42	0.00	0.56	0.56	0.56
time (sec)	N/A	0.234	0.185	0.105	0.045	0.069	0.000	0.138	0.158	10.404

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	43	36	117	78	0	37	35	35
N.S.	1	1.00	0.57	0.47	1.54	1.03	0.00	0.49	0.46	0.46
time (sec)	N/A	0.384	0.340	0.135	0.037	0.074	0.000	0.157	0.172	10.358

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	120	55	49	177	104	0	51	48	61
N.S.	1	1.03	0.47	0.42	1.53	0.90	0.00	0.44	0.41	0.53
time (sec)	N/A	0.557	3.411	0.155	0.041	0.070	0.000	0.190	0.173	10.586

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	164	65	62	197	128	0	65	61	106
N.S.	1	1.04	0.41	0.39	1.25	0.81	0.00	0.41	0.39	0.67
time (sec)	N/A	0.728	5.097	0.184	0.045	0.078	0.000	0.190	0.159	10.507

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	157	102	209	368	177	0	197	326	251
N.S.	1	0.92	0.60	1.22	2.15	1.04	0.00	1.15	1.91	1.47
time (sec)	N/A	0.481	1.089	0.719	0.039	0.094	0.000	0.234	0.176	14.137

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	139	91	195	321	161	0	178	248	219
N.S.	1	0.93	0.61	1.30	2.14	1.07	0.00	1.19	1.65	1.46
time (sec)	N/A	0.447	0.790	0.514	0.038	0.089	0.000	0.203	0.161	13.993

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	81	82	143	227	145	0	159	234	187
N.S.	1	0.86	0.87	1.52	2.41	1.54	0.00	1.69	2.49	1.99
time (sec)	N/A	0.346	0.385	0.381	0.037	0.090	0.000	0.197	0.181	15.426

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	67	80	79	150	99	0	87	168	155
N.S.	1	0.92	1.10	1.08	2.05	1.36	0.00	1.19	2.30	2.12
time (sec)	N/A	0.452	0.029	0.286	0.043	0.090	0.000	0.164	0.158	14.371

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	56	54	96	108	103	0	111	150	113
N.S.	1	0.92	0.89	1.57	1.77	1.69	0.00	1.82	2.46	1.85
time (sec)	N/A	0.294	0.089	0.237	0.036	0.107	0.000	0.156	0.175	13.072

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	69	65	82	225	108	0	100	147	77
N.S.	1	0.93	0.88	1.11	3.04	1.46	0.00	1.35	1.99	1.04
time (sec)	N/A	0.486	0.479	0.181	0.039	0.123	0.000	0.190	0.172	11.143

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	109	65	201	128	0	84	87	63
N.S.	1	1.00	1.22	0.73	2.26	1.44	0.00	0.94	0.98	0.71
time (sec)	N/A	0.472	0.173	0.177	0.039	0.118	0.000	0.162	0.163	11.086

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	189	83	0	22	22	22
N.S.	1	1.00	0.66	0.61	4.97	2.18	0.00	0.58	0.58	0.58
time (sec)	N/A	0.265	0.077	0.124	0.041	0.092	0.000	0.175	0.169	10.748

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	47	38	270	114	0	41	37	37
N.S.	1	1.00	0.59	0.48	3.38	1.42	0.00	0.51	0.46	0.46
time (sec)	N/A	0.427	0.286	0.164	0.044	0.095	0.000	0.174	0.157	11.083

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	126	59	51	269	140	0	57	50	67
N.S.	1	1.04	0.49	0.42	2.22	1.16	0.00	0.47	0.41	0.55
time (sec)	N/A	0.617	0.465	0.203	0.075	0.097	0.000	0.217	0.169	10.780

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	172	69	64	389	168	0	73	63	108
N.S.	1	1.06	0.42	0.39	2.39	1.03	0.00	0.45	0.39	0.66
time (sec)	N/A	0.832	1.079	0.253	0.056	0.106	0.000	0.229	0.158	10.749

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	208	122	253	443	209	0	235	408	316
N.S.	1	0.92	0.54	1.11	1.95	0.92	0.00	1.04	1.80	1.39
time (sec)	N/A	0.574	2.739	1.062	0.039	0.134	0.000	0.286	0.174	14.418

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	190	111	239	408	193	0	216	312	284
N.S.	1	0.92	0.54	1.16	1.98	0.94	0.00	1.05	1.51	1.38
time (sec)	N/A	0.518	1.890	0.783	0.041	0.138	0.000	0.264	0.163	14.103

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	104	102	176	368	177	0	197	316	252
N.S.	1	0.86	0.84	1.45	3.04	1.46	0.00	1.63	2.61	2.08
time (sec)	N/A	0.394	0.840	0.581	0.041	0.125	0.000	0.230	0.170	14.245

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	94	125	142	244	115	0	103	232	220
N.S.	1	0.94	1.25	1.42	2.44	1.15	0.00	1.03	2.32	2.20
time (sec)	N/A	0.549	0.034	0.445	0.036	0.124	0.000	0.186	0.168	14.178

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	81	81	143	227	145	0	159	234	188
N.S.	1	0.86	0.86	1.52	2.41	1.54	0.00	1.69	2.49	2.00
time (sec)	N/A	0.345	0.438	0.375	0.037	0.121	0.000	0.193	0.160	15.094

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	111	133	117	0	128	182	146
N.S.	1	1.00	0.81	1.29	1.55	1.36	0.00	1.49	2.12	1.70
time (sec)	N/A	0.360	0.345	0.303	0.035	0.128	0.000	0.167	0.167	13.877

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	98	65	106	387	125	0	118	224	105
N.S.	1	0.98	0.65	1.06	3.87	1.25	0.00	1.18	2.24	1.05
time (sec)	N/A	0.627	0.519	0.264	0.039	0.119	0.000	0.202	0.154	12.423

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	117	65	97	349	165	0	116	165	93
N.S.	1	0.98	0.55	0.82	2.93	1.39	0.00	0.97	1.39	0.78
time (sec)	N/A	0.713	0.555	0.286	0.040	0.120	0.000	0.183	0.169	11.345

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	135	139	78	309	176	0	102	100	78
N.S.	1	1.02	1.05	0.59	2.34	1.33	0.00	0.77	0.76	0.59
time (sec)	N/A	0.713	0.174	0.213	0.046	0.115	0.000	0.202	0.163	10.919

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	356	111	0	22	22	22
N.S.	1	1.00	0.66	0.61	9.37	2.92	0.00	0.58	0.58	0.58
time (sec)	N/A	0.256	0.104	0.146	0.048	0.107	0.000	0.191	0.175	10.616

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	47	38	357	140	0	41	37	37
N.S.	1	1.00	0.59	0.48	4.46	1.75	0.00	0.51	0.46	0.46
time (sec)	N/A	0.437	0.277	0.195	0.054	0.101	0.000	0.224	0.159	10.684

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	126	59	51	518	168	0	57	50	67
N.S.	1	1.04	0.49	0.42	4.28	1.39	0.00	0.47	0.41	0.55
time (sec)	N/A	0.619	0.381	0.204	0.065	0.113	0.000	0.246	0.157	10.708

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	172	69	64	517	194	0	73	63	108
N.S.	1	1.06	0.43	0.40	3.19	1.20	0.00	0.45	0.39	0.67
time (sec)	N/A	0.841	4.057	0.254	0.063	0.107	0.000	0.303	0.170	11.319

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	116	53	138	591	153	0	132	193	112
N.S.	1	0.96	0.44	1.14	4.88	1.26	0.00	1.09	1.60	0.93
time (sec)	N/A	0.456	0.813	0.409	0.044	0.118	0.000	0.176	0.158	11.410

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	98	53	106	386	140	0	116	151	96
N.S.	1	0.98	0.53	1.06	3.86	1.40	0.00	1.16	1.51	0.96
time (sec)	N/A	0.636	0.427	0.299	0.038	0.112	0.000	0.177	0.168	10.961

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	70	62	80	224	119	0	97	98	77
N.S.	1	0.95	0.84	1.08	3.03	1.61	0.00	1.31	1.32	1.04
time (sec)	N/A	0.495	0.347	0.226	0.037	0.125	0.000	0.158	0.153	10.804

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	77	47	101	70	0	58	46	31
N.S.	1	1.00	1.88	1.15	2.46	1.71	0.00	1.41	1.12	0.76
time (sec)	N/A	0.293	0.109	0.115	0.037	0.121	0.000	0.139	0.167	10.418

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	18	18	0	18	18	18
N.S.	1	1.00	1.00	1.06	1.12	1.12	0.00	1.12	1.12	1.12
time (sec)	N/A	0.309	0.019	0.100	0.032	0.092	0.000	0.131	0.149	10.339

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	51	53	48	77	50	0	56	50	50
N.S.	1	0.86	0.90	0.81	1.31	0.85	0.00	0.95	0.85	0.85
time (sec)	N/A	0.340	0.394	0.142	0.041	0.105	0.000	0.144	0.168	10.384

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	74	61	60	97	74	0	69	63	63
N.S.	1	0.95	0.78	0.77	1.24	0.95	0.00	0.88	0.81	0.81
time (sec)	N/A	0.398	0.448	0.166	0.036	0.102	0.000	0.168	0.166	10.540

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	109	79	74	117	102	0	82	76	83
N.S.	1	0.91	0.66	0.62	0.98	0.85	0.00	0.68	0.63	0.69
time (sec)	N/A	0.466	1.282	0.200	0.041	0.109	0.000	0.167	0.159	10.933

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	162	75	155	765	210	0	156	294	170
N.S.	1	0.99	0.46	0.95	4.66	1.28	0.00	0.95	1.79	1.04
time (sec)	N/A	0.673	2.256	0.503	0.047	0.120	0.000	0.249	0.161	10.544

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	146	75	125	531	197	0	140	218	136
N.S.	1	0.97	0.50	0.83	3.54	1.31	0.00	0.93	1.45	0.91
time (sec)	N/A	0.895	1.525	0.384	0.047	0.125	0.000	0.189	0.158	10.551

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	118	75	95	341	178	0	121	142	104
N.S.	1	0.99	0.63	0.80	2.87	1.50	0.00	1.02	1.19	0.87
time (sec)	N/A	0.731	0.382	0.290	0.041	0.122	0.000	0.175	0.174	10.426

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	109	65	196	138	0	89	64	46
N.S.	1	1.00	1.24	0.74	2.23	1.57	0.00	1.01	0.73	0.52
time (sec)	N/A	0.500	0.134	0.187	0.059	0.115	0.000	0.161	0.157	10.376

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	23	21	94	53	0	20	20	20
N.S.	1	1.00	0.64	0.58	2.61	1.47	0.00	0.56	0.56	0.56
time (sec)	N/A	0.238	0.052	0.104	0.035	0.102	0.000	0.133	0.177	10.327

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	51	59	48	76	49	0	69	50	61
N.S.	1	0.86	1.00	0.81	1.29	0.83	0.00	1.17	0.85	1.03
time (sec)	N/A	0.348	0.615	0.132	0.033	0.097	0.000	0.152	0.168	10.436

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	32	33	28	31	50	0	31	31	28
N.S.	1	0.84	0.87	0.74	0.82	1.32	0.00	0.82	0.82	0.74
time (sec)	N/A	0.349	0.034	0.146	0.032	0.103	0.000	0.148	0.156	10.912

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	69	79	76	121	109	0	96	76	76
N.S.	1	0.86	0.99	0.95	1.51	1.36	0.00	1.20	0.95	0.95
time (sec)	N/A	0.365	0.701	0.171	0.041	0.099	0.000	0.176	0.172	11.368

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	90	89	87	140	120	0	109	89	89
N.S.	1	0.92	0.91	0.89	1.43	1.22	0.00	1.11	0.91	0.91
time (sec)	N/A	0.417	1.487	0.210	0.039	0.108	0.000	0.194	0.160	12.098

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	129	99	99	161	163	0	122	102	102
N.S.	1	0.91	0.70	0.70	1.14	1.16	0.00	0.87	0.72	0.72
time (sec)	N/A	0.473	3.340	0.247	0.041	0.114	0.000	0.249	0.174	13.306

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	210	75	168	935	263	0	176	307	193
N.S.	1	0.98	0.35	0.78	4.35	1.22	0.00	0.82	1.43	0.90
time (sec)	N/A	0.896	4.459	0.618	0.064	0.146	0.000	0.279	0.163	10.881

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	194	75	136	680	250	0	159	231	159
N.S.	1	1.01	0.39	0.70	3.52	1.30	0.00	0.82	1.20	0.82
time (sec)	N/A	1.184	2.929	0.487	0.058	0.120	0.000	0.251	0.166	10.784

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	166	75	108	470	231	0	141	155	126
N.S.	1	1.01	0.46	0.66	2.87	1.41	0.00	0.86	0.95	0.77
time (sec)	N/A	0.980	0.829	0.356	0.047	0.129	0.000	0.213	0.170	10.874

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	134	139	76	304	192	0	109	77	61
N.S.	1	1.02	1.06	0.58	2.32	1.47	0.00	0.83	0.59	0.47
time (sec)	N/A	0.717	0.160	0.230	0.045	0.125	0.000	0.188	0.159	10.722

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	185	82	0	22	22	22
N.S.	1	1.00	0.66	0.61	4.87	2.16	0.00	0.58	0.58	0.58
time (sec)	N/A	0.268	0.069	0.153	0.040	0.105	0.000	0.159	0.170	10.558

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	43	36	115	79	0	37	35	35
N.S.	1	1.00	0.57	0.47	1.51	1.04	0.00	0.49	0.46	0.46
time (sec)	N/A	0.390	0.168	0.142	0.058	0.105	0.000	0.147	0.160	10.644

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	74	67	59	95	76	0	87	61	74
N.S.	1	0.95	0.86	0.76	1.22	0.97	0.00	1.12	0.78	0.95
time (sec)	N/A	0.403	0.428	0.152	0.034	0.102	0.000	0.159	0.179	10.347

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	69	79	74	120	109	0	103	76	111
N.S.	1	0.86	0.99	0.92	1.50	1.36	0.00	1.29	0.95	1.39
time (sec)	N/A	0.353	0.644	0.175	0.039	0.099	0.000	0.177	0.163	10.863

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	41	50	41	41	76	0	41	41	38
N.S.	1	0.69	0.85	0.69	0.69	1.29	0.00	0.69	0.69	0.64
time (sec)	N/A	0.361	0.055	0.191	0.038	0.101	0.000	0.181	0.157	10.730

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	81	99	99	159	163	0	128	102	129
N.S.	1	0.82	1.00	1.00	1.61	1.65	0.00	1.29	1.03	1.30
time (sec)	N/A	0.367	2.263	0.225	0.044	0.104	0.000	0.226	0.179	11.430

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	110	109	110	181	190	0	142	115	109
N.S.	1	0.92	0.91	0.92	1.51	1.58	0.00	1.18	0.96	0.91
time (sec)	N/A	0.435	4.719	0.298	0.042	0.116	0.000	0.239	0.172	11.710

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	145	119	121	200	217	0	155	128	120
N.S.	1	0.90	0.73	0.75	1.23	1.34	0.00	0.96	0.79	0.74
time (sec)	N/A	0.513	5.122	0.330	0.042	0.111	0.000	0.255	0.201	12.258

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	171	70	239	0	119	0	107	100	483
N.S.	1	1.05	0.43	1.47	0.00	0.73	0.00	0.66	0.61	2.96
time (sec)	N/A	0.804	1.868	4.812	0.000	0.121	0.000	0.436	0.238	17.316

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	126	60	213	0	105	0	80	98	384
N.S.	1	1.03	0.49	1.75	0.00	0.86	0.00	0.66	0.80	3.15
time (sec)	N/A	0.598	0.589	4.954	0.000	0.118	0.000	0.390	0.229	14.394

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	50	183	0	82	0	56	50	120
N.S.	1	1.00	0.62	2.26	0.00	1.01	0.00	0.69	0.62	1.48
time (sec)	N/A	0.410	0.349	2.085	0.000	0.114	0.000	0.256	0.182	14.286

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	38	45	0	65	0	31	47	87
N.S.	1	1.00	0.97	1.15	0.00	1.67	0.00	0.79	1.21	2.23
time (sec)	N/A	0.246	0.144	0.835	0.000	0.116	0.000	0.233	0.194	11.500

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	0	0	272	0	61	71	0
N.S.	1	1.00	1.01	0.00	0.00	3.53	0.00	0.79	0.92	0.00
time (sec)	N/A	0.391	0.203	0.000	0.000	0.167	0.000	0.376	0.190	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	81	894	0	342	0	72	90	0
N.S.	1	1.00	1.07	11.76	0.00	4.50	0.00	0.95	1.18	0.00
time (sec)	N/A	0.390	0.674	2.118	0.000	0.198	0.000	0.470	0.200	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	117	60	708	0	405	0	104	111	0
N.S.	1	1.04	0.53	6.27	0.00	3.58	0.00	0.92	0.98	0.00
time (sec)	N/A	0.528	0.412	2.120	0.000	0.191	0.000	0.619	0.224	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	179	88	378	0	147	0	109	148	606
N.S.	1	1.05	0.51	2.21	0.00	0.86	0.00	0.64	0.87	3.54
time (sec)	N/A	0.958	2.126	15.467	0.000	0.118	0.000	0.542	0.242	22.246

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	132	78	339	0	131	0	82	76	503
N.S.	1	1.03	0.61	2.65	0.00	1.02	0.00	0.64	0.59	3.93
time (sec)	N/A	0.711	1.439	11.987	0.000	0.128	0.000	0.383	0.219	16.657

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	294	0	105	0	58	98	384
N.S.	1	1.00	0.78	3.46	0.00	1.24	0.00	0.68	1.15	4.52
time (sec)	N/A	0.487	0.876	1.409	0.000	0.123	0.000	0.374	0.225	14.866

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	49	0	84	0	33	73	93
N.S.	1	1.00	1.34	1.20	0.00	2.05	0.00	0.80	1.78	2.27
time (sec)	N/A	0.287	0.528	0.639	0.000	0.119	0.000	0.288	0.203	14.820

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	123	107	95	0	343	0	82	110	0
N.S.	1	1.05	0.91	0.81	0.00	2.93	0.00	0.70	0.94	0.00
time (sec)	N/A	0.628	0.482	0.543	0.000	0.160	0.000	0.480	0.228	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	113	126	64	1251	0	372	0	109	136	0
N.S.	1	1.12	0.57	11.07	0.00	3.29	0.00	0.96	1.20	0.00
time (sec)	N/A	0.606	0.361	1.420	0.000	0.181	0.000	0.585	0.229	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	150	1062	0	429	0	106	170	0
N.S.	1	1.09	1.28	9.08	0.00	3.67	0.00	0.91	1.45	0.00
time (sec)	N/A	0.631	0.999	1.418	0.000	0.207	0.000	0.676	0.223	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	164	168	64	368	0	517	0	133	196	0
N.S.	1	1.02	0.39	2.24	0.00	3.15	0.00	0.81	1.20	0.00
time (sec)	N/A	0.791	0.517	4.744	0.000	0.187	0.000	0.560	0.244	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	179	88	126	0	163	0	109	102	710
N.S.	1	1.05	0.51	0.74	0.00	0.95	0.00	0.64	0.60	4.15
time (sec)	N/A	0.979	3.346	54.018	0.000	0.131	0.000	0.554	0.245	24.069

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	132	78	113	0	147	0	82	144	607
N.S.	1	1.03	0.61	0.88	0.00	1.15	0.00	0.64	1.12	4.74
time (sec)	N/A	0.733	0.924	53.786	0.000	0.130	0.000	0.545	0.250	22.762

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	98	0	119	0	58	100	471
N.S.	1	1.00	0.78	1.15	0.00	1.40	0.00	0.68	1.18	5.54
time (sec)	N/A	0.487	0.585	2.838	0.000	0.125	0.000	0.404	0.221	22.163

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	49	0	97	0	33	97	375
N.S.	1	1.00	1.34	1.20	0.00	2.37	0.00	0.80	2.37	9.15
time (sec)	N/A	0.292	0.399	0.678	0.000	0.122	0.000	0.370	0.244	15.853

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	168	119	118	0	377	0	111	144	0
N.S.	1	1.02	0.73	0.72	0.00	2.30	0.00	0.68	0.88	0.00
time (sec)	N/A	0.855	0.660	0.456	0.000	0.189	0.000	0.589	0.224	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	171	64	498	0	432	0	124	180	0
N.S.	1	1.02	0.38	2.96	0.00	2.57	0.00	0.74	1.07	0.00
time (sec)	N/A	0.895	0.515	3.174	0.000	0.199	0.000	0.568	0.246	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	176	64	383	0	441	0	133	224	0
N.S.	1	1.01	0.37	2.20	0.00	2.53	0.00	0.76	1.29	0.00
time (sec)	N/A	0.877	0.545	2.733	0.000	0.180	0.000	0.521	0.252	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	72	124	163	88	0	108	142	164
N.S.	1	1.00	0.51	0.87	1.15	0.62	0.00	0.76	1.00	1.15
time (sec)	N/A	0.722	0.766	4.418	0.119	0.119	0.000	0.434	0.227	17.426

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	107	62	111	137	77	0	86	106	125
N.S.	1	0.99	0.57	1.03	1.27	0.71	0.00	0.80	0.98	1.16
time (sec)	N/A	0.577	0.478	3.589	0.119	0.114	0.000	0.346	0.227	13.931

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	50	78	110	50	0	60	72	77
N.S.	1	1.00	0.69	1.08	1.53	0.69	0.00	0.83	1.00	1.07
time (sec)	N/A	0.426	0.277	2.572	0.123	0.113	0.000	0.254	0.217	12.749

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	31	84	45	0	59	36	40
N.S.	1	1.00	0.74	0.79	2.15	1.15	0.00	1.51	0.92	1.03
time (sec)	N/A	0.296	0.095	0.408	0.117	0.119	0.000	0.208	0.191	12.569

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	52	467	0	269	0	64	42	0
N.S.	1	1.00	0.58	5.25	0.00	3.02	0.00	0.72	0.47	0.00
time (sec)	N/A	0.444	0.266	1.935	0.000	0.189	0.000	0.267	0.208	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	124	58	477	0	329	0	100	59	0
N.S.	1	1.02	0.48	3.91	0.00	2.70	0.00	0.82	0.48	0.00
time (sec)	N/A	0.591	0.461	2.618	0.000	0.191	0.000	0.325	0.213	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	156	160	58	493	0	401	0	128	60	0
N.S.	1	1.03	0.37	3.16	0.00	2.57	0.00	0.82	0.38	0.00
time (sec)	N/A	0.738	0.508	2.293	0.000	0.209	0.000	0.358	0.197	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	72	261	188	103	0	121	182	188
N.S.	1	1.00	0.46	1.68	1.21	0.66	0.00	0.78	1.17	1.21
time (sec)	N/A	0.819	1.247	9.405	0.122	0.137	0.000	0.462	0.262	15.465

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	122	64	235	163	82	0	93	136	136
N.S.	1	0.99	0.52	1.91	1.33	0.67	0.00	0.76	1.11	1.11
time (sec)	N/A	0.677	0.775	9.190	0.120	0.115	0.000	0.337	0.225	15.043

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	88	54	170	110	72	0	60	92	134
N.S.	1	0.99	0.61	1.91	1.24	0.81	0.00	0.67	1.03	1.51
time (sec)	N/A	0.521	0.313	4.220	0.126	0.117	0.000	0.294	0.213	15.105

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	49	109	60	0	62	46	94
N.S.	1	1.00	1.34	1.20	2.66	1.46	0.00	1.51	1.12	2.29
time (sec)	N/A	0.293	0.101	0.467	0.122	0.120	0.000	0.211	0.210	14.939

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	137	60	0	0	331	0	93	58	0
N.S.	1	0.99	0.43	0.00	0.00	2.40	0.00	0.67	0.42	0.00
time (sec)	N/A	0.675	0.274	0.000	0.000	0.198	0.000	0.389	0.197	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	169	172	64	968	0	369	0	129	51	0
N.S.	1	1.02	0.38	5.73	0.00	2.18	0.00	0.76	0.30	0.00
time (sec)	N/A	0.880	0.433	4.285	0.000	0.193	0.000	0.330	0.211	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	203	208	64	999	0	487	0	160	78	0
N.S.	1	1.02	0.32	4.92	0.00	2.40	0.00	0.79	0.38	0.00
time (sec)	N/A	1.029	0.638	4.173	0.000	0.205	0.000	0.337	0.210	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	172	74	409	214	109	0	126	222	492
N.S.	1	1.02	0.44	2.42	1.27	0.64	0.00	0.75	1.31	2.91
time (sec)	N/A	0.917	0.654	21.391	0.132	0.118	0.000	0.514	0.233	19.686

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	138	64	370	189	104	0	90	166	456
N.S.	1	1.02	0.47	2.74	1.40	0.77	0.00	0.67	1.23	3.38
time (sec)	N/A	0.752	1.028	22.603	0.128	0.116	0.000	0.378	0.239	17.765

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	54	273	163	88	0	58	112	446
N.S.	1	1.00	0.61	3.10	1.85	1.00	0.00	0.66	1.27	5.07
time (sec)	N/A	0.514	0.453	3.435	0.125	0.123	0.000	0.289	0.219	16.893

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	49	136	74	0	62	56	441
N.S.	1	1.00	1.34	1.20	3.32	1.80	0.00	1.51	1.37	10.76
time (sec)	N/A	0.292	0.098	0.470	0.121	0.121	0.000	0.243	0.189	17.884

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	185	60	0	0	401	0	119	60	0
N.S.	1	1.02	0.33	0.00	0.00	2.22	0.00	0.66	0.33	0.00
time (sec)	N/A	0.938	0.678	0.000	0.000	0.182	0.000	0.303	0.198	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	212	220	64	1470	0	483	0	154	75	0
N.S.	1	1.04	0.30	6.93	0.00	2.28	0.00	0.73	0.35	0.00
time (sec)	N/A	1.124	0.709	3.441	0.000	0.198	0.000	0.384	0.202	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	246	256	64	1516	0	461	0	184	62	0
N.S.	1	1.04	0.26	6.16	0.00	1.87	0.00	0.75	0.25	0.00
time (sec)	N/A	1.356	0.935	3.398	0.000	0.254	0.000	0.505	0.195	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	70	126	638	93	0	107	102	136
N.S.	1	1.00	1.63	2.93	14.84	2.16	0.00	2.49	2.37	3.16
time (sec)	N/A	0.315	0.702	2.424	0.189	0.125	0.000	0.763	0.284	12.858

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	60	109	298	78	0	83	69	78
N.S.	1	1.00	1.40	2.53	6.93	1.81	0.00	1.93	1.60	1.81
time (sec)	N/A	0.319	0.522	1.816	0.183	0.125	0.000	0.578	0.272	11.566

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	51	52	55	56	0	54	34	47
N.S.	1	1.00	1.24	1.27	1.34	1.37	0.00	1.32	0.83	1.15
time (sec)	N/A	0.319	0.287	0.464	0.129	0.112	0.000	0.397	0.230	11.268

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	180	92	0	0	0	48	0
N.S.	1	1.00	1.06	3.53	1.80	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.329	0.261	2.260	0.129	0.000	0.000	0.000	0.189	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	103	514	79	0	58	57	118
N.S.	1	1.00	1.00	2.45	12.24	1.88	0.00	1.38	1.36	2.81
time (sec)	N/A	0.330	0.443	1.681	0.203	0.126	0.000	0.654	0.193	12.007

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	56	122	758	106	0	86	68	203
N.S.	1	1.00	1.30	2.84	17.63	2.47	0.00	2.00	1.58	4.72
time (sec)	N/A	0.338	0.461	2.047	0.240	0.115	0.000	0.872	0.191	15.651

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	127	1680	112	0	132	138	294
N.S.	1	1.00	0.94	1.43	18.88	1.26	0.00	1.48	1.55	3.30
time (sec)	N/A	0.561	2.545	2.108	0.198	0.124	0.000	0.871	0.312	15.619

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	87	127	1105	112	0	110	136	195
N.S.	1	1.00	0.98	1.43	12.42	1.26	0.00	1.24	1.53	2.19
time (sec)	N/A	0.561	1.007	1.758	0.195	0.134	0.000	0.781	0.320	15.665

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	64	136	550	82	0	86	70	108
N.S.	1	1.00	0.72	1.53	6.18	0.92	0.00	0.97	0.79	1.21
time (sec)	N/A	0.570	0.595	1.740	0.189	0.122	0.000	0.700	0.254	13.578

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	66	115	56	76	0	56	67	76
N.S.	1	1.00	1.53	2.67	1.30	1.77	0.00	1.30	1.56	1.77
time (sec)	N/A	0.325	0.342	1.810	0.129	0.116	0.000	0.611	0.255	12.280

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	62	278	275	0	0	0	91	0
N.S.	1	1.00	0.65	2.93	2.89	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	0.556	0.374	1.867	0.207	0.000	0.000	0.000	0.200	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	229	122	0	0	0	110	0
N.S.	1	1.00	0.71	2.31	1.23	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.568	0.508	1.834	0.144	0.000	0.000	0.000	0.224	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	116	533	95	0	61	131	165
N.S.	1	1.00	1.00	2.76	12.69	2.26	0.00	1.45	3.12	3.93
time (sec)	N/A	0.335	0.766	1.863	0.206	0.116	0.000	1.004	0.189	14.195

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	136	1559	133	0	89	150	273
N.S.	1	1.00	0.77	1.55	17.72	1.51	0.00	1.01	1.70	3.10
time (sec)	N/A	0.590	1.504	1.796	0.573	0.119	0.000	0.864	0.201	16.326

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	68	149	2608	158	0	113	171	340
N.S.	1	1.00	0.74	1.62	28.35	1.72	0.00	1.23	1.86	3.70
time (sec)	N/A	0.586	3.788	1.894	2.822	0.121	0.000	0.907	0.195	16.480

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	68	162	3906	184	0	137	190	407
N.S.	1	1.00	0.74	1.76	42.46	2.00	0.00	1.49	2.07	4.42
time (sec)	N/A	0.585	5.336	2.000	16.313	0.124	0.000	0.798	0.215	16.744

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	138	92	142	2454	152	0	134	204	307
N.S.	1	1.03	0.69	1.06	18.31	1.13	0.00	1.00	1.52	2.29
time (sec)	N/A	0.819	3.767	2.202	0.210	0.117	0.000	0.853	0.354	15.422

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	138	76	142	1526	106	0	112	105	215
N.S.	1	1.03	0.57	1.06	11.39	0.79	0.00	0.84	0.78	1.60
time (sec)	N/A	0.828	1.108	2.056	0.202	0.122	0.000	0.787	0.262	15.004

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	88	140	1106	112	0	88	136	195
N.S.	1	1.00	0.99	1.57	12.43	1.26	0.00	0.99	1.53	2.19
time (sec)	N/A	0.572	0.747	2.256	0.199	0.123	0.000	0.805	0.301	14.829

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	70	131	58	93	0	60	102	136
N.S.	1	1.00	1.63	3.05	1.35	2.16	0.00	1.40	2.37	3.16
time (sec)	N/A	0.321	0.378	2.201	0.134	0.115	0.000	0.807	0.250	13.257

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	142	76	322	737	0	0	0	139	0
N.S.	1	1.01	0.54	2.28	5.23	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.818	0.309	2.701	0.227	0.000	0.000	0.000	0.205	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	145	143	78	295	2035	0	0	0	165	0
N.S.	1	0.99	0.54	2.03	14.03	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.824	0.566	2.706	0.387	0.000	0.000	0.000	0.210	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	149	81	258	169	0	0	0	199	0
N.S.	1	1.03	0.56	1.78	1.17	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	0.837	0.759	2.670	0.124	0.000	0.000	0.000	0.214	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	133	1815	126	0	65	225	199
N.S.	1	1.00	1.00	3.17	43.21	3.00	0.00	1.55	5.36	4.74
time (sec)	N/A	0.333	1.582	2.625	0.200	0.132	0.000	0.919	0.237	15.394

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	78	151	2719	166	0	91	259	350
N.S.	1	1.00	0.89	1.72	30.90	1.89	0.00	1.03	2.94	3.98
time (sec)	N/A	0.578	3.539	2.452	3.059	0.130	0.000	1.121	0.226	15.729

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	138	78	164	4108	194	0	115	285	419
N.S.	1	1.04	0.59	1.23	30.89	1.46	0.00	0.86	2.14	3.15
time (sec)	N/A	0.872	5.359	2.876	16.659	0.127	0.000	0.869	0.233	16.042

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	141	74	350	737	0	0	161	135	0
N.S.	1	1.01	0.53	2.52	5.30	0.00	0.00	1.16	0.97	0.00
time (sec)	N/A	0.817	0.559	2.403	0.221	0.000	0.000	0.894	0.203	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	60	283	276	0	0	129	92	0
N.S.	1	1.00	0.64	3.01	2.94	0.00	0.00	1.37	0.98	0.00
time (sec)	N/A	0.538	0.403	2.309	0.198	0.000	0.000	0.651	0.176	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	78	64	0	0	59	47	0
N.S.	1	1.00	1.00	1.56	1.28	0.00	0.00	1.18	0.94	0.00
time (sec)	N/A	0.317	0.205	0.638	0.126	0.000	0.000	0.548	0.179	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	177	44	212	0	70	53	0
N.S.	1	1.00	1.00	3.77	0.94	4.51	0.00	1.49	1.13	0.00
time (sec)	N/A	0.383	0.258	2.167	0.188	0.191	0.000	0.669	0.172	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	220	406	390	0	103	70	0
N.S.	1	1.00	0.79	2.32	4.27	4.11	0.00	1.08	0.74	0.00
time (sec)	N/A	0.670	0.404	2.068	0.204	0.202	0.000	0.899	0.184	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	145	83	246	1201	464	0	122	71	0
N.S.	1	1.04	0.59	1.76	8.58	3.31	0.00	0.87	0.51	0.00
time (sec)	N/A	0.946	0.810	2.258	0.267	0.218	0.000	0.766	0.170	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	141	84	358	2035	0	0	165	165	0
N.S.	1	0.99	0.59	2.52	14.33	0.00	0.00	1.16	1.16	0.00
time (sec)	N/A	0.821	0.588	2.395	0.310	0.000	0.000	0.835	0.208	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	70	229	99	0	0	79	112	0
N.S.	1	1.00	0.74	2.41	1.04	0.00	0.00	0.83	1.18	0.00
time (sec)	N/A	0.577	0.366	2.421	0.128	0.000	0.000	0.846	0.195	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	86	54	78	0	36	57	50
N.S.	1	1.00	1.00	2.05	1.29	1.86	0.00	0.86	1.36	1.19
time (sec)	N/A	0.327	0.944	3.420	0.138	0.126	0.000	0.594	0.165	11.704

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	60	219	397	388	0	95	69	0
N.S.	1	1.00	0.63	2.31	4.18	4.08	0.00	1.00	0.73	0.00
time (sec)	N/A	0.661	0.286	2.222	0.206	0.185	0.000	0.892	0.186	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	77	64	252	567	410	0	131	62	0
N.S.	1	0.74	0.62	2.42	5.45	3.94	0.00	1.26	0.60	0.00
time (sec)	N/A	0.510	0.349	2.372	0.201	0.198	0.000	1.086	0.169	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	122	102	295	0	552	0	153	89	0
N.S.	1	0.84	0.70	2.02	0.00	3.78	0.00	1.05	0.61	0.00
time (sec)	N/A	0.796	0.528	2.440	0.000	0.213	0.000	0.942	0.185	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	88	272	133	0	0	106	195	0
N.S.	1	1.00	0.61	1.88	0.92	0.00	0.00	0.73	1.34	0.00
time (sec)	N/A	0.818	0.614	2.525	0.131	0.000	0.000	0.654	0.197	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	86	98	95	0	59	132	119
N.S.	1	1.00	1.21	2.05	2.33	2.26	0.00	1.40	3.14	2.83
time (sec)	N/A	0.333	0.448	2.296	0.135	0.117	0.000	0.823	0.199	12.785

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	111	58	104	0	37	67	120
N.S.	1	1.00	1.65	2.58	1.35	2.42	0.00	0.86	1.56	2.79
time (sec)	N/A	0.326	1.012	2.326	0.134	0.118	0.000	0.740	0.169	12.468

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	145	70	246	1191	464	0	124	71	0
N.S.	1	1.04	0.50	1.76	8.51	3.31	0.00	0.89	0.51	0.00
time (sec)	N/A	0.952	0.308	2.441	0.250	0.218	0.000	0.754	0.177	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	122	102	295	0	544	0	163	86	0
N.S.	1	0.84	0.70	2.02	0.00	3.73	0.00	1.12	0.59	0.00
time (sec)	N/A	0.782	0.417	2.484	0.000	0.233	0.000	0.928	0.185	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	102	77	310	1659	490	0	182	73	0
N.S.	1	0.64	0.48	1.94	10.37	3.06	0.00	1.14	0.46	0.00
time (sec)	N/A	0.626	0.485	2.367	0.350	0.272	0.000	0.865	0.171	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	95	0	0	0	0	0	34	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.326	0.594	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	0	0	74	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.323	0.142	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	0	0	0	0	0	49	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.286	0.109	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	36	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.317	0.000	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	45	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.313	0.000	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	159	109	0	228	191	0	0	109	0
N.S.	1	0.99	0.68	0.00	1.42	1.19	0.00	0.00	0.68	0.00
time (sec)	N/A	0.804	0.725	0.000	0.139	0.157	0.000	0.000	0.280	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	102	72	0	171	112	0	0	73	154
N.S.	1	1.02	0.72	0.00	1.71	1.12	0.00	0.00	0.73	1.54
time (sec)	N/A	0.506	0.468	0.000	0.133	0.142	0.000	0.000	0.224	12.182

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	A	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	163	0	114	70	0	0	35	0
N.S.	1	1.00	3.54	0.00	2.48	1.52	0.00	0.00	0.76	0.00
time (sec)	N/A	0.290	1.452	0.000	0.144	0.117	0.000	0.000	0.209	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	0	0	0	49	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.330	0.319	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0	58	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.330	0.380	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0	69	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.335	0.416	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	168	104	0	156	120	0	0	110	290
N.S.	1	0.99	0.62	0.00	0.92	0.71	0.00	0.00	0.65	1.72
time (sec)	N/A	0.814	1.713	0.000	0.126	0.153	0.000	0.000	0.174	18.775

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	106	76	108	107	93	0	0	84	145
N.S.	1	1.02	0.73	1.04	1.03	0.89	0.00	0.00	0.81	1.39
time (sec)	N/A	0.555	0.943	2.994	0.121	0.131	0.000	0.000	0.186	16.118

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	76	62	72	0	0	64	105
N.S.	1	1.00	1.04	1.62	1.32	1.53	0.00	0.00	1.36	2.23
time (sec)	N/A	0.308	0.667	2.886	0.118	0.148	0.000	0.000	0.185	11.419

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	104	0	0	0	0	0	36	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.345	0.467	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	105	0	0	0	0	0	79	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.353	1.097	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	105	0	0	0	0	0	119	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.345	2.435	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	104	68	137	172	131	0	145	242	175
N.S.	1	0.99	0.65	1.30	1.64	1.25	0.00	1.38	2.30	1.67
time (sec)	N/A	0.404	0.210	2.891	0.034	0.122	0.000	0.172	0.164	15.006

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	78	57	126	160	117	0	128	182	146
N.S.	1	0.91	0.66	1.47	1.86	1.36	0.00	1.49	2.12	1.70
time (sec)	N/A	0.361	0.127	2.786	0.034	0.133	0.000	0.167	0.162	13.691

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	34	36	35	51	15	35	15
N.S.	1	1.00	1.00	2.00	2.12	2.06	3.00	0.88	2.06	0.88
time (sec)	N/A	0.287	0.012	0.292	0.031	0.105	0.896	0.129	0.160	10.682

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	57	154	78	194	105	0	87	96	71
N.S.	1	1.02	2.75	1.39	3.46	1.88	0.00	1.55	1.71	1.27
time (sec)	N/A	0.485	0.542	0.236	0.035	0.135	0.000	0.156	0.165	10.766

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	73	77	60	144	123	0	81	60	44
N.S.	1	1.04	1.10	0.86	2.06	1.76	0.00	1.16	0.86	0.63
time (sec)	N/A	0.566	0.295	0.202	0.040	0.124	0.000	0.163	0.168	10.918

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	43	36	115	78	0	37	35	35
N.S.	1	1.02	0.50	0.42	1.34	0.91	0.00	0.43	0.41	0.41
time (sec)	N/A	0.531	1.206	0.191	0.037	0.105	0.000	0.153	0.165	10.796

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	138	325	0	0	0	0	0	78	0
N.S.	1	0.99	2.32	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.428	2.159	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	0	0	0	0	0	38	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.327	0.231	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	181	3396	0	0	0	0	0	59	0
N.S.	1	1.01	18.87	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.728	14.601	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	229	187	0	0	0	0	0	79	0
N.S.	1	1.01	0.83	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.055	1.742	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	99	73	88	0	260	0	132	42	0
N.S.	1	1.22	0.90	1.09	0.00	3.21	0.00	1.63	0.52	0.00
time (sec)	N/A	0.409	1.466	0.493	0.000	0.159	0.000	0.447	0.176	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	127	162	182	979	331	0	0	45	0
N.S.	1	1.22	1.56	1.75	9.41	3.18	0.00	0.00	0.43	0.00
time (sec)	N/A	0.450	3.665	2.760	0.253	0.224	0.000	0.000	0.174	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	157	724	224	1310	459	0	235	49	0
N.S.	1	1.12	5.17	1.60	9.36	3.28	0.00	1.68	0.35	0.00
time (sec)	N/A	0.469	10.503	1.645	0.241	0.185	0.000	1.126	0.178	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	135	236	112	536	330	0	0	50	0
N.S.	1	1.16	2.03	0.97	4.62	2.84	0.00	0.00	0.43	0.00
time (sec)	N/A	0.474	3.208	0.813	0.223	0.137	0.000	0.000	0.173	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	198	328	557	1400	560	0	0	54	0
N.S.	1	1.11	1.83	3.11	7.82	3.13	0.00	0.00	0.30	0.00
time (sec)	N/A	0.515	2.676	2.201	0.258	0.299	0.000	0.000	0.183	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	94	228	56	255	0	0	55	0
N.S.	1	1.00	2.04	4.96	1.22	5.54	0.00	0.00	1.20	0.00
time (sec)	N/A	0.470	1.659	2.504	0.207	0.248	0.000	0.000	0.183	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	98	403	0	357	0	136	36	0
N.S.	1	1.00	1.51	6.20	0.00	5.49	0.00	2.09	0.55	0.00
time (sec)	N/A	0.350	0.517	9.296	0.000	0.342	0.000	0.295	0.179	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	252	158	237	379	281	0	566	1271	361
N.S.	1	1.07	0.67	1.00	1.61	1.19	0.00	2.40	5.39	1.53
time (sec)	N/A	1.400	2.284	3.411	0.044	0.148	0.000	0.209	0.189	14.540

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	182	100	185	266	211	0	380	801	255
N.S.	1	1.06	0.58	1.08	1.56	1.23	0.00	2.22	4.68	1.49
time (sec)	N/A	1.034	0.574	1.072	0.043	0.146	0.000	0.216	0.185	14.277

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	114	80	118	165	150	0	232	450	196
N.S.	1	1.06	0.74	1.09	1.53	1.39	0.00	2.15	4.17	1.81
time (sec)	N/A	0.687	0.350	0.677	0.034	0.145	0.000	0.160	0.180	13.609

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	59	75	75	88	96	0	124	208	111
N.S.	1	1.05	1.34	1.34	1.57	1.71	0.00	2.21	3.71	1.98
time (sec)	N/A	0.437	0.023	0.415	0.046	0.122	0.000	0.164	0.190	11.615

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	107	90	0	255	0	127	122	195
N.S.	1	1.00	1.55	1.30	0.00	3.70	0.00	1.84	1.77	2.83
time (sec)	N/A	0.492	0.632	0.230	0.000	0.188	0.000	0.161	0.169	10.909

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	90	75	105	0	357	0	137	331	85
N.S.	1	1.14	0.95	1.33	0.00	4.52	0.00	1.73	4.19	1.08
time (sec)	N/A	0.475	0.626	0.195	0.000	0.149	0.000	0.146	0.177	10.378

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	149	167	178	0	736	0	263	823	171
N.S.	1	1.14	1.27	1.36	0.00	5.62	0.00	2.01	6.28	1.31
time (sec)	N/A	0.737	1.329	0.358	0.000	0.174	0.000	0.211	0.173	12.408

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	216	247	271	0	1278	0	449	1718	321
N.S.	1	1.14	1.31	1.43	0.00	6.76	0.00	2.38	9.09	1.70
time (sec)	N/A	1.075	3.206	0.558	0.000	0.216	0.000	0.210	0.179	14.653

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	390	192	361	683	387	0	736	1491	484
N.S.	1	1.19	0.59	1.10	2.09	1.18	0.00	2.25	4.56	1.48
time (sec)	N/A	0.562	5.791	4.618	0.057	0.160	0.000	0.246	0.175	14.450

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	302	152	269	469	294	0	506	1033	394
N.S.	1	1.25	0.63	1.11	1.94	1.21	0.00	2.09	4.27	1.63
time (sec)	N/A	0.468	4.807	2.569	0.042	0.141	0.000	0.223	0.169	14.125

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	269	112	202	324	209	0	320	603	237
N.S.	1	1.53	0.64	1.15	1.84	1.19	0.00	1.82	3.43	1.35
time (sec)	N/A	0.406	1.463	0.951	0.037	0.133	0.000	0.189	0.172	14.980

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	92	72	120	167	138	0	178	299	161
N.S.	1	0.89	0.70	1.17	1.62	1.34	0.00	1.73	2.90	1.56
time (sec)	N/A	0.579	0.736	0.627	0.034	0.121	0.000	0.165	0.174	13.976

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	201	329	150	0	398	0	196	291	529
N.S.	1	2.12	3.46	1.58	0.00	4.19	0.00	2.06	3.06	5.57
time (sec)	N/A	0.413	2.493	0.326	0.000	0.315	0.000	0.197	0.174	12.280

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	230	312	157	0	567	0	230	549	2563
N.S.	1	1.97	2.67	1.34	0.00	4.85	0.00	1.97	4.69	21.91
time (sec)	N/A	0.426	2.192	0.375	0.000	0.302	0.000	0.183	0.181	13.473

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	224	249	167	0	622	0	211	496	158
N.S.	1	1.72	1.92	1.28	0.00	4.78	0.00	1.62	3.82	1.22
time (sec)	N/A	0.367	1.738	0.355	0.000	0.159	0.000	0.216	0.176	13.103

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	307	211	228	0	1234	0	403	1430	286
N.S.	1	1.44	0.99	1.07	0.00	5.79	0.00	1.89	6.71	1.34
time (sec)	N/A	0.436	3.608	0.606	0.000	0.226	0.000	0.228	0.188	14.410

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	462	322	352	0	1908	0	710	2582	438
N.S.	1	1.67	1.17	1.28	0.00	6.91	0.00	2.57	9.36	1.59
time (sec)	N/A	0.590	6.703	0.938	0.000	0.256	0.000	0.303	0.195	14.880

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	345	182	355	701	337	0	584	1173	411
N.S.	1	1.20	0.63	1.23	2.43	1.17	0.00	2.03	4.07	1.43
time (sec)	N/A	0.474	9.150	4.509	0.046	0.141	0.000	0.257	0.182	15.454

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	309	142	257	459	245	0	376	757	287
N.S.	1	1.20	0.55	1.00	1.79	0.95	0.00	1.46	2.95	1.12
time (sec)	N/A	0.421	3.819	1.977	0.039	0.137	0.000	0.217	0.173	15.116

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	110	91	175	262	161	0	212	379	203
N.S.	1	0.88	0.73	1.40	2.10	1.29	0.00	1.70	3.03	1.62
time (sec)	N/A	0.409	1.335	0.818	0.035	0.120	0.000	0.173	0.193	14.543

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	265	419	224	0	532	0	285	799	1902
N.S.	1	1.73	2.74	1.46	0.00	3.48	0.00	1.86	5.22	12.43
time (sec)	N/A	0.496	3.095	0.427	0.000	0.463	0.000	0.206	0.185	12.020

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	284	455	216	0	859	0	317	1262	3135
N.S.	1	1.76	2.83	1.34	0.00	5.34	0.00	1.97	7.84	19.47
time (sec)	N/A	0.485	4.418	0.516	0.000	0.527	0.000	0.202	0.185	14.474

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	330	393	227	0	1176	0	376	1575	4131
N.S.	1	1.76	2.09	1.21	0.00	6.26	0.00	2.00	8.38	21.97
time (sec)	N/A	0.518	3.781	0.649	0.000	0.518	0.000	0.250	0.186	16.949

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	288	398	227	0	1012	0	307	933	264
N.S.	1	1.62	2.24	1.28	0.00	5.69	0.00	1.72	5.24	1.48
time (sec)	N/A	0.398	3.479	0.621	0.000	0.192	0.000	0.238	0.209	13.481

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	371	274	303	0	1714	0	601	2232	385
N.S.	1	1.39	1.03	1.14	0.00	6.44	0.00	2.26	8.39	1.45
time (sec)	N/A	0.455	5.607	1.092	0.000	0.253	0.000	0.326	0.221	13.568

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	305	148	275	596	297	0	344	867	211
N.S.	1	1.67	0.81	1.50	3.26	1.62	0.00	1.88	4.74	1.15
time (sec)	N/A	0.494	4.782	0.734	0.043	0.140	0.000	0.192	0.209	11.128

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	215	98	181	388	216	0	219	525	139
N.S.	1	1.84	0.84	1.55	3.32	1.85	0.00	1.87	4.49	1.19
time (sec)	N/A	0.430	1.533	0.476	0.047	0.123	0.000	0.170	0.192	10.710

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	169	237	103	223	155	0	136	218	85
N.S.	1	2.49	3.49	1.51	3.28	2.28	0.00	2.00	3.21	1.25
time (sec)	N/A	0.339	2.491	0.290	0.059	0.125	0.000	0.157	0.179	10.862

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	109	53	99	74	0	70	60	41
N.S.	1	1.00	2.53	1.23	2.30	1.72	0.00	1.63	1.40	0.95
time (sec)	N/A	0.368	0.762	0.144	0.036	0.125	0.000	0.141	0.178	10.445

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	154	160	74	0	353	0	110	107	110
N.S.	1	1.86	1.93	0.89	0.00	4.25	0.00	1.33	1.29	1.33
time (sec)	N/A	0.365	0.940	0.198	0.000	0.159	0.000	0.159	0.195	10.399

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	234	286	146	0	691	0	221	635	187
N.S.	1	1.61	1.97	1.01	0.00	4.77	0.00	1.52	4.38	1.29
time (sec)	N/A	0.424	2.816	0.286	0.000	0.172	0.000	0.159	0.174	10.516

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	328	1422	221	0	1331	0	362	1559	379
N.S.	1	1.58	6.87	1.07	0.00	6.43	0.00	1.75	7.53	1.83
time (sec)	N/A	0.506	7.245	0.424	0.000	0.220	0.000	0.225	0.208	11.725

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	392	446	382	772	456	0	506	1404	268
N.S.	1	1.52	1.73	1.48	2.99	1.77	0.00	1.96	5.44	1.04
time (sec)	N/A	0.607	6.454	0.990	0.048	0.142	0.000	0.283	0.220	10.733

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	302	310	275	536	361	0	359	829	193
N.S.	1	1.56	1.61	1.42	2.78	1.87	0.00	1.86	4.30	1.00
time (sec)	N/A	0.492	4.622	0.704	0.051	0.126	0.000	0.203	0.211	10.703

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	222	294	175	342	268	0	250	397	136
N.S.	1	1.67	2.21	1.32	2.57	2.02	0.00	1.88	2.98	1.02
time (sec)	N/A	0.443	3.258	0.434	0.040	0.130	0.000	0.187	0.205	10.759

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	178	181	84	195	155	0	158	131	89
N.S.	1	2.00	2.03	0.94	2.19	1.74	0.00	1.78	1.47	1.00
time (sec)	N/A	0.362	1.851	0.205	0.046	0.129	0.000	0.170	0.210	10.364

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	64	76	46	93	58	0	60	51	45
N.S.	1	0.98	1.17	0.71	1.43	0.89	0.00	0.92	0.78	0.69
time (sec)	N/A	0.341	0.685	0.144	0.035	0.111	0.000	0.154	0.214	10.418

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	219	209	122	0	598	0	249	207	168
N.S.	1	1.70	1.62	0.95	0.00	4.64	0.00	1.93	1.60	1.30
time (sec)	N/A	0.423	1.720	0.250	0.000	0.151	0.000	0.190	0.217	10.882

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	309	376	203	0	1242	0	474	861	314
N.S.	1	1.46	1.78	0.96	0.00	5.89	0.00	2.25	4.08	1.49
time (sec)	N/A	0.494	3.333	0.332	0.000	0.197	0.000	0.199	0.216	11.173

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	420	2220	280	0	2030	0	751	1915	505
N.S.	1	1.48	7.82	0.99	0.00	7.15	0.00	2.64	6.74	1.78
time (sec)	N/A	0.598	7.950	0.489	0.000	0.267	0.000	0.255	0.220	11.272

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	501	1338	491	946	620	0	672	1638	327
N.S.	1	1.38	3.69	1.35	2.61	1.71	0.00	1.85	4.51	0.90
time (sec)	N/A	0.703	9.666	1.239	0.066	0.148	0.000	0.321	0.206	10.596

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	400	439	360	689	502	0	504	1007	252
N.S.	1	1.39	1.53	1.25	2.40	1.75	0.00	1.76	3.51	0.88
time (sec)	N/A	0.613	5.266	0.904	0.056	0.174	0.000	0.274	0.195	10.549

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	310	292	247	475	385	0	374	538	195
N.S.	1	1.51	1.42	1.20	2.32	1.88	0.00	1.82	2.62	0.95
time (sec)	N/A	0.514	3.574	0.600	0.050	0.134	0.000	0.236	0.207	10.707

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	225	295	117	307	248	0	259	216	147
N.S.	1	1.69	2.22	0.88	2.31	1.86	0.00	1.95	1.62	1.11
time (sec)	N/A	0.412	2.435	0.299	0.042	0.124	0.000	0.198	0.204	10.588

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	192	84	74	184	113	0	129	111	79
N.S.	1	1.67	0.73	0.64	1.60	0.98	0.00	1.12	0.97	0.69
time (sec)	N/A	0.364	0.734	0.236	0.040	0.118	0.000	0.178	0.214	10.431

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	99	135	56	115	93	0	75	66	66
N.S.	1	0.97	1.32	0.55	1.13	0.91	0.00	0.74	0.65	0.65
time (sec)	N/A	0.481	0.847	0.195	0.035	0.100	0.000	0.160	0.207	10.304

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	294	345	203	0	1001	0	471	325	228
N.S.	1	1.62	1.91	1.12	0.00	5.53	0.00	2.60	1.80	1.26
time (sec)	N/A	0.502	2.749	0.312	0.000	0.169	0.000	0.191	0.212	10.756

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	392	1772	284	0	1693	0	918	1060	464
N.S.	1	1.36	6.15	0.99	0.00	5.88	0.00	3.19	3.68	1.61
time (sec)	N/A	0.598	7.665	0.404	0.000	0.224	0.000	0.224	0.219	11.077

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	508	1096	365	0	2677	0	1369	2231	655
N.S.	1	1.38	2.98	0.99	0.00	7.27	0.00	3.72	6.06	1.78
time (sec)	N/A	0.721	8.445	0.570	0.000	0.308	0.000	0.298	0.201	11.239

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	102	311	0	307	0	0	44	0
N.S.	1	1.00	1.67	5.10	0.00	5.03	0.00	0.00	0.72	0.00
time (sec)	N/A	0.323	0.621	1.164	0.000	0.318	0.000	0.000	0.224	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	187	549	0	1048	0	0	45	0
N.S.	1	1.00	1.34	3.92	0.00	7.49	0.00	0.00	0.32	0.00
time (sec)	N/A	0.842	13.634	1.078	0.000	0.370	0.000	0.000	0.195	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	107	135	0	246	0	0	65	0
N.S.	1	1.00	1.37	1.73	0.00	3.15	0.00	0.00	0.83	0.00
time (sec)	N/A	0.340	0.702	0.738	0.000	0.186	0.000	0.000	0.217	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	171	401	0	1100	0	0	67	0
N.S.	1	1.00	1.21	2.84	0.00	7.80	0.00	0.00	0.48	0.00
time (sec)	N/A	0.874	0.752	1.125	0.000	0.441	0.000	0.000	0.199	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	94	424	0	343	0	139	33	0
N.S.	1	1.00	1.54	6.95	0.00	5.62	0.00	2.28	0.54	0.00
time (sec)	N/A	0.307	0.337	7.507	0.000	0.335	0.000	0.304	0.187	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	188	841	0	1108	0	0	43	0
N.S.	1	1.00	1.26	5.64	0.00	7.44	0.00	0.00	0.29	0.00
time (sec)	N/A	0.945	1.536	1.910	0.000	6.038	0.000	0.000	0.201	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	144	505	0	963	0	0	54	0
N.S.	1	1.00	1.18	4.14	0.00	7.89	0.00	0.00	0.44	0.00
time (sec)	N/A	0.666	1.274	8.507	0.000	0.367	0.000	0.000	0.181	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	141	505	0	1041	0	0	56	0
N.S.	1	1.00	1.14	4.07	0.00	8.40	0.00	0.00	0.45	0.00
time (sec)	N/A	0.727	0.637	3.705	0.000	0.518	0.000	0.000	0.197	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	198	832	0	1103	0	0	64	0
N.S.	1	1.00	1.19	4.98	0.00	6.60	0.00	0.00	0.38	0.00
time (sec)	N/A	0.939	1.631	1.775	0.000	0.812	0.000	0.000	0.233	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	231	245	155	3073	0	1579	0	0	68	0
N.S.	1	1.06	0.67	13.30	0.00	6.84	0.00	0.00	0.29	0.00
time (sec)	N/A	1.407	1.380	2.325	0.000	77.670	0.000	0.000	0.208	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	266	206	237	379	281	0	850	1323	555
N.S.	1	1.06	0.82	0.95	1.52	1.12	0.00	3.40	5.29	2.22
time (sec)	N/A	1.435	4.367	1.550	0.039	0.162	0.000	0.219	0.195	14.794

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	191	145	185	266	211	0	586	838	395
N.S.	1	1.06	0.81	1.03	1.48	1.17	0.00	3.26	4.66	2.19
time (sec)	N/A	1.063	0.912	1.148	0.035	0.148	0.000	0.205	0.188	14.597

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	121	93	118	165	150	0	294	469	227
N.S.	1	1.05	0.81	1.03	1.43	1.30	0.00	2.56	4.08	1.97
time (sec)	N/A	0.706	0.523	0.777	0.041	0.139	0.000	0.159	0.216	14.189

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	64	75	75	88	96	0	153	218	104
N.S.	1	1.05	1.23	1.23	1.44	1.57	0.00	2.51	3.57	1.70
time (sec)	N/A	0.439	0.015	0.471	0.032	0.125	0.000	0.155	0.185	11.897

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	92	0	316	0	127	193	573
N.S.	1	1.00	1.47	1.21	0.00	4.16	0.00	1.67	2.54	7.54
time (sec)	N/A	0.479	0.436	0.277	0.000	0.649	0.000	0.173	0.186	11.945

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	110	97	132	0	389	0	172	333	106
N.S.	1	1.11	0.98	1.33	0.00	3.93	0.00	1.74	3.36	1.07
time (sec)	N/A	0.499	0.396	0.229	0.000	0.163	0.000	0.152	0.180	11.156

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	191	172	236	0	752	0	399	977	250
N.S.	1	1.15	1.04	1.42	0.00	4.53	0.00	2.40	5.89	1.51
time (sec)	N/A	0.777	1.032	0.378	0.000	0.203	0.000	0.203	0.198	14.143

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	278	405	376	0	1238	0	693	1900	439
N.S.	1	1.17	1.71	1.59	0.00	5.22	0.00	2.92	8.02	1.85
time (sec)	N/A	1.176	1.845	0.610	0.000	0.230	0.000	0.228	0.238	15.595

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	580	480	0	1093	0	606	2357	9987
N.S.	1	1.00	2.35	1.94	0.00	4.43	0.00	2.45	9.54	40.43
time (sec)	N/A	0.743	5.235	0.892	0.000	150.809	0.000	0.221	0.199	20.050

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	389	289	0	779	0	339	1366	6730
N.S.	1	1.00	2.29	1.70	0.00	4.58	0.00	1.99	8.04	39.59
time (sec)	N/A	0.627	2.981	0.622	0.000	32.150	0.000	0.223	0.198	18.324

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	135	165	0	518	0	195	445	3559
N.S.	1	1.00	1.31	1.60	0.00	5.03	0.00	1.89	4.32	34.55
time (sec)	N/A	0.549	2.285	0.416	0.000	4.687	0.000	0.182	0.210	16.488

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	92	0	309	0	127	193	571
N.S.	1	1.00	1.47	1.21	0.00	4.07	0.00	1.67	2.54	7.51
time (sec)	N/A	0.478	0.431	0.283	0.000	0.506	0.000	0.198	0.240	12.318

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	119	108	0	1040	0	522	291	2665
N.S.	1	1.00	0.98	0.89	0.00	8.60	0.00	4.31	2.40	22.02
time (sec)	N/A	0.552	0.399	0.500	0.000	2.380	0.000	0.262	0.200	13.789

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	221	229	208	0	2863	0	331	1678	20827
N.S.	1	1.18	1.22	1.11	0.00	15.23	0.00	1.76	8.93	110.78
time (sec)	N/A	0.987	1.213	0.944	0.000	121.841	0.000	0.196	0.216	24.713

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	784	689	0	0	0	857	9042	17256
N.S.	1	1.00	2.07	1.82	0.00	0.00	0.00	2.26	23.86	45.53
time (sec)	N/A	0.995	9.486	1.588	0.000	0.000	0.000	0.308	0.811	26.354

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	511	465	0	1925	0	551	5286	12483
N.S.	1	1.00	1.72	1.57	0.00	6.48	0.00	1.86	17.80	42.03
time (sec)	N/A	0.826	5.026	1.184	0.000	172.174	0.000	0.247	0.238	23.771

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	362	314	0	1326	0	539	2455	7958
N.S.	1	1.00	1.59	1.38	0.00	5.82	0.00	2.36	10.77	34.90
time (sec)	N/A	0.729	3.191	0.748	0.000	37.830	0.000	0.206	0.209	20.649

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	180	215	0	798	0	267	921	4926
N.S.	1	1.00	0.91	1.09	0.00	4.03	0.00	1.35	4.65	24.88
time (sec)	N/A	0.648	1.119	0.487	0.000	5.327	0.000	0.210	0.230	19.254

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	111	97	132	0	394	0	173	333	106
N.S.	1	1.11	0.97	1.32	0.00	3.94	0.00	1.73	3.33	1.06
time (sec)	N/A	0.489	0.406	0.213	0.000	0.112	0.000	0.175	0.195	11.753

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	218	176	210	0	2852	0	330	1678	20827
N.S.	1	1.17	0.95	1.13	0.00	15.33	0.00	1.77	9.02	111.97
time (sec)	N/A	0.951	1.456	0.930	0.000	58.928	0.000	0.231	0.209	24.919

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	183	316	0	0	0	0	32	0
N.S.	1	1.00	0.86	1.48	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.725	3.805	6.808	0.000	0.000	0.000	0.000	0.165	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	44664	321	0	0	0	0	43	0
N.S.	1	1.00	227.88	1.64	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.446	33.369	10.644	0.000	0.000	0.000	0.000	0.199	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	233	191	0	0	0	0	66	0
N.S.	1	1.00	1.21	0.99	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.431	2.817	7.842	0.000	0.000	0.000	0.000	0.167	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	176	148	0	0	0	0	53	0
N.S.	1	1.00	1.60	1.35	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.328	2.341	2.531	0.000	0.000	0.000	0.000	0.164	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	176	142	0	0	0	0	55	0
N.S.	1	1.00	1.41	1.14	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.329	0.591	2.345	0.000	0.000	0.000	0.000	0.172	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	39359	262	0	0	0	0	68	0
N.S.	1	1.00	99.39	0.66	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.076	36.240	12.947	0.000	0.000	0.000	0.000	0.179	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	223	445	0	0	0	0	43	0
N.S.	1	1.00	1.31	2.62	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	2.061	25.155	6.267	0.000	0.000	0.000	0.000	0.248	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	223	0	0	0	0	66	0
N.S.	1	1.00	1.00	2.69	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.730	1.227	5.776	0.000	0.000	0.000	0.000	0.213	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	174	222	442	0	0	0	0	36	0
N.S.	1	1.04	1.32	2.63	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	2.279	24.905	6.115	0.000	0.000	0.000	0.000	0.214	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	264	123	0	0	0	0	34	0
N.S.	1	1.00	2.78	1.29	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.329	5.719	3.283	0.000	0.000	0.000	0.000	0.199	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	311	0	275	0	0	0	0	44	0
N.S.	1	1.05	0.00	0.93	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	2.688	0.000	3.394	0.000	0.000	0.000	0.000	0.235	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	375	190	0	0	0	0	54	0
N.S.	1	1.00	1.79	0.91	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.723	12.506	1.766	0.000	0.000	0.000	0.000	0.195	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	156	192	0	0	0	0	56	0
N.S.	1	1.00	0.73	0.90	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.774	4.880	5.753	0.000	0.000	0.000	0.000	0.188	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	230	1019	207	0	517	0	0	64	0
N.S.	1	1.00	4.45	0.90	0.00	2.26	0.00	0.00	0.28	0.00
time (sec)	N/A	1.211	8.290	1.843	0.000	0.099	0.000	0.000	0.213	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	310	0	323	0	0	0	0	68	0
N.S.	1	0.99	0.00	1.04	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	2.535	0.000	5.020	0.000	0.000	0.000	0.000	0.220	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	183	316	0	0	0	0	32	0
N.S.	1	1.00	0.86	1.48	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.692	0.224	6.716	0.000	0.000	0.000	0.000	0.179	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	223	445	0	0	0	0	43	0
N.S.	1	1.00	1.31	2.62	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	2.015	25.179	6.067	0.000	0.000	0.000	0.000	0.212	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	187	205	0	0	0	0	55	0
N.S.	1	1.00	1.83	2.01	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.322	8.376	6.566	0.000	0.000	0.000	0.000	0.189	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	165	205	0	0	0	0	57	0
N.S.	1	1.00	0.79	0.98	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.771	3.992	7.148	0.000	0.000	0.000	0.000	0.178	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	221	0	0	0	0	66	0
N.S.	1	1.00	1.00	2.66	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.721	1.248	5.743	0.000	0.000	0.000	0.000	0.235	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	246	313	0	0	0	0	70	0
N.S.	1	1.00	1.48	1.89	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	1.995	25.252	7.540	0.000	0.000	0.000	0.000	0.214	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	59	151	49	68	121	0	47	47	47
N.S.	1	0.88	2.25	0.73	1.01	1.81	0.00	0.70	0.70	0.70
time (sec)	N/A	0.421	3.307	15.823	0.036	0.081	0.000	0.475	0.188	11.909

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	75	175	62	88	146	0	60	60	60
N.S.	1	0.84	1.97	0.70	0.99	1.64	0.00	0.67	0.67	0.67
time (sec)	N/A	0.451	4.909	25.401	0.040	0.080	0.000	0.457	0.180	12.154

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [274] had the largest ratio of [.5384619999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	0.99	30	0.100
2	A	3	3	1.00	30	0.100
3	A	3	3	0.92	30	0.100
4	A	6	6	1.00	28	0.214
5	A	4	4	1.00	30	0.133
6	A	2	2	1.00	30	0.067
7	A	4	4	1.00	30	0.133
8	A	6	6	1.03	30	0.200
9	A	8	8	1.04	30	0.267
10	A	3	3	0.92	32	0.094
11	A	3	3	0.93	32	0.094
12	A	3	3	0.86	32	0.094
13	A	8	8	0.92	32	0.250
14	A	3	3	0.92	30	0.100
15	A	9	8	0.93	32	0.250
16	A	6	6	1.00	32	0.188
17	A	2	2	1.00	32	0.062
18	A	4	4	1.00	32	0.125
19	A	6	6	1.04	32	0.188
20	A	8	8	1.06	32	0.250
21	A	3	3	0.92	32	0.094

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	0.92	32	0.094
23	A	3	3	0.86	32	0.094
24	A	10	10	0.94	32	0.312
25	A	3	3	0.86	32	0.094
26	A	3	3	1.00	30	0.100
27	A	11	10	0.98	32	0.312
28	A	11	10	0.98	32	0.312
29	A	8	8	1.02	32	0.250
30	A	2	2	1.00	32	0.062
31	A	4	4	1.00	32	0.125
32	A	6	6	1.04	32	0.188
33	A	8	8	1.06	32	0.250
34	A	5	5	0.96	32	0.156
35	A	11	10	0.98	32	0.312
36	A	9	8	0.95	32	0.250
37	A	4	4	1.00	30	0.133
38	A	7	6	1.00	32	0.188
39	A	3	3	0.86	32	0.094
40	A	3	3	0.95	32	0.094
41	A	3	3	0.91	32	0.094
42	A	7	7	0.99	32	0.219
43	A	13	12	0.97	32	0.375
44	A	11	10	0.99	32	0.312
45	A	6	6	1.00	32	0.188
46	A	2	2	1.00	30	0.067
47	A	3	3	0.86	32	0.094
48	A	7	6	0.84	32	0.188
49	A	3	3	0.86	32	0.094
50	A	3	3	0.92	32	0.094
51	A	3	3	0.91	32	0.094
52	A	9	9	0.98	32	0.281
53	A	15	14	1.01	32	0.438

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	13	12	1.01	32	0.375
55	A	8	8	1.02	32	0.250
56	A	2	2	1.00	32	0.062
57	A	4	4	1.00	30	0.133
58	A	3	3	0.95	32	0.094
59	A	3	3	0.86	32	0.094
60	A	8	7	0.69	32	0.219
61	A	3	3	0.82	32	0.094
62	A	3	3	0.92	32	0.094
63	A	3	3	0.90	32	0.094
64	A	8	8	1.05	32	0.250
65	A	6	6	1.03	32	0.188
66	A	4	4	1.00	32	0.125
67	A	2	2	1.00	32	0.062
68	A	6	5	1.00	32	0.156
69	A	6	5	1.00	32	0.156
70	A	8	7	1.04	32	0.219
71	A	8	8	1.05	34	0.235
72	A	6	6	1.03	34	0.176
73	A	4	4	1.00	34	0.118
74	A	2	2	1.00	34	0.059
75	A	8	7	1.05	34	0.206
76	A	8	7	1.12	34	0.206
77	A	8	7	1.09	34	0.206
78	A	10	9	1.02	34	0.265
79	A	8	8	1.05	34	0.235
80	A	6	6	1.03	34	0.176
81	A	4	4	1.00	34	0.118
82	A	2	2	1.00	34	0.059
83	A	10	9	1.02	34	0.265
84	A	10	9	1.02	34	0.265
85	A	10	9	1.01	34	0.265

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	8	8	1.00	34	0.235
87	A	6	6	0.99	34	0.176
88	A	4	4	1.00	34	0.118
89	A	2	2	1.00	34	0.059
90	A	6	5	1.00	34	0.147
91	A	8	7	1.02	34	0.206
92	A	10	9	1.03	34	0.265
93	A	8	8	1.00	34	0.235
94	A	6	6	0.99	34	0.176
95	A	4	4	0.99	34	0.118
96	A	2	2	1.00	34	0.059
97	A	8	7	0.99	34	0.206
98	A	10	9	1.02	34	0.265
99	A	12	11	1.02	34	0.324
100	A	8	8	1.02	34	0.235
101	A	6	6	1.02	34	0.176
102	A	4	4	1.00	34	0.118
103	A	2	2	1.00	34	0.059
104	A	10	9	1.02	34	0.265
105	A	12	11	1.04	34	0.324
106	A	14	13	1.04	34	0.382
107	A	2	2	1.00	36	0.056
108	A	2	2	1.00	36	0.056
109	A	2	2	1.00	36	0.056
110	A	2	2	1.00	36	0.056
111	A	2	2	1.00	36	0.056
112	A	2	2	1.00	36	0.056
113	A	4	4	1.00	36	0.111
114	A	4	4	1.00	36	0.111
115	A	4	4	1.00	36	0.111
116	A	2	2	1.00	36	0.056
117	A	4	4	1.00	36	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	4	1.00	36	0.111
119	A	2	2	1.00	36	0.056
120	A	4	4	1.00	36	0.111
121	A	4	4	1.00	36	0.111
122	A	4	4	1.00	36	0.111
123	A	6	6	1.03	36	0.167
124	A	6	6	1.03	36	0.167
125	A	4	4	1.00	36	0.111
126	A	2	2	1.00	36	0.056
127	A	6	6	1.01	36	0.167
128	A	6	6	0.99	36	0.167
129	A	6	6	1.03	36	0.167
130	A	2	2	1.00	36	0.056
131	A	4	4	1.00	36	0.111
132	A	6	6	1.04	36	0.167
133	A	6	6	1.01	36	0.167
134	A	4	4	1.00	36	0.111
135	A	2	2	1.00	36	0.056
136	A	5	5	1.00	36	0.139
137	A	7	7	1.00	36	0.194
138	A	9	9	1.04	36	0.250
139	A	6	6	0.99	36	0.167
140	A	4	4	1.00	36	0.111
141	A	2	2	1.00	36	0.056
142	A	7	7	1.00	36	0.194
143	A	7	7	0.74	36	0.194
144	A	9	9	0.84	36	0.250
145	A	6	6	1.00	36	0.167
146	A	2	2	1.00	36	0.056
147	A	2	2	1.00	36	0.056
148	A	9	9	1.04	36	0.250
149	A	9	9	0.84	36	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	9	9	0.64	36	0.250
151	A	5	4	1.00	32	0.125
152	A	5	4	1.00	32	0.125
153	A	5	4	1.00	30	0.133
154	A	5	4	1.00	32	0.125
155	A	5	4	1.00	32	0.125
156	A	6	6	0.99	34	0.176
157	A	4	4	1.02	34	0.118
158	A	2	2	1.00	34	0.059
159	A	5	4	1.00	34	0.118
160	A	5	4	1.00	34	0.118
161	A	5	4	1.00	34	0.118
162	A	6	6	0.99	36	0.167
163	A	4	4	1.02	36	0.111
164	A	2	2	1.00	36	0.056
165	A	5	4	1.00	34	0.118
166	A	5	4	1.00	36	0.111
167	A	5	4	1.00	36	0.111
168	A	3	3	0.99	32	0.094
169	A	3	3	0.91	32	0.094
170	A	6	5	1.00	30	0.167
171	A	10	9	1.02	32	0.281
172	A	8	8	1.04	32	0.250
173	A	7	7	1.02	32	0.219
174	A	3	3	0.99	34	0.088
175	A	4	4	1.00	32	0.125
176	A	8	8	1.01	34	0.235
177	A	10	10	1.01	34	0.294
178	A	7	6	1.22	36	0.167
179	A	6	5	1.22	40	0.125
180	A	11	10	1.12	38	0.263
181	A	7	6	1.16	40	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	10	9	1.11	40	0.225
183	A	7	6	1.00	38	0.158
184	A	4	3	1.00	34	0.088
185	A	15	14	1.07	29	0.483
186	A	13	12	1.06	29	0.414
187	A	11	10	1.06	29	0.345
188	A	9	8	1.05	27	0.296
189	A	9	8	1.00	29	0.276
190	A	10	9	1.14	29	0.310
191	A	13	12	1.14	29	0.414
192	A	15	14	1.14	29	0.483
193	A	14	13	1.19	31	0.419
194	A	11	10	1.25	31	0.323
195	A	11	10	1.53	31	0.323
196	A	11	10	0.89	29	0.345
197	B	10	9	2.12	31	0.290
198	A	10	9	1.97	31	0.290
199	A	7	6	1.72	31	0.194
200	A	8	7	1.44	31	0.226
201	A	14	13	1.67	31	0.419
202	A	12	11	1.20	31	0.355
203	A	12	11	1.20	31	0.355
204	A	5	5	0.88	29	0.172
205	A	13	12	1.73	31	0.387
206	A	12	11	1.76	31	0.355
207	A	12	11	1.76	31	0.355
208	A	8	7	1.62	31	0.226
209	A	9	8	1.39	31	0.258
210	A	12	11	1.67	31	0.355
211	A	9	8	1.84	31	0.258
212	B	8	7	2.49	31	0.226
213	A	5	5	1.00	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	6	5	1.86	31	0.161
215	A	9	8	1.61	31	0.258
216	A	11	10	1.58	31	0.323
217	A	14	13	1.52	31	0.419
218	A	11	10	1.56	31	0.323
219	A	9	8	1.67	31	0.258
220	A	8	7	2.00	31	0.226
221	A	4	4	0.98	29	0.138
222	A	10	9	1.70	31	0.290
223	A	12	11	1.46	31	0.355
224	A	14	13	1.48	31	0.419
225	A	16	15	1.38	31	0.484
226	A	13	12	1.39	31	0.387
227	A	11	10	1.51	31	0.323
228	A	9	8	1.69	31	0.258
229	A	7	6	1.67	31	0.194
230	A	6	6	0.97	29	0.207
231	A	13	12	1.62	31	0.387
232	A	15	14	1.36	31	0.452
233	A	17	16	1.38	31	0.516
234	A	4	3	1.00	35	0.086
235	A	8	7	1.00	35	0.200
236	A	4	3	1.00	35	0.086
237	A	8	7	1.00	37	0.189
238	A	4	3	1.00	33	0.091
239	A	8	7	1.00	39	0.179
240	A	8	7	1.00	33	0.212
241	A	8	7	1.00	35	0.200
242	A	8	7	1.00	39	0.179
243	A	12	11	1.06	39	0.282
244	A	15	14	1.06	29	0.483
245	A	13	12	1.06	29	0.414

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	11	10	1.05	29	0.345
247	A	9	8	1.05	27	0.296
248	A	9	8	1.00	29	0.276
249	A	10	9	1.11	29	0.310
250	A	12	11	1.15	29	0.379
251	A	15	14	1.17	29	0.483
252	A	5	5	1.00	31	0.161
253	A	5	5	1.00	31	0.161
254	A	5	5	1.00	31	0.161
255	A	9	8	1.00	29	0.276
256	A	8	7	1.00	31	0.226
257	A	11	10	1.18	31	0.323
258	A	5	5	1.00	31	0.161
259	A	5	5	1.00	31	0.161
260	A	5	5	1.00	31	0.161
261	A	5	5	1.00	31	0.161
262	A	10	9	1.11	29	0.310
263	A	11	10	1.17	31	0.323
264	A	5	5	1.00	33	0.152
265	A	2	2	1.00	35	0.057
266	A	2	2	1.00	35	0.057
267	A	2	2	1.00	35	0.057
268	A	2	2	1.00	35	0.057
269	A	5	5	1.00	37	0.135
270	A	13	13	1.00	39	0.333
271	A	6	6	1.00	39	0.154
272	A	15	15	1.04	39	0.385
273	A	2	2	1.00	33	0.061
274	A	21	21	1.05	39	0.538
275	A	5	5	1.00	33	0.152
276	A	5	5	1.00	35	0.143
277	A	14	14	1.00	39	0.359

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	21	21	0.99	39	0.538
279	A	5	5	1.00	33	0.152
280	A	13	13	1.00	39	0.333
281	A	2	2	1.00	33	0.061
282	A	5	5	1.00	35	0.143
283	A	6	6	1.00	39	0.154
284	A	13	13	1.00	39	0.333
285	A	6	5	0.88	28	0.179
286	A	6	5	0.84	28	0.179

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 30, antiderivative size = 105

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx\\&=\frac{7ac^4\operatorname{arctanh}(\sin(e+fx))}{8f}-\frac{ac^4\sec(e+fx)\tan(e+fx)}{8f}\\&\quad-\frac{3ac^4\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{4ac^4\tan^3(e+fx)}{3f}+\frac{ac^4\tan^5(e+fx)}{5f}\end{aligned}$$

output
```
7/8*a*c^4*arctanh(sin(f*x+e))/f-1/8*a*c^4*sec(f*x+e)*tan(f*x+e)/f-3/4*a*c^
4*sec(f*x+e)^3*tan(f*x+e)/f+4/3*a*c^4*tan(f*x+e)^3/f+1/5*a*c^4*tan(f*x+e)^
5/f
```

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.47

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$$
$$= \frac{ac^{7/2}\left(-210\arcsin\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\sqrt{c-c\sec(e+fx)}+\sqrt{c}\sqrt{1+\sec(e+fx)}(136-121\sec(e+fx)-}{120f(-1+\sec(e+fx))\sqrt{1}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]
```

output
```
(a*c^(7/2)*(-210*ArcSin[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Sqrt[c
 - c*Sec[e + f*x]] + Sqrt[c]*Sqrt[1 + Sec[e + f*x]]*(136 - 121*Sec[e + f*x
] - 127*Sec[e + f*x]^2 + 202*Sec[e + f*x]^3 - 114*Sec[e + f*x]^4 + 24*Sec[
e + f*x]^5))*Tan[e + f*x])/(120*f*(-1 + Sec[e + f*x])*Sqrt[1 + Sec[e + f*x
]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4446}$$

$$-ac\int \left(-c^3\tan^2(e+fx)\sec^4(e+fx)+3c^3\tan^2(e+fx)\sec^3(e+fx)-3c^3\tan^2(e+fx)\sec^2(e+fx)+c^3\tan$$

$$\Big\downarrow \text{2009}$$

$$-ac\left(-\frac{7c^3\text{arctanh}(\sin(e+fx))}{8f}-\frac{c^3\tan^5(e+fx)}{5f}-\frac{4c^3\tan^3(e+fx)}{3f}+\frac{3c^3\tan(e+fx)\sec^3(e+fx)}{4f}+\frac{c^3\tan$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]
```

output
```
-(a*c*((-7*c^3*ArcTanh[Sin[e + f*x]])/(8*f) + (c^3*Sec[e + f*x]*Tan[e + f*
x])/(8*f) + (3*c^3*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (4*c^3*Tan[e + f*x
]^3)/(3*f) - (c^3*Tan[e + f*x]^5)/(5*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.51

method	result
norman	$\frac{\frac{7 a c^4 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{4 f}-\frac{49 a c^4 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^3}{6 f}+\frac{224 a c^4 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^5}{15 f}-\frac{79 a c^4 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^7}{6 f}-\frac{7 a c^4 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^9}{4 f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^2-1\right)^5}-\frac{7 a c^4 \ln\Big(}{}$
risch	$\frac{i a c^4 \left(15 \mathrm{e}^{9 i (f x+e)}-360 \mathrm{e}^{8 i (f x+e)}+390 \mathrm{e}^{7 i (f x+e)}-960 \mathrm{e}^{6 i (f x+e)}-400 \mathrm{e}^{4 i (f x+e)}-390 \mathrm{e}^{3 i (f x+e)}-320 \mathrm{e}^{2 i (f x+e)}-15 \mathrm{e}^{i (}\right.}{60 f \left(\mathrm{e}^{2 i (f x+e)}+1\right)^5}$
parts	$\frac{2 a c^4 \ln(\sec(f x+e)+\tan(f x+e))}{f}-\frac{a c^4 \left(-\frac{8}{15}-\frac{\sec(f x+e)^4}{5}-\frac{4 \sec(f x+e)^2}{15}\right) \tan(f x+e)}{f}-\frac{3 a c^4 \tan(f x+e)}{f}+\frac{a c^4}{}$
parallelrisch	$-\frac{13 a c^4 \left(\left(\left(\frac{35 \cos(f x+e)}{26}+\frac{35 \cos(3 f x+3 e)}{52}+\frac{7 \cos(5 f x+5 e)}{52}\right) \ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{35 \cos(f x+e)}{26}-\frac{35 \cos(3 f x+3 e)}{52}-\right.\right.\right.}{2 f (\cos(5 f x+5 e)+5 \cos(3 f x}$
derivativedivides	$\frac{a c^4 \ln(\sec(f x+e)+\tan(f x+e))-3 a c^4 \tan(f x+e)+2 a c^4 \left(\frac{\sec(f x+e) \tan(f x+e)}{2}+\frac{\ln(\sec(f x+e)+\tan(f x+e))}{2}\right)-2 a c^4 \left(-\frac{2}{3}\right.}{}$
default	$\frac{a c^4 \ln(\sec(f x+e)+\tan(f x+e))-3 a c^4 \tan(f x+e)+2 a c^4 \left(\frac{\sec(f x+e) \tan(f x+e)}{2}+\frac{\ln(\sec(f x+e)+\tan(f x+e))}{2}\right)-2 a c^4 \left(-\frac{2}{3}\right.}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE
)
```

output
```
(7/4*a*c^4/f*tan(1/2*f*x+1/2*e)-49/6*a*c^4/f*tan(1/2*f*x+1/2*e)^3+224/15*a
*c^4/f*tan(1/2*f*x+1/2*e)^5-79/6*a*c^4/f*tan(1/2*f*x+1/2*e)^7-7/4*a*c^4/f*
tan(1/2*f*x+1/2*e)^9)/(tan(1/2*f*x+1/2*e)^2-1)^5-7/8*a*c^4/f*ln(tan(1/2*f*
x+1/2*e)-1)+7/8*a*c^4/f*ln(tan(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$$
$$=\frac{105\,ac^4\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-105\,ac^4\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)-2\,\left(136\,ac^4\right.}{240\,f\cos}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="fri
cas")
```

output

```
1/240*(105*a*c^4*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 105*a*c^4*cos(f*x
+ e)^5*log(-sin(f*x + e) + 1) - 2*(136*a*c^4*cos(f*x + e)^4 + 15*a*c^4*cos
(f*x + e)^3 - 112*a*c^4*cos(f*x + e)^2 + 90*a*c^4*cos(f*x + e) - 24*a*c^4)
*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$$
$$= ac^4\left(\int \sec\left(e+fx\right)dx+\int\left(-3\sec^2\left(e+fx\right)\right)\,dx+\int 2\sec^3\left(e+fx\right)dx\right.$$
$$\left.+\int 2\sec^4\left(e+fx\right)dx+\int\left(-3\sec^5\left(e+fx\right)\right)\,dx+\int\sec^6\left(e+fx\right)dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**4,x)
```

output

```
a*c**4*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + Inte
gral(2*sec(e + f*x)**3, x) + Integral(2*sec(e + f*x)**4, x) + Integral(-3*
sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(95) = 190$.

Time $= 0.04$ (sec) , antiderivative size $= 215$, normalized size of antiderivative $= 2.05$

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$$
$$= \frac{16\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)ac^4+160\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ac}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="max
ima")
```

output
```
1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a*c^4 +
 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^4 + 45*a*c^4*(2*(3*sin(f*x + e)
^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f
*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a*c^4*(2*sin(f*x + e)/(sin(f
*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^
4*log(sec(f*x + e) + tan(f*x + e)) - 720*a*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.38

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx = \frac{105\,ac^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,ac^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+790\,ac^4}{}}{120\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="gia
c")
```

output
```
1/120*(105*a*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a*c^4*log(abs(ta
n(1/2*f*x + 1/2*e) - 1)) - 2*(105*a*c^4*tan(1/2*f*x + 1/2*e)^9 + 790*a*c^4
*tan(1/2*f*x + 1/2*e)^7 - 896*a*c^4*tan(1/2*f*x + 1/2*e)^5 + 490*a*c^4*tan
(1/2*f*x + 1/2*e)^3 - 105*a*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e
)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 15.87 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.68

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx = \frac{7\,a\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f} - \frac{\frac{7\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}+\frac{79\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{6}-\frac{224\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}+\frac{49\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{6}-\frac{7\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

input

```
int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

output

```
(7*a*c^4*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((49*a*c^4*tan(e/2 + (f*x)/2)^
3)/6 - (7*a*c^4*tan(e/2 + (f*x)/2))/4 - (224*a*c^4*tan(e/2 + (f*x)/2)^5)/1
5 + (79*a*c^4*tan(e/2 + (f*x)/2)^7)/6 + (7*a*c^4*tan(e/2 + (f*x)/2)^9)/4)/
(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/
2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.30

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$$

$$= \frac{a\,c^4\left(-105\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^4+210\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\right.}{}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x)
```

output

```
(a*c**4*( - 105*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 + 2
10*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 - 105*cos(e + f*
x)*log(tan((e + f*x)/2) - 1) + 105*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*
sin(e + f*x)**4 - 210*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)*
*2 + 105*cos(e + f*x)*log(tan((e + f*x)/2) + 1) + 15*cos(e + f*x)*sin(e +
f*x)**3 - 105*cos(e + f*x)*sin(e + f*x) - 136*sin(e + f*x)**5 + 160*sin(e
+ f*x)**3))/(120*cos(e + f*x)*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.2 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx &= \frac{5ac^3\text{arctanh}(\sin(e+fx))}{8f}-\frac{3ac^3\sec(e+fx)\tan(e+fx)}{8f}\\ &\quad -\frac{ac^3\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{2ac^3\tan^3(e+fx)}{3f}\end{aligned}$$

output
```
5/8*a*c^3*arctanh(sin(f*x+e))/f-3/8*a*c^3*sec(f*x+e)*tan(f*x+e)/f-1/4*a*c^
3*sec(f*x+e)^3*tan(f*x+e)/f+2/3*a*c^3*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx =$$

$$-\frac{ac^{5/2}\Big(30\arcsin\Big(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\Big)\sqrt{c-c\sec(e+fx)}+\sqrt{c}\sqrt{1+\sec(e+fx)}(-16+7\sec(e+fx)+25}{24f(-1+\sec(e+fx))\sqrt{1+\sec(e+f}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

output
```
-1/24*(a*c^(5/2)*(30*ArcSin[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Sq
rt[c - c*Sec[e + f*x]] + Sqrt[c]*Sqrt[1 + Sec[e + f*x]]*(-16 + 7*Sec[e + f
*x] + 25*Sec[e + f*x]^2 - 22*Sec[e + f*x]^3 + 6*Sec[e + f*x]^4))*Tan[e + f
*x])/(f*(-1 + Sec[e + f*x])*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4446}$$

$$-ac\int \left(c^2\tan^2(e+fx)\sec^3(e+fx)-2c^2\tan^2(e+fx)\sec^2(e+fx)+c^2\tan^2(e+fx)\sec(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ac\Bigg(-\frac{5c^2\text{arctanh}(\sin(e+fx))}{8f}-\frac{2c^2\tan^3(e+fx)}{3f}+\frac{c^2\tan(e+fx)\sec^3(e+fx)}{4f}+\frac{3c^2\tan(e+fx)\sec(e+f}{8f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

output
```
-(a*c*((-5*c^2*ArcTanh[Sin[e + f*x]])/(8*f) + (3*c^2*Sec[e + f*x]*Tan[e +
f*x])/(8*f) + (c^2*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (2*c^2*Tan[e + f*x
]^3)/(3*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a c^3 \ln(\sec(fx+e)+\tan(fx+e))-2a c^3 \tan(fx+e)-2a c^3 (-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}) \tan(fx+e)-a c^3 (-(-\frac{\sec(fx+e)^3}{4}-3 \sec}{f}$
default	$\frac{a c^3 \ln(\sec(fx+e)+\tan(fx+e))-2a c^3 \tan(fx+e)-2a c^3 (-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}) \tan(fx+e)-a c^3 (-(-\frac{\sec(fx+e)^3}{4}-3 \sec}{f}$
parts	$\frac{a c^3 \ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{2a c^3 \tan(fx+e)}{f}-\frac{2a c^3 (-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}) \tan(fx+e)}{f}-a c^3 (-(-\frac{\sec(fx+e}{4}$
norman	$\frac{-\frac{5a c^3 \tan(\frac{fx}{2}+\frac{e}{2})}{4f}+\frac{55a c^3 \tan(\frac{fx}{2}+\frac{e}{2})^3}{12f}-\frac{73a c^3 \tan(\frac{fx}{2}+\frac{e}{2})^5}{12f}-\frac{5a c^3 \tan(\frac{fx}{2}+\frac{e}{2})^7}{4f}}{(\tan(\frac{fx}{2}+\frac{e}{2})^2-1)^4}-\frac{5a c^3 \ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{8f}+5$
risch	$\frac{i a c^3 (9 e^{7i(fx+e)}-48 e^{6i(fx+e)}+33 e^{5i(fx+e)}-48 e^{4i(fx+e)}-33 e^{3i(fx+e)}-16 e^{2i(fx+e)}-9 e^{i(fx+e)}-16)}{12f(e^{2i(fx+e)}+1)^4}-5a c^3 \ln($
parallelrisch	$-\frac{3a((\frac{10\cos(2fx+2e)}{3}+\frac{5\cos(4fx+4e)}{6}+\frac{5}{2}) \ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)+(-\frac{10\cos(2fx+2e)}{3}-\frac{5\cos(4fx+4e)}{6}-\frac{5}{2}) \ln(\tan(\frac{fx}{2}+\frac{e}{2}}{4f(\cos(4fx+4e)+4\cos(2fx+2e)+3)}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/f*(a*c^3*ln(sec(f*x+e)+tan(f*x+e))-2*a*c^3*tan(f*x+e)-2*a*c^3*(-2/3-1/3*
sec(f*x+e)^2)*tan(f*x+e)-a*c^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*
x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$= \frac{15\,ac^3\cos{(fx+e)}^4\log{(\sin{(fx+e)}+1)}-15\,ac^3\cos{(fx+e)}^4\log{(-\sin{(fx+e)}+1)}-2\,(16\,ac^3\cos}{48\,f\cos{(fx+e)}^4}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
1/48*(15*a*c^3*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a*c^3*cos(f*x + e
)^4*log(-sin(f*x + e) + 1) - 2*(16*a*c^3*cos(f*x + e)^3 + 9*a*c^3*cos(f*x
+ e)^2 - 16*a*c^3*cos(f*x + e) + 6*a*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$= -ac^3\left(\int\left(-\sec{(e+fx)}\right)dx+\int 2\sec^2{(e+fx)}\,dx+\int\left(-2\sec^4{(e+fx)}\right)dx\right.$$
$$\left.+\int\sec^5{(e+fx)}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**3,x)
```

output
```
-a*c**3*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + Int
egral(-2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.55

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$=\frac{32\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ac^3+3\,ac^3\left(\frac{2\left(3\,\sin(fx+e)^3-5\,\sin(fx+e)\right)}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\mathrm{l}\right.}{48\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="max
ima")
```

output
```
1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^3 + 3*a*c^3*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(si
n(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 48*a*c^3*log(sec(f*x + e) + t
an(f*x + e)) - 96*a*c^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$=\frac{15\,ac^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,ac^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+73\,ac^3\tan\left(\frac{1}{2}\right.\right.}{\left(\mathrm{t}\right.}}{24\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="gia
c")
```

output
```
1/24*(15*a*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a*c^3*log(abs(tan(1
/2*f*x + 1/2*e) - 1)) - 2*(15*a*c^3*tan(1/2*f*x + 1/2*e)^7 + 73*a*c^3*tan(
1/2*f*x + 1/2*e)^5 - 55*a*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a*c^3*tan(1/2*f*
x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 14.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.70

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$=\frac{5\,a\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{5\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}+\frac{73\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}-\frac{55\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{5\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

input
```
int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

output
```
(5*a*c^3*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((5*a*c^3*tan(e/2 + (f*x)/2))/
4 - (55*a*c^3*tan(e/2 + (f*x)/2)^3)/12 + (73*a*c^3*tan(e/2 + (f*x)/2)^5)/1
2 + (5*a*c^3*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e
/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.12

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$=\frac{a\,c^3\left(16\cos\left(fx+e\right)\sin\left(fx+e\right)^3-15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^4+30\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\mathrm{si}}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x)
```

output

```
(a*c**3*(16*cos(e + f*x)*sin(e + f*x)**3 - 15*log(tan((e + f*x)/2) - 1)*si
n(e + f*x)**4 + 30*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 - 15*log(tan(
(e + f*x)/2) - 1) + 15*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4 - 30*log(
tan((e + f*x)/2) + 1)*sin(e + f*x)**2 + 15*log(tan((e + f*x)/2) + 1) + 9*s
in(e + f*x)**3 - 15*sin(e + f*x)))/(24*f*(sin(e + f*x)**4 - 2*sin(e + f*x)
**2 + 1))
```

3.3 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 30, antiderivative size = 61

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = \frac{ac^2\text{arctanh}(\sin(e+fx))}{2f}-\frac{ac^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{ac^2\tan^3(e+fx)}{3f}$$

output
```
1/2*a*c^2*arctanh(sin(f*x+e))/f-1/2*a*c^2*sec(f*x+e)*tan(f*x+e)/f+1/3*a*c^
2*tan(f*x+e)^3/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 132 vs. 2(61) = 122.

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.16

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = \frac{ac^{3/2}\left(-6\arcsin\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\sqrt{c-c\sec(e+fx)}+\sqrt{c}\sqrt{1+\sec(e+fx)}(2+\sec(e+fx)-5\sec^2(e}{6f(-1+\sec(e+fx))\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

output
```
(a*c^(3/2)*(-6*ArcSin[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Sqrt[c -
 c*Sec[e + f*x]] + Sqrt[c]*Sqrt[1 + Sec[e + f*x]]*(2 + Sec[e + f*x] - 5*Se
c[e + f*x]^2 + 2*Sec[e + f*x]^3))*Tan[e + f*x])/(6*f*(-1 + Sec[e + f*x])*S
qrt[1 + Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4446}$$

$$-ac\int \left(c\sec(e+fx)\tan^2(e+fx)-c\sec^2(e+fx)\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ac\left(-\frac{c\,\text{arctanh}(\sin(e+fx))}{2f}-\frac{c\tan^3(e+fx)}{3f}+\frac{c\tan(e+fx)\sec(e+fx)}{2f}\right)$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

output
```
-(a*c*(-1/2*(c*ArcTanh[Sin[e + f*x]])/f + (c*Sec[e + f*x]*Tan[e + f*x])/(2
*f) - (c*Tan[e + f*x]^3)/(3*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{a c^2 \ln(\sec(fx+e)+\tan(fx+e))-a c^2 \tan(fx+e)-a c^2 \left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a c^2 \left(-\frac{2}{3}-\mathrm{sec}\right.}{f}$
default	$\frac{a c^2 \ln(\sec(fx+e)+\tan(fx+e))-a c^2 \tan(fx+e)-a c^2 \left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a c^2 \left(-\frac{2}{3}-\mathrm{sec}\right.}{f}$
risch	$\frac{i a c^2 \left(3\,\mathrm{e}^{5i(fx+e)}-6\,\mathrm{e}^{4i(fx+e)}-3\,\mathrm{e}^{i(fx+e)}-2\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}+\frac{a c^2 \ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{2f}-\frac{a c^2 \ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{2f}$
parts	$\frac{a c^2 \ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a c^2 \left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{a c^2 \tan(fx+e)}{f}-\frac{a c^2 \left(\frac{\sec(fx+e)\tan(fx-}{2}\right.}{}$
norman	$\frac{\frac{a c^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{8a c^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{a c^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{a c^2 \ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a c^2 \ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$
parallelrisch	$-\frac{a\left(\frac{3(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\sin(3fx+3e)-3\right.}{3f(\cos(3fx+3e)+3\cos(fx+e))}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/f*(a*c^2*ln(sec(f*x+e)+tan(f*x+e))-a*c^2*tan(f*x+e)-a*c^2*(1/2*sec(f*x+e
)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a*c^2*(-2/3-1/3*sec(f*x+e)^2)*
tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.69

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$$
$$=\frac{3\,ac^2\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,ac^2\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)-2\left(2\,ac^2\cos\left(fx\right.\right.}{12\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output
```
1/12*(3*a*c^2*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a*c^2*cos(f*x + e)^
3*log(-sin(f*x + e) + 1) - 2*(2*a*c^2*cos(f*x + e)^2 + 3*a*c^2*cos(f*x + e
) - 2*a*c^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$$
$$=ac^2\left(\int\sec\left(e+fx\right)dx+\int\left(-\sec^2\left(e+fx\right)\right)\,dx+\int\left(-\sec^3\left(e+fx\right)\right)\,dx\right.$$
$$\left.+\int\sec^4\left(e+fx\right)dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**2,x)
```

output
```
a*c**2*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Integr
al(-sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.77

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$$
$$= \frac{4\,(\tan{(fx+e)}^3+3\,\tan{(fx+e)})ac^2+3\,ac^2\Big(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log{(\sin{(fx+e)}+1)}+\log{(\sin{(fx+e)}-}}{12\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2 + 3*a*c^2*(2*sin(f*x + e)/
(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 12
*a*c^2*log(sec(f*x + e) + tan(f*x + e)) - 12*a*c^2*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(55) = 110$.

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$$
$$= \frac{3\,ac^2\log{\left(\left|\tan{\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+1\right|\right)}-3\,ac^2\log{\left(\left|\tan{\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}-1\right|\right)}-\frac{2\,\Big(3\,ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+8\,ac^2\tan\left(\frac{1}{2}\,fx+\right.}{\Big(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-}}{6\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
1/6*(3*a*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a*c^2*log(abs(tan(1/2*
f*x + 1/2*e) - 1)) - 2*(3*a*c^2*tan(1/2*f*x + 1/2*e)^5 + 8*a*c^2*tan(1/2*f
*x + 1/2*e)^3 - 3*a*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)
^3)/f
```

Mupad [B] (verification not implemented)

Time = 13.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.87

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx \\
&= \frac{a\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f} \\
&\quad - \frac{a\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\frac{8\,a\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}-a\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}
\end{aligned}
$$

input

```
int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

output

```
(a*c^2*atanh(tan(e/2 + (f*x)/2)))/f - ((8*a*c^2*tan(e/2 + (f*x)/2)^3)/3 -
a*c^2*tan(e/2 + (f*x)/2) + a*c^2*tan(e/2 + (f*x)/2)^5)/(f*(3*tan(e/2 + (f*
x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.46

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx \\
&= \frac{a\,c^2\left(-3\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^2+3\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+3\cos\right.}{6\cos}
\end{aligned}
$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x)
```

output

```
(a*c**2*( - 3*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 + 3*c
os(e + f*x)*log(tan((e + f*x)/2) - 1) + 3*cos(e + f*x)*log(tan((e + f*x)/2
) + 1)*sin(e + f*x)**2 - 3*cos(e + f*x)*log(tan((e + f*x)/2) + 1) + 3*cos(
e + f*x)*sin(e + f*x) - 2*sin(e + f*x)**3))/(6*cos(e + f*x)*f*(sin(e + f*x
)**2 - 1))
```

3.4 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 38

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = \frac{ac\,\text{arctanh}(\sin(e+fx))}{2f} - \frac{ac\sec(e+fx)\tan(e+fx)}{2f}$$

output

```
1/2*a*c*arctanh(sin(f*x+e))/f-1/2*a*c*sec(f*x+e)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -ac\left(-\frac{\text{arctanh}(\sin(e+fx))}{2f} + \frac{\sec(e+fx)\tan(e+fx)}{2f}\right)$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

output
```
-(a*c*(-1/2*ArcTanh[Sin[e + f*x]]/f + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4446, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4446}$$

$$-ac\int \sec(e+fx)\tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \sec(e+fx)\tan(e+fx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$-ac\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int \sec(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-ac\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$-ac\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{\text{arctanh}(\sin(e+fx))}{2f}\right)$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

output

```
-(a*c*(-1/2*ArcTanh[Sin[e + f*x]]/f + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4446

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

method	result	size
derivativedivides	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))-ac\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	58
default	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))-ac\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	58
parts	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{ac\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	60
parallelrisch	$\frac{a\left((-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-2\sin(fx+e)\right)c}{2f(1+\cos(2fx+2e))}$	80
risch	$\frac{iac\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}+\frac{ac\ln\left(e^{i(fx+e)}+i\right)}{2f}-\frac{ac\ln\left(e^{i(fx+e)}-i\right)}{2f}$	84
norman	$\frac{-\frac{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{ac\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{ac\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$	91

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*c*ln(sec(f*x+e)+tan(f*x+e))-a*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(s
ec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\frac{ac\cos{(fx+e)}^2\log{(\sin{(fx+e)}+1)}-ac\cos{(fx+e)}^2\log{(-\sin{(fx+e)}+1)}-2\,ac\sin{(fx+e)}}{4\,f\cos{(fx+e)}^2}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="frica
s")
```

output
```
1/4*(a*c*cos(f*x + e)^2*log(sin(f*x + e) + 1) - a*c*cos(f*x + e)^2*log(-si
n(f*x + e) + 1) - 2*a*c*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$= -ac\left(\int\left(-\sec\left(e+fx\right)\right)\,dx+\int\sec^{3}\left(e+fx\right)dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

output
```
-a*c*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$= \frac{ac\left(\frac{2\sin(fx+e)}{\sin(fx+e)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right.}{4\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
1/4*(a*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) + 4*a*c*log(sec(f*x + e) + tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\frac{ac\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-ac\log\left(\left|\sin\left(fx+e\right)-1\right|\right)+\frac{2\,ac\sin(fx+e)}{\sin(fx+e)^{2}-1}}{4\,f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac"
)
```

output

```
1/4*(a*c*log(abs(sin(f*x + e) + 1)) - a*c*log(abs(sin(f*x + e) - 1)) + 2*a
*c*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Mupad [B] (verification not implemented)

Time = 11.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\frac{a\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}+a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)}$$

input

```
int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

output

```
(a*c*atanh(tan(e/2 + (f*x)/2)))/f - (a*c*tan(e/2 + (f*x)/2)^3 + a*c*tan(e/
2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$= \frac{ac\bigl(-\log\bigl(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\bigr)\sin\left(fx+e\right)^2+\log\bigl(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\bigr)+\log\bigl(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\bigr)\sin\left(fx+e\right)^2}{2f\left(\sin\left(fx+e\right)^2-1\right)}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

output
```
(a*c*( - log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 + log(tan((e + f*x)/2)
- 1) + log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 - log(tan((e + f*x)/2) +
1) + sin(e + f*x)))/(2*f*(sin(e + f*x)**2 - 1))
```

3.5 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\operatorname{arctanh}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

output
```
-a*arctanh(sin(f*x+e))/c/f-2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.83

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\left(-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)}{f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
```

output
```
-((a*((-2*Cot[(e + f*x)/2])/f - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f
 + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]/f))/c)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4445, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{c-c\sec(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{4445}$$
$$-\frac{a\int \sec(e+fx)dx}{c}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{c}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$
$$\Big\downarrow \text{4257}$$
$$-\frac{a\text{arctanh}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
```

output
```
-((a*ArcTanh[Sin[e + f*x]])/(c*f)) - (2*a*Tan[e + f*x])/(f*(c - c*Sec[e +
f*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

method	result	size
parallelrisch	$\frac{a\left(\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+2\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}$	47
derivativedivides	$\frac{2a\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$	50
default	$\frac{2a\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$	50
risch	$\frac{4ia}{fc\left(\mathrm{e}^{i(fx+e)}-1\right)}+\frac{a\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{cf}-\frac{a\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{cf}$	68
norman	$\frac{-\frac{2a}{cf}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{a\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$	100

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/c/f*a*(ln(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*e)+1)+2*cot(1/2*f*x+1
/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-a\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-4\,a\cos\left(fx+e\right)-4\,a}{2\,cf\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="frica
s")
```

output
```
-1/2*(a*log(sin(f*x + e) + 1)*sin(f*x + e) - a*log(-sin(f*x + e) + 1)*sin(
f*x + e) - 4*a*cos(f*x + e) - 4*a)/(c*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\left(\int \frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

output
```
-a*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**
2/(sec(e + f*x) - 1), x))/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(43) = 86$.

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c} - \frac{\cos(fx+e)+1}{c\sin(fx+e)}\right) - \frac{a(\cos(fx+e)+1)}{c\sin(fx+e)}}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
-(a*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*
x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a*(cos(f*x + e
) + 1)/(c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c} - \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c} - \frac{2\,a}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
-(a*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - a*log(abs(tan(1/2*f*x + 1/2*e)
- 1))/c - 2*a/(c*tan(1/2*f*x + 1/2*e)))/f
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{2\,a\,\left(\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

input
```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

output
```
-(2*a*(atanh(tan(e/2 + (f*x)/2)) - cot(e/2 + (f*x)/2)))/(c*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = \frac{a\left(\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cf}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

output
```
(a*(log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2) - log(tan((e + f*x)/2) + 1)
*tan((e + f*x)/2) + 2))/(tan((e + f*x)/2)*c*f)
```

3.6 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}$$

output
```
-1/3*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{a\cot^3\left(\frac{1}{2}(e+fx)\right)}{3c^2f}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]
```

output
```
-1/3*(a*Cot[(e + f*x)/2]^3)/(c^2*f)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 4438$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{3f(c-c\sec(e+fx))^2}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]
```

output
```
-1/3*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$-\frac{a}{3 f c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	21
default	$-\frac{a}{3 f c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	21
parallelrisch	$-\frac{a \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 f c^{2}}$	21
risch	$\frac{2 i a \left(3 \,\mathrm{e}^{2 i (f x+e)}+1\right)}{3 f c^{2} \left(\mathrm{e}^{i (f x+e)}-1\right)^{3}}$	37
norman	$\frac{\frac{a}{3 c f}-\frac{a \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{3 c f}}{c \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	61

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output

```
-1/3/f*a/c^2/tan(1/2*f*x+1/2*e)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx=\frac{a\cos\left(fx+e\right)^{2}+2\,a\cos\left(fx+e\right)+a}{3\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output

```
1/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)/((c^2*f*cos(f*x + e) - c^2*f
)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = \frac{a\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx+\int \frac{\sec^2(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx\right)}{c^2}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

output
```
a*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{\frac{a\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}-\frac{a\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}}{6f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
-1/6*(a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(
c^2*sin(f*x + e)^3) - a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f
*x + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{a}{3\,c^2f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="gia
c")
```

output

```
-1/3*a/(c^2*f*tan(1/2*f*x + 1/2*e)^3)
```

Mupad [B] (verification not implemented)

Time = 10.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,c^2\,f}$$

input

```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

output

```
-(a*cot(e/2 + (f*x)/2)^3)/(3*c^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{a}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^2f}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x)
```

output

```
( - a)/(3*tan((e + f*x)/2)**3*c**2*f)
```

3.7 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{15cf(c-c\sec(e+fx))^2}$$

output
```
-1/5*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^3-1/15*(a+a*sec(f*x+e)
)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{a(-4+\sec(e+fx))(1+\sec(e+fx))\tan(e+fx)}{15c^3f(-1+\sec(e+fx))^3}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
```

output
```
-1/15*(a*(-4 + Sec[e + f*x])*(1 + Sec[e + f*x])*Tan[e + f*x])/(c^3*f*(-1 +
 Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{(c-c\sec(e+fx))^2}dx}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{15cf(c-c\sec(e+fx))^2}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
```

output
```
-1/5*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^3) - ((a
+ a*Sec[e + f*x])*Tan[e + f*x])/(15*c*f*(c - c*Sec[e + f*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

method	result	size
parallelrisch	$\frac{a\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-5\right)}{30c^{3}f}$	36
derivativedivides	$\frac{a\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}\right)}{2f\,c^{3}}$	37
default	$\frac{a\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}\right)}{2f\,c^{3}}$	37
risch	$\frac{2ia\left(15\,{\mathrm e}^{4i(fx+e)}-15\,{\mathrm e}^{3i(fx+e)}+25\,{\mathrm e}^{2i(fx+e)}-5\,{\mathrm e}^{i(fx+e)}+4\right)}{15f\,c^{3}\left({\mathrm e}^{i(fx+e)}-1\right)^{5}}$	70
norman	$\frac{-\frac{a}{10cf}+\frac{4a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{15cf}-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{6cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$	81

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/30*a*cot(1/2*f*x+1/2*e)^3*(3*cot(1/2*f*x+1/2*e)^2-5)/c^3/f
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = \frac{4\,a\cos\left(fx+e\right)^3+7\,a\cos\left(fx+e\right)^2+2\,a\cos\left(fx+e\right)-a}{15\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
1/15*(4*a*cos(f*x + e)^3 + 7*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/((c^
3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{a\left(\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx\right)}{c^3}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
```

output
```
-a*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e + f*x
)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{\frac{a\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}+\frac{3a\left(\frac{5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}}{60f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="max
ima")
```

output

```
-1/60*(a*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 3*a*(5*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x +
 e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{5\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,a}{30\,c^3f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="gia
c")
```

output

```
-1/30*(5*a*tan(1/2*f*x + 1/2*e)^2 - 3*a)/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = \frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-5\right)}{30\,c^3\,f}$$

input
```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

output
```
(a*cot(e/2 + (f*x)/2)^3*(3*cot(e/2 + (f*x)/2)^2 - 5))/(30*c^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = \frac{a\left(-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+3\right)}{30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^3f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x)
```

output
```
(a*( - 5*tan((e + f*x)/2)**2 + 3))/(30*tan((e + f*x)/2)**5*c**3*f)
```

3.8 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 116

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{35cf(c-c\sec(e+fx))^3} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{105f\left(c^2-c^2\sec(e+fx)\right)^2}$$

output
```
-1/7*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^4-2/35*(a+a*sec(f*x+e)
)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^3-2/105*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c
^2-c^2*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{a(1+\sec(e+fx))\left(23-10\sec(e+fx)+2\sec^2(e+fx)\right)\tan(e+fx)}{105c^4f(-1+\sec(e+fx))^4}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

output
```
-1/105*(a*(1 + Sec[e + f*x])*(23 - 10*Sec[e + f*x] + 2*Sec[e + f*x]^2)*Tan
[e + f*x])/(c^4*f*(-1 + Sec[e + f*x])^4)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4439, 3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{2\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{(c-c\sec(e+fx))^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

$$\Big\downarrow \text{4439}$$

$$\frac{2\left(\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{(c-c\sec(e+fx))^2}dx}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}\right)}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}\right)}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

$$\Big\downarrow \text{4438}$$

$$\frac{2\left(-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{15cf(c-c\sec(e+fx))^2}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}\right)}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

output

```
-1/7*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4) + (2*(
-1/5*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^3) - ((a
+ a*Sec[e + f*x])*Tan[e + f*x])/(15*c*f*(c - c*Sec[e + f*x])^2)))/(7*c)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

method	result	size
parallelrisch	$-\frac{a\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-42\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+35\right)}{420c^{4}f}$	49
derivativedivides	$\frac{a\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}\right)}{4f\,c^{4}}$	50
default	$\frac{a\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}\right)}{4f\,c^{4}}$	50
risch	$\frac{2ia\left(105\,\mathrm{e}^{6i(fx+e)}-210\,\mathrm{e}^{5i(fx+e)}+455\,\mathrm{e}^{4i(fx+e)}-350\,\mathrm{e}^{3i(fx+e)}+273\,\mathrm{e}^{2i(fx+e)}-56\,\mathrm{e}^{i(fx+e)}+23\right)}{105f\,c^{4}\left(\mathrm{e}^{i(fx+e)}-1\right)^{7}}$	92
norman	$\frac{\frac{a}{28cf}-\frac{19a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{140cf}+\frac{11a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{60cf}-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{12cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$	101

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE
)
```

output

```
-1/420*a*cot(1/2*f*x+1/2*e)^3*(15*cot(1/2*f*x+1/2*e)^4-42*cot(1/2*f*x+1/2*
e)^2+35)/c^4/f
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{4}}\,dx$$
$$=\frac{23\,a\cos\left(fx+e\right)^{4}+36\,a\cos\left(fx+e\right)^{3}+5\,a\cos\left(fx+e\right)^{2}-6\,a\cos\left(fx+e\right)+2\,a}{105\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-c^{4}f\right)\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fri
cas")
```

output
```
1/105*(23*a*cos(f*x + e)^4 + 36*a*cos(f*x + e)^3 + 5*a*cos(f*x + e)^2 - 6*
a*cos(f*x + e) + 2*a)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*
c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{a\Bigg(\int \frac{\sec(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{\sec^2(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx}{c^4}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

output
```
a*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e +
f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)
**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**
4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{\frac{a\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}+\frac{3a\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos}{c^4\sin(fx+e)^7}}{840f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="max
ima")
```

output
```
1/840*(a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x +
 e) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,a}{420\,c^4f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="gia
c")
```

output
```
-1/420*(35*a*tan(1/2*f*x + 1/2*e)^4 - 42*a*tan(1/2*f*x + 1/2*e)^2 + 15*a)/
(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = \frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{10\,c^4\,f}-\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12\,c^4\,f}-\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{28\,c^4\,f}$$

input
```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

output
```
(a*cot(e/2 + (f*x)/2)^5)/(10*c^4*f) - (a*cot(e/2 + (f*x)/2)^3)/(12*c^4*f)
- (a*cot(e/2 + (f*x)/2)^7)/(28*c^4*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.41

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx=\frac{a\left(-35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+42\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-15\right)}{420\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7c^4f}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x)
```

output

```
(a*( - 35*tan((e + f*x)/2)**4 + 42*tan((e + f*x)/2)**2 - 15))/(420*tan((e
+ f*x)/2)**7*c**4*f)
```

3.9 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 158

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{105c^2f(c-c\sec(e+fx))^3} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{315cf\left(c^2-c^2\sec(e+fx)\right)^2}$$

output

```
-1/9*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-1/21*(a+a*sec(f*x+e)
)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4-2/105*(a+a*sec(f*x+e))*tan(f*x+e)/c^2/
f/(c-c*sec(f*x+e))^3-2/315*(a+a*sec(f*x+e))*tan(f*x+e)/c/f/(c^2-c^2*sec(f*
x+e))^2
```

Mathematica [A] (verified)

Time = 5.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.41

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{a(1+\sec(e+fx))\left(-58+33\sec(e+fx)-12\sec^2(e+fx)+2\sec^3(e+fx)\right)\tan(e+fx)}{315c^5f(-1+\sec(e+fx))^5}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

output

```
-1/315*(a*(1 + Sec[e + f*x])*(-58 + 33*Sec[e + f*x] - 12*Sec[e + f*x]^2 +
2*Sec[e + f*x]^3)*Tan[e + f*x])/(c^5*f*(-1 + Sec[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 4439, 3042, 4439, 3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{(c-c\sec(e+fx))^4}dx}{3c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx}{3c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{4439}$$

$$\frac{\frac{2\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{(c-c\sec(e+fx))^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}}{3c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}}{3c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{4439}$$

$$\frac{\frac{2\left(\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{(c-c\sec(e+fx))^2}dx}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}\right)}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}}{3c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}\right)}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}}{3c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{4438}$$

$$\frac{\frac{2\left(-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{15cf(c-c\sec(e+fx))^2}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}\right)}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}}{3c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{9f(c-c\sec(e+fx))^5}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

output
```
-1/9*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) + (-1/
7*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4) + (2*(-1/
5*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^3) - ((a + a
*Sec[e + f*x])*Tan[e + f*x])/(15*c*f*(c - c*Sec[e + f*x])^2)))/(7*c))/(3*c
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

method	result
parallelrisch	$\frac{a\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-135\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+189\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-105\right)}{2520c^{5}f}$
derivativedivides	$\frac{a\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}\right)}{8f\,c^{5}}$
default	$\frac{a\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}\right)}{8f\,c^{5}}$
risch	$\frac{2ia(315\,\mathrm{e}^{8i(fx+e)}-945\,\mathrm{e}^{7i(fx+e)}+2625\,\mathrm{e}^{6i(fx+e)}-3465\,\mathrm{e}^{5i(fx+e)}+3843\,\mathrm{e}^{4i(fx+e)}-2247\,\mathrm{e}^{3i(fx+e)}+1143\,\mathrm{e}^{2i(fx+e)}-}{315f\,c^{5}\left(\mathrm{e}^{i(fx+e)}-1\right)^{9}}$
norman	$\frac{-\frac{a}{72cf}+\frac{17a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{252cf}-\frac{9a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{70cf}+\frac{7a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{60cf}-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{24cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE
)
```

output
```
1/2520*a*cot(1/2*f*x+1/2*e)^3*(35*cot(1/2*f*x+1/2*e)^6-135*cot(1/2*f*x+1/2
*e)^4+189*cot(1/2*f*x+1/2*e)^2-105)/c^5/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx$$
$$=\frac{58\,a\cos\left(fx+e\right)^{5}+83\,a\cos\left(fx+e\right)^{4}+4\,a\cos\left(fx+e\right)^{3}-11\,a\cos\left(fx+e\right)^{2}+8\,a\cos\left(fx+e\right)-2\,a}{315\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}+6\,c^{5}f\cos\left(fx+e\right)^{2}-4\,c^{5}f\cos\left(fx+e\right)+c^{5}f\right)\sin\left(fx+e\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="fri
cas")
```

output
```
1/315*(58*a*cos(f*x + e)^5 + 83*a*cos(f*x + e)^4 + 4*a*cos(f*x + e)^3 - 11
*a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - 2*a)/((c^5*f*cos(f*x + e)^4 - 4*c^5
*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)
*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx =$$

$$-\frac{a\Big(\int \frac{\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx+\int \frac{\sec^2(e+f}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx}}{c^5}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**5,x)
```

output
```
-a*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e
+ f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e
+ f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*s
ec(e + f*x)**2 + 5*sec(e + f*x) - 1), x))/c**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.25

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx =$$

$$-\frac{\frac{a\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{5a\Big(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63}{(\cos}}{c^5\sin(fx+e}}{5040f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="max
ima")
```

output
```
-1/5040*(a*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)
^9) + 5*a*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x +
e) + 1)^9/(c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.41

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{105\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-189\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+135\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-35\,a}{2520\,c^5f\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="gia
c")
```

output
```
-1/2520*(105*a*tan(1/2*f*x + 1/2*e)^6 - 189*a*tan(1/2*f*x + 1/2*e)^4 + 135
*a*tan(1/2*f*x + 1/2*e)^2 - 35*a)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

Mupad [B] (verification not implemented)

Time = 10.51 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = \frac{a\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(35\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-135\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+189\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\right.}{2520\,c^5\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}$$

input
```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

output
```
(a*cos(e/2 + (f*x)/2)^3*(35*cos(e/2 + (f*x)/2)^6 - 105*sin(e/2 + (f*x)/2)^
6 + 189*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 135*cos(e/2 + (f*x)/2)
^4*sin(e/2 + (f*x)/2)^2))/(2520*c^5*f*sin(e/2 + (f*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{a\left(-105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+189\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-135\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+35\right)}{2520\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9c^5f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x)
```

output
```
(a*( - 105*tan((e + f*x)/2)**6 + 189*tan((e + f*x)/2)**4 - 135*tan((e + f*
x)/2)**2 + 35))/(2520*tan((e + f*x)/2)**9*c**5*f)
```

3.10 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$

Optimal result

Integrand size = 32, antiderivative size = 171

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx &= \frac{9a^2c^5\operatorname{arctanh}(\sin(e+fx))}{16f}-\frac{3a^2c^5\sec(e+fx)\tan(e+fx)}{16f}\\&\quad-\frac{3a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f}\\&\quad+\frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{2f}-\frac{4a^2c^5\tan^5(e+fx)}{5f}-\frac{a^2c^5\tan^7(e+fx)}{7f}\end{aligned}$$

output
```
9/16*a^2*c^5*arctanh(sin(f*x+e))/f-3/16*a^2*c^5*sec(f*x+e)*tan(f*x+e)/f-3/
8*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+1/4*a^2*c^5*sec(f*x+e)*tan(f*x+e)^3/f+
1/2*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)^3/f-4/5*a^2*c^5*tan(f*x+e)^5/f-1/7*a^2
*c^5*tan(f*x+e)^7/f
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$=\frac{a^2c^5(10080\text{arctanh}(\sin(e+fx))-\sec^7(e+fx)(2520\sin(e+fx)-455\sin(2(e+fx))-616\sin(3(e+}{17920f}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

output

```
(a^2*c^5*(10080*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(2520*Sin[e + f*x]
- 455*Sin[2*(e + f*x)] - 616*Sin[3*(e + f*x)] + 2380*Sin[4*(e + f*x)] - 39
2*Sin[5*(e + f*x)] + 245*Sin[6*(e + f*x)] + 184*Sin[7*(e + f*x)])))/(17920
*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^5\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5dx$$
$$\Big\downarrow \; 4446$$
$$a^2c^2\int\left(-c^3\sec^4(e+fx)\tan^4(e+fx)+3c^3\sec^3(e+fx)\tan^4(e+fx)-3c^3\sec^2(e+fx)\tan^4(e+fx)+c^3\sec\right.$$
$$\Big\downarrow \; 2009$$

$$a^2c^2\left(\frac{9c^3\text{arctanh}(\sin(e+fx))}{16f}-\frac{c^3\tan^7(e+fx)}{7f}-\frac{4c^3\tan^5(e+fx)}{5f}+\frac{c^3\tan^3(e+fx)\sec^3(e+fx)}{2f}-3c^3\tan($$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

output
```
a^2*c^2*((9*c^3*ArcTanh[Sin[e + f*x]])/(16*f) - (3*c^3*Sec[e + f*x]*Tan[e
+ f*x])/(16*f) - (3*c^3*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (c^3*Sec[e +
f*x]*Tan[e + f*x]^3)/(4*f) + (c^3*Sec[e + f*x]^3*Tan[e + f*x]^3)/(2*f) - (
4*c^3*Tan[e + f*x]^5)/(5*f) - (c^3*Tan[e + f*x]^7)/(7*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.22

method	result
risch	$\frac{i a^2 c^5 \left(245\, \mathrm{e}^{13 i (f x+e)}-1680\, \mathrm{e}^{12 i (f x+e)}+2380\, \mathrm{e}^{11 i (f x+e)}-4480\, \mathrm{e}^{10 i (f x+e)}-455\, \mathrm{e}^{9 i (f x+e)}-3920\, \mathrm{e}^{8 i (f x+e)}-8960\, \mathrm{e}^{6 i (f x}{280 f \left(\mathrm{e}^{2 i (f x+e)}+1\right)^7}$
norman	$\frac{\frac{9 a^2 c^5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{8 f}-\frac{15 a^2 c^5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^3}{2 f}+\frac{849 a^2 c^5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^5}{40 f}-\frac{1152 a^2 c^5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^7}{35 f}+\frac{1199 a^2 c^5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^9}{40 f}+15 a^2}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^2-1\right)^7}$
parallelrisch	$-\frac{9 a^2 c^5 \left(\frac{(\cos(7 f x+7 e)+21 \cos(3 f x+3 e)+35 \cos(f x+e)+7 \cos(5 f x+5 e)) \ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{16}+(-\cos(7 f x+7 e)-7 \cos(5 f x+5 e)-\right.}{f(\cos(7 f x+7}$
derivativedivides	$a^2 c^5 \ln(\sec(f x+e)+\tan(f x+e))-3 a^2 c^5 \tan(f x+e)+a^2 c^5 \left(\frac{\sec(f x+e) \tan(f x+e)}{2}+\frac{\ln(\sec(f x+e)+\tan(f x+e))}{2}\right)-5 a^2 c^5 \left(-\right.$
default	$a^2 c^5 \ln(\sec(f x+e)+\tan(f x+e))-3 a^2 c^5 \tan(f x+e)+a^2 c^5 \left(\frac{\sec(f x+e) \tan(f x+e)}{2}+\frac{\ln(\sec(f x+e)+\tan(f x+e))}{2}\right)-5 a^2 c^5 \left(-\right.$
parts	$\frac{a^2 c^5 \ln(\sec(f x+e)+\tan(f x+e))}{f}+\frac{a^2 c^5 \left(\frac{\sec(f x+e) \tan(f x+e)}{2}+\frac{\ln(\sec(f x+e)+\tan(f x+e))}{2}\right)}{f}-\frac{3 a^2 c^5 \tan(f x+e)}{f}-5$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBO
SE)
```

output
```
1/280*I*a^2*c^5*(245*exp(13*I*(f*x+e))-1680*exp(12*I*(f*x+e))+2380*exp(11*
I*(f*x+e))-4480*exp(10*I*(f*x+e))-455*exp(9*I*(f*x+e))-3920*exp(8*I*(f*x+e
))-8960*exp(6*I*(f*x+e))+455*exp(5*I*(f*x+e))-3248*exp(4*I*(f*x+e))-2380*e
xp(3*I*(f*x+e))-896*exp(2*I*(f*x+e))-245*exp(I*(f*x+e))-368)/f/(exp(2*I*(f
*x+e))+1)^7+9/16*a^2*c^5/f*ln(exp(I*(f*x+e))+I)-9/16*a^2*c^5/f*ln(exp(I*(f
*x+e))-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$

$$= \frac{315\, a^2 c^5 \cos\left(fx+e\right)^7 \log\left(\sin\left(fx+e\right)+1\right) - 315\, a^2 c^5 \cos\left(fx+e\right)^7 \log\left(-\sin\left(fx+e\right)+1\right) - 2\left(368\, a^2\right.}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="f
ricas")
```

output

```
1/1120*(315*a^2*c^5*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 315*a^2*c^5*cos
(f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(368*a^2*c^5*cos(f*x + e)^6 + 245*a
^2*c^5*cos(f*x + e)^5 - 656*a^2*c^5*cos(f*x + e)^4 + 350*a^2*c^5*cos(f*x +
 e)^3 + 208*a^2*c^5*cos(f*x + e)^2 - 280*a^2*c^5*cos(f*x + e) + 80*a^2*c^5
)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx\\&=-a^2c^5\left(\int(-\sec(e+fx))\,dx+\int 3\sec^2(e+fx)\,dx+\int\left(-\sec^3(e+fx)\right)\,dx\right.\\&\quad+\int\left(-5\sec^4(e+fx)\right)\,dx+\int 5\sec^5(e+fx)\,dx+\int\sec^6(e+fx)\,dx\\&\quad\left.+\int\left(-3\sec^7(e+fx)\right)\,dx+\int\sec^8(e+fx)\,dx\right)\end{aligned}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

output

```
-a**2*c**5*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**2, x) +
Integral(-sec(e + f*x)**3, x) + Integral(-5*sec(e + f*x)**4, x) + Integral
(5*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x) + Integral(-3*sec(e
+ f*x)**7, x) + Integral(sec(e + f*x)**8, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(157) = 314$.

Time = 0.04 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx =$$

$$-\frac{96\left(5\tan\left(fx+e\right)^7+21\tan\left(fx+e\right)^5+35\tan\left(fx+e\right)^3+35\tan\left(fx+e\right)\right)a^2c^5+224\left(3\tan\left(fx+\right.\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="m
axima")
```

output
```
-1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35
*tan(f*x + e))*a^2*c^5 + 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*ta
n(f*x + e))*a^2*c^5 - 5600*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 105
*a^2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin
(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x +
e) + 1) + 15*log(sin(f*x + e) - 1)) - 1050*a^2*c^5*(2*(3*sin(f*x + e)^3 -
5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x +
e) + 1) + 3*log(sin(f*x + e) - 1)) + 840*a^2*c^5*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3360*a^2*c^
5*log(sec(f*x + e) + tan(f*x + e)) + 10080*a^2*c^5*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$

$$=\frac{315\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-315\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(315\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}-210\right.}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="g
iac")
```

output
```
1/560*(315*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 315*a^2*c^5*log(ab
s(tan(1/2*f*x + 1/2*e) - 1)) - 2*(315*a^2*c^5*tan(1/2*f*x + 1/2*e)^13 - 21
00*a^2*c^5*tan(1/2*f*x + 1/2*e)^11 - 8393*a^2*c^5*tan(1/2*f*x + 1/2*e)^9 +
 9216*a^2*c^5*tan(1/2*f*x + 1/2*e)^7 - 5943*a^2*c^5*tan(1/2*f*x + 1/2*e)^5
 + 2100*a^2*c^5*tan(1/2*f*x + 1/2*e)^3 - 315*a^2*c^5*tan(1/2*f*x + 1/2*e))
/(tan(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.47

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$= \frac{-\frac{9\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{8}+\frac{15\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{2}+\frac{1199\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{40}-\frac{1152\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{35}+\frac{849\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{40}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-21\right.}$$
$$+\frac{9\,a^2\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

input
```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5)/cos(e + f*x),x)
```

output
```
((849*a^2*c^5*tan(e/2 + (f*x)/2)^5)/40 - (15*a^2*c^5*tan(e/2 + (f*x)/2)^3)
/2 - (1152*a^2*c^5*tan(e/2 + (f*x)/2)^7)/35 + (1199*a^2*c^5*tan(e/2 + (f*x
)/2)^9)/40 + (15*a^2*c^5*tan(e/2 + (f*x)/2)^11)/2 - (9*a^2*c^5*tan(e/2 + (
f*x)/2)^13)/8 + (9*a^2*c^5*tan(e/2 + (f*x)/2))/8)/(f*(7*tan(e/2 + (f*x)/2)
^2 - 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x
)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f
*x)/2)^14 - 1)) + (9*a^2*c^5*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.91

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$= \frac{a^2c^5\left(-315\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^6+945\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin}{}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x)
```

output

```
(a**2*c**5*( - 315*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6
+ 945*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 - 945*cos(e +
 f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 + 315*cos(e + f*x)*log(tan
((e + f*x)/2) - 1) + 315*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*
x)**6 - 945*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4 + 945*c
os(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 - 315*cos(e + f*x)*l
og(tan((e + f*x)/2) + 1) + 245*cos(e + f*x)*sin(e + f*x)**5 - 840*cos(e +
f*x)*sin(e + f*x)**3 + 315*cos(e + f*x)*sin(e + f*x) - 368*sin(e + f*x)**7
 + 448*sin(e + f*x)**5))/(560*cos(e + f*x)*f*(sin(e + f*x)**6 - 3*sin(e +
f*x)**4 + 3*sin(e + f*x)**2 - 1))
```

3.11 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 32, antiderivative size = 150

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx &= \frac{7a^2c^4\operatorname{arctanh}(\sin(e+fx))}{16f} - \frac{5a^2c^4\sec(e+fx)\tan(e+fx)}{16f} \\ &\quad - \frac{a^2c^4\sec^3(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{4f} \\ &\quad + \frac{a^2c^4\sec^3(e+fx)\tan^3(e+fx)}{6f} - \frac{2a^2c^4\tan^5(e+fx)}{5f}\end{aligned}$$

output

```
7/16*a^2*c^4*arctanh(sin(f*x+e))/f-5/16*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f-1/
8*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)/f+1/4*a^2*c^4*sec(f*x+e)*tan(f*x+e)^3/f+
1/6*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)^3/f-2/5*a^2*c^4*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.61

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$=\frac{a^2c^4(1680\text{arctanh}(\sin(e+fx))+\sec^6(e+fx)(330\sin(e+fx)-240\sin(2(e+fx))-445\sin(3(e+fx}{3840f}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

output

```
(a^2*c^4*(1680*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^6*(330*Sin[e + f*x] -
240*Sin[2*(e + f*x)] - 445*Sin[3*(e + f*x)] + 192*Sin[4*(e + f*x)] - 135*S
in[5*(e + f*x)] - 48*Sin[6*(e + f*x)])))/(3840*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4446}$$

$$a^2c^2\int \left(c^2\sec^3(e+fx)\tan^4(e+fx)-2c^2\sec^2(e+fx)\tan^4(e+fx)+c^2\sec(e+fx)\tan^4(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(\frac{7c^2\text{arctanh}(\sin(e+fx))}{16f}-\frac{2c^2\tan^5(e+fx)}{5f}+\frac{c^2\tan^3(e+fx)\sec^3(e+fx)}{6f}-\frac{c^2\tan(e+fx)\sec^3(e+fx}{8f}\right.$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

output

```
a^2*c^2*((7*c^2*ArcTanh[Sin[e + f*x]])/(16*f) - (5*c^2*Sec[e + f*x]*Tan[e
+ f*x])/(16*f) - (c^2*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (c^2*Sec[e + f*
x]*Tan[e + f*x]^3)/(4*f) + (c^2*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6*f) - (2*
c^2*Tan[e + f*x]^5)/(5*f))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.30

method	result
norman	$\frac{-\frac{7a^2c^4\tan(\frac{fx}{2}+\frac{e}{2})}{8f}+\frac{119a^2c^4\tan(\frac{fx}{2}+\frac{e}{2})^3}{24f}-\frac{231a^2c^4\tan(\frac{fx}{2}+\frac{e}{2})^5}{20f}+\frac{281a^2c^4\tan(\frac{fx}{2}+\frac{e}{2})^7}{20f}+\frac{119a^2c^4\tan(\frac{fx}{2}+\frac{e}{2})^9}{24f}-7a^2}{(\tan(\frac{fx}{2}+\frac{e}{2})^2-1)^6}$
risch	$\frac{ic^4a^2(135\,\mathrm{e}^{11i(fx+e)}-480\,\mathrm{e}^{10i(fx+e)}+445\,\mathrm{e}^{9i(fx+e)}-480\,\mathrm{e}^{8i(fx+e)}-330\,\mathrm{e}^{7i(fx+e)}-960\,\mathrm{e}^{6i(fx+e)}+330\,\mathrm{e}^{5i(fx+e)}-96}{120f(\mathrm{e}^{2i(fx+e)}+1)^6}$
parallelrisch	$-\frac{2a^2(\frac{7(5+\frac{\cos(6fx+6e)}{2}+3\cos(4fx+4e)+\frac{15\cos(2fx+2e)}{2})\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{16}+\frac{7(-5-\frac{15\cos(2fx+2e)}{2}-3\cos(4fx+4e)-\mathrm{co}}{16}}{f(10+\cos(6fx+6e)+6\,\mathrm{c}}$
derivativedivides	$\frac{a^2c^4\ln(\sec(fx+e)+\tan(fx+e))-2a^2c^4\tan(fx+e)-a^2c^4(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2})-4a^2c^4(-}{}$
default	$\frac{a^2c^4\ln(\sec(fx+e)+\tan(fx+e))-2a^2c^4\tan(fx+e)-a^2c^4(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2})-4a^2c^4(-}{}$
parts	$\frac{a^2c^4\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{a^2c^4(-(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16})\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\mathrm{t}}{16}}{f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBO
SE)
```

output
```
(-7/8*a^2*c^4/f*tan(1/2*f*x+1/2*e)+119/24*a^2*c^4/f*tan(1/2*f*x+1/2*e)^3-2
31/20*a^2*c^4/f*tan(1/2*f*x+1/2*e)^5+281/20*a^2*c^4/f*tan(1/2*f*x+1/2*e)^7
+119/24*a^2*c^4/f*tan(1/2*f*x+1/2*e)^9-7/8*a^2*c^4/f*tan(1/2*f*x+1/2*e)^11
)/(tan(1/2*f*x+1/2*e)^2-1)^6-7/16*a^2*c^4/f*ln(tan(1/2*f*x+1/2*e)-1)+7/16*
a^2*c^4/f*ln(tan(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$=\frac{105\,a^2c^4\cos(fx+e)^6\log(\sin(fx+e)+1)-105\,a^2c^4\cos(fx+e)^6\log(-\sin(fx+e)+1)-2\,(96\,a^2c}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="f
ricas")
```

output
```
1/480*(105*a^2*c^4*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 105*a^2*c^4*cos(
f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(96*a^2*c^4*cos(f*x + e)^5 + 135*a^2
*c^4*cos(f*x + e)^4 - 192*a^2*c^4*cos(f*x + e)^3 + 10*a^2*c^4*cos(f*x + e)
^2 + 96*a^2*c^4*cos(f*x + e) - 40*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^6
)
```

Sympy [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx &= a^2c^4\bigg(\int \sec\left(e+fx\right)dx+\int\left(-2\sec^2\left(e+fx\right)\right)dx+\int\left(-\sec^3\left(e+fx\right)\right)dx\\ &\quad+\int 4\sec^4\left(e+fx\right)dx+\int\left(-\sec^5\left(e+fx\right)\right)dx+\int\left(-2\sec^6\left(e+fx\right)\right)dx\\ &\quad+\int\sec^7\left(e+fx\right)dx\bigg)\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

output
```
a**2*c**4*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + I
ntegral(-sec(e + f*x)**3, x) + Integral(4*sec(e + f*x)**4, x) + Integral(-
sec(e + f*x)**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(sec(e + f
*x)**7, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(138) = 276$.

Time = 0.04 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.14

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx =$$

$$-\frac{64\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^2c^4-640\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="m
axima")
```

output
```
-1/480*(64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^
4 - 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 5*a^2*c^4*(2*(15*sin(f
*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f
*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(
f*x + e) - 1)) - 30*a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*
x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x
 + e) - 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f
*x + e) + 1) + log(sin(f*x + e) - 1)) - 480*a^2*c^4*log(sec(f*x + e) + tan
(f*x + e)) + 960*a^2*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$

$$=\frac{105\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}-595\right.}{}}{240\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="g
iac")
```

output
```
1/240*(105*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a^2*c^4*log(ab
s(tan(1/2*f*x + 1/2*e) - 1)) - 2*(105*a^2*c^4*tan(1/2*f*x + 1/2*e)^11 - 59
5*a^2*c^4*tan(1/2*f*x + 1/2*e)^9 - 1686*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 1
386*a^2*c^4*tan(1/2*f*x + 1/2*e)^5 - 595*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 +
105*a^2*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.46

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{-\frac{7\,a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}+\frac{119\,a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{281\,a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{20}-\frac{231\,a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{20}+\frac{119\,a^2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24}-}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-6\,\mathrm{ta}\right.}$$
$$+\frac{7\,a^2\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

input
```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

output
```
((119*a^2*c^4*tan(e/2 + (f*x)/2)^3)/24 - (231*a^2*c^4*tan(e/2 + (f*x)/2)^5
)/20 + (281*a^2*c^4*tan(e/2 + (f*x)/2)^7)/20 + (119*a^2*c^4*tan(e/2 + (f*x
)/2)^9)/24 - (7*a^2*c^4*tan(e/2 + (f*x)/2)^11)/8 - (7*a^2*c^4*tan(e/2 + (f
*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e
/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(
e/2 + (f*x)/2)^12 + 1)) + (7*a^2*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.65

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{a^2c^4\big(96\cos(fx+e)\sin(fx+e)^5-105\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\big)\sin(fx+e)^6+315\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x)
```

output
```
(a**2*c**4*(96*cos(e + f*x)*sin(e + f*x)**5 - 105*log(tan((e + f*x)/2) - 1
)*sin(e + f*x)**6 + 315*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 - 315*lo
g(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 + 105*log(tan((e + f*x)/2) - 1) +
105*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**6 - 315*log(tan((e + f*x)/2) +
 1)*sin(e + f*x)**4 + 315*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 - 105*
log(tan((e + f*x)/2) + 1) + 135*sin(e + f*x)**5 - 280*sin(e + f*x)**3 + 10
5*sin(e + f*x)))/(240*f*(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e + f
*x)**2 - 1))
```

3.12 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 32, antiderivative size = 94

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx \\ = \frac{3a^2c^3\text{arctanh}(\sin(e+fx))}{8f} - \frac{3a^2c^3\sec(e+fx)\tan(e+fx)}{8f} \\ + \frac{a^2c^3\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{a^2c^3\tan^5(e+fx)}{5f}\end{aligned}$$

output
```
3/8*a^2*c^3*arctanh(sin(f*x+e))/f-3/8*a^2*c^3*sec(f*x+e)*tan(f*x+e)/f+1/4*
a^2*c^3*sec(f*x+e)*tan(f*x+e)^3/f-1/5*a^2*c^3*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx \\ = \frac{a^2c^3(120\text{arctanh}(\sin(e+fx)) - \sec^5(e+fx)(40\sin(e+fx) + 10\sin(2(e+fx)) - 20\sin(3(e+fx)) +}{320f}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^2*c^3*(120*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^5*(40*Sin[e + f*x] + 10
*Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] + 25*Sin[4*(e + f*x)] + 4*Sin[5*(e
 + f*x)])))/(320*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4446}$$

$$a^2c^2\int \left(c\sec(e+fx)\tan^4(e+fx)-c\sec^2(e+fx)\tan^4(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(\frac{3c\text{arctanh}(\sin(e+fx))}{8f}-\frac{c\tan^5(e+fx)}{5f}+\frac{c\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3c\tan(e+fx)\sec(e+fx)}{8f}\right)$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

output
```
a^2*c^2*((3*c*ArcTanh[Sin[e + f*x]])/(8*f) - (3*c*Sec[e + f*x]*Tan[e + f*x
])/(8*f) + (c*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) - (c*Tan[e + f*x]^5)/(5*f
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

method	result
risch	$\frac{i a^2 c^3 (25\,e^{9i(fx+e)}-40\,e^{8i(fx+e)}+10\,e^{7i(fx+e)}-80\,e^{4i(fx+e)}-10\,e^{3i(fx+e)}-25\,e^{i(fx+e)}-8)}{20f\,(e^{2i(fx+e)}+1)^5}+\frac{3a^2c^3\ln(e^{i(fx+e)}+i}{8f}$
parts	$\frac{a^2c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}-\frac{a^2c^3\tan(fx+e)}{f}-\frac{a^2c^3\sec(fx+e}{f}$
norman	$\frac{\frac{3a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{7a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2f}+\frac{32a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5f}+\frac{7a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{2f}-\frac{3a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}-\frac{3a^2c^3\ln(}{}$
parallelrisch	$-\frac{a^2\left(\left(\left(\frac{15\cos(fx+e)}{2}+\frac{15\cos(3fx+3e)}{4}+\frac{3\cos(5fx+5e)}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{15\cos(fx+e)}{2}-\frac{15\cos(3fx+3e)}{4}-\frac{3\cos(5f}{4}\right.\right.\right.}{2f(\cos(5fx+5e)+5\cos(3fx+}$
derivativedivides	$\frac{a^2c^3\ln(\sec(fx+e)+\tan(fx+e))-a^2c^3\tan(fx+e)-2a^2c^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-2a^2c^3\left(-\right.}{}$
default	$\frac{a^2c^3\ln(\sec(fx+e)+\tan(fx+e))-a^2c^3\tan(fx+e)-2a^2c^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-2a^2c^3\left(-\right.}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/20*I*a^2*c^3*(25*exp(9*I*(f*x+e))-40*exp(8*I*(f*x+e))+10*exp(7*I*(f*x+e)
)-80*exp(4*I*(f*x+e))-10*exp(3*I*(f*x+e))-25*exp(I*(f*x+e))-8)/f/(exp(2*I*
(f*x+e))+1)^5+3/8*a^2*c^3/f*ln(exp(I*(f*x+e))+I)-3/8*a^2*c^3/f*ln(exp(I*(f
*x+e))-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= \frac{15\,a^2c^3\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right) - 15\,a^2c^3\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right) - 2\,\big(8\,a^2c^3\,\mathrm{co}}{80\,f\cos\big(}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/80*(15*a^2*c^3*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^2*c^3*cos(f*x
 + e)^5*log(-sin(f*x + e) + 1) - 2*(8*a^2*c^3*cos(f*x + e)^4 + 25*a^2*c^3*
cos(f*x + e)^3 - 16*a^2*c^3*cos(f*x + e)^2 - 10*a^2*c^3*cos(f*x + e) + 8*a
^2*c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= -a^2c^3\left(\int\left(-\sec\left(e+fx\right)\right)\,dx + \int\sec^2\left(e+fx\right)dx + \int 2\sec^3\left(e+fx\right)dx + \int\left(-2\sec^4\left(e+fx\right)\right)\,dx + \int\left(-\sec^5\left(e+fx\right)\right)\,dx + \int\sec^6\left(e+fx\right)dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)
```

output
```
-a**2*c**3*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + In
tegral(2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(
-sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(86) = 172.

Time = 0.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.41

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx =$$

$$-\frac{16\,(3\,\tan(fx+e)^5+10\,\tan(fx+e)^3+15\,\tan(fx+e))a^2c^3-160\,(\tan(fx+e)^3+3\,\tan(fx+e)}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
-1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^
3 - 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 15*a^2*c^3*(2*(3*sin(f
*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*lo
g(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^3*(2*sin(f*x +
e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) -
 240*a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 240*a^2*c^3*tan(f*x + e))/
f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.69

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$

$$=\frac{15\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\,(15\,a^2c^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^9-70\,a^2c^3}{}}{40\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
1/40*(15*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^2*c^3*log(abs(t
an(1/2*f*x + 1/2*e) - 1)) - 2*(15*a^2*c^3*tan(1/2*f*x + 1/2*e)^9 - 70*a^2*
c^3*tan(1/2*f*x + 1/2*e)^7 - 128*a^2*c^3*tan(1/2*f*x + 1/2*e)^5 + 70*a^2*c
^3*tan(1/2*f*x + 1/2*e)^3 - 15*a^2*c^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x
+ 1/2*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 15.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.99

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= \frac{-\frac{3\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}+\frac{7\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{2}+\frac{32\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{7\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{2}+\frac{3\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$
$$+\frac{3\,a^2\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

input
```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

output
```
((32*a^2*c^3*tan(e/2 + (f*x)/2)^5)/5 - (7*a^2*c^3*tan(e/2 + (f*x)/2)^3)/2
+ (7*a^2*c^3*tan(e/2 + (f*x)/2)^7)/2 - (3*a^2*c^3*tan(e/2 + (f*x)/2)^9)/4
+ (3*a^2*c^3*tan(e/2 + (f*x)/2))/4)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/
2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/
2 + (f*x)/2)^10 - 1)) + (3*a^2*c^3*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.49

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$

$$= \frac{a^2c^3\left(-15\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^4+30\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(}{}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x)
```

output

```
(a**2*c**3*( - 15*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 +
 30*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 - 15*cos(e + f*
x)*log(tan((e + f*x)/2) - 1) + 15*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*s
in(e + f*x)**4 - 30*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2
 + 15*cos(e + f*x)*log(tan((e + f*x)/2) + 1) + 25*cos(e + f*x)*sin(e + f*x
)**3 - 15*cos(e + f*x)*sin(e + f*x) - 8*sin(e + f*x)**5))/(40*cos(e + f*x)
*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.13 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 73

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{3a^2c^2\text{arctanh}(\sin(e+fx))}{8f} - \frac{3a^2c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^2\sec(e+fx)\tan^3(e+fx)}{4f}$$

output
```
3/8*a^2*c^2*arctanh(sin(f*x+e))/f-3/8*a^2*c^2*sec(f*x+e)*tan(f*x+e)/f+1/4*
a^2*c^2*sec(f*x+e)*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2\left(\frac{3\text{arctanh}(\sin(e+fx))}{8f} + \frac{3\sec(e+fx)\tan(e+fx)}{8f} - \frac{3\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{\sec(e+fx)\tan^3(e+fx)}{f}\right)$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

output
```
a^2*c^2*((3*ArcTanh[Sin[e + f*x]])/(8*f) + (3*Sec[e + f*x]*Tan[e + f*x])/(
8*f) - (3*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (Sec[e + f*x]*Tan[e + f*x]^
3)/f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4446, 3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4446}$$

$$a^2c^2\int \sec(e+fx)\tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \sec(e+fx)\tan(e+fx)^4dx$$

$$\Big\downarrow \text{3091}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\int \sec(e+fx)\tan^2(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\int \sec(e+fx)\tan(e+fx)^2dx\right)$$

$$\Big\downarrow \text{3091}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int\sec(e+fx)dx\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int\csc\left(e+fx+\frac{\pi}{2}\right)dx\right)\right)$$

$$\Big\downarrow \text{4257}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{\text{arctanh}(\sin(e+fx))}{2f}\right)\right)$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

output
```
a^2*c^2*((Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) - (3*(-1/2*ArcTanh[Sin[e + f*
x]]/f + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))/4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

method	result
parts	$\frac{a^{2} c^{2} \left(-\left(-\frac{\sec\left(f x +e \right)^{3}}{4}-\frac{3 \sec\left(f x +e \right)}{8}\right) \tan\left(f x +e \right)+\frac{3 \ln\left(\sec\left(f x +e \right)+\tan\left(f x +e \right)\right)}{8}\right)}{f}-\frac{a^{2} c^{2} \sec\left(f x +e \right) \tan\left(f x +e \right)}{f}$
derivativedivides	$a^{2} c^{2} \ln\left(\sec\left(f x +e \right)+\tan\left(f x +e \right)\right)-2 a^{2} c^{2} \left(\frac{\sec\left(f x +e \right) \tan\left(f x +e \right)}{2}+\frac{\ln\left(\sec\left(f x +e \right)+\tan\left(f x +e \right)\right)}{2}\right)+a^{2} c^{2} (-(-\frac{\sec\left(f x +e \right)^{3}}{4}- 3\,\mathrm{se}$ … $/\,f$
default	$a^{2} c^{2} \ln\left(\sec\left(f x +e \right)+\tan\left(f x +e \right)\right)-2 a^{2} c^{2} \left(\frac{\sec\left(f x +e \right) \tan\left(f x +e \right)}{2}+\frac{\ln\left(\sec\left(f x +e \right)+\tan\left(f x +e \right)\right)}{2}\right)+a^{2} c^{2} (-(-\frac{\sec\left(f x +e \right)^{3}}{4}- 3\,\mathrm{se}$ … $/\,f$
risch	$\frac{i a^{2} c^{2} \left(5 \,{\mathrm e}^{7 i \left(f x +e \right)}-3 \,{\mathrm e}^{5 i \left(f x +e \right)}+3 \,{\mathrm e}^{3 i \left(f x +e \right)}-5 \,{\mathrm e}^{i \left(f x +e \right)}\right)}{4 f \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)^{4}}-\frac{3 a^{2} c^{2} \ln\left({\mathrm e}^{i \left(f x +e \right)}-i \right)}{8 f}+\frac{3 a^{2} c^{2} \ln\left({\mathrm e}^{i \left(f x +e \right)}+i \right)}{8 f}$
parallelrisch	$-\,3 a^{2} \left(\left(\frac{3}{4}+\frac{\cos\left(4 f x +4 e \right)}{4}+\cos\left(2 f x +2 e \right)\right) \ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)+\left(-\cos\left(2 f x +2 e \right)-\frac{\cos\left(4 f x +4 e \right)}{4}-\frac{3}{4}\right) \ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+\right.\right.$ … $/\,\left(2 f \left(\cos\left(4 f x +4 e \right)+4 \cos\left(2 f x +2 e \right)+3\right)\right)$
norman	$\frac{-\frac{3 a^{2} c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{4 f}+\frac{11 a^{2} c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{4 f}+\frac{11 a^{2} c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{4 f}-\frac{3 a^{2} c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{4 f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{4}}-\frac{3 a^{2} c^{2} \ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{8 f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^2/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-a^2*c^2*sec(f*x+e)*tan(f*x+e)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$$
$$=\frac{3\,a^2c^2\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c^2\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)-2\left(5\,a^2c^2\cos\left(\right.\right.}{16\,f\cos\left(fx+e\right)^4}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
1/16*(3*a^2*c^2*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c^2*cos(f*x +
 e)^4*log(-sin(f*x + e) + 1) - 2*(5*a^2*c^2*cos(f*x + e)^2 - 2*a^2*c^2)*si
n(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$$
$$=a^2c^2\left(\int\sec\left(e+fx\right)dx+\int\left(-2\sec^3\left(e+fx\right)\right)\,dx+\int\sec^5\left(e+fx\right)dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
```

output

```
a**2*c**2*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**3, x) + I
ntegral(sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(67) = 134$.

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.05

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = -\frac{a^2c^2\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)-8a^2c^2\left(\frac{2\sin(fx}{\sin(fx+e}\right.}{16f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
-1/16*(a^2*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*
sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) -
 8*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) +
log(sin(f*x + e) - 1)) - 16*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{3a^2c^2\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3a^2c^2\log\left(\left|\sin\left(fx+e\right)-1\right|\right)+\frac{2\left(5a^2c^2\sin(fx+e)^3-3a^2c^2\sin(fx+e)\right)}{\left(\sin(fx+e)^2-1\right)^2}}{16f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/16*(3*a^2*c^2*log(abs(sin(f*x + e) + 1)) - 3*a^2*c^2*log(abs(sin(f*x + e
) - 1)) + 2*(5*a^2*c^2*sin(f*x + e)^3 - 3*a^2*c^2*sin(f*x + e))/(sin(f*x +
 e)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.12

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$$
$$=\frac{-\frac{3a^2c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7}{4}+\frac{11a^2c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{4}+\frac{11a^2c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{4}-\frac{3a^2c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{4}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}$$
$$+\frac{3a^2c^2\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{4f}$$

input
```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

output
```
((11*a^2*c^2*tan(e/2 + (f*x)/2)^3)/4 + (11*a^2*c^2*tan(e/2 + (f*x)/2)^5)/4
 - (3*a^2*c^2*tan(e/2 + (f*x)/2)^7)/4 - (3*a^2*c^2*tan(e/2 + (f*x)/2))/4)/
(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)
^6 + tan(e/2 + (f*x)/2)^8 + 1)) + (3*a^2*c^2*atanh(tan(e/2 + (f*x)/2)))/(4
*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.30

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$$
$$=\frac{a^2c^2\left(-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^4+6\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^2-3\log\left(\tan\left(\frac{fx}{2}+\right.\right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x)
```

output
```
(a**2*c**2*( - 3*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 + 6*log(tan((e
+ f*x)/2) - 1)*sin(e + f*x)**2 - 3*log(tan((e + f*x)/2) - 1) + 3*log(tan((
e + f*x)/2) + 1)*sin(e + f*x)**4 - 6*log(tan((e + f*x)/2) + 1)*sin(e + f*x
)**2 + 3*log(tan((e + f*x)/2) + 1) + 5*sin(e + f*x)**3 - 3*sin(e + f*x)))/
(8*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.14 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 61

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c\text{arctanh}(\sin(e+fx))}{2f}-\frac{a^2c\sec(e+fx)\tan(e+fx)}{2f}-\frac{a^2c\tan^3(e+fx)}{3f}$$

output
```
1/2*a^2*c*arctanh(sin(f*x+e))/f-1/2*a^2*c*sec(f*x+e)*tan(f*x+e)/f-1/3*a^2*
c*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c\left(6\coth^{-1}(\sin(e+fx))-3\text{arctanh}(\sin(e+fx))-3\sec(e+fx)\tan(e+fx)-2\tan^3(e+fx)\right)}{6f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

output
```
(a^2*c*(6*ArcCoth[Sin[e + f*x]] - 3*ArcTanh[Sin[e + f*x]] - 3*Sec[e + f*x]
*Tan[e + f*x] - 2*Tan[e + f*x]^3))/(6*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4446}$$

$$-ac\int \left(a\sec^2(e+fx)\tan^2(e+fx)+a\sec(e+fx)\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ac\left(-\frac{a\text{arctanh}(\sin(e+fx))}{2f}+\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)\sec(e+fx)}{2f}\right)$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

output
```
-(a*c*(-1/2*(a*ArcTanh[Sin[e + f*x]])/f + (a*Sec[e + f*x]*Tan[e + f*x])/(2
*f) + (a*Tan[e + f*x]^3)/(3*f)))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2c\tan(fx+e)-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^2c\left(-\frac{2}{3}-\sec(\right.}{f}$
default	$\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2c\tan(fx+e)-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^2c\left(-\frac{2}{3}-\sec(\right.}{f}$
risch	$\frac{ia^2c\left(3\,\mathrm{e}^{5i(fx+e)}+6\,\mathrm{e}^{4i(fx+e)}-3\,\mathrm{e}^{i(fx+e)}+2\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}+\frac{a^2c\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{2f}-\frac{a^2c\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{2f}$
parts	$\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{a^2c\tan(fx+e)}{f}-\frac{a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{a^2c\left(-\right.}{}$
norman	$\frac{\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{8a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$
parallelrisch	$\frac{a^2\left(\frac{3(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\sin(3fx+3e)-3\,\mathrm{s}\right.}{3f(\cos(3fx+3e)+3\cos(fx+e))}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output

```
1/f*(a^2*c*ln(sec(f*x+e)+tan(f*x+e))+a^2*c*tan(f*x+e)-a^2*c*(1/2*sec(f*x+e
)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^2*c*(-2/3-1/3*sec(f*x+e)^2)*
tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.69

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{3\,a^2c\cos{(fx+e)}^3\log{(\sin{(fx+e)}+1)} - 3\,a^2c\cos{(fx+e)}^3\log{(-\sin{(fx+e)}+1)} + 2\,(2\,a^2c\cos{(fx}}{12\,f\cos{(fx+e)}^3}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
1/12*(3*a^2*c*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^
3*log(-sin(f*x + e) + 1) + 2*(2*a^2*c*cos(f*x + e)^2 - 3*a^2*c*cos(f*x + e
) - 2*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= -a^2c\left(\int(-\sec{(e+fx)})\,dx + \int\left(-\sec^2{(e+fx)}\right)\,dx + \int\sec^3{(e+fx)}\,dx + \int\sec^4{(e+fx)}\,dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

output

```
-a**2*c*(Integral(-sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Inte
gral(sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.77

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx =$$

$$-\frac{4\,(\tan(fx+e)^3+3\,\tan(fx+e))a^2c-3\,a^2c(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)}{12\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*sin(f*x + e)
/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 1
2*a^2*c*log(sec(f*x + e) + tan(f*x + e)) - 12*a^2*c*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(55) = 110$.

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$

$$=\frac{3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\,(3\,a^2c\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5-8\,a^2c\tan(\frac{1}{2}\,fx+}{(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-}}{6\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
1/6*(3*a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^2*c*log(abs(tan(1/2*
f*x + 1/2*e) - 1)) - 2*(3*a^2*c*tan(1/2*f*x + 1/2*e)^5 - 8*a^2*c*tan(1/2*f
*x + 1/2*e)^3 - 3*a^2*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)
^3)/f
```

Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.85

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=\frac{-c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\frac{8\,c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$
$$+\frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

input

```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

output

```
(a^2*c*tan(e/2 + (f*x)/2) + (8*a^2*c*tan(e/2 + (f*x)/2)^3)/3 - a^2*c*tan(e
/2 + (f*x)/2)^5)/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan
(e/2 + (f*x)/2)^6 - 1)) + (a^2*c*atanh(tan(e/2 + (f*x)/2)))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.46

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=\frac{a^2c(-3\cos(fx+e)\log(\tan(\frac{fx}{2}+\frac{e}{2})-1)\sin(fx+e)^2+3\cos(fx+e)\log(\tan(\frac{fx}{2}+\frac{e}{2})-1)+3\cos(}{6\cos}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

output

```
(a**2*c*( - 3*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 + 3*c
os(e + f*x)*log(tan((e + f*x)/2) - 1) + 3*cos(e + f*x)*log(tan((e + f*x)/2
) + 1)*sin(e + f*x)**2 - 3*cos(e + f*x)*log(tan((e + f*x)/2) + 1) + 3*cos(
e + f*x)*sin(e + f*x) + 2*sin(e + f*x)**3))/(6*cos(e + f*x)*f*(sin(e + f*x
)**2 - 1))
```

3.15 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{3a^2\text{arctanh}(\sin(e+fx))}{cf}-\frac{3a^2\tan(e+fx)}{cf}-\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))}$$

output
```
-3*a^2*arctanh(sin(f*x+e))/c/f-3*a^2*tan(f*x+e)/c/f-2*(a^2+a^2*sec(f*x+e))
*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{a^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^2\tan(e+fx)}{5cf\sqrt{2-2\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
```

output
```
-1/5*(a^2*Hypergeometric2F1[3/2, 5/2, 7/2, (1 + Sec[e + f*x])/2]*(1 + Sec[
e + f*x])^2*Tan[e + f*x])/(c*f*Sqrt[2 - 2*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4445, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{3a\int \sec(e+fx)(\sec(e+fx)a+a)dx}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)dx}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{3a\left(a\int \sec^2(e+fx)dx+a\int \sec(e+fx)dx\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\left(a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+a\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{3a\left(a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{a\int 1d(-\tan(e+fx))}{f}\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{24}$$

$$-\frac{3a\left(a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{a\tan(e+fx)}{f}\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}-\frac{3a\left(\frac{a\text{arctanh}(\sin(e+fx))}{f}+\frac{a\tan(e+fx)}{f}\right)}{c}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
```

output

```
(-2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])) - (3*a
*((a*ArcTanh[Sin[e + f*x]])/f + (a*Tan[e + f*x])/f))/c
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{4a^2\left(\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$
default	$\frac{4a^2\left(\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$
parallelrisch	$-\frac{a^2 (-5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\cos(fx+e)-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{cf\cos(fx+e)}$
risch	$\frac{2ia^2\left(4\,\mathrm{e}^{2i(fx+e)}-\mathrm{e}^{i(fx+e)}+5\right)}{fc\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)}+\frac{3a^2\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{cf}-\frac{3a^2\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{cf}$
norman	$\frac{\frac{4a^2}{cf}-\frac{10a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}+\frac{6a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{3a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{3a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
4/f*a^2/c*(1/4/(tan(1/2*f*x+1/2*e)+1)-3/4*ln(tan(1/2*f*x+1/2*e)+1)+1/4/(ta
n(1/2*f*x+1/2*e)-1)+3/4*ln(tan(1/2*f*x+1/2*e)-1)+1/tan(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{3\,a^2\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-3\,a^2\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx}{2\,cf\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
-1/2*(3*a^2*cos(f*x + e)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*a^2*cos(f*
x + e)*log(-sin(f*x + e) + 1)*sin(f*x + e) - 10*a^2*cos(f*x + e)^2 - 8*a^2
*cos(f*x + e) + 2*a^2)/(c*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{a^2\left(\int \frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{2\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

output
```
-a**2*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(2*sec(e + f
*x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) - 1
), x))/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(75) = 150$.

Time = 0.04 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.04

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx =$$

$$-\frac{a^2\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+2a^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(f}{\cos(fx}\right.}{c}\right.}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-(a^2*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*
x + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(c
os(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) + 2*
a^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*
x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a^2*(cos(f*x +
 e) + 1)/(c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{\frac{3a^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{c}-\frac{3a^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{c}-\frac{2\left(3a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-2a^2\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)c}}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="gia
c")
```

output

```
-(3*a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - 3*a^2*log(abs(tan(1/2*f*x +
 1/2*e) - 1))/c - 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^2)/((tan(1/2*f*x +
 1/2*e)^3 - tan(1/2*f*x + 1/2*e))*c))/f
```

Mupad [B] (verification not implemented)

Time = 11.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = \frac{6\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-4\,a^2}{c\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)} - \frac{6\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

input

```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

output

```
(6*a^2*tan(e/2 + (f*x)/2)^2 - 4*a^2)/(c*f*tan(e/2 + (f*x)/2)*(tan(e/2 + (f
*x)/2)^2 - 1)) - (6*a^2*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = \frac{a^2\left(3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.\right.}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cf\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

output

```
(a**2*(3*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**3 - 3*log(tan((e + f*
x)/2) - 1)*tan((e + f*x)/2) - 3*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)
**3 + 3*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2) + 6*tan((e + f*x)/2)**2
 - 4))/(tan((e + f*x)/2)*c*f*(tan((e + f*x)/2)**2 - 1))
```

3.16 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\text{arctanh}(\sin(e+fx))}{c^2f} - \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{2a^2\tan(e+fx)}{f\left(c^2-c^2\sec(e+fx)\right)}$$

output
```
a^2*arctanh(sin(f*x+e))/c^2/f-2/3*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/f/(c-c*s
ec(f*x+e))^2+2*a^2*tan(f*x+e)/f/(c^2-c^2*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\left(-\frac{4\cot\left(\frac{1}{2}(e+fx)\right)}{3f} - \frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{3f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c^2}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(a^2*((-4*Cot[(e + f*x)/2])/(3*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2
)/(3*f) - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]]/f))/c^2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4445, 3042, 4445, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{c-c\sec(e+fx)}dx}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{4445}$$

$$-\frac{a\left(-\frac{a\int\sec(e+fx)dx}{c}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{c}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow 4257$$

$$-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}-\frac{a\left(-\frac{a\,\text{arctanh}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}\right)}{c}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2) -
(a*(-((a*ArcTanh[Sin[e + f*x]])/(c*f)) - (2*a*Tan[e + f*x])/(f*(c - c*Sec[
e + f*x]))))/c
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{a^{2}\left(-2\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-6\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3c^{2}f}$
derivativedivides	$\frac{2a^{2}\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^{2}}$
default	$\frac{2a^{2}\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^{2}}$
risch	$\frac{8ia^{2}\left(3\,e^{i(fx+e)}-1\right)}{3f\,c^{2}\left(e^{i(fx+e)}-1\right)^{3}}+\frac{a^{2}\ln\left(e^{i(fx+e)}+i\right)}{c^{2}f}-\frac{a^{2}\ln\left(e^{i(fx+e)}-i\right)}{c^{2}f}$
norman	$\frac{-\frac{2a^{2}}{3cf}-\frac{2a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3cf}+\frac{10a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{3cf}-\frac{2a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{a^{2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{c^{2}f}-\frac{a^{2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}{c^{2}f}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output

```
1/3*a^2*(-2*cot(1/2*f*x+1/2*e)^3+3*ln(tan(1/2*f*x+1/2*e)+1)-3*ln(tan(1/2*f
*x+1/2*e)-1)-6*cot(1/2*f*x+1/2*e))/c^2/f
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.44

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx =$$
$$-\frac{8\,a^{2}\cos\left(fx+e\right)^{2}-8\,a^{2}\cos\left(fx+e\right)-3\left(a^{2}\cos\left(fx+e\right)-a^{2}\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+3}{6\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+\right.}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
-1/6*(8*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) - 3*(a^2*cos(f*x + e) - a^
2)*log(sin(f*x + e) + 1)*sin(f*x + e) + 3*(a^2*cos(f*x + e) - a^2)*log(-si
n(f*x + e) + 1)*sin(f*x + e) - 16*a^2)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f
*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^{2} \left(\int \frac{\sec{\left(e+fx \right)}}{\sec^{2}{\left(e+fx \right)}-2\sec{\left(e+fx \right)}+1}\,dx+\int \frac{2\sec^{2}{\left(e+fx \right)}}{\sec^{2}{\left(e+fx \right)}-2\sec{\left(e+fx \right)}+1}\,dx+\int \frac{\sec^{3}{\left(e+fx \right)}}{\sec^{2}{\left(e+fx \right)}-2\sec{\left(e+fx \right)}+1}\,dx\right)}{c^{2}}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

output

```
a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + I
ntegral(2*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Int
egral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(89) = 178$.

Time = 0.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^{2} \left(\frac{6 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{2}} - \frac{6 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{2}} - \frac{\left(\frac{9 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)(\cos(fx+e)+1)^{3}}{c^{2} \sin(fx+e)^{3}}\right) - \frac{2 a^{2} \left(\frac{3 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)(\cos(fx+}{c^{2} \sin(fx+e)^{3}}}{6 f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
1/6*(a^2*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 2*a^2*(3*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + a^2*(
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f
*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^2}-\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^2}-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^2\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/3*(3*a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 3*a^2*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/c^2 - 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 + a^2)/(c^2*tan(1
/2*f*x + 1/2*e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 11.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^2\,f} - \frac{2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\frac{2\,a^2}{3}}{c^2\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

output
```
(2*a^2*atanh(tan(e/2 + (f*x)/2)))/(c^2*f) - (2*a^2*tan(e/2 + (f*x)/2)^2 +
(2*a^2)/3)/(c^2*f*tan(e/2 + (f*x)/2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$$
$$= \frac{a^2\left(-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 -\right.}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^2f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

output
```
(a**2*( - 3*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**3 + 3*log(tan((e +
 f*x)/2) + 1)*tan((e + f*x)/2)**3 - 6*tan((e + f*x)/2)**2 - 2))/(3*tan((e
+ f*x)/2)**3*c**2*f)
```

3.17 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3}$$

output
```
-1/5*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2\cot^5\left(\frac{1}{2}(e+fx)\right)}{5c^3f}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^2*Cot[(e + f*x)/2]^5)/(5*c^3*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
-1/5*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{a^{2}}{5 f \,c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}$	23
default	$\frac{a^{2}}{5 f \,c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}$	23
parallelrisch	$\frac{a^{2} \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5 f \,c^{3}}$	23
risch	$\frac{2 i a^{2} \left(5 \,{\mathrm e}^{4 i \left(f x +e \right)}+10 \,{\mathrm e}^{2 i \left(f x +e \right)}+1\right)}{5 f \,c^{3} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{5}}$	50
norman	$\frac{\frac{a^{2}}{5 c f}-\frac{2 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{5 c f}+\frac{a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{5 c f}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}$	87

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output

```
1/5/f*a^2/c^3/tan(1/2*f*x+1/2*e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.18

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{a^{2} \cos \left(f x + e\right)^{3} + 3 \, a^{2} \cos \left(f x + e\right)^{2} + 3 \, a^{2} \cos \left(f x + e\right) + a^{2}}{5 \, {\left(c^{3} f \cos \left(f x + e\right)^{2} - 2 \, c^{3} f \cos \left(f x + e\right) + c^{3} f\right)} \sin \left(f x + e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)
/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^2\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{2\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{\sec}{\sec^3(e+fx)-3\sec}\right.}{c^3}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

output
```
-a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(
e + f*x) - 1), x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*
x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(37) = 74$.

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{\frac{a^2\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5} - \frac{a^2\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5} - 6\,a^2}{60\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/60*(a^2*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - a^2*(10*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - 6*a^2*(5*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2}{5\,c^3 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
1/5*a^2/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,c^3\,f}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

output
```
(a^2*cot(e/2 + (f*x)/2)^5)/(5*c^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 c^3 f}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

output

```
a**2/(5*tan((e + f*x)/2)**5*c**3*f)
```

3.18 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}$$

output
```
-1/7*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^4-1/35*(a+a*sec(f*x+
e))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^3
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2(-6+\sec(e+fx))(1+\sec(e+fx))^2\tan(e+fx)}{35c^4f(-1+\sec(e+fx))^4}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^2*(-6 + Sec[e + f*x])*(1 + Sec[e + f*x])^2*Tan[e + f*x])/(35*c^4*f*(-1
+ Sec[e + f*x])^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{(c-c\sec(e+fx))^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
-1/7*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4) - ((
a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.48

method	result	size
parallelrisch	$-\frac{a^{2} \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{5} \left(5 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-7\right)}{70 c^{4} f}$	38
derivativedivides	$\frac{a^{2} \left(-\frac{1}{7 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{5 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}\right)}{2 f \,c^{4}}$	39
default	$\frac{a^{2} \left(-\frac{1}{7 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{5 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}\right)}{2 f \,c^{4}}$	39
risch	$\frac{2 i a^{2} \left(35 \,{\mathrm e}^{6 i \left(f x +e \right)}-35 \,{\mathrm e}^{5 i \left(f x +e \right)}+140 \,{\mathrm e}^{4 i \left(f x +e \right)}-70 \,{\mathrm e}^{3 i \left(f x +e \right)}+91 \,{\mathrm e}^{2 i \left(f x +e \right)}-7 \,{\mathrm e}^{i \left(f x +e \right)}+6\right)}{35 f \,c^{4} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{7}}$	94
norman	$\frac{-\frac{a^{2}}{14 c f}+\frac{17 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{70 c f}-\frac{19 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{70 c f}+\frac{a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{10 c f}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}$	109

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBO
SE)
```

output
```
-1/70*a^2*cot(1/2*f*x+1/2*e)^5*(5*cot(1/2*f*x+1/2*e)^2-7)/c^4/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{6\,a^2\cos\left(fx+e\right)^4+17\,a^2\cos\left(fx+e\right)^3+15\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)-a^2}{35\,\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="f
ricas")
```

output
```
1/35*(6*a^2*cos(f*x + e)^4 + 17*a^2*cos(f*x + e)^3 + 15*a^2*cos(f*x + e)^2
 + 3*a^2*cos(f*x + e) - a^2)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)
^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2\left(\int\frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,d\right.}{c^4}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

output
```
a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
 + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(2*sec(e + f*x)**2/(sec(e +
 f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)
 + Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
 + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(78) = 156$.

Time = 0.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{\frac{2\,a^2\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}+\frac{3\,a^2\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(}{c^4\sin(fx+e)^7}}{840\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="m
axima")
```

output
```
1/840*(2*a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f
*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a^2*(21*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^2*(21
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(
c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{7\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-5\,a^2}{70\,c^4 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="g
iac")
```

output
```
1/70*(7*a^2*tan(1/2*f*x + 1/2*e)^2 - 5*a^2)/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

Mupad [B] (verification not implemented)

Time = 11.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = -\frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(5\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-7\right)}{70\,c^4\,f}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

output
```
-(a^2*cot(e/2 + (f*x)/2)^5*(5*cot(e/2 + (f*x)/2)^2 - 7))/(70*c^4*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2\left(7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-5\right)}{70\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7c^4f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

output
```
(a**2*(7*tan((e + f*x)/2)**2 - 5))/(70*tan((e + f*x)/2)**7*c**4*f)
```

3.19 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = &-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5}\\ &-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}\\ &-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{315c^2f(c-c\sec(e+fx))^3}\end{aligned}$$

output

```
-1/9*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-2/63*(a+a*sec(f*x+
e))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4-2/315*(a+a*sec(f*x+e))^2*tan(f*x+e
)/c^2/f/(c-c*sec(f*x+e))^3
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.49

$$\begin{aligned}&\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx\\ &=\frac{a^2(1+\sec(e+fx))^2\left(47-14\sec(e+fx)+2\sec^2(e+fx)\right)\tan(e+fx)}{315c^5f(-1+\sec(e+fx))^5}\end{aligned}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^2*(1 + Sec[e + f*x])^2*(47 - 14*Sec[e + f*x] + 2*Sec[e + f*x]^2)*Tan[e
+ f*x])/(315*c^5*f*(-1 + Sec[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4439, 3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{2\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{(c-c\sec(e+fx))^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{4439}$$

$$\frac{2\left(\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{(c-c\sec(e+fx))^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}\right)}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}\right)}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{4438}$$

$$\frac{2\left(-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}\right)}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

output

```
-1/9*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) + (2
*(-1/7*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4) -
((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3)))/(9
*c)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

method	result
parallelrisch	$\frac{a^{2} \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{5} \left(35 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-90 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+63\right)}{1260 c^{5} f}$
derivativedivides	$\frac{a^{2} \left(-\frac{2}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}+\frac{1}{5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}\right)}{4 f\, c^{5}}$
default	$\frac{a^{2} \left(-\frac{2}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}+\frac{1}{5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}\right)}{4 f\, c^{5}}$
risch	$\frac{2 i a^{2} (315\, e^{8 i (f x+e)}-630\, e^{7 i (f x+e)}+2310\, e^{6 i (f x+e)}-2520\, e^{5 i (f x+e)}+3402\, e^{4 i (f x+e)}-1638\, e^{3 i (f x+e)}+1062\, e^{2 i (f x+e)}-}{315 f\, c^{5} \left(e^{i (f x+e)}-1\right)^{9}}$
norman	$\frac{\frac{a^{2}}{36 c f}-\frac{8 a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{63 c f}+\frac{139 a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{630 c f}-\frac{6 a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{35 c f}+\frac{a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{8}}{20 c f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} c^{4} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBO
SE)
```

output
```
1/1260*a^2*cot(1/2*f*x+1/2*e)^5*(35*cot(1/2*f*x+1/2*e)^4-90*cot(1/2*f*x+1/
2*e)^2+63)/c^5/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{47\,a^2\cos\left(fx+e\right)^5+127\,a^2\cos\left(fx+e\right)^4+101\,a^2\cos\left(fx+e\right)^3+11\,a^2\cos\left(fx+e\right)^2-8\,a^2\cos\left(fx+e\right.}{315\,\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="f
ricas")
```

output
```
1/315*(47*a^2*cos(f*x + e)^5 + 127*a^2*cos(f*x + e)^4 + 101*a^2*cos(f*x +
e)^3 + 11*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) + 2*a^2)/((c^5*f*cos(f*x
 + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*
x + e) + c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx =$$
$$-\frac{a^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int \frac{2\sec^2\left(e+\right.}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+f\right.}\right.}{c^5}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

output
```
-a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec
(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(2*s
ec(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 -
 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(
sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)
**2 + 5*sec(e + f*x) - 1), x))/c**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(118) = 236$.

Time = 0.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.22

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx =$$

$$-\frac{\frac{a^2\left(\frac{180\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\,\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{10\,a^2\left(\frac{18\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\right.}{c^5\sin(fx}}{5040\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="m
axima")
```

output
```
-1/5040*(a^2*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x +
e)^9) + 10*a^2*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f
*x + e) + 1)^9/(c^5*sin(f*x + e)^9) + 7*a^2*(18*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)
^9/(c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{63\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-90\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^2}{1260\,c^5 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="g
iac")
```

output
```
1/1260*(63*a^2*tan(1/2*f*x + 1/2*e)^4 - 90*a^2*tan(1/2*f*x + 1/2*e)^2 + 35
*a^2)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20\,c^5\,f} - \frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{14\,c^5\,f} + \frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{36\,c^5\,f}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

output
```
(a^2*cot(e/2 + (f*x)/2)^5)/(20*c^5*f) - (a^2*cot(e/2 + (f*x)/2)^7)/(14*c^5
*f) + (a^2*cot(e/2 + (f*x)/2)^9)/(36*c^5*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.41

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2\left(63\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+35\right)}{1260\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9 c^5 f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)
```

output
```
(a**2*(63*tan((e + f*x)/2)**4 - 90*tan((e + f*x)/2)**2 + 35))/(1260*tan((e
 + f*x)/2)**9*c**5*f)
```

3.20 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 163

$$
\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = &-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} \\
&-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} \\
&-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{231c^2f(c-c\sec(e+fx))^4} \\
&-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\left(c^2-c^2\sec(e+fx)\right)^3}
\end{aligned}
$$

output

```
-1/11*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^6-1/33*(a+a*sec(f*x
+e))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^5-2/231*(a+a*sec(f*x+e))^2*tan(f*x+
e)/c^2/f/(c-c*sec(f*x+e))^4-2/1155*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c^2-c^
2*sec(f*x+e))^3
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{a^2(1+\sec(e+fx))^2\left(-152+61\sec(e+fx)-16\sec^2(e+fx)+2\sec^3(e+fx)\right)\tan(e+fx)}{1155c^6f(-1+\sec(e+fx))^6}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
```

output

```
(a^2*(1 + Sec[e + f*x])^2*(-152 + 61*Sec[e + f*x] - 16*Sec[e + f*x]^2 + 2*
Sec[e + f*x]^3)*Tan[e + f*x])/(1155*c^6*f*(-1 + Sec[e + f*x])^6)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4439, 3042, 4439, 3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{3\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{(c-c\sec(e+fx))^5}dx}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{ 4439}$$

$$\frac{3\left(\frac{2\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{(c-c\sec(e+fx))^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{ 4439}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{(c-c\sec(e+fx))^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}\right)}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{7c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}\right)}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{ 4438}$$

$$\frac{3\left(\frac{2\left(-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}\right)}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{11f(c-c\sec(e+fx))^6}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
```

output

```
-1/11*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^6) + (
3*(-1/9*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) +
 (2*(-1/7*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4)
 - ((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3)))
/(9*c)))/(11*c)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

method	result
parallelrisch	$-\frac{a^{2} \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{5} \left(105 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-385 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+495 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-231\right)}{9240 c^{6} f}$
derivativedivides	$\frac{a^{2} \left(\frac{1}{5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{11 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{3}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{8 f\, c^{6}}$
default	$\frac{a^{2} \left(\frac{1}{5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{11 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{3}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{8 f\, c^{6}}$
risch	$\frac{2 i a^{2} (1155\, \mathrm{e}^{10 i (f x+e)}-3465\, \mathrm{e}^{9 i (f x+e)}+13860\, \mathrm{e}^{8 i (f x+e)}-23100\, \mathrm{e}^{7 i (f x+e)}+37422\, \mathrm{e}^{6 i (f x+e)}-32802\, \mathrm{e}^{5 i (f x+e)}+27060\, \mathrm{e}}{1155 f\, c^{6} \left(\mathrm{e}^{i (f x+e)}-1\right)^{11}}$
norman	$\frac{-\frac{a^{2}}{88 c f}+\frac{17 a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{264 c f}-\frac{137 a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{924 c f}+\frac{73 a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{420 c f}-\frac{29 a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{8}}{280 c f}+\frac{a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{10}}{40 c f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} c^{5} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{11}}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBO
SE)
```

output
```
-1/9240*a^2*cot(1/2*f*x+1/2*e)^5*(105*cot(1/2*f*x+1/2*e)^6-385*cot(1/2*f*x
+1/2*e)^4+495*cot(1/2*f*x+1/2*e)^2-231)/c^6/f
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx$$
$$=\frac{152\,a^{2}\cos\left(fx+e\right)^{6}+395\,a^{2}\cos\left(fx+e\right)^{5}+289\,a^{2}\cos\left(fx+e\right)^{4}+15\,a^{2}\cos\left(fx+e\right)^{3}-19\,a^{2}\cos(fx+}{1155\,(c^{6}f\cos\left(fx+e\right)^{5}-5\,c^{6}f\cos\left(fx+e\right)^{4}+10\,c^{6}f\cos\left(fx+e\right)^{3}-10\,c^{6}f\cos\left(fx+e\right)^{2}+5\,c^{6}f\cos}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="f
ricas")
```

output

```
1/1155*(152*a^2*cos(f*x + e)^6 + 395*a^2*cos(f*x + e)^5 + 289*a^2*cos(f*x
+ e)^4 + 15*a^2*cos(f*x + e)^3 - 19*a^2*cos(f*x + e)^2 + 10*a^2*cos(f*x +
e) - 2*a^2)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos
(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(
f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{a^2\left(\int \frac{\sec(e+fx)}{\sec^6(e+fx)-6\sec^5(e+fx)+15\sec^4(e+fx)-20\sec^3(e+fx)+15\sec^2(e+fx)-6\sec(e+fx)+1}\,dx + \int \frac{}{\sec^6(e+fx)-6\sec^5(e+fx)+}\right.}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**6,x)
```

output

```
a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(
e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1
), x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 +
15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f
*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**6 - 6*sec(e + f*x)*
*5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(
e + f*x) + 1), x))/c**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(159) = 318$.

Time = 0.06 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{\frac{a^2\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{990\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{1386\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{3465\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-315\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}} + \frac{6a^2\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\right.}{}}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="m
axima")
```

output

```
1/110880*(a^2*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 6*a^2*(385*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 462*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x +
 e) + 1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x +
e) + 1)^11/(c^6*sin(f*x + e)^11) + 5*a^2*(385*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(f*
x + e)^10/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x +
 e)^11))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = \frac{231\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-495\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+385\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-105\,a^2}{9240\,c^6f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="g
iac")
```

output

```
1/9240*(231*a^2*tan(1/2*f*x + 1/2*e)^6 - 495*a^2*tan(1/2*f*x + 1/2*e)^4 +
385*a^2*tan(1/2*f*x + 1/2*e)^2 - 105*a^2)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = -\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(105\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-385\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+495\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\right.\right.}{9240\,c^6\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^6),x)
```

output
```
-(a^2*cos(e/2 + (f*x)/2)^5*(105*cos(e/2 + (f*x)/2)^6 - 231*sin(e/2 + (f*x)
/2)^6 + 495*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 385*cos(e/2 + (f*x
)/2)^4*sin(e/2 + (f*x)/2)^2))/(9240*c^6*f*sin(e/2 + (f*x)/2)^11)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = \frac{a^2\left(231\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-495\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+385\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-105\right)}{9240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}c^6f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x)
```

output
```
(a**2*(231*tan((e + f*x)/2)**6 - 495*tan((e + f*x)/2)**4 + 385*tan((e + f*
x)/2)**2 - 105))/(9240*tan((e + f*x)/2)**11*c**6*f)
```

3.21 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$

Optimal result

Integrand size = 32, antiderivative size = 227

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx &= \frac{55a^3c^6\operatorname{arctanh}(\sin(e+fx))}{128f}-\frac{25a^3c^6\sec(e+fx)\tan(e+fx)}{128f}\\ &\quad-\frac{15a^3c^6\sec^3(e+fx)\tan(e+fx)}{64f}+\frac{5a^3c^6\sec(e+fx)\tan^3(e+fx)}{24f}\\ &\quad+\frac{5a^3c^6\sec^3(e+fx)\tan^3(e+fx)}{16f}-\frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f}\\ &\quad-\frac{3a^3c^6\sec^3(e+fx)\tan^5(e+fx)}{8f}+\frac{4a^3c^6\tan^7(e+fx)}{7f}+\frac{a^3c^6\tan^9(e+fx)}{9f}\end{aligned}$$

output

```
55/128*a^3*c^6*arctanh(sin(f*x+e))/f-25/128*a^3*c^6*sec(f*x+e)*tan(f*x+e)/
f-15/64*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)/f+5/24*a^3*c^6*sec(f*x+e)*tan(f*x+
e)^3/f+5/16*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)^3/f-1/6*a^3*c^6*sec(f*x+e)*tan
(f*x+e)^5/f-3/8*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)^5/f+4/7*a^3*c^6*tan(f*x+e)
^7/f+1/9*a^3*c^6*tan(f*x+e)^9/f
```

Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$$
$$= \frac{a^3c^6(443520\text{arctanh}(\sin(e+fx)) - \sec^9(e+fx)(-88704\sin(e+fx) + 88074\sin(2(e+fx)) + 37632\,\text{s}}{}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```

output
```
(a^3*c^6*(443520*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^9*(-88704*Sin[e + f*
x] + 88074*Sin[2*(e + f*x)] + 37632*Sin[3*(e + f*x)] - 2142*Sin[4*(e + f*x
)] + 2304*Sin[5*(e + f*x)] + 39858*Sin[6*(e + f*x)] - 7488*Sin[7*(e + f*x)
] + 4599*Sin[8*(e + f*x)] + 1856*Sin[9*(e + f*x)])))/(1032192*f)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^6\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6dx$$

$$\Big\downarrow \text{4446}$$

$$-a^3c^3\int\left(-c^3\sec^4(e+fx)\tan^6(e+fx)+3c^3\sec^3(e+fx)\tan^6(e+fx)-3c^3\sec^2(e+fx)\tan^6(e+fx)+c^3\,\text{se}\right.$$

$$\Big\downarrow \text{2009}$$

$$-a^3c^3\left(-\frac{55c^3\operatorname{arctanh}(\sin(e+fx))}{128f}-\frac{c^3\tan^9(e+fx)}{9f}-\frac{4c^3\tan^7(e+fx)}{7f}+\frac{3c^3\tan^5(e+fx)\sec^3(e+fx)}{8f}-5$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```

output
```
-(a^3*c^3*((-55*c^3*ArcTanh[Sin[e + f*x]])/(128*f) + (25*c^3*Sec[e + f*x]*
Tan[e + f*x])/(128*f) + (15*c^3*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) - (5*c
^3*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (5*c^3*Sec[e + f*x]^3*Tan[e + f*x
]^3)/(16*f) + (c^3*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) + (3*c^3*Sec[e + f*x
]^3*Tan[e + f*x]^5)/(8*f) - (4*c^3*Tan[e + f*x]^7)/(7*f) - (c^3*Tan[e + f*
x]^9)/(9*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11

method	result
risch	$ia^3c^6\left(4599\,\mathrm{e}^{17i(fx+e)}-24192\,\mathrm{e}^{16i(fx+e)}+39858\,\mathrm{e}^{15i(fx+e)}-64512\,\mathrm{e}^{14i(fx+e)}-2142\,\mathrm{e}^{13i(fx+e)}-118272\,\mathrm{e}^{12i(fx+e)}+\right.$
parallelrisch	$-55a^3\left(\left(\cos(9fx+9e)+9\cos(7fx+7e)+36\cos(5fx+5e)+84\cos(3fx+3e)+126\cos(fx+e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+(-\right.$
derivativedivides	$-a^3c^6\left(-\frac{128}{315}-\frac{\sec(fx+e)^8}{9}-\frac{8\sec(fx+e)^6}{63}-\frac{16\sec(fx+e)^4}{105}-\frac{64\sec(fx+e)^2}{315}\right)\tan(fx+e)-3a^3c^6\left(-\left(-\frac{\sec(fx+e)^7}{8}-\right.\right.$
default	$-a^3c^6\left(-\frac{128}{315}-\frac{\sec(fx+e)^8}{9}-\frac{8\sec(fx+e)^6}{63}-\frac{16\sec(fx+e)^4}{105}-\frac{64\sec(fx+e)^2}{315}\right)\tan(fx+e)-3a^3c^6\left(-\left(-\frac{\sec(fx+e)^7}{8}-\right.\right.$
parts	$\frac{a^3c^6\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a^3c^6\left(-\frac{128}{315}-\frac{\sec(fx+e)^8}{9}-\frac{8\sec(fx+e)^6}{63}-\frac{16\sec(fx+e)^4}{105}-\frac{64\sec(fx+e)^2}{315}\right)\tan(fx+}{f}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x,method=_RETURNVERBO
SE)
```

output

```
1/4032*I*a^3*c^6*(4599*exp(17*I*(f*x+e))-24192*exp(16*I*(f*x+e))+39858*exp
(15*I*(f*x+e))-64512*exp(14*I*(f*x+e))-2142*exp(13*I*(f*x+e))-118272*exp(1
2*I*(f*x+e))+88074*exp(11*I*(f*x+e))-322560*exp(10*I*(f*x+e))-145152*exp(8
*I*(f*x+e))-88074*exp(7*I*(f*x+e))-193536*exp(6*I*(f*x+e))+2142*exp(5*I*(f
*x+e))-69120*exp(4*I*(f*x+e))-39858*exp(3*I*(f*x+e))-9216*exp(2*I*(f*x+e))
-4599*exp(I*(f*x+e))-3712)/f/(exp(2*I*(f*x+e))+1)^9+55/128*a^3*c^6/f*ln(ex
p(I*(f*x+e))+I)-55/128*a^3*c^6/f*ln(exp(I*(f*x+e))-I)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.92

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$$

$$=\frac{3465\,a^3c^6\cos\left(fx+e\right)^9\log\left(\sin\left(fx+e\right)+1\right)-3465\,a^3c^6\cos\left(fx+e\right)^9\log\left(-\sin\left(fx+e\right)+1\right)-2\,(371}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="f
ricas")
```

output
```
1/16128*(3465*a^3*c^6*cos(f*x + e)^9*log(sin(f*x + e) + 1) - 3465*a^3*c^6*
cos(f*x + e)^9*log(-sin(f*x + e) + 1) - 2*(3712*a^3*c^6*cos(f*x + e)^8 + 4
599*a^3*c^6*cos(f*x + e)^7 - 10240*a^3*c^6*cos(f*x + e)^6 + 3066*a^3*c^6*c
os(f*x + e)^5 + 8448*a^3*c^6*cos(f*x + e)^4 - 7224*a^3*c^6*cos(f*x + e)^3
- 1024*a^3*c^6*cos(f*x + e)^2 + 3024*a^3*c^6*cos(f*x + e) - 896*a^3*c^6)*s
in(f*x + e))/(f*cos(f*x + e)^9)
```

Sympy [F]

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx\\
&= a^3c^6\bigg(\int \sec(e+fx)\,dx+\int\left(-3\sec^2(e+fx)\right)\,dx+\int 8\sec^4(e+fx)\,dx\\
&\quad+\int\left(-6\sec^5(e+fx)\right)\,dx+\int\left(-6\sec^6(e+fx)\right)\,dx+\int 8\sec^7(e+fx)\,dx\\
&\quad+\int\left(-3\sec^9(e+fx)\right)\,dx+\int \sec^{10}(e+fx)\,dx\bigg)
\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**6,x)
```

output
```
a**3*c**6*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + I
ntegral(8*sec(e + f*x)**4, x) + Integral(-6*sec(e + f*x)**5, x) + Integral
(-6*sec(e + f*x)**6, x) + Integral(8*sec(e + f*x)**7, x) + Integral(-3*sec
(e + f*x)**9, x) + Integral(sec(e + f*x)**10, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(209) = 418.

Time = 0.04 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.95

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$$

$$= \frac{256\,(35\tan(fx+e)^9+180\tan(fx+e)^7+378\tan(fx+e)^5+420\tan(fx+e)^3+315\tan(fx+e)}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="m
axima")
```

output
```
1/80640*(256*(35*tan(f*x + e)^9 + 180*tan(f*x + e)^7 + 378*tan(f*x + e)^5
+ 420*tan(f*x + e)^3 + 315*tan(f*x + e))*a^3*c^6 - 32256*(3*tan(f*x + e)^5
 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^6 + 215040*(tan(f*x + e)^3 +
 3*tan(f*x + e))*a^3*c^6 + 315*a^3*c^6*(2*(105*sin(f*x + e)^7 - 385*sin(f*
x + e)^5 + 511*sin(f*x + e)^3 - 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(
f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x +
e) + 1) + 105*log(sin(f*x + e) - 1)) - 6720*a^3*c^6*(2*(15*sin(f*x + e)^5
- 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4
+ 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e) -
 1)) + 30240*a^3*c^6*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^
4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) -
 1)) + 80640*a^3*c^6*log(sec(f*x + e) + tan(f*x + e)) - 241920*a^3*c^6*tan
(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.04

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$$

$$= \frac{3465\,a^3c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3465\,a^3c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(3465\,a^3c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{17}\right.}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="g
iac")
```

output
```
1/8064*(3465*a^3*c^6*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3465*a^3*c^6*log
(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(3465*a^3*c^6*tan(1/2*f*x + 1/2*e)^17
- 30030*a^3*c^6*tan(1/2*f*x + 1/2*e)^15 + 115038*a^3*c^6*tan(1/2*f*x + 1/2
*e)^13 + 334602*a^3*c^6*tan(1/2*f*x + 1/2*e)^11 - 360448*a^3*c^6*tan(1/2*f
*x + 1/2*e)^9 + 255222*a^3*c^6*tan(1/2*f*x + 1/2*e)^7 - 115038*a^3*c^6*tan
(1/2*f*x + 1/2*e)^5 + 30030*a^3*c^6*tan(1/2*f*x + 1/2*e)^3 - 3465*a^3*c^6*
tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^9)/f
```

Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.39

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx = \frac{55\,a^3\,c^6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{64\,f}$$
$$-\frac{\frac{55\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{17}}{64}-\frac{715\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{15}}{96}+\frac{913\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{32}+\frac{18589\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{224}-\frac{5632\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f}{}\right.}{63}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{18}-9\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16}+36\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-84\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+126\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}\right.}$$

input
```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6)/cos(e + f*x),x)
```

output
```
(55*a^3*c^6*atanh(tan(e/2 + (f*x)/2)))/(64*f) - ((715*a^3*c^6*tan(e/2 + (f
*x)/2)^3)/96 - (913*a^3*c^6*tan(e/2 + (f*x)/2)^5)/32 + (14179*a^3*c^6*tan(
e/2 + (f*x)/2)^7)/224 - (5632*a^3*c^6*tan(e/2 + (f*x)/2)^9)/63 + (18589*a^
3*c^6*tan(e/2 + (f*x)/2)^11)/224 + (913*a^3*c^6*tan(e/2 + (f*x)/2)^13)/32
- (715*a^3*c^6*tan(e/2 + (f*x)/2)^15)/96 + (55*a^3*c^6*tan(e/2 + (f*x)/2)^
17)/64 - (55*a^3*c^6*tan(e/2 + (f*x)/2))/64)/(f*(9*tan(e/2 + (f*x)/2)^2 -
36*tan(e/2 + (f*x)/2)^4 + 84*tan(e/2 + (f*x)/2)^6 - 126*tan(e/2 + (f*x)/2)
^8 + 126*tan(e/2 + (f*x)/2)^10 - 84*tan(e/2 + (f*x)/2)^12 + 36*tan(e/2 + (
f*x)/2)^14 - 9*tan(e/2 + (f*x)/2)^16 + tan(e/2 + (f*x)/2)^18 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.80

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$$
$$=\frac{a^3c^6\left(-3465\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^8+13860\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x)
```

output

```
(a**3*c**6*( - 3465*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**8
 + 13860*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6 - 20790*co
s(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 + 13860*cos(e + f*x)*
log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 - 3465*cos(e + f*x)*log(tan((e +
 f*x)/2) - 1) + 3465*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**
8 - 13860*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**6 + 20790*c
os(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4 - 13860*cos(e + f*x)
*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 + 3465*cos(e + f*x)*log(tan((e
+ f*x)/2) + 1) + 4599*cos(e + f*x)*sin(e + f*x)**7 - 16863*cos(e + f*x)*si
n(e + f*x)**5 + 12705*cos(e + f*x)*sin(e + f*x)**3 - 3465*cos(e + f*x)*sin
(e + f*x) - 3712*sin(e + f*x)**9 + 4608*sin(e + f*x)**7))/(8064*cos(e + f*
x)*f*(sin(e + f*x)**8 - 4*sin(e + f*x)**6 + 6*sin(e + f*x)**4 - 4*sin(e +
f*x)**2 + 1))
```

3.22 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$

Optimal result

Integrand size = 32, antiderivative size = 206

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx \\
&= \frac{45a^3c^5\text{arctanh}(\sin(e+fx))}{128f} - \frac{35a^3c^5\sec(e+fx)\tan(e+fx)}{128f} \\
&\quad - \frac{5a^3c^5\sec^3(e+fx)\tan(e+fx)}{64f} + \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{24f} \\
&\quad + \frac{5a^3c^5\sec^3(e+fx)\tan^3(e+fx)}{48f} - \frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f} \\
&\quad - \frac{a^3c^5\sec^3(e+fx)\tan^5(e+fx)}{8f} + \frac{2a^3c^5\tan^7(e+fx)}{7f}
\end{aligned}$$

output

```
45/128*a^3*c^5*arctanh(sin(f*x+e))/f-35/128*a^3*c^5*sec(f*x+e)*tan(f*x+e)/
f-5/64*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)/f+5/24*a^3*c^5*sec(f*x+e)*tan(f*x+e
)^3/f+5/48*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)^3/f-1/6*a^3*c^5*sec(f*x+e)*tan(
f*x+e)^5/f-1/8*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)^5/f+2/7*a^3*c^5*tan(f*x+e)^
7/f
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx =$$

$$-\frac{a^3c^5(-20160\text{arctanh}(\sin(e+fx))+\sec^8(e+fx)(5705\sin(e+fx)-1792\sin(2(e+fx))+21\sin(3(}{}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

output

```
-1/57344*(a^3*c^5*(-20160*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^8*(5705*Sin
[e + f*x] - 1792*Sin[2*(e + f*x)] + 21*Sin[3*(e + f*x)] + 1792*Sin[4*(e +
f*x)] + 2065*Sin[5*(e + f*x)] - 768*Sin[6*(e + f*x)] + 581*Sin[7*(e + f*x)
] + 128*Sin[8*(e + f*x)])))/f
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5dx$$

$$\Big\downarrow \text{4446}$$

$$-a^3c^3\int \left(c^2\sec^3(e+fx)\tan^6(e+fx)-2c^2\sec^2(e+fx)\tan^6(e+fx)+c^2\sec(e+fx)\tan^6(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-a^3c^3\left(-\frac{45c^2\text{arctanh}(\sin(e+fx))}{128f}-\frac{2c^2\tan^7(e+fx)}{7f}+\frac{c^2\tan^5(e+fx)\sec^3(e+fx)}{8f}-\frac{5c^2\tan^3(e+fx)\sec^3(}{48f}\right.$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

output

```
-(a^3*c^3*((-45*c^2*ArcTanh[Sin[e + f*x]])/(128*f) + (35*c^2*Sec[e + f*x]*
Tan[e + f*x])/(128*f) + (5*c^2*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) - (5*c^
2*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (5*c^2*Sec[e + f*x]^3*Tan[e + f*x]
^3)/(48*f) + (c^2*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) + (c^2*Sec[e + f*x]^3
*Tan[e + f*x]^5)/(8*f) - (2*c^2*Tan[e + f*x]^7)/(7*f)))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.16

method	result
norman	$\frac{-\frac{45a^3c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64f}+\frac{345a^3c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{64f}-\frac{1149a^3c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{64f}+\frac{15159a^3c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{448f}-\frac{17609a^3c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{448f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^8}$
risch	$ic^5a^3(581\,e^{15i(fx+e)}-1792\,e^{14i(fx+e)}+2065\,e^{13i(fx+e)}-1792\,e^{12i(fx+e)}+21\,e^{11i(fx+e)}-8960\,e^{10i(fx+e)}+5705\,e^{9i(f}$
parallelrisch	$-\frac{815a^3\left(\frac{9\left(\frac{35}{2}+28\cos(2fx+2e)+14\cos(4fx+4e)+4\cos(6fx+6e)+\frac{\cos(8fx+8e)}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{163}+\frac{9(-\frac{35}{2}-28\cos(2fx+2e}{}\right.}{}$
parts	$-\frac{2a^3c^5\tan(fx+e)}{f}-\frac{a^3c^5\sec(fx+e)\tan(fx+e)}{f}-\frac{6a^3c^5\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{6a^3c^5(-\frac{8}{15}-\frac{\sec(fx}{5}}{}$
derivativedivides	$\frac{a^3c^5\ln(\sec(fx+e)+\tan(fx+e))-2a^3c^5\tan(fx+e)-2a^3c^5\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-6a^3c^5(-}{}$
default	$\frac{a^3c^5\ln(\sec(fx+e)+\tan(fx+e))-2a^3c^5\tan(fx+e)-2a^3c^5\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-6a^3c^5(-}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBO
SE)
```

output
```
(-45/64*a^3*c^5/f*tan(1/2*f*x+1/2*e)+345/64*a^3*c^5/f*tan(1/2*f*x+1/2*e)^3
-1149/64*a^3*c^5/f*tan(1/2*f*x+1/2*e)^5+15159/448*a^3*c^5/f*tan(1/2*f*x+1/
2*e)^7-17609/448*a^3*c^5/f*tan(1/2*f*x+1/2*e)^9-1149/64*a^3*c^5/f*tan(1/2*
f*x+1/2*e)^11+345/64*a^3*c^5/f*tan(1/2*f*x+1/2*e)^13-45/64*a^3*c^5/f*tan(1
/2*f*x+1/2*e)^15)/(tan(1/2*f*x+1/2*e)^2-1)^8-45/128*a^3*c^5/f*ln(tan(1/2*f
*x+1/2*e)-1)+45/128*a^3*c^5/f*ln(tan(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$= \frac{315\,a^3c^5\cos\left(fx+e\right)^8\log\left(\sin\left(fx+e\right)+1\right)-315\,a^3c^5\cos\left(fx+e\right)^8\log\left(-\sin\left(fx+e\right)+1\right)-2\left(256\,a^3\right.}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="f
ricas")
```

output

```
1/1792*(315*a^3*c^5*cos(f*x + e)^8*log(sin(f*x + e) + 1) - 315*a^3*c^5*cos
(f*x + e)^8*log(-sin(f*x + e) + 1) - 2*(256*a^3*c^5*cos(f*x + e)^7 + 581*a
^3*c^5*cos(f*x + e)^6 - 768*a^3*c^5*cos(f*x + e)^5 - 210*a^3*c^5*cos(f*x +
 e)^4 + 768*a^3*c^5*cos(f*x + e)^3 - 168*a^3*c^5*cos(f*x + e)^2 - 256*a^3*
c^5*cos(f*x + e) + 112*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^8)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$= -a^3c^5\left(\int\left(-\sec\left(e+fx\right)\right)\,dx+\int 2\sec^2\left(e+fx\right)dx+\int 2\sec^3\left(e+fx\right)dx\right.$$
$$+\int\left(-6\sec^4\left(e+fx\right)\right)\,dx+\int 6\sec^6\left(e+fx\right)dx+\int\left(-2\sec^7\left(e+fx\right)\right)\,dx$$
$$\left.+\int\left(-2\sec^8\left(e+fx\right)\right)\,dx+\int\sec^9\left(e+fx\right)dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
```

output

```
-a**3*c**5*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) +
Integral(2*sec(e + f*x)**3, x) + Integral(-6*sec(e + f*x)**4, x) + Integra
l(6*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7, x) + Integral(-2*se
c(e + f*x)**8, x) + Integral(sec(e + f*x)**9, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(190) = 380$.

Time = 0.04 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.98

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$

$$= \frac{1536\left(5\tan\left(fx+e\right)^7+21\tan\left(fx+e\right)^5+35\tan\left(fx+e\right)^3+35\tan\left(fx+e\right)\right)a^3c^5-10752\left(3\tan\left(fx\right.\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="m
axima")
```

output
```
1/26880*(1536*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 +
35*tan(f*x + e))*a^3*c^5 - 10752*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 1
5*tan(f*x + e))*a^3*c^5 + 53760*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^5
+ 35*a^3*c^5*(2*(105*sin(f*x + e)^7 - 385*sin(f*x + e)^5 + 511*sin(f*x + e
)^3 - 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e
)^4 - 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x + e) + 1) + 105*log(sin(f*x
+ e) - 1)) - 560*a^3*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*si
n(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 1
5*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) + 13440*a^3*c^5*(2*sin
(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e)
- 1)) + 26880*a^3*c^5*log(sec(f*x + e) + tan(f*x + e)) - 53760*a^3*c^5*tan
(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.05

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$

$$= \frac{315\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-315\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(315\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{15}-241\right.}{}}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="g
iac")
```

output

```
1/896*(315*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 315*a^3*c^5*log(ab
s(tan(1/2*f*x + 1/2*e) - 1)) - 2*(315*a^3*c^5*tan(1/2*f*x + 1/2*e)^15 - 24
15*a^3*c^5*tan(1/2*f*x + 1/2*e)^13 + 8043*a^3*c^5*tan(1/2*f*x + 1/2*e)^11
 + 17609*a^3*c^5*tan(1/2*f*x + 1/2*e)^9 - 15159*a^3*c^5*tan(1/2*f*x + 1/2*e
)^7 + 8043*a^3*c^5*tan(1/2*f*x + 1/2*e)^5 - 2415*a^3*c^5*tan(1/2*f*x + 1/2
*e)^3 + 315*a^3*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^8)/
f
```

Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.38

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = \frac{45\,a^3\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{64\,f} - \frac{\frac{45\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{15}}{64} - \frac{345\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{64} + \frac{1149\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{64} + \frac{17609\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{448} - \frac{15159\,a^3\,c^5\,\tan\left(\frac{e}{2}+\right.}{448}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16} - 8\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14} + 28\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12} - 56\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10} + 70\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}\right.}$$

input

```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5)/cos(e + f*x),x)
```

output

```
(45*a^3*c^5*atanh(tan(e/2 + (f*x)/2)))/(64*f) - ((1149*a^3*c^5*tan(e/2 + (
f*x)/2)^5)/64 - (345*a^3*c^5*tan(e/2 + (f*x)/2)^3)/64 - (15159*a^3*c^5*tan
(e/2 + (f*x)/2)^7)/448 + (17609*a^3*c^5*tan(e/2 + (f*x)/2)^9)/448 + (1149*
a^3*c^5*tan(e/2 + (f*x)/2)^11)/64 - (345*a^3*c^5*tan(e/2 + (f*x)/2)^13)/64
 + (45*a^3*c^5*tan(e/2 + (f*x)/2)^15)/64 + (45*a^3*c^5*tan(e/2 + (f*x)/2))
/64)/(f*(28*tan(e/2 + (f*x)/2)^4 - 8*tan(e/2 + (f*x)/2)^2 - 56*tan(e/2 + (
f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 - 56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/
2 + (f*x)/2)^12 - 8*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*x)/2)^16 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.51

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$

$$= \frac{a^3c^5\bigl(256\cos(fx+e)\sin(fx+e)^7-315\log\bigl(\tan\bigl(\frac{fx}{2}+\frac{e}{2}\bigr)-1\bigr)\sin(fx+e)^8+1260\log\bigl(\tan\bigl(\frac{fx}{2}+\frac{e}{2}\bigr)-}{}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x)
```

output

```
(a**3*c**5*(256*cos(e + f*x)*sin(e + f*x)**7 - 315*log(tan((e + f*x)/2) -
1)*sin(e + f*x)**8 + 1260*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6 - 1890
*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 + 1260*log(tan((e + f*x)/2) - 1
)*sin(e + f*x)**2 - 315*log(tan((e + f*x)/2) - 1) + 315*log(tan((e + f*x)/
2) + 1)*sin(e + f*x)**8 - 1260*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**6 +
 1890*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4 - 1260*log(tan((e + f*x)/2
) + 1)*sin(e + f*x)**2 + 315*log(tan((e + f*x)/2) + 1) + 581*sin(e + f*x)*
*7 - 1533*sin(e + f*x)**5 + 1155*sin(e + f*x)**3 - 315*sin(e + f*x)))/(896
*f*(sin(e + f*x)**8 - 4*sin(e + f*x)**6 + 6*sin(e + f*x)**4 - 4*sin(e + f*
x)**2 + 1))
```

3.23 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx &= \frac{5a^3c^4\text{arctanh}(\sin(e+fx))}{16f}-\frac{5a^3c^4\sec(e+fx)\tan(e+fx)}{16f}\\ &\quad+\frac{5a^3c^4\sec(e+fx)\tan^3(e+fx)}{24f}\\ &\quad-\frac{a^3c^4\sec(e+fx)\tan^5(e+fx)}{6f}+\frac{a^3c^4\tan^7(e+fx)}{7f}\end{aligned}$$

output
```
5/16*a^3*c^4*arctanh(sin(f*x+e))/f-5/16*a^3*c^4*sec(f*x+e)*tan(f*x+e)/f+5/
24*a^3*c^4*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^3*c^4*sec(f*x+e)*tan(f*x+e)^5/f
+1/7*a^3*c^4*tan(f*x+e)^7/f
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$= \frac{a^3c^4(3360\text{arctanh}(\sin(e+fx))-\sec^7(e+fx)(-840\sin(e+fx)+595\sin(2(e+fx))+504\sin(3(e+}{10752f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^3*c^4*(3360*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(-840*Sin[e + f*x] +
 595*Sin[2*(e + f*x)] + 504*Sin[3*(e + f*x)] + 196*Sin[4*(e + f*x)] - 168*
Sin[5*(e + f*x)] + 231*Sin[6*(e + f*x)] + 24*Sin[7*(e + f*x)])))/(10752*f)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$
$$\Big\downarrow \text{4446}$$
$$-a^3c^3\int \left(c\sec(e+fx)\tan^6(e+fx)-c\sec^2(e+fx)\tan^6(e+fx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-a^3c^3\left(-\frac{5c\text{arctanh}(\sin(e+fx))}{16f}-\frac{c\tan^7(e+fx)}{7f}+\frac{c\tan^5(e+fx)\sec(e+fx)}{6f}-\frac{5c\tan^3(e+fx)\sec(e+fx)}{24f}\right.$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

output
```
-(a^3*c^3*((-5*c*ArcTanh[Sin[e + f*x]])/(16*f) + (5*c*Sec[e + f*x]*Tan[e +
 f*x])/(16*f) - (5*c*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (c*Sec[e + f*x]
*Tan[e + f*x]^5)/(6*f) - (c*Tan[e + f*x]^7)/(7*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.45

method	result
risch	$\frac{ia^3c^4\left(231\,\mathrm{e}^{13i(fx+e)}-336\,\mathrm{e}^{12i(fx+e)}+196\,\mathrm{e}^{11i(fx+e)}+595\,\mathrm{e}^{9i(fx+e)}-1680\,\mathrm{e}^{8i(fx+e)}-595\,\mathrm{e}^{5i(fx+e)}-1008\,\mathrm{e}^{4i(fx+e)}-\right.}{168f\left(\mathrm{e}^{2i(fx+e)}+1\right)^7}$
norman	$\frac{\frac{5a^3c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}-\frac{25a^3c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{6f}+\frac{283a^3c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{24f}-\frac{128a^3c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{7f}-\frac{283a^3c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{24f}+\frac{25a^3c^4}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^7}$
parallelrisch	$\frac{5a^3c^4\left(\frac{(-\cos(7fx+7e)-7\cos(5fx+5e)-21\cos(3fx+3e)-35\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{(\cos(7fx+7e)+21\cos(3fx+3e)+35}{}\right.}{f(\cos(7fx+7e)+2}$
derivativedivides	$\frac{a^3c^4\ln(\sec(fx+e)+\tan(fx+e))-a^3c^4\tan(fx+e)-3a^3c^4\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-3a^3c^4\left(-\right.}{}$
default	$\frac{a^3c^4\ln(\sec(fx+e)+\tan(fx+e))-a^3c^4\tan(fx+e)-3a^3c^4\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-3a^3c^4\left(-\right.}{}$
parts	$\frac{a^3c^4\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a^3c^4\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}-\frac{a^3c^4\tan(f}{f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/168*I*a^3*c^4*(231*exp(13*I*(f*x+e))-336*exp(12*I*(f*x+e))+196*exp(11*I*
(f*x+e))+595*exp(9*I*(f*x+e))-1680*exp(8*I*(f*x+e))-595*exp(5*I*(f*x+e))-1
008*exp(4*I*(f*x+e))-196*exp(3*I*(f*x+e))-231*exp(I*(f*x+e))-48)/f/(exp(2*
I*(f*x+e))+1)^7+5/16*a^3*c^4/f*ln(exp(I*(f*x+e))+I)-5/16*a^3*c^4/f*ln(exp(
I*(f*x+e))-I)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$=\frac{105\,a^3c^4\cos\left(fx+e\right)^7\log\left(\sin\left(fx+e\right)+1\right)-105\,a^3c^4\cos\left(fx+e\right)^7\log\left(-\sin\left(fx+e\right)+1\right)-2\left(48\,a^3c\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="f
ricas")
```

output
```
1/672*(105*a^3*c^4*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 105*a^3*c^4*cos(
f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(48*a^3*c^4*cos(f*x + e)^6 + 231*a^3
*c^4*cos(f*x + e)^5 - 144*a^3*c^4*cos(f*x + e)^4 - 182*a^3*c^4*cos(f*x + e
)^3 + 144*a^3*c^4*cos(f*x + e)^2 + 56*a^3*c^4*cos(f*x + e) - 48*a^3*c^4)*s
in(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx \\ = a^3c^4\bigg(\int \sec(e+fx)\,dx + \int \left(-\sec^2(e+fx)\right)\,dx + \int \left(-3\sec^3(e+fx)\right)\,dx \\ + \int 3\sec^4(e+fx)\,dx + \int 3\sec^5(e+fx)\,dx + \int \left(-3\sec^6(e+fx)\right)\,dx \\ + \int \left(-\sec^7(e+fx)\right)\,dx + \int \sec^8(e+fx)\,dx\bigg)\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

output
```
a**3*c**4*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Int
egral(-3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)**4, x) + Integral(3
*sec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x) + Integral(-sec(e +
 f*x)**7, x) + Integral(sec(e + f*x)**8, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(111) = 222$.

Time = 0.04 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.04

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$

$$= \frac{96\,(5\tan(fx+e)^7+21\tan(fx+e)^5+35\tan(fx+e)^3+35\tan(fx+e))a^3c^4-672\,(3\tan(fx+e}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="m
axima")
```

output
```
1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*
tan(f*x + e))*a^3*c^4 - 672*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan
(f*x + e))*a^3*c^4 + 3360*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 + 35*a
^3*c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f
*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e)
 + 1) + 15*log(sin(f*x + e) - 1)) - 630*a^3*c^4*(2*(3*sin(f*x + e)^3 - 5*s
in(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e)
+ 1) + 3*log(sin(f*x + e) - 1)) + 2520*a^3*c^4*(2*sin(f*x + e)/(sin(f*x +
e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^4*
log(sec(f*x + e) + tan(f*x + e)) - 3360*a^3*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.63

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$

$$= \frac{105\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}-700}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="g
iac")
```

output
```
1/336*(105*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a^3*c^4*log(ab
s(tan(1/2*f*x + 1/2*e) - 1)) - 2*(105*a^3*c^4*tan(1/2*f*x + 1/2*e)^13 - 70
0*a^3*c^4*tan(1/2*f*x + 1/2*e)^11 + 1981*a^3*c^4*tan(1/2*f*x + 1/2*e)^9 +
3072*a^3*c^4*tan(1/2*f*x + 1/2*e)^7 - 1981*a^3*c^4*tan(1/2*f*x + 1/2*e)^5
+ 700*a^3*c^4*tan(1/2*f*x + 1/2*e)^3 - 105*a^3*c^4*tan(1/2*f*x + 1/2*e))/(
tan(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

Mupad [B] (verification not implemented)

Time = 14.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.08

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = \frac{5\,a^3\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

$$-\frac{\frac{5\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{8}-\frac{25\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{6}+\frac{283\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{128\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{7}-\frac{283\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{24}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-2\right.}$$

input
```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

output
```
(5*a^3*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f) - ((25*a^3*c^4*tan(e/2 + (f*x)
/2)^3)/6 - (283*a^3*c^4*tan(e/2 + (f*x)/2)^5)/24 + (128*a^3*c^4*tan(e/2 +
(f*x)/2)^7)/7 + (283*a^3*c^4*tan(e/2 + (f*x)/2)^9)/24 - (25*a^3*c^4*tan(e/
2 + (f*x)/2)^11)/6 + (5*a^3*c^4*tan(e/2 + (f*x)/2)^13)/8 - (5*a^3*c^4*tan(
e/2 + (f*x)/2))/8)/(f*(7*tan(e/2 + (f*x)/2)^2 - 21*tan(e/2 + (f*x)/2)^4 +
35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^
10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^14 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.61

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$

$$= \frac{a^3c^4\left(-105\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^6+315\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x)
```

output

```
(a**3*c**4*( - 105*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6
+ 315*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 - 315*cos(e +
 f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 + 105*cos(e + f*x)*log(tan
((e + f*x)/2) - 1) + 105*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*
x)**6 - 315*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4 + 315*c
os(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 - 105*cos(e + f*x)*l
og(tan((e + f*x)/2) + 1) + 231*cos(e + f*x)*sin(e + f*x)**5 - 280*cos(e +
f*x)*sin(e + f*x)**3 + 105*cos(e + f*x)*sin(e + f*x) - 48*sin(e + f*x)**7)
)/(336*cos(e + f*x)*f*(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e + f*x
)**2 - 1))
```

3.24 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 32, antiderivative size = 100

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx &= \frac{5a^3c^3\operatorname{arctanh}(\sin(e+fx))}{16f} - \frac{5a^3c^3\sec(e+fx)\tan(e+fx)}{16f}\\ &\quad + \frac{5a^3c^3\sec(e+fx)\tan^3(e+fx)}{24f} - \frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f}\end{aligned}$$

output

```
5/16*a^3*c^3*arctanh(sin(f*x+e))/f-5/16*a^3*c^3*sec(f*x+e)*tan(f*x+e)/f+5/
24*a^3*c^3*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^3*c^3*sec(f*x+e)*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx\\ =-a^3c^3\bigg(-\frac{5\operatorname{arctanh}(\sin(e+fx))}{16f}-\frac{5\sec(e+fx)\tan(e+fx)}{16f}\\ -\frac{5\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{5\sec^5(e+fx)\tan(e+fx)}{6f}\\ -\frac{5\sec^3(e+fx)\tan^3(e+fx)}{3f}+\frac{\sec(e+fx)\tan^5(e+fx)}{f}\bigg)\end{aligned}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

output

```
-(a^3*c^3*((-5*ArcTanh[Sin[e + f*x]])/(16*f) - (5*Sec[e + f*x]*Tan[e + f*x
])/(16*f) - (5*Sec[e + f*x]^3*Tan[e + f*x])/(24*f) + (5*Sec[e + f*x]^5*Tan
[e + f*x])/(6*f) - (5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(3*f) + (Sec[e + f*x]
*Tan[e + f*x]^5)/f))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4446, 3042, 3091, 3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4446}$$

$$-a^3c^3\int \sec(e+fx)\tan^6(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \sec(e+fx)\tan(e+fx)^6dx$$

$$\Big\downarrow \text{3091}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)\sec(e+fx)}{6f}-\frac{5}{6}\int \sec(e+fx)\tan^4(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)\sec(e+fx)}{6f}-\frac{5}{6}\int \sec(e+fx)\tan(e+fx)^4dx\right)$$

$$\Big\downarrow \text{3091}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)\sec(e+fx)}{6f}-\frac{5}{6}\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\int \sec(e+fx)\tan^2(e+fx)dx\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)\sec(e+fx)}{6f}-\frac{5}{6}\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\int \sec(e+fx)\tan(e+fx)^2dx\right)\right)$$

$$\Big\downarrow \text{3091}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)\sec(e+fx)}{6f}-\frac{5}{6}\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int \sec(e+\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)\sec(e+fx)}{6f}-\frac{5}{6}\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int \csc\left(e+\right.\right.\right.\right.$$

$$\Big\downarrow \text{4257}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)\sec(e+fx)}{6f}-\frac{5}{6}\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{\text{arctanh}(\sin(}{2f}\right.\right.\right.$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

output
```
-(a^3*c^3*((Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (5*((Sec[e + f*x]*Tan[e +
 f*x]^3)/(4*f) - (3*(-1/2*ArcTanh[Sin[e + f*x]]/f + (Sec[e + f*x]*Tan[e +
f*x])/(2*f)))/4))/6))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.42

method	result
risch	$\frac{ia^3c^3\left(33\,\mathrm{e}^{11i(fx+e)}-5\,\mathrm{e}^{9i(fx+e)}+90\,\mathrm{e}^{7i(fx+e)}-90\,\mathrm{e}^{5i(fx+e)}+5\,\mathrm{e}^{3i(fx+e)}-33\,\mathrm{e}^{i(fx+e)}\right)}{24f\left(\mathrm{e}^{2i(fx+e)}+1\right)^6}+\frac{5a^3c^3\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{16f}-$
parallelrisch	$\frac{5a^3\left(\left(-\frac{45\cos(2fx+2e)}{2}-9\cos(4fx+4e)-\frac{3\cos(6fx+6e)}{2}-15\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(\frac{3\cos(6fx+6e)}{2}+9\cos(4fx+4e)+45\right.\right.}{24f(10+\cos(6fx+6e)+6\cos(4fx+4e)+15}$
derivativedivides	$\frac{a^3c^3\ln(\sec(fx+e)+\tan(fx+e))-3a^3c^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+3a^3c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-3\right.\right.}{}$
default	$\frac{a^3c^3\ln(\sec(fx+e)+\tan(fx+e))-3a^3c^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+3a^3c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-3\right.\right.}{}$
parts	$\frac{a^3c^3\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{3a^3c^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{3a^3c^3\left(-\left(-\frac{\sec(fx+e}{4}\right.\right.}{}$
norman	$\frac{-\frac{5a^3c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{85a^3c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{24f}-\frac{33a^3c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}-\frac{33a^3c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{85a^3c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{24f}-\frac{5a^3c^3\,\mathrm{ta}}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/24*I*a^3*c^3/f/(exp(2*I*(f*x+e))+1)^6*(33*exp(11*I*(f*x+e))-5*exp(9*I*(f
*x+e))+90*exp(7*I*(f*x+e))-90*exp(5*I*(f*x+e))+5*exp(3*I*(f*x+e))-33*exp(I
*(f*x+e)))+5/16*a^3*c^3/f*ln(exp(I*(f*x+e))+I)-5/16*a^3*c^3/f*ln(exp(I*(f*
x+e))-I)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$=\frac{15\,a^3c^3\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c^3\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)-2\left(33\,a^3c^3\,\mathrm{c}\right.}{96\,f\cos\left(fx+e\right)^6}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/96*(15*a^3*c^3*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*a^3*c^3*cos(f*x
 + e)^6*log(-sin(f*x + e) + 1) - 2*(33*a^3*c^3*cos(f*x + e)^4 - 26*a^3*c^3
*cos(f*x + e)^2 + 8*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$= -a^3c^3\left(\int\left(-\sec\left(e+fx\right)\right)\,dx+\int 3\sec^3\left(e+fx\right)dx+\int\left(-3\sec^5\left(e+fx\right)\right)\,dx + \int\sec^7\left(e+fx\right)dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

output
```
-a**3*c**3*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**3, x) +
Integral(-3*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**7, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(92) = 184$.

Time = 0.04 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.44

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$= \frac{a^3c^3\left(\frac{2\left(15\sin(fx+e)^5-40\sin(fx+e)^3+33\sin(fx+e)\right)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}-15\log\left(\sin\left(fx+e\right)+1\right)+15\log\left(\sin\left(fx+e\right)-1\right)\right)-1}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/96*(a^3*c^3*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))
/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f
*x + e) + 1) + 15*log(sin(f*x + e) - 1)) - 18*a^3*c^3*(2*(3*sin(f*x + e)^3
 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x
 + e) + 1) + 3*log(sin(f*x + e) - 1)) + 72*a^3*c^3*(2*sin(f*x + e)/(sin(f*
x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 96*a^3*c^
3*log(sec(f*x + e) + tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$

$$= \frac{15\,a^3c^3\log\left(\left|\sin\left(fx+e\right)+1\right|\right) - 15\,a^3c^3\log\left(\left|\sin\left(fx+e\right)-1\right|\right) + \frac{2\left(33\,a^3c^3\sin(fx+e)^5 - 40\,a^3c^3\sin(fx+e)^3 + 15\,a^3c^3}{\left(\sin(fx+e)^2-1\right)^3}}{96\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
1/96*(15*a^3*c^3*log(abs(sin(f*x + e) + 1)) - 15*a^3*c^3*log(abs(sin(f*x +
 e) - 1)) + 2*(33*a^3*c^3*sin(f*x + e)^5 - 40*a^3*c^3*sin(f*x + e)^3 + 15*
a^3*c^3*sin(f*x + e))/(sin(f*x + e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.20

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = \frac{5\,a^3\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

$$-\frac{\frac{5\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8} - \frac{85\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24} + \frac{33\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{4} + \frac{33\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{4} - \frac{85\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24} +}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12} - 6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10} + 15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8} - 20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6} + 15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4} - 6}$$

input
```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

output
```
(5*a^3*c^3*atanh(tan(e/2 + (f*x)/2)))/(8*f) - ((33*a^3*c^3*tan(e/2 + (f*x)
/2)^5)/4 - (85*a^3*c^3*tan(e/2 + (f*x)/2)^3)/24 + (33*a^3*c^3*tan(e/2 + (f
*x)/2)^7)/4 - (85*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (5*a^3*c^3*tan(e/2 +
(f*x)/2)^11)/8 + (5*a^3*c^3*tan(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/
2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*
x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.32

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$

$$= \frac{a^3c^3\left(-15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^6+45\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^4-45\log\left(\tan\left(\frac{fx}{2}\right.\right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x)
```

output
```
(a**3*c**3*( - 15*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6 + 45*log(tan((
e + f*x)/2) - 1)*sin(e + f*x)**4 - 45*log(tan((e + f*x)/2) - 1)*sin(e + f*
x)**2 + 15*log(tan((e + f*x)/2) - 1) + 15*log(tan((e + f*x)/2) + 1)*sin(e
+ f*x)**6 - 45*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4 + 45*log(tan((e +
 f*x)/2) + 1)*sin(e + f*x)**2 - 15*log(tan((e + f*x)/2) + 1) + 33*sin(e +
f*x)**5 - 40*sin(e + f*x)**3 + 15*sin(e + f*x)))/(48*f*(sin(e + f*x)**6 -
3*sin(e + f*x)**4 + 3*sin(e + f*x)**2 - 1))
```

3.25 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 94

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx\\ =\frac{3a^3c^2\text{arctanh}(\sin(e+fx))}{8f}-\frac{3a^3c^2\sec(e+fx)\tan(e+fx)}{8f}\\ +\frac{a^3c^2\sec(e+fx)\tan^3(e+fx)}{4f}+\frac{a^3c^2\tan^5(e+fx)}{5f}\end{aligned}$$

output

```
3/8*a^3*c^2*arctanh(sin(f*x+e))/f-3/8*a^3*c^2*sec(f*x+e)*tan(f*x+e)/f+1/4*
a^3*c^2*sec(f*x+e)*tan(f*x+e)^3/f+1/5*a^3*c^2*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx\\ =\frac{a^3c^2(120\text{arctanh}(\sin(e+fx))+\sec^5(e+fx)(40\sin(e+fx)-10\sin(2(e+fx))-20\sin(3(e+fx))-}{320f}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

output
```
(a^3*c^2*(120*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^5*(40*Sin[e + f*x] - 10
*Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] - 25*Sin[4*(e + f*x)] + 4*Sin[5*(e
 + f*x)])))/(320*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4446}$$

$$a^2c^2\int \left(a\sec^2(e+fx)\tan^4(e+fx)+a\sec(e+fx)\tan^4(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(\frac{3a\text{arctanh}(\sin(e+fx))}{8f}+\frac{a\tan^5(e+fx)}{5f}+\frac{a\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a\tan(e+fx)\sec(e+fx)}{8f}\right)$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

output
```
a^2*c^2*((3*a*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a*Sec[e + f*x]*Tan[e + f*x
])/(8*f) + (a*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a*Tan[e + f*x]^5)/(5*f
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

method	result
risch	$\frac{i a^3 c^2 (25\,e^{9i(fx+e)}+40\,e^{8i(fx+e)}+10\,e^{7i(fx+e)}+80\,e^{4i(fx+e)}-10\,e^{3i(fx+e)}-25\,e^{i(fx+e)}+8)}{20f(e^{2i(fx+e)}+1)^5}-\frac{3a^3c^2\ln(e^{i(fx+e)}-i}{8f}$
parts	$\frac{a^3c^2\tan(fx+e)}{f}+\frac{a^3c^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}-\frac{a^3c^2(-\frac{8}{15}-}{}$
norman	$\frac{\frac{3a^3c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{7a^3c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2f}-\frac{32a^3c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5f}+\frac{7a^3c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{2f}-\frac{3a^3c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}-\frac{3a^3c^2\ln(}{}$
parallelrisch	$-\frac{a^3\left(\left(\frac{15\cos(fx+e)}{2}+\frac{15\cos(3fx+3e)}{4}+\frac{3\cos(5fx+5e)}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+(-\frac{15\cos(fx+e)}{2}-\frac{15\cos(3fx+3e)}{4}-\frac{3\cos(5fx}{4}}{2f(\cos(5fx+5e)+5\cos(3fx+}$
derivativedivides	$\frac{a^3c^2\ln(\sec(fx+e)+\tan(fx+e))+a^3c^2\tan(fx+e)-2a^3c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^3c^2(-}{}$
default	$\frac{a^3c^2\ln(\sec(fx+e)+\tan(fx+e))+a^3c^2\tan(fx+e)-2a^3c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^3c^2(-}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/20*I*a^3*c^2*(25*exp(9*I*(f*x+e))+40*exp(8*I*(f*x+e))+10*exp(7*I*(f*x+e)
)+80*exp(4*I*(f*x+e))-10*exp(3*I*(f*x+e))-25*exp(I*(f*x+e))+8)/f/(exp(2*I*
(f*x+e))+1)^5-3/8*a^3*c^2/f*ln(exp(I*(f*x+e))-I)+3/8*a^3*c^2/f*ln(exp(I*(f
*x+e))+I)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= \frac{15\,a^3c^2\cos{(fx+e)}^5\log{(\sin{(fx+e)}+1)} - 15\,a^3c^2\cos{(fx+e)}^5\log{(-\sin{(fx+e)}+1)} + 2\,(8\,a^3c^2\,\mathrm{co}}{80\,f\cos{(}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
1/80*(15*a^3*c^2*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c^2*cos(f*x
 + e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^3*c^2*cos(f*x + e)^4 - 25*a^3*c^2*
cos(f*x + e)^3 - 16*a^3*c^2*cos(f*x + e)^2 + 10*a^3*c^2*cos(f*x + e) + 8*a
^3*c^2)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= a^3c^2\left(\int \sec{(e+fx)}\,dx + \int \sec^2{(e+fx)}\,dx + \int \left(-2\sec^3{(e+fx)}\right)\,dx + \int \left(-2\sec^4{(e+fx)}\right)\,dx + \int \sec^5{(e+fx)}\,dx + \int \sec^6{(e+fx)}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)
```

output
```
a**3*c**2*(Integral(sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + Inte
gral(-2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(s
ec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(86) = 172$.

Time = 0.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.41

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$

$$= \frac{16\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^3c^2-160\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^2
 - 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 - 15*a^3*c^2*(2*(3*sin(f*
x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log
(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 120*a^3*c^2*(2*sin(f*x + e
)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) +
240*a^3*c^2*log(sec(f*x + e) + tan(f*x + e)) + 240*a^3*c^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.69

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$

$$= \frac{15\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-70\,a^3c^2\,\mathrm{t}}{}}{40\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/40*(15*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c^2*log(abs(t
an(1/2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c^2*tan(1/2*f*x + 1/2*e)^9 - 70*a^3*
c^2*tan(1/2*f*x + 1/2*e)^7 + 128*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 70*a^3*c
^2*tan(1/2*f*x + 1/2*e)^3 - 15*a^3*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x
+ 1/2*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 15.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \frac{3\,a^3\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f} - \frac{\frac{3\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}-\frac{7\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{2}+\frac{32\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}+\frac{7\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{2}-\frac{3\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

input
```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

output
```
(3*a^3*c^2*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((7*a^3*c^2*tan(e/2 + (f*x)/
2)^3)/2 + (32*a^3*c^2*tan(e/2 + (f*x)/2)^5)/5 - (7*a^3*c^2*tan(e/2 + (f*x)
/2)^7)/2 + (3*a^3*c^2*tan(e/2 + (f*x)/2)^9)/4 - (3*a^3*c^2*tan(e/2 + (f*x)
/2))/4)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2
+ (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.49

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$

$$= \frac{a^3c^2\left(-15\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^4+30\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(\right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x)
```

output
```
(a**3*c**2*( - 15*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 +
 30*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 - 15*cos(e + f*
x)*log(tan((e + f*x)/2) - 1) + 15*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*s
in(e + f*x)**4 - 30*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2
 + 15*cos(e + f*x)*log(tan((e + f*x)/2) + 1) + 25*cos(e + f*x)*sin(e + f*x
)**3 - 15*cos(e + f*x)*sin(e + f*x) + 8*sin(e + f*x)**5))/(40*cos(e + f*x)
*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.26 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx\\ =\frac{5a^3c\text{arctanh}(\sin(e+fx))}{8f}-\frac{3a^3c\sec(e+fx)\tan(e+fx)}{8f}\\ -\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{4f}-\frac{2a^3c\tan^3(e+fx)}{3f}\end{aligned}$$

output
```
5/8*a^3*c*arctanh(sin(f*x+e))/f-3/8*a^3*c*sec(f*x+e)*tan(f*x+e)/f-1/4*a^3*
c*sec(f*x+e)^3*tan(f*x+e)/f-2/3*a^3*c*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx\\ &=\frac{a^3c(60\text{arctanh}(\sin(e+fx))-\sec^4(e+fx)(33\sin(e+fx)+16\sin(2(e+fx))+9\sin(3(e+fx))-8\,\mathrm{si}}{96f}\end{aligned}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

output

```
(a^3*c*(60*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^4*(33*Sin[e + f*x] + 16*Si
n[2*(e + f*x)] + 9*Sin[3*(e + f*x)] - 8*Sin[4*(e + f*x)])))/(96*f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4446}$$

$$-ac\int \left(a^2\tan^2(e+fx)\sec^3(e+fx)+2a^2\tan^2(e+fx)\sec^2(e+fx)+a^2\tan^2(e+fx)\sec(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ac\Bigg(-\frac{5a^2\text{arctanh}(\sin(e+fx))}{8f}+\frac{2a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)\sec^3(e+fx)}{4f}+\frac{3a^2\tan(e+fx)\sec(e+f}{8f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

output

```
-(a*c*((-5*a^2*ArcTanh[Sin[e + f*x]])/(8*f) + (3*a^2*Sec[e + f*x]*Tan[e +
f*x])/(8*f) + (a^2*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (2*a^2*Tan[e + f*x
]^3)/(3*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^3 c \ln(\sec(fx+e)+\tan(fx+e))+2a^3 c \tan(fx+e)+2a^3 c (-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}) \tan(fx+e)-a^3 c (-(-\frac{\sec(fx+e)^3}{4}-\frac{3 \sec(f}{8}}{f}$
default	$\frac{a^3 c \ln(\sec(fx+e)+\tan(fx+e))+2a^3 c \tan(fx+e)+2a^3 c (-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}) \tan(fx+e)-a^3 c (-(-\frac{\sec(fx+e)^3}{4}-\frac{3 \sec(f}{8}}{f}$
parts	$\frac{a^3 c \ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{2a^3 c \tan(fx+e)}{f}+\frac{2a^3 c (-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}) \tan(fx+e)}{f}-\frac{a^3 c (-(-\frac{\sec(fx+e)^3}{4}}{}$
norman	$\frac{-\frac{5a^3 c \tan(\frac{fx}{2}+\frac{e}{2})}{4f}-\frac{73a^3 c \tan(\frac{fx}{2}+\frac{e}{2})^3}{12f}+\frac{55a^3 c \tan(\frac{fx}{2}+\frac{e}{2})^5}{12f}-\frac{5a^3 c \tan(\frac{fx}{2}+\frac{e}{2})^7}{4f}}{(\tan(\frac{fx}{2}+\frac{e}{2})^2-1)^4}-\frac{5a^3 c \ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{8f}+5a$
risch	$\frac{i a^3 c (9 e^{7i(fx+e)}+48 e^{6i(fx+e)}+33 e^{5i(fx+e)}+48 e^{4i(fx+e)}-33 e^{3i(fx+e)}+16 e^{2i(fx+e)}-9 e^{i(fx+e)}+16)}{12f (e^{2i(fx+e)}+1)^4}-5a^3 c \ln(e$
parallelrisch	$-\frac{3((\frac{10 \cos(2fx+2e)}{3}+\frac{5 \cos(4fx+4e)}{6}+\frac{5}{2}) \ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)+(-\frac{10 \cos(2fx+2e)}{3}-\frac{5 \cos(4fx+4e)}{6}-\frac{5}{2}) \ln(\tan(\frac{fx}{2}+\frac{e}{2}}{4f(\cos(4fx+4e)+4\cos(2fx+2e)+3)}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
1/f*(a^3*c*ln(sec(f*x+e)+tan(f*x+e))+2*a^3*c*tan(f*x+e)+2*a^3*c*(-2/3-1/3*
sec(f*x+e)^2)*tan(f*x+e)-a^3*c*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*
x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$= \frac{15\,a^3c\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right) - 15\,a^3c\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right) + 2\,(16\,a^3c\cos}{48\,f\cos\left(fx+e\right)^4}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
1/48*(15*a^3*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x + e
)^4*log(-sin(f*x + e) + 1) + 2*(16*a^3*c*cos(f*x + e)^3 - 9*a^3*c*cos(f*x
+ e)^2 - 16*a^3*c*cos(f*x + e) - 6*a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$= -a^3c\left(\int\left(-\sec\left(e+fx\right)\right)\,dx + \int\left(-2\sec^2\left(e+fx\right)\right)\,dx + \int 2\sec^4\left(e+fx\right)dx + \int\sec^5\left(e+fx\right)dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

output
```
-a**3*c*(Integral(-sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + In
tegral(2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.55

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx =$$

$$-\frac{32\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^3c-3a^3c\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+\right.}{48f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 3*a^3*c*(2*(3*sin(f*x
+ e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(s
in(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 48*a^3*c*log(sec(f*x + e) +
tan(f*x + e)) - 96*a^3*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$

$$=\frac{15a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-15a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(15a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-55a^3c\tan\left(\frac{1}{2}\right.\right.}{\left(\right.}}{24f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
1/24*(15*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c*log(abs(tan(1
/2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c*tan(1/2*f*x + 1/2*e)^7 - 55*a^3*c*tan(
1/2*f*x + 1/2*e)^5 + 73*a^3*c*tan(1/2*f*x + 1/2*e)^3 + 15*a^3*c*tan(1/2*f*
x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.70

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{5\,a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$
$$-\frac{\frac{5\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{55\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{73\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{5\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

input
```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

output
```
(5*a^3*c*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((5*a^3*c*tan(e/2 + (f*x)/2))/
4 + (73*a^3*c*tan(e/2 + (f*x)/2)^3)/12 - (55*a^3*c*tan(e/2 + (f*x)/2)^5)/1
2 + (5*a^3*c*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e
/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.12

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{a^3c\left(-16\cos\left(fx+e\right)\sin\left(fx+e\right)^3-15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^4+30\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x)
```

output
```
(a**3*c*( - 16*cos(e + f*x)*sin(e + f*x)**3 - 15*log(tan((e + f*x)/2) - 1)
*sin(e + f*x)**4 + 30*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 - 15*log(t
an((e + f*x)/2) - 1) + 15*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4 - 30*l
og(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 + 15*log(tan((e + f*x)/2) + 1) +
9*sin(e + f*x)**3 - 15*sin(e + f*x)))/(24*f*(sin(e + f*x)**4 - 2*sin(e + f
*x)**2 + 1))
```

3.27 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{15a^3\text{arctanh}(\sin(e+fx))}{2cf} - \frac{10a^3\tan(e+fx)}{cf} - \frac{5a^3\sec(e+fx)\tan(e+fx)}{2cf} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))}$$

output

```
-15/2*a^3*arctanh(sin(f*x+e))/c/f-10*a^3*tan(f*x+e)/c/f-5/2*a^3*sec(f*x+e)
*tan(f*x+e)/c/f-2*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.65

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{7}{2},\frac{9}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^3\tan(e+fx)}{7cf\sqrt{2-2\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
```

output

```
-1/7*(a^3*Hypergeometric2F1[3/2, 7/2, 9/2, (1 + Sec[e + f*x])/2]*(1 + Sec[
e + f*x])^3*Tan[e + f*x])/(c*f*Sqrt[2 - 2*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4445, 3042, 4275, 3042, 4254, 24, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{5a\int \sec(e+fx)(\sec(e+fx)a+a)^2dx}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2dx}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{4275}$$

$$-\frac{5a\left(2a^2\int \sec^2(e+fx)dx+\int \sec(e+fx)\left(\sec^2(e+fx)a^2+a^2\right)dx\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a\left(2a^2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx+\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{5a\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\int 1d(-\tan(e+fx))}{f}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{24}$$

$$-\frac{5a\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\tan(e+fx)}{f}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{4534}$$

$$-\frac{5a\left(\frac{3}{2}a^2\int \sec(e+fx)dx+\frac{2a^2\tan(e+fx)}{f}+\frac{a^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a\left(\frac{3}{2}a^2\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{2a^2\tan(e+fx)}{f}+\frac{a^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{5a\left(\frac{3a^2\text{arctanh}(\sin(e+fx))}{2f}+\frac{2a^2\tan(e+fx)}{f}+\frac{a^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
```

output

```
(-2*a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])) - (5*a
*((3*a^2*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a^2*Tan[e + f*x])/f + (a^2*Sec[
e + f*x]*Tan[e + f*x])/(2*f)))/c
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4445

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

rule 4534

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

method	result
parallelrisch	$\frac{15a^3\left(\left(1+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-1-\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\frac{14\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)-\right.}{15}\right.}{2cf\left(1+\cos(2fx+2e)\right)}$
derivativedivides	$\frac{8a^3\left(\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{15}{}\right.}{fc}$
default	$\frac{8a^3\left(\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{15}{}\right.}{fc}$
risch	$\frac{ia^3\left(17\,\mathrm{e}^{4i(fx+e)}-9\,\mathrm{e}^{3i(fx+e)}+39\,\mathrm{e}^{2i(fx+e)}-7\,\mathrm{e}^{i(fx+e)}+24\right)}{fc\left(\mathrm{e}^{i(fx+e)}-1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}-\frac{15a^3\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{2cf}+\frac{15a^3\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{2cf}$
norman	$\frac{-\frac{8a^3}{cf}+\frac{33a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}-\frac{40a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}+\frac{15a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{15a^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2cf}-\frac{15a^3\ln\left(\tan\left(\frac{fx}{2}\right.\right.}{2cf}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output

```
15/2*a^3*((1+cos(2*f*x+2*e))*ln(tan(1/2*f*x+1/2*e)-1)+(-1-cos(2*f*x+2*e))*
ln(tan(1/2*f*x+1/2*e)+1)-14/15*cot(1/2*f*x+1/2*e)*(cos(f*x+e)-12/7*cos(2*f
*x+2*e)-11/7))/c/f/(1+cos(2*f*x+2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx =$$

$$-\frac{15\,a^3\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-15\,a^3\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)\sin}{4\,cf\cos\left(fx+e\right)^2\sin\left(fx\right.}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
-1/4*(15*a^3*cos(f*x + e)^2*log(sin(f*x + e) + 1)*sin(f*x + e) - 15*a^3*co
s(f*x + e)^2*log(-sin(f*x + e) + 1)*sin(f*x + e) - 48*a^3*cos(f*x + e)^3 -
 34*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 2*a^3)/(c*f*cos(f*x + e)^2*
sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$$

$$= -\frac{a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{3\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

output

```
-a**3*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f
*x)**2/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x) -
 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) - 1), x))/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(97) = 194$.

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.87

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\left(\frac{2\left(\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+6a^3\left(\frac{\frac{3\sin(fx+}{(\cos(fx+e)}}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-}\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-1/2*(a^3*(2*(5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 1)/(c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*c*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*log
(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - 3*log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/c) + 6*a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(
f*x + e)/(cos(f*x + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log
(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) +
 1) - 1)/c) + 6*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e)))
- 2*a^3*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{\frac{15a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{c}-\frac{15a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{c}-\frac{16a^3}{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}-\frac{2\left(7a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-9a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2c}}{2f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
-1/2*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - 15*a^3*log(abs(tan(1/2
*f*x + 1/2*e) - 1))/c - 16*a^3/(c*tan(1/2*f*x + 1/2*e)) - 2*(7*a^3*tan(1/2
*f*x + 1/2*e)^3 - 9*a^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1
)^2*c))/f
```

Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{15\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-25\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+8\,a^3}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)} - \frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

output
```
(15*a^3*tan(e/2 + (f*x)/2)^4 - 25*a^3*tan(e/2 + (f*x)/2)^2 + 8*a^3)/(f*(c*
tan(e/2 + (f*x)/2) - 2*c*tan(e/2 + (f*x)/2)^3 + c*tan(e/2 + (f*x)/2)^5)) -
 (15*a^3*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.24

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$$

$$= a^3\Big(15\log\Big(\tan\Big(\frac{fx}{2}+\frac{e}{2}\Big)-1\Big)\tan\Big(\frac{fx}{2}+\frac{e}{2}\Big)^5-30\log\Big(\tan\Big(\frac{fx}{2}+\frac{e}{2}\Big)-1\Big)\tan\Big(\frac{fx}{2}+\frac{e}{2}\Big)^3+15\log\Big(\tan\Big(\frac{fx}{2}+$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

output

```
(a**3*(15*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**5 - 30*log(tan((e +
f*x)/2) - 1)*tan((e + f*x)/2)**3 + 15*log(tan((e + f*x)/2) - 1)*tan((e + f
*x)/2) - 15*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**5 + 30*log(tan((e
+ f*x)/2) + 1)*tan((e + f*x)/2)**3 - 15*log(tan((e + f*x)/2) + 1)*tan((e +
 f*x)/2) + 30*tan((e + f*x)/2)**4 - 50*tan((e + f*x)/2)**2 + 16))/(2*tan((
e + f*x)/2)*c*f*(tan((e + f*x)/2)**4 - 2*tan((e + f*x)/2)**2 + 1))
```

3.28 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 119

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{5a^3\text{arctanh}(\sin(e+fx))}{c^2f} + \frac{5a^3\tan(e+fx)}{c^2f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\left(c^2-c^2\sec(e+fx)\right)}$$

output
```
5*a^3*arctanh(sin(f*x+e))/c^2/f+5*a^3*tan(f*x+e)/c^2/f-2/3*a*(a+a*sec(f*x+
e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^2+10/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)
/f/(c^2-c^2*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.55

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{a^3\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{7}{2},\frac{9}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^3\tan(e+fx)}{14c^2f\sqrt{2-2\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
```

output
```
-1/14*(a^3*Hypergeometric2F1[5/2, 7/2, 9/2, (1 + Sec[e + f*x])/2]*(1 + Sec
[e + f*x])^3*Tan[e + f*x])/(c^2*f*Sqrt[2 - 2*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4445, 3042, 4445, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{5a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{c-c\sec(e+fx)}dx}{3c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{4445}$$

$$-\frac{5a\left(-\frac{3a\int \sec(e+fx)(\sec(e+fx)a+a)dx}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}\right)}{3c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a\left(-\frac{3a\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)dx}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}\right)}{3c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{5a\left(-\frac{3a\left(a\int \sec^2(e+fx)dx+a\int \sec(e+fx)dx\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}\right)}{3c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a\left(-\frac{3a\left(a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+a\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}\right)}{3c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{5a\left(-\frac{3a\left(a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{a\int 1d(-\tan(e+fx))}{f}\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}\right)}{3c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{5a\left(-\frac{3a\left(a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{a\tan(e+fx)}{f}\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}\right)}{3c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{5a\left(-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}-\frac{3a\left(\frac{a\text{arctanh}(\sin(e+fx))}{f}+\frac{a\tan(e+fx)}{f}\right)}{c}\right)}{3c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
```

output

```
(-2*a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2) -
(5*a*((-2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x]))
- (3*a*((a*ArcTanh[Sin[e + f*x]])/f + (a*Tan[e + f*x])/f))/c))/(3*c)
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4445

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{4a^3\left(-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{2}{\tan\left(\frac{fx}{2}+\right.}\right.}{f\,c^2}$
default	$\frac{4a^3\left(-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{2}{\tan\left(\frac{fx}{2}+\right.}\right.}{f\,c^2}$
parallelrisch	$-\frac{5a^3\left(\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+\frac{17\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\cos(fx+e)-\frac{23}{}\right.}{15}\right.}{\cos(fx+e)c^2f}$
risch	$-\frac{2ia^3\left(12\,\mathrm{e}^{4i(fx+e)}-51\,\mathrm{e}^{3i(fx+e)}+41\,\mathrm{e}^{2i(fx+e)}-57\,\mathrm{e}^{i(fx+e)}+23\right)}{3f\,c^2\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)^3}-\frac{5a^3\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{c^2f}+\frac{5a^3\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{c^2f}$
norman	$\frac{\frac{4a^3}{3cf}+\frac{4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}-\frac{22a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}+\frac{80a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3cf}-\frac{10a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{5a^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{c^2f}+\frac{5}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
4/f*a^3/c^2*(-1/4/(tan(1/2*f*x+1/2*e)+1)+5/4*ln(tan(1/2*f*x+1/2*e)+1)-1/4/
(tan(1/2*f*x+1/2*e)-1)-5/4*ln(tan(1/2*f*x+1/2*e)-1)-1/3/tan(1/2*f*x+1/2*e)
^3-2/tan(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{46\,a^3\cos\left(fx+e\right)^3-22\,a^3\cos\left(fx+e\right)^2-62\,a^3\cos\left(fx+e\right)+6\,a^3-15\left(a^3\cos\left(fx+e\right)^2-a^3\cos\left(f\right.\right.}{6\left(c^2f\cos\left(fx+\right.\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
-1/6*(46*a^3*cos(f*x + e)^3 - 22*a^3*cos(f*x + e)^2 - 62*a^3*cos(f*x + e)
+ 6*a^3 - 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(sin(f*x + e) + 1)
*sin(f*x + e) + 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(-sin(f*x +
e) + 1)*sin(f*x + e))/((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x
 + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$$
$$=\frac{a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^{2}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^{3}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec}{\sec^{2}\left(e+fx\right)}\right.}{c^2}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

output
```
a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + I
ntegral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Int
egral(3*sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**4/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(117) = 234.

Time = 0.04 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.93

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx =$$
$$-\frac{a^3\left(\frac{\frac{14\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{27\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+1}{\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2}\right)-3a^3\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2}\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
-1/6*(a^3*((14*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 27*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 1)/(c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - c^2*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 12*log(sin(f*x + e)/(cos(f*x + e) + 1)
+ 1)/c^2 + 12*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^2) - 3*a^3*(6*log
(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(cos(f*x +
e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x +
e) + 1)^3/(c^2*sin(f*x + e)^3)) + 3*a^3*(3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) - a^3*(3*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^2} - \frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^2} - \frac{6\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)c^2} - \frac{4\left(6\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^3\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/3*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 15*a^3*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/c^2 - 6*a^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*
e)^2 - 1)*c^2) - 4*(6*a^3*tan(1/2*f*x + 1/2*e)^2 + a^3)/(c^2*tan(1/2*f*x +
 1/2*e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 11.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{10\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{c^2\,f} + \frac{-10\,a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+\frac{20\,a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{3}+\frac{4\,a^3}{3}}{c^2\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

output
```
(10*a^3*atanh(tan(e/2 + (f*x)/2)))/(c^2*f) + ((20*a^3*tan(e/2 + (f*x)/2)^2
)/3 - 10*a^3*tan(e/2 + (f*x)/2)^4 + (4*a^3)/3)/(c^2*f*tan(e/2 + (f*x)/2)^3
*(tan(e/2 + (f*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$$
$$=\frac{a^3\left(-15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+15\log\left(\tan\left(\frac{fx}{2}\right.\right.\right.}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

output
```
(a**3*( - 15*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**5 + 15*log(tan((e
 + f*x)/2) - 1)*tan((e + f*x)/2)**3 + 15*log(tan((e + f*x)/2) + 1)*tan((e
+ f*x)/2)**5 - 15*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**3 - 30*tan((
e + f*x)/2)**4 + 20*tan((e + f*x)/2)**2 + 4))/(3*tan((e + f*x)/2)**3*c**2*
f*(tan((e + f*x)/2)**2 - 1))
```

3.29 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 132

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = &-\frac{a^3\text{arctanh}(\sin(e+fx))}{c^3f}\\&-\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3}\\&+\frac{2(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf(c-c\sec(e+fx))^2}\\&-\frac{2a^3\tan(e+fx)}{f\,(c^3-c^3\sec(e+fx))}\end{aligned}$$

output
```
-a^3*arctanh(sin(f*x+e))/c^3/f-2/5*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*
sec(f*x+e))^3+2/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^2-2
*a^3*tan(f*x+e)/f/(c^3-c^3*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx =$$
$$-\frac{a^3\left(-\frac{26\cot\left(\frac{1}{2}(e+fx)\right)}{15f}+\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{15f}-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)}{5f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}}{c^3}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

output
```
-((a^3*((-26*Cot[(e + f*x)/2])/(15*f) + (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/
2]^2)/(15*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4)/(5*f) - Log[Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
/f))/c^3)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4445, 3042, 4445, 3042, 4445, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{4445}$$
$$-\frac{a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{(c-c\sec(e+fx))^2}dx}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

$$\Big\downarrow \ 4445$$

$$-\frac{a\left(-\frac{a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{c-c\sec(e+fx)}dx}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\left(-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

$$\Big\downarrow \ 4445$$

$$-\frac{a\left(-\frac{a\left(-\frac{a\int \sec(e+fx)dx}{c}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\left(-\frac{a\left(-\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{c}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}\right)}{c}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

$$\Big\downarrow \ 4257$$

$$-\frac{a\left(-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}-\frac{a\left(-\frac{a\,\text{arctanh}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}\right)}{c}\right)}{c}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(-2*a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^3) -
(a*((-2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2
) - (a*(-((a*ArcTanh[Sin[e + f*x]])/(c*f)) - (2*a*Tan[e + f*x])/(f*(c - c*
Sec[e + f*x]))))/c))/c
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{2a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^3}$
default	$\frac{2a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^3}$
parallelrisch	$\frac{a^3\left(6\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+10\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+30\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15c^3f}$
risch	$\frac{4ia^3\left(15\,\mathrm{e}^{4i(fx+e)}-30\,\mathrm{e}^{3i(fx+e)}+100\,\mathrm{e}^{2i(fx+e)}-50\,\mathrm{e}^{i(fx+e)}+13\right)}{15f\,c^3\left(\mathrm{e}^{i(fx+e)}-1\right)^5}+\frac{a^3\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{c^3f}-\frac{a^3\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{c^3f}$
norman	$\frac{-\frac{2a^3}{5cf}+\frac{8a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15cf}-\frac{6a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{5cf}+\frac{22a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{5cf}-\frac{16a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{3cf}+\frac{2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+ a^3\ln\Big(\tan$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
2/f*a^3/c^3*(-1/2*ln(tan(1/2*f*x+1/2*e)+1)+1/2*ln(tan(1/2*f*x+1/2*e)-1)+1/
5/tan(1/2*f*x+1/2*e)^5+1/3/tan(1/2*f*x+1/2*e)^3+1/tan(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.33

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{52\,a^3\cos(fx+e)^3-44\,a^3\cos(fx+e)^2-4\,a^3\cos(fx+e)+92\,a^3-15\,\Big(a^3\cos(fx+e)^2-2\,a^3\cos(fx}{30\,\Big(c^3f\cos(fx+}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/30*(52*a^3*cos(f*x + e)^3 - 44*a^3*cos(f*x + e)^2 - 4*a^3*cos(f*x + e) +
 92*a^3 - 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*log(sin(f*x +
 e) + 1)*sin(f*x + e) + 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)
*log(-sin(f*x + e) + 1)*sin(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos
(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^3\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{3\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{3\,\mathrm{se}}{\sec^3(e+fx)-3\,\mathrm{sec}}\right.}{c^3}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

output
```
-a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(
e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e +
f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f
*x)**4/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**
3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(131) = 262$.

Time = 0.05 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.34

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^3\left(\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^3} - \frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^3} - \frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right) - \frac{3a^3\left(\frac{10\,\mathrm{sin}}{(\cos(f}\right.}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
-1/60*(a^3*(60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^3 - 60*log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/c^3 - (20*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c
^3*sin(f*x + e)^5)) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x
+ e)^5) + a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 9*a^
3*(5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*si
n(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^3}-\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^3}-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^3\right)}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
-1/15*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^3 - 15*a^3*log(abs(tan(
1/2*f*x + 1/2*e) - 1))/c^3 - 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 + 5*a^3*tan(
1/2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 10.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+\frac{2\,a^3}{5}}{c^3\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5} - \frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^3\,f}$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

output
```
((2*a^3*tan(e/2 + (f*x)/2)^2)/3 + 2*a^3*tan(e/2 + (f*x)/2)^4 + (2*a^3)/5)/
(c^3*f*tan(e/2 + (f*x)/2)^5) - (2*a^3*atanh(tan(e/2 + (f*x)/2)))/(c^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{a^3\left(15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\right.}{15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^3f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

output
```
(a**3*(15*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**5 - 15*log(tan((e +
f*x)/2) + 1)*tan((e + f*x)/2)**5 + 30*tan((e + f*x)/2)**4 + 10*tan((e + f*
x)/2)**2 + 6))/(15*tan((e + f*x)/2)**5*c**3*f)
```

3.30 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx=-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{7f(c-c\sec(e+fx))^4}$$

output
```
-1/7*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx=-\frac{a^3\cot^7\left(\frac{1}{2}(e+fx)\right)}{7c^4f}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
```

output
```
-1/7*(a^3*Cot[(e + f*x)/2]^7)/(c^4*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \ 4438$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{7f(c-c\sec(e+fx))^4}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
```

output
```
-1/7*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$-\frac{a^{3}}{7 f \,c^{4} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}$	23
default	$-\frac{a^{3}}{7 f \,c^{4} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}$	23
parallelrisch	$-\frac{a^{3} \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{7 f \,c^{4}}$	23
risch	$\frac{2 i a^{3} \left(7 \,{\mathrm e}^{6 i \left(f x +e \right)}+35 \,{\mathrm e}^{4 i \left(f x +e \right)}+21 \,{\mathrm e}^{2 i \left(f x +e \right)}+1\right)}{7 f \,c^{4} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{7}}$	61
norman	$\frac{\frac{a^{3}}{7 c f}-\frac{3 a^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{7 c f}+\frac{3 a^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{7 c f}-\frac{a^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{7 c f}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3} c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}$	109

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBO
SE)
```

output
```
-1/7/f*a^3/c^4/tan(1/2*f*x+1/2*e)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(37) = 74.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{a^{3} \cos \left(f x + e\right)^{4} + 4 \, a^{3} \cos \left(f x + e\right)^{3} + 6 \, a^{3} \cos \left(f x + e\right)^{2} + 4 \, a^{3} \cos \left(f x + e\right) + a^{3}}{7 \, {\left(c^{4} f \cos \left(f x + e\right)^{3} - 3 \, c^{4} f \cos \left(f x + e\right)^{2} + 3 \, c^{4} f \cos \left(f x + e\right) - c^{4} f\right)} \sin \left(f x + e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="f
ricas")
```

output
```
1/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*
a^3*cos(f*x + e) + a^3)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 +
3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^3\Bigg(\int \frac{\sec(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx+\int \frac{3\sec^2(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,d}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

output
```
a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
 + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e +
 f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)
 + Integral(3*sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec
(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**4/(sec(e +
 f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)
)/c**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(37) = 74$.

Time = 0.05 (sec) , antiderivative size = 356, normalized size of antiderivative = 9.37

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = -\frac{\frac{a^3\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+5\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}-\frac{a^3\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)}{c^4\sin(fx+e)^7}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="m
axima")
```

output

```
-1/280*(a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5)*(cos(f*x
+ e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*s
in(f*x + e)^7) + a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15
)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = -\frac{a^3}{7\,c^4 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="g
iac")
```

output

```
-1/7*a^3/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

Mupad [B] (verification not implemented)

Time = 10.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = -\frac{a^3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{7\,c^4\,f}$$

input

```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

output

```
-(a^3*cot(e/2 + (f*x)/2)^7)/(7*c^4*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx=-\frac{a^3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7c^4f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

output
```
( - a**3)/(7*tan((e + f*x)/2)**7*c**4*f)
```

3.31 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))^3\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}$$

output
```
-1/9*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-1/63*(a+a*sec(f*x+
e))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{a^3(-8+\sec(e+fx))(1+\sec(e+fx))^3\tan(e+fx)}{63c^5f(-1+\sec(e+fx))^5}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

output
```
-1/63*(a^3*(-8 + Sec[e + f*x])*(1 + Sec[e + f*x])^3*Tan[e + f*x])/(c^5*f*(
-1 + Sec[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{3042, 4439, 3042, 4438\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^3}{(c-c\sec(e+fx))^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

output
```
-1/9*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) - ((
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.48

method	result
parallelrisch	$\frac{a^{3} \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{7} \left(7 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-9\right)}{126 c^{5} f}$
derivativedivides	$\frac{a^{3} \left(\frac{1}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{2 f \,c^{5}}$
default	$\frac{a^{3} \left(\frac{1}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{2 f \,c^{5}}$
risch	$\frac{2 i a^{3} (63 \,\mathrm{e}^{8 i (f x+e)}-63 \,\mathrm{e}^{7 i (f x+e)}+483 \,\mathrm{e}^{6 i (f x+e)}-315 \,\mathrm{e}^{5 i (f x+e)}+693 \,\mathrm{e}^{4 i (f x+e)}-189 \,\mathrm{e}^{3 i (f x+e)}+225 \,\mathrm{e}^{2 i (f x+e)}-9 \,\mathrm{e}^{i (f x+}}{63 f \,c^{5} \left(\mathrm{e}^{i (f x+e)}-1\right)^{9}}$
norman	$\frac{-\frac{a^{3}}{18 c f}+\frac{5 a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{21 c f}-\frac{8 a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{21 c f}+\frac{17 a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{63 c f}-\frac{a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{8}}{14 c f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3} c^{4} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBO
SE)
```

output

```
1/126*a^3*cot(1/2*f*x+1/2*e)^7*(7*cot(1/2*f*x+1/2*e)^2-9)/c^5/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.75

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{8\,a^3\cos\left(fx+e\right)^5+31\,a^3\cos\left(fx+e\right)^4+44\,a^3\cos\left(fx+e\right)^3+26\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)-}{63\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right.}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="f
ricas")
```

output

```
1/63*(8*a^3*cos(f*x + e)^5 + 31*a^3*cos(f*x + e)^4 + 44*a^3*cos(f*x + e)^3
 + 26*a^3*cos(f*x + e)^2 + 4*a^3*cos(f*x + e) - a^3)/((c^5*f*cos(f*x + e)^
4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e)
 + c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int \frac{3\sec^2\left(e+\right.}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+f\right.}\right.}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

output
```
-a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec
(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*s
ec(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 -
 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3
/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*
x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**
5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e
+ f*x) - 1), x))/c**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(78) = 156$.

Time = 0.05 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{a^3\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{15\,a^3\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\right.}{c^5\sin(fx}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="m
axima")
```

output
```
-1/5040*(a^3*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x +
e)^9) + 15*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f
*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^3*(18*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 63*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8 + 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) + 21*a^3*
(18*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{9\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-7\,a^3}{126\,c^5f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="g
iac")
```

output
```
-1/126*(9*a^3*tan(1/2*f*x + 1/2*e)^2 - 7*a^3)/(c^5*f*tan(1/2*f*x + 1/2*e)^
9)
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\left(7\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-9\right)}{126\,c^5\,f}$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

output
```
(a^3*cot(e/2 + (f*x)/2)^7*(7*cot(e/2 + (f*x)/2)^2 - 9))/(126*c^5*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3\left(-9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+7\right)}{126\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9c^5f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

output
```
(a**3*( - 9*tan((e + f*x)/2)**2 + 7))/(126*tan((e + f*x)/2)**9*c**5*f)
```

3.32 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{99cf(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{693c^2f(c-c\sec(e+fx))^4}$$

output

```
-1/11*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^6-2/99*(a+a*sec(f*x
+e))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^5-2/693*(a+a*sec(f*x+e))^3*tan(f*x+
e)/c^2/f/(c-c*sec(f*x+e))^4
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.49

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = -\frac{a^3(1+\sec(e+fx))^3\left(79-18\sec(e+fx)+2\sec^2(e+fx)\right)\tan(e+fx)}{693c^6f(-1+\sec(e+fx))^6}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

output
```
-1/693*(a^3*(1 + Sec[e + f*x])^3*(79 - 18*Sec[e + f*x] + 2*Sec[e + f*x]^2)
*Tan[e + f*x])/(c^6*f*(-1 + Sec[e + f*x])^6)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4439, 3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{2\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^3}{(c-c\sec(e+fx))^5}dx}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{4439}$$

$$\frac{2\left(\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^3}{(c-c\sec(e+fx))^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

$$\Big\downarrow \text{4438}$$

$$\frac{2\left(-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

output
```
-1/11*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^6) + (
2*(-1/9*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) -
 ((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4)))/(
11*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

method	result
parallelrisch	$-\frac{a^{3} \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{7} \left(63 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-154 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+99\right)}{2772 c^{6} f}$
derivativedivides	$\frac{a^{3} \left(-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}+\frac{2}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{11 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{11}}\right)}{4 f\, c^{6}}$
default	$\frac{a^{3} \left(-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}+\frac{2}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{11 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{11}}\right)}{4 f\, c^{6}}$
risch	$\frac{2 i a^{3} (693\, \mathrm{e}^{10 i (f x+e)}-1386\, \mathrm{e}^{9 i (f x+e)}+8085\, \mathrm{e}^{8 i (f x+e)}-10626\, \mathrm{e}^{7 i (f x+e)}+21252\, \mathrm{e}^{6 i (f x+e)}-15246\, \mathrm{e}^{5 i (f x+e)}+15444\, \mathrm{e}^{4 i (}}{693 f\, c^{6} \left(\mathrm{e}^{i (f x+e)}-1\right)^{11}}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBO
SE)
```

output
```
-1/2772*a^3*cot(1/2*f*x+1/2*e)^7*(63*cot(1/2*f*x+1/2*e)^4-154*cot(1/2*f*x+
1/2*e)^2+99)/c^6/f
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$$

$$= \frac{79\,a^3\cos\left(fx+e\right)^6+298\,a^3\cos\left(fx+e\right)^5+404\,a^3\cos\left(fx+e\right)^4+216\,a^3\cos\left(fx+e\right)^3+19\,a^3\cos\left(fx+\right.}{693\,\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-10\,c^6f\cos\left(fx+e\right)^2+5\,c^6f\cos\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="f
ricas")
```

output
```
1/693*(79*a^3*cos(f*x + e)^6 + 298*a^3*cos(f*x + e)^5 + 404*a^3*cos(f*x +
e)^4 + 216*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 10*a^3*cos(f*x + e
) + 2*a^3)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(
f*x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f
*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$$

$$= \frac{a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+15\sec^4\left(e+fx\right)-20\sec^3\left(e+fx\right)+15\sec^2\left(e+fx\right)-6\sec\left(e+fx\right)+1}\,dx+\int \frac{}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+}\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

output

```
a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(
e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1
), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 +
15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f
*x) + 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x)**6 - 6*sec(e + f*x
)**5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*se
c(e + f*x) + 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**6 - 6*sec(e
+ f*x)**5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 -
 6*sec(e + f*x) + 1), x))/c**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(118) = 236$.

Time = 0.06 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.28

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="m
axima")
```

output

```
1/110880*(3*a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 9*a^3*(385*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 - 462*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x
+ e) + 1)^11/(c^6*sin(f*x + e)^11) + 5*a^3*(385*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(
f*x + e)^10/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x
 + e)^11) - a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 990*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10 + 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11))/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$$
$$=-\frac{99\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-154\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+63\,a^3}{2772\,c^6 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="g
iac")
```

output
```
-1/2772*(99*a^3*tan(1/2*f*x + 1/2*e)^4 - 154*a^3*tan(1/2*f*x + 1/2*e)^2 +
63*a^3)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx=\frac{a^3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{18\,c^6\,f}-\frac{a^3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{28\,c^6\,f}$$
$$-\frac{a^3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{44\,c^6\,f}$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^6),x)
```

output
```
(a^3*cot(e/2 + (f*x)/2)^9)/(18*c^6*f) - (a^3*cot(e/2 + (f*x)/2)^7)/(28*c^6
*f) - (a^3*cot(e/2 + (f*x)/2)^11)/(44*c^6*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.41

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = \frac{a^3\left(-99\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+154\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-63\right)}{2772\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}c^6f}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x)
```

output

```
(a**3*( - 99*tan((e + f*x)/2)**4 + 154*tan((e + f*x)/2)**2 - 63))/(2772*ta
n((e + f*x)/2)**11*c**6*f)
```

3.33 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 162

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = &-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7}\\ &-\frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6}\\ &-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{429c^2f(c-c\sec(e+fx))^5}\\ &-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{3003c^3f(c-c\sec(e+fx))^4}\end{aligned}$$

output
```
-1/13*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^7-3/143*(a+a*sec(f*
x+e))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^6-2/429*(a+a*sec(f*x+e))^3*tan(f*x
+e)/c^2/f/(c-c*sec(f*x+e))^5-2/3003*(a+a*sec(f*x+e))^3*tan(f*x+e)/c^3/f/(c
-c*sec(f*x+e))^4
```

Mathematica [A] (verified)

Time = 4.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{a^3(1+\sec(e+fx))^3\left(-310+97\sec(e+fx)-20\sec^2(e+fx)+2\sec^3(e+fx)\right)\tan(e+fx)}{3003c^7f(-1+\sec(e+fx))^7}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

output

```
-1/3003*(a^3*(1 + Sec[e + f*x])^3*(-310 + 97*Sec[e + f*x] - 20*Sec[e + f*x
]^2 + 2*Sec[e + f*x]^3)*Tan[e + f*x])/(c^7*f*(-1 + Sec[e + f*x])^7)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4439, 3042, 4439, 3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^7}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^7}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{3\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^3}{(c-c\sec(e+fx))^6}dx}{13c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{13f(c-c\sec(e+fx))^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}dx}{13c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{13f(c-c\sec(e+fx))^7}$$

$$\Big\downarrow 4439$$

$$\frac{3\left(\frac{2\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^3}{(c-c\sec(e+fx))^5}dx}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}\right)}{13c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{13f(c-c\sec(e+fx))^7}$$

$$\Big\downarrow 3042$$

$$\frac{3\left(\frac{2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}\right)}{13c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{13f(c-c\sec(e+fx))^7}$$

$$\Big\downarrow 4439$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^3}{(c-c\sec(e+fx))^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}\right)}{13c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{13f(c-c\sec(e+fx))^7}$$

$$\Big\downarrow 3042$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx}{9c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}\right)}{13c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{13f(c-c\sec(e+fx))^7}$$

$$\Big\downarrow 4438$$

$$\frac{3\left(\frac{2\left(-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}\right)}{11c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}\right)}{13c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{13f(c-c\sec(e+fx))^7}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

output

```
-1/13*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^7) + (
3*(-1/11*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^6)
+ (2*(-1/9*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5
) - ((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4))
)/(11*c)))/(13*c)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

method	result
parallelrisch	$\frac{a^{3} \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{7} \left(231 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-819 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+1001 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-429\right)}{24024 c^{7} f}$
derivativedivides	$\frac{a^{3} \left(\frac{1}{13 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{13}}-\frac{3}{11 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{8 f\, c^{7}}$
default	$\frac{a^{3} \left(\frac{1}{13 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{13}}-\frac{3}{11 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{8 f\, c^{7}}$
risch	$\frac{2 i a^{3} \left(3003\, {\mathrm e}^{12 i (f x+e)}-9009\, {\mathrm e}^{11 i (f x+e)}+51051\, {\mathrm e}^{10 i (f x+e)}-99099\, {\mathrm e}^{9 i (f x+e)}+216216\, {\mathrm e}^{8 i (f x+e)}-246246\, {\mathrm e}^{7 i (f x+e)}+285}{3003 f\, c^{7} \left({\mathrm e}^{i (f x+}\right.}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x,method=_RETURNVERBO
SE)
```

output
```
1/24024*a^3*cot(1/2*f*x+1/2*e)^7*(231*cot(1/2*f*x+1/2*e)^6-819*cot(1/2*f*x
+1/2*e)^4+1001*cot(1/2*f*x+1/2*e)^2-429)/c^7/f
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.20

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx$$
$$=\frac{310\,a^{3}\cos\left(fx+e\right)^{7}+1143\,a^{3}\cos\left(fx+e\right)^{6}+1492\,a^{3}\cos\left(fx+e\right)^{5}+736\,a^{3}\cos\left(fx+e\right)^{4}+34\,a^{3}\cos\left(f\right.}{3003\left(c^{7}f\cos\left(fx+e\right)^{6}-6\,c^{7}f\cos\left(fx+e\right)^{5}+15\,c^{7}f\cos\left(fx+e\right)^{4}-20\,c^{7}f\cos\left(fx+e\right)^{3}+15\,c^{7}f\,c\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="f
ricas")
```

output
```
1/3003*(310*a^3*cos(f*x + e)^7 + 1143*a^3*cos(f*x + e)^6 + 1492*a^3*cos(f*
x + e)^5 + 736*a^3*cos(f*x + e)^4 + 34*a^3*cos(f*x + e)^3 - 29*a^3*cos(f*x
 + e)^2 + 12*a^3*cos(f*x + e) - 2*a^3)/((c^7*f*cos(f*x + e)^6 - 6*c^7*f*co
s(f*x + e)^5 + 15*c^7*f*cos(f*x + e)^4 - 20*c^7*f*cos(f*x + e)^3 + 15*c^7*
f*cos(f*x + e)^2 - 6*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx =$$

$$-\frac{a^3\left(\int \frac{\sec(e+fx)}{\sec^7(e+fx)-7\sec^6(e+fx)+21\sec^5(e+fx)-35\sec^4(e+fx)+35\sec^3(e+fx)-21\sec^2(e+fx)+7\sec(e+fx)-1}\,dx+\int \frac{}{\sec^7(e+}\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**7,x)
```

output
```
-a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec
(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**
2 + 7*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**7
- 7*sec(e + f*x)**6 + 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e +
 f*x)**3 - 21*sec(e + f*x)**2 + 7*sec(e + f*x) - 1), x) + Integral(3*sec(e
 + f*x)**3/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec(e + f*x)**5 - 35*
sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2 + 7*sec(e + f*x)
 - 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**7 - 7*sec(e + f*x)**6
+ 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e
+ f*x)**2 + 7*sec(e + f*x) - 1), x))/c**7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(158) = 316$.

Time = 0.06 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.19

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="m
axima")
```

output
```
-1/960960*(a^3*(8190*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5005*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 25740*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 90
09*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30030*sin(f*x + e)^10/(cos(f*x +
e) + 1)^10 - 45045*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 3465)*(cos(f*x
+ e) + 1)^13/(c^7*sin(f*x + e)^13) + 5*a^3*(1638*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 5005*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 8580*sin(f*x + e)^
6/(cos(f*x + e) + 1)^6 - 9009*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 6006*s
in(f*x + e)^10/(cos(f*x + e) + 1)^10 - 3003*sin(f*x + e)^12/(cos(f*x + e)
+ 1)^12 - 231)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13) + 35*a^3*(468*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 715*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 1287*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 1716*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 1287*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 99)*(cos(f
*x + e) + 1)^13/(c^7*sin(f*x + e)^13) + 77*a^3*(65*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 117*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^
12/(cos(f*x + e) + 1)^12 - 15)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13)
)/f
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{429\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 1001\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 819\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 231\,a^3}{24024\,c^7 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="g
iac")
```

output
```
-1/24024*(429*a^3*tan(1/2*f*x + 1/2*e)^6 - 1001*a^3*tan(1/2*f*x + 1/2*e)^4
 + 819*a^3*tan(1/2*f*x + 1/2*e)^2 - 231*a^3)/(c^7*f*tan(1/2*f*x + 1/2*e)^1
3)
```

Mupad [B] (verification not implemented)

Time = 11.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = \frac{a^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\left(231\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-819\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1001\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right.\right.}{24024\,c^7\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^7),x)
```

output
```
(a^3*cos(e/2 + (f*x)/2)^7*(231*cos(e/2 + (f*x)/2)^6 - 429*sin(e/2 + (f*x)/
2)^6 + 1001*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 819*cos(e/2 + (f*x
)/2)^4*sin(e/2 + (f*x)/2)^2))/(24024*c^7*f*sin(e/2 + (f*x)/2)^13)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = \frac{a^3\left(-429\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+1001\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-819\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+231\right)}{24024\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}c^7f}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x)
```

output
```
(a**3*( - 429*tan((e + f*x)/2)**6 + 1001*tan((e + f*x)/2)**4 - 819*tan((e
+ f*x)/2)**2 + 231))/(24024*tan((e + f*x)/2)**13*c**7*f)
```

3.34 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = &-\frac{35c^4\text{arctanh}(\sin(e+fx))}{2af}+\frac{28c^4\tan(e+fx)}{af}\\ &-\frac{21c^4\sec(e+fx)\tan(e+fx)}{2af}\\ &+\frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))}\\ &+\frac{7c^4\tan^3(e+fx)}{3af}\end{aligned}$$

output

```
-35/2*c^4*arctanh(sin(f*x+e))/a/f+28*c^4*tan(f*x+e)/a/f-21/2*c^4*sec(f*x+e
)*tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))+7/3*
c^4*tan(f*x+e)^3/a/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.81 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{16c^4\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}}{af}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

output
```
(-16*c^4*Cot[e + f*x]*Hypergeometric2F1[-7/2, -1/2, 1/2, (1 + Sec[e + f*x]
)/2]*Sqrt[2 - 2*Sec[e + f*x]])/(a*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4445, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\int \sec(e+fx)(c-c\sec(e+fx))^3dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx}{a}$$

$$\Big\downarrow \text{4278}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\int \left(-c^3\sec^4(e+fx)+3c^3\sec^3(e+fx)-3c^3\sec^2(e+fx)+c^3\sec(e+fx)\right)dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\left(\frac{5c^3\text{arctanh}(\sin(e+fx))}{2f}-\frac{c^3\tan^3(e+fx)}{3f}-\frac{4c^3\tan(e+fx)}{f}+\frac{3c^3\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

output

```
(2*c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) - (7*c*
((5*c^3*ArcTanh[Sin[e + f*x]])/(2*f) - (4*c^3*Tan[e + f*x])/f + (3*c^3*Sec
[e + f*x]*Tan[e + f*x])/(2*f) - (c^3*Tan[e + f*x]^3)/(3*f)))/a
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4445

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{105c^4\left(\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\cos(fx+e)-\frac{\cos(3fx+3e)}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{446\tan}{}\right.}{2af(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{16c^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{3}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{29}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{32}-\frac{}{48\left(\tan\right.}\right.}{fa}$
default	$\frac{16c^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{3}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{29}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{32}-\frac{}{48\left(\tan\right.}\right.}{fa}$
risch	$\frac{ic^4\left(111\,\mathrm{e}^{6i(fx+e)}+81\,\mathrm{e}^{5i(fx+e)}+354\,\mathrm{e}^{4i(fx+e)}+144\,\mathrm{e}^{3i(fx+e)}+417\,\mathrm{e}^{2i(fx+e)}+55\,\mathrm{e}^{i(fx+e)}+166\right)}{3af\left(\mathrm{e}^{2i(fx+e)}+1\right)^3\left(\mathrm{e}^{i(fx+e)}+1\right)}+\frac{35c^4\ln\left(\mathrm{e}^{i(fx+e}\right.}{2af}$
norman	$\frac{\frac{35c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{385c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}+\frac{511c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3af}-\frac{93c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}+\frac{16c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}+\frac{35c^4\ln\left(\tan\left(\right.\right.}{2af}$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output

```
105/2*c^4*((cos(f*x+e)+1/3*cos(3*f*x+3*e))*ln(tan(1/2*f*x+1/2*e)-1)+(-cos(
f*x+e)-1/3*cos(3*f*x+3*e))*ln(tan(1/2*f*x+1/2*e)+1)+446/315*tan(1/2*f*x+1/
2*e)*(cos(f*x+e)+55/223*cos(2*f*x+2*e)+83/223*cos(3*f*x+3*e)+59/223))/a/f/
(cos(3*f*x+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.26

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right.\right.}{12\left(af\cos\left(fx\right.\right.}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
-1/12*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(sin(f*x + e) + 1)
 - 105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) -
2*(166*c^4*cos(f*x + e)^3 + 55*c^4*cos(f*x + e)^2 - 13*c^4*cos(f*x + e) +
2*c^4)*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{c^4\left(\int\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{6\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^5\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right.}{a}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
```

output

```
c**4*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-4*sec(e + f
*x)**2/(sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e + f*x) +
 1), x) + Integral(-4*sec(e + f*x)**4/(sec(e + f*x) + 1), x) + Integral(se
c(e + f*x)**5/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(116) = 232$.

Time = 0.04 (sec) , antiderivative size = 591, normalized size of antiderivative = 4.88

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="max
ima")
```

output

```
1/6*(c^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(
a*(cos(f*x + e) + 1))) + 12*c^4*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f
*x + e) + 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin
(f*x + e)/(a*(cos(f*x + e) + 1))) - 36*c^4*(log(sin(f*x + e)/(cos(f*x + e)
 + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)
/((a - a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*
x + e)/(a*(cos(f*x + e) + 1))) - 24*c^4*(log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(
cos(f*x + e) + 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx =$$

$$-\frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{96\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}+\frac{2\left(87\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-136\,c^4\tan\left(\frac{1}{2}\,f\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}}{6\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
-1/6*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 105*c^4*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a - 96*c^4*tan(1/2*f*x + 1/2*e)/a + 2*(87*c^4*tan(1/
2*f*x + 1/2*e)^5 - 136*c^4*tan(1/2*f*x + 1/2*e)^3 + 57*c^4*tan(1/2*f*x + 1
/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a))/f
```

Mupad [B] (verification not implemented)

Time = 11.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$
$$= \frac{16\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f} - \frac{29\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5 - \frac{136\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3} + 19\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}$$
$$- \frac{35\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

input
```
int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output
```
(16*c^4*tan(e/2 + (f*x)/2))/(a*f) - (29*c^4*tan(e/2 + (f*x)/2)^5 - (136*c^
4*tan(e/2 + (f*x)/2)^3)/3 + 19*c^4*tan(e/2 + (f*x)/2))/(a*f*(tan(e/2 + (f*
x)/2)^2 - 1)^3) - (35*c^4*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.60

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$
$$= \frac{c^4\left(105\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^3 - 105\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx\right.}{}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x)
```

output
```
(c**4*(105*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**3 - 105*co
s(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x) - 105*cos(e + f*x)*log(t
an((e + f*x)/2) + 1)*sin(e + f*x)**3 + 105*cos(e + f*x)*log(tan((e + f*x)/
2) + 1)*sin(e + f*x) + 111*cos(e + f*x)*sin(e + f*x)**2 - 96*cos(e + f*x)
+ 166*sin(e + f*x)**4 - 264*sin(e + f*x)**2 + 96))/(6*cos(e + f*x)*sin(e +
 f*x)*a*f*(sin(e + f*x)**2 - 1))
```

3.35 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{15c^3\text{arctanh}(\sin(e+fx))}{2af}+\frac{10c^3\tan(e+fx)}{af}-\frac{5c^3\sec(e+fx)\tan(e+fx)}{2af}+\frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output
```
-15/2*c^3*arctanh(sin(f*x+e))/a/f+10*c^3*tan(f*x+e)/a/f-5/2*c^3*sec(f*x+e)
*tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.53

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{8c^3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}}{af}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

output
```
(-8*c^3*Cot[e + f*x]*Hypergeometric2F1[-5/2, -1/2, 1/2, (1 + Sec[e + f*x])
/2]*Sqrt[2 - 2*Sec[e + f*x]])/(a*f)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4445, 3042, 4275, 3042, 4254, 24, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\int \sec(e+fx)(c-c\sec(e+fx))^2dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx}{a}$$

$$\Big\downarrow \text{ 4275}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\int \sec(e+fx)\left(\sec^2(e+fx)c^2+c^2\right)dx-2c^2\int \sec^2(e+fx)dx\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2c^2+c^2\right)dx-2c^2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)}{a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{2c^2\int 1d(-\tan(e+fx))}{f}+\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2c^2+c^2\right)dx\right)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2c^2+c^2\right)dx-\frac{2c^2\tan(e+fx)}{f}\right)}{a}$$

$$\Big\downarrow \text{4534}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{3}{2}c^2\int \sec(e+fx)dx-\frac{2c^2\tan(e+fx)}{f}+\frac{c^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{3}{2}c^2\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{2c^2\tan(e+fx)}{f}+\frac{c^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{3c^2\text{arctanh}(\sin(e+fx))}{2f}-\frac{2c^2\tan(e+fx)}{f}+\frac{c^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

output
```
(2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) - (5*c*
((3*c^2*ArcTanh[Sin[e + f*x]])/(2*f) - (2*c^2*Tan[e + f*x])/f + (c^2*Sec[e
 + f*x]*Tan[e + f*x])/(2*f)))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{15\left((-1-\cos(2fx+2e))\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)+(1+\cos(2fx+2e))\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)-\frac{14\tan(\frac{fx}{2}+\frac{e}{2})(\cos(fx+e)+}{15}\right.}{2af(1+\cos(2fx+2e))}$
derivativedivides	$\frac{8c^3\left(\tan(\frac{fx}{2}+\frac{e}{2})+\frac{1}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}-\frac{9}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)}-\frac{15\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{16}-\frac{1}{16(\tan(\frac{fx}{2}+\frac{e}{2})-1)^2}-\right.}{fa}$
default	$\frac{8c^3\left(\tan(\frac{fx}{2}+\frac{e}{2})+\frac{1}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}-\frac{9}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)}-\frac{15\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{16}-\frac{1}{16(\tan(\frac{fx}{2}+\frac{e}{2})-1)^2}-\right.}{fa}$
risch	$\frac{ic^3(17\,e^{4i(fx+e)}+9\,e^{3i(fx+e)}+39\,e^{2i(fx+e)}+7\,e^{i(fx+e)}+24)}{fa(e^{i(fx+e)}+1)(e^{2i(fx+e)}+1)^2}-\frac{15c^3\ln(e^{i(fx+e)}+i)}{2af}+\frac{15c^3\ln(e^{i(fx+e)}-i)}{2af}$
norman	$\frac{-\frac{15c^3\tan(\frac{fx}{2}+\frac{e}{2})}{af}+\frac{40c^3\tan(\frac{fx}{2}+\frac{e}{2})^3}{af}-\frac{33c^3\tan(\frac{fx}{2}+\frac{e}{2})^5}{af}+\frac{8c^3\tan(\frac{fx}{2}+\frac{e}{2})^7}{af}}{(\tan(\frac{fx}{2}+\frac{e}{2})^2-1)^3}+\frac{15c^3\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{2af}-$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
-15/2*((-1-cos(2*f*x+2*e))*ln(tan(1/2*f*x+1/2*e)-1)+(1+cos(2*f*x+2*e))*ln(
tan(1/2*f*x+1/2*e)+1)-14/15*tan(1/2*f*x+1/2*e)*(cos(f*x+e)+12/7*cos(2*f*x+
2*e)+11/7))*c^3/a/f/(1+cos(2*f*x+2*e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.40

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2}{4\left(af\cos\left(fx+e\right)^3+af\,\mathrm{co}\right.}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
-1/4*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(sin(f*x + e) + 1) -
 15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(
24*c^3*cos(f*x + e)^2 + 7*c^3*cos(f*x + e) - c^3)*sin(f*x + e))/(a*f*cos(f
*x + e)^3 + a*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{c^3\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{3\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

output

```
-c**3*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(3*sec(e +
f*x)**2/(sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e + f*x)
 + 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(97) = 194$.

Time = 0.04 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.86

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$$
$$= \frac{c^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-6c^3\left(\frac{\log\left(\frac{\sin(}{\cos(f}\right.}{}\right.}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
1/2*(c^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a + 3
*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x +
 e) + 1))) - 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e)
+ 1))) - 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c
^3*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx =$$
$$-\frac{\frac{15c^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{15c^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}-\frac{16c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}+\frac{2\left(9c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-7c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.\right.}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a}}{2f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
-1/2*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 15*c^3*log(abs(tan(1/2
*f*x + 1/2*e) - 1))/a - 16*c^3*tan(1/2*f*x + 1/2*e)/a + 2*(9*c^3*tan(1/2*f
*x + 1/2*e)^3 - 7*c^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^
2*a))/f
```

Mupad [B] (verification not implemented)

Time = 10.96 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \frac{8\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f} - \frac{9\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-7\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)^2} - \frac{15\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

input
```
int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output
```
(8*c^3*tan(e/2 + (f*x)/2))/(a*f) - (9*c^3*tan(e/2 + (f*x)/2)^3 - 7*c^3*tan
(e/2 + (f*x)/2))/(a*f*(tan(e/2 + (f*x)/2)^2 - 1)^2) - (15*c^3*atanh(tan(e/
2 + (f*x)/2)))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$$

$$= \frac{c^3\left(-24\cos\left(fx+e\right)\sin\left(fx+e\right)^2+16\cos\left(fx+e\right)+15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^3-15\log\left(\mathrm{ta}\right.\right.}{2\,\mathrm{s}}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x)
```

output

```
(c**3*( - 24*cos(e + f*x)*sin(e + f*x)**2 + 16*cos(e + f*x) + 15*log(tan((
e + f*x)/2) - 1)*sin(e + f*x)**3 - 15*log(tan((e + f*x)/2) - 1)*sin(e + f*
x) - 15*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**3 + 15*log(tan((e + f*x)/2
) + 1)*sin(e + f*x) + 17*sin(e + f*x)**2 - 16))/(2*sin(e + f*x)*a*f*(sin(e
 + f*x)**2 - 1))
```

3.36 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 74

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{3c^2\operatorname{arctanh}(\sin(e+fx))}{af} + \frac{3c^2\tan(e+fx)}{af} + \frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output
```
-3*c^2*arctanh(sin(f*x+e))/a/f+3*c^2*tan(f*x+e)/a/f+2*(c^2-c^2*sec(f*x+e))
*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{4\sqrt{2}c^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan\left(\frac{1}{2}(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

output
```
(4*Sqrt[2]*c^2*Hypergeometric2F1[-3/2, -1/2, 1/2, (1 + Sec[e + f*x])/2]*Ta
n[(e + f*x)/2])/(a*f*Sqrt[1 - Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4445, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4445}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\int \sec(e+fx)(c-c\sec(e+fx))dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(c\int \sec(e+fx)dx-c\int \sec^2(e+fx)dx\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(c\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-c\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)}{a}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(\frac{c\int 1d(-\tan(e+fx))}{f}+c\int \csc\left(e+fx+\frac{\pi}{2}\right)dx\right)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(c\int\csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{c\tan(e+fx)}{f}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(\frac{c\,\text{arctanh}(\sin(e+fx))}{f}-\frac{c\tan(e+fx)}{f}\right)}{a}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

output
```
(2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) - (3*c*
((c*ArcTanh[Sin[e + f*x]])/f - (c*Tan[e + f*x])/f))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4445

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{4c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{fa}$
default	$\frac{4c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{fa}$
parallelrisch	$\frac{c^2\left(3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cos(fx+e)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af\cos(fx+e)}$
risch	$\frac{2ic^2\left(4\,\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}+5\right)}{fa\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}+1\right)}-\frac{3c^2\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{af}+\frac{3c^2\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{af}$
norman	$\frac{\frac{6c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{10c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}+\frac{4c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}+\frac{3c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{af}-\frac{3c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output

```
4/f*c^2/a*(tan(1/2*f*x+1/2*e)-1/4/(tan(1/2*f*x+1/2*e)+1)-3/4*ln(tan(1/2*f*
x+1/2*e)+1)-1/4/(tan(1/2*f*x+1/2*e)-1)+3/4*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\log}{2\left(af\cos\left(fx+e\right)^2+af\cos\left(fx+e\right)\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
-1/2*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*
(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(5*c^2*
cos(f*x + e) + c^2)*sin(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{c^2\left(\int \frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{2\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

output
```
c**2*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-2*sec(e + f
*x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) + 1
), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(75) = 150$.

Time = 0.04 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.03

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx =$$

$$-\frac{c^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+2\,c^2\Bigg(}{f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-(c^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(
f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c^
2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
+ e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)
/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx =$$

$$-\frac{\frac{3\,c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{3\,c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{4\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}+\frac{2\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a}}{f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="gia
c")
```

output

```
-(3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 3*c^2*log(abs(tan(1/2*f*x +
 1/2*e) - 1))/a - 4*c^2*tan(1/2*f*x + 1/2*e)/a + 2*c^2*tan(1/2*f*x + 1/2*e
)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{4\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f} + \frac{2\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)} - \frac{6\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

input

```
int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output

```
(4*c^2*tan(e/2 + (f*x)/2))/(a*f) + (2*c^2*tan(e/2 + (f*x)/2))/(f*(a - a*ta
n(e/2 + (f*x)/2)^2)) - (6*c^2*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{c^2\left(3\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)-3\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\sin\left(fx+e\right)}{\cos\left(fx+e\right)\sin\left(fx+e\right)af}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x)
```

output

```
(c**2*(3*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x) - 3*cos(e + f
*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x) + 4*cos(e + f*x) + 5*sin(e + f*
x)**2 - 4))/(cos(e + f*x)*sin(e + f*x)*a*f)
```

3.37 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\operatorname{arctanh}(\sin(e+fx))}{af}+\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output

```
-c*arctanh(sin(f*x+e))/a/f+2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.88

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{f}\right)}{a}$$

input

```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

output

```
-((c*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]]/f - (2*Tan[(e + f*x)/2])/f))/a)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4445, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4445}$$

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\int \sec(e+fx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\,\text{arctanh}(\sin(e+fx))}{af}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

output

```
-((c*ArcTanh[Sin[e + f*x]])/(a*f)) + (2*c*Tan[e + f*x])/(f*(a + a*Sec[e +
f*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$\frac{c\left(\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$	47
derivativedivides	$\frac{2c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{fa}$	48
default	$\frac{2c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{fa}$	48
risch	$\frac{4ic}{fa\left(\mathrm{e}^{i(fx+e)}+1\right)}-\frac{c\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{fa}+\frac{c\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{fa}$	68
norman	$\frac{-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}+\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{fa}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}+\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{fa}-\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{fa}$	98

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/a/f*c*(ln(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*e)+1)+2*tan(1/2*f*x+1
/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{(c\cos(fx+e)+c)\log(\sin(fx+e)+1)-(c\cos(fx+e)+c)\log(-\sin(fx+e)+1)-4\,c\sin(fx}{2\,(af\cos(fx+e)+af)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="frica
s")
```

output
```
-1/2*((c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) - (c*cos(f*x + e) + c)*lo
g(-sin(f*x + e) + 1) - 4*c*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(\int\left(-\frac{\sec(e+fx)}{\sec(e+fx)+1}\right)dx+\int\frac{\sec^2(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

output
```
-c*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)*
*2/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-\frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}}{f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
-(c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*
x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c*sin(f*x + e)
/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}-\frac{2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}}{f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
-(c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - c*log(abs(tan(1/2*f*x + 1/2*e)
- 1))/a - 2*c*tan(1/2*f*x + 1/2*e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 10.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{2\,c\,\left(\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

input

```
int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output

```
-(2*c*(atanh(tan(e/2 + (f*x)/2)) - tan(e/2 + (f*x)/2)))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{c\left(\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

output

```
(c*(log(tan((e + f*x)/2) - 1) - log(tan((e + f*x)/2) + 1) + 2*tan((e + f*x
)/2)))/(a*f)
```

3.38 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 16

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx=\frac{\csc(e+fx)}{acf}$$

output
```
csc(f*x+e)/a/c/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx=\frac{\csc(e+fx)}{acf}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]
```

output
```
Csc[e + f*x]/(a*c*f)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4446, 3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4446}$$

$$-\frac{\int \cot(e+fx)\csc(e+fx)dx}{ac}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int -\sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)dx}{ac}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \sec\left(\frac{1}{2}(2e-\pi)+fx\right)\tan\left(\frac{1}{2}(2e-\pi)+fx\right)dx}{ac}$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\int 1d\csc(e+fx)}{acf}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\csc(e+fx)}{acf}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]
```

output
```
Csc[e + f*x]/(a*c*f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{\csc(fx+e)}{acf}$	17
parallelrisch	$\frac{\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)}{2fac}$	30
norman	$\frac{\frac{1}{2fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2fac}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	47
risch	$\frac{2i\,\mathrm{e}^{i(fx+e)}}{fac\left(\mathrm{e}^{i(fx+e)}-1\right)\left(\mathrm{e}^{i(fx+e)}+1\right)}$	48

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
csc(f*x+e)/a/c/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = \frac{1}{acf\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="frica
s")
```

output
```
1/(a*c*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-1}\,dx}{ac}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(sec(e + f*x)/(sec(e + f*x)**2 - 1), x)/(a*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = \frac{1}{acf\sin{(fx+e)}}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
1/(a*c*f*sin(f*x + e))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = \frac{1}{acf\sin{(fx+e)}}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
1/(a*c*f*sin(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 10.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = \frac{1}{a\,c\,f\,\sin{(e+f\,x)}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))),x)
```

output
```
1/(a*c*f*sin(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = \frac{1}{\sin\left(fx+e\right)acf}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

output
```
1/(sin(e + f*x)*a*c*f)
```

3.39 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 59

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}$$

output
```
-1/3*cot(f*x+e)^3/a/c^2/f+csc(f*x+e)/a/c^2/f-1/3*csc(f*x+e)^3/a/c^2/f
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = -\frac{(-3+4\cos(e+fx)+\cos(2(e+fx)))\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)}{24ac^2f}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
```

output
```
-1/24*((-3 + 4*Cos[e + f*x] + Cos[2*(e + f*x)])*Csc[(e + f*x)/2]^3*Sec[(e
+ f*x)/2])/(a*c^2*f)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4446}$$

$$\frac{\int \left(a\csc(e+fx)\cot^3(e+fx)+a\csc^2(e+fx)\cot^2(e+fx)\right)dx}{a^2c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a\cot^3(e+fx)}{3f}-\frac{a\csc^3(e+fx)}{3f}+\frac{a\csc(e+fx)}{f}}{a^2c^2}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
```

output
```
(-1/3*(a*Cot[e + f*x]^3)/f + (a*Csc[e + f*x])/f - (a*Csc[e + f*x]^3)/(3*f)
)/(a^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4fa\,c^{2}}$	48
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4fa\,c^{2}}$	48
parallelrisch	$\frac{3\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-4\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{12fa\,c^{2}}$	48
norman	$\frac{-\frac{1}{12fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{4fac}}{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}$	72
risch	$\frac{2i\left(3\,\mathrm{e}^{3i(fx+e)}-3\,\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}+1\right)}{3fa\,c^{2}\left(\mathrm{e}^{i(fx+e)}-1\right)^{3}\left(\mathrm{e}^{i(fx+e)}+1\right)}$	72

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/4/f/a/c^2*(tan(1/2*f*x+1/2*e)-1/3/tan(1/2*f*x+1/2*e)^3+2/tan(1/2*f*x+1/2
*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{\cos\left(fx+e\right)^2+2\,\cos\left(fx+e\right)-2}{3\left(ac^2f\cos\left(fx+e\right)-ac^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output
```
1/3*(cos(f*x + e)^2 + 2*cos(f*x + e) - 2)/((a*c^2*f*cos(f*x + e) - a*c^2*f
)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-\sec^2\left(e+fx\right)-\sec\left(e+fx\right)+1}\,dx}{ac^2}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(sec(e + f*x)/(sec(e + f*x)**3 - sec(e + f*x)**2 - sec(e + f*x) +
1), x)/(a*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{\frac{\left(\frac{6\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{ac^2\sin(fx+e)^3}+\frac{3\,\sin(fx+e)}{ac^2(\cos(fx+e)+1)}}{12\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
1/12*((6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a*
c^2*sin(f*x + e)^3) + 3*sin(f*x + e)/(a*c^2*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx=\frac{\frac{3\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{ac^2}+\frac{6\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1}{ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{12\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
1/12*(3*tan(1/2*f*x + 1/2*e)/(a*c^2) + (6*tan(1/2*f*x + 1/2*e)^2 - 1)/(a*c
^2*tan(1/2*f*x + 1/2*e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx=\frac{3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1}{12\,a\,c^2\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^2),x)
```

output
```
(6*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^4 - 1)/(12*a*c^2*f*tan(e/2
+ (f*x)/2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}{12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 a\,c^2 f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x)
```

output
```
(3*tan((e + f*x)/2)**4 + 6*tan((e + f*x)/2)**2 - 1)/(12*tan((e + f*x)/2)**
3*a*c**2*f)
```

3.40 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{2\cot^5(e+fx)}{5ac^3f}+\frac{\csc(e+fx)}{ac^3f}-\frac{\csc^3(e+fx)}{ac^3f}+\frac{2\csc^5(e+fx)}{5ac^3f}$$

output
```
2/5*cot(f*x+e)^5/a/c^3/f+csc(f*x+e)/a/c^3/f-csc(f*x+e)^3/a/c^3/f+2/5*csc(f
*x+e)^5/a/c^3/f
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{(5-5\cos(e+fx)+\cos(2(e+fx))+\cos(3(e+fx)))\csc^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)}{80ac^3f}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
```

output
```
((5 - 5*Cos[e + f*x] + Cos[2*(e + f*x)] + Cos[3*(e + f*x)])*Csc[(e + f*x)/
2]^5*Sec[(e + f*x)/2])/(80*a*c^3*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4446}$$

$$-\frac{\int \left(a^2\csc(e+fx)\cot^5(e+fx)+2a^2\csc^2(e+fx)\cot^4(e+fx)+a^2\csc^3(e+fx)\cot^3(e+fx)\right)dx}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2a^2\cot^5(e+fx)}{5f}-\frac{2a^2\csc^5(e+fx)}{5f}+\frac{a^2\csc^3(e+fx)}{f}-\frac{a^2\csc(e+fx)}{f}}{a^3c^3}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
```

output
```
-(((-2*a^2*Cot[e + f*x]^5)/(5*f) - (a^2*Csc[e + f*x])/f + (a^2*Csc[e + f*x
]^3)/f - (2*a^2*Csc[e + f*x]^5)/(5*f))/(a^3*c^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\left(5+\cos(3fx+3e)+\cos(2fx+2e)-5\cos(fx+e)\right)}{80fac^{3}}$	60
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8fac^{3}}$	61
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8fac^{3}}$	61
risch	$\frac{2i\left(5\,e^{5i(fx+e)}-10\,e^{4i(fx+e)}+10\,e^{3i(fx+e)}-3\,e^{i(fx+e)}+2\right)}{5fac^{3}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{5}}$	85
norman	$\frac{\frac{1}{40fac}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{8fac}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{8fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{8fac}}{c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$	94

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/80/f/a/c^3*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^5*(5+cos(3*f*x+3*e)+cos
(2*f*x+2*e)-5*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{2\,\cos\left(fx+e\right)^3+\cos\left(fx+e\right)^2-4\,\cos\left(fx+e\right)+2}{5\,\left(ac^3f\cos\left(fx+e\right)^2-2\,ac^3f\cos\left(fx+e\right)+ac^3f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
1/5*(2*cos(f*x + e)^3 + cos(f*x + e)^2 - 4*cos(f*x + e) + 2)/((a*c^3*f*cos
(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-2\sec^3\left(e+fx\right)+2\sec\left(e+fx\right)-1}\,dx}{ac^3}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*
x) - 1), x)/(a*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = -\frac{\frac{\left(\frac{5\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{ac^3\sin(fx+e)^5}-\frac{5\,\sin(fx+e)}{ac^3(\cos(fx+e)+1)}}{40\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="max
ima")
```

output
```
-1/40*((5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a*c^3*sin(f*x + e)^5) - 5*sin(f*x
+ e)/(a*c^3*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{\frac{5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^3}+\frac{15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1}{ac^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}}{40f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="gia
c")
```

output
```
1/40*(5*tan(1/2*f*x + 1/2*e)/(a*c^3) + (15*tan(1/2*f*x + 1/2*e)^4 - 5*tan(
1/2*f*x + 1/2*e)^2 + 1)/(a*c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1}{40\,a\,c^3\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^3),x)
```

output
```
(15*tan(e/2 + (f*x)/2)^4 - 5*tan(e/2 + (f*x)/2)^2 + 5*tan(e/2 + (f*x)/2)^6
 + 1)/(40*a*c^3*f*tan(e/2 + (f*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1}{40\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 a\,c^3 f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x)
```

output
```
(5*tan((e + f*x)/2)**6 + 15*tan((e + f*x)/2)**4 - 5*tan((e + f*x)/2)**2 +
1)/(40*tan((e + f*x)/2)**5*a*c**3*f)
```

3.41 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx &= -\frac{\cot^5(e+fx)}{5ac^4f}-\frac{4\cot^7(e+fx)}{7ac^4f}+\frac{\csc(e+fx)}{ac^4f}\\ &\quad -\frac{2\csc^3(e+fx)}{ac^4f}+\frac{9\csc^5(e+fx)}{5ac^4f}-\frac{4\csc^7(e+fx)}{7ac^4f}\end{aligned}$$

output

```
-1/5*cot(f*x+e)^5/a/c^4/f-4/7*cot(f*x+e)^7/a/c^4/f+csc(f*x+e)/a/c^4/f-2*cs
c(f*x+e)^3/a/c^4/f+9/5*csc(f*x+e)^5/a/c^4/f-4/7*csc(f*x+e)^7/a/c^4/f
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\begin{aligned}&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx\\ &=\frac{(-13+4\sec(e+fx)+20\sec^2(e+fx)-24\sec^3(e+fx)+8\sec^4(e+fx))\tan(e+fx)}{35ac^4f(-1+\sec(e+fx))^4(1+\sec(e+fx))}\end{aligned}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

output
```
((-13 + 4*Sec[e + f*x] + 20*Sec[e + f*x]^2 - 24*Sec[e + f*x]^3 + 8*Sec[e +
 f*x]^4)*Tan[e + f*x])/(35*a*c^4*f*(-1 + Sec[e + f*x])^4*(1 + Sec[e + f*x]
))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4446}$$

$$\frac{\int \left(a^3\csc(e+fx)\cot^7(e+fx)+3a^3\csc^2(e+fx)\cot^6(e+fx)+3a^3\csc^3(e+fx)\cot^5(e+fx)+a^3\csc^4(e+f\right.}{a^4c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{4a^3\cot^7(e+fx)}{7f}-\frac{a^3\cot^5(e+fx)}{5f}-\frac{4a^3\csc^7(e+fx)}{7f}+\frac{9a^3\csc^5(e+fx)}{5f}-\frac{2a^3\csc^3(e+fx)}{f}+\frac{a^3\csc(e+fx)}{f}}{a^4c^4}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

output
```
(-1/5*(a^3*Cot[e + f*x]^5)/f - (4*a^3*Cot[e + f*x]^7)/(7*f) + (a^3*Csc[e +
 f*x])/f - (2*a^3*Csc[e + f*x]^3)/f + (9*a^3*Csc[e + f*x]^5)/(5*f) - (4*a^
3*Csc[e + f*x]^7)/(7*f))/(a^4*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{16fa\,c^{4}}$	74
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{16fa\,c^{4}}$	74
parallelrisch	$-\frac{\left(-105+13\cos(4fx+4e)-8\cos(3fx+3e)-28\cos(2fx+2e)+168\cos(fx+e)\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{4480fa\,c^{4}}$	75
norman	$\frac{-\frac{1}{112fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{20fac}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{8fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{4fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{16fac}}{c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$	116
risch	$\frac{2i\left(35\,\mathrm{e}^{7i(fx+e)}-105\,\mathrm{e}^{6i(fx+e)}+175\,\mathrm{e}^{5i(fx+e)}-105\,\mathrm{e}^{4i(fx+e)}-7\,\mathrm{e}^{3i(fx+e)}+77\,\mathrm{e}^{2i(fx+e)}-43\,\mathrm{e}^{i(fx+e)}+13\right)}{35fa\,c^{4}\left(\mathrm{e}^{i(fx+e)}-1\right)^{7}\left(\mathrm{e}^{i(fx+e)}+1\right)}$	118

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE
)
```

output
```
1/16/f/a/c^4*(tan(1/2*f*x+1/2*e)-2/tan(1/2*f*x+1/2*e)^3+4/tan(1/2*f*x+1/2*
e)-1/7/tan(1/2*f*x+1/2*e)^7+4/5/tan(1/2*f*x+1/2*e)^5)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = \frac{13\cos\left(fx+e\right)^4-4\cos\left(fx+e\right)^3-20\cos\left(fx+e\right)^2+24\cos\left(fx+e\right)-8}{35\left(ac^4f\cos\left(fx+e\right)^3-3\,ac^4f\cos\left(fx+e\right)^2+3\,ac^4f\cos\left(fx+e\right)-ac^4f\right)\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fri
cas")
```

output

```
1/35*(13*cos(f*x + e)^4 - 4*cos(f*x + e)^3 - 20*cos(f*x + e)^2 + 24*cos(f*
x + e) - 8)/((a*c^4*f*cos(f*x + e)^3 - 3*a*c^4*f*cos(f*x + e)^2 + 3*a*c^4*
f*cos(f*x + e) - a*c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-3\sec^4\left(e+fx\right)+2\sec^3\left(e+fx\right)+2\sec^2\left(e+fx\right)-3\sec\left(e+fx\right)+1}\,dx}{ac^4}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

output

```
Integral(sec(e + f*x)/(sec(e + f*x)**5 - 3*sec(e + f*x)**4 + 2*sec(e + f*x
)**3 + 2*sec(e + f*x)**2 - 3*sec(e + f*x) + 1), x)/(a*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = \frac{\frac{\left(\frac{28\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{70\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{140\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{ac^4\sin(fx+e)^7}+\frac{35\sin(fx+e)}{ac^4(\cos(fx+e)+1)}}{560\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="max
ima")
```

output
```
1/560*((28*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 140*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e)
 + 1)^7/(a*c^4*sin(f*x + e)^7) + 35*sin(f*x + e)/(a*c^4*(cos(f*x + e) + 1)
))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = \frac{\frac{35\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{ac^4}+\frac{140\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-70\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+28\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-5}{ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{560\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="gia
c")
```

output
```
1/560*(35*tan(1/2*f*x + 1/2*e)/(a*c^4) + (140*tan(1/2*f*x + 1/2*e)^6 - 70*
tan(1/2*f*x + 1/2*e)^4 + 28*tan(1/2*f*x + 1/2*e)^2 - 5)/(a*c^4*tan(1/2*f*x
 + 1/2*e)^7))/f
```

Mupad [B] (verification not implemented)

Time = 10.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{16\,a\,c^4\,f}+\frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6}{4}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{8}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{20}-\frac{1}{112}}{a\,c^4\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^4),x)
```

output
```
tan(e/2 + (f*x)/2)/(16*a*c^4*f) + (tan(e/2 + (f*x)/2)^2/20 - tan(e/2 + (f*
x)/2)^4/8 + tan(e/2 + (f*x)/2)^6/4 - 1/112)/(a*c^4*f*tan(e/2 + (f*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.63

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+140\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-70\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+28\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-5}{560\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7a\,c^4f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x)
```

output
```
(35*tan((e + f*x)/2)**8 + 140*tan((e + f*x)/2)**6 - 70*tan((e + f*x)/2)**4
 + 28*tan((e + f*x)/2)**2 - 5)/(560*tan((e + f*x)/2)**7*a*c**4*f)
```

3.42 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 164

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx &= \frac{105c^5\text{arctanh}(\sin(e+fx))}{2a^2f} - \frac{84c^5\tan(e+fx)}{a^2f} \\ &\quad + \frac{63c^5\sec(e+fx)\tan(e+fx)}{2a^2f} \\ &\quad - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))} \\ &\quad + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\ &\quad - \frac{7c^5\tan^3(e+fx)}{a^2f}\end{aligned}$$

output

```
105/2*c^5*arctanh(sin(f*x+e))/a^2/f-84*c^5*tan(f*x+e)/a^2/f+63/2*c^5*sec(f
*x+e)*tan(f*x+e)/a^2/f-6*c^2*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a^2+a^2*sec(
f*x+e))+2/3*c*(c-c*sec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-7*c^5*tan
(f*x+e)^3/a^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{32c^5\,\text{Hypergeometric2F1}\left(-\frac{9}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{3a^2f(-1+\sec(e+fx))(1+\sec(e+fx))^2}$$

input

```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

output

```
(-32*c^5*Hypergeometric2F1[-9/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[2
- 2*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f*(-1 + Sec[e + f*x])*(1 + Sec[e +
f*x])^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 4445, 3042, 4445, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{\sec(e+fx)a+a}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{a}$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\int \sec(e+fx)(c-c\sec(e+fx))^3dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4278}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\int \left(-c^3\sec^4(e+fx)+3c^3\sec^3(e+fx)-3c^3\sec^2(e+fx)+c^3\sec(e+fx)\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\left(\frac{5c^3\text{arctanh}(\sin(e+fx))}{2f}-\frac{c^3\tan^3(e+fx)}{3f}-\frac{4c^3\tan(e+fx)}{f}+\frac{3c^3\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}\right)}{a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

output

```
(2*c*(c - c*Sec[e + f*x])^4*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (
3*c*((2*c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) -
(7*c*((5*c^3*ArcTanh[Sin[e + f*x]])/(2*f) - (4*c^3*Tan[e + f*x])/f + (3*c^
3*Sec[e + f*x]*Tan[e + f*x])/(2*f) - (c^3*Tan[e + f*x]^3)/(3*f)))/a))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{16c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{55}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{105\ln\left(\tan\left(\frac{fx}{2}\right.\right.}{32}\right.}{f a^2}$
default	$\frac{16c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{55}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{105\ln\left(\tan\left(\frac{fx}{2}\right.\right.}{32}\right.}{f a^2}$
parallelrisch	$-\frac{1969\left(\frac{630(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{1969}+\frac{630(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{1969}+\tan\left(\frac{fx}{2}\right.\right.}{12f a^2(\cos(3fx+3e)+3\cos}$
risch	$-\frac{ic^5\left(309\,\mathrm{e}^{8i(fx+e)}+969\,\mathrm{e}^{7i(fx+e)}+1693\,\mathrm{e}^{6i(fx+e)}+3027\,\mathrm{e}^{5i(fx+e)}+2901\,\mathrm{e}^{4i(fx+e)}+3247\,\mathrm{e}^{3i(fx+e)}+1995\,\mathrm{e}^{2i(fx+e)}\right.}{3f a^2\left(\mathrm{e}^{2i(fx+e)}+1\right)^3\left(\mathrm{e}^{i(fx+e)}+1\right)^3}$
norman	$\frac{\frac{105c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{490c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}+\frac{896c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}-\frac{790c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}+\frac{965c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3af}-\frac{112c^5\tan\left(\frac{fx}{2}+\right.}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5 a}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
16/f*c^5/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-4*tan(1/2*f*x+1/2*e)+1/48/(tan(1/2
*f*x+1/2*e)+1)^3-1/4/(tan(1/2*f*x+1/2*e)+1)^2+55/32/(tan(1/2*f*x+1/2*e)+1)
+105/32*ln(tan(1/2*f*x+1/2*e)+1)+1/48/(tan(1/2*f*x+1/2*e)-1)^3+1/4/(tan(1/
2*f*x+1/2*e)-1)^2+55/32/(tan(1/2*f*x+1/2*e)-1)-105/32*ln(tan(1/2*f*x+1/2*e
)-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.28

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{315\left(c^5\cos(fx+e)^5+2c^5\cos(fx+e)^4+c^5\cos(fx+e)^3\right)\log(\sin(fx+e)+1)-315\left(c^5\cos(fx+e\right.}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
1/12*(315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 + c^5*cos(f*x + e)^3)
*log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 +
c^5*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) - 2*(494*c^5*cos(f*x + e)^4 + 6
79*c^5*cos(f*x + e)^3 + 102*c^5*cos(f*x + e)^2 - 17*c^5*cos(f*x + e) + 2*c
^5)*sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)^4 + a^2*f*c
os(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx =$$

$$-\frac{c^5\left(\int\left(-\frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{5\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{10\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx-\right.}{a^2}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

output

```
-c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(5*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(-10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(10*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-5*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(160) = 320$.

Time = 0.05 (sec) , antiderivative size = 765, normalized size of antiderivative = 4.66

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
-1/6*(c^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 - a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x +
 e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/
a^2) + 5*c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1)
+ sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x
+ e) + 1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 10
*c^5*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(s
in(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/a^2) + 5*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)/a^2 - c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin
(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$

$$= \frac{\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}+\frac{2\,\left(165\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-280\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+123\,c^5\tan\left(\frac{1}{2}\,fx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^2}}{6\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/6*(315*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 315*c^5*log(abs(tan(
1/2*f*x + 1/2*e) - 1))/a^2 + 2*(165*c^5*tan(1/2*f*x + 1/2*e)^5 - 280*c^5*t
an(1/2*f*x + 1/2*e)^3 + 123*c^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*
e)^2 - 1)^3*a^2) - 32*(a^4*c^5*tan(1/2*f*x + 1/2*e)^3 + 12*a^4*c^5*tan(1/2
*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{55\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{280\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+41\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)} - \frac{64\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} - \frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} + \frac{105\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

input
```
int((c - c/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output
```
(55*c^5*tan(e/2 + (f*x)/2)^5 - (280*c^5*tan(e/2 + (f*x)/2)^3)/3 + 41*c^5*t
an(e/2 + (f*x)/2))/(f*(3*a^2*tan(e/2 + (f*x)/2)^2 - 3*a^2*tan(e/2 + (f*x)/
2)^4 + a^2*tan(e/2 + (f*x)/2)^6 - a^2)) - (64*c^5*tan(e/2 + (f*x)/2))/(a^2
*f) - (16*c^5*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) + (105*c^5*atanh(tan(e/2 + (
f*x)/2)))/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.79

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^5\left(-315\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+945\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-945\log\left(\tan\right.\right.}{}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x)
```

output

```
(c**5*( - 315*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**6 + 945*log(tan(
(e + f*x)/2) - 1)*tan((e + f*x)/2)**4 - 945*log(tan((e + f*x)/2) - 1)*tan(
(e + f*x)/2)**2 + 315*log(tan((e + f*x)/2) - 1) + 315*log(tan((e + f*x)/2)
 + 1)*tan((e + f*x)/2)**6 - 945*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)
**4 + 945*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2 - 315*log(tan((e +
 f*x)/2) + 1) - 32*tan((e + f*x)/2)**9 - 288*tan((e + f*x)/2)**7 + 1386*ta
n((e + f*x)/2)**5 - 1680*tan((e + f*x)/2)**3 + 630*tan((e + f*x)/2)))/(6*a
**2*f*(tan((e + f*x)/2)**6 - 3*tan((e + f*x)/2)**4 + 3*tan((e + f*x)/2)**2
 - 1))
```

3.43 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 150

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx &= \frac{35c^4\operatorname{arctanh}(\sin(e+fx))}{2a^2f}-\frac{70c^4\tan(e+fx)}{3a^2f}\\ &\quad+\frac{35c^4\sec(e+fx)\tan(e+fx)}{6a^2f}\\ &\quad+\frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2}\\ &\quad-\frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}\end{aligned}$$

output

```
35/2*c^4*arctanh(sin(f*x+e))/a^2/f-70/3*c^4*tan(f*x+e)/a^2/f+35/6*c^4*sec(
f*x+e)*tan(f*x+e)/a^2/f+2/3*c*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x
+e))^2-14/3*(c^2-c^2*sec(f*x+e))^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.50

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{16c^4\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{3a^2f(-1+\sec(e+fx))(1+\sec(e+fx))^2}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

output
```
(-16*c^4*Hypergeometric2F1[-7/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[2
- 2*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f*(-1 + Sec[e + f*x])*(1 + Sec[e +
f*x])^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4445, 3042, 4445, 3042, 4275, 3042, 4254, 24, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{\sec(e+fx)a+a}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}$$

$$\Big\downarrow \ 4445$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\int \sec(e+fx)(c-c\sec(e+fx))^2dx}{a}\right)}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx}{a}\right)}{3a}$$

$$\Big\downarrow \ 4275$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c(\int \sec(e+fx)\left(\sec^2(e+fx)c^2+c^2\right)dx-2c^2\int \sec^2(e+fx)dx)}{a}\right)}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2c^2+c^2\right)dx-2c^2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)}{a}\right)}{3a}$$

$$\Big\downarrow \ 4254$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{2c^2\int 1d(-\tan(e+fx))}{f}+\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2c^2+c^2\right)dx\right)}{a}\right)}{3a}$$

$$\Big\downarrow \ 24$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2c^2+c^2\right)dx-\frac{2c^2\tan(e+fx)}{f}\right)}{a}\right)}{3a}$$

$$\Big\downarrow \text{ 4534}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{3}{2}c^2\int \sec(e+fx)dx-\frac{2c^2\tan(e+fx)}{f}+\frac{c^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}\right)}{3a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{3}{2}c^2\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{2c^2\tan(e+fx)}{f}+\frac{c^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}\right)}{3a}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{3c^2\text{arctanh}(\sin(e+fx))}{2f}-\frac{2c^2\tan(e+fx)}{f}+\frac{c^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}\right)}{3a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

output

```
(2*c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (
7*c*((2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) -
(5*c*((3*c^2*ArcTanh[Sin[e + f*x]])/(2*f) - (2*c^2*Tan[e + f*x])/f + (c^2*
Sec[e + f*x]*Tan[e + f*x])/(2*f)))/a))/(3*a)
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4445

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

rule 4534

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{8c^4\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{13}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f\,a^2}$
default	$\frac{8c^4\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{13}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f\,a^2}$
parallelrisch	$-\frac{51\left(\frac{35(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{51}+\frac{35(-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{51}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\cos\right.\right.}{2f\,a^2(1+\cos(2fx+2e))}$
risch	$-\frac{ic^4\left(99\,\mathrm{e}^{6i(fx+e)}+333\,\mathrm{e}^{5i(fx+e)}+434\,\mathrm{e}^{4i(fx+e)}+714\,\mathrm{e}^{3i(fx+e)}+487\,\mathrm{e}^{2i(fx+e)}+393\,\mathrm{e}^{i(fx+e)}+164\right)}{3f\,a^2\left(\mathrm{e}^{2i(fx+e)}+1\right)^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3}-\frac{35c^4\ln(\mathrm{e}^{i(f}}{2a^2f}$
norman	$\frac{-\frac{35c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{385c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{511c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3af}+\frac{93c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}-\frac{40c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3af}-\frac{8c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^1}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4a}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
8/f*c^4/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-3*tan(1/2*f*x+1/2*e)-1/16/(tan(1/2*
f*x+1/2*e)+1)^2+13/16/(tan(1/2*f*x+1/2*e)+1)+35/16*ln(tan(1/2*f*x+1/2*e)+1
)+1/16/(tan(1/2*f*x+1/2*e)-1)^2+13/16/(tan(1/2*f*x+1/2*e)-1)-35/16*ln(tan(
1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.31

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$

$$=\frac{105\left(c^4\cos\left(fx+e\right)^4+2\,c^4\cos\left(fx+e\right)^3+c^4\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right.\right.}{12\left(a^2f\right.}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
1/12*(105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)
*log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 +
c^4*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(164*c^4*cos(f*x + e)^3 + 2
29*c^4*cos(f*x + e)^2 + 30*c^4*cos(f*x + e) - 3*c^4)*sin(f*x + e))/(a^2*f*
cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{c^4\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int\left(-\frac{4\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{6\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int\left(-\right.\right.}{a^2}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

output
```
c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(-4*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(6*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(-4*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(142) = 284$.

Time = 0.05 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.54

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
-1/6*(c^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + s
in(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 4*c^4
*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^
3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x
+ e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^4*((9*sin(f*x + e)/(c
os(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^2) + 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}+\frac{6\left(13\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-11\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^2}-16\,(a^4c^4\tan(\tfrac{1}{2}\,f}{6\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/6*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 105*c^4*log(abs(tan(
1/2*f*x + 1/2*e) - 1))/a^2 + 6*(13*c^4*tan(1/2*f*x + 1/2*e)^3 - 11*c^4*tan
(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^2) - 16*(a^4*c^4*tan(
1/2*f*x + 1/2*e)^3 + 9*a^4*c^4*tan(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{13\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-11\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2\right)} - \frac{24\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} - \frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} + \frac{35\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

input

```
int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output

```
(13*c^4*tan(e/2 + (f*x)/2)^3 - 11*c^4*tan(e/2 + (f*x)/2))/(f*(a^2*tan(e/2
+ (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) - (24*c^4*tan(e/2 + (f*x
)/2))/(a^2*f) - (8*c^4*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) + (35*c^4*atanh(tan
(e/2 + (f*x)/2)))/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.45

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{c^4\Bigl(-105\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+210\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-105\log(\tan}{}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x)
```

output

```
(c**4*( - 105*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4 + 210*log(tan(
(e + f*x)/2) - 1)*tan((e + f*x)/2)**2 - 105*log(tan((e + f*x)/2) - 1) + 10
5*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4 - 210*log(tan((e + f*x)/2)
 + 1)*tan((e + f*x)/2)**2 + 105*log(tan((e + f*x)/2) + 1) - 16*tan((e + f*
x)/2)**7 - 112*tan((e + f*x)/2)**5 + 350*tan((e + f*x)/2)**3 - 210*tan((e
+ f*x)/2)))/(6*a**2*f*(tan((e + f*x)/2)**4 - 2*tan((e + f*x)/2)**2 + 1))
```

3.44 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 119

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{5c^3\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{5c^3\tan(e+fx)}{a^2f} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10(c^3-c^3\sec(e+fx))\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))}$$

output
```
5*c^3*arctanh(sin(f*x+e))/a^2/f-5*c^3*tan(f*x+e)/a^2/f+2/3*c*(c-c*sec(f*x+
e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-10/3*(c^3-c^3*sec(f*x+e))*tan(f*x+e)
/f/(a^2+a^2*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{8c^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{3a^2f(-1+\sec(e+fx))(1+\sec(e+fx))^2}$$

input

```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

output

```
(-8*c^3*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[2 -
 2*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f
*x])^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 4445, 3042, 4445, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow 4445$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{\sec(e+fx)a+a}dx}{3a}$$

$$\Big\downarrow 3042$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}$$

$$\Big\downarrow 4445$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\int \sec(e+fx)(c-c\sec(e+fx))dx}{a}\right)}{3a}$$

$$\Big\downarrow 3042$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\int\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a}\right)}{3a}$$

$$\Big\downarrow \ 4274$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(c\int\sec(e+fx)dx-c\int\sec^2(e+fx)dx\right)}{a}\right)}{3a}$$

$$\Big\downarrow \ 3042$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(c\int\csc\left(e+fx+\frac{\pi}{2}\right)dx-c\int\csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)}{a}\right)}{3a}$$

$$\Big\downarrow \ 4254$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(\frac{c\int 1d(-\tan(e+fx))}{f}+c\int\csc\left(e+fx+\frac{\pi}{2}\right)dx\right)}{a}\right)}{3a}$$

$$\Big\downarrow \ 24$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(c\int\csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{c\tan(e+fx)}{f}\right)}{a}\right)}{3a}$$

$$\Big\downarrow \ 4257$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(\frac{c\,\text{arctanh}(\sin(e+fx))}{f}-\frac{c\tan(e+fx)}{f}\right)}{a}\right)}{3a}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

output
```
(2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (
5*c*((2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) -
(3*c*((c*ArcTanh[Sin[e + f*x]])/f - (c*Tan[e + f*x])/f))/a))/(3*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{4c^3 (-\frac{\tan(\frac{fx}{2}+\frac{e}{2})^3}{3} - 2\tan(\frac{fx}{2}+\frac{e}{2}) + \frac{1}{4\tan(\frac{fx}{2}+\frac{e}{2})+4} + \frac{5\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{4} + \frac{1}{4\tan(\frac{fx}{2}+\frac{e}{2})-4} - \frac{5\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{4}}{f\,a^2}$
default	$\frac{4c^3 (-\frac{\tan(\frac{fx}{2}+\frac{e}{2})^3}{3} - 2\tan(\frac{fx}{2}+\frac{e}{2}) + \frac{1}{4\tan(\frac{fx}{2}+\frac{e}{2})+4} + \frac{5\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{4} + \frac{1}{4\tan(\frac{fx}{2}+\frac{e}{2})-4} - \frac{5\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{4}}{f\,a^2}$
parallelrisch	$-\frac{5c^3 (\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)\cos(fx+e) - \ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)\cos(fx+e) + \frac{17\sec(\frac{fx}{2}+\frac{e}{2})^2 (\cos(fx+e)+\frac{23\cos(2fx+2e)}{68}}{15}}{f\,a^2\cos(fx+e)}$
risch	$-\frac{2ic^3(12\,\mathrm{e}^{4i(fx+e)}+51\,\mathrm{e}^{3i(fx+e)}+41\,\mathrm{e}^{2i(fx+e)}+57\,\mathrm{e}^{i(fx+e)}+23)}{3f\,a^2(\mathrm{e}^{i(fx+e)}+1)^3(\mathrm{e}^{2i(fx+e)}+1)} + \frac{5c^3\ln(\mathrm{e}^{i(fx+e)}+i)}{a^2f} - \frac{5c^3\ln(\mathrm{e}^{i(fx+e)}-i)}{a^2f}$
norman	$\frac{\frac{10c^3\tan(\frac{fx}{2}+\frac{e}{2})}{af} - \frac{80c^3\tan(\frac{fx}{2}+\frac{e}{2})^3}{3af} + \frac{22c^3\tan(\frac{fx}{2}+\frac{e}{2})^5}{af} - \frac{4c^3\tan(\frac{fx}{2}+\frac{e}{2})^7}{af} - \frac{4c^3\tan(\frac{fx}{2}+\frac{e}{2})^9}{3af}}{(\tan(\frac{fx}{2}+\frac{e}{2})^2-1)^3 a} - \frac{5c^3\ln(\tan(\frac{fx}{2}+\frac{e}{2}}{a^2f}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
4/f*c^3/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-2*tan(1/2*f*x+1/2*e)+1/4/(tan(1/2*f
*x+1/2*e)+1)+5/4*ln(tan(1/2*f*x+1/2*e)+1)+1/4/(tan(1/2*f*x+1/2*e)-1)-5/4*l
n(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{15\,(c^3\cos(fx+e)^3+2\,c^3\cos(fx+e)^2+c^3\cos(fx+e))\log(\sin(fx+e)+1)-15\,(c^3\cos(fx+e)^3+}{6\,(a^2f\cos(fx+e)^3}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
1/6*(15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*log
(sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*c
os(f*x + e))*log(-sin(f*x + e) + 1) - 2*(23*c^3*cos(f*x + e)^2 + 34*c^3*co
s(f*x + e) + 3*c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x
+ e)^2 + a^2*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx =$$

$$-\frac{c^3\Bigg(\int\bigg(-\frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\bigg)\,dx+\int\frac{3\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\bigg(-\frac{3\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\bigg)\,dx -}{a^2}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

output

```
-c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(-3*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(117) = 234.

Time = 0.04 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.87

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx =$$

$$-\frac{c^3\Bigg(\frac{\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\,\log\Big(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\Big)}{a^2}+\frac{12\,\log\Big(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\Big)}{a^2}+\frac{12\,\sin(fx+e)}{\Big(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\Big)(\cos(fx+e)+1)}\Bigg)}{}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
-1/6*(c^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log
(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 3*c^3*((9*sin(f*
x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*l
og(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x
+ e) + 1) - 1)/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) -
 sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$$

$$= \frac{\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} - \frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} + \frac{6\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2} - \frac{4\,(a^4c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6\,a^4c^3\tan(\frac{1}{2}\,fx-}{a^6}}{3\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/3*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 15*c^3*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a^2 + 6*c^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*
e)^2 - 1)*a^2) - 4*(a^4*c^3*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^3*tan(1/2*f*x
 + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{10\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f} - \frac{4\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} - \frac{8\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} + \frac{2\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}$$

input

```
int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output

```
(10*c^3*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (4*c^3*tan(e/2 + (f*x)/2)^3)/
(3*a^2*f) - (8*c^3*tan(e/2 + (f*x)/2))/(a^2*f) + (2*c^3*tan(e/2 + (f*x)/2)
)/(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^3\Bigl(-15\log\bigl(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\bigr)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+15\log\bigl(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\bigr)+15\log\bigl(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\bigr)\tan}{3a^2f\Bigl(\tan\bigl(\frac{fx}{2}+}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x)
```

output

```
(c**3*( - 15*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2 + 15*log(tan((e
 + f*x)/2) - 1) + 15*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2 - 15*lo
g(tan((e + f*x)/2) + 1) - 4*tan((e + f*x)/2)**5 - 20*tan((e + f*x)/2)**3 +
 30*tan((e + f*x)/2)))/(3*a**2*f*(tan((e + f*x)/2)**2 - 1))
```

3.45 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 88

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\operatorname{arctanh}(\sin(e+fx))}{a^2f} - \frac{2c^2\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

output
```
c^2*arctanh(sin(f*x+e))/a^2/f-2*c^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+2/3*
(c^2-c^2*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\left(-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{4\tan\left(\frac{1}{2}(e+fx)\right)}{3f}-\frac{2\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{3f}\right)}{a^2}$$

input

```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

output

```
(c^2*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]]/f - (4*Tan[(e + f*x)/2])/(3*f) - (2*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/(3*f)))/a^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4445, 3042, 4445, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4445}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{\sec(e+fx)a+a}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{a}$$

$$\Big\downarrow \text{4445}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\left(\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\int \sec(e+fx)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\left(\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\int\csc\left(e+fx+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4257$$

$$\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\left(\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\,\mathrm{arctanh}(\sin(e+fx))}{af}\right)}{a}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

output
```
(2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (
c*(-((c*ArcTanh[Sin[e + f*x]])/(a*f)) + (2*c*Tan[e + f*x])/(f*(a + a*Sec[e
 + f*x]))))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2c^{2}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^{2}}$
default	$\frac{2c^{2}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^{2}}$
parallelrisch	$\frac{c^{2}\left(-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^{2}f}$
risch	$-\frac{8ic^{2}\left(3\,\mathrm{e}^{i(fx+e)}+1\right)}{3f\,a^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}}-\frac{c^{2}\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{a^{2}f}+\frac{c^{2}\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{a^{2}f}$
norman	$\frac{-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{10c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3af}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{3af}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}a}+\frac{c^{2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{a^{2}f}-\frac{c^{2}\ln\Big(\tan\Big(}{a}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
2/f*c^2/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-tan(1/2*f*x+1/2*e)-1/2*ln(tan(1/2*f
*x+1/2*e)-1)+1/2*ln(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.57

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{3\left(c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\cos\left(fx+e\right)+c^{2}\right)\log\left(\sin\left(fx+e\right)+1\right)-3\,\Big(c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\cos\,(fx+e}{6\,\Big(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
1/6*(3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(sin(f*x + e) +
1) - 3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(-sin(f*x + e) +
 1) - 8*(c^2*cos(f*x + e) + 2*c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2
*a^2*f*cos(f*x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{2\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int \frac{\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

output

```
c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(-2*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(87) = 174$.

Time = 0.06 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.23

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{2c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-}{6f}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="m
axima")
```

output

```
-1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 2*c^2*(3*sin(f*x + e)/(cos(f*x +
 e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/
(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{3\,c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} - \frac{3\,c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} - \frac{2\left(a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="g
iac")
```

output

```
1/3*(3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 3*c^2*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/a^2 - 2*(a^4*c^2*tan(1/2*f*x + 1/2*e)^3 + 3*a^4*c^2*tan
(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{2\,c^2\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)}{3\,a^2\,f}$$

input

```
int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output
```
-(2*c^2*(3*tan(e/2 + (f*x)/2) - 3*atanh(tan(e/2 + (f*x)/2)) + tan(e/2 + (f
*x)/2)^3))/(3*a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\left(-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^2f}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x)
```

output
```
(c**2*( - 3*log(tan((e + f*x)/2) - 1) + 3*log(tan((e + f*x)/2) + 1) - 2*ta
n((e + f*x)/2)**3 - 6*tan((e + f*x)/2)))/(3*a**2*f)
```

3.46 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\frac{(c-c\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

output
```
1/3*(c-c*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=-\frac{c\tan^3\left(\frac{1}{2}(e+fx)\right)}{3a^2f}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(c*Tan[(e + f*x)/2]^3)/(a^2*f)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \displaystyle \int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a\sec(e+fx)+a)^2}dx \\ \Big\downarrow \text{3042} \\ \displaystyle \int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx \\ \Big\downarrow \text{4438} \\ \displaystyle \frac{\tan(e+fx)(c-c\sec(e+fx))}{3f(a\sec(e+fx)+a)^2} \end{array}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

output
```
((c - c*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$-\frac{c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 f \,a^{2}}$	21
default	$-\frac{c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 f \,a^{2}}$	21
parallelrisch	$-\frac{c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 f \,a^{2}}$	21
risch	$\frac{2 i c \left(3 \,{\mathrm e}^{2 i \left(f x +e \right)}+1\right)}{3 f \,a^{2} \left({\mathrm e}^{i \left(f x +e \right)}+1\right)^{3}}$	37
norman	$\frac{\frac{c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 f a}-\frac{c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{3 f a}}{a \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)}$	61

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
-1/3/f*c/a^2*tan(1/2*f*x+1/2*e)^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\frac{\left(c\cos\left(fx+e\right)-c\right)\sin\left(fx+e\right)}{3\,\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)+a^{2}f\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output
```
1/3*(c*cos(f*x + e) - c)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(
f*x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\left(\int \left(-\frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

output
```
-c*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(35) = 70$.

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.61

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2} - \frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
-1/6*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3)/a^2 - c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{3\,a^2f}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="gia
c")
```

output

```
-1/3*c*tan(1/2*f*x + 1/2*e)^3/(a^2*f)
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=-\frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{3\,a^2\,f}$$

input

```
int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output

```
-(c*tan(e/2 + (f*x)/2)^3)/(3*a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c}{3a^2f}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

output

```
( - tan((e + f*x)/2)**3*c)/(3*a**2*f)
```

3.47 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 59

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\cot^3(e+fx)}{3a^2cf}+\frac{\csc(e+fx)}{a^2cf}-\frac{\csc^3(e+fx)}{3a^2cf}$$

output
```
1/3*cot(f*x+e)^3/a^2/c/f+csc(f*x+e)/a^2/c/f-1/3*csc(f*x+e)^3/a^2/c/f
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{(-1+2\sec(e+fx)+2\sec^2(e+fx))\tan(e+fx)}{3a^2cf(-1+\sec(e+fx))(1+\sec(e+fx))^2}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

output

```
((-1 + 2*Sec[e + f*x] + 2*Sec[e + f*x]^2)*Tan[e + f*x])/(3*a^2*c*f*(-1 + S
ec[e + f*x])*(1 + Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4446}$$

$$\frac{\int \left(c\cot^3(e+fx)\csc(e+fx)-c\cot^2(e+fx)\csc^2(e+fx)\right)dx}{a^2c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c\cot^3(e+fx)}{3f}-\frac{c\csc^3(e+fx)}{3f}+\frac{c\csc(e+fx)}{f}}{a^2c^2}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

output

```
((c*Cot[e + f*x]^3)/(3*f) + (c*Csc[e + f*x])/f - (c*Csc[e + f*x]^3)/(3*f))
/(a^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	48
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	48
parallelrisch	$\frac{-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{12f\,a^{2}c}$	48
norman	$\frac{\frac{1}{4fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2fac}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{12fac}}{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	72
risch	$\frac{2i\left(3\,\mathrm{e}^{3i(fx+e)}+3\,\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}-1\right)}{3f\,a^{2}c\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)}$	72

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
1/4/f/a^2/c*(-1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)+1/tan(1/2*f*x+
1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\cos\left(fx+e\right)^2-2\,\cos\left(fx+e\right)-2}{3\left(a^2cf\cos\left(fx+e\right)+a^2cf\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
-1/3*(cos(f*x + e)^2 - 2*cos(f*x + e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*
f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+\sec^2\left(e+fx\right)-\sec\left(e+fx\right)-1}\,dx}{a^2c}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(sec(e + f*x)/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) -
 1), x)/(a**2*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\frac{\frac{6\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c}+\frac{3(\cos(fx+e)+1)}{a^2c\sin(fx+e)}}{12f}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="max
ima")
```

output

```
1/12*((6*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/(a^2*c) + 3*(cos(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\frac{3}{a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}-\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3}}{12f}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="gia
c")
```

output

```
1/12*(3/(a^2*c*tan(1/2*f*x + 1/2*e)) - (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 6
*a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-8\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1}{12\,a^2\,c\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))),x)
```

output
```
-(4*cos(e/2 + (f*x)/2)^4 - 8*cos(e/2 + (f*x)/2)^2 + 1)/(12*a^2*c*f*cos(e/2
 + (f*x)/2)^3*sin(e/2 + (f*x)/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+3}{12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a^2cf}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

output
```
( - tan((e + f*x)/2)**4 + 6*tan((e + f*x)/2)**2 + 3)/(12*tan((e + f*x)/2)*
a**2*c*f)
```

3.48 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\csc(e+fx)}{a^2c^2f} - \frac{\csc^3(e+fx)}{3a^2c^2f}$$

output
```
csc(f*x+e)/a^2/c^2/f-1/3*csc(f*x+e)^3/a^2/c^2/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\frac{\csc(e+fx)}{f} - \frac{\csc^3(e+fx)}{3f}}{a^2c^2}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

output
```
(Csc[e + f*x]/f - Csc[e + f*x]^3/(3*f))/(a^2*c^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4446, 3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4446}$$

$$\frac{\int \cot^3(e+fx)\csc(e+fx)dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)^3dx}{a^2c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \sec\left(\frac{1}{2}(2e-\pi)+fx\right)\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^3dx}{a^2c^2}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\int \left(\csc^2(e+fx)-1\right)d\csc(e+fx)}{a^2c^2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}\csc^3(e+fx)-\csc(e+fx)}{a^2c^2f}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

output

```
-((-Csc[e + f*x] + Csc[e + f*x]^3/3)/(a^2*c^2*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{-\frac{\csc\left(f x +e \right)^{3}}{3}+\csc\left(f x +e \right)}{a^{2} c^{2} f}$	28
parallelrisch	$-\frac{\left(-1+3 \cos\left(2 f x +2 e \right)\right) \sec\left(\frac{f x}{2}+\frac{e}{2}\right)^{3} \csc\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{48 f \,a^{2} c^{2}}$	47
risch	$\frac{2 i \left(3 \,{\mathrm e}^{5 i \left(f x +e \right)}-2 \,{\mathrm e}^{3 i \left(f x +e \right)}+3 \,{\mathrm e}^{i \left(f x +e \right)}\right)}{3 f \,a^{2} c^{2} \left({\mathrm e}^{i \left(f x +e \right)}+1\right)^{3} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{3}}$	73
norman	$\frac{-\frac{1}{24 f a c}+\frac{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{8 f a c}+\frac{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{8 f a c}-\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{24 f a c}}{a c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	97

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/a^2/c^2/f*(-1/3*csc(f*x+e)^3+csc(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{3\cos{(fx+e)}^2-2}{3\,(a^2c^2f\cos{(fx+e)}^2-a^2c^2f)\sin{(fx+e)}}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
1/3*(3*cos(f*x + e)^2 - 2)/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x
 + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec{(e+fx)}}{\sec^4{(e+fx)}-2\sec^2{(e+fx)}+1}\,dx}{a^2c^2}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*
c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{3\,\sin\left(fx+e\right)^2-1}{3\,a^2c^2f\sin\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{3\,\sin\left(fx+e\right)^2-1}{3\,a^2c^2f\sin\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 10.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\sin\left(e+f\,x\right)^2-\frac{1}{3}}{a^2\,c^2\,f\sin\left(e+f\,x\right)^3}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2),x)
```

output
```
(sin(e + f*x)^2 - 1/3)/(a^2*c^2*f*sin(e + f*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{3\sin\left(fx+e\right)^2-1}{3\sin\left(fx+e\right)^3 a^2c^2f}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

output

```
(3*sin(e + f*x)**2 - 1)/(3*sin(e + f*x)**3*a**2*c**2*f)
```

3.49 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\begin{aligned}&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx\\&=\frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}\end{aligned}$$

output

```
1/5*cot(f*x+e)^5/a^2/c^3/f+csc(f*x+e)/a^2/c^3/f-2/3*csc(f*x+e)^3/a^2/c^3/f
+1/5*csc(f*x+e)^5/a^2/c^3/f
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\begin{aligned}&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx\\&=\frac{(3+12\sec(e+fx)-12\sec^2(e+fx)-8\sec^3(e+fx)+8\sec^4(e+fx))\tan(e+fx)}{15a^2c^3f(-1+\sec(e+fx))^3(1+\sec(e+fx))^2}\end{aligned}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

output

```
((3 + 12*Sec[e + f*x] - 12*Sec[e + f*x]^2 - 8*Sec[e + f*x]^3 + 8*Sec[e + f
*x]^4)*Tan[e + f*x])/(15*a^2*c^3*f*(-1 + Sec[e + f*x])^3*(1 + Sec[e + f*x]
)^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4446}$$

$$-\frac{\int \left(a\csc(e+fx)\cot^5(e+fx)+a\csc^2(e+fx)\cot^4(e+fx)\right)dx}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{a\cot^5(e+fx)}{5f}-\frac{a\csc^5(e+fx)}{5f}+\frac{2a\csc^3(e+fx)}{3f}-\frac{a\csc(e+fx)}{f}}{a^3c^3}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

output

```
-((-1/5*(a*Cot[e + f*x]^5)/f - (a*Csc[e + f*x])/f + (2*a*Csc[e + f*x]^3)/(
3*f) - (a*Csc[e + f*x]^5)/(5*f))/(a^3*c^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,a^{2}c^{3}}$	76
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,a^{2}c^{3}}$	76
parallelrisch	$\frac{\left(25+3\cos\left(4fx+4e\right)+24\cos\left(3fx+3e\right)-36\cos\left(2fx+2e\right)+8\cos\left(fx+e\right)\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{1920f\,a^{2}c^{3}}$	77
risch	$\frac{2i\left(15\,{\mathrm e}^{7i\left(fx+e\right)}-15\,{\mathrm e}^{6i\left(fx+e\right)}-5\,{\mathrm e}^{5i\left(fx+e\right)}+25\,{\mathrm e}^{4i\left(fx+e\right)}+13\,{\mathrm e}^{3i\left(fx+e\right)}-21\,{\mathrm e}^{2i\left(fx+e\right)}+9\,{\mathrm e}^{i\left(fx+e\right)}+3\right)}{15f\,a^{2}c^{3}\left({\mathrm e}^{i\left(fx+e\right)}+1\right)^{3}\left({\mathrm e}^{i\left(fx+e\right)}-1\right)^{5}}$	118
norman	$\frac{\frac{1}{80fac}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{12fac}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{8fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{4fac}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{48fac}}{a\,c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$	119

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/16/f/a^2/c^3*(-1/3*tan(1/2*f*x+1/2*e)^3+4*tan(1/2*f*x+1/2*e)-4/3/tan(1/2
*f*x+1/2*e)^3+1/5/tan(1/2*f*x+1/2*e)^5+6/tan(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{3\cos\left(fx+e\right)^4+12\cos\left(fx+e\right)^3-12\cos\left(fx+e\right)^2-8\cos\left(fx+e\right)+8}{15\left(a^2c^3f\cos\left(fx+e\right)^3-a^2c^3f\cos\left(fx+e\right)^2-a^2c^3f\cos\left(fx+e\right)+a^2c^3f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/15*(3*cos(f*x + e)^4 + 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 - 8*cos(f*x
 + e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3
*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-\sec^4\left(e+fx\right)-2\sec^3\left(e+fx\right)+2\sec^2\left(e+fx\right)+\sec\left(e+fx\right)-1}\,dx}{a^2c^3}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(sec(e + f*x)/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)
**3 + 2*sec(e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\frac{5\left(\frac{12\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{90\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{a^2c^3\sin(fx+e)^5}}{240\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/240*(5*(12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/(a^2*c^3) - (20*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 90*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x +
e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\frac{90\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-20\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3}{a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}-\frac{5\left(a^4c^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-12a^4c^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6c^9}}{240\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
1/240*((90*tan(1/2*f*x + 1/2*e)^4 - 20*tan(1/2*f*x + 1/2*e)^2 + 3)/(a^2*c^
3*tan(1/2*f*x + 1/2*e)^5) - 5*(a^4*c^6*tan(1/2*f*x + 1/2*e)^3 - 12*a^4*c^6
*tan(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

Mupad [B] (verification not implemented)

Time = 11.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+60\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+90\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3}{240\,a^2\,c^3\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3),x)
```

output
```
(90*tan(e/2 + (f*x)/2)^4 - 20*tan(e/2 + (f*x)/2)^2 + 60*tan(e/2 + (f*x)/2)
^6 - 5*tan(e/2 + (f*x)/2)^8 + 3)/(240*a^2*c^3*f*tan(e/2 + (f*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+60\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+3}{240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5a^2c^3f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

output
```
( - 5*tan((e + f*x)/2)**8 + 60*tan((e + f*x)/2)**6 + 90*tan((e + f*x)/2)**
4 - 20*tan((e + f*x)/2)**2 + 3)/(240*tan((e + f*x)/2)**5*a**2*c**3*f)
```

3.50 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 98

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc^5(e+fx)}{a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}$$

output

```
-2/7*cot(f*x+e)^7/a^2/c^4/f+csc(f*x+e)/a^2/c^4/f-4/3*csc(f*x+e)^3/a^2/c^4/
f+csc(f*x+e)^5/a^2/c^4/f-2/7*csc(f*x+e)^7/a^2/c^4/f
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{(-6-9\sec(e+fx)+24\sec^2(e+fx)-4\sec^3(e+fx)-16\sec^4(e+fx)+8\sec^5(e+fx))\tan(e+fx}{21a^2c^4f(-1+\sec(e+fx))^4(1+\sec(e+fx))^2}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

output
```
((-6 - 9*Sec[e + f*x] + 24*Sec[e + f*x]^2 - 4*Sec[e + f*x]^3 - 16*Sec[e +
f*x]^4 + 8*Sec[e + f*x]^5)*Tan[e + f*x])/(21*a^2*c^4*f*(-1 + Sec[e + f*x])
^4*(1 + Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4446}$$

$$\frac{\int \left(a^2\csc(e+fx)\cot^7(e+fx)+2a^2\csc^2(e+fx)\cot^6(e+fx)+a^2\csc^3(e+fx)\cot^5(e+fx)\right)dx}{a^4c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2a^2\cot^7(e+fx)}{7f}-\frac{2a^2\csc^7(e+fx)}{7f}+\frac{a^2\csc^5(e+fx)}{f}-\frac{4a^2\csc^3(e+fx)}{3f}+\frac{a^2\csc(e+fx)}{f}}{a^4c^4}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

output
```
((-2*a^2*Cot[e + f*x]^7)/(7*f) + (a^2*Csc[e + f*x])/f - (4*a^2*Csc[e + f*x
]^3)/(3*f) + (a^2*Csc[e + f*x]^5)/f - (2*a^2*Csc[e + f*x]^7)/(7*f))/(a^4*c
^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{10}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{10}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}}{32f\,a^{2}c^{4}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{10}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{10}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}}{32f\,a^{2}c^{4}}$
parallelrisch	$-\frac{(-21+3\cos(5fx+5e)+9\cos(4fx+4e)-33\cos(3fx+3e)+52\cos(2fx+2e)+14\cos(fx+e))\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\csc\big(\frac{fx}{2}+\frac{e}{2}}{5376f\,a^{2}c^{4}}$
risch	$\frac{2i\big(21\,\mathrm{e}^{9i(fx+e)}-42\,\mathrm{e}^{8i(fx+e)}+28\,\mathrm{e}^{7i(fx+e)}+56\,\mathrm{e}^{6i(fx+e)}-42\,\mathrm{e}^{5i(fx+e)}-28\,\mathrm{e}^{4i(fx+e)}+76\,\mathrm{e}^{3i(fx+e)}-24\,\mathrm{e}^{2i(fx+e)}-3}{21f\,a^{2}c^{4}\left(\mathrm{e}^{i(fx+e)}-1\right)^{7}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}}$
norman	$\frac{-\frac{1}{224fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{32fac}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{48fac}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{16fac}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{32fac}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{96fac}}{a\,c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/32/f/a^2/c^4*(-1/3*tan(1/2*f*x+1/2*e)^3+5*tan(1/2*f*x+1/2*e)-10/3/tan(1/
2*f*x+1/2*e)^3+10/tan(1/2*f*x+1/2*e)+1/tan(1/2*f*x+1/2*e)^5-1/7/tan(1/2*f*
x+1/2*e)^7)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{6\cos\left(fx+e\right)^5+9\cos\left(fx+e\right)^4-24\cos\left(fx+e\right)^3+4\cos\left(fx+e\right)^2+16\cos\left(fx+e\right)-8}{21\left(a^2c^4f\cos\left(fx+e\right)^4-2\,a^2c^4f\cos\left(fx+e\right)^3+2\,a^2c^4f\cos\left(fx+e\right)-a^2c^4f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="f
ricas")
```

output
```
1/21*(6*cos(f*x + e)^5 + 9*cos(f*x + e)^4 - 24*cos(f*x + e)^3 + 4*cos(f*x
+ e)^2 + 16*cos(f*x + e) - 8)/((a^2*c^4*f*cos(f*x + e)^4 - 2*a^2*c^4*f*cos
(f*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{\sec^6\left(e+fx\right)-2\sec^5\left(e+fx\right)-\sec^4\left(e+fx\right)+4\sec^3\left(e+fx\right)-\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx}{a^2c^4}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

output
```
Integral(sec(e + f*x)/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)*
*4 + 4*sec(e + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c
**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.43

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\frac{7\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^4}+\frac{\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{70\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{210\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)(\cos(fx+e)+1)^7}{a^2c^4\sin(fx+e)^7}}{672\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="m
axima")
```

output
```
1/672*(7*(15*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/(a^2*c^4) + (21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 210*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3
)*(cos(f*x + e) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\frac{210\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+21\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3}{a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}-\frac{7\left(a^4c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-15\,a^4c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6c^{12}}}{672\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="g
iac")
```

output
```
1/672*((210*tan(1/2*f*x + 1/2*e)^6 - 70*tan(1/2*f*x + 1/2*e)^4 + 21*tan(1/
2*f*x + 1/2*e)^2 - 3)/(a^2*c^4*tan(1/2*f*x + 1/2*e)^7) - 7*(a^4*c^8*tan(1/
2*f*x + 1/2*e)^3 - 15*a^4*c^8*tan(1/2*f*x + 1/2*e))/(a^6*c^12))/f
```

Mupad [B] (verification not implemented)

Time = 12.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+105\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+210\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-70\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-3}{672\,a^2\,c^4\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4),x)
```

output
```
(21*tan(e/2 + (f*x)/2)^2 - 70*tan(e/2 + (f*x)/2)^4 + 210*tan(e/2 + (f*x)/2
)^6 + 105*tan(e/2 + (f*x)/2)^8 - 7*tan(e/2 + (f*x)/2)^10 - 3)/(672*a^2*c^4
*f*tan(e/2 + (f*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+210\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-70\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3}{672\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}a^2c^4f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

output
```
( - 7*tan((e + f*x)/2)**10 + 105*tan((e + f*x)/2)**8 + 210*tan((e + f*x)/2
)**6 - 70*tan((e + f*x)/2)**4 + 21*tan((e + f*x)/2)**2 - 3)/(672*tan((e +
f*x)/2)**7*a**2*c**4*f)
```

3.51 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 141

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx\\ =\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}\\ +\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}\end{aligned}$$

output

```
1/7*cot(f*x+e)^7/a^2/c^5/f+4/9*cot(f*x+e)^9/a^2/c^5/f+csc(f*x+e)/a^2/c^5/f
-7/3*csc(f*x+e)^3/a^2/c^5/f+3*csc(f*x+e)^5/a^2/c^5/f-13/7*csc(f*x+e)^7/a^2
/c^5/f+4/9*csc(f*x+e)^9/a^2/c^5/f
```

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx\\ =\frac{(19+6\sec(e+fx)-66\sec^2(e+fx)+56\sec^3(e+fx)+24\sec^4(e+fx)-48\sec^5(e+fx)+16\sec^6(e}{63a^2c^5f(-1+\sec(e+fx))^5(1+\sec(e+fx))^2}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

output
```
((19 + 6*Sec[e + f*x] - 66*Sec[e + f*x]^2 + 56*Sec[e + f*x]^3 + 24*Sec[e +
 f*x]^4 - 48*Sec[e + f*x]^5 + 16*Sec[e + f*x]^6)*Tan[e + f*x])/(63*a^2*c^5
*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4446}$$

$$-\frac{\int (a^3\csc(e+fx)\cot^9(e+fx)+3a^3\csc^2(e+fx)\cot^8(e+fx)+3a^3\csc^3(e+fx)\cot^7(e+fx)+a^3\csc^4(e+}{a^5c^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{4a^3\cot^9(e+fx)}{9f}-\frac{a^3\cot^7(e+fx)}{7f}-\frac{4a^3\csc^9(e+fx)}{9f}+\frac{13a^3\csc^7(e+fx)}{7f}-\frac{3a^3\csc^5(e+fx)}{f}+\frac{7a^3\csc^3(e+fx)}{3f}-\frac{a^3\csc(e+fx)}{f}}{a^5c^5}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

output
```
-((-1/7*(a^3*Cot[e + f*x]^7)/f - (4*a^3*Cot[e + f*x]^9)/(9*f) - (a^3*Csc[e
 + f*x])/f + (7*a^3*Csc[e + f*x]^3)/(3*f) - (3*a^3*Csc[e + f*x]^5)/f + (13
*a^3*Csc[e + f*x]^7)/(7*f) - (4*a^3*Csc[e + f*x]^9)/(9*f))/(a^5*c^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

method	result
parallelrisch	$-\frac{(-294+150\cos(4fx+4e)+387\cos(2fx+2e)+72\cos(fx+e)-19\cos(6fx+6e)-12\cos(5fx+5e)-508\cos(3fx+3e))}{129024f\,a^2c^5}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{20}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{15}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{6}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}}{64f\,a^2c^5}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{20}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{15}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{6}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}}{64f\,a^2c^5}$
risch	$\frac{2i\left(63\,e^{11i(fx+e)}-189\,e^{10i(fx+e)}+273\,e^{9i(fx+e)}+63\,e^{8i(fx+e)}-378\,e^{7i(fx+e)}+294\,e^{6i(fx+e)}+306\,e^{5i(fx+e)}-450\,e^{4i(}\right.}{63f\,a^2c^5\left(e^{i(fx+e)}-1\right)^9\left(e^{i(fx+e)}+1\right)^3}$
norman	$\frac{\frac{1}{576fac}-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{224fac}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{64fac}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{48fac}+\frac{15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{64fac}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{32fac}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{192fac}}{a\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBO
SE)
```

output
```
-1/129024*(-294+150*cos(4*f*x+4*e)+387*cos(2*f*x+2*e)+72*cos(f*x+e)-19*cos
(6*f*x+6*e)-12*cos(5*f*x+5*e)-508*cos(3*f*x+3*e))*sec(1/2*f*x+1/2*e)^3*csc
(1/2*f*x+1/2*e)^9/f/a^2/c^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{19\,\cos{(fx+e)}^6 + 6\,\cos{(fx+e)}^5 - 66\,\cos{(fx+e)}^4 + 56\,\cos{(fx+e)}^3 + 24\,\cos{(fx+e)}^2}{63\,(a^2c^5f\cos{(fx+e)}^5 - 3\,a^2c^5f\cos{(fx+e)}^4 + 2\,a^2c^5f\cos{(fx+e)}^3 + 2\,a^2c^5f\cos{(fx+e)}^2 - 3\,a^2c^5f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="f
ricas")
```

output
```
1/63*(19*cos(f*x + e)^6 + 6*cos(f*x + e)^5 - 66*cos(f*x + e)^4 + 56*cos(f*
x + e)^3 + 24*cos(f*x + e)^2 - 48*cos(f*x + e) + 16)/((a^2*c^5*f*cos(f*x +
 e)^5 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^
5*f*cos(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$$
$$= -\frac{\int \frac{\sec{(e+fx)}}{\sec^7{(e+fx)}-3\sec^6{(e+fx)}+\sec^5{(e+fx)}+5\sec^4{(e+fx)}-5\sec^3{(e+fx)}-\sec^2{(e+fx)}+3\sec{(e+fx)}-1}\,dx}{a^2c^5}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

output
```
-Integral(sec(e + f*x)/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)
**5 + 5*sec(e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e +
f*x) - 1), x)/(a**2*c**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.14

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{\frac{21\left(\frac{18\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^5}-\frac{\left(\frac{54\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{189\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{945\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx+e)+1)^9}{a^2c^5\sin(fx+e)^9}}{4032\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="m
axima")
```

output
```
1/4032*(21*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a^2*c^5) - (54*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- 945*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(a^2*c
^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{\frac{945\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-420\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+189\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-54\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7}{a^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}-\frac{21\left(a^4c^{10}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-18a^4c^{10}\tan\left(\frac{1}{2}fx\right.\right.}{a^6c^{15}}}{4032\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="g
iac")
```

output
```
1/4032*((945*tan(1/2*f*x + 1/2*e)^8 - 420*tan(1/2*f*x + 1/2*e)^6 + 189*tan
(1/2*f*x + 1/2*e)^4 - 54*tan(1/2*f*x + 1/2*e)^2 + 7)/(a^2*c^5*tan(1/2*f*x
+ 1/2*e)^9) - 21*(a^4*c^10*tan(1/2*f*x + 1/2*e)^3 - 18*a^4*c^10*tan(1/2*f*
x + 1/2*e))/(a^6*c^15))/f
```

Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{-21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+378\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+945\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-420\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+189\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-}{4032\,a^2\,c^5\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5),x)
```

output
```
(189*tan(e/2 + (f*x)/2)^4 - 54*tan(e/2 + (f*x)/2)^2 - 420*tan(e/2 + (f*x)/
2)^6 + 945*tan(e/2 + (f*x)/2)^8 + 378*tan(e/2 + (f*x)/2)^10 - 21*tan(e/2 +
 (f*x)/2)^12 + 7)/(4032*a^2*c^5*f*tan(e/2 + (f*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{-21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}+378\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+945\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}-420\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+189\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-}{4032\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}a^2c^5f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)
```

output
```
( - 21*tan((e + f*x)/2)**12 + 378*tan((e + f*x)/2)**10 + 945*tan((e + f*x)
/2)**8 - 420*tan((e + f*x)/2)**6 + 189*tan((e + f*x)/2)**4 - 54*tan((e + f
*x)/2)**2 + 7)/(4032*tan((e + f*x)/2)**9*a**2*c**5*f)
```

3.52 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 215

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = &-\frac{231c^6\operatorname{arctanh}(\sin(e+fx))}{2a^3f}\\ &+\frac{924c^6\tan(e+fx)}{5a^3f}\\ &-\frac{693c^6\sec(e+fx)\tan(e+fx)}{10a^3f}\\ &-\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\\ &+\frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\ &+\frac{66\left(c^2-c^2\sec(e+fx)\right)^3\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}\\ &+\frac{77c^6\tan^3(e+fx)}{5a^3f}\end{aligned}$$

output
```
-231/2*c^6*arctanh(sin(f*x+e))/a^3/f+924/5*c^6*tan(f*x+e)/a^3/f-693/10*c^6
*sec(f*x+e)*tan(f*x+e)/a^3/f-22/15*c^2*(c-c*sec(f*x+e))^4*tan(f*x+e)/a/f/(
a+a*sec(f*x+e))^2+2/5*c*(c-c*sec(f*x+e))^5*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
+66/5*(c^2-c^2*sec(f*x+e))^3*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+77/5*c^6*ta
n(f*x+e)^3/a^3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.35

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = -\frac{64c^6\,\text{Hypergeometric2F1}\left(-\frac{11}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{5a^3f(-1+\sec(e+fx))(1+\sec(e+fx))^3}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

output
```
(-64*c^6*Hypergeometric2F1[-11/2, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[2
 - 2*Sec[e + f*x]]*Tan[e + f*x])/(5*a^3*f*(-1 + Sec[e + f*x])*(1 + Sec[e +
 f*x])^3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3042, 4445, 3042, 4445, 3042, 4445, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}-\frac{11c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(\sec(e+fx)a+a)^2}dx}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}-\frac{11c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}$$

$$\Big\downarrow \text{ 4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}-\frac{11c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\int\frac{\sec(e+fx)(c-c\sec(e+fx))^4}{\sec(e+fx)a+a}dx}{a}\right)}{5a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}-\frac{11c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{a}\right)}{5a}$$

$$\Big\downarrow \text{ 4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}-\frac{11c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\int\sec(e+fx)(c-c\sec(e+fx))^3dx}{a}\right)}{a}\right)}{5a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}-\frac{11c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\int\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx}{a}\right)}{a}\right)}{5a}$$

$$\Big\downarrow \text{ 4278}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}-\frac{11c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\int\left(-c^3\sec^4(e+fx)+3c^3\sec^3(e+fx)-3c^3\sec^2(e+fx)+c^3\sec(e+fx)\right)dx}{a}\right.}{a}\right.}{5a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}-\frac{11c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}-\frac{3c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{7c\left(\frac{5c^3\text{arctanh}(\sin(e+fx))}{2f}-\frac{c^3\tan^3(e+fx)}{3f}-\frac{4c^3\tan(e+fx)}{f}+\frac{3c^3\tan}{}\right.}{a}\right.}{a}\right.}{5a}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

output
```
(2*c*(c - c*Sec[e + f*x])^5*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (
11*c*((2*c*(c - c*Sec[e + f*x])^4*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^
2) - (3*c*((2*c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x
])) - (7*c*((5*c^3*ArcTanh[Sin[e + f*x]])/(2*f) - (4*c^3*Tan[e + f*x])/f +
 (3*c^3*Sec[e + f*x]*Tan[e + f*x])/(2*f) - (c^3*Tan[e + f*x]^3)/(3*f)))/a)
)/a))/(5*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{16c^{6}\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{5}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{89}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.}\right.}{f}$
default	$\frac{16c^{6}\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{5}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{89}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.}\right.}{f}$
parallelrisch	$\frac{2723c^{6}\left(\frac{3960(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{389}+\frac{3960(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{389}+\tan\left(\right.\right.}{240f\,a^{3}(\cos(3fx}$
risch	$\frac{ic^{6}\left(3495\,\mathrm{e}^{10i(fx+e)}+17205\,\mathrm{e}^{9i(fx+e)}+44480\,\mathrm{e}^{8i(fx+e)}+79450\,\mathrm{e}^{7i(fx+e)}+120176\,\mathrm{e}^{6i(fx+e)}+130340\,\mathrm{e}^{5i(fx+e)}+12749\right.}{15a^{3}f\left(\mathrm{e}^{2i(fx+e)}+1\right)^{3}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
16/f*c^6/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+4/3*tan(1/2*f*x+1/2*e)^3+10*tan(1/2
*f*x+1/2*e)-1/48/(tan(1/2*f*x+1/2*e)+1)^3+5/16/(tan(1/2*f*x+1/2*e)+1)^2-89
/32/(tan(1/2*f*x+1/2*e)+1)-231/32*ln(tan(1/2*f*x+1/2*e)+1)-1/48/(tan(1/2*f
*x+1/2*e)-1)^3-5/16/(tan(1/2*f*x+1/2*e)-1)^2-89/32/(tan(1/2*f*x+1/2*e)-1)+
231/32*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{3465\left(c^6\cos\left(fx+e\right)^6+3\,c^6\cos\left(fx+e\right)^5+3\,c^6\cos\left(fx+e\right)^4+c^6\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+\right.}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
-1/60*(3465*(c^6*cos(f*x + e)^6 + 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e
)^4 + c^6*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 3465*(c^6*cos(f*x + e)^6
 + 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e)^4 + c^6*cos(f*x + e)^3)*log(-
sin(f*x + e) + 1) - 2*(5446*c^6*cos(f*x + e)^5 + 12843*c^6*cos(f*x + e)^4
+ 8711*c^6*cos(f*x + e)^3 + 815*c^6*cos(f*x + e)^2 - 105*c^6*cos(f*x + e)
+ 10*c^6)*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)^5 + 3
*a^3*f*cos(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$$

$$= \frac{c^6\left(\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{6\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int \frac{}{\sec^3\left(e+fx\right)+3}\right.}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

output
```
c**6*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) + 1), x) + Integral(-6*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*sec(e + f*x)**3/(sec(e +
 f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-20*sec(
e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x)
 + Integral(15*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(-6*sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec
(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**7/(sec(e +
 f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(204) = 408$.

Time = 0.06 (sec) , antiderivative size = 935, normalized size of antiderivative = 4.35

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/60*(c^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*
x + e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3
 + 690*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^6*(60*(5*sin(f*
x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3 -
2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(si
n(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x + e
) + 1) - 1)/a^3) + 45*c^6*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin
(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 20*c^6*((105*sin(f*x + e)/(cos(f*
x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3
+ 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^6*(15*sin(f*x...
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{\frac{3465\,c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{3465\,c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{10\left(267\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-472\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+213\,c^6\tan}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^3}}{30\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
-1/30*(3465*c^6*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 3465*c^6*log(abs(
tan(1/2*f*x + 1/2*e) - 1))/a^3 + 10*(267*c^6*tan(1/2*f*x + 1/2*e)^5 - 472*
c^6*tan(1/2*f*x + 1/2*e)^3 + 213*c^6*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x +
 1/2*e)^2 - 1)^3*a^3) - 32*(3*a^12*c^6*tan(1/2*f*x + 1/2*e)^5 + 20*a^12*c^
6*tan(1/2*f*x + 1/2*e)^3 + 150*a^12*c^6*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.90

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \frac{160\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f} - \frac{89\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{472\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+71\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)} + \frac{64\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f} + \frac{16\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f} - \frac{231\,c^6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}$$

input
```
int((c - c/cos(e + f*x))^6/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(160*c^6*tan(e/2 + (f*x)/2))/(a^3*f) - (89*c^6*tan(e/2 + (f*x)/2)^5 - (472
*c^6*tan(e/2 + (f*x)/2)^3)/3 + 71*c^6*tan(e/2 + (f*x)/2))/(f*(3*a^3*tan(e/
2 + (f*x)/2)^2 - 3*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^6 - a
^3)) + (64*c^6*tan(e/2 + (f*x)/2)^3)/(3*a^3*f) + (16*c^6*tan(e/2 + (f*x)/2
)^5)/(5*a^3*f) - (231*c^6*atanh(tan(e/2 + (f*x)/2)))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.43

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$$

$$= \frac{c^6 \Big(3465 \log \big(\tan \left(\frac{fx}{2}+\frac{e}{2}\right)-1\big) \tan \left(\frac{fx}{2}+\frac{e}{2}\right)^6 - 10395 \log \big(\tan \left(\frac{fx}{2}+\frac{e}{2}\right)-1\big) \tan \left(\frac{fx}{2}+\frac{e}{2}\right)^4 + 10395 \log \big(}{}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x)
```

output
```
(c**6*(3465*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**6 - 10395*log(tan(
(e + f*x)/2) - 1)*tan((e + f*x)/2)**4 + 10395*log(tan((e + f*x)/2) - 1)*ta
n((e + f*x)/2)**2 - 3465*log(tan((e + f*x)/2) - 1) - 3465*log(tan((e + f*x
)/2) + 1)*tan((e + f*x)/2)**6 + 10395*log(tan((e + f*x)/2) + 1)*tan((e + f
*x)/2)**4 - 10395*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2 + 3465*log
(tan((e + f*x)/2) + 1) + 96*tan((e + f*x)/2)**11 + 352*tan((e + f*x)/2)**9
 + 3168*tan((e + f*x)/2)**7 - 15246*tan((e + f*x)/2)**5 + 18480*tan((e + f
*x)/2)**3 - 6930*tan((e + f*x)/2)))/(30*a**3*f*(tan((e + f*x)/2)**6 - 3*ta
n((e + f*x)/2)**4 + 3*tan((e + f*x)/2)**2 - 1))
```

3.53 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 193

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = &-\frac{63c^5\operatorname{arctanh}(\sin(e+fx))}{2a^3f}+\frac{42c^5\tan(e+fx)}{a^3f}\\ &-\frac{21c^5\sec(e+fx)\tan(e+fx)}{2a^3f}\\ &-\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2}\\ &+\frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\ &+\frac{42c\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}\end{aligned}$$

output
```
-63/2*c^5*arctanh(sin(f*x+e))/a^3/f+42*c^5*tan(f*x+e)/a^3/f-21/2*c^5*sec(f
*x+e)*tan(f*x+e)/a^3/f-6/5*c^2*(c-c*sec(f*x+e))^3*tan(f*x+e)/a/f/(a+a*sec(
f*x+e))^2+2/5*c*(c-c*sec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+42/5*c*
(c^2-c^2*sec(f*x+e))^2*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.93 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.39

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{32c^5\,\text{Hypergeometric2F1}\left(-\frac{9}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{5a^3f(-1+\sec(e+fx))(1+\sec(e+fx))^3}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

output
```
(-32*c^5*Hypergeometric2F1[-9/2, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[2
- 2*Sec[e + f*x]]*Tan[e + f*x])/(5*a^3*f*(-1 + Sec[e + f*x])*(1 + Sec[e +
f*x])^3)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 4445, 3042, 4445, 3042, 4445, 3042, 4275, 3042, 4254, 24, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-\frac{9c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(\sec(e+fx)a+a)^2}dx}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-\frac{9c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-\frac{9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\int\frac{\sec(e+fx)(c-c\sec(e+fx))^3}{\sec(e+fx)a+a}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-\frac{9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-\frac{9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\int\sec(e+fx)(c-c\sec(e+fx))^2dx}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-\frac{9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\int\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{4275}$$

$$\frac{\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\int \sec(e+fx)\left(\sec^2(e+fx)c^2+c^2\right)dx-2c^2\int \sec^2(e+fx)dx\right)}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2c^2+c^2\right)dx-2c^2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)}{a}\right.}{3a}\right.}{5a}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{2c^2\int 1d(-\tan(e+fx))}{f}+\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2c^2+c^2\right)dx\right)}{a}\right.}{3a}\right.}{5a}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2c^2+c^2\right)dx-\frac{2c^2\tan(e+fx)}{f}\right)}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \; 4534$$

$$\frac{\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-}{}\frac{9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{3}{2}c^2\int\sec(e+fx)dx-\frac{2c^2\tan(e+fx)}{f}+\frac{c^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-}{}\frac{9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{3}{2}c^2\int\csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{2c^2\tan(e+fx)}{f}+\frac{c^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}\right)}{3a}\right.}{5a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}-}{}\frac{9c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}-\frac{5c\left(\frac{3c^2\text{arctanh}(\sin(e+fx))}{2f}-\frac{2c^2\tan(e+fx)}{f}+\frac{c^2\tan(e+fx)\sec(e+fx)}{2f}\right)}{a}\right.}{3a}\right.}{5a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

output

```
(2*c*(c - c*Sec[e + f*x])^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (
9*c*((2*c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2
) - (7*c*((2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]
)) - (5*c*((3*c^2*ArcTanh[Sin[e + f*x]])/(2*f) - (2*c^2*Tan[e + f*x])/f +
(c^2*Sec[e + f*x]*Tan[e + f*x])/(2*f)))/a))/(3*a)))/(5*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{8c^5\bigg(\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}+\tan(\frac{fx}{2}+\frac{e}{2})^3+6\tan(\frac{fx}{2}+\frac{e}{2})+\frac{1}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}-\frac{17}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)}-\frac{63\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{16}-}{f\,a^3}$
default	$\frac{8c^5\bigg(\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}+\tan(\frac{fx}{2}+\frac{e}{2})^3+6\tan(\frac{fx}{2}+\frac{e}{2})+\frac{1}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}-\frac{17}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)}-\frac{63\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{16}-}{f\,a^3}$
parallelrisch	$\frac{3749\bigg(\frac{2520(1+\cos(2fx+2e))\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{3749}+\frac{2520(-1-\cos(2fx+2e))\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{3749}+\tan(\frac{fx}{2}+\frac{e}{2})\sec(\frac{fx}{2}+\frac{e}{2})^4\bigg(}{80f\,a^3(1+\cos(2fx+2e))}$
risch	$\frac{ic^5(325\,\mathrm{e}^{8i(fx+e)}+1545\,\mathrm{e}^{7i(fx+e)}+3805\,\mathrm{e}^{6i(fx+e)}+5545\,\mathrm{e}^{5i(fx+e)}+7351\,\mathrm{e}^{4i(fx+e)}+6115\,\mathrm{e}^{3i(fx+e)}+4407\,\mathrm{e}^{2i(fx+e)}+}{5f\,a^3(\mathrm{e}^{i(fx+e)}+1)^5(\mathrm{e}^{2i(fx+e)}+1)^2}$
norman	$\frac{-\frac{63c^5\tan(\frac{fx}{2}+\frac{e}{2})}{af}+\frac{294c^5\tan(\frac{fx}{2}+\frac{e}{2})^3}{af}-\frac{2688c^5\tan(\frac{fx}{2}+\frac{e}{2})^5}{5af}+\frac{474c^5\tan(\frac{fx}{2}+\frac{e}{2})^7}{af}-\frac{193c^5\tan(\frac{fx}{2}+\frac{e}{2})^9}{af}+\frac{24c^5\tan(\frac{fx}{2}+}{af}}{\left(\tan(\frac{fx}{2}+\frac{e}{2})^2-1\right)^5a^2}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
8/f*c^5/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+tan(1/2*f*x+1/2*e)^3+6*tan(1/2*f*x+1
/2*e)+1/16/(tan(1/2*f*x+1/2*e)+1)^2-17/16/(tan(1/2*f*x+1/2*e)+1)-63/16*ln(
tan(1/2*f*x+1/2*e)+1)-1/16/(tan(1/2*f*x+1/2*e)-1)^2-17/16/(tan(1/2*f*x+1/2
*e)-1)+63/16*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.30

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{315\left(c^5\cos(fx+e)^5+3\,c^5\cos(fx+e)^4+3\,c^5\cos(fx+e)^3+c^5\cos(fx+e)^2\right)\log(\sin(fx+e)+1}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
-1/20*(315*(c^5*cos(f*x + e)^5 + 3*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)
^3 + c^5*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 +
 3*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + c^5*cos(f*x + e)^2)*log(-si
n(f*x + e) + 1) - 2*(496*c^5*cos(f*x + e)^4 + 1163*c^5*cos(f*x + e)^3 + 80
1*c^5*cos(f*x + e)^2 + 65*c^5*cos(f*x + e) - 5*c^5)*sin(f*x + e))/(a^3*f*c
os(f*x + e)^5 + 3*a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*co
s(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{c^5\left(\int\left(-\frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int\frac{5\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{}{\sec^3(e-}\right.\right.}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

output
```
-c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(5*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
 + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**3/(sec(e
 + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec
(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x
) + Integral(-5*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*s
ec(e + f*x) + 1), x) + Integral(sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e
 + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(186) = 372$.

Time = 0.06 (sec) , antiderivative size = 680, normalized size of antiderivative = 3.52

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output

```
1/60*(c^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) +
 40*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^5*(40*sin(f*x + e)/((a^3 -
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x
 + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^5*((1
05*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(
f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3)
+ 10*c^5*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^5*(15*sin(f*x
 + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 15*c^5*(5*sin(f*x + e)/(cos(f*x + e
) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{10\left(17\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^3}-\frac{16\,(a^{12}c^5\,\mathrm{ta}}{}}{10\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
-1/10*(315*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 315*c^5*log(abs(ta
n(1/2*f*x + 1/2*e) - 1))/a^3 + 10*(17*c^5*tan(1/2*f*x + 1/2*e)^3 - 15*c^5*
tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^3) - 16*(a^12*c^5*
tan(1/2*f*x + 1/2*e)^5 + 5*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^5*t
an(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx &= \frac{48\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f} - \frac{17\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-15\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^3\right)}\\&\quad+\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{a^3\,f}+\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}-\frac{63\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}\end{aligned}$$

input
```
int((c - c/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(48*c^5*tan(e/2 + (f*x)/2))/(a^3*f) - (17*c^5*tan(e/2 + (f*x)/2)^3 - 15*c^
5*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (f*x)/2)^4 - 2*a^3*tan(e/2 + (f*x)
/2)^2 + a^3)) + (8*c^5*tan(e/2 + (f*x)/2)^3)/(a^3*f) + (8*c^5*tan(e/2 + (f
*x)/2)^5)/(5*a^3*f) - (63*c^5*atanh(tan(e/2 + (f*x)/2)))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.20

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$$

$$= \frac{c^5\Big(315\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\big)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-630\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\big)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+315\log\big(\tan\big(}{}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x)
```

output

```
(c**5*(315*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4 - 630*log(tan((e
+ f*x)/2) - 1)*tan((e + f*x)/2)**2 + 315*log(tan((e + f*x)/2) - 1) - 315*l
og(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4 + 630*log(tan((e + f*x)/2) +
1)*tan((e + f*x)/2)**2 - 315*log(tan((e + f*x)/2) + 1) + 16*tan((e + f*x)/
2)**9 + 48*tan((e + f*x)/2)**7 + 336*tan((e + f*x)/2)**5 - 1050*tan((e + f
*x)/2)**3 + 630*tan((e + f*x)/2)))/(10*a**3*f*(tan((e + f*x)/2)**4 - 2*tan
((e + f*x)/2)**2 + 1))
```

3.54 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 164

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx &= -\frac{7c^4\operatorname{arctanh}(\sin(e+fx))}{a^3f}+\frac{7c^4\tan(e+fx)}{a^3f}\\ &\quad+\frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\ &\quad-\frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\\ &\quad+\frac{14(c^4-c^4\sec(e+fx))\tan(e+fx)}{3f\left(a^3+a^3\sec(e+fx)\right)}\end{aligned}$$

output

```
-7*c^4*arctanh(sin(f*x+e))/a^3/f+7*c^4*tan(f*x+e)/a^3/f+2/5*c*(c-c*sec(f*x
+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^3-14/15*(c^2-c^2*sec(f*x+e))^2*tan(f*
x+e)/a/f/(a+a*sec(f*x+e))^2+14/3*(c^4-c^4*sec(f*x+e))*tan(f*x+e)/f/(a^3+a^
3*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.83 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{16c^4\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{5a^3f(-1+\sec(e+fx))(1+\sec(e+fx))^3}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

output
```
(-16*c^4*Hypergeometric2F1[-7/2, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[2
- 2*Sec[e + f*x]]*Tan[e + f*x])/(5*a^3*f*(-1 + Sec[e + f*x])*(1 + Sec[e +
f*x])^3)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4445, 3042, 4445, 3042, 4445, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(\sec(e+fx)a+a)^2}dx}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\int\frac{\sec(e+fx)(c-c\sec(e+fx))^2}{\sec(e+fx)a+a}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\int\sec(e+fx)(c-c\sec(e+fx))dx}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\int\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{4274}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(c\int\sec(e+fx)dx-c\int\sec^2(e+fx)dx\right)}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(c\int\csc\left(e+fx+\frac{\pi}{2}\right)dx-c\int\csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(\frac{c\int 1d(-\tan(e+fx))}{f}+c\int\csc\left(e+fx+\frac{\pi}{2}\right)dx\right)}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{24}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(c\int\csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{c\tan(e+fx)}{f}\right)}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}-\frac{7c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}-\frac{5c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}-\frac{3c\left(\frac{c\,\text{arctanh}(\sin(e+fx))}{f}-\frac{c\tan(e+fx)}{f}\right)}{a}\right)}{3a}\right)}{5a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

output
```
(2*c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (
7*c*((2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2
) - (5*c*((2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]
)) - (3*c*((c*ArcTanh[Sin[e + f*x]])/f - (c*Tan[e + f*x])/f))/a))/(3*a)))/
(5*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{4c^4 (\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}+\frac{2\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+3\tan(\frac{fx}{2}+\frac{e}{2})-\frac{1}{4(\tan(\frac{fx}{2}+\frac{e}{2})+1)}-\frac{7\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{4}-\frac{1}{4(\tan(\frac{fx}{2}+\frac{e}{2})-1)}+}{f\,a^3}$
default	$\frac{4c^4 (\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}+\frac{2\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+3\tan(\frac{fx}{2}+\frac{e}{2})-\frac{1}{4(\tan(\frac{fx}{2}+\frac{e}{2})+1)}-\frac{7\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{4}-\frac{1}{4(\tan(\frac{fx}{2}+\frac{e}{2})-1)}+}{f\,a^3}$
parallelrisch	$\frac{1609 (\frac{1680\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)\cos(fx+e)}{1609}-\frac{1680\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)\cos(fx+e)}{1609}+\tan(\frac{fx}{2}+\frac{e}{2})\sec(\frac{fx}{2}+\frac{e}{2})^4(\cos(fx+e)+}{240f\,a^3\cos(fx+e)}$
risch	$\frac{2ic^4\left(120\,\mathrm{e}^{6i(fx+e)}+495\,\mathrm{e}^{5i(fx+e)}+1235\,\mathrm{e}^{4i(fx+e)}+1270\,\mathrm{e}^{3i(fx+e)}+1342\,\mathrm{e}^{2i(fx+e)}+715\,\mathrm{e}^{i(fx+e)}+167\right)}{15f\,a^3\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}+1\right)^5}+$
norman	$\frac{\frac{14c^4\tan(\frac{fx}{2}+\frac{e}{2})}{af}-\frac{154c^4\tan(\frac{fx}{2}+\frac{e}{2})^3}{3af}+\frac{1022c^4\tan(\frac{fx}{2}+\frac{e}{2})^5}{15af}-\frac{186c^4\tan(\frac{fx}{2}+\frac{e}{2})^7}{5af}+\frac{92c^4\tan(\frac{fx}{2}+\frac{e}{2})^9}{15af}-}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4a^2}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
4/f*c^4/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+2/3*tan(1/2*f*x+1/2*e)^3+3*tan(1/2*f
*x+1/2*e)-1/4/(tan(1/2*f*x+1/2*e)+1)-7/4*ln(tan(1/2*f*x+1/2*e)+1)-1/4/(tan
(1/2*f*x+1/2*e)-1)+7/4*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.41

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{105\left(c^4\cos\left(fx+e\right)^4+3\,c^4\cos\left(fx+e\right)^3+3\,c^4\cos\left(fx+e\right)^2+c^4\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
-1/30*(105*(c^4*cos(f*x + e)^4 + 3*c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)
^2 + c^4*cos(f*x + e))*log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 3
*c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*log(-sin(f*
x + e) + 1) - 2*(167*c^4*cos(f*x + e)^3 + 381*c^4*cos(f*x + e)^2 + 277*c^4
*cos(f*x + e) + 15*c^4)*sin(f*x + e))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(
f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$

$$= \frac{c^4\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{4\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int\frac{}{\sec^3(e+fx)+3}\right.}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

output
```
c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) + 1), x) + Integral(-4*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e
+ f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(161) = 322$.

Time = 0.05 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.87

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$

$$= \frac{3c^4\left(\frac{40\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}+\frac{\frac{85\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin}{\cos(}\right.}{a}\right.}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/60*(3*c^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 -
60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a^3) + 4*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 2
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^4*(15*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 -
12*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) +
 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{30\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3}-\frac{4\left(3\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+10\,a^{12}c^4}{15\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
-1/15*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 105*c^4*log(abs(ta
n(1/2*f*x + 1/2*e) - 1))/a^3 + 30*c^4*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x +
 1/2*e)^2 - 1)*a^3) - 4*(3*a^12*c^4*tan(1/2*f*x + 1/2*e)^5 + 10*a^12*c^4*t
an(1/2*f*x + 1/2*e)^3 + 45*a^12*c^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 10.87 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.77

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{12\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}+\frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f} + \frac{4\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f} - \frac{14\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f} - \frac{2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}$$

input

```
int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output

```
(12*c^4*tan(e/2 + (f*x)/2))/(a^3*f) + (8*c^4*tan(e/2 + (f*x)/2)^3)/(3*a^3*
f) + (4*c^4*tan(e/2 + (f*x)/2)^5)/(5*a^3*f) - (14*c^4*atanh(tan(e/2 + (f*x
)/2)))/(a^3*f) - (2*c^4*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (f*x)/2)^2 -
 a^3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4\Big(105\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\big)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-105\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\big)-105\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\big)\,\mathrm{ta}}{15a^3}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x)
```

output

```
(c**4*(105*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2 - 105*log(tan((e
+ f*x)/2) - 1) - 105*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2 + 105*l
og(tan((e + f*x)/2) + 1) + 12*tan((e + f*x)/2)**7 + 28*tan((e + f*x)/2)**5
 + 140*tan((e + f*x)/2)**3 - 210*tan((e + f*x)/2)))/(15*a**3*f*(tan((e + f
*x)/2)**2 - 1))
```

3.55 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 131

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = & -\frac{c^3\operatorname{arctanh}(\sin(e+fx))}{a^3f} \\ & +\frac{2c^3\tan(e+fx)}{f\,(a^3+a^3\sec(e+fx))} \\ & +\frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} \\ & -\frac{2(c^3-c^3\sec(e+fx))\tan(e+fx)}{3af(a+a\sec(e+fx))^2}\end{aligned}$$

output

```
-c^3*arctanh(sin(f*x+e))/a^3/f+2*c^3*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+2/5
*c*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3-2/3*(c^3-c^3*sec(f*x
+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{c^3\left(-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{26\tan\left(\frac{1}{2}(e+fx)\right)}{15f}+\frac{2\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+f\right.}{15f}\right.}{a^3}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

output
```
-((c^3*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]]/f - (26*Tan[(e + f*x)/2])/(15*f) + (2*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/(15*f) - (2*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])/(
5*f)))/a^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4445, 3042, 4445, 3042, 4445, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}-\frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(\sec(e+fx)a+a)^2}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}-\frac{c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a}$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}-\frac{c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\int\frac{\sec(e+fx)(c-c\sec(e+fx))}{\sec(e+fx)a+a}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}-\frac{c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4445}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}-\frac{c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\left(\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\int\sec(e+fx)dx}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}-\frac{c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\left(\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\int\csc\left(e+fx+\frac{\pi}{2}\right)dx}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}-\frac{c\left(\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}-\frac{c\left(\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\,\text{arctanh}(\sin(e+fx))}{af}\right)}{a}\right)}{a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

output
```
(2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (
c*((2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
- (c*(-((c*ArcTanh[Sin[e + f*x]])/(a*f)) + (2*c*Tan[e + f*x])/(f*(a + a*Se
c[e + f*x]))))/a))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{2c^3\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^3}$
default	$\frac{2c^3\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^3}$
parallelrisch	$\frac{c^3\left(6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15a^3f}$
risch	$\frac{4ic^3\left(15\,\mathrm{e}^{4i(fx+e)}+30\,\mathrm{e}^{3i(fx+e)}+100\,\mathrm{e}^{2i(fx+e)}+50\,\mathrm{e}^{i(fx+e)}+13\right)}{15f\,a^3\left(\mathrm{e}^{i(fx+e)}+1\right)^5}+\frac{c^3\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{a^3f}-\frac{c^3\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{a^3f}$
norman	$\frac{-\frac{2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{16c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{22c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5af}+\frac{6c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{5af}-\frac{8c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{15af}+\frac{2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{5af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3a^2}+$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
2/f*c^3/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+1/3*tan(1/2*f*x+1/2*e)^3+tan(1/2*f*x
+1/2*e)+1/2*ln(tan(1/2*f*x+1/2*e)-1)-1/2*ln(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.47

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{15\left(c^3\cos\left(fx+e\right)^3+3\,c^3\cos\left(fx+e\right)^2+3\,c^3\cos\left(fx+e\right)+c^3\right)\log\left(\sin\left(fx+e\right)+1\right)-15\,(c^3\cos\,(f}{30\,(a^3f\cos\,(fx}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
-1/30*(15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e)
+ c^3)*log(sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)
^2 + 3*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 4*(13*c^3*cos(f*x
+ e)^2 + 24*c^3*cos(f*x + e) + 23*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3
 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^3\left(\int\left(-\frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int\frac{3\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{}{\sec^3(e-}\right.\right.}{a^3}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

output
```
-c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
 + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e
+ f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e +
 f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a
**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(129) = 258.

Time = 0.05 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.32

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+\frac{3c^3\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\mathrm{s}}{(\cos}\right.}{}}{60f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/a^3) + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^3
*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^3*(5*sin(f*x + e)/(
cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}-\frac{2\,\bigl(3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^{12}c^3\tan\bigl(}{a^{15}}}{15\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
-1/15*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 15*c^3*log(abs(tan(
1/2*f*x + 1/2*e) - 1))/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 + 5*a^12
*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 10.72 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx \\ = \frac{2\,c^3\,\left(15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-15\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\right)}{15\,a^3\,f}$$

input
```
int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(2*c^3*(15*tan(e/2 + (f*x)/2) - 15*atanh(tan(e/2 + (f*x)/2)) + 5*tan(e/2 +
 (f*x)/2)^3 + 3*tan(e/2 + (f*x)/2)^5))/(15*a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx \\ = \frac{c^3\Big(15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+30\tan}{15a^3f}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x)
```

output
```
(c**3*(15*log(tan((e + f*x)/2) - 1) - 15*log(tan((e + f*x)/2) + 1) + 6*tan
((e + f*x)/2)**5 + 10*tan((e + f*x)/2)**3 + 30*tan((e + f*x)/2)))/(15*a**3
*f)
```

3.56 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx=\frac{(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

output
```
1/5*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx=\frac{c^2\tan^5\left(\frac{1}{2}(e+fx)\right)}{5a^3f}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c^2*Tan[(e + f*x)/2]^5)/(5*a^3*f)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \; 4438$$

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

output
```
((c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5 f \,a^{3}}$	23
default	$\frac{c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5 f \,a^{3}}$	23
parallelrisch	$\frac{c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5 f \,a^{3}}$	23
risch	$\frac{2 i c^{2} \left(5 \,{\mathrm e}^{4 i (f x +e)}+10 \,{\mathrm e}^{2 i (f x +e)}+1\right)}{5 f \,a^{3} \left({\mathrm e}^{i (f x +e)}+1\right)^{5}}$	50
norman	$\frac{\frac{c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5 a f}-\frac{2 c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{5 a f}+\frac{c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}{5 a f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} a^{2}}$	87

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output

```
1/5/f*c^2/a^3*tan(1/2*f*x+1/2*e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\left(c^{2} \cos\left(fx+e\right)^{2} - 2\,c^{2} \cos\left(fx+e\right) + c^{2}\right) \sin\left(fx+e\right)}{5\,\left(a^{3} f \cos\left(fx+e\right)^{3} + 3\,a^{3} f \cos\left(fx+e\right)^{2} + 3\,a^{3} f \cos\left(fx+e\right) + a^{3} f\right)}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/5*(c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) + c^2)*sin(f*x + e)/(a^3*f*co
s(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx + \int \left(-\frac{2\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx + \int \frac{}{\sec^3\left(e+fx\right)+3}\right.}{a^3}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

output
```
c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) + 1), x) + Integral(-2*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*
x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(37) = 74$.

Time $= 0.04$ (sec) , antiderivative size $= 185$, normalized size of antiderivative $= 4.87$

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\frac{c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{6c^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx}{(\cos(fx+}\right.}{a^3}}{60\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/60*(c^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*
x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 6*c^2*(5*sin(f*x + e)/(cos(f*x + e
) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx=\frac{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}{5\,a^3f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
1/5*c^2*tan(1/2*f*x + 1/2*e)^5/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 10.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx=\frac{c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}$$

input
```
int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(c^2*tan(e/2 + (f*x)/2)^5)/(5*a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 c^2}{5a^3f}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x)
```

output

```
(tan((e + f*x)/2)**5*c**2)/(5*a**3*f)
```

3.57 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-c\sec(e+fx))\tan(e+fx)}{15af(a+a\sec(e+fx))^2}$$

output

```
1/5*(c-c*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+1/15*(c-c*sec(f*x+e))
*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{c(-1+\sec(e+fx))(4+\sec(e+fx))\tan(e+fx)}{15a^3f(1+\sec(e+fx))^3}$$

input

```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

output
```
-1/15*(c*(-1 + Sec[e + f*x])*(4 + Sec[e + f*x])*Tan[e + f*x])/(a^3*f*(1 +
Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{3042, 4439, 3042, 4438\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(\sec(e+fx)a+a)^2}dx}{5a}+\frac{\tan(e+fx)(c-c\sec(e+fx))}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(e+fx)(c-c\sec(e+fx))}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{4438}$$

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{15af(a\sec(e+fx)+a)^2}+\frac{\tan(e+fx)(c-c\sec(e+fx))}{5f(a\sec(e+fx)+a)^3}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

output
```
((c - c*Sec[e + f*x])*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - c
*Sec[e + f*x])*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

method	result	size
parallelrisch	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-5\right)}{30a^{3}f}$	36
derivativedivides	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}\right)}{2f\,a^{3}}$	37
default	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}\right)}{2f\,a^{3}}$	37
risch	$\frac{2ic\left(15\,e^{4i(fx+e)}+15\,e^{3i(fx+e)}+25\,e^{2i(fx+e)}+5\,e^{i(fx+e)}+4\right)}{15f\,a^{3}\left(e^{i(fx+e)}+1\right)^{5}}$	70
norman	$\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{6fa}-\frac{4c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{15fa}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{10fa}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)a^{2}}$	81

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/30*c*tan(1/2*f*x+1/2*e)^3*(3*tan(1/2*f*x+1/2*e)^2-5)/a^3/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\left(4\,c\cos\left(fx+e\right)^2-3\,c\cos\left(fx+e\right)-c\right)\sin\left(fx+e\right)}{15\,\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
1/15*(4*c*cos(f*x + e)^2 - 3*c*cos(f*x + e) - c)*sin(f*x + e)/(a^3*f*cos(f
*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{c\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

output
```
-c*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*
x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{3c\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="max
ima")
```

output
```
1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x +
 e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-5c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}{30a^3f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="gia
c")
```

output
```
1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 - 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 10.64 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx=\frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-5\right)}{30\,a^3\,f}$$

input
```
int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(c*tan(e/2 + (f*x)/2)^3*(3*tan(e/2 + (f*x)/2)^2 - 5))/(30*a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx=\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 c\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-5\right)}{30a^3f}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

output
```
(tan((e + f*x)/2)**3*c*(3*tan((e + f*x)/2)**2 - 5))/(30*a**3*f)
```

3.58 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{2\cot^5(e+fx)}{5a^3cf}+\frac{\csc(e+fx)}{a^3cf}-\frac{\csc^3(e+fx)}{a^3cf}+\frac{2\csc^5(e+fx)}{5a^3cf}$$

output
```
-2/5*cot(f*x+e)^5/a^3/c/f+csc(f*x+e)/a^3/c/f-csc(f*x+e)^3/a^3/c/f+2/5*csc(
f*x+e)^5/a^3/c/f
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{(-2+\sec(e+fx)+4\sec^2(e+fx)+2\sec^3(e+fx))\tan(e+fx)}{5a^3cf(-1+\sec(e+fx))(1+\sec(e+fx))^3}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

output
```
((-2 + Sec[e + f*x] + 4*Sec[e + f*x]^2 + 2*Sec[e + f*x]^3)*Tan[e + f*x])/(
5*a^3*c*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4446}$$

$$-\frac{\int \left(c^2\csc(e+fx)\cot^5(e+fx)-2c^2\csc^2(e+fx)\cot^4(e+fx)+c^2\csc^3(e+fx)\cot^3(e+fx)\right)dx}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2c^2\cot^5(e+fx)}{5f}-\frac{2c^2\csc^5(e+fx)}{5f}+\frac{c^2\csc^3(e+fx)}{f}-\frac{c^2\csc(e+fx)}{f}}{a^3c^3}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

output
```
-(((2*c^2*Cot[e + f*x]^5)/(5*f) - (c^2*Csc[e + f*x])/f + (c^2*Csc[e + f*x]
^3)/f - (2*c^2*Csc[e + f*x]^5)/(5*f))/(a^3*c^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{40 f\,a^{3} c}$	59
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8 f\,a^{3} c}$	61
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8 f\,a^{3} c}$	61
risch	$\frac{2 i \left(5\,{\mathrm e}^{5 i (f x+e)}+10\,{\mathrm e}^{4 i (f x+e)}+10\,{\mathrm e}^{3 i (f x+e)}-3\,{\mathrm e}^{i (f x+e)}-2\right)}{5 f\,a^{3} c \left({\mathrm e}^{i (f x+e)}+1\right)^{5} \left({\mathrm e}^{i (f x+e)}-1\right)}$	85
norman	$\frac{\frac{1}{8 f a c}+\frac{3 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{8 f a c}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{8 f a c}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{40 f a c}}{a^{2} \tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	94

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
1/40*(tan(1/2*f*x+1/2*e)^5-5*tan(1/2*f*x+1/2*e)^3+5*cot(1/2*f*x+1/2*e)+15*
tan(1/2*f*x+1/2*e))/f/a^3/c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{2\,\cos\left(fx+e\right)^3-\cos\left(fx+e\right)^2-4\,\cos\left(fx+e\right)-2}{5\,\left(a^3cf\cos\left(fx+e\right)^2+2\,a^3cf\cos\left(fx+e\right)+a^3cf\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
-1/5*(2*cos(f*x + e)^3 - cos(f*x + e)^2 - 4*cos(f*x + e) - 2)/((a^3*c*f*co
s(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)+2\sec^3\left(e+fx\right)-2\sec\left(e+fx\right)-1}\,dx}{a^3c}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(sec(e + f*x)/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*
x) - 1), x)/(a**3*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{\frac{\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c}+\frac{5\,(\cos(fx+e)+1)}{a^3c\sin(fx+e)}}{40\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
1/40*((15*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) + 5*(cos(f*x + e) +
 1)/(a^3*c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{\frac{5}{a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\frac{a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-5\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}c^5}}{40\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
1/40*(5/(a^3*c*tan(1/2*f*x + 1/2*e)) + (a^12*c^4*tan(1/2*f*x + 1/2*e)^5 -
5*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e))/(a^1
5*c^5))/f
```

Mupad [B] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$$
$$= -\frac{16\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-28\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+8\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1}{40\,a^3\,c\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))),x)
```

output
```
-(8*cos(e/2 + (f*x)/2)^2 - 28*cos(e/2 + (f*x)/2)^4 + 16*cos(e/2 + (f*x)/2)
^6 - 1)/(40*a^3*c*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$$
$$= \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+5}{40\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a^3cf}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

output
```
(tan((e + f*x)/2)**6 - 5*tan((e + f*x)/2)**4 + 15*tan((e + f*x)/2)**2 + 5)
/(40*tan((e + f*x)/2)*a**3*c*f)
```

3.59 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}$$

output

```
-1/5*cot(f*x+e)^5/a^3/c^2/f+csc(f*x+e)/a^3/c^2/f-2/3*csc(f*x+e)^3/a^3/c^2/
f+1/5*csc(f*x+e)^5/a^3/c^2/f
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{(3-12\sec(e+fx)-12\sec^2(e+fx)+8\sec^3(e+fx)+8\sec^4(e+fx))\tan(e+fx)}{15a^3c^2f(-1+\sec(e+fx))^2(1+\sec(e+fx))^3}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

output

```
((3 - 12*Sec[e + f*x] - 12*Sec[e + f*x]^2 + 8*Sec[e + f*x]^3 + 8*Sec[e + f
*x]^4)*Tan[e + f*x])/(15*a^3*c^2*f*(-1 + Sec[e + f*x])^2*(1 + Sec[e + f*x]
)^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4446}$$

$$-\frac{\int \left(c\cot^5(e+fx)\csc(e+fx)-c\cot^4(e+fx)\csc^2(e+fx)\right)dx}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{c\cot^5(e+fx)}{5f}-\frac{c\csc^5(e+fx)}{5f}+\frac{2c\csc^3(e+fx)}{3f}-\frac{c\csc(e+fx)}{f}}{a^3c^3}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

output

```
-(((c*Cot[e + f*x]^5)/(5*f) - (c*Csc[e + f*x])/f + (2*c*Csc[e + f*x]^3)/(3
*f) - (c*Csc[e + f*x]^5)/(5*f))/(a^3*c^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-5 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}-20 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+60 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)+90 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{240 f \,a^{3} c^{2}}$	74
derivativedivides	$\frac{\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{4 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}+6 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)-\frac{1}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}}{16 f \,a^{3} c^{2}}$	76
default	$\frac{\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{4 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}+6 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)-\frac{1}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}}{16 f \,a^{3} c^{2}}$	76
risch	$\frac{2 i \left(15 \,{\mathrm e}^{7 i \left(f x +e \right)}+15 \,{\mathrm e}^{6 i \left(f x +e \right)}-5 \,{\mathrm e}^{5 i \left(f x +e \right)}-25 \,{\mathrm e}^{4 i \left(f x +e \right)}+13 \,{\mathrm e}^{3 i \left(f x +e \right)}+21 \,{\mathrm e}^{2 i \left(f x +e \right)}+9 \,{\mathrm e}^{i \left(f x +e \right)}-3\right)}{15 f \,a^{3} c^{2} \left({\mathrm e}^{i \left(f x +e \right)}+1\right)^{5} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{3}}$	118
norman	$\frac{-\frac{1}{48 f a c}+\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{4 f a c}+\frac{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{8 f a c}-\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{12 f a c}+\frac{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{8}}{80 f a c}}{a^{2} c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	119

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/240*(3*tan(1/2*f*x+1/2*e)^5-5*cot(1/2*f*x+1/2*e)^3-20*tan(1/2*f*x+1/2*e)
^3+60*cot(1/2*f*x+1/2*e)+90*tan(1/2*f*x+1/2*e))/f/a^3/c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{3\cos\left(fx+e\right)^4-12\cos\left(fx+e\right)^3-12\cos\left(fx+e\right)^2+8\cos\left(fx+e\right)+8}{15\left(a^3c^2f\cos\left(fx+e\right)^3+a^3c^2f\cos\left(fx+e\right)^2-a^3c^2f\cos\left(fx+e\right)-a^3c^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
-1/15*(3*cos(f*x + e)^4 - 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 + 8*cos(f*
x + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^
2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)+\sec^4\left(e+fx\right)-2\sec^3\left(e+fx\right)-2\sec^2\left(e+fx\right)+\sec\left(e+fx\right)+1}\,dx}{a^3c^2}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(sec(e + f*x)/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)*
*3 - 2*sec(e + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.50

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\frac{\frac{90\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c^2}+\frac{5\left(\frac{12\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{a^3c^2\sin(fx+e)^3}}{240\,f}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="m
axima")
```

output

```
1/240*((90*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) + 5*(12*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x
+ e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\frac{5\left(12\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}{a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}+\frac{3a^{12}c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-20a^{12}c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+90a^{12}c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^{10}}}{240\,f}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="g
iac")
```

output

```
1/240*(5*(12*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^2*tan(1/2*f*x + 1/2*e)^3)
+ (3*a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 20*a^12*c^8*tan(1/2*f*x + 1/2*e)^3
+ 90*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))/f
```

Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{48\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-192\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+168\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-32\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3}{240\,a^3\,c^2\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2),x)
```

output
```
(168*cos(e/2 + (f*x)/2)^4 - 32*cos(e/2 + (f*x)/2)^2 - 192*cos(e/2 + (f*x)/
2)^6 + 48*cos(e/2 + (f*x)/2)^8 + 3)/(240*a^3*c^2*f*(cos(e/2 + (f*x)/2)^5*s
in(e/2 + (f*x)/2) - cos(e/2 + (f*x)/2)^7*sin(e/2 + (f*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+60\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-5}{240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3a^3c^2f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

output
```
(3*tan((e + f*x)/2)**8 - 20*tan((e + f*x)/2)**6 + 90*tan((e + f*x)/2)**4 +
 60*tan((e + f*x)/2)**2 - 5)/(240*tan((e + f*x)/2)**3*a**3*c**2*f)
```

3.60 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 59

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\csc(e+fx)}{a^3c^3f}-\frac{2\csc^3(e+fx)}{3a^3c^3f}+\frac{\csc^5(e+fx)}{5a^3c^3f}$$

output
```
csc(f*x+e)/a^3/c^3/f-2/3*csc(f*x+e)^3/a^3/c^3/f+1/5*csc(f*x+e)^5/a^3/c^3/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{-\frac{\csc(e+fx)}{f}+\frac{2\csc^3(e+fx)}{3f}-\frac{\csc^5(e+fx)}{5f}}{a^3c^3}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

output
```
-((-(Csc[e + f*x]/f) + (2*Csc[e + f*x]^3)/(3*f) - Csc[e + f*x]^5/(5*f))/(a
^3*c^3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3042, 4446, 3042, 25, 3086, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \ 4446$$

$$-\frac{\int \cot^5(e+fx)\csc(e+fx)dx}{a^3c^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int -\sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)^5dx}{a^3c^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \sec\left(\frac{1}{2}(2e-\pi)+fx\right)\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^5dx}{a^3c^3}$$

$$\Big\downarrow \ 3086$$

$$\frac{\int \left(\csc^2(e+fx)-1\right)^2d\csc(e+fx)}{a^3c^3f}$$

$$\Big\downarrow \ 210$$

$$\frac{\int \left(\csc^4(e+fx)-2\csc^2(e+fx)+1\right)d\csc(e+fx)}{a^3c^3f}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{5}\csc^5(e+fx)-\frac{2}{3}\csc^3(e+fx)+\csc(e+fx)}{a^3c^3f}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

output
```
(Csc[e + f*x] - (2*Csc[e + f*x]^3)/3 + Csc[e + f*x]^5/5)/(a^3*c^3*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{-\frac{\csc(fx+e)^{5}}{5}+\frac{2\csc(fx+e)^{3}}{3}-\csc(fx+e)}{a^{3}c^{3}f}$	41
parallelrisch	$\frac{(15\cos(4fx+4e)-20\cos(2fx+2e)+29)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{3840f\,a^{3}c^{3}}$	58
risch	$\frac{2i\left(15\,\mathrm{e}^{9i(fx+e)}-20\,\mathrm{e}^{7i(fx+e)}+58\,\mathrm{e}^{5i(fx+e)}-20\,\mathrm{e}^{3i(fx+e)}+15\,\mathrm{e}^{i(fx+e)}\right)}{15f\,a^{3}c^{3}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}\left(\mathrm{e}^{i(fx+e)}-1\right)^{5}}$	95
norman	$\frac{\frac{1}{160fac}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{96fac}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{16fac}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{16fac}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{96fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{160fac}}{a^{2}c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$	141

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
-1/a^3/c^3/f*(-1/5*csc(f*x+e)^5+2/3*csc(f*x+e)^3-csc(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$$
$$=\frac{15\cos(fx+e)^4-20\cos(fx+e)^2+8}{15\left(a^3c^3f\cos(fx+e)^4-2\,a^3c^3f\cos(fx+e)^2+a^3c^3f\right)\sin(fx+e)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/15*(15*cos(f*x + e)^4 - 20*cos(f*x + e)^2 + 8)/((a^3*c^3*f*cos(f*x + e)^
4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sec^6(e+fx)-3\sec^4(e+fx)+3\sec^2(e+fx)-1}\,dx}{a^3c^3}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(sec(e + f*x)/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*
x)**2 - 1), x)/(a**3*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{15\,\sin\left(fx+e\right)^4-10\,\sin\left(fx+e\right)^2+3}{15\,a^3c^3f\,\sin\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5
)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{15\,\sin\left(fx+e\right)^4-10\,\sin\left(fx+e\right)^2+3}{15\,a^3c^3f\,\sin\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5
)
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\sin\left(e+f\,x\right)^4-\frac{2\sin\left(e+f\,x\right)^2}{3}+\frac{1}{5}}{a^3\,c^3\,f\sin\left(e+f\,x\right)^5}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3),x)
```

output
```
(sin(e + f*x)^4 - (2*sin(e + f*x)^2)/3 + 1/5)/(a^3*c^3*f*sin(e + f*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{15\sin\left(fx+e\right)^4-10\sin\left(fx+e\right)^2+3}{15\sin\left(fx+e\right)^5a^3c^3f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

output
```
(15*sin(e + f*x)**4 - 10*sin(e + f*x)**2 + 3)/(15*sin(e + f*x)**5*a**3*c**
3*f)
```

3.61 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 99

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{\cot^7(e+fx)}{7a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}$$

output

```
-1/7*cot(f*x+e)^7/a^3/c^4/f+csc(f*x+e)/a^3/c^4/f-csc(f*x+e)^3/a^3/c^4/f+3/
5*csc(f*x+e)^5/a^3/c^4/f-1/7*csc(f*x+e)^7/a^3/c^4/f
```

Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{(-5-30\sec(e+fx)+30\sec^2(e+fx)+40\sec^3(e+fx)-40\sec^4(e+fx)-16\sec^5(e+fx)+16\sec^6}{35a^3c^4f(-1+\sec(e+fx))^4(1+\sec(e+fx))^3}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

output
```
((-5 - 30*Sec[e + f*x] + 30*Sec[e + f*x]^2 + 40*Sec[e + f*x]^3 - 40*Sec[e
+ f*x]^4 - 16*Sec[e + f*x]^5 + 16*Sec[e + f*x]^6)*Tan[e + f*x])/(35*a^3*c^
4*f*(-1 + Sec[e + f*x])^4*(1 + Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{ 4446}$$

$$\frac{\int \left(a\csc(e+fx)\cot^7(e+fx)+a\csc^2(e+fx)\cot^6(e+fx)\right)dx}{a^4c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{a\cot^7(e+fx)}{7f}-\frac{a\csc^7(e+fx)}{7f}+\frac{3a\csc^5(e+fx)}{5f}-\frac{a\csc^3(e+fx)}{f}+\frac{a\csc(e+fx)}{f}}{a^4c^4}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

output
```
(-1/7*(a*Cot[e + f*x]^7)/f + (a*Csc[e + f*x])/f - (a*Csc[e + f*x]^3)/f + (
3*a*Csc[e + f*x]^5)/(5*f) - (a*Csc[e + f*x]^7)/(7*f))/(a^4*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

method	result
parallelrisch	$-\frac{\left(-182+5\cos(6fx+6e)+60\cos(5fx+5e)-90\cos(4fx+4e)-20\cos(3fx+3e)+235\cos(2fx+2e)+152\cos(fx+e)\right)\sec}{71680f\,a^{3}c^{4}}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{6}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{5}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{20}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{64f\,a^{3}c^{4}}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{6}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{5}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{20}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{64f\,a^{3}c^{4}}$
risch	$\frac{2i\left(35\,\mathrm{e}^{11i(fx+e)}-35\,\mathrm{e}^{10i(fx+e)}-35\,\mathrm{e}^{9i(fx+e)}+105\,\mathrm{e}^{8i(fx+e)}+126\,\mathrm{e}^{7i(fx+e)}-182\,\mathrm{e}^{6i(fx+e)}+26\,\mathrm{e}^{5i(fx+e)}+130\,\mathrm{e}^{4i(fx}\right.}{35f\,a^{3}c^{4}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}\left(\mathrm{e}^{i(fx+e)}-1\right)^{7}}$
norman	$\frac{-\frac{1}{448fac}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{160fac}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{64fac}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{16fac}+\frac{15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{64fac}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{32fac}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{320fac}}{a^{2}c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBO
SE)
```

output
```
-1/71680*(-182+5*cos(6*f*x+6*e)+60*cos(5*f*x+5*e)-90*cos(4*f*x+4*e)-20*cos
(3*f*x+3*e)+235*cos(2*f*x+2*e)+152*cos(f*x+e))*sec(1/2*f*x+1/2*e)^5*csc(1/
2*f*x+1/2*e)^7/f/a^3/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.65

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{5\cos\left(fx+e\right)^{6}+30\cos\left(fx+e\right)^{5}-30\cos\left(fx+e\right)^{4}-40\cos\left(fx+e\right)^{3}+40\cos\left(fx+e\right)^{2}+}{35\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}-a^{3}c^{4}f\cos\left(fx+e\right)^{4}-2\,a^{3}c^{4}f\cos\left(fx+e\right)^{3}+2\,a^{3}c^{4}f\cos\left(fx+e\right)^{2}+a^{3}c^{4}f\,\mathrm{co}\right.}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="f
ricas")
```

output
```
1/35*(5*cos(f*x + e)^6 + 30*cos(f*x + e)^5 - 30*cos(f*x + e)^4 - 40*cos(f*
x + e)^3 + 40*cos(f*x + e)^2 + 16*cos(f*x + e) - 16)/((a^3*c^4*f*cos(f*x +
 e)^5 - a^3*c^4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*
f*cos(f*x + e)^2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{\int \frac{\sec\left(e+fx\right)}{\sec^{7}\left(e+fx\right)-\sec^{6}\left(e+fx\right)-3\sec^{5}\left(e+fx\right)+3\sec^{4}\left(e+fx\right)+3\sec^{3}\left(e+fx\right)-3\sec^{2}\left(e+fx\right)-\sec\left(e+fx\right)+1}\,dx}{a^{3}c^{4}}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

output
```
Integral(sec(e + f*x)/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)*
*5 + 3*sec(e + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f
*x) + 1), x)/(a**3*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.61

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\frac{7\left(\frac{75\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^4}+\frac{\left(\frac{42\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{175\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{700\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{a^3c^4\sin(fx+e)^7}}{2240\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="m
axima")
```

output
```
1/2240*(7*(75*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) + (42*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
700*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(a^3*c^4
*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\frac{700\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-175\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+42\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-5}{a^3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}+\frac{7\,(a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-10\,a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+75\,a^{12}c^{16}\tan}{a^{15}c^{20}}}{2240\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="g
iac")
```

output
```
1/2240*((700*tan(1/2*f*x + 1/2*e)^6 - 175*tan(1/2*f*x + 1/2*e)^4 + 42*tan(
1/2*f*x + 1/2*e)^2 - 5)/(a^3*c^4*tan(1/2*f*x + 1/2*e)^7) + 7*(a^12*c^16*ta
n(1/2*f*x + 1/2*e)^5 - 10*a^12*c^16*tan(1/2*f*x + 1/2*e)^3 + 75*a^12*c^16*
tan(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

Mupad [B] (verification not implemented)

Time = 11.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\left(2\sin\left(\frac{e}{4}+\frac{fx}{4}\right)^2-1\right)\left(\frac{235\sin(e+fx)^2}{16}-\frac{45\sin(2e+2fx)^2}{8}+\frac{19\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{2}+\frac{5\sin(3e+3fx)^2}{16}-\frac{5\sin\left(\frac{3e}{2}+\frac{3fx}{2}\right)^2}{4}+\right.}{2240\,a^3c^4f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)^3}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4),x)
```

output
```
((2*sin(e/4 + (f*x)/4)^2 - 1)*((19*sin(e/2 + (f*x)/2)^2)/2 - (45*sin(2*e +
 2*f*x)^2)/8 + (5*sin(3*e + 3*f*x)^2)/16 - (5*sin((3*e)/2 + (3*f*x)/2)^2)/
4 + (15*sin((5*e)/2 + (5*f*x)/2)^2)/4 + (235*sin(e + f*x)^2)/16 - 5))/(224
0*a^3*c^4*f*sin(e/2 + (f*x)/2)^7*(sin(e/2 + (f*x)/2)^2 - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}-70\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+525\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+700\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-175\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+42}{2240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}a^3c^4f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

output
```
(7*tan((e + f*x)/2)**12 - 70*tan((e + f*x)/2)**10 + 525*tan((e + f*x)/2)**
8 + 700*tan((e + f*x)/2)**6 - 175*tan((e + f*x)/2)**4 + 42*tan((e + f*x)/2
)**2 - 5)/(2240*tan((e + f*x)/2)**7*a**3*c**4*f)
```

3.62 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx &= \frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}\\ &\quad+\frac{9\csc^5(e+fx)}{5a^3c^5f}-\frac{\csc^7(e+fx)}{a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}\end{aligned}$$

output
```
2/9*cot(f*x+e)^9/a^3/c^5/f+csc(f*x+e)/a^3/c^5/f-5/3*csc(f*x+e)^3/a^3/c^5/f
+9/5*csc(f*x+e)^5/a^3/c^5/f-csc(f*x+e)^7/a^3/c^5/f+2/9*csc(f*x+e)^9/a^3/c^
5/f
```

Mathematica [A] (verified)

Time = 4.72 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

$$\begin{aligned}&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx\\ &= \frac{(10+25\sec(e+fx)-60\sec^2(e+fx)-10\sec^3(e+fx)+80\sec^4(e+fx)-24\sec^5(e+fx)-32\sec^6}{45a^3c^5f(-1+\sec(e+fx))^5(1+\sec(e+fx))^3}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

output
```
((10 + 25*Sec[e + f*x] - 60*Sec[e + f*x]^2 - 10*Sec[e + f*x]^3 + 80*Sec[e
+ f*x]^4 - 24*Sec[e + f*x]^5 - 32*Sec[e + f*x]^6 + 16*Sec[e + f*x]^7)*Tan[
e + f*x])/(45*a^3*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4446}$$

$$-\frac{\int \left(a^2\csc(e+fx)\cot^9(e+fx)+2a^2\csc^2(e+fx)\cot^8(e+fx)+a^2\csc^3(e+fx)\cot^7(e+fx)\right)dx}{a^5c^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2a^2\cot^9(e+fx)}{9f}-\frac{2a^2\csc^9(e+fx)}{9f}+\frac{a^2\csc^7(e+fx)}{f}-\frac{9a^2\csc^5(e+fx)}{5f}+\frac{5a^2\csc^3(e+fx)}{3f}-\frac{a^2\csc(e+fx)}{f}}{a^5c^5}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

output
```
-(((-2*a^2*Cot[e + f*x]^9)/(9*f) - (a^2*Csc[e + f*x])/f + (5*a^2*Csc[e + f
*x]^3)/(3*f) - (9*a^2*Csc[e + f*x]^5)/(5*f) + (a^2*Csc[e + f*x]^7)/f - (2*
a^2*Csc[e + f*x]^9)/(9*f))/(a^5*c^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

method	result
parallelrisch	$-\frac{\left(-258-110\cos\left(4fx+4e\right)+169\cos\left(2fx+2e\right)+129\cos\left(fx+e\right)-25\cos\left(6fx+6e\right)-5\cos\left(7fx+7e\right)+85\cos\left(5fx+5e\right)-1\right.}{184320 f\,a^{3}c^{5}}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{21}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{35}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}+\frac{35}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-}{128 f\,a^{3}c^{5}}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{21}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{35}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}+\frac{35}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-}{128 f\,a^{3}c^{5}}$
risch	$\frac{2i\left(45\,\mathrm{e}^{13i(fx+e)}-90\,\mathrm{e}^{12i(fx+e)}+30\,\mathrm{e}^{11i(fx+e)}+240\,\mathrm{e}^{10i(fx+e)}-69\,\mathrm{e}^{9i(fx+e)}-354\,\mathrm{e}^{8i(fx+e)}+516\,\mathrm{e}^{7i(fx+e)}+96\,\mathrm{e}^{6i(fx}\right.}{45 f\,a^{3}c^{5}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}\left(\mathrm{e}^{i(fx+e)}-1\right)^{9}}$
norman	$\frac{\frac{1}{1152fac}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{128fac}+\frac{21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{640fac}-\frac{35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{384fac}+\frac{35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{128fac}+\frac{21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{128fac}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{384fac}+}{a^{2}c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBO
SE)
```

output
```
-1/184320*(-258-110*cos(4*f*x+4*e)+169*cos(2*f*x+2*e)+129*cos(f*x+e)-25*co
s(6*f*x+6*e)-5*cos(7*f*x+7*e)+85*cos(5*f*x+5*e)-145*cos(3*f*x+3*e))*sec(1/
2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)^9/f/a^3/c^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.58

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{10\cos\left(fx+e\right)^7+25\cos\left(fx+e\right)^6-60\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^4+80\cos\left(fx+e\right)}{45\left(a^3c^5f\cos\left(fx+e\right)^6-2\,a^3c^5f\cos\left(fx+e\right)^5-a^3c^5f\cos\left(fx+e\right)^4+4\,a^3c^5f\cos\left(fx+e\right)^3-a^3c^5f\cos\right.}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="f
ricas")
```

output
```
1/45*(10*cos(f*x + e)^7 + 25*cos(f*x + e)^6 - 60*cos(f*x + e)^5 - 10*cos(f
*x + e)^4 + 80*cos(f*x + e)^3 - 24*cos(f*x + e)^2 - 32*cos(f*x + e) + 16)/
((a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5 - a^3*c^5*f*cos(f*
x + e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f*x + e)^2 - 2*a^3*c
^5*f*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= -\frac{\int \frac{\sec\left(e+fx\right)}{\sec^8\left(e+fx\right)-2\sec^7\left(e+fx\right)-2\sec^6\left(e+fx\right)+6\sec^5\left(e+fx\right)-6\sec^3\left(e+fx\right)+2\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)-1}\,dx}{a^3c^5}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

output
```
-Integral(sec(e + f*x)/(sec(e + f*x)**8 - 2*sec(e + f*x)**7 - 2*sec(e + f*
x)**6 + 6*sec(e + f*x)**5 - 6*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + 2*sec(
e + f*x) - 1), x)/(a**3*c**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{\frac{3\left(\frac{315\sin(fx+e)}{\cos(fx+e)+1}-\frac{35\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^5}-\frac{\left(\frac{45\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{189\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{525\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1575\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-5\right)(\cos(fx+e)}{a^3c^5\sin(fx+e)^9}}{5760\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="m
axima")
```

output
```
1/5760*(3*(315*sin(f*x + e)/(cos(f*x + e) + 1) - 35*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - (45*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 525*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1575*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.18

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{\frac{1575\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-525\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+189\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+5}{a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}+\frac{3(3a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-35a^{12}c^{20}\tan(\frac{1}{2}}{a^{15}c^{25}}}{5760\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="g
iac")
```

output
```
1/5760*((1575*tan(1/2*f*x + 1/2*e)^8 - 525*tan(1/2*f*x + 1/2*e)^6 + 189*ta
n(1/2*f*x + 1/2*e)^4 - 45*tan(1/2*f*x + 1/2*e)^2 + 5)/(a^3*c^5*tan(1/2*f*x
 + 1/2*e)^9) + 3*(3*a^12*c^20*tan(1/2*f*x + 1/2*e)^5 - 35*a^12*c^20*tan(1/
2*f*x + 1/2*e)^3 + 315*a^12*c^20*tan(1/2*f*x + 1/2*e))/(a^15*c^25))/f
```

Mupad [B] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{145\cos(3e+3fx)}{32} - \frac{169\cos(2e+2fx)}{32} - \frac{129\cos(e+fx)}{32} + \frac{55\cos(4e+4fx)}{16} - \frac{85\cos(5e+5fx)}{32} + \frac{25\cos(6e+6fx)}{32} + \frac{5\cos(7e+}{32}}{5760\,a^3c^5f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5),x)
```

output
```
((145*cos(3*e + 3*f*x))/32 - (169*cos(2*e + 2*f*x))/32 - (129*cos(e + f*x)
)/32 + (55*cos(4*e + 4*f*x))/16 - (85*cos(5*e + 5*f*x))/32 + (25*cos(6*e +
 6*f*x))/32 + (5*cos(7*e + 7*f*x))/32 + 129/16)/(5760*a^3*c^5*f*cos(e/2 +
(f*x)/2)^5*sin(e/2 + (f*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.96

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14} - 105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12} + 945\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10} + 1575\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8} - 525\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6} +}{5760\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9 a^3c^5f}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

output
```
(9*tan((e + f*x)/2)**14 - 105*tan((e + f*x)/2)**12 + 945*tan((e + f*x)/2)*
*10 + 1575*tan((e + f*x)/2)**8 - 525*tan((e + f*x)/2)**6 + 189*tan((e + f*
x)/2)**4 - 45*tan((e + f*x)/2)**2 + 5)/(5760*tan((e + f*x)/2)**9*a**3*c**5
*f)
```

3.63 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 162

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx &= -\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\csc(e+fx)}{a^3c^6f}-\frac{8\csc^3(e+fx)}{3a^3c^6f}\\ &\quad+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{17\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}\end{aligned}$$

output
```
-1/9*cot(f*x+e)^9/a^3/c^6/f-4/11*cot(f*x+e)^11/a^3/c^6/f+csc(f*x+e)/a^3/c^
6/f-8/3*csc(f*x+e)^3/a^3/c^6/f+22/5*csc(f*x+e)^5/a^3/c^6/f-4*csc(f*x+e)^7/
a^3/c^6/f+17/9*csc(f*x+e)^9/a^3/c^6/f-4/11*csc(f*x+e)^11/a^3/c^6/f
```

Mathematica [A] (verified)

Time = 5.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{(-125-120\sec(e+fx)+680\sec^2(e+fx)-400\sec^3(e+fx)-720\sec^4(e+fx)+832\sec^5(e+fx)}{495a^3c^6f(-1+\sec(e+fx))^6(1+\sec(e}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

output

```
((-125 - 120*Sec[e + f*x] + 680*Sec[e + f*x]^2 - 400*Sec[e + f*x]^3 - 720*
Sec[e + f*x]^4 + 832*Sec[e + f*x]^5 + 64*Sec[e + f*x]^6 - 384*Sec[e + f*x]
^7 + 128*Sec[e + f*x]^8)*Tan[e + f*x])/(495*a^3*c^6*f*(-1 + Sec[e + f*x])^
6*(1 + Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^6}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}dx$$

$$\Big\downarrow \ 4446$$

$$\frac{\int \left(a^3\csc(e+fx)\cot^{11}(e+fx)+3a^3\csc^2(e+fx)\cot^{10}(e+fx)+3a^3\csc^3(e+fx)\cot^9(e+fx)+a^3\csc^4(e+\right.}{a^6c^6}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{4a^3\cot^{11}(e+fx)}{11f}-\frac{a^3\cot^9(e+fx)}{9f}-\frac{4a^3\csc^{11}(e+fx)}{11f}+\frac{17a^3\csc^9(e+fx)}{9f}-\frac{4a^3\csc^7(e+fx)}{f}+\frac{22a^3\csc^5(e+fx)}{5f}-\frac{8a^3\csc^3(e+fx)}{3f}+}{a^6c^6}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

output
```
(-1/9*(a^3*Cot[e + f*x]^9)/f - (4*a^3*Cot[e + f*x]^11)/(11*f) + (a^3*Csc[e
 + f*x])/f - (8*a^3*Csc[e + f*x]^3)/(3*f) + (22*a^3*Csc[e + f*x]^5)/(5*f)
- (4*a^3*Csc[e + f*x]^7)/f + (17*a^3*Csc[e + f*x]^9)/(9*f) - (4*a^3*Csc[e
+ f*x]^11)/(11*f))/(a^6*c^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4446
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m
Int[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m
), x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

method	result
parallelrisch	$-\frac{(-8745-1300\cos(4fx+4e)+8184\cos(2fx+2e)+9680\cos(fx+e)-1720\cos(6fx+6e)+240\cos(7fx+7e)+4880\cos(}{16220160 f\,a^3c^6}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+28\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{56}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{70}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{}{9}}{256 f\,a^3c^6}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+28\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{56}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{70}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{}{9}}{256 f\,a^3c^6}$
risch	$\frac{2i(495\,\mathrm{e}^{15i(fx+e)}-1485\,\mathrm{e}^{14i(fx+e)}+1815\,\mathrm{e}^{13i(fx+e)}+2475\,\mathrm{e}^{12i(fx+e)}-4917\,\mathrm{e}^{11i(fx+e)}-33\,\mathrm{e}^{10i(fx+e)}+11715\,\mathrm{e}^{9i(fx+}{495 f\,a^3c^6(}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBO
SE)
```

output
```
-1/16220160*(-8745-1300*cos(4*f*x+4*e)+8184*cos(2*f*x+2*e)+9680*cos(f*x+e)
-1720*cos(6*f*x+6*e)+240*cos(7*f*x+7*e)+4880*cos(5*f*x+5*e)-5584*cos(3*f*x
+3*e)+125*cos(8*f*x+8*e))*sec(1/2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)^11/f/a^3
/c^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{125\,\cos\left(fx+e\right)^{8}+120\,\cos\left(fx+e\right)^{7}-680\,\cos\left(fx+e\right)^{6}+400\,\cos\left(fx+e\right)^{5}+720\,\cos\left(fx\right.}{495\,\left(a^{3}c^{6}f\cos\left(fx+e\right)^{7}-3\,a^{3}c^{6}f\cos\left(fx+e\right)^{6}+a^{3}c^{6}f\cos\left(fx+e\right)^{5}+5\,a^{3}c^{6}f\cos\left(fx+e\right)^{4}-5\,a^{3}c^{6}f\right.}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="f
ricas")
```

output
```
1/495*(125*cos(f*x + e)^8 + 120*cos(f*x + e)^7 - 680*cos(f*x + e)^6 + 400*
cos(f*x + e)^5 + 720*cos(f*x + e)^4 - 832*cos(f*x + e)^3 - 64*cos(f*x + e)
^2 + 384*cos(f*x + e) - 128)/((a^3*c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(
f*x + e)^6 + a^3*c^6*f*cos(f*x + e)^5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3
*c^6*f*cos(f*x + e)^3 - a^3*c^6*f*cos(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e
) - a^3*c^6*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{\sec^{9}\left(e+fx\right)-3\sec^{8}\left(e+fx\right)+8\sec^{6}\left(e+fx\right)-6\sec^{5}\left(e+fx\right)-6\sec^{4}\left(e+fx\right)+8\sec^{3}\left(e+fx\right)-3\sec\left(e+fx\right)+1}\,dx}{a^{3}c^{6}}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

output
```
Integral(sec(e + f*x)/(sec(e + f*x)**9 - 3*sec(e + f*x)**8 + 8*sec(e + f*x
)**6 - 6*sec(e + f*x)**5 - 6*sec(e + f*x)**4 + 8*sec(e + f*x)**3 - 3*sec(e
 + f*x) + 1), x)/(a**3*c**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.23

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\frac{33\left(\frac{420\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^6}+\frac{\left(\frac{440\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1980\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5544\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{11550\,\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{27720\,\sin(fx}{(\cos(fx+e)+}}{a^3c^6\sin(fx+e)^{11}}}{126720\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="m
axima")
```

output
```
1/126720*(33*(420*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) + (440
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1980*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + 5544*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 11550*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 + 27720*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 45)*(co
s(f*x + e) + 1)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\frac{27720\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-11550\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}+5544\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-1980\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+440\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-45}{a^3c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}+\frac{33\left(3\,a^{12}c^{24}\,\mathrm{ta}\right.}{}}{126720\,f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="g
iac")
```

output
```
1/126720*((27720*tan(1/2*f*x + 1/2*e)^10 - 11550*tan(1/2*f*x + 1/2*e)^8 +
5544*tan(1/2*f*x + 1/2*e)^6 - 1980*tan(1/2*f*x + 1/2*e)^4 + 440*tan(1/2*f*
x + 1/2*e)^2 - 45)/(a^3*c^6*tan(1/2*f*x + 1/2*e)^11) + 33*(3*a^12*c^24*tan
(1/2*f*x + 1/2*e)^5 - 40*a^12*c^24*tan(1/2*f*x + 1/2*e)^3 + 420*a^12*c^24*
tan(1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = -\frac{\frac{605\cos(e+f\,x)}{8}+\frac{1023\cos(2\,e+2\,f\,x)}{16}-\frac{349\cos(3\,e+3\,f\,x)}{8}-\frac{325\cos(4\,e+4\,f\,x)}{32}+\frac{305\cos(5\,e+5\,f\,x)}{8}-\frac{215\cos(6\,e+6\,f\,x)}{16}+15}{126720\,a^3\,c^6\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6),x)
```

output
```
-((605*cos(e + f*x))/8 + (1023*cos(2*e + 2*f*x))/16 - (349*cos(3*e + 3*f*x
))/8 - (325*cos(4*e + 4*f*x))/32 + (305*cos(5*e + 5*f*x))/8 - (215*cos(6*e
 + 6*f*x))/16 + (15*cos(7*e + 7*f*x))/8 + (125*cos(8*e + 8*f*x))/128 - 874
5/128)/(126720*a^3*c^6*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^11)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.79

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{99\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{16}-1320\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}+13860\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}+27720\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-11550\tan}{126720\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}a^3c^6f}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x)
```

output

```
(99*tan((e + f*x)/2)**16 - 1320*tan((e + f*x)/2)**14 + 13860*tan((e + f*x)
/2)**12 + 27720*tan((e + f*x)/2)**10 - 11550*tan((e + f*x)/2)**8 + 5544*ta
n((e + f*x)/2)**6 - 1980*tan((e + f*x)/2)**4 + 440*tan((e + f*x)/2)**2 - 4
5)/(126720*tan((e + f*x)/2)**11*a**3*c**6*f)
```

3.64 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 163

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = &-\frac{256c^4(a+a\sec(e+fx))\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}}\\ &-\frac{64c^3(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{105f}\\ &-\frac{8c^2(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{21f}\\ &-\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{9f}\end{aligned}$$

output
```
-256/315*c^4*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/105*c
^3*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f-8/21*c^2*(a+a*sec(
f*x+e))*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/9*c*(a+a*sec(f*x+e))*(c-c*se
c(f*x+e))^(5/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.43

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx=\frac{2ac^4(1+\sec(e+fx))\left(-319+321\sec(e+fx)-165\sec^2(e+fx)+35\sec^3(e+fx)\right)\tan(e}{315f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
(2*a*c^4*(1 + Sec[e + f*x])*(-319 + 321*Sec[e + f*x] - 165*Sec[e + f*x]^2
+ 35*Sec[e + f*x]^3)*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4443, 3042, 4443, 3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{4}{3}c\int \sec(e+fx)(\sec(e+fx)a+a)(c-c\sec(e+fx))^{5/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}{9f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}{9f}$$

$$\Big\downarrow \text{ 4443}$$

$$\frac{4}{3}c\Bigg(\frac{8}{7}c\int \sec(e+fx)(\sec(e+fx)a+a)(c-c\sec(e+fx))^{3/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(}{7f}$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}{9f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{3}c\Bigg(\frac{8}{7}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2c\tan(e+fx)(a\sec(e}{}$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}{9f}$$

$$\Big\downarrow \text{ 4443}$$

$$\frac{4}{3}c\Bigg(\frac{8}{7}c\Bigg(\frac{4}{5}c\int \sec(e+fx)(\sec(e+fx)a+a)\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-cs}}{5f}$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}{9f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{3}c\Bigg(\frac{8}{7}c\Bigg(\frac{4}{5}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)(a\sec(}{}$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}{9f}$$

$$\Big\downarrow \text{ 4441}$$

$$\frac{4}{3}c\Bigg(\frac{8}{7}c\Bigg(-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}\Bigg)-\frac{2c\,\mathrm{ta}}{}$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}{9f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
```

output

```
(-2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(9*f)
+ (4*c*((-2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(7*f) + (8*c*((-8*c^2*(a + a*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[c -
c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[
e + f*x])/(5*f)))/7))/3
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.47

method	result
default	$a\left(-\frac{16\left(364\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}-819\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+711\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-285\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+45\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{2}\,c^{3}\cot\left(\frac{fx}{2}+\right.}{45f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{4}}\right.$
parts	$\frac{16a\left(177\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-301\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+175\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-35\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{2}\,c^{3}\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{35f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}-\frac{16a\left(364\cos\left(\frac{fx}{2}\right.\right.}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(-16/45/f*(364*cos(1/2*f*x+1/2*e)^8-819*cos(1/2*f*x+1/2*e)^6+711*cos(1/2
*f*x+1/2*e)^4-285*cos(1/2*f*x+1/2*e)^2+45)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*
sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(1/2)*c^3/(2*cos(1/2*f*x+1/2*e)^2-1)^4*cot(1
/2*f*x+1/2*e)+16/35/f*(177*cos(1/2*f*x+1/2*e)^6-301*cos(1/2*f*x+1/2*e)^4+1
75*cos(1/2*f*x+1/2*e)^2-35)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*2^(1/2)*c^3/(2*cos(1/2*f*x+1/2*e)^2-1)^3*cot(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = \frac{2\left(319\,ac^{3}\cos\left(fx+e\right)^{5}+317\,ac^{3}\cos\left(fx+e\right)^{4}-158\,ac^{3}\cos\left(fx+e\right)^{3}-26\,ac^{3}\cos\left(fx+e\right)\right.}{315\,f\cos\left(fx+e\right)^{4}\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm=
"fricas")
```

output
```
2/315*(319*a*c^3*cos(f*x + e)^5 + 317*a*c^3*cos(f*x + e)^4 - 158*a*c^3*cos
(f*x + e)^3 - 26*a*c^3*cos(f*x + e)^2 + 95*a*c^3*cos(f*x + e) - 35*a*c^3)*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx=\int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{7}{2}}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm=
"maxima")
```

output
```
-2/315*(315*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/4)*(5*(a*c^3*f*cos(2*f*x + 2*e)^4 + a*c^3*f*sin(2*f*x + 2*e)^4 + 4
*a*c^3*f*cos(2*f*x + 2*e)^3 + 6*a*c^3*f*cos(2*f*x + 2*e)^2 + 4*a*c^3*f*cos
(2*f*x + 2*e) + a*c^3*f + 2*(a*c^3*f*cos(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(2*
f*x + 2*e) + a*c^3*f)*sin(2*f*x + 2*e)^2)*integrate((cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos
(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
 sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e)
 - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e)
)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*
e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*
e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x +
4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x +
2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 ...
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = \frac{32\sqrt{2}\left(105\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{3}c^{2}+189\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}c^{3}+135\left(c\tan\left(\frac{1}{2}\right.\right.\right.}{315\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{9}{2}}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm=
"giac")
```

output
```
32/315*sqrt(2)*(105*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^2 + 189*(c*tan(1/2*
f*x + 1/2*e)^2 - c)^2*c^3 + 135*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 35*c^
5)*a*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/2)*f)
```

Mupad [B] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.96

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input

```
int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

output

```
((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*2i)
/f + (a*c^3*exp(e*1i + f*x*1i)*638i)/(315*f)))/(exp(e*1i + f*x*1i) - 1) -
((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*32i
)/(9*f) + (a*c^3*exp(e*1i + f*x*1i)*32i)/(9*f)))/((exp(e*1i + f*x*1i) - 1)
*(exp(e*2i + f*x*2i) + 1)^4) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i
+ f*x*1i)/2))^(1/2)*((a*c^3*96i)/(7*f) + (a*c^3*exp(e*1i + f*x*1i)*32i)/(6
3*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3) - ((c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*64i)/(5*f) - (
a*c^3*exp(e*1i + f*x*1i)*736i)/(105*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*
2i + f*x*2i) + 1)^2) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i
)/2))^(1/2)*((a*c^3*8i)/(3*f) - (a*c^3*exp(e*1i + f*x*1i)*1256i)/(315*f)))
/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1))
```

Reduce [F]

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))(c \\
&-c\sec(e+fx))^{7/2}\,dx = \sqrt{c}\,a\,c^3\left(-\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^5\,dx\right)\right. \\
&+2\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4\,dx\right) \\
&-2\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right) \\
&\left.+\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)
\end{aligned}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x)
```

output

```
sqrt(c)*a*c**3*( - int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**5,x) + 2*in
t(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4,x) - 2*int(sqrt( - sec(e + f*x
) + 1)*sec(e + f*x)**2,x) + int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.65 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 122

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = \\ -\frac{64c^3(a+a\sec(e+fx))\tan(e+fx)}{105f\sqrt{c-c\sec(e+fx)}} \\ -\frac{16c^2(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{35f} \\ -\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{7f}\end{aligned}$$

output

```
-64/105*c^3*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/35*c^2
*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f-2/7*c*(a+a*sec(f*x+e
))*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.49

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2ac^3(1+\sec(e+fx))\left(71-54\sec(e+fx)+15\sec^2(e+fx)\right)\tan(e+fx)}{105f\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
```

output

```
(-2*a*c^3*(1 + Sec[e + f*x])*(71 - 54*Sec[e + f*x] + 15*Sec[e + f*x]^2)*Ta
n[e + f*x])/(105*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4443, 3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{8}{7}c\int \sec(e+fx)(\sec(e+fx)a+a)(c-c\sec(e+fx))^{3/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{7}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}{7f}$$

$$\Big\downarrow \; 4443$$

$$\frac{8}{7}c\left(\frac{4}{5}c\int \sec(e+fx)(\sec(e+fx)a+a)\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e}}{5f}\right.$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}{7f}$$

$$\Big\downarrow \; 3042$$

$$\frac{8}{7}c\left(\frac{4}{5}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)(a\sec(e+}{}\right.$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}{7f}$$

$$\Big\downarrow \; 4441$$

$$\frac{8}{7}c\left(-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}\right)-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}{7f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
```

output

```
(-2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(7*f)
+ (8*c*((-8*c^2*(a + a*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e
+ f*x]]) - (2*c*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x]
)/(5*f)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.75

method	result
default	$a\Bigg(-\frac{8\left(92\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-161\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+98\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-21\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{2}\,c^{2}\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{21f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}+\frac{8\Big(43\cos\Big(\frac{fx}{2}+}{}$
parts	$\frac{8a\left(43\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-50\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+15\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{2}\,c^{2}\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{15f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}-\frac{8a\Big(92\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-161\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{2}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(-8/21/f*(92*cos(1/2*f*x+1/2*e)^6-161*cos(1/2*f*x+1/2*e)^4+98*cos(1/2*f*
x+1/2*e)^2-21)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*
2^(1/2)*c^2/(2*cos(1/2*f*x+1/2*e)^2-1)^3*cot(1/2*f*x+1/2*e)+8/15/f*(43*cos
(1/2*f*x+1/2*e)^4-50*cos(1/2*f*x+1/2*e)^2+15)*(-c/(2*cos(1/2*f*x+1/2*e)^2-
1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(1/2)*c^2/(2*cos(1/2*f*x+1/2*e)^2-1)^2*co
t(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = \frac{2\left(71\,ac^2\cos\left(fx+e\right)^4+88\,ac^2\cos\left(fx+e\right)^3-22\,ac^2\cos\left(fx+e\right)^2-24\,ac^2\cos\left(fx+e\right)+\right.}{105\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm=
"fricas")
```

output
```
2/105*(71*a*c^2*cos(f*x + e)^4 + 88*a*c^2*cos(f*x + e)^3 - 22*a*c^2*cos(f*
x + e)^2 - 24*a*c^2*cos(f*x + e) + 15*a*c^2)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx=\int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm=
"maxima")
```

output

```
-2/105*(105*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(3/4)*(3*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2
*a*c^2*f*cos(2*f*x + 2*e) + a*c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f
*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin
(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin
(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - c
os(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*si
n(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) +
 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) -
 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)
*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e)
 + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x +
 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + ...
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx=\frac{16\sqrt{2}\left(35\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}c^{4}+42\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c^{5}+15c^{6}\right)a}{105\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{7}{2}}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm=
"giac")
```

output
```
16/105*sqrt(2)*(35*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^4 + 42*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^5 + 15*c^6)*a/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.15

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx &= \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^{2}\,2\mathrm{i}}{f}+\frac{a\,c^{2}\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,142\mathrm{i}}{105\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}\\ &+\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^{2}\,16\mathrm{i}}{7\,f}-\frac{a\,c^{2}\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^{3}}\\ &-\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^{2}\,8\mathrm{i}}{5\,f}-\frac{a\,c^{2}\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,184\mathrm{i}}{35\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^{2}}\\ &-\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^{2}\,4\mathrm{i}}{3\,f}+\frac{a\,c^{2}\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,244\mathrm{i}}{105\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}\end{aligned}$$

input
```
int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

output

```
((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*2i)
/f + (a*c^2*exp(e*1i + f*x*1i)*142i)/(105*f)))/(exp(e*1i + f*x*1i) - 1) +
((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*16i
)/(7*f) - (a*c^2*exp(e*1i + f*x*1i)*16i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)
*(exp(e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i
+ f*x*1i)/2))^(1/2)*((a*c^2*8i)/(5*f) - (a*c^2*exp(e*1i + f*x*1i)*184i)/(3
5*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) - ((c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*4i)/(3*f) + (a
*c^2*exp(e*1i + f*x*1i)*244i)/(105*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2
i + f*x*2i) + 1))
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = \sqrt{c}\,a\,c^{2}\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^{4}\,dx - \left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^{3}\,dx\right) - \left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^{2}\,dx\right) + \int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x)
```

output

```
sqrt(c)*a*c**2*(int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4,x) - int(sqr
t( - sec(e + f*x) + 1)*sec(e + f*x)**3,x) - int(sqrt( - sec(e + f*x) + 1)*
sec(e + f*x)**2,x) + int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.66 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 81

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2(a+a\sec(e+fx))\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f}$$

output

```
-8/15*c^2*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/5*c*(a+a*
sec(f*x+e))*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = \frac{2ac^2(1+\sec(e+fx))(-7+3\sec(e+fx))\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a*c^2*(1 + Sec[e + f*x])*(-7 + 3*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt
[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$
$$\Big\downarrow \text{4443}$$
$$\frac{4}{5}c\int \sec(e+fx)(\sec(e+fx)a+a)\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}$$
$$\Big\downarrow \text{3042}$$
$$\frac{4}{5}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}$$
$$\Big\downarrow \text{4441}$$
$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(-8*c^2*(a + a*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f*x]])
 - (2*c*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(73) = 146$.

Time = 2.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.26

method	result
default	$a\left(-\frac{4\left(12\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-15\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+5\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\sqrt{2}\,c\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{5f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}+\frac{4\left(-3+5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\sqrt{-\frac{c\sin(}{2\cos(}}}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.$
parts	$\frac{4a\left(-3+5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\sqrt{2}\,c\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}-\frac{4a\left(12\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-15\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+5\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\right.}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right.}}}{5f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(-4/5/f*(12*cos(1/2*f*x+1/2*e)^4-15*cos(1/2*f*x+1/2*e)^2+5)*(-c/(2*cos(1
/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(1/2)*c/(2*cos(1/2*f*x+1/
2*e)^2-1)^2*cot(1/2*f*x+1/2*e)+4/3/f*(-3+5*cos(1/2*f*x+1/2*e)^2)*(-c/(2*co
s(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(1/2)*c/(2*cos(1/2*f*x
+1/2*e)^2-1)*cot(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx=\frac{2\left(7\,ac\cos\left(fx+e\right)^3+11\,ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)-3\,ac\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{15\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm=
"fricas")
```

output
```
2/15*(7*a*c*cos(f*x + e)^3 + 11*a*c*cos(f*x + e)^2 + a*c*cos(f*x + e) - 3*
a*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e
))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = a\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx + \int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)\right)dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
a*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(-c*sqr
t(-c*sec(e + f*x) + c)*sec(e + f*x)**3, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = \int (a\sec\left(fx+e\right)+a)(-c\sec\left(fx+e\right)+c)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm=
"maxima")
```

output

```
-2/15*(15*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)^(1/4)*((a*c*f*cos(2*f*x + 2*e)^2 + a*c*f*sin(2*f*x + 2*e)^2 + 2*a*c*f*c
os(2*f*x + 2*e) + a*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2
*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)
*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)
^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*
e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*
e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x
+ 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x
+ 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x +
2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*
x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*...
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = \frac{8\sqrt{2}\left(5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^3+3c^4\right)a}{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm=
"giac")
```

output

```
8/15*sqrt(2)*(5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3 + 3*c^4)*a/((c*tan(1/2*
f*x + 1/2*e)^2 - c)^(5/2)*f)
```

Mupad [B] (verification not implemented)

Time = 14.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2\,a\,c\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^3\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(7+7\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-6\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\right)}{15\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2}$$

input
```
int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

output
```
-(2*a*c*(exp(e*1i + f*x*1i)*1i + 1i)^3*(c - c/(exp(- e*1i - f*x*1i)/2 + ex
p(e*1i + f*x*1i)/2))^(1/2)*(7*exp(e*2i + f*x*2i) - 6*exp(e*1i + f*x*1i) +
7))/(15*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c - c\sec(e+fx))^{3/2}\,dx = \sqrt{c}\,ac\left(-\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right) + \int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*a*c*( - int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3,x) + int(sqr
t( - sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.67 $\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

output
```
-2/3*c*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2ac(1+\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(-2*a*c*(1 + Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4441}$$

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)}{3f\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
```

output

```
(-2*c*(a + a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

method	result	s
default	$\frac{a\sqrt{2}\,(\cos(fx+e)+1)^{2}\sqrt{-2c(-1+\sec(fx+e))}\,\sec(fx+e)\csc(fx+e)}{3f}$	4
parts	$-\frac{a\sqrt{2}\,\sqrt{-2c(-1+\sec(fx+e))}\,\sin(fx+e)}{f(\cos(fx+e)-1)}-\frac{a\sqrt{2}\,\sqrt{-2c(-1+\sec(fx+e))}\,(2\cot(fx+e)+\csc(fx+e)-\sec(fx+e)\csc(fx+e))}{3f}$	9

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/3*a/f*2^(1/2)*(cos(f*x+e)+1)^2*(-2*c*(-1+sec(f*x+e)))^(1/2)*sec(f*x+e)*c
sc(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2\left(a\cos\left(fx+e\right)^{2}+2\,a\cos\left(fx+e\right)+a\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm=
"fricas")
```

output
```
2/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx \\
&= a\left(\int \sqrt{-c\sec(e+fx)+c}\sec(e+fx)\,dx\right. \\
&\qquad \left.+\int \sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)\,dx\right)
\end{aligned}
$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
a*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(sqrt(-c*
sec(e + f*x) + c)*sec(e + f*x)**2, x))
```

Maxima [F]

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx \\
&= \int (a\sec(fx+e)+a)\sqrt{-c\sec(fx+e)+c}\sec(fx+e)\,dx
\end{aligned}
$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm=
"maxima")
```

output

```
2/3*(3*(a*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x +
 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x +
 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x
 + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x
 + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4
*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2
*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6
*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2
*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/
(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*x
+ 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + co
s(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6
*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f...
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx=\frac{4\sqrt{2}ac^2}{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm=
"giac")
```

output

```
4/3*sqrt(2)*a*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*f)
```

Mupad [B] (verification not implemented)

Time = 11.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = \frac{2\,a\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\left(2\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{3\,f\,\left(8\,\cos\left(2\,e+2\,f\,x\right)-12\,\cos\left(e+f\,x\right)-4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+7\right)}$$

input
```
int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

output
```
(2*a*(c - c/cos(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(
3*f*(8*cos(2*e + 2*f*x) - 12*cos(e + f*x) - 4*cos(3*e + 3*f*x) + cos(4*e +
 4*f*x) + 7))
```

Reduce [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx \\ = \sqrt{c}\,a\bigg(\int \sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^{2}dx \\ + \int \sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx\bigg)\end{aligned}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*a*(int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2,x) + int(sqrt( -
sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.68 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 77

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

output
```
-2*2^(1/2)*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))
/c^(1/2)/f+2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a\left(-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\sec(e+fx)}}{\sqrt{2}}\right)+\sqrt{1+\sec(e+fx)}\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
```

output

```
(2*a*(-(Sqrt[2]*ArcTanh[Sqrt[1 + Sec[e + f*x]]/Sqrt[2]]) + Sqrt[1 + Sec[e
+ f*x]])*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4444, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4444}$$

$$2a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{4a\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
```

output

```
(-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*
x]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4444

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Simp[2*c*((2*n - 1)/(2*n - 1))   Int[Csc[e + f*x]*((c + d*Csc[e + f*x
])^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\sec(fx+e)\,(a+a\sec(fx+e))}{\sqrt{c-c\sec(fx+e)}}\,dx$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x)
```

output

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \left[\frac{\sqrt{2}ac\sqrt{-\frac{1}{c}}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^{2}+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)-2\,(a\,c}{cf\sin(fx+e)}\right.$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm=
"fricas")
```

output

```
[(sqrt(2)*a*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)*s
in(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(a*cos(f*
x + e) + a)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e)), 2
*(sqrt(2)*a*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))
*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - (a*cos(f*x + e) + a)*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = a\left(\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx + \int \frac{\sec^{2}(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
a*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e +
f*x)**2/sqrt(-c*sec(e + f*x) + c), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec{(fx+e)}+a)\sec{(fx+e)}}{\sqrt{-c\sec{(fx+e)}+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2\,a\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}\right)}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm=
"giac")
```

output
```
2*a*(sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c) +
sqrt(2)/sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{a+\frac{a}{\cos(e+f\,x)}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input

```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

output

```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,a\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)-1}dx+\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)-1}dx\right)}{c}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x)
```

output

```
( - sqrt(c)*a*(int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*
x) - 1),x) + int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) -
1),x)))/c
```

3.69 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 76

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

output
```
1/2*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/c^(3/2)/f-a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{a\left(\cot\left(\frac{1}{2}(e+fx)\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{1+\sec(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{2}}\right)}{cf\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x
]
```

output
```
(a*(Cot[(e + f*x)/2] + (ArcTanh[Sqrt[1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[1 + S
ec[e + f*x]]*Tan[(e + f*x)/2])/Sqrt[2]))/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3042, 4445, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 4445$$

$$-\frac{a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{2c}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 4282$$

$$\frac{a\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{cf}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 216$$

$$\frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x]
```

output

```
(a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(Sqr
t[2]*c^(3/2)*f) - (a*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4445

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(66) = 132$.

Time = 2.12 (sec) , antiderivative size = 894, normalized size of antiderivative = 11.76

method	result	size
default	Expression too large to display	894
parts	Expression too large to display	894

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVER
BOSE)
```

output

```
a*(-1/8/f*2^(1/2)/c/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)*(1-cos(1/2*f*x+1/2*e))*(-3*arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*
f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))
*(1-cos(1/2*f*x+1/2*e))^2*csc(1/2*f*x+1/2*e)^2+3*ln(2*(((2*cos(1/2*f*x+1/2
*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*
x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos
(1/2*f*x+1/2*e)+1))*(1-cos(1/2*f*x+1/2*e))^2*csc(1/2*f*x+1/2*e)^2+((2*cos(
1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*(1-cos(1/2*f*x+1/2*e))
^2*csc(1/2*f*x+1/2*e)^2-((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)
^2)^(1/2))*(cos(1/2*f*x+1/2*e)+1)^2/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f
*x+1/2*e)+1)^2)^(1/2)*csc(1/2*f*x+1/2*e)^3+1/8/f*(cos(1/2*f*x+1/2*e)*arcta
nh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^
2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))-cos(1/2*f*x+1/2*e)*ln(2*(((2*cos(1/2
*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*co
s(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)
-1)/(cos(1/2*f*x+1/2*e)+1))+2*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2
*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)-arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1
/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/
2))+ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(
1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(66) = 132$.

Time = 0.20 (sec) , antiderivative size = 342, normalized size of antiderivative = 4.50

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = \left[\frac{\sqrt{2}(ac\cos(fx+e)-ac)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos}{\cos}}}{(\cos(fx+}\right.}{4}\right.$$

$$-\frac{\frac{\sqrt{2}(ac\cos(fx+e)-ac)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)}{\sqrt{c}}-2\left(a\cos(fx+e)^2+a\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-}{\cos(fx+e)}}}{2\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm=
"fricas")
```

output
```
[1/4*(sqrt(2)*(a*c*cos(f*x + e) - a*c)*sqrt(-1/c)*log((2*sqrt(2)*(cos(f*x
+ e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c)
+ (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*si
n(f*x + e) + 4*(a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), -1/2*(sqrt(
2)*(a*c*cos(f*x + e) - a*c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)/sqrt(c) - 2*(a*
cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/
((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = a\left(\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)-}}\right.$$

$$\left.+\int \frac{\sec^{2}\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
a*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sq
rt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**2/(-c*sqrt(-c*sec(e
+ f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a\sec\left(fx+e\right)+a\right)\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2), x
)
```

Giac [A] (verification not implemented)

Time $= 0.47$ (sec) , antiderivative size $= 72$, normalized size of antiderivative $= 0.95$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}a\left(\frac{\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}\right)}{2\,f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm=
"giac")
```

output

```
-1/2*sqrt(2)*a*(arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(3/2)
 + sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/(c^2*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{a+\frac{a}{\cos(e+f\,x)}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input

```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

output

```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,a\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2-2\sec(fx+e)+1}dx + \int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2-2\sec(fx+e)+1}dx\right)}{c^2}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x)
```

output

```
(sqrt(c)*a*(int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)*
*2 - 2*sec(e + f*x) + 1),x) + int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))
/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x)))/c**2
```

3.70 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 113

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} - \frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}}$$

output

```
1/16*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/
2)/c^(5/2)/f-1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+1/8*a*tan(f*x+e)/c/
f/(c-c*sec(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\,\text{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))\tan(e+fx)}{12c^2f\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x
]
```

output

```
-1/12*(a*Hypergeometric2F1[3/2, 3, 5/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e +
 f*x])*Tan[e + f*x])/(c^2*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3042, 4445, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{a\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}dx}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4283}$$

$$-\frac{a\left(\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 4282$$

$$-\frac{a\left(-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{2cf}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 216$$

$$-\frac{a\left(-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x]
```

output

```
-1/2*(a*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5/2)) - (a*(-1/2*ArcTan[(Sq
rt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2]*c^(3/2)*f
) - Tan[e + f*x]/(2*f*(c - c*Sec[e + f*x])^(3/2))))/(4*c)
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(94) = 188$.

Time = 2.12 (sec) , antiderivative size = 708, normalized size of antiderivative = 6.27

method	result	size
default	Expression too large to display	708
parts	Expression too large to display	708

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(1/64/f*2^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/c^2*(arctan
h((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2
-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(-5*cot(1/2*f*x+1/2*e)+5*csc(1/2*f*x+
1/2*e))+ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*
cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(
1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(5*cot(1/2*f*x+1/2*e)
-5*csc(1/2*f*x+1/2*e))+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^
2)^(1/2)*(2*cot(1/2*f*x+1/2*e)^3-6*cot(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^2
))-1/64/f*2^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/c^2*(arctan
h((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2
-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(-3*cot(1/2*f*x+1/2*e)+3*csc(1/2*f*x+
1/2*e))+ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*
cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(
1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(3*cot(1/2*f*x+1/2*e)
-3*csc(1/2*f*x+1/2*e))+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^
2)^(1/2)*(14*cot(1/2*f*x+1/2*e)^3-10*cot(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)
^2)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{\sqrt{2}\left(a\cos\left(fx+e\right)^2-2\,a\cos\left(fx+e\right)+a\right)\sqrt{-c}\log\left(-\frac{2\,\sqrt{2}\left(\cos(\right.}{}\right.}{}\right.$$

$$-\frac{\sqrt{2}\left(a\cos\left(fx+e\right)^2-2\,a\cos\left(fx+e\right)+a\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin\left(fx+e\right)-2\left(3\,a\cos\right.}{16\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm=
"fricas")
```

output

```
[-1/32*(sqrt(2)*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-c)*log(-(2
*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e)) - (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)
*sin(f*x + e)))*sin(f*x + e) - 4*(3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2
+ a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x
 + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), -1/16*(sqrt(2)*(a*c
os(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f
*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x +
e) - 2*(3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x
+ e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = a\Bigg(\int \frac{\sec(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx}}$$

$$+\int \frac{\sec^2(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+}}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)
```

output

```
a*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 -
 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x)
 + c)), x) + Integral(sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(
e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c
*sec(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2), x
)
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{2}\left(a\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}ac-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,ac^2}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}\right)}{16\,c^3f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm=
"giac")
```

output
```
-1/16*sqrt(2)*(a*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)
) + ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a*c - sqrt(c*tan(1/2*f*x + 1/2*e
)^2 - c)*a*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))/(c^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{a+\frac{a}{\cos(e+f\,x)}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input

```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

output

```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,a\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx+\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)}{c^3}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x)
```

output

```
( - sqrt(c)*a*(int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*
x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x) + int((sqrt( - sec(e +
f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f
*x) - 1),x)))/c**3
```

3.71 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 171

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx =&-\frac{256c^4(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\sqrt{c-c\sec(e+fx)}}\\&-\frac{64c^3(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{231f}\\&-\frac{8c^2(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{33f}\\&-\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{11f}\end{aligned}$$

output

```
-256/1155*c^4*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/23
1*c^3*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f-8/33*c^2*(a+a
*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/11*c*(a+a*sec(f*x+e))
^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.51

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = \frac{2a^2c^3\cos^4\left(\frac{1}{2}(e+fx)\right)\left(-1930+3419\cos(e+fx)-1510\cos(2(e+fx))+533\cos(3(e+f}{1155f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x
]
```

output
```
(2*a^2*c^3*Cos[(e + f*x)/2]^4*(-1930 + 3419*Cos[e + f*x] - 1510*Cos[2*(e +
 f*x)] + 533*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*
Sec[e + f*x]])/(1155*f)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 4443, 3042, 4443, 3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{12}{11}c\int \sec(e+fx)(\sec(e+fx)a+a)^2(c-c\sec(e+fx))^{5/2}dx - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}{11f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{11}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}{11f}$$

$$\Big\downarrow \; 4443$$

$$\frac{12}{11}c\left(\frac{8}{9}c\int \sec(e+fx)(\sec(e+fx)a+a)^2(c-c\sec(e+fx))^{3/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-cs}{9f}\right. \\ \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}{11f}$$

$$\Big\downarrow \; 3042$$

$$\frac{12}{11}c\left(\frac{8}{9}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2c\tan(e+fx)(a\sec}{}\right. \\ \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}{11f}$$

$$\Big\downarrow \; 4443$$

$$\frac{12}{11}c\left(\frac{8}{9}c\left(\frac{4}{7}c\int \sec(e+fx)(\sec(e+fx)a+a)^2\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-}}{7f}\right.\right. \\ \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}{11f}$$

$$\Big\downarrow \; 3042$$

$$\frac{12}{11}c\left(\frac{8}{9}c\left(\frac{4}{7}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)(a\se}{}\right.\right. \\ \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}{11f}$$

$$\Big\downarrow \; 4441$$

$$\frac{12}{11}c\left(\frac{8}{9}c\left(-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}\right)-2\right. \\ \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}{11f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
(-2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(11*
f) + (12*c*((-2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e
+ f*x])/(9*f) + (8*c*((-8*c^2*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*S
qrt[c - c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e +
f*x]]*Tan[e + f*x])/(7*f)))/9))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(155) = 310$.

Time = 15.47 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.21

method	result
default	$a^2 \Bigg(\frac{16\left(48416\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-133144\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+149787\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-87087\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+26565\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3465\right)\surd}{3465f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{5}}$
parts	$\frac{16a^2\left(177\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-301\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+175\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-35\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{2}\,c^3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{35f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}} + \frac{16a^2\left(48416\cos\right.}{}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNV
ERBOSE)
```

output

```
a^2*(16/3465/f*(48416*cos(1/2*f*x+1/2*e)^10-133144*cos(1/2*f*x+1/2*e)^8+14
9787*cos(1/2*f*x+1/2*e)^6-87087*cos(1/2*f*x+1/2*e)^4+26565*cos(1/2*f*x+1/2
*e)^2-3465)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(
1/2)*c^3/(2*cos(1/2*f*x+1/2*e)^2-1)^5*cot(1/2*f*x+1/2*e)+16/35/f*(177*cos(
1/2*f*x+1/2*e)^6-301*cos(1/2*f*x+1/2*e)^4+175*cos(1/2*f*x+1/2*e)^2-35)*(-c
/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(1/2)*c^3/(2*cos
(1/2*f*x+1/2*e)^2-1)^3*cot(1/2*f*x+1/2*e)-32/45/f*(364*cos(1/2*f*x+1/2*e)^
8-819*cos(1/2*f*x+1/2*e)^6+711*cos(1/2*f*x+1/2*e)^4-285*cos(1/2*f*x+1/2*e)
^2+45)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(1/2)*
c^3/(2*cos(1/2*f*x+1/2*e)^2-1)^4*cot(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.86

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = \frac{2\left(533\,a^2c^3\cos\left(fx+e\right)^6+844\,a^2c^3\cos\left(fx+e\right)^5-211\,a^2c^3\cos\left(fx+e\right)^4-472\,a^2c^3\cos\left(f\right.\right.}{1155\,f\cos\left(fx+e\right)^5\,\mathrm{si}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorith
m="fricas")
```

output
```
2/1155*(533*a^2*c^3*cos(f*x + e)^6 + 844*a^2*c^3*cos(f*x + e)^5 - 211*a^2*
c^3*cos(f*x + e)^4 - 472*a^2*c^3*cos(f*x + e)^3 + 295*a^2*c^3*cos(f*x + e)
^2 + 140*a^2*c^3*cos(f*x + e) - 105*a^2*c^3)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.64

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx =$$

$$-\frac{64\sqrt{2}\left(231\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^3c^3+495\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^4+385\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\right.\right.}{1155\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{11}{2}}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorith
m="giac")
```

output
```
-64/1155*sqrt(2)*(231*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^3 + 495*(c*tan(1/
2*f*x + 1/2*e)^2 - c)^2*c^4 + 385*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 105
*c^6)*a^2*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11/2)*f)
```

Mupad [B] (verification not implemented)

Time = 22.25 (sec) , antiderivative size = 606, normalized size of antiderivative = 3.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

output

```
(((a^2*c^3*2i)/f + (a^2*c^3*exp(e*1i + f*x*1i)*1066i)/(1155*f))*(c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) -
 1) + (((a^2*c^3*64i)/(11*f) - (a^2*c^3*exp(e*1i + f*x*1i)*64i)/(11*f))*(c
 - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i +
f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^5) - (((a^2*c^3*32i)/(3*f) - (a^2*c^
3*exp(e*1i + f*x*1i)*608i)/(33*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*
1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)
^4) - (((a^2*c^3*4i)/f + (a^2*c^3*exp(e*1i + f*x*1i)*2932i)/(1155*f))*(c -
 c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*
x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + (((a^2*c^3*16i)/(5*f) + (a^2*c^3*ex
p(e*1i + f*x*1i)*4272i)/(385*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i
 + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2
) + (((a^2*c^3*32i)/(7*f) - (a^2*c^3*exp(e*1i + f*x*1i)*4640i)/(231*f))*(c
 - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i +
f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
```

Reduce [F]

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2(c\\
&-c\sec(e+fx))^{7/2}\,dx=\sqrt{c}\,a^2c^3\left(-\left(\int\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^6dx\right)\right.\\
&+\int\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^5dx\\
&+2\left(\int\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^4dx\right)\\
&-2\left(\int\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^3dx\right)\\
&-\left(\int\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^2dx\right)\\
&\left.+\int\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx\right)
\end{aligned}
$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x)
```

output

```
sqrt(c)*a**2*c**3*( - int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**6,x) + i
nt(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**5,x) + 2*int(sqrt( - sec(e + f*
x) + 1)*sec(e + f*x)**4,x) - 2*int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)*
*3,x) - int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2,x) + int(sqrt( - sec
(e + f*x) + 1)*sec(e + f*x),x))
```

3.72 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 128

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3(a+a\sec(e+fx))^2\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{63f} - \frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{9f}$$

output

```
-64/315*c^3*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/63*c
^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f-2/9*c*(a+a*sec(f
*x+e))^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = \frac{4a^2c^2\cos^4\left(\frac{1}{2}(e+fx)\right)(177-220\cos(e+fx)+107\cos(2(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e}{315f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x
]
```

output
```
(4*a^2*c^2*Cos[(e + f*x)/2]^4*(177 - 220*Cos[e + f*x] + 107*Cos[2*(e + f*x
)])*Cot[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(315*f)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4443, 3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{8}{9}c\int \sec(e+fx)(\sec(e+fx)a+a)^2(c-c\sec(e+fx))^{3/2}dx - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}{9f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{9}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}{9f}$$

$$\Big\downarrow \text{4443}$$

$$\frac{8}{9}c\left(\frac{4}{7}c\int \sec(e+fx)(\sec(e+fx)a+a)^2\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec}}{7f}\right. \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}{9f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{9}c\left(\frac{4}{7}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)(a\sec(e+}{}\right. \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}{9f}$$

$$\Big\downarrow \text{4441}$$

$$\frac{8}{9}c\left(-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}\right)-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}{9f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x]
```

output

```
(-2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(9*f
) + (8*c*((-8*c^2*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[c - c*Se
c[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e
+ f*x])/(7*f)))/9
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(116) = 232$.

Time = 11.99 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.65

method	result
default	$a^{2} \Bigg(\frac{8 \left(2336 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{8}-5256 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}+4599 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-1890 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+315\right) \sqrt{2}\, \sqrt{-\frac{c \sin\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, c^{2} \cot(}{315 f \left(2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{4}}$
parts	$\frac{8 a^{2} \left(43 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-50 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+15\right) \sqrt{-\frac{c \sin\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, \sqrt{2}\, c^{2} \cot\left(\frac{f x}{2}+\frac{e}{2}\right)}{15 f \left(2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}+\frac{8 a^{2} \left(2336 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{8}-5256 \cos(\frac{f x}{2}}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(8/315/f*(2336*cos(1/2*f*x+1/2*e)^8-5256*cos(1/2*f*x+1/2*e)^6+4599*cos
(1/2*f*x+1/2*e)^4-1890*cos(1/2*f*x+1/2*e)^2+315)*2^(1/2)*(-c/(2*cos(1/2*f*
x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^2/(2*cos(1/2*f*x+1/2*e)^2-1)^4
*cot(1/2*f*x+1/2*e)+8/15/f*(43*cos(1/2*f*x+1/2*e)^4-50*cos(1/2*f*x+1/2*e)^
2+15)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(1/2)*c
^2/(2*cos(1/2*f*x+1/2*e)^2-1)^2*cot(1/2*f*x+1/2*e)-16/21/f*(92*cos(1/2*f*x
+1/2*e)^6-161*cos(1/2*f*x+1/2*e)^4+98*cos(1/2*f*x+1/2*e)^2-21)*(-c/(2*cos(
1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(1/2)*c^2/(2*cos(1/2*f*x
+1/2*e)^2-1)^3*cot(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = \frac{2(107\,a^2c^2\cos(fx+e)^5+211\,a^2c^2\cos(fx+e)^4+26\,a^2c^2\cos(fx+e)^3-118\,a^2c^2\cos(fx}{315\,f\cos(fx+e)^4\sin(fx+e)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorith
m="fricas")
```

output
```
2/315*(107*a^2*c^2*cos(f*x + e)^5 + 211*a^2*c^2*cos(f*x + e)^4 + 26*a^2*c^
2*cos(f*x + e)^3 - 118*a^2*c^2*cos(f*x + e)^2 - 5*a^2*c^2*cos(f*x + e) + 3
5*a^2*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f
*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = a^2\left(\int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx + \int\left(-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)\right)\,dx + \int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^5\left(e+fx\right)dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
a**2*(Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(
-2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3, x) + Integral(c**2*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**5, x))
```

Maxima [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.64

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = -\frac{32\sqrt{2}\left(63\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^5+90\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^6+35c^7\right)a^2}{315\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{9}{2}}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorith
m="giac")
```

output
```
-32/315*sqrt(2)*(63*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^5 + 90*(c*tan(1/2*f
*x + 1/2*e)^2 - c)*c^6 + 35*c^7)*a^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/2)
*f)
```

Mupad [B] (verification not implemented)

Time = 16.66 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.93

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

output

```
(((a^2*c^2*2i)/f + (a^2*c^2*exp(e*1i + f*x*1i)*214i)/(315*f))*(c - c/(exp(
- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1
) + (((a^2*c^2*32i)/(9*f) + (a^2*c^2*exp(e*1i + f*x*1i)*32i)/(9*f))*(c - c
/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*
1i) - 1)*(exp(e*2i + f*x*2i) + 1)^4) - (((a^2*c^2*64i)/(7*f) + (a^2*c^2*ex
p(e*1i + f*x*1i)*320i)/(63*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
 f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
+ (((a^2*c^2*48i)/(5*f) + (a^2*c^2*exp(e*1i + f*x*1i)*368i)/(105*f))*(c -
c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x
*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) - (((a^2*c^2*16i)/(3*f) + (a^2*c^2*e
xp(e*1i + f*x*1i)*208i)/(315*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i
 + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1))
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx=\sqrt{c}\,a^2c^2\left(\int\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^5\,dx-2\left(\int\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)+\int\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x)
```

output

```
sqrt(c)*a**2*c**2*(int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**5,x) - 2*in
t(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3,x) + int(sqrt( - sec(e + f*x)
+ 1)*sec(e + f*x),x))
```

3.73 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 85

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2(a+a\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{7f}$$

output
```
-8/35*c^2*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/7*c*(a+
a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = \frac{8a^2c\cos^4\left(\frac{1}{2}(e+fx)\right)(-5+9\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{35f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x
]
```

output
```
(8*a^2*c*Cos[(e + f*x)/2]^4*(-5 + 9*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e +
 f*x]^3*Sqrt[c - c*Sec[e + f*x]])/(35*f)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{4}{7}c\int \sec(e+fx)(\sec(e+fx)a+a)^2\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{7}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}$$

$$\Big\downarrow \text{4441}$$

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(-8*c^2*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[c - c*Sec[e + f*x]
]) - (2*c*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(7
*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(77) = 154$.

Time = 1.41 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.46

method	result
default	$a^{2}\left(\frac{4\left(416\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-728\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+455\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-105\right)\sqrt{2}\,\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{105f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}+\frac{4\left(-3+5\cos\left(\right.\right.}{}\right.$
parts	$\frac{4a^{2}\left(-3+5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{2}\,c\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}+\frac{4a^{2}\left(416\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-728\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+455\cos\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{105f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(4/105/f*(416*cos(1/2*f*x+1/2*e)^6-728*cos(1/2*f*x+1/2*e)^4+455*cos(1/
2*f*x+1/2*e)^2-105)*2^(1/2)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*c/(2*cos(1/2*f*x+1/2*e)^2-1)^3*cot(1/2*f*x+1/2*e)+4/3/f*(-3+5
*cos(1/2*f*x+1/2*e)^2)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2
)^(1/2)*2^(1/2)*c/(2*cos(1/2*f*x+1/2*e)^2-1)*cot(1/2*f*x+1/2*e)-8/5/f*(12*
cos(1/2*f*x+1/2*e)^4-15*cos(1/2*f*x+1/2*e)^2+5)*(-c/(2*cos(1/2*f*x+1/2*e)^
2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*2^(1/2)*c/(2*cos(1/2*f*x+1/2*e)^2-1)^2*co
t(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \sec(e+fx)(a+a\sec(e+fx))^{2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{2\left(9\,a^{2}c\cos\left(fx+e\right)^{4}+22\,a^{2}c\cos\left(fx+e\right)^{3}+12\,a^{2}c\cos\left(fx+e\right)^{2}-6\,a^{2}c\cos\left(fx+e\right)-5\,a\right.}{35\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorith
m="fricas")
```

output
```
2/35*(9*a^2*c*cos(f*x + e)^4 + 22*a^2*c*cos(f*x + e)^3 + 12*a^2*c*cos(f*x
+ e)^2 - 6*a^2*c*cos(f*x + e) - 5*a^2*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F]

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2(c\\
&-c\sec(e+fx))^{3/2}\,dx = a^2\left(\int c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)\,dx\right.\\
&+\int c\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)\,dx\\
&+\int\left(-c\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)\right)dx\\
&\left.+\int\left(-c\sqrt{-c\sec(e+fx)+c}\sec^4(e+fx)\right)dx\right)
\end{aligned}
$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
a**2*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(c*s
qrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-c*sqrt(-c*sec(e +
 f*x) + c)*sec(e + f*x)**3, x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec
(e + f*x)**4, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx=\int (a\sec{(fx+e)}+a)^2(-c\sec{(fx+e)}+c)^{\frac{3}{2}}\sec{(fx+e)}\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorith
m="maxima")
```

output

```
2/35*(35*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(3/4)*((a^2*c*f*cos(2*f*x + 2*e)^2 + a^2*c*f*sin(2*f*x + 2*e)^2 + 2*a^2*
c*f*cos(2*f*x + 2*e) + a^2*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2
*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x +
 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f
*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f
*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2
*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos
(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos
(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(
7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3
*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) -
 cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) -
 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)
*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)
^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*
e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*...
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = -\frac{16\,\sqrt{2}\left(7\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^4+5\,c^5\right)a^2}{35\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{7}{2}}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorith
m="giac")
```

output
```
-16/35*sqrt(2)*(7*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 5*c^5)*a^2/((c*tan(
1/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

Mupad [B] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.52

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c & \\ -c\sec(e+fx))^{3/2}\,dx &= \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^2\,c\,2\mathrm{i}}{f}+\frac{a^2\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,18\mathrm{i}}{35\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1} \\ &\quad -\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^2\,c\,16\mathrm{i}}{7\,f}-\frac{a^2\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3} \\ &\quad -\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^2\,c\,4\mathrm{i}}{f}-\frac{a^2\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,44\mathrm{i}}{35\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} \\ &\quad +\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^2\,c\,24\mathrm{i}}{5\,f}-\frac{a^2\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,72\mathrm{i}}{35\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}\end{aligned}$$

input
```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

output

```
((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*2i)
/f + (a^2*c*exp(e*1i + f*x*1i)*18i)/(35*f)))/(exp(e*1i + f*x*1i) - 1) - ((
c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*16i)/
(7*f) - (a^2*c*exp(e*1i + f*x*1i)*16i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(
exp(e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
f*x*1i)/2))^(1/2)*((a^2*c*4i)/f - (a^2*c*exp(e*1i + f*x*1i)*44i)/(35*f)))/
((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i
- f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*24i)/(5*f) - (a^2*c*exp
(e*1i + f*x*1i)*72i)/(35*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i
) + 1)^2)
```

Reduce [F]

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2(c \\
&-c\sec(e+fx))^{3/2}\,dx = \sqrt{c}\,a^2c\left(-\left(\int \sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^4 dx\right)\right. \\
&-\left(\int \sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^3 dx\right) \\
&+\int \sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^2 dx \\
&\left.+\int \sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right) dx\right)
\end{aligned}
$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x)
```

output

```
sqrt(c)*a**2*c*( - int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4,x) - int(
sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3,x) + int(sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2,x) + int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.74 $\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}}$$

output
```
-2/5*c*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = \frac{8a^2\cos^4\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{5f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
```

output

```
(8*a^2*Cos[(e + f*x)/2]^4*Cot[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[c - c*Sec[e
 + f*x]])/(5*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{3042, 4441\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4441}$$

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
```

output

```
(-2*c*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

method	result
default	$\frac{a^{2}\sqrt{2}\,(\cos(fx+e)+1)^{3}\sqrt{-2c(-1+\sec(fx+e))}\,\sec(fx+e)^{2}\csc(fx+e)}{5f}$
parts	$-\frac{a^{2}\sqrt{2}\,\sqrt{-2c(-1+\sec(fx+e))}\,\sin(fx+e)}{f(\cos(fx+e)-1)}+\frac{a^{2}\sqrt{2}\left(8\cos(fx+e)^{3}+4\cos(fx+e)^{2}-\cos(fx+e)+3\right)\sqrt{-2c(-1+\sec(fx+e))}\,\sec(f}{15f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/5*a^2/f*2^(1/2)*(cos(f*x+e)+1)^3*(-2*c*(-1+sec(f*x+e)))^(1/2)*sec(f*x+e)
^2*csc(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.05

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2\left(a^{2}\cos\left(fx+e\right)^{3}+3\,a^{2}\cos\left(fx+e\right)^{2}+3\,a^{2}\cos\left(fx+e\right)+a^{2}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{5\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorith
m="fricas")
```

output
```
2/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [F]

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx \\
&= a^2\left(\int \sqrt{-c\sec(e+fx)+c}\sec(e+fx)\,dx\right. \\
&\quad + \int 2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)\,dx \\
&\quad \left.+ \int \sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)\,dx\right)
\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
a**2*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(2*sqr
t(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(sqrt(-c*sec(e + f*x)
 + c)*sec(e + f*x)**3, x))
```

Maxima [F]

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx \\
&= \int (a\sec(fx+e)+a)^2\sqrt{-c\sec(fx+e)+c}\sec(fx+e)\,dx
\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorith
m="maxima")
```

output

```
2/5*(5*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(1/4)*(3*(a^2*f*cos(2*f*x + 2*e)^2 + a^2*f*sin(2*f*x + 2*e)^2 + 2*a^2*f*co
s(2*f*x + 2*e) + a^2*f)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3
*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) +
 cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*
e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*
e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x +
2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x
+ 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x
+ 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*
f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*
f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(
4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*
f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + si
n(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), c...
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = -\frac{8\sqrt{2}a^2c^3}{5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorith
m="giac")
```

output

```
-8/5*sqrt(2)*a^2*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*f)
```

Mupad [B] (verification not implemented)

Time = 14.82 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.27

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2\,a^2\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^5\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{5\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}$$

input
```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

output
```
(2*a^2*(exp(e*1i + f*x*1i)*1i + 1i)^5*(c - c/(exp(- e*1i - f*x*1i)/2 + exp
(e*1i + f*x*1i)/2))^(1/2))/(5*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2
i) + 1)^2)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\sqrt{c}\,a^2\Bigg(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3\,dx$$
$$+2\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)$$
$$+\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\Bigg)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*a**2*(int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3,x) + 2*int(sqr
t( - sec(e + f*x) + 1)*sec(e + f*x)**2,x) + int(sqrt( - sec(e + f*x) + 1)*
sec(e + f*x),x))
```

3.75 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 117

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{4\sqrt{2}a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{16a^2\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{2a^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3cf}$$

output

```
-4*2^(1/2)*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2
))/c^(1/2)/f+16/3*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/3*a^2*(c-c*sec
(f*x+e))^(1/2)*tan(f*x+e)/c/f
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= \frac{2a^2\left(-6\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)+\sqrt{a(1+\sec(e+fx))}(7+\sec(e+fx))\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x
]
```

output

```
(2*a^2*(-6*Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqr
t[a])] + Sqrt[a*(1 + Sec[e + f*x])]*(7 + Sec[e + f*x]))*Tan[e + f*x])/(3*f
*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4444, 3042, 4444, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4444}$$

$$2a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4444}$$

$$2a\left(2a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4282}$$

$$2a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{4a\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{f}\right)+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}+2a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}\right)$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]]) +
 2*a*((-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[
e + f*x]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]])
)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4444
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Simp[2*c*((2*n - 1)/(2*n - 1))   Int[Csc[e + f*x]*((c + d*Csc[e + f*x
])^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{a^{2} \sqrt{2}\, \tan \left(f x +e \right) \left(-12 \sqrt{-\frac{\cos \left(f x +e \right)}{\cos \left(f x +e \right)+1}}\, \arctan \left(\frac{\sqrt{2}}{2 \sqrt{-\frac{\cos \left(f x +e \right)}{\cos \left(f x +e \right)+1}}}\right)+\sqrt{2}\, \sec \left(f x +e \right)+7 \sqrt{2}\right)}{3 f \sqrt{-c \left(-1+\sec \left(f x +e \right)\right)}}$	95

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNV
ERBOSE)
```

output

```
1/3*a^2/f*2^(1/2)*tan(f*x+e)*(-12*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arcta
n(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+2^(1/2)*sec(f*x+e)+7*2^(
1/2))/(-c*(-1+sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.93

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \left[\frac{2\left(3\sqrt{2}a^2c\sqrt{-\frac{1}{c}}\cos(fx+e)\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\mathrm{si}}{3\,cf\cos(fx+e)\sin(fx+e)}\right.$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorith
m="fricas")
```

output

```
[2/3*(3*sqrt(2)*a^2*c*sqrt(-1/c)*cos(f*x + e)*log(-(2*sqrt(2)*(cos(f*x + e
)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (
3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f
*x + e) - (7*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x + e)*sin(f*x + e)), 2/3*(6*sqrt(2)
*a^2*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*
x + e)/(sqrt(c)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - (7*a^2*cos(f*x
+ e)^2 + 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))
)/(c*f*cos(f*x + e)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = a^2\left(\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx + \int \frac{2\sec^2(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx + \int \frac{\sec^3(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
a**2*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(2*sec
(e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**3/sqrt
(-c*sec(e + f*x) + c), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^2\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x
)
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{4\,a^2\left(\frac{3\,\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-4\,c\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}}\right)}{3\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorith
m="giac")
```

output
```
4/3*a^2*(3*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt
(c) + sqrt(2)*(3*c*tan(1/2*f*x + 1/2*e)^2 - 4*c)/(c*tan(1/2*f*x + 1/2*e)^2
 - c)^(3/2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\sqrt{c}\,a^2\left(-\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)-1}dx\right)-2\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)-1}dx\right)-\Big(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)-1}dx\right.}{c}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x)
```

output

```
(sqrt(c)*a**2*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e +
 f*x) - 1),x) - 2*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e +
 f*x) - 1),x) - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)
 - 1),x)))/c
```

3.76 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 113

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{3\sqrt{2}a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{2a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{2a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}$$

output
```
3*2^(1/2)*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2)
)/c^(3/2)/f-2*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-2*a^2*tan(f*x+e)/c/f
/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^2\,\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^2\tan(e+fx)}{10cf\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2)
,x]
```

output
```
-1/10*(a^2*Hypergeometric2F1[2, 5/2, 7/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e
 + f*x])^2*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4445, 3042, 4444, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{3a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{\sqrt{c-c\sec(e+fx)}}dx}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4444}$$

$$-\frac{3a\left(2a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\left(2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{3a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{4a\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{f}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}-\frac{3a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}\right)}{2c}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2),x]
```

output

```
-(((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)))
- (3*a*((-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Se
c[e + f*x]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]
])))/(2*c)
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4444
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Simp[2*c*((2*n - 1)/(2*n - 1))   Int[Csc[e + f*x]*((c + d*Csc[e + f*x
])^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(100) = 200$.

Time = 1.42 (sec) , antiderivative size = 1251, normalized size of antiderivative = 11.07

method	result	size
default	Expression too large to display	1251
parts	Expression too large to display	1256

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNV
ERBOSE)
```

output

```
a^2*(-1/8/f*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)/c/(-c/(2*cos(1/2*f*x+1/2*e)
^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*((-7*cos(1/2*f*x+1/2*e)-7)*sin(1/2*f*x+1
/2*e)*arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*
f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*((2*cos(1/2*f*x+1/2*e)^2-
1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)+(7*cos(1/2*f*x+1/2*e)+7)*sin(1/2*f*x+1/
2*e)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*ln(2*(((2
*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e
)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*
x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))+(-20*cos(1/2*f*x+1/2*e)^2+18)*cot(1/2*
f*x+1/2*e))+1/8/f*(cos(1/2*f*x+1/2*e)*arctanh((2*cos(1/2*f*x+1/2*e)-1)/(co
s(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^
(1/2))-cos(1/2*f*x+1/2*e)*ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1
/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*
f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))+2*(
(2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2
*e)-arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*
x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))+ln(2*(((2*cos(1/2*f*x+1/2*e
)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+
1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1
/2*f*x+1/2*e)+1)))*2^(1/2)/c/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x...
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.29

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = \left[\frac{3\sqrt{2}(a^2c\cos(fx+e)-a^2c)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)}{(\cos}\right.}{}\right.$$

$$-\frac{\frac{3\sqrt{2}(a^2c\cos(fx+e)-a^2c)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)}{\sqrt{c}}-2\left(2a^2\cos(fx+e)^2+a^2\cos(fx+e)-a^2\right)}{(c^2f\cos(fx+e)-c^2f)\sin(fx+e)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorith
m="fricas")
```

output

```
[1/2*(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*sqrt(-1/c)*log((2*sqrt(2)*(co
s(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(
-1/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e
)))*sin(f*x + e) + 4*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt(
(c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x
+ e)), -(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*arctan(sqrt(2)*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x
+ e)/sqrt(c) - 2*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)
)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = a^2\left(\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx}}\right.$$

$$+\int \frac{2\sec^2(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx$$

$$\left.+\int \frac{\sec^3(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)
```

output

```
a**2*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c
*sqrt(-c*sec(e + f*x) + c)), x) + Integral(2*sec(e + f*x)**2/(-c*sqrt(-c*s
ec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integra
l(sec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*s
ec(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^2\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
 x)
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^2\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{2}\left(3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}+\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c\right)c}\right)}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorith
m="giac")
```

output
```
-a^2*(3*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(3/2)
 + sqrt(2)*(3*c*tan(1/2*f*x + 1/2*e)^2 - c)/(((c*tan(1/2*f*x + 1/2*e)^2 -
c)^(3/2) + sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c)*c))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,a^2\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^2-2\sec(fx+e)+1}dx+2\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2-2\sec(fx+e)+1}\right.\right.}{c^2}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*a**2*(int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*
x)**2 - 2*sec(e + f*x) + 1),x) + 2*int((sqrt( - sec(e + f*x) + 1)*sec(e +
f*x)**2)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x) + int((sqrt( - sec(e +
f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x)))/c**2
```

3.77 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 117

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{3a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} - \frac{a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{5/2}} + \frac{5a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}$$

output
```
-3/8*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(
1/2)/c^(5/2)/f-a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+5/4*a^2*tan(f*x+e)/
c/f/(c-c*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^{3/2}\left(\sqrt{a}(-1+4\sec(e+fx)+5\sec^2(e+fx))+6\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)\sec^2(e+fx)\sqrt{a(1+\sec}\right.}{4c^2f(-1+\sec(e+fx))^2(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2)
,x]
```

output
```
-1/4*(a^(3/2)*(Sqrt[a]*(-1 + 4*Sec[e + f*x] + 5*Sec[e + f*x]^2) + 6*Sqrt[2
]*ArcTanh[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*Sec[e + f*x]^2*Sqr
t[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^4)*Tan[e + f*x])/(c^2*f*(-1 + Sec
[e + f*x])^2*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {3042, 4445, 3042, 4445, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{3a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{(c-c\sec(e+fx))^{3/2}}dx}{4c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4445}$$

$$-\frac{3a\left(-\frac{a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{2c}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\left(-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{3a\left(\frac{a\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{cf}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}-\frac{3a\left(\frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}\right)}{4c}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2),x]
```

output

```
-1/2*((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5/2)
) - (3*a*((a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x
]])])/(Sqrt[2]*c^(3/2)*f) - (a*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)
)))/(4*c)
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(100) = 200$.

Time = 1.42 (sec) , antiderivative size = 1062, normalized size of antiderivative = 9.08

method	result	size
default	Expression too large to display	1062
parts	Expression too large to display	1067

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(1/64/f*2^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/c^2*(ln(2
*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+
1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1
/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(-19*cot(1/2*f*x+1/2*e)+19*csc(1/
2*f*x+1/2*e))+arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*
cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(19*cot(1/2*f*x+1
/2*e)-19*csc(1/2*f*x+1/2*e))+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*
e)+1)^2)^(1/2)*(18*cot(1/2*f*x+1/2*e)^3-22*cot(1/2*f*x+1/2*e)*csc(1/2*f*x+
1/2*e)^2))-1/64/f*2^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e
)^2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/c^2
*(arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+
1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(-3*cot(1/2*f*x+1/2*e)+3*csc(
1/2*f*x+1/2*e))+ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2
)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)
+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(3*cot(1/2*f*
x+1/2*e)-3*csc(1/2*f*x+1/2*e))+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/
2*e)+1)^2)^(1/2)*(14*cot(1/2*f*x+1/2*e)^3-10*cot(1/2*f*x+1/2*e)*csc(1/2*f*
x+1/2*e)^2))+1/32/f*2^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.67

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^2-2\,a^2\cos\left(fx+e\right)+a^2\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}}{}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorith
m="fricas")
```

output
```
[-1/16*(3*sqrt(2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-c)
*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x +
e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*
x + e)^2 - 5*a^2*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((
c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/8*(3
*sqrt(2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(c)*arctan(sq
rt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*
x + e)))*sin(f*x + e) - 2*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*x + e)^2 - 5*a
^2*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x
+ e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = a^2\left(\int \frac{\sec(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\,\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+f}}\right.$$

$$+\int \frac{2\sec^2(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\,\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+}}$$

$$+\int \frac{\sec^3(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\,\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
a**2*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**
2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f
*x) + c)), x) + Integral(2*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)
*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sq
rt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**3/(c**2*sqrt(-c*sec(
e + f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f
*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx=\frac{\sqrt{2}\left(3\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c+5\sqrt{}}{c^2\tan\left(\frac{1}{2}fx+\right.}\right.}{8c^3f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorith
m="giac")
```

output
```
1/8*sqrt(2)*(3*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))
+ (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 5*sqrt(c*tan(1/2*f*x + 1/2*e
)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))*a^2/(c^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,a^2\left(-\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)-2\left(\int \frac{\sqrt{-\mathrm{s}}}{\sec(fx+e)^3}\right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*a**2*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e +
 f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x) - 2*int((sqrt( - sec
(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*s
ec(e + f*x) - 1),x) - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e
+ f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x)))/c**3
```

3.78 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 164

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} - \frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}}$$

output
```
-1/32*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^
(1/2)/c^(7/2)/f-1/3*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/
2)+1/4*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)-1/16*a^2*tan(f*x+e)/c^2/f
/(c-c*sec(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^2\,\text{Hypergeometric2F1}\left(\frac{5}{2},4,\frac{7}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^2\tan(e+fx)}{40c^3f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2)
,x]
```

output
```
-1/40*(a^2*Hypergeometric2F1[5/2, 4, 7/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e
 + f*x])^2*Tan[e + f*x])/(c^3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4445, 3042, 4445, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{(c-c\sec(e+fx))^{5/2}}dx}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4445}$$

$$-\frac{a\left(-\frac{a\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}dx}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4283}$$

$$-\frac{a\left(-\frac{a\left(\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-\frac{a\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{a\left(-\frac{a\left(-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{2cf}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}-\frac{a\left(-\frac{a\left(-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}\right)}{2c}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/3*((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(7/2)
) - (a*(-1/2*(a*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5/2)) - (a*(-1/2*Ar
cTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2]*c
^(3/2)*f) - Tan[e + f*x]/(2*f*(c - c*Sec[e + f*x])^(3/2))))/(4*c)))/(2*c)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(141) = 282$.

Time = 4.74 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.24

method	result
default	$\frac{a^{2}\sqrt{2}\left(\left(14\cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+8\cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-6\right)\sqrt{\frac{2\cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}}\,\cot\left(\frac{f x}{2}+\frac{e}{2}\right)\csc\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+\ln\left(\frac{2\sqrt{\frac{2\cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}}\,\cos\left(\frac{f x}{2}\right.}{}\right.\right.}{192c^{3}f}$
parts	Expression too large to display

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/192*a^2/c^3*2^(1/2)/f/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^
2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*((14*
cos(1/2*f*x+1/2*e)^4+8*cos(1/2*f*x+1/2*e)^2-6)*((2*cos(1/2*f*x+1/2*e)^2-1)
/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cot(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^4+l
n(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f
*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*co
s(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(-3*cot(1/2*f*x+1/2*e)+3*csc(1
/2*f*x+1/2*e))+arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2
*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(3*cot(1/2*f*x+1
/2*e)-3*csc(1/2*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.15

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = \Bigg[-\frac{3\sqrt{2}\left(a^2\cos(fx+e)^3-3a^2\cos(fx+e)^2+3a^2\cos(fx+e)\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorith
m="fricas")
```

output
```
[-1/192*(3*sqrt(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(
f*x + e) - a^2)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sq
rt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*si
n(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*(7*a^2*cos
(f*x + e)^4 + 29*a^2*cos(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*
x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 -
3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/96
*(3*sqrt(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e
) - a^2)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*co
s(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) + 2*(7*a^2*cos(f*x + e)^4
+ 29*a^2*cos(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e))*sqrt
((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(
f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = a^2\left(\int \frac{}{-c^3\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3c^3\sqrt{-c\sec(e+}\right.$$
$$+\int \frac{2\sec^2(e+fx)}{-c^3\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3c^3\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-3c^3\sqrt{-c\sec(e+f}$$
$$+\int \frac{\sec^3(e+fx)}{-c^3\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3c^3\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-3c^3\sqrt{-c\sec(e+f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(7/2),x)
```

output
```
a**2*(Integral(sec(e + f*x)/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)*
*3 + 3*c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x) + Integr
al(2*sec(e + f*x)**2/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*
c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e + f*
x) + c)*sec(e + f*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x) + Integral(sec(
e + f*x)**3/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*c**3*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e + f*x) + c)*s
ec(e + f*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{2}\left(\frac{3\,a^2\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{7}{2}}} + \frac{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}a^2+8\left(c\tan(\right.}{c^6\,\mathrm{ta}}\right.}{96\,f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorith
m="giac")
```

output

```
1/96*sqrt(2)*(3*a^2*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(
7/2) + (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*a^2 + 8*(c*tan(1/2*f*x + 1/
2*e)^2 - c)^(3/2)*a^2*c - 3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^2*c^2)/(c
^6*tan(1/2*f*x + 1/2*e)^6))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input

```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

output

```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = \sqrt{c}\,a^2\bigg(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^4-4\sec(fx+e)^3+6\sec(fx+e)^2-4\sec(fx+e)+1}\,dx + 2\bigg(\int$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*a**2*(int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*
x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1),x) + 2
*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**4 - 4*sec(
e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1),x) + int((sqrt( - se
c(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec
(e + f*x)**2 - 4*sec(e + f*x) + 1),x)))/c**4
```

3.79 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 171

$$
\begin{aligned}
\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx =\\
&-\frac{256c^4(a+a\sec(e+fx))^3\tan(e+fx)}{3003f\sqrt{c-c\sec(e+fx)}}\\
&-\frac{64c^3(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{429f}\\
&-\frac{24c^2(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{143f}\\
&-\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{13f}
\end{aligned}
$$

output

```
-256/3003*c^4*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/42
9*c^3*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f-24/143*c^2*(a
+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/13*c*(a+a*sec(f*x+e
))^3*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 3.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.51

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = \frac{4a^3c^3\cos^6(\frac{1}{2}(e+fx))(-3766+6285\cos(e+fx)-2842\cos(2(e+fx))+835\cos(3(e+f}{3003f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x
]
```

output
```
(4*a^3*c^3*Cos[(e + f*x)/2]^6*(-3766 + 6285*Cos[e + f*x] - 2842*Cos[2*(e +
 f*x)] + 835*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^6*Sqrt[c - c*
Sec[e + f*x]])/(3003*f)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 4443, 3042, 4443, 3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{12}{13}c\int \sec(e+fx)(\sec(e+fx)a+a)^3(c-c\sec(e+fx))^{5/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}{13f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{13}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx-$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}{13f}$$

$$\Big\downarrow \text{ 4443}$$

$$\frac{12}{13}c\Bigg(\frac{8}{11}c\int \sec(e+fx)(\sec(e+fx)a+a)^3(c-c\sec(e+fx))^{3/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c}{11f}$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}{13f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{12}{13}c\Bigg(\frac{8}{11}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2c\tan(e+fx)(a\,\mathrm{se}}{}$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}{13f}$$

$$\Big\downarrow \text{ 4443}$$

$$\frac{12}{13}c\Bigg(\frac{8}{11}c\Bigg(\frac{4}{9}c\int \sec(e+fx)(\sec(e+fx)a+a)^3\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c}}{9f}$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}{13f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{12}{13}c\Bigg(\frac{8}{11}c\Bigg(\frac{4}{9}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)(a\,\mathrm{s}}{}$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}{13f}$$

$$\Big\downarrow \text{ 4441}$$

$$\frac{12}{13}c\Bigg(\frac{8}{11}c\Bigg(-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^3}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}\Bigg)-$$
$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}{13f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x]
```

output

```
(-2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(13*
f) + (12*c*((-2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[e
+ f*x])/(11*f) + (8*c*((-8*c^2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*f*
Sqrt[c - c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e +
 f*x]]*Tan[e + f*x])/(9*f)))/11))/13
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 54.02 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.74

method	result
default	$\frac{128a^3\sqrt{2}\,c^3\left(835\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6-1963\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+1573\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-429\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{3003f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$
parts	$\frac{16a^3\left(177\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6-301\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+175\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-35\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,\sqrt{2}\,c^3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{35f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{16a^3\left(53632\cos\right.}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
128/3003*a^3*2^(1/2)*c^3/f*(835*cos(1/2*f*x+1/2*e)^6-1963*cos(1/2*f*x+1/2*
e)^4+1573*cos(1/2*f*x+1/2*e)^2-429)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2
*f*x+1/2*e)^2)^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)^6*cos(1/2*f*x+1/2*e)^6*cot
(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = \frac{2\left(835\,a^3c^3\cos\left(fx+e\right)^7+1919\,a^3c^3\cos\left(fx+e\right)^6+271\,a^3c^3\cos\left(fx+e\right)^5-1637\,a^3c^3\cos\left(\right.\right.}{3003\,f\,\mathrm{co}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorith
m="fricas")
```

output

```
2/3003*(835*a^3*c^3*cos(f*x + e)^7 + 1919*a^3*c^3*cos(f*x + e)^6 + 271*a^3
*c^3*cos(f*x + e)^5 - 1637*a^3*c^3*cos(f*x + e)^4 - 103*a^3*c^3*cos(f*x +
e)^3 + 973*a^3*c^3*cos(f*x + e)^2 + 21*a^3*c^3*cos(f*x + e) - 231*a^3*c^3)
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^6*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx=\text{Timed out}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(7/2),x)
```

output

```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx=\text{Timed out}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorith
m="maxima")
```

output

```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.64

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = \frac{128\sqrt{2}\Big(429\,\big(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\big)^3c^4+1001\,\big(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\big)^2c^5+819\,\big(c\tan}{3003\,\big(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\big)^{\frac{13}{2}}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorith
m="giac")
```

output
```
128/3003*sqrt(2)*(429*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^4 + 1001*(c*tan(1
/2*f*x + 1/2*e)^2 - c)^2*c^5 + 819*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^6 + 23
1*c^7)*a^3*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(13/2)*f)
```

Mupad [B] (verification not implemented)

Time = 24.07 (sec) , antiderivative size = 710, normalized size of antiderivative = 4.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

output
```
(((a^3*c^3*2i)/f + (a^3*c^3*exp(e*1i + f*x*1i)*1670i)/(3003*f))*(c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) -
 1) + (((a^3*c^3*128i)/(13*f) + (a^3*c^3*exp(e*1i + f*x*1i)*128i)/(13*f))*
(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i
+ f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^6) - (((a^3*c^3*384i)/(11*f) + (a^
3*c^3*exp(e*1i + f*x*1i)*3456i)/(143*f))*(c - c/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i
) + 1)^5) - (((a^3*c^3*8i)/f + (a^3*c^3*exp(e*1i + f*x*1i)*2168i)/(3003*f)
)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1
i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + (((a^3*c^3*24i)/f + (a^3*c^3*
exp(e*1i + f*x*1i)*5464i)/(1001*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e
*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1
)^2) + (((a^3*c^3*160i)/(3*f) + (a^3*c^3*exp(e*1i + f*x*1i)*11360i)/(429*f
))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*
1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^4) - (((a^3*c^3*320i)/(7*f) + (
a^3*c^3*exp(e*1i + f*x*1i)*46400i)/(3003*f))*(c - c/(exp(- e*1i - f*x*1i)/
2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*
x*2i) + 1)^3)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = \sqrt{c}\,a^3c^3\left(-\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^7\,dx\right)+3\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^5\,dx\right)-3\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)+\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x)
```

output

```
sqrt(c)*a**3*c**3*( - int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**7,x) + 3
*int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**5,x) - 3*int(sqrt( - sec(e +
f*x) + 1)*sec(e + f*x)**3,x) + int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x),
x))
```

3.80 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 128

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3(a+a\sec(e+fx))^3\tan(e+fx)}{693f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{99f} - \frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{11f}$$

output

```
-64/693*c^3*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/99*c
^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f-2/11*c*(a+a*sec(
f*x+e))^3*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = \frac{8a^3c^2\cos^6\left(\frac{1}{2}(e+fx)\right)(277-364\cos(e+fx)+151\cos(2(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^5(e}{693f}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x
]
```

output

```
(8*a^3*c^2*Cos[(e + f*x)/2]^6*(277 - 364*Cos[e + f*x] + 151*Cos[2*(e + f*x
)])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*Sec[e + f*x]])/(693*f)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4443, 3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{8}{11}c\int \sec(e+fx)(\sec(e+fx)a+a)^3(c-c\sec(e+fx))^{3/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}{11f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{11}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}{11f}$$

$$\Big\downarrow \text{4443}$$

$$\frac{8}{11}c\left(\frac{4}{9}c\int \sec(e+fx)(\sec(e+fx)a+a)^3\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\,\mathrm{se}}}{9f}\right.$$

$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}{11f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{11}c\left(\frac{4}{9}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)(a\sec(e-}{}\right.$$

$$\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}{11f}$$

$$\Big\downarrow \text{4441}$$

$$\frac{8}{11}c\left(-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^3}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}\right)-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}{11f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x]
```

output

```
(-2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(11*
f) + (8*c*((-8*c^2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*f*Sqrt[c - c*S
ec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e
 + f*x])/(9*f)))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 53.79 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

method	result
default	$\frac{64a^{3}\sqrt{2}\,c^{2}\left(151\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-242\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+99\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{693f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{5}}$
parts	$\frac{8a^{3}\left(43\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-50\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+15\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{2}\,c^{2}\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{15f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}-\frac{8a^{3}\left(9088\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-24992\cos\right.}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNV
ERBOSE)
```

output

```
64/693*a^3*2^(1/2)*c^2/f*(151*cos(1/2*f*x+1/2*e)^4-242*cos(1/2*f*x+1/2*e)^
2+99)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/(2*cos(1/
2*f*x+1/2*e)^2-1)^5*cos(1/2*f*x+1/2*e)^6*cot(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = \frac{2\left(151\,a^3c^2\cos\left(fx+e\right)^6+422\,a^3c^2\cos\left(fx+e\right)^5+241\,a^3c^2\cos\left(fx+e\right)^4-236\,a^3c^2\cos\left(fx\right.\right.}{693\,f\cos\left(fx+e\right)^5\sin}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorith
m="fricas")
```

output

```
2/693*(151*a^3*c^2*cos(f*x + e)^6 + 422*a^3*c^2*cos(f*x + e)^5 + 241*a^3*c
^2*cos(f*x + e)^4 - 236*a^3*c^2*cos(f*x + e)^3 - 199*a^3*c^2*cos(f*x + e)^
2 + 70*a^3*c^2*cos(f*x + e) + 63*a^3*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(5/2),x)
```

output

```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.64

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx=\frac{64\sqrt{2}\left(99\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^6+154\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^7+63c^8\right)a^3}{693\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{11}{2}}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorith
m="giac")
```

output
```
64/693*sqrt(2)*(99*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^6 + 154*(c*tan(1/2*f
*x + 1/2*e)^2 - c)*c^7 + 63*c^8)*a^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11/2
)*f)
```

Mupad [B] (verification not implemented)

Time = 22.76 (sec) , antiderivative size = 607, normalized size of antiderivative = 4.74

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

output
```
(((a^3*c^2*2i)/f + (a^3*c^2*exp(e*1i + f*x*1i)*302i)/(693*f))*(c - c/(exp(
- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1
) - (((a^3*c^2*64i)/(11*f) - (a^3*c^2*exp(e*1i + f*x*1i)*64i)/(11*f))*(c -
 c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*
x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^5) + (((a^3*c^2*16i)/f - (a^3*c^2*exp(
e*1i + f*x*1i)*944i)/(231*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) +
 (((a^3*c^2*160i)/(9*f) - (a^3*c^2*exp(e*1i + f*x*1i)*1120i)/(99*f))*(c -
c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x
*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^4) - (((a^3*c^2*20i)/(3*f) - (a^3*c^2*e
xp(e*1i + f*x*1i)*844i)/(693*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i
 + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1))
- (((a^3*c^2*160i)/(7*f) - (a^3*c^2*exp(e*1i + f*x*1i)*6880i)/(693*f))*(c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f
*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = \sqrt{c}\,a^3c^2\left(\int\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^6\,dx + \int\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^5\,dx - 2\left(\int\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4\,dx\right) - 2\left(\int\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right) + \int\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx + \int\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x)
```

output

```
sqrt(c)*a**3*c**2*(int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**6,x) + int(
sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**5,x) - 2*int(sqrt( - sec(e + f*x)
+ 1)*sec(e + f*x)**4,x) - 2*int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3,
x) + int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2,x) + int(sqrt( - sec(e
+ f*x) + 1)*sec(e + f*x),x))
```

3.81 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 85

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2(a+a\sec(e+fx))^3\tan(e+fx)}{63f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{9f}$$

output
```
-8/63*c^2*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/9*c*(a+
a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \frac{16a^3c\cos^6\left(\frac{1}{2}(e+fx)\right)(-7+11\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+f}{63f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x
]
```

output
```
(16*a^3*c*Cos[(e + f*x)/2]^6*(-7 + 11*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e
 + f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(63*f)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{4}{9}c\int \sec(e+fx)(\sec(e+fx)a+a)^3\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{9}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}$$

$$\Big\downarrow \text{4441}$$

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^3}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(-8*c^2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*f*Sqrt[c - c*Sec[e + f*x]
]) - (2*c*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(9
*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

method	result
default	$\frac{32a^3\sqrt{2}\,c\left(11\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-9\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{63f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$
parts	$\frac{4a^3\left(-3+5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,\sqrt{2}\,c\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)} - \frac{4a^3\left(2176\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^8-4896\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6+4284\cos\left(\frac{fx}{2}\right.\right.}{315f\left(\right.}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNV
ERBOSE)
```

output

```
32/63*a^3*2^(1/2)*c/f*(11*cos(1/2*f*x+1/2*e)^2-9)*(-c/(2*cos(1/2*f*x+1/2*e
)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)^4*cos(1/2*f*
x+1/2*e)^6*cot(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \frac{2\left(11\,a^3c\cos\left(fx+e\right)^5+37\,a^3c\cos\left(fx+e\right)^4+38\,a^3c\cos\left(fx+e\right)^3+2\,a^3c\cos\left(fx+e\right)^2-\right.}{63\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorith
m="fricas")
```

output

```
2/63*(11*a^3*c*cos(f*x + e)^5 + 37*a^3*c*cos(f*x + e)^4 + 38*a^3*c*cos(f*x
 + e)^3 + 2*a^3*c*cos(f*x + e)^2 - 17*a^3*c*cos(f*x + e) - 7*a^3*c)*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = a^3\left(\int c\sqrt{-c\sec{(e+fx)}+c}\sec{(e+fx)}\,dx + \int 2c\sqrt{-c\sec{(e+fx)}+c}\sec^2{(e+fx)}\,dx + \int\left(-2c\sqrt{-c\sec{(e+fx)}+c}\sec^4{(e+fx)}\right)dx + \int\left(-c\sqrt{-c\sec{(e+fx)}+c}\sec^5{(e+fx)}\right)dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
a**3*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(2*c
*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-2*c*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x)**4, x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)
*sec(e + f*x)**5, x))
```

Maxima [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \frac{32\sqrt{2}\left(9\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+7c^6\right)a^3}{63\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{9}{2}}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorith
m="giac")
```

output
```
32/63*sqrt(2)*(9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 7*c^6)*a^3/((c*tan(1
/2*f*x + 1/2*e)^2 - c)^(9/2)*f)
```

Mupad [B] (verification not implemented)

Time = 22.16 (sec) , antiderivative size = 471, normalized size of antiderivative = 5.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

output
```
((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*2i)
/f + (a^3*c*exp(e*1i + f*x*1i)*22i)/(63*f)))/(exp(e*1i + f*x*1i) - 1) - ((
c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*32i)/
(9*f) + (a^3*c*exp(e*1i + f*x*1i)*32i)/(9*f)))/((exp(e*1i + f*x*1i) - 1)*(
exp(e*2i + f*x*2i) + 1)^4) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
f*x*1i)/2))^(1/2)*((a^3*c*8i)/(3*f) - (a^3*c*exp(e*1i + f*x*1i)*200i)/(63*
f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(-
e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*32i)/(7*f) + (a^3*
c*exp(e*1i + f*x*1i)*608i)/(63*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i +
f*x*2i) + 1)^3) - (a^3*c*exp(e*1i + f*x*1i)*(c - c/(exp(- e*1i - f*x*1i)/2
 + exp(e*1i + f*x*1i)/2))^(1/2)*160i)/(21*f*(exp(e*1i + f*x*1i) - 1)*(exp(
e*2i + f*x*2i) + 1)^2)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \sqrt{c}\,a^3c\left(-\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^5\,dx\right) - 2\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4\,dx\right) + 2\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right) + \int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x)
```

output

```
sqrt(c)*a**3*c*( - int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**5,x) - 2*in
t(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4,x) + 2*int(sqrt( - sec(e + f*x
) + 1)*sec(e + f*x)**2,x) + int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.82 $\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{c-c\sec(e+fx)}}$$

output

```
-2/7*c*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = \frac{16a^3\cos^6\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{7f}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
```

output

```
(16*a^3*Cos[(e + f*x)/2]^6*Cot[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[c - c*Sec[
e + f*x]])/(7*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4441}$$

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3}{7f\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
```

output

```
(-2*c*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

method	result
default	$\frac{a^{3}\sqrt{2}\,(\cos(fx+e)+1)^{4}\sqrt{-2c(-1+\sec(fx+e))}\,\sec(fx+e)^{3}\csc(fx+e)}{7f}$
parts	$-\frac{a^{3}\sqrt{2}\,\sqrt{-2c(-1+\sec(fx+e))}\,\sin(fx+e)}{f(\cos(fx+e)-1)}-\frac{a^{3}\sqrt{2}\left(16\cos(fx+e)^{4}+8\cos(fx+e)^{3}-2\cos(fx+e)^{2}+\cos(fx+e)-5\right)\sqrt{-2c(-1+s}}{35f}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNV
ERBOSE)
```

output

```
1/7*a^3/f*2^(1/2)*(cos(f*x+e)+1)^4*(-2*c*(-1+sec(f*x+e)))^(1/2)*sec(f*x+e)
^3*csc(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.37

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2\left(a^{3}\cos\left(fx+e\right)^{4}+4\,a^{3}\cos\left(fx+e\right)^{3}+6\,a^{3}\cos\left(fx+e\right)^{2}+4\,a^{3}\cos\left(fx+e\right)+a^{3}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{7\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorith
m="fricas")
```

output
```
2/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*
a^3*cos(f*x + e) + a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x
 + e)^3*sin(f*x + e))
```

Sympy [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx\\ = a^3\left(\int \sqrt{-c\sec(e+fx)+c}\sec(e+fx)\,dx\right.\\ +\int 3\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)\,dx\\ +\int 3\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)\,dx\\ \left.+\int \sqrt{-c\sec(e+fx)+c}\sec^4(e+fx)\,dx\right)\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
a**3*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(3*sqr
t(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(3*sqrt(-c*sec(e + f*
x) + c)*sec(e + f*x)**3, x) + Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f
*x)**4, x))
```

Maxima [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx\\ =\int (a\sec(fx+e)+a)^3\sqrt{-c\sec(fx+e)+c}\sec(fx+e)\,dx\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorith
m="maxima")
```

output

```
2/7*(7*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(3/4)*(5*(a^3*f*cos(2*f*x + 2*e)^2 + a^3*f*sin(2*f*x + 2*e)^2 + 2*a^3*f*co
s(2*f*x + 2*e) + a^3*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) +
 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x +
 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*
x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*
x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*
f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*co
s(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) -
6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))
*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10
*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x +
 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x
 + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f
*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos
(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + ...
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = \frac{16\sqrt{2}a^3c^4}{7\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{7}{2}}f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorith
m="giac")
```

output

```
16/7*sqrt(2)*a^3*c^4/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

Mupad [B] (verification not implemented)

Time = 15.85 (sec) , antiderivative size = 375, normalized size of antiderivative = 9.15

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx \\
&= \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,2\mathrm{i}}{f}+\frac{a^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2\mathrm{i}}{7\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1} \\
&\quad - \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,8\mathrm{i}}{f}+\frac{a^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,8\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2} \\
&\quad + \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,4\mathrm{i}}{f}+\frac{a^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,36\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} \\
&\quad + \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,16\mathrm{i}}{7\,f}-\frac{a^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}
\end{aligned}
$$

input

```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

output

```
((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*2i)/f
 + (a^3*exp(e*1i + f*x*1i)*2i)/(7*f)))/(exp(e*1i + f*x*1i) - 1) - ((c - c/
(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*8i)/f + (a^3*
exp(e*1i + f*x*1i)*8i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2
i) + 1)^2) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2
)*((a^3*4i)/f + (a^3*exp(e*1i + f*x*1i)*36i)/(7*f)))/((exp(e*1i + f*x*1i)
- 1)*(exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1
i + f*x*1i)/2))^(1/2)*((a^3*16i)/(7*f) - (a^3*exp(e*1i + f*x*1i)*16i)/(7*f
)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
```

Reduce [F]

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx \\
&= \sqrt{c}\,a^3\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4\,dx\right. \\
&\quad + 3\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right) \\
&\quad + 3\left(\int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right) \\
&\quad \left. + \int \sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)
\end{aligned}
$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x)
```

output

```
sqrt(c)*a**3*(int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4,x) + 3*int(sqr
t( - sec(e + f*x) + 1)*sec(e + f*x)**3,x) + 3*int(sqrt( - sec(e + f*x) + 1
)*sec(e + f*x)**2,x) + int(sqrt( - sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.83 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 164

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx = &-\frac{8\sqrt{2}a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}\\ &+\frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\\ &+\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}}\\ &+\frac{4(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output

```
-8*2^(1/2)*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2
))/c^(1/2)/f+8*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+2/5*a*(a+a*sec(f*x+
e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+4/3*(a^3+a^3*sec(f*x+e))*tan(f*x
+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= \frac{2a^3\left(-60\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)+\sqrt{a(1+\sec(e+fx))}(73+16\sec(e+fx)+3\sec^2(e+fx))}{15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x
]
```

output

```
(2*a^3*(-60*Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sq
rt[a])] + Sqrt[a*(1 + Sec[e + f*x])]*(73 + 16*Sec[e + f*x] + 3*Sec[e + f*x
]^2))*Tan[e + f*x])/(15*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*
x]])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4444, 3042, 4444, 3042, 4444, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{\sqrt{c-c\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4444}$$
$$2a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4444}$$

$$2a\left(2a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4444}$$

$$2a\left(2a\left(2a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2a\left(2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4282}$$

$$2a\left(2a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{4a\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{f}\right)+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$2a\left(\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}+2a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}\right)\right)+\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(2*a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]]) +
 2*a*((2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*
x]]) + 2*a*((-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c -
c*Sec[e + f*x]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e +
f*x]])))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4444

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Simp[2*c*((2*n - 1)/(2*n - 1))   Int[Csc[e + f*x]*((c + d*Csc[e + f*x
])^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.72

method	result
default	$-\frac{a^{3}\sqrt{2}\left(120\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\tan(fx+e)+\left(-73\cos(fx+e)^{2}-16\cos(fx+e)-3\right)\sqrt{2}\,\tan(fx+e)\sec(}{15f\sqrt{-c(-1+\sec(fx+e))}}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNV
ERBOSE)
```

output

```
-1/15*a^3/f*2^(1/2)/(-c*(-1+sec(f*x+e)))^(1/2)*(120*(-cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*tan(f*
x+e)+(-73*cos(f*x+e)^2-16*cos(f*x+e)-3)*2^(1/2)*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.30

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx$$

$$=\left[\frac{2\left(30\sqrt{2}a^{3}c\sqrt{-\frac{1}{c}}\cos\left(fx+e\right)^{2}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^{2}+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)}{15\,cf\cos\left(fx+e\right.}\right.$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorith
m="fricas")
```

output

```
[2/15*(30*sqrt(2)*a^3*c*sqrt(-1/c)*cos(f*x + e)^2*log(-(2*sqrt(2)*(cos(f*x
 + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c)
 - (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*s
in(f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19*a^3*cos(
f*x + e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x +
e)^2*sin(f*x + e)), 2/15*(60*sqrt(2)*a^3*sqrt(c)*arctan(sqrt(2)*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*cos(f*x
 + e)^2*sin(f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19
*a^3*cos(f*x + e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*c
os(f*x + e)^2*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx = a^3\left(\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx + \int \frac{3\sec^2\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx + \int \frac{3\sec^3\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx + \int \frac{\sec^4\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

output

```
a**3*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec
(e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec(e + f*x)**3/sq
rt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**4/sqrt(-c*sec(e + f*x
) + c), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^3\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorith
m="maxima")
```

output

```
integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x
)
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.68

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{8\,a^3\left(\frac{15\,\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\left(15\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2-5\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c+3\,c^2\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}}\right)}{15\,f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorith
m="giac")
```

output

```
8/15*a^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sq
rt(c) + sqrt(2)*(15*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 - 5*(c*tan(1/2*f*x +
1/2*e)^2 - c)*c + 3*c^2)/(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^3}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\frac{\sqrt{c}\,a^3\left(-\left(\int\frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4}{\sec(fx+e)-1}dx\right)-3\left(\int\frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)-1}dx\right)-3\left(\int\frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)-1}\right.\right.}{c}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*a**3*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4)/(sec(e +
 f*x) - 1),x) - 3*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e +
 f*x) - 1),x) - 3*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e +
 f*x) - 1),x) - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)
 - 1),x)))/c
```

3.84 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 168

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx &= \frac{10\sqrt{2}a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}\\ &\quad-\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}\\ &\quad-\frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
10*2^(1/2)*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2
))/c^(3/2)/f-a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-10*a
^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(1/2)-5/3*(a^3+a^3*sec(f*x+e))*tan(f*x+
e)/c/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^3\,\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^3\tan(e+fx)}{14cf\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2)
,x]
```

output
```
-1/14*(a^3*Hypergeometric2F1[2, 7/2, 9/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e
 + f*x])^3*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4445, 3042, 4444, 3042, 4444, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{5a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{\sqrt{c-c\sec(e+fx)}}dx}{2c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4444$$

$$-\frac{5a\left(2a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)}{2c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a\left(2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)}{2c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4444$$

$$-\frac{5a\left(2a\left(2a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)}{2c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a\left(2a\left(2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)}{2c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4282$$

$$-\frac{5a\left(2a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{4a\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{f}\right)+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}\right)}{2c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 216$$

$$-\frac{5a\left(\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}+2a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}\right)\right)}{2c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2),x]
```

output

```
-((a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)))
- (5*a*((2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e +
f*x]]) + 2*a*((-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c
- c*Sec[e + f*x]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e
+ f*x]]))))/(2*c)
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4444

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Simp[2*c*((2*n - 1)/(2*n - 1))   Int[Csc[e + f*x]*((c + d*Csc[e + f*x
])^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(151) = 302$.

Time = 3.17 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.96

method	result
default	$-\frac{a^3\sqrt{2}\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\left(76\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-100\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+30\right)\sqrt{\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}+\left(-60\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^5+60\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+6\right.\right.}{}$
parts	Expression too large to display

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/3*a^3/c*2^(1/2)/f/(3+2*2^(1/2))^3/(-3+2*2^(1/2))^3*(cos(1/2*f*x+1/2*e)*
(76*cos(1/2*f*x+1/2*e)^4-100*cos(1/2*f*x+1/2*e)^2+30)*((2*cos(1/2*f*x+1/2*
e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)+(-60*cos(1/2*f*x+1/2*e)^5+60*cos(1
/2*f*x+1/2*e)^4+60*cos(1/2*f*x+1/2*e)^3-60*cos(1/2*f*x+1/2*e)^2-15*cos(1/2
*f*x+1/2*e)+15)*arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((
2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))+(60*cos(1/2*f*x
+1/2*e)^5-60*cos(1/2*f*x+1/2*e)^4-60*cos(1/2*f*x+1/2*e)^3+60*cos(1/2*f*x+1
/2*e)^2+15*cos(1/2*f*x+1/2*e)-15)*ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1
/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(
cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)
+1)))/(4*cos(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x+1/2*e)^2+1)/(-c/(2*cos(1/2*f*x
+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(
1/2*f*x+1/2*e)+1)^2)^(1/2)*csc(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = \left[\frac{15\sqrt{2}\left(a^3c\cos(fx+e)^2-a^3c\cos(fx+e)\right)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos(f\right.}{}\right.}{}\right.$$

$$-\frac{2\left(\frac{15\sqrt{2}\left(a^3c\cos(fx+e)^2-a^3c\cos(fx+e)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)}{\sqrt{c}}-\left(19\,a^3\cos(fx+e)^3+7\,a^3\cos(\right.\right.}{3\left(c^2f\cos(fx+e)^2-c^2f\cos(fx+e)\right)\sin(fx+e)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorith
m="fricas")
```

output

```
[1/3*(15*sqrt(2)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*sqrt(-1/c)*lo
g((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))*sqrt(-1/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e)
- 1)*sin(f*x + e)))*sin(f*x + e) + 2*(19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*
x + e)^2 - 13*a^3*cos(f*x + e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e)))/((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e)), -2/3*(15*
sqrt(2)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*arctan(sqrt(2)*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(
f*x + e)/sqrt(c) - (19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*x + e)^2 - 13*a^3*
cos(f*x + e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f
*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = a^3\left(\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)}}\right.$$

$$+\int \frac{3\sec^2(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx$$

$$+\int \frac{3\sec^3(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx$$

$$\left.+\int \frac{\sec^4(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

output

```
a**3*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c
*sqrt(-c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**2/(-c*sqrt(-c*s
ec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integra
l(3*sec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c
*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(-c*sqrt(-c*sec(e + f*x
) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^3\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
 x)
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{2\,a^3\left(\frac{15\,\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\,\sqrt{2}\left(6\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-7\,c\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}c}+\frac{3\,\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}\right)}{3\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorith
m="giac")
```

output
```
-2/3*a^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^
(3/2) + 2*sqrt(2)*(6*c*tan(1/2*f*x + 1/2*e)^2 - 7*c)/((c*tan(1/2*f*x + 1/2
*e)^2 - c)^(3/2)*c) + 3*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/(c^2*ta
n(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^3}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input

```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

output

```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx= \sqrt{c}\,a^3\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4}{\sec(fx+e)^2-2\sec(fx+e)+1}dx+3\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^2-2\sec(fx+e)+1}d\right.\right.$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x)
```

output

```
(sqrt(c)*a**3*(int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4)/(sec(e + f*
x)**2 - 2*sec(e + f*x) + 1),x) + 3*int((sqrt( - sec(e + f*x) + 1)*sec(e +
f*x)**3)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x) + 3*int((sqrt( - sec(e
+ f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x) + i
nt((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 - 2*sec(e + f
*x) + 1),x)))/c**2
```

3.85 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 174

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{15a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} - \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}}$$

output

```
-15/4*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^
(1/2)/c^(5/2)/f-1/2*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/
2)+5/4*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(3/2)+15/4*a^3
*tan(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.37

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^3 \text{Hypergeometric2F1}\left(3,\frac{7}{2},\frac{9}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^3\tan(e+fx)}{28c^2f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2)
,x]
```

output
```
-1/28*(a^3*Hypergeometric2F1[3, 7/2, 9/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e
 + f*x])^3*Tan[e + f*x])/(c^2*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4445, 3042, 4445, 3042, 4444, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4445}$$

$$-\frac{5a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^2}{(c-c\sec(e+fx))^{3/2}}dx}{4c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 4445$$

$$-\frac{5a\left(-\frac{3a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)}{\sqrt{c-c\sec(e+fx)}}dx}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a\left(-\frac{3a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 4444$$

$$-\frac{5a\left(-\frac{3a\left(2a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5a\left(-\frac{3a\left(2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 4282$$

$$-\frac{5a\left(-\frac{3a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{4a\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{f}\right)}{2c}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}\right)}{4c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{5a\left(-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}-\frac{3a\left(\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}\right)}{2c}\right)}{4c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5/2)
) - (5*a*(-(((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x]
)^(3/2))) - (3*a*((-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[c - c*Sec[e + f*x]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Se
c[e + f*x]])))/(2*c)))/(4*c)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4444
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Simp[2*c*((2*n - 1)/(2*n - 1))   Int[Csc[e + f*x]*((c + d*Csc[e + f*x
])^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

rule 4445
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^
(-1)] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(151) = 302$.

Time = 2.73 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.20

method	result
default	$-\frac{a^{3}\sqrt{2}\,\Bigg(\left(15\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+15\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\operatorname{arctanh}\left(\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\sqrt{\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}}\right)\sqrt{\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}+\Big(-15\cos\Big(\frac{fx}{2}}{}$
parts	Expression too large to display

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNV
ERBOSE)
```

output

```
-1/8*a^3/c^2*2^(1/2)/f/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2
)^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)*((15*cos(1/2*f*x+1/2*e)+15)*sin(1/2*f*x
+1/2*e)*arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/
2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*((2*cos(1/2*f*x+1/2*e)^
2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)+(-15*cos(1/2*f*x+1/2*e)-15)*sin(1/2*f
*x+1/2*e)*ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2
)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)
^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*((2*cos(1/2*f*x+1/2
*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)+(-36*cos(1/2*f*x+1/2*e)^4+70*cos(
1/2*f*x+1/2*e)^2-30)*cot(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = \Bigg[-\frac{15\sqrt{2}\left(a^3\cos\left(fx+e\right)^2-2\,a^3\cos\left(fx+e\right)+a^3\right)\sqrt{-c}\log\Bigg(\frac{2\sqrt{}}{}}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorith
m="fricas")
```

output

```
[-1/8*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(-c)
*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x +
e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(
f*x + e)^2 - 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x +
e)), 1/4*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(
c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sq
rt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*
x + e)^2 - 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x
+ e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)
)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = a^3\left(\int \frac{\sec\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+f\right.}}\right.$$

$$+\int \frac{3\sec^2\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+}}$$

$$+\int \frac{3\sec^3\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+}}$$

$$+\int \frac{\sec^4\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
a**3*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**
2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f
*x) + c)), x) + Integral(3*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)
*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sq
rt(-c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**3/(c**2*sqrt(-c*se
c(e + f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e +
 f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(c*
*2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x)
 + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^3\left(\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{8\sqrt{2}}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}c^2}+\frac{7\sqrt{2}\left(c\tan\right.}{}\right.}{4f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorith
m="giac")
```

output
```
1/4*a^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(
5/2) + 8*sqrt(2)/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2) + (7*sqrt(2)*(c*
tan(1/2*f*x + 1/2*e)^2 - c)^(3/2) + 9*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^
2 - c)*c)/(c^4*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^3}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,a^3\bigg(-\bigg(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\bigg)-3\bigg(\int \frac{\sqrt{-\mathrm{s}}}{\sec(fx+e)^3}}{}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*a**3*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4)/(sec(e +
 f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x) - 3*int((sqrt( - sec
(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*s
ec(e + f*x) - 1),x) - 3*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(s
ec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x) - int((sqrt( -
 sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*
sec(e + f*x) - 1),x)))/c**3
```

3.86 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 142

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}} + \frac{32c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5af} + \frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output

```
128/5*c^4*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(1/2)+32/5*c^3*(c-c*sec(f*x+e))^
(1/2)*tan(f*x+e)/a/f+12/5*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/a/f+2*c*(c
-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{2c^4(91+43\sec(e+fx)-7\sec^2(e+fx)+\sec^3(e+fx))\tan(e}{5af(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x
]
```

output
```
(2*c^4*(91 + 43*Sec[e + f*x] - 7*Sec[e + f*x]^2 + Sec[e + f*x]^3)*Tan[e +
f*x])/(5*a*f*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 4442, 3042, 4280, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4442}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{f(a\sec(e+fx)+a)}-\frac{6c\int \sec(e+fx)(c-c\sec(e+fx))^{5/2}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{f(a\sec(e+fx)+a)}-\frac{6c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a}$$

$$\Big\downarrow \text{ 4280}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{f(a\sec(e+fx)+a)}-\frac{6c\left(\frac{8}{5}c\int \sec(e+fx)(c-c\sec(e+fx))^{3/2}dx-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f}\right)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{f(a\sec(e+fx)+a)}-\frac{6c\left(\frac{8}{5}c\int\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f}\right)}{a}$$

$$\Big\downarrow 4280$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{f(a\sec(e+fx)+a)}-\frac{6c\left(\frac{8}{5}c\left(\frac{4}{3}c\int\sec(e+fx)\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}\right)-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f}\right)}{a}$$

$$\Big\downarrow 3042$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{f(a\sec(e+fx)+a)}-\frac{6c\left(\frac{8}{5}c\left(\frac{4}{3}c\int\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}\right)-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3}}{5f}\right.}{a}$$

$$\Big\downarrow 4279$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{f(a\sec(e+fx)+a)}-\frac{6c\left(\frac{8}{5}c\left(-\frac{8c^2\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}\right)-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f}\right)}{a}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x]
```

output
```
(2*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) - (
6*c*((-2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f) + (8*c*((-8*c^2*
Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]]) - (2*c*Sqrt[c - c*Sec[e + f*x
]]*Tan[e + f*x])/(3*f)))/5))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{8\left(91\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-115\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+45\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-5\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{3}\sqrt{2}\,\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)}{5af\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}$	124

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output

```
-8/5/a/f*(91*cos(1/2*f*x+1/2*e)^6-115*cos(1/2*f*x+1/2*e)^4+45*cos(1/2*f*x+
1/2*e)^2-5)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^3
*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)^2*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e
)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = -\frac{2\left(91\,c^{3}\cos\left(fx+e\right)^{3}+43\,c^{3}\cos\left(fx+e\right)^{2}-7\,c^{3}\cos\left(fx+e\right)+c^{3}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{5\,af\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm=
"fricas")
```

output

```
-2/5*(91*c^3*cos(f*x + e)^3 + 43*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) +
 c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)^2*sin(f*x
+ e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \text{Timed out}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e)),x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{8\left(16\sqrt{2}c^{\frac{7}{2}} - \frac{56\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{70\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{35\sqrt{2}c^{\frac{7}{2}}\sin(fx+}{(\cos(fx+e)+1}\right.}{5\,af\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{7}{2}}}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
8/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(
2)*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8)/(a*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(
7/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = -\frac{8\sqrt{2}c^3\left(\frac{5\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{a} - \frac{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c+5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^2+c^3}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}a}\right)}{5f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
-8/5*sqrt(2)*c^3*(5*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/a - (15*(c*tan(1/2*
f*x + 1/2*e)^2 - c)^2*c + 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2 + c^3)/((c*
tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*a))/f
```

Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx =$$

$$-\frac{2\,c^3\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,86\mathrm{i}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,245\mathrm{i}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,180\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,245\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}}{5\,a\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}$$

input
```
int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output
```
-(2*c^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp
(e*1i + f*x*1i)*86i + exp(e*2i + f*x*2i)*245i + exp(e*3i + f*x*3i)*180i +
exp(e*4i + f*x*4i)*245i + exp(e*5i + f*x*5i)*86i + exp(e*6i + f*x*6i)*91i
+ 91i))/(5*a*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{\sqrt{c}\,c^3\Bigl(-\Bigl(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4}{\sec(fx+e)+1}dx\Bigr)+3\Bigl(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(}{\sec(fx+e)+1}}{}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x)
```

output
```
(sqrt(c)*c**3*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4)/(sec(e +
 f*x) + 1),x) + 3*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e +
 f*x) + 1),x) - 3*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e +
 f*x) + 1),x) + int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)
 + 1),x)))/a
```

3.87 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 108

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx=\frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}}+\frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3af}+\frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output

```
32/3*c^3*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(1/2)+8/3*c^2*(c-c*sec(f*x+e))^(1
/2)*tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e)
)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx=-\frac{2c^3(-23-10\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{3af(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x
]
```

output
```
(-2*c^3*(-23 - 10*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e + f*x])/(3*a*f*(1 +
 Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4442, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\int \sec(e+fx)(c-c\sec(e+fx))^{3/2}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a}$$

$$\Big\downarrow \text{4280}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\left(\frac{4}{3}c\int \sec(e+fx)\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\left(\frac{4}{3}c\int\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}\right)}{a}$$

$$\Big\downarrow \text{4279}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\left(-\frac{8c^2\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}\right)}{a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x]
```

output

```
(2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) - (
4*c*((-8*c^2*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]]) - (2*c*Sqrt[c -
c*Sec[e + f*x]]*Tan[e + f*x])/(3*f)))/a
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{4\left(23\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-18\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+3\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{2}\sqrt{2}\,\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)}{3af\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$	111

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
-4/3/a/f*(23*cos(1/2*f*x+1/2*e)^4-18*cos(1/2*f*x+1/2*e)^2+3)*(-c/(2*cos(1/
2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^2*2^(1/2)/(2*cos(1/2*f*x+1
/2*e)^2-1)*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = -\frac{2\left(23\,c^{2}\cos\left(fx+e\right)^{2}+10\,c^{2}\cos\left(fx+e\right)-c^{2}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,af\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm=
"fricas")
```

output

```
-2/3*(23*c^2*cos(f*x + e)^2 + 10*c^2*cos(f*x + e) - c^2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec(e+fx)+1}\,dx + \int \left(-\frac{2c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)}{\sec(e+fx)+1}\right.}{a}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e)),x)
```

output

```
(Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1),
x) + Integral(-2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/(sec(e + f
*x) + 1), x) + Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3/(se
c(e + f*x) + 1), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = -\frac{4\left(8\sqrt{2}c^{\frac{5}{2}} - \frac{20\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{3\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{3\,af\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm=
"maxima")
```

output

```
-4/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*sqrt(2
)*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a*f*(sin(f*x + e)/(cos(f*x
 + e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = -\frac{4\sqrt{2}\left(\frac{3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2}{a}-\frac{6\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^3+c^4}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}a}\right)}{3f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
-4/3*sqrt(2)*(3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2/a - (6*(c*tan(1/2*f
*x + 1/2*e)^2 - c)*c^3 + c^4)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a))/f
```

Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{2\,c^2\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,\left(2\,\sin\left(e+f\,x\right)-44\,\sin\left(2\,e+2\,f\,x\right)+25\,\mathrm{s}\right.}{3\,a\,f\,\left(\cos\left(3\,e+3\,f\,x\right)-2\,\cos\left(e+f\,x\right)-2\right.}$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output
```
(2*c^2*((c*(cos(e + f*x) - 1))/cos(e + f*x))^(1/2)*(2*sin(e + f*x) - 44*si
n(2*e + 2*f*x) + 25*sin(3*e + 3*f*x) - 26*sin(4*e + 4*f*x) + 23*sin(5*e +
5*f*x)))/(3*a*f*(cos(3*e + 3*f*x) - 2*cos(e + f*x) - 2*cos(4*e + 4*f*x) +
cos(5*e + 5*f*x) + 2))
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{\sqrt{c}\,c^2\Bigg(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)+1}dx - 2\Bigg(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)+1}}{a}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x)
```

output
```
(sqrt(c)*c**2*(int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*
x) + 1),x) - 2*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*
x) + 1),x) + int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) +
1),x)))/a
```

3.88 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 72

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output
```
4*c^2*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(1/2)+2*c*(c-c*sec(f*x+e))^(1/2)*tan
(f*x+e)/f/(a+a*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\frac{2c^2(3+\sec(e+fx))\tan(e+fx)}{af(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x
]
```

output
```
(2*c^2*(3 + Sec[e + f*x])*Tan[e + f*x])/(a*f*(1 + Sec[e + f*x])*Sqrt[c - c
*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{3042, 4442, 3042, 4279\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}-\frac{2c\int \sec(e+fx)\sqrt{c-c\sec(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}-\frac{2c\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{4279}$$

$$\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x]
```

output
```
(4*c^2*Tan[e + f*x])/(a*f*Sqrt[c - c*Sec[e + f*x]]) + (2*c*Sqrt[c - c*Sec[
e + f*x]]*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4442

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2\sqrt{2}\,c\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\left(3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$	78

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output

```
-2/a/f*2^(1/2)*c*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2
)*(3*cot(1/2*f*x+1/2*e)-sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = -\frac{2\,(3\,c\cos\left(fx+e\right)+c)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{af\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
-2*(3*c*cos(f*x + e) + c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*sin
(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx + \int \left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)}{a}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

output
```
(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x)
+ Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/(sec(e + f*x) + 1)
, x))/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.53

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{2\left(2\sqrt{2}c^{\frac{3}{2}} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}} + \frac{\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}\right)}{af\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
2*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a*f*(sin(f*x +
e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/
2))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = -\frac{2\sqrt{2}\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}c}{a}-\frac{c^{2}}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}a}\right)}{f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
-2*sqrt(2)*(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c/a - c^2/(sqrt(c*tan(1/2*f
*x + 1/2*e)^2 - c)*a))/f
```

Mupad [B] (verification not implemented)

Time = 12.75 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = -\frac{c\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\left(2\,\sin\left(e+f\,x\right)+6\,\sin\left(2\,e+2\,f\,x\right)+2\,\sin\left(3\,e+3\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)\right)}{a\,f\,{\sin\left(2\,e+2\,f\,x\right)}^{2}}$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output
```
-(c*(c - c/cos(e + f*x))^(1/2)*(2*sin(e + f*x) + 6*sin(2*e + 2*f*x) + 2*si
n(3*e + 3*f*x) + 3*sin(4*e + 4*f*x)))/(a*f*sin(2*e + 2*f*x)^2)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{\sqrt{c}\,c\left(-\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)+1}dx\right)+\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e}{\sec(fx+e)+1}}{a}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x)
```

output
```
(sqrt(c)*c*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*
x) + 1),x) + int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) +
1),x)))/a
```

3.89 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 39

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

output

```
2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\sqrt{c-c\sec(e+fx)}}{af}$$

input

```
Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
```

output

```
(-2*Cot[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4441}$$

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
```

output
```
(2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{-2c(-1+\sec(fx+e))}\,\cot(fx+e)}{af}$	31

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
-1/a/f*2^(1/2)*(-2*c*(-1+sec(f*x+e)))^(1/2)*cot(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=-\frac{2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{af\sin(fx+e)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
-2*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(a*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\frac{\int \frac{\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
```

output
```
Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.15

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{\sqrt{2}\sqrt{c}-\frac{\sqrt{2}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}{af\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
-(sqrt(2)*sqrt(c) - sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/(
a*f*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x +
 e) + 1) - 1))
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}(\cos(fx+e))}{af}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
-sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*sgn(tan(1/2*f*x + 1/2*e)^3 + t
an(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a*f)
```

Mupad [B] (verification not implemented)

Time = 12.57 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=-\frac{\sin\left(2\,e+2\,f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{a\,f\,{\sin\left(e+f\,x\right)}^2}$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output
```
-(sin(2*e + 2*f*x)*(c - c/cos(e + f*x))^(1/2))/(a*f*sin(e + f*x)^2)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x)
```

output
```
(sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x
))/a
```

3.90 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 89

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx\\ = -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}+\frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output

```
-1/2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)
/a/c^(1/2)/f+tan(f*x+e)/f/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx\\ = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan\left(\frac{1}{2}(e+fx)\right)}{af\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

output

```
(Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[e + f*x])/2]*Tan[(e + f*x)/2])/(
a*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3042, 4448, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{af}$$

$$\Big\downarrow \text{216}$$

$$\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
-(ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[
2]*a*Sqrt[c]*f)) + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e +
 f*x]])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(78) = 156$.

Time = 1.94 (sec) , antiderivative size = 467, normalized size of antiderivative = 5.25

$$-\frac{\sqrt{2}\left(3\ln\left(\frac{2\sqrt{\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}\,\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+2\sqrt{\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}-4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)+\operatorname{arctanh}\left(\frac{}{\left(\cos\left(\frac{fx}{2}+\right.\right.}\right.\right.}{}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
-1/4/a/f*2^(1/2)/(cos(1/2*f*x+1/2*e)+1)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin
(1/2*f*x+1/2*e)^2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1
)^2)^(1/2)*(3*ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^
(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1
)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2
*e)+arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*
x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*sin(1/2*f*x+1/2*e)-4*ln(4*(
((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/
2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2
*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)-2*((2*cos(1/2*f*
x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*sin(1/2*f*x+1/2*e)-2*((2*cos
(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*tan(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.02

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx$$

$$= \left[\frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^{2}+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)-4\sqrt{\frac{c\,\mathrm{co}}{c}}}{4acf\sin(fx+e)}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm=
"fricas")
```

output
```
[1/4*(sqrt(2)*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)
*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(
sqrt(2)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos
(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+\sqrt{-c\sec(e+fx)+c}}\,dx}{a}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*se
c(e + f*x) + c)), x)/a
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)), x
)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\sqrt{2}\left(\frac{\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{c}\right)}{2\,af}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm=
"giac")
```

output
```
1/2*sqrt(2)*(arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c) -
sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/c)/(a*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2-1}dx\right)}{ac}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x)
```

output

```
( - sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2
- 1),x))/(a*c)
```

3.91 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 122

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} - \frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}$$

output
```
-3/8*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)
/a/c^(3/2)/f-3/4*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(3/2)+tan(f*x+e)/f/(a+a*s
ec(f*x+e))/(c-c*sec(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan (}{2acf\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x
]
```

output
```
(Hypergeometric2F1[-1/2, 2, 1/2, (1 + Sec[e + f*x])/2]*Tan[(e + f*x)/2])/(
2*a*c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4448, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{3\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\left(\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{3\left(-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{2cf}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{3\left(-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x]
```

output

```
Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)) + (3*(-1/
2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[
2]*c^(3/2)*f) - Tan[e + f*x]/(2*f*(c - c*Sec[e + f*x])^(3/2))))/(2*a)
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(105) = 210$.

Time = 2.62 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.91

method	result
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{4\sqrt{\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}\,\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+4\sqrt{\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}-8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-4}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\left(24\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-24\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\mathrm{arctan}}{}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/16/a/f*2^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/c*(ln(4*(((
2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*
e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f
*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(24*cot(1/2*f*x+1/2*e)-24*csc(1/2*f*x
+1/2*e))+arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1
/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(-3*cot(1/2*f*x+1/2*e)
+3*csc(1/2*f*x+1/2*e))+ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*
e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x
+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(-21*c
ot(1/2*f*x+1/2*e)+21*csc(1/2*f*x+1/2*e))+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(
1/2*f*x+1/2*e)+1)^2)^(1/2)*(2*cot(1/2*f*x+1/2*e)-4*sec(1/2*f*x+1/2*e)*csc(
1/2*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.70

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \Bigg[-\frac{3\sqrt{2}\sqrt{-c}(\cos(fx+e)-1)\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx\right.}{}\right.}{}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm=
"fricas")
```

output
```
[-1/16*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) - 1)*log((2*sqrt(2)*(cos(f*x + e)
^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c
*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*
x + e) + 4*(cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/8*(3*sqrt(2
)*sqrt(c)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(
f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(cos(f*x +
 e)^2 - 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^2*f
*cos(f*x + e) - a*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx}{a}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sq
rt(-c*sec(e + f*x) + c)), x)/a
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)),
 x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\frac{3\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{2\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{c^2} - \right.}{8af}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm=
"giac")
```

output
```
1/8*sqrt(2)*(3*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(3/2)
- 2*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/c^2 - sqrt(c*tan(1/2*f*x + 1/2*e)^2
 - c)/(c^2*tan(1/2*f*x + 1/2*e)^2))/(a*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^{3}-\sec(fx+e)^{2}-\sec(fx+e)+1}dx\right)}{a\,c^{2}}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 - s
ec(e + f*x)**2 - sec(e + f*x) + 1),x))/(a*c**2)
```

3.92 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 156

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{15\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} - \frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}} - \frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}}$$

output

```
-15/64*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/
2)/a/c^(5/2)/f-5/8*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(5/2)+tan(f*x+e)/f/(a+a
*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2)-15/32*tan(f*x+e)/a/c/f/(c-c*sec(f*x+e)
)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.37

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan (}{4ac^2f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x
]
```

output
```
(Hypergeometric2F1[-1/2, 3, 1/2, (1 + Sec[e + f*x])/2]*Tan[(e + f*x)/2])/(
4*a*c^2*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4448, 3042, 4283, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{5\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{5/2}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 4283$$

$$\frac{5\left(\frac{3\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}dx}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{5\left(\frac{3\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 4283$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 4282$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{2cf}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow 216$$

$$\frac{5\left(\frac{3\left(-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)) + (5*(-1/
4*Tan[e + f*x]/(f*(c - c*Sec[e + f*x])^(5/2)) + (3*(-1/2*ArcTan[(Sqrt[c]*T
an[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2]*c^(3/2)*f) - Tan
[e + f*x]/(2*f*(c - c*Sec[e + f*x])^(3/2))))/(8*c)))/(2*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(135) = 270$.

Time = 2.29 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.16

method	result
default	$-\frac{\sqrt{2}\,\Bigg(\ln\Bigg(\frac{2\sqrt{\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}\,\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+2\sqrt{\frac{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}{\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}-4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\Bigg)\left(-225\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+225\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\mathrm{ar}}{}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/128/a/f*2^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1
/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/c^2*(ln(2*
(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1
/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/
2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(-225*cot(1/2*f*x+1/2*e)+225*csc(1
/2*f*x+1/2*e))+arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2
*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(-15*cot(1/2*f*x
+1/2*e)+15*csc(1/2*f*x+1/2*e))+ln(4*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*
f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos
(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)
)*(240*cot(1/2*f*x+1/2*e)-240*csc(1/2*f*x+1/2*e))+(6*cos(1/2*f*x+1/2*e)^4+
14*cos(1/2*f*x+1/2*e)^2-16)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e
)+1)^2)^(1/2)*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^3)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.57

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{15\sqrt{2}\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sqrt{-c}\log\left(\right.}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm=
"fricas")
```

output
```
[-1/128*(15*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*
sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*
sin(f*x + e)))*sin(f*x + e) - 4*(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15
*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x
+ e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)), 1/64*(15*sqrt(2)
*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(
f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x +
 e) + 2*(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f
*x + e) + a*c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)-c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-c^2}}{a}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c*
*2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - c**2*sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x)/a
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(5/2)),
 x)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(15\sqrt{c}\arctan\left(\frac{\sqrt{c\tan(\frac{1}{2}fx+\frac{1}{2}e)^2-c}}{\sqrt{c}}\right)-8\sqrt{c\tan(\frac{1}{2}f}\right.}{64}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm=
"giac")
```

output
```
1/64*sqrt(2)*(15*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)
) - 8*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c) - (9*(c*tan(1/2*f*x + 1/2*e)^2 -
c)^(3/2)*c + 7*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x +
1/2*e)^4))/(a*c^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^4-2\sec(fx+e)^3+2\sec(fx+e)-1}dx\right)}{a\,c^3}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**4
- 2*sec(e + f*x)**3 + 2*sec(e + f*x) - 1),x))/(a*c**3)
```

3.93 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 155

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx &= \\ &-\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3a^2f} \\ &-\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)}+\frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}\end{aligned}$$

output

```
-64/3*c^4*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(1/2)-16/3*c^3*(c-c*sec(f*x+e)
)^(1/2)*tan(f*x+e)/a^2/f-4*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a^2+a^
2*sec(f*x+e))+2/3*c*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx=\frac{2c^4(-45-69\sec(e+fx)-15\sec^2(e+fx)+\sec^3(e+fx))\tan}{3a^2f(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2
,x]
```

output
```
(2*c^4*(-45 - 69*Sec[e + f*x] - 15*Sec[e + f*x]^2 + Sec[e + f*x]^3)*Tan[e
+ f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 4442, 3042, 4442, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sec(e+fx)a+a}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{a}$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\int \sec(e+fx)(c-c\sec(e+fx))^{3/2}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4280}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\left(\frac{4}{3}c\int \sec(e+fx)\sqrt{c-c\sec(e+fx)}dx-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\left(\frac{4}{3}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{4279}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}-\frac{4c\left(-\frac{8c^2\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}\right)}{a}\right)}{a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2,x]
```

output

```
(2*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
 - (2*c*((2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(a + a*Sec[e + f
*x])) - (4*c*((-8*c^2*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]]) - (2*c*
Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(3*f)))/a))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [A] (verified)

Time = 9.40 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.68

method	result
default	$\frac{\frac{4\left(996\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}-1245\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+465\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-35\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-5\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{3}\sqrt{2}\,\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\csc\left(\frac{fx}{2}+\frac{e}{2}\right)}{15f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}} - 8(}{a^{2}}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/a^2*(4/15/f*(996*cos(1/2*f*x+1/2*e)^8-1245*cos(1/2*f*x+1/2*e)^6+465*cos(
1/2*f*x+1/2*e)^4-35*cos(1/2*f*x+1/2*e)^2-5)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^3*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)^2*sec(
1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e)-8/5/f*(91*cos(1/2*f*x+1/2*e)^6-115*cos
(1/2*f*x+1/2*e)^4+45*cos(1/2*f*x+1/2*e)^2-5)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1
)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^3*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)^2*sec
(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,(45\,c^3\cos(fx+e)^3+69\,c^3\cos(fx+e)^2+15\,c^3\cos(fx+e)-}{3\,(a^2f\cos(fx+e)^2+a^2f\cos(fx+e))\sin(fx+}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorith
m="fricas")
```

output
```
2/3*(45*c^3*cos(f*x + e)^3 + 69*c^3*cos(f*x + e)^2 + 15*c^3*cos(f*x + e) -
 c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e)^2 + a^2
*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{4\left(16\sqrt{2}c^{\frac{7}{2}}-\frac{56\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{70\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{35\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{4\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)}{3\,a^2f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{7}{2}}}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorith
m="maxima")
```

output
```
-4/3*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt
(2)*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*sqrt(2)*c^(7/2)*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 + sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{4\sqrt{2}c^3\left(\frac{9\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c+c^2}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}a^2}-\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}a^4c^2+9\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,a^4c^3}{a^6c^3}\right)}{3f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorith
m="giac")
```

output
```
-4/3*sqrt(2)*c^3*((9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + c^2)/((c*tan(1/2*f
*x + 1/2*e)^2 - c)^(3/2)*a^2) - ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a^4*
c^2 + 9*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^4*c^3)/(a^6*c^3))/f
```

Mupad [B] (verification not implemented)

Time = 15.46 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,c^3\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,138\mathrm{i}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,195\mathrm{i}+\mathrm{e}\right.}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^3\,\left(\mathrm{e}^{e\,1\mathrm{i}+}\right.}$$

input
```
int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output
```
(2*c^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(
e*1i + f*x*1i)*138i + exp(e*2i + f*x*2i)*195i + exp(e*3i + f*x*3i)*268i +
exp(e*4i + f*x*4i)*195i + exp(e*5i + f*x*5i)*138i + exp(e*6i + f*x*6i)*45i
 + 45i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1)^3*(exp(e*1i + f*x*1i) - exp(e*2
i + f*x*2i) + exp(e*3i + f*x*3i) - 1))
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{\sqrt{c}\,c^3\left(-\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)+3\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(f}{\sec(fx+e)^2+2\sec(fx+}\right.\right.}{}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x)
```

output
```
(sqrt(c)*c**3*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4)/(sec(e +
 f*x)**2 + 2*sec(e + f*x) + 1),x) + 3*int((sqrt( - sec(e + f*x) + 1)*sec(e
 + f*x)**3)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) - 3*int((sqrt( - sec
(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)
+ int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1),x)))/a**2
```

3.94 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 123

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

output
```
-16/3*c^3*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(1/2)-8/3*c^2*(c-c*sec(f*x+e))
^(1/2)*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+2/3*c*(c-c*sec(f*x+e))^(3/2)*tan(
f*x+e)/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c^3(11+18\sec(e+fx)+3\sec^2(e+fx))\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2
,x]
```

output
```
(-2*c^3*(11 + 18*Sec[e + f*x] + 3*Sec[e + f*x]^2)*Tan[e + f*x])/(3*a^2*f*(
1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4442, 3042, 4442, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sec(e+fx)a+a}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\left(\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}-\frac{2c\int \sec(e+fx)\sqrt{c-c\sec(e+fx)}dx}{a}\right)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\left(\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}-\frac{2c\int\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a}\right)}{3a}$$

$$\Big\downarrow \text{4279}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\left(\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}\right)}{3a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2,x]
```

output

```
(2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
 - (4*c*((4*c^2*Tan[e + f*x])/(a*f*Sqrt[c - c*Sec[e + f*x]]) + (2*c*Sqrt[c
 - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))))/(3*a)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4442

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(111) = 222.

Time = 9.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.91

method	result
default	$\frac{\frac{2\left(68\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-51\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{2}\sqrt{2}\,\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\csc\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}-\frac{4\left(23\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-18\cos\right.}{}}{a^{2}}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/a^2*(2/3/f*(68*cos(1/2*f*x+1/2*e)^6-51*cos(1/2*f*x+1/2*e)^4+6*cos(1/2*f*
x+1/2*e)^2+1)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c
^2*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)*sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2
*e)-4/3/f*(23*cos(1/2*f*x+1/2*e)^4-18*cos(1/2*f*x+1/2*e)^2+3)*(-c/(2*cos(1
/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^2*2^(1/2)/(2*cos(1/2*f*x+
1/2*e)^2-1)*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx=\frac{2\left(11\,c^2\cos\left(fx+e\right)^2+18\,c^2\cos\left(fx+e\right)+3\,c^2\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\left(a^2f\cos\left(fx+e\right)+a^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorith
m="fricas")
```

output
```
2/3*(11*c^2*cos(f*x + e)^2 + 18*c^2*cos(f*x + e) + 3*c^2)*sqrt((c*cos(f*x
+ e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\bigg(8\sqrt{2}c^{\frac{5}{2}} - \frac{20\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{2\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)}{(\cos(fx+e)+1)^6}}{3a^2f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorith
m="maxima")
```

output
```
2/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*sqrt(2)
*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - sqrt(2)*c^(5/2)*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/
2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2\sqrt{2}\left(\frac{3\,c^3}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}a^2}-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}a^4c+6\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}a^4c^2}{a^6}\right)}{3\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorith
m="giac")
```

output
```
-2/3*sqrt(2)*(3*c^3/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^2) - ((c*tan(1/2
*f*x + 1/2*e)^2 - c)^(3/2)*a^4*c + 6*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^
4*c^2)/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,c^2\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,36\mathrm{i}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,34\mathrm{i}+\mathrm{e}^{e\,3}\right.}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right.}$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output
```
(2*c^2*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(
e*1i + f*x*1i)*36i + exp(e*2i + f*x*2i)*34i + exp(e*3i + f*x*3i)*36i + exp
(e*4i + f*x*4i)*11i + 11i))/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i +
f*x*1i) + 1)^3)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{\sqrt{c}\,c^2\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^2+2\sec(fx+e)+1}dx-2\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2+2\sec(fx+e)+1}\right.\right.}{a^2}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x)
```

output
```
(sqrt(c)*c**2*(int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*
x)**2 + 2*sec(e + f*x) + 1),x) - 2*int((sqrt( - sec(e + f*x) + 1)*sec(e +
f*x)**2)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) + int((sqrt( - sec(e +
f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)))/a**2
```

3.95 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 89

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{4c^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

output
```
-4/3*c^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)+2/3*c*(c
-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c^2(1+3\sec(e+fx))\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2
,x]
```

output
```
(-2*c^2*(1 + 3*Sec[e + f*x])*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2*S
qrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3042, 4442, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sec(e+fx)a+a}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}$$

$$\Big\downarrow \text{4441}$$

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{4c^2\tan(e+fx)}{3af(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2,x]
```

output
```
(-4*c^2*Tan[e + f*x])/(3*a*f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]
) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^
2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(81) = 162$.

Time = 4.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.91

method	result
default	$\frac{\frac{\left(20\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\, c\sqrt{2}\,\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\csc\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f}-\frac{2\sqrt{2}\,c\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\left(3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-\mathrm{s}\right.}{f}}{a^{2}}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/a^2*(1/3/f*(20*cos(1/2*f*x+1/2*e)^4-5*cos(1/2*f*x+1/2*e)^2-1)*(-c/(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c*2^(1/2)*sec(1/2*f*x+1/2
*e)^3*csc(1/2*f*x+1/2*e)-2/f*2^(1/2)*c*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(
1/2*f*x+1/2*e)^2)^(1/2)*(3*cot(1/2*f*x+1/2*e)-sec(1/2*f*x+1/2*e)*csc(1/2*f
*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx=\frac{2\left(c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\left(a^2f\cos\left(fx+e\right)+a^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorith
m="fricas")
```

output
```
2/3*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx=\frac{\int\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)}{a^2}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**2,x)
```

output
```
(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*se
c(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**
2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2\sqrt{2}c^{\frac{3}{2}} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}{3\,a^2f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorith
m="maxima")
```

output
```
-1/3*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + sqrt(2)*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^2*f*(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1
)^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}}{a^2} + \frac{3\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,c}{a^2}}{3\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorith
m="giac")
```

output
```
1/3*(sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)/a^2 + 3*sqrt(2)*sqrt(c*t
an(1/2*f*x + 1/2*e)^2 - c)*c/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,c\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,6\mathrm{i}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{e\,3\mathrm{i}+f}\right.}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^3}$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output
```
(2*c*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e*
1i + f*x*1i)*6i + exp(e*2i + f*x*2i)*2i + exp(e*3i + f*x*3i)*6i + exp(e*4i
 + f*x*4i)*1i + 1i))/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i)
 + 1)^3)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{\sqrt{c}\,c\left(-\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)+\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e}{\sec(fx+e)^2+2\sec(fx+e)+1}\right.}{a^2}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x)
```

output
```
(sqrt(c)*c*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*
x)**2 + 2*sec(e + f*x) + 1),x) + int((sqrt( - sec(e + f*x) + 1)*sec(e + f*
x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)))/a**2
```

3.96 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx=\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

output
```
2/3*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\begin{aligned}&\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx\\&=-\frac{\cos^2(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{6a^2f}\end{aligned}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x
]
```

output
```
-1/6*(Cos[e + f*x]^2*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e
+ f*x]])/(a^2*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{3042, 4441\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4441}$$

$$\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x]
```

output
```
(2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\sqrt{2}\,\cos(fx+e)\sqrt{-2c(-1+\sec(fx+e))}\,\cot(fx+e)}{a^2 f(3\cos(fx+e)+3)}$	49

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/a^2/f*2^(1/2)*cos(f*x+e)/(3*cos(f*x+e)+3)*(-2*c*(-1+sec(f*x+e)))^(1/2)*
cot(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos\left(fx+e\right)^2}{3\left(a^2f\cos\left(fx+e\right)+a^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorith
m="fricas")
```

output

```
-2/3*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2/((a^2*f*cos(f*
x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{\sqrt{-c\sec{(e+fx)}+c}\sec{(e+fx)}}{\sec^2{(e+fx)}+2\sec{(e+fx)}+1}\,dx}{a^2}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**2,x)
```

output

```
Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e
 + f*x) + 1), x)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(37) = 74.

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.66

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{2}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{2}\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{6\,a^2f\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorith
m="maxima")
```

output

```
-1/6*(sqrt(2)*sqrt(c) - 2*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a^2*f*sqrt(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1
))
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}(\cos\left(fx+e\right))}{6\,a^2cf}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorith
m="giac")
```

output

```
1/6*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^
3 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a^2*c*f)
```

Mupad [B] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.29

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx=\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^2\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^3}$$

input

```
int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output

```
((exp(e*2i + f*x*2i)*1i + 1i)^2*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i
+ f*x*1i)/2))^(1/2)*2i)/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*
1i) + 1)^3)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx=\frac{\sqrt{c}\left(\int\frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x)
```

output
```
(sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 + 2
*sec(e + f*x) + 1),x))/a**2
```

3.97 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 138

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\\ &\quad+\frac{\tan(e+fx)}{2f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output

```
-1/4*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)
/a^2/c^(1/2)/f+1/3*tan(f*x+e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+
1/2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.43

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x
]
```

output

```
(Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(3*f
*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4448, 3042, 4448, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)\sqrt{c-c\sec(e+fx)}}dx}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4448}$$

$$\frac{\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4282}$$

$$\frac{\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{af}}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x]
```

output

```
Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]) + (-(Ar
cTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2]*a
*Sqrt[c]*f)) + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x
]]))/(2*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [F]

$$\int \frac{\sec (fx+e)}{(a+a\sec (fx+e))^2 \sqrt{c-c\sec (fx+e)}}dx$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{3\,\sqrt{2}\sqrt{-c}(\cos\left(fx+e\right)+1)\log\left(\frac{2\,\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3\,c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin}{24\left(a^2cf\cos\left(fx+e\right)+a^2cf\right)\sin\left(fx+e\right.}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorith
m="fricas")
```

output
```
[-1/24*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) + 1)*log((2*sqrt(2)*(cos(f*x + e)
^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c
*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*
x + e) + 4*(5*cos(f*x + e)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/12*(3*sqr
t(2)*sqrt(c)*(cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(
f*x + e)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^
2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)+2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a^2}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + 2*sqrt(
-c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**2
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^2\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*sqrt(-c*sec(f*x + e) + c)),
 x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\sqrt{2}\left(\frac{3\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^4-3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^5}{c^6}\right)}{12\,a^2f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorith
m="giac")
```

output
```
1/12*sqrt(2)*(3*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c)
 + ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^4 - 3*sqrt(c*tan(1/2*f*x + 1/2*
e)^2 - c)*c^5)/c^6)/(a^2*f)
```

Mupad [F(-1)]

Timed out.

$$
\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx \\
&= \int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^2\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx
\end{aligned}
$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$
\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx \\
&= -\frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3+\sec(fx+e)^2-\sec(fx+e)-1}dx\right)}{a^2c}
\end{aligned}
$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3
+ sec(e + f*x)**2 - sec(e + f*x) - 1),x))/(a**2*c)
```

3.98 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 169

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{5\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} \\ &\quad -\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} \\ &\quad +\frac{5\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}}\end{aligned}$$

output

```
-5/16*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/a^2/c^(3/2)/f-5/8*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(3/2)+1/3*tan(f*x+e)
/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2)+5/6*tan(f*x+e)/f/(a^2+a^2*sec
(f*x+e))/(c-c*sec(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.38

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\text{ta}}{6a^2cf(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2))
,x]
```

output

```
(Hypergeometric2F1[-3/2, 2, -1/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(6*a
^2*c*f*(1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4448, 3042, 4448, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{5\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)(c-c\sec(e+fx))^{3/2}}dx}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4448}$$

$$\frac{5\left(\frac{3\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4283}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{2cf}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 216$$

$$\frac{5\left(\frac{3\left(-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)) + (5*
(Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)) + (3*(-1
/2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt
[2]*c^(3/2)*f) - Tan[e + f*x]/(2*f*(c - c*Sec[e + f*x])^(3/2))))/(2*a)))/(
6*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(146) = 292$.

Time = 4.28 (sec) , antiderivative size = 968, normalized size of antiderivative = 5.73

method	result	size
default	Expression too large to display	968

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/a^2*(-1/96/f*2^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^
2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c*(ln(
2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x
+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(
1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(117*cot(1/2*f*x+1/2*e)-117*csc(
1/2*f*x+1/2*e))+arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((
2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(3*cot(1/2*f*x+
1/2*e)-3*csc(1/2*f*x+1/2*e))+ln(4*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*
x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1
/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*
(-120*cot(1/2*f*x+1/2*e)+120*csc(1/2*f*x+1/2*e))+(14*cos(1/2*f*x+1/2*e)^4-
12*cos(1/2*f*x+1/2*e)^2+4)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)
+1)^2)^(1/2)*sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e))+1/16/f*2^(1/2)/((2*c
os(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/(-c/(2*cos(1/2*f*x+
1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c*(ln(2*(((2*cos(1/2*f*x+1/2*e)^2-
1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*
e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f
*x+1/2*e)+1))*(21*cot(1/2*f*x+1/2*e)-21*csc(1/2*f*x+1/2*e))+arctanh((2*cos
(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos
(1/2*f*x+1/2*e)+1)^2)^(1/2))*(3*cot(1/2*f*x+1/2*e)-3*csc(1/2*f*x+1/2*e)...
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.18

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\mathrm{co}\right.}{}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorith
m="fricas")
```

output
```
[-1/96*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x +
 e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (
3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin
(f*x + e) + 4*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^
2*f)*sin(f*x + e)), 1/48*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(c)*arctan(s
qrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f
*x + e)))*sin(f*x + e) - 2*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x +
e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)-c\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+c}}{a^2}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c*sq
rt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sqrt(-c*sec(e + f*x) + c)*sec(
e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x)/a**2
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorith
m="maxima")
```

output
```
integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^(3/2)
), x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\frac{15\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2} + \right.}{48\,a^2 f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorith
m="giac")
```

output
```
1/48*sqrt(2)*(15*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(3/2
) - 3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/(c^2*tan(1/2*f*x + 1/2*e)^2) + 2*
((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^6 - 6*sqrt(c*tan(1/2*f*x + 1/2*e)^
2 - c)*c^7)/c^9)/(a^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^2\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^4-2\sec(fx+e)^2+1}dx\right)}{a^2c^2}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x)
```

output

```
(sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**4 - 2
*sec(e + f*x)**2 + 1),x))/(a**2*c**2)
```

3.99 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 203

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx &= -\frac{35\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} \\ &\quad -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} \\ &\quad +\frac{7\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}-\frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}\end{aligned}$$

output

```
-35/128*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1
/2)/a^2/c^(5/2)/f-35/48*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(5/2)+1/3*tan(f*
x+e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2)+7/6*tan(f*x+e)/f/(a^2+a^2
*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2)-35/64*tan(f*x+e)/a^2/c/f/(c-c*sec(f*x+
e))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.32

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx=\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\text{ta}}{12a^2c^2f(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2))
,x]
```

output
```
(Hypergeometric2F1[-3/2, 3, -1/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(12*
a^2*c^2*f*(1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4448, 3042, 4448, 3042, 4283, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{7\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)(c-c\sec(e+fx))^{5/2}}dx}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{\csc \left(e+fx+\frac{\pi }{2}\right)}{\left(\csc \left(e+fx+\frac{\pi }{2}\right)a+a\right)\left(c-c\csc \left(e+fx+\frac{\pi }{2}\right)\right)^{5/2}}dx}{6a}+\frac{\tan (e+fx)}{3f(a\sec (e+fx)+a)^2(c-c\sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{4448}$$

$$\frac{7\left(\frac{5\int \frac{\sec (e+fx)}{(c-c\sec (e+fx))^{5/2}}dx}{2a}+\frac{\tan (e+fx)}{f(a\sec (e+fx)+a)(c-c\sec (e+fx))^{5/2}}\right)}{6a}+\frac{\tan (e+fx)}{3f(a\sec (e+fx)+a)^2(c-c\sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\int \frac{\csc \left(e+fx+\frac{\pi }{2}\right)}{\left(c-c\csc \left(e+fx+\frac{\pi }{2}\right)\right)^{5/2}}dx}{2a}+\frac{\tan (e+fx)}{f(a\sec (e+fx)+a)(c-c\sec (e+fx))^{5/2}}\right)}{6a}+\frac{\tan (e+fx)}{3f(a\sec (e+fx)+a)^2(c-c\sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{4283}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{\sec (e+fx)}{(c-c\sec (e+fx))^{3/2}}dx}{8c}-\frac{\tan (e+fx)}{4f(c-c\sec (e+fx))^{5/2}}\right)}{2a}+\frac{\tan (e+fx)}{f(a\sec (e+fx)+a)(c-c\sec (e+fx))^{5/2}}\right)}{6a}+\frac{\tan (e+fx)}{3f(a\sec (e+fx)+a)^2(c-c\sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{\csc \left(e+fx+\frac{\pi }{2}\right)}{\left(c-c\csc \left(e+fx+\frac{\pi }{2}\right)\right)^{3/2}}dx}{8c}-\frac{\tan (e+fx)}{4f(c-c\sec (e+fx))^{5/2}}\right)}{2a}+\frac{\tan (e+fx)}{f(a\sec (e+fx)+a)(c-c\sec (e+fx))^{5/2}}\right)}{6a}+\frac{\tan (e+fx)}{3f(a\sec (e+fx)+a)^2(c-c\sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{4283}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{2cf}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow 216$$

$$\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)) + (7*
(Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)) + (5*(-1
/4*Tan[e + f*x]/(f*(c - c*Sec[e + f*x])^(5/2)) + (3*(-1/2*ArcTan[(Sqrt[c]*
Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2]*c^(3/2)*f) - Ta
n[e + f*x]/(2*f*(c - c*Sec[e + f*x])^(3/2))))/(8*c)))/(2*a)))/(6*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(176) = 352$.

Time = 4.17 (sec) , antiderivative size = 999, normalized size of antiderivative = 4.92

method	result	size
default	Expression too large to display	999

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/a^2*(1/768/f*2^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^
2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^2*((
122*cos(1/2*f*x+1/2*e)^6-206*cos(1/2*f*x+1/2*e)^4+112*cos(1/2*f*x+1/2*e)^2
-16)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*sec(1/2*f
*x+1/2*e)^3*csc(1/2*f*x+1/2*e)^3+ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/
2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(c
os(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+
1))*(-2655*cot(1/2*f*x+1/2*e)+2655*csc(1/2*f*x+1/2*e))+arctanh((2*cos(1/2*
f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*
f*x+1/2*e)+1)^2)^(1/2))*(15*cot(1/2*f*x+1/2*e)-15*csc(1/2*f*x+1/2*e))+ln(4
*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+
1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1
/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(2640*cot(1/2*f*x+1/2*e)-2640*csc
(1/2*f*x+1/2*e)))-1/128/f*2^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f
*x+1/2*e)^2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(
1/2)/c^2*(ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2
)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)
^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(-225*cot(1/2*f*x+1
/2*e)+225*csc(1/2*f*x+1/2*e))+arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*
x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{105\sqrt{2}\left(\cos(fx+e)^3-\cos(fx+e)^2-\cos(fx+e\right.}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorith
m="fricas")
```

output

```
[-1/768*(105*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*
sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((co
s(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(43*cos(f*x + e)^4 - 161*c
os(f*x + e)^3 - 35*cos(f*x + e)^2 + 105*cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2
 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)), 1/384*(105*sqrt(2)*(
cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e
)))*sin(f*x + e) - 2*(43*cos(f*x + e)^4 - 161*cos(f*x + e)^3 - 35*cos(f*x
+ e)^2 + 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*
c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) +
 a^2*c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^4(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+}}{a^2}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

output

```
Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*
c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c**2*sqrt(-c*sec(e + f*x)
 + c)), x)/a**2
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorith
m="maxima")
```

output
```
integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^(5/2)
), x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.79

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(105\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{8\left(\left(c\tan\left(\frac{1}{2}fx\right.\right.\right.}{}\right.\right.}{}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorith
m="giac")
```

output
```
1/384*sqrt(2)*(105*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(
c)) + 8*((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^2 - 9*sqrt(c*tan(1/2*f*x +
 1/2*e)^2 - c)*c^3)/c^3 - 3*(13*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 1
1*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))/(a
^2*c^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^2\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^5-\sec(fx+e)^4-2\sec(fx+e)^3+2\sec(fx+e)^2+\sec(fx+e)-1}dx\right)}{a^2c^3}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x)
```

output

```
( - sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**5
- sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + sec(e + f*x) -
 1),x))/(a**2*c**3)
```

3.100 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 169

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx &= \frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}}\\ &\quad+\frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}-\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2}\\ &\quad+\frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\end{aligned}$$

output

```
32/5*c^4*tan(f*x+e)/a^3/f/(c-c*sec(f*x+e))^(1/2)+16/5*c^3*(c-c*sec(f*x+e))
^(1/2)*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))-4/5*c^2*(c-c*sec(f*x+e))^(3/2)*ta
n(f*x+e)/a/f/(a+a*sec(f*x+e))^2+2/5*c*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/
(a+a*sec(f*x+e))^3
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.44

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c^4(23+55\sec(e+fx)+45\sec^2(e+fx)+5\sec^3(e+fx))\tan}{5a^3f(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3
,x]
```

output
```
(2*c^4*(23 + 55*Sec[e + f*x] + 45*Sec[e + f*x]^2 + 5*Sec[e + f*x]^3)*Tan[e
 + f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 4442, 3042, 4442, 3042, 4442, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{5f(a\sec(e+fx)+a)^3}-\frac{6c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(\sec(e+fx)a+a)^2}dx}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{5f(a\sec(e+fx)+a)^3}-\frac{6c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}$$

$$\Big\downarrow \; 4442$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{5f(a\sec(e+fx)+a)^3}-\frac{6c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sec(e+fx)a+a}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{5f(a\sec(e+fx)+a)^3}-\frac{6c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \; 4442$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{5f(a\sec(e+fx)+a)^3}-\frac{6c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\left(\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}-\frac{2c\int\sec(e+fx)\sqrt{c-c\sec(e+fx)}dx}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{5f(a\sec(e+fx)+a)^3}-\frac{6c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\left(\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}-\frac{2c\int\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \; 4279$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{5f(a\sec(e+fx)+a)^3}-\frac{6c\left(\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}-\frac{4c\left(\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}\right)}{3a}\right)}{5a}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3,x]
```

output
```
(2*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
 - (6*c*((2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e +
 f*x])^2) - (4*c*((4*c^2*Tan[e + f*x])/(a*f*Sqrt[c - c*Sec[e + f*x]]) + (2
*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))))/(3*a)
))/(5*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(153) = 306$.

Time = 21.39 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.42

method	result
default	$-\frac{2\left(3168\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-3960\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+1485\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-125\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{3}\sqrt{2}\,\sec\left(\frac{fx}{2}\right.}{15f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/a^3*(-2/15/f*(3168*cos(1/2*f*x+1/2*e)^10-3960*cos(1/2*f*x+1/2*e)^8+1485*
cos(1/2*f*x+1/2*e)^6-125*cos(1/2*f*x+1/2*e)^4-5*cos(1/2*f*x+1/2*e)^2-3)*(-
c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^3*2^(1/2)/(2*co
s(1/2*f*x+1/2*e)^2-1)^2*sec(1/2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)+8/15/f*(99
6*cos(1/2*f*x+1/2*e)^8-1245*cos(1/2*f*x+1/2*e)^6+465*cos(1/2*f*x+1/2*e)^4-
35*cos(1/2*f*x+1/2*e)^2-5)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*
e)^2)^(1/2)*c^3*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)^2*sec(1/2*f*x+1/2*e)^3*
csc(1/2*f*x+1/2*e)-8/5/f*(91*cos(1/2*f*x+1/2*e)^6-115*cos(1/2*f*x+1/2*e)^4
+45*cos(1/2*f*x+1/2*e)^2-5)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2
*e)^2)^(1/2)*c^3*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)^2*sec(1/2*f*x+1/2*e)*c
sc(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.64

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx =$$
$$-\frac{2\left(23\,c^{3}\cos\left(fx+e\right)^{3}+55\,c^{3}\cos\left(fx+e\right)^{2}+45\,c^{3}\cos\left(fx+e\right)+5\,c^{3}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{5\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-2/5*(23*c^3*cos(f*x + e)^3 + 55*c^3*cos(f*x + e)^2 + 45*c^3*cos(f*x + e)
+ 5*c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 +
2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.27

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2\left(16\sqrt{2}c^{\frac{7}{2}} - \frac{56\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{70\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{35\sqrt{2}c^{\frac{7}{2}}\sin(fx+}{(\cos(fx+e)+1}\right.}{5a^3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}(}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorith
m="maxima")
```

output
```
2/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(
2)*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8 - sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x +
e) + 1)^10 + sqrt(2)*c^(7/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)/(a^3*f
*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x + e)/(cos(f*x + e) +
 1) - 1)^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.75

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2\sqrt{2}c^3\left(\frac{5\,c}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}a^3} - \frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}a^{12}c^8+5\left(c\tan\left(\frac{1}{2}\,f\right.\right.}{}\right.}{5\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorith
m="giac")
```

output
```
2/5*sqrt(2)*c^3*(5*c/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^3) - ((c*tan(1/
2*f*x + 1/2*e)^2 - c)^(5/2)*a^12*c^8 + 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3
/2)*a^12*c^9 + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^12*c^10)/(a^15*c^10
))/f
```

Mupad [B] (verification not implemented)

Time = 19.69 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.91

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
f*x*1i)/2))^(1/2)*128i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*
1i) + 1)^4) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2
 + exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(ex
p(e*1i + f*x*1i) + 1)^2) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*48i)/(5*a^3*f*(exp(e*1i + f*x
*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(1/2)*((c^3*46i)/(5*a^3*f) + (c^3*exp(e*1i + f*x*1i
)*4i)/(a^3*f) + (c^3*exp(e*2i + f*x*2i)*46i)/(5*a^3*f)))/((exp(e*1i + f*x*
1i) - 1)*(exp(e*1i + f*x*1i) + 1)) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/
(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*64i)/(5*a^3*f*(exp(
e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{\sqrt{c}\,c^3\left(-\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)+3\left(\int \frac{\sqrt{-}}{\sec(fx+e)}\right.\right.}{}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x)
```

output
```
(sqrt(c)*c**3*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**4)/(sec(e +
 f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) + 3*int((sqrt( - sec
(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*s
ec(e + f*x) + 1),x) - 3*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(s
ec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) + int((sqrt( -
 sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*
sec(e + f*x) + 1),x)))/a**3
```

3.101 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 135

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{16c^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

output

```
16/15*c^3*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)-8/15*c^
2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+2/5*c*(c-c*sec(
f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c^3\left(7+10\sec(e+fx)+15\sec^2(e+fx)\right)\tan(e+fx)}{15a^3f(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3
,x]
```

output

```
(2*c^3*(7 + 10*Sec[e + f*x] + 15*Sec[e + f*x]^2)*Tan[e + f*x])/(15*a^3*f*(
1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4442, 3042, 4442, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f(a\sec(e+fx)+a)^3}-\frac{4c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(\sec(e+fx)a+a)^2}dx}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f(a\sec(e+fx)+a)^3}-\frac{4c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}$$

$$\Big\downarrow \text{4442}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f(a\sec(e+fx)+a)^3}-\frac{4c\left(\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sec(e+fx)a+a}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f(a\sec(e+fx)+a)^3}-\frac{4c\left(\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{2c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{4441}$$

$$\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f(a\sec(e+fx)+a)^3}-\frac{4c\left(\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{4c^2\tan(e+fx)}{3af(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}\right)}{5a}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3,x]
```

output

```
(2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
 - (4*c*((-4*c^2*Tan[e + f*x])/(3*a*f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[
e + f*x]]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a + a*Sec[e
 + f*x])^2)))/(5*a)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(123) = 246$.

Time = 22.60 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.74

method	result
default	$-\frac{\left(928\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}-696\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+87\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+3\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{2}\sqrt{2}\,\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\csc\left(\frac{fx}{2}+\frac{e}{2}\right)}{15f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}+ 4\left(68\right.$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/a^3*(-1/15/f*(928*cos(1/2*f*x+1/2*e)^8-696*cos(1/2*f*x+1/2*e)^6+87*cos(1
/2*f*x+1/2*e)^4+6*cos(1/2*f*x+1/2*e)^2+3)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*s
in(1/2*f*x+1/2*e)^2)^(1/2)*c^2*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)*sec(1/2*
f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)+4/3/f*(68*cos(1/2*f*x+1/2*e)^6-51*cos(1/2*
f*x+1/2*e)^4+6*cos(1/2*f*x+1/2*e)^2+1)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(
1/2*f*x+1/2*e)^2)^(1/2)*c^2*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)*sec(1/2*f*x
+1/2*e)^3*csc(1/2*f*x+1/2*e)-4/3/f*(23*cos(1/2*f*x+1/2*e)^4-18*cos(1/2*f*x
+1/2*e)^2+3)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^
2*2^(1/2)/(2*cos(1/2*f*x+1/2*e)^2-1)*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)
)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.77

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2\left(7\,c^2\cos\left(fx+e\right)^3+10\,c^2\cos\left(fx+e\right)^2+15\,c^2\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{15\left(a^3f\cos\left(fx+e\right)^2+2\,a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-2/15*(7*c^2*cos(f*x + e)^3 + 10*c^2*cos(f*x + e)^2 + 15*c^2*cos(f*x + e))
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*
cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.40

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{8\sqrt{2}c^{\frac{5}{2}} - \frac{20\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{5\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{5\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8} - \frac{3\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}}{15\,a^3 f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-1/15*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sqrt(
2)*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(5/2)*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8 - 3*sqrt(2)*c^(5/2)*sin(f*x + e)^10/(cos(f*x
+ e) + 1)^10)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/2)*(sin(f*x
+ e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\frac{15\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2}{a^3} + \frac{3\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}+10\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c}{a^3}}{15\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorith
m="giac")
```

output
```
-1/15*(15*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2/a^3 + (3*sqrt(2)*
(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 10*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^
2 - c)^(3/2)*c)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.38

$$
\begin{aligned}
&\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \\
&-\frac{c^2\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,14\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)} \\
&+\frac{c^2\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^2} \\
&-\frac{c^2\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,112\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^3} \\
&+\frac{c^2\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,64\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^4} \\
&-\frac{c^2\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,32\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^5}
\end{aligned}
$$

input

```
int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output

```
(c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
f*x*1i)/2))^(1/2)*16i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*
1i) + 1)^2) - (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2
 + exp(e*1i + f*x*1i)/2))^(1/2)*14i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(e
xp(e*1i + f*x*1i) + 1)) - (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i
 - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*112i)/(15*a^3*f*(exp(e*1i + f*
x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^3) + (c^2*(exp(e*2i + f*x*2i) + 1)*(c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*64i)/(5*a^3*f*(
exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^4) - (c^2*(exp(e*2i + f*x
*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*32
i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{\sqrt{c}\,c^2\Big(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx - 2\Big(\int \frac{\sqrt{-\sec(fx}}{\sec(fx+e)^3+3\sec}}{a}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x)
```

output

```
(sqrt(c)*c**2*(int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*
x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) - 2*int((sqrt( - sec(e
+ f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1),x) + int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f
*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)))/a**3
```

3.102 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 88

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{4c^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

output
```
-4/15*c^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+2/5*c*(
c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2c^2(-1+5\sec(e+fx))\tan(e+fx)}{15a^3f(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3
,x]
```

output

```
(-2*c^2*(-1 + 5*Sec[e + f*x])*Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^3
*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 4442, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 4442}$$

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a\sec(e+fx)+a)^3}-\frac{2c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(\sec(e+fx)a+a)^2}dx}{5a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a\sec(e+fx)+a)^3}-\frac{2c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}$$

$$\Big\downarrow \text{ 4441}$$

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a\sec(e+fx)+a)^3}-\frac{4c^2\tan(e+fx)}{15af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3,x]
```

output
```
(-4*c^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*
x]]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x
])^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(80) = 160$.

Time = 3.44 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.10

method	result
default	$-\frac{\left(224\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6-56\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-7\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,c\sqrt{2}\,\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^5\csc\left(\frac{fx}{2}+\frac{e}{2}\right)}{30f}+\frac{2\left(20\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-5\cos\right.}{}$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output

```
1/a^3*(-1/30/f*(224*cos(1/2*f*x+1/2*e)^6-56*cos(1/2*f*x+1/2*e)^4-7*cos(1/2
*f*x+1/2*e)^2-3)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2
)*c*2^(1/2)*sec(1/2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)+2/3/f*(20*cos(1/2*f*x+
1/2*e)^4-5*cos(1/2*f*x+1/2*e)^2-1)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)*c*2^(1/2)*sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e)-2/f*
2^(1/2)*c*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(3*co
t(1/2*f*x+1/2*e)-sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2\left(c\cos\left(fx+e\right)^3-5\,c\cos\left(fx+e\right)^2\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{15\left(a^3f\cos\left(fx+e\right)^2+2\,a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorith
m="fricas")
```

output

```
-2/15*(c*cos(f*x + e)^3 - 5*c*cos(f*x + e)^2)*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x
 + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \frac{c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \Bigl(-\frac{c\sqrt{-c\sec(e+fx)-}}{\sec^3(e+fx)+3\sec^2(e+}}{a^3}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**3,x)
```

output
```
(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*se
c(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x)
 + c)*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x
) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(80) = 160$.

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.85

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2\sqrt{2}c^{\frac{3}{2}} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{7\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}{30\,a^3 f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-1/30*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7*sqrt(2)
*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3*sqrt(2)*c^(3/2)*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(
3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\sqrt{2}\left(3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}+5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}c\right)}{30\,a^3cf}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorith
m="giac")
```

output
```
-1/30*sqrt(2)*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 5*(c*tan(1/2*f*x +
 1/2*e)^2 - c)^(3/2)*c)/(a^3*c*f)
```

Mupad [B] (verification not implemented)

Time = 16.89 (sec) , antiderivative size = 446, normalized size of antiderivative = 5.07

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = &-\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}\\ &+\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,28\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^2}\\ &-\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,76\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^3}\\ &+\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,32\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^4}\\ &-\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^5}\end{aligned}$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output

```
(c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*
x*1i)/2))^(1/2)*28i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^2) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + e
xp(e*1i + f*x*1i)/2))^(1/2)*2i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*
1i + f*x*1i) + 1)) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*
1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*76i)/(15*a^3*f*(exp(e*1i + f*x*1i) -
1)*(exp(e*1i + f*x*1i) + 1)^3) + (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*32i)/(5*a^3*f*(exp(e*1i +
 f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^4) - (c*(exp(e*2i + f*x*2i) + 1)*(c
 - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*
(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx=\frac{\sqrt{c}\,c\left(-\left(\int\frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)+\int\frac{\sqrt{-\sec(}{\sec(fx+e)^3+3\,\mathrm{s}}\right.}{a^3}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x)
```

output

```
(sqrt(c)*c*( - int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*
x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) + int((sqrt( - sec(e +
f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1),x)))/a**3
```

3.103 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx=\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

output
```
2/5*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\begin{aligned}&\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx\\&=-\frac{\cos^3(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{20a^3f}\end{aligned}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x
]
```

output
```
-1/20*(Cos[e + f*x]^3*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^5*Sqrt[c - c*Sec[e
 + f*x]])/(a^3*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a\sec(e+fx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \text{4441}$$
$$\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x]
```

output
```
(2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{-2c(-1+\sec(fx+e))}\,\cos(fx+e)^{2}\cot(fx+e)}{5a^{3}f(\cos(fx+e)+1)^{2}}$	49

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-1/5/a^3/f*2^(1/2)*(-2*c*(-1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)^2*cos(f*x+e
)^2*cot(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos\left(fx+e\right)^{3}}{5\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-2/5*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^3/((a^3*f*cos(f*
x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \frac{\sqrt{-c\sec{(e+fx)}+c}\sec{(e+fx)}}{\sec^3{(e+fx)}+3\sec^2{(e+fx)}+3\sec{(e+fx)}+1}\,dx}{a^3}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**3,x)
```

output
```
Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e
 + f*x)**2 + 3*sec(e + f*x) + 1), x)/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.32

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\sqrt{2}\sqrt{c}-\frac{3\sqrt{2}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sqrt{2}\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{\sqrt{2}\sqrt{c}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}{20\,a^3f\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-1/20*(sqrt(2)*sqrt(c) - 3*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 3*sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - sqrt(2)*sqr
t(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^3*f*sqrt(sin(f*x + e)/(cos(f*
x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}(\cos\left(fx+e\right))}{20\,a^3c^2f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorith
m="giac")
```

output
```
-1/20*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*sgn(tan(1/2*f*x + 1/2*e
)^3 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a^3*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 441, normalized size of antiderivative = 10.76

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)} + \frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,8\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^2} - \frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^3} + \frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^4} - \frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,8\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^5}$$

input

```
int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output

```
((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*
1i)/2))^(1/2)*8i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) +
1)^2) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1
i + f*x*1i)/2))^(1/2)*2i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*
x*1i) + 1)) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + e
xp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*
1i + f*x*1i) + 1)^3) + ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*
1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1
)*(exp(e*1i + f*x*1i) + 1)^4) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*
1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*8i)/(5*a^3*f*(exp(e*1i + f*x
*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)}{a^3}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x)
```

output

```
(sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 + 3
*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x))/a**3
```

3.104 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 181

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\\ &\quad+\frac{\tan(e+fx)}{6af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\\ &\quad+\frac{\tan(e+fx)}{4f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output

```
-1/8*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)
/a^3/c^(1/2)/f+1/5*tan(f*x+e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2)+
1/6*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+1/4*tan(f*x+e
)/f/(a^3+a^3*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.33

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x
]
```

output
```
(Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(5*f
*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4448, 3042, 4448, 3042, 4448, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)^2\sqrt{c-c\sec(e+fx)}}dx}{2a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4448}$$

$$\frac{\frac{\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)\sqrt{c-c\sec(e+fx)}}dx}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4448}$$

$$\frac{\frac{\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\frac{\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{af}}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}}{2a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x]
```

output

```
Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]) + (Tan[
e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]) + (-(ArcTan
[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2]*a*Sqr
t[c]*f)) + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]))
/(2*a))/(2*a)
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [F]

$$\int \frac{\sec \left(fx+e\right)}{\left(a+a\sec \left(fx+e\right)\right)^{3}\sqrt{c-c\sec \left(fx+e\right)}}dx$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.22

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \Bigg[-\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^{2}+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3\,c\cos(fx}{(\cos(fx+e)-1)\sin(fx+e)}\right.}{240\left(a^{3}cf\cos\left(fx+e\right)^{2}+2\,a^{3}cf\right.}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorith
m="fricas")
```

output
```
[-1/240*(15*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*
sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*
sin(f*x + e)))*sin(f*x + e) + 4*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 1
5*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x
 + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)), 1/120*(15*sqrt(
2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x
 + e) - 2*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*co
s(f*x + e) + a^3*c*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\frac{\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+3\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+\sqrt{-c\sec(e+fx)+c}}\,dx}{a^3}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*sqrt(
-c*sec(e + f*x) + c)*sec(e + f*x)**2 + 3*sqrt(-c*sec(e + f*x) + c)*sec(e +
 f*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**3
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^3\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorith
m="maxima")
```

output

```
integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*sqrt(-c*sec(f*x + e) + c)),
 x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\sqrt{2}\left(\frac{15\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}c^{12}-5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^{13}+15\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^{14}}{c^{15}}\right)}{120\,a^3f}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorith
m="giac")
```

output

```
1/120*sqrt(2)*(15*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(
c) - (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^12 - 5*(c*tan(1/2*f*x + 1/2
*e)^2 - c)^(3/2)*c^13 + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^14)/c^15)/
(a^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^3\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input

```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2)),x)
```

output

```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^4+2\sec(fx+e)^3-2\sec(fx+e)-1}dx\right)}{a^3c}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x)
```

output

```
( - sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**4
+ 2*sec(e + f*x)**3 - 2*sec(e + f*x) - 1),x))/(a**3*c)
```

3.105 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 212

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{7\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} \\ &\quad - \frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} \\ &\quad + \frac{7\tan(e+fx)}{30af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} \\ &\quad + \frac{7\tan(e+fx)}{12f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}}\end{aligned}$$

output
```
-7/32*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/a^3/c^(3/2)/f-7/16*tan(f*x+e)/a^3/f/(c-c*sec(f*x+e))^(3/2)+1/5*tan(f*x+e
)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2)+7/30*tan(f*x+e)/a/f/(a+a*sec
(f*x+e))^2/(c-c*sec(f*x+e))^(3/2)+7/12*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))/(
c-c*sec(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.30

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},2,-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\text{ta}}{10a^3cf(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2))
,x]
```

output
```
(Hypergeometric2F1[-5/2, 2, -3/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(10*
a^3*c*f*(1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4448, 3042, 4448, 3042, 4448, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4448}$$

$$\frac{7\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)^2(c-c\sec(e+fx))^{3/2}}dx}{10a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{10a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4448}$$

$$\frac{7\left(\frac{5\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)(c-c\sec(e+fx))^{3/2}}dx}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}\right)}{10a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7\left(\frac{5\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}\right)}{10a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4448}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}\right)}{10a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}\right)}{10a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4283}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}\right)}{10a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}\right)}{10a}$$

$$\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4282$$

$$\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{2cf}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec}\right.}{10a}$$

$$\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 216$$

$$\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))}\right.}{10a}$$

$$\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)) + (7*
(Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)) + (5
*(Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)) + (3*(-
1/2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqr
t[2]*c^(3/2)*f) - Tan[e + f*x]/(2*f*(c - c*Sec[e + f*x])^(3/2))))/(2*a)))/
(6*a)))/(10*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(185) = 370$.

Time = 3.44 (sec) , antiderivative size = 1470, normalized size of antiderivative = 6.93

method	result	size
default	Expression too large to display	1470

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/a^3*(-1/960/f*2^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)
^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c*(ar
ctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*
e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(15*cot(1/2*f*x+1/2*e)-15*csc(1/2
*f*x+1/2*e))+ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(
1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)
^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(-855*cot(1/2*f*
x+1/2*e)+855*csc(1/2*f*x+1/2*e))+ln(4*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/
2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(c
os(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+
1))*(840*cot(1/2*f*x+1/2*e)-840*csc(1/2*f*x+1/2*e))+(-122*cos(1/2*f*x+1/2*
e)^6+84*cos(1/2*f*x+1/2*e)^4+20*cos(1/2*f*x+1/2*e)^2-12)*((2*cos(1/2*f*x+1
/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*sec(1/2*f*x+1/2*e)^5*csc(1/2*f*
x+1/2*e))-1/48/f*2^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1
)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c*(l
n(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f
*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*co
s(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(117*cot(1/2*f*x+1/2*e)-117*cs
c(1/2*f*x+1/2*e))+arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/
((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(3*cot(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.28

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^3+\cos\left(fx+e\right)^2-\cos\left(fx+e\right.\right.}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorith
m="fricas")
```

output

```
[-1/960*(105*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*
sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((co
s(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(139*cos(f*x + e)^4 + 21*c
os(f*x + e)^3 - 175*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^
2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)), 1/480*(105*sqrt(2)*
(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(c)*arctan(sqrt(2
)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x +
e)))*sin(f*x + e) - 2*(139*cos(f*x + e)^4 + 21*cos(f*x + e)^3 - 175*cos(f*
x + e)^2 - 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^
3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e)
 - a^3*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec^4(e+fx)-2c\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+2}}{a^3}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

output

```
Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c*
sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 2*c*sqrt(-c*sec(e + f*x) + c)*
sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x)/a**3
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^3(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorith
m="maxima")
```

output
```
integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^(3/2)
), x)
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\frac{105\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{15\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}\right.}{}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorith
m="giac")
```

output
```
1/480*sqrt(2)*(105*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(3
/2) - 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/(c^2*tan(1/2*f*x + 1/2*e)^2) -
 2*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^16 - 10*(c*tan(1/2*f*x + 1/2*
e)^2 - c)^(3/2)*c^17 + 45*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^18)/c^20)/(
a^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^3\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^5+\sec(fx+e)^4-2\sec(fx+e)^3-2\sec(fx+e)^2+\sec(fx+e}}{a^3c^2}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**5 + s
ec(e + f*x)**4 - 2*sec(e + f*x)**3 - 2*sec(e + f*x)**2 + sec(e + f*x) + 1)
,x))/(a**3*c**2)
```

3.106 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 246

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx &= -\frac{63\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} \\ &\quad -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} \\ &\quad +\frac{3\tan(e+fx)}{10af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} \\ &\quad +\frac{21\tan(e+fx)}{20f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}-\frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}}\end{aligned}$$

output
```
-63/256*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1
/2)/a^3/c^(5/2)/f-21/32*tan(f*x+e)/a^3/f/(c-c*sec(f*x+e))^(5/2)+1/5*tan(f*
x+e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2)+3/10*tan(f*x+e)/a/f/(a+a*
sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2)+21/20*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e
))/(c-c*sec(f*x+e))^(5/2)-63/128*tan(f*x+e)/a^3/c/f/(c-c*sec(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.94 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.26

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},3,-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\text{ta}}{20a^3c^2f(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2))
,x]
```

output
```
(Hypergeometric2F1[-5/2, 3, -3/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(20*
a^3*c^2*f*(1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {3042, 4448, 3042, 4448, 3042, 4448, 3042, 4283, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{9\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)^2(c-c\sec(e+fx))^{5/2}}dx}{10a}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\int \frac{\csc \left(e+fx+\frac{\pi }{2}\right)}{\left(\csc \left(e+fx+\frac{\pi }{2}\right)a+a\right)^2 \left(c-c \csc \left(e+fx+\frac{\pi }{2}\right)\right)^{5/2}}dx}{10 a}+\frac{\tan (e+fx)}{5 f (a \sec (e+fx)+a)^3 (c-c \sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{4448}$$

$$\frac{9 \left(\frac{7 \int \frac{\sec (e+fx)}{(\sec (e+fx) a+a) (c-c \sec (e+fx))^{5/2}}dx}{6 a}+\frac{\tan (e+fx)}{3 f (a \sec (e+fx)+a)^2 (c-c \sec (e+fx))^{5/2}}\right)}{10 a}+\frac{\tan (e+fx)}{5 f (a \sec (e+fx)+a)^3 (c-c \sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9 \left(\frac{7 \int \frac{\csc \left(e+fx+\frac{\pi }{2}\right)}{\left(\csc \left(e+fx+\frac{\pi }{2}\right) a+a\right) \left(c-c \csc \left(e+fx+\frac{\pi }{2}\right)\right)^{5/2}}dx}{6 a}+\frac{\tan (e+fx)}{3 f (a \sec (e+fx)+a)^2 (c-c \sec (e+fx))^{5/2}}\right)}{10 a}+\frac{\tan (e+fx)}{5 f (a \sec (e+fx)+a)^3 (c-c \sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{4448}$$

$$\frac{9 \left(\frac{7 \left(\frac{5 \int \frac{\sec (e+fx)}{(c-c \sec (e+fx))^{5/2}}dx}{2 a}+\frac{\tan (e+fx)}{f (a \sec (e+fx)+a) (c-c \sec (e+fx))^{5/2}}\right)}{6 a}+\frac{\tan (e+fx)}{3 f (a \sec (e+fx)+a)^2 (c-c \sec (e+fx))^{5/2}}\right)}{10 a}+\frac{\tan (e+fx)}{5 f (a \sec (e+fx)+a)^3 (c-c \sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9 \left(\frac{7 \left(\frac{5 \int \frac{\csc \left(e+fx+\frac{\pi }{2}\right)}{\left(c-c \csc \left(e+fx+\frac{\pi }{2}\right)\right)^{5/2}}dx}{2 a}+\frac{\tan (e+fx)}{f (a \sec (e+fx)+a) (c-c \sec (e+fx))^{5/2}}\right)}{6 a}+\frac{\tan (e+fx)}{3 f (a \sec (e+fx)+a)^2 (c-c \sec (e+fx))^{5/2}}\right)}{10 a}+\frac{\tan (e+fx)}{5 f (a \sec (e+fx)+a)^3 (c-c \sec (e+fx))^{5/2}}$$

$$\Big\downarrow \text{4283}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}dx}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}\right)}{10a}$$

$$\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))}\right.}{10a}$$

$$\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \; 4283$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\frac{\tan(}{3f(a\sec(e+fx)+a)}}{10a}$$

$$\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{4c}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\frac{}{3f(a\sec(e+fx)}}{10a}$$

$$\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{1}{\frac{c^2\tan^2(e+fx)}{c-c\sec(e+fx)}+2c}d\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{2cf}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\right.}{10a}$$

$$\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}-\frac{\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}\right)}{8c}-\frac{\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}\right)}{2a}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}\right)}{6a}+\frac{}{3f(a\sec(e}\right)}{10a}$$

$$\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)) + (9*
(Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)) + (7
*(Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)) + (5*(-
1/4*Tan[e + f*x]/(f*(c - c*Sec[e + f*x])^(5/2)) + (3*(-1/2*ArcTan[(Sqrt[c]
*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2]*c^(3/2)*f) - T
an[e + f*x]/(2*f*(c - c*Sec[e + f*x])^(3/2))))/(8*c)))/(2*a)))/(6*a)))/(10
*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1515 vs. $2(215) = 430$.

Time $= 3.40$ (sec) , antiderivative size $= 1516$, normalized size of antiderivative $= 6.16$

method	result	size
default	Expression too large to display	1516

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/a^3*(-1/7680/f*2^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1
)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^2*
((538*cos(1/2*f*x+1/2*e)^8-814*cos(1/2*f*x+1/2*e)^6+176*cos(1/2*f*x+1/2*e)
^4+208*cos(1/2*f*x+1/2*e)^2-48)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1
/2*e)+1)^2)^(1/2)*sec(1/2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)^3+ln(2*(((2*cos(
1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)+((2
*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)-2*cos(1/2*f*x+1/2
*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(-30735*cot(1/2*f*x+1/2*e)+30735*csc(1/2*f*
x+1/2*e))+ln(4*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2
)*cos(1/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)
^(1/2)-2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1))*(30480*cot(1/2*f*x+
1/2*e)-30480*csc(1/2*f*x+1/2*e))+arctanh((2*cos(1/2*f*x+1/2*e)-1)/(cos(1/2
*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)
)*(255*cot(1/2*f*x+1/2*e)-255*csc(1/2*f*x+1/2*e)))+1/384/f*2^(1/2)/((2*cos
(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/
2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^2*((122*cos(1/2*f*x+1/2*e)^6-206*c
os(1/2*f*x+1/2*e)^4+112*cos(1/2*f*x+1/2*e)^2-16)*((2*cos(1/2*f*x+1/2*e)^2-
1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e)
^3+ln(2*(((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1
/2*f*x+1/2*e)+((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/...
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.87

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{315\sqrt{2}\left(\cos(fx+e)^4-2\cos(fx+e)^2+1\right)\sqrt{-c}\,\mathrm{lo}}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorith
m="fricas")
```

output
```
[-1/2560*(315*sqrt(2)*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-c)*log
((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) -
 1)*sin(f*x + e)))*sin(f*x + e) + 4*(257*cos(f*x + e)^5 - 354*cos(f*x + e)
^4 - 588*cos(f*x + e)^3 + 210*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*c
os(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e)), 1/1280*(315*sqrt(2)*(cos(f*x + e
)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(25
7*cos(f*x + e)^5 - 354*cos(f*x + e)^4 - 588*cos(f*x + e)^3 + 210*cos(f*x +
 e)^2 + 315*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c
^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e)
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)^3\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorith
m="maxima")
```

output
```
integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^(5/2)
), x)
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.75

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\Bigg(315\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-\frac{5\Big(17\Big(c\tan\big(\frac{1}{2}}{}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorith
m="giac")
```

output
```
1/1280*sqrt(2)*(315*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt
(c)) - 5*(17*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 15*sqrt(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4) - 8*((c*tan(1/2*f*x +
1/2*e)^2 - c)^(5/2)*c^8 - 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^9 + 30*
sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^10)/c^10)/(a^3*c^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^3\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^6-3\sec(fx+e)^4+3\sec(fx+e)^2-1}dx\right)}{a^3c^3}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*int((sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**6
- 3*sec(e + f*x)**4 + 3*sec(e + f*x)**2 - 1),x))/(a**3*c**3)
```

3.107 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\frac{a(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

output

```
1/3*a*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=-\frac{ac^3\sec(e+fx)\left(3-3\sec(e+fx)+\sec^2(e+fx)\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)
,x]
```

output
```
-1/3*(a*c^3*Sec[e + f*x]*(3 - 3*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e + f*x
])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4441}$$

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(37) = 74$.

Time = 2.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.93

method	result	size
default	$\frac{8\left(-1+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{2}\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}$	126
risch	$\frac{2ic^{2}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(3\,\mathrm{e}^{5i(fx+e)}-6\,\mathrm{e}^{4i(fx+e)}+10\,\mathrm{e}^{3i(fx+e)}-6\,\mathrm{e}^{2i(fx+e)}+3\,\mathrm{e}^{i(fx+e)}\right)}{3\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)^{2}f}$	165

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
8/3/f*(-1+cos(1/2*f*x+1/2*e))^2*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+
1/2*e)^2)^(1/2)*c^2*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)*(cos(1/2*f*x+1/2*e)+1)^2/(2*cos(1/2*f*x+1/2*e)^2-1)^2*tan(1/2*f*x+1/2
*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(37) = 74$.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \frac{\left(3\,c^{2}\cos\left(fx+e\right)^{2}-3\,c^{2}\cos\left(fx+e\right)+c^{2}\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
1/3*(3*c^2*cos(f*x + e)^2 - 3*c^2*cos(f*x + e) + c^2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e
)^2*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(37) = 74$.

Time = 0.19 (sec) , antiderivative size = 638, normalized size of antiderivative = 14.84

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output

```
2/3*(30*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 9*c^2*cos(2*f*x + 2*e)*sin
(f*x + e) - 3*c^2*sin(f*x + e) - (3*c^2*sin(5*f*x + 5*e) - 6*c^2*sin(4*f*x
 + 4*e) + 10*c^2*sin(3*f*x + 3*e) - 6*c^2*sin(2*f*x + 2*e) + 3*c^2*sin(f*x
 + e))*cos(6*f*x + 6*e) + 9*(c^2*sin(4*f*x + 4*e) + c^2*sin(2*f*x + 2*e))*
cos(5*f*x + 5*e) - 3*(10*c^2*sin(3*f*x + 3*e) + 3*c^2*sin(f*x + e))*cos(4*
f*x + 4*e) + (3*c^2*cos(5*f*x + 5*e) - 6*c^2*cos(4*f*x + 4*e) + 10*c^2*cos
(3*f*x + 3*e) - 6*c^2*cos(2*f*x + 2*e) + 3*c^2*cos(f*x + e))*sin(6*f*x + 6
*e) - 3*(3*c^2*cos(4*f*x + 4*e) + 3*c^2*cos(2*f*x + 2*e) + c^2)*sin(5*f*x
+ 5*e) + 3*(10*c^2*cos(3*f*x + 3*e) + 3*c^2*cos(f*x + e) + 2*c^2)*sin(4*f*
x + 4*e) - 10*(3*c^2*cos(2*f*x + 2*e) + c^2)*sin(3*f*x + 3*e) + 3*(3*c^2*c
os(f*x + e) + 2*c^2)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x +
4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(
3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*
f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(37) = 74$.

Time = 0.76 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.49

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \frac{8\left(3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^4+3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+c^6\right)\sqrt{-ac}|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\right.\right.}{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^3c^2f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
8/3*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^4 + 3*(c*tan(1/2*f*x + 1/2*e)^2
- c)*c^5 + c^6)*sqrt(-a*c)*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x
 + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 12.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.16

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \frac{2\,c^2\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,(10\,\sin\left(e+f\,x\right)-12\,\sin\left(2\,e+2\,f\,x\right)+13\,\sin\left(3\,e+3\right.}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)-\cos\left(6\,e+6\right.\right.}$$

input
```
int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x
)
```

output
```
(2*c^2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))
/cos(e + f*x))^(1/2)*(10*sin(e + f*x) - 12*sin(2*e + 2*f*x) + 13*sin(3*e +
 3*f*x) - 6*sin(4*e + 4*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*e + 2*f*x)
 - 2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

Reduce [F]

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \sqrt{c}\,\sqrt{a}\,c^2\left(\int \sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^3dx - 2\left(\int \sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^2dx\right) + \int \sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*c**2*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)
*sec(e + f*x)**3,x) - 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x),x))
```

3.108 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

output
```
1/2*a*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{ac^2(-2+\sec(e+fx))\sec(e+fx)\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)
,x]
```

output
```
(a*c^2*(-2 + Sec[e + f*x])*Sec[e + f*x]*Tan[e + f*x])/(2*f*Sqrt[a*(1 + Sec
[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4441}$$

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(37) = 74.

Time = 1.82 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.53

method	result	size
default	$-\frac{2\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$	109
risch	$\frac{2ic\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{3i(fx+e)}-\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}\right)}{\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)f}$	137

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/f*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(-c/(2*cos(
1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c/(2*cos(1/2*f*x+1/2*e)^2-
1)*sin(1/2*f*x+1/2*e)^2*tan(1/2*f*x+1/2*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.81

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c - c\sec(e+fx))^{3/2}\,dx = \frac{(2\,c\cos\left(fx+e\right)-c)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
1/2*(2*c*cos(f*x + e) - c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx = \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c(\sec\left(e+fx\right)-1)\right)^{\frac{3}{2}}\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)*sec(e +
 f*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(37) = 74.

Time = 0.18 (sec) , antiderivative size = 298, normalized size of antiderivative = 6.93

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx = \frac{2\,(2\,c\cos\left(3\,fx+3\,e\right)\sin\left(2\,fx+2\,e\right)-2\,c\cos\left(2\,fx+2\,e\right)\sin\left(fx+e\right)-(c\sin\left(3\,fx+3\,e\right)}{(2\,(2\,\cos\left(2\,fx+2\,e\right)+}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
2*(2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*c*cos(2*f*x + 2*e)*sin(f*x +
e) - (c*sin(3*f*x + 3*e) - c*sin(2*f*x + 2*e) + c*sin(f*x + e))*cos(4*f*x
+ 4*e) + (c*cos(3*f*x + 3*e) - c*cos(2*f*x + 2*e) + c*cos(f*x + e))*sin(4*
f*x + 4*e) - (2*c*cos(2*f*x + 2*e) + c)*sin(3*f*x + 3*e) + (2*c*cos(f*x +
e) + c)*sin(2*f*x + 2*e) - c*sin(f*x + e))*sqrt(a)*sqrt(c)/((2*(2*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2
 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x
+ 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(37) = 74$.

Time = 0.58 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.93

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx = \frac{2\left(2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^3+c^4\right)\sqrt{-ac}|c|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^2f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
2*(2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3 + c^4)*sqrt(-a*c)*abs(c)*sgn(tan(1
/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^
2*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 11.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.81

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx = \frac{c\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,(\sin(e+f\,x)-\sin(2\,e+2\,f\,x)+\sin(3\,e+3\,f\,x))}{f\,\sin(2\,e+2\,f\,x)^2}$$

input
```
int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x
)
```

output
```
(c*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*
(sin(e + f*x) - sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(f*sin(2*e + 2*f*x)^
2)
```

Reduce [F]

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)+\int\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*sqrt(a)*c*( - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)
*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)
*sec(e + f*x),x))
```

3.109 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx$

Optimal result

Integrand size = 36, antiderivative size = 41

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

output

```
-c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x
]
```

output
```
-((c*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[c -
 c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow 4441$$

$$-\frac{c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-((c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

method	result	size
default	$-\frac{\sqrt{2}\,\sin(fx+e)\sqrt{a(1+\sec(fx+e))}\,\sqrt{-2c(-1+\sec(fx+e))}}{f(2\cos(fx+e)-2)}$	52
risch	$\frac{2i\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\mathrm{e}^{i(fx+e)}}{\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)f}$	102

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/f*2^(1/2)*sin(f*x+e)/(2*cos(f*x+e)-2)*(a*(1+sec(f*x+e)))^(1/2)*(-2*c*(-
1+sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx=\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{f\sin(fx+e)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
+ e))/(f*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*
x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2\sqrt{-a}\sqrt{c}}{f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
2*sqrt(-a)*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/
(cos(f*x + e) + 1) - 1))
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \frac{2\sqrt{-ac}|c|\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
2*sqrt(-a*c)*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c
*tan(1/2*f*x + 1/2*e)^2 - c)*f)
```

Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}}{f\,\sin\left(e+f\,x\right)}$$

input
```
int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x
)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2))/(
f*sin(e + f*x))
```

Reduce [F]

$$\begin{aligned}&\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx\\&=\sqrt{c}\,\sqrt{a}\left(\int \sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx\right)\end{aligned}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*sqrt(a)*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e
 + f*x),x)
```

3.110 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 51

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\frac{a\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output

```
a*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\frac{\log(1-\sec(e+fx))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]]
,x]
```

output
```
(Log[1 - Sec[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqr
t[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{3042, 4440\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a\sec(e+fx)+a}}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4440}$$

$$\frac{a\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(47) = 94$.

Time = 2.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.53

method	result
default	$\frac{\left(2\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\ln\left(\frac{-2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}}}{f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}}$
risch	$-\frac{2i\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,f}+\frac{i\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/f*(2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e))-ln(2*(-cos(1/2*f*x+1/2*e
)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))-ln(-2*(cos(1/2*f*x+1/2*e)+si
n(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1)))*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*c
os(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*
e)^2)^(1/2)*tan(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/
(c*sec(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1
)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= -\frac{\frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{c}} + \frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}} - \frac{2\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}}}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
-(sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(c) + sqrt(-a)*log
(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - 2*sqrt(-a)*log(sin(f*x + e
)/(cos(f*x + e) + 1))/sqrt(c))/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x
)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)-1}dx\right)}{c}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x) - 1),x))/c
```

3.111 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}$$

output

```
-1/2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\cot(e+fx)\sqrt{a(1+\sec(e+fx))}}{cf\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/
2),x]
```

output

```
(Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{2f(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
-1/2*(Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(36) = 72$.

Time = 1.68 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.45

method	result	size
default	$\frac{\sqrt{\frac{a \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, \left(\cot\left(\frac{f x}{2}+\frac{e}{2}\right)+\sec\left(\frac{f x}{2}+\frac{e}{2}\right) \csc\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{4 f \sqrt{-\frac{c \sin\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, c}$	103
risch	$\frac{2 i \sqrt{\frac{a \left(\mathrm{e}^{i (f x+e)}+1\right)^{2}}{\mathrm{e}^{2 i (f x+e)}+1}}\, \mathrm{e}^{i (f x+e)}}{c \left(\mathrm{e}^{i (f x+e)}+1\right) \left(\mathrm{e}^{i (f x+e)}-1\right) \sqrt{\frac{c \left(\mathrm{e}^{i (f x+e)}-1\right)^{2}}{\mathrm{e}^{2 i (f x+e)}+1}}\, f}$	105

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4/f*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c*(cot(1/2*f*x+1/2*e)+sec
(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos\left(fx+e\right)}{\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
+ e))*cos(f*x + e)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**
(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(36) = 72.

Time = 0.20 (sec) , antiderivative size = 514, normalized size of antiderivative = 12.24

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{}{\left(c^2\cos\left(4\,fx+4\,e\right)^2+4\,c^2\cos\left(3\,fx+3\,e\right)^2+4\,c^2\cos\left(2\,fx+2\,e\right)^2+4\,c^2\cos\left(fx+e\right)^2+c^2\sin\left(4\,fx+4\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
-2*((sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) - (cos(3*f*x + 3*e)
 + cos(f*x + e))*sin(4*f*x + 4*e) + (2*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3
*e) - 2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*cos(f*x + e)*sin(2*f*x + 2*e
) + 2*cos(2*f*x + 2*e)*sin(f*x + e) + sin(f*x + e))*sqrt(a)*sqrt(c)/((c^2*
cos(4*f*x + 4*e)^2 + 4*c^2*cos(3*f*x + 3*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 +
 4*c^2*cos(f*x + e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(3*f*x + 3*e)^2
+ 4*c^2*sin(2*f*x + 2*e)^2 - 8*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2*s
in(f*x + e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(3*f*x + 3*e) - 2*c
^2*cos(2*f*x + 2*e) + 2*c^2*cos(f*x + e) - c^2)*cos(4*f*x + 4*e) - 4*(2*c^
2*cos(2*f*x + 2*e) - 2*c^2*cos(f*x + e) + c^2)*cos(3*f*x + 3*e) - 4*(2*c^2
*cos(f*x + e) - c^2)*cos(2*f*x + 2*e) - 4*(c^2*sin(3*f*x + 3*e) - c^2*sin(
2*f*x + 2*e) + c^2*sin(f*x + e))*sin(4*f*x + 4*e) - 8*(c^2*sin(2*f*x + 2*e
) - c^2*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{a^2\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}-1\right)}{2\sqrt{-ac}cf|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
1/2*a^2*(1/tan(1/2*f*x + 1/2*e)^2 - 1)/(sqrt(-a*c)*c*f*abs(a)*sgn(tan(1/2*
f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 12.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.81

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,\left(\sin\left(e+f\,x\right)-2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)}{c^2\,f\,\left(4\,\cos\left(e+f\,x\right)+4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x
)
```

output
```
-(2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/co
s(e + f*x))^(1/2)*(sin(e + f*x) - 2*sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/
(c^2*f*(4*cos(e + f*x) + 4*cos(2*e + 2*f*x) - 4*cos(3*e + 3*f*x) + cos(4*e
 + 4*f*x) - 5))
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2-2\sec(fx+e)+1}dx\right)}{c^2}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec
(e + f*x))/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x))/c**2
```

3.112 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}$$

output
```
-1/2*a*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{2c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/
2),x]
```

output
```
(a*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(2*c^3*f*(-1 + Sec[e + f*x])^3*S
qrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4441}$$

$$-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/
2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(37) = 74$.

Time = 2.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.84

method	result	size
default	$-\frac{\left(13\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-11\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{64f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{2}}$	122
risch	$\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{3i(fx+e)}-e^{2i(fx+e)}+e^{i(fx+e)}\right)}{c^{2}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{3}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}$	126

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
-1/64/f*(13*cos(1/2*f*x+1/2*e)^4+6*cos(1/2*f*x+1/2*e)^2-11)*(a/(2*cos(1/2*
f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^2*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\left(2\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
1/2*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3
*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**
(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(37) = 74$.

Time = 0.24 (sec) , antiderivative size = 758, normalized size of antiderivative = 17.63

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
2*((sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + (sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - cos(2*f*x + 2*e)*sin(4*f*x +
4*e) + cos(4*f*x + 4*e)*sin(2*f*x + 2*e) - (cos(4*f*x + 4*e) + 2*cos(2*f*x
 + 2*e) + 1)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(4
*f*x + 4*e) + 2*cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((c^3*cos(4*f*x + 4*e
)^2 + 36*c^3*cos(2*f*x + 2*e)^2 + 16*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 16*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))^2 + c^3*sin(4*f*x + 4*e)^2 + 12*c^3*sin(4*f*x + 4*e)*sin(2*f*x +
 2*e) + 36*c^3*sin(2*f*x + 2*e)^2 + 16*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 16*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 12*c^3*cos(2*f*x + 2*e) + c^3 + 2*(6*c^3*cos(2*f*x + 2*e)
+ c^3)*cos(4*f*x + 4*e) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e)
 - 4*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*cos(4*f*x + 4*e)
+ 6*c^3*cos(2*f*x + 2*e) + c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e) - 4*c^3*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(37) = 74$.

Time = 0.87 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^2\left(\frac{2\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)a+a^2}{a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}-1\right)}{8\sqrt{-ac}c^2f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
1/8*a^2*((2*(a*tan(1/2*f*x + 1/2*e)^2 - a)*a + a^2)/(a^2*tan(1/2*f*x + 1/2
*e)^4) - 1)/(sqrt(-a*c)*c^2*f*abs(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*
f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 15.65 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.72

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\Big(\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,4\mathrm{i}}{c^3\,f}+\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+}}{c^3\,f}}{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sin\left(e+f\,x\right)\,10\mathrm{i}-\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,8}$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x
)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((exp(e*3i + f*x*3i)*(a + a/cos(e + f*x))^(1/2
)*4i)/(c^3*f) + (exp(e*3i + f*x*3i)*cos(2*e + 2*f*x)*(a + a/cos(e + f*x))^
(1/2)*4i)/(c^3*f) - (cos(e + f*x)*exp(e*3i + f*x*3i)*(a + a/cos(e + f*x))^
(1/2)*4i)/(c^3*f)))/(exp(e*3i + f*x*3i)*sin(e + f*x)*10i - exp(e*3i + f*x*
3i)*sin(2*e + 2*f*x)*8i + exp(e*3i + f*x*3i)*sin(3*e + 3*f*x)*2i)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)}{c^3}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x
))/c**3
```

3.113 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a^2(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{10f\sqrt{a+a\sec(e+fx)}} + \frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{5f}$$

output

```
1/10*a^2*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/5*a*
(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a^2c^4\sec(e+fx)\left(-10+10\sec(e+fx)-5\sec^3(e+fx)+2\sec^4(e+fx)\right)\tan(e+}{10f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/
2),x]
```

output

```
(a^2*c^4*Sec[e + f*x]*(-10 + 10*Sec[e + f*x] - 5*Sec[e + f*x]^3 + 2*Sec[e
+ f*x]^4)*Tan[e + f*x])/(10*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
+ f*x]])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{2}{5}a\int \sec(e+fx)\sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{7/2}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}a\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{5f}$$

$$\Big\downarrow \text{4441}$$

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{10f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{5f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
(a^2*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(10*f*Sqrt[a + a*Sec[e + f*x
]]) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])
/(5*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.43

method	result
default	$-\dfrac{8\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\left(6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)a\,c^{3}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{5f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{4}}$
risch	$\dfrac{2ia\,c^{3}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,(5\,\mathrm{e}^{9i(fx+e)}-10\,\mathrm{e}^{8i(fx+e)}+20\,\mathrm{e}^{7i(fx+e)}-10\,\mathrm{e}^{6i(fx+e)}+14\,\mathrm{e}^{5i(fx+e)}-10\,\mathrm{e}^{4i(fx+e)}+20}{5\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)^{4}\left(\mathrm{e}^{i(fx+e)}-1\right)f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-8/5/f*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(a/(2*co
s(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(6*cos(1/2*f*x+1/2*e)^2-
1)*a*c^3/(2*cos(1/2*f*x+1/2*e)^2-1)^4*sin(1/2*f*x+1/2*e)^6*tan(1/2*f*x+1/2
*e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx=\frac{\left(10\,ac^{3}\cos\left(fx+e\right)^{4}-10\,ac^{3}\cos\left(fx+e\right)^{3}+5\,ac^{3}\cos\left(fx+e\right)-2\,ac^{3}\right)\sqrt{\frac{a\cos(fx+}{\cos(fx+}}}{10\,f\cos\left(fx+e\right)^{4}\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algo
rithm="fricas")
```

output
```
1/10*(10*a*c^3*cos(f*x + e)^4 - 10*a*c^3*cos(f*x + e)^3 + 5*a*c^3*cos(f*x
+ e) - 2*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx=\text{Timed out}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(7/2),x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. $2(77) = 154$.

Time $= 0.20$ (sec) , antiderivative size $= 1680$, normalized size of antiderivative $= 18.88$

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx=\text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algo
rithm="maxima")
```

output

```
2/5*(100*a*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 25*a*c^3*cos(2*f*x + 2*
e)*sin(f*x + e) - 5*a*c^3*sin(f*x + e) - (5*a*c^3*sin(9*f*x + 9*e) - 10*a*
c^3*sin(8*f*x + 8*e) + 20*a*c^3*sin(7*f*x + 7*e) - 10*a*c^3*sin(6*f*x + 6*
e) + 14*a*c^3*sin(5*f*x + 5*e) - 10*a*c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(
3*f*x + 3*e) - 10*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3*sin(f*x + e))*cos(10*f*
x + 10*e) + 25*(a*c^3*sin(8*f*x + 8*e) + 2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^
3*sin(4*f*x + 4*e) + a*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 5*(20*a*c^
3*sin(7*f*x + 7*e) + 10*a*c^3*sin(6*f*x + 6*e) + 14*a*c^3*sin(5*f*x + 5*e)
 + 10*a*c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(3*f*x + 3*e) + 5*a*c^3*sin(f*x
 + e))*cos(8*f*x + 8*e) + 100*(2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^3*sin(4*f*
x + 4*e) + a*c^3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(14*a*c^3*sin(5*f
*x + 5*e) + 20*a*c^3*sin(3*f*x + 3*e) - 5*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3
*sin(f*x + e))*cos(6*f*x + 6*e) + 70*(2*a*c^3*sin(4*f*x + 4*e) + a*c^3*sin
(2*f*x + 2*e))*cos(5*f*x + 5*e) - 50*(4*a*c^3*sin(3*f*x + 3*e) - a*c^3*sin
(2*f*x + 2*e) + a*c^3*sin(f*x + e))*cos(4*f*x + 4*e) + (5*a*c^3*cos(9*f*x
+ 9*e) - 10*a*c^3*cos(8*f*x + 8*e) + 20*a*c^3*cos(7*f*x + 7*e) - 10*a*c^3*
cos(6*f*x + 6*e) + 14*a*c^3*cos(5*f*x + 5*e) - 10*a*c^3*cos(4*f*x + 4*e) +
 20*a*c^3*cos(3*f*x + 3*e) - 10*a*c^3*cos(2*f*x + 2*e) + 5*a*c^3*cos(f*x +
 e))*sin(10*f*x + 10*e) - 5*(5*a*c^3*cos(8*f*x + 8*e) + 10*a*c^3*cos(6*f*x
 + 6*e) + 10*a*c^3*cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)...
```

Giac [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.48

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx =$$

$$-\frac{8\left(10\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^3c^3+20\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^4+15\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^5+}{5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^5f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algo
rithm="giac")
```

output
```
-8/5*(10*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^3 + 20*(c*tan(1/2*f*x + 1/2*e)
^2 - c)^2*c^4 + 15*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 4*c^6)*sqrt(-a*c)*
a*c*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2*
f*x + 1/2*e)^2 - c)^5*f)
```

Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.30

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c -c\sec(e+fx))^{7/2}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\left(\frac{a\,c^3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,28\mathrm{i}}{5\,f}-\frac{a\,c^3\,\cos(e+f\,x)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,8\mathrm{i}}{f}+\right.}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,4\mathrm{i}+}$$

input
```
int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x
)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a*c^3*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x)
)^(1/2)*28i)/(5*f) - (a*c^3*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e +
 f*x))^(1/2)*8i)/f + (a*c^3*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a/cos
(e + f*x))^(1/2)*16i)/f - (a*c^3*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x)*(a +
a/cos(e + f*x))^(1/2)*8i)/f + (a*c^3*exp(e*5i + f*x*5i)*cos(4*e + 4*f*x)*(
a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*5i + f*x*5i)*sin(e + f*x)*4i + ex
p(e*5i + f*x*5i)*sin(3*e + 3*f*x)*6i + exp(e*5i + f*x*5i)*sin(5*e + 5*f*x)
*2i)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c -c\sec(e+fx))^{7/2}\,dx = \sqrt{c}\,\sqrt{a}\,a\,c^3\left(-\left(\int\sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^5dx\right)+2\left(\right.\right.$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x)
```

output

```
sqrt(c)*sqrt(a)*a*c**3*( - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**5,x) + 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*
x) + 1)*sec(e + f*x)**4,x) - 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e +
f*x) + 1)*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e +
f*x) + 1)*sec(e + f*x),x))
```

3.114 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{a+a\sec(e+fx)}}+\frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{4f}$$

output
```
1/6*a^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/4*a*(
a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=$$

$$-\frac{ac^3(5\cos(e+fx)-3\cos(2(e+fx))+3\cos(3(e+fx)))\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+f\right.}{12f\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2),x]
```

output

```
-1/12*(a*c^3*(5*Cos[e + f*x] - 3*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])*Se
c[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Se
c[e + f*x]])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{1}{2}a\int \sec(e+fx)\sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{5/2}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}{4f}$$

$$\Big\downarrow \text{4441}$$

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{6f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}{4f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*Sqrt[a + a*Sec[e + f*x]
]) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/
(4*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{4\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\left(5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)a\,c^{2}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$	127
risch	$\frac{2ia\,c^{2}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(3\,\mathrm{e}^{7i(fx+e)}-3\,\mathrm{e}^{6i(fx+e)}+5\,\mathrm{e}^{5i(fx+e)}+5\,\mathrm{e}^{3i(fx+e)}-3\,\mathrm{e}^{2i(fx+e)}+3\,\mathrm{e}^{i(fx+e)}\right)}{3\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)f}$	177

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
4/3/f*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(-c/(2*cos
(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(5*cos(1/2*f*x+1/2*e)^2-1
)*a*c^2/(2*cos(1/2*f*x+1/2*e)^2-1)^3*sin(1/2*f*x+1/2*e)^4*tan(1/2*f*x+1/2*
e)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\frac{\left(12\,ac^{2}\cos\left(fx+e\right)^{3}-6\,ac^{2}\cos\left(fx+e\right)^{2}-4\,ac^{2}\cos\left(fx+e\right)+3\,ac^{2}\right)\sqrt{\frac{a\cos(fx+e)}{\cos(fx+e}}}{12\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output

```
1/12*(12*a*c^2*cos(f*x + e)^3 - 6*a*c^2*cos(f*x + e)^2 - 4*a*c^2*cos(f*x +
 e) + 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(77) = 154$.

Time = 0.20 (sec) , antiderivative size = 1105, normalized size of antiderivative = 12.42

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
2/3*(20*a*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a*c^2*cos(2*f*x + 2*e
)*sin(f*x + e) - 3*a*c^2*sin(f*x + e) - (3*a*c^2*sin(7*f*x + 7*e) - 3*a*c^
2*sin(6*f*x + 6*e) + 5*a*c^2*sin(5*f*x + 5*e) + 5*a*c^2*sin(3*f*x + 3*e) -
 3*a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*
a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*
e))*cos(7*f*x + 7*e) - 2*(10*a*c^2*sin(5*f*x + 5*e) + 9*a*c^2*sin(4*f*x +
4*e) + 10*a*c^2*sin(3*f*x + 3*e) + 6*a*c^2*sin(f*x + e))*cos(6*f*x + 6*e)
+ 10*(3*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e
) - 6*(5*a*c^2*sin(3*f*x + 3*e) - 3*a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f
*x + e))*cos(4*f*x + 4*e) + (3*a*c^2*cos(7*f*x + 7*e) - 3*a*c^2*cos(6*f*x
+ 6*e) + 5*a*c^2*cos(5*f*x + 5*e) + 5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos
(2*f*x + 2*e) + 3*a*c^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a*c^2*cos(6*
f*x + 6*e) + 6*a*c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*
sin(7*f*x + 7*e) + (20*a*c^2*cos(5*f*x + 5*e) + 18*a*c^2*cos(4*f*x + 4*e)
+ 20*a*c^2*cos(3*f*x + 3*e) + 12*a*c^2*cos(f*x + e) + 3*a*c^2)*sin(6*f*x +
 6*e) - 5*(6*a*c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*si
n(5*f*x + 5*e) + 6*(5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos(2*f*x + 2*e) +
3*a*c^2*cos(f*x + e))*sin(4*f*x + 4*e) - 5*(4*a*c^2*cos(2*f*x + 2*e) + a*c
^2)*sin(3*f*x + 3*e) + 3*(4*a*c^2*cos(f*x + e) + a*c^2)*sin(2*f*x + 2*e))*
sqrt(a)*sqrt(c)/((2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*...
```

Giac [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx =$$

$$-\frac{4\left(6\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}c^{5}+8\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c^{6}+3c^{7}\right)\sqrt{-ac}\,a|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.}{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{4}c^{2}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
-4/3*(6*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^5 + 8*(c*tan(1/2*f*x + 1/2*e)^2
 - c)*c^6 + 3*c^7)*sqrt(-a*c)*a*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/
2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^4*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.66 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\left(\frac{a\,c^2\,\cos(e+f\,x)\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,20{}\mathrm{i}}{3\,f}-\frac{a\,c^2\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{}{\cos(}}}{f}\right.}{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,4{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(4\,e\right.}$$

input
```
int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x
)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a*c^2*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a
/cos(e + f*x))^(1/2)*20i)/(3*f) - (a*c^2*exp(e*4i + f*x*4i)*cos(2*e + 2*f*
x)*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a*c^2*exp(e*4i + f*x*4i)*cos(3*e +
3*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*4i + f*x*4i)*sin(2*e + 2*
f*x)*4i + exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \sqrt{c}\,\sqrt{a}\,a\,c^2\left(\left(\int\sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^4dx\right)-\left(\left(\int\sqrt{\sec\right.\right.\right.$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*a*c**2*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**4,x) - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**3,x) - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x),x))
```

3.115 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{c(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f}$$

output

```
-1/3*c^2*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/3*c*
(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c - c\sec(e+fx))^{3/2}\,dx = \frac{a^2c^2\sec(e+fx)\left(-3+\sec^2(e+fx)\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/
2),x]
```

output
```
(a^2*c^2*Sec[e + f*x]*(-3 + Sec[e + f*x]^2)*Tan[e + f*x])/(3*f*Sqrt[a*(1 +
 Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{2}{3}c\int \sec(e+fx)(\sec(e+fx)a+a)^{3/2}\sqrt{c-c\sec(e+fx)}dx-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{4441}$$

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{3f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output

```
-1/3*(c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f
*x]]) - (c*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(3*f)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.53

method	result	size
default	$-\frac{4\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)ac\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}$	136
risch	$\frac{2iac\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(3\,\mathrm{e}^{5i(fx+e)}+2\,\mathrm{e}^{3i(fx+e)}+3\,\mathrm{e}^{i(fx+e)}\right)}{3\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)^{2}\left(\mathrm{e}^{i(fx+e)}-1\right)f}$	142

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output

```
-4/3/f*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(a/(2*co
s(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(2*cos(1/2*f*x+1/2*e)-1)
*(2*cos(1/2*f*x+1/2*e)+1)*a*c/(2*cos(1/2*f*x+1/2*e)^2-1)^2*sin(1/2*f*x+1/2
*e)^2*tan(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{\left(3\,ac\cos\left(fx+e\right)^{2}-ac\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output

```
1/3*(3*a*c*cos(f*x + e)^2 - a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(77) = 154$.

Time $= 0.19$ (sec) , antiderivative size $= 550$, normalized size of antiderivative $= 6.18$

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
2/3*(6*a*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 9*a*c*cos(f*x + e)*sin(2*f*
x + 2*e) - 9*a*c*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a*c*sin(f*x + e) - (3*a
*c*sin(5*f*x + 5*e) + 2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(6*f
*x + 6*e) + 9*(a*c*sin(4*f*x + 4*e) + a*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*
e) - 3*(2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(4*f*x + 4*e) + (3
*a*c*cos(5*f*x + 5*e) + 2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(6
*f*x + 6*e) - 3*(3*a*c*cos(4*f*x + 4*e) + 3*a*c*cos(2*f*x + 2*e) + a*c)*si
n(5*f*x + 5*e) + 3*(2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(4*f*x
 + 4*e) - 2*(3*a*c*cos(2*f*x + 2*e) + a*c)*sin(3*f*x + 3*e))*sqrt(a)*sqrt(
c)/((2*(3*cos(4*f*x + 4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + co
s(6*f*x + 6*e)^2 + 6*(3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f
*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e
))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) +
 1)*f)
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{4\left(3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^4+2\,c^5\right)\sqrt{-ac}a|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^3c^2f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
-4/3*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 2*c^5)*sqrt(-a*c)*a*abs(c)*sg
n(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^
2 - c)^3*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c -c\sec(e+fx))^{3/2}\,dx = \frac{2\,a\,c\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,(2\,\sin(e+f\,x)+5\,\sin(3\,e+3\,f\,x)+3\,\sin(5}{3\,f\,\,(\cos(2\,e+2\,f\,x)-2\,\cos(4\,e+4\,f\,x)-\cos(6\,e+6\,f\,x)+2)}$$

input
```
int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x
)
```

output
```
(2*a*c*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1
/2)*(2*sin(e + f*x) + 5*sin(3*e + 3*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(
2*e + 2*f*x) - 2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \sqrt{c}\,\sqrt{a}\,ac\left(-\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)+\int \sqrt{}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x)
```

output

```
sqrt(c)*sqrt(a)*a*c*( - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**3,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x),x))
```

3.116 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

output

```
-1/2*c*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{ac(1+2\cos(e+fx))\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{2f\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]
,x]
```

output
```
-1/2*(a*c*(1 + 2*Cos[e + f*x])*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*T
an[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4441}$$

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-1/2*(c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x
]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(37) = 74.

Time = 1.81 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.67

method	result	size
default	$\frac{\sqrt{2}\left(3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{-\frac{2c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$	115
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{3i(fx+e)}+e^{2i(fx+e)}+e^{i(fx+e)}\right)}{\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$	135

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2^(1/2)/f*(3*cos(1/2*f*x+1/2*e)^2-1)*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2
*f*x+1/2*e)^2)^(1/2)*(-2*c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2
)^(1/2)*a/(2*cos(1/2*f*x+1/2*e)^2-1)*tan(1/2*f*x+1/2*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{(2\,a\cos(fx+e)+a)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\,f\cos(fx+e)\sin(fx+e)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output

```
1/2*(2*a*cos(f*x + e) + a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = \int (a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}\sec(e+fx)\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
```

output

```
Integral((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))*sec(e +
 f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2\sqrt{-a}a\sqrt{c}}{f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^2}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
-2*sqrt(-a)*a*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x
+ e)/(cos(f*x + e) + 1) - 1)^2)
```

Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2\sqrt{-ac}ac|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
-2*sqrt(-a*c)*a*c*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)
)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*f)
```

Mupad [B] (verification not implemented)

Time = 12.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\frac{a\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,\left(\sin\left(e+f\,x\right)+\sin\left(2\,e+2\,f\,x\right)+\sin\left(\right.\right.}{f\,\sin\left(2\,e+2\,f\,x\right)^2}$$

input

```
int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x
)
```

output

```
(a*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*
(sin(e + f*x) + sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(f*sin(2*e + 2*f*x)^
2)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\sqrt{c}\,\sqrt{a}\,a\left(\int\sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^2dx+\int\sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx\right)$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x)
```

output

```
sqrt(c)*sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*se
c(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*se
c(e + f*x),x))
```

3.117 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 95

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

output

```
2*a^2*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^2(2\log(1-\sec(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x
]],x]
```

output
```
(a^2*(2*Log[1 - Sec[e + f*x]] + Sec[e + f*x])*Tan[e + f*x])/(f*Sqrt[a*(1 +
 Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4443, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{3/2}}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4443}$$

$$2a\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4440}$$

$$\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) + (a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt
[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.93

method	result
risch	$-\frac{2ia\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(2\ln(e^{i(fx+e)}-1)e^{3i(fx+e)}-\ln(e^{2i(fx+e)}+1)e^{3i(fx+e)}+2\,e^{i(fx+e)}\ln(e^{i(fx+e)}-1)-e^{i(fx+e)}\ln(e^{2i(fx+e}}{(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}}{e^{2i(fx+e}}}}$
default	$-\frac{((7\cos(\frac{fx}{2}+\frac{e}{2})^2-2\cos(\frac{fx}{2}+\frac{e}{2})-5)\ln(-\frac{2(\cos(\frac{fx}{2}+\frac{e}{2})-\sin(\frac{fx}{2}+\frac{e}{2}))}{\cos(\frac{fx}{2}+\frac{e}{2})+1})+(-16\cos(\frac{fx}{2}+\frac{e}{2})^2+8)\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2*I*a*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(2*ln(exp(I*(f*
x+e))-1)*exp(3*I*(f*x+e))-ln(exp(2*I*(f*x+e))+1)*exp(3*I*(f*x+e))+2*exp(I*
(f*x+e))*ln(exp(I*(f*x+e))-1)-exp(I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)-2*exp(
2*I*(f*x+e))*ln(exp(I*(f*x+e))-1)+exp(2*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)-
exp(I*(f*x+e))+exp(2*I*(f*x+e))-2*ln(exp(I*(f*x+e))-1)+ln(exp(2*I*(f*x+e))
+1))/(exp(I*(f*x+e))+1)/(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2
)/f/(exp(2*I*(f*x+e))+1)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-(a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqr
t(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}\sec{(e+fx)}}{\sqrt{-c\,(\sec{(e+fx)}-1)}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/sqrt(-c*(sec(e + f*x)
- 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(87) = 174.

Time = 0.21 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=$$

$$-\frac{2\left(a\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\sin\left(2fx+2e\right)+\left(a\cos\left(2fx+2e\right)^{2}+a\sin\left(2fx\right.\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
-2*(a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e
) + (a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) +
a)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*(a*cos(2*f*x + 2*e)
^2 + a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 1) - (a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c*cos(2*f*x + 2*e)^2 +
c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x
)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^{2}}{\sec(fx+e)-1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)-1}dx\right)}{c}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x) - 1),x) + int((sqrt(sec(e + f*x) + 1)*sq
rt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) - 1),x)))/c
```

3.118 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{a^2\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-a^2*ln(1-sec
(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^2\left(\log(1-\sec(e+fx))-\frac{2}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(
3/2),x]
```

output
```
-((a^2*(Log[1 - Sec[e + f*x]] - 2/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(c*f*
Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4442, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4442}$$

$$-\frac{a\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4440}$$

$$-\frac{a^2\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(3/2),x
]
```

output

```
-((a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2))
) - (a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4442

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.31

method	result
risch	$\frac{ia\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(2\,\mathrm{e}^{2i(fx+e)}\ln\left(\mathrm{e}^{i(fx+e)}-1\right)-\mathrm{e}^{2i(fx+e)}\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)-4\,\mathrm{e}^{i(fx+e)}\ln\left(\mathrm{e}^{i(fx+e)}-1\right)+2\,\mathrm{e}^{i(fx+e)}\ln\left(\mathrm{e}^{2i(fx+e)}\right.}{c\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$
default	$-\frac{\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,a\left(4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2\right.}{2f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output

```
I*a*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(2*exp(2*I*(f*x+e)
)*ln(exp(I*(f*x+e))-1)-exp(2*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)-4*exp(I*(f*
x+e))*ln(exp(I*(f*x+e))-1)+2*exp(I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)+4*exp(I
*(f*x+e))+2*ln(exp(I*(f*x+e))-1)-ln(exp(2*I*(f*x+e))+1))/c/(exp(I*(f*x+e))
+1)/(exp(I*(f*x+e))-1)/(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)
/f
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output

```
integral((a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt
(-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sec(e+fx)}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/(-c*(sec(e + f*x) - 1)
)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.23

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\frac{\sqrt{-a}a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{\frac{3}{2}}} + \frac{\sqrt{-a}a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}} - \frac{2\sqrt{-a}a\log\left(\frac{\sin(fx+}{\cos(fx+e}\right.}{c^{\frac{3}{2}}}}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
(sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(3/2) + sqrt(-a)*a*
log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 2*sqrt(-a)*a*log(sin(f*
x + e)/(cos(f*x + e) + 1))/c^(3/2) + sqrt(-a)*a*(cos(f*x + e) + 1)^2/(c^(3
/2)*sin(f*x + e)^2))/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x
)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^{2}}{\sec(fx+e)^{2}-2\sec(fx+e)+1}dx+\int$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x)**2)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x) + int((sqrt(sec
(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 -
2*sec(e + f*x) + 1),x)))/c**2
```

3.119 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=-\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}$$

output
```
-1/4*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=-\frac{(a(1+\sec(e+fx)))^{3/2}\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(
5/2),x]
```

output
```
-1/4*((a*(1 + Sec[e + f*x]))^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(
5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{4f(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(5/2),x
]
```

output
```
-1/4*((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5/
2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.76

method	result	size
risch	$\frac{2ia\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{3i(fx+e)}+\mathrm{e}^{i(fx+e)}\right)}{c^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)^{3}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\, f}$	116
default	$-\frac{\left(5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3\right)a\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{32f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\, c^{2}}$	123

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2*I*a/c^2/(exp(I*(f*x+e))+1)*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))
^(1/2)/(exp(I*(f*x+e))-1)^3/(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^
(1/2)/f*(exp(3*I*(f*x+e))+exp(I*(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(36) = 72.

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos\left(fx+e\right)^{2}}{\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))*cos(f*x + e)^2/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3
*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}\sec{(e+fx)}}{(-c(\sec{(e+fx)}-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/(-c*(sec(e + f*x) - 1)
)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(36) = 72$.

Time = 0.21 (sec) , antiderivative size = 533, normalized size of antiderivative = 12.69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\frac{}{(c^3\cos{(4fx+4e)}^2+16c^3\cos{(3fx+3e)}^2+36c^3\cos{(2fx+}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output

```
2*(6*a*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 6*a*cos(f*x + e)*sin(2*f*x + 2*
e) - 6*a*cos(2*f*x + 2*e)*sin(f*x + e) - (a*sin(3*f*x + 3*e) + a*sin(f*x +
 e))*cos(4*f*x + 4*e) + (a*cos(3*f*x + 3*e) + a*cos(f*x + e))*sin(4*f*x +
4*e) - (6*a*cos(2*f*x + 2*e) + a)*sin(3*f*x + 3*e) - a*sin(f*x + e))*sqrt(
a)*sqrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 16*c^3*cos(3*f*x + 3*e)^2 + 36*c^3*c
os(2*f*x + 2*e)^2 + 16*c^3*cos(f*x + e)^2 + c^3*sin(4*f*x + 4*e)^2 + 16*c^
3*sin(3*f*x + 3*e)^2 + 36*c^3*sin(2*f*x + 2*e)^2 - 48*c^3*sin(2*f*x + 2*e)
*sin(f*x + e) + 16*c^3*sin(f*x + e)^2 - 8*c^3*cos(f*x + e) + c^3 - 2*(4*c^
3*cos(3*f*x + 3*e) - 6*c^3*cos(2*f*x + 2*e) + 4*c^3*cos(f*x + e) - c^3)*co
s(4*f*x + 4*e) - 8*(6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(f*x + e) + c^3)*cos
(3*f*x + 3*e) - 12*(4*c^3*cos(f*x + e) - c^3)*cos(2*f*x + 2*e) - 4*(2*c^3*
sin(3*f*x + 3*e) - 3*c^3*sin(2*f*x + 2*e) + 2*c^3*sin(f*x + e))*sin(4*f*x
+ 4*e) - 16*(3*c^3*sin(2*f*x + 2*e) - 2*c^3*sin(f*x + e))*sin(3*f*x + 3*e)
)*f)
```

Giac [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\left(a-\frac{a}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}\right)a^{2}}{4\,\sqrt{-ac}c^{2}f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output

```
1/4*(a - a/tan(1/2*f*x + 1/2*e)^4)*a^2/(sqrt(-a*c)*c^2*f*abs(a)*sgn(tan(1/
2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.93

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{2\,a\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,\left(6\,\sin\left(e+f\,x\right)-8\,\sin\left(2\,e+2\,f\,x\right)+7\,\sin\left(3\,e+3\,f\,x\right)-4\,\sin\left(4\,e\right.\right.}{c^3\,f\,\left(48\,\cos\left(e+f\,x\right)+15\,\cos\left(2\,e+2\,f\,x\right)-40\,\cos\left(3\,e+3\,f\,x\right)+26\,\cos\left(4\,e+4\,f\,x\right)-8\,\cos\left(5\,e+5\right.\right.}$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x
)
```

output
```
-(2*a*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/
cos(e + f*x))^(1/2)*(6*sin(e + f*x) - 8*sin(2*e + 2*f*x) + 7*sin(3*e + 3*f
*x) - 4*sin(4*e + 4*f*x) + sin(5*e + 5*f*x)))/(c^3*f*(48*cos(e + f*x) + 15
*cos(2*e + 2*f*x) - 40*cos(3*e + 3*f*x) + 26*cos(4*e + 4*f*x) - 8*cos(5*e
+ 5*f*x) + cos(6*e + 6*f*x) - 42))
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)}{c^3}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x)
- 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*
x))/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x)))/c**3
```

3.120 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 88

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}-\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{24cf(c-c\sec(e+fx))^{5/2}}$$

output

```
-1/6*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)-1/24*(a+a*
sec(f*x+e))^(3/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{a^2(1+3\sec(e+fx))\tan(e+fx)}{6c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+f}}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(
7/2),x]
```

output
```
(a^2*(1 + 3*Sec[e + f*x])*Tan[e + f*x])/(6*c^3*f*(-1 + Sec[e + f*x])^3*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{3042, 4439, 3042, 4438\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^{3/2}}{(c-c\sec(e+fx))^{5/2}}dx}{6c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{6c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{24cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(7/2),x
]
```

output
```
-1/6*((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(7/
2)) - ((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(24*c*f*(c - c*Sec[e + f*x
])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\left(13 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}+9 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-21 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+7\right) a \sqrt{\frac{a \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, \sec \left(\frac{f x}{2}+\frac{e}{2}\right) \csc \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{96 f \sqrt{-\frac{c \sin \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, c^{3}}$	136
risch	$\frac{2 i a \sqrt{\frac{a \left({\mathrm e}^{i \left(f x+e\right)}+1\right)^{2}}{{\mathrm e}^{2 i \left(f x+e\right)}+1}}\, \left(3 \,{\mathrm e}^{5 i \left(f x+e\right)}-3 \,{\mathrm e}^{4 i \left(f x+e\right)}+8 \,{\mathrm e}^{3 i \left(f x+e\right)}-3 \,{\mathrm e}^{2 i \left(f x+e\right)}+3 \,{\mathrm e}^{i \left(f x+e\right)}\right)}{3 c^{3} \left({\mathrm e}^{i \left(f x+e\right)}+1\right) \left({\mathrm e}^{i \left(f x+e\right)}-1\right)^{5} \sqrt{\frac{c \left({\mathrm e}^{i \left(f x+e\right)}-1\right)^{2}}{{\mathrm e}^{2 i \left(f x+e\right)}+1}}\, f}$	153

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
1/96/f*(13*cos(1/2*f*x+1/2*e)^6+9*cos(1/2*f*x+1/2*e)^4-21*cos(1/2*f*x+1/2*
e)^2+7)*a*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2
*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^3*sec(1/2*f*x+1/2*e
)*csc(1/2*f*x+1/2*e)^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\left(6\,a\cos\left(fx+e\right)^3-3\,a\cos\left(fx+e\right)^2+a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx}{\cos(fx}}}{6\,\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algo
rithm="fricas")
```

output
```
1/6*(6*a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4
*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)
*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(76) = 152$.

Time = 0.57 (sec) , antiderivative size = 1559, normalized size of antiderivative = 17.72

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algo
rithm="maxima")
```

output

```
2/3*(3*(a*sin(4*f*x + 4*e) + a*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 3*(a*s
in(6*f*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e) - 4*a*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 4*(2*a*sin(6*f*x + 6*e) + 15*a*sin(4*f*x + 4*
e) + 15*a*sin(2*f*x + 2*e) + 3*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(a*s
in(6*f*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e))*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a*cos(4*f*x + 4*e) + a*cos(
2*f*x + 2*e))*sin(6*f*x + 6*e) + 3*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*x + 2*
e) - 3*(a*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) -
 4*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(2*a*cos(6*f*x + 6*e) + 15*a
*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + 3*a*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 2*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 3*(a*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*
x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a
)*sqrt(c)/((c^4*cos(6*f*x + 6*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 225*c^4*
cos(2*f*x + 2*e)^2 + 36*c^4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 400*c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ 36*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^4*s...
```

Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\left(a-\frac{3\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)a^3+a^4}{a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6}\right)a^2}{24\sqrt{-ac}c^3f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algo
rithm="giac")
```

output
```
1/24*(a - (3*(a*tan(1/2*f*x + 1/2*e)^2 - a)*a^3 + a^4)/(a^3*tan(1/2*f*x + 
1/2*e)^6))*a^2/(sqrt(-a*c)*c^3*f*abs(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1
/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 16.33 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.10

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,4\mathrm{i}}{c^4\,f}-\frac{a\,\cos(e+f\,x)\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a-}}{3\,c^4\,f}\right.}{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(e+f\,x\right)\,28\mathrm{i}-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(2\,e+2\right.}$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x
)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a*exp(e*4i + f*x*4i)*(a + a/cos(e + f*x))^(1
/2)*4i)/(c^4*f) - (a*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/cos(e + f*x))^
(1/2)*44i)/(3*c^4*f) + (a*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)*(a + a/cos(e
 + f*x))^(1/2)*4i)/(c^4*f) - (a*exp(e*4i + f*x*4i)*cos(3*e + 3*f*x)*(a + a
/cos(e + f*x))^(1/2)*4i)/(c^4*f)))/(exp(e*4i + f*x*4i)*sin(e + f*x)*28i - 
exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)*28i + exp(e*4i + f*x*4i)*sin(3*e + 3*f
*x)*12i - exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^{2}}{\sec(fx+e)^{4}-4\sec(fx+e)^{3}+6\sec(fx+e)^{2}-4\sec(fx+e)+1}dx+\int\right.}{c^{4}}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x)**2)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2
- 4*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x
) + 1)*sec(e + f*x))/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)
**2 - 4*sec(e + f*x) + 1),x)))/c**4
```

3.121 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}} + \frac{a^2\tan(e+fx)}{12cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}$$

output

```
-1/4*a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)+1/12*a^2
*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 3.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^2(1+2\sec(e+fx))\tan(e+fx)}{6c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(
9/2),x]
```

output

```
-1/6*(a^2*(1 + 2*Sec[e + f*x])*Tan[e + f*x])/(c^4*f*(-1 + Sec[e + f*x])^4*
Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4442, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4442}$$

$$-\frac{a\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{7/2}}dx}{4c}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{4f(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{4c}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{4f(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{4441}$$

$$\frac{a^2\tan(e+fx)}{12cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{4f(c-c\sec(e+fx))^{9/2}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(9/2),x
]
```

output

```
-1/4*(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(9/
2)) + (a^2*Tan[e + f*x])/(12*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f
*x])^(7/2))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4442

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

method	result
default	$-\frac{\left(1463 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{8}+292 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-3510 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+2852 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-713\right) a \sqrt{\frac{a \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, \sec\left(\frac{f x}{2}+\frac{e}{2}\right) \csc(}{12288 f \sqrt{-\frac{c \sin\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, c^{4}}$
risch	$\frac{2 i a \sqrt{\frac{a \left(e^{i (f x+e)}+1\right)^{2}}{e^{2 i (f x+e)}+1}}\, \left(3 e^{7 i (f x+e)}-6 e^{6 i (f x+e)}+17 e^{5 i (f x+e)}-16 e^{4 i (f x+e)}+17 e^{3 i (f x+e)}-6 e^{2 i (f x+e)}+3 e^{i (f x+e)}\right)}{3 c^{4} \left(e^{i (f x+e)}+1\right) \left(e^{i (f x+e)}-1\right)^{7} \sqrt{\frac{c \left(e^{i (f x+e)}-1\right)^{2}}{e^{2 i (f x+e)}+1}}\, f}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RET
URNVERBOSE)
```

output

```
-1/12288/f*(1463*cos(1/2*f*x+1/2*e)^8+292*cos(1/2*f*x+1/2*e)^6-3510*cos(1/
2*f*x+1/2*e)^4+2852*cos(1/2*f*x+1/2*e)^2-713)*a*(a/(2*cos(1/2*f*x+1/2*e)^2
-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)/c^4*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^7
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.72

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{\left(6\,a\cos\left(fx+e\right)^4-6\,a\cos\left(fx+e\right)^3+4\,a\cos\left(fx+e\right)^2-a\,\mathrm{co}}{6\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2\right.}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algo
rithm="fricas")
```

output

```
1/6*(6*a*cos(f*x + e)^4 - 6*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 - a*cos(
f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*
cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(9/2),x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2608 vs. $2(80) = 160$.

Time = 2.82 (sec) , antiderivative size = 2608, normalized size of antiderivative = 28.35

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algo
rithm="maxima")
```

output
```
2/3*(28*a*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 28*a*cos(4*f*x + 4*e)*sin(2*
f*x + 2*e) + 2*(3*a*sin(6*f*x + 6*e) + 8*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*
x + 2*e))*cos(8*f*x + 8*e) + (3*a*sin(8*f*x + 8*e) + 36*a*sin(6*f*x + 6*e)
 + 82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x + 2*e) - 32*a*sin(5/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*sin(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + (17*a*sin(8*f*x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e
) + 140*a*sin(2*f*x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a
*sin(8*f*x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e) + 140*
a*sin(2*f*x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a*sin(8*f*
x + 8*e) + 36*a*sin(6*f*x + 6*e) + 82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x
+ 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(3*a*cos(
6*f*x + 6*e) + 8*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e))*sin(8*f*x + 8*
e) - 2*(14*a*cos(4*f*x + 4*e) - 3*a)*sin(6*f*x + 6*e) + 4*(7*a*cos(2*f*x +
 2*e) + 4*a)*sin(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - (3*a*cos(8*f*x + 8*
e) + 36*a*cos(6*f*x + 6*e) + 82*a*cos(4*f*x + 4*e) + 36*a*cos(2*f*x + 2*e)
 - 32*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*cos(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*a)*sin(7/2*arctan2(s...
```

Giac [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{\left(a-\frac{6\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2a^3+4\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)a^4+a^5}{a^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8}\right)a^2}{96\sqrt{-ac}c^4f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algo
rithm="giac")
```

output
```
1/96*(a - (6*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*a^3 + 4*(a*tan(1/2*f*x + 1/2
*e)^2 - a)*a^4 + a^5)/(a^4*tan(1/2*f*x + 1/2*e)^8))*a^2/(sqrt(-a*c)*c^4*f*
abs(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 16.48 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.70

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+fx)}}\left(\frac{a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,68\mathrm{i}}{3\,c^5\,f}-\frac{a\,\cos(e+f\,x)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a}}{3\,c^5\,f}\right.}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,84\mathrm{i}-\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(2\,e\right.}$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(9/2)),x
)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))^(1
/2)*68i)/(3*c^5*f) - (a*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x
))^(1/2)*88i)/(3*c^5*f) + (a*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a/co
s(e + f*x))^(1/2)*80i)/(3*c^5*f) - (a*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x)*
(a + a/cos(e + f*x))^(1/2)*8i)/(c^5*f) + (a*exp(e*5i + f*x*5i)*cos(4*e + 4
*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f)))/(exp(e*5i + f*x*5i)*sin(e +
 f*x)*84i - exp(e*5i + f*x*5i)*sin(2*e + 2*f*x)*96i + exp(e*5i + f*x*5i)*s
in(3*e + 3*f*x)*54i - exp(e*5i + f*x*5i)*sin(4*e + 4*f*x)*16i + exp(e*5i +
 f*x*5i)*sin(5*e + 5*f*x)*2i)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx =$$

$$-\frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^5-5\sec(fx+e)^4+10\sec(fx+e)^3-10\sec(fx+e)^2+5\sec(fx+e)-1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(}}{\sec(fx+e)^5-5\sec(fx+e)^4+10\sec(fx+}\right.}{c^5}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)
**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1),x) + int((sqrt(sec(e + f*x)
 + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**5 - 5*sec(e +
 f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1),x
)))/c**5
```

3.122 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}} + \frac{a^2\tan(e+fx)}{20cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}}$$

output
```
-1/5*a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)+1/20*a^
2*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(9/2)
```

Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{a^2(3+5\sec(e+fx))\tan(e+fx)}{20c^5f(-1+\sec(e+fx))^5\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(
11/2),x]
```

output
```
(a^2*(3 + 5*Sec[e + f*x])*Tan[e + f*x])/(20*c^5*f*(-1 + Sec[e + f*x])^5*Sq
rt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4442, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{4442}$$

$$-\frac{a\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{9/2}}dx}{5c}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{5f(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{5c}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{5f(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{4441}$$

$$\frac{a^2\tan(e+fx)}{20cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{5f(c-c\sec(e+fx))^{11/2}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(11/2),
x]
```

output
```
-1/5*(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(11
/2)) + (a^2*Tan[e + f*x])/(20*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e +
f*x])^(9/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(80) = 160$.

Time = 2.00 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.76

method	result
default	$\frac{\left(4341 \cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-1225 \cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}-12910 \cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+18030 \cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-9655 \cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+1931\right) a \sqrt{\frac{a \cos\left(\frac{fx}{2}+\right.}{2 \cos\left(\frac{fx}{2}+\frac{e}{2}\right.}}}{40960 f \sqrt{-\frac{c \sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\, c^{5}}$
risch	$\frac{2 i a \sqrt{\frac{a \left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\, \left(5 e^{9i(fx+e)}-15 e^{8i(fx+e)}+50 e^{7i(fx+e)}-75 e^{6i(fx+e)}+102 e^{5i(fx+e)}-75 e^{4i(fx+e)}+50 e^{3i(fx+e)}-15 e^{2i(fx\right.}{5 c^{5} \left(e^{i(fx+e)}+1\right) \left(e^{i(fx+e)}-1\right)^{9} \sqrt{\frac{c \left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\, f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
1/40960/f*(4341*cos(1/2*f*x+1/2*e)^10-1225*cos(1/2*f*x+1/2*e)^8-12910*cos(
1/2*f*x+1/2*e)^6+18030*cos(1/2*f*x+1/2*e)^4-9655*cos(1/2*f*x+1/2*e)^2+1931
)*a*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1
/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^5*sec(1/2*f*x+1/2*e)*csc(
1/2*f*x+1/2*e)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(80) = 160$.

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\left(20\, a \cos\left(fx+e\right)^{5} - 30\, a \cos\left(fx+e\right)^{4} + 30\, a \cos\left(fx+e\right)^{3} - 1\right.}{20\,\left(c^{6} f \cos\left(fx+e\right)^{5} - 5\, c^{6} f \cos\left(fx+e\right)^{4} + 10\, c^{6} f \cos\left(fx+e\right.\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, alg
orithm="fricas")
```

output
```
1/20*(20*a*cos(f*x + e)^5 - 30*a*cos(f*x + e)^4 + 30*a*cos(f*x + e)^3 - 15
*a*cos(f*x + e)^2 + 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)^5 - 5*c^6
*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*
c^6*f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3906 vs. $2(80) = 160$.

Time $= 16.31$ (sec) , antiderivative size $= 3906$, normalized size of antiderivative $= 42.46$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, alg
orithm="maxima")
```

output

```
-2/5*(225*a*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 225*a*cos(4*f*x + 4*e)*sin
(2*f*x + 2*e) - 15*(a*sin(8*f*x + 8*e) + 5*a*sin(6*f*x + 6*e) + 5*a*sin(4*
f*x + 4*e) + a*sin(2*f*x + 2*e))*cos(10*f*x + 10*e) - 225*(a*sin(6*f*x + 6
*e) + a*sin(4*f*x + 4*e))*cos(8*f*x + 8*e) - 5*(a*sin(10*f*x + 10*e) + 15*
a*sin(8*f*x + 8*e) + 60*a*sin(6*f*x + 6*e) + 60*a*sin(4*f*x + 4*e) + 15*a*
sin(2*f*x + 2*e) - 20*a*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) - 48*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*a*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 10*(5*a*sin(10*f*x + 10*e) + 45*a*sin(8*f*x
 + 8*e) + 150*a*sin(6*f*x + 6*e) + 150*a*sin(4*f*x + 4*e) + 45*a*sin(2*f*x
 + 2*e) - 36*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*a
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(17*a*sin(10*f*x + 10*e) + 135*a*sin
(8*f*x + 8*e) + 420*a*sin(6*f*x + 6*e) + 420*a*sin(4*f*x + 4*e) + 135*a*si
n(2*f*x + 2*e) + 60*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 40*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 50*(a*sin(10*f*x + 10*e) + 9*a*
sin(8*f*x + 8*e) + 30*a*sin(6*f*x + 6*e) + 30*a*sin(4*f*x + 4*e) + 9*a*sin
(2*f*x + 2*e) + 2*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*(a*sin(10*f*x ...
```

Giac [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.49

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\left(a-\frac{10\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^3a^3+10\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2a^4+5\left(a\tan\left(\frac{1}{2}fx\right.\right.}{a^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}}\right.}{320\sqrt{-ac}c^5f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\right.\right.}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, alg
orithm="giac")
```

output

```
1/320*(a - (10*(a*tan(1/2*f*x + 1/2*e)^2 - a)^3*a^3 + 10*(a*tan(1/2*f*x +
1/2*e)^2 - a)^2*a^4 + 5*(a*tan(1/2*f*x + 1/2*e)^2 - a)*a^5 + a^6)/(a^5*tan
(1/2*f*x + 1/2*e)^10))*a^2/(sqrt(-a*c)*c^5*f*abs(a)*sgn(tan(1/2*f*x + 1/2*
e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{a\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,60\mathrm{i}}{c^6\,f}-\frac{a\,\cos(e+f\,x)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a}}{5\,c^6\,f}\right.}{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(e+f\,x\right)\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin(2}$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(11/2)),
x)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(1
/2)*60i)/(c^6*f) - (a*cos(e + f*x)*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))
^(1/2)*608i)/(5*c^6*f) + (a*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a/cos
(e + f*x))^(1/2)*72i)/(c^6*f) - (a*exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*(a
+ a/cos(e + f*x))^(1/2)*44i)/(c^6*f) + (a*exp(e*6i + f*x*6i)*cos(4*e + 4*f
*x)*(a + a/cos(e + f*x))^(1/2)*12i)/(c^6*f) - (a*exp(e*6i + f*x*6i)*cos(5*
e + 5*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^6*f)))/(exp(e*6i + f*x*6i)*si
n(e + f*x)*264i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*
x*6i)*sin(3*e + 3*f*x)*220i - exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i + ex
p(e*6i + f*x*6i)*sin(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x
)*2i)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^6-6\sec(fx+e)^5+15\sec(fx+e)^4-20\sec(fx+e)^3+15\sec(fx}\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x)**2)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e + f*x)**4
 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),x) + int(
(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f
*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*
sec(e + f*x)**2 - 6*sec(e + f*x) + 1),x)))/c**6
```

3.123 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 134

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a^3(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f} + \frac{a(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{6f}$$

output
```
1/15*a^3*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/15*a
^2*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f+1/6*a*(a+a*s
ec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 3.77 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{4a^2c^4(21+28\cos(e+fx)+11\cos(2(e+fx)))\sec^6(e+fx)\sqrt{a(1+\sec(e+fx))}}{15f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/
2),x]
```

output
```
(4*a^2*c^4*(21 + 28*Cos[e + f*x] + 11*Cos[2*(e + f*x)])*Sec[e + f*x]^6*Sqr
t[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^8*Tan[(e + f*x)/2])/(15*f*Sqrt[c
- c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4443, 3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{2}{3}a\int \sec(e+fx)(\sec(e+fx)a+a)^{3/2}(c-c\sec(e+fx))^{7/2}dx+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$

$$\Big\downarrow \ 4443$$

$$\frac{2}{3}a\Bigg(\frac{2}{5}a\int \sec(e+fx)\sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{7/2}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(}{5f}$$
$$\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{3}a\Bigg(\frac{2}{5}a\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e}{}$$
$$\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$

$$\Big\downarrow \ 4441$$

$$\frac{2}{3}a\left(\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{10f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{5f}\right)+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

output

```
(a*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(6*
f) + (2*a*((a^2*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(10*f*Sqrt[a + a*
Sec[e + f*x]]) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Ta
n[e + f*x])/(5*f)))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06

$$-\frac{16\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\left(22\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a^2c^3\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^6\tan\left(\frac{fx}{2}\right.}{15f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x)
```

output
```
-16/15/f*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(a/(2*
cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(22*cos(1/2*f*x+1/2*e)
^4-8*cos(1/2*f*x+1/2*e)^2+1)*a^2*c^3/(2*cos(1/2*f*x+1/2*e)^2-1)^5*sin(1/2*
f*x+1/2*e)^6*tan(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{(30\,a^2c^3\cos(fx+e)^5 - 15\,a^2c^3\cos(fx+e)^4 - 20\,a^2c^3\cos(fx+e)^3 + 15\,a^2c^3\cos}{30\,f\cos(fx+e)^5\sin(}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algo
rithm="fricas")
```

output
```
1/30*(30*a^2*c^3*cos(f*x + e)^5 - 15*a^2*c^3*cos(f*x + e)^4 - 20*a^2*c^3*c
os(f*x + e)^3 + 15*a^2*c^3*cos(f*x + e)^2 + 6*a^2*c^3*cos(f*x + e) - 5*a^2
*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2454 vs. $2(116) = 232$.

Time $= 0.21$ (sec) , antiderivative size $= 2454$, normalized size of antiderivative $= 18.31$

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algo
rithm="maxima")
```

output

```
2/15*(210*a^2*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 90*a^2*c^3*cos(2*f*x
 + 2*e)*sin(f*x + e) - 15*a^2*c^3*sin(f*x + e) - (15*a^2*c^3*sin(11*f*x +
11*e) - 15*a^2*c^3*sin(10*f*x + 10*e) + 35*a^2*c^3*sin(9*f*x + 9*e) + 78*a
^2*c^3*sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*sin(5*f
*x + 5*e) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e) + 15
*a^2*c^3*sin(f*x + e))*cos(12*f*x + 12*e) + 15*(6*a^2*c^3*sin(10*f*x + 10*
e) + 15*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c^
3*sin(4*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(11*f*x + 11*e) - 3*(7
0*a^2*c^3*sin(9*f*x + 9*e) + 75*a^2*c^3*sin(8*f*x + 8*e) + 156*a^2*c^3*sin
(7*f*x + 7*e) + 156*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e)
 + 70*a^2*c^3*sin(3*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(10*f*x + 10*
e) + 35*(15*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^
2*c^3*sin(4*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 15
*(78*a^2*c^3*sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*s
in(5*f*x + 5*e) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e
) + 15*a^2*c^3*sin(f*x + e))*cos(8*f*x + 8*e) + 78*(20*a^2*c^3*sin(6*f*x +
 6*e) + 15*a^2*c^3*sin(4*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(7*f*
x + 7*e) - 10*(156*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e)
+ 70*a^2*c^3*sin(3*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(6*f*x + 6*e)
+ 234*(5*a^2*c^3*sin(4*f*x + 4*e) + 2*a^2*c^3*sin(2*f*x + 2*e))*cos(5*f...
```

Giac [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c -c\sec(e+fx))^{7/2}\,dx = \frac{16\left(20\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^3c^4+45\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^5+36\left(c\tan\right.\right.}{15\left(c\tan\left(\frac{1}{2}f\right.\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algo
rithm="giac")
```

output
```
16/15*(20*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^4 + 45*(c*tan(1/2*f*x + 1/2*e
)^2 - c)^2*c^5 + 36*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^6 + 10*c^7)*sqrt(-a*c
)*a^2*c*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(
1/2*f*x + 1/2*e)^2 - c)^6*f)
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.29

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c -c\sec(e+fx))^{7/2}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(-\frac{a^2\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,20\mathrm{i}}{3\,f}+\frac{a^2\,c^3\,\cos(e+f\,x)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,104}{5\,f}\right.}{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)}$$

input
```
int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x
)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a^2*c^3*cos(e + f*x)*exp(e*6i + f*x*6i)*(a +
 a/cos(e + f*x))^(1/2)*104i)/(5*f) - (a^2*c^3*exp(e*6i + f*x*6i)*(a + a/co
s(e + f*x))^(1/2)*20i)/(3*f) + (a^2*c^3*exp(e*6i + f*x*6i)*cos(3*e + 3*f*x
)*(a + a/cos(e + f*x))^(1/2)*28i)/(3*f) - (a^2*c^3*exp(e*6i + f*x*6i)*cos(
4*e + 4*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a^2*c^3*exp(e*6i + f*x*6i
)*cos(5*e + 5*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*6i + f*x*6i)*
sin(2*e + 2*f*x)*10i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*8i + exp(e*6i +
 f*x*6i)*sin(6*e + 6*f*x)*2i)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx=\sqrt{c}\,\sqrt{a}\,a^2c^3\left(-\left(\int\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^6\,dx\right)+\int$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*c**3*( - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f
*x) + 1)*sec(e + f*x)**6,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f
*x) + 1)*sec(e + f*x)**5,x) + 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e +
 f*x) + 1)*sec(e + f*x)**4,x) - 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e
 + f*x) + 1)*sec(e + f*x)**3,x) - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e
 + f*x) + 1)*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e
 + f*x) + 1)*sec(e + f*x),x))
```

3.124 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 134

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c^3(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} - \frac{c^2(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f} - \frac{c(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f}$$

output
```
-2/15*c^3*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/5*c
^2*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f-1/5*c*(a+a*s
ec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.57

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{a^3c^3\sec(e+fx)\left(15-10\sec^2(e+fx)+3\sec^4(e+fx)\right)\tan(e+fx)}{15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/
2),x]
```

output
```
-1/15*(a^3*c^3*Sec[e + f*x]*(15 - 10*Sec[e + f*x]^2 + 3*Sec[e + f*x]^4)*Ta
n[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4443, 3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \ 4443$$

$$\frac{4}{5}c\int \sec(e+fx)(\sec(e+fx)a+a)^{5/2}(c-c\sec(e+fx))^{3/2}dx-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \ 3042$$

$$\frac{4}{5}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \; 4443$$

$$\frac{4}{5}c\left(\frac{1}{2}c\int \sec(e+fx)(\sec(e+fx)a+a)^{5/2}\sqrt{c-c\sec(e+fx)}dx-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-cs}}{4f}\right. \\ \frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \; 3042$$

$$\frac{4}{5}c\left(\frac{1}{2}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{c\tan(e+fx)(a\sec(e+}{}\right. \\ \frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \; 4441$$

$$\frac{4}{5}c\left(-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}\right)- \\ \frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}{5f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output

```
-1/5*(c*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/f + (4*c*(-1/6*(c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*Sqrt[c -
 c*Sec[e + f*x]]) - (c*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
*Tan[e + f*x])/(4*f)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06

$$\frac{16\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\left(16\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-7\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a^2c^2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^4\tan\left(\frac{fx}{2}+\right.}{15f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x)
```

output
```
16/15/f*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(-c/(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(16*cos(1/2*f*x+1/2*e)^
4-7*cos(1/2*f*x+1/2*e)^2+1)*a^2*c^2/(2*cos(1/2*f*x+1/2*e)^2-1)^4*sin(1/2*f
*x+1/2*e)^4*tan(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{\left(15\,a^2c^2\cos\left(fx+e\right)^4-10\,a^2c^2\cos\left(fx+e\right)^2+3\,a^2c^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{15\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
1/15*(15*a^2*c^2*cos(f*x + e)^4 - 10*a^2*c^2*cos(f*x + e)^2 + 3*a^2*c^2)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1526 vs. 2(116) = 232.

Time = 0.20 (sec) , antiderivative size = 1526, normalized size of antiderivative = 11.39

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output

```
2/15*(100*a^2*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 75*a^2*c^2*cos(f*x +
 e)*sin(2*f*x + 2*e) - 75*a^2*c^2*cos(2*f*x + 2*e)*sin(f*x + e) - 15*a^2*c
^2*sin(f*x + e) - (15*a^2*c^2*sin(9*f*x + 9*e) + 20*a^2*c^2*sin(7*f*x + 7*
e) + 58*a^2*c^2*sin(5*f*x + 5*e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^
2*sin(f*x + e))*cos(10*f*x + 10*e) + 75*(a^2*c^2*sin(8*f*x + 8*e) + 2*a^2*
c^2*sin(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*
e))*cos(9*f*x + 9*e) - 5*(20*a^2*c^2*sin(7*f*x + 7*e) + 58*a^2*c^2*sin(5*f
*x + 5*e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(8*f
*x + 8*e) + 100*(2*a^2*c^2*sin(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) +
 a^2*c^2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(58*a^2*c^2*sin(5*f*x + 5
*e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(6*f*x + 6
*e) + 290*(2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*e))*cos(5*f*
x + 5*e) - 50*(4*a^2*c^2*sin(3*f*x + 3*e) + 3*a^2*c^2*sin(f*x + e))*cos(4*
f*x + 4*e) + (15*a^2*c^2*cos(9*f*x + 9*e) + 20*a^2*c^2*cos(7*f*x + 7*e) +
58*a^2*c^2*cos(5*f*x + 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos
(f*x + e))*sin(10*f*x + 10*e) - 15*(5*a^2*c^2*cos(8*f*x + 8*e) + 10*a^2*c^
2*cos(6*f*x + 6*e) + 10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c^2*cos(2*f*x + 2
*e) + a^2*c^2)*sin(9*f*x + 9*e) + 5*(20*a^2*c^2*cos(7*f*x + 7*e) + 58*a^2*
c^2*cos(5*f*x + 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x +
e))*sin(8*f*x + 8*e) - 20*(10*a^2*c^2*cos(6*f*x + 6*e) + 10*a^2*c^2*cos...
```

Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{16\left(10\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^6+15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^7+6c^8\right)\sqrt{-a}}{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^5c^2f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
16/15*(10*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^6 + 15*(c*tan(1/2*f*x + 1/2*e
)^2 - c)*c^7 + 6*c^8)*sqrt(-a*c)*a^2*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + t
an(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^5*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.00 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.60

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c -c\sec(e+fx))^{5/2}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\Bigg(\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,116\mathrm{i}}{15\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,16}{3\,f}}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,6\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+}}$$

input
```
int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x
)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a^2*c^2*exp(e*5i + f*x*5i)*(a + a/cos(e + f*
x))^(1/2)*116i)/(15*f) + (a^2*c^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a +
 a/cos(e + f*x))^(1/2)*16i)/(3*f) + (a^2*c^2*exp(e*5i + f*x*5i)*cos(4*e +
4*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*5i + f*x*5i)*sin(e + f*x)
*4i + exp(e*5i + f*x*5i)*sin(3*e + 3*f*x)*6i + exp(e*5i + f*x*5i)*sin(5*e
+ 5*f*x)*2i)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c -c\sec(e+fx))^{5/2}\,dx = \sqrt{c}\,\sqrt{a}\,a^2c^2\Bigg(\int \sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^5 dx-2\Bigg(\int \sqrt{s}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*c**2*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**5,x) - 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*
x) + 1)*sec(e + f*x)**3,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*
x) + 1)*sec(e + f*x),x))
```

3.125 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{c-c\sec(e+fx)}} - \frac{c(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{4f}$$

output

```
-1/6*c^2*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/4*c*
(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{a^2c^2(5\cos(e+fx)+3(\cos(2(e+fx))+\cos(3(e+fx))))\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\big(\frac{1}{2}(e+}{12f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2),x]
```

output
```
-1/12*(a^2*c^2*(5*Cos[e + f*x] + 3*(Cos[2*(e + f*x)] + Cos[3*(e + f*x)]))*
Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[c - c*
Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{1}{2}c\int \sec(e+fx)(\sec(e+fx)a+a)^{5/2}\sqrt{c-c\sec(e+fx)}dx-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$

$$\Big\downarrow \text{4441}$$

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
-1/6*(c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f
*x]]) - (c*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(4*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

method	result	size
default	$-\frac{4\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\left(11\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)a^{2}c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$	140
risch	$\frac{2ia^{2}c\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(3\,\mathrm{e}^{7i(fx+e)}+3\,\mathrm{e}^{6i(fx+e)}+5\,\mathrm{e}^{5i(fx+e)}+5\,\mathrm{e}^{3i(fx+e)}+3\,\mathrm{e}^{2i(fx+e)}+3\,\mathrm{e}^{i(fx+e)}\right)}{3\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)f}$	177

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-4/3/f*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(-c/(2*co
s(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(11*cos(1/2*f*x+1/2*e)^4
-6*cos(1/2*f*x+1/2*e)^2+1)*a^2*c/(2*cos(1/2*f*x+1/2*e)^2-1)^3*sin(1/2*f*x+
1/2*e)^2*tan(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{\left(12\,a^{2}c\cos\left(fx+e\right)^{3}+6\,a^{2}c\cos\left(fx+e\right)^{2}-4\,a^{2}c\cos\left(fx+e\right)-3\,a^{2}c\right)\sqrt{\frac{a\cos(fx+e)}{\cos(fx+e}}}{12\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
1/12*(12*a^2*c*cos(f*x + e)^3 + 6*a^2*c*cos(f*x + e)^2 - 4*a^2*c*cos(f*x +
 e) - 3*a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1106 vs. $2(77) = 154$.

Time = 0.20 (sec) , antiderivative size = 1106, normalized size of antiderivative = 12.43

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output

```
2/3*(20*a^2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a^2*c*cos(2*f*x + 2*e
)*sin(f*x + e) - 3*a^2*c*sin(f*x + e) - (3*a^2*c*sin(7*f*x + 7*e) + 3*a^2*
c*sin(6*f*x + 6*e) + 5*a^2*c*sin(5*f*x + 5*e) + 5*a^2*c*sin(3*f*x + 3*e) +
 3*a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*
a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*
e))*cos(7*f*x + 7*e) - 2*(10*a^2*c*sin(5*f*x + 5*e) - 9*a^2*c*sin(4*f*x +
4*e) + 10*a^2*c*sin(3*f*x + 3*e) + 6*a^2*c*sin(f*x + e))*cos(6*f*x + 6*e)
+ 10*(3*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*e
) - 6*(5*a^2*c*sin(3*f*x + 3*e) + 3*a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f
*x + e))*cos(4*f*x + 4*e) + (3*a^2*c*cos(7*f*x + 7*e) + 3*a^2*c*cos(6*f*x
+ 6*e) + 5*a^2*c*cos(5*f*x + 5*e) + 5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos
(2*f*x + 2*e) + 3*a^2*c*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a^2*c*cos(6*
f*x + 6*e) + 6*a^2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*
sin(7*f*x + 7*e) + (20*a^2*c*cos(5*f*x + 5*e) - 18*a^2*c*cos(4*f*x + 4*e)
+ 20*a^2*c*cos(3*f*x + 3*e) + 12*a^2*c*cos(f*x + e) - 3*a^2*c)*sin(6*f*x +
 6*e) - 5*(6*a^2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*si
n(5*f*x + 5*e) + 6*(5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos(2*f*x + 2*e) +
3*a^2*c*cos(f*x + e))*sin(4*f*x + 4*e) - 5*(4*a^2*c*cos(2*f*x + 2*e) + a^2
*c)*sin(3*f*x + 3*e) + 3*(4*a^2*c*cos(f*x + e) - a^2*c)*sin(2*f*x + 2*e))*
sqrt(a)*sqrt(c)/((2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*...
```

Giac [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{4\left(4\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+3c^6\right)\sqrt{-ac}a^2|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}\right.\right.}{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^4c^2f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output

```
4/3*(4*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 3*c^6)*sqrt(-a*c)*a^2*abs(c)*s
gn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)
^2 - c)^4*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 14.83 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{a^2\,c\,\cos(e+f\,x)\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,20\mathrm{i}}{3\,f}+\frac{a^2\,c\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{a}{\cos(}}}{f}\right.}{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(4\,e\right.}$$

input
```
int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x
)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a^2*c*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a
/cos(e + f*x))^(1/2)*20i)/(3*f) + (a^2*c*exp(e*4i + f*x*4i)*cos(2*e + 2*f*
x)*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a^2*c*exp(e*4i + f*x*4i)*cos(3*e +
3*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*4i + f*x*4i)*sin(2*e + 2*
f*x)*4i + exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\sqrt{c}\,\sqrt{a}\,a^2c\left(-\left(\int\sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^4dx\right)-\left(\int\right.\right.$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*c*( - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**4,x) - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**3,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x),x))
```

3.126 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

output

```
-1/3*c*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{a^3c\sec(e+fx)\left(3+3\sec(e+fx)+\sec^2(e+fx)\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
,x]
```

output
```
-1/3*(a^3*c*Sec[e + f*x]*(3 + 3*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e + f*x
])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4441}$$

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-1/3*(c*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x
]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(37) = 74$.

Time = 2.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.05

method	result	size
default	$\frac{4\sqrt{2}\left(7\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{-\frac{2c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}$	131
risch	$\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(3\,e^{5i(fx+e)}+6\,e^{4i(fx+e)}+10\,e^{3i(fx+e)}+6\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}+1\right)^{2}\left(e^{i(fx+e)}-1\right)f}$	165

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
4/3*2^(1/2)/f*(7*cos(1/2*f*x+1/2*e)^4-5*cos(1/2*f*x+1/2*e)^2+1)*(a/(2*cos(
1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(-2*c/(2*cos(1/2*f*x+1/2*e
)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a^2/(2*cos(1/2*f*x+1/2*e)^2-1)^2*tan(1/
2*f*x+1/2*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{\left(3\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)+a^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output

```
1/3*(3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e
)^2*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \text{Timed out}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\frac{8\sqrt{-a}a^2\sqrt{c}}{3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^3\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^3}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output

```
8/3*sqrt(-a)*a^2*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^3*(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)^3)
```

Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\frac{8\sqrt{-ac}a^2c^2|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^3f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output

```
8/3*sqrt(-a*c)*a^2*c^2*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1
/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^3*f)
```

Mupad [B] (verification not implemented)

Time = 13.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.16

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{2\,a^2\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,\left(10\,\sin\left(e+f\,x\right)+12\,\sin\left(2\,e+2\,f\right.\right.}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-2\,\cos\left(4\,e+\right.\right.}$$

input
```
int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x
)
```

output
```
(2*a^2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))
/cos(e + f*x))^(1/2)*(10*sin(e + f*x) + 12*sin(2*e + 2*f*x) + 13*sin(3*e +
 3*f*x) + 6*sin(4*e + 4*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*e + 2*f*x)
 - 2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \sqrt{c}\,\sqrt{a}\,a^2\left(\int \sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^3dx + 2\left(\int \sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^2dx\right) + \int \sqrt{\sec\left(fx+e\right)+1}\,\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)
*sec(e + f*x)**3,x) + 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x),x))
```

3.127 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 141

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &\quad+\frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
4*a^3*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+2*a^2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+1
/2*a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^3(1+8\log(1-\sec(e+fx))+6\sec(e+fx)+\sec^2(e+fx))\,\mathrm{t}}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x
]],x]
```

output

```
(a^3*(1 + 8*Log[1 - Sec[e + f*x]] + 6*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e
 + f*x])/(2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4443, 3042, 4443, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{5/2}}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4443}$$

$$2a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^{3/2}}{\sqrt{c-c\sec(e+fx)}}dx+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4443}$$

$$2a\left(2a\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4440}$$

$$2a\left(\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[c - c*Sec[e + f*x]])
 + 2*a*((2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]]) + (a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])
/(f*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(127) = 254$.

Time = 2.70 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.28

method	result
default	$-\frac{2\left(\left(8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+2\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-16\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+16\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-4\right)\ln(-}{f\sqrt{\;}}$
risch	$-\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(3e^{2i(fx+e)}+e^{i(fx+e)}+3\right)\left(e^{2i(fx+e)}-e^{i(fx+e)}\right)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f\left(e^{2i(fx+e)}+1\right)^{2}}-\frac{8ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln(e^{i(fx+e)}-1}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/f*((8*cos(1/2*f*x+1/2*e)^4-8*cos(1/2*f*x+1/2*e)^2+2)*ln(-2*(cos(1/2*f*x
+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(-16*cos(1/2*f*x+1/2*e
)^4+16*cos(1/2*f*x+1/2*e)^2-4)*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e))+
(8*cos(1/2*f*x+1/2*e)^4-8*cos(1/2*f*x+1/2*e)^2+2)*ln(-2*(cos(1/2*f*x+1/2*e
)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(-7*cos(1/2*f*x+1/2*e)^2+3)*
sin(1/2*f*x+1/2*e)^2)*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^
(1/2)*a^2/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/(4*co
s(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x+1/2*e)^2+1)*tan(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec{(fx+e)}+a)^{\frac{5}{2}}\sec{(fx+e)}}{\sqrt{-c\sec{(fx+e)}+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*s
qrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(127) = 254$.

Time = 0.23 (sec) , antiderivative size = 737, normalized size of antiderivative = 5.23

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output

```
-2*(a^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - a^2*cos(4*f*x + 4*e)*sin(2*f*x
 + 2*e) - a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f
*x + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*
cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*si
n(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e
) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) +
 3*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 3*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x
+ 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a^2*cos(
4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 3*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e)
 + a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt
(c)/((c*cos(4*f*x + 4*e)^2 + 4*c*cos(2*f*x + 2*e)^2 + c*sin(4*f*x + 4*e)^2
 + 4*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c*sin(2*f*x + 2*e)^2 + 2*(2*c
*cos(2*f*x + 2*e) + c)*cos(4*f*x + 4*e) + 4*c*cos(2*f*x + 2*e) + c)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x
)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^{2}\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^{3}}{\sec(fx+e)-1}dx\right)-2\left(\int \right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**3)/(sec(e + f*x) - 1),x) - 2*int((sqrt(sec(e + f*x) +
1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x) - 1),x) - int(
(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f
*x) - 1),x)))/c
```

3.128 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}$$

output

```
-a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-4*a^3*ln(1-s
ec(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-2*
a^2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^3\left(4\log(1-\sec(e+fx)) - \frac{4}{-1+\sec(e+fx)} + \sec(e+fx)\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(
3/2),x]
```

output
```
-((a^3*(4*Log[1 - Sec[e + f*x]] - 4/(-1 + Sec[e + f*x]) + Sec[e + f*x])*Ta
n[e + f*x])/(c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4442, 3042, 4443, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4442}$$

$$-\frac{2a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^{3/2}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4443}$$

$$-\frac{2a\left(2a\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\left(2a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4440}$$

$$-\frac{2a\left(\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(3/2),x
]
```

output

```
-((a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2
))) - (2*a*((2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f
*x])/(f*Sqrt[c - c*Sec[e + f*x]])))/c
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(133) = 266$.

Time = 2.71 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.03

method	result
default	$-\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,a^{2}\left(\left(-8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+4\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+4\right)\ln\left(-\right.\right.$ $f\left(2c\right.$
risch	$\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(5\,e^{3i(fx+e)}-2\,e^{2i(fx+e)}+5\,e^{i(fx+e)}\right)}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f\left(e^{2i(fx+e)}+1\right)}+\frac{8ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}-4ia^{2}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/f*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*a^2/(2*cos(
1/2*f*x+1/2*e)^2-1)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(
1/2)/c*((-8*cos(1/2*f*x+1/2*e)^2+4)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+
1/2*e))/(cos(1/2*f*x+1/2*e)+1))*tan(1/2*f*x+1/2*e)+(-8*cos(1/2*f*x+1/2*e)^
2+4)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))
*tan(1/2*f*x+1/2*e)+(16*cos(1/2*f*x+1/2*e)^2-8)*ln(-cot(1/2*f*x+1/2*e)+csc
(1/2*f*x+1/2*e))*tan(1/2*f*x+1/2*e)-5*cot(1/2*f*x+1/2*e)+3*sec(1/2*f*x+1/2
*e)*csc(1/2*f*x+1/2*e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c
^2*sec(f*x + e) + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2035 vs. $2(133) = 266$.

Time = 0.39 (sec) , antiderivative size = 2035, normalized size of antiderivative = 14.03

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output

```
2*(8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 - 2*a^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*a^2*cos(4*f*x + 4*e)*sin(2*
f*x + 2*e) + 2*a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*co
s(2*f*x + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2
*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^
2 + 4*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*
e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2
+ 4*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*sin
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(2*f*x + 2*
e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) - 4*(a^2*cos(
4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) - 2*a^2*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x
)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^2-2\sec(fx+e)+1}dx+2\left(\int \frac{\sqrt{\sec(f}}{}\right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**3)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x) + 2*int((sqr
t(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*
x)**2 - 2*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e
 + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x)))/c**
2
```

3.129 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(1-\sec(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output

```
-1/2*a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+a^2*(a+a
*sec(f*x+e))^(1/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(3/2)+a^3*ln(1-sec(f*x+
e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^3\left(-\log(1-\sec(e+fx))+\frac{-2+4\sec(e+fx)}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(
5/2),x]
```

output
```
-((a^3*(-Log[1 - Sec[e + f*x]] + (-2 + 4*Sec[e + f*x])/(-1 + Sec[e + f*x])
^2)*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x
]]))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4442, 3042, 4442, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4442}$$

$$-\frac{a\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^{3/2}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4442}$$

$$-\frac{a\left(-\frac{a\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4440}$$

$$-\frac{a\left(-\frac{a^2\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(5/2),x
]
```

output

```
-1/2*(a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(
5/2)) - (a*(-((a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e +
f*x])^(3/2))) - (a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*S
ec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])))/c
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.78

method	result
default	$-\frac{\left(16\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-32\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+16\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(\right.\right.}{16f\sqrt{-\frac{c\sin\left(\frac{fx}{2}\right.}{2\cos\left(\frac{fx}{2}\right.}}}$
risch	$\frac{8ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,e^{2i(fx+e)}}{c^{2}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{3}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}-\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{c^{2}\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}+\frac{ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}\right.}{e^{2i(fx+e)}}}}{c^{2}\left(e^{i(fx\right.}}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/16/f*(16*sin(1/2*f*x+1/2*e)^4*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2
*e))/(cos(1/2*f*x+1/2*e)+1))-32*sin(1/2*f*x+1/2*e)^4*ln(-cot(1/2*f*x+1/2*e
)+csc(1/2*f*x+1/2*e))+16*sin(1/2*f*x+1/2*e)^4*ln(-2*(cos(1/2*f*x+1/2*e)+si
n(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))-3*cos(1/2*f*x+1/2*e)^4+6*cos(1/2
*f*x+1/2*e)^2+5)*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)
*a^2/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^2*sec(1/
2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^3
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*s
qrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*
c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = $$

$$-\frac{\frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{\frac{5}{2}}}+\frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{5}{2}}}-\frac{4\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}}+\frac{\left(\sqrt{-a}a^2\sqrt{c}+\frac{2\sqrt{-a}a^2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(c}{c^3\sin(fx+e)^4}}{2f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
-1/2*(2*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(5/2) + 2*
sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 4*sqrt(-a)
*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) + (sqrt(-a)*a^2*sqrt(c)
+ 2*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e
) + 1)^4/(c^3*sin(f*x + e)^4))/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x
)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)-2\left(\int\right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**3)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*
x) - 1),x) - 2*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e
 + f*x)**2)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x)
- int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec
(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x)))/c**3
```

3.130 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}$$

output
```
-1/6*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=-\frac{(a(1+\sec(e+fx)))^{5/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(
7/2),x]
```

output
```
-1/6*((a*(1 + Sec[e + f*x]))^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(
7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(7/2),x
]
```

output
```
-1/6*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(7/
2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.17

method	result	size
risch	$\frac{2 i a^{2} \sqrt{\frac{a \left({\mathrm e}^{i (f x +e)}+1\right)^{2}}{{\mathrm e}^{2 i (f x +e)}+1}}\, \left(3 \,{\mathrm e}^{5 i (f x +e)}+10 \,{\mathrm e}^{3 i (f x +e)}+3 \,{\mathrm e}^{i (f x +e)}\right)}{3 c^{3} \left({\mathrm e}^{i (f x +e)}+1\right) \left({\mathrm e}^{i (f x +e)}-1\right)^{5} \sqrt{\frac{c \left({\mathrm e}^{i (f x +e)}-1\right)^{2}}{{\mathrm e}^{2 i (f x +e)}+1}}\, f}$	133
default	$\frac{\left(11 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}+15 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-15 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+5\right) a^{2} \sqrt{\frac{a \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, \sec \left(\frac{f x}{2}+\frac{e}{2}\right) \csc \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{96 f \sqrt{-\frac{c \sin \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, c^{3}}$	138

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*I*a^2/c^3/(exp(I*(f*x+e))+1)*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))
+1))^(1/2)/(exp(I*(f*x+e))-1)^5/(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*(f*x+e))+
1))^(1/2)/f*(3*exp(5*I*(f*x+e))+10*exp(3*I*(f*x+e))+3*exp(I*(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\left(3\,a^{2}\cos\left(fx+e\right)^{3}+a^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{3\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algo
rithm="fricas")
```

output
```
1/3*(3*a^2*cos(f*x + e)^3 + a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4*f*cos(f*x + e)^3
 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1815 vs. $2(36) = 72$.

Time $= 0.20$ (sec) , antiderivative size $= 1815$, normalized size of antiderivative $= 43.21$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algo
rithm="maxima")
```

output
```
2/3*(208*a^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 48*a^2*cos(f*x + e)*sin(2
*f*x + 2*e) - 48*a^2*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a^2*sin(f*x + e) -
(3*a^2*sin(7*f*x + 7*e) + 13*a^2*sin(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e
) + 3*a^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(8*a^2*sin(6*f*x + 6*e) + 15*
a^2*sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(13*a
^2*sin(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(6*
f*x + 6*e) + 26*(15*a^2*sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(5*f
*x + 5*e) - 30*(13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(4*f*x +
4*e) + (3*a^2*cos(7*f*x + 7*e) + 13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*
x + 3*e) + 3*a^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(16*a^2*cos(6*f*x + 6*
e) + 30*a^2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x + 2*e) + a^2)*sin(7*f*x +
7*e) + 16*(13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(f
*x + e))*sin(6*f*x + 6*e) - 13*(30*a^2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x
 + 2*e) + a^2)*sin(5*f*x + 5*e) + 30*(13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(
f*x + e))*sin(4*f*x + 4*e) - 13*(16*a^2*cos(2*f*x + 2*e) + a^2)*sin(3*f*x
+ 3*e))*sqrt(a)*sqrt(c)/((c^4*cos(8*f*x + 8*e)^2 + 36*c^4*cos(7*f*x + 7*e)
^2 + 256*c^4*cos(6*f*x + 6*e)^2 + 676*c^4*cos(5*f*x + 5*e)^2 + 900*c^4*cos
(4*f*x + 4*e)^2 + 676*c^4*cos(3*f*x + 3*e)^2 + 256*c^4*cos(2*f*x + 2*e)^2
+ 36*c^4*cos(f*x + e)^2 + c^4*sin(8*f*x + 8*e)^2 + 36*c^4*sin(7*f*x + 7*e)
^2 + 256*c^4*sin(6*f*x + 6*e)^2 + 676*c^4*sin(5*f*x + 5*e)^2 + 900*c^4*...
```

Giac [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.55

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{\left(a^2-\frac{a^2}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6}\right)a^2}{6\,\sqrt{-ac}c^3f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algo
rithm="giac")
```

output
```
-1/6*(a^2 - a^2/tan(1/2*f*x + 1/2*e)^6)*a^2/(sqrt(-a*c)*c^3*f*abs(a)*sgn(t
an(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{a^2\,\cos(e+f\,x)\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,52\mathrm{i}}{3\,c^4\,f}+\frac{a^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\cos(3\,e+3\,f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,4\mathrm{i}}{c^4\,f}\right)}{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin(e+f\,x)\,28\mathrm{i}-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin(2\,e+2\,f\,x)\,28\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,12\mathrm{i}-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin}$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x
)
```

output
```
-((c - c/cos(e + f*x))^(1/2)*((a^2*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/
cos(e + f*x))^(1/2)*52i)/(3*c^4*f) + (a^2*exp(e*4i + f*x*4i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^4*f)))/(exp(e*4i + f*x*4i)*sin(e + f
*x)*28i - exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)*28i + exp(e*4i + f*x*4i)*sin
(3*e + 3*f*x)*12i - exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \sqrt{c}\,\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^4-4\sec(fx+e)^3+6\sec(fx+e)^2-4\sec(fx+e)+1}dx+2\Big(\right.$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**3)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)*
*2 - 4*sec(e + f*x) + 1),x) + 2*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e
+ f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(
e + f*x)**2 - 4*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( -
 sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*
sec(e + f*x)**2 - 4*sec(e + f*x) + 1),x)))/c**4
```

3.131 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 88

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}}-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{48cf(c-c\sec(e+fx))^{7/2}}$$

output
```
-1/8*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)-1/48*(a+a*
sec(f*x+e))^(5/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 3.54 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^3(1+2\sec(e+fx)+3\sec^2(e+fx))\tan(e+fx)}{6c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(
9/2),x]
```

output
```
-1/6*(a^3*(1 + 2*Sec[e + f*x] + 3*Sec[e + f*x]^2)*Tan[e + f*x])/(c^4*f*(-1
 + Sec[e + f*x])^4*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^{5/2}}{(c-c\sec(e+fx))^{7/2}}dx}{8c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{8c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{48cf(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(9/2),x
]
```

output
```
-1/8*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(9/
2)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(48*c*f*(c - c*Sec[e + f*x
])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.72

method	result
default	$-\frac{\left(631 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{8}+548 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-1590 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+1060 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-265\right) a^{2} \sqrt{\frac{a \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, \sec\left(\frac{f x}{2}+\frac{e}{2}\right) \csc(}{6144 f \sqrt{-\frac{c \sin\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, c^{4}}$
risch	$\frac{2 i a^{2} \sqrt{\frac{a \left(\mathrm{e}^{i (f x+e)}+1\right)^{2}}{\mathrm{e}^{2 i (f x+e)}+1}}\, \left(3 \,\mathrm{e}^{7 i (f x+e)}-3 \,\mathrm{e}^{6 i (f x+e)}+17 \,\mathrm{e}^{5 i (f x+e)}-10 \,\mathrm{e}^{4 i (f x+e)}+17 \,\mathrm{e}^{3 i (f x+e)}-3 \,\mathrm{e}^{2 i (f x+e)}+3 \,\mathrm{e}^{i (f x+e)}\right)}{3 c^{4} \left(\mathrm{e}^{i (f x+e)}+1\right) \left(\mathrm{e}^{i (f x+e)}-1\right)^{7} \sqrt{\frac{c \left(\mathrm{e}^{i (f x+e)}-1\right)^{2}}{\mathrm{e}^{2 i (f x+e)}+1}}\, f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/6144/f*(631*cos(1/2*f*x+1/2*e)^8+548*cos(1/2*f*x+1/2*e)^6-1590*cos(1/2*
f*x+1/2*e)^4+1060*cos(1/2*f*x+1/2*e)^2-265)*a^2*(a/(2*cos(1/2*f*x+1/2*e)^2
-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)/c^4*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(76) = 152$.

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{\left(6\,a^{2}\cos\left(fx+e\right)^{4}-3\,a^{2}\cos\left(fx+e\right)^{3}+4\,a^{2}\cos\left(fx+e\right)^{2}-a}{6\,\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}+6\,c^{5}f\cos\left(fx+e\right)^{2}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algo
rithm="fricas")
```

output
```
1/6*(6*a^2*cos(f*x + e)^4 - 3*a^2*cos(f*x + e)^3 + 4*a^2*cos(f*x + e)^2 -
a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
+ e) - c)/cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 +
6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2719 vs. $2(76) = 152$.

Time = 3.06 (sec) , antiderivative size = 2719, normalized size of antiderivative = 30.90

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algo
rithm="maxima")
```

output

```
2/3*(70*a^2*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 70*a^2*cos(4*f*x + 4*e)*si
n(2*f*x + 2*e) + 3*a^2*sin(2*f*x + 2*e) + (3*a^2*sin(6*f*x + 6*e) + 10*a^2
*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) + (3*a^2*sin(
8*f*x + 8*e) + 60*a^2*sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2
*sin(2*f*x + 2*e) - 32*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 32*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*e)
 + 308*a^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f*x
 + 2*e) + 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*
e) + 308*a^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f
*x + 2*e) + 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*c
os(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a^2*sin(8*f*x + 8
*e) + 60*a^2*sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2*sin(2*f*
x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (3*a^2*co
s(6*f*x + 6*e) + 10*a^2*cos(4*f*x + 4*e) + 3*a^2*cos(2*f*x + 2*e))*sin(8*f
*x + 8*e) - (70*a^2*cos(4*f*x + 4*e) - 3*a^2)*sin(6*f*x + 6*e) + 10*(7*a^2
*cos(2*f*x + 2*e) + a^2)*sin(4*f*x + 4*e) - (3*a^2*cos(8*f*x + 8*e) + 60*a
^2*cos(6*f*x + 6*e) + 130*a^2*cos(4*f*x + 4*e) + 60*a^2*cos(2*f*x + 2*e) -
 32*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a^2*c...
```

Giac [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{\left(a^2-\frac{4\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)a^5+a^6}{a^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8}\right)a^2}{48\,\sqrt{-ac}c^4f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algo
rithm="giac")
```

output
```
-1/48*(a^2 - (4*(a*tan(1/2*f*x + 1/2*e)^2 - a)*a^5 + a^6)/(a^4*tan(1/2*f*x
 + 1/2*e)^8))*a^2/(sqrt(-a*c)*c^4*f*abs(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + ta
n(1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 15.73 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.98

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\left(\frac{a^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,68\mathrm{i}}{3\,c^5\,f}-\frac{a^2\,\cos(e+f\,x)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}}{3\,c^5\,f}\right.}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,84\mathrm{i}-\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(2\right.}$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(9/2)),x
)
```

output

```
((c - c/cos(e + f*x))^(1/2)*((a^2*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))^
(1/2)*68i)/(3*c^5*f) - (a^2*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e +
 f*x))^(1/2)*52i)/(3*c^5*f) + (a^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a
+ a/cos(e + f*x))^(1/2)*80i)/(3*c^5*f) - (a^2*exp(e*5i + f*x*5i)*cos(3*e +
 3*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f) + (a^2*exp(e*5i + f*x*5i)*c
os(4*e + 4*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f)))/(exp(e*5i + f*x*5
i)*sin(e + f*x)*84i - exp(e*5i + f*x*5i)*sin(2*e + 2*f*x)*96i + exp(e*5i +
 f*x*5i)*sin(3*e + 3*f*x)*54i - exp(e*5i + f*x*5i)*sin(4*e + 4*f*x)*16i +
exp(e*5i + f*x*5i)*sin(5*e + 5*f*x)*2i)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^{2}\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^{3}}{\sec(fx+e)^{5}-5\sec(fx+e)^{4}+10\sec(fx+e)^{3}-10\sec(fx+e)^{2}+5\sec}\right.\right.}{}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**3)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f
*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1),x) - 2*int((sqrt(sec(e +
 f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**5 - 5
*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x
) - 1),x) - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e +
f*x))/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e
 + f*x)**2 + 5*sec(e + f*x) - 1),x)))/c**5
```

3.132 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 133

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{40cf(c-c\sec(e+fx))^{9/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{240c^2f(c-c\sec(e+fx))^{7/2}}$$

output
```
-1/10*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)-1/40*(a+
a*sec(f*x+e))^(5/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(9/2)-1/240*(a+a*sec(f
*x+e))^(5/2)*tan(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 5.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{a^3(2+5\sec(e+fx)+5\sec^2(e+fx))\tan(e+fx)}{15c^5f(-1+\sec(e+fx))^5\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(
11/2),x]
```

output
```
(a^3*(2 + 5*Sec[e + f*x] + 5*Sec[e + f*x]^2)*Tan[e + f*x])/(15*c^5*f*(-1 +
 Sec[e + f*x])^5*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4439, 3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{4439}$$

$$\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^{5/2}}{(c-c\sec(e+fx))^{9/2}}dx}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{4439}$$

$$\frac{\frac{\int \frac{\sec(e+fx)(\sec(e+fx)a+a)^{5/2}}{(c-c\sec(e+fx))^{7/2}}dx}{8c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{8c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow 4438$$

$$\frac{-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{48cf(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}}{5c}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(11/2),
x]
```

output

```
-1/10*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(1
1/2)) + (-1/8*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e +
f*x])^(9/2)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(48*c*f*(c - c*Se
c[e + f*x])^(7/2)))/(5*c)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.23

method	result
default	$\frac{\left(1427\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+545\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}-4930\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+5570\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-2785\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+557\right)a^{2}\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-}}}{15360f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{5}}$
risch	$\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(15\,e^{9i(fx+e)}-30\,e^{8i(fx+e)}+140\,e^{7i(fx+e)}-170\,e^{6i(fx+e)}+282\,e^{5i(fx+e)}-170\,e^{4i(fx+e)}+140\,e^{3i(fx+e)}-3\right.}{15c^{5}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{9}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
1/15360/f*(1427*cos(1/2*f*x+1/2*e)^10+545*cos(1/2*f*x+1/2*e)^8-4930*cos(1/
2*f*x+1/2*e)^6+5570*cos(1/2*f*x+1/2*e)^4-2785*cos(1/2*f*x+1/2*e)^2+557)*a^
2*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2
*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^5*sec(1/2*f*x+1/2*e)*csc(1/
2*f*x+1/2*e)^9
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\left(15\,a^2\cos\left(fx+e\right)^5 - 15\,a^2\cos\left(fx+e\right)^4 + 20\,a^2\cos\left(fx+e\right)^3}{15\left(c^6f\cos\left(fx+e\right)^5 - 5\,c^6f\cos\left(fx+e\right)^4 + 10\,c^6f\cos\left(fx+\right.\right.}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, alg
orithm="fricas")
```

output
```
1/15*(15*a^2*cos(f*x + e)^5 - 15*a^2*cos(f*x + e)^4 + 20*a^2*cos(f*x + e)^
3 - 10*a^2*cos(f*x + e)^2 + 2*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)
^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x +
 e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4108 vs. 2(115) = 230.

Time = 16.66 (sec) , antiderivative size = 4108, normalized size of antiderivative = 30.89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, alg
orithm="maxima")
```

output
```
-2/15*(1350*a^2*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 1350*a^2*cos(4*f*x + 4
*e)*sin(2*f*x + 2*e) - 30*a^2*sin(2*f*x + 2*e) - 10*(3*a^2*sin(8*f*x + 8*e
) + 17*a^2*sin(6*f*x + 6*e) + 17*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x +
2*e))*cos(10*f*x + 10*e) - 1350*(a^2*sin(6*f*x + 6*e) + a^2*sin(4*f*x + 4*
e))*cos(8*f*x + 8*e) - 5*(3*a^2*sin(10*f*x + 10*e) + 75*a^2*sin(8*f*x + 8*
e) + 290*a^2*sin(6*f*x + 6*e) + 290*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*
x + 2*e) - 80*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
92*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 80*a^2*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 20*(7*a^2*sin(10*f*x + 10*e) + 135*a^2*sin(8
*f*x + 8*e) + 450*a^2*sin(6*f*x + 6*e) + 450*a^2*sin(4*f*x + 4*e) + 135*a^
2*sin(2*f*x + 2*e) - 72*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 20*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(
7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(47*a^2*sin(10*f*x +
10*e) + 855*a^2*sin(8*f*x + 8*e) + 2730*a^2*sin(6*f*x + 6*e) + 2730*a^2*si
n(4*f*x + 4*e) + 855*a^2*sin(2*f*x + 2*e) + 240*a^2*sin(3/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 160*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 20*(7*a^2*sin(10*f*x + 10*e) + 135*a^2*sin(8*f*x + 8*e) + 450*a^2*sin(6*
f*x + 6*e) + 450*a^2*sin(4*f*x + 4*e) + 135*a^2*sin(2*f*x + 2*e) + 20*a...
```

Giac [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{\left(a^2-\frac{10\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)^2a^5+5\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a\right)a^6+a^7}{a^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}}\right)a^2}{240\sqrt{-ac}c^5f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, alg
orithm="giac")
```

output
```
-1/240*(a^2 - (10*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*a^5 + 5*(a*tan(1/2*f*x
+ 1/2*e)^2 - a)*a^6 + a^7)/(a^5*tan(1/2*f*x + 1/2*e)^10))*a^2/(sqrt(-a*c)*
c^5*f*abs(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B] (verification not implemented)

Time = 16.04 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.15

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\left(\frac{a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,136\mathrm{i}}{3\,c^6\,f}-\frac{a^2\,\cos(e+f\,x)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}}{15\,c^6}\right.}{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(e+f\,x\right)\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}}$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(11/2)),
x)
```

output
```
((c - c/cos(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^
(1/2)*136i)/(3*c^6*f) - (a^2*cos(e + f*x)*exp(e*6i + f*x*6i)*(a + a/cos(e
+ f*x))^(1/2)*1688i)/(15*c^6*f) + (a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)
*(a + a/cos(e + f*x))^(1/2)*160i)/(3*c^6*f) - (a^2*exp(e*6i + f*x*6i)*cos(
3*e + 3*f*x)*(a + a/cos(e + f*x))^(1/2)*124i)/(3*c^6*f) + (a^2*exp(e*6i +
f*x*6i)*cos(4*e + 4*f*x)*(a + a/cos(e + f*x))^(1/2)*8i)/(c^6*f) - (a^2*exp
(e*6i + f*x*6i)*cos(5*e + 5*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^6*f)))/
(exp(e*6i + f*x*6i)*sin(e + f*x)*264i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x
)*330i + exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*220i - exp(e*6i + f*x*6i)*sin
(4*e + 4*f*x)*88i + exp(e*6i + f*x*6i)*sin(5*e + 5*f*x)*20i - exp(e*6i + f
*x*6i)*sin(6*e + 6*f*x)*2i)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(\int\frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^6-6\sec(fx+e)^5+15\sec(fx+e)^4-20\sec(fx+e)^3+15\sec(f}\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**3)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e + f*x)
**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),x) + 2
*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(s
ec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)*
*3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x)
+ 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**6 - 6*sec(e +
f*x)**5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6
*sec(e + f*x) + 1),x)))/c**6
```

3.133 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 139

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

output

```
-4*c^3*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e
))^(1/2)-2*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-
1/2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c^3(1+8\log(1+\sec(e+fx))-6\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x
]],x]
```

output
```
-1/2*(c^3*(1 + 8*Log[1 + Sec[e + f*x]] - 6*Sec[e + f*x] + Sec[e + f*x]^2)*
Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4443, 3042, 4443, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4443}$$

$$2c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{\sec(e+fx)a+a}}dx-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$2c\int \frac{\csc \left(e+fx+\frac{\pi}{2}\right) \left(c-c\csc \left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\csc \left(e+fx+\frac{\pi}{2}\right) a+a}}dx-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \textcolor{blue}{4443}$$

$$2c\left(2c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$2c\left(2c\int \frac{\csc \left(e+fx+\frac{\pi}{2}\right) \sqrt{c-c\csc \left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc \left(e+fx+\frac{\pi}{2}\right) a+a}}dx-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \textcolor{blue}{4440}$$

$$2c\left(-\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-1/2*(c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x
]]) + 2*c*((-2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f
*x])/(f*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.52

method	result
risch	$-\frac{2 i c^{2} \sqrt{\frac{c \left(\mathrm{e}^{i (f x+e)}-1\right)^{2}}{\mathrm{e}^{2 i (f x+e)}+1}}\, \left(3 \,\mathrm{e}^{2 i (f x+e)}-\mathrm{e}^{i (f x+e)}+3\right) \left(\mathrm{e}^{i (f x+e)}+\mathrm{e}^{2 i (f x+e)}\right)}{\sqrt{\frac{a \left(\mathrm{e}^{i (f x+e)}+1\right)^{2}}{\mathrm{e}^{2 i (f x+e)}+1}}\, \left(\mathrm{e}^{i (f x+e)}-1\right) f \left(\mathrm{e}^{2 i (f x+e)}+1\right)^{2}}+\frac{8 i c^{2} \left(\mathrm{e}^{i (f x+e)}+1\right) \sqrt{\frac{c \left(\mathrm{e}^{i (f x+e)}-1\right)^{2}}{\mathrm{e}^{2 i (f x+e)}+1}}\, \ln(\mathrm{e}^{i (f x+e)}+1}{\sqrt{\frac{a \left(\mathrm{e}^{i (f x+e)}+1\right)^{2}}{\mathrm{e}^{2 i (f x+e)}+1}}\, \left(\mathrm{e}^{i (f x+e)}-1\right) f}$
default	$\frac{2 ((16 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-16 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+4) \ln (-\cot \left(\frac{f x}{2}+\frac{e}{2}\right)+\csc \left(\frac{f x}{2}+\frac{e}{2}\right)+1)+(-8 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+8 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-2) \ln (-}{}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2*I*c^2/(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)/(exp(I*(f*x+e
))-1)*(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(3*exp(2*I*(f*x+
e))-exp(I*(f*x+e))+3)/f/(exp(2*I*(f*x+e))+1)^2*(exp(I*(f*x+e))+exp(2*I*(f*
x+e)))+8*I*c^2/(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(exp(I*
(f*x+e))+1)/(exp(I*(f*x+e))-1)*(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*(f*x+e))+1
))^(1/2)/f*ln(exp(I*(f*x+e))+1)-4*I*c^2/(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(
f*x+e))+1))^(1/2)*(exp(I*(f*x+e))+1)/(exp(I*(f*x+e))-1)*(c*(exp(I*(f*x+e))
-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)/f*ln(exp(2*I*(f*x+e))+1)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec{(fx+e)}+c)^{\frac{5}{2}}\sec{(fx+e)}}{\sqrt{a\sec{(fx+e)}+a}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sq
rt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(125) = 250$.

Time $= 0.22$ (sec) , antiderivative size $= 737$, normalized size of antiderivative $= 5.30$

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output

```
2*(c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - c^2*cos(4*f*x + 4*e)*sin(2*f*x
+ 2*e) - c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*
x + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*c
os(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^
2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin
(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e)
 + c^2)*cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) -
3*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 3*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x +
 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(c^2*cos(4
*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 3*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e)
+ c^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(
c)/((a*cos(4*f*x + 4*e)^2 + 4*a*cos(2*f*x + 2*e)^2 + a*sin(4*f*x + 4*e)^2
+ 4*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a*sin(2*f*x + 2*e)^2 + 2*(2*a*
cos(2*f*x + 2*e) + a)*cos(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*f)
```

Giac [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\left(\frac{2\sqrt{-ac}c^3\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{a|c|} - \frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}c^3+4\left(ct\right.}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}\right.}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
2*(2*sqrt(-a*c)*c^3*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a*abs(c)) - (3*(c*t
an(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*c^3 + 4*(c*tan(1/2*f*x + 1/2*e)^2
- c)*sqrt(-a*c)*c^4 + sqrt(-a*c)*c^5)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*a*
abs(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x
)
```

output
```
int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)+1}dx-2\left(\int \frac{\sqrt{\sec(fx}{}\right.\right.}{}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)
```

output

```
(sqrt(c)*sqrt(a)*c**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**3)/(sec(e + f*x) + 1),x) - 2*int((sqrt(sec(e + f*x) + 1)*
sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x) + 1),x) + int((sq
rt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)
 + 1),x)))/a
```

3.134 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 94

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output

```
-2*c^2*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e
))^(1/2)-c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.64

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^2(-2\log(1+\sec(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x
]],x]
```

output
```
(c^2*(-2*Log[1 + Sec[e + f*x]] + Sec[e + f*x])*Tan[e + f*x])/(f*Sqrt[a*(1
+ Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4443, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4443}$$

$$2c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$2c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4440}$$

$$-\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(-2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sq
rt[c - c*Sec[e + f*x]]) - (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqr
t[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.01

method	result
risch	$\frac{2ic\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,(2\ln(e^{i(fx+e)}+1)e^{3i(fx+e)}-\ln(e^{2i(fx+e)}+1)e^{3i(fx+e)}+2\,e^{i(fx+e)}\ln(e^{i(fx+e)}+1)-e^{i(fx+e)}\ln(e^{2i(fx+e)}+}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-}$
default	$-\frac{2((2\cos(\frac{fx}{2}+\frac{e}{2})^{2}-1)\ln(-\frac{2(\cos(\frac{fx}{2}+\frac{e}{2})+\sin(\frac{fx}{2}+\frac{e}{2}))}{\cos(\frac{fx}{2}+\frac{e}{2})+1})+(-4\cos(\frac{fx}{2}+\frac{e}{2})^{2}+2)\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2})+1)+(-}{}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2*I*c*(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(2*ln(exp(I*(f*x
+e))+1)*exp(3*I*(f*x+e))-ln(exp(2*I*(f*x+e))+1)*exp(3*I*(f*x+e))+2*exp(I*(
f*x+e))*ln(exp(I*(f*x+e))+1)-exp(I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)+2*exp(2
*I*(f*x+e))*ln(exp(I*(f*x+e))+1)-exp(2*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)-e
xp(I*(f*x+e))-exp(2*I*(f*x+e))+2*ln(exp(I*(f*x+e))+1)-ln(exp(2*I*(f*x+e))+
1))/(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)/(exp(I*(f*x+e))-1)
/f/(exp(2*I*(f*x+e))+1)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}\sec(fx+e)}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(-c*sec(f*x + e) + c)/sq
rt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c(\sec{(e+fx)}-1))^{\frac{3}{2}}\sec{(e+fx)}}{\sqrt{a\left(\sec{(e+fx)}+1\right)}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/sqrt(a*(sec(e + f*x)
+ 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(86)=172$.

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.94

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=$$

$$-\frac{2\left(c\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\sin\left(2fx+2e\right)-\left(c\cos\left(2fx+2e\right)^2+c\sin\left(2fx\right.\right.\right.}{}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
-2*(c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e
) - (c*cos(2*f*x + 2*e)^2 + c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) +
c)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(c*cos(2*f*x + 2*e)
^2 + c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 1) - (c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a*cos(2*f*x + 2*e)^2 +
a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*f)
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\left(\frac{\sqrt{-ac}c^2\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{a|c|}-\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}c^2+\sqrt{-ac}c^3}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)a|c|}\right.}{f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
2*(sqrt(-a*c)*c^2*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a*abs(c)) - ((c*tan(1
/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*c^2 + sqrt(-a*c)*c^3)/((c*tan(1/2*f*x +
1/2*e)^2 - c)*a*abs(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)
)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x
)
```

output
```
int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)+1}dx\right)+\int \frac{\sqrt{\sec(f}}{}\right.}{a}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x)
```

output

```
(sqrt(c)*sqrt(a)*c*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sq
rt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x)))/a
```

3.135 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 50

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=-\frac{c\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-c*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=-\frac{c\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]]
,x]
```

output
```
-((c*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqr
t[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \ {\color{blue}3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \ {\color{blue}4440}$$

$$-\frac{c\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

output

```
-((c*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[
c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\sqrt{2}\,(\ln(-\cot(fx+e)+\csc(fx+e)-1)+\ln(-\cot(fx+e)+\csc(fx+e)+1))\sqrt{-2c(-1+\sec(fx+e))}\,\sqrt{a(1+\sec(fx+e))}\,\cot(fx+e)}{2fa}$
risch	$\frac{2i\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,\ln\left(e^{i(fx+e)}+1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}-\frac{i\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,\ln\left(e^{2i(fx+e)}+1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
-1/2/f*2^(1/2)/a*(ln(-cot(f*x+e)+csc(f*x+e)-1)+ln(-cot(f*x+e)+csc(f*x+e)+1
))*(-2*c*(-1+sec(f*x+e)))^(1/2)*(a*(1+sec(f*x+e)))^(1/2)*cot(f*x+e)
```

Fricas [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{-c\sec\left(fx+e\right)+c}\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output

```
integral(sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a),
x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1
)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}}+\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}}}{f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
-(sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(-a) + sqrt(c)*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(-a))/f
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c^2\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sqrt{-ac}f|c|}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
-c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 - c))*sgn(tan(1/2*f*x + 1/2*e)^3 + t
an(1/2*f*x + 1/2*e))/(sqrt(-a*c)*f*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x
)
```

output
```
int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec
(e + f*x))/(sec(e + f*x) + 1),x))/a
```

3.136 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 47

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output

```
-arctanh(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\operatorname{arctanh}(\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
,x]
```

output
```
-((ArcTanh[Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[
c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3042, 4447, 25, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4447}$$
$$-\frac{\tan(e+fx)\int -\csc(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$
$$\Big\downarrow \text{25}$$
$$\frac{\tan(e+fx)\int \csc(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\tan(e+fx)\int \csc(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$
$$\Big\downarrow \text{4257}$$
$$-\frac{\tan(e+fx)\text{arctanh}(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
-((ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4447
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   In
t[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(43) = 86$.

Time = 2.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.77

method	result
default	$-\frac{\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right) \mathrm{c}}{f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}}$
risch	$-\frac{i\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,f}+\frac{i\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\ln\left(\mathrm{e}^{i(fx+e)}+1\right)}{\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/f*sin(1/2*f*x+1/2*e)*(ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)+ln(-
cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)-ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*
x+1/2*e)))*cos(1/2*f*x+1/2*e)/(2*cos(1/2*f*x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x
+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*si
n(1/2*f*x+1/2*e)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(43) = 86$.

Time = 0.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 4.51

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^2+\left(ac\cos(fx+e)^2+ac\right)\sin(fx+e)\right)}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right)}{2\,acf}, \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
[-1/2*sqrt(-a*c)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x
+ e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f),
sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2/(a*c*sin(f*x + e)))/(a*c*f
)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\int \frac{\sec{(e+fx)}}{\sqrt{a(\sec{(e+fx)}+1)}\sqrt{-c(\sec{(e+fx)}-1)}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) -
1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=-\frac{\arctan{(\sin{(fx+e)},\cos{(fx+e)}+1)}-\arctan{(\sin{(fx+e)},\cos{(fx+e)}-1)}}{\sqrt{a}\sqrt{c}f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
-(arctan2(sin(f*x + e), cos(f*x + e) + 1) - arctan2(sin(f*x + e), cos(f*x
+ e) - 1))/(sqrt(a)*sqrt(c)*f)
```

Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{c^2\left(\frac{\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c}-\frac{\log(|c|)}{c}\right)}{2\sqrt{-ac}f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
-1/2*c^2*(log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c - log(abs(c))/c)/(sqrt(-a*c
)*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2-1}dx\right)}{ac}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**2 - 1),x))/(a*c)
```

3.137 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 95

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} - \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-1/2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)-1/2*arctan
h(cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(\frac{\operatorname{arctanh}(\sec(e+fx))}{2c^2}+\frac{1}{2c^2(1-\sec(e+fx))}\right)\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)),x]
```

output

```
-((c*(ArcTanh[Sec[e + f*x]]/(2*c^2) + 1/(2*c^2*(1 - Sec[e + f*x])))*Tan[e
+ f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 4448, 3042, 4447, 25, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}}dx}{2c}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4447}$$

$$-\frac{\tan(e+fx)\int -\csc(e+fx)dx}{2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\int \csc(e+fx)dx}{2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\int \csc(e+fx)dx}{2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\tan(e+fx)\text{arctanh}(\cos(e+fx))}{2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
-1/2*Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2))
- (ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4447
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   In
t[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(83) = 166$.

Time = 2.07 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.32

method	result
default	$\frac{(-4\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2})-1)\sin(\frac{fx}{2}+\frac{e}{2})^2+4\sin(\frac{fx}{2}+\frac{e}{2})^2\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2}))-4\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+}{8f(2\cos(\frac{fx}{2}+\frac{e}{2})^2-1)\sqrt{\frac{a\cos(\frac{fx}{2}+\frac{e}{2})^2}{2\cos(\frac{fx}{2}+\frac{e}{2})^2-1}}\,c\sqrt{-\frac{c\sin(\frac{fx}{2}+\frac{e}{2})^2}{2\cos(\frac{fx}{2}+\frac{e}{2})^2-1}}}$
risch	$\frac{i(\ln(e^{i(fx+e)}+1)e^{3i(fx+e)}-\ln(e^{i(fx+e)}-1)e^{3i(fx+e)}-e^{2i(fx+e)}\ln(e^{i(fx+e)}+1)+e^{2i(fx+e)}\ln(e^{i(fx+e)}-1)-e^{i(fx+e)}\ln(e^{i(fx+e)}}{2c\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}(e^{2i(fx+e)}+1)(e^{i(fx+e)}-1)}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/f*(-4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*sin(1/2*f*x+1/2*e)^
2+4*sin(1/2*f*x+1/2*e)^2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e))-4*ln(-
cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)*sin(1/2*f*x+1/2*e)^2+cos(1/2*f*x+
1/2*e)^2+1)/(2*cos(1/2*f*x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1
/2*f*x+1/2*e)^2)^(1/2)/c/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)
^2)^(1/2)*cot(1/2*f*x+1/2*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(83) = 166$.

Time = 0.20 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.11

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \Bigg[-\frac{\sqrt{-ac}(\cos(fx+e)-1)\log\Bigg(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\right.}{}}{}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
[-1/4*(sqrt(-a*c)*(cos(f*x + e) - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x +
e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(
f*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a*c^2*f*cos(f*x + e) - a
*c^2*f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) - 1)*arctan(sqrt(a*c)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
+ e))*cos(f*x + e)^2/(a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
)/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\,(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))*
*(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(83) = 166$.

Time = 0.20 (sec) , antiderivative size = 406, normalized size of antiderivative = 4.27

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{((2(2\cos(fx+e)-1)\cos(2fx+2e)-\cos(2fx+2}{}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
1/2*((2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos
(f*x + e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin
(f*x + e)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1)
- (2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*
x + e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*
x + e)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2
*cos(f*x + e)*sin(2*f*x + 2*e) - 2*cos(2*f*x + 2*e)*sin(f*x + e) - 2*sin(f
*x + e))*sqrt(a)*sqrt(c)/((a*c^2*cos(2*f*x + 2*e)^2 + 4*a*c^2*cos(f*x + e)
^2 + a*c^2*sin(2*f*x + 2*e)^2 - 4*a*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*
a*c^2*sin(f*x + e)^2 - 4*a*c^2*cos(f*x + e) + a*c^2 - 2*(2*a*c^2*cos(f*x +
 e) - a*c^2)*cos(2*f*x + 2*e))*f)
```

Giac [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{c^2\left(\frac{\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c^2}-\frac{\log(|c|)}{c^2}-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}\right)}{4\sqrt{-ac}f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
-1/4*c^2*(log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c^2 - log(abs(c))/c^2 - (c*ta
n(1/2*f*x + 1/2*e)^2 - c)/(c^3*tan(1/2*f*x + 1/2*e)^2))/(sqrt(-a*c)*f*abs(
c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^{3}-\sec(fx+e)^{2}-\sec(fx+e)+1}dx\right)}{a\,c^{2}}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec
(e + f*x))/(sec(e + f*x)**3 - sec(e + f*x)**2 - sec(e + f*x) + 1),x))/(a*c
**2)
```

3.138 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 140

$$\begin{aligned}
\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx &= \\
&-\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} \\
&-\frac{\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} \\
&-\frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{4c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

output

```
-1/4*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2)-1/4*tan(f*
x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)-1/4*arctanh(cos(f*x
+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(2+\operatorname{arctanh}(\sec(e+fx))(-1+\sec(e+fx))^2-\sec(e+fx))\tan(e+fx)}{4c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
)),x]
```

output

```
-1/4*((2 + ArcTanh[Sec[e + f*x]]*(-1 + Sec[e + f*x])^2 - Sec[e + f*x])*Tan
[e + f*x])/(c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
- c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4448, 3042, 4448, 3042, 4447, 25, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{3/2}}dx}{2c}-\frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2c}-\frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 4448$$

$$\frac{\frac{\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}}dx}{2c}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{2c}-\frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{2c}-\frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 4447$$

$$\frac{-\frac{\tan(e+fx)\int -\csc(e+fx)dx}{2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{2c}-\frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\tan(e+fx)\int \csc(e+fx)dx}{2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{2c}-\frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\tan(e+fx)\int \csc(e+fx)dx}{2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{2c}-\frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 4257$$

$$\frac{-\frac{\tan(e+fx)\operatorname{arctanh}(\cos(e+fx))}{2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{2c}-\frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

input

```
Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

output

```
-1/4*Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2))
+ (-1/2*Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2
)) - (ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c - c*Sec[e + f*x]]))/(2*c)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4447

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   In
t[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(122) = 244$.

Time = 2.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.76

method	result
default	$\frac{(32\sin(\frac{fx}{2}+\frac{e}{2})^4\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2}))-32\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2})-1)\sin(\frac{fx}{2}+\frac{e}{2})^4-32\ln(-\cot(\frac{fx}{2}+\frac{e}{2})}{128f(2\cos(\frac{fx}{2}+\frac{e}{2})^2-1)\sqrt{\frac{a\cos(\frac{fx}{2}+\frac{e}{2})^2}{2\cos(\frac{fx}{2}+\frac{e}{2})^2-1}}\sqrt{-}}$
risch	$\frac{i(3\,\mathrm{e}^{2i(fx+e)}-4\,\mathrm{e}^{i(fx+e)}+3)(\mathrm{e}^{i(fx+e)}+\mathrm{e}^{2i(fx+e)})}{2c^2\sqrt{\frac{a(\mathrm{e}^{i(fx+e)}+1)^2}{\mathrm{e}^{2i(fx+e)}+1}}(\mathrm{e}^{i(fx+e)}-1)^3\sqrt{\frac{c(\mathrm{e}^{i(fx+e)}-1)^2}{\mathrm{e}^{2i(fx+e)}+1}}(\mathrm{e}^{2i(fx+e)}+1)f}-\frac{i(\mathrm{e}^{i(fx+e)}+1)(\mathrm{e}^{i(fx+e)}-1)\ln(\mathrm{e}^{i(fx+e)}-1)}{4c^2\sqrt{\frac{a(\mathrm{e}^{i(fx+e)}+1)^2}{\mathrm{e}^{2i(fx+e)}+1}}\sqrt{\frac{c(\mathrm{e}^{i(fx+e)}-1)^2}{\mathrm{e}^{2i(fx+e)}+1}}(\mathrm{e}^{2i(fx+e)}+}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/128/f*(32*sin(1/2*f*x+1/2*e)^4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)
)-32*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*sin(1/2*f*x+1/2*e)^4-32*
ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)*sin(1/2*f*x+1/2*e)^4-21*cos(1
/2*f*x+1/2*e)^4-6*cos(1/2*f*x+1/2*e)^2+19)/(2*cos(1/2*f*x+1/2*e)^2-1)/(a/(
2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1
/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^2*cot(1/2*f*x+1/2*e)*csc(1/2*f*x+
1/2*e)^2
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.31

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2-2\,\cos\left(fx+e\right)+1\right)\log\left(-\frac{4(}{}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
[-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*
c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f
*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 - 2*cos(f
*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*
f)*sin(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*arc
tan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))*cos(f*x + e)^2/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*c
os(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*
f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))*
*(5/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(122) = 244$.

Time = 0.27 (sec) , antiderivative size = 1201, normalized size of antiderivative = 8.58

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output

```
1/4*((2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos
(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e
) + 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) - 1)*
cos(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(3*
f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(4
*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e)
- 16*sin(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(
f*x + e) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), c
os(f*x + e) + 1) - (2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x
 + e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) -
 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(
f*x + e) - 1)*cos(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2
 + 4*(2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x
+ 4*e) - sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin
(3*f*x + 3*e) - 16*sin(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f
*x + 2*e)*sin(f*x + e) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(s
in(f*x + e), cos(f*x + e) - 1) - 2*(3*sin(3*f*x + 3*e) - 4*sin(2*f*x + 2*e
) + 3*sin(f*x + e))*cos(4*f*x + 4*e) + 2*(3*cos(3*f*x + 3*e) - 4*cos(2*f*x
 + 2*e) + 3*cos(f*x + e))*sin(4*f*x + 4*e) - 2*(2*cos(2*f*x + 2*e) + 3)*si
n(3*f*x + 3*e) + 4*(cos(f*x + e) + 2)*sin(2*f*x + 2*e) + 4*cos(3*f*x + ...
```

Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\frac{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2+2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4} - 2\,\log\left(|c|\tan\right.}{16\,\sqrt{-ac}cf|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\right.}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
1/16*((3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 2*(c*tan(1/2*f*x + 1/2*e)^2 -
c)*c)/(c^2*tan(1/2*f*x + 1/2*e)^4) - 2*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)
+ 2*log(abs(c)))/(sqrt(-a*c)*c*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1
/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^4-2\sec(fx+e)^3+2\sec(fx+e)-1}dx\right)}{a\,c^3}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x) - 1),x
))/(a*c**3)
```

3.139 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 142

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

output

```
4*c^3*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)+2*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/
2)+c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-4c^2\log(1+\sec(e+fx))+c^2\sec(e+fx)-\frac{4c^2}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(
3/2),x]
```

output
```
-((c*(-4*c^2*Log[1 + Sec[e + f*x]] + c^2*Sec[e + f*x] - (4*c^2)/(1 + Sec[e
 + f*x]))*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
 f*x]]))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4442, 3042, 4443, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)^{3/2}}-\frac{2c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{\sec(e+fx)a+a}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)^{3/2}}-\frac{2c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}$$

$$\Big\downarrow \text{4443}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)^{3/2}}-\frac{2c\left(2c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)^{3/2}}-\frac{2c\left(2c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)}{a}$$

$$\Big\downarrow \text{4440}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)^{3/2}}-\frac{2c\left(-\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)}{a}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(3/2),x
]
```

output
```
(c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2))
 - (2*c*((-2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(f*Sqrt[a + a*Sec[e + f*x]])))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(130) = 260$.

Time = 2.40 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.52

method	result
default	$2\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{2}\left(\left(4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-2\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-2\right)\ln\left(-\frac{2(\right.\right.$
risch	$\frac{2ic^{2}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(5\,\mathrm{e}^{3i(fx+e)}+2\,\mathrm{e}^{2i(fx+e)}+5\,\mathrm{e}^{i(fx+e)}\right)}{a\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)f\left(\mathrm{e}^{2i(fx+e)}+1\right)}-\frac{8ic^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\ln\left(\mathrm{e}^{i(fx+e)}+1\right)}{a\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)f}+\frac{4ic^{2}(}{}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2/f*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^2/(2*cos(
1/2*f*x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1
/2)/a*((4*cos(1/2*f*x+1/2*e)^2-2)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/
2*e))/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*e)+(4*cos(1/2*f*x+1/2*e)^2-2
)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*co
t(1/2*f*x+1/2*e)+ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*(-8*cos(1/2*
f*x+1/2*e)^2*cot(1/2*f*x+1/2*e)+4*cot(1/2*f*x+1/2*e))+ln(-cot(1/2*f*x+1/2*
e)+csc(1/2*f*x+1/2*e)+1)*(-8*cos(1/2*f*x+1/2*e)^2*cot(1/2*f*x+1/2*e)+4*cot
(1/2*f*x+1/2*e))+(3*cos(1/2*f*x+1/2*e)^2-1)*tan(1/2*f*x+1/2*e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a
^2*sec(f*x + e) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2035 vs. $2(130) = 260$.

Time $= 0.31$ (sec) , antiderivative size $= 2035$, normalized size of antiderivative $= 14.33$

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output

```
-2*(8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 - 2*c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*c^2*cos(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 2*c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*c
os(2*f*x + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(
2*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)
^2 + 4*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4
*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2
 + 4*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(2*f*x + 2
*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e) + 4*(c^2*cos
(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + 2*c^2*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + c^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(...
```

Giac [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{2\left(\frac{2\sqrt{-ac}c^3\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{a^2|c|}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}c^2}{a^2|c|}-\frac{2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}c^3+\sqrt{-ac}c^4}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)a^2|c|}\right)\operatorname{sgn}\Big(\tan\big(}{f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
-2*(2*sqrt(-a*c)*c^3*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a^2*abs(c)) + (c*t
an(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*c^2/(a^2*abs(c)) - (2*(c*tan(1/2*f*x
 + 1/2*e)^2 - c)*sqrt(-a*c)*c^3 + sqrt(-a*c)*c^4)/((c*tan(1/2*f*x + 1/2*e)
^2 - c)*a^2*abs(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x
)
```

output
```
int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^2+2\sec(fx+e)+1}dx - 2\left(\int \frac{\sqrt{\sec(fx}{}\right.\right.}{}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**3)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) - 2*int((sqr
t(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*
x)**2 + 2*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e
 + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)))/a**
2
```

3.140 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 95

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{c^2\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

output
```
c^2*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=-\frac{c\left(-c\log(1+\sec(e+fx))-\frac{2c}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(
3/2),x]
```

output
```
-((c*(-(c*Log[1 + Sec[e + f*x]]) - (2*c)/(1 + Sec[e + f*x]))*Tan[e + f*x])
/(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4442, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}-\frac{c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}-\frac{c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}$$

$$\Big\downarrow \text{4440}$$

$$\frac{c^2\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}$$

input

```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(3/2),x
]
```

output

```
(c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) + (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a +
 a*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4442

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.42 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.41

method	result
risch	$-\frac{ic\sqrt{\frac{c(\mathrm{e}^{i(fx+e)}-1)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,(2\,\mathrm{e}^{2i(fx+e)}\ln(\mathrm{e}^{i(fx+e)}+1)-\mathrm{e}^{2i(fx+e)}\ln(\mathrm{e}^{2i(fx+e)}+1)+4\,\mathrm{e}^{i(fx+e)}\ln(\mathrm{e}^{i(fx+e)}+1)-2\,\mathrm{e}^{i(fx+e)}\ln(\mathrm{e}^{2i(fx+}}{a(\mathrm{e}^{i(fx+e)}+1)\sqrt{\frac{a(\mathrm{e}^{i(fx+e)}+1)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,(\mathrm{e}^{i(fx+e)}-1)f}$
default	$-\frac{\sqrt{-\frac{c\sin(\frac{fx}{2}+\frac{e}{2})^{2}}{2\cos(\frac{fx}{2}+\frac{e}{2})^{2}-1}}\,c\,(2\cot(\frac{fx}{2}+\frac{e}{2})\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2})-1)+2\cot(\frac{fx}{2}+\frac{e}{2})\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2})+1}{f\sqrt{\frac{a\cos(\frac{fx}{2}+}{2\cos(\frac{fx}{2}+\frac{e}{2}}}}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-I*c*(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(2*exp(2*I*(f*x+e
))*ln(exp(I*(f*x+e))+1)-exp(2*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)+4*exp(I*(f
*x+e))*ln(exp(I*(f*x+e))+1)-2*exp(I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)-4*exp(
I*(f*x+e))+2*ln(exp(I*(f*x+e))+1)-ln(exp(2*I*(f*x+e))+1))/a/(exp(I*(f*x+e)
)+1)/(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)/(exp(I*(f*x+e))-1
)/f
```

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec{(fx+e)}+c)^{\frac{3}{2}}\sec{(fx+e)}}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqr
t(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output

```
Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/(a*(sec(e + f*x) + 1)
)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}a} + \frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}a} + \frac{c^{\frac{3}{2}}\sin(fx+e)^2}{\sqrt{-a}a(\cos(fx+e)+1)^2}}{f}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output

```
(c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a) + c^(3/2)*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a) + c^(3/2)*sin(f*x + e
)^2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + c\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)-c\right)c^2\mathrm{sgn}}{\sqrt{-ac}af|c|}$$

input

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output

```
(c*tan(1/2*f*x + 1/2*e)^2 + c*log(c*tan(1/2*f*x + 1/2*e)^2 - c) - c)*c^2*s
gn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*a*f*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input

```
int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x
)
```

output

```
int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)+\int \frac{\sqrt{\sec(f}}{}}{a^2}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x)
```

output

```
(sqrt(c)*sqrt(a)*c*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) + int((sqrt(
sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2
 + 2*sec(e + f*x) + 1),x)))/a**2
```

3.141 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

output
```
1/2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\csc(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/
2),x]
```

output
```
(Csc[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4438}$$

$$\frac{\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{2f(a\sec(e+fx)+a)^{3/2}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(36) = 72$.

Time = 3.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-\frac{2c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4af\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}}$	86
risch	$\frac{2i\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,e^{i(fx+e)}}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}$	105

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/4*2^(1/2)/a/f*(-2*c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*tan(1/2*f*x+1
/2*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos\left(fx+e\right)}{\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
+ e))*cos(f*x + e)/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/(a*(sec(e + f*x) + 1))**
(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{2\sqrt{-a}af}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
1/2*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(cos(f*x +
 e) + 1) - 1)/(sqrt(-a)*a*f)
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c}{2\sqrt{-ac}af|c|}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
1/2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c/(sqrt(-a*c)*a*f*abs(c))
```

Mupad [B] (verification not implemented)

Time = 11.70 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{a\,f\,\sin\left(e+f\,x\right)\,\sqrt{\frac{a\,\left(\cos(e+f\,x)+1\right)}{\cos(e+f\,x)}}}$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x
)
```

output
```
(c - c/cos(e + f*x))^(1/2)/(a*f*sin(e + f*x)*((a*(cos(e + f*x) + 1))/cos(e
 + f*x))^(1/2))
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec
(e + f*x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x))/a**2
```

3.142 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
1/2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)-1/2*arctanh
(cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(-1+\operatorname{arctanh}(\sec(e+fx))(1+\sec(e+fx)))\tan(e+fx)}{2f(a(1+\sec(e+fx)))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]
]),x]
```

output
```
-1/2*((-1 + ArcTanh[Sec[e + f*x]]*(1 + Sec[e + f*x]))*Tan[e + f*x])/(f*(a*
(1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3042, 4448, 3042, 4447, 25, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}}dx}{2a}+\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4447}$$

$$\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\int -\csc(e+fx)dx}{2a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\int \csc(e+fx)dx}{2a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\int\csc(e+fx)dx}{2a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 4257$$

$$\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\text{arctanh}(\cos(e+fx))}{2af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
Tan[e + f*x]/(2*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (
ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4447
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   In
t[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(83) = 166$.

Time = 2.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.31

method	result
default	$-\frac{\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln}{4f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}}$
risch	$\frac{i\left(\ln\left(e^{i(fx+e)}+1\right)e^{3i(fx+e)}-\ln\left(e^{i(fx+e)}-1\right)e^{3i(fx+e)}+e^{2i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)-e^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-e^{i(fx+e)}\ln\left(e^{i(fx+e)}\right.\right.}{2a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/4/f*(2*cos(1/2*f*x+1/2*e)^2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1
)+2*cos(1/2*f*x+1/2*e)^2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)-2*co
s(1/2*f*x+1/2*e)^2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e))+sin(1/2*f*x+
1/2*e)^2)/(2*cos(1/2*f*x+1/2*e)^2-1)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^
(1/2)/a*tan(1/2*f*x+1/2*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(83) = 166$.

Time = 0.19 (sec) , antiderivative size = 388, normalized size of antiderivative = 4.08

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{\sqrt{-ac}(\cos(fx+e)+1)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\right.}{}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
[-1/4*(sqrt(-a*c)*(cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x +
e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(
f*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a^2*c*f*cos(f*x + e) + a
^2*c*f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) + 1)*arctan(sqrt(a*c)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
+ e))*cos(f*x + e)^2/(a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
)/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x)
 - 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(83) = 166$.

Time = 0.21 (sec) , antiderivative size = 397, normalized size of antiderivative = 4.18

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx =$$

$$-\frac{((2\,(2\,\cos\left(fx+e\right)+1)\cos\left(2\,fx+2\,e\right)+\cos\left(2\,fx+2\,e\right)^2+4\,\cos\left(fx+e\right)^2+\sin\left(2\,fx+2\,e\right)^2+4\,\sin}{}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output

```
-1/2*((2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*co
s(f*x + e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*si
n(f*x + e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1)
 - (2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f
*x + e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f
*x + e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) -
2*cos(f*x + e)*sin(2*f*x + 2*e) + 2*cos(2*f*x + 2*e)*sin(f*x + e) + 2*sin(
f*x + e))*sqrt(a)*sqrt(c)/((a^2*c*cos(2*f*x + 2*e)^2 + 4*a^2*c*cos(f*x + e
)^2 + a^2*c*sin(2*f*x + 2*e)^2 + 4*a^2*c*sin(2*f*x + 2*e)*sin(f*x + e) + 4
*a^2*c*sin(f*x + e)^2 + 4*a^2*c*cos(f*x + e) + a^2*c + 2*(2*a^2*c*cos(f*x
+ e) + a^2*c)*cos(2*f*x + 2*e))*f)
```

Giac [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{c^2\left(\frac{\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c}-\frac{\log(|c|)}{c}-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}{c^2}\right)}{4\,\sqrt{-ac}af|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
-1/4*c^2*(log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c - log(abs(c))/c - (c*tan(1/
2*f*x + 1/2*e)^2 - c)/c^2)/(sqrt(-a*c)*a*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)
^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^{3}+\sec(fx+e)^{2}-\sec(fx+e)-1}dx\right)}{a^{2}c}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) - 1),x))/(
a**2*c)
```

3.143 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 104

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
1/2*csc(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*arc
tanh(cos(f*x+e))*tan(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\csc(e+fx)-\operatorname{arctanh}(\sec(e+fx))\tan(e+fx)}{2acf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3
/2)),x]
```

output
```
(Csc[e + f*x] - ArcTanh[Sec[e + f*x]]*Tan[e + f*x])/(2*a*c*f*Sqrt[a*(1 + S
ec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 4447, 25, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4447}$$

$$\frac{\tan(e+fx)\int -\cot^2(e+fx)\csc(e+fx)dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(e+fx)\int \cot^2(e+fx)\csc(e+fx)dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tan(e+fx)\int \sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)^2dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3091}$$

$$-\frac{\tan(e+fx)\left(-\frac{1}{2}\int \csc(e+fx)dx-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tan(e+fx)\left(-\frac{1}{2}\int\csc(e+fx)dx-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\tan(e+fx)\left(\frac{\text{arctanh}(\cos(e+fx))}{2f}-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x
]
```

output
```
-(((ArcTanh[Cos[e + f*x]]/(2*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*f))*Tan[e
 + f*x])/(a*c*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4447
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   In
t[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{i(e^{4i(fx+e)}\ln(e^{i(fx+e)}-1)-e^{4i(fx+e)}\ln(e^{i(fx+e)}+1)-2\,e^{2i(fx+e)}\ln(e^{i(fx+e)}-1)+2\,e^{2i(fx+e)}\ln(e^{i(fx+e)}+1)-2\,e^{3i(fx+e)}-2}{2ac(e^{i(fx+e)}+1)\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f(e^{2i(fx+e)}+1}$
default	$-\frac{(8\cos(\frac{fx}{2}+\frac{e}{2})^2\sin(\frac{fx}{2}+\frac{e}{2})^2\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2})+1)-8\cos(\frac{fx}{2}+\frac{e}{2})^2\sin(\frac{fx}{2}+\frac{e}{2})^2\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+}{16f(2\cos(\frac{fx}{2}+\frac{e}{2})^2-1)\sqrt{-}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*I*(exp(4*I*(f*x+e))*ln(exp(I*(f*x+e))-1)-exp(4*I*(f*x+e))*ln(exp(I*(f
*x+e))+1)-2*exp(2*I*(f*x+e))*ln(exp(I*(f*x+e))-1)+2*exp(2*I*(f*x+e))*ln(ex
p(I*(f*x+e))+1)-2*exp(3*I*(f*x+e))-2*exp(I*(f*x+e))+ln(exp(I*(f*x+e))-1)-l
n(exp(I*(f*x+e))+1))/a/c/(exp(I*(f*x+e))+1)/(a*(exp(I*(f*x+e))+1)^2/(exp(2
*I*(f*x+e))+1))^(1/2)/(exp(I*(f*x+e))-1)/(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*
(f*x+e))+1))^(1/2)/f/(exp(2*I*(f*x+e))+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(92) = 184$.

Time = 0.20 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.94

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e}{\cos(fx+}}}{}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
[-1/4*(sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x
+ e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*si
n(f*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2)/((a^2*c^2*f*cos(f*x +
e)^2 - a^2*c^2*f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e)^2 - 1)*arcta
n(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
 c)/cos(f*x + e))*cos(f*x + e)^2/(a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*
cos(f*x + e)^2)/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1
))**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(92) = 184$.

Time = 0.20 (sec) , antiderivative size = 567, normalized size of antiderivative = 5.45

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output

```
1/2*((2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4
*cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(f*x + e)
, cos(f*x + e) + 1) - (2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4
*f*x + 4*e)^2 - 4*cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arc
tan2(sin(f*x + e), cos(f*x + e) - 1) - 2*(sin(3*f*x + 3*e) + sin(f*x + e))
*cos(4*f*x + 4*e) + 2*(cos(3*f*x + 3*e) + cos(f*x + e))*sin(4*f*x + 4*e) +
 2*(2*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - 4*cos(3*f*x + 3*e)*sin(2*f*
x + 2*e) - 4*cos(f*x + e)*sin(2*f*x + 2*e) + 4*cos(2*f*x + 2*e)*sin(f*x +
e) - 2*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*
c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 4*a^2*c^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f
*x + 2*e) + a^2*c^2 - 2*(2*a^2*c^2*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x +
 4*e))*f)
```

Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.26

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{c^2 \Bigg(\frac{2\log(|c|)-1}{c^2} - \frac{2\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c^2} + \frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 -}{c^3}}{8\sqrt{-ac}\,af|c|\mathrm{sgn}\Big(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + \tan\big(}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
1/8*c^2*((2*log(abs(c)) - 1)/c^2 - 2*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c^
2 + (c*tan(1/2*f*x + 1/2*e)^2 - c)/c^3 + (2*c*tan(1/2*f*x + 1/2*e)^2 - c)/
(c^3*tan(1/2*f*x + 1/2*e)^2))/(sqrt(-a*c)*a*f*abs(c)*sgn(tan(1/2*f*x + 1/2
*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^4-2\sec(fx+e)^2+1}dx\right)}{a^2c^2}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec
(e + f*x))/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1),x))/(a**2*c**2)
```

3.144 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 146

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx=\frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}-\frac{3\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output

```
3/8*csc(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4*t
an(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2)-3/8*arctanh(cos(
f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx=-\frac{(2+3\sec(e+fx)-3\sec^2(e+fx)+3\operatorname{arctanh}(\sec(e+fx))(-1+\sec(e+fx))^2(1+\sec(e+fx)))\tan(}{8c^2f(-1+\sec(e+fx))^2(a(1+\sec(e+fx)))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5
/2)),x]
```

output
```
-1/8*((2 + 3*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 3*ArcTanh[Sec[e + f*x]]*(-1
 + Sec[e + f*x])^2*(1 + Sec[e + f*x]))*Tan[e + f*x])/(c^2*f*(-1 + Sec[e +
f*x])^2*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4448, 3042, 4447, 25, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{3\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)^{3/2}(c-c\sec(e+fx))^{3/2}}dx}{4c}-\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4c}-\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4447}$$

$$\frac{3\tan(e+fx)\int -\cot^2(e+fx)\csc(e+fx)dx}{4ac^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\tan(e+fx)\int\cot^2(e+fx)\csc(e+fx)dx}{4ac^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow\ 3042$$

$$-\frac{3\tan(e+fx)\int\sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)^2dx}{4ac^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow\ 3091$$

$$-\frac{3\tan(e+fx)\left(-\frac{1}{2}\int\csc(e+fx)dx-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)}{4ac^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow\ 3042$$

$$-\frac{3\tan(e+fx)\left(-\frac{1}{2}\int\csc(e+fx)dx-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)}{4ac^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow\ 4257$$

$$-\frac{3\tan(e+fx)\left(\frac{\text{arctanh}(\cos(e+fx))}{2f}-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)}{4ac^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x
]
```

output
```
-1/4*Tan[e + f*x]/(f*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)
) - (3*(ArcTanh[Cos[e + f*x]]/(2*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*f))*T
an[e + f*x])/(4*a*c^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4447
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   In
t[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(128) = 256$.

Time = 2.44 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.02

method	result
default	$\frac{(96\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2}))\sin(\frac{fx}{2}+\frac{e}{2})^4\cos(\frac{fx}{2}+\frac{e}{2})^2-96\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2})-1)\sin(\frac{fx}{2}+\frac{e}{2})^4\cos(\frac{fx}{2}+}{256f(2\cos(\frac{fx}{2}+\frac{e}{2})}$
risch	$\frac{i(5\,e^{5i(fx+e)}-2\,e^{4i(fx+e)}+2\,e^{3i(fx+e)}-2\,e^{2i(fx+e)}+5\,e^{i(fx+e)})}{4a\,c^2(e^{2i(fx+e)}+1)(e^{i(fx+e)}+1)\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)^3\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{3i(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)\ln}{8a\,c^2(e^{2i(fx+e)}+1)\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/256/f*(96*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^
4*cos(1/2*f*x+1/2*e)^2-96*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*sin
(1/2*f*x+1/2*e)^4*cos(1/2*f*x+1/2*e)^2-96*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f
*x+1/2*e)+1)*sin(1/2*f*x+1/2*e)^4*cos(1/2*f*x+1/2*e)^2-13*cos(1/2*f*x+1/2*
e)^6-54*cos(1/2*f*x+1/2*e)^4+75*cos(1/2*f*x+1/2*e)^2-16)/(2*cos(1/2*f*x+1/
2*e)^2-1)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/(a/(2
*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a/c^2*sec(1/2*f*x+1/2
*e)*csc(1/2*f*x+1/2*e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(128) = 256$.

Time = 0.21 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \Bigg[-\frac{3\,(\cos(fx+e)^3-\cos(fx+e)^2-\cos(fx+e)+}{}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
[-1/16*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a*c)*
log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f
*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin
(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*
x + e)^3 - cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e
)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f
*x + e)), 1/8*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt
(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2/(a*c*sin(f*x + e)))*sin(f*x +
e) + (5*cos(f*x + e)^3 - cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3
*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^
2*c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\frac{2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)}{c}+\frac{9\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}+12\bigl(c\tan\bigl(\frac{1}{2}\,fx}{c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}}{32\sqrt{-ac}acf|c|\mathrm{sgn}\bigl(\tan\bigl(}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
1/32*(2*(c*tan(1/2*f*x + 1/2*e)^2 - c)/c + (9*(c*tan(1/2*f*x + 1/2*e)^2 -
c)^2 + 12*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 4*c^2)/(c^2*tan(1/2*f*x + 1/2
*e)^4) - 6*log(abs(c)*tan(1/2*f*x + 1/2*e)^2) + 6*log(abs(c)) - 4)/(sqrt(-
a*c)*a*c*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^{5}-\sec(fx+e)^{4}-2\sec(fx+e)^{3}+2\sec(fx+e)^{2}+\sec(fx+e)-1}dx\right)}{a^{2}c^{3}}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*s
ec(e + f*x)**2 + sec(e + f*x) - 1),x))/(a**2*c**3)
```

3.145 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^3\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}}$$

output
```
-c^3*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x
+e))^(1/2)-c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/2
)+1/2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c\left(c^2\log(1+\sec(e+fx)) - \frac{2c^2}{(1+\sec(e+fx))^2} + \frac{4c^2}{1+\sec(e+fx)}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(
5/2),x]
```

output
```
-((c*(c^2*Log[1 + Sec[e + f*x]] - (2*c^2)/(1 + Sec[e + f*x])^2 + (4*c^2)/(
1 + Sec[e + f*x]))*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
- c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4442, 3042, 4442, 3042, 4440}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4442}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}-\frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(\sec(e+fx)a+a)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}-\frac{c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{4442}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}-\frac{c\left(\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}-\frac{c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}-\frac{c\left(\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}-\frac{c\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{4440}$$

$$\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}-\frac{c\left(\frac{c^2\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}\right)}{a}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(5/2),x
]
```

output
```
(c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2
)) - (c*((c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x]
)/(f*(a + a*Sec[e + f*x])^(3/2))))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4440
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

rule 4442
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Simp[d*((2*n - 1)/(b*(2*m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*
x])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[
m, -2^(-1)]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(131) = 262$.

Time = 2.52 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.88

method	result
default	$-\frac{c^{2} \sqrt{-\frac{c \sin\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}}\, \left(2 \ln\left(-\frac{2 \left(\cos\left(\frac{f x}{2}+\frac{e}{2}\right)+\sin\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{f x}{2}+\frac{e}{2}\right)+1}\right) \cot\left(\frac{f x}{2}+\frac{e}{2}\right)-4 \cot\left(\frac{f x}{2}+\frac{e}{2}\right) \ln\left(-\cot\left(\frac{f x}{2}+\frac{e}{2}\right)+\csc\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right.\right.}{2}$
risch	$-\frac{8 i c^{2} \sqrt{\frac{c \left(e^{i (f x+e)}-1\right)^{2}}{e^{2 i (f x+e)}+1}}\, e^{2 i (f x+e)}}{a^{2} \left(e^{i (f x+e)}+1\right)^{3} \sqrt{\frac{a \left(e^{i (f x+e)}+1\right)^{2}}{e^{2 i (f x+e)}+1}}\, \left(e^{i (f x+e)}-1\right) f}+\frac{2 i c^{2} \left(e^{i (f x+e)}+1\right) \sqrt{\frac{c \left(e^{i (f x+e)}-1\right)^{2}}{e^{2 i (f x+e)}+1}}\, \ln\left(e^{i (f x+e)}+1\right)}{a^{2} \sqrt{\frac{a \left(e^{i (f x+e)}+1\right)^{2}}{e^{2 i (f x+e)}+1}}\, \left(e^{i (f x+e)}-1\right) f}-\frac{i c^{2} \left(e^{i (f x+e)}+\right.}{a^{2} \sqrt{a}}$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2/f*c^2*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/(a/(
2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a^2*(2*ln(-2*(cos(1/
2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*e
)-4*cot(1/2*f*x+1/2*e)*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)-4*cot(
1/2*f*x+1/2*e)*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)+2*ln(-2*(cos(1
/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*
e)-cot(1/2*f*x+1/2*e)+sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a
^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}a^{2}} + \frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}a^{2}} - \frac{\frac{2\,\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}} + \frac{\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}}{a^{3}}}{2\,f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
-1/2*(2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a^2) +
2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a^2) - (2*sqr
t(-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(-a)*c^(5/2)*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.73

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(2\,c^2\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)+\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2+4\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c\right)c^2\mathrm{sgn}(}{2\sqrt{-ac}a^2f|c|}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
-1/2*(2*c^2*log(c*tan(1/2*f*x + 1/2*e)^2 - c) + (c*tan(1/2*f*x + 1/2*e)^2
- c)^2 + 4*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c)*c^2*sgn(tan(1/2*f*x + 1/2*e)^
3 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*a^2*f*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x
)
```

output
```
int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx - 2\left(\int \frac{\sqrt{\sec(fx}}{\sec(fx-}\right.\right.}{}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**3)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1),x) - 2*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e +
f*x)**2)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) + i
nt((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e
+ f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)))/a**3
```

3.146 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

output
```
1/4*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c(-1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{4f(a(1+\sec(e+fx)))^{5/2}}$$

input
```
Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(
5/2),x]
```

output
```
-1/4*(c*(-1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a*(
1 + Sec[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a\sec(e+fx)+a)^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$
$$\Big\downarrow \text{4438}$$
$$\frac{\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{4f(a\sec(e+fx)+a)^{5/2}}$$

input
```
Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(5/2),x
]
```

output
```
((c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(36) = 72$.

Time = 2.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.05

method	result	size
default	$-\frac{c\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{4f\,a^{2}\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}}$	86
risch	$\frac{2ic\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{a^{2}\left(e^{i(fx+e)}+1\right)^{3}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}$	116

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/4/f*c*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a^2/(a
/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*tan(1/2*f*x+1/2*e)
^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(36) = 72$.

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos\left(fx+e\right)^{2}}{\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))*cos(f*x + e)^2/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3
*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(-c(\sec\left(e+fx\right)-1)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/(a*(sec(e + f*x) + 1)
)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.33

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{-a}c^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\sin\left(fx+e\right)^4}{4\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)f(\cos\left(fx+e\right)+1)^4}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
-1/4*sqrt(-a)*c^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)*sin(f*x + e)^4/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2)*f*(cos(f*x + e) + 1)^4)
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c\right)c}{4\sqrt{-ac}a^2f|c|}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
-1/4*((c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*
c)*c/(sqrt(-a*c)*a^2*f*abs(c))
```

Mupad [B] (verification not implemented)

Time = 12.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{2\,c\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,(\sin\left(e+f\,x\right)+2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right))}{a^2\,f\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,(4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(e+f\,x\right)+4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5)}$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x
)
```

output
```
-(2*c*((c*(cos(e + f*x) - 1))/cos(e + f*x))^(1/2)*(sin(e + f*x) + 2*sin(2*
e + 2*f*x) + sin(3*e + 3*f*x)))/(a^2*f*((a*(cos(e + f*x) + 1))/cos(e + f*x
))^(1/2)*(4*cos(2*e + 2*f*x) - 4*cos(e + f*x) + 4*cos(3*e + 3*f*x) + cos(4
*e + 4*f*x) - 5))
```

Reduce [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)+\int \frac{\sqrt{\sec(f}}{\sec(fx}\right.}{a^3}$$

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*
x))/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)))/a**3
```

3.147 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

output
```
1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{(1+2\cos(e+fx))\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e}}{8a^2f\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/
2),x]
```

output
```
((1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e
 + f*x]])/(8*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4441}$$

$$\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/2),x]
```

output

```
(c*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(37) = 74$.

Time = 2.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.58

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-\frac{2c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}{16f\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,a^{2}}$	111
risch	$\frac{2i\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{3i(fx+e)}+\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}\right)}{a^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)f}$	124

input
```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/16*2^(1/2)/f*(-2*c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2
)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a^2*(3*tan(1/2
*f*x+1/2*e)-tan(1/2*f*x+1/2*e)*sec(1/2*f*x+1/2*e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\left(2\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
1/2*(2*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3
*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/(a*(sec(e + f*x) + 1))**
(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{2}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{2}}{8\sqrt{-a}a^{2}f}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
-1/8*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x + e)/(cos(f*
x + e) + 1) - 1)^2/(sqrt(-a)*a^2*f)
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2}{8\,\sqrt{-ac}a^2f|c|}$$

input
```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
-1/8*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2/(sqrt(-a*c)*a^2*f*abs(c))
```

Mupad [B] (verification not implemented)

Time = 12.47 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.79

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=$$
$$-\frac{2\left(3\,\sin\left(e+f\,x\right)+3\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)\,\sqrt{\frac{c\,\left(\cos(e+f\,x)-1\right)}{\cos(e+f\,x)}}}{a^2\,f\,\sqrt{\frac{a\,\left(\cos(e+f\,x)+1\right)}{\cos(e+f\,x)}}\,\left(4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(e+f\,x\right)+4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x
)
```

output
```
-(2*(3*sin(e + f*x) + 3*sin(2*e + 2*f*x) + sin(3*e + 3*f*x))*((c*(cos(e +
f*x) - 1))/cos(e + f*x))^(1/2))/(a^2*f*((a*(cos(e + f*x) + 1))/cos(e + f*x
))^(1/2)*(4*cos(2*e + 2*f*x) - 4*cos(e + f*x) + 4*cos(3*e + 3*f*x) + cos(4
*e + 4*f*x) - 5))
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)}{a^3}$$

input

```
int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec
(e + f*x))/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x))/
a**3
```

3.148 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 140

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\\ &+ \frac{\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\\ &- \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output

```
1/4*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)+1/4*tan(f*x
+e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+
e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

$$\begin{aligned}&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx =\\ &-\frac{(-2-\sec(e+fx)+\operatorname{arctanh}(\sec(e+fx))(1+\sec(e+fx))^2)\tan(e+fx)}{4f(a(1+\sec(e+fx)))^{5/2}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]
]),x]
```

output
```
-1/4*((-2 - Sec[e + f*x] + ArcTanh[Sec[e + f*x]]*(1 + Sec[e + f*x])^2)*Tan
[e + f*x])/(f*(a*(1 + Sec[e + f*x]))^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4448, 3042, 4448, 3042, 4447, 25, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4448}$$

$$\frac{\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)^{3/2}\sqrt{c-c\sec(e+fx)}}dx}{2a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4448}$$

$$\frac{\frac{\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}}dx}{2a}+\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 4447$$

$$\frac{\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\int -\csc(e+fx)dx}{2a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\tan(e+fx)\int \csc(e+fx)dx}{2a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\tan(e+fx)\int \csc(e+fx)dx}{2a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 4257$$

$$\frac{\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\text{arctanh}(\cos(e+fx))}{2af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}}{2a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output

```
Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + (
Tan[e + f*x]/(2*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (
ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4447
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   In
t[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(122) = 244$.

Time = 2.44 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.76

method	result
default	$\frac{\left(4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\mathrm{cs}\right.\right.}{16f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\sqrt{-\frac{c}{2\mathrm{c}}}}$
risch	$\frac{i\left(3\,\mathrm{e}^{2i(fx+e)}+4\,\mathrm{e}^{i(fx+e)}+3\right)\left(\mathrm{e}^{2i(fx+e)}-\mathrm{e}^{i(fx+e)}\right)}{2a^{2}\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,f}+\frac{i\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\ln\left(\mathrm{e}^{i(fx+e)}+1\right)}{4a^{2}\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)}{\mathrm{e}^{2i(fx+e)}+1}}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/16/f*(4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e))*cos(1/2*f*x+1/2*e)^4-
4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)*cos(1/2*f*x+1/2*e)^4-4*ln(-
cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*cos(1/2*f*x+1/2*e)^4+5*cos(1/2*f*
x+1/2*e)^4-6*cos(1/2*f*x+1/2*e)^2+1)/(2*cos(1/2*f*x+1/2*e)^2-1)/(a/(2*cos(
1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^
2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a^2*tan(1/2*f*x+1/2*e)*sec(1/2*f*x+1/2*e)
^2
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.31

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx=\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\,(}{}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output

```
[-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*
c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f
*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 + 2*cos(f
*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*
f)*sin(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*arc
tan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))*cos(f*x + e)^2/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*c
os(f*x + e)^2 + 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*
f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\sec{(e+fx)}}{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}\sqrt{-c(\sec{(e+fx)}-1)}}\,dx$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output

```
Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(5/2)*sqrt(-c*(sec(e + f*x)
 - 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(122) = 244$.

Time $= 0.25$ (sec) , antiderivative size $= 1191$, normalized size of antiderivative $= 8.51$

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx=\text{Too large to display}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
-1/4*((2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*co
s(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*x +
e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) + 1)
*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin(3
*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin(
4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e)
 + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin
(f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e),
cos(f*x + e) + 1) - (2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*
x + e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e)
+ 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos
(f*x + e) + 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^
2 + 4*(2*sin(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x
 + 4*e) + sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*si
n(3*f*x + 3*e) + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*
f*x + 2*e)*sin(f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(
sin(f*x + e), cos(f*x + e) - 1) + 2*(3*sin(3*f*x + 3*e) + 4*sin(2*f*x + 2*
e) + 3*sin(f*x + e))*cos(4*f*x + 4*e) - 2*(3*cos(3*f*x + 3*e) + 4*cos(2*f*
x + 2*e) + 3*cos(f*x + e))*sin(4*f*x + 4*e) + 2*(2*cos(2*f*x + 2*e) + 3)*s
in(3*f*x + 3*e) - 4*(cos(f*x + e) - 2)*sin(2*f*x + 2*e) - 4*cos(3*f*x +...
```

Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{c^2\left(\frac{2\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c}-\frac{2\,\log(|c|)}{c}+\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^3-2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^4}{c^6}\right)}{16\sqrt{-ac}a^2f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
-1/16*c^2*(2*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c - 2*log(abs(c))/c + ((c*
tan(1/2*f*x + 1/2*e)^2 - c)^2*c^3 - 2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4)/
c^6)/(sqrt(-a*c)*a^2*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1
/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^4+2\sec(fx+e)^3-2\sec(fx+e)-1}dx\right)}{a^3c}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x) - 1),x
))/(a**3*c)
```

3.149 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 146

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} - \frac{3\text{arctanh}(\cos(e+fx))\tan(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output

```
3/8*csc(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+1/4*t
an(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2)-3/8*arctanh(cos(
f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{(2-3\sec(e+fx)-3\sec^2(e+fx)+3\text{arctanh}(\sec(e+fx))(-1+\sec(e+fx))(1+\sec(e+fx))^2)\tan(}{8cf(-1+\sec(e+fx))(a(1+\sec(e+fx)))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3
/2)),x]
```

output
```
-1/8*((2 - 3*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 3*ArcTanh[Sec[e + f*x]]*(-1
 + Sec[e + f*x])*(1 + Sec[e + f*x])^2)*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*
x])*(a*(1 + Sec[e + f*x]))^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4448, 3042, 4447, 25, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4448}$$

$$\frac{3\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)^{3/2}(c-c\sec(e+fx))^{3/2}}dx}{4a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4a}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4447}$$

$$\frac{3\tan(e+fx)\int -\cot^2(e+fx)\csc(e+fx)dx}{4a^2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}-\frac{3\tan(e+fx)\int\cot^2(e+fx)\csc(e+fx)dx}{4a^2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}-\frac{3\tan(e+fx)\int\sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)^2dx}{4a^2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3091$$

$$\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}-\frac{3\tan(e+fx)\left(-\frac{1}{2}\int\csc(e+fx)dx-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)}{4a^2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}-\frac{3\tan(e+fx)\left(-\frac{1}{2}\int\csc(e+fx)dx-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)}{4a^2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 4257$$

$$\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}-\frac{3\tan(e+fx)\left(\frac{\text{arctanh}(\cos(e+fx))}{2f}-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)}{4a^2c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x
]
```

output

```
Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)) -
 (3*(ArcTanh[Cos[e + f*x]]/(2*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*f))*Tan[
e + f*x])/(4*a^2*c*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4447
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   In
t[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

rule 4448
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[
(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
 + a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (IL
tQ[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(128) = 256$.

Time = 2.48 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.02

method	result
default	$-\frac{(-12\cos(\frac{fx}{2}+\frac{e}{2})^4\sin(\frac{fx}{2}+\frac{e}{2})^2\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}+\frac{e}{2}))+12\cos(\frac{fx}{2}+\frac{e}{2})^4\sin(\frac{fx}{2}+\frac{e}{2})^2\ln(-\cot(\frac{fx}{2}+\frac{e}{2})+\csc(\frac{fx}{2}}{32f(2\cos(\frac{fx}{2}+\frac{e}{2}}$
risch	$\frac{i(5\,e^{5i(fx+e)}+2\,e^{4i(fx+e)}+2\,e^{3i(fx+e)}+2\,e^{2i(fx+e)}+5\,e^{i(fx+e)})}{4a^2c(e^{2i(fx+e)}+1)(e^{i(fx+e)}+1)^3\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{3i(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)\ln(}{8a^2c(e^{2i(fx+e)}+1)\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/32/f*(-12*cos(1/2*f*x+1/2*e)^4*sin(1/2*f*x+1/2*e)^2*ln(-cot(1/2*f*x+1/2
*e)+csc(1/2*f*x+1/2*e))+12*cos(1/2*f*x+1/2*e)^4*sin(1/2*f*x+1/2*e)^2*ln(-c
ot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)+12*cos(1/2*f*x+1/2*e)^4*sin(1/2*f*
x+1/2*e)^2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)+6*cos(1/2*f*x+1/2*
e)^6-16*cos(1/2*f*x+1/2*e)^4+9*cos(1/2*f*x+1/2*e)^2-1)/(2*cos(1/2*f*x+1/2*
e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a^2/c*sec(1/2*f*x+1/2*e
)^3*csc(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.73

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{3\,(\cos(fx+e)^3+\cos(fx+e)^2-\cos(fx+e)-}{}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
[-1/16*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a*c)*
log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f
*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin
(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*
x + e)^3 + cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e
)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f
*x + e)), 1/8*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt
(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2/(a*c*sin(f*x + e)))*sin(f*x +
e) + (5*cos(f*x + e)^3 + cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2
*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^
3*c^2*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{c^2\left(\frac{2\,(3\,\log(|c|)-2)}{c^2} - \frac{6\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c^2} + \frac{2\,\left(3\,c\tan\left(\frac{1}{2}\,fx\right.\right.}{c^3\tan\left(\frac{1}{2}\,fx\right.}\right.}{32\,\sqrt{-ac}a^2f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,f\right.\right.}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
1/32*c^2*(2*(3*log(abs(c)) - 2)/c^2 - 6*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)
/c^2 + 2*(3*c*tan(1/2*f*x + 1/2*e)^2 - c)/(c^3*tan(1/2*f*x + 1/2*e)^2) - (
(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^4 - 4*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^
5)/c^8)/(sqrt(-a*c)*a^2*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x
+ 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^5+\sec(fx+e)^4-2\sec(fx+e)^3-2\sec(fx+e)^2+\mathrm{se}}\right.}{a^3c^2}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output

```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec
(e + f*x))/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**3 - 2*sec(
e + f*x)**2 + sec(e + f*x) + 1),x))/(a**3*c**2)
```

3.150 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 160

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{3\text{arctanh}(\cos(e+fx))\tan(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output

```
3/8*csc(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4
*cot(f*x+e)^2*csc(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)-3/8*arctanh(cos(f*x+e))*tan(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)
/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.48

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{(3-2\cot^2(e+fx))\csc(e+fx)-3\text{arctanh}(\sec(e+}{8a^2c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e-}}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5
/2)),x]
```

output
```
((3 - 2*Cot[e + f*x]^2)*Csc[e + f*x] - 3*ArcTanh[Sec[e + f*x]]*Tan[e + f*x
])/(8*a^2*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.64, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4447, 25, 3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4447$$

$$-\frac{\tan(e+fx)\int -\cot^4(e+fx)\csc(e+fx)dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\tan(e+fx)\int \cot^4(e+fx)\csc(e+fx)dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(e+fx)\int \sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)^4dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan(e+fx)\left(-\frac{3}{4}\int \cot^2(e+fx)\csc(e+fx)dx-\frac{\cot^3(e+fx)\csc(e+fx)}{4f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\left(-\frac{3}{4}\int \sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)^2dx-\frac{\cot^3(e+fx)\csc(e+fx)}{4f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan(e+fx)\left(-\frac{3}{4}\left(-\frac{1}{2}\int \csc(e+fx)dx-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)-\frac{\cot^3(e+fx)\csc(e+fx)}{4f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\left(-\frac{3}{4}\left(-\frac{1}{2}\int \csc(e+fx)dx-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)-\frac{\cot^3(e+fx)\csc(e+fx)}{4f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan(e+fx)\left(-\frac{3}{4}\left(\frac{\text{arctanh}(\cos(e+fx))}{2f}-\frac{\cot(e+fx)\csc(e+fx)}{2f}\right)-\frac{\cot^3(e+fx)\csc(e+fx)}{4f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x
]
```

output
```
((-1/4*(Cot[e + f*x]^3*Csc[e + f*x])/f - (3*(ArcTanh[Cos[e + f*x]]/(2*f) -
 (Cot[e + f*x]*Csc[e + f*x])/(2*f)))/4)*Tan[e + f*x])/(a^2*c^2*Sqrt[a + a*
Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4447
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   In
t[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(142) = 284$.

Time = 2.37 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left(192\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-192\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\right.\right.}{512f\left(\right.}$
risch	$\frac{i\left(5\,\mathrm{e}^{7i(fx+e)}+3\,\mathrm{e}^{5i(fx+e)}+3\,\mathrm{e}^{3i(fx+e)}+5\,\mathrm{e}^{i(fx+e)}\right)}{4a^{2}c^{2}\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)^{3}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,f}-\frac{3i\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)}{8a^{2}c^{2}\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right.}{\mathrm{e}^{2i(fx+e)}+1}}}$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
-1/512/f*(192*sin(1/2*f*x+1/2*e)^4*cos(1/2*f*x+1/2*e)^4*ln(-cot(1/2*f*x+1/
2*e)+csc(1/2*f*x+1/2*e)-1)-192*sin(1/2*f*x+1/2*e)^4*cos(1/2*f*x+1/2*e)^4*l
n(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e))+192*sin(1/2*f*x+1/2*e)^4*cos(1/2
*f*x+1/2*e)^4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)-35*cos(1/2*f*x+
1/2*e)^8+230*cos(1/2*f*x+1/2*e)^6-275*cos(1/2*f*x+1/2*e)^4+96*cos(1/2*f*x+
1/2*e)^2-8)/(2*cos(1/2*f*x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1
/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2
)^(1/2)/a^2/c^2*sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e)^3
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.06

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \Bigg[-\frac{3\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{-ac}\log\Big(}{}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output

```
[-1/16*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-4*(2*sqr
t(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((
cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e)^4 - 3*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x +
e)^2 + a^3*c^3*f)*sin(f*x + e)), 1/8*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2
 + 1)*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2/(a*c*sin(f*x + e)))*
sin(f*x + e) + (5*cos(f*x + e)^4 - 3*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*co
s(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. 2(142) = 284.

Time = 0.35 (sec) , antiderivative size = 1659, normalized size of antiderivative = 10.37

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
1/8*(3*(2*(4*cos(6*f*x + 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) -
1)*cos(8*f*x + 8*e) - cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2
*f*x + 2*e) + 1)*cos(6*f*x + 6*e) - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*
x + 2*e) - 1)*cos(4*f*x + 4*e) - 36*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*
e)^2 + 4*(2*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*si
n(8*f*x + 8*e) - sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x
 + 2*e))*sin(6*f*x + 6*e) - 16*sin(6*f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2
+ 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f
*x + 2*e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 3*(2*(4*cos(6*f*x
 + 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) -
cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6
*f*x + 6*e) - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*
x + 4*e) - 36*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x
+ 6*e) - 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8
*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6
*e) - 16*sin(6*f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2
(sin(f*x + e), cos(f*x + e) - 1) - 2*(5*sin(7*f*x + 7*e) + 3*sin(5*f*x + 5
*e) + 3*sin(3*f*x + 3*e) + 5*sin(f*x + e))*cos(8*f*x + 8*e) - 20*(2*sin(6*
f*x + 6*e) - 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) ...
```

Giac [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^2-6\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^3}{c^4}-\frac{18\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2+28\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c+11\,c^2}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}+12\,\log\Big(|c|}{64\sqrt{-ac}a^2cf|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algo
rithm="giac")
```

output

```
-1/64*(((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^2 - 6*(c*tan(1/2*f*x + 1/2*e)^2
 - c)*c^3)/c^4 - (18*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 28*(c*tan(1/2*f*x
+ 1/2*e)^2 - c)*c + 11*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4) + 12*log(abs(c)*t
an(1/2*f*x + 1/2*e)^2) - 12*log(abs(c)) + 11)/(sqrt(-a*c)*a^2*c*f*abs(c)*s
gn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}$$

input

```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2))
,x)
```

output

```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^6-3\sec(fx+e)^4+3\sec(fx+e)^2-1}dx\right)}{a^3c^3}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output

```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x)**2 - 1
),x))/(a**3*c**3)
```

3.151 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 32, antiderivative size = 101

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = -\frac{2^{\frac{1}{2}+n}c\,\text{Hypergeometric2F1}\left(\frac{1}{2}+m,\frac{1}{2}-n,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec}{f(1+2m)}$$

output
```
-2^(1/2+n)*c*hypergeom([1/2-n, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f
*x+e))^(1/2-n)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(1+
2*m)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e}{f+2fn}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output

```
(2^(1/2 + m)*Hypergeometric2F1[1/2 - m, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x
])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e
+ f*x])^n*Tan[e + f*x])/(f + 2*f*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4449, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \ 4449$$

$$-\frac{ac\tan(e+fx)\int(\sec(e+fx)a+a)^{m-\frac{1}{2}}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 80$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}(\sec(e+fx)a+a)^{m}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 79$$

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\,(m+}{f(2m+1)}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output
```
-((2^(1/2 + n)*c*Hypergeometric2F1[1/2 + m, 1/2 - n, 3/2 + m, (1 + Sec[e +
 f*x])/2]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^n\,dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="f
ricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a(\sec(e+fx)+1))^m\,(-c(\sec(e+fx)-1))^n\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n*sec(e + f*x)
, x)
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="m
axima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="g
iac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n)/cos(e + f*x),x)
```

output
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n)/cos(e + f*x), x)
```

Reduce [F]

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx\\&=\int (\sec(fx+e)\,a+a)^m\,(-\sec(fx+e)\,c+c)^n\sec(fx+e)\,dx\end{aligned}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

output
```
int((sec(e + f*x)*a + a)**m*( - sec(e + f*x)*c + c)**n*sec(e + f*x),x)
```

3.152 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 92

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx = \frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+fx))^{-}}{5f}$$

output
```
1/5*2^(1/2+m)*a*hypergeom([5/2, 1/2-m],[7/2],1/2-1/2*sec(f*x+e))*(1+sec(f*
x+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*(c-c*sec(f*x+e))^2*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx = \frac{2^{\frac{1}{2}+m}c^2\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1-\sec(e+fx))\right)(-1+\sec(e+fx))^2(1+\sec(e+fx))^{-\frac{1}{2}-}}{5f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^2,x]
```

output

```
(2^(1/2 + m)*c^2*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2
]*(-1 + Sec[e + f*x])^2*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]
))^m*Tan[e + f*x])/(5*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4449, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(c-c\sec(e+fx))^2(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4449}$$

$$-\frac{ac\tan(e+fx)\int(\sec(e+fx)a+a)^{m-\frac{1}{2}}(c-c\sec(e+fx))^{3/2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$-\frac{ac2^{m-\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\int\left(\frac{1}{2}\sec(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}(c-c\sec(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))^2(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-\right.}{5f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^2,x]
```

output
```
(2^(1/2 + m)*a*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2]*
(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e +
f*x])^2*Tan[e + f*x])/(5*f)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec(fx+e)\left(a+a\sec(fx+e)\right)^m\left(c-c\sec(fx+e)\right)^2dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

Fricas [F]

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx\\&=\int (c\sec(fx+e)-c)^2(a\sec(fx+e)+a)^m\sec(fx+e)\,dx\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*(a
*sec(f*x + e) + a)^m, x)
```

Sympy [F]

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx\\&=c^2\left(\int (a\sec(e+fx)+a)^m\sec(e+fx)\,dx\right.\\&\quad+\int \left(-2(a\sec(e+fx)+a)^m\sec^2(e+fx)\right)dx\\&\quad\left.+\int (a\sec(e+fx)+a)^m\sec^3(e+fx)\,dx\right)\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**2,x)
```

output
```
c**2*(Integral((a*sec(e + f*x) + a)**m*sec(e + f*x), x) + Integral(-2*(a*s
ec(e + f*x) + a)**m*sec(e + f*x)**2, x) + Integral((a*sec(e + f*x) + a)**m
*sec(e + f*x)**3, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$$
$$=\int (c\sec(fx+e)-c)^2(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$$
$$=\int (c\sec(fx+e)-c)^2(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

output
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^2)/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$$
$$=c^2\left(\int(\sec\left(fx+e\right)a+a)^m\sec\left(fx+e\right)^3dx-2\left(\int(\sec\left(fx+e\right)a+a)^m\sec\left(fx+e\right)^2dx\right)+\int(\sec\left(fx+e\right)a+a)^m\sec\left(fx+e\right)dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

output
```
c**2*(int((sec(e + f*x)*a + a)**m*sec(e + f*x)**3,x) - 2*int((sec(e + f*x)
*a + a)**m*sec(e + f*x)**2,x) + int((sec(e + f*x)*a + a)**m*sec(e + f*x),x
))
```

3.153 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 90

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$$
$$=\frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+fx))^{-}}{3f}$$

output
```
1/3*2^(1/2+m)*a*hypergeom([3/2, 1/2-m],[5/2],1/2-1/2*sec(f*x+e))*(1+sec(f*
x+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*(c-c*sec(f*x+e))*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx =$$
$$-\frac{2^{\frac{1}{2}+m}c\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\sec(e+fx))\right)(-1+\sec(e+fx))(1+\sec(e+fx))^{-\frac{1}{2}}}{3f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

output
```
-1/3*(2^(1/2 + m)*c*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x]
)/2]*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x
]))^m*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4449, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(c-c\sec(e+fx))(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4449}$$

$$-\frac{ac\tan(e+fx)\int(\sec(e+fx)a+a)^{m-\frac{1}{2}}\sqrt{c-c\sec(e+fx)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$-\frac{ac2^{m-\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\int\left(\frac{1}{2}\sec(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}\sqrt{c-c\sec(e+f}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-\right.}{3f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

output
```
(2^(1/2 + m)*a*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])/2]*
(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e +
f*x])*Tan[e + f*x])/(3*f)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec (fx+e)\,(a+a\sec (fx+e))^m\,(c-c\sec (fx+e))\,dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = \int -(c\sec (fx+e)-c)(a\sec (fx+e)+a)^m\sec (fx+e)\;dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx \\ = -c\left(\int (-(a\sec (e+fx)+a)^m\sec (e+fx))\;dx \right.\\ \left. + \int (a\sec (e+fx)+a)^m\sec^2 (e+fx)\,dx\right)\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e)),x)
```

output
```
-c*(Integral(-(a*sec(e + f*x) + a)**m*sec(e + f*x), x) + Integral((a*sec(e
 + f*x) + a)**m*sec(e + f*x)**2, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$$
$$=\int -(c\sec{(fx+e)}-c)(a\sec{(fx+e)}+a)^m\sec{(fx+e)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-integrate((c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$$
$$=\int -(c\sec{(fx+e)}-c)(a\sec{(fx+e)}+a)^m\sec{(fx+e)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
integrate(-(c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

output
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x)))/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = c\left(-\left(\int \left(\sec\left(fx+e\right)a+a\right)^m \sec\left(fx+e\right)^2 dx\right) + \int \left(\sec\left(fx+e\right)a+a\right)^m \sec\left(fx+e\right)dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)
```

output
```
c*( - int((sec(e + f*x)*a + a)**m*sec(e + f*x)**2,x) + int((sec(e + f*x)*a
 + a)**m*sec(e + f*x),x))
```

3.154 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 90

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx = -\frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+f}{f(c-c\sec(e+fx))}$$

output
```
-2^(1/2+m)*a*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+
e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Mathematica [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx = \int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
```

output
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]), x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4449, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^m}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4449}$$

$$-\frac{ac\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{m-\frac{1}{2}}}{(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$-\frac{ac2^{m-\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\int \frac{\left(\frac{1}{2}\sec(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sec\right.}{f(c-c\sec(e+fx))}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
```

output
```
-((2^(1/2 + m)*a*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sec[e + f*x])/
2]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*x]
)/(f*(c - c*Sec[e + f*x])))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec (fx+e) (a+a\sec (fx+e))^m}{c-c\sec (fx+e)}dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=\int -\frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{c\sec(fx+e)-c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
integral(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=-\frac{\int \frac{(a\sec(e+fx)+a)^m\sec(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e)),x)
```

output
```
-Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x) - 1), x)/c
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=\int -\frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{c\sec(fx+e)-c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=\int -\frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{c\sec(fx+e)-c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
integrate(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=-\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{c-c\,\cos(e+f\,x)}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

output
```
-int((a + a/cos(e + f*x))^m/(c - c*cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=-\frac{\int \frac{(\sec(fx+e)a+a)^m\sec(fx+e)}{\sec(fx+e)-1}dx}{c}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
```

output
```
( - int(((sec(e + f*x)*a + a)**m*sec(e + f*x))/(sec(e + f*x) - 1),x))/c
```

3.155 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2} dx =$$
$$-\frac{2^{\frac{1}{2}+m} a \,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+}{3f(c-c\sec(e+fx))^2}$$

output
```
-1/3*2^(1/2+m)*a*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sec(f*x+e))*(1+sec
(f*x+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*tan(f*x+e)/f/(c-c*sec(f*x+e))^2
```

Mathematica [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2} dx = \int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2} dx$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
```

output
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2, x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4449, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^m}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4449}$$

$$-\frac{ac\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{m-\frac{1}{2}}}{(c-c\sec(e+fx))^{5/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$-\frac{ac2^{m-\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\int \frac{\left(\frac{1}{2}\sec(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{(c-c\sec(e+fx))^{5/2}}d\sec(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\text{s}\right.}{3f(c-c\sec(e+fx))^2}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
```

output

```
-1/3*(2^(1/2 + m)*a*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sec[e + f*
x])/2]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e +
f*x])/(f*(c - c*Sec[e + f*x])^2)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec (fx+e)\,(a+a\sec (fx+e))^m}{(c-c\sec (fx+e))^2}dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = \int \frac{(a\sec\left(fx+e\right)+a)^m\sec\left(fx+e\right)}{\left(c\sec\left(fx+e\right)-c\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^2*sec(f*x + e)^2 - 2*c^2*s
ec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{(a\sec{(e+fx)}+a)^m\sec{(e+fx)}}{\sec^2{(e+fx)}-2\sec{(e+fx)}+1}\,dx}{c^2}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e +
 f*x) + 1), x)/c**2
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = \int \frac{(a\sec\left(fx+e\right)+a)^m\sec\left(fx+e\right)}{\left(c\sec\left(fx+e\right)-c\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(c\sec(fx+e)-c)^2}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

output
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{(\sec(fx+e)a+a)^m\sec(fx+e)}{\sec(fx+e)^2-2\sec(fx+e)+1}dx}{c^2}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
```

output
```
int(((sec(e + f*x)*a + a)**m*sec(e + f*x))/(sec(e + f*x)**2 - 2*sec(e + f*
x) + 1),x)/c**2
```

3.156 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 160

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx =\\ -\frac{64c^3(a+a\sec(e+fx))^m\tan(e+fx)}{f(5+2m)\left(3+8m+4m^2\right)\sqrt{c-c\sec(e+fx)}}\\ -\frac{16c^2(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\left(15+16m+4m^2\right)}\\ -\frac{2c(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(5+2m)}\end{aligned}$$

output
```
-64*c^3*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(5+2*m)/(4*m^2+8*m+3)/(c-c*sec(f*x
+e))^(1/2)-16*c^2*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(
4*m^2+16*m+15)-2*c*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/
(5+2*m)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.68

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx =$$
$$-\frac{2c^3(a(1+\sec(e+fx)))^m\left(43+24m+4m^2-2(7+16m+4m^2)\sec(e+fx)+(3+8m+4m^2)\sec^2(e-\right.}{f(1+2m)(3+2m)(5+2m)\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x
]
```

output
```
(-2*c^3*(a*(1 + Sec[e + f*x]))^m*(43 + 24*m + 4*m^2 - 2*(7 + 16*m + 4*m^2)
*Sec[e + f*x] + (3 + 8*m + 4*m^2)*Sec[e + f*x]^2)*Tan[e + f*x])/(f*(1 + 2*
m)*(3 + 2*m)*(5 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 4443, 3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(c-c\sec(e+fx))^{5/2}(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{8c\int \sec(e+fx)(\sec(e+fx)a+a)^m(c-c\sec(e+fx))^{3/2}dx}{2m+5}-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m}{f(2m+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx}{2m+5}-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m}{f(2m+5)}$$

$$\Big\downarrow \text{4443}$$

$$\frac{8c\left(\frac{4c\int \sec(e+fx)(\sec(e+fx)a+a)^m\sqrt{c-c\sec(e+fx)}dx}{2m+3}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m}{f(2m+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8c\left(\frac{4c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2m+3}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m}{f(2m+5)}$$

$$\Big\downarrow \text{4441}$$

$$\frac{8c\left(-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+1)(2m+3)\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m}{f(2m+5)}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x]
```

output

```
(-2*c*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(
5 + 2*m)) + (8*c*((-8*c^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m
)*(3 + 2*m)*Sqrt[c - c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^m*Sqrt[c
 - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(3 + 2*m))))/(5 + 2*m)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [F]

$$\int \sec (fx+e) (a+a\sec (fx+e))^m (c-c\sec (fx+e))^{\frac{5}{2}} dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e$$

$$+fx))^{5/2}\,dx = \frac{2\left(4\,c^2m^2+(4\,c^2m^2+24\,c^2m+43\,c^2)\cos\left(fx+e\right)^3+8\,c^2m-(4\,c^2m^2+8\,c^2m-29\,c^2)\cos\right.}{(8\,fm^3+36\,fm^2+46\,fm+15}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorith
m="fricas")
```

output
```
2*(4*c^2*m^2 + (4*c^2*m^2 + 24*c^2*m + 43*c^2)*cos(f*x + e)^3 + 8*c^2*m -
(4*c^2*m^2 + 8*c^2*m - 29*c^2)*cos(f*x + e)^2 + 3*c^2 - (4*c^2*m^2 + 24*c^
2*m + 11*c^2)*cos(f*x + e))*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(
f*x + e)^2*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.42

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx =$$

$$-\frac{2\left(\frac{\sqrt{2}(2^{m+5}m+5\cdot 2^{m+4})(-a)^m c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{\sqrt{2}(2^{m+4}m^2+2^{m+6}m+15\cdot 2^{m+2})(-a)^m c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-2^{m+\frac{11}{2}}(-a)^m c^{\frac{5}{2}}\right)e^{(-}}{(8\,m^3+36\,m^2+46\,m+15)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorith
m="maxima")
```

output

```
-2*(sqrt(2)*(2^(m + 5)*m + 5*2^(m + 4))*(-a)^m*c^(5/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - sqrt(2)*(2^(m + 4)*m^2 + 2^(m + 6)*m + 15*2^(m + 2))*(-
a)^m*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2^(m + 11/2)*(-a)^m*c^(
5/2))*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1))/((8*m^3 + 36*m^2 + 46*m + 15)*f*(sin(f*x + e)/(co
s(f*x + e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = \int (-c\sec(fx+e)+c)^{\frac{5}{2}}(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorith
m="giac")
```

output

```
integrate((-c*sec(f*x + e) + c)^(5/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

output
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx=\sqrt{c}\,c^2\left(\int \left(\sec\left(fx+e\right)a+a\right)^m\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^3dx-2\left(\int \left(\sec\left(fx+e\right)a+a\right)^m\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^2dx\right)+\int \left(\sec\left(fx+e\right)a+a\right)^m\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*c**2*(int((sec(e + f*x)*a + a)**m*sqrt( - sec(e + f*x) + 1)*sec(e
+ f*x)**3,x) - 2*int((sec(e + f*x)*a + a)**m*sqrt( - sec(e + f*x) + 1)*sec
(e + f*x)**2,x) + int((sec(e + f*x)*a + a)**m*sqrt( - sec(e + f*x) + 1)*se
c(e + f*x),x))
```

3.157 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 100

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2(a+a\sec(e+fx))^m\tan(e+fx)}{f\,(3+8m+4m^2)\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(3+2m)}$$

output

```
-8*c^2*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(4*m^2+8*m+3)/(c-c*sec(f*x+e))^(1/2
)-2*c*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(3+2*m)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = \frac{2c^2(a(1+\sec(e+fx)))^m(-5-2m+(1+2m)\sec(e+fx))\tan(e+fx)}{f(1+2m)(3+2m)\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x
]
```

output
```
(2*c^2*(a*(1 + Sec[e + f*x]))^m*(-5 - 2*m + (1 + 2*m)*Sec[e + f*x])*Tan[e
+ f*x])/(f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3042, 4443, 3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(c-c\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4443}$$

$$\frac{4c\int \sec(e+fx)(\sec(e+fx)a+a)^m\sqrt{c-c\sec(e+fx)}dx}{2m+3}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f(2m+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4c\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2m+3}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f(2m+3)}$$

$$\Big\downarrow \text{4441}$$

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+1)(2m+3)\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f(2m+3)}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(-8*c^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*(3 + 2*m)*Sqrt[c
 - c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]]
*Tan[e + f*x])/(f*(3 + 2*m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

rule 4443
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Simp[c*((2*n - 1)/(m + n))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c +
d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)]
&&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Maple [F]

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{\frac{3}{2}}dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = \frac{2\left((2\,cm+5\,c)\cos\left(fx+e\right)^{2}-2\,cm+4\,c\cos\left(fx+e\right)-c\right)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^{m}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{\left(4\,fm^{2}+8\,fm+3\,f\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorith
m="fricas")
```

output
```
2*((2*c*m + 5*c)*cos(f*x + e)^2 - 2*c*m + 4*c*cos(f*x + e) - c)*((a*cos(f*
x + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((4*f*
m^2 + 8*f*m + 3*f)*cos(f*x + e)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.71

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2\left(\sqrt{2}2^{m+2}(-a)^m\,c^{\frac{3}{2}}-\frac{\sqrt{2}(2^{m+2}m+3\cdot 2^{m+1})(-a)^m c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)e^{\left(-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\right)}}{(4\,m^2+8\,m+3)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorith
m="maxima")
```

output
```
-2*(sqrt(2)*2^(m + 2)*(-a)^m*c^(3/2) - sqrt(2)*(2^(m + 2)*m + 3*2^(m + 1))
*(-a)^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*e^(-m*log(sin(f*x + e
)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/((
4*m^2 + 8*m + 3)*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = \int (-c\sec(fx+e)+c)^{\frac{3}{2}}(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorith
m="giac")
```

output
```
integrate((-c*sec(f*x + e) + c)^(3/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
 x)
```

Mupad [B] (verification not implemented)

Time = 12.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx =$$

$$-\frac{2\,c\,\left(\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}\right)^m\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,(5\,\sin\left(e+f\,x\right)-2\,\sin\left(2\,e+2\,f\,x\right)+5\,\sin\left(3\,e+3\,f\,x\right)+2\,m\,\sin(}{f\,\left(4\,m^2+8\,m+3\right)\,(3\,\cos\left(e+f\,x\right)-2\,\cos\left(2\,e+2\,f\,x\right)+\cos(}$$

input
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

output
```
-(2*c*((a*(cos(e + f*x) + 1))/cos(e + f*x))^m*((c*(cos(e + f*x) - 1))/cos(
e + f*x))^(1/2)*(5*sin(e + f*x) - 2*sin(2*e + 2*f*x) + 5*sin(3*e + 3*f*x)
+ 2*m*sin(e + f*x) - 4*m*sin(2*e + 2*f*x) + 2*m*sin(3*e + 3*f*x)))/(f*(8*m
 + 4*m^2 + 3)*(3*cos(e + f*x) - 2*cos(2*e + 2*f*x) + cos(3*e + 3*f*x) - 2)
)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = \sqrt{c}\,c\left(-\left(\int \left(\sec\left(fx+e\right)a+a\right)^m\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)^2dx\right)+\int \left(\sec\left(fx+e\right)a+a\right)^m\sqrt{-\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*c*( - int((sec(e + f*x)*a + a)**m*sqrt( - sec(e + f*x) + 1)*sec(e
+ f*x)**2,x) + int((sec(e + f*x)*a + a)**m*sqrt( - sec(e + f*x) + 1)*sec(e
 + f*x),x))
```

3.158 $\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}$

Optimal result

Integrand size = 34, antiderivative size = 46

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

output
```
-2*c*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$= \frac{\sqrt{2}e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\csc\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^{-m}(a(1+\sec(}{(f+2fm)\sqrt{\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(Sqrt[2]*(1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x
)))]*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^m*Csc[(e + f*x)/2
]*(a*(1 + Sec[e + f*x]))^m*Sqrt[c - c*Sec[e + f*x]])/(E^((I/2)*(e + f*x))*
(f + 2*f*m)*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^m)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3042, 4441}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4441}$$

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(-2*c*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e +
 f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4441
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Maple [F]

$$\int \sec{(fx+e)}\,(a+a\sec{(fx+e)})^m\sqrt{c-c\sec{(fx+e)}}dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx = \frac{2\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^m\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}(\cos{(fx+e)}+1)}{(2\,fm+f)\sin{(fx+e)}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorith
m="fricas")
```

output
```
2*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*(cos(f*x + e) + 1)/((2*f*m + f)*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int (a(\sec(e+fx)+1))^m\sqrt{-c(\sec(e+fx)-1)}\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(44) = 88.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2^{m+\frac{3}{2}}(-a)^m\sqrt{c}e^{\left(-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\right)}}{f(2m+1)\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorith
m="maxima")
```

output
```
2^(m + 3/2)*(-a)^m*sqrt(c)*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
- m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/(f*(2*m + 1)*sqrt(sin(f*x +
e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int \sqrt{-c\sec\left(fx+e\right)+c}\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

output
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\sqrt{c}\left(\int (\sec (fx+e)\,a+a)^m\sqrt{-\sec (fx+e)+1}\,\sec (fx+e)\,dx\right)$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

output

```
sqrt(c)*int((sec(e + f*x)*a + a)**m*sqrt( - sec(e + f*x) + 1)*sec(e + f*x)
,x)
```

3.159 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

output
```
-hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*tan(f
*x+e)/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a(1+\sec(e+fx)))^m\tan(e+fx)}{(f+2fm)\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x
]
```

output
```
-((Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a*(1 + Se
c[e + f*x]))^m*Tan[e + f*x])/((f + 2*f*m)*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3042, 4449, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^m}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4449}$$

$$-\frac{ac\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{m-\frac{1}{2}}}{c(1-\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{m-\frac{1}{2}}}{1-\sec(e+fx)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{78}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m \text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-((Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec
[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec(fx+e)\,(a+a\sec(fx+e))^m}{\sqrt{c-c\sec(fx+e)}}\,dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorith
m="fricas")
```

output
```
integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c
*sec(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a(\sec(e+fx)+1))^m\sec(e+fx)}{\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1)
), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x
)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec\left(fx+e\right)+a)^m\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorith
m="giac")
```

output

```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input

```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

output

```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx=-\frac{\sqrt{c}\left(\int \frac{(\sec(fx+e)a+a)^m\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)-1}dx\right)}{c}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*int(((sec(e + f*x)*a + a)**m*sqrt( - sec(e + f*x) + 1)*sec(e +
 f*x))/(sec(e + f*x) - 1),x))/c
```

3.160 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{2cf(1+2m)\sqrt{c-c\sec(e+fx)}}$$

output

```
-1/2*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*t
an(f*x+e)/c/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a(1+\sec(e+fx)))^m\tan(e+fx)}{4cf\left(\frac{1}{2}+m\right)\sqrt{c-c\sec(e+fx)}}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2)
,x]
```

output

```
-1/4*(Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a*(1 +
 Sec[e + f*x]))^m*Tan[e + f*x])/(c*f*(1/2 + m)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3042, 4449, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^m}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4449}$$

$$-\frac{ac\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{m-\frac{1}{2}}}{c^2(1-\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{m-\frac{1}{2}}}{(1-\sec(e+fx))^2}d\sec(e+fx)}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 78}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m \text{Hypergeometric2F1}\left(2,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),x]
```

output

```
-1/2*(Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*
Sec[e + f*x])^m*Tan[e + f*x])/(c*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec(fx+e)\,(a+a\sec(fx+e))^m}{(c-c\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorith
m="fricas")
```

output
```
integral(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^
2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a(\sec(e+fx)+1))^m\sec(e+fx)}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(
3/2), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
 x)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorith
m="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{(\sec(fx+e)a+a)^m\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2-2\sec(fx+e)+1}dx\right)}{c^2}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*int(((sec(e + f*x)*a + a)**m*sqrt( - sec(e + f*x) + 1)*sec(e + f*
x))/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x))/c**2
```

3.161 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(3,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{4c^2f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

output
```
-1/4*hypergeom([3, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*t
an(f*x+e)/c^2/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(3,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a(1+\sec(e+fx)))^m\tan(e+fx)}{8c^2f\left(\frac{1}{2}+m\right)\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2)
,x]
```

output
```
-1/8*(Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a*(1 +
 Sec[e + f*x]))^m*Tan[e + f*x])/(c^2*f*(1/2 + m)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 4449, 27, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^m}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4449}$$

$$-\frac{ac\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{m-\frac{1}{2}}}{c^3(1-\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{m-\frac{1}{2}}}{(1-\sec(e+fx))^3}d\sec(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{78}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m \text{Hypergeometric2F1}\left(3,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/4*(Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*
Sec[e + f*x])^m*Tan[e + f*x])/(c^2*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\sec \left(fx+e\right) \left(a+a\sec \left(fx+e\right)\right)^{m}}{\left(c-c\sec \left(fx+e\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorith
m="fricas")
```

output
```
integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c
^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2),
 x)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorith
m="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx=-\frac{\sqrt{c}\left(\int \frac{(\sec(fx+e)a+a)^m\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)}{c^3}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*int(((sec(e + f*x)*a + a)**m*sqrt( - sec(e + f*x) + 1)*sec(e +
 f*x))/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x))/c**3
```

3.162 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 169

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx\\ =-\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{f(1+2m)}\\ +\frac{2(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{af\,(3+8m+4m^2)}\\ -\frac{2(a+a\sec(e+fx))^{2+m}(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{a^2f(1+2m)\,(15+16m+4m^2)}\end{aligned}$$

output

```
-(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/f/(1+2*m)+2*(a+a*se
c(f*x+e))^(1+m)*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/a/f/(4*m^2+8*m+3)-2*(a+
a*sec(f*x+e))^(2+m)*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/a^2/f/(1+2*m)/(4*m^
2+16*m+15)
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.62

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx$$
$$=\frac{(a(1+\sec(e+fx)))^m(c-c\sec(e+fx))^{-m}\left(7+12m+4m^2-2(3+2m)\sec(e+fx)+2\sec^2(e+fx)\right.}{c^3f(1+2m)(3+2m)(5+2m)(-1+\sec(e+fx))^3}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m
),x]
```

output
```
((a*(1 + Sec[e + f*x]))^m*(7 + 12*m + 4*m^2 - 2*(3 + 2*m)*Sec[e + f*x] + 2
*Sec[e + f*x]^2)*Tan[e + f*x])/(c^3*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(-1 +
Sec[e + f*x])^3*(c - c*Sec[e + f*x])^m)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4439, 3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m-3}dx$$

$$\Big\downarrow \text{4439}$$

$$-\frac{2\int \sec(e+fx)(\sec(e+fx)a+a)^{m+1}(c-c\sec(e+fx))^{-m-3}dx}{a(2m+1)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-3}}{f(2m+1)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{m+1}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m-3}dx}{a(2m+1)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-3}}{f(2m+1)}$$

$$\Big\downarrow \text{ 4439}$$

$$-\frac{2\left(-\frac{\int \sec(e+fx)(\sec(e+fx)a+a)^{m+2}(c-c\sec(e+fx))^{-m-3}dx}{a(2m+3)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-3}}{f(2m+3)}\right)}{a(2m+1)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-3}}{f(2m+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\left(-\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{m+2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m-3}dx}{a(2m+3)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-3}}{f(2m+3)}\right)}{a(2m+1)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-3}}{f(2m+1)}$$

$$\Big\downarrow \text{ 4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-3}}{f(2m+1)}-\frac{2\left(\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+2}(c-c\sec(e+fx))^{-m-3}}{af(2m+3)(2m+5)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-3}}{f(2m+3)}\right)}{a(2m+1)}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m),x]
```

output

```
-(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(f*(
1 + 2*m))) - (2*(-(((a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-3
 - m)*Tan[e + f*x])/(f*(3 + 2*m))) + ((a + a*Sec[e + f*x])^(2 + m)*(c - c*S
ec[e + f*x])^(-3 - m)*Tan[e + f*x])/(a*f*(3 + 2*m)*(5 + 2*m))))/(a*(1 + 2*
m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [F]

$$\int \sec (fx+e) (a+a\sec (fx+e))^m (c-c\sec (fx+e))^{-3-m}\, dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = -\frac{\left((4\,m^2+12\,m+7)\cos\left(fx+e\right)^2-2\,(2\,m+3)\cos\left(fx+e\right)+2\right)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^m\left(\frac{c\cos(fx+e)-c}{\cos(fx+e)}\right)^{-m-3}}{\left(8\,fm^3+36\,fm^2+46\,fm+15\,f\right)\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorit
hm="fricas")
```

output
```
-((4*m^2 + 12*m + 7)*cos(f*x + e)^2 - 2*(2*m + 3)*cos(f*x + e) + 2)*((a*co
s(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))^(-m -
3)*sin(f*x + e)/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = \int \left(a(\sec\left(e+fx\right)+1)\right)^m\left(-c(\sec\left(e+fx\right)-1)\right)^{-m-3}\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-3-m),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(-m - 3)*sec(e
 + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = \frac{\left((4\,m^2+8\,m+3)(-a)^m - \frac{2\,(4\,m^2+12\,m+5)\,(-a)^m\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{(4\,m^2+16\,m+15)\,(-a)^m\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)c^{-m-3}(\cos\,(fx+}{4\,(8\,m^3+36\,m^2+46\,m+15)f\,\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{2\,m}\sin\,(fx+e)^5}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorit
hm="maxima")
```

output
```
1/4*((4*m^2 + 8*m + 3)*(-a)^m - 2*(4*m^2 + 12*m + 5)*(-a)^m*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + (4*m^2 + 16*m + 15)*(-a)^m*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4)*c^(-m - 3)*(cos(f*x + e) + 1)^5/((8*m^3 + 36*m^2 + 46*m + 15
)*f*(sin(f*x + e)/(cos(f*x + e) + 1))^(2*m)*sin(f*x + e)^5)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = \int (a\sec\,(fx+e)+a)^m(-c\sec\,(fx+e)+c)^{-m-3}\sec\,(fx+e)\,\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorit
hm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 3)*sec(f*x +
e), x)
```

Mupad [B] (verification not implemented)

Time = 18.78 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.72

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx =$$

$$-\frac{\left(\cos\left(3\,e+3\,f\,x\right)-\sin\left(3\,e+3\,f\,x\right)\,1\mathrm{i}\right)\,\left(\frac{\sin(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,\left(\cos(3\,e+3\,f\,x)+\sin(3\,e+3\,f\,x)\,1\mathrm{i}\right)\,\left(4\,m^2+12\,m+15\right)}{f\,\left(m^3\,8\mathrm{i}+m^2\,36\mathrm{i}+m\,46\mathrm{i}+15\mathrm{i}\right)}\right.}{8\,\mathrm{co}}$$

input
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 3)),x)
```

output
```
-((cos(3*e + 3*f*x) - sin(3*e + 3*f*x)*1i)*((sin(e + f*x)*(a + a/cos(e + f
*x))^m*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*(12*m + 4*m^2 + 15)*2i)/(f
*(m*46i + m^2*36i + m^3*8i + 15i)) - (sin(2*e + 2*f*x)*(8*m + 12)*(a + a/c
os(e + f*x))^m*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*2i)/(f*(m*46i + m^
2*36i + m^3*8i + 15i)) + (sin(3*e + 3*f*x)*(a + a/cos(e + f*x))^m*(cos(3*e
 + 3*f*x) + sin(3*e + 3*f*x)*1i)*(12*m + 4*m^2 + 7)*2i)/(f*(m*46i + m^2*36
i + m^3*8i + 15i))))/(8*cos(e + f*x)^3*(c - c/cos(e + f*x))^(m + 3))
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx =$$

$$-\frac{\int \frac{(\sec(fx+e)a+a)^m\sec(fx+e)}{(-\sec(fx+e)c+c)^m\sec(fx+e)^3-3(-\sec(fx+e)c+c)^m\sec(fx+e)^2+3(-\sec(fx+e)c+c)^m\sec(fx+e)-(-\sec(fx+e)c+c)^m}dx}{c^3}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

output
```
( - int(((sec(e + f*x)*a + a)**m*sec(e + f*x))/(( - sec(e + f*x)*c + c)**m
*sec(e + f*x)**3 - 3*( - sec(e + f*x)*c + c)**m*sec(e + f*x)**2 + 3*( - se
c(e + f*x)*c + c)**m*sec(e + f*x) - ( - sec(e + f*x)*c + c)**m),x))/c**3
```

3.163 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 104

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx\\ =-\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{f(1+2m)}\\ +\frac{(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{af\,(3+8m+4m^2)}\end{aligned}$$

output
```
-(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m)*tan(f*x+e)/f/(1+2*m)+(a+a*sec(
f*x+e))^(1+m)*(c-c*sec(f*x+e))^(-2-m)*tan(f*x+e)/a/f/(4*m^2+8*m+3)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.73

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx\\ =\frac{(a(1+\sec(e+fx)))^m(-2(1+m)+\sec(e+fx))(c-c\sec(e+fx))^{-m}\tan(e+fx)}{c^2f(1+2m)(3+2m)(-1+\sec(e+fx))^2}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m
),x]
```

output
```
((a*(1 + Sec[e + f*x]))^m*(-2*(1 + m) + Sec[e + f*x])*Tan[e + f*x])/(c^2*f
*(1 + 2*m)*(3 + 2*m)*(-1 + Sec[e + f*x])^2*(c - c*Sec[e + f*x])^m)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4439, 3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m-2}dx$$

$$\Big\downarrow \text{ 4439}$$

$$-\frac{\int \sec(e+fx)(\sec(e+fx)a+a)^{m+1}(c-c\sec(e+fx))^{-m-2}dx}{a(2m+1)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}}{f(2m+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{m+1}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m-2}dx}{a(2m+1)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}}{f(2m+1)}$$

$$\Big\downarrow \text{ 4438}$$

$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-2}}{af(2m+1)(2m+3)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}}{f(2m+1)}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m),x]
```

output
```
-(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)*Tan[e + f*x])/(f*(
1 + 2*m))) + ((a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-2 - m)*T
an[e + f*x])/(a*f*(1 + 2*m)*(3 + 2*m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

rule 4439
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp
[(m + n + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ
[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0
] &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$\frac{\left(\frac{c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}\right)^{-m} \left(-2 m-1+\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2} \left(3+2 m\right)\right) \left(-\frac{a}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1}\right)^{m} \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{2 f \left(1+2 m\right) \left(3+2 m\right) c^{2}}$	108

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x,method=_RETURN
VERBOSE)
```

output
```
1/2/f/(1+2*m)/(3+2*m)/c^2*(1/(tan(1/2*f*x+1/2*e)^2-1)*c*tan(1/2*f*x+1/2*e)
^2)^(-m)*(-2*m-1+tan(1/2*f*x+1/2*e)^2*(3+2*m))*(-1/(tan(1/2*f*x+1/2*e)^2-1
)*a)^m*cot(1/2*f*x+1/2*e)^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx = -\frac{(2\,(m+1)\cos\left(fx+e\right)-1)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^{m}\left(\frac{c\cos(fx+e)-c}{\cos(fx+e)}\right)^{-m-2}\sin\left(fx+e\right)}{(4\,fm^2+8\,fm+3\,f)\cos\left(fx+e\right)^2}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorit
hm="fricas")
```

output
```
-(2*(m + 1)*cos(f*x + e) - 1)*((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*co
s(f*x + e) - c)/cos(f*x + e))^(-m - 2)*sin(f*x + e)/((4*f*m^2 + 8*f*m + 3*
f)*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx = \int \left(a(\sec\left(e+fx\right)+1)\right)^{m}\left(-c(\sec\left(e+fx\right)-1)\right)^{-m-2}\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-2-m),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(-m - 2)*sec(e
 + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$$
$$= -\frac{\left((-a)^m(2m+1) - \frac{(-a)^m(2m+3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)c^{-m-2}(\cos(fx+e)+1)^3}{2(4m^2+8m+3)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{2m}\sin(fx+e)^3}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorit
hm="maxima")
```

output
```
-1/2*((-a)^m*(2*m + 1) - (-a)^m*(2*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2)*c^(-m - 2)*(cos(f*x + e) + 1)^3/((4*m^2 + 8*m + 3)*f*(sin(f*x + e)/(c
os(f*x + e) + 1))^(2*m)*sin(f*x + e)^3)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$$
$$= \int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m-2}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorit
hm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 2)*sec(f*x +
e), x)
```

Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.39

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$$
$$=\frac{\sin\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,1{\mathrm{i}}}{f\,\cos\left(e+f\,x\right)^2\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{m+2}\,\left(m^2\,4{\mathrm{i}}+m\,8{\mathrm{i}}+3{\mathrm{i}}\right)}-\frac{\sin\left(2\,e+2\,f\,x\right)\,\left(2\,m+2\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,1{\mathrm{i}}}{2\,f\,\cos\left(e+f\,x\right)^2\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{m+2}\,\left(m^2\,4{\mathrm{i}}+m\,8{\mathrm{i}}+3{\mathrm{i}}\right)}$$

input
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 2)),x)
```

output
```
(sin(e + f*x)*(a + a/cos(e + f*x))^m*1i)/(f*cos(e + f*x)^2*(c - c/cos(e +
f*x))^(m + 2)*(m*8i + m^2*4i + 3i)) - (sin(2*e + 2*f*x)*(2*m + 2)*(a + a/c
os(e + f*x))^m*1i)/(2*f*cos(e + f*x)^2*(c - c/cos(e + f*x))^(m + 2)*(m*8i
+ m^2*4i + 3i))
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$$
$$=\frac{\int \frac{(\sec(fx+e)a+a)^m\sec(fx+e)}{(-\sec(fx+e)c+c)^m\sec(fx+e)^2-2(-\sec(fx+e)c+c)^m\sec(fx+e)+(-\sec(fx+e)c+c)^m}dx}{c^2}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x)
```

output
```
int(((sec(e + f*x)*a + a)**m*sec(e + f*x))/(( - sec(e + f*x)*c + c)**m*sec
(e + f*x)**2 - 2*( - sec(e + f*x)*c + c)**m*sec(e + f*x) + ( - sec(e + f*x
)*c + c)**m),x)/c**2
```

3.164 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 47

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\tan(e+fx)}{f(1+2m)}$$

output
```
-(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m)*tan(f*x+e)/f/(1+2*m)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx = -\frac{(a(1+\sec(e+fx)))^m(c-c\sec(e+fx))^{-1-m}\tan(e+fx)}{2f\left(\frac{1}{2}+m\right)}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m
),x]
```

output
```
-1/2*((a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e + f*x])^(-1 - m)*Tan[e + f*x])
/(f*(1/2 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3042, 4438}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m-1}dx$$

$$\Big\downarrow \text{4438}$$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}}{f(2m+1)}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m),x]
```

output
```
-(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)*Tan[e + f*x])/(f*(
1 + 2*m)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4438

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

method	result	size
parallelrisch	$\frac{\left(\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}\right)^{-m}\left(-\frac{a}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}\right)^{m}\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{f(1+2m)c}$	76

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x,method=_RETURN
VERBOSE)
```

output

```
1/f/(1+2*m)/c*(1/(tan(1/2*f*x+1/2*e)^2-1)*c*tan(1/2*f*x+1/2*e)^2)^(-m)*(-1
/(tan(1/2*f*x+1/2*e)^2-1)*a)^m*cot(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$$
$$=-\frac{\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^{m}\left(\frac{c\cos(fx+e)-c}{\cos(fx+e)}\right)^{-m-1}\sin\left(fx+e\right)}{\left(2\,fm+f\right)\cos\left(fx+e\right)}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorit
hm="fricas")
```

output

```
-((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))
^(-m - 1)*sin(f*x + e)/((2*f*m + f)*cos(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$$
$$=\int (a(\sec{(e+fx)}+1))^m\,(-c(\sec{(e+fx)}-1))^{-m-1}\sec{(e+fx)}\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-1-m),x)
```

output

```
Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(-m - 1)*sec(e
 + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$$
$$=\frac{(-a)^m\,c^{-m-1}(\cos{(fx+e)}+1)}{f(2\,m+1)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{2\,m}\sin{(fx+e)}}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorit
hm="maxima")
```

output

```
(-a)^m*c^(-m - 1)*(cos(f*x + e) + 1)/(f*(2*m + 1)*(sin(f*x + e)/(cos(f*x +
 e) + 1))^(2*m)*sin(f*x + e))
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$$
$$=\int (a\sec{(fx+e)}+a)^m(-c\sec{(fx+e)}+c)^{-m-1}\sec{(fx+e)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorit
hm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 1)*sec(f*x +
e), x)
```

Mupad [B] (verification not implemented)

Time = 11.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx =$$
$$-\frac{\left(\sin{(e+f\,x)}+\sin{(3\,e+3\,f\,x)}\right)\left(\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}\right)^m}{c\,f\,\left(2\,m+1\right)\left(\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}\right)^m\left(3\,\cos{(e+f\,x)}-2\,\cos{(2\,e+2\,f\,x)}+\cos{(3\,e+3\,f\,x)}-2\right)}$$

input
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 1)),x)
```

output
```
-((sin(e + f*x) + sin(3*e + 3*f*x))*((a*(cos(e + f*x) + 1))/cos(e + f*x))^
m)/(c*f*(2*m + 1)*((c*(cos(e + f*x) - 1))/cos(e + f*x))^m*(3*cos(e + f*x)
- 2*cos(2*e + 2*f*x) + cos(3*e + 3*f*x) - 2))
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$$
$$= -\frac{\int \frac{(\sec(fx+e)a+a)^m \sec(fx+e)}{(-\sec(fx+e)c+c)^m \sec(fx+e)-(-\sec(fx+e)c+c)^m}dx}{c}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x)
```

output

```
( - int(((sec(e + f*x)*a + a)**m*sec(e + f*x))/(( - sec(e + f*x)*c + c)**m
*sec(e + f*x) - ( - sec(e + f*x)*c + c)**m),x))/c
```

3.165 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 101

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx =$$

$$-\frac{2^{\frac{1}{2}-m}c\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}+m,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{\frac{1}{2}+m}(a+a\,\mathrm{se}}{f(1+2m)}$$

output
```
-2^(1/2-m)*c*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f
*x+e))^(1/2+m)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m)*tan(f*x+e)/f/(1+
2*m)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx =$$

$$-\frac{2^{\frac{1}{2}+m}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1}{2}-m,\frac{3}{2}-m,\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\mathrm{s}}{f(-1+2m)}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

output

```
-((2^(1/2 + m)*Hypergeometric2F1[1/2 - m, 1/2 - m, 3/2 - m, (1 - Sec[e + f
*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*Tan[e + f*x
])/(f*(-1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 4449, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}dx$$

$$\Big\downarrow \text{4449}$$

$$-\frac{ac\tan(e+fx)\int(\sec(e+fx)a+a)^{m-\frac{1}{2}}(c-c\sec(e+fx))^{-m-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$-\frac{ac2^{-m-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{m+\frac{1}{2}}(c-c\sec(e+fx))^{-m-1}\int\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{-m-\frac{1}{2}}(\sec(e+fx)a}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{c2^{\frac{1}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m+\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}\text{Hypergeometric2F1}\left(m\right.}{f(2m+1)}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

output
```
-((2^(1/2 - m)*c*Hypergeometric2F1[1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
 f*x])/2]*(1 - Sec[e + f*x])^(1/2 + m)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e
 + f*x])^(-1 - m)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec (fx+e) (a+a\sec (fx+e))^m (-c(-1+\sec (fx+e)))^{-m} \, dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = \int \frac{(a\sec (fx+e)+a)^m \sec (fx+e)}{(-c\sec (fx+e)+c)^m}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm=
"fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/((c-c*sec(f*x+e))**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx$$
$$=\int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^m}\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm=
"maxima")
```

output

```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx$$
$$=\int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^m}\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm=
"giac")
```

output

```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^m}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^m),x)
```

output
```
int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^m), x)
```

Reduce [F]

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx\\&=\int \frac{(\sec(fx+e)\,a+a)^m\sec(fx+e)}{(-\sec(fx+e)\,c+c)^m}dx\end{aligned}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

output
```
int(((sec(e + f*x)*a + a)**m*sec(e + f*x))/( - sec(e + f*x)*c + c)**m,x)
```

3.166 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx = -\frac{2^{\frac{3}{2}-m}c\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+m,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}+m}(a+}{f(1+2m)}$$

output
```
-2^(3/2-m)*c*hypergeom([-1/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(
f*x+e))^(-1/2+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/((c-c*sec(f*x+e))
^m)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx = \frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{3}{2}-m,\frac{5}{2}-m,\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec}{f(3-2m)}$$

input

```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m)
,x]
```

output

```
(2^(1/2 + m)*Hypergeometric2F1[1/2 - m, 3/2 - m, 5/2 - m, (1 - Sec[e + f*x
])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e
+ f*x])^(1 - m)*Tan[e + f*x])/(f*(3 - 2*m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4449, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{1-m}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-m}dx$$

$$\Big\downarrow \ 4449$$

$$-\frac{ac\tan(e+fx)\int(\sec(e+fx)a+a)^{m-\frac{1}{2}}(c-c\sec(e+fx))^{\frac{1}{2}-m}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 80$$

$$-\frac{ac2^{\frac{1}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(c-c\sec(e+fx))^{-m}\int\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{\frac{1}{2}-m}(\sec(e+fx)a+a)}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 79$$

$$-\frac{c2^{\frac{3}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,\text{Hypergeometric2F1}\,(m-}{f(2m+1)}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),x]
```

output
```
-((2^(3/2 - m)*c*Hypergeometric2F1[-1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e
+ f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^{m}\,(c-c\sec(fx+e))^{1-m}\,dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$= \int (a\sec(fx+e)+a)^{m}(-c\sec(fx+e)+c)^{-m+1}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorith
m="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e
), x)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$= \int (a(\sec(e+fx)+1))^{m}\,(-c(\sec(e+fx)-1))^{1-m}\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1-m),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(1 - m)*sec(e
+ f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+1}\sec(fx+e)\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorith
m="maxima")
```

output

```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x +
e), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+1}\sec(fx+e)\,dx$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorith
m="giac")
```

output

```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x +
e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^{1-m}}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1 - m))/cos(e + f*x),x)
```

output
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1 - m))/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx = c\left(-\left(\int \frac{\left(\sec\left(fx+e\right)a+a\right)^m\sec\left(fx+e\right)^2}{\left(-\sec\left(fx+e\right)c+c\right)^m}dx\right) + \int \frac{\left(\sec\left(fx+e\right)a+a\right)^m\sec\left(fx+e\right)}{\left(-\sec\left(fx+e\right)c+c\right)^m}dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

output
```
c*( - int(((sec(e + f*x)*a + a)**m*sec(e + f*x)**2)/( - sec(e + f*x)*c + c
)**m,x) + int(((sec(e + f*x)*a + a)**m*sec(e + f*x))/( - sec(e + f*x)*c +
c)**m,x))
```

3.167 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 101

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx = -\frac{2^{\frac{5}{2}-m}c\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+m,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{3}{2}+m}(a+}{f(1+2m)}$$

output
```
-2^(5/2-m)*c*hypergeom([-3/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(
f*x+e))^(-3/2+m)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m)*tan(f*x+e)/f/(1
+2*m)
```

Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx = \frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{5}{2}-m,\frac{7}{2}-m,\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec}{f(5-2m)}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m)
,x]
```

output
```
(2^(1/2 + m)*Hypergeometric2F1[1/2 - m, 5/2 - m, 7/2 - m, (1 - Sec[e + f*x
])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e
+ f*x])^(2 - m)*Tan[e + f*x])/(f*(5 - 2*m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4449, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{2-m}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{2-m}dx$$

$$\Big\downarrow \; 4449$$

$$-\frac{ac\tan(e+fx)\int(\sec(e+fx)a+a)^{m-\frac{1}{2}}(c-c\sec(e+fx))^{\frac{3}{2}-m}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 80$$

$$-\frac{ac^2 2^{\frac{3}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(c-c\sec(e+fx))^{-m}\int\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{\frac{3}{2}-m}(\sec(e+fx)a+a}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 79$$

$$-\frac{c^2 2^{\frac{5}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\text{Hypergeometric2F1}\left(m\right.}{f(2m+1)}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),x]
```

output

```
-((2^(5/2 - m)*c^2*Hypergeometric2F1[-3/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[
e + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e +
f*x])/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4449

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a +
 b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{2-m}dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{m}(c-c\sec(e+fx))^{2-m}\,dx = \int (a\sec\left(fx+e\right)+a)^{m}(-c\sec\left(fx+e\right)+c)^{-m+2}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorith
m="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e
), x)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{m}(c-c\sec(e+fx))^{2-m}\,dx = \int \left(a(\sec\left(e+fx\right)+1)\right)^{m}\left(-c(\sec\left(e+fx\right)-1)\right)^{2-m}\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(2-m),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(2 - m)*sec(e
+ f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+2}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorith
m="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x +
e), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+2}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorith
m="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x +
e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{2-m}}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(2 - m))/cos(e + f*x),x)
```

output
```
int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(2 - m))/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$=c^2\left(\int \frac{\left(\sec\left(fx+e\right)a+a\right)^m\sec\left(fx+e\right)^3}{\left(-\sec\left(fx+e\right)c+c\right)^m}dx\right.$$
$$-2\left(\int \frac{\left(\sec\left(fx+e\right)a+a\right)^m\sec\left(fx+e\right)^2}{\left(-\sec\left(fx+e\right)c+c\right)^m}dx\right)$$
$$\left.+\int \frac{\left(\sec\left(fx+e\right)a+a\right)^m\sec\left(fx+e\right)}{\left(-\sec\left(fx+e\right)c+c\right)^m}dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
```

output
```
c**2*(int(((sec(e + f*x)*a + a)**m*sec(e + f*x)**3)/( - sec(e + f*x)*c + c
)**m,x) - 2*int(((sec(e + f*x)*a + a)**m*sec(e + f*x)**2)/( - sec(e + f*x)
*c + c)**m,x) + int(((sec(e + f*x)*a + a)**m*sec(e + f*x))/( - sec(e + f*x
)*c + c)**m,x))
```

3.168 $\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 105

$$\begin{aligned}
&\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx \\
&= \frac{a^3 c\operatorname{arctanh}(\sin(e+fx))}{4f} + \frac{a^3 c\sec(e+fx)\tan(e+fx)}{4f} \\
&\quad - \frac{a^3 c\sec^3(e+fx)\tan(e+fx)}{2f} - \frac{2a^3 c\tan^3(e+fx)}{3f} - \frac{a^3 c\tan^5(e+fx)}{5f}
\end{aligned}$$

output

```
1/4*a^3*c*arctanh(sin(f*x+e))/f+1/4*a^3*c*sec(f*x+e)*tan(f*x+e)/f-1/2*a^3*
c*sec(f*x+e)^3*tan(f*x+e)/f-2/3*a^3*c*tan(f*x+e)^3/f-1/5*a^3*c*tan(f*x+e)^
5/f
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{a^3c(15\text{arctanh}(\sin(e+fx))-\tan(e+fx)\left(-15\sec(e+fx)+30\sec^3(e+fx)+40\tan^2(e+fx)+12\,\text{t}}{60f}$$

input

```
Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

output

```
(a^3*c*(15*ArcTanh[Sin[e + f*x]] - Tan[e + f*x]*(-15*Sec[e + f*x] + 30*Sec
[e + f*x]^3 + 40*Tan[e + f*x]^2 + 12*Tan[e + f*x]^4)))/(60*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.094$, Rules used = {3042, 4450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{4450}$$
$$-ac\int \left(a^2\tan^2(e+fx)\sec^4(e+fx)+2a^2\tan^2(e+fx)\sec^3(e+fx)+a^2\tan^2(e+fx)\sec^2(e+fx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-ac\left(-\frac{a^2\text{arctanh}(\sin(e+fx))}{4f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{2a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)\sec^3(e+fx)}{2f}-\frac{a^2\tan(}{}\right.$$

input
```
Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

output
```
-(a*c*(-1/4*(a^2*ArcTanh[Sin[e + f*x]])/f - (a^2*Sec[e + f*x]*Tan[e + f*x]
)/(4*f) + (a^2*Sec[e + f*x]^3*Tan[e + f*x])/(2*f) + (2*a^2*Tan[e + f*x]^3)
/(3*f) + (a^2*Tan[e + f*x]^5)/(5*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4450
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp
[((-a)*c)^m   Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*
csc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x]
&& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m,
0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^3c\tan(fx+e)+2a^3c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-2a^3c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+}{f}$
default	$\frac{a^3c\tan(fx+e)+2a^3c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-2a^3c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+}{f}$
parts	$\frac{a^3c\tan(fx+e)}{f}+\frac{a^3c\sec(fx+e)\tan(fx+e)}{f}+\frac{a^3c\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{2a^3c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx}{8}}{}$
norman	$\frac{\frac{a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}+\frac{25a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{64a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}+\frac{7a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3f}-\frac{a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{2f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}-\frac{a^3c\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{4f}$
risch	$-\frac{ia^3c(15\,\mathrm{e}^{9i(fx+e)}-60\,\mathrm{e}^{8i(fx+e)}-90\,\mathrm{e}^{7i(fx+e)}-240\,\mathrm{e}^{6i(fx+e)}-40\,\mathrm{e}^{4i(fx+e)}+90\,\mathrm{e}^{3i(fx+e)}-80\,\mathrm{e}^{2i(fx+e)}-15\,\mathrm{e}^{i(fx+e}}{30f\left(\mathrm{e}^{2i(fx+e)}+1\right)^5}$
parallelrisch	$-\frac{3a^3c\left(\left(\frac{5\cos(fx+e)}{6}+\frac{5\cos(3fx+3e)}{12}+\frac{\cos(5fx+5e)}{12}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{5\cos(fx+e)}{6}-\frac{5\cos(3fx+3e)}{12}-\frac{\cos(5fx+5e)}{12}\right.\right.}{f(\cos(5fx+5e)+5\cos(3fx+3e)+}$

input
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBO
SE)
```

output
```
1/f*(a^3*c*tan(f*x+e)+2*a^3*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)
+tan(f*x+e)))-2*a^3*c*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*
ln(sec(f*x+e)+tan(f*x+e)))+a^3*c*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2
)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{15\,a^3c\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)+2\left(28\,a^3c\cos\left(\right.\right.}{120\,f\cos\left(fx\right.}$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="f
ricas")
```

output
```
1/120*(15*a^3*c*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x +
e)^5*log(-sin(f*x + e) + 1) + 2*(28*a^3*c*cos(f*x + e)^4 + 15*a^3*c*cos(f*
x + e)^3 - 16*a^3*c*cos(f*x + e)^2 - 30*a^3*c*cos(f*x + e) - 12*a^3*c)*sin
(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -a^3c\left(\int\left(-\sec^2(e+fx)\right)dx+\int\left(-2\sec^3(e+fx)\right)dx+\int 2\sec^5(e+fx)\,dx + \int \sec^6(e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

output
```
-a**3*c*(Integral(-sec(e + f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) +
 Integral(2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.64

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\frac{8\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c-15\,a^3c\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\,\mathrm{lo}\right.}{}$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="m
axima")
```

output
```
-1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c -
 15*a^3*c*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 60*a
^3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(si
n(f*x + e) - 1)) - 120*a^3*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.38

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$

$$=\frac{15\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-70\,a^3c\tan\left(\frac{1}{2}\right.\right.}{}}{60\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="g
iac")
```

output
```
1/60*(15*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c*log(abs(tan(1
/2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c*tan(1/2*f*x + 1/2*e)^9 - 70*a^3*c*tan(
1/2*f*x + 1/2*e)^7 + 128*a^3*c*tan(1/2*f*x + 1/2*e)^5 - 250*a^3*c*tan(1/2*
f*x + 1/2*e)^3 - 15*a^3*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 -
1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 15.01 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$

$$=\frac{-\frac{c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{2}+\frac{7\,c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3}-\frac{64\,c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}+\frac{25\,c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+\frac{c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$
$$+\frac{a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}$$

input
```
int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

output
```
((a^3*c*tan(e/2 + (f*x)/2))/2 + (25*a^3*c*tan(e/2 + (f*x)/2)^3)/3 - (64*a^
3*c*tan(e/2 + (f*x)/2)^5)/15 + (7*a^3*c*tan(e/2 + (f*x)/2)^7)/3 - (a^3*c*t
an(e/2 + (f*x)/2)^9)/2)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)
^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)
^10 - 1)) + (a^3*c*atanh(tan(e/2 + (f*x)/2)))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.30

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$= \frac{a^3c\left(-15\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^4+30\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(f\right.}{}$$

input
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x)
```

output
```
(a**3*c*( - 15*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 + 30
*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 - 15*cos(e + f*x)*
log(tan((e + f*x)/2) - 1) + 15*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(
e + f*x)**4 - 30*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 +
15*cos(e + f*x)*log(tan((e + f*x)/2) + 1) - 15*cos(e + f*x)*sin(e + f*x)**
3 - 15*cos(e + f*x)*sin(e + f*x) + 28*sin(e + f*x)**5 - 40*sin(e + f*x)**3
))/(60*cos(e + f*x)*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.169 $\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 86

$$\begin{aligned}\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx\\ =\frac{a^2c\,\text{arctanh}(\sin(e+fx))}{8f}+\frac{a^2c\sec(e+fx)\tan(e+fx)}{8f}\\ -\frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f}-\frac{a^2c\tan^3(e+fx)}{3f}\end{aligned}$$

output
```
1/8*a^2*c*arctanh(sin(f*x+e))/f+1/8*a^2*c*sec(f*x+e)*tan(f*x+e)/f-1/4*a^2*
c*sec(f*x+e)^3*tan(f*x+e)/f-1/3*a^2*c*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\begin{aligned}\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx\\ =\frac{a^2c(3\text{arctanh}(\sin(e+fx))+\tan(e+fx)\,(3\sec(e+fx)-6\sec^3(e+fx)-8\tan^2(e+fx)))}{24f}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

output
```
(a^2*c*(3*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*Sec[e + f*x] - 6*Sec[e +
 f*x]^3 - 8*Tan[e + f*x]^2)))/(24*f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 4450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4450}$$

$$-ac\int \left(a\tan^2(e+fx)\sec^3(e+fx)+a\tan^2(e+fx)\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ac\left(-\frac{a\text{arctanh}(\sin(e+fx))}{8f}+\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{a\tan(e+fx)\sec(e+fx)}{8f}\right)$$

input
```
Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

output
```
-(a*c*(-1/8*(a*ArcTanh[Sin[e + f*x]])/f - (a*Sec[e + f*x]*Tan[e + f*x])/(8
*f) + (a*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (a*Tan[e + f*x]^3)/(3*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4450
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp
[((-a)*c)^m   Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*
csc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x]
&& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m,
0] && GtQ[m*n, 0]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{a^2 c \tan(fx+e)+a^2 c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^2 c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-a^2 c(-(-}{f}$
default	$\frac{a^2 c \tan(fx+e)+a^2 c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^2 c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-a^2 c(-(-}{f}$
parts	$\frac{a^2 c \tan(fx+e)}{f}+\frac{a^2 c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{a^2 c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{a^2}{}$
norman	$\frac{-\frac{a^2 c \tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{53 a^2 c \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{12f}+\frac{11 a^2 c \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}-\frac{a^2 c \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{a^2 c \ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8f}+\frac{a^2 c \ln}{}$
risch	$-\frac{i a^2 c\left(3\,\mathrm{e}^{7i(fx+e)}-24\,\mathrm{e}^{6i(fx+e)}-21\,\mathrm{e}^{5i(fx+e)}-24\,\mathrm{e}^{4i(fx+e)}+21\,\mathrm{e}^{3i(fx+e)}-8\,\mathrm{e}^{2i(fx+e)}-3\,\mathrm{e}^{i(fx+e)}-8\right)}{12f\left(\mathrm{e}^{2i(fx+e)}+1\right)^4}+\frac{a^2 c \ln(\mathrm{e}^{i}}{8}$
parallelrisch	$\frac{a^2\left(\left(-2\cos(2fx+2e)-\frac{\cos(4fx+4e)}{2}-\frac{3}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(2\cos(2fx+2e)+\frac{\cos(4fx+4e)}{2}+\frac{3}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.}{4f(\cos(4fx+4e)+4\cos(2fx+2e)+3)}$

input
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBO
SE)
```

output
```
1/f*(a^2*c*tan(f*x+e)+a^2*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+t
an(f*x+e)))+a^2*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-a^2*c*(-(-1/4*sec(f*x
+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{3\,a^2c\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)+2\,(8\,a^2c\cos\,(fx}{48\,f\cos\left(fx+e\right)^4}$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="f
ricas")
```

output
```
1/48*(3*a^2*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^
4*log(-sin(f*x + e) + 1) + 2*(8*a^2*c*cos(f*x + e)^3 + 3*a^2*c*cos(f*x + e
)^2 - 8*a^2*c*cos(f*x + e) - 6*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= -a^2c\left(\int\left(-\sec^2\left(e+fx\right)\right)\,dx+\int\left(-\sec^3\left(e+fx\right)\right)\,dx+\int\sec^4\left(e+fx\right)dx+\int\sec^5\left(e+fx\right)dx\right)$$

input
```
integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

output
```
-a**2*c*(Integral(-sec(e + f*x)**2, x) + Integral(-sec(e + f*x)**3, x) + I
ntegral(sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(78) = 156$.

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.86

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx =$$

$$-\frac{16\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2c-3a^2c\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+\right.}{}$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="m
axima")
```

output
```
-1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*(3*sin(f*x
+ e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(s
in(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 12*a^2*c*(2*sin(f*x + e)/(si
n(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^
2*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$

$$=\frac{3a^2c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-3a^2c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(3a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-11a^2c\tan\left(\frac{1}{2}fx+\right.\right.}{\left(\tan\left(\right.\right.}}{24f}$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="g
iac")
```

output
```
1/24*(3*a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^2*c*log(abs(tan(1/2
*f*x + 1/2*e) - 1)) - 2*(3*a^2*c*tan(1/2*f*x + 1/2*e)^7 - 11*a^2*c*tan(1/2
*f*x + 1/2*e)^5 + 53*a^2*c*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*tan(1/2*f*x +
1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.70

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f} - \frac{\frac{c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{11\,c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{53\,c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

input
```
int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

output
```
(a^2*c*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((a^2*c*tan(e/2 + (f*x)/2))/4 +
(53*a^2*c*tan(e/2 + (f*x)/2)^3)/12 - (11*a^2*c*tan(e/2 + (f*x)/2)^5)/12 +
(a^2*c*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (
f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.12

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{a^2c\left(-8\cos\left(fx+e\right)\sin\left(fx+e\right)^3-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^4+6\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\right.}{}$$

input
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

output
```
(a**2*c*( - 8*cos(e + f*x)*sin(e + f*x)**3 - 3*log(tan((e + f*x)/2) - 1)*s
in(e + f*x)**4 + 6*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 - 3*log(tan((
e + f*x)/2) - 1) + 3*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4 - 6*log(tan
((e + f*x)/2) + 1)*sin(e + f*x)**2 + 3*log(tan((e + f*x)/2) + 1) - 3*sin(e
 + f*x)**3 - 3*sin(e + f*x)))/(24*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 +
 1))
```

3.170 $\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac\tan^3(e+fx)}{3f}$$

output
```
-1/3*a*c*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac\tan^3(e+fx)}{3f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

output
```
-1/3*(a*c*Tan[e + f*x]^3)/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4450, 3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4450}$$

$$-ac\int \sec^2(e+fx)\tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \sec(e+fx)^2\tan(e+fx)^2dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{ac\int \tan^2(e+fx)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{15}$$

$$-\frac{ac\tan^3(e+fx)}{3f}$$

input
```
Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

output
```
-1/3*(a*c*Tan[e + f*x]^3)/f
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 4450
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp
[((-a)*c)^m   Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*
csc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x]
&& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m,
0] && GtQ[m*n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

method	result	size
norman	$\frac{8ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$	34
risch	$\frac{2iac\left(3\,\mathrm{e}^{4i(fx+e)}+1\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^{3}}$	35
derivativedivides	$\frac{ac\tan(fx+e)+ac\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$	36
default	$\frac{ac\tan(fx+e)+ac\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$	36
parts	$\frac{ac\tan(fx+e)}{f}+\frac{ac\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$	38
parallelrisch	$\frac{8ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}$	45

input

```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output

```
8/3/f*a*c*tan(1/2*f*x+1/2*e)^3/(tan(1/2*f*x+1/2*e)^2-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \sec^{2}(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\frac{\left(ac\cos\left(fx+e\right)^{2}-ac\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^{3}}$$

input

```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
1/3*(a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

Time = 0.90 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.00

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = \begin{cases} \frac{-ac\left(\frac{\tan^3(e+fx)}{3}+\tan(e+fx)\right)+ac\tan(e+fx)}{f} & \text{for } f\neq 0 \\ x(a\sec(e)+a)(-c\sec(e)+c)\sec^2(e) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

output
```
Piecewise(((-a*c*(tan(e + f*x)**3/3 + tan(e + f*x)) + a*c*tan(e + f*x))/f,
 Ne(f, 0)), (x*(a*sec(e) + a)*(-c*sec(e) + c)*sec(e)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{\left(\tan(fx+e)^3+3\tan(fx+e)\right)ac-3\,ac\tan(fx+e)}{3\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a*c - 3*a*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx=-\frac{ac\tan{(fx+e)}^3}{3\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
-1/3*a*c*tan(f*x + e)^3/f
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx=-\frac{a\,c\tan(e+f\,x)^3}{3\,f}$$

input
```
int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

output
```
-(a*c*tan(e + f*x)^3)/(3*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\frac{\sin{(fx+e)}^3\,ac}{3\cos{(fx+e)}\,f\left(\sin{(fx+e)}^2-1\right)}$$

input
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

output
```
(sin(e + f*x)**3*a*c)/(3*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.171 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 56

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx=\frac{2c\,\text{arctanh}(\sin(e+fx))}{af}-\frac{c\tan(e+fx)}{af}-\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output

```
2*c*arctanh(sin(f*x+e))/a/f-c*tan(f*x+e)/a/f-2*c*tan(f*x+e)/f/(a+a*sec(f*x
+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(56) = 112.

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.75

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\Bigl(\frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{\sin}{f(\cos(\frac{1}{2}(e+f}}{a}$$

input

```
Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

output
```
-((c*((2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])/f - (2*Log[Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]])/f + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])) + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
) + (2*Tan[(e + f*x)/2])/f))/a)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3042, 4496, 25, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4496}$$

$$-\frac{\int -\sec(e+fx)(2ac-ac\sec(e+fx))dx}{a^2}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sec(e+fx)(2ac-ac\sec(e+fx))dx}{a^2}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(2ac-ac\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{2ac\int \sec(e+fx)dx-ac\int \sec^2(e+fx)dx}{a^2}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2ac\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-ac\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{ac\int 1d(-\tan(e+fx))}{f}+2ac\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{a^2}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{2ac\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{ac\tan(e+fx)}{f}}{a^2}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{2ac\text{arctanh}(\sin(e+fx))}{f}-\frac{ac\tan(e+fx)}{f}}{a^2}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

input
```
Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

output
```
(-2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) + ((2*a*c*ArcTanh[Sin[e + f*x
]])/f - (a*c*Tan[e + f*x])/f)/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{2c\left(-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2}-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2}\right)}{fa}$
default	$\frac{2c\left(-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2}-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2}\right)}{fa}$
parallelrisch	$-\frac{c\left(2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cos(fx+e)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af\cos(fx+e)}$
risch	$-\frac{2ic(2\,\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}+3)}{fa\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)}-\frac{2c\ln(\mathrm{e}^{i(fx+e)}-i)}{fa}+\frac{2c\ln(\mathrm{e}^{i(fx+e)}+i)}{fa}$
norman	$\frac{-\frac{4c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}+\frac{6c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{fa}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{fa}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}-\frac{2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{fa}+\frac{2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{fa}$

input
```
int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
2/f*c/a*(-tan(1/2*f*x+1/2*e)+1/2/(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2
*e)-1)+ln(tan(1/2*f*x+1/2*e)+1)+1/2/(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.88

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$$

$$= \frac{(c\cos(fx+e)^2 + c\cos(fx+e))\log(\sin(fx+e)+1) - (c\cos(fx+e)^2 + c\cos(fx+e))\log(-\sin(f}{af\cos(fx+e)^2 + af\cos(fx+e)}$$

input
```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
((c*cos(f*x + e)^2 + c*cos(f*x + e))*log(sin(f*x + e) + 1) - (c*cos(f*x +
e)^2 + c*cos(f*x + e))*log(-sin(f*x + e) + 1) - (3*c*cos(f*x + e) + c)*sin
(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(\int \left(-\frac{\sec^2(e+fx)}{\sec(e+fx)+1}\right)dx + \int \frac{\sec^3(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

input
```
integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

output
```
-c*(Integral(-sec(e + f*x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*
x)**3/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(56) = 112$.

Time = 0.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.46

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$$

$$= \frac{c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a} - \frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)} - \frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right) + c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right.}{a}\right.}{f}$$

input
```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
(c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) +
1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*(log(
sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$$

$$= \frac{2\left(\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a} - \frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a} - \frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a} + \frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a}\right)}{f}$$

input
```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
2*(c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - c*log(abs(tan(1/2*f*x + 1/2*e)
 - 1))/a - c*tan(1/2*f*x + 1/2*e)/a + c*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x
 + 1/2*e)^2 - 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{4\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f} - \frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)} - \frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}$$

input
```
int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))),x)
```

output
```
(4*c*atanh(tan(e/2 + (f*x)/2)))/(a*f) - (2*c*tan(e/2 + (f*x)/2))/(f*(a - a
*tan(e/2 + (f*x)/2)^2)) - (2*c*tan(e/2 + (f*x)/2))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{c(-2\cos(fx+e)\log(\tan(\frac{fx}{2}+\frac{e}{2})-1)\sin(fx+e)+2\cos(fx+e)\log(\tan(\frac{fx}{2}+\frac{e}{2})+1)\sin(fx+e}{\cos(fx+e)\sin(fx+e)\,af}$$

input
```
int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

output
```
(c*( - 2*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x) + 2*cos(e + f
*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x) - 2*cos(e + f*x) - 3*sin(e + f*
x)**2 + 2))/(cos(e + f*x)*sin(e + f*x)*a*f)
```

3.172 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 70

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\operatorname{arctanh}(\sin(e+fx))}{a^2f} + \frac{7c\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

output
```
-c*arctanh(sin(f*x+e))/a^2/f+7/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-2/3*c*t
an(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\left(\coth^{-1}(\sin(e+fx))(1+\sec(e+fx))^2+2\operatorname{arctanh}(\sin(e+fx))(1+\sec(e+fx))^2-(5+7\sec(e+\right.}{3a^2f(1+\sec(e+fx))^2}$$

input
```
Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(c*(ArcCoth[Sin[e + f*x]]*(1 + Sec[e + f*x])^2 + 2*ArcTanh[Sin[e + f*
x]]*(1 + Sec[e + f*x])^2 - (5 + 7*Sec[e + f*x])*Tan[e + f*x]))/(a^2*f*(1 +
 Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4496, 25, 3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4496}$$

$$-\frac{\int -\frac{\sec(e+fx)(4ac-3ac\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(e+fx)(4ac-3ac\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(4ac-3ac\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{4486}$$

$$\frac{7ac\int \frac{\sec(e+fx)}{\sec(e+fx)a+a}dx-3c\int \sec(e+fx)dx}{3a^2}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{7ac\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-3c\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{3a^2}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \textcolor{blue}{4257}$$

$$\frac{7ac\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{3c\operatorname{arctanh}(\sin(e+fx))}{f}}{3a^2}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \textcolor{blue}{4281}$$

$$\frac{\frac{7ac\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{3c\operatorname{arctanh}(\sin(e+fx))}{f}}{3a^2}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

input

```
Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

output

```
(-2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((-3*c*ArcTanh[Sin[e +
f*x]])/f + (7*a*c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])))/(3*a^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)}{f\,a^{2}}$
default	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)}{f\,a^{2}}$
parallelrisch	$\frac{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f\,a^{2}}$
risch	$\frac{2ic\left(3\,\mathrm{e}^{2i(fx+e)}+12\,\mathrm{e}^{i(fx+e)}+5\right)}{3f\,a^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}}-\frac{c\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{f\,a^{2}}+\frac{c\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{f\,a^{2}}$
norman	$\frac{\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}-\frac{11c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3fa}+\frac{4c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{3fa}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{3fa}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}a}+\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{f\,a^{2}}-\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}{f\,a^{2}}$

input
```
int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/f*c/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)+ln(tan(1/2*f*x+1/
2*e)-1)-ln(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{3\,(c\cos{(fx+e)}^2+2\,c\cos{(fx+e)}+c)\log{(\sin{(fx+e)}+1)}-3\,(c\cos{(fx+e)}^2+2\,c\cos{(fx+e)}}{6\,(a^2f\cos{(fx+e)}^2+2\,a^2f\cos{(fx+e)}}$$

input
```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
-1/6*(3*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) -
3*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(-sin(f*x + e) + 1) - 2*(5*
c*cos(f*x + e) + 7*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*
x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\left(\int\left(-\frac{\sec^2{(e+fx)}}{\sec^2{(e+fx)}+2\sec{(e+fx)}+1}\right)dx+\int\frac{\sec^3{(e+fx)}}{\sec^2{(e+fx)}+2\sec{(e+fx)}+1}\,dx\right)}{a^2}$$

input
```
integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

output
```
-c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(66) = 132$.

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{c\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right) + \frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

input
```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + c*(3*sin(f*x + e)/(cos(f*x + e) + 1
) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$
$$= -\frac{\frac{3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} - \frac{3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} - \frac{a^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6a^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6}}{3f}$$

input
```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
-1/3*(3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 3*c*log(abs(tan(1/2*f*x
 + 1/2*e) - 1))/a^2 - (a^4*c*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c*tan(1/2*f*x
+ 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.92 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{c\left(6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\right)}{3\,a^2\,f}$$

input
```
int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))^2),x)
```

output
```
(c*(6*tan(e/2 + (f*x)/2) - 6*atanh(tan(e/2 + (f*x)/2)) + tan(e/2 + (f*x)/2
)^3))/(3*a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{c\left(3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^2f}$$

input
```
int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

output
```
(c*(3*log(tan((e + f*x)/2) - 1) - 3*log(tan((e + f*x)/2) + 1) + tan((e + f
*x)/2)**3 + 6*tan((e + f*x)/2)))/(3*a**2*f)
```

3.173 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{4c\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}$$

output

```
-2/5*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+11/15*c*tan(f*x+e)/a/f/(a+a*sec(f*x
+e))^2-4/15*c*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.50

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{c(4+\cos(e+fx))\sec^2\left(\frac{1}{2}(e+fx)\right)\tan^3\left(\frac{1}{2}(e+fx)\right)}{30a^3f}$$

input
```
Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

output
```
-1/30*(c*(4 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/(a^3*f)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 4496, 25, 3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4496}$$

$$-\frac{\int -\frac{\sec(e+fx)(6ac-5ac\sec(e+fx))}{(\sec(e+fx)a+a)^2}dx}{5a^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(e+fx)(6ac-5ac\sec(e+fx))}{(\sec(e+fx)a+a)^2}dx}{5a^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(6ac-5ac\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\frac{11ac\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}-\frac{4}{3}c\int \frac{\sec(e+fx)}{\sec(e+fx)a+a}dx}{5a^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{11ac\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}-\frac{4}{3}c\int \frac{\csc(e+fx+\frac{\pi}{2})}{\csc(e+fx+\frac{\pi}{2})a+a}dx}{5a^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\frac{11ac\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}-\frac{4c\tan(e+fx)}{3f(a\sec(e+fx)+a)}}{5a^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

input

```
Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

output

```
(-2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((11*a*c*Tan[e + f*x])/
(3*f*(a + a*Sec[e + f*x])^2) - (4*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x]
)))/(5*a^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4488

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.42

method	result	size
parallelrisch	$-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+5\right)}{30f\,a^{3}}$	36
derivativedivides	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}\right)}{2f\,a^{3}}$	37
default	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}\right)}{2f\,a^{3}}$	37
risch	$\frac{2ic\left(15\,\mathrm{e}^{3i(fx+e)}-5\,\mathrm{e}^{2i(fx+e)}+5\,\mathrm{e}^{i(fx+e)}+1\right)}{15f\,a^{3}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}}$	59
norman	$\frac{-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{6fa}+\frac{7c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{30fa}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{30fa}-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{10fa}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}a^{2}}$	101

input
```
int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
-1/30*c*tan(1/2*f*x+1/2*e)^3*(3*tan(1/2*f*x+1/2*e)^2+5)/f/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\left(c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)-4\,c\right)\sin\left(fx+e\right)}{15\,\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

input
```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/15*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e) - 4*c)*sin(f*x + e)/(a^3*f*cos(f
*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{c\left(\int\left(-\frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

input
```
integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

output
```
-c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec
(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{3c\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

input
```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
-1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x
+ e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.43

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+5c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}{30a^3f}$$

input
```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
-1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 + 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.41

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+5\right)}{30\,a^3\,f}$$

input
```
int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))^3),x)
```

output
```
-(c*tan(e/2 + (f*x)/2)^3*(3*tan(e/2 + (f*x)/2)^2 + 5))/(30*a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.41

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 c\left(-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-5\right)}{30a^3f}$$

input
```
int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

output
```
(tan((e + f*x)/2)**3*c*( - 3*tan((e + f*x)/2)**2 - 5))/(30*a**3*f)
```

3.174 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 140

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^p\tan^3(e+fx)}{3f} - \frac{a^2c\cos^2(e+fx)^{\frac{4+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{4+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^{1+p}\tan^3(e+fx)}{3fg}$$

output

```
-1/3*a^2*c*(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([3/2, 3/2+1/2*p],[5/2],sin
(f*x+e)^2)*(g*sec(f*x+e))^p*tan(f*x+e)^3/f-1/3*a^2*c*(cos(f*x+e)^2)^(2+1/2
*p)*hypergeom([3/2, 2+1/2*p],[5/2],sin(f*x+e)^2)*(g*sec(f*x+e))^(p+1)*tan(
f*x+e)^3/f/g
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 325 vs. $2(140) = 280$.

Time = 2.16 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.32

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$

$$= \frac{a^2\csc^2\left(\frac{1}{2}(e+fx)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^p(1+\sec(e+fx))^2(c-c\sec(e+fx))\Big(-2\cos^3(e+}{}$$

input
```
Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x
]
```

output
```
(a^2*Csc[(e + f*x)/2]^2*Sec[(e + f*x)/2]^4*(g*Sec[e + f*x])^p*(1 + Sec[e +
 f*x])^2*(c - c*Sec[e + f*x])*(-2*Cos[e + f*x]^3*(Cos[e + f*x]^2)^(p/2)*(2
*Hypergeometric2F1[1/2, (2 + p)/2, 3/2, Sin[e + f*x]^2] - Hypergeometric2F
1[1/2, (4 + p)/2, 3/2, Sin[e + f*x]^2])*Sin[e + f*x] - ((Sec[e + f*x]^2)^(
-1 - p/2)*((4 + p)*Hypergeometric2F1[1/2, 1 - p/2, 3/2, -Tan[e + f*x]^2] -
 3*(Sec[e + f*x]^2)^(p/2))*Sin[e + f*x])/(1 + p) - (Hypergeometric2F1[1/2,
 (2 + p)/2, (4 + p)/2, Sec[e + f*x]^2]*Sin[e + f*x])/((2 + p)*Sqrt[-Tan[e
+ f*x]^2]) + (2*Cot[e + f*x]*Hypergeometric2F1[1/2, (3 + p)/2, (5 + p)/2,
Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2])/(3 + p)))/(32*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {3042, 4450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))(g\sec(e+fx))^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a \csc \left(e+fx+\frac{\pi}{2}\right)+a\right)^2 \left(c-c \csc \left(e+fx+\frac{\pi}{2}\right)\right) \left(g \csc \left(e+fx+\frac{\pi}{2}\right)\right)^p dx$$

$$\Big\downarrow \text{4450}$$

$$-ac\int \left(a \tan^2(e+fx)(g\sec(e+fx))^p+a\sec(e+fx)\tan^2(e+fx)(g\sec(e+fx))^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ac\left(\frac{a\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p \text{Hypergeometric2F1}\left(\frac{3}{2},\frac{p+3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f}+\frac{a\tan^3(e}{}\right.$$

input
```
Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

output
```
-(a*c*((a*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5
/2, Sin[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/(3*f) + (a*(Cos[e +
 f*x]^2)^((4 + p)/2)*Hypergeometric2F1[3/2, (4 + p)/2, 5/2, Sin[e + f*x]^2
]*(g*Sec[e + f*x])^(1 + p)*Tan[e + f*x]^3)/(3*f*g)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4450
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp
[((-a)*c)^m   Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*
csc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x]
&& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m,
0] && GtQ[m*n, 0]
```

Maple [F]

$$\int (g\sec(fx+e))^{p}(a+a\sec(fx+e))^{2}(c-c\sec(fx+e))\,dx$$

input

```
int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

output

```
int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

Fricas [F]

$$\int (g\sec(e+fx))^{p}(a+a\sec(e+fx))^{2}(c-c\sec(e+fx))\,dx$$
$$=\int -(a\sec(fx+e)+a)^{2}(c\sec(fx+e)-c)(g\sec(fx+e))^{p}\,dx$$

input

```
integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorith
m="fricas")
```

output

```
integral(-(a^2*c*sec(f*x + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e
) - a^2*c)*(g*sec(f*x + e))^p, x)
```

Sympy [F]

$$\int (g\sec(e+fx))^{p}(a+a\sec(e+fx))^{2}(c-c\sec(e+fx))\,dx$$
$$=-a^{2}c\left(\int\left(-(g\sec(e+fx))^{p}\right)dx+\int\left(-(g\sec(e+fx))^{p}\sec(e+fx)\right)dx\right.$$
$$\left.+\int(g\sec(e+fx))^{p}\sec^{2}(e+fx)\,dx+\int(g\sec(e+fx))^{p}\sec^{3}(e+fx)\,dx\right)$$

input

```
integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

output
```
-a**2*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral(-(g*sec(e + f*x))**p
*sec(e + f*x), x) + Integral((g*sec(e + f*x))**p*sec(e + f*x)**2, x) + Int
egral((g*sec(e + f*x))**p*sec(e + f*x)**3, x))
```

Maxima [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \int -(a\sec(fx+e)+a)^2(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

input
```
integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorith
m="maxima")
```

output
```
-integrate((a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
 x)
```

Giac [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \int -(a\sec(fx+e)+a)^2(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

input
```
integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorith
m="giac")
```

output
```
integrate(-(a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
 x)
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx\\&=\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^2\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)\left(\frac{g}{\cos\left(e+f\,x\right)}\right)^p\,dx\end{aligned}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p,x)
```

output
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p, x)
```

Reduce [F]

$$\begin{aligned}&\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx\\&=g^pa^2c\Bigg(\int \sec\left(fx+e\right)^p dx-\left(\int \sec\left(fx+e\right)^p\sec\left(fx+e\right)^3 dx\right)\\&\quad-\left(\int \sec\left(fx+e\right)^p\sec\left(fx+e\right)^2 dx\right)+\int \sec\left(fx+e\right)^p\sec\left(fx+e\right)dx\Bigg)\end{aligned}$$

input
```
int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

output
```
g**p*a**2*c*(int(sec(e + f*x)**p,x) - int(sec(e + f*x)**p*sec(e + f*x)**3,
x) - int(sec(e + f*x)**p*sec(e + f*x)**2,x) + int(sec(e + f*x)**p*sec(e +
f*x),x))
```

3.175 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 65

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac\cos^2(e+fx)^{\frac{3+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^p\tan^3(e+fx)}{3f}$$

output

```
-1/3*a*c*(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([3/2, 3/2+1/2*p],[5/2],sin(f
*x+e)^2)*(g*sec(f*x+e))^p*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac(g\sec(e+fx))^p\tan(e+fx)\left(p+\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p}{2},\frac{2+p}{2},\sec^2(e+fx)\right)}{\sqrt{-\tan^2(e+fx)}}\right)}{fp(1+p)}$$

input
```
Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

output
```
-((a*c*(g*Sec[e + f*x])^p*Tan[e + f*x]*(p + Hypergeometric2F1[1/2, p/2, (2
 + p)/2, Sec[e + f*x]^2]/Sqrt[-Tan[e + f*x]^2]))/(f*p*(1 + p)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4450, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)(c-c\sec(e+fx))(g\sec(e+fx))^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^p\,dx$$

$$\Big\downarrow \text{4450}$$

$$-ac\int (g\sec(e+fx))^p\tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int (g\sec(e+fx))^p\tan(e+fx)^2dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{ac\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{p+3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

output
```
-1/3*(a*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5
/2, Sin[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 4450
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp
[((-a)*c)^m   Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*
csc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x]
&& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m,
0] && GtQ[m*n, 0]
```

Maple [F]

$$\int (g\sec(fx+e))^p (a+a\sec(fx+e)) (c-c\sec(fx+e))\,dx$$

input
```
int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

output
```
int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

Fricas [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\int -(a\sec(fx+e)+a)(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

input
```
integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral(-(a*c*sec(f*x + e)^2 - a*c)*(g*sec(f*x + e))^p, x)
```

Sympy [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=-ac\left(\int(-(g\sec(e+fx))^p)\,dx+\int(g\sec(e+fx))^p\sec^2(e+fx)\,dx\right)$$

input
```
integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

output
```
-a*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral((g*sec(e + f*x))**p*sec
(e + f*x)**2, x))
```

Maxima [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\int -(a\sec(fx+e)+a)(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

input
```
integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
-integrate((a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x
)
```

Giac [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\int -(a\sec{(fx+e)}+a)(c\sec{(fx+e)}-c)(g\sec{(fx+e)})^p\,dx$$

input
```
integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
integrate(-(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x
)
```

Mupad [F(-1)]

Timed out.

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\int \left(a+\frac{a}{\cos{(e+f\,x)}}\right)\left(c-\frac{c}{\cos{(e+f\,x)}}\right)\left(\frac{g}{\cos{(e+f\,x)}}\right)^p\,dx$$

input
```
int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p,x)
```

output
```
int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p, x)
```

Reduce [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$= g^p ac\left(\int \sec(fx+e)^p\,dx-\left(\int \sec(fx+e)^p\sec(fx+e)^2\,dx\right)\right)$$

input
```
int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

output
```
g**p*a*c*(int(sec(e + f*x)**p,x) - int(sec(e + f*x)**p*sec(e + f*x)**2,x))
```

3.176 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 180

$$\begin{aligned}
\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = & -\frac{cg(1-2p)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^{-1+p}\sin(e+fx)}{af(1-p)\sqrt{\sin^2(e+fx)}} \\
& +\frac{2c\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(e+fx)}{af\sqrt{\sin^2(e+fx)}} \\
& -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))}
\end{aligned}$$

output

```
-c*g*(1-2*p)*hypergeom([1/2, 1/2-1/2*p],[3/2-1/2*p],cos(f*x+e)^2)*(g*sec(f
*x+e))^(-1+p)*sin(f*x+e)/a/f/(1-p)/(sin(f*x+e)^2)^(1/2)+2*c*hypergeom([1/2
, -1/2*p],[1-1/2*p],cos(f*x+e)^2)*(g*sec(f*x+e))^p*sin(f*x+e)/a/f/(sin(f*x
+e)^2)^(1/2)-2*c*(g*sec(f*x+e))^p*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 14.60 (sec) , antiderivative size = 3396, normalized size of antiderivative = 18.87

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x
]
```

output
```
(-6*c*Sec[e + f*x]^p*(g*Sec[e + f*x])^p*Tan[(e + f*x)/2]^3*(-((AppellF1[1/
2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2
]^2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/(a*f*(3*Sec[(e + f
*x)/2]^2*Sec[e + f*x]^p*(-((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, p, 1 - p, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p,
2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 +
 p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]
^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/
(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*
p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2)) + 6*p*Sec[e + f*x]^(1 + p)*Sin[e + f*x]*Tan[(e + f*...
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 4508, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))(g\sec(e+fx))^p}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^p}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \ 4508$$

$$\frac{\int (g\sec(e+fx))^p(ac(1-2p)+2acp\sec(e+fx))dx}{a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^p\left(ac(1-2p)+2acp\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \ 4274$$

$$\frac{ac(1-2p)\int (g\sec(e+fx))^pdx+\frac{2acp\int (g\sec(e+fx))^{p+1}dx}{g}}{a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \ 3042$$

$$\frac{ac(1-2p)\int \left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^pdx+\frac{2acp\int \left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{p+1}dx}{g}}{a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \ 4259$$

$$\frac{\frac{2acp\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\int\left(\frac{\cos(e+fx)}{g}\right)^{-p-1}dx}{g}+ac(1-2p)\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\int\left(\frac{\cos(e+fx)}{g}\right)^{-p}dx}{a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{\frac{2acp\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{g}\right)^{-p-1}dx}{g}+ac(1-2p)\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{g}\right)^{-p}d}{a^2}$$
$$\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{f(a\sec(e+fx)+a)}$$

$$\Big\downarrow \textcolor{blue}{3122}$$

$$\frac{\frac{2ac\sin(e+fx)(g\sec(e+fx))^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}-\frac{acg(1-2p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,\text{Hypergeometric2F1}\Big(\frac{1}{2},}{f(1-p)\sqrt{\sin^2(e+fx)}}}{a^2}$$
$$\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{f(a\sec(e+fx)+a)}$$

input
```
Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

output
```
(-((a*c*g*(1 - 2*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f
*x]^2]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(f*(1 - p)*Sqrt[Sin[e + f*x
]^2])) + (2*a*c*Hypergeometric2F1[1/2, -1/2*p, (2 - p)/2, Cos[e + f*x]^2]*
(g*Sec[e + f*x])^p*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2]))/a^2 - (2*c*(g*S
ec[e + f*x])^p*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [F]

$$\int \frac{(g\sec(fx+e))^{p}\,(c-c\sec(fx+e))}{a+a\sec(fx+e)}dx$$

input
```
int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

output
```
int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx=\int -\frac{(c\sec(fx+e)-c)(g\sec(fx+e))^p}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$$
$$=-\frac{c\left(\int\left(-\frac{(g\sec(e+fx))^p}{\sec(e+fx)+1}\right)dx+\int\frac{(g\sec(e+fx))^p\sec(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

input
```
integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

output
```
-c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x) + 1), x) + Integral((g*sec
(e + f*x))**p*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx=\int -\frac{(c\sec(fx+e)-c)(g\sec(fx+e))^p}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%%{2,[0,0,0,1]%%%}
Error: Ba
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)\,{\left(\frac{g}{\cos(e+f\,x)}\right)}^p}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x)),x)
```

output
```
int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x)), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx\\&=\frac{g^p c\left(\int \frac{\sec(fx+e)^p}{\sec(fx+e)+1}dx-\left(\int \frac{\sec(fx+e)^p\sec(fx+e)}{\sec(fx+e)+1}dx\right)\right)}{a}\end{aligned}$$

input
```
int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

output
```
(g**p*c*(int(sec(e + f*x)**p/(sec(e + f*x) + 1),x) - int((sec(e + f*x)**p*
sec(e + f*x))/(sec(e + f*x) + 1),x)))/a
```

3.177 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 226

$$\begin{aligned}\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = &-\frac{cg(3-4p)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^{-1+p}\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}\\ &+\frac{c(5-4p)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}\\ &-\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2}\end{aligned}$$

output
```
-1/3*c*g*(3-4*p)*hypergeom([1/2, 1/2-1/2*p],[3/2-1/2*p],cos(f*x+e)^2)*(g*s
ec(f*x+e))^(-1+p)*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)+1/3*c*(5-4*p)*hype
rgeom([1/2, -1/2*p],[1-1/2*p],cos(f*x+e)^2)*(g*sec(f*x+e))^p*sin(f*x+e)/a^
2/f/(sin(f*x+e)^2)^(1/2)-1/3*c*(5-4*p)*(g*sec(f*x+e))^p*tan(f*x+e)/a^2/f/(
1+sec(f*x+e))-2/3*c*(g*sec(f*x+e))^p*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.83

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx =$$

$$-\frac{c(g\sec(e+fx))^p\left(2p(1+p)\tan(e+fx)+(1+\sec(e+fx))\left(-p(1+p)(-5+4p)\tan(e+fx)-((\right.\right.}{}$$

input
```
Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2
,x]
```

output
```
-1/3*(c*(g*Sec[e + f*x])^p*(2*p*(1 + p)*Tan[e + f*x] + (1 + Sec[e + f*x])*
(-(p*(1 + p)*(-5 + 4*p)*Tan[e + f*x]) - ((-1 + p)*(1 + p)*(-3 + 4*p)*Cot[e
 + f*x]*Hypergeometric2F1[1/2, p/2, (2 + p)/2, Sec[e + f*x]^2] + (5 - 4*p)
*p^2*Csc[e + f*x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sec[e + f*x
]^2])*(1 + Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2])))/(a^2*f*p*(1 + p)*(1 + Se
c[e + f*x])^2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4508, 3042, 4508, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))(g\sec(e+fx))^p}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^p}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \; 4508$$

$$\frac{\int \frac{(g\sec(e+fx))^p(ac(3-2p)-2ac(1-p)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^p\left(ac(3-2p)-2ac(1-p)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{ 4508}$$

$$\frac{\frac{\int (g\sec(e+fx))^p\left(c(3-4p)(1-p)a^2+c(5-4p)p\sec(e+fx)a^2\right)dx}{a^2}-\frac{c(5-4p)\tan(e+fx)(g\sec(e+fx))^p}{f(\sec(e+fx)+1)}}{3a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^p\left(c(3-4p)(1-p)a^2+c(5-4p)p\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{a^2}-\frac{c(5-4p)\tan(e+fx)(g\sec(e+fx))^p}{f(\sec(e+fx)+1)}}{3a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{ 4274}$$

$$\frac{\frac{a^2c(3-4p)(1-p)\int (g\sec(e+fx))^pdx+\frac{a^2c(5-4p)p\int (g\sec(e+fx))^{p+1}dx}{g}}{a^2}-\frac{c(5-4p)\tan(e+fx)(g\sec(e+fx))^p}{f(\sec(e+fx)+1)}}{3a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{a^2c(3-4p)(1-p)\int \left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^pdx+\frac{a^2c(5-4p)p\int \left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{p+1}dx}{g}}{a^2}-\frac{c(5-4p)\tan(e+fx)(g\sec(e+fx))^p}{f(\sec(e+fx)+1)}}{3a^2}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{ 4259}$$

$$\frac{\frac{\frac{a^2c(5-4p)p\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\int \left(\frac{\cos(e+fx)}{g}\right)^{-p-1}dx}{g}+a^2c(3-4p)(1-p)\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\int \left(\frac{\cos(e+fx)}{g}\right)^{-p}dx}{a^2}-\frac{c(5-4p)\tan}{f(}}{3a^2}$$

$$\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{a^2c(5-4p)p\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{g}\right)^{-p-1}dx}{g}+a^2c(3-4p)(1-p)\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{g}\right)^{-p}dx}{a^2}-c(}{3a^2}$$

$$\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow 3122$$

$$\frac{\frac{\frac{a^2c(5-4p)\sin(e+fx)(g\sec(e+fx))^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}-\frac{a^2cg(3-4p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\right.}{f\sqrt{\sin^2(e+fx)}}}{a^2}}{3a^2}$$

$$\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

input

```
Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

output

```
(-2*c*(g*Sec[e + f*x])^p*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((-(
(a^2*c*g*(3 - 4*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*
x]^2]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])) +
(a^2*c*(5 - 4*p)*Hypergeometric2F1[1/2, -1/2*p, (2 - p)/2, Cos[e + f*x]^2]
*(g*Sec[e + f*x])^p*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2]))/a^2 - (c*(5 -
4*p)*(g*Sec[e + f*x])^p*Tan[e + f*x])/(f*(1 + Sec[e + f*x])))/(3*a^2)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [F]

$$\int \frac{(g\sec(fx+e))^{p}(c-c\sec(fx+e))}{(a+a\sec(fx+e))^{2}}dx$$

input
```
int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

output
```
int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\int -\frac{(c\sec(fx+e)-c)(g\sec(fx+e))^{p}}{(a\sec(fx+e)+a)^{2}}\,dx$$

input
```
integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorith
m="fricas")
```

output
```
integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a^2*sec(f*x + e)^2 + 2*
a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$
$$= -\frac{c\left(\int\left(-\frac{(g\sec(e+fx))^p}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{(g\sec(e+fx))^p\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

input
```
integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

output
```
-c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
x) + Integral((g*sec(e + f*x))**p*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x))/a**2
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\int -\frac{(c\sec(fx+e)-c)(g\sec(fx+e))^p}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorith
m="maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a)^2,
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\text{Exception raised: TypeError}$$

input

```
integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorith
m="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,4,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%%{4,[0,0,0,2]%%%}
 Error: B
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)\,\left(\frac{g}{\cos(e+f\,x)}\right)^p}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

input

```
int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x))^2,x)
```

output

```
int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{g^p c\left(\int \frac{\sec(fx+e)^p}{\sec(fx+e)^2+2\sec(fx+e)+1}dx-\left(\int \frac{\sec(fx+e)^p\sec(fx+e)}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)\right)}{a^2}$$

input

```
int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

output

```
(g**p*c*(int(sec(e + f*x)**p/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) - i
nt((sec(e + f*x)**p*sec(e + f*x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x
)))/a**2
```

3.178 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 81

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf}+\frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf}$$

output
```
-1/2*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)
/a^(1/2)/c/f+cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(-2+\sqrt{2}\arctan\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{-1+\sec(e+fx)}\right)}{2cf\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x
]
```

output
```
-1/2*(Cot[(e + f*x)/2]*(-2 + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2
]]*Sqrt[-1 + Sec[e + f*x]]))/(c*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4452, 27, 87, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4452}$$

$$-\frac{ac\tan(e+fx)\int \frac{\sec(e+fx)}{a(\sec(e+fx)+1)(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\tan(e+fx)\int \frac{\sec(e+fx)}{(\sec(e+fx)+1)(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{c\tan(e+fx)\left(\frac{1}{c\sqrt{c-c\sec(e+fx)}}-\frac{\int \frac{1}{(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{c\tan(e+fx)\left(\frac{\int \frac{1}{2-\frac{c-c\sec(e+fx)}{c}}d\sqrt{c-c\sec(e+fx)}}{c^2}+\frac{1}{c\sqrt{c-c\sec(e+fx)}}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 219$$

$$-\frac{c\tan(e+fx)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}+\frac{1}{c\sqrt{c-c\sec(e+fx)}}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

output

```
-((c*(ArcTanh[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]/(Sqrt[2]*c^(3/2)
) + 1/(c*Sqrt[c - c*Sec[e + f*x]]))*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*
x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4452
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Cs
c[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\left(-\frac{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)}{2}+\cot(fx+e)\right)\sqrt{a(1+\sec(fx+e))}}{cfa}$	88

input
```
int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
1/c/f*(-1/2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((-2*cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))+cot(f*x+e))/a*(a*(1+sec(f*x+e)))^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.21

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$
$$= \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+4\sqrt{\frac{a\cos(f}{\cos(}}}{4acf\sin(fx+e)} \right.$$

input
```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorith
m="fricas")
```

output
```
[1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*
x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*
(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$
$$= -\frac{\int \frac{\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

input
```
integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(sec(e + f*x)**2/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*
sec(e + f*x) + a)), x)/c
```

Maxima [F]

$$\begin{aligned}&\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx\\&=\int -\frac{\sec\left(fx+e\right)^{2}}{\sqrt{a\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)-c)}\,dx\end{aligned}$$

input
```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorith
m="maxima")
```

output
```
-integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)),
 x)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.63

$$\begin{aligned}&\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx\\&=\frac{\frac{\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2}\right)}{\sqrt{-a}\operatorname{csgn}(\cos(fx+e))}-\frac{4\sqrt{2}\sqrt{-a}}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}\right)^{2}-a\right)\operatorname{csgn}(\cos(fx+e}}{4\,f}\end{aligned}$$

input
```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorith
m="giac")
```

output
```
1/4*(sqrt(2)*log((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/
2*e)^2 + a))^2)/(sqrt(-a)*c*sgn(cos(f*x + e))) - 4*sqrt(2)*sqrt(-a)/(((sqr
t(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*c
*sgn(cos(f*x + e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$
$$=-\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c\,\cos\left(e+f\,x\right)-c\,\left(\frac{\cos(2\,e+2\,f\,x)}{2}+\frac{1}{2}\right)\right)}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))),x)
```

output
```
-int(1/((a + a/cos(e + f*x))^(1/2)*(c*cos(e + f*x) - c*(cos(2*e + 2*f*x)/2
 + 1/2))), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx=-\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2-1}dx\right)}{ac}$$

input
```
int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
( - sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**2
- 1),x))/(a*c)
```

3.179 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 104

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = -\frac{2\sqrt{a}g^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{2g\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{cf}$$

output
```
-2*a^(1/2)*g^(3/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)
/(a+a*sec(f*x+e))^(1/2))/c/f+2*g*cot(f*x+e)*(g*sec(f*x+e))^(1/2)*(a+a*sec(
f*x+e))^(1/2)/c/f
```

Mathematica [A] (verified)

Time = 3.67 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.56

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx=\frac{2\cot\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^{3/2}\sqrt{a(1+\sec(e+fx))}\Big(}{}$$

input
```
Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e +
 f*x]),x]
```

output
```
(2*Cot[(e + f*x)/2]*(g*Sec[e + f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*(Sqr
t[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (Log[1 + Sec[e + f*x]] - Log[Sqrt
[Sec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e +
f*x]^2]])*Sqrt[Tan[e + f*x]^2]))/(c*f*Sec[e + f*x]^(3/2)*(1 + Sec[e + f*x]
)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4452, 57, 65, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}(g\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4452$$

$$-\frac{acg\tan(e+fx)\int \frac{\sqrt{g\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 57$$

$$-\frac{acg\tan(e+fx)\left(\frac{2\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{g\int\frac{1}{\sqrt{g\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)}{c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 65$$

$$-\frac{acg\tan(e+fx)\left(\frac{2\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{2g\int\frac{1}{\frac{c\sec(e+fx)g}{c-c\sec(e+fx)}+g}d\frac{\sqrt{g\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}}{c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 218$$

$$-\frac{acg\tan(e+fx)\left(\frac{2\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])
,x]
```

output

```
-((a*c*g*((-2*Sqrt[g]*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqrt[g]*Sqrt[
c - c*Sec[e + f*x]])])/c^(3/2) + (2*Sqrt[g*Sec[e + f*x]])/(c*Sqrt[c - c*Se
c[e + f*x]]))*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e +
 f*x]]))
```

Defintions of rubi rules used

rule 57

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4452
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Cs
c[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(88) = 176$.

Time = 2.76 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.75

method	result
default	$\frac{\sqrt{2}\, g \left(2 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right) \left(\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{2}\, \left(\cot\left(\frac{f x}{2}+\frac{e}{2}\right)-\csc\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{2}\right) \sin\left(\frac{f x}{2}+\frac{e}{2}\right)+\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{2}\, \left(\cot\left(\frac{f x}{2}+\frac{e}{2}\right)-\csc\left(\frac{f x}{2}+\right.\right.}{2}\right.\right.}{c f}$

input
```
int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
1/c*2^(1/2)*g/f*(2*cos(1/2*f*x+1/2*e)^2-1)*(2^(1/2)*arctanh(1/2*2^(1/2)*(c
ot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)+2^(1/2)*arctan
h(1/2*2^(1/2)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e)-1))*sin(1/2*f*x+1/2*e
)+1)*(g/(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*co
s(1/2*f*x+1/2*e)^2)^(1/2)*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.18

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx=\left[\frac{\sqrt{ag}g\log\left(\frac{ag\cos(fx+e)^3-7\,ag\cos(fx+e)^2+4\sqrt{ag}\left(\cos(fx+e)^2-2\,\cos(f}{\cos(fx+e)^3+\cos(f}\right.}{}\right.$$

$$-\frac{\sqrt{-ag}g\arctan\left(\frac{\sqrt{-ag}\left(\cos(fx+e)^2-2\,\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}}{2\,ag\sin(fx+e)}\right)\sin(fx+e)-2\,g\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{}{\cos(f}}}{cf\sin(fx+e)}$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,
algorithm="fricas")
```

output
```
[1/2*(sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*cos(f*x + e)^2 + 4*sqrt(
a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos(f*x + e)^3 + cos(f*x
+ e)^2))*sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g
/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), -(sqrt(-a*g)*g*arctan(1/2
*sqrt(-a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt(g/cos(f*x + e))/(a*g*sin(f*x + e)))*sin(f*x + e) - 2*g*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(
c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = -\frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx}{c}$$

input
```
integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x
)
```

output
```
-Integral((g*sec(e + f*x))**(3/2)*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) -
 1), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 979 vs. $2(88) = 176$.

Time = 0.25 (sec) , antiderivative size = 979, normalized size of antiderivative = 9.41

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,
algorithm="maxima")
```

output
```
1/2*(4*sqrt(2)*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sqrt(2)*g*cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 4*sqrt(2)*g*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ g*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 2) + (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2
) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x...
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx=\int -\frac{\sqrt{a\sec\left(fx+e\right)+a}\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{c\sec\left(fx+e\right)-c}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,
algorithm="giac")
```

output
```
integrate(-sqrt(a*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e)
 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c - c/cos(e + f*x
)),x)
```

output
```
int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c - c/cos(e + f*x
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx=-\frac{\sqrt{g}\,\sqrt{a}\,\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)-1}dx\right)g}{c}$$

input
```
int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
( - sqrt(g)*sqrt(a)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x) + 1)*sec(e +
 f*x))/(sec(e + f*x) - 1),x)*g)/c
```

3.180 $\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 140

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx &= -\frac{2\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(e+fx)}\sin(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} \\
&\quad +\frac{\csc(e+fx)\sqrt{a+a\sec(e+fx)}}{acf\sqrt{\sec(e+fx)}}
\end{aligned}$$

output

```
-2*arcsinh(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/c/f+1/2*arct
anh(1/2*a^(1/2)*sec(f*x+e)^(1/2)*sin(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)
)*2^(1/2)/a^(1/2)/c/f+csc(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f/sec(f*x+e)^(
1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 724 vs. $2(140) = 280$.

Time = 10.50 (sec) , antiderivative size = 724, normalized size of antiderivative = 5.17

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$

$$= \frac{\sec^{\frac{3}{2}}(e+fx)\sqrt{(1+\cos(e+fx))\sec(e+fx)}\sqrt{1+\sec(e+fx)}\left(-\frac{2\cot(e)}{f}+\frac{\csc\left(\frac{e}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{fx}{2}\right)}{f}+\right.}{\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))}$$

$$+\frac{\cos(e+fx)\left(\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)-2\sqrt{2}\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}\sqrt{-1+\right.\right.}{2f(1+}$$

$$+\frac{\cos(e+fx)\left(-8\log(1+\sec(e+fx))+8\log\left(\sqrt{\sec(e+fx)}+\sec^{\frac{3}{2}}(e+fx)+\sqrt{1+\sec(e+fx)}\right.\right.}{}$$

input

```
Integrate[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]
)),x]
```

output

```
(Sec[e + f*x]^(3/2)*Sqrt[(1 + Cos[e + f*x])*Sec[e + f*x]]*Sqrt[1 + Sec[e +
 f*x]]*((-2*Cot[e])/f + (Csc[e/2]*Csc[e/2 + (f*x)/2]*Sin[(f*x)/2])/f + (Se
c[e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/f)*Sin[e/2 + (f*x)/2]^2)/(Sqrt[a*(
1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])) + (Cos[e + f*x]*(Log[1 - 2*Sec[e
+ f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e +
f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] - Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]
^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e +
 f*x]^2]])*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e + f*x]^2]*Sin[e/2 + (f
*x)/2]^2*Sin[e + f*x])/(2*f*(1 + Cos[e + f*x])*Sqrt[2 - 2*Cos[e + f*x]^2]*
Sqrt[1 - Cos[e + f*x]^2]*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x]))
+ (Cos[e + f*x]*(-8*Log[1 + Sec[e + f*x]] + 8*Log[Sqrt[Sec[e + f*x]] + Sec
[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Sqrt
[2]*(-Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f
*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Log[1 - 2*Sec[e +
 f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f
*x]]*Sqrt[-1 + Sec[e + f*x]^2]]))*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e
 + f*x]^2]*Sin[e/2 + (f*x)/2]^2*Sin[e + f*x])/(2*f*(1 + Cos[e + f*x])*(1 -
 Cos[e + f*x]^2)*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4452, 27, 109, 27, 175, 64, 104, 216, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4452}$$

$$-\frac{ac\tan(e+fx)\int \frac{\sec^{\frac{3}{2}}(e+fx)}{a(\sec(e+fx)+1)(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\tan(e+fx)\int \frac{\sec^{\frac{3}{2}}(e+fx)}{(\sec(e+fx)+1)(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{c\tan(e+fx)\left(\frac{\sqrt{\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{\int \frac{c(2\sec(e+fx)+1)}{2\sqrt{\sec(e+fx)}(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)}{c^2}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\tan(e+fx)\left(\frac{\sqrt{\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{\int \frac{2\sec(e+fx)+1}{\sqrt{\sec(e+fx)}(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{175}$$

$$-\frac{c\tan(e+fx)\left(\frac{\sqrt{\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{2\int\frac{1}{\sqrt{\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)-\int\frac{1}{\sqrt{\sec(e+fx)}(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 64$$

$$-\frac{c\tan(e+fx)\left(\frac{\sqrt{\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{-\int\frac{1}{\sqrt{\sec(e+fx)}(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)-\frac{4\int\frac{1}{\sqrt{1-\frac{c-c\sec(e+fx)}{c}}}d\sqrt{c-c\sec(e+fx)}}{c}}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 104$$

$$-\frac{c\tan(e+fx)\left(\frac{\sqrt{\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{-2\int\frac{1}{\frac{2c\sec(e+fx)}{c-c\sec(e+fx)}+1}d\frac{\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}-\frac{4\int\frac{1}{\sqrt{1-\frac{c-c\sec(e+fx)}{c}}}d\sqrt{c-c\sec(e+fx)}}{c}}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 216$$

$$-\frac{c\tan(e+fx)\left(\frac{\sqrt{\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{-\frac{4\int\frac{1}{\sqrt{1-\frac{c-c\sec(e+fx)}{c}}}d\sqrt{c-c\sec(e+fx)}}{c}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}}}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 223$$

$$-\frac{c\tan(e+fx)\left(\frac{\sqrt{\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{-\frac{4\arcsin\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}}}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

output

```
-((c*(-1/2*((-4*ArcSin[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]])/Sqrt[c] - (Sqrt[
2]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]]])/
Sqrt[c])/c + Sqrt[Sec[e + f*x]]/(c*Sqrt[c - c*Sec[e + f*x]]))*Tan[e + f*x]
)/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 64

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4452
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Cs
c[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sec(fx+e)^{\frac{5}{2}}\sqrt{a(1+\sec(fx+e))}\left(\arctan\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{2\sqrt{-\frac{1}{\cos(fx+e)+1}}}\right)\sqrt{2}\,\sin(fx+e)-2\arctan\left(\frac{-\csc(fx+e)+\cot(fx+e)+1}{2\sqrt{-\frac{1}{\cos(fx+e)+1}}}\right)\sin(fx}{2cfa(\cos(fx+e)+1)\sqrt{-}}$

input
```
int(sec(f*x+e)^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
1/2/c/f/a*sec(f*x+e)^(5/2)*(a*(1+sec(f*x+e)))^(1/2)*(arctan(1/2*2^(1/2)/(-
1/(cos(f*x+e)+1))^(1/2)*(cot(f*x+e)-csc(f*x+e)))*2^(1/2)*sin(f*x+e)-2*arct
an(1/2/(-1/(cos(f*x+e)+1))^(1/2)*(-csc(f*x+e)+cot(f*x+e)+1))*sin(f*x+e)-2*
arctan(1/2/(-1/(cos(f*x+e)+1))^(1/2)*(-csc(f*x+e)+cot(f*x+e)-1))*sin(f*x+e
)+(-2/(cos(f*x+e)+1))^(1/2)*2^(1/2)*(cos(f*x+e)+1))/(cos(f*x+e)+1)/(-1/(co
s(f*x+e)+1))^(1/2)*cos(f*x+e)^2*cot(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.28

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$

$$= \Bigg[\frac{\sqrt{2}\sqrt{a}\log\Bigg(-\frac{\cos(fx+e)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\cos(fx+e)}\sin(fx+e)}{\sqrt{a}}-2\cos(fx+e)-3}{\cos(fx+e)^2+2\cos(fx+e)+1}\Bigg)\sin(fx+e)+2\sqrt{a}\log\Bigg(\frac{a\cos(fx}{\quad}}{4acf}$$

$$-\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(fx+e)}}{\sin(fx+e)}\right)\sin(fx+e)+2\sqrt{-a}\arctan\Bigg(\frac{\Big(\cos(fx+e)^2-2\cos(fx+e)}{2a\sqrt{\cos(fx+e)}}}{2acf\sin(fx+e)}$$

input
```
integrate(sec(f*x+e)^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algo
rithm="fricas")
```

output

```
[1/4*(sqrt(2)*sqrt(a)*log(-(cos(f*x + e)^2 - 2*sqrt(2)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt(cos(f*x + e))*sin(f*x + e)/sqrt(a) - 2*cos(f*x +
 e) - 3)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 2*sqrt(a)*l
og((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2*cos(f*x
+ e))*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)/sqrt(co
s(f*x + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))*sin(f*x + e) + 4*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*f*sin(f*x +
e)), -1/2*(sqrt(2)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sqrt(-1/a)*sqrt(cos(f*x + e))/sin(f*x + e))*sin(f*x + e) + 2*
sqrt(-a)*arctan(1/2*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(-a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))/(a*sqrt(cos(f*x + e))*sin(f*x + e)))*sin(f*x
+ e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*
f*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Timed out}$$

input

```
integrate(sec(f*x+e)**(5/2)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(120) = 240$.

Time = 0.24 (sec) , antiderivative size = 1310, normalized size of antiderivative = 9.36

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algo
rithm="maxima")
```

output

```
-1/2*((sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sq
rt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2))*log(2*cos(1
/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 2) - (sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sq
rt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*si
n(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) + (sqrt(2)*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2
*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arc...
```

Giac [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$
$$= Degree extension factorisation inside mismatch over$$
$$-\frac{\frac{\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\right)^2\right)}{\sqrt{a}c}+\frac{4\,\sqrt{a}\log\left(\left|\frac{2\,\left(\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\right)^2-4\,\sqrt{2}|a|-6\,a}{2\,\left(\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\right)^2+4\,\sqrt{2}|a|-6\,a}\right|\right)}{c|a|}+}{4\,f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input

```
integrate(sec(f*x+e)^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algo
rithm="giac")
```

output

```
Degree*extension*factorisation*inside*mismatch*over - 1/4*(sqrt(2)*log((sq
rt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2)/(sqrt(
a)*c) + 4*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2
*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*f
*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6
*a))/(c*abs(a)) + 4*sqrt(2)*sqrt(a)/(((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt
(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*c))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input

```
int((1/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)
)),x)
```

output

```
int((1/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)
)), x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2-1}dx\right)}{ac}$$

input

```
int(sec(f*x+e)^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

output

```
( - sqrt(a)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2
)/(sec(e + f*x)**2 - 1),x))/(a*c)
```

3.181 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 116

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{g^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{g\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{acf}$$

output

```
-1/2*g^(3/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2^(1/2)/(g*sec(f*x+e))
^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(1/2)/c/f+g*cot(f*x+e)*(g*sec(f*x
+e))^(1/2)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 236 vs. $2(116) = 232$.

Time = 3.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.03

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx =$$

$$-\frac{a\cos\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^{5/2}\sin^3\left(\frac{1}{2}(e+fx)\right)\left(-4-4\sec(e+fx)+\frac{\left(\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)-\right.\right.}{}\right.}{cfg(-1+\sec(e+f}$$

input

```
Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e +
f*x])),x]
```

output

```
-((a*Cos[(e + f*x)/2]*(g*Sec[e + f*x])^(5/2)*Sin[(e + f*x)/2]^3*(-4 - 4*Se
c[e + f*x] + ((Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[
Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log[1 - 2*Sec
[e + f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e
 + f*x]]*Sqrt[Tan[e + f*x]^2]])*Sqrt[Tan[e + f*x]^2])/Sqrt[Sec[(e + f*x)/2
]^2]))/(c*f*g*(-1 + Sec[e + f*x])^2*(a*(1 + Sec[e + f*x]))^(3/2)))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4452, 27, 105, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow 4452$$

$$-\frac{acg\tan(e+fx)\int \frac{\sqrt{g\sec(e+fx)}}{a(\sec(e+fx)+1)(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 27$$

$$-\frac{cg\tan(e+fx)\int \frac{\sqrt{g\sec(e+fx)}}{(\sec(e+fx)+1)(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 105$$

$$-\frac{cg\tan(e+fx)\left(\frac{\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{g\int \frac{1}{\sqrt{g\sec(e+fx)}(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 104$$

$$-\frac{cg\tan(e+fx)\left(\frac{\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{g\int \frac{1}{\frac{2c\sec(e+fx)g}{c-c\sec(e+fx)}+g}d\frac{\sqrt{g\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}}{c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 218$$

$$-\frac{cg\tan(e+fx)\left(\frac{\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{\sqrt{g}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input

```
Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]))
,x]
```

output

```
-((c*g*(-((Sqrt[g]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqrt[g]*
Sqrt[c - c*Sec[e + f*x]])])/(Sqrt[2]*c^(3/2))) + Sqrt[g*Sec[e + f*x]]/(c*S
qrt[c - c*Sec[e + f*x]]))*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4452
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Cs
c[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

method	result
default	$\frac{g\sqrt{g\sec(fx+e)}\,\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,\operatorname{arcsinh}(\cot(fx+e)-\csc(fx+e))\sin(fx+e)+2(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\right)\cot(fx+e)}{2cfa(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}}$

input
```
int((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
1/2/c*g/f/a*(g*sec(f*x+e))^(1/2)*(a*(1+sec(f*x+e)))^(1/2)*(2^(1/2)*arcsinh
(cot(f*x+e)-csc(f*x+e))*sin(f*x+e)+2*(cos(f*x+e)+1)*(1/(cos(f*x+e)+1))^(1/
2))/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2)*cot(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.84

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \left[\frac{\sqrt{2}ag\sqrt{\frac{g}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)\sin(f}{\cos(fx+e)^2+2\cos(f}\right.}{}\right.$$

input
```
integrate((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,
algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*a*g*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*cos
(f*x + e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1
))*sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f
*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*a*g*sqrt(-g/a)*a
rctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/co
s(f*x + e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x + e) + 2*g*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(
f*x + e))]
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

input

```
integrate((g*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x
)
```

output

```
-Integral((g*sec(e + f*x))**(3/2)/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) -
 sqrt(a*sec(e + f*x) + a)), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(97) = 194$.

Time = 0.22 (sec) , antiderivative size = 536, normalized size of antiderivative = 4.62

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Too large to display}$$

input

```
integrate((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,
algorithm="maxima")
```

output

```
1/2*(4*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (g
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 -
 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*g*sin(1/4
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(g)/((sqrt(2)*c*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*c)*sqrt(a)*f)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int -\frac{(g\sec(fx+e))^{\frac{3}{2}}}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)}\,dx$$

input

```
integrate((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,
algorithm="giac")
```

output

```
integrate(-(g*sec(f*x + e))^(3/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e
) - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)
)),x)
```

output
```
int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\sqrt{g}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2-1}dx\right)g}{ac}$$

input
```
int((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
( - sqrt(g)*sqrt(a)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x) + 1)*sec(e +
 f*x))/(sec(e + f*x)**2 - 1),x)*g)/(a*c)
```

3.182 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 179

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{2g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} + \frac{g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{g^2\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{acf}$$

output
```
-2*g^(5/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*se
c(f*x+e))^(1/2))/a^(1/2)/c/f+1/2*g^(5/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f
*x+e)*2^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(1/2)
/c/f+g^2*cot(f*x+e)*(g*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.83

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx =$$
$$-\frac{(g\sec(e+fx))^{5/2}\sqrt{1+\sec(e+fx)}\sin^3(e+fx)\left(8\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}+\left(16\log(1+\sec(e\right.\right.}{}$$

input
```
Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e +
f*x])),x]
```

output
```
-1/8*((g*Sec[e + f*x])^(5/2)*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]^3*(8*Sqrt
[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (16*Log[1 + Sec[e + f*x]] - 16*Log
[Sqrt[Sec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan
[e + f*x]^2]] + Sqrt[2]*(Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqr
t[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log
[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt
[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]]))*Sqrt[Tan[e + f*x]^2]))/(c*f*(-1
 + Cos[e + f*x])*(1 + Cos[e + f*x])^2*(-1 + Sec[e + f*x])*Sec[e + f*x]^(5/
2)*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 4452, 27, 109, 27, 175, 65, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow 4452$$

$$-\frac{acg\tan(e+fx)\int \frac{(g\sec(e+fx))^{3/2}}{a(\sec(e+fx)+1)(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 27$$

$$-\frac{cg\tan(e+fx)\int \frac{(g\sec(e+fx))^{3/2}}{(\sec(e+fx)+1)(c-c\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 109$$

$$-\frac{cg\tan(e+fx)\left(\frac{g\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{\int \frac{cg^2(2\sec(e+fx)+1)}{2\sqrt{g\sec(e+fx)}(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)}{c^2}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 27$$

$$-\frac{cg\tan(e+fx)\left(\frac{g\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{g^2\int \frac{2\sec(e+fx)+1}{\sqrt{g\sec(e+fx)}(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 175$$

$$-\frac{cg\tan(e+fx)\Bigg(\frac{g\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{g^2\Big(2\int \frac{1}{\sqrt{g\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}d\sec(e+fx)-\int \frac{1}{\sqrt{g\sec(e+fx)}(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec}{2c}}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 65$$

$$-\frac{cg\tan(e+fx)\Bigg(\frac{g\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{g^2\Big(4\int \frac{1}{\frac{c\sec(e+fx)g}{c-c\sec(e+fx)}+g}d\frac{\sqrt{g\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}-\int \frac{1}{\sqrt{g\sec(e+fx)}(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}d\sec(e+fx}{2c}}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 104$$

$$-\frac{cg\tan(e+fx)\left(\frac{g\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{g^2\left(4\int\frac{1}{\frac{c\sec(e+fx)g}{c-c\sec(e+fx)}+g}d\frac{\sqrt{g\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}-2\int\frac{1}{\frac{2c\sec(e+fx)g}{c-c\sec(e+fx)}+g}d\frac{\sqrt{g\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{cg\tan(e+fx)\left(\frac{g\sqrt{g\sec(e+fx)}}{c\sqrt{c-c\sec(e+fx)}}-\frac{g^2\left(\frac{4\arctan\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\sqrt{g}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\sqrt{g}}\right)}{2c}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]))
,x]
```

output
```
-((c*g*(-1/2*(g^2*((4*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqrt[g]*Sqrt[
c - c*Sec[e + f*x]])])/(Sqrt[c]*Sqrt[g]) - (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[c
]*Sqrt[g*Sec[e + f*x]])/(Sqrt[g]*Sqrt[c - c*Sec[e + f*x]])])/(Sqrt[c]*Sqrt
[g])))/c + (g*Sqrt[g*Sec[e + f*x]])/(c*Sqrt[c - c*Sec[e + f*x]]))*Tan[e +
f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4452
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Cs
c[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(149) = 298.

Time = 2.20 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.11

method	result
default	$\frac{\sqrt{2}\left(-\frac{\sqrt{\frac{g}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\, g^{2}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\left(\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}\right)\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\left(\cot\left(f\right.\right.}{}\right.\right.}{}\right.\right.}{}$

input
```
int((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)/c*(-1/f*(g/(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*g^2/(2*cos(1/2*f*
x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(2
^(1/2)*arctanh(1/2*2^(1/2)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e)-1))*(2*c
os(1/2*f*x+1/2*e)^3-cos(1/2*f*x+1/2*e))+2^(1/2)*arctanh(1/2*2^(1/2)*(cot(1
/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e)+1))*(2*cos(1/2*f*x+1/2*e)^3-cos(1/2*f*x+1
/2*e))+ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*(2*cos(1/2*f*x+1/2*e)^
3-cos(1/2*f*x+1/2*e))+ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)*(-2*cos
(1/2*f*x+1/2*e)^3+cos(1/2*f*x+1/2*e))+4*cos(1/2*f*x+1/2*e)^2*cot(1/2*f*x+1
/2*e)-3*cot(1/2*f*x+1/2*e))+1/f*(g/(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*g^2/(
2*cos(1/2*f*x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)
^2)^(1/2)*(2^(1/2)*arctanh(1/2*2^(1/2)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2
*e)-1))*(6*cos(1/2*f*x+1/2*e)^3-3*cos(1/2*f*x+1/2*e))+2^(1/2)*arctanh(1/2*
2^(1/2)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e)+1))*(6*cos(1/2*f*x+1/2*e)^3
-3*cos(1/2*f*x+1/2*e))+6*cos(1/2*f*x+1/2*e)^2*cot(1/2*f*x+1/2*e)-4*cot(1/2
*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 560, normalized size of antiderivative = 3.13

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \left[\frac{\sqrt{2}ag^2\sqrt{\frac{g}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)\sin(fx}{\cos(fx+e)^2+2\cos(fx}\right.}{} \right.$$

$$-\frac{\sqrt{2}ag^2\sqrt{-\frac{g}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)}{g\sin(fx+e)}\right)\sin(fx+e)+2ag^2\sqrt{-\frac{g}{a}}\arctan\left(\frac{\left(\cos(fx+e)^2\right.}{}\right.}{2acf\sin(fx}$$

input
```
integrate((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,
algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*a*g^2*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*co
s(f*x + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) +
1))*sin(f*x + e) + 2*a*g^2*sqrt(g/a)*log((g*cos(f*x + e)^3 + 4*(cos(f*x +
e)^2 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt(g/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g)/(cos(f*x + e)
^3 + cos(f*x + e)^2))*sin(f*x + e) + 4*g^2*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), -1/2*(sq
rt(2)*a*g^2*sqrt(-g/a)*arctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x
 + e) + 2*a*g^2*sqrt(-g/a)*arctan(1/2*(cos(f*x + e)^2 - 2*cos(f*x + e))*sq
rt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(g*s
in(f*x + e)))*sin(f*x + e) - 2*g^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Timed out}$$

input

```
integrate((g*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x
)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(149) = 298$.

Time $= 0.26$ (sec) , antiderivative size $= 1400$, normalized size of antiderivative $= 7.82$

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Too large to display}$$

input

```
integrate((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,
algorithm="maxima")
```

output

```
1/2*(4*g^2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g^2*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 4*g^2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (s
qrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2
)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g
^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log
(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 2) + (sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sq
rt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)
*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*...
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx=\int -\frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)}\,dx$$

input

```
integrate((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,
algorithm="giac")
```

output

```
integrate(-(g*sec(f*x + e))^(5/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e
) - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)
)),x)
```

output
```
int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\sqrt{g}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2-1}dx\right)g^2}{ac}$$

input
```
int((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
( - sqrt(g)*sqrt(a)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x) + 1)*sec(e +
 f*x)**2)/(sec(e + f*x)**2 - 1),x)*g**2)/(a*c)
```

3.183 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 46

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\log(\tan(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(tan(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\frac{4i\left(-1+e^{i(e+fx)}\right)\operatorname{arctanh}\left(e^{2i(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}{\left(1+e^{i(e+fx)}\right)f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]
]),x]
```

output
```
((4*I)*(-1 + E^(I*(e + f*x)))*ArcTanh[E^((2*I)*(e + f*x))]*Cos[(e + f*x)/2
]^2*Sec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt
[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4451, 25, 3042, 3100, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4451}$$

$$-\frac{\tan(e+fx)\int -\csc(e+fx)\sec(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\int \csc(e+fx)\sec(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\int \csc(e+fx)\sec(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3100}$$

$$\frac{\tan(e+fx)\int \cot(e+fx)d\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{14}$$

$$\frac{\tan(e+fx)\log(\tan(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
(Log[Tan[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Se
c[e + f*x]])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4451
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Simp[(
(-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e
+ f*x]]))   Int[(g*Csc[e + f*x])^p*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a,
b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && Integ
erQ[m + 1/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.50 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.96

method	result
risch	$-\frac{i(\mathrm{e}^{i(fx+e)}+1)(\mathrm{e}^{i(fx+e)}-1)\ln(\mathrm{e}^{2i(fx+e)}-1)}{\sqrt{\frac{a(\mathrm{e}^{i(fx+e)}+1)^2}{\mathrm{e}^{2i(fx+e)}+1}}(\mathrm{e}^{2i(fx+e)}+1)\sqrt{\frac{c(\mathrm{e}^{i(fx+e)}-1)^2}{\mathrm{e}^{2i(fx+e)}+1}}\,f}+\frac{i(\mathrm{e}^{i(fx+e)}+1)(\mathrm{e}^{i(fx+e)}-1)\ln(\mathrm{e}^{2i(fx+e)}+1)}{\sqrt{\frac{a(\mathrm{e}^{i(fx+e)}+1)^2}{\mathrm{e}^{2i(fx+e)}+1}}(\mathrm{e}^{2i(fx+e)}+1)\sqrt{\frac{c(\mathrm{e}^{i(fx+e)}-1)^2}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$
default	$\frac{\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\bigg(\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\ln\bigg(-}{f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\sqrt{-\frac{c\sin\bigg(}{2\cos\bigg(}}}$

input

```
int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output

```
-I/(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)/(exp(2*I*(f*x+e))+1
)*(exp(I*(f*x+e))+1)/(c*(exp(I*(f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(
exp(I*(f*x+e))-1)/f*ln(exp(2*I*(f*x+e))-1)+I/(a*(exp(I*(f*x+e))+1)^2/(exp(
2*I*(f*x+e))+1))^(1/2)/(exp(2*I*(f*x+e))+1)*(exp(I*(f*x+e))+1)/(c*(exp(I*(
f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(exp(I*(f*x+e))-1)/f*ln(exp(2*I*(
f*x+e))+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(42) = 84.

Time = 0.25 (sec) , antiderivative size = 255, normalized size of antiderivative = 5.54

$$\int\frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$$

$$=\Bigg[-\frac{\sqrt{-ac}\log\Bigg(-\frac{8\left(\left(2\cos(fx+e)^3-\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+\left(2ac\cos(fx+e)^4-2ac\cos(fx+e)^2+ac\right)\sin(f}{\left(\cos(fx+e)^4-\cos(fx+e)^2\right)\sin(fx+e)}}{2acf}$$

input
```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
[-1/2*sqrt(-a*c)*log(-8*((2*cos(f*x + e)^3 - cos(f*x + e))*sqrt(-a*c)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)
) + (2*a*c*cos(f*x + e)^4 - 2*a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((co
s(f*x + e)^4 - cos(f*x + e)^2)*sin(f*x + e)))/(a*c*f), sqrt(a*c)*arctan(sq
rt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e))*cos(f*x + e)/((2*a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)))/(a
*c*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\int \frac{\sec^2(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x)
 - 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx =$$
$$-\frac{\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)-1\right)}{\sqrt{a}\sqrt{c}f}$$

input
```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
-(arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) - 1))/(sqrt(a)*sqrt(c)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2
)),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2-1}dx\right)}{ac}$$

input
```
int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x)**2)/(sec(e + f*x)**2 - 1),x))/(a*c)
```

3.184 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 65

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{c-d}\sqrt{d}f}$$

output
```
2*a^(1/2)*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(c-d)^(1/2)/(a+a*sec(f*x+e))^
(1/2))/(c-d)^(1/2)/d^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c-d}\sqrt{\cos(e+fx)}}\right)\sqrt{\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{c-d}\sqrt{d}f}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
```

output
```
(Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c - d]*Sqrt[Cos[
e + f*x]])]*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]
)/(Sqrt[c - d]*Sqrt[d]*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {3042, 4455, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a\sec(e+fx)+a}}{c-d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{c-d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4455}$$

$$-\frac{2a\int \frac{1}{a(c-d)-\frac{a^2d\tan^2(e+fx)}{\sec(e+fx)a+a}}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c-d}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
```

output
```
(2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c - d]*Sqrt[a + a*
Sec[e + f*x]])])/(Sqrt[c - d]*Sqrt[d]*f)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(51) = 102$.

Time = 9.30 (sec) , antiderivative size = 403, normalized size of antiderivative = 6.20

method	result
default	$-\frac{\left(\ln\left(-\frac{2\left(-\sqrt{-\frac{2d}{c+d}}\sqrt{(-\cos(fx+e)+1)^2\csc(fx+e)^2-1}\,c-\sqrt{-\frac{2d}{c+d}}\sqrt{(-\cos(fx+e)+1)^2\csc(fx+e)^2-1}\,d+\sqrt{(c-d)(c+d)}\,(-\cot(fx+e)+\csc(\right.}{c(-\cot(fx+e)+\csc(fx+e))+d(-\cot(fx+e)+\csc(fx+e))+\sqrt{(c-d)(c+d)}}\right.\right.}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
-1/f/((c-d)*(c+d))^(1/2)/(-2*d/(c+d))^(1/2)*(ln(-2*(-(-2*d/(c+d))^(1/2)*((
-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*c-(-2*d/(c+d))^(1/2)*((-cos(f*x+e)+
1)^2*csc(f*x+e)^2-1)^(1/2)*d+((c-d)*(c+d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+
c+d)/(c*(-cot(f*x+e)+csc(f*x+e))+d*(-cot(f*x+e)+csc(f*x+e))+((c-d)*(c+d))^
(1/2)))-ln(-2*((-2*d/(c+d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)
*c+(-2*d/(c+d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*d+((c-d)*(c
+d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c-d)/(-c*(-cot(f*x+e)+csc(f*x+e))-d*(-
cot(f*x+e)+csc(f*x+e))+((c-d)*(c+d))^(1/2))))*((-cos(f*x+e)+1)^2*csc(f*x+e
)^2-1)^(1/2)*(-2*a/((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(51) = 102$.

Time = 0.34 (sec) , antiderivative size = 357, normalized size of antiderivative = 5.49

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx$$

$$= \Bigg[\frac{\sqrt{\frac{a}{cd-d^2}}\log\Bigg(-\frac{(ac^2-8\,acd+8\,ad^2)\cos(fx+e)^3+ad^2+(ac^2-2\,acd)\cos(fx+e)^2+4\,\Big((c^2d-3\,cd^2+2\,d^3)\cos(fx+e)^2+(cd^2-d^3)\cos(fx+}{c^2\cos(fx+e)^3+(c^2-2\,cd)\cos(fx+e)^2+d^2-(2\,cd-d^2)\cos(}}{2\,f}$$

$$-\frac{\sqrt{-\frac{a}{cd-d^2}}\arctan\left(\frac{2\,(cd-d^2)\sqrt{-\frac{a}{cd-d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{(ac-2\,ad)\cos(fx+e)^2+ad+(ac-ad)\cos(fx+e)}\right)}{f}\Bigg]$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
[1/2*sqrt(a/(c*d - d^2))*log(-((a*c^2 - 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3
+ a*d^2 + (a*c^2 - 2*a*c*d)*cos(f*x + e)^2 + 4*((c^2*d - 3*c*d^2 + 2*d^3)*
cos(f*x + e)^2 + (c*d^2 - d^3)*cos(f*x + e))*sqrt(a/(c*d - d^2))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (6*a*c*d - 7*a*d^2)*cos(f*x
+ e))/(c^2*cos(f*x + e)^3 + (c^2 - 2*c*d)*cos(f*x + e)^2 + d^2 - (2*c*d -
d^2)*cos(f*x + e)))/f, -sqrt(-a/(c*d - d^2))*arctan(2*(c*d - d^2)*sqrt(-a/
(c*d - d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e)/((a*c - 2*a*d)*cos(f*x + e)^2 + a*d + (a*c - a*d)*cos(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{c-d\sec\left(e+fx\right)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c - d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = \int -\frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)-c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
-integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) - c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.09

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx$$
$$= \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2c+\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2d+ac-}{4\sqrt{cd-d^2}a}\right.}{\sqrt{cd-d^2}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
2*sqrt(-a)*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2*c + (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c - 3*a*d)/(sqrt(c*d - d^2)*a))*sgn(c
os(f*x + e))/(sqrt(c*d - d^2)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = -\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{d-c\,\cos\left(e+f\,x\right)}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - d/cos(e + f*x))),x)
```

output
```
-int((a + a/cos(e + f*x))^(1/2)/(d - c*cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = -\sqrt{a}\left(\int \frac{\sqrt{\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)d-c}dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x)
```

output
```
 - sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)*d - c),
x)
```

3.185 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 29, antiderivative size = 236

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx \\
&= \frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\operatorname{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4)\tan(e+fx)}{30f} \\
&\quad + \frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sec(e+fx)\tan(e+fx)}{120f} \\
&\quad + \frac{a(12c^2+35cd+16d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60f} \\
&\quad + \frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f}
\end{aligned}$$

output
```
1/8*a*(8*c^4+16*c^3*d+24*c^2*d^2+12*c*d^3+3*d^4)*arctanh(sin(f*x+e))/f+1/3
0*a*(12*c^4+95*c^3*d+112*c^2*d^2+80*c*d^3+16*d^4)*tan(f*x+e)/f+1/120*a*d*(
24*c^3+130*c^2*d+116*c*d^2+45*d^3)*sec(f*x+e)*tan(f*x+e)/f+1/60*a*(12*c^2+
35*c*d+16*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/20*a*(4*c+5*d)*(c+d*sec(f
*x+e))^3*tan(f*x+e)/f+1/5*a*(c+d*sec(f*x+e))^4*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.67

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$
$$=\frac{a\big(120c^4\coth^{-1}(\sin(e+fx))+15d(16c^3+24c^2d+12cd^2+3d^3)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\,(}{}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

output
```
(a*(120*c^4*ArcCoth[Sin[e + f*x]] + 15*d*(16*c^3 + 24*c^2*d + 12*c*d^2 + 3
*d^3)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(120*(c + d)^4 + 15*d*(16*c^3 +
 24*c^2*d + 12*c*d^2 + 3*d^3)*Sec[e + f*x] + 30*d^3*(4*c + d)*Sec[e + f*x]
^3 + 80*d^2*(3*c^2 + 2*c*d + d^2)*Tan[e + f*x]^2 + 24*d^4*Tan[e + f*x]^4))
)/(120*f)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {3042, 4490, 3042, 4490, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c+d\sec(e+fx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$
$$\Big\downarrow \text{4490}$$
$$\frac{1}{5}\int \sec(e+fx)(c+d\sec(e+fx))^3(a(5c+4d)+a(4c+5d)\sec(e+fx))dx+\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(a(5c+4d)+a(4c+5d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{4490}$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\int \sec(e+fx)(c+d\sec(e+fx))^2\left(a\left(20c^2+28dc+15d^2\right)+a\left(12c^2+35dc+16d^2\right)\sec(e+fx)\right)dx+a(4$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\Big(a\left(20c^2+28dc+15d^2\right)+a\left(12c^2+35dc+16d^2\right)\csc\Big(e+$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{4490}$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\int \sec(e+fx)(c+d\sec(e+fx))\Big(a\left(60c^3+108dc^2+115d^2c+32d^3\right)+a\big(24c^3+130dc^2+116d^2c+45$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)\Big(a\left(60c^3+108dc^2+115d^2c+32d^3\right)+a\big(24c^3+130dc^2$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\int \sec(e+fx)\Big(15a\left(8c^4+16dc^3+24d^2c^2+12d^3c+3d^4\right)+4a\big(12c^4+95dc^3+112d^2c^2+80d^3c+$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)\Big(15a\big(8c^4+16dc^3+24d^2c^2+12d^3c+3d^4\big)+4a\big(12c^4+95dc^3+112d^2c^2+80$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow 4274$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(4a\big(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4\big)\int \sec^2(e+fx)dx+15a\big(8c^4+16c^3d+24c^2d^2+12c$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(15a\big(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\big)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+4a\big(12c^4+95c^3d+112c^2d^2+$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow 4254$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(15a\big(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\big)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{4a\big(12c^4+95c^3d+112c^2d^2+}{}$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow 24$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(15a\big(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\big)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{4a\big(12c^4+95c^3d+112c^2d^2+}{f}$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow 4257$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(\frac{15a\big(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\big)\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{4a\big(12c^4+95c^3d+112c^2d^2+8}{f}$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

output

```
(a*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f) + ((a*(4*c + 5*d)*(c + d*Sec
[e + f*x])^3*Tan[e + f*x])/(4*f) + ((a*(12*c^2 + 35*c*d + 16*d^2)*(c + d*S
ec[e + f*x])^2*Tan[e + f*x])/(3*f) + ((a*d*(24*c^3 + 130*c^2*d + 116*c*d^2
 + 45*d^3)*Sec[e + f*x]*Tan[e + f*x])/(2*f) + ((15*a*(8*c^4 + 16*c^3*d + 2
4*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f*x]])/f + (4*a*(12*c^4 + 95
*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Tan[e + f*x])/f)/2)/3)/4)/5
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00

method	result
parts	$\frac{(4acd^3+ad^4)\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}-\frac{(6ac^2d^2+4acd^3)(-\frac{2}{3}-}{f}$
derivativedivides	$ac^4\ln(\sec(fx+e)+\tan(fx+e))+4ac^3d\tan(fx+e)+6ac^2d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4acd^3($
default	$ac^4\ln(\sec(fx+e)+\tan(fx+e))+4ac^3d\tan(fx+e)+6ac^2d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4acd^3($
norman	$-\frac{a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4f}-\frac{a\left(8c^4+48c^3d+72c^2d^2+52cd^3+13d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{4a(45c^4+180c^3d+1}{}$
parallelrisch	$2a(-5(c^4+2c^3d+3c^2d^2+\frac{3}{2}cd^3+\frac{3}{8}d^4)\left(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+3e)}{2}+\cos(fx+e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+5(c^4+2c^3d+3c$
risch	$ia(480c^3d+480c^2d^2+320cd^3+64d^4+120c^4+840cd^3e^{3i(fx+e)}+1920c^3de^{2i(fx+e)}+2400c^2d^2e^{2i(fx+e)}+1600cd^3e^{2i(f$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE
)
```

output
```
(4*a*c*d^3+a*d^4)/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln
(sec(f*x+e)+tan(f*x+e)))-(6*a*c^2*d^2+4*a*c*d^3)/f*(-2/3-1/3*sec(f*x+e)^2)
*tan(f*x+e)+(4*a*c^3*d+6*a*c^2*d^2)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(se
c(f*x+e)+tan(f*x+e)))+(a*c^4+4*a*c^3*d)/f*tan(f*x+e)+a*c^4/f*ln(sec(f*x+e)
+tan(f*x+e))-a*d^4/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$

$$= \frac{15\,(8\,ac^4+16\,ac^3d+24\,ac^2d^2+12\,acd^3+3\,ad^4)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,(8\,ac^4+16\,ac}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fri
cas")
```

output
```
1/240*(15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos
(f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^
2 + 12*a*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*a*
d^4 + 8*(15*a*c^4 + 60*a*c^3*d + 60*a*c^2*d^2 + 40*a*c*d^3 + 8*a*d^4)*cos(
f*x + e)^4 + 15*(16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos(f*x
 + e)^3 + 16*(15*a*c^2*d^2 + 10*a*c*d^3 + 2*a*d^4)*cos(f*x + e)^2 + 30*(4*
a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx \\
&= a\bigg(\int c^4\sec(e+fx)\,dx+\int c^4\sec^2(e+fx)\,dx+\int d^4\sec^5(e+fx)\,dx \\
&\quad+\int d^4\sec^6(e+fx)\,dx+\int 4cd^3\sec^4(e+fx)\,dx+\int 4cd^3\sec^5(e+fx)\,dx \\
&\quad+\int 6c^2d^2\sec^3(e+fx)\,dx+\int 6c^2d^2\sec^4(e+fx)\,dx+\int 4c^3d\sec^2(e+fx)\,dx \\
&\quad+\int 4c^3d\sec^3(e+fx)\,dx\bigg)
\end{aligned}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

output

```
a*(Integral(c**4*sec(e + f*x), x) + Integral(c**4*sec(e + f*x)**2, x) + In
tegral(d**4*sec(e + f*x)**5, x) + Integral(d**4*sec(e + f*x)**6, x) + Inte
gral(4*c*d**3*sec(e + f*x)**4, x) + Integral(4*c*d**3*sec(e + f*x)**5, x)
+ Integral(6*c**2*d**2*sec(e + f*x)**3, x) + Integral(6*c**2*d**2*sec(e +
f*x)**4, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integral(4*c**3*d*se
c(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.61

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$

$$= \frac{480\,\big(\tan(fx+e)^3+3\,\tan(fx+e)\big)ac^2d^2+320\,\big(\tan(fx+e)^3+3\,\tan(fx+e)\big)acd^3+16\,\big(3\,\tan(fx}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="max
ima")
```

output
```
1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2*d^2 + 320*(tan(f*x + e)
^3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 +
15*tan(f*x + e))*a*d^4 - 60*a*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))
/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log
(sin(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin
(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(
f*x + e) - 1)) - 240*a*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(si
n(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240
*a*c^4*log(sec(f*x + e) + tan(f*x + e)) + 240*a*c^4*tan(f*x + e) + 960*a*c
^3*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(224) = 448$.

Time = 0.21 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.40

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="gia
c")
```

output
```
1/120*(15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*log
(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2
+ 12*a*c*d^3 + 3*a*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(120*a*c^4*
tan(1/2*f*x + 1/2*e)^9 + 240*a*c^3*d*tan(1/2*f*x + 1/2*e)^9 + 360*a*c^2*d^
2*tan(1/2*f*x + 1/2*e)^9 + 180*a*c*d^3*tan(1/2*f*x + 1/2*e)^9 + 45*a*d^4*t
an(1/2*f*x + 1/2*e)^9 - 480*a*c^4*tan(1/2*f*x + 1/2*e)^7 - 1440*a*c^3*d*ta
n(1/2*f*x + 1/2*e)^7 - 1200*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 1160*a*c*d^
3*tan(1/2*f*x + 1/2*e)^7 - 130*a*d^4*tan(1/2*f*x + 1/2*e)^7 + 720*a*c^4*ta
n(1/2*f*x + 1/2*e)^5 + 2880*a*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 2400*a*c^2*d^
2*tan(1/2*f*x + 1/2*e)^5 + 1600*a*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 464*a*d^4
*tan(1/2*f*x + 1/2*e)^5 - 480*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 2400*a*c^3*d*
tan(1/2*f*x + 1/2*e)^3 - 2640*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 1400*a*c*
d^3*tan(1/2*f*x + 1/2*e)^3 - 190*a*d^4*tan(1/2*f*x + 1/2*e)^3 + 120*a*c^4*
tan(1/2*f*x + 1/2*e) + 720*a*c^3*d*tan(1/2*f*x + 1/2*e) + 1080*a*c^2*d^2*t
an(1/2*f*x + 1/2*e) + 780*a*c*d^3*tan(1/2*f*x + 1/2*e) + 195*a*d^4*tan(1/2
*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.53

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$

$$= \frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{2\,\left(4\,c^4+8\,c^3\,d+12\,c^2\,d^2+6\,c\,d^3+\frac{3\,d^4}{2}\right)}\right)\,\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{4\,f}$$

$$- \frac{\left(2\,a\,c^4+4\,a\,c^3\,d+6\,a\,c^2\,d^2+3\,a\,c\,d^3+\frac{3\,a\,d^4}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+\left(-8\,a\,c^4-24\,a\,c^3\,d-20\,a\,c^2\,d^2-\frac{58\,a\,c}{3}\right.}{}$$

input
```
int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

output

```
(a*atanh((tan(e/2 + (f*x)/2)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2
*d^2))/(2*(6*c*d^3 + 8*c^3*d + 4*c^4 + (3*d^4)/2 + 12*c^2*d^2)))*(12*c*d^3
 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2))/(4*f) - (tan(e/2 + (f*x)/2)^9*(
2*a*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*c*d^3 + 4*a*c^3*d) - tan(e/2 + (
f*x)/2)^7*(8*a*c^4 + (13*a*d^4)/6 + 20*a*c^2*d^2 + (58*a*c*d^3)/3 + 24*a*c
^3*d) - tan(e/2 + (f*x)/2)^3*(8*a*c^4 + (19*a*d^4)/6 + 44*a*c^2*d^2 + (70*
a*c*d^3)/3 + 40*a*c^3*d) + tan(e/2 + (f*x)/2)^5*(12*a*c^4 + (116*a*d^4)/15
 + 40*a*c^2*d^2 + (80*a*c*d^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*(2*a*c
^4 + (13*a*d^4)/4 + 18*a*c^2*d^2 + 13*a*c*d^3 + 12*a*c^3*d))/(f*(5*tan(e/2
 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(
e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1271, normalized size of antiderivative = 5.39

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x)
```

output

```
(a*( - 120*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**4 - 2
40*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**3*d - 360*cos
(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**2*d**2 - 180*cos(e
+ f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c*d**3 - 45*cos(e + f*x)*
log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*d**4 + 240*cos(e + f*x)*log(tan(
(e + f*x)/2) - 1)*sin(e + f*x)**2*c**4 + 480*cos(e + f*x)*log(tan((e + f*x
)/2) - 1)*sin(e + f*x)**2*c**3*d + 720*cos(e + f*x)*log(tan((e + f*x)/2) -
 1)*sin(e + f*x)**2*c**2*d**2 + 360*cos(e + f*x)*log(tan((e + f*x)/2) - 1)
*sin(e + f*x)**2*c*d**3 + 90*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e
+ f*x)**2*d**4 - 120*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c**4 - 240*cos
(e + f*x)*log(tan((e + f*x)/2) - 1)*c**3*d - 360*cos(e + f*x)*log(tan((e +
 f*x)/2) - 1)*c**2*d**2 - 180*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c*d**
3 - 45*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*d**4 + 120*cos(e + f*x)*log(
tan((e + f*x)/2) + 1)*sin(e + f*x)**4*c**4 + 240*cos(e + f*x)*log(tan((e +
 f*x)/2) + 1)*sin(e + f*x)**4*c**3*d + 360*cos(e + f*x)*log(tan((e + f*x)/
2) + 1)*sin(e + f*x)**4*c**2*d**2 + 180*cos(e + f*x)*log(tan((e + f*x)/2)
+ 1)*sin(e + f*x)**4*c*d**3 + 45*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*si
n(e + f*x)**4*d**4 - 240*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*
x)**2*c**4 - 480*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*c*
*3*d - 720*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*c**2*...
```

3.186 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 171

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx &= \frac{a(8c^3+12c^2d+12cd^2+3d^3)\operatorname{arctanh}(\sin(e+fx))}{8f} \\ &\quad+\frac{a(3c^3+16c^2d+12cd^2+4d^3)\tan(e+fx)}{6f} \\ &\quad+\frac{ad(6c^2+20cd+9d^2)\sec(e+fx)\tan(e+fx)}{24f} \\ &\quad+\frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f}\end{aligned}$$

output
```
1/8*a*(8*c^3+12*c^2*d+12*c*d^2+3*d^3)*arctanh(sin(f*x+e))/f+1/6*a*(3*c^3+1
6*c^2*d+12*c*d^2+4*d^3)*tan(f*x+e)/f+1/24*a*d*(6*c^2+20*c*d+9*d^2)*sec(f*x
+e)*tan(f*x+e)/f+1/12*a*(3*c+4*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/4*a*(c
+d*sec(f*x+e))^3*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.58

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$= \frac{a\left(24c^3\coth^{-1}(\sin(e+fx)) + 9d(2c+d)^2\operatorname{arctanh}(\sin(e+fx)) + \tan(e+fx)\left(24(c+d)^3 + 9d(2c+d)^2\right.\right.}{24f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

output
```
(a*(24*c^3*ArcCoth[Sin[e + f*x]] + 9*d*(2*c + d)^2*ArcTanh[Sin[e + f*x]] +
 Tan[e + f*x]*(24*(c + d)^3 + 9*d*(2*c + d)^2*Sec[e + f*x] + 6*d^3*Sec[e +
 f*x]^3 + 8*d^2*(3*c + d)*Tan[e + f*x]^2)))/(24*f)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {3042, 4490, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c+d\sec(e+fx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \text{4490}$$
$$\frac{1}{4}\int \sec(e+fx)(c+d\sec(e+fx))^2(a(4c+3d)+a(3c+4d)\sec(e+fx))dx+\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(a(4c+3d)+a(3c+4d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{4490}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \sec(e+fx)(c+d\sec(e+fx))\left(a\left(12c^2+15dc+8d^2\right)+a\left(6c^2+20dc+9d^2\right)\sec(e+fx)\right)dx+\frac{a(3c+4}{}\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(a\left(12c^2+15dc+8d^2\right)+a\left(6c^2+20dc+9d^2\right)\csc\left(e+fx+\right.\right.\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(e+fx)\left(3a\left(8c^3+12dc^2+12d^2c+3d^3\right)+4a\left(3c^3+16dc^2+12d^2c+4d^3\right)\sec(e+fx)\right)dx+\frac{ad(}{}\right.\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(3a\left(8c^3+12dc^2+12d^2c+3d^3\right)+4a\left(3c^3+16dc^2+12d^2c+4d^3\right)\csc\left(e+fx+\right.\right.\right.\right.\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(4a\left(3c^3+16c^2d+12cd^2+4d^3\right)\int \sec^2(e+fx)dx+3a\left(8c^3+12c^2d+12cd^2+3d^3\right)\int \sec(e+fx)dx\right)\right.\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3a\left(8c^3+12c^2d+12cd^2+3d^3\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+4a\left(3c^3+16c^2d+12cd^2+4d^3\right)\int \csc\left(e+\right.\right.\right.\right.$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3a\left(8c^3+12c^2d+12cd^2+3d^3\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{4a\left(3c^3+16c^2d+12cd^2+4d^3\right)\int 1d(-\tan(}{f}\right.\right.\right.\right.$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3a\left(8c^3+12c^2d+12cd^2+3d^3\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{4a\left(3c^3+16c^2d+12cd^2+4d^3\right)\tan(e+fx)}{f}\right.\right.\right.\right.$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{3a\left(8c^3+12c^2d+12cd^2+3d^3\right)\text{arctanh}(\sin(e+fx))}{f}+\frac{4a\left(3c^3+16c^2d+12cd^2+4d^3\right)\tan(e+fx)}{f}\right)\right.\right.\right.$$

$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

output
```
(a*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(4*f) + ((a*(3*c + 4*d)*(c + d*Sec
[e + f*x])^2*Tan[e + f*x])/(3*f) + ((a*d*(6*c^2 + 20*c*d + 9*d^2)*Sec[e +
f*x]*Tan[e + f*x])/(2*f) + ((3*a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*Arc
Tanh[Sin[e + f*x]])/f + (4*a*(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e +
 f*x])/f)/2)/3)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08

method	result
parts	$-\frac{(3acd^2+ad^3)\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{(3ac^2d+3acd^2)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
derivativedivides	$ac^3\ln(\sec(fx+e)+\tan(fx+e))+3ac^2d\tan(fx+e)+3acd^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-ad^3\Big(-$
default	$ac^3\ln(\sec(fx+e)+\tan(fx+e))+3ac^2d\tan(fx+e)+3acd^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-ad^3\Big(-$
parallelrisch	$2\Bigg(-2\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(\frac{3}{2}c^2d+\frac{3}{2}cd^2+\frac{3}{8}d^3+c^3\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+2\Big(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2$
norman	$\dfrac{-\frac{a(8c^3+12c^2d+12cd^2+3d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{a(8c^3+36c^2d+36cd^2+13d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{a(72c^3+180c^2d+84cd^2+49d^3)\tan\Big(}{12f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$
risch	$ia\big(72c^2d+48cd^2+16d^3+24c^3+9d^3e^{i(fx+e)}+24c^3e^{6i(fx+e)}-9d^3e^{7i(fx+e)}+33d^3e^{3i(fx+e)}+72c^3e^{2i(fx+e)}+64d^3e^{2i(f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output
```
-(3*a*c*d^2+a*d^3)/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+(3*a*c^2*d+3*a*c*d
^2)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+(a*c^3+3*a
*c^2*d)/f*tan(f*x+e)+a*c^3/f*ln(sec(f*x+e)+tan(f*x+e))+a*d^3/f*(-(-1/4*sec
(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.23

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$

$$=\frac{3\,(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3)\cos(fx+e)^4\log(\sin(fx+e)+1)-3\,(8\,ac^3+12\,ac^2d+12\,acd^2+}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
1/48*(3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^4*log(s
in(f*x + e) + 1) - 3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x
 + e)^4*log(-sin(f*x + e) + 1) + 2*(6*a*d^3 + 8*(3*a*c^3 + 9*a*c^2*d + 6*a
*c*d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*a*c^2*d + 4*a*c*d^2 + a*d^3)*cos(f
*x + e)^2 + 8*(3*a*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
 e)^4)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$= a\left(\int c^3\sec(e+fx)\,dx+\int c^3\sec^2(e+fx)\,dx+\int d^3\sec^4(e+fx)\,dx + \int d^3\sec^5(e+fx)\,dx+\int 3cd^2\sec^3(e+fx)\,dx+\int 3cd^2\sec^4(e+fx)\,dx + \int 3c^2d\sec^2(e+fx)\,dx+\int 3c^2d\sec^3(e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)
```

output
```
a*(Integral(c**3*sec(e + f*x), x) + Integral(c**3*sec(e + f*x)**2, x) + In
tegral(d**3*sec(e + f*x)**4, x) + Integral(d**3*sec(e + f*x)**5, x) + Inte
gral(3*c*d**2*sec(e + f*x)**3, x) + Integral(3*c*d**2*sec(e + f*x)**4, x)
+ Integral(3*c**2*d*sec(e + f*x)**2, x) + Integral(3*c**2*d*sec(e + f*x)**
3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.56

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$

$$= \frac{48\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)acd^2+16\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ad^3-3\,ad^3\left(\frac{2\left(3\,\sin(fx+e)^3\right.}{\sin(fx+e)^4-2}\right.}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="max
ima")
```

output

```
1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c*d^2 + 16*(tan(f*x + e)^3 +
3*tan(f*x + e))*a*d^3 - 3*a*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(si
n(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 36*a*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(si
n(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin
(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c
^3*log(sec(f*x + e) + tan(f*x + e)) + 48*a*c^3*tan(f*x + e) + 144*a*c^2*d*
tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(161) = 322.

Time = 0.22 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.22

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$

$$= \frac{3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="gia
c")
```

output
```
1/24*(3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*log(abs(tan(1/2*f*x
+ 1/2*e) + 1)) - 3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*log(abs(t
an(1/2*f*x + 1/2*e) - 1)) - 2*(24*a*c^3*tan(1/2*f*x + 1/2*e)^7 + 36*a*c^2*
d*tan(1/2*f*x + 1/2*e)^7 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 9*a*d^3*tan
(1/2*f*x + 1/2*e)^7 - 72*a*c^3*tan(1/2*f*x + 1/2*e)^5 - 180*a*c^2*d*tan(1/
2*f*x + 1/2*e)^5 - 84*a*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 49*a*d^3*tan(1/2*f*
x + 1/2*e)^5 + 72*a*c^3*tan(1/2*f*x + 1/2*e)^3 + 252*a*c^2*d*tan(1/2*f*x +
 1/2*e)^3 + 156*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 31*a*d^3*tan(1/2*f*x + 1/
2*e)^3 - 24*a*c^3*tan(1/2*f*x + 1/2*e) - 108*a*c^2*d*tan(1/2*f*x + 1/2*e)
- 108*a*c*d^2*tan(1/2*f*x + 1/2*e) - 39*a*d^3*tan(1/2*f*x + 1/2*e))/(tan(1
/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.49

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$

$$=\frac{\left(-2\,a\,c^3-3\,a\,c^2\,d-3\,a\,c\,d^2-\frac{3\,a\,d^3}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(6\,a\,c^3+15\,a\,c^2\,d+7\,a\,c\,d^2+\frac{49\,a\,d^3}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right.}$$

$$+\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{2\,\left(4\,c^3+6\,c^2\,d+6\,c\,d^2+\frac{3\,d^3}{2}\right)}\right)\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{4\,f}$$

input
```
int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

output
```
(tan(e/2 + (f*x)/2)*(2*a*c^3 + (13*a*d^3)/4 + 9*a*c*d^2 + 9*a*c^2*d) - tan
(e/2 + (f*x)/2)^7*(2*a*c^3 + (3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d) - tan(e/
2 + (f*x)/2)^3*(6*a*c^3 + (31*a*d^3)/12 + 13*a*c*d^2 + 21*a*c^2*d) + tan(e
/2 + (f*x)/2)^5*(6*a*c^3 + (49*a*d^3)/12 + 7*a*c*d^2 + 15*a*c^2*d))/(f*(6*
tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + t
an(e/2 + (f*x)/2)^8 + 1)) + (a*atanh((tan(e/2 + (f*x)/2)*(12*c*d^2 + 12*c^
2*d + 8*c^3 + 3*d^3))/(2*(6*c*d^2 + 6*c^2*d + 4*c^3 + (3*d^3)/2)))*(12*c*d
^2 + 12*c^2*d + 8*c^3 + 3*d^3))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 801, normalized size of antiderivative = 4.68

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx=\text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x)
```

output

```
(a*( - 24*cos(e + f*x)*sin(e + f*x)**3*c**3 - 72*cos(e + f*x)*sin(e + f*x)
**3*c**2*d - 48*cos(e + f*x)*sin(e + f*x)**3*c*d**2 - 16*cos(e + f*x)*sin(
e + f*x)**3*d**3 + 24*cos(e + f*x)*sin(e + f*x)*c**3 + 72*cos(e + f*x)*sin
(e + f*x)*c**2*d + 72*cos(e + f*x)*sin(e + f*x)*c*d**2 + 24*cos(e + f*x)*s
in(e + f*x)*d**3 - 24*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**3 - 36*
log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**2*d - 36*log(tan((e + f*x)/2)
 - 1)*sin(e + f*x)**4*c*d**2 - 9*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4
*d**3 + 48*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c**3 + 72*log(tan((e
+ f*x)/2) - 1)*sin(e + f*x)**2*c**2*d + 72*log(tan((e + f*x)/2) - 1)*sin(e
 + f*x)**2*c*d**2 + 18*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*d**3 - 24
*log(tan((e + f*x)/2) - 1)*c**3 - 36*log(tan((e + f*x)/2) - 1)*c**2*d - 36
*log(tan((e + f*x)/2) - 1)*c*d**2 - 9*log(tan((e + f*x)/2) - 1)*d**3 + 24*
log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*c**3 + 36*log(tan((e + f*x)/2) +
 1)*sin(e + f*x)**4*c**2*d + 36*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*
c*d**2 + 9*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*d**3 - 48*log(tan((e
+ f*x)/2) + 1)*sin(e + f*x)**2*c**3 - 72*log(tan((e + f*x)/2) + 1)*sin(e +
 f*x)**2*c**2*d - 72*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*c*d**2 - 18
*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*d**3 + 24*log(tan((e + f*x)/2)
+ 1)*c**3 + 36*log(tan((e + f*x)/2) + 1)*c**2*d + 36*log(tan((e + f*x)/2)
+ 1)*c*d**2 + 9*log(tan((e + f*x)/2) + 1)*d**3 - 36*sin(e + f*x)**3*c**...
```

3.187 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 108

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx &= \frac{a(2c^2+2cd+d^2)\operatorname{arctanh}(\sin(e+fx))}{2f}+\frac{2a(c^2+3cd+d^2)\tan(e+fx)}{3f}\\ &\quad+\frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f}\end{aligned}$$

output
```
1/2*a*(2*c^2+2*c*d+d^2)*arctanh(sin(f*x+e))/f+2/3*a*(c^2+3*c*d+d^2)*tan(f*
x+e)/f+1/6*a*d*(2*c+3*d)*sec(f*x+e)*tan(f*x+e)/f+1/3*a*(c+d*sec(f*x+e))^2*
tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{a\left(6c^2\coth^{-1}(\sin(e+fx))+3d(2c+d)\text{arctanh}(\sin(e+fx))+\tan(e+fx)\left(3d(2c+d)\sec(e+fx)+2\right.\right.}{6f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

output
```
(a*(6*c^2*ArcCoth[Sin[e + f*x]] + 3*d*(2*c + d)*ArcTanh[Sin[e + f*x]] + Ta
n[e + f*x]*(3*d*(2*c + d)*Sec[e + f*x] + 2*(3*(c + d)^2 + d^2*Tan[e + f*x]
^2))))/(6*f)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c+d\sec(e+fx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \text{4490}$$
$$\frac{1}{3}\int \sec(e+fx)(c+d\sec(e+fx))(a(3c+2d)+a(2c+3d)\sec(e+fx))dx+\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(a(3c+2d)+a(2c+3d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 4485$$

$$\frac{1}{3}\left(\frac{1}{2}\int \sec(e+fx)\left(3a\left(2c^2+2dc+d^2\right)+4a\left(c^2+3dc+d^2\right)\sec(e+fx)\right)dx+\frac{ad(2c+3d)\tan(e+fx)\sec(e+}{2f}\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(3a\left(2c^2+2dc+d^2\right)+4a\left(c^2+3dc+d^2\right)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{ad(2c+3d)\tan(e}{2}\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 4274$$

$$\frac{1}{3}\left(\frac{1}{2}\left(4a\left(c^2+3cd+d^2\right)\int \sec^2(e+fx)dx+3a\left(2c^2+2cd+d^2\right)\int \sec(e+fx)dx\right)+\frac{ad(2c+3d)\tan(e+fx)}{2f}\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3a\left(2c^2+2cd+d^2\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+4a\left(c^2+3cd+d^2\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)+\frac{ad(2c+3d)}{}\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 4254$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3a\left(2c^2+2cd+d^2\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{4a\left(c^2+3cd+d^2\right)\int 1d(-\tan(e+fx))}{f}\right)+\frac{ad(2c+3d)\,\mathrm{ta}}{}\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3a\left(2c^2+2cd+d^2\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{4a\left(c^2+3cd+d^2\right)\tan(e+fx)}{f}\right)+\frac{ad(2c+3d)\tan(e+fx}{2f}\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3a\left(2c^2+2cd+d^2\right)\text{arctanh}(\sin(e+fx))}{f}+\frac{4a\left(c^2+3cd+d^2\right)\tan(e+fx)}{f}\right)+\frac{ad(2c+3d)\tan(e+fx)s}{2f}\right.$$
$$\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

output
```
(a*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f) + ((a*d*(2*c + 3*d)*Sec[e +
f*x]*Tan[e + f*x])/(2*f) + ((3*a*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x
]])/f + (4*a*(c^2 + 3*c*d + d^2)*Tan[e + f*x])/f)/2)/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

method	result
parts	$\frac{(2acd+a\,d^2)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{(a\,c^2+2acd)\tan(fx+e)}{f}+\frac{a\,c^2\ln(\sec(fx+e)+\tan}{f}$
derivativedivides	$\frac{a\,c^2\ln(\sec(fx+e)+\tan(fx+e))+2acd\tan(fx+e)+a\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a\,c^2\tan(fx-}{f}$
default	$\frac{a\,c^2\ln(\sec(fx+e)+\tan(fx+e))+2acd\tan(fx+e)+a\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a\,c^2\tan(fx-}{f}$
norman	$\frac{-\frac{a\left(2c^2+2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{a\left(2c^2+6cd+3d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{4a\left(3c^2+6cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{a(2c^2+2cd+d^2)\ln(}{2f}$
parallelrisch	$\frac{2a\left(-\frac{3\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\left(c^2+cd+\frac{1}{2}d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\left(c^2+cd+\frac{1}{2}d^2\right)\ln(\tan(\frac{fx}{2}}{2}\right.}{f(\cos(3fx+3e)+3\cos(fx+e}$
risch	$-\frac{ia(6cd\,e^{5i(fx+e)}+3d^2e^{5i(fx+e)}-6c^2e^{4i(fx+e)}-12cd\,e^{4i(fx+e)}-12c^2e^{2i(fx+e)}-24cd\,e^{2i(fx+e)}-12d^2e^{2i(fx+e)}-6d}{3f\left(e^{2i(fx+e)}+1\right)^3}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
(2*a*c*d+a*d^2)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))
)+(a*c^2+2*a*c*d)/f*tan(f*x+e)+a*c^2/f*ln(sec(f*x+e)+tan(f*x+e))-a*d^2/f*(
-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.39

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{3\left(2\,ac^2+2\,acd+ad^2\right)\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\left(2\,ac^2+2\,acd+ad^2\right)\cos\left(fx+e\right)^3\log\left(-\right.}{12\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output

```
1/12*(3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
 3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(
2*a*d^2 + 2*(3*a*c^2 + 6*a*c*d + 2*a*d^2)*cos(f*x + e)^2 + 3*(2*a*c*d + a*
d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= a\left(\int c^2\sec(e+fx)\,dx+\int c^2\sec^2(e+fx)\,dx+\int d^2\sec^3(e+fx)\,dx\right.$$
$$\left.+\int d^2\sec^4(e+fx)\,dx+\int 2cd\sec^2(e+fx)\,dx+\int 2cd\sec^3(e+fx)\,dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
```

output

```
a*(Integral(c**2*sec(e + f*x), x) + Integral(c**2*sec(e + f*x)**2, x) + In
tegral(d**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**4, x) + Inte
gral(2*c*d*sec(e + f*x)**2, x) + Integral(2*c*d*sec(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.53

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= \frac{4\left(\tan(fx+e)^3+3\tan(fx+e)\right)ad^2-6\,acd\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-\right.}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*d^2 - 6*a*c*d*(2*sin(f*x + e)/
(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*
a*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(s
in(f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*a*c^2*t
an(f*x + e) + 24*a*c*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(100) = 200.

Time = 0.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.15

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$

$$= \frac{3\left(2\,ac^2+2\,acd+ad^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(2\,ac^2+2\,acd+ad^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
1/6*(3*(2*a*c^2 + 2*a*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*
(2*a*c^2 + 2*a*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(6*a*c^
2*tan(1/2*f*x + 1/2*e)^5 + 6*a*c*d*tan(1/2*f*x + 1/2*e)^5 + 3*a*d^2*tan(1/
2*f*x + 1/2*e)^5 - 12*a*c^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*c*d*tan(1/2*f*x
+ 1/2*e)^3 - 4*a*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*c^2*tan(1/2*f*x + 1/2*e)
 + 18*a*c*d*tan(1/2*f*x + 1/2*e) + 9*a*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*
f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.81

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{a\,\mathrm{atanh}\left(\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)}{4\,c^2+4\,c\,d+2\,d^2}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)}{f}$$
$$-\frac{\left(2\,a\,c^2+2\,a\,c\,d+a\,d^2\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-4\,a\,c^2-8\,a\,c\,d-\frac{4\,a\,d^2}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(2\,a\,c^2+6\,a\,c\,d+\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

input
```
int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

output
```
(a*atanh((2*tan(e/2 + (f*x)/2)*(2*c*d + 2*c^2 + d^2))/(4*c*d + 4*c^2 + 2*d
^2))*(2*c*d + 2*c^2 + d^2))/f - (tan(e/2 + (f*x)/2)*(2*a*c^2 + 3*a*d^2 + 6
*a*c*d) + tan(e/2 + (f*x)/2)^5*(2*a*c^2 + a*d^2 + 2*a*c*d) - tan(e/2 + (f*
x)/2)^3*(4*a*c^2 + (4*a*d^2)/3 + 8*a*c*d))/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*
tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.17

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{a\left(-6\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^2c^2-6\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx\right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x)
```

output

```
(a*( - 6*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c**2 - 6*c
os(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c*d - 3*cos(e + f*x)
*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*d**2 + 6*cos(e + f*x)*log(tan((
e + f*x)/2) - 1)*c**2 + 6*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c*d + 3*c
os(e + f*x)*log(tan((e + f*x)/2) - 1)*d**2 + 6*cos(e + f*x)*log(tan((e + f
*x)/2) + 1)*sin(e + f*x)**2*c**2 + 6*cos(e + f*x)*log(tan((e + f*x)/2) + 1
)*sin(e + f*x)**2*c*d + 3*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f
*x)**2*d**2 - 6*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*c**2 - 6*cos(e + f*
x)*log(tan((e + f*x)/2) + 1)*c*d - 3*cos(e + f*x)*log(tan((e + f*x)/2) + 1
)*d**2 - 6*cos(e + f*x)*sin(e + f*x)*c*d - 3*cos(e + f*x)*sin(e + f*x)*d**
2 + 6*sin(e + f*x)**3*c**2 + 12*sin(e + f*x)**3*c*d + 4*sin(e + f*x)**3*d*
*2 - 6*sin(e + f*x)*c**2 - 12*sin(e + f*x)*c*d - 6*sin(e + f*x)*d**2))/(6*
cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.188 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 56

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{a(2c+d)\text{arctanh}(\sin(e+fx))}{2f}+\frac{a(c+d)\tan(e+fx)}{f}+\frac{ad\sec(e+fx)\tan(e+fx)}{2f}$$

output
```
1/2*a*(2*c+d)*arctanh(sin(f*x+e))/f+a*(c+d)*tan(f*x+e)/f+1/2*a*d*sec(f*x+e
)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{ac\coth^{-1}(\sin(e+fx))}{f}+\frac{ad\text{arctanh}(\sin(e+fx))}{2f} + \frac{ac\tan(e+fx)}{f}+\frac{ad\tan(e+fx)}{f}+\frac{ad\sec(e+fx)\tan(e+fx)}{2f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

output
```
(a*c*ArcCoth[Sin[e + f*x]])/f + (a*d*ArcTanh[Sin[e + f*x]])/(2*f) + (a*c*T
an[e + f*x])/f + (a*d*Tan[e + f*x])/f + (a*d*Sec[e + f*x]*Tan[e + f*x])/(2
*f)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{2}\int \sec(e+fx)(a(2c+d)+2a(c+d)\sec(e+fx))dx+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a(2c+d)+2a(c+d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{2}\left(2a(c+d)\int \sec^2(e+fx)dx+a(2c+d)\int \sec(e+fx)dx\right)+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(a(2c+d)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+2a(c+d)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \textbf{4254}$$

$$\frac{1}{2}\left(a(2c+d)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{2a(c+d)\int 1d(-\tan(e+fx))}{f}\right)+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{1}{2}\left(a(2c+d)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{2a(c+d)\tan(e+fx)}{f}\right)+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \textbf{4257}$$

$$\frac{1}{2}\left(\frac{a(2c+d)\text{arctanh}(\sin(e+fx))}{f}+\frac{2a(c+d)\tan(e+fx)}{f}\right)+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

output
```
(a*d*Sec[e + f*x]*Tan[e + f*x])/(2*f) + ((a*(2*c + d)*ArcTanh[Sin[e + f*x]
])/f + (2*a*(c + d)*Tan[e + f*x])/f)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)+ac\tan(fx+e)+ad\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
default	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)+ac\tan(fx+e)+ad\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parts	$\frac{(ac+ad)\tan(fx+e)}{f}+\frac{ac\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{ad\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parallelrisch	$-\frac{a((c+\frac{d}{2})(1+\cos(2fx+2e))\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)-(c+\frac{d}{2})(1+\cos(2fx+2e))\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)+(-c-d)\sin(2fx}{f(1+\cos(2fx+2e))}$
norman	$\frac{\frac{a(2c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{a(2c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}-\frac{a(2c+d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a(2c+d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$
risch	$-\frac{ia\left(d\,\mathrm{e}^{3i(fx+e)}-2c\,\mathrm{e}^{2i(fx+e)}-2d\,\mathrm{e}^{2i(fx+e)}-d\,\mathrm{e}^{i(fx+e)}-2c-2d\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^{2}}+\frac{ac\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{f}+\frac{a\ln\left(\mathrm{e}^{i(fx+e)}+i\right)d}{2f}-$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*c*ln(sec(f*x+e)+tan(f*x+e))+a*d*tan(f*x+e)+a*c*tan(f*x+e)+a*d*(1/2*
sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$= \frac{(2\,ac+ad)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2\,ac+ad)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\,(}{4\,f\cos\left(fx+e\right)^2}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="frica
s")
```

output
```
1/4*((2*a*c + a*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + a*d)*co
s(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a*d + 2*(a*c + a*d)*cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$= a\left(\int c\sec\left(e+fx\right)dx+\int c\sec^2\left(e+fx\right)dx+\int d\sec^2\left(e+fx\right)dx\right.$$
$$\left.+\int d\sec^3\left(e+fx\right)dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

output
```
a*(Integral(c*sec(e + f*x), x) + Integral(c*sec(e + f*x)**2, x) + Integral
(d*sec(e + f*x)**2, x) + Integral(d*sec(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.57

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx =$$

$$-\frac{ad\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log\big(\sin(fx+e)+1\big)+\log\big(\sin(fx+e)-1\big)\right)-4\,ac\log\big(\sec(fx+e)+\tan(fx+}{4\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
-1/4*(a*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + l
og(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*a*c*tan
(f*x + e) - 4*a*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(52) = 104.

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.21

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$$

$$=\frac{(2\,ac+ad)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-(2\,ac+ad)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\big(2\,ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a}{}}{2\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
1/2*((2*a*c + a*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - (2*a*c + a*d)*log(
abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(2*a*c*tan(1/2*f*x + 1/2*e)^3 + a*d*tan
(1/2*f*x + 1/2*e)^3 - 2*a*c*tan(1/2*f*x + 1/2*e) - 3*a*d*tan(1/2*f*x + 1/2
*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 11.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.98

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$=\frac{a\,\mathrm{atanh}\left(\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c+d)}{4\,c+2\,d}\right)\,(2\,c+d)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,(2\,a\,c+a\,d)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,a\,c+3\,a\,d)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

input
```
int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

output
```
(a*atanh((2*tan(e/2 + (f*x)/2)*(2*c + d))/(4*c + 2*d))*(2*c + d))/f - (tan
(e/2 + (f*x)/2)^3*(2*a*c + a*d) - tan(e/2 + (f*x)/2)*(2*a*c + 3*a*d))/(f*(
tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.71

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$=\frac{a\left(-2\cos\left(fx+e\right)\sin\left(fx+e\right)c-2\cos\left(fx+e\right)\sin\left(fx+e\right)d-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^{2}c\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

output
```
(a*( - 2*cos(e + f*x)*sin(e + f*x)*c - 2*cos(e + f*x)*sin(e + f*x)*d - 2*l
og(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c - log(tan((e + f*x)/2) - 1)*sin
(e + f*x)**2*d + 2*log(tan((e + f*x)/2) - 1)*c + log(tan((e + f*x)/2) - 1)
*d + 2*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*c + log(tan((e + f*x)/2)
+ 1)*sin(e + f*x)**2*d - 2*log(tan((e + f*x)/2) + 1)*c - log(tan((e + f*x)
/2) + 1)*d - sin(e + f*x)*d))/(2*f*(sin(e + f*x)**2 - 1))
```

3.189 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\operatorname{arctanh}(\sin(e+fx))}{df} - \frac{2a\sqrt{c-d}\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d\sqrt{c+d}f}$$

output
```
a*arctanh(sin(f*x+e))/d/f-2*a*(c-d)^(1/2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+
1/2*e)/(c+d)^(1/2))/d/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.55

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\Bigg(\frac{2(c-d)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+}{df}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

output
```
(a*((2*(c - d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[
c^2 - d^2] - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]]))/(d*f)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4486}$$

$$\frac{a\int \sec(e+fx)dx}{d}-\frac{a(c-d)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{d}-\frac{a(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(e+fx))}{df}-\frac{a(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{d}$$

$$\Big\downarrow \text{4318}$$

$$\frac{a\text{arctanh}(\sin(e+fx))}{df}-\frac{a(c-d)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\text{arctanh}(\sin(e+fx))}{df}-\frac{a(c-d)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{d^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{a\text{arctanh}(\sin(e+fx))}{df}-\frac{2a(c-d)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{d^2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\text{arctanh}(\sin(e+fx))}{df}-\frac{2a\sqrt{c-d}\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c+d}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

output

```
(a*ArcTanh[Sin[e + f*x]])/(d*f) - (2*a*Sqrt[c - d]*ArcTanh[(Sqrt[c - d]*Ta
n[(e + f*x)/2])/Sqrt[c + d]])/(d*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{4 a \left(\frac{\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{4 d}-\frac{\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{4 d}-\frac{(c-d) \operatorname{arctanh}\left(\frac{(c-d) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{2 d \sqrt{(c-d)(c+d)}}\right)}{f}$
default	$\frac{4 a \left(\frac{\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{4 d}-\frac{\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{4 d}-\frac{(c-d) \operatorname{arctanh}\left(\frac{(c-d) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{2 d \sqrt{(c-d)(c+d)}}\right)}{f}$
risch	$\frac{\sqrt{(c-d)(c+d)}\, a \ln\left(\mathrm{e}^{i(f x+e)}-\frac{i\sqrt{(c-d)(c+d)}-d}{c}\right)}{(c+d) f d}-\frac{\sqrt{(c-d)(c+d)}\, a \ln\left(\mathrm{e}^{i(f x+e)}+\frac{i\sqrt{(c-d)(c+d)}+d}{c}\right)}{(c+d) f d}+\frac{a \ln\left(\mathrm{e}^{i(f x+}\right.}{d f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
4/f*a*(1/4/d*ln(tan(1/2*f*x+1/2*e)+1)-1/4/d*ln(tan(1/2*f*x+1/2*e)-1)-1/2*(
c-d)/d/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.70

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx$$

$$= \Bigg[\frac{a\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\,(c^2+cd+(cd+d^2)\cos(fx+e))\sqrt{\frac{c-d}{c+d}}\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)+a\log\big(\sin\big(f}{2\,df}$$

$$-\frac{2\,a\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{(d\cos(fx+e)+c)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right)-a\log\left(\sin\left(fx+e\right)+1\right)+a\log\left(-\sin\left(fx+e\right)+1\right)}{2\,df}\Bigg]$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="frica
s")
```

output
```
[1/2*(a*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(
f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d)
)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d
^2)) + a*log(sin(f*x + e) + 1) - a*log(-sin(f*x + e) + 1))/(d*f), -1/2*(2*
a*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d
))/((c - d)*sin(f*x + e))) - a*log(sin(f*x + e) + 1) + a*log(-sin(f*x + e)
 + 1))/(d*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = a\left(\int \frac{\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx + \int \frac{\sec^2\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output

```
a*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)*
*2/(c + d*sec(e + f*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxim
a")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(60) = 120$.

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.84

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx$$

$$= \frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d} - \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d} + \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)(ac}{\sqrt{-c^2+d^2}d}}{f}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
(a*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d - a*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/d + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*ta
n(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(a*c - a*d
)/(sqrt(-c^2 + d^2)*d))/f
```

Mupad [B] (verification not implemented)

Time = 10.91 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.83

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c+d\right)} + \frac{2\,a\,\left(\mathrm{atanh}\left(\frac{d^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,d^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c^2\,d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c^2-d^2\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}\,\left(d^2+c\,d\right)}\right)\,\sqrt{c^2-d^2}+c\,\mathrm{at}\right.}{d\,f\,\left(c+d\right)}$$

input
```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

output
```
(2*a*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*a*(ata
nh((d^3*sin(e/2 + (f*x)/2) - c^3*sin(e/2 + (f*x)/2) + c*d^2*sin(e/2 + (f*x
)/2) - c^2*d*sin(e/2 + (f*x)/2) + c*sin(e/2 + (f*x)/2)*(c^2 - d^2))/(cos(e
/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*(c*d + d^2)))*(c^2 - d^2)^(1/2) + c*atanh(
sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))))/(d*f*(c + d))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.77

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\left(-2\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)c-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d+\right.}{df\,(c+d)}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output
```
(a*( - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*
d)/sqrt( - c**2 + d**2)) - log(tan((e + f*x)/2) - 1)*c - log(tan((e + f*x)
/2) - 1)*d + log(tan((e + f*x)/2) + 1)*c + log(tan((e + f*x)/2) + 1)*d))/(
d*f*(c + d))
```

3.190 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 79

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2a\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}(c+d)^{3/2}f} + \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))}$$

output

```
2*a*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(1/2)/(c+d)^
(3/2)/f+a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{a\left(-\frac{2\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{\sin(e+fx)}{d+c\cos(e+fx)}\right)}{(c+d)f}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

output
```
(a*((-2*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d
^2] + Sin[e + f*x]/(d + c*Cos[e + f*x])))/((c + d)*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3042, 4491, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4491}$$

$$\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}-\frac{\int -\frac{a(c-d)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(c-d)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(c-d)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c^2-d^2}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{4318}$$

$$\frac{a(c-d)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{d\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{a(c-d)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{d\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

$$\Big\downarrow \textbf{3138}$$

$$\frac{2a(c-d)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{df\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{2a\sqrt{c-d}\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c+d}\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

output
```
(2*a*Sqrt[c - d]*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqr
t[c + d]*(c^2 - d^2)*f) + (a*Tan[e + f*x])/((c + d)*f*(c + d*Sec[e + f*x])
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{4a\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}+\frac{\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{2(c+d)\sqrt{(c-d)(c+d)}}\right)}{f}$
default	$\frac{4a\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}+\frac{\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{2(c+d)\sqrt{(c-d)(c+d)}}\right)}{f}$
risch	$\frac{2ia\left(d\,\mathrm{e}^{i(fx+e)}+c\right)}{cf(c+d)\left(c\,\mathrm{e}^{2i(fx+e)}+2d\,\mathrm{e}^{i(fx+e)}+c\right)}+\frac{a\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^{2}-id^{2}+d\sqrt{c^{2}-d^{2}}}{\sqrt{c^{2}-d^{2}}\,c}\right)}{\sqrt{c^{2}-d^{2}}\,(c+d)f}-\frac{a\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^{2}-id^{2}-d\sqrt{c^{2}-d^{2}}}{\sqrt{c^{2}-d^{2}}\,c}\right.}{\sqrt{c^{2}-d^{2}}\,(c+d)f}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
4/f*a*(-1/2*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1
/2*e)^2*d-c-d)+1/2/(c+d)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2
*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(70) = 140$.

Time = 0.15 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.52

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$$
$$= \left[\frac{(ac\cos(fx+e)+ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)}{2\,((c^4+c^3d-c^2d^2-cd^3)f\cos(fx+e)+(c^3d+c^2d^2-cd^3-d^4)f)}\right.$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output
```
[1/2*((a*c*cos(f*x + e) + a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (
c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f
*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) +
2*(a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d^3)*f*cos(f*x
 + e) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f), ((a*c*cos(f*x + e) + a*d)*sqrt
(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*si
n(f*x + e))) + (a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d
^3)*f*cos(f*x + e) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = a\left(\int \frac{\sec(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx + \int \frac{\sec^2(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

output
```
a*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**2
), x) + Integral(sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e +
 f*x)**2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.73

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a}{\sqrt{-c^2+d^2}(c+d)}+\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(c+d)}\right)}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
-2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f
*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a/(sqrt(-c^2 + d^
2)*(c + d)) + a*tan(1/2*f*x + 1/2*e)/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/
2*f*x + 1/2*e)^2 - c - d)*(c + d)))/f
```

Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(c+d\right)\,\left(\left(d-c\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+c+d\right)} + \frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)}{f\,{\left(c+d\right)}^{3/2}\,\sqrt{c-d}}$$

input
```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

output
```
(2*a*tan(e/2 + (f*x)/2))/(f*(c + d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d))
) + (2*a*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c +
d)^(3/2)*(c - d)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 331, normalized size of antiderivative = 4.19

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{2a\left(\sqrt{-c^2+d^2}\, atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c-\sqrt{-c^2+d^2}\, atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right.}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c^4-\right.}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

output

```
(2*a*(sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*c - sqrt( - c**2 + d**2)*atan((t
an((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x
)/2)**2*d - sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/
2)*d)/sqrt( - c**2 + d**2))*c - sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2
)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*d - tan((e + f*x)/2)*c**2
 + tan((e + f*x)/2)*d**2))/(f*(tan((e + f*x)/2)**2*c**4 - 2*tan((e + f*x)/2
)**2*c**2*d**2 + tan((e + f*x)/2)**2*d**4 - c**4 - 2*c**3*d + 2*c*d**3 + d
**4))
```

3.191 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 131

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{a(2c-d)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{5/2}f} + \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))}$$

output

```
a*(2*c-d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3/2)/
(c+d)^(5/2)/f+1/2*a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^2+1/2*a*(c-2*d)*ta
n(f*x+e)/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.27

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{a(1+\cos(e+fx))\sec^2\left(\frac{1}{2}(e+fx)\right)\Big(-2(2c-d)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^2+\sqrt{c}}{4(c-d)(c+d)^2\sqrt{c^2-d^2}f(d+c\cos(e+fx}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

output
```
(a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(-2*(2*c - d)*ArcTanh[((-c + d)*T
an[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2 + Sqrt[c^2 - d^2]
*((c - 2*d)*d + (2*c^2 - 2*c*d - d^2)*Cos[e + f*x])*Sin[e + f*x]))/(4*(c -
 d)*(c + d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {3042, 4491, 25, 3042, 4491, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4491}$$

$$\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}-\frac{\int -\frac{\sec(e+fx)(2a(c-d)+a\sec(e+fx)(c-d))}{(c+d\sec(e+fx))^2}dx}{2\left(c^2-d^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(e+fx)(2a(c-d)+a\sec(e+fx)(c-d))}{(c+d\sec(e+fx))^2}dx}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(2a(c-d)+a\csc\left(e+fx+\frac{\pi}{2}\right)(c-d)\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{4491}$$

$$\frac{\frac{a(c-2d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}-\frac{\int -\frac{a(c-d)(2c-d)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a(c-d)(2c-d)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}+\frac{a(c-2d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{a(c-d)(2c-d)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}+\frac{a(c-2d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{a(c-d)(2c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c^2-d^2}+\frac{a(c-2d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \ 4318$$

$$\frac{\frac{a(c-d)(2c-d)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{d(c^2-d^2)}+\frac{a(c-2d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{a(c-d)(2c-d)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{d(c^2-d^2)}+\frac{a(c-2d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \ 3138$$

$$\frac{\frac{2a(c-d)(2c-d)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{df(c^2-d^2)}+\frac{a(c-2d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{2a\sqrt{c-d}(2c-d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c+d}(c^2-d^2)}+\frac{a(c-2d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

output
```
(a*Tan[e + f*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])^2) + ((2*a*Sqrt[c - d]*
(2*c - d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c + d
]*(c^2 - d^2)*f) + (a*(c - 2*d)*Tan[e + f*x])/((c + d)*f*(c + d*Sec[e + f*
x])))/(2*(c^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4491

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{4a\left(\frac{-\frac{(2c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{4\left(c^{2}+2cd+d^{2}\right)}+\frac{(2c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c-d)(c+d)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}+\frac{(2c-d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{4\left(c^{3}+c^{2}d-c\,d^{2}-d^{3}\right)\sqrt{(c-d)(c+d)}}\right)}{f}$
default	$\frac{4a\left(\frac{-\frac{(2c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{4\left(c^{2}+2cd+d^{2}\right)}+\frac{(2c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c-d)(c+d)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}+\frac{(2c-d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{4\left(c^{3}+c^{2}d-c\,d^{2}-d^{3}\right)\sqrt{(c-d)(c+d)}}\right)}{f}$
risch	$\frac{ia(-3c^{3}d\,\mathrm{e}^{3i(fx+e)}+2c^{2}d^{2}\mathrm{e}^{3i(fx+e)}+2c\,d^{3}\mathrm{e}^{3i(fx+e)}-2c^{4}\mathrm{e}^{2i(fx+e)}+2c^{3}d\,\mathrm{e}^{2i(fx+e)}-3c^{2}d^{2}\mathrm{e}^{2i(fx+e)}+4c\,d^{3}\mathrm{e}^{2i(fx+e)}}{c^{2}(-c^{2}+d^{2})f\left(c\,\mathrm{e}^{2i(fx+e)}+2d\,\mathrm{e}^{i(fx+e)}+c\right)^{2}}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output

```
4/f*a*((-1/4*(2*c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/4*(2*c-3*d)/(c
-d)/(c+d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2
*d-c-d)^2+1/4*(2*c-d)/(c^3+c^2*d-c*d^2-d^3)/((c-d)*(c+d))^(1/2)*arctanh((c
-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(118) = 236$.

Time = 0.17 (sec) , antiderivative size = 736, normalized size of antiderivative = 5.62

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
[1/4*((2*a*c*d^2 - a*d^3 + (2*a*c^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2
*d - a*c*d^2)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2
 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x
+ e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(
a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c^4 - 2*a*c^3*d - 3*a*c^2
*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/((c^7 + c^6*d - 2*c^
5*d^2 - 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^6*d + c^5*d
^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^5*d^2 +
c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f), 1/2*((2*a*c*d^2 - a*d^3
 + (2*a*c^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2*d - a*c*d^2)*cos(f*x +
e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
 d^2)*sin(f*x + e))) + (a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c
^4 - 2*a*c^3*d - 3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x +
e))/((c^7 + c^6*d - 2*c^5*d^2 - 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x +
 e)^2 + 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*co
s(f*x + e) + (c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f)]
```

Sympy [F]

$$\begin{aligned}&\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx\\&= a\left(\int \frac{\sec(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx\right.\\&\qquad\left.+\int \frac{\sec^2(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx\right)\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

output
```
a*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e +
f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**2/(c**3 + 3*c
**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(118) = 236.

Time = 0.21 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.01

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(2ac-ad)}{(c^3+c^2d-cd^2-d^3)\sqrt{-c^2+d^2}} - \frac{2ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+ac}{(c^3+c^2d-cd^2-}}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="gia
c")
```

output
```
((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*a*c - a*d)/((c^
3 + c^2*d - c*d^2 - d^3)*sqrt(-c^2 + d^2)) - (2*a*c^2*tan(1/2*f*x + 1/2*e)
^3 - 3*a*c*d*tan(1/2*f*x + 1/2*e)^3 + a*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*c
^2*tan(1/2*f*x + 1/2*e) + a*c*d*tan(1/2*f*x + 1/2*e) + 3*a*d^2*tan(1/2*f*x
 + 1/2*e))/((c^3 + c^2*d - c*d^2 - d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(
1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 12.41 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$
$$=\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)\,\left(2\,c-d\right)}{f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{3/2}}-\frac{\frac{{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,a\,c-a\,d\right)}{{\left(c+d\right)}^2}-\frac{a\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-3\,d\right)}{\left(c+d\right)\,\left(c-d\right)}}{f\,\left(2\,c\,d-{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2-2\,d^2\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(c^2-2\,c\,d+d^2\right)+c^2+d^2\right)}$$

input
```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

output
```
(a*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(2*c - d))/(f*(
c + d)^(5/2)*(c - d)^(3/2)) - ((tan(e/2 + (f*x)/2)^3*(2*a*c - a*d))/(c + d
)^2 - (a*tan(e/2 + (f*x)/2)*(2*c - 3*d))/((c + d)*(c - d)))/(f*(2*c*d - ta
n(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d
^2) + c^2 + d^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 823, normalized size of antiderivative = 6.28

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

output

```
(a*(8*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*cos(e + f*x)*c**2*d - 4*sqrt( - c**2 + d**2)*atan((t
an((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)
*c*d**2 - 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/
2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**3 + 2*sqrt( - c**2 + d**2)*
atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e
 + f*x)**2*c**2*d + 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan(
(e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**3 - 2*sqrt( - c**2 + d**2)*atan((
tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**2*d + 4*
sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt(
- c**2 + d**2))*c*d**2 - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
 tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*d**3 + 2*cos(e + f*x)*sin(e + f
*x)*c**4 - 2*cos(e + f*x)*sin(e + f*x)*c**3*d - 3*cos(e + f*x)*sin(e + f*x
)*c**2*d**2 + 2*cos(e + f*x)*sin(e + f*x)*c*d**3 + cos(e + f*x)*sin(e + f*
x)*d**4 + sin(e + f*x)*c**3*d - 2*sin(e + f*x)*c**2*d**2 - sin(e + f*x)*c*
d**3 + 2*sin(e + f*x)*d**4))/(2*f*(2*cos(e + f*x)*c**6*d + 2*cos(e + f*x)*
c**5*d**2 - 4*cos(e + f*x)*c**4*d**3 - 4*cos(e + f*x)*c**3*d**4 + 2*cos(e
+ f*x)*c**2*d**5 + 2*cos(e + f*x)*c*d**6 - sin(e + f*x)**2*c**7 - sin(e +
f*x)**2*c**6*d + 2*sin(e + f*x)**2*c**5*d**2 + 2*sin(e + f*x)**2*c**4*d**3
 - sin(e + f*x)**2*c**3*d**4 - sin(e + f*x)**2*c**2*d**5 + c**7 + c**6*...
```

3.192 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 189

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{a(2c^2-2cd+d^2)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{7/2}f} + \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \frac{a(c-4d)(2c-d)\tan(e+fx)}{6(c-d)^2(c+d)^3f(c+d\sec(e+fx))}$$

output
```
a*(2*c^2-2*c*d+d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c
-d)^(5/2)/(c+d)^(7/2)/f+1/3*a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^3+1/6*a*
(2*c-3*d)*tan(f*x+e)/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))^2+1/6*a*(c-4*d)*(2*c
-d)*tan(f*x+e)/(c-d)^2/(c+d)^3/f/(c+d*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx =$$

$$-\frac{a(1+\cos(e+fx))\sec^2\left(\frac{1}{2}(e+fx)\right)\Bigl(6(2c^2-2cd+d^2)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx}{}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

output

```
-1/12*(a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(6*(2*c^2 - 2*c*d + d^2)*Ar
cTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3
- (Sqrt[c^2 - d^2]*(6*c^4 - 12*c^3*d + 2*c^2*d^2 - 15*c*d^3 + 10*d^4 + 6*d
*(2*c^3 - 7*c^2*d + 2*c*d^2 + d^3)*Cos[e + f*x] + (6*c^4 - 12*c^3*d - 2*c^
2*d^2 + 3*c*d^3 + 2*d^4)*Cos[2*(e + f*x)])*Sin[e + f*x])/2))/((c - d)^2*(c
 + d)^3*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {3042, 4491, 25, 3042, 4491, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)}{(c+d\sec(e+fx))^4}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \; 4491$$

$$\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}-\frac{\int -\frac{\sec(e+fx)(3a(c-d)+2a\sec(e+fx)(c-d))}{(c+d\sec(e+fx))^3}dx}{3\left(c^2-d^2\right)}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{\sec(e+fx)(3a(c-d)+2a\sec(e+fx)(c-d))}{(c+d\sec(e+fx))^3}dx}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(3a(c-d)+2a\csc\left(e+fx+\frac{\pi}{2}\right)(c-d)\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow 4491$$

$$\frac{\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}-\frac{\int -\frac{\sec(e+fx)(2a(3c-2d)(c-d)+a(2c-3d)\sec(e+fx)(c-d))}{(c+d\sec(e+fx))^2}dx}{2(c^2-d^2)}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\int \frac{\sec(e+fx)(2a(3c-2d)(c-d)+a(2c-3d)\sec(e+fx)(c-d))}{(c+d\sec(e+fx))^2}dx}{2(c^2-d^2)}+\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(2a(3c-2d)(c-d)+a(2c-3d)\csc\left(e+fx+\frac{\pi}{2}\right)(c-d)\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2(c^2-d^2)}+\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow 4491$$

$$\frac{\frac{\frac{a(c-4d)(2c-d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}-\frac{\int -\frac{3a(c-d)\left(2c^2-2dc+d^2\right)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}}{2(c^2-d^2)}+\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{3a(c-d)\left(2c^2-2cd+d^2\right)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3a(c-d)\left(2c^2-2cd+d^2\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c^2-d^2}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{3a(c-d)\left(2c^2-2cd+d^2\right)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{d\left(c^2-d^2\right)}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3a(c-d)\left(2c^2-2cd+d^2\right)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{d\left(c^2-d^2\right)}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{6a(c-d)\left(2c^2-2cd+d^2\right)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{df\left(c^2-d^2\right)}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{6a\sqrt{c-d}\left(2c^2-2cd+d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c+d}\left(c^2-d^2\right)}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{a(2c-3d)\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

output

```
(a*Tan[e + f*x])/(3*(c + d)*f*(c + d*Sec[e + f*x])^3) + ((a*(2*c - 3*d)*Ta
n[e + f*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])^2) + ((6*a*Sqrt[c - d]*(2*c^
2 - 2*c*d + d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqr
t[c + d]*(c^2 - d^2)*f) + (a*(c - 4*d)*(2*c - d)*Tan[e + f*x])/((c + d)*f*
(c + d*Sec[e + f*x])))/(2*(c^2 - d^2)))/(3*(c^2 - d^2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\dfrac{4a\left(\dfrac{-\dfrac{\left(2c^{2}-2cd+d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{4\left(c^{3}+3c^{2}d+3c\,d^{2}+d^{3}\right)}+\dfrac{\left(3c^{2}-6cd+d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3(c-d)\left(c^{2}+2cd+d^{2}\right)}-\dfrac{\left(2c^{2}-6cd+3d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c^{2}-2cd+d^{2}\right)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{3}}+\dfrac{\left(2c^{2}-2cd+d^{2}\right)\operatorname{arctanh}}{4\left(c^{5}+c^{4}d-2c^{3}d^{2}-2c^{2}d^{3}\right.}\right.}{f}$
default	$\dfrac{4a\left(\dfrac{-\dfrac{\left(2c^{2}-2cd+d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{4\left(c^{3}+3c^{2}d+3c\,d^{2}+d^{3}\right)}+\dfrac{\left(3c^{2}-6cd+d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3(c-d)\left(c^{2}+2cd+d^{2}\right)}-\dfrac{\left(2c^{2}-6cd+3d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c^{2}-2cd+d^{2}\right)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{3}}+\dfrac{\left(2c^{2}-2cd+d^{2}\right)\operatorname{arctanh}}{4\left(c^{5}+c^{4}d-2c^{3}d^{2}-2c^{2}d^{3}\right.}\right.}{f}$
risch	Expression too large to display

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE
)
```

output
```
4/f*a*((-1/4*(2*c^2-2*c*d+d^2)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e
)^5+1/3*(3*c^2-6*c*d+d^2)/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/4*(
2*c^2-6*c*d+3*d^2)/(c+d)/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*
x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)^3+1/4*(2*c^2-2*c*d+d^2)/(c^5+c^4*d-
2*c^3*d^2-2*c^2*d^3+c*d^4+d^5)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f
*x+1/2*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(174) = 348$.

Time = 0.22 (sec) , antiderivative size = 1278, normalized size of antiderivative = 6.76

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fri
cas")
```

output
```
[1/12*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^5 - 2*a*c^4*d + a*c^3*d
^2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a*c^2*d^3)*cos(f*x + e)^
2 + 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e))*sqrt(c^2 - d^2)*
log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)
*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*
c*d*cos(f*x + e) + d^2)) + 2*(2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*
a*c*d^5 - 4*a*d^6 + (6*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4
*a*c^2*d^4 - 3*a*c*d^5 - 2*a*d^6)*cos(f*x + e)^2 + 3*(2*a*c^5*d - 7*a*c^4*
d^2 + 8*a*c^2*d^4 - 2*a*c*d^5 - a*d^6)*cos(f*x + e))*sin(f*x + e))/((c^10
+ c^9*d - 3*c^8*d^2 - 3*c^7*d^3 + 3*c^6*d^4 + 3*c^5*d^5 - c^4*d^6 - c^3*d^
7)*f*cos(f*x + e)^3 + 3*(c^9*d + c^8*d^2 - 3*c^7*d^3 - 3*c^6*d^4 + 3*c^5*d
^5 + 3*c^4*d^6 - c^3*d^7 - c^2*d^8)*f*cos(f*x + e)^2 + 3*(c^8*d^2 + c^7*d^
3 - 3*c^6*d^4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos
(f*x + e) + (c^7*d^3 + c^6*d^4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2
*d^8 - c*d^9 - d^10)*f), 1/6*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^
5 - 2*a*c^4*d + a*c^3*d^2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a
*c^2*d^3)*cos(f*x + e)^2 + 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x
 + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^
2 - d^2)*sin(f*x + e))) + (2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c
*d^5 - 4*a*d^6 + (6*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4...
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = a\left(\int \frac{\sec(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx + \int \frac{\sec^2(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
```

output
```
a*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e
 + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integr
al(sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x
)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(174) = 348$.

Time = 0.21 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx =$$

$$-\frac{\frac{3\,(2\,ac^2-2\,acd+ad^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^5+c^4d-2\,c^3d^2-2\,c^2d^3+cd^4+d^5)\sqrt{-c^2+d^2}}+\frac{6\,ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-18\,ac^3d\tan\left(\frac{1}{2}\right.}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="gia
c")
```

output
```
-1/3*(3*(2*a*c^2 - 2*a*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt
(-c^2 + d^2)))/((c^5 + c^4*d - 2*c^3*d^2 - 2*c^2*d^3 + c*d^4 + d^5)*sqrt(-
c^2 + d^2)) + (6*a*c^4*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^3*d*tan(1/2*f*x + 1
/2*e)^5 + 21*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a*c*d^3*tan(1/2*f*x + 1
/2*e)^5 + 3*a*d^4*tan(1/2*f*x + 1/2*e)^5 - 12*a*c^4*tan(1/2*f*x + 1/2*e)^3
 + 24*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 8*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3
- 24*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 4*a*d^4*tan(1/2*f*x + 1/2*e)^3 + 6*a
*c^4*tan(1/2*f*x + 1/2*e) - 6*a*c^3*d*tan(1/2*f*x + 1/2*e) - 21*a*c^2*d^2*
tan(1/2*f*x + 1/2*e) + 9*a*d^4*tan(1/2*f*x + 1/2*e))/((c^5 + c^4*d - 2*c^3
*d^2 - 2*c^2*d^3 + c*d^4 + d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x
+ 1/2*e)^2 - c - d)^3))/f
```

Mupad [B] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.70

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(2\,a\,c^2-2\,a\,c\,d+a\,d^2\right)}{{\left(c+d\right)}^3}+\frac{a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2-6\,c\,d+3\,d^2\right)}{\left(c+d\right)\,\left(c^2-2\,c\,d+d^2\right)}-4\,a\,\mathrm{tan}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3\right)+3\,c\,d\right.}$$

$$+\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-2\,d\right)\,\left(c^2-2\,c\,d+d^2\right)}{2\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}}\right)\,\left(2\,c^2-2\,c\,d+d^2\right)}{f\,{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{5/2}}$$

input

```
int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

output

```
((tan(e/2 + (f*x)/2)^5*(2*a*c^2 + a*d^2 - 2*a*c*d))/(c + d)^3 + (a*tan(e/2
 + (f*x)/2)*(2*c^2 - 6*c*d + 3*d^2))/((c + d)*(c^2 - 2*c*d + d^2)) - (4*a*
tan(e/2 + (f*x)/2)^3*(3*c^2 - 6*c*d + d^2))/(3*(c + d)^2*(c - d)))/(f*(tan
(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)
^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - t
an(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) + (a*atanh((tan(e/2
+ (f*x)/2)*(2*c - 2*d)*(c^2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*(c - d)^(5/2)
))*(2*c^2 - 2*c*d + d^2))/(f*(c + d)^(7/2)*(c - d)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1718, normalized size of antiderivative = 9.09

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x)
```

output

```
(a*(12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)
/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*c**5 - 12*sqrt( - c**2
 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**
2))*cos(e + f*x)*sin(e + f*x)**2*c**4*d + 6*sqrt( - c**2 + d**2)*atan((tan
((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*s
in(e + f*x)**2*c**3*d**2 - 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*
c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c**5 + 12*sqrt(
 - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**
2 + d**2))*cos(e + f*x)*c**4*d - 42*sqrt( - c**2 + d**2)*atan((tan((e + f*
x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c**3*d**2
 + 36*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*cos(e + f*x)*c**2*d**3 - 18*sqrt( - c**2 + d**2)*ata
n((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e +
f*x)*c*d**4 + 36*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e +
f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**4*d - 36*sqrt( - c**2
+ d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2
))*sin(e + f*x)**2*c**3*d**2 + 18*sqrt( - c**2 + d**2)*atan((tan((e + f*x)
/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**2*d**
3 - 36*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)
/sqrt( - c**2 + d**2))*c**4*d + 36*sqrt( - c**2 + d**2)*atan((tan((e + ...
```

3.193 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 31, antiderivative size = 327

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx \\
&= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\,\mathrm{arctanh}(\sin(e+fx))}{16f} \\
&\quad - \frac{a^2(4c^5-48c^4d-311c^3d^2-448c^2d^3-288cd^4-64d^5)\tan(e+fx)}{60df} \\
&\quad - \frac{a^2(8c^4-96c^3d-438c^2d^2-464cd^3-165d^4)\sec(e+fx)\tan(e+fx)}{240f} \\
&\quad - \frac{a^2(4c^3-48c^2d-123cd^2-64d^3)\,(c+d\sec(e+fx))^2\tan(e+fx)}{120df} \\
&\quad - \frac{a^2(4c^2-48cd-55d^2)\,(c+d\sec(e+fx))^3\tan(e+fx)}{120df} \\
&\quad - \frac{a^2(c-12d)(c+d\sec(e+fx))^4\tan(e+fx)}{30df} + \frac{a^2(c+d\sec(e+fx))^5\tan(e+fx)}{6df}
\end{aligned}
$$

output
```
1/16*a^2*(24*c^4+64*c^3*d+84*c^2*d^2+48*c*d^3+11*d^4)*arctanh(sin(f*x+e))/
f-1/60*a^2*(4*c^5-48*c^4*d-311*c^3*d^2-448*c^2*d^3-288*c*d^4-64*d^5)*tan(f
*x+e)/d/f-1/240*a^2*(8*c^4-96*c^3*d-438*c^2*d^2-464*c*d^3-165*d^4)*sec(f*x
+e)*tan(f*x+e)/f-1/120*a^2*(4*c^3-48*c^2*d-123*c*d^2-64*d^3)*(c+d*sec(f*x+
e))^2*tan(f*x+e)/d/f-1/120*a^2*(4*c^2-48*c*d-55*d^2)*(c+d*sec(f*x+e))^3*ta
n(f*x+e)/d/f-1/30*a^2*(c-12*d)*(c+d*sec(f*x+e))^4*tan(f*x+e)/d/f+1/6*a^2*(
c+d*sec(f*x+e))^5*tan(f*x+e)/d/f
```

Mathematica [A] (verified)

Time = 5.79 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.59

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$$
$$= \frac{a^2\left(240c^4\coth^{-1}(\sin(e+fx))+15(8c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\operatorname{arctanh}(\sin(e+fx))+\tan(e\right.}{}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

output
```
(a^2*(240*c^4*ArcCoth[Sin[e + f*x]] + 15*(8*c^4 + 64*c^3*d + 84*c^2*d^2 +
48*c*d^3 + 11*d^4)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(480*(c + d)^4 + 1
5*(8*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Sec[e + f*x] + 10*d^
2*(36*c^2 + 48*c*d + 11*d^2)*Sec[e + f*x]^3 + 40*d^4*Sec[e + f*x]^5 + 320*
d*(c + d)^3*Tan[e + f*x]^2 + 96*d^3*(2*c + d)*Tan[e + f*x]^4)))/(240*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4475, 111, 25, 27, 170, 25, 27, 164, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int\frac{(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^4}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{111}$$

$$-\frac{a^2\tan(e+fx)\Bigg(-\frac{\int-\frac{a^2(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^2\left(6c^2+2dc+3d^2+d(9c+2d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{6a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e}{}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\int\frac{a^2(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^2\left(6c^2+2dc+3d^2+d(9c+2d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{6a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx}{6a^2}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\int\frac{(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^2(6c^2+2dc+3d^2+d(9c+2d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{d\sqrt{a-a\sec(e+fx}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{170}$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{1}{6}\Bigg(-\frac{\int-\frac{a^2(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))\left(30c^3+28dc^2+37d^2c+4d^3+d\left(48c^2+32dc+19d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a^2}-\frac{d(9}{}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{1}{6}\Bigg(\frac{\int\frac{a^2(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))\left(30c^3+28dc^2+37d^2c+4d^3+d\left(48c^2+32dc+19d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a^2}-\frac{d(9c+2}{}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\,\mathrm{se}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\left(\frac{1}{5}\int\frac{(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))\left(30c^3+28dc^2+37d^2c+4d^3+d\left(48c^2+32dc+19d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+f\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{}}$$

$$\Big\downarrow \text{164}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{5}{4}\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\int\frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{d\sqrt{a-a\sec(e}}{}\right.\right.\right.}{}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{5}{4}\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\left(\frac{3}{2}a\int\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{\sqrt{a-a\sec}}{}\right.\right.\right.\right.}{}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{5}{4}\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\left(\frac{3}{2}a\left(a\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+\right.\right.\right.\right.\right.}{}$$

$$\Big\downarrow \text{45}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{5}{4}\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\left(\frac{3}{2}a\left(2a\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}-\right.\right.\right.\right.\right.}{}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\left(\frac{1}{5}\left(\frac{5}{4}\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\left(\frac{3}{2}a\left(-2\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)-\frac{\sqrt{a-a\sec(}}{}\right.\right.\right.\right.\right.}{}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

output

```
-((a^2*(-1/6*(d*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(5/2)*(c + d
*Sec[e + f*x])^3)/a^2 + (-1/5*(d*(9*c + 2*d)*Sqrt[a - a*Sec[e + f*x]]*(a +
 a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2)/a^2 + (-1/4*(d*Sqrt[a - a*S
ec[e + f*x]]*(a + a*Sec[e + f*x])^(5/2)*(2*(52*c^3 + 56*c^2*d + 48*c*d^2 +
 9*d^3) + d*(48*c^2 + 32*c*d + 19*d^2)*Sec[e + f*x]))/a^2 + (5*(24*c^4 + 6
4*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*(-1/2*(Sqrt[a - a*Sec[e + f*x]]*
(a + a*Sec[e + f*x])^(3/2))/a + (3*a*(-2*ArcTan[Sqrt[a - a*Sec[e + f*x]]/S
qrt[a + a*Sec[e + f*x]]] - (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*
x]])/a))/2))/4)/5)/6)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a
*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.10

method	result
parts	$-\frac{(4a^2cd^3+2a^2d^4)\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}+\frac{(2a^2c^4+4a^2c^3d)\tan(fx+e)}{f}+\frac{(6a^2c^2d^2+8a}{}$
norman	$\frac{\frac{17a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{24f}-\frac{a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{8f}+\frac{a^2\left(40c^4+1\right.}{}}{}$
parallelrisch	$\frac{2a^2\left(-\frac{45\left(c^4+\frac{8}{3}c^3d+\frac{7}{2}c^2d^2+2cd^3+\frac{11}{24}d^4\right)\left(\frac{2}{3}+\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4fx+4e)}{5}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{45\left(c^4+\frac{8}{3}c\right.}{}\right.}{}$
derivativedivides	$\frac{a^2c^4\ln(\sec(fx+e)+\tan(fx+e))+4a^2c^3d\tan(fx+e)+6a^2c^2d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4a^2c}{}$
default	$\frac{a^2c^4\ln(\sec(fx+e)+\tan(fx+e))+4a^2c^3d\tan(fx+e)+6a^2c^2d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4a^2c}{}$
risch	Expression too large to display

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBO
SE)
```

output
```
-(4*a^2*c*d^3+2*a^2*d^4)/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(
f*x+e)+(2*a^2*c^4+4*a^2*c^3*d)/f*tan(f*x+e)+(6*a^2*c^2*d^2+8*a^2*c*d^3+a^2
*d^4)/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+
tan(f*x+e)))-(4*a^2*c^3*d+12*a^2*c^2*d^2+4*a^2*c*d^3)/f*(-2/3-1/3*sec(f*x+
e)^2)*tan(f*x+e)+(a^2*c^4+8*a^2*c^3*d+6*a^2*c^2*d^2)/f*(1/2*sec(f*x+e)*tan
(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^2*c^4/f*ln(sec(f*x+e)+tan(f*x+e))
+a^2*d^4/f*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x
+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.18

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$$
$$= \frac{15\,(24\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4)\cos(fx+e)^6\log(\sin(fx+e)+1)-15\,(24\,a^2c}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="f
ricas")
```

output
```
1/480*(15*(24*a^2*c^4 + 64*a^2*c^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*
a^2*d^4)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*(24*a^2*c^4 + 64*a^2*c^
3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^6*log(-sin(
f*x + e) + 1) + 2*(40*a^2*d^4 + 32*(15*a^2*c^4 + 50*a^2*c^3*d + 60*a^2*c^2
*d^2 + 36*a^2*c*d^3 + 8*a^2*d^4)*cos(f*x + e)^5 + 15*(8*a^2*c^4 + 64*a^2*c
^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^4 + 64*(5*
a^2*c^3*d + 15*a^2*c^2*d^2 + 9*a^2*c*d^3 + 2*a^2*d^4)*cos(f*x + e)^3 + 10*
(36*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^2 + 96*(2*a^2*c*
d^3 + a^2*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx \\
&= a^2\left(\int c^4\sec(e+fx)\,dx+\int 2c^4\sec^2(e+fx)\,dx+\int c^4\sec^3(e+fx)\,dx\right. \\
&\quad+\int d^4\sec^5(e+fx)\,dx+\int 2d^4\sec^6(e+fx)\,dx+\int d^4\sec^7(e+fx)\,dx \\
&\quad+\int 4cd^3\sec^4(e+fx)\,dx+\int 8cd^3\sec^5(e+fx)\,dx+\int 4cd^3\sec^6(e+fx)\,dx \\
&\quad+\int 6c^2d^2\sec^3(e+fx)\,dx+\int 12c^2d^2\sec^4(e+fx)\,dx+\int 6c^2d^2\sec^5(e+fx)\,dx \\
&\quad\left.+\int 4c^3d\sec^2(e+fx)\,dx+\int 8c^3d\sec^3(e+fx)\,dx+\int 4c^3d\sec^4(e+fx)\,dx\right)
\end{aligned}
$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**4,x)
```

output
```
a**2*(Integral(c**4*sec(e + f*x), x) + Integral(2*c**4*sec(e + f*x)**2, x)
 + Integral(c**4*sec(e + f*x)**3, x) + Integral(d**4*sec(e + f*x)**5, x) +
 Integral(2*d**4*sec(e + f*x)**6, x) + Integral(d**4*sec(e + f*x)**7, x) +
 Integral(4*c*d**3*sec(e + f*x)**4, x) + Integral(8*c*d**3*sec(e + f*x)**5
, x) + Integral(4*c*d**3*sec(e + f*x)**6, x) + Integral(6*c**2*d**2*sec(e
+ f*x)**3, x) + Integral(12*c**2*d**2*sec(e + f*x)**4, x) + Integral(6*c**
2*d**2*sec(e + f*x)**5, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integ
ral(8*c**3*d*sec(e + f*x)**3, x) + Integral(4*c**3*d*sec(e + f*x)**4, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(313) = 626$.

Time = 0.06 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.09

$$
\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx = \text{Too large to display}
$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="m
axima")
```

output

```
1/480*(640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3*d + 1920*(tan(f*x + e
)^3 + 3*tan(f*x + e))*a^2*c^2*d^2 + 128*(3*tan(f*x + e)^5 + 10*tan(f*x + e
)^3 + 15*tan(f*x + e))*a^2*c*d^3 + 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a
^2*c*d^3 + 64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2
*d^4 - 5*a^2*d^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x +
e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(si
n(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) - 180*a^2*c^2*d^2*(2*(3*sin(f*
x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log
(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 240*a^2*c*d^3*(2*(3*sin(f*
x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log
(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 30*a^2*d^4*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(si
n(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 960
*a^2*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) +
log(sin(f*x + e) - 1)) - 720*a^2*c^2*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 -
 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^2*c^4*log(sec
(f*x + e) + tan(f*x + e)) + 960*a^2*c^4*tan(f*x + e) + 1920*a^2*c^3*d*tan(
f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(313) = 626$.

Time = 0.25 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.25

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="g
iac")
```

output
```
1/240*(15*(24*a^2*c^4 + 64*a^2*c^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*
a^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(24*a^2*c^4 + 64*a^2*c^3*
d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*log(abs(tan(1/2*f*x + 1/2*
e) - 1)) - 2*(360*a^2*c^4*tan(1/2*f*x + 1/2*e)^11 + 960*a^2*c^3*d*tan(1/2*
f*x + 1/2*e)^11 + 1260*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^11 + 720*a^2*c*d^3
*tan(1/2*f*x + 1/2*e)^11 + 165*a^2*d^4*tan(1/2*f*x + 1/2*e)^11 - 2040*a^2*
c^4*tan(1/2*f*x + 1/2*e)^9 - 5440*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^9 - 7140*
a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^9 - 4080*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^9
 - 935*a^2*d^4*tan(1/2*f*x + 1/2*e)^9 + 4560*a^2*c^4*tan(1/2*f*x + 1/2*e)^
7 + 13440*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^7 + 15480*a^2*c^2*d^2*tan(1/2*f*x
 + 1/2*e)^7 + 10272*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^7 + 1986*a^2*d^4*tan(1/
2*f*x + 1/2*e)^7 - 5040*a^2*c^4*tan(1/2*f*x + 1/2*e)^5 - 17280*a^2*c^3*d*t
an(1/2*f*x + 1/2*e)^5 - 19080*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 11232*a
^2*c*d^3*tan(1/2*f*x + 1/2*e)^5 - 3006*a^2*d^4*tan(1/2*f*x + 1/2*e)^5 + 27
60*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 + 11200*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^3
 + 13980*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 7440*a^2*c*d^3*tan(1/2*f*x +
 1/2*e)^3 + 1305*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 600*a^2*c^4*tan(1/2*f*x
+ 1/2*e) - 2880*a^2*c^3*d*tan(1/2*f*x + 1/2*e) - 4500*a^2*c^2*d^2*tan(1/2*
f*x + 1/2*e) - 3120*a^2*c*d^3*tan(1/2*f*x + 1/2*e) - 795*a^2*d^4*tan(1/2*f
*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 14.45 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.48

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$$

$$= \frac{\left(-3\,a^2\,c^4-8\,a^2\,c^3\,d-\frac{21\,a^2\,c^2\,d^2}{2}-6\,a^2\,c\,d^3-\frac{11\,a^2\,d^4}{8}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+\left(17\,a^2\,c^4+\frac{136\,a^2\,c^3\,d}{3}+\frac{119\,a^2\,c^2\,d^2}{2}-\right.}{}$$

$$+\frac{a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(24\,c^4+64\,c^3\,d+84\,c^2\,d^2+48\,c\,d^3+11\,d^4\right)}{4\,\left(6\,c^4+16\,c^3\,d+21\,c^2\,d^2+12\,c\,d^3+\frac{11\,d^4}{4}\right)}\right)\,\left(24\,c^4+64\,c^3\,d+84\,c^2\,d^2+48\,c\,d^3+11\,d^4\right)}{8\,f}$$

input
```
int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

output

```
(tan(e/2 + (f*x)/2)*(5*a^2*c^4 + (53*a^2*d^4)/8 + 26*a^2*c*d^3 + 24*a^2*c^
3*d + (75*a^2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^11*(3*a^2*c^4 + (11*a^2*d^4
)/8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (21*a^2*c^2*d^2)/2) + tan(e/2 + (f*x)/2)
^9*(17*a^2*c^4 + (187*a^2*d^4)/24 + 34*a^2*c*d^3 + (136*a^2*c^3*d)/3 + (11
9*a^2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^3*(23*a^2*c^4 + (87*a^2*d^4)/8 + 62
*a^2*c*d^3 + (280*a^2*c^3*d)/3 + (233*a^2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)
^7*(38*a^2*c^4 + (331*a^2*d^4)/20 + (428*a^2*c*d^3)/5 + 112*a^2*c^3*d + 12
9*a^2*c^2*d^2) + tan(e/2 + (f*x)/2)^5*(42*a^2*c^4 + (501*a^2*d^4)/20 + (46
8*a^2*c*d^3)/5 + 144*a^2*c^3*d + 159*a^2*c^2*d^2))/(f*(15*tan(e/2 + (f*x)/
2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*
x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (a^2*ata
nh((tan(e/2 + (f*x)/2)*(48*c*d^3 + 64*c^3*d + 24*c^4 + 11*d^4 + 84*c^2*d^2
))/(4*(12*c*d^3 + 16*c^3*d + 6*c^4 + (11*d^4)/4 + 21*c^2*d^2)))*(48*c*d^3
+ 64*c^3*d + 24*c^4 + 11*d^4 + 84*c^2*d^2))/(8*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1491, normalized size of antiderivative = 4.56

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x)
```

output

```
(a**2*( - 480*cos(e + f*x)*sin(e + f*x)**5*c**4 - 1600*cos(e + f*x)*sin(e
+ f*x)**5*c**3*d - 1920*cos(e + f*x)*sin(e + f*x)**5*c**2*d**2 - 1152*cos(
e + f*x)*sin(e + f*x)**5*c*d**3 - 256*cos(e + f*x)*sin(e + f*x)**5*d**4 +
960*cos(e + f*x)*sin(e + f*x)**3*c**4 + 3520*cos(e + f*x)*sin(e + f*x)**3*
c**3*d + 4800*cos(e + f*x)*sin(e + f*x)**3*c**2*d**2 + 2880*cos(e + f*x)*s
in(e + f*x)**3*c*d**3 + 640*cos(e + f*x)*sin(e + f*x)**3*d**4 - 480*cos(e
+ f*x)*sin(e + f*x)*c**4 - 1920*cos(e + f*x)*sin(e + f*x)*c**3*d - 2880*co
s(e + f*x)*sin(e + f*x)*c**2*d**2 - 1920*cos(e + f*x)*sin(e + f*x)*c*d**3
- 480*cos(e + f*x)*sin(e + f*x)*d**4 - 360*log(tan((e + f*x)/2) - 1)*sin(e
 + f*x)**6*c**4 - 960*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6*c**3*d - 1
260*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6*c**2*d**2 - 720*log(tan((e +
 f*x)/2) - 1)*sin(e + f*x)**6*c*d**3 - 165*log(tan((e + f*x)/2) - 1)*sin(e
 + f*x)**6*d**4 + 1080*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**4 + 28
80*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**3*d + 3780*log(tan((e + f*
x)/2) - 1)*sin(e + f*x)**4*c**2*d**2 + 2160*log(tan((e + f*x)/2) - 1)*sin(
e + f*x)**4*c*d**3 + 495*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*d**4 -
1080*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c**4 - 2880*log(tan((e + f*
x)/2) - 1)*sin(e + f*x)**2*c**3*d - 3780*log(tan((e + f*x)/2) - 1)*sin(e +
 f*x)**2*c**2*d**2 - 2160*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c*d**3
 - 495*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*d**4 + 360*log(tan((e ...
```

3.194 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 31, antiderivative size = 242

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx\\ =\frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\operatorname{arctanh}(\sin(e+fx))}{8f}\\ -\frac{a^2(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)\tan(e+fx)}{10df}\\ -\frac{a^2(2c^3-20c^2d-57cd^2-30d^3)\sec(e+fx)\tan(e+fx)}{40f}\\ -\frac{a^2\left(c^2-10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{20df}\\ -\frac{a^2(c-10d)(c+d\sec(e+fx))^3\tan(e+fx)}{20df}+\frac{a^2(c+d\sec(e+fx))^4\tan(e+fx)}{5df}\end{aligned}$$

output

```
3/8*a^2*(2*c+d)*(2*c^2+3*c*d+2*d^2)*arctanh(sin(f*x+e))/f-1/10*a^2*(c^4-10
*c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)*tan(f*x+e)/d/f-1/40*a^2*(2*c^3-20*c^2*d
-57*c*d^2-30*d^3)*sec(f*x+e)*tan(f*x+e)/f-1/20*a^2*(c^2-10*c*d-12*d^2)*(c+
d*sec(f*x+e))^2*tan(f*x+e)/d/f-1/20*a^2*(c-10*d)*(c+d*sec(f*x+e))^3*tan(f*
x+e)/d/f+1/5*a^2*(c+d*sec(f*x+e))^4*tan(f*x+e)/d/f
```

Mathematica [A] (verified)

Time = 4.81 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.63

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$
$$=\frac{a^2\left(40c^3\coth^{-1}(\sin(e+fx))+5(4c^3+24c^2d+21cd^2+6d^3)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\,(5(4c^3\right.}{}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

output
```
(a^2*(40*c^3*ArcCoth[Sin[e + f*x]] + 5*(4*c^3 + 24*c^2*d + 21*c*d^2 + 6*d^
3)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(5*(4*c^3 + 24*c^2*d + 21*c*d^2 +
6*d^3)*Sec[e + f*x] + 10*d^2*(3*c + 2*d)*Sec[e + f*x]^3 + 8*(10*(c + d)^3
+ 5*d*(c + d)^2*Tan[e + f*x]^2 + d^3*Tan[e + f*x]^4))))/(40*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3042, 4475, 111, 25, 27, 164, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 111}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))\left(5c^2+2dc+2d^2+d(7c+2d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+}{5}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))\left(5c^2+2dc+2d^2+d(7c+2d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)}{5a^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{5}\int \frac{(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))\left(5c^2+2dc+2d^2+d(7c+2d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{d\sqrt{a-a\sec(e+fx)}(}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 164$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{5}\left(\frac{5}{4}(2c+d)\left(2c^2+3cd+2d^2\right)\int\frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)}{}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

$$\Big\downarrow 60$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{5}\left(\frac{5}{4}(2c+d)\left(2c^2+3cd+2d^2\right)\left(\frac{3}{2}a\int\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+}{2a}\right.\right.\right.}{f\sqrt{a-a\sec(}}$$

$$\Big\downarrow 60$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{5}\left(\frac{5}{4}(2c+d)\left(2c^2+3cd+2d^2\right)\left(\frac{3}{2}a\left(a\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)-\frac{\sqrt{a-a\sec(}}{}\right.\right.\right.\right.}{}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{5}\left(\frac{5}{4}(2c+d)\left(2c^2+3cd+2d^2\right)\left(\frac{3}{2}a\left(2a\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{}}{a}\right.\right.\right.\right.}{}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{1}{5}\Bigg(\frac{5}{4}(2c+d)\left(2c^2+3cd+2d^2\right)\Bigg(\frac{3}{2}a\Bigg(-2\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}{a}}{}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^2*(-1/5*(d*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(5/2)*(c + d
*Sec[e + f*x])^2)/a^2 + (-1/4*(d*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f
*x])^(5/2)*(2*(8*c^2 + 5*c*d + 2*d^2) + d*(7*c + 2*d)*Sec[e + f*x]))/a^2 +
 (5*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*(-1/2*(Sqrt[a - a*Sec[e + f*x]]*(a +
 a*Sec[e + f*x])^(3/2))/a + (3*a*(-2*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[
a + a*Sec[e + f*x]]] - (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
/a))/2))/4)/5)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.11

method	result
parts	$\frac{(3a^2c\,d^2+2a^2d^3)\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{(2a^2c^3+3a^2c^2d)\tan(}{f}$
norman	$\frac{\frac{7a^2\left(4c^3+8c^2d+7c\,d^2+2d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{2f}-\frac{3a^2\left(4c^3+8c^2d+7c\,d^2+2d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4f}-\frac{8a^2\left(15c^3+35c^2d+25c\,d^2+9d^3\right)\tan\Big(}{5f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}$
parallelrisch	$\frac{4a^2\Bigg(-\frac{15\left(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+3e)}{2}+\cos(fx+e)\right)\left(c^2+\frac{3}{2}cd+d^2\right)\left(c+\frac{d}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{15\Big(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+3e)}{2}}{}}{}$
derivativedivides	$\frac{a^2c^3\ln(\sec(fx+e)+\tan(fx+e))+3a^2c^2d\tan(fx+e)+3a^2c\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2d^3\Big(}{}$
default	$\frac{a^2c^3\ln(\sec(fx+e)+\tan(fx+e))+3a^2c^2d\tan(fx+e)+3a^2c\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2d^3\Big(}{}$
risch	$-\frac{ia^2\big(-200c^2d-160c\,d^2-48d^3-80c^3-30d^3\mathrm{e}^{i(fx+e)}-320c^3\mathrm{e}^{6i(fx+e)}+140d^3\mathrm{e}^{7i(fx+e)}-140d^3\mathrm{e}^{3i(fx+e)}-320c^3\mathrm{e}^{2i(fx}}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
(3*a^2*c*d^2+2*a^2*d^3)/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+
3/8*ln(sec(f*x+e)+tan(f*x+e)))+(2*a^2*c^3+3*a^2*c^2*d)/f*tan(f*x+e)-(3*a^2
*c^2*d+6*a^2*c*d^2+a^2*d^3)/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+(a^2*c^3+
6*a^2*c^2*d+3*a^2*c*d^2)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+ta
n(f*x+e)))+a^2*c^3/f*ln(sec(f*x+e)+tan(f*x+e))-a^2*d^3/f*(-8/15-1/5*sec(f*
x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.21

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$
$$= \frac{15\,(4\,a^2c^3+8\,a^2c^2d+7\,a^2cd^2+2\,a^2d^3)\cos{(fx+e)}^5\log{(\sin{(fx+e)}+1)}-15\,(4\,a^2c^3+8\,a^2c^2d+7\,a^2}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/80*(15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^
5*log(sin(f*x + e) + 1) - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^
2*d^3)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^2*d^3 + 8*(10*a^2*c^
3 + 25*a^2*c^2*d + 20*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^4 + 5*(4*a^2*c^3
 + 24*a^2*c^2*d + 21*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^3 + 8*(5*a^2*c^2*
d + 10*a^2*c*d^2 + 3*a^2*d^3)*cos(f*x + e)^2 + 10*(3*a^2*c*d^2 + 2*a^2*d^3
)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx\\
&= a^2\left(\int c^3\sec{(e+fx)}\,dx+\int 2c^3\sec^2{(e+fx)}\,dx+\int c^3\sec^3{(e+fx)}\,dx\right.\\
&\quad+\int d^3\sec^4{(e+fx)}\,dx+\int 2d^3\sec^5{(e+fx)}\,dx+\int d^3\sec^6{(e+fx)}\,dx\\
&\quad+\int 3cd^2\sec^3{(e+fx)}\,dx+\int 6cd^2\sec^4{(e+fx)}\,dx+\int 3cd^2\sec^5{(e+fx)}\,dx\\
&\quad\left.+\int 3c^2d\sec^2{(e+fx)}\,dx+\int 6c^2d\sec^3{(e+fx)}\,dx+\int 3c^2d\sec^4{(e+fx)}\,dx\right)
\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**3,x)
```

output
```
a**2*(Integral(c**3*sec(e + f*x), x) + Integral(2*c**3*sec(e + f*x)**2, x)
 + Integral(c**3*sec(e + f*x)**3, x) + Integral(d**3*sec(e + f*x)**4, x) +
 Integral(2*d**3*sec(e + f*x)**5, x) + Integral(d**3*sec(e + f*x)**6, x) +
 Integral(3*c*d**2*sec(e + f*x)**3, x) + Integral(6*c*d**2*sec(e + f*x)**4
, x) + Integral(3*c*d**2*sec(e + f*x)**5, x) + Integral(3*c**2*d*sec(e + f
*x)**2, x) + Integral(6*c**2*d*sec(e + f*x)**3, x) + Integral(3*c**2*d*sec
(e + f*x)**4, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(230) = 460.

Time = 0.04 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.94

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$

$$= \frac{240\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2c^2d+480\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2cd^2+16\,(3\tan(fx}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/240*(240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2*d + 480*(tan(f*x + e)
^3 + 3*tan(f*x + e))*a^2*c*d^2 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3
+ 15*tan(f*x + e))*a^2*d^3 + 80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d^3
- 45*a^2*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*
sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) -
 30*a^2*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin
(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 60
*a^2*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 360*a^2*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1)
- log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 180*a^2*c*d^2*(2*sin(f*
x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1
)) + 240*a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 480*a^2*c^3*tan(f*x +
e) + 720*a^2*c^2*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(230) = 460$.

Time = 0.22 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.09

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
1/40*(15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*log(abs(tan(1
/2*f*x + 1/2*e) + 1)) - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*
d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(60*a^2*c^3*tan(1/2*f*x + 1/2*
e)^9 + 120*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^9 + 105*a^2*c*d^2*tan(1/2*f*x +
1/2*e)^9 + 30*a^2*d^3*tan(1/2*f*x + 1/2*e)^9 - 280*a^2*c^3*tan(1/2*f*x + 1
/2*e)^7 - 560*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^7 - 490*a^2*c*d^2*tan(1/2*f*x
 + 1/2*e)^7 - 140*a^2*d^3*tan(1/2*f*x + 1/2*e)^7 + 480*a^2*c^3*tan(1/2*f*x
 + 1/2*e)^5 + 1120*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 800*a^2*c*d^2*tan(1/
2*f*x + 1/2*e)^5 + 288*a^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 360*a^2*c^3*tan(1/
2*f*x + 1/2*e)^3 - 1040*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 790*a^2*c*d^2*t
an(1/2*f*x + 1/2*e)^3 - 180*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 100*a^2*c^3*t
an(1/2*f*x + 1/2*e) + 360*a^2*c^2*d*tan(1/2*f*x + 1/2*e) + 375*a^2*c*d^2*t
an(1/2*f*x + 1/2*e) + 130*a^2*d^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2
*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.63

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$

$$= \frac{3\,a^2\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c+d)\,\left(2\,c^2+3\,c\,d+2\,d^2\right)}{2\,\left(6\,c^3+12\,c^2\,d+\frac{21\,c\,d^2}{2}+3\,d^3\right)}\right)\,(2\,c+d)\,\left(2\,c^2+3\,c\,d+2\,d^2\right)}{4\,f} - \frac{\left(3\,a^2\,c^3+6\,a^2\,c^2\,d+\frac{21\,a^2\,c\,d^2}{4}+\frac{3\,a^2\,d^3}{2}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+\left(-14\,a^2\,c^3-28\,a^2\,c^2\,d-\frac{49\,a^2\,c\,d^2}{2}-7\,a^2\,d^3\right)\,\mathrm{ta}}{f\,\left(\mathrm{ta}\right.}$$

input
```
int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

output
```
(3*a^2*atanh((3*tan(e/2 + (f*x)/2)*(2*c + d)*(3*c*d + 2*c^2 + 2*d^2))/(2*(
(21*c*d^2)/2 + 12*c^2*d + 6*c^3 + 3*d^3)))*(2*c + d)*(3*c*d + 2*c^2 + 2*d^
2))/(4*f) - (tan(e/2 + (f*x)/2)^9*(3*a^2*c^3 + (3*a^2*d^3)/2 + (21*a^2*c*d
^2)/4 + 6*a^2*c^2*d) - tan(e/2 + (f*x)/2)^7*(14*a^2*c^3 + 7*a^2*d^3 + (49*
a^2*c*d^2)/2 + 28*a^2*c^2*d) - tan(e/2 + (f*x)/2)^3*(18*a^2*c^3 + 9*a^2*d^
3 + (79*a^2*c*d^2)/2 + 52*a^2*c^2*d) + tan(e/2 + (f*x)/2)^5*(24*a^2*c^3 +
(72*a^2*d^3)/5 + 40*a^2*c*d^2 + 56*a^2*c^2*d) + tan(e/2 + (f*x)/2)*(5*a^2*
c^3 + (13*a^2*d^3)/2 + (75*a^2*c*d^2)/4 + 18*a^2*c^2*d))/(f*(5*tan(e/2 + (
f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2
+ (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1033, normalized size of antiderivative = 4.27

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x)
```

output

```
(a**2*( - 60*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**3 -
 120*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**2*d - 105*c
os(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c*d**2 - 30*cos(e +
f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*d**3 + 120*cos(e + f*x)*log
(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c**3 + 240*cos(e + f*x)*log(tan((e
+ f*x)/2) - 1)*sin(e + f*x)**2*c**2*d + 210*cos(e + f*x)*log(tan((e + f*x)
/2) - 1)*sin(e + f*x)**2*c*d**2 + 60*cos(e + f*x)*log(tan((e + f*x)/2) - 1
)*sin(e + f*x)**2*d**3 - 60*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c**3 -
120*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c**2*d - 105*cos(e + f*x)*log(t
an((e + f*x)/2) - 1)*c*d**2 - 30*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*d*
*3 + 60*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*c**3 + 120*
cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*c**2*d + 105*cos(e
+ f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*c*d**2 + 30*cos(e + f*x)*
log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*d**3 - 120*cos(e + f*x)*log(tan(
(e + f*x)/2) + 1)*sin(e + f*x)**2*c**3 - 240*cos(e + f*x)*log(tan((e + f*x
)/2) + 1)*sin(e + f*x)**2*c**2*d - 210*cos(e + f*x)*log(tan((e + f*x)/2) +
 1)*sin(e + f*x)**2*c*d**2 - 60*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin
(e + f*x)**2*d**3 + 60*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*c**3 + 120*c
os(e + f*x)*log(tan((e + f*x)/2) + 1)*c**2*d + 105*cos(e + f*x)*log(tan((e
 + f*x)/2) + 1)*c*d**2 + 30*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*d**3...
```

3.195 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 176

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx &= \frac{a^2(12c^2+16cd+7d^2)\operatorname{arctanh}(\sin(e+fx))}{8f}\\ &\quad-\frac{a^2(c^3-8c^2d-20cd^2-8d^3)\tan(e+fx)}{6df}\\ &\quad-\frac{a^2(2c(c-8d)-21d^2)\sec(e+fx)\tan(e+fx)}{24f}\\ &\quad-\frac{a^2(c-8d)(c+d\sec(e+fx))^2\tan(e+fx)}{12df}+\frac{a^2(c+d\sec(e+fx))^3\tan(e+fx)}{4df}\end{aligned}$$

output
```
1/8*a^2*(12*c^2+16*c*d+7*d^2)*arctanh(sin(f*x+e))/f-1/6*a^2*(c^3-8*c^2*d-2
0*c*d^2-8*d^3)*tan(f*x+e)/d/f-1/24*a^2*(2*c*(c-8*d)-21*d^2)*sec(f*x+e)*tan
(f*x+e)/f-1/12*a^2*(c-8*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/d/f+1/4*a^2*(c+d*
sec(f*x+e))^3*tan(f*x+e)/d/f
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$
$$=\frac{a^2\left(24c^2\coth^{-1}(\sin(e+fx))+3(4c^2+16cd+7d^2)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(3(4c^2+16cd+\right.\right.}{24f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

output
```
(a^2*(24*c^2*ArcCoth[Sin[e + f*x]] + 3*(4*c^2 + 16*c*d + 7*d^2)*ArcTanh[Si
n[e + f*x]] + Tan[e + f*x]*(3*(4*c^2 + 16*c*d + 7*d^2)*Sec[e + f*x] + 6*d^
2*Sec[e + f*x]^3 + 16*(c + d)*(3*(c + d) + d*Tan[e + f*x]^2))))/(24*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.53, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3042, 4475, 101, 25, 27, 90, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int\frac{(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{101}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2(\sec(e+fx)a+a)^{3/2}\left(4c^2+2dc+d^2+d(5c+2d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{4a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{5/2}(c+}{4a^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(\sec(e+fx)a+a)^{3/2}\left(4c^2+2dc+d^2+d(5c+2d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{4a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{5/2}(c+d\sec}{4a^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{4}\int \frac{(\sec(e+fx)a+a)^{3/2}(4c^2+2dc+d^2+d(5c+2d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{5}}{4a^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 90$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{4}\left(\frac{1}{3}(12c^2+16cd+7d^2)\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{d(5c+2d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)}{3a^2}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 60$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{4}\left(\frac{1}{3}(12c^2+16cd+7d^2)\left(\frac{3}{2}a\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2a}\right)\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e}$$

$$\Big\downarrow 60$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{4}\left(\frac{1}{3}(12c^2+16cd+7d^2)\left(\frac{3}{2}a\left(a\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{}}{a}\right.\right.\right.\right.}{f\sqrt{a-}}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{4}\left(\frac{1}{3}(12c^2+16cd+7d^2)\left(\frac{3}{2}a\left(2a\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e}}{a}\right.\right.\right.\right.}{f\sqrt{a-a}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\Big(\frac{1}{4}\Big(\frac{1}{3}(12c^2+16cd+7d^2)\Big(\frac{3}{2}a\Big(-2\arctan\Big(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\Big)-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a}\Big)-}{f\sqrt{a-a\sec(e}}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^2*(-1/4*(d*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(5/2)*(c + d
*Sec[e + f*x]))/a^2 + (-1/3*(d*(5*c + 2*d)*Sqrt[a - a*Sec[e + f*x]]*(a + a
*Sec[e + f*x])^(5/2))/a^2 + ((12*c^2 + 16*c*d + 7*d^2)*(-1/2*(Sqrt[a - a*S
ec[e + f*x]]*(a + a*Sec[e + f*x])^(3/2))/a + (3*a*(-2*ArcTan[Sqrt[a - a*Se
c[e + f*x]]/Sqrt[a + a*Sec[e + f*x]]] - (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]])/a))/2))/3)/4)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

method	result
parts	$-\frac{(2a^2cd+2a^2d^2)\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{(2a^2c^2+2a^2cd)\tan(fx+e)}{f}+\frac{(a^2c^2+4a^2cd+a^2d^2)\left(\frac{\sec(fx+e)}{}\right.}{}$
parallelrisch	$\frac{4a^2\left(-\frac{3\left(c^2+\frac{4}{3}cd+\frac{7}{12}d^2\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3\left(c^2+\frac{4}{3}cd+\frac{7}{12}d^2\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2\right.}{2}\right.}{f(\cos(4}$
norman	$\frac{\frac{11a^2\left(12c^2+16cd+7d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}-\frac{a^2\left(12c^2+16cd+7d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{a^2\left(20c^2+48cd+25d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{a^2\left(156c^2+2\right.}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$
derivativedivides	$\frac{a^2c^2\ln(\sec(fx+e)+\tan(fx+e))+2a^2cd\tan(fx+e)+a^2d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^2c^2\tan}{}$
default	$\frac{a^2c^2\ln(\sec(fx+e)+\tan(fx+e))+2a^2cd\tan(fx+e)+a^2d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2a^2c^2\tan}{}$
risch	$-\frac{ia^2(12c^2e^{7i(fx+e)}+48cd\,e^{7i(fx+e)}+21d^2e^{7i(fx+e)}-48c^2e^{6i(fx+e)}-48cd\,e^{6i(fx+e)}+12c^2e^{5i(fx+e)}+48cd\,e^{5i(fx+e)}}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
-(2*a^2*c*d+2*a^2*d^2)/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+(2*a^2*c^2+2*a
^2*c*d)/f*tan(f*x+e)+(a^2*c^2+4*a^2*c*d+a^2*d^2)/f*(1/2*sec(f*x+e)*tan(f*x
+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+1/f*ln(sec(f*x+e)+tan(f*x+e))*a^2*c^2+a
^2*d^2/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)
+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$

$$=\frac{3\left(12a^2c^2+16a^2cd+7a^2d^2\right)\cos(fx+e)^4\log(\sin(fx+e)+1)-3\left(12a^2c^2+16a^2cd+7a^2d^2\right)\cos(f}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
1/48*(3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(sin(f*x +
 e) + 1) - 3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(-sin
(f*x + e) + 1) + 2*(6*a^2*d^2 + 16*(3*a^2*c^2 + 5*a^2*c*d + 2*a^2*d^2)*cos
(f*x + e)^3 + 3*(4*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^2 + 16*(
a^2*c*d + a^2*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx \\
&= a^2\left(\int c^2\sec(e+fx)\,dx+\int 2c^2\sec^2(e+fx)\,dx+\int c^2\sec^3(e+fx)\,dx\right. \\
&\quad+\int d^2\sec^3(e+fx)\,dx+\int 2d^2\sec^4(e+fx)\,dx+\int d^2\sec^5(e+fx)\,dx \\
&\quad\left.+\int 2cd\sec^2(e+fx)\,dx+\int 4cd\sec^3(e+fx)\,dx+\int 2cd\sec^4(e+fx)\,dx\right)
\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**2,x)
```

output
```
a**2*(Integral(c**2*sec(e + f*x), x) + Integral(2*c**2*sec(e + f*x)**2, x)
 + Integral(c**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**3, x) +
 Integral(2*d**2*sec(e + f*x)**4, x) + Integral(d**2*sec(e + f*x)**5, x) +
 Integral(2*c*d*sec(e + f*x)**2, x) + Integral(4*c*d*sec(e + f*x)**3, x) +
 Integral(2*c*d*sec(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.84

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$

$$= \frac{32\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2cd+32\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2d^2-3\,a^2d^2\left(\frac{2\left(3\,\sin(fx+\right.}{\sin(fx+e)^4-}\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c*d + 32*(tan(f*x + e)^3 +
3*tan(f*x + e))*a^2*d^2 - 3*a^2*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))
/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log
(sin(f*x + e) - 1)) - 12*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - lo
g(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^2*c*d*(2*sin(f*x + e)/
(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12
*a^2*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) + 48*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)) + 96*a^
2*c^2*tan(f*x + e) + 96*a^2*c*d*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.82

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$

$$= \frac{3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,f\right.\right.\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/24*(3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*log(abs(tan(1/2*f*x + 1/2*e)
 + 1)) - 3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*log(abs(tan(1/2*f*x + 1/2
*e) - 1)) - 2*(36*a^2*c^2*tan(1/2*f*x + 1/2*e)^7 + 48*a^2*c*d*tan(1/2*f*x
+ 1/2*e)^7 + 21*a^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 132*a^2*c^2*tan(1/2*f*x +
 1/2*e)^5 - 176*a^2*c*d*tan(1/2*f*x + 1/2*e)^5 - 77*a^2*d^2*tan(1/2*f*x +
1/2*e)^5 + 156*a^2*c^2*tan(1/2*f*x + 1/2*e)^3 + 272*a^2*c*d*tan(1/2*f*x +
1/2*e)^3 + 83*a^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 60*a^2*c^2*tan(1/2*f*x + 1/
2*e) - 144*a^2*c*d*tan(1/2*f*x + 1/2*e) - 75*a^2*d^2*tan(1/2*f*x + 1/2*e))
/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 14.98 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.35

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$

$$= \frac{\left(-3\,a^2\,c^2-4\,a^2\,c\,d-\frac{7\,a^2\,d^2}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(11\,a^2\,c^2+\frac{44\,a^2\,c\,d}{3}+\frac{77\,a^2\,d^2}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-13\,a^2\,c^2\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\frac{e}{2}+\right.\right.}$$

$$+\frac{a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(12\,c^2+16\,c\,d+7\,d^2\right)}{2\,\left(6\,c^2+8\,c\,d+\frac{7\,d^2}{2}\right)}\right)\,\left(12\,c^2+16\,c\,d+7\,d^2\right)}{4\,f}$$

input
```
int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

output
```
(tan(e/2 + (f*x)/2)*(5*a^2*c^2 + (25*a^2*d^2)/4 + 12*a^2*c*d) - tan(e/2 +
(f*x)/2)^7*(3*a^2*c^2 + (7*a^2*d^2)/4 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^5*
(11*a^2*c^2 + (77*a^2*d^2)/12 + (44*a^2*c*d)/3) - tan(e/2 + (f*x)/2)^3*(13
*a^2*c^2 + (83*a^2*d^2)/12 + (68*a^2*c*d)/3))/(f*(6*tan(e/2 + (f*x)/2)^4 -
 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 +
1)) + (a^2*atanh((tan(e/2 + (f*x)/2)*(16*c*d + 12*c^2 + 7*d^2))/(2*(8*c*d
+ 6*c^2 + (7*d^2)/2)))*(16*c*d + 12*c^2 + 7*d^2))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.43

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x)
```

output

```
(a**2*( - 48*cos(e + f*x)*sin(e + f*x)**3*c**2 - 80*cos(e + f*x)*sin(e + f
*x)**3*c*d - 32*cos(e + f*x)*sin(e + f*x)**3*d**2 + 48*cos(e + f*x)*sin(e
+ f*x)*c**2 + 96*cos(e + f*x)*sin(e + f*x)*c*d + 48*cos(e + f*x)*sin(e + f
*x)*d**2 - 36*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**2 - 48*log(tan(
(e + f*x)/2) - 1)*sin(e + f*x)**4*c*d - 21*log(tan((e + f*x)/2) - 1)*sin(e
 + f*x)**4*d**2 + 72*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c**2 + 96*l
og(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c*d + 42*log(tan((e + f*x)/2) - 1
)*sin(e + f*x)**2*d**2 - 36*log(tan((e + f*x)/2) - 1)*c**2 - 48*log(tan((e
 + f*x)/2) - 1)*c*d - 21*log(tan((e + f*x)/2) - 1)*d**2 + 36*log(tan((e +
f*x)/2) + 1)*sin(e + f*x)**4*c**2 + 48*log(tan((e + f*x)/2) + 1)*sin(e + f
*x)**4*c*d + 21*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*d**2 - 72*log(ta
n((e + f*x)/2) + 1)*sin(e + f*x)**2*c**2 - 96*log(tan((e + f*x)/2) + 1)*si
n(e + f*x)**2*c*d - 42*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*d**2 + 36
*log(tan((e + f*x)/2) + 1)*c**2 + 48*log(tan((e + f*x)/2) + 1)*c*d + 21*lo
g(tan((e + f*x)/2) + 1)*d**2 - 12*sin(e + f*x)**3*c**2 - 48*sin(e + f*x)**
3*c*d - 21*sin(e + f*x)**3*d**2 + 12*sin(e + f*x)*c**2 + 48*sin(e + f*x)*c
*d + 27*sin(e + f*x)*d**2))/(24*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1
))
```

3.196 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 103

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx\\ =\frac{a^2(3c+2d)\text{arctanh}(\sin(e+fx))}{2f}+\frac{2a^2(3c+2d)\tan(e+fx)}{3f}\\ +\frac{a^2(3c+2d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f}\end{aligned}$$

output
```
1/2*a^2*(3*c+2*d)*arctanh(sin(f*x+e))/f+2/3*a^2*(3*c+2*d)*tan(f*x+e)/f+1/6
*a^2*(3*c+2*d)*sec(f*x+e)*tan(f*x+e)/f+1/3*d*(a+a*sec(f*x+e))^2*tan(f*x+e)
/f
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.70

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$=\frac{a^2\left(6c\coth^{-1}(\sin(e+fx))+3(c+2d)\text{arctanh}(\sin(e+fx))+\tan(e+fx)\left(12(c+d)+3(c+2d)\sec(e\right.\right.}{6f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]
```

output
```
(a^2*(6*c*ArcCoth[Sin[e + f*x]] + 3*(c + 2*d)*ArcTanh[Sin[e + f*x]] + Tan[
e + f*x]*(12*(c + d) + 3*(c + 2*d)*Sec[e + f*x] + 2*d*Tan[e + f*x]^2)))/(6
*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {3042, 4489, 3042, 4275, 3042, 4254, 24, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4489}$$

$$\frac{1}{3}(3c+2d)\int \sec(e+fx)(\sec(e+fx)a+a)^2dx+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(3c+2d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2dx+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{ 4275}$$

$$\frac{1}{3}(3c+2d)\left(2a^2\int \sec^2(e+fx)dx+\int \sec(e+fx)\left(\sec^2(e+fx)a^2+a^2\right)dx\right)+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}(3c+2d)\left(2a^2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx+\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx\right)+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{1}{3}(3c+2d)\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\int 1d(-\tan(e+fx))}{f}\right)+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{3}(3c+2d)\left(\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\tan(e+fx)}{f}\right)+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{ 4534}$$

$$\frac{1}{3}(3c+2d)\left(\frac{3}{2}a^2\int \sec(e+fx)dx+\frac{2a^2\tan(e+fx)}{f}+\frac{a^2\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}(3c+2d)\left(\frac{3}{2}a^2\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{2a^2\tan(e+fx)}{f}+\frac{a^2\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}(3c+2d)\left(\frac{3a^2\text{arctanh}(\sin(e+fx))}{2f}+\frac{2a^2\tan(e+fx)}{f}+\frac{a^2\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]
```

output
```
(d*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*f) + ((3*c + 2*d)*((3*a^2*ArcTa
nh[Sin[e + f*x]])/(2*f) + (2*a^2*Tan[e + f*x])/f + (a^2*Sec[e + f*x]*Tan[e
 + f*x])/(2*f)))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17

method	result
parts	$\frac{(a^2c+2a^2d)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{(2a^2c+a^2d)\tan(fx+e)}{f}+\frac{a^2c\ln(\sec(fx+e)+\tan(}{f}$
derivativedivides	$\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2d\tan(fx+e)+2a^2c\tan(fx+e)+2a^2d\Big(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}}{f}$
default	$\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))+a^2d\tan(fx+e)+2a^2c\tan(fx+e)+2a^2d\Big(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}}{f}$
parallelrisch	$\frac{a^2\Bigg(-\frac{9\left(c+\frac{2d}{3}\right)\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{9\left(c+\frac{2d}{3}\right)\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+(}{f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{\frac{8a^2(3c+2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{a^2(3c+2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{a^2(5c+6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{a^2(3c+2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a^2(}{}$
risch	$-\frac{ia^2(3c\,\mathrm{e}^{5i(fx+e)}+6d\,\mathrm{e}^{5i(fx+e)}-12c\,\mathrm{e}^{4i(fx+e)}-6d\,\mathrm{e}^{4i(fx+e)}-24c\,\mathrm{e}^{2i(fx+e)}-24d\,\mathrm{e}^{2i(fx+e)}-3\,\mathrm{e}^{i(fx+e)}c-6d\,\mathrm{e}^{i(fx+e}}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
(a^2*c+2*a^2*d)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))
)+(2*a^2*c+a^2*d)/f*tan(f*x+e)+a^2*c/f*ln(sec(f*x+e)+tan(f*x+e))-a^2*d/f*(
-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$= \frac{3\,(3\,a^2c+2\,a^2d)\cos\,(fx+e)^3\log\,(\sin\,(fx+e)+1)-3\,(3\,a^2c+2\,a^2d)\cos\,(fx+e)^3\log\,(-\sin\,(fx+e)}{12\,f\cos\,(fx+}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
1/12*(3*(3*a^2*c + 2*a^2*d)*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*(3*a^
2*c + 2*a^2*d)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*a^2*d + 2*(6*a
^2*c + 5*a^2*d)*cos(f*x + e)^2 + 3*(a^2*c + 2*a^2*d)*cos(f*x + e))*sin(f*x
 + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$= a^2\bigg(\int c\sec\,(e+fx)\,dx+\int 2c\sec^2\,(e+fx)\,dx+\int c\sec^3\,(e+fx)\,dx$$
$$+\int d\sec^2\,(e+fx)\,dx+\int 2d\sec^3\,(e+fx)\,dx+\int d\sec^4\,(e+fx)\,dx\bigg)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e)),x)
```

output
```
a**2*(Integral(c*sec(e + f*x), x) + Integral(2*c*sec(e + f*x)**2, x) + Int
egral(c*sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**2, x) + Integral(2*
d*sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.62

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$

$$= \frac{4\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2d-3\,a^2c\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-\right.\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d - 3*a^2*c*(2*sin(f*x + e)/
(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 6*
a^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(s
in(f*x + e) - 1)) + 12*a^2*c*log(sec(f*x + e) + tan(f*x + e)) + 24*a^2*c*t
an(f*x + e) + 12*a^2*d*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.73

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$

$$= \frac{3\left(3\,a^2c+2\,a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(3\,a^2c+2\,a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(9\,a^2c\tan(\right.}{}}{6\,f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="gia
c")
```

output

```
1/6*(3*(3*a^2*c + 2*a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(3*a^2*c
 + 2*a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(9*a^2*c*tan(1/2*f*x +
1/2*e)^5 + 6*a^2*d*tan(1/2*f*x + 1/2*e)^5 - 24*a^2*c*tan(1/2*f*x + 1/2*e)^
3 - 16*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c*tan(1/2*f*x + 1/2*e) + 18*a
^2*d*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.56

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$=\frac{2\,a^2\,\mathrm{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,c}{2}+d\right)}{6\,c+4\,d}\right)\,\left(\frac{3\,c}{2}+d\right)}{f}-\frac{\left(3\,a^2\,c+2\,a^2\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-8\,a^2\,c-\frac{16\,a^2\,d}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(5\,a^2\,c+6\,a^2\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

input

```
int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

output

```
(2*a^2*atanh((4*tan(e/2 + (f*x)/2)*((3*c)/2 + d))/(6*c + 4*d))*((3*c)/2 +
d))/f - (tan(e/2 + (f*x)/2)*(5*a^2*c + 6*a^2*d) + tan(e/2 + (f*x)/2)^5*(3*
a^2*c + 2*a^2*d) - tan(e/2 + (f*x)/2)^3*(8*a^2*c + (16*a^2*d)/3))/(f*(3*ta
n(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.90

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$=\frac{a^2\left(-9\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^2c-6\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+\right.\right.}{}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x)
```

output

```
(a**2*( - 9*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c - 6*c
os(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*d + 9*cos(e + f*x)*l
og(tan((e + f*x)/2) - 1)*c + 6*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*d +
9*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*c + 6*cos(e + f*x
)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*d - 9*cos(e + f*x)*log(tan((e
+ f*x)/2) + 1)*c - 6*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*d - 3*cos(e +
f*x)*sin(e + f*x)*c - 6*cos(e + f*x)*sin(e + f*x)*d + 12*sin(e + f*x)**3*c
 + 10*sin(e + f*x)**3*d - 12*sin(e + f*x)*c - 12*sin(e + f*x)*d))/(6*cos(e
 + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.197 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx = -\frac{a^2(c-2d)\text{arctanh}(\sin(e+fx))}{d^2f} + \frac{2a^2(c-d)^{3/2}\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2\sqrt{c+d}f} + \frac{a^2\tan(e+fx)}{df}$$

output
```
-a^2*(c-2*d)*arctanh(sin(f*x+e))/d^2/f+2*a^2*(c-d)^(3/2)*arctanh((c-d)^(1/
2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/d^2/(c+d)^(1/2)/f+a^2*tan(f*x+e)/d/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$$

$$= \frac{a^2\cos(e+fx)(d+c\cos(e+fx))\sec^4\big(\tfrac{1}{2}(e+fx)\big)\,(1+\sec(e+fx))^2\,\Bigg((c-2d)\log\big(\cos\big(\tfrac{1}{2}(e+fx)\big)-}{}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

output
```
(a^2*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x
])^2*((c - 2*d)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - (c - 2*d)*Log[C
os[(e + f*x)/2] + Sin[(e + f*x)/2]] - ((2*I)*(c - d)^2*ArcTan[((I*Cos[e] +
 Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[
(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e
] - I*Sin[e])^2]) + (d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])) + (d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]))))/(4*d^2*f*(c + d*Sec[e + f*x]))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(95) = 190.

Time = 0.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 4475, 113, 25, 27, 175, 45, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow 4475$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 113$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^3(d-(c-2d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{ad}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{d}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^3(d-(c-2d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{ad}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{d}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\int \frac{d-(c-2d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{d}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 175$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c-d)^2\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{d}-\frac{(c-2d)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+f}{d}\right.}{d}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c-d)^2\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{d}-\frac{2(c-2d)\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}\right.}{d}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{2(c-d)^2\int\frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{d}-\frac{2(c-2d)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}\right)}{d}-\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{2(c-d)^{3/2}\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{ad\sqrt{c+d}}+\frac{2(c-2d)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)}{ad}\right)}{d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}{d}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

output
```
-((a^2*((a^2*((2*(c - 2*d)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[
e + f*x]]])/(a*d) + (2*(c - d)^(3/2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e
+ f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])])/(a*d*Sqrt[c + d])))/d -
(Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])/d)*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{8a^{2} \Bigg(-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{(c-2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8d^{2}}-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{(-c+2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^{2}}- \big(-c^{2}+2cd-}{f}$
default	$\frac{8a^{2} \Bigg(-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{(c-2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8d^{2}}-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{(-c+2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^{2}}- \big(-c^{2}+2cd-}{f}$
risch	$\frac{2ia^{2}}{fd\left(\mathrm{e}^{2i(fx+e)}+1\right)}-\frac{a^{2}\ln\left(\mathrm{e}^{i(fx+e)}+i\right)c}{d^{2}f}+\frac{2a^{2}\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{df}+\frac{\sqrt{(c-d)(c+d)}\,a^{2}\ln\Big(\mathrm{e}^{i(fx+e)}+\frac{i\sqrt{(c-d)(c+d)}+d}{c}}{(c+d)f\,d^{2}}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
8/f*a^2*(-1/8/d/(tan(1/2*f*x+1/2*e)-1)+1/8*(c-2*d)/d^2*ln(tan(1/2*f*x+1/2*
e)-1)-1/8/d/(tan(1/2*f*x+1/2*e)+1)+1/8/d^2*(-c+2*d)*ln(tan(1/2*f*x+1/2*e)+
1)-1/4/d^2*(-c^2+2*c*d-d^2)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+
1/2*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.19

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$$

$$= \left[\frac{2\,a^2d\sin\left(fx+e\right)-\left(a^2c-a^2d\right)\sqrt{\frac{c-d}{c+d}}\cos\left(fx+e\right)\log\left(\frac{2\,cd\cos(fx+e)-\left(c^2-2\,d^2\right)\cos(fx+e)^2-2\,\left(c^2+cd+\left(cd+d^2\right)\cos}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
[1/2*(2*a^2*d*sin(f*x + e) - (a^2*c - a^2*d)*sqrt((c - d)/(c + d))*cos(f*x
 + e)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*
d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 -
 d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (a^2*c - 2*a^2*d)
*cos(f*x + e)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-
sin(f*x + e) + 1))/(d^2*f*cos(f*x + e)), 1/2*(2*a^2*d*sin(f*x + e) + 2*(a^
2*c - a^2*d)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c
- d)/(c + d))/((c - d)*sin(f*x + e)))*cos(f*x + e) - (a^2*c - 2*a^2*d)*cos
(f*x + e)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-sin(
f*x + e) + 1))/(d^2*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx = a^2\left(\int \frac{\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx + \int \frac{2\sec^2\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx + \int \frac{\sec^3\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

output
```
a**2*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(2*sec(e +
f*x)**2/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)**3/(c + d*sec(e +
 f*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(86) = 172.

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.06

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx =$$

$$-\frac{\frac{2\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)d}+\frac{\left(a^2c-2\,a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d^2}-\frac{\left(a^2c-2\,a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d^2}+\frac{2\left(a^2c^2-2\,a^2cd+a^2d^2\right)\Bigg(}{}}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
-(2*a^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*d) + (a^2*c - 2
*a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^2 - (a^2*c - 2*a^2*d)*log(abs
(tan(1/2*f*x + 1/2*e) - 1))/d^2 + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(pi*fl
oor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e
) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/(sqrt(-c^2 + d^2)*d^2))/f
```

Mupad [B] (verification not implemented)

Time = 12.28 (sec) , antiderivative size = 529, normalized size of antiderivative = 5.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$$

$$= \frac{2\,a^2\left(\frac{\sin(e+f\,x)}{2}+2\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{f\,\cos\left(e+f\,x\right)\,\left(c+d\right)}$$

$$+\frac{2\,a^2\left(\frac{c\,\sin(e+f\,x)}{2}+c\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{d\,f\,\cos\left(e+f\,x\right)\,\left(c+d\right)}$$

$$-\frac{2\,a^2\left(c^2\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)+\cos\left(e+f\,x\right)\,\mathrm{atan}\left(\frac{\left(2\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c^4-2\,c^3\,d+2\,c\,d^3-d^4\right)^{3/2}-2\,c^5}{}\right.\right.}{}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

output
```
(2*a^2*(sin(e + f*x)/2 + 2*cos(e + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 +
 (f*x)/2))))/(f*cos(e + f*x)*(c + d)) + (2*a^2*((c*sin(e + f*x))/2 + c*cos
(e + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))))/(d*f*cos(e + f*x)
*(c + d)) - (2*a^2*(c^2*cos(e + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f
*x)/2)) + cos(e + f*x)*atan(((2*c*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d +
c^4 - d^4)^(3/2) - 2*c^5*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4
)^(1/2) + 5*d^5*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) -
 c*d^4*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) + 4*c^4*d*
sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) - 9*c^2*d^3*sin(e
/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) + 3*c^3*d^2*sin(e/2 +
(f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2))*1i)/(d*cos(e/2 + (f*x)/2)*
(c + d)*(8*c*d^4 + 3*c^4*d - 5*d^5 + 2*c^2*d^3 - 8*c^3*d^2)))*((c + d)*(c
- d)^3)^(1/2)*1i))/(d^2*f*cos(e + f*x)*(c + d))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.06

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$$

$$= \frac{a^2\left(2\sqrt{-c^2+d^2}\, atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)\cos\left(fx+e\right)c-2\sqrt{-c^2+d^2}\, atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{-c^2+d^2}}\right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x)
```

output
```
(a**2*(2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*
d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c - 2*sqrt( - c**2 + d**2)*atan((tan
((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*d
 + cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c**2 - cos(e + f*x)*log(tan((e +
 f*x)/2) - 1)*c*d - 2*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*d**2 - cos(e
+ f*x)*log(tan((e + f*x)/2) + 1)*c**2 + cos(e + f*x)*log(tan((e + f*x)/2)
+ 1)*c*d + 2*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*d**2 + sin(e + f*x)*c*
d + sin(e + f*x)*d**2))/(cos(e + f*x)*d**2*f*(c + d))
```

3.198 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 117

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx &= \frac{a^2\text{arctanh}(\sin(e+fx))}{d^2f} \\ &\quad - \frac{2a^2\sqrt{c-d}(c+2d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2(c+d)^{3/2}f} \\ &\quad - \frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))}\end{aligned}$$

output

```
a^2*arctanh(sin(f*x+e))/d^2/f-2*a^2*(c-d)^(1/2)*(c+2*d)*arctanh((c-d)^(1/2
)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/d^2/(c+d)^(3/2)/f-a^2*(c-d)*tan(f*x+e)/d
/(c+d)/f/(c+d*sec(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.67

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{a^2(d+c\cos(e+fx))\sec^4\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^2\left(-\left((d+c\cos(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

output
```
(a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^2*(-((d +
c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (d + c*Cos[e +
 f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (2*(c^2 + c*d - 2*d^2)*A
rcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqr
t[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*x])*(I*Cos[e]
+ Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((c - d
)*d*(d*Sin[e] - c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] +
Sin[e/2]))))/(4*d^2*f*(c + d*Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 4475, 109, 25, 27, 175, 45, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c+d\sec(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 4475$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 109$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{d(c+d)(c+d\sec(e+fx))}-\frac{\int -\frac{a^3(2d+(c+d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{ad(c+d)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^3(2d+(c+d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{ad(c+d)}+\frac{(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{d(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\int \frac{2d+(c+d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{d(c+d)}+\frac{(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{d(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 175$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c+d)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{d}-\frac{(c-d)(c+2d)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e}{d}\right.}{d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{2(c+d)\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}-\frac{(c-d)(c+2d)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+}{d}\right.}{d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{2(c+d)\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}-\frac{2(c-d)(c+2d)\int \frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{d}\right)}{d(c+d)}-\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(-\frac{2(c+d)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)}{ad}-\frac{2\sqrt{c-d}(c+2d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{ad\sqrt{c+d}}\right)}{d(c+d)}+\frac{(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec}}{d(c+d)(c+d\sec(e+fx}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^2*((a^2*((-2*(c + d)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e
 + f*x]]])/(a*d) - (2*Sqrt[c - d]*(c + 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a
*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])])/(a*d*Sqrt[c + d])
))/(d*(c + d)) + ((c - d)*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])/(d*(c + d)*(c + d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f
*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{8a^2\Bigg(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^2}+\frac{(c-d)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(c+2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c+d)\sqrt{(c-d)(c+d)}}\right)}{4d^2}-}{f}$
default	$\frac{8a^2\Bigg(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^2}+\frac{(c-d)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(c+2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c+d)\sqrt{(c-d)(c+d)}}\right)}{4d^2}-}{f}$
risch	$-\frac{2ia^2(c-d)\left(d\,e^{i(fx+e)}+c\right)}{df(c+d)c\left(c\,e^{2i(fx+e)}+2d\,e^{i(fx+e)}+c\right)}+\frac{\sqrt{(c-d)(c+d)}\,a^2\ln\left(e^{i(fx+e)}-\frac{i\sqrt{(c-d)(c+d)}-d}{c}\right)c}{(c+d)^2fd^2}+\frac{2\sqrt{(c-d)(c+d)}\,a}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
8/f*a^2*(1/8/d^2*ln(tan(1/2*f*x+1/2*e)+1)+1/4*(c-d)/d^2*(d/(c+d)*tan(1/2*f
*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)-(c+2*d)/(c+d
)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)
))-1/8/d^2*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(108) = 216$.

Time = 0.30 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.85

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$

$$= \left[\frac{\left(a^2cd + 2\,a^2d^2 + (a^2c^2 + 2\,a^2cd)\cos(fx+e)\right)\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2\,cd\cos(fx+e) - (c^2-2\,d^2)\cos(fx+e)^2 - 2\,(c^2+cd+(cd+d^2)\,c}{c^2\cos(fx+e)^2 + 2\,cd\cos(fx+e}\right.}{} \right.$$

$$- \frac{2\left(a^2cd + 2\,a^2d^2 + (a^2c^2 + 2\,a^2cd)\cos(fx+e)\right)\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{(d\cos(fx+e)+c)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right) - (a^2cd + a^2d}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
[1/2*((a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(f*x + e))*sqrt((c -
 d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c
^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) +
2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (a^2*c*d +
 a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(sin(f*x + e) + 1) - (a^2*
c*d + a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(-sin(f*x + e) + 1) -
 2*(a^2*c*d - a^2*d^2)*sin(f*x + e))/((c^2*d^2 + c*d^3)*f*cos(f*x + e) + (
c*d^3 + d^4)*f), -1/2*(2*(a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(
f*x + e))*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d
)/(c + d))/((c - d)*sin(f*x + e))) - (a^2*c*d + a^2*d^2 + (a^2*c^2 + a^2*c
*d)*cos(f*x + e))*log(sin(f*x + e) + 1) + (a^2*c*d + a^2*d^2 + (a^2*c^2 +
a^2*c*d)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^2*c*d - a^2*d^2)*sin(
f*x + e))/((c^2*d^2 + c*d^3)*f*cos(f*x + e) + (c*d^3 + d^4)*f)]
```

Sympy [F]

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx &= a^2\left(\int \frac{\sec(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right.\\ &\quad+\int \frac{2\sec^2(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\\ &\quad\left.+\int \frac{\sec^3(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right)\end{aligned}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

output

```
a**2*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)
**2), x) + Integral(2*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*se
c(e + f*x)**2), x) + Integral(sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x) +
 d**2*sec(e + f*x)**2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(108) = 216$.

Time = 0.18 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d^2} - \frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d^2} + \frac{2\,(a^2c^2+a^2cd-2\,a^2d^2)\Bigg(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,c-2\,d)+\arctan\Bigg(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-}{\sqrt{-c}}}{(cd^2+d^3)\sqrt{-c^2+d^2}}}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
(a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^2 - a^2*log(abs(tan(1/2*f*x + 1/
2*e) - 1))/d^2 + 2*(a^2*c^2 + a^2*c*d - 2*a^2*d^2)*(pi*floor(1/2*(f*x + e)
/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x
 + 1/2*e))/sqrt(-c^2 + d^2)))/((c*d^2 + d^3)*sqrt(-c^2 + d^2)) + 2*(a^2*c*
tan(1/2*f*x + 1/2*e) - a^2*d*tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e
)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)*(c*d + d^2)))/f
```

Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 2563, normalized size of antiderivative = 21.91

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

output
```
(a^2*atan(((a^2*((a^2*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^
2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*a^2*tan(e/2 + (f
*x)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2)))
)/d^2 + (32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*
c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2))*1i)/d^2 - (a^2*((a^2*
((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^
4))/(2*c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^
3*d^6 + 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d^2 - (32*tan(e/2 +
(f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^
2))/(2*c*d^3 + d^4 + c^2*d^2))*1i)/d^2)/((64*(2*a^6*d^4 - a^6*c^4 - 5*a^6*
c*d^3 + a^6*c^3*d + 3*a^6*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((a^2
*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d
^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*a^2*tan(e/2 + (f*x)/2)*(2*c*d^8 - 4*c
^3*d^6 + 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d^2 + (32*tan(e/2 +
 (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d
^2))/(2*c*d^3 + d^4 + c^2*d^2)))/d^2 + (a^2*((a^2*((32*(3*a^2*d^8 - 2*a^2*
c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2
*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/(d^2
*(2*c*d^3 + d^4 + c^2*d^2))))/d^2 - (32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*
a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 549, normalized size of antiderivative = 4.69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x)
```

output

```
(a**2*( - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/
2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c**2 - 4*sqrt( - c**2 + d**2)*ata
n((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e +
f*x)*c*d - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)
/2)*d)/sqrt( - c**2 + d**2))*c*d - 4*sqrt( - c**2 + d**2)*atan((tan((e + f
*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*d**2 - cos(e + f*x)*l
og(tan((e + f*x)/2) - 1)*c**3 - 2*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c
**2*d - cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c*d**2 + cos(e + f*x)*log(t
an((e + f*x)/2) + 1)*c**3 + 2*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*c**2*
d + cos(e + f*x)*log(tan((e + f*x)/2) + 1)*c*d**2 - log(tan((e + f*x)/2) -
 1)*c**2*d - 2*log(tan((e + f*x)/2) - 1)*c*d**2 - log(tan((e + f*x)/2) - 1
)*d**3 + log(tan((e + f*x)/2) + 1)*c**2*d + 2*log(tan((e + f*x)/2) + 1)*c*
d**2 + log(tan((e + f*x)/2) + 1)*d**3 - sin(e + f*x)*c**2*d + sin(e + f*x)
*d**3))/(d**2*f*(cos(e + f*x)*c**3 + 2*cos(e + f*x)*c**2*d + cos(e + f*x)*
c*d**2 + c**2*d + 2*c*d**2 + d**3))
```

3.199 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 130

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \frac{3a^2\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}(c+d)^{5/2}f} + \frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2\tan(e+fx)}{2(c+d)^2f(c+d\sec(e+fx))}$$

output

```
3*a^2*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(1/2)/(c+d
)^(5/2)/f+1/2*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^2+3
/2*a^2*tan(f*x+e)/(c+d)^2/f/(c+d*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{a^2(d+c\cos(e+fx))\sec^4\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(1+\sec(e+fx))^2\left(-\frac{6i\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin}}\right.}{\sqrt{c^2-d^2}}\right.}{8(c+}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
(a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Sec[e + f*x]*(1 + Sec[e + f*x
])^2*(((-6*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[
(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f
*x])^2*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])
+ ((c - d)*(c + d)*Sec[e]*(-(d*Sin[e]) + c*Sin[f*x]))/c^2 + ((d + c*Cos[e
+ f*x])*Sec[e]*((c^2 - 4*c*d - 2*d^2)*Sin[e] + c*(4*c + d)*Sin[f*x]))/c^2)
)/(8*(c + d)^2*f*(c + d*Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 4475, 105, 105, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \ 4475$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 105$$

$$-\frac{a^2\tan(e+fx)\left(\frac{3a\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2a(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 105$$

$$-\frac{a^2\tan(e+fx)\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{c+d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx}}{2a(c+d)(c}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{3a\left(\frac{2a\int \frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{c+d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(}{2a(c+d)(c+d\sec(e}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{3a\left(\frac{2\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{\sqrt{c-d}(c+d)^{3/2}}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2a(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^2*(-1/2*(Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(3/2))/(a*(c +
 d)*(c + d*Sec[e + f*x])^2) + (3*a*((2*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[
e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])])/(Sqrt[c - d]*(c + d)^(
3/2)) - (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])/(a*(c + d)*(c
+ d*Sec[e + f*x]))))/(2*(c + d)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]
]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{8a^2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}+\frac{-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}+\frac{3\operatorname{arctanh}\left(\frac{(c-d)}{\sqrt{(c}}\right)}{8(c+d)\sqrt{(c-}}}{c+d}\right)}{f}$
default	$\frac{8a^2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}+\frac{-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}+\frac{3\operatorname{arctanh}\left(\frac{(c-d)}{\sqrt{(c}}\right)}{8(c+d)\sqrt{(c-}}}{c+d}\right)}{f}$
risch	$\frac{ia^2\left(-c^3e^{3i(fx+e)}+4c^2d\,e^{3i(fx+e)}+2c\,d^2e^{3i(fx+e)}+4c^3e^{2i(fx+e)}+c^2d\,e^{2i(fx+e)}+8c\,d^2e^{2i(fx+e)}+2d^3e^{2i(fx+e)}+c^3e^{i}\right.}{c^2(c+d)^2f\left(c\,e^{2i(fx+e)}+2d\,e^{i(fx+e)}+c\right)^2}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
8/f*a^2*(1/4*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+
1/2*e)^2*d-c-d)^2+3/4/(c+d)*(-1/2*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*x+
1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)+1/2/(c+d)/((c-d)*(c+d))^(1/2)*arctanh
((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(117) = 234$.

Time = 0.16 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.78

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$
$$= \left[\frac{3\left(a^2c^2\cos\left(fx+e\right)^2+2\,a^2cd\cos\left(fx+e\right)+a^2d^2\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}(}{c^2\cos(fx+e)^2+2\,cd\cos(}\right.}{4\left((c^6+2\,c^5d-2\,c^3d^3-c^2d^4)f\cos\left(fx+e\right)^2+2\,(\right.}\right.$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
[1/4*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(f*x + e) + a^2*d^2)*sqrt(c
^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(
c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
 e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2
- 4*a^2*d^3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e)
)*sin(f*x + e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 +
2*(c^5*d + 2*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^
3*d^3 - 2*c*d^5 - d^6)*f), 1/2*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(
f*x + e) + a^2*d^2)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x +
 e) + c)/((c^2 - d^2)*sin(f*x + e))) + (a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2
- 4*a^2*d^3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e)
)*sin(f*x + e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 +
2*(c^5*d + 2*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^
3*d^3 - 2*c*d^5 - d^6)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = a^2\left(\int \frac{\sec(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx + \int \frac{2\sec^2(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx + \int \frac{\sec^3(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx\right)$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

output

```
a**2*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
 + f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(2*sec(e + f*x)**2/(c**3
+ 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3)
, x) + Integral(sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*s
ec(e + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="m
axima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.62

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx =$$

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^2}{(c^2+2cd+d^2)\sqrt{-c^2+d^2}}+\frac{3a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5a^2c\,\mathrm{ta}}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\right.}}{f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
-(3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f
*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^2/((c^2 + 2*c*d
 + d^2)*sqrt(-c^2 + d^2)) + (3*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a^2*d*tan(
1/2*f*x + 1/2*e)^3 - 5*a^2*c*tan(1/2*f*x + 1/2*e) - 5*a^2*d*tan(1/2*f*x +
1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2*(
c^2 + 2*c*d + d^2)))/f
```

Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{\frac{5a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+d}-\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(a^2c-a^2d)}{(c+d)^2}}{f\left(2cd-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(2c^2-2d^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(c^2-2cd+d^2)+c^2+d^2\right)}$$

$$+\frac{3a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)}{f\,(c+d)^{5/2}\sqrt{c-d}}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

output
```
((5*a^2*tan(e/2 + (f*x)/2))/(c + d) - (3*tan(e/2 + (f*x)/2)^3*(a^2*c - a^2
*d))/(c + d)^2)/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2
 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) + (3*a^2*atanh((tan(e/2 +
(f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)^(5/2)*(c - d)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.82

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{a^2\left(12\sqrt{-c^2+d^2}\, atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)\cos\left(fx+e\right)cd-6\sqrt{-c^2+d^2}\, atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\right.}{\sqrt{-c^2+d^2}}\right.\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x)
```

output
```
(a**2*(12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)
*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c*d - 6*sqrt( - c**2 + d**2)*atan((
tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x
)**2*c**2 + 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x
)/2)*d)/sqrt( - c**2 + d**2))*c**2 + 6*sqrt( - c**2 + d**2)*atan((tan((e +
 f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*d**2 + 4*cos(e + f*
x)*sin(e + f*x)*c**3 + cos(e + f*x)*sin(e + f*x)*c**2*d - 4*cos(e + f*x)*s
in(e + f*x)*c*d**2 - cos(e + f*x)*sin(e + f*x)*d**3 + sin(e + f*x)*c**3 +
4*sin(e + f*x)*c**2*d - sin(e + f*x)*c*d**2 - 4*sin(e + f*x)*d**3))/(2*f*(
2*cos(e + f*x)*c**5*d + 4*cos(e + f*x)*c**4*d**2 - 4*cos(e + f*x)*c**2*d**
4 - 2*cos(e + f*x)*c*d**5 - sin(e + f*x)**2*c**6 - 2*sin(e + f*x)**2*c**5*
d + 2*sin(e + f*x)**2*c**3*d**3 + sin(e + f*x)**2*c**2*d**4 + c**6 + 2*c**
5*d + c**4*d**2 - c**2*d**4 - 2*c*d**5 - d**6))
```

3.200 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 213

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx &= \frac{a^2(3c-2d)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{7/2}f} \\ &\quad - \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} \\ &\quad + \frac{(3c-2d)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} \\ &\quad + \frac{a^2(3c-2d)\tan(e+fx)}{2(c-d)(c+d)^3f(c+d\sec(e+fx))}\end{aligned}$$

output

```
a^2*(3*c-2*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3
/2)/(c+d)^(7/2)/f-1/3*d*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec
(f*x+e))^3+1/6*(3*c-2*d)*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/(c-d)/(c+d)^2/f/(
c+d*sec(f*x+e))^2+1/2*a^2*(3*c-2*d)*tan(f*x+e)/(c-d)/(c+d)^3/f/(c+d*sec(f*
x+e))
```

Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$
$$= \frac{a^2(c-d)^2\left(24(3c-2d)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^3-2\sqrt{c^2-d^2}(12c^3-5c^2d+6c}{24(-c+d)^3(c+d)^3\sqrt{}}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

output

```
(a^2*(c - d)^2*(24*(3*c - 2*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^
2 - d^2]]*(d + c*Cos[e + f*x])^3 - 2*Sqrt[c^2 - d^2]*(12*c^3 - 5*c^2*d + 6
*c*d^2 - 22*d^3 + 6*(c^3 + 6*c^2*d - 7*c*d^2 - 2*d^3)*Cos[e + f*x] + (12*c
^3 - 7*c^2*d - 6*c*d^2 - 2*d^3)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(24*(-c +
 d)^3*(c + d)^3*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.44, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3042, 4475, 107, 105, 105, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c+d\sec(e+fx))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \text{4475}$$
$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^4}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{107}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(3c-2d)\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{3(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{5/2}}{3a^2(c^2-d^2)(c+d\sec(e+fx))^3}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(3c-2d)\left(\frac{3a\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2a(c+d)(c+d\sec(e+fx))^2}\right)}{3(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}(}{3a^2(c^2-d^2)(c+}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(3c-2d)\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{c+d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-a\sec}}{2a(c}\right.}{3(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(3c-2d)\left(\frac{3a\left(\frac{2a\int \frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{c+d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(}{2a(c+d)(c+d}\right.}{3(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a^2 \tan(e+fx) \left(\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{5/2}}{3a^2(c^2-d^2)(c+d\sec(e+fx))^3} + \frac{(3c-2d)\left(\frac{3a\left(\frac{2\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}} \right)}{\sqrt{c-d}(c+d)^{3/2}} - \frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)}}{a(c+d)(c+d\sec(e+fx))} \right.}{2(c+d)} \right.}{3(c^2-d^2)} \right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

output

```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(5/2))/(3*a^2*(c^
2 - d^2)*(c + d*Sec[e + f*x])^3) + ((3*c - 2*d)*(-1/2*(Sqrt[a - a*Sec[e +
f*x]]*(a + a*Sec[e + f*x])^(3/2))/(a*(c + d)*(c + d*Sec[e + f*x])^2) + (3*
a*((2*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a -
a*Sec[e + f*x]])])/(Sqrt[c - d]*(c + d)^(3/2)) - (Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]])/(a*(c + d)*(c + d*Sec[e + f*x]))))/(2*(c + d)))
)/(3*(c^2 - d^2)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Se
c[e + f*x]]))
```

Defintions of rubi rules used

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{8a^2\left(-\frac{\frac{(3c-2d)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{8c^3+24c^2d+24c\,d^2+8d^3}-\frac{(3c-2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3\left(c^2+2cd+d^2\right)}+\frac{(5c-6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c-d)(c+d)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}+\frac{(3c-2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{8\left(c^4+2c^3d-2c\,d^3-d^4\right)\sqrt{(c-d)(c+d)}}\right)}{f}$
default	$\frac{8a^2\left(-\frac{\frac{(3c-2d)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{8c^3+24c^2d+24c\,d^2+8d^3}-\frac{(3c-2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3\left(c^2+2cd+d^2\right)}+\frac{(5c-6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c-d)(c+d)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}+\frac{(3c-2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{8\left(c^4+2c^3d-2c\,d^3-d^4\right)\sqrt{(c-d)(c+d)}}\right)}{f}$
risch	$i a^2\left(2c^3d^3-12c^6+6c^4d^2+7c^5d-12c^3d^3\mathrm{e}^{3i(fx+e)}+40c^2d^4\mathrm{e}^{3i(fx+e)}+36c^2d^4\mathrm{e}^{2i(fx+e)}+12c\,d^5\mathrm{e}^{2i(fx+e)}-18c^5d\,\mathrm{e}^{5i(fx+}\right.$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBO
SE)
```

output
```
8/f*a^2*(-(1/8*(3*c-2*d)*(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e
)^5-1/3*(3*c-2*d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/8*(5*c-6*d)/(c-d)
/(c+d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-
c-d)^3+1/8*(3*c-2*d)/(c^4+2*c^3*d-2*c*d^3-d^4)/((c-d)*(c+d))^(1/2)*arctanh
((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(198) = 396.

Time = 0.23 (sec) , antiderivative size = 1234, normalized size of antiderivative = 5.79

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="f
ricas")
```

output
```
[1/12*(3*(3*a^2*c*d^3 - 2*a^2*d^4 + (3*a^2*c^4 - 2*a^2*c^3*d)*cos(f*x + e)
^3 + 3*(3*a^2*c^3*d - 2*a^2*c^2*d^2)*cos(f*x + e)^2 + 3*(3*a^2*c^2*d^2 - 2
*a^2*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x +
e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^
2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^2*d^5 + (12*
a^2*c^5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 6*a^2*c*d^4 + 2*a
^2*d^5)*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*
c^2*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e))*sin(f*x + e))/((c^9 + 2*c
^8*d - c^7*d^2 - 4*c^6*d^3 - c^5*d^4 + 2*c^4*d^5 + c^3*d^6)*f*cos(f*x + e)
^3 + 3*(c^8*d + 2*c^7*d^2 - c^6*d^3 - 4*c^5*d^4 - c^4*d^5 + 2*c^3*d^6 + c^
2*d^7)*f*cos(f*x + e)^2 + 3*(c^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c
^3*d^6 + 2*c^2*d^7 + c*d^8)*f*cos(f*x + e) + (c^6*d^3 + 2*c^5*d^4 - c^4*d^
5 - 4*c^3*d^6 - c^2*d^7 + 2*c*d^8 + d^9)*f), 1/6*(3*(3*a^2*c*d^3 - 2*a^2*d
^4 + (3*a^2*c^4 - 2*a^2*c^3*d)*cos(f*x + e)^3 + 3*(3*a^2*c^3*d - 2*a^2*c^2
*d^2)*cos(f*x + e)^2 + 3*(3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*x + e))*sqrt(
-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin
(f*x + e))) + (a^2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 +
10*a^2*d^5 + (12*a^2*c^5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 +
6*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*...
```

Sympy [F]

$$\begin{aligned}
&\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx \\
&= a^2\left(\int \frac{\sec(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right. \\
&\quad+\int \frac{2\sec^2(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx \\
&\quad\left.+\int \frac{\sec^3(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right)
\end{aligned}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

output
```
a**2*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*se
c(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Int
egral(2*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e
+ f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integra
l(sec(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)
**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(198) = 396$.

Time = 0.23 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = -\frac{\frac{3\,(3\,a^2c-2\,a^2d)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4+2\,c^3d-2\,cd^3-d^4)\sqrt{-c^2+d^2}}+\frac{9\,a^2c^3\tan\big(\frac{1}{2}\,fx+\frac{1}{2}\,e\big)^5-24\,a^2c^2d\tan\big(\frac{1}{2}\,fx+}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="g
iac")
```

output
```
-1/3*(3*(3*a^2*c - 2*a^2*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*
d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 +
d^2)))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*sqrt(-c^2 + d^2)) + (9*a^2*c^3*tan
(1/2*f*x + 1/2*e)^5 - 24*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 21*a^2*c*d^2*t
an(1/2*f*x + 1/2*e)^5 - 6*a^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 24*a^2*c^3*tan(
1/2*f*x + 1/2*e)^3 + 16*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 24*a^2*c*d^2*ta
n(1/2*f*x + 1/2*e)^3 - 16*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^3*tan(
1/2*f*x + 1/2*e) + 12*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - 21*a^2*c*d^2*tan(1/
2*f*x + 1/2*e) - 18*a^2*d^3*tan(1/2*f*x + 1/2*e))/((c^4 + 2*c^3*d - 2*c*d^
3 - d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))
/f
```

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(3\,a^2\,c^2-5\,a^2\,c\,d+2\,a^2\,d^2\right)}{{\left(c+d\right)}^3}-\frac{8\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\,a^2\,c-2\,a^2\,d\right)}{3\,{\left(c+d\right)}^2}+}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3\right)+3\,c\,d\right.}$$

$$+\frac{2\,a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)\,\left(\frac{3\,c}{2}-d\right)}{f\,{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{3/2}}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

output
```
((tan(e/2 + (f*x)/2)^5*(3*a^2*c^2 + 2*a^2*d^2 - 5*a^2*c*d))/(c + d)^3 - (8
*tan(e/2 + (f*x)/2)^3*(3*a^2*c - 2*a^2*d))/(3*(c + d)^2) + (a^2*tan(e/2 +
(f*x)/2)*(5*c - 6*d))/((c + d)*(c - d)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2
 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*
c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d
^2 - 3*c^2*d + c^3 - d^3))) + (2*a^2*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/
2))/(c + d)^(1/2))*((3*c)/2 - d))/(f*(c + d)^(7/2)*(c - d)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1430, normalized size of antiderivative = 6.71

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x)
```

output

```
(a**2*(18*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)
*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*c**4 - 12*sqrt( - c
**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 +
d**2))*cos(e + f*x)*sin(e + f*x)**2*c**3*d - 18*sqrt( - c**2 + d**2)*atan(
(tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*
x)*c**4 + 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)
/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c**3*d - 54*sqrt( - c**2 + d**2)
*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(
e + f*x)*c**2*d**2 + 36*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - ta
n((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c*d**3 + 54*sqrt( - c
**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 +
d**2))*sin(e + f*x)**2*c**3*d - 36*sqrt( - c**2 + d**2)*atan((tan((e + f*x
)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**2*d*
*2 - 54*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*c**3*d + 36*sqrt( - c**2 + d**2)*atan((tan((e + f*
x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**2*d**2 - 18*sqrt( -
 c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2
+ d**2))*c*d**3 + 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((
e + f*x)/2)*d)/sqrt( - c**2 + d**2))*d**4 - 3*cos(e + f*x)*sin(e + f*x)*c*
*5 - 18*cos(e + f*x)*sin(e + f*x)*c**4*d + 24*cos(e + f*x)*sin(e + f*x)...
```

3.201 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 276

$$\begin{aligned}
&\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx \\
&= \frac{a^2(12c^2-16cd+7d^2)\,\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4(c-d)^{5/2}(c+d)^{9/2}f} - \frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} \\
&\quad + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2(2c^2+16cd-21d^2)\tan(e+fx)}{24(c-d)d(c+d)^3f(c+d\sec(e+fx))^2} \\
&\quad + \frac{a^2(2c^3+16c^2d-59cd^2+32d^3)\tan(e+fx)}{24(c-d)^2d(c+d)^4f(c+d\sec(e+fx))}
\end{aligned}$$

output

```
1/4*a^2*(12*c^2-16*c*d+7*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)
^(1/2))/(c-d)^(5/2)/(c+d)^(9/2)/f-1/4*a^2*(c-d)*tan(f*x+e)/d/(c+d)/f/(c+d*
sec(f*x+e))^4+1/12*a^2*(c+8*d)*tan(f*x+e)/d/(c+d)^2/f/(c+d*sec(f*x+e))^3+1
/24*a^2*(2*c^2+16*c*d-21*d^2)*tan(f*x+e)/(c-d)/d/(c+d)^3/f/(c+d*sec(f*x+e)
)^2+1/24*a^2*(2*c^3+16*c^2*d-59*c*d^2+32*d^3)*tan(f*x+e)/(c-d)^2/d/(c+d)^4
/f/(c+d*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 6.70 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.17

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$$

$$= \frac{a^2\left(-\frac{24(12c^2-16cd+7d^2)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{(24c^5+192c^4d-446c^3d^2+128c^2d^3-148cd^4+160d^5+(144c^5-172c^4d+20}{}\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

output
```
(a^2*((-24*(12*c^2 - 16*c*d + 7*d^2)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + ((24*c^5 + 192*c^4*d - 446*c^3*d^2 + 12
8*c^2*d^3 - 148*c*d^4 + 160*d^5 + (144*c^5 - 172*c^4*d + 208*c^3*d^2 - 785
*c^2*d^3 + 368*c*d^4 + 102*d^5)*Cos[e + f*x] + 2*(12*c^5 + 96*c^4*d - 227*
c^3*d^2 + 32*c^2*d^3 + 44*c*d^4 + 16*d^5)*Cos[2*(e + f*x)] + 48*c^5*Cos[3*
(e + f*x)] - 68*c^4*d*Cos[3*(e + f*x)] - 16*c^3*d^2*Cos[3*(e + f*x)] + 5*c
^2*d^3*Cos[3*(e + f*x)] + 16*c*d^4*Cos[3*(e + f*x)] + 6*d^5*Cos[3*(e + f*x
)])*Sin[e + f*x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)^2*(c + d)^4*f)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.67, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {3042, 4475, 109, 25, 27, 168, 27, 168, 27, 168, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^2}{(c+d\sec(e+fx))^5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^5}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{4d(c+d)(c+d\sec(e+fx))^4}-\frac{\int -\frac{a^3(8d+(c+7d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))^4}d\sec(e+fx)}{4ad(c+d)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^3(8d+(c+7d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))^4}d\sec(e+fx)}{4ad(c+d)}+\frac{(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{4d(c+d)(c+d\sec(e+fx))^4}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\int \frac{8d+(c+7d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))^4}d\sec(e+fx)}{4d(c+d)}+\frac{(c-d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{4d(c+d)(c+d\sec(e+fx))^4}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{\int \frac{a^2(c-d)(21d+2(c+8d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))^3}d\sec(e+fx)}{3a^2\left(c^2-d^2\right)}-\frac{(c+8d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{3a^2(c+d)(c+d\sec(e+fx))^3}\right)}{4d(c+d)}+\frac{(c-d)\sqrt{a-}}{4d(}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c-d)\int \frac{21d+2(c+8d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))^3}d\sec(e+fx)}{3\left(c^2-d^2\right)}-\frac{(c+8d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{3a^2(c+d)(c+d\sec(e+fx))^3}\right)}{4d(c+d)}+(c-d\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c-d)\left(\frac{\int\frac{a^2\left(2(19c-16d)d-\left(21d^2-2c(c+8d)\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2a^2\left(c^2-d^2\right)}-\frac{\left(2c^2+16cd-21d^2\right)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}{2a^2\left(c^2-d^2\right)(c+d\sec(e+fx))^2}\right.}{3\left(c^2-d^2\right)}\right.}{4d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c-d)\left(\frac{\int\frac{2(19c-16d)d-\left(21d^2-2c(c+8d)\right)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2\left(c^2-d^2\right)}-\frac{\left(2c^2+16cd-21d^2\right)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}{2a^2\left(c^2-d^2\right)(c+d\sec(e+fx))^2}\right.}{3\left(c^2-d^2\right)}\right.}{4d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}$$

$$\Big\downarrow 168$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c-d)\left(\frac{\int\frac{3a^2d\left(12c^2-16dc+7d^2\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a^2\left(c^2-d^2\right)}-\frac{\left(2c^3+16c^2d-59cd^2+32d^3\right)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e}{a^2\left(c^2-d^2\right)(c+d\sec(e+fx))}\right.}{2\left(c^2-d^2\right)}\right.}{3\left(c^2-d^2\right)}\right.}{4d(c+d)}\right.}{f\sqrt{a-a\sec(e+}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c-d)\left(\frac{3d\left(12c^2-16cd+7d^2\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{c^2-d^2}-\frac{\left(2c^3+16c^2d-59cd^2+32d^3\right)\sqrt{a-}}{a^2\left(c^2-d^2\right)(c+}\right.}{2\left(c^2-d^2\right)}\right.}{3\left(c^2-d^2\right)}\right.}{4d(}\right.}{f\sqrt{a-a\,\mathrm{s}}}$$

$$\Big\downarrow 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c-d)\left(\frac{\frac{6d\left(12c^2-16cd+7d^2\right)\int\frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{c^2-d^2}-\frac{\left(2c^3+16c^2d-59cd^2+32d^3\right)\sqrt{a-a\sec(e+}}{a^2\left(c^2-d^2\right)(c+d\sec(e+f}\right.}{2\left(c^2-d^2\right)}\right.}{3\left(c^2-d^2\right)}\right.}{4d(c+d)}\right.}{f\sqrt{a-a\sec(}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{(c-d)\left(\frac{\frac{6d\left(12c^2-16cd+7d^2\right)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a\sqrt{c-d}\sqrt{c+d}\left(c^2-d^2\right)}-\frac{\left(2c^3+16c^2d-59cd^2+32d^3\right)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a^2\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}\right.}{3\left(c^2-d^2\right)}\right.}{4d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

output
```
-((a^2*(((c - d)*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])/(4*d*(
c + d)*(c + d*Sec[e + f*x])^4) + (a^2*(-1/3*((c + 8*d)*Sqrt[a - a*Sec[e +
f*x]]*Sqrt[a + a*Sec[e + f*x]])/(a^2*(c + d)*(c + d*Sec[e + f*x])^3) + ((c
 - d)*(-1/2*((2*c^2 + 16*c*d - 21*d^2)*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a
*Sec[e + f*x]])/(a^2*(c^2 - d^2)*(c + d*Sec[e + f*x])^2) + ((6*d*(12*c^2 -
 16*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d
]*Sqrt[a - a*Sec[e + f*x]])])/(a*Sqrt[c - d]*Sqrt[c + d]*(c^2 - d^2)) - ((
2*c^3 + 16*c^2*d - 59*c*d^2 + 32*d^3)*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*
Sec[e + f*x]])/(a^2*(c^2 - d^2)*(c + d*Sec[e + f*x])))/(2*(c^2 - d^2))))/(
3*(c^2 - d^2))))/(4*d*(c + d))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{8a^2\left(-\frac{\frac{\left(12c^2-16cd+7d^2\right)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{32c^4+128c^3d+192c^2d^2+128c\,d^3+32d^4}-\frac{11\left(12c^2-16cd+7d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{96\left(c^3+3c^2d+3c\,d^2+d^3\right)}+\frac{\left(156c^2-272cd+83d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{96(c-d)\left(c^2+2cd+d^2\right)}-(2}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^4}\right.}{f}$
default	$\frac{8a^2\left(-\frac{\frac{\left(12c^2-16cd+7d^2\right)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{32c^4+128c^3d+192c^2d^2+128c\,d^3+32d^4}-\frac{11\left(12c^2-16cd+7d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{96\left(c^3+3c^2d+3c\,d^2+d^3\right)}+\frac{\left(156c^2-272cd+83d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{96(c-d)\left(c^2+2cd+d^2\right)}-(2}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^4}\right.}{f}$
risch	Expression too large to display

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBO
SE)
```

output
```
8/f*a^2*(-(1/32*(12*c^2-16*c*d+7*d^2)*(c-d)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3
+d^4)*tan(1/2*f*x+1/2*e)^7-11/96*(12*c^2-16*c*d+7*d^2)/(c^3+3*c^2*d+3*c*d^
2+d^3)*tan(1/2*f*x+1/2*e)^5+1/96*(156*c^2-272*c*d+83*d^2)/(c-d)/(c^2+2*c*d
+d^2)*tan(1/2*f*x+1/2*e)^3-1/32*(20*c^2-48*c*d+25*d^2)/(c+d)/(c^2-2*c*d+d^
2)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)
^4+1/32*(12*c^2-16*c*d+7*d^2)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+2*c*d
^5+d^6)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(257) = 514$.

Time = 0.26 (sec) , antiderivative size = 1908, normalized size of antiderivative = 6.91

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="f
ricas")
```

output

```
[1/48*(3*(12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^6 - 16*a^2
*c^5*d + 7*a^2*c^4*d^2)*cos(f*x + e)^4 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2
+ 7*a^2*c^3*d^3)*cos(f*x + e)^3 + 6*(12*a^2*c^4*d^2 - 16*a^2*c^3*d^3 + 7*a
^2*c^2*d^4)*cos(f*x + e)^2 + 4*(12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*
d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)
*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*
c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^2*c^5
*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^2*c*d^6 - 3
2*a^2*d^7 + (48*a^2*c^7 - 68*a^2*c^6*d - 64*a^2*c^5*d^2 + 73*a^2*c^4*d^3 +
 32*a^2*c^3*d^4 + a^2*c^2*d^5 - 16*a^2*c*d^6 - 6*a^2*d^7)*cos(f*x + e)^3 +
 (12*a^2*c^7 + 96*a^2*c^6*d - 239*a^2*c^5*d^2 - 64*a^2*c^4*d^3 + 271*a^2*c
^3*d^4 - 16*a^2*c^2*d^5 - 44*a^2*c*d^6 - 16*a^2*d^7)*cos(f*x + e)^2 + (8*a
^2*c^6*d + 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 221*a^2*c^2
*d^5 - 80*a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e))*sin(f*x + e))/((c^12 + 2*c
^11*d - 2*c^10*d^2 - 6*c^9*d^3 + 6*c^7*d^5 + 2*c^6*d^6 - 2*c^5*d^7 - c^4*d
^8)*f*cos(f*x + e)^4 + 4*(c^11*d + 2*c^10*d^2 - 2*c^9*d^3 - 6*c^8*d^4 + 6*
c^6*d^6 + 2*c^5*d^7 - 2*c^4*d^8 - c^3*d^9)*f*cos(f*x + e)^3 + 6*(c^10*d^2
+ 2*c^9*d^3 - 2*c^8*d^4 - 6*c^7*d^5 + 6*c^5*d^7 + 2*c^4*d^8 - 2*c^3*d^9 -
c^2*d^10)*f*cos(f*x + e)^2 + 4*(c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 - 6*c^6*d^
6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e) + (c^8*...
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$$

$$= a^2\left(\int \frac{\sec(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5}\right.$$

$$+\int \frac{2\sec^2(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5}$$

$$+\int \frac{\sec^3(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**5,x)
```

output
```
a**2*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*s
ec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 +
 d**5*sec(e + f*x)**5), x) + Integral(2*sec(e + f*x)**2/(c**5 + 5*c**4*d*s
ec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3
+ 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(sec(e +
f*x)**3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*
c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**
5), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(257) = 514$.

Time = 0.30 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="g
iac")
```

output
```
1/12*(3*(12*a^2*c^2 - 16*a^2*c*d + 7*a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi +
 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
1/2*e))/sqrt(-c^2 + d^2)))/((c^6 + 2*c^5*d - c^4*d^2 - 4*c^3*d^3 - c^2*d^4
 + 2*c*d^5 + d^6)*sqrt(-c^2 + d^2)) - (36*a^2*c^5*tan(1/2*f*x + 1/2*e)^7 -
 156*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^7 + 273*a^2*c^3*d^2*tan(1/2*f*x + 1/2*
e)^7 - 243*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^7 + 111*a^2*c*d^4*tan(1/2*f*x
+ 1/2*e)^7 - 21*a^2*d^5*tan(1/2*f*x + 1/2*e)^7 - 132*a^2*c^5*tan(1/2*f*x +
 1/2*e)^5 + 308*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 121*a^2*c^3*d^2*tan(1/2
*f*x + 1/2*e)^5 - 231*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 253*a^2*c*d^4*t
an(1/2*f*x + 1/2*e)^5 - 77*a^2*d^5*tan(1/2*f*x + 1/2*e)^5 + 156*a^2*c^5*ta
n(1/2*f*x + 1/2*e)^3 - 116*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 345*a^2*c^3*
d^2*tan(1/2*f*x + 1/2*e)^3 + 199*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 189*
a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 83*a^2*d^5*tan(1/2*f*x + 1/2*e)^3 - 60*
a^2*c^5*tan(1/2*f*x + 1/2*e) - 36*a^2*c^4*d*tan(1/2*f*x + 1/2*e) + 177*a^2
*c^3*d^2*tan(1/2*f*x + 1/2*e) + 147*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 81*
a^2*c*d^4*tan(1/2*f*x + 1/2*e) - 75*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 +
2*c^5*d - c^4*d^2 - 4*c^3*d^3 - c^2*d^4 + 2*c*d^5 + d^6)*(c*tan(1/2*f*x +
1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^4))/f
```

Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.59

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$$

$$= \frac{\frac{11\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(12a^2c^2-16a^2cd+7a^2d^2\right)}{12(c+d)^3} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(12a^2c^3-28a^2c^2d-\right.}{4(c+d)^4}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(6c^4-12c^2d^2+6d^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(-4c^4-8c^3d+8cd^3+4d^4\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}$$

$$+\frac{a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2c-2d)\left(c^2-2cd+d^2\right)}{2\sqrt{c+d}\,(c-d)^{5/2}}\right)\left(12c^2-16cd+7d^2\right)}{4f(c+d)^{9/2}(c-d)^{5/2}}$$

input
```
int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^5),x)
```

output

```
((11*tan(e/2 + (f*x)/2)^5*(12*a^2*c^2 + 7*a^2*d^2 - 16*a^2*c*d))/(12*(c +
d)^3) - (tan(e/2 + (f*x)/2)^7*(12*a^2*c^3 - 7*a^2*d^3 + 23*a^2*c*d^2 - 28*
a^2*c^2*d))/(4*(c + d)^4) - (a^2*tan(e/2 + (f*x)/2)^3*(156*c^2 - 272*c*d +
 83*d^2))/(12*(c + d)^2*(c - d)) + (a^2*tan(e/2 + (f*x)/2)*(20*c^2 - 48*c*
d + 25*d^2))/(4*(c + d)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f*x)/2)^4*(6*
c^4 + 6*d^4 - 12*c^2*d^2) + tan(e/2 + (f*x)/2)^2*(8*c*d^3 - 8*c^3*d - 4*c^
4 + 4*d^4) - tan(e/2 + (f*x)/2)^6*(8*c*d^3 - 8*c^3*d + 4*c^4 - 4*d^4) + ta
n(e/2 + (f*x)/2)^8*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2) + 4*c*d^3 +
 4*c^3*d + c^4 + d^4 + 6*c^2*d^2)) + (a^2*atanh((tan(e/2 + (f*x)/2)*(2*c -
 2*d)*(c^2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*(c - d)^(5/2)))*(12*c^2 - 16*c
*d + 7*d^2))/(4*f*(c + d)^(9/2)*(c - d)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2582, normalized size of antiderivative = 9.36

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x)
```

output

```
(a**2*(288*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2
)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*c**5*d - 384*sqrt(
 - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**
2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*c**4*d**2 + 168*sqrt( - c**2 + d**
2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*co
s(e + f*x)*sin(e + f*x)**2*c**3*d**3 - 288*sqrt( - c**2 + d**2)*atan((tan(
(e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c*
*5*d + 384*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2
)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c**4*d**2 - 456*sqrt( - c**2 + d**
2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*co
s(e + f*x)*c**3*d**3 + 384*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
 tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c**2*d**4 - 168*sq
rt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( -
c**2 + d**2))*cos(e + f*x)*c*d**5 - 72*sqrt( - c**2 + d**2)*atan((tan((e +
 f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**4*c**
6 + 96*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)
/sqrt( - c**2 + d**2))*sin(e + f*x)**4*c**5*d - 42*sqrt( - c**2 + d**2)*at
an((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e +
 f*x)**4*c**4*d**2 + 144*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - t
an((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**6 - 192*sqr...
```

3.202 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 31, antiderivative size = 288

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx \\
&= \frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\operatorname{arctanh}(\sin(e+fx))}{16f} \\
&\quad + \frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f} \\
&\quad + \frac{(40c^3+90c^2d+78cd^2+23d^3)\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{48f} \\
&\quad + \frac{a(3c+8d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{30f} \\
&\quad + \frac{a(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} \\
&\quad + \frac{a(a+a\sec(e+fx))^2\,(2(4c^3+74c^2d+66cd^2+21d^3)+d(6c^2+62cd+31d^2)\sec(e+fx))\tan(e+fx}{120f}
\end{aligned}$$

output
```
1/16*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)*arctanh(sin(f*x+e))/f+1/16*a^3*
(40*c^3+90*c^2*d+78*c*d^2+23*d^3)*tan(f*x+e)/f+1/48*(40*c^3+90*c^2*d+78*c*
d^2+23*d^3)*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f+1/30*a*(3*c+8*d)*(a+a*sec(f*
x+e))^2*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/6*a*(a+a*sec(f*x+e))^2*(c+d*sec(
f*x+e))^3*tan(f*x+e)/f+1/120*a*(a+a*sec(f*x+e))^2*(8*c^3+148*c^2*d+132*c*d
^2+42*d^3+d*(6*c^2+62*c*d+31*d^2)*sec(f*x+e))*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 9.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.63

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$$
$$= \frac{a^3\left(240c^3\coth^{-1}(\sin(e+fx))+15\left(24c^3+90c^2d+78cd^2+23d^3\right)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\,(1}{}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

output
```
(a^3*(240*c^3*ArcCoth[Sin[e + f*x]] + 15*(24*c^3 + 90*c^2*d + 78*c*d^2 + 2
3*d^3)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(15*(24*c^3 + 90*c^2*d + 78*c*
d^2 + 23*d^3)*Sec[e + f*x] + 10*d*(18*c^2 + 54*c*d + 23*d^2)*Sec[e + f*x]^
3 + 40*d^3*Sec[e + f*x]^5 + 16*(c + d)*(60*(c + d)^2 + 5*(c^2 + 8*c*d + 7*
d^2)*Tan[e + f*x]^2 + 9*d^2*Tan[e + f*x]^4))))/(240*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4475, 111, 25, 27, 164, 60, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))^3\,dx$$

$$\Big\downarrow 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \; 4475$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 111$$

$$-\frac{a^2\tan(e+fx)\Bigg(-\frac{\int -\frac{a^2(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))\left(6c^2+3dc+2d^2+d(8c+3d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{6a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+}{6}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\int \frac{a^2(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))\left(6c^2+3dc+2d^2+d(8c+3d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{6a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)}{6a^2}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\int \frac{(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))(6c^2+3dc+2d^2+d(8c+3d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{d\sqrt{a-a\sec(e+fx)}}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 164$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\left(\frac{3}{20}\left(40c^3+90c^2d+78cd^2+23d^3\right)\int \frac{(\sec(e+fx)a+a)^{5/2}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+}{}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(}$$

$$\Big\downarrow \; 60$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{6}\left(\frac{3}{20}\left(40c^3+90c^2d+78cd^2+23d^3\right)\left(\frac{5}{3}a\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{\sqrt{a-a\sec(e+fx)}(a\sec}{3a}\right.\right.\right.}{f\sqrt{a-a}}$$

$$\Big\downarrow \; 60$$

$$-\frac{a^2\tan(e+fx)\bigg(\frac{1}{6}\bigg(\frac{3}{20}\left(40c^3+90c^2d+78cd^2+23d^3\right)\bigg(\frac{5}{3}a\bigg(\frac{3}{2}a\int\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{\sqrt{a-a\sec(e+fx)}(a}{2a}}{}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a^2\tan(e+fx)\bigg(\frac{1}{6}\bigg(\frac{3}{20}\left(40c^3+90c^2d+78cd^2+23d^3\right)\bigg(\frac{5}{3}a\bigg(\frac{3}{2}a\bigg(a\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)-}{}$$

$$\Big\downarrow \text{45}$$

$$-\frac{a^2\tan(e+fx)\bigg(\frac{1}{6}\bigg(\frac{3}{20}\left(40c^3+90c^2d+78cd^2+23d^3\right)\bigg(\frac{5}{3}a\bigg(\frac{3}{2}a\bigg(2a\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}-\frac{\sqrt{a-a}}{}}{}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a^2\tan(e+fx)\bigg(\frac{1}{6}\bigg(\frac{3}{20}\left(40c^3+90c^2d+78cd^2+23d^3\right)\bigg(\frac{5}{3}a\bigg(\frac{3}{2}a\bigg(-2\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a}}{a}}{}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^2*(-1/6*(d*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(7/2)*(c + d
*Sec[e + f*x])^2)/a^2 + (-1/20*(d*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e +
f*x])^(7/2)*(70*c^2 + 54*c*d + 19*d^2 + 4*d*(8*c + 3*d)*Sec[e + f*x]))/a^2
 + (3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*(-1/3*(Sqrt[a - a*Sec[e + f*
x]]*(a + a*Sec[e + f*x])^(5/2))/a + (5*a*(-1/2*(Sqrt[a - a*Sec[e + f*x]]*(
a + a*Sec[e + f*x])^(3/2))/a + (3*a*(-2*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sq
rt[a + a*Sec[e + f*x]]] - (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]])/a))/2))/3))/20)/6)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.23

method	result
norman	$-\frac{33a^3(40c^3+90c^2d+78c\,d^2+23d^3)\tan(\frac{fx}{2}+\frac{e}{2})^7}{20f}+\frac{17a^3(40c^3+90c^2d+78c\,d^2+23d^3)\tan(\frac{fx}{2}+\frac{e}{2})^9}{24f}-\frac{a^3(40c^3+90c^2d+78c\,d^2+}{8f}$
parallelrisch	$6\Big(-\frac{25(\frac{2}{3}+\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4fx+4e)}{5}+\cos(2fx+2e))(c^3+\frac{9}{4}c^2d+\frac{39}{20}c\,d^2+\frac{23}{40}d^3)\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{4}+\frac{25(\frac{2}{3}+\frac{\cos(6fx+6e}{15}}{}$
parts	$-\frac{(3a^3c\,d^2+3a^3d^3)(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15})\tan(fx+e)}{f}+\frac{(3a^3c^3+3a^3c^2d)\tan(fx+e)}{f}+\frac{(3a^3c^2d+9a^3}{}$
derivativedivides	$\frac{a^3c^3\ln(\sec(fx+e)+\tan(fx+e))+3a^3c^2d\tan(fx+e)+3a^3c\,d^2(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2})-a^3d^3(}{}$
default	$\frac{a^3c^3\ln(\sec(fx+e)+\tan(fx+e))+3a^3c^2d\tan(fx+e)+3a^3c\,d^2(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2})-a^3d^3(}{}$
risch	$-\frac{ia^3(-2160c^2d-1824c\,d^2-544d^3-880c^3-8800c^3e^{6i(fx+e)}+2250d^3e^{7i(fx+e)}-4560c^3e^{2i(fx+e)}-3264d^3e^{2i(fx+e)}-}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
(-33/20*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)/f*tan(1/2*f*x+1/2*e)^7+17/24
*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)/f*tan(1/2*f*x+1/2*e)^9-1/8*a^3*(40*
c^3+90*c^2*d+78*c*d^2+23*d^3)/f*tan(1/2*f*x+1/2*e)^11+1/8*a^3*(88*c^3+294*
c^2*d+306*c*d^2+105*d^3)/f*tan(1/2*f*x+1/2*e)+3/20*a^3*(520*c^3+1250*c^2*d
+998*c*d^2+323*d^3)/f*tan(1/2*f*x+1/2*e)^5-1/24*a^3*(1112*c^3+3078*c^2*d+2
514*c*d^2+633*d^3)/f*tan(1/2*f*x+1/2*e)^3)/(tan(1/2*f*x+1/2*e)^2-1)^6-1/16
*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)/f*ln(tan(1/2*f*x+1/2*e)-1)+1/16*a^3
*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)/f*ln(tan(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.17

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$$
$$= \frac{15\left(40\,a^3c^3+90\,a^3c^2d+78\,a^3cd^2+23\,a^3d^3\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\left(40\,a^3c^3+90\,a^3c^2d\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/480*(15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x
+ e)^6*log(sin(f*x + e) + 1) - 15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^
2 + 23*a^3*d^3)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(40*a^3*d^3 + 16
*(55*a^3*c^3 + 135*a^3*c^2*d + 114*a^3*c*d^2 + 34*a^3*d^3)*cos(f*x + e)^5
+ 15*(24*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^
4 + 16*(5*a^3*c^3 + 45*a^3*c^2*d + 57*a^3*c*d^2 + 17*a^3*d^3)*cos(f*x + e)
^3 + 10*(18*a^3*c^2*d + 54*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^2 + 144*(a
^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx \\
&= a^3\bigg(\int c^3\sec(e+fx)\,dx+\int 3c^3\sec^2(e+fx)\,dx+\int 3c^3\sec^3(e+fx)\,dx \\
&\quad+\int c^3\sec^4(e+fx)\,dx+\int d^3\sec^4(e+fx)\,dx+\int 3d^3\sec^5(e+fx)\,dx \\
&\quad+\int 3d^3\sec^6(e+fx)\,dx+\int d^3\sec^7(e+fx)\,dx+\int 3cd^2\sec^3(e+fx)\,dx \\
&\quad+\int 9cd^2\sec^4(e+fx)\,dx+\int 9cd^2\sec^5(e+fx)\,dx+\int 3cd^2\sec^6(e+fx)\,dx \\
&\quad+\int 3c^2d\sec^2(e+fx)\,dx+\int 9c^2d\sec^3(e+fx)\,dx+\int 9c^2d\sec^4(e+fx)\,dx \\
&\quad+\int 3c^2d\sec^5(e+fx)\,dx\bigg)
\end{aligned}
$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**3,x)
```

output
```
a**3*(Integral(c**3*sec(e + f*x), x) + Integral(3*c**3*sec(e + f*x)**2, x)
 + Integral(3*c**3*sec(e + f*x)**3, x) + Integral(c**3*sec(e + f*x)**4, x)
 + Integral(d**3*sec(e + f*x)**4, x) + Integral(3*d**3*sec(e + f*x)**5, x)
 + Integral(3*d**3*sec(e + f*x)**6, x) + Integral(d**3*sec(e + f*x)**7, x)
 + Integral(3*c*d**2*sec(e + f*x)**3, x) + Integral(9*c*d**2*sec(e + f*x)*
*4, x) + Integral(9*c*d**2*sec(e + f*x)**5, x) + Integral(3*c*d**2*sec(e +
 f*x)**6, x) + Integral(3*c**2*d*sec(e + f*x)**2, x) + Integral(9*c**2*d*s
ec(e + f*x)**3, x) + Integral(9*c**2*d*sec(e + f*x)**4, x) + Integral(3*c*
*2*d*sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(273) = 546$.

Time = 0.05 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.43

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="m
axima")
```

output

```
1/480*(160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 + 1440*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a^3*c^2*d + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 +
 15*tan(f*x + e))*a^3*c*d^2 + 1440*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c
*d^2 + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*d^3
 + 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^3 - 5*a^3*d^3*(2*(15*sin(f*
x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*
x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f
*x + e) - 1)) - 90*a^3*c^2*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f
*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*
x + e) - 1)) - 270*a^3*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f
*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*
x + e) - 1)) - 90*a^3*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x
+ e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
 e) - 1)) - 360*a^3*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x
 + e) + 1) + log(sin(f*x + e) - 1)) - 1080*a^3*c^2*d*(2*sin(f*x + e)/(sin(
f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 360*a^3
*c*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(
sin(f*x + e) - 1)) + 480*a^3*c^3*log(sec(f*x + e) + tan(f*x + e)) + 1440*a
^3*c^3*tan(f*x + e) + 1440*a^3*c^2*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(273) = 546$.

Time = 0.26 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.03

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="g
iac")
```

output

```
1/240*(15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*log(abs(
tan(1/2*f*x + 1/2*e) + 1)) - 15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2
+ 23*a^3*d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(600*a^3*c^3*tan(1/2*
f*x + 1/2*e)^11 + 1350*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^11 + 1170*a^3*c*d^2*
tan(1/2*f*x + 1/2*e)^11 + 345*a^3*d^3*tan(1/2*f*x + 1/2*e)^11 - 3400*a^3*c
^3*tan(1/2*f*x + 1/2*e)^9 - 7650*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^9 - 6630*a
^3*c*d^2*tan(1/2*f*x + 1/2*e)^9 - 1955*a^3*d^3*tan(1/2*f*x + 1/2*e)^9 + 79
20*a^3*c^3*tan(1/2*f*x + 1/2*e)^7 + 17820*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^7
 + 15444*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 4554*a^3*d^3*tan(1/2*f*x + 1/2
*e)^7 - 9360*a^3*c^3*tan(1/2*f*x + 1/2*e)^5 - 22500*a^3*c^2*d*tan(1/2*f*x
+ 1/2*e)^5 - 17964*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 5814*a^3*d^3*tan(1/2
*f*x + 1/2*e)^5 + 5560*a^3*c^3*tan(1/2*f*x + 1/2*e)^3 + 15390*a^3*c^2*d*ta
n(1/2*f*x + 1/2*e)^3 + 12570*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 3165*a^3*d
^3*tan(1/2*f*x + 1/2*e)^3 - 1320*a^3*c^3*tan(1/2*f*x + 1/2*e) - 4410*a^3*c
^2*d*tan(1/2*f*x + 1/2*e) - 4590*a^3*c*d^2*tan(1/2*f*x + 1/2*e) - 1575*a^3
*d^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 15.45 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.43

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$$
$$= \frac{a^3 \operatorname{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(40\,c^3+90\,c^2\,d+78\,c\,d^2+23\,d^3\right)}{4\left(10\,c^3+\frac{45\,c^2\,d}{2}+\frac{39\,c\,d^2}{2}+\frac{23\,d^3}{4}\right)}\right)\left(40\,c^3+90\,c^2\,d+78\,c\,d^2+23\,d^3\right)}{8\,f}$$
$$- \frac{\left(5\,a^3\,c^3+\frac{45\,a^3\,c^2\,d}{4}+\frac{39\,a^3\,c\,d^2}{4}+\frac{23\,a^3\,d^3}{8}\right)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+\left(-\frac{85\,a^3\,c^3}{3}-\frac{255\,a^3\,c^2\,d}{4}-\frac{221\,a^3\,c\,d^2}{4}-\frac{391\,a^3\,d^3}{24}\right)\,\mathrm{t}}{}$$

input
```
int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

output
```
(a^3*atanh((tan(e/2 + (f*x)/2)*(78*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/(4
*((39*c*d^2)/2 + (45*c^2*d)/2 + 10*c^3 + (23*d^3)/4)))*(78*c*d^2 + 90*c^2*
d + 40*c^3 + 23*d^3))/(8*f) - (tan(e/2 + (f*x)/2)^11*(5*a^3*c^3 + (23*a^3*
d^3)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4) - tan(e/2 + (f*x)/2)^9*((85*
a^3*c^3)/3 + (391*a^3*d^3)/24 + (221*a^3*c*d^2)/4 + (255*a^3*c^2*d)/4) + t
an(e/2 + (f*x)/2)^3*((139*a^3*c^3)/3 + (211*a^3*d^3)/8 + (419*a^3*c*d^2)/4
 + (513*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^7*(66*a^3*c^3 + (759*a^3*d^3)/2
0 + (1287*a^3*c*d^2)/10 + (297*a^3*c^2*d)/2) - tan(e/2 + (f*x)/2)^5*(78*a^
3*c^3 + (969*a^3*d^3)/20 + (1497*a^3*c*d^2)/10 + (375*a^3*c^2*d)/2) - tan(
e/2 + (f*x)/2)*(11*a^3*c^3 + (105*a^3*d^3)/8 + (153*a^3*c*d^2)/4 + (147*a^
3*c^2*d)/4))/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan
(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + ta
n(e/2 + (f*x)/2)^12 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1173, normalized size of antiderivative = 4.07

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x)
```

output

```
(a**3*( - 880*cos(e + f*x)*sin(e + f*x)**5*c**3 - 2160*cos(e + f*x)*sin(e
+ f*x)**5*c**2*d - 1824*cos(e + f*x)*sin(e + f*x)**5*c*d**2 - 544*cos(e +
f*x)*sin(e + f*x)**5*d**3 + 1840*cos(e + f*x)*sin(e + f*x)**3*c**3 + 5040*
cos(e + f*x)*sin(e + f*x)**3*c**2*d + 4560*cos(e + f*x)*sin(e + f*x)**3*c*
d**2 + 1360*cos(e + f*x)*sin(e + f*x)**3*d**3 - 960*cos(e + f*x)*sin(e + f
*x)*c**3 - 2880*cos(e + f*x)*sin(e + f*x)*c**2*d - 2880*cos(e + f*x)*sin(e
 + f*x)*c*d**2 - 960*cos(e + f*x)*sin(e + f*x)*d**3 - 600*log(tan((e + f*x
)/2) - 1)*sin(e + f*x)**6*c**3 - 1350*log(tan((e + f*x)/2) - 1)*sin(e + f*
x)**6*c**2*d - 1170*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6*c*d**2 - 345
*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6*d**3 + 1800*log(tan((e + f*x)/2
) - 1)*sin(e + f*x)**4*c**3 + 4050*log(tan((e + f*x)/2) - 1)*sin(e + f*x)*
*4*c**2*d + 3510*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c*d**2 + 1035*l
og(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*d**3 - 1800*log(tan((e + f*x)/2)
- 1)*sin(e + f*x)**2*c**3 - 4050*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2
*c**2*d - 3510*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c*d**2 - 1035*log
(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*d**3 + 600*log(tan((e + f*x)/2) - 1
)*c**3 + 1350*log(tan((e + f*x)/2) - 1)*c**2*d + 1170*log(tan((e + f*x)/2)
 - 1)*c*d**2 + 345*log(tan((e + f*x)/2) - 1)*d**3 + 600*log(tan((e + f*x)/
2) + 1)*sin(e + f*x)**6*c**3 + 1350*log(tan((e + f*x)/2) + 1)*sin(e + f*x)
**6*c**2*d + 1170*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**6*c*d**2 + 34...
```

3.203 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 257

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx \\
&= \frac{a^3(20c^2+30cd+13d^2)\operatorname{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{a^3(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)\tan(e+fx)}{30d^2f} \\
&\quad + \frac{a^3(4c^3-30c^2d+146cd^2+195d^3)\sec(e+fx)\tan(e+fx)}{120df} \\
&\quad + \frac{a^3(2c^2-15cd+76d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60d^2f} \\
&\quad - \frac{a^3(2c-11d)(c+d\sec(e+fx))^3\tan(e+fx)}{20d^2f} \\
&\quad + \frac{(a^3+a^3\sec(e+fx))\,(c+d\sec(e+fx))^3\tan(e+fx)}{5df}
\end{aligned}$$

output
```
1/8*a^3*(20*c^2+30*c*d+13*d^2)*arctanh(sin(f*x+e))/f+1/30*a^3*(2*c^4-15*c^
3*d+72*c^2*d^2+180*c*d^3+76*d^4)*tan(f*x+e)/d^2/f+1/120*a^3*(4*c^3-30*c^2*
d+146*c*d^2+195*d^3)*sec(f*x+e)*tan(f*x+e)/d/f+1/60*a^3*(2*c^2-15*c*d+76*d
^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/d^2/f-1/20*a^3*(2*c-11*d)*(c+d*sec(f*x+e
))^3*tan(f*x+e)/d^2/f+1/5*(a^3+a^3*sec(f*x+e))*(c+d*sec(f*x+e))^3*tan(f*x+
e)/d/f
```

Mathematica [A] (verified)

Time = 3.82 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.55

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$
$$= \frac{a^3\left(120c^2\coth^{-1}(\sin(e+fx))+15(12c^2+30cd+13d^2)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(15(12c^2+\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

output
```
(a^3*(120*c^2*ArcCoth[Sin[e + f*x]] + 15*(12*c^2 + 30*c*d + 13*d^2)*ArcTan
h[Sin[e + f*x]] + Tan[e + f*x]*(15*(12*c^2 + 30*c*d + 13*d^2)*Sec[e + f*x]
 + 30*d*(2*c + 3*d)*Sec[e + f*x]^3 + 8*(60*(c + d)^2 + 5*(c^2 + 6*c*d + 5*
d^2)*Tan[e + f*x]^2 + 3*d^2*Tan[e + f*x]^4))))/(120*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4475, 101, 25, 27, 90, 60, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))^2\,dx$$

$$\Big\downarrow 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \ 4475$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 101$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2(\sec(e+fx)a+a)^{5/2}\left(5c^2+3dc+d^2+3d(2c+d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{7/2}(c+}{5a^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(\sec(e+fx)a+a)^{5/2}\left(5c^2+3dc+d^2+3d(2c+d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a^2}-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{7/2}(c+d\sec}{5a^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{5}\int \frac{(\sec(e+fx)a+a)^{5/2}(5c^2+3dc+d^2+3d(2c+d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{7}}{5a^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 90$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{5}\left(\frac{1}{4}\left(20c^2+30cd+13d^2\right)\int \frac{(\sec(e+fx)a+a)^{5/2}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{3d(2c+d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a}{4a^2}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 60$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{5}\left(\frac{1}{4}\left(20c^2+30cd+13d^2\right)\left(\frac{5}{3}a\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{5/}}{3a}\right.\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(}}$$

$$\Big\downarrow \ 60$$

$$-\frac{a^2\tan(e+fx)\left(\frac{1}{5}\left(\frac{1}{4}\left(20c^2+30cd+13d^2\right)\left(\frac{5}{3}a\left(\frac{3}{2}a\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)}{2a}\right.\right.\right.\right.}{f\sqrt{a-a\,\mathrm{se}}}$$

$$\Big\downarrow 60$$

$$-\frac{a^2\tan(e+fx)\,\Bigg(\frac{1}{5}\bigg(\frac{1}{4}\left(20c^2+30cd+13d^2\right)\bigg(\frac{5}{3}a\bigg(\frac{3}{2}a\bigg(a\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)-\frac{\sqrt{a-a\sec(e+}}{}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\,\Bigg(\frac{1}{5}\Bigg(\frac{1}{4}\left(20c^2+30cd+13d^2\right)\Bigg(\frac{5}{3}a\Bigg(\frac{3}{2}a\Bigg(2a\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{}}{a}}{}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\,\Bigg(\frac{1}{5}\bigg(\frac{1}{4}\left(20c^2+30cd+13d^2\right)\bigg(\frac{5}{3}a\bigg(\frac{3}{2}a\bigg(-2\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a}}{}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

output

```
-((a^2*(-1/5*(d*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(7/2)*(c + d
*Sec[e + f*x]))/a^2 + ((-3*d*(2*c + d)*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec
[e + f*x])^(7/2))/(4*a^2) + ((20*c^2 + 30*c*d + 13*d^2)*(-1/3*(Sqrt[a - a*
Sec[e + f*x]]*(a + a*Sec[e + f*x])^(5/2))/a + (5*a*(-1/2*(Sqrt[a - a*Sec[e
 + f*x]]*(a + a*Sec[e + f*x])^(3/2))/a + (3*a*(-2*ArcTan[Sqrt[a - a*Sec[e
+ f*x]]/Sqrt[a + a*Sec[e + f*x]]] - (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]])/a))/2))/3))/4)/5)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00

method	result
norman	$\frac{-\frac{32a^{3}(20c^{2}+30cd+13d^{2})\tan(\frac{fx}{2}+\frac{e}{2})^{5}}{15f}+\frac{7a^{3}(20c^{2}+30cd+13d^{2})\tan(\frac{fx}{2}+\frac{e}{2})^{7}}{6f}-\frac{a^{3}(20c^{2}+30cd+13d^{2})\tan(\frac{fx}{2}+\frac{e}{2})^{9}}{4f}-a^{3}(44}{(\tan(\frac{fx}{2}+\frac{e}{2})^{2}-1)^{5}}$
parts	$\frac{(2a^{3}cd+3a^{3}d^{2})(-(-\frac{\sec(fx+e)^{3}}{4}-\frac{3\sec(fx+e)}{8})\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8})}{f}+\frac{(3a^{3}c^{2}+2a^{3}cd)\tan(fx}{f}$
parallelrisch	$26a^{3}(-\frac{75(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+3e)}{2}+\cos(fx+e))(c^{2}+\frac{3}{2}cd+\frac{13}{20}d^{2})\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{26}+75(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+3e)}{2}+$
derivativedivides	$a^{3}c^{2}\ln(\sec(fx+e)+\tan(fx+e))+2a^{3}cd\tan(fx+e)+a^{3}d^{2}(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2})+3a^{3}c^{2}\tan$
default	$a^{3}c^{2}\ln(\sec(fx+e)+\tan(fx+e))+2a^{3}cd\tan(fx+e)+a^{3}d^{2}(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2})+3a^{3}c^{2}\tan$
risch	$-ia^{3}(-720cd-304d^{2}-440c^{2}+450cd\,e^{9i(fx+e)}-240cd\,e^{8i(fx+e)}-720d^{2}e^{6i(fx+e)}-360c^{2}e^{8i(fx+e)}+180c^{2}e^{9i(fx+e)}+$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
(-32/15*a^3*(20*c^2+30*c*d+13*d^2)/f*tan(1/2*f*x+1/2*e)^5+7/6*a^3*(20*c^2+
30*c*d+13*d^2)/f*tan(1/2*f*x+1/2*e)^7-1/4*a^3*(20*c^2+30*c*d+13*d^2)/f*tan
(1/2*f*x+1/2*e)^9-1/4*a^3*(44*c^2+98*c*d+51*d^2)/f*tan(1/2*f*x+1/2*e)+1/6*
a^3*(212*c^2+366*c*d+133*d^2)/f*tan(1/2*f*x+1/2*e)^3)/(tan(1/2*f*x+1/2*e)^
2-1)^5-1/8*a^3*(20*c^2+30*c*d+13*d^2)/f*ln(tan(1/2*f*x+1/2*e)-1)+1/8*a^3*(
20*c^2+30*c*d+13*d^2)/f*ln(tan(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.95

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$
$$= \frac{15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\mathrm{co}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
1/240*(15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log(sin(f*
x + e) + 1) - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log
(-sin(f*x + e) + 1) + 2*(24*a^3*d^2 + 8*(55*a^3*c^2 + 90*a^3*c*d + 38*a^3*
d^2)*cos(f*x + e)^4 + 15*(12*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x +
e)^3 + 8*(5*a^3*c^2 + 30*a^3*c*d + 19*a^3*d^2)*cos(f*x + e)^2 + 30*(2*a^3*
c*d + 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx \\
&= a^3\left(\int c^2\sec(e+fx)\,dx+\int 3c^2\sec^2(e+fx)\,dx+\int 3c^2\sec^3(e+fx)\,dx\right. \\
&\quad+\int c^2\sec^4(e+fx)\,dx+\int d^2\sec^3(e+fx)\,dx+\int 3d^2\sec^4(e+fx)\,dx \\
&\quad+\int 3d^2\sec^5(e+fx)\,dx+\int d^2\sec^6(e+fx)\,dx+\int 2cd\sec^2(e+fx)\,dx \\
&\quad\left.+\int 6cd\sec^3(e+fx)\,dx+\int 6cd\sec^4(e+fx)\,dx+\int 2cd\sec^5(e+fx)\,dx\right)
\end{aligned}
$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**2,x)
```

output

```
a**3*(Integral(c**2*sec(e + f*x), x) + Integral(3*c**2*sec(e + f*x)**2, x)
 + Integral(3*c**2*sec(e + f*x)**3, x) + Integral(c**2*sec(e + f*x)**4, x)
 + Integral(d**2*sec(e + f*x)**3, x) + Integral(3*d**2*sec(e + f*x)**4, x)
 + Integral(3*d**2*sec(e + f*x)**5, x) + Integral(d**2*sec(e + f*x)**6, x)
 + Integral(2*c*d*sec(e + f*x)**2, x) + Integral(6*c*d*sec(e + f*x)**3, x)
 + Integral(6*c*d*sec(e + f*x)**4, x) + Integral(2*c*d*sec(e + f*x)**5, x)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.79

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$

$$= \frac{80\,(\tan(fx+e)^3+3\,\tan(fx+e))a^3c^2+480\,(\tan(fx+e)^3+3\,\tan(fx+e))a^3cd+16\,(3\,\tan(fx+}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
1/240*(80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 480*(tan(f*x + e)^3
+ 3*tan(f*x + e))*a^3*c*d + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*
tan(f*x + e))*a^3*d^2 + 240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^2 - 30
*a^3*c*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*
x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 45*a^
3*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x +
 e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 180*a^3*
c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin
(f*x + e) - 1)) - 360*a^3*c*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(s
in(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 60*a^3*d^2*(2*sin(f*x + e)/(si
n(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a
^3*c^2*log(sec(f*x + e) + tan(f*x + e)) + 720*a^3*c^2*tan(f*x + e) + 480*a
^3*c*d*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.46

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$

$$= \frac{15\,(20\,a^3c^2+30\,a^3cd+13\,a^3d^2)\log\big(\big|\tan\big(\tfrac{1}{2}\,fx+\tfrac{1}{2}\,e\big)+1\big|\big)-15\,(20\,a^3c^2+30\,a^3cd+13\,a^3d^2)\log\big(\big|\tan}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/120*(15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2
*e) + 1)) - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*log(abs(tan(1/2*f*x
+ 1/2*e) - 1)) - 2*(300*a^3*c^2*tan(1/2*f*x + 1/2*e)^9 + 450*a^3*c*d*tan(1
/2*f*x + 1/2*e)^9 + 195*a^3*d^2*tan(1/2*f*x + 1/2*e)^9 - 1400*a^3*c^2*tan(
1/2*f*x + 1/2*e)^7 - 2100*a^3*c*d*tan(1/2*f*x + 1/2*e)^7 - 910*a^3*d^2*tan
(1/2*f*x + 1/2*e)^7 + 2560*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 3840*a^3*c*d*t
an(1/2*f*x + 1/2*e)^5 + 1664*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 2120*a^3*c^2
*tan(1/2*f*x + 1/2*e)^3 - 3660*a^3*c*d*tan(1/2*f*x + 1/2*e)^3 - 1330*a^3*d
^2*tan(1/2*f*x + 1/2*e)^3 + 660*a^3*c^2*tan(1/2*f*x + 1/2*e) + 1470*a^3*c*
d*tan(1/2*f*x + 1/2*e) + 765*a^3*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x +
1/2*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 15.12 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.12

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$

$$= \frac{a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(20\,c^2+30\,c\,d+13\,d^2)}{2\,\left(10\,c^2+15\,c\,d+\frac{13\,d^2}{2}\right)}\right)\,(20\,c^2+30\,c\,d+13\,d^2)}{4\,f}$$

$$-\frac{\left(5\,a^3\,c^2+\frac{15\,a^3\,c\,d}{2}+\frac{13\,a^3\,d^2}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+\left(-\frac{70\,a^3\,c^2}{3}-35\,a^3\,c\,d-\frac{91\,a^3\,d^2}{6}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(\frac{128\,a^3\,c^2}{3}\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.\right.}$$

input
```
int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

output
```
(a^3*atanh((tan(e/2 + (f*x)/2)*(30*c*d + 20*c^2 + 13*d^2))/(2*(15*c*d + 10
*c^2 + (13*d^2)/2)))*(30*c*d + 20*c^2 + 13*d^2))/(4*f) - (tan(e/2 + (f*x)/
2)*(11*a^3*c^2 + (51*a^3*d^2)/4 + (49*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^9*(
5*a^3*c^2 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^7*((70*a
^3*c^2)/3 + (91*a^3*d^2)/6 + 35*a^3*c*d) - tan(e/2 + (f*x)/2)^3*((106*a^3*
c^2)/3 + (133*a^3*d^2)/6 + 61*a^3*c*d) + tan(e/2 + (f*x)/2)^5*((128*a^3*c^
2)/3 + (416*a^3*d^2)/15 + 64*a^3*c*d))/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan
(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan
(e/2 + (f*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.95

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x)
```

output

```
(a**3*( - 300*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c**2
- 450*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c*d - 195*cos
(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*d**2 + 600*cos(e + f*x
)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c**2 + 900*cos(e + f*x)*log(ta
n((e + f*x)/2) - 1)*sin(e + f*x)**2*c*d + 390*cos(e + f*x)*log(tan((e + f*
x)/2) - 1)*sin(e + f*x)**2*d**2 - 300*cos(e + f*x)*log(tan((e + f*x)/2) -
1)*c**2 - 450*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c*d - 195*cos(e + f*x
)*log(tan((e + f*x)/2) - 1)*d**2 + 300*cos(e + f*x)*log(tan((e + f*x)/2) +
 1)*sin(e + f*x)**4*c**2 + 450*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(
e + f*x)**4*c*d + 195*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)*
*4*d**2 - 600*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*c**2
- 900*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*c*d - 390*cos
(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*d**2 + 300*cos(e + f*x
)*log(tan((e + f*x)/2) + 1)*c**2 + 450*cos(e + f*x)*log(tan((e + f*x)/2) +
 1)*c*d + 195*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*d**2 - 180*cos(e + f*
x)*sin(e + f*x)**3*c**2 - 450*cos(e + f*x)*sin(e + f*x)**3*c*d - 195*cos(e
 + f*x)*sin(e + f*x)**3*d**2 + 180*cos(e + f*x)*sin(e + f*x)*c**2 + 510*co
s(e + f*x)*sin(e + f*x)*c*d + 285*cos(e + f*x)*sin(e + f*x)*d**2 + 440*sin
(e + f*x)**5*c**2 + 720*sin(e + f*x)**5*c*d + 304*sin(e + f*x)**5*d**2 - 9
20*sin(e + f*x)**3*c**2 - 1680*sin(e + f*x)**3*c*d - 760*sin(e + f*x)**...
```

3.204 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 125

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx &= \frac{5a^3(4c+3d)\operatorname{arctanh}(\sin(e+fx))}{8f}+\frac{a^3(4c+3d)\tan(e+fx)}{f}\\&\quad+\frac{3a^3(4c+3d)\sec(e+fx)\tan(e+fx)}{8f}\\&\quad+\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{a^3(4c+3d)\tan^3(e+fx)}{12f}\end{aligned}$$

output
```
5/8*a^3*(4*c+3*d)*arctanh(sin(f*x+e))/f+a^3*(4*c+3*d)*tan(f*x+e)/f+3/8*a^3
*(4*c+3*d)*sec(f*x+e)*tan(f*x+e)/f+1/4*d*(a+a*sec(f*x+e))^3*tan(f*x+e)/f+1
/12*a^3*(4*c+3*d)*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$
$$=\frac{a^3\left(24c\coth^{-1}(\sin(e+fx))+9(4c+5d)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(96(c+d)+9(4c+5d)\,\mathrm{se}\right.\right.}{24f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]
```

output
```
(a^3*(24*c*ArcCoth[Sin[e + f*x]] + 9*(4*c + 5*d)*ArcTanh[Sin[e + f*x]] + T
an[e + f*x]*(96*(c + d) + 9*(4*c + 5*d)*Sec[e + f*x] + 6*d*Sec[e + f*x]^3
+ 8*(c + 3*d)*Tan[e + f*x]^2)))/(24*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3042, 4489, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4489}$$

$$\frac{1}{4}(4c+3d)\int \sec(e+fx)(\sec(e+fx)a+a)^3dx+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4c+3d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3dx+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3}{4f}$$

$$\Big\downarrow \text{ 4278}$$

$$\frac{1}{4}(4c+3d)\int \left(a^3\sec^4(e+fx)+3a^3\sec^3(e+fx)+3a^3\sec^2(e+fx)+a^3\sec(e+fx)\right)dx+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3}{4f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}(4c+3d)\left(\frac{5a^3\text{arctanh}(\sin(e+fx))}{2f}+\frac{a^3\tan^3(e+fx)}{3f}+\frac{4a^3\tan(e+fx)}{f}+\frac{3a^3\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3}{4f}$$

input

```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]
```

output

```
(d*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(4*f) + ((4*c + 3*d)*((5*a^3*ArcTa
nh[Sin[e + f*x]])/(2*f) + (4*a^3*Tan[e + f*x])/f + (3*a^3*Sec[e + f*x]*Tan
[e + f*x])/(2*f) + (a^3*Tan[e + f*x]^3)/(3*f)))/4
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

method	result
norman	$\frac{-\frac{73a^3(4c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{12f}+\frac{55a^3(4c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}-\frac{5a^3(4c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{a^3(49d+44c)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{5a^3(4c}{}$
parallelrisch	$\frac{26\left(-\frac{15\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(c+\frac{3d}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{13}+\frac{15\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(c+\frac{3d}{4}\right)\ln\left(\tan\left(\frac{f}{}\right)\right)}{13}\right)}{3f(\cos(4fx+4e)+4\cos(2}$
parts	$-\frac{(a^3c+3a^3d)\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{(3a^3c+a^3d)\tan(fx+e)}{f}+\frac{(3a^3c+3a^3d)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\ln\right)}{f}$
derivativedivides	$\frac{a^3c\ln(\sec(fx+e)+\tan(fx+e))+a^3d\tan(fx+e)+3a^3c\tan(fx+e)+3a^3d\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e)}{2}\right)}{}$
default	$\frac{a^3c\ln(\sec(fx+e)+\tan(fx+e))+a^3d\tan(fx+e)+3a^3c\tan(fx+e)+3a^3d\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e)}{2}\right)}{}$
risch	$-\frac{ia^3\left(36c\,e^{7i(fx+e)}+45d\,e^{7i(fx+e)}-72c\,e^{6i(fx+e)}-24d\,e^{6i(fx+e)}+36c\,e^{5i(fx+e)}+69d\,e^{5i(fx+e)}-264c\,e^{4i(fx+e)}-216\right)}{12f\left(e^{2i(f}\right)}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
(-73/12*a^3*(4*c+3*d)/f*tan(1/2*f*x+1/2*e)^3+55/12*a^3*(4*c+3*d)/f*tan(1/2
*f*x+1/2*e)^5-5/4*a^3*(4*c+3*d)/f*tan(1/2*f*x+1/2*e)^7+1/4*a^3*(49*d+44*c)
/f*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2-1)^4-5/8*a^3*(4*c+3*d)/f*ln(t
an(1/2*f*x+1/2*e)-1)+5/8*a^3*(4*c+3*d)/f*ln(tan(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.29

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$
$$= \frac{15\left(4\,a^3c+3\,a^3d\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-15\left(4\,a^3c+3\,a^3d\right)\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+\right.\right.}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
1/48*(15*(4*a^3*c + 3*a^3*d)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*(4*
a^3*c + 3*a^3*d)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*(6*a^3*d + 8*(1
1*a^3*c + 9*a^3*d)*cos(f*x + e)^3 + 9*(4*a^3*c + 5*a^3*d)*cos(f*x + e)^2 +
 8*(a^3*c + 3*a^3*d)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx\\ = a^3\bigg(\int c\sec\left(e+fx\right)dx+\int 3c\sec^2\left(e+fx\right)dx+\int 3c\sec^3\left(e+fx\right)dx\\ +\int c\sec^4\left(e+fx\right)dx+\int d\sec^2\left(e+fx\right)dx+\int 3d\sec^3\left(e+fx\right)dx\\ +\int 3d\sec^4\left(e+fx\right)dx+\int d\sec^5\left(e+fx\right)dx\bigg)\end{aligned}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e)),x)
```

output

```
a**3*(Integral(c*sec(e + f*x), x) + Integral(3*c*sec(e + f*x)**2, x) + Int
egral(3*c*sec(e + f*x)**3, x) + Integral(c*sec(e + f*x)**4, x) + Integral(
d*sec(e + f*x)**2, x) + Integral(3*d*sec(e + f*x)**3, x) + Integral(3*d*se
c(e + f*x)**4, x) + Integral(d*sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(117) = 234$.

Time = 0.04 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.10

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$

$$= \frac{16\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^3c+48\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^3d-3\,a^3d\Bigg(\frac{2\left(3\,\sin(fx+e)^3-\right.}{\sin(fx+e)^4-2\,\mathrm{s}}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c + 48*(tan(f*x + e)^3 + 3*
tan(f*x + e))*a^3*d - 3*a^3*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(
f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f
*x + e) - 1)) - 36*a^3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a^3*d*(2*sin(f*x + e)/(sin(f*x +
 e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a^3*c*log
(sec(f*x + e) + tan(f*x + e)) + 144*a^3*c*tan(f*x + e) + 48*a^3*d*tan(f*x
+ e))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.70

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$

$$= \frac{15\,(4\,a^3c+3\,a^3d)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,(4\,a^3c+3\,a^3d)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(60\,a^3c\right.}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
1/24*(15*(4*a^3*c + 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(4*a^
3*c + 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(60*a^3*c*tan(1/2*f*
x + 1/2*e)^7 + 45*a^3*d*tan(1/2*f*x + 1/2*e)^7 - 220*a^3*c*tan(1/2*f*x + 1
/2*e)^5 - 165*a^3*d*tan(1/2*f*x + 1/2*e)^5 + 292*a^3*c*tan(1/2*f*x + 1/2*e
)^3 + 219*a^3*d*tan(1/2*f*x + 1/2*e)^3 - 132*a^3*c*tan(1/2*f*x + 1/2*e) -
147*a^3*d*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.62

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$
$$= \frac{\left(-5\,a^3\,c-\frac{15\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(\frac{55\,a^3\,c}{3}+\frac{55\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-\frac{73\,a^3\,c}{3}-\frac{73\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\right.}$$
$$+\frac{5\,a^3\,\mathrm{atanh}\left(\frac{5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(4\,c+3\,d)}{2\,\left(10\,c+\frac{15\,d}{2}\right)}\right)\,(4\,c+3\,d)}{4\,f}$$

input
```
int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

output
```
(tan(e/2 + (f*x)/2)*(11*a^3*c + (49*a^3*d)/4) - tan(e/2 + (f*x)/2)^7*(5*a^
3*c + (15*a^3*d)/4) + tan(e/2 + (f*x)/2)^5*((55*a^3*c)/3 + (55*a^3*d)/4) -
 tan(e/2 + (f*x)/2)^3*((73*a^3*c)/3 + (73*a^3*d)/4))/(f*(6*tan(e/2 + (f*x)
/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/
2)^8 + 1)) + (5*a^3*atanh((5*tan(e/2 + (f*x)/2)*(4*c + 3*d))/(2*(10*c + (1
5*d)/2)))*(4*c + 3*d))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.03

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$
$$=\frac{a^3\left(-88\cos\left(fx+e\right)\sin\left(fx+e\right)^3c-72\cos\left(fx+e\right)\sin\left(fx+e\right)^3d+96\cos\left(fx+e\right)\sin\left(fx+e\right)c+9\right.}{}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x)
```

output
```
(a**3*( - 88*cos(e + f*x)*sin(e + f*x)**3*c - 72*cos(e + f*x)*sin(e + f*x)
**3*d + 96*cos(e + f*x)*sin(e + f*x)*c + 96*cos(e + f*x)*sin(e + f*x)*d -
60*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*c - 45*log(tan((e + f*x)/2) -
 1)*sin(e + f*x)**4*d + 120*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c +
90*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*d - 60*log(tan((e + f*x)/2) -
 1)*c - 45*log(tan((e + f*x)/2) - 1)*d + 60*log(tan((e + f*x)/2) + 1)*sin(
e + f*x)**4*c + 45*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*d - 120*log(t
an((e + f*x)/2) + 1)*sin(e + f*x)**2*c - 90*log(tan((e + f*x)/2) + 1)*sin(
e + f*x)**2*d + 60*log(tan((e + f*x)/2) + 1)*c + 45*log(tan((e + f*x)/2) +
 1)*d - 36*sin(e + f*x)**3*c - 45*sin(e + f*x)**3*d + 36*sin(e + f*x)*c +
51*sin(e + f*x)*d))/(24*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.205 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = \frac{a^3\text{arctanh}(\sin(e+fx))}{2df} + \frac{a^3(c^2-3cd+3d^2)\,\text{arctanh}(\sin(e+fx))}{d^3f} - \frac{2a^3(c-d)^{5/2}\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3\sqrt{c+d}f} - \frac{a^3(c-3d)\tan(e+fx)}{d^2f} + \frac{a^3\sec(e+fx)\tan(e+fx)}{2df}$$

output

```
1/2*a^3*arctanh(sin(f*x+e))/d/f+a^3*(c^2-3*c*d+3*d^2)*arctanh(sin(f*x+e))/
d^3/f-2*a^3*(c-d)^(5/2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2)
)/d^3/(c+d)^(1/2)/f-a^3*(c-3*d)*tan(f*x+e)/d^2/f+1/2*a^3*sec(f*x+e)*tan(f*
x+e)/d/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$$

$$= \frac{a^3\cos^2(e+fx)(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\Bigg(-2(2c^2-6cd+7d^2)\log\big(\cos\big(}{}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

output
```
(a^3*Cos[e + f*x]^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f
*x])^3*(-2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (
8*(c - d)^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f
*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(I*Cos[e] + Sin[e]
))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + d^2/(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])) - d^2/(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]))))/(32*d^3*f*(c + d*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.73, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4475, 113, 25, 27, 171, 25, 27, 175, 45, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{c+d\sec(e+fx)}\,dx$$

$$\Big\downarrow 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow 4475$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{5/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 113$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^3\sqrt{\sec(e+fx)a+a}(c+2d-(2c-5d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2ad}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^3\sqrt{\sec(e+fx)a+a}(c+2d-(2c-5d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2ad}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}(c+2d-(2c-5d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2d}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 171$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{a^2\left(\frac{(2c-5d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{ad}-\frac{\int -\frac{a^2\left(d(c+2d)+\left(2c^2-6dc+7d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{ad}\right)}{2d}-\sqrt{a-a\,\mathrm{se}}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{\int\frac{a^2\left(d(c+2d)+\left(2c^2-6dc+7d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{ad}+\frac{(2c-5d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{ad}\right)}{2d}-\frac{\sqrt{a-a\sec(}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{a\int\frac{d(c+2d)+\left(2c^2-6dc+7d^2\right)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{d}+\frac{(2c-5d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{ad}\right)}{2d}-\frac{\sqrt{a-a\,\mathrm{se}}}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 175$$

$$-\frac{a^2\tan(e+fx)\left(\frac{a^2\left(\frac{a\left(\frac{\left(2c^2-6cd+7d^2\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{d}-\frac{2(c-d)^3\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+f}}{d}\right.\right.}{d}\right.}{2d}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx}}$$

$$\Big\downarrow \; 45$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{a^2\Bigg(\frac{a\Big(\frac{2\left(2c^2-6cd+7d^2\right)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}-\frac{2(c-d)^3\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx}}{d}}{d}}{2d}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx}}$$

$$\Big\downarrow \; 104$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{a^2\Bigg(\frac{a\Big(\frac{2\left(2c^2-6cd+7d^2\right)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}-\frac{4(c-d)^3\int\frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a}}{\sqrt{a-a\sec(e+}}}{d}}{d}}{2d}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}$$

$$\Big\downarrow \; 218$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{a^2\Bigg(\frac{a\left(-\frac{2\left(2c^2-6cd+7d^2\right)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)}{ad}-\frac{4(c-d)^{5/2}\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{ad\sqrt{c+d}}\right)}{d}+\frac{(2c-5d)\sqrt{a-a\sec(e+fx)}}{ad}}{2d}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

output
```
-((a^2*(-1/2*(Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(3/2))/d + (a^
2*((a*((-2*(2*c^2 - 6*c*d + 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a
+ a*Sec[e + f*x]]])/(a*d) - (4*(c - d)^(5/2)*ArcTan[(Sqrt[c + d]*Sqrt[a +
a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])])/(a*d*Sqrt[c + d]
)))/d + ((2*c - 5*d)*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])/(a
*d)))/(2*d))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{16a^3\left(-\frac{1}{32d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{5d-2c}{32d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{\left(2c^2-6cd+7d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{32d^3}-\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\operatorname{arctan}}{8d^3\sqrt{(c-d}}\right.}{f}$
default	$\frac{16a^3\left(-\frac{1}{32d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{5d-2c}{32d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{\left(2c^2-6cd+7d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{32d^3}-\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\operatorname{arctan}}{8d^3\sqrt{(c-d}}\right.}{f}$
risch	$-\frac{i a^3\left(d\,\mathrm{e}^{3i(fx+e)}+2c\,\mathrm{e}^{2i(fx+e)}-6d\,\mathrm{e}^{2i(fx+e)}-d\,\mathrm{e}^{i(fx+e)}+2c-6d\right)}{f\,d^2\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}-\frac{a^3\ln\left(\mathrm{e}^{i(fx+e)}-i\right)c^2}{f\,d^3}+\frac{3a^3\ln\left(\mathrm{e}^{i(fx+e)}-i\right)c}{f\,d^2}$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output

```
16/f*a^3*(-1/32/d/(tan(1/2*f*x+1/2*e)+1)^2-1/32*(5*d-2*c)/d^2/(tan(1/2*f*x
+1/2*e)+1)+1/32*(2*c^2-6*c*d+7*d^2)/d^3*ln(tan(1/2*f*x+1/2*e)+1)-1/8*(c^3-
3*c^2*d+3*c*d^2-d^3)/d^3/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2
*e)/((c-d)*(c+d))^(1/2))+1/32/d/(tan(1/2*f*x+1/2*e)-1)^2-1/32*(5*d-2*c)/d^
2/(tan(1/2*f*x+1/2*e)-1)+1/32/d^3*(-2*c^2+6*c*d-7*d^2)*ln(tan(1/2*f*x+1/2*
e)-1))
```

Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 532, normalized size of antiderivative = 3.48

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$$

$$= \Bigg[\frac{2\,(a^3c^2-2\,a^3cd+a^3d^2)\sqrt{\frac{c-d}{c+d}}\cos{(fx+e)}^2\log\Bigg(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\,(c^2+cd+(cd+d^2)\cos(fx+e))}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}}{}$$

$$-\frac{4\,(a^3c^2-2\,a^3cd+a^3d^2)\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{(d\cos(fx+e)+c)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right)\cos{(fx+e)}^2-(2\,a^3c^2-6\,a^3cd+7\,a}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
[1/4*(2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt((c - d)/(c + d))*cos(f*x + e)
^2*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d +
 (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^
2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (2*a^3*c^2 - 6*a^3*c
*d + 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a^3*c^2 - 6*a^3*
c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a^3*d^2 - 2*(a
^3*c*d - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(d^3*f*cos(f*x + e)^2), -1
/4*(4*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt(-(c - d)/(c + d))*arctan(-(d*co
s(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e)))*cos(f*x + e
)^2 - (2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e)
+ 1) + (2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e
) + 1) - 2*(a^3*d^2 - 2*(a^3*c*d - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/
(d^3*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = a^3\left(\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx + \int \frac{3\sec^2(e+fx)}{c+d\sec(e+fx)}\,dx + \int \frac{3\sec^3(e+fx)}{c+d\sec(e+fx)}\,dx + \int \frac{\sec^4(e+fx)}{c+d\sec(e+fx)}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

output
```
a**3*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(3*sec(e +
f*x)**2/(c + d*sec(e + f*x)), x) + Integral(3*sec(e + f*x)**3/(c + d*sec(e
 + f*x)), x) + Integral(sec(e + f*x)**4/(c + d*sec(e + f*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(140) = 280$.

Time = 0.21 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.86

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$$

$$= \frac{\frac{(2a^3c^2-6a^3cd+7a^3d^2)\log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)+1|)}{d^3} - \frac{(2a^3c^2-6a^3cd+7a^3d^2)\log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)-1|)}{d^3} + \frac{4(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
1/2*((2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1)
)/d^3 - (2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/d^3 + 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*t
an(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/(sqrt(-c^2 + d^2)*d^3) + 2*(2*a^3*
c*tan(1/2*f*x + 1/2*e)^3 - 5*a^3*d*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*c*tan(1/
2*f*x + 1/2*e) + 7*a^3*d*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 -
1)^2*d^2))/f
```

Mupad [B] (verification not implemented)

Time = 12.02 (sec) , antiderivative size = 1902, normalized size of antiderivative = 12.43

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

output

```
(atanh((18824*a^9*c^2*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 -
(16680*a^9*c^3)/d + (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6
)/d^4 - 11560*a^9*c*d) - (16680*a^9*c^3*tan(e/2 + (f*x)/2))/(2968*a^9*d^3
- 16680*a^9*c^3 - 11560*a^9*c*d^2 + 18824*a^9*c^2*d + (8608*a^9*c^4)/d - (
2480*a^9*c^5)/d^2 + (320*a^9*c^6)/d^3) + (8608*a^9*c^4*tan(e/2 + (f*x)/2))
/(8608*a^9*c^4 + 2968*a^9*d^4 - 11560*a^9*c*d^3 - 16680*a^9*c^3*d + 18824*
a^9*c^2*d^2 - (2480*a^9*c^5)/d + (320*a^9*c^6)/d^2) - (2480*a^9*c^5*tan(e/
2 + (f*x)/2))/(2968*a^9*d^5 - 2480*a^9*c^5 - 11560*a^9*c*d^4 + 8608*a^9*c^
4*d + 18824*a^9*c^2*d^3 - 16680*a^9*c^3*d^2 + (320*a^9*c^6)/d) + (320*a^9*
c^6*tan(e/2 + (f*x)/2))/(320*a^9*c^6 + 2968*a^9*d^6 - 11560*a^9*c*d^5 - 24
80*a^9*c^5*d + 18824*a^9*c^2*d^4 - 16680*a^9*c^3*d^3 + 8608*a^9*c^4*d^2) +
 (2968*a^9*d^2*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (16680*
a^9*c^3)/d + (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/d^4 -
 11560*a^9*c*d) - (11560*a^9*c*d*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968
*a^9*d^2 - (16680*a^9*c^3)/d + (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (
320*a^9*c^6)/d^4 - 11560*a^9*c*d))*(2*a^3*c^2 + 7*a^3*d^2 - 6*a^3*c*d))/(d
^3*f) - ((tan(e/2 + (f*x)/2)*(2*a^3*c - 7*a^3*d))/d^2 - (a^3*tan(e/2 + (f*
x)/2)^3*(2*c - 5*d))/d^2)/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^
2 + 1)) - (a^3*atan(((a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)
*(8*a^6*c^7 - 53*a^6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 799, normalized size of antiderivative = 5.22

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x)
```

output

```
(a**3*( - 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/
2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**2 + 8*sqrt( - c**2 + d**2)*
atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e
 + f*x)**2*c*d - 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e
+ f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*d**2 + 4*sqrt( - c**2 +
 d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2)
)*c**2 - 8*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2
)*d)/sqrt( - c**2 + d**2))*c*d + 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x
)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*d**2 + 2*cos(e + f*x)*s
in(e + f*x)*c**2*d - 4*cos(e + f*x)*sin(e + f*x)*c*d**2 - 6*cos(e + f*x)*s
in(e + f*x)*d**3 - 2*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c**3 + 4*lo
g(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*c**2*d - log(tan((e + f*x)/2) - 1)
*sin(e + f*x)**2*c*d**2 - 7*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*d**3
 + 2*log(tan((e + f*x)/2) - 1)*c**3 - 4*log(tan((e + f*x)/2) - 1)*c**2*d +
 log(tan((e + f*x)/2) - 1)*c*d**2 + 7*log(tan((e + f*x)/2) - 1)*d**3 + 2*l
og(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*c**3 - 4*log(tan((e + f*x)/2) + 1
)*sin(e + f*x)**2*c**2*d + log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*c*d**
2 + 7*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*d**3 - 2*log(tan((e + f*x)
/2) + 1)*c**3 + 4*log(tan((e + f*x)/2) + 1)*c**2*d - log(tan((e + f*x)/2)
+ 1)*c*d**2 - 7*log(tan((e + f*x)/2) + 1)*d**3 - sin(e + f*x)*c*d**2 - ...
```

3.206 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 161

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx &= -\frac{a^3(2c-3d)\operatorname{arctanh}(\sin(e+fx))}{d^3f} \\ &\quad+\frac{2a^3(c-d)^{3/2}(2c+3d)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3(c+d)^{3/2}f} \\ &\quad+\frac{2a^3c\tan(e+fx)}{d^2(c+d)f}-\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))}\end{aligned}$$

output
```
-a^3*(2*c-3*d)*arctanh(sin(f*x+e))/d^3/f+2*a^3*(c-d)^(3/2)*(2*c+3*d)*arcta
nh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/d^3/(c+d)^(3/2)/f+2*a^3*c*t
an(f*x+e)/d^2/(c+d)/f-(c-d)*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/d/(c+d)/f/(c+d
*sec(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.42 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.83

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{a^3\cos(e+fx)(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left((2c-3d)(d+c\cos(e+fx))\log\right.}{}$$

input

```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]
```

output

```
(a^3*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x
])^3*((2*c - 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]] + (-2*c + 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]] - ((2*I)*(c - d)^2*(2*c + 3*d)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin
[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin
[e])^2])]*(d + c*Cos[e + f*x])*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c^2 - d^
2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((c - d)^2*d*(-(d*Sin[e]) + c*Sin[f*x]))
/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])) + (d*(d + c*Cos[e
 + f*x])*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])) + (d*(d + c*Cos[e + f*x])*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))))/(8*d^3*f*(c + d*Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4475, 109, 27, 171, 25, 27, 175, 45, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{5/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{d(c+d)(c+d\sec(e+fx))}-\frac{\int \frac{a^3\sqrt{\sec(e+fx)a+a}(-2\sec(e+fx)c+c-3d)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{ad(c+d)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{d(c+d)(c+d\sec(e+fx))}-\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}(-2\sec(e+fx)c+c-3d)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{d(c+d)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{171}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{d(c+d)(c+d\sec(e+fx))}-\frac{a^2\left(\frac{2c\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{ad}-\frac{\int -\frac{a^2((c-3d)d+(2c-3d)(c+d}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+}}}{ad}\right.}{d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{d(c+d)(c+d\sec(e+fx))}-\frac{a^2\left(\frac{\int \frac{a^2((c-3d)d+(2c-3d)(c+d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{ad}+\frac{2c\sqrt{a-}}{}\right.}{d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{d(c+d)(c+d\sec(e+fx))}-\frac{a^2\left(\frac{a\int\frac{(c-3d)d+(2c-3d)(c+d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{d}+\frac{2c\sqrt{a}}{}\right.}{d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{175}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{d(c+d)(c+d\sec(e+fx))}-\frac{a^2\left(\frac{a\left(\frac{(2c-3d)(c+d)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{d}-\frac{(c-d)^2}{}\right.}{}\right.}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

$$\Big\downarrow \text{45}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{d(c+d)(c+d\sec(e+fx))}-\frac{a^2\left(\frac{a\left(\frac{2(2c-3d)(c+d)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}-\frac{(c-d)^2(}{}\right.}{d}\right.}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+f}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{d(c+d)(c+d\sec(e+fx))}-\frac{a^2\left(\frac{a\left(\frac{2(2c-3d)(c+d)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}-\frac{2(c-d)^2}{}\right.}{d}\right.}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)}}$$

$$\Big\downarrow \ 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{d(c+d)(c+d\sec(e+fx))}-\frac{a^2\left(\frac{a\left(-\frac{2(2c+3d)(c-d)^{3/2}\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{ad\sqrt{c+d}}-\frac{2(2c-3d)(c+d)}{}\right.}{d}\right.}{d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^2*(((c - d)*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(3/2))/(d*(
c + d)*(c + d*Sec[e + f*x])) - (a^2*((a*((-2*(2*c - 3*d)*(c + d)*ArcTan[Sq
rt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]]])/(a*d) - (2*(c - d)^(3/2)
*(2*c + 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sq
rt[a - a*Sec[e + f*x]])])/(a*d*Sqrt[c + d])))/d + (2*c*Sqrt[a - a*Sec[e +
f*x]]*Sqrt[a + a*Sec[e + f*x]])/(a*d)))/(d*(c + d)))*Tan[e + f*x])/(f*Sqrt
[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{16a^3\left(-\frac{\left(c^2-2cd+d^2\right)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(2c+3d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{2(c+d)\sqrt{(c-d)(c+d)}}\right)}{4d^3}-\frac{}{16d^2\left(\mathrm{ta}\right.}\right.}{f}$
default	$\frac{16a^3\left(-\frac{\left(c^2-2cd+d^2\right)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(2c+3d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{2(c+d)\sqrt{(c-d)(c+d)}}\right)}{4d^3}-\frac{}{16d^2\left(\mathrm{ta}\right.}\right.}{f}$
risch	$\frac{2ia^3\left(c^2d\,\mathrm{e}^{3i(fx+e)}-2c\,d^2\mathrm{e}^{3i(fx+e)}+d^3\mathrm{e}^{3i(fx+e)}+2c^3\mathrm{e}^{2i(fx+e)}-c^2d\,\mathrm{e}^{2i(fx+e)}+c\,d^2\mathrm{e}^{2i(fx+e)}+3c^2d\,\mathrm{e}^{i(fx+e)}+d^3\mathrm{e}^{i(fx}\right.}{f\,d^2\left(\mathrm{e}^{2i(fx+e)}+1\right)(c+d)c\left(c\,\mathrm{e}^{2i(fx+e)}+2d\,\mathrm{e}^{i(fx+e)}+c\right)}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
16/f*a^3*(-1/4*(c^2-2*c*d+d^2)/d^3*(1/2*d/(c+d)*tan(1/2*f*x+1/2*e)/(c*tan(
1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)-1/2*(2*c+3*d)/(c+d)/((c-d)*(c
+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)))-1/16/d^2
/(tan(1/2*f*x+1/2*e)+1)+1/16/d^3*(-2*c+3*d)*ln(tan(1/2*f*x+1/2*e)+1)-1/16/
d^2/(tan(1/2*f*x+1/2*e)-1)+1/16*(2*c-3*d)/d^3*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(152) = 304$.

Time = 0.53 (sec) , antiderivative size = 859, normalized size of antiderivative = 5.34

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
[-1/2*(((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*
d + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*
cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*c
os(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*
x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*
d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*
log(sin(f*x + e) + 1) - ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e
)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(-sin(f*x + e
) + 1) - 2*(a^3*c*d^2 + a^3*d^3 + (2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(
f*x + e))*sin(f*x + e))/((c^2*d^3 + c*d^4)*f*cos(f*x + e)^2 + (c*d^4 + d^5
)*f*cos(f*x + e)), 1/2*(2*((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x +
 e)^2 + (2*a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt(-(c - d)/
(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(
f*x + e))) - ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^
3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) + ((2
*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*
d^2 - 3*a^3*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^3*c*d^2 + a^3
*d^3 + (2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/((c
^2*d^3 + c*d^4)*f*cos(f*x + e)^2 + (c*d^4 + d^5)*f*cos(f*x + e))]
```

Sympy [F]

$$
\begin{aligned}
&\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx \\
&= a^3\left(\int \frac{\sec(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right. \\
&\quad + \int \frac{3\sec^2(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx \\
&\quad + \int \frac{3\sec^3(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx \\
&\quad \left. + \int \frac{\sec^4(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right)
\end{aligned}
$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)
```

output

```
a**3*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)
**2), x) + Integral(3*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*se
c(e + f*x)**2), x) + Integral(3*sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x)
 + d**2*sec(e + f*x)**2), x) + Integral(sec(e + f*x)**4/(c**2 + 2*c*d*sec(
e + f*x) + d**2*sec(e + f*x)**2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="m
axima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(152) = 304$.

Time = 0.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = -\frac{\frac{2\,(2\,a^3c^3-a^3c^2d-4\,a^3cd^2+3\,a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(cd^3+d^4)\sqrt{-c^2+d^2}}+\frac{4\left(a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}}{}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="g
iac")
```

output

```
-(2*(2*a^3*c^3 - a^3*c^2*d - 4*a^3*c*d^2 + 3*a^3*d^3)*(pi*floor(1/2*(f*x +
 e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*
f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c*d^3 + d^4)*sqrt(-c^2 + d^2)) + 4*(a^3
*c^2*tan(1/2*f*x + 1/2*e)^3 - a^3*c*d*tan(1/2*f*x + 1/2*e)^3 - a^3*c^2*tan
(1/2*f*x + 1/2*e) - a^3*d^2*tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)
^4 - d*tan(1/2*f*x + 1/2*e)^4 - 2*c*tan(1/2*f*x + 1/2*e)^2 + c + d)*(c*d^2
 + d^3)) + (2*a^3*c - 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^3 - (2
*a^3*c - 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/d^3)/f
```

Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 3135, normalized size of antiderivative = 19.47

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

output
```
(a^3*atan(((a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*
d^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^
6*c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^
10 - 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7
+ d^8 + c^2*d^6) - (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*
d^8 + c^5*d^6))/(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c -
3*d)*1i)/d^3 + (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^
6*c*d^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3
+ a^6*c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*
c*d^10 - 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*
d^7 + d^8 + c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*
c^3*d^8 + c^5*d^6))/(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*
c - 3*d)*1i)/d^3)/((128*(4*a^9*c^7 - 9*a^9*c*d^6 - 16*a^9*c^6*d + 36*a^9*c
^2*d^5 - 50*a^9*c^3*d^4 + 20*a^9*c^4*d^3 + 15*a^9*c^5*d^2))/(2*c*d^7 + d^8
 + c^2*d^6) + (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6
*c*d^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 +
 a^6*c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c
*d^10 - 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d
^7 + d^8 + c^2*d^6) - (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c
^3*d^8 + c^5*d^6))/(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1262, normalized size of antiderivative = 7.84

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x)
```

output

```
(a**3*(4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*
d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c**2*d + 2*sqrt( - c**2 + d**2)*atan
((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f
*x)*c*d**2 - 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*
x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*d**3 - 4*sqrt( - c**2 + d**2)*
atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e
 + f*x)**2*c**3 - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e
 + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**2*d + 6*sqrt( - c**
2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d*
*2))*sin(e + f*x)**2*c*d**2 + 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2
)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**3 + 2*sqrt( - c**2 + d*
*2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c
**2*d - 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)
*d)/sqrt( - c**2 + d**2))*c*d**2 + 2*cos(e + f*x)*log(tan((e + f*x)/2) - 1
)*c**3*d + cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c**2*d**2 - 4*cos(e + f*
x)*log(tan((e + f*x)/2) - 1)*c*d**3 - 3*cos(e + f*x)*log(tan((e + f*x)/2)
- 1)*d**4 - 2*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*c**3*d - cos(e + f*x)
*log(tan((e + f*x)/2) + 1)*c**2*d**2 + 4*cos(e + f*x)*log(tan((e + f*x)/2)
 + 1)*c*d**3 + 3*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*d**4 + 2*cos(e + f
*x)*sin(e + f*x)*c**3*d + cos(e + f*x)*sin(e + f*x)*c**2*d**2 + cos(e +...
```

3.207 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 188

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx &= \frac{a^3\text{arctanh}(\sin(e+fx))}{d^3f} - \frac{a^3\sqrt{c-d}(2c^2+6cd+7d^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3(c+d)^{5/2}f}\\ &\quad - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))}\end{aligned}$$

output

```
a^3*arctanh(sin(f*x+e))/d^3/f-a^3*(c-d)^(1/2)*(2*c^2+6*c*d+7*d^2)*arctanh(
(c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/d^3/(c+d)^(5/2)/f-1/2*(c-d)*(a
^3+a^3*sec(f*x+e))*tan(f*x+e)/d/(c+d)/f/(c+d*sec(f*x+e))^2-1/2*a^3*(c-d)*(
2*c+5*d)*tan(f*x+e)/d^2/(c+d)^2/f/(c+d*sec(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.09

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{a^3(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left(-4(d+c\cos(e+fx))^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

output
```
(a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x])^3*(-4*(d +
 c*Cos[e + f*x])^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*(d + c*Cos
[e + f*x])^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (4*(2*c^3 + 4*c^2*
d + c*d^2 - 7*d^3)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])
*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[
e + f*x])^2*(I*Cos[e] + Sin[e]))/((c + d)^2*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] -
 I*Sin[e])^2]) + ((c - d)*d*Sec[e]*((2*c^4 + 6*c^3*d + 5*c^2*d^2 + 12*c*d^
3 + 2*d^4)*Sin[e] - c*(d*(7*c^2 + 18*c*d + 2*d^2)*Sin[f*x] - d*(c^2 + 6*c*
d + 2*d^2)*Sin[2*e + f*x] + c*(2*c^2 + 6*c*d + d^2)*Sin[e + 2*f*x])))/(c^2
*(c + d)^2)))/(32*d^3*f*(c + d*Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4475, 109, 27, 166, 25, 27, 175, 45, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c+d\sec(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{5/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d(c+d)(c+d\sec(e+fx))^2}-\frac{\int \frac{a^3\sqrt{\sec(e+fx)a+a}(c-5d-2(c+d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2ad(c+d)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d(c+d)(c+d\sec(e+fx))^2}-\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}(c-5d-2(c+d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2d(c+d)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{166}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d(c+d)(c+d\sec(e+fx))^2}-\frac{a^2\left(\frac{\int -\frac{a^2\left(2\sec(e+fx)(c+d)^2+d(c+7d)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{ad(c+d)}-(c-d)\right.}{2d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d(c+d)(c+d\sec(e+fx))^2}-\frac{a^2\left(-\frac{\int \frac{a^2\left(2\sec(e+fx)(c+d)^2+d(c+7d)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{ad(c+d)}-(c-d)\right.}{2d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d(c+d)(c+d\sec(e+fx))^2}-\frac{a^2\left(-\frac{a\int\frac{2\sec(e+fx)(c+d)^2+d(c+7d)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{d(c+d)}-\frac{(c-}{}\right.}{2d(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 175$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d(c+d)(c+d\sec(e+fx))^2}-\frac{a^2\left(-\frac{a\left(\frac{2(c+d)^2\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{d}-\frac{\left(2c(c+d)^2\right.}{}\right.}{}\right.}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec}}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d(c+d)(c+d\sec(e+fx))^2}-\frac{a^2\left(-\frac{a\left(\frac{4(c+d)^2\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}-\frac{\left(2c(c+d)^2-\right.}{}\right.}{d}\right.}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(}}$$

$$\Big\downarrow 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d(c+d)(c+d\sec(e+fx))^2}-\frac{a^2\left(-\frac{a\left(\frac{4(c+d)^2\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{d}-\frac{2\left(2c(c+d)^2-\right.}{}\right.}{d(c+d}\right.}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(c-d)\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2d(c+d)(c+d\sec(e+fx))^2}-\frac{a^2\left(-\frac{a\left(-\frac{2\left(2c(c+d)^2-d^2(c+7d)\right)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{ad\sqrt{c-d}\sqrt{c+d}}-\frac{4(c+d)}{}\right.}{d(c+d)}\right.}{2d}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

output

```
-((a^2*(((c - d)*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(3/2))/(2*d
*(c + d)*(c + d*Sec[e + f*x])^2) - (a^2*(-((a*((-4*(c + d)^2*ArcTan[Sqrt[a
 - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]]])/(a*d) - (2*(2*c*(c + d)^2 -
d^2*(c + 7*d))*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*
Sqrt[a - a*Sec[e + f*x]])])/(a*Sqrt[c - d]*d*Sqrt[c + d])))/(d*(c + d))) -
 ((c - d)*(2*c + 5*d)*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])/(
a*d*(c + d)*(c + d*Sec[e + f*x]))))/(2*d*(c + d)))*Tan[e + f*x])/(f*Sqrt[a
 - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{16a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16d^3}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16d^3}+\frac{(c-d)\left(\frac{\frac{d\left(2c^2+3cd-5d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c^2+4cd+2d^2}-\frac{d(2c+7d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{(2c^2+}{}\right.}{8d^3}\right.}{f}$
default	$\frac{16a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16d^3}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16d^3}+\frac{(c-d)\left(\frac{\frac{d\left(2c^2+3cd-5d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c^2+4cd+2d^2}-\frac{d(2c+7d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{(2c^2+}{}\right.}{8d^3}\right.}{f}$
risch	$\frac{ia^3(-c^4d\,e^{3i(fx+e)}-5c^3d^2e^{3i(fx+e)}+4c^2d^3e^{3i(fx+e)}+2c\,d^4e^{3i(fx+e)}-2c^5e^{2i(fx+e)}-4c^4d\,e^{2i(fx+e)}+c^3d^2e^{2i(fx+e)}-}{c^2f(c+d)^2}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
16/f*a^3*(-1/16/d^3*ln(tan(1/2*f*x+1/2*e)-1)+1/16/d^3*ln(tan(1/2*f*x+1/2*e
)+1)+1/8*(c-d)/d^3*((1/2*d*(2*c^2+3*c*d-5*d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x
+1/2*e)^3-1/2*d*(2*c+7*d)/(c+d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^
2-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-1/2*(2*c^2+6*c*d+7*d^2)/(c^2+2*c*d+d^2)/((
c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(175) = 350$.

Time = 0.52 (sec) , antiderivative size = 1176, normalized size of antiderivative = 6.26

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output

```
[1/4*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 + (2*a^3*c^4 + 6*a^3*c^3*d
+ 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2*d^2 + 7*a^3*c
*d^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 -
 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c
- d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(
f*x + e) + d^2)) + 2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a
^3*c^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^
3*c*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) - 2*(a^3*c^2*d^2 + 2*a^3*c*d^
3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^
3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1)
- 2*(3*a^3*c^2*d^2 + 3*a^3*c*d^3 - 6*a^3*d^4 + (2*a^3*c^3*d + 4*a^3*c^2*d^
2 - 5*a^3*c*d^3 - a^3*d^4)*cos(f*x + e))*sin(f*x + e))/((c^4*d^3 + 2*c^3*d
^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x
 + e) + (c^2*d^5 + 2*c*d^6 + d^7)*f), -1/2*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 +
 7*a^3*d^4 + (2*a^3*c^4 + 6*a^3*c^3*d + 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*
(2*a^3*c^3*d + 6*a^3*c^2*d^2 + 7*a^3*c*d^3)*cos(f*x + e))*sqrt(-(c - d)/(c
 + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*
x + e))) - (a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3*d +
 a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3)*c
os(f*x + e))*log(sin(f*x + e) + 1) + (a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d...
```

Sympy [F]

$$
\begin{aligned}
&\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx \\
&= a^3\left(\int \frac{\sec(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx\right. \\
&\quad+\int \frac{3\sec^2(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx \\
&\quad+\int \frac{3\sec^3(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx \\
&\quad\left.+\int \frac{\sec^4(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx\right)
\end{aligned}
$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

output
```
a**3*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
 + f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(3*sec(e + f*x)**2/(c**3
+ 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3)
, x) + Integral(3*sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2
*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**4/(c
**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)
**3), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(175) = 350.

Time = 0.25 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{\frac{a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d^3} - \frac{a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d^3} + \frac{(2a^3c^3+4a^3c^2d+a^3cd^2-7a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan(}{}\right.\right.}{(c^2d^3+2cd^4+d^5)\sqrt{-c^2+d^2}}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
(a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^3 - a^3*log(abs(tan(1/2*f*x + 1/
2*e) - 1))/d^3 + (2*a^3*c^3 + 4*a^3*c^2*d + a^3*c*d^2 - 7*a^3*d^3)*(pi*flo
or(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e)
 - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*s
qrt(-c^2 + d^2)) + (2*a^3*c^3*tan(1/2*f*x + 1/2*e)^3 + a^3*c^2*d*tan(1/2*f
*x + 1/2*e)^3 - 8*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 5*a^3*d^3*tan(1/2*f*x
 + 1/2*e)^3 - 2*a^3*c^3*tan(1/2*f*x + 1/2*e) - 7*a^3*c^2*d*tan(1/2*f*x + 1
/2*e) + 2*a^3*c*d^2*tan(1/2*f*x + 1/2*e) + 7*a^3*d^3*tan(1/2*f*x + 1/2*e))
/((c^2*d^2 + 2*c*d^3 + d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/
2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 16.95 (sec) , antiderivative size = 4131, normalized size of antiderivative = 21.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

output

```
- ((a^3*tan(e/2 + (f*x)/2)*(5*c*d + 2*c^2 - 7*d^2))/(d^2*(c + d)) - (a^3*t
an(e/2 + (f*x)/2)^3*(c^2*d - 8*c*d^2 + 2*c^3 + 5*d^3))/(d^2*(c + d)^2))/(f
*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2
 - 2*c*d + d^2) + c^2 + d^2)) - (a^3*atan(((a^3*((8*tan(e/2 + (f*x)/2)*(8*
a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a
^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 +
 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2
*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))
/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*a^3*tan(e/2 + (f*
x)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c
^6*d^7 + 8*c^7*d^6))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4
))))/d^3)*1i)/d^3 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 +
59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4
*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) -
 (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9
+ 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^
2*d^8 + 4*c^3*d^7 + c^4*d^6) + (8*a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 + 16*c^
2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/(d
^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3)*1i)/d^3)/((16
*(4*a^9*c^6 - 35*a^9*d^6 + 61*a^9*c*d^5 + 10*a^9*c^5*d + 5*a^9*c^2*d^4 ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1575, normalized size of antiderivative = 8.38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x)
```

output

```
(a**3*( - 8*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/
2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c**3*d - 24*sqrt( - c**2 + d**2)*
atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e
 + f*x)*c**2*d**2 - 28*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan
((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c*d**3 + 4*sqrt( - c**
2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d*
*2))*sin(e + f*x)**2*c**4 + 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)
*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**3*d + 14
*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt(
 - c**2 + d**2))*sin(e + f*x)**2*c**2*d**2 - 4*sqrt( - c**2 + d**2)*atan((
tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**4 - 12*s
qrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( -
 c**2 + d**2))*c**3*d - 18*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
 tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**2*d**2 - 12*sqrt( - c**2 + d
**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*
c*d**3 - 14*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/
2)*d)/sqrt( - c**2 + d**2))*d**4 - 4*cos(e + f*x)*log(tan((e + f*x)/2) - 1
)*c**4*d - 12*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*c**3*d**2 - 12*cos(e
+ f*x)*log(tan((e + f*x)/2) - 1)*c**2*d**3 - 4*cos(e + f*x)*log(tan((e + f
*x)/2) - 1)*c*d**4 + 4*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*c**4*d + ...
```

3.208 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 178

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}(c+d)^{7/2}f} + \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} - \frac{5a^3(c-d)\tan(e+fx)}{6d(c+d)^2f(c+d\sec(e+fx))^2} + \frac{5a^3(c+4d)\tan(e+fx)}{6d(c+d)^3f(c+d\sec(e+fx))}$$

output
```
5*a^3*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(1/2)/(c+d
)^(7/2)/f+1/3*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^3-5
/6*a^3*(c-d)*tan(f*x+e)/d/(c+d)^2/f/(c+d*sec(f*x+e))^2+5/6*a^3*(c+4*d)*tan
(f*x+e)/d/(c+d)^3/f/(c+d*sec(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.48 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.24

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{a^3(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(1+\sec(e+fx))^3\left(-\frac{120i\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i}}\right.}{\sqrt{c^2-d^2}}\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

output
```
(a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*Sec[e + f*x]*(1 + Sec[e + f*x
])^3*(((-120*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Ta
n[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e +
 f*x])^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]
) + (c*Sec[e]*(6*(8*c^4 + 6*c^3*d + 30*c^2*d^2 + 9*c*d^3 + 2*d^4)*Sin[f*x]
 - 3*(6*c^4 - 3*c^3*d + 30*c^2*d^2 + 18*c*d^3 + 4*d^4)*Sin[2*e + f*x] + c*
(3*(3*c^3 + 38*c^2*d + 12*c*d^2 + 2*d^3)*Sin[e + 2*f*x] + 3*(3*c^3 - 6*c^2
*d - 6*c*d^2 - 2*d^3)*Sin[3*e + 2*f*x] + c*(22*c^2 + 9*c*d + 2*d^2)*Sin[2*
e + 3*f*x])) - 2*d*(66*c^4 + 27*c^3*d + 50*c^2*d^2 + 18*c*d^3 + 4*d^4)*Tan
[e])/c^3))/(192*(c + d)^3*f*(c + d*Sec[e + f*x])^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.62, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 4475, 105, 105, 105, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c+d\sec(e+fx))^4}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{ 4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{5/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^4}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{5a\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{3(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{5/2}}{3a(c+d)(c+d\sec(e+fx))^3}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{5a\left(\frac{3a\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2a(c+d)(c+d\sec(e+fx))^2}\right)}{3(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e-}{3a(c+d)(c+d\sec(e+}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{5a\left(\frac{3a\left(\frac{a\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{c+d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx}{2a(c+d)(c-}\right.}{3(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{5a\left(\frac{3a\left(\frac{2a\int\frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{c+d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e-}{2a(c+d)(c+d\sec(e+}\right)}{3(c+d)}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{5a\left(\frac{3a\left(\frac{2\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{\sqrt{c-d}(c+d)^{3/2}}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2a(c+d)(c+d\sec(e+fx))^2}\right)}{3(c+d)}-\sqrt{}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

output

```
-((a^2*(-1/3*(Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(5/2))/(a*(c +
 d)*(c + d*Sec[e + f*x])^3) + (5*a*(-1/2*(Sqrt[a - a*Sec[e + f*x]]*(a + a*
Sec[e + f*x])^(3/2))/(a*(c + d)*(c + d*Sec[e + f*x])^2) + (3*a*((2*ArcTan[
(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x
]])])/(Sqrt[c - d]*(c + d)^(3/2)) - (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]])/(a*(c + d)*(c + d*Sec[e + f*x]))))/(2*(c + d))))/(3*(c + d))
)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{16a^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{6(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}-\frac{5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{3\left(-\frac{}{2(c+}\right.}{}\right.}{}\right.}{f}$
default	$\frac{16a^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{6(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}-\frac{5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{3\left(-\frac{}{2(c+}\right.}{}\right.}{}\right.}{f}$
risch	$\frac{ia^3\left(22c^5+9c^4d-9c^5e^{5i(fx+e)}+8d^5e^{3i(fx+e)}+9c^5e^{i(fx+e)}+18c^5e^{4i(fx+e)}+2c^3d^2+132c^4d\,e^{3i(fx+e)}+54c^3d^2e^{3i(fx+e)}\right.}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBO
SE)
```

output

```
16/f*a^3*(-1/6*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*
x+1/2*e)^2*d-c-d)^3-5/6/(c+d)*(-1/4*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*
x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-3/4/(c+d)*(-1/2*tan(1/2*f*x+1/2*e
)/(c+d)/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)+1/2/(c+d)/((c-
d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(163) = 326$.

Time = 0.19 (sec) , antiderivative size = 1012, normalized size of antiderivative = 5.69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="f
ricas")
```

output

```
[1/12*(15*(a^3*c^3*cos(f*x + e)^3 + 3*a^3*c^2*d*cos(f*x + e)^2 + 3*a^3*c*d
^2*cos(f*x + e) + a^3*d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2
- 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x +
 e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2
*a^3*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4 + (22*a
^3*c^4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*d^4)*cos(f*x +
 e)^2 + 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x +
e))*sin(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 - 3*c^4*d^4 - c^
3*d^5)*f*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 - 2*c^4*d^4 - 3
*c^3*d^5 - c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4
- 2*c^3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e) + (c^5*d^3 + 3*c^4*d^4 + 2
*c^3*d^5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f), 1/6*(15*(a^3*c^3*cos(f*x + e)^3
+ 3*a^3*c^2*d*cos(f*x + e)^2 + 3*a^3*c*d^2*cos(f*x + e) + a^3*d^3)*sqrt(-c
^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) + (2*a^3*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a
^3*d^4 + (22*a^3*c^4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*
d^4)*cos(f*x + e)^2 + 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d
^4)*cos(f*x + e))*sin(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 -
3*c^4*d^4 - c^3*d^5)*f*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 -
 2*c^4*d^4 - 3*c^3*d^5 - c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5...
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$
$$= a^3\left(\int \frac{\sec(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right.$$
$$+\int \frac{3\sec^2(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx$$
$$+\int \frac{3\sec^3(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx$$
$$\left.+\int \frac{\sec^4(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right)$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
```

output
```
a**3*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*se
c(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Int
egral(3*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e
+ f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integra
l(3*sec(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*
x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(se
c(e + f*x)**4/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2
+ 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.72

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^3}{(c^3+3c^2d+3cd^2+d^3)\sqrt{-c^2+d^2}}+\frac{15a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-30a^3cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+15}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="g
iac")
```

output
```
-1/3*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^3/((c^3 +
3*c^2*d + 3*c*d^2 + d^3)*sqrt(-c^2 + d^2)) + (15*a^3*c^2*tan(1/2*f*x + 1/2
*e)^5 - 30*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 + 15*a^3*d^2*tan(1/2*f*x + 1/2*e
)^5 - 40*a^3*c^2*tan(1/2*f*x + 1/2*e)^3 + 40*a^3*d^2*tan(1/2*f*x + 1/2*e)^
3 + 33*a^3*c^2*tan(1/2*f*x + 1/2*e) + 66*a^3*c*d*tan(1/2*f*x + 1/2*e) + 33
*a^3*d^2*tan(1/2*f*x + 1/2*e))/((c^3 + 3*c^2*d + 3*c*d^2 + d^3)*(c*tan(1/2
*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.48

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{\frac{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(a^3c^2-2a^3cd+a^3d^2)}{(c+d)^3}+\frac{11a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+d}-\frac{40\tan\left(\frac{e}{}\right.}{}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(-3c^3-3c^2d+3cd^2+3d^3)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(-3c^3+3c^2d+3cd^2-3d^3)+3cd\right.}$$

$$+\frac{5a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)}{f(c+d)^{7/2}\sqrt{c-d}}$$

input

```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

output

```
((5*tan(e/2 + (f*x)/2)^5*(a^3*c^2 + a^3*d^2 - 2*a^3*c*d))/(c + d)^3 + (11*
a^3*tan(e/2 + (f*x)/2))/(c + d) - (40*tan(e/2 + (f*x)/2)^3*(a^3*c - a^3*d)
)/(3*(c + d)^2))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d
^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 +
 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3
))) + (5*a^3*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(
c + d)^(7/2)*(c - d)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 933, normalized size of antiderivative = 5.24

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x)
```

output

```
(a**3*(30*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)
*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*c**3 - 30*sqrt( - c
**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 +
d**2))*cos(e + f*x)*c**3 - 90*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*
c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c*d**2 + 90*sqr
t( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c
**2 + d**2))*sin(e + f*x)**2*c**2*d - 90*sqrt( - c**2 + d**2)*atan((tan((e
 + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**2*d - 30*sqrt(
 - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**
2 + d**2))*d**3 - 9*cos(e + f*x)*sin(e + f*x)*c**4 - 48*cos(e + f*x)*sin(e
 + f*x)*c**3*d + 48*cos(e + f*x)*sin(e + f*x)*c*d**3 + 9*cos(e + f*x)*sin(
e + f*x)*d**4 + 22*sin(e + f*x)**3*c**4 + 9*sin(e + f*x)**3*c**3*d - 20*si
n(e + f*x)**3*c**2*d**2 - 9*sin(e + f*x)**3*c*d**3 - 2*sin(e + f*x)**3*d**
4 - 24*sin(e + f*x)*c**4 - 18*sin(e + f*x)*c**3*d + 18*sin(e + f*x)*c*d**3
 + 24*sin(e + f*x)*d**4))/(6*f*(cos(e + f*x)*sin(e + f*x)**2*c**8 + 3*cos(
e + f*x)*sin(e + f*x)**2*c**7*d + 2*cos(e + f*x)*sin(e + f*x)**2*c**6*d**2
 - 2*cos(e + f*x)*sin(e + f*x)**2*c**5*d**3 - 3*cos(e + f*x)*sin(e + f*x)*
*2*c**4*d**4 - cos(e + f*x)*sin(e + f*x)**2*c**3*d**5 - cos(e + f*x)*c**8
- 3*cos(e + f*x)*c**7*d - 5*cos(e + f*x)*c**6*d**2 - 7*cos(e + f*x)*c**5*d
**3 - 3*cos(e + f*x)*c**4*d**4 + 7*cos(e + f*x)*c**3*d**5 + 9*cos(e + f...
```

3.209 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 266

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx &= \frac{5a^3(4c-3d)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4(c-d)^{3/2}(c+d)^{9/2}f}\\ &\quad-\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4}\\ &\quad+\frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3}\\ &\quad-\frac{5a^3(4c-3d)\tan(e+fx)}{24d(c+d)^3f(c+d\sec(e+fx))^2}\\ &\quad+\frac{5a^3(4c-3d)(c+4d)\tan(e+fx)}{24(c-d)d(c+d)^4f(c+d\sec(e+fx))}\end{aligned}$$

output
```
5/4*a^3*(4*c-3*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d
)^(3/2)/(c+d)^(9/2)/f-1/4*d*(a+a*sec(f*x+e))^3*tan(f*x+e)/(c^2-d^2)/f/(c+d
*sec(f*x+e))^4+1/12*a*(4*c-3*d)*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c-d)/(c+d)^
2/f/(c+d*sec(f*x+e))^3-5/24*a^3*(4*c-3*d)*tan(f*x+e)/d/(c+d)^3/f/(c+d*sec(
f*x+e))^2+5/24*a^3*(4*c-3*d)*(c+4*d)*tan(f*x+e)/(c-d)/d/(c+d)^4/f/(c+d*sec
(f*x+e))
```

Mathematica [A] (verified)

Time = 5.61 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$

$$= \frac{a^3\left(-\frac{120(4c-3d)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \frac{(-72c^4-478c^3d+336c^2d^2+28cd^3+336d^4+(-296c^4-84c^3d-577c^2d^2+984cd^3+1}{}\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]
```

output
```
(a^3*((-120*(4*c - 3*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2
]])/Sqrt[c^2 - d^2] - ((-72*c^4 - 478*c^3*d + 336*c^2*d^2 + 28*c*d^3 + 336
*d^4 + (-296*c^4 - 84*c^3*d - 577*c^2*d^2 + 984*c*d^3 + 198*d^4)*Cos[e + f
*x] + (-72*c^4 - 470*c^3*d + 384*c^2*d^2 + 200*c*d^3 + 48*d^4)*Cos[2*(e +
f*x)] - 88*c^4*Cos[3*(e + f*x)] + 36*c^3*d*Cos[3*(e + f*x)] + 37*c^2*d^2*C
os[3*(e + f*x)] + 24*c*d^3*Cos[3*(e + f*x)] + 6*d^4*Cos[3*(e + f*x)])*Sin[
e + f*x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)*(c + d)^4*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.39, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 4475, 107, 105, 105, 105, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a\sec(e+fx)+a)^3}{(c+d\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \; 4475$$

$$-\frac{a^2\tan(e+fx)\int \frac{(\sec(e+fx)a+a)^{5/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^5}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 107$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(4c-3d)\int \frac{(\sec(e+fx)a+a)^{5/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^4}d\sec(e+fx)}{4(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{7/2}}{4a^2(c^2-d^2)(c+d\sec(e+fx))^4}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 105$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(4c-3d)\left(\frac{5a\int \frac{(\sec(e+fx)a+a)^{3/2}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{3(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{5/2}}{3a(c+d)(c+d\sec(e+fx))^3}\right)}{4(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}(}{4a^2(c^2-d^2)(c+}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 105$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(4c-3d)\left(\frac{5a\left(\frac{3a\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{3/2}}{2a(c+d)(c+d\sec(e+fx))^2}\right)}{3(c+d)}-\frac{\sqrt{a-a\sec(e+fx)}(a\,\mathrm{s}}{3a(c+d)(c+d\,\mathrm{se}}\right.}{4(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 105$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(4c-3d)\left(\frac{5a\left(\frac{3a\left(\frac{a\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{c+d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-}}{}\right.}{3(c+d)}\right.}{4(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e}}$$

$$\Big\downarrow 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{(4c-3d)\left(\frac{5a\left(\frac{3a\left(\frac{2a\int\frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{c+d}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a(c+d)(c+d\sec(e+fx))}\right)}{2(c+d)}-\frac{\sqrt{a-a\sec(e+f}}{2a(c+d)}\right.}{3(c+d)}\right.}{4(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+f}}$$

$$\Big\downarrow {\color{blue}218}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\sqrt{a-a\sec(e+fx)}(a\sec(e+fx)+a)^{7/2}}{4a^2(c^2-d^2)(c+d\sec(e+fx))^4}+\frac{(4c-3d)\left(\frac{5a\left(\frac{3a\left(\frac{2\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{\sqrt{c-d}(c+d)^{3/2}}-\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}{a(c+d)(c+d\sec(e+fx}}{2(c+d)}}{3(c+d)}\right)}{4}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]
```

output

```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(7/2))/(4*a^2*(c^
2 - d^2)*(c + d*Sec[e + f*x])^4) + ((4*c - 3*d)*(-1/3*(Sqrt[a - a*Sec[e +
f*x]]*(a + a*Sec[e + f*x])^(5/2))/(a*(c + d)*(c + d*Sec[e + f*x])^3) + (5*
a*(-1/2*(Sqrt[a - a*Sec[e + f*x]]*(a + a*Sec[e + f*x])^(3/2))/(a*(c + d)*(
c + d*Sec[e + f*x])^2) + (3*a*((2*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f
*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])])/(Sqrt[c - d]*(c + d)^(3/2))
 - (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])/(a*(c + d)*(c + d*S
ec[e + f*x]))))/(2*(c + d))))/(3*(c + d))))/(4*(c^2 - d^2)))*Tan[e + f*x])
/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{16a^3\left(\frac{-\frac{5(4c-3d)\left(c^2-2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{64\left(c^4+4c^3d+6c^2d^2+4c\,d^3+d^4\right)}+\frac{55(c-d)(4c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{192\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{73(4c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{192\left(c^2+2cd+d^2\right)}+\frac{(44c-49d)\tan\left(\frac{fx}{2}\right.}{64(c-d)(c+d)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^4}\right.}{f}$
default	$\frac{16a^3\left(\frac{-\frac{5(4c-3d)\left(c^2-2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{64\left(c^4+4c^3d+6c^2d^2+4c\,d^3+d^4\right)}+\frac{55(c-d)(4c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{192\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{73(4c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{192\left(c^2+2cd+d^2\right)}+\frac{(44c-49d)\tan\left(\frac{fx}{2}\right.}{64(c-d)(c+d)}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^4}\right.}{f}$
risch	Expression too large to display

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBO
SE)
```

output
```
16/f*a^3*((-5/64*(4*c-3*d)*(c^2-2*c*d+d^2)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+
d^4)*tan(1/2*f*x+1/2*e)^7+55/192*(c-d)*(4*c-3*d)/(c^3+3*c^2*d+3*c*d^2+d^3)
*tan(1/2*f*x+1/2*e)^5-73/192*(4*c-3*d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^
3+1/64*(44*c-49*d)/(c-d)/(c+d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2
-tan(1/2*f*x+1/2*e)^2*d-c-d)^4+5/64*(4*c-3*d)/(c^5+3*c^4*d+2*c^3*d^2-2*c^2
*d^3-3*c*d^4-d^5)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c
-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(247) = 494$.

Time = 0.25 (sec) , antiderivative size = 1714, normalized size of antiderivative = 6.44

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="f
ricas")
```

output

```
[1/48*(15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4*d)*cos(f*x + e
)^4 + 4*(4*a^3*c^4*d - 3*a^3*c^3*d^2)*cos(f*x + e)^3 + 6*(4*a^3*c^3*d^2 -
3*a^3*c^2*d^3)*cos(f*x + e)^2 + 4*(4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x +
e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2
 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2
*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^3*c^5*d + 12*a^3*c^4
*d^2 + 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6 + (88*a
^3*c^6 - 36*a^3*c^5*d - 125*a^3*c^4*d^2 + 12*a^3*c^3*d^3 + 31*a^3*c^2*d^4
+ 24*a^3*c*d^5 + 6*a^3*d^6)*cos(f*x + e)^3 + (36*a^3*c^6 + 235*a^3*c^5*d -
 228*a^3*c^4*d^2 - 335*a^3*c^3*d^3 + 168*a^3*c^2*d^4 + 100*a^3*c*d^5 + 24*
a^3*d^6)*cos(f*x + e)^2 + (8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2 - 27
6*a^3*c^3*d^3 - 217*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x + e)
)*sin(f*x + e))/((c^11 + 3*c^10*d + c^9*d^2 - 5*c^8*d^3 - 5*c^7*d^4 + c^6*
d^5 + 3*c^5*d^6 + c^4*d^7)*f*cos(f*x + e)^4 + 4*(c^10*d + 3*c^9*d^2 + c^8*
d^3 - 5*c^7*d^4 - 5*c^6*d^5 + c^5*d^6 + 3*c^4*d^7 + c^3*d^8)*f*cos(f*x + e
)^3 + 6*(c^9*d^2 + 3*c^8*d^3 + c^7*d^4 - 5*c^6*d^5 - 5*c^5*d^6 + c^4*d^7 +
 3*c^3*d^8 + c^2*d^9)*f*cos(f*x + e)^2 + 4*(c^8*d^3 + 3*c^7*d^4 + c^6*d^5
- 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x + e) + (
c^7*d^4 + 3*c^6*d^5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*d^10
 + d^11)*f), 1/24*(15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4...
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$

$$= a^3\left(\int \frac{\sec(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec}\right.$$

$$+\int \frac{3\sec^2(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(}$$

$$+\int \frac{3\sec^3(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(}$$

$$+\int \frac{\sec^4(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(}$$

input

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

output
```
a**3*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*s
ec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 +
 d**5*sec(e + f*x)**5), x) + Integral(3*sec(e + f*x)**2/(c**5 + 5*c**4*d*s
ec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3
+ 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(3*sec(e
+ f*x)**3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 1
0*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)
**5), x) + Integral(sec(e + f*x)**4/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**
3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f
*x)**4 + d**5*sec(e + f*x)**5), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(247) = 494.

Time = 0.33 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="g
iac")
```

output
```
1/12*(15*(4*a^3*c - 3*a^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2
 + d^2)))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*sqrt(-c
^2 + d^2)) - (60*a^3*c^4*tan(1/2*f*x + 1/2*e)^7 - 225*a^3*c^3*d*tan(1/2*f*
x + 1/2*e)^7 + 315*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 195*a^3*c*d^3*tan(
1/2*f*x + 1/2*e)^7 + 45*a^3*d^4*tan(1/2*f*x + 1/2*e)^7 - 220*a^3*c^4*tan(1
/2*f*x + 1/2*e)^5 + 385*a^3*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 55*a^3*c^2*d^2*
tan(1/2*f*x + 1/2*e)^5 - 385*a^3*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 165*a^3*d^
4*tan(1/2*f*x + 1/2*e)^5 + 292*a^3*c^4*tan(1/2*f*x + 1/2*e)^3 + 73*a^3*c^3
*d*tan(1/2*f*x + 1/2*e)^3 - 511*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 73*a^
3*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 219*a^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 132*
a^3*c^4*tan(1/2*f*x + 1/2*e) - 249*a^3*c^3*d*tan(1/2*f*x + 1/2*e) + 45*a^3
*c^2*d^2*tan(1/2*f*x + 1/2*e) + 309*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + 147*a
^3*d^4*tan(1/2*f*x + 1/2*e))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c
*d^4 - d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^
4))/f
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.45

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$

$$= \frac{\frac{55\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(4\,a^3\,c^2-7\,a^3\,c\,d+3\,a^3\,d^2\right)}{12\,{\left(c+d\right)}^3}-\frac{73\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(4\,a^3\,c-3}{12\,{\left(c+d\right)}^2}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(6\,c^4-12\,c^2\,d^2+6\,d^4\right)+{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(-4\,c^4-8\,c^3\,d+8\,c\,d^3+4\,d^4\right)-\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right.}$$
$$+\frac{5\,a^3\,\mathrm{atanh}\left(\frac{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)\,\left(4\,c-3\,d\right)}{4\,f\,{\left(c+d\right)}^{9/2}\,{\left(c-d\right)}^{3/2}}$$

input
```
int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^5),x)
```

output

```
((55*tan(e/2 + (f*x)/2)^5*(4*a^3*c^2 + 3*a^3*d^2 - 7*a^3*c*d))/(12*(c + d)
^3) - (73*tan(e/2 + (f*x)/2)^3*(4*a^3*c - 3*a^3*d))/(12*(c + d)^2) - (5*ta
n(e/2 + (f*x)/2)^7*(4*a^3*c^3 - 3*a^3*d^3 + 10*a^3*c*d^2 - 11*a^3*c^2*d))/
(4*(c + d)^4) + (a^3*tan(e/2 + (f*x)/2)*(44*c - 49*d))/(4*(c + d)*(c - d))
)/(f*(tan(e/2 + (f*x)/2)^4*(6*c^4 + 6*d^4 - 12*c^2*d^2) + tan(e/2 + (f*x)/
2)^2*(8*c*d^3 - 8*c^3*d - 4*c^4 + 4*d^4) - tan(e/2 + (f*x)/2)^6*(8*c*d^3 -
 8*c^3*d + 4*c^4 - 4*d^4) + tan(e/2 + (f*x)/2)^8*(c^4 - 4*c^3*d - 4*c*d^3
+ d^4 + 6*c^2*d^2) + 4*c*d^3 + 4*c^3*d + c^4 + d^4 + 6*c^2*d^2)) + (5*a^3*
atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(4*c - 3*d))/(4*f*
(c + d)^(9/2)*(c - d)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2232, normalized size of antiderivative = 8.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x)
```

output

```
(a**3*(480*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2
)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*c**4*d - 360*sqrt(
 - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**
2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*c**3*d**2 - 480*sqrt( - c**2 + d**
2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*co
s(e + f*x)*c**4*d + 360*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - ta
n((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c**3*d**2 - 480*sqrt(
 - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**
2 + d**2))*cos(e + f*x)*c**2*d**3 + 360*sqrt( - c**2 + d**2)*atan((tan((e
+ f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*c*d**
4 - 120*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*sin(e + f*x)**4*c**5 + 90*sqrt( - c**2 + d**2)*ata
n((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e +
f*x)**4*c**4*d + 240*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((
e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**5 - 180*sqrt( - c*
*2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d
**2))*sin(e + f*x)**2*c**4*d + 720*sqrt( - c**2 + d**2)*atan((tan((e + f*x
)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**3*d*
*2 - 540*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*
d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*c**2*d**3 - 120*sqrt( - c**2 +...
```

3.210 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 183

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx \\
&= \frac{d(8c^3-12c^2d+12cd^2-3d^3)\operatorname{arctanh}(\sin(e+fx))}{2af} \\
&\quad - \frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} \\
&\quad - \frac{d(4(3c^3-16c^2d+12cd^2-4d^3)+d(6c^2-20cd+9d^2)\sec(e+fx))\tan(e+fx)}{6af}
\end{aligned}$$

output

```
1/2*d*(8*c^3-12*c^2*d+12*c*d^2-3*d^3)*arctanh(sin(f*x+e))/a/f-1/3*(3*c-4*d
)*d*(c+d*sec(f*x+e))^2*tan(f*x+e)/a/f+(c-d)*(c+d*sec(f*x+e))^3*tan(f*x+e)/
f/(a+a*sec(f*x+e))-1/6*d*(12*c^3-64*c^2*d+48*c*d^2-16*d^3+d*(6*c^2-20*c*d+
9*d^2)*sec(f*x+e))*tan(f*x+e)/a/f
```

Mathematica [A] (verified)

Time = 4.78 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.81

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$
$$= 6(2c-d)d(2c^2-2cd+d^2)\coth^{-1}(\sin(e+fx))+3(4c-d)d^3\text{arctanh}(\sin(e+fx))+6(c-d)^4\tan(\tfrac{1}{2}(e$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

output
```
(6*(2*c - d)*d*(2*c^2 - 2*c*d + d^2)*ArcCoth[Sin[e + f*x]] + 3*(4*c - d)*d
^3*ArcTanh[Sin[e + f*x]] + 6*(c - d)^4*Tan[(e + f*x)/2] + 6*d^2*(6*c^2 - 4
*c*d + d^2)*Tan[e + f*x] + 3*(4*c - d)*d^3*Sec[e + f*x]*Tan[e + f*x] + 2*d
^4*Tan[e + f*x]*(3 + Tan[e + f*x]^2))/(6*a*f)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.67, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4475, 109, 25, 27, 170, 25, 27, 164, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a\sec(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(c+d\sec(e+fx))^4}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2d(4c-3d-(3c-4d)\sec(e+fx))(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2d(4c-3d-(3c-4d)\sec(e+fx))(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\int \frac{(4c-3d-(3c-4d)\sec(e+fx))(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 170$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(\frac{(3c-4d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}{3a^2}-\frac{\int -\frac{a^2(c+d\sec(e+fx))\left(12c^2-15dc+8d^2-\left(6c^2-20dc+9d^2\right)\sec(e+}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}}{3a^2}\right.}{a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(\frac{\int \frac{a^2(c+d\sec(e+fx))\left(12c^2-15dc+8d^2-\left(6c^2-20dc+9d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{3a^2}+\frac{(3c-4d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}(c-}{3a^2}\right.}{a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(\frac{1}{3}\int \frac{(c+d\sec(e+fx))\left(12c^2-15dc+8d^2-\left(6c^2-20dc+9d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)+\frac{(3c-4d)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}(c}{3a^2}\right.}{a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 164$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(\frac{1}{3}\left(\frac{3}{2}\left(8c^3-12c^2d+12cd^2-3d^3\right)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(6c^2-\right.\right.}{}\right.\right.}{a}\right.}{f\sqrt{a-a\sec(e}}$$

$$\Big\downarrow \; 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(\frac{1}{3}\left(3\left(8c^3-12c^2d+12cd^2-3d^3\right)\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(6c^2-20cd\right.\right.}{2}\right.\right.}{a}\right.}{f\sqrt{a-a\sec(e+}}$$

$$\Big\downarrow \; 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(6c^2-20cd+9d^2\right)\sec(e+fx)+4\left(3c^3-16c^2d+12cd^2-4d^3\right)\right)}{2a^2}-\frac{3\left(8c^3-12c^2d+12cd^2-3d^3\right)}{}\right.\right.}{a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{}}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

output

```
-((a^2*(-(((c - d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3)/(a^2*S
qrt[a + a*Sec[e + f*x]])) + (d*(((3*c - 4*d)*Sqrt[a - a*Sec[e + f*x]]*Sqrt
[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2)/(3*a^2) + ((-3*(8*c^3 - 12*c^
2*d + 12*c*d^2 - 3*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e +
 f*x]]])/a + (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]*(4*(3*c^3
- 16*c^2*d + 12*c*d^2 - 4*d^3) + d*(6*c^2 - 20*c*d + 9*d^2)*Sec[e + f*x]))
/(2*a^2))/3))/a)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.50

method	result
parallelrisch	$\frac{-12d\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\left(c^3-\frac{3}{2}c^2d+\frac{3}{2}c\,d^2-\frac{3}{8}d^3\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+12d\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\left(c^3-\frac{3}{2}\right.}{}$
derivativedivides	$\frac{c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4c^3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+6c^2d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4c\,d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-}{}$
default	$\frac{c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4c^3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+6c^2d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4c\,d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-}{}$
norman	$\frac{\frac{\left(c^4-4c^3d+6c^2d^2-4c\,d^3+d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{fa}+\frac{\left(c^4-4c^3d+18c^2d^2-8c\,d^3+4d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}-\frac{\left(4c^4-16c^3d+36c^2d^2-28c\,d^3+9d^4\right.}{fa}}{\left(\tan\left(\frac{fx}{2}\right.\right.}$
risch	$\frac{i\left(-24c^3d+72c^2d^2-48c\,d^3+16d^4+6c^4-48c\,d^3\mathrm{e}^{3i(fx+e)}-72c^3d\,\mathrm{e}^{2i(fx+e)}+36c^2d^2\mathrm{e}^{5i(fx+e)}+180c^2d^2\mathrm{e}^{2i(fx+e)}-108c\right.}{}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
3*(-4*d*(cos(f*x+e)+1/3*cos(3*f*x+3*e))*(c^3-3/2*c^2*d+3/2*c*d^2-3/8*d^3)*
ln(tan(1/2*f*x+1/2*e)-1)+4*d*(cos(f*x+e)+1/3*cos(3*f*x+3*e))*(c^3-3/2*c^2*
d+3/2*c*d^2-3/8*d^3)*ln(tan(1/2*f*x+1/2*e)+1)+((1/3*c^4-4/3*c^3*d+4*c^2*d^
2-8/3*c*d^3+8/9*d^4)*cos(3*f*x+3*e)+(4*c^2*d^2-4/3*c*d^3+7/9*d^4)*cos(2*f*
x+2*e)+(c^4-4*c^3*d+12*c^2*d^2-16/3*c*d^3+22/9*d^4)*cos(f*x+e)+4*c^2*d^2-4
/3*c*d^3+11/9*d^4)*tan(1/2*f*x+1/2*e))/a/f/(cos(3*f*x+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.62

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$
$$= \frac{3\left(\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3\right)\log\left(\mathrm{s}\right.}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
1/12*(3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3
*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(sin(f*x + e) + 1)
- 3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3*d -
 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2
*(2*d^4 + 2*(3*c^4 - 12*c^3*d + 36*c^2*d^2 - 24*c*d^3 + 8*d^4)*cos(f*x + e
)^3 + (36*c^2*d^2 - 12*c*d^3 + 7*d^4)*cos(f*x + e)^2 + (12*c*d^3 - d^4)*co
s(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c^4\sec(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{d^4\sec^5(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{4cd^3\sec^4(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{6c^2d^2\sec^3(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{4c^3d\sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
```

output
```
(Integral(c**4*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**4*sec(e +
 f*x)**5/(sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/(sec(e
 + f*x) + 1), x) + Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x) + 1)
, x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(174) = 348$.

Time = 0.04 (sec) , antiderivative size = 596, normalized size of antiderivative = 3.26

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
1/6*(d^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(
a*(cos(f*x + e) + 1))) - 12*c*d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*s
in(f*x + e)/(a*(cos(f*x + e) + 1))) - 36*c^2*d^2*(log(sin(f*x + e)/(cos(f*
x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*
x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) -
sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 24*c^3*d*(log(sin(f*x + e)/(cos(f*x
 + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x +
 e)/(a*(cos(f*x + e) + 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.88

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{\frac{3\left(8\,c^{3}d-12\,c^{2}d^{2}+12\,cd^{3}-3\,d^{4}\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{3\left(8\,c^{3}d-12\,c^{2}d^{2}+12\,cd^{3}-3\,d^{4}\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}+\frac{6\left(c^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e}{}}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="gia
c")
```

output

```
1/6*(3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*x + 1/2
*e) + 1))/a - 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/a + 6*(c^4*tan(1/2*f*x + 1/2*e) - 4*c^3*d*tan(1/2*f*x +
 1/2*e) + 6*c^2*d^2*tan(1/2*f*x + 1/2*e) - 4*c*d^3*tan(1/2*f*x + 1/2*e) +
d^4*tan(1/2*f*x + 1/2*e))/a - 2*(36*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 36*c*
d^3*tan(1/2*f*x + 1/2*e)^5 + 15*d^4*tan(1/2*f*x + 1/2*e)^5 - 72*c^2*d^2*ta
n(1/2*f*x + 1/2*e)^3 + 48*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 16*d^4*tan(1/2*f*
x + 1/2*e)^3 + 36*c^2*d^2*tan(1/2*f*x + 1/2*e) - 12*c*d^3*tan(1/2*f*x + 1/
2*e) + 9*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a))/f
```

Mupad [B] (verification not implemented)

Time = 11.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$

$$= \frac{\left(12\,c^2\,d^2-12\,c\,d^3+5\,d^4\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-24\,c^2\,d^2+16\,c\,d^3-\frac{16\,d^4}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(12\,c^2\,d^2-4\,c\,d\right.}{f\,\left(-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\right)}$$

$$+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c-d\right)^4}{a\,f}+\frac{d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(8\,c^3-12\,c^2\,d+12\,c\,d^2-3\,d^3\right)}{a\,f}$$

input

```
int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output

```
(tan(e/2 + (f*x)/2)*(3*d^4 - 4*c*d^3 + 12*c^2*d^2) + tan(e/2 + (f*x)/2)^5*
(5*d^4 - 12*c*d^3 + 12*c^2*d^2) - tan(e/2 + (f*x)/2)^3*((16*d^4)/3 - 16*c*
d^3 + 24*c^2*d^2))/(f*(a - 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (f*x)/
2)^4 - a*tan(e/2 + (f*x)/2)^6)) + (tan(e/2 + (f*x)/2)*(c - d)^4)/(a*f) + (
d*atanh(tan(e/2 + (f*x)/2))*(12*c*d^2 - 12*c^2*d + 8*c^3 - 3*d^3))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 867, normalized size of antiderivative = 4.74

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x)
```

output

```
( - 24*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**3*c**3*d + 36*
cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**3*c**2*d**2 - 36*cos(
e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**3*c*d**3 + 9*cos(e + f*x)
*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**3*d**4 + 24*cos(e + f*x)*log(tan(
(e + f*x)/2) - 1)*sin(e + f*x)*c**3*d - 36*cos(e + f*x)*log(tan((e + f*x)/
2) - 1)*sin(e + f*x)*c**2*d**2 + 36*cos(e + f*x)*log(tan((e + f*x)/2) - 1)
*sin(e + f*x)*c*d**3 - 9*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*
x)*d**4 + 24*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**3*c**3*d
 - 36*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**3*c**2*d**2 + 3
6*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**3*c*d**3 - 9*cos(e
+ f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**3*d**4 - 24*cos(e + f*x)*lo
g(tan((e + f*x)/2) + 1)*sin(e + f*x)*c**3*d + 36*cos(e + f*x)*log(tan((e +
 f*x)/2) + 1)*sin(e + f*x)*c**2*d**2 - 36*cos(e + f*x)*log(tan((e + f*x)/2
) + 1)*sin(e + f*x)*c*d**3 + 9*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(
e + f*x)*d**4 + 6*cos(e + f*x)*sin(e + f*x)**2*c**4 - 24*cos(e + f*x)*sin(
e + f*x)**2*c**3*d + 36*cos(e + f*x)*sin(e + f*x)**2*c**2*d**2 - 36*cos(e
+ f*x)*sin(e + f*x)**2*c*d**3 + 9*cos(e + f*x)*sin(e + f*x)**2*d**4 - 6*co
s(e + f*x)*c**4 + 24*cos(e + f*x)*c**3*d - 36*cos(e + f*x)*c**2*d**2 + 24*
cos(e + f*x)*c*d**3 - 6*cos(e + f*x)*d**4 + 6*sin(e + f*x)**4*c**4 - 24*si
n(e + f*x)**4*c**3*d + 72*sin(e + f*x)**4*c**2*d**2 - 48*sin(e + f*x)**...
```

3.211 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 117

$$\begin{aligned}&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx\\&=\frac{3d(2c^2-2cd+d^2)\,\mathrm{arctanh}(\sin(e+fx))}{2af}+\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}\\&\quad-\frac{d(4(c^2-3cd+d^2)+(2c-3d)d\sec(e+fx))\tan(e+fx)}{2af}\end{aligned}$$

output
```
3/2*d*(2*c^2-2*c*d+d^2)*arctanh(sin(f*x+e))/a/f+(c-d)*(c+d*sec(f*x+e))^2*t
an(f*x+e)/f/(a+a*sec(f*x+e))-1/2*d*(4*c^2-12*c*d+4*d^2+(2*c-3*d)*d*sec(f*x
+e))*tan(f*x+e)/a/f
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\begin{aligned}&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx\\&=\frac{2d(3c^2-3cd+d^2)\coth^{-1}(\sin(e+fx))+d^3\mathrm{arctanh}(\sin(e+fx))+2(c-d)^3\tan\left(\frac{1}{2}(e+fx)\right)+2(3c-}{2af}\end{aligned}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

output
```
(2*d*(3*c^2 - 3*c*d + d^2)*ArcCoth[Sin[e + f*x]] + d^3*ArcTanh[Sin[e + f*x
]] + 2*(c - d)^3*Tan[(e + f*x)/2] + 2*(3*c - d)*d^2*Tan[e + f*x] + d^3*Sec
[e + f*x]*Tan[e + f*x])/(2*a*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 4475, 109, 25, 27, 164, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2d(3c-2d-(2c-3d)\sec(e+fx))(c+d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2d(3c-2d-(2c-3d)\sec(e+fx))(c+d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\int \frac{(3c-2d-(2c-3d)\sec(e+fx))(c+d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 164$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(\frac{3}{2}\left(2c^2-2cd+d^2\right)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(4\left(c^2-3cd+d^2\right)+d(2c-}{2a^2}}{a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(3\left(2c^2-2cd+d^2\right)\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(4\left(c^2-3cd+d^2\right)+d(2c-3d)\,\mathrm{se}}{2a^2}}{a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(4\left(c^2-3cd+d^2\right)+d(2c-3d)\sec(e+fx)\right)}{2a^2}-\frac{3\left(2c^2-2cd+d^2\right)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)}{a}\right)}{a}-\frac{(c}{}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

output

```
-((a^2*(-(((c - d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2)/(a^2*S
qrt[a + a*Sec[e + f*x]])) + (d*((-3*(2*c^2 - 2*c*d + d^2)*ArcTan[Sqrt[a -
a*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]]])/a + (Sqrt[a - a*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]]*(4*(c^2 - 3*c*d + d^2) + (2*c - 3*d)*d*Sec[e + f*x
]))/(2*a^2)))/a)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.55

method	result
parallelrisch	$\frac{-3\left(c^{2}-cd+\frac{1}{2}d^{2}\right)d\left(1+\cos\left(2fx+2e\right)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+3\left(c^{2}-cd+\frac{1}{2}d^{2}\right)d\left(1+\cos\left(2fx+2e\right)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+t}{fa\left(1+\cos\left(2fx+2e\right)\right)}$
derivativedivides	$\frac{c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3c^{2}d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c\,d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{d^{3}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{3d\left(2c^{2}-2cd+d^{2}\right)\ln\left(\tan\right.}{2}}{fa}$
default	$\frac{c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3c^{2}d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c\,d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{d^{3}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{3d\left(2c^{2}-2cd+d^{2}\right)\ln\left(\tan\right.}{2}}{fa}$
norman	$\frac{\frac{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{fa}+\frac{\left(3c^{3}-9c^{2}d+21c\,d^{2}-7d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{fa}-\frac{3\left(c^{3}-3c^{2}d+5c\,d^{2}-2d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{fa}-\frac{\left(c^{3}-\right.}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$
risch	$\frac{i\left(2c^{3}\mathrm{e}^{4i(fx+e)}-6c^{2}d\,\mathrm{e}^{4i(fx+e)}+6c\,d^{2}\mathrm{e}^{4i(fx+e)}-3d^{3}\mathrm{e}^{4i(fx+e)}+6c\,d^{2}\mathrm{e}^{3i(fx+e)}-3d^{3}\mathrm{e}^{3i(fx+e)}+4c^{3}\mathrm{e}^{2i(fx+e)}-12c^{2}d\,\mathrm{e}\right.}{fa\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{2i(fx+e)}-\right.}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
(-3*(c^2-c*d+1/2*d^2)*d*(1+cos(2*f*x+2*e))*ln(tan(1/2*f*x+1/2*e)-1)+3*(c^2
-c*d+1/2*d^2)*d*(1+cos(2*f*x+2*e))*ln(tan(1/2*f*x+1/2*e)+1)+tan(1/2*f*x+1/
2*e)*((c^3-3*c^2*d+6*c*d^2-2*d^3)*cos(2*f*x+2*e)+(6*c*d^2-d^3)*cos(f*x+e)+
c^3-3*c^2*d+6*c*d^2-d^3))/f/a/(1+cos(2*f*x+2*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.85

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$$
$$= \frac{3\left(\left(2\,c^2d-2\,cd^2+d^3\right)\cos\left(fx+e\right)^3+\left(2\,c^2d-2\,cd^2+d^3\right)\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(\left(2\,c\right.\right.}{}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
1/4*(3*((2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^
3)*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 3*((2*c^2*d - 2*c*d^2 + d^3)*co
s(f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^2)*log(-sin(f*x + e)
 + 1) + 2*(d^3 + 2*(c^3 - 3*c^2*d + 6*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (6*c
*d^2 - d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x
+ e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$$
$$= \frac{\int \frac{c^3\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{d^3\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{3cd^2\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{3c^2d\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

output

```
(Integral(c**3*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**3*sec(e +
 f*x)**4/(sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/(sec(e
 + f*x) + 1), x) + Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x
))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(114) = 228$.

Time = 0.05 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.32

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{d^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+6cd^2\left(\frac{\log(}{}\right.}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-1/2*(d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a +
3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x
+ e) + 1))) + 6*c*d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x +
e) + 1))) - 6*c^2*d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1)))
- 2*c^3*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.87

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \frac{\frac{3\left(2c^2d-2cd^2+d^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{3\left(2c^2d-2cd^2+d^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}+\frac{2\left(c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.}{}}{2f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
1/2*(3*(2*c^2*d - 2*c*d^2 + d^3)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 3*
(2*c^2*d - 2*c*d^2 + d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a + 2*(c^3*ta
n(1/2*f*x + 1/2*e) - 3*c^2*d*tan(1/2*f*x + 1/2*e) + 3*c*d^2*tan(1/2*f*x +
1/2*e) - d^3*tan(1/2*f*x + 1/2*e))/a - 2*(6*c*d^2*tan(1/2*f*x + 1/2*e)^3 -
 3*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*c*d^2*tan(1/2*f*x + 1/2*e) + d^3*tan(1/2
*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a))/f
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx &= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(6\,c\,d^2-d^3\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\,\left(6\,c\,d^2-3\,d^3\right)}{f\,\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-2\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a\right)}\\&\quad+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(c-d\right)^3}{a\,f}+\frac{3\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\,\left(2\,c^2-2\,c\,d+d^2\right)}{a\,f}\end{aligned}$$

input
```
int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output
```
(tan(e/2 + (f*x)/2)*(6*c*d^2 - d^3) - tan(e/2 + (f*x)/2)^3*(6*c*d^2 - 3*d^
3))/(f*(a - 2*a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4)) + (tan(e/2
 + (f*x)/2)*(c - d)^3)/(a*f) + (3*d*atanh(tan(e/2 + (f*x)/2))*(2*c^2 - 2*c
*d + d^2))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 525, normalized size of antiderivative = 4.49

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x)
```

output

```
( - 2*cos(e + f*x)*sin(e + f*x)**2*c**3 + 6*cos(e + f*x)*sin(e + f*x)**2*c
**2*d - 12*cos(e + f*x)*sin(e + f*x)**2*c*d**2 + 4*cos(e + f*x)*sin(e + f*
x)**2*d**3 + 2*cos(e + f*x)*c**3 - 6*cos(e + f*x)*c**2*d + 6*cos(e + f*x)*
c*d**2 - 2*cos(e + f*x)*d**3 - 6*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**3
*c**2*d + 6*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**3*c*d**2 - 3*log(tan((
e + f*x)/2) - 1)*sin(e + f*x)**3*d**3 + 6*log(tan((e + f*x)/2) - 1)*sin(e
+ f*x)*c**2*d - 6*log(tan((e + f*x)/2) - 1)*sin(e + f*x)*c*d**2 + 3*log(ta
n((e + f*x)/2) - 1)*sin(e + f*x)*d**3 + 6*log(tan((e + f*x)/2) + 1)*sin(e
+ f*x)**3*c**2*d - 6*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**3*c*d**2 + 3*
log(tan((e + f*x)/2) + 1)*sin(e + f*x)**3*d**3 - 6*log(tan((e + f*x)/2) +
1)*sin(e + f*x)*c**2*d + 6*log(tan((e + f*x)/2) + 1)*sin(e + f*x)*c*d**2 -
 3*log(tan((e + f*x)/2) + 1)*sin(e + f*x)*d**3 + 2*sin(e + f*x)**2*c**3 -
6*sin(e + f*x)**2*c**2*d + 6*sin(e + f*x)**2*c*d**2 - 3*sin(e + f*x)**2*d*
*3 - 2*c**3 + 6*c**2*d - 6*c*d**2 + 2*d**3)/(2*sin(e + f*x)*a*f*(sin(e + f
*x)**2 - 1))
```

3.212 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 68

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{(2c-d)d\operatorname{arctanh}(\sin(e+fx))}{af} + \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output
```
(2*c-d)*d*arctanh(sin(f*x+e))/a/f+d^2*tan(f*x+e)/a/f+(c-d)^2*tan(f*x+e)/f/
(a+a*sec(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(68) = 136.

Time = 2.49 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.49

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$$

$$= 2\cos\left(\frac{1}{2}(e+fx)\right)\cos(e+fx)(c+d\sec(e+fx))^2\Big((c-d)^2\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+d\cos\left(\frac{1}{2}(e+fx)\right)\Big(-((2c$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

output
```
(2*Cos[(e + f*x)/2]*Cos[e + f*x]*(c + d*Sec[e + f*x])^2*((c - d)^2*Sec[e/2
]*Sin[(f*x)/2] + d*Cos[(e + f*x)/2]*(-((2*c - d)*(Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + (d*Sin[f*x
])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))))/(a*f*(d + c*Cos[e +
f*x])^2*(1 + Sec[e + f*x]))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(68) = 136.

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.49, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 4475, 100, 27, 90, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^3d(2c-d+d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^4}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\int\frac{2c-d+d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{90}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left((2c-d)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)-\frac{d\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a^2}\right)}{a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{a^2\sqrt{a\sec(e+fx)+a}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{45}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(2(2c-d)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}-\frac{d\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a^2}\right)}{a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{d\left(-\frac{d\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}{a^2}-\frac{2(2c-d)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)}{a}\right)}{a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

output
```
-((a^2*(-(((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(a^2*Sqrt[a + a*Sec[e + f*x
]])) + (d*((-2*(2*c - d)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e
+ f*x]]])/a - (d*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])/a^2))/
a)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.51

method	result
parallelrisch	$\frac{-2d\cos(fx+e)(c-\frac{d}{2})\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)+2d\cos(fx+e)(c-\frac{d}{2})\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)+((c^2-2cd+2d^2)\cos(fx+e)+}{\cos(fx+e)af}$
derivativedivides	$\frac{c^2\tan(\frac{fx}{2}+\frac{e}{2})-2\tan(\frac{fx}{2}+\frac{e}{2})cd+d^2\tan(\frac{fx}{2}+\frac{e}{2})-\frac{d^2}{\tan(\frac{fx}{2}+\frac{e}{2})-1}-d(2c-d)\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)-\frac{d^2}{\tan(\frac{fx}{2}+\frac{e}{2})+1}+}{fa}$
default	$\frac{c^2\tan(\frac{fx}{2}+\frac{e}{2})-2\tan(\frac{fx}{2}+\frac{e}{2})cd+d^2\tan(\frac{fx}{2}+\frac{e}{2})-\frac{d^2}{\tan(\frac{fx}{2}+\frac{e}{2})-1}-d(2c-d)\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)-\frac{d^2}{\tan(\frac{fx}{2}+\frac{e}{2})+1}+}{fa}$
norman	$\frac{\frac{(c^2-2cd+d^2)\tan(\frac{fx}{2}+\frac{e}{2})^5}{fa}+\frac{(c^2-2cd+3d^2)\tan(\frac{fx}{2}+\frac{e}{2})}{fa}-\frac{2(c^2-2cd+2d^2)\tan(\frac{fx}{2}+\frac{e}{2})^3}{fa}}{(\tan(\frac{fx}{2}+\frac{e}{2})^2-1)^2}+\frac{d(2c-d)\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1}{fa}$
risch	$\frac{2i(c^2e^{2i(fx+e)}-2cd\,e^{2i(fx+e)}+d^2e^{2i(fx+e)}+d^2e^{i(fx+e)}+c^2-2cd+2d^2)}{fa(e^{2i(fx+e)}+1)(e^{i(fx+e)}+1)}+\frac{2d\ln(e^{i(fx+e)}+i)c}{fa}-\frac{d^2\ln(e^{i(fx+e)}+i}{fa}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
(-2*d*cos(f*x+e)*(c-1/2*d)*ln(tan(1/2*f*x+1/2*e)-1)+2*d*cos(f*x+e)*(c-1/2*
d)*ln(tan(1/2*f*x+1/2*e)+1)+((c^2-2*c*d+2*d^2)*cos(f*x+e)+d^2)*tan(1/2*f*x
+1/2*e))/cos(f*x+e)/a/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.28

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{((2\,cd-d^2)\cos{(fx+e)}^2+(2\,cd-d^2)\cos{(fx+e)})\log{(\sin{(fx+e)}+1)}-((2\,cd-d^2)\cos{(fx+e)}^2}{2\,(af\cos{(fx+e)}^2}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
1/2*(((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))*log(sin(f
*x + e) + 1) - ((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))
*log(-sin(f*x + e) + 1) + 2*(d^2 + (c^2 - 2*c*d + 2*d^2)*cos(f*x + e))*sin
(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c^2\sec{(e+fx)}}{\sec{(e+fx)}+1}\,dx + \int \frac{d^2\sec^3{(e+fx)}}{\sec{(e+fx)}+1}\,dx + \int \frac{2cd\sec^2{(e+fx)}}{\sec{(e+fx)}+1}\,dx}{a}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

output
```
(Integral(c**2*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**2*sec(e +
 f*x)**3/(sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(sec(e +
f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(68) = 136$.

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.28

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx =$$

$$-\frac{d^2\Bigg(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\,\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\Bigg)-2\,cd\Bigg(}{f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
-(d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(
f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - 2*c*
d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
+ e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)
/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.00

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$$

$$=\frac{\frac{(2\,cd-d^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{(2\,cd-d^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a}+\frac{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,cd\tan\left(\frac{1}{2}\,fx\right.}{a}}{f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
((2*c*d - d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - (2*c*d - d^2)*log(ab
s(tan(1/2*f*x + 1/2*e) - 1))/a - 2*d^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x
+ 1/2*e)^2 - 1)*a) + (c^2*tan(1/2*f*x + 1/2*e) - 2*c*d*tan(1/2*f*x + 1/2*e
) + d^2*tan(1/2*f*x + 1/2*e))/a)/f
```

Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(c-d\right)}^2}{a\,f} + \frac{2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\right)} + \frac{2\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,c-d\right)}{a\,f}$$

input
```
int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output
```
(tan(e/2 + (f*x)/2)*(c - d)^2)/(a*f) + (2*d^2*tan(e/2 + (f*x)/2))/(f*(a -
a*tan(e/2 + (f*x)/2)^2)) + (2*d*atanh(tan(e/2 + (f*x)/2))*(2*c - d))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.21

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$$
$$= \frac{-2\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)\,cd+\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)}{}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x)
```

output
```
( - 2*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)*c*d + cos(e + f*
x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)*d**2 + 2*cos(e + f*x)*log(tan((e
 + f*x)/2) + 1)*sin(e + f*x)*c*d - cos(e + f*x)*log(tan((e + f*x)/2) + 1)*
sin(e + f*x)*d**2 + cos(e + f*x)*c**2 - 2*cos(e + f*x)*c*d + cos(e + f*x)*
d**2 + sin(e + f*x)**2*c**2 - 2*sin(e + f*x)**2*c*d + 2*sin(e + f*x)**2*d*
*2 - c**2 + 2*c*d - d**2)/(cos(e + f*x)*sin(e + f*x)*a*f)
```

3.213 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 43

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx=\frac{d\operatorname{arctanh}(\sin(e+fx))}{af}+\frac{(c-d)\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output
```
d*arctanh(sin(f*x+e))/a/f+(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(43) = 86.

Time = 0.76 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.53

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx$$
$$=\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\Bigl(d\cos\left(\frac{1}{2}(e+fx)\right)\Bigl(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\Bigl(\cos\left(\frac{1}{2}(e+fx)\right)}{af(1+\cos(e+fx))}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

output

```
(2*Cos[(e + f*x)/2]*(d*Cos[(e + f*x)/2]*(-Log[Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Sec[e/2]*Si
n[(f*x)/2]))/(a*f*(1 + Cos[e + f*x]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4486}$$

$$(c-d)\int \frac{\sec(e+fx)}{\sec(e+fx)a+a}dx+\frac{d\int \sec(e+fx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+\frac{d\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+\frac{d\text{arctanh}(\sin(e+fx))}{af}$$

$$\Big\downarrow \text{4281}$$

$$\frac{d\text{arctanh}(\sin(e+fx))}{af}+\frac{(c-d)\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

output
```
(d*ArcTanh[Sin[e + f*x]])/(a*f) + ((c - d)*Tan[e + f*x])/(f*(a + a*Sec[e +
 f*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

method	result	size
parallelrisch	$\frac{-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(c-d)}{fa}$	53
derivativedivides	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d}{fa}$	61
default	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d}{fa}$	61
risch	$\frac{2ic}{fa\left(\mathrm{e}^{i(fx+e)}+1\right)}-\frac{2id}{fa\left(\mathrm{e}^{i(fx+e)}+1\right)}+\frac{d\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{fa}-\frac{d\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{fa}$	91
norman	$\frac{\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{fa}-\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}+\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{fa}-\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{fa}$	105

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output

```
(-ln(tan(1/2*f*x+1/2*e)-1)*d+ln(tan(1/2*f*x+1/2*e)+1)*d+tan(1/2*f*x+1/2*e)
*(c-d))/f/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx$$
$$=\frac{(d\cos{(fx+e)}+d)\log{(\sin{(fx+e)}+1)}-(d\cos{(fx+e)}+d)\log{(-\sin{(fx+e)}+1)}+2\,(c-d)\sin}{2\,(af\cos{(fx+e)}+af)}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="frica
s")
```

output

```
1/2*((d*cos(f*x + e) + d)*log(sin(f*x + e) + 1) - (d*cos(f*x + e) + d)*log
(-sin(f*x + e) + 1) + 2*(c - d)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx=\frac{\int \frac{c\sec(e+fx)}{\sec(e+fx)+1}\,dx+\int \frac{d\sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

output
```
(Integral(c*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d*sec(e + f*x)*
*2/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(43) = 86$.

Time $= 0.04$ (sec) , antiderivative size $= 99$, normalized size of antiderivative $= 2.30$

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx$$
$$=\frac{d\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+\frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}}{f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
(d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*sin(f*x + e)/
(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{\frac{d\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{d\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}+\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}}{f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
(d*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - d*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/a + (c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/a)/f
```

Mupad [B] (verification not implemented)

Time = 10.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(c-d)}{a\,f}+\frac{2\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

input
```
int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

output
```
(tan(e/2 + (f*x)/2)*(c - d))/(a*f) + (2*d*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d+\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{af}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

output
```
( - log(tan((e + f*x)/2) - 1)*d + log(tan((e + f*x)/2) + 1)*d + tan((e + f
*x)/2)*c - tan((e + f*x)/2)*d)/(a*f)
```

3.214 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 83

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = -\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a(c-d)^{3/2}\sqrt{c+d}f}+\frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))}$$

output
```
-2*d*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a/(c-d)^(3/2)/(c+
d)^(1/2)/f+tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.93

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\frac{2d\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos\left(\frac{1}{2}(e+fx)\right)(i\cos(e)+\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right.\right.}{a(c-d)f(1+\cos(e+fx))}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

output
```
(2*Cos[(e + f*x)/2]*((2*d*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*
Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[
(e + f*x)/2]*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e]
)^2]) + Sec[e/2]*Sin[(f*x)/2]))/(a*(c - d)*f*(1 + Cos[e + f*x]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4475, 107, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{107}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{d\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{2d\int \frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{2d\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2(c-d)^{3/2}\sqrt{c+d}}-\frac{\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

output

```
-((a^2*((-2*d*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*S
qrt[a - a*Sec[e + f*x]])])/(a^2*(c - d)^(3/2)*Sqrt[c + d]) - Sqrt[a - a*Se
c[e + f*x]]/(a^2*(c - d)*Sqrt[a + a*Sec[e + f*x]]))*Tan[e + f*x])/(f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c-d}-\frac{2d\,\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c-d)\sqrt{(c-d)(c+d)}}}{fa}$	74
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c-d}-\frac{2d\,\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c-d)\sqrt{(c-d)(c+d)}}}{fa}$	74
risch	$\frac{2i}{fa(c-d)\left(e^{i(fx+e)}+1\right)}+\frac{d\ln\left(e^{i(fx+e)}-\frac{ic^2-id^2-d\sqrt{c^2-d^2}}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(c-d)fa}-\frac{d\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+d\sqrt{c^2-d^2}}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(c-d)fa}$	190

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f/a*(tan(1/2*f*x+1/2*e)/(c-d)-2*d/(c-d)/((c-d)*(c+d))^(1/2)*arctanh((c-d
)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.25

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx$$
$$=\left[-\frac{\sqrt{c^2-d^2}(d\cos(fx+e)+d)\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)}{2\,((ac^3-ac^2d-acd^2+ad^3)f\cos(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f)}\right.$$
$$\left.-\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+d)\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)-(c^2-d^2)\sin(fx+e)}{(ac^3-ac^2d-acd^2+ad^3)f\cos(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f}\right]$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="frica
s")
```

output
```
[-1/2*(sqrt(c^2 - d^2)*(d*cos(f*x + e) + d)*log((2*c*d*cos(f*x + e) - (c^2
 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x
+ e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(
c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x +
e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f), -(sqrt(-c^2 + d^2)*(d*cos(f*x
 + e) + d)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(
f*x + e))) - (c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3
)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx=\frac{\int \frac{\sec(e+fx)}{c\sec(e+fx)+c+d\sec^2(e+fx)+d\sec(e+fx)}\,dx}{a}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sec(e + f*x)/(c*sec(e + f*x) + c + d*sec(e + f*x)**2 + d*sec(e +
f*x)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d}{(ac-ad)\sqrt{-c^2+d^2}}+\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac-ad}}{f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
(2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d/((a*c - a*d)*sqr
t(-c^2 + d^2)) + tan(1/2*f*x + 1/2*e)/(a*c - a*d))/f
```

Mupad [B] (verification not implemented)

Time = 10.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,\left(c-d\right)}-\frac{2\,d\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2-2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^2}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c+d}\,{\left(c-d\right)}^{3/2}}\right)}{a\,f\,\sqrt{c+d}\,{\left(c-d\right)}^{3/2}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))),x)
```

output
```
tan(e/2 + (f*x)/2)/(a*f*(c - d)) - (2*d*atanh((c^2*sin(e/2 + (f*x)/2) + d^
2*sin(e/2 + (f*x)/2) - 2*c*d*sin(e/2 + (f*x)/2))/(cos(e/2 + (f*x)/2)*(c +
d)^(1/2)*(c - d)^(3/2))))/(a*f*(c + d)^(1/2)*(c - d)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{-2\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)d+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2}{af\left(c^3-c^2d-c\,d^2+d^3\right)}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output
```
( - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*d + tan((e + f*x)/2)*c**2 - tan((e + f*x)/2)*d**2)/(
a*f*(c**3 - c**2*d - c*d**2 + d**3))
```

3.215 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 145

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx &= -\frac{2d(2c+d)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a(c-d)^{5/2}(c+d)^{3/2}f}+\frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))}\\ &\quad-\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))(c+d\sec(e+fx))}\end{aligned}$$

output
```
-2*d*(2*c+d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a/(c-d)^(
5/2)/(c+d)^(3/2)/f+(c+2*d)*tan(f*x+e)/(c-d)^2/(c+d)/f/(a+a*sec(f*x+e))-d*t
an(f*x+e)/(c^2-d^2)/f/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.97

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^3(e+fx)\left(\frac{2d(2c+d)\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin}\right.}{a(c-d)^2f(1+\sec(e+}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

output
```
(2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^3*((2*d*(2*c + d)*Ar
cTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt
[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]*(d + c*Cos[e +
f*x])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e
])^2]) + (d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (d^2*Cos[(e + f*x)/2
]*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] +
 Sin[e/2]))))/(a*(c - d)^2*f*(1 + Sec[e + f*x])*(c + d*Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.61, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 4475, 114, 27, 169, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)(c+d\sec(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int\frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 114$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int\frac{a^2(c+d-d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{a^2(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}}{a^2(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int\frac{c+d-d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{c^2-d^2}+\frac{d\sqrt{a-a\sec(e+fx)}}{a^2(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{\int\frac{a^2d(2c+d)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a^3(c-d)}-\frac{(c+2d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{c^2-d^2}+\frac{d\sqrt{a-a\sec(e+f}}{a^2(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{d(2c+d)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a(c-d)}-\frac{(c+2d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{c^2-d^2}+\frac{d\sqrt{a-a\,s}}{a^2(c^2-d^2)\sqrt{a\sec(e+f}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{2d(2c+d)\int\frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{a(c-d)}-\frac{(c+2d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{c^2-d^2}+\frac{d\sqrt{a-a\sec(e+fx}}{a^2(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{2d(2c+d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2(c-d)^{3/2}\sqrt{c+d}}-\frac{(c+2d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{c^2-d^2}+\frac{d\sqrt{a-a\sec(e+fx)}}{a^2(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

output
```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c^2 - d^2)*Sqrt[a + a*Sec[e + f
*x]]*(c + d*Sec[e + f*x])) + ((-2*d*(2*c + d)*ArcTan[(Sqrt[c + d]*Sqrt[a +
 a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]])/(a^2*(c - d)^(3
/2)*Sqrt[c + d]) - ((c + 2*d)*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*Sqrt[
a + a*Sec[e + f*x]]))/(c^2 - d^2))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x
]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^{2}-2cd+d^{2}}+\frac{4d\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}-\frac{(2c+d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{2(c+d)\sqrt{(c-d)(c+d)}}\right)}{(c-d)^{2}}}{fa}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^{2}-2cd+d^{2}}+\frac{4d\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}-\frac{(2c+d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{2(c+d)\sqrt{(c-d)(c+d)}}\right)}{(c-d)^{2}}}{fa}$
risch	$\frac{2i\left(c^{3}\mathrm{e}^{2i(fx+e)}+c^{2}d\,\mathrm{e}^{2i(fx+e)}+d^{3}\mathrm{e}^{2i(fx+e)}+2c^{2}d\,\mathrm{e}^{i(fx+e)}+3c\,d^{2}\mathrm{e}^{i(fx+e)}+d^{3}\mathrm{e}^{i(fx+e)}+c^{3}+c^{2}d+c\,d^{2}\right)}{\left(\mathrm{e}^{i(fx+e)}+1\right)\left(c\,\mathrm{e}^{2i(fx+e)}+2d\,\mathrm{e}^{i(fx+e)}+c\right)f(c-d)^{2}ac(c+d)}+\frac{2d\ln\left(\mathrm{e}^{i(f}\right.}{\sqrt{c}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/f/a*(tan(1/2*f*x+1/2*e)/(c^2-2*c*d+d^2)+4*d/(c-d)^2*(-1/2*d/(c+d)*tan(1/
2*f*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)-1/2*(2*c+
d)/(c+d)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d)
)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(136) = 272$.

Time = 0.17 (sec) , antiderivative size = 691, normalized size of antiderivative = 4.77

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$$

$$= \left[\frac{\left(2\,cd^2+d^3+\left(2\,c^2d+cd^2\right)\cos\left(fx+e\right)^2+\left(2\,c^2d+3\,cd^2+d^3\right)\cos\left(fx+e\right)\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)}{}\right.}{2\left(\left(ac^6-ac^5d-2\,ac^4d^2+2\,ac^3d^3+ac^2d^4-acd^5\right)f\cos\left(fx+e\right)^2+\right.}\right.$$

$$-\frac{\left(2\,cd^2+d^3+\left(2\,c^2d+cd^2\right)\cos\left(fx+e\right)^2+\left(2\,c^2d+3\,cd^2+d^3\right)\cos\left(fx+e\right)\right)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-}}{}\right.}{\left(ac^6-ac^5d-2\,ac^4d^2+2\,ac^3d^3+ac^2d^4-acd^5\right)f\cos\left(fx+e\right)^2+\left(ac^6-3\,ac^4d^2+3\right.}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output
```
[1/2*((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d
^2 + d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2
*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(c^3*
d + 2*c^2*d^2 - c*d^3 - 2*d^4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*
sin(f*x + e))/((a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 -
a*c*d^5)*f*cos(f*x + e)^2 + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*
cos(f*x + e) + (a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5
- a*d^6)*f), -((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*
d + 3*c*d^2 + d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)
*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (c^3*d + 2*c^2*d^2 - c
*d^3 - 2*d^4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*sin(f*x + e))/((a
*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^5)*f*cos(f*
x + e)^2 + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*cos(f*x + e) + (a
*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec{(e+fx)}}{c^2\sec{(e+fx)}+c^2+2cd\sec^2{(e+fx)}+2cd\sec{(e+fx)}+d^2\sec^3{(e+fx)}+d^2\sec^2{(e+fx)}}\,dx}{a}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(sec(e + f*x)/(c**2*sec(e + f*x) + c**2 + 2*c*d*sec(e + f*x)**2 +
2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.52

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx =$$

$$-\frac{\frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{(ac^3-ac^2d-acd^2+ad^3)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}-\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\right.}{\sqrt{-c^2+d^2}}\right.\right.}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{-c^2+d^2}}}{f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
-(2*d^2*tan(1/2*f*x + 1/2*e)/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*(c*tan(1
/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) - 2*(pi*floor(1/2*(
f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan
(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*c*d + d^2)/((a*c^3 - a*c^2*d - a*
c*d^2 + a*d^3)*sqrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^2 - 2*a*c*d +
 a*d^2))/f
```

Mupad [B] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a\,f\,(c-d)^2} - \frac{2\,d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,\left(c+d\right)\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\,\left(a\,c^3-3\,a\,c^2\,d+3\,a\,c\,d^2-a\,d^3\right)-a\,d^3-a\,c^3+a\,c\,d^2+a\,c^2\,d\right)} - \frac{2\,d\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2\,c-2\,d)\,\left(a\,c^2-2\,a\,c\,d+a\,d^2\right)}{2\,a\,\sqrt{c+d}\,(c-d)^{5/2}}\right)\,(2\,c+d)}{a\,f\,(c+d)^{3/2}\,(c-d)^{5/2}}$$

input

```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))^2),x)
```

output

```
tan(e/2 + (f*x)/2)/(a*f*(c - d)^2) - (2*d^2*tan(e/2 + (f*x)/2))/(f*(c + d)
*(tan(e/2 + (f*x)/2)^2*(a*c^3 - a*d^3 + 3*a*c*d^2 - 3*a*c^2*d) - a*d^3 - a
*c^3 + a*c*d^2 + a*c^2*d)) - (2*d*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(a
*c^2 + a*d^2 - 2*a*c*d))/(2*a*(c + d)^(1/2)*(c - d)^(5/2)))*(2*c + d))/(a*
f*(c + d)^(3/2)*(c - d)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 635, normalized size of antiderivative = 4.38

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

output

```
( - 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*c**2*d + 2*sqrt( - c**2 + d**2)*
atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((
e + f*x)/2)**2*c*d**2 + 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*d**3 + 4*sqr
t( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c
**2 + d**2))*c**2*d + 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - ta
n((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c*d**2 + 2*sqrt( - c**2 + d**2)*at
an((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*d**3 +
tan((e + f*x)/2)**3*c**4 - 2*tan((e + f*x)/2)**3*c**2*d**2 + tan((e + f*x)
/2)**3*d**4 - tan((e + f*x)/2)*c**4 - 2*tan((e + f*x)/2)*c**3*d - 2*tan((e
 + f*x)/2)*c**2*d**2 + 2*tan((e + f*x)/2)*c*d**3 + 3*tan((e + f*x)/2)*d**4
)/(a*f*(tan((e + f*x)/2)**2*c**6 - 2*tan((e + f*x)/2)**2*c**5*d - tan((e +
 f*x)/2)**2*c**4*d**2 + 4*tan((e + f*x)/2)**2*c**3*d**3 - tan((e + f*x)/2)
**2*c**2*d**4 - 2*tan((e + f*x)/2)**2*c*d**5 + tan((e + f*x)/2)**2*d**6 -
c**6 + 3*c**4*d**2 - 3*c**2*d**4 + d**6))
```

3.216 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 207

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx\\ = -\frac{3d(2c^2+2cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a(c-d)^{7/2}(c+d)^{5/2}f}\\ +\frac{d(2c+3d)\tan(e+fx)}{2a(c-d)^2(c+d)f(c+d\sec(e+fx))^2}\\ +\frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\\ +\frac{d(2c+d)(c+4d)\tan(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sec(e+fx))}\end{aligned}$$

output

```
-3*d*(2*c^2+2*c*d+d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))
/a/(c-d)^(7/2)/(c+d)^(5/2)/f+1/2*d*(2*c+3*d)*tan(f*x+e)/a/(c-d)^2/(c+d)/f/
(c+d*sec(f*x+e))^2+tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2+
1/2*d*(2*c+d)*(c+4*d)*tan(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sec(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.24 (sec) , antiderivative size = 1422, normalized size of antiderivative = 6.87

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

output
```
((2*c^2 + 2*c*d + d^2)*Cos[e/2 + (f*x)/2]^2*(d + c*Cos[e + f*x])^3*Sec[e +
 f*x]^4*(((-6*I)*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2
*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e
]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2
]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (6*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[
c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt
[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Si
n[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^3*(c + d
)^2*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f*x)/2]*(d
+ c*Cos[e + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^4*(8*c^5*d*Sin[(f*x)/2] + 1
0*c^4*d^2*Sin[(f*x)/2] - 11*c^3*d^3*Sin[(f*x)/2] - 17*c^2*d^4*Sin[(f*x)/2]
 - 2*c*d^5*Sin[(f*x)/2] + 2*d^6*Sin[(f*x)/2] - 8*c^5*d*Sin[(3*f*x)/2] - 22
*c^4*d^2*Sin[(3*f*x)/2] - 27*c^3*d^3*Sin[(3*f*x)/2] - 5*c^2*d^4*Sin[(3*f*x
)/2] + 2*c*d^5*Sin[(3*f*x)/2] + 4*c^6*Sin[e - (f*x)/2] + 8*c^5*d*Sin[e - (
f*x)/2] + 18*c^4*d^2*Sin[e - (f*x)/2] + 35*c^3*d^3*Sin[e - (f*x)/2] + 25*c
^2*d^4*Sin[e - (f*x)/2] + 2*c*d^5*Sin[e - (f*x)/2] - 2*d^6*Sin[e - (f*x)/2
] - 4*c^6*Sin[e + (f*x)/2] - 8*c^5*d*Sin[e + (f*x)/2] - 6*c^4*d^2*Sin[e +
(f*x)/2] - 7*c^3*d^3*Sin[e + (f*x)/2] + 5*c^2*d^4*Sin[e + (f*x)/2] + 2*c*d
^5*Sin[e + (f*x)/2] - 2*d^6*Sin[e + (f*x)/2] + 8*c^5*d*Sin[2*e + (f*x)/2]
+ 22*c^4*d^2*Sin[2*e + (f*x)/2] + 17*c^3*d^3*Sin[2*e + (f*x)/2] + 13*c^...
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.58, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4475, 114, 27, 168, 27, 169, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4475$$

$$-\frac{a^2\tan(e+fx)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 114$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(2c+d-2d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2a^2(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2a^2(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{2c+d-2d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2a^2(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{\int \frac{a^2((c+d)(2c+3d)-d(4c+d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{a^2\left(c^2-d^2\right)}+\frac{d(4c+d)\sqrt{a-a\sec(e+fx)}}{a^2\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{}{2a^2(c^2-d^2}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{(c+d)(2c+3d)-d(4c+d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{c^2-d^2}+\frac{d(4c+d)\sqrt{a-a\sec(e+fx)}}{a^2\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{}{2a^2(c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{\int \frac{3a^2d\left(2c^2+2dc+d^2\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a^3(c-d)}-\frac{(2c+d)(c+4d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{c^2-d^2}+\frac{d(4c+d)\sqrt{a-a\sec(e-}}{a^2\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}(c}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{3d\left(2c^2+2cd+d^2\right)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a(c-d)}-\frac{(2c+d)(c+4d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{c^2-d^2}+\frac{d(}{a^2\left(c^2-d^2\right)}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{6d\left(2c^2+2cd+d^2\right)\int \frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{a(c-d)}-\frac{(2c+d)(c+4d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{c^2-d^2}+\frac{d(4c+d)\sqrt{a-}}{a^2\left(c^2-d^2\right)\sqrt{a\sec(e+}}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{6d\left(2c^2+2cd+d^2\right)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2(c-d)^{3/2}\sqrt{c+d}}-\frac{(2c+d)(c+4d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{c^2-d^2}+\frac{d(4c+d)\sqrt{a-a\sec(e+fx)}}{a^2\left(c^2-d^2\right)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

output
```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]])/(2*a^2*(c^2 - d^2)*Sqrt[a + a*Sec[e +
 f*x]]*(c + d*Sec[e + f*x])^2) + ((d*(4*c + d)*Sqrt[a - a*Sec[e + f*x]])/(
a^2*(c^2 - d^2)*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) + ((-6*d*(2
*c^2 + 2*c*d + d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c
- d]*Sqrt[a - a*Sec[e + f*x]])])/(a^2*(c - d)^(3/2)*Sqrt[c + d]) - ((2*c +
 d)*(c + 4*d)*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*Sqrt[a + a*Sec[e + f*
x]]))/(c^2 - d^2))/(2*(c^2 - d^2)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*
x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^{3}-3c^{2}d+3c\,d^{2}-d^{3}}+\frac{2d\Bigg(\frac{-\frac{3d\left(2c^{2}-cd-d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2\left(c^{2}+2cd+d^{2}\right)}+\frac{d(6c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c+2d}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}-\frac{3\left(2c^{2}+2cd+d^{2}\right)\operatorname{arctanh}\Bigg(\frac{(c-d)\tan\bigl(\frac{fx}{2}+}{\sqrt{(c-d)(c+d)}}}{2\left(c^{2}+2cd+d^{2}\right)\sqrt{(c-d)(c+d)}}}{(c-d)^{3}}}{fa}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^{3}-3c^{2}d+3c\,d^{2}-d^{3}}+\frac{2d\Bigg(\frac{-\frac{3d\left(2c^{2}-cd-d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2\left(c^{2}+2cd+d^{2}\right)}+\frac{d(6c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c+2d}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}-\frac{3\left(2c^{2}+2cd+d^{2}\right)\operatorname{arctanh}\Bigg(\frac{(c-d)\tan\bigl(\frac{fx}{2}+}{\sqrt{(c-d)(c+d)}}}{2\left(c^{2}+2cd+d^{2}\right)\sqrt{(c-d)(c+d)}}}{(c-d)^{3}}}{fa}$
risch	$i\bigl(-2c^{3}d^{3}-2c^{6}-8c^{4}d^{2}-4c^{5}d-13c^{2}d^{4}\mathrm{e}^{3i(fx+e)}-18c^{4}d^{2}\mathrm{e}^{2i(fx+e)}+d^{4}c^{2}-4c^{5}d\,\mathrm{e}^{4i(fx+e)}-2c^{4}d^{2}\mathrm{e}^{4i(fx+e)}-7c^{3}d^{3}\mathrm{e}^{4i(f}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/f/a*(tan(1/2*f*x+1/2*e)/(c^3-3*c^2*d+3*c*d^2-d^3)+2*d/(c-d)^3*((-3/2*d*(
2*c^2-c*d-d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(6*c+d)/(c+d)*ta
n(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-3/
2*(2*c^2+2*c*d+d^2)/(c^2+2*c*d+d^2)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(
1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(194) = 388$.

Time = 0.22 (sec) , antiderivative size = 1331, normalized size of antiderivative = 6.43

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
[-1/4*(3*(2*c^2*d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(
f*x + e)^3 + (2*c^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 +
(4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log(
(2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*
cos(f*x + e) + d^2)) - 2*(2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*
d^6 + (2*c^6 + 4*c^5*d + 6*c^4*d^2 - 2*c^3*d^3 - 9*c^2*d^4 - 2*c*d^5 + d^6
)*cos(f*x + e)^2 + (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4 - 11*c*d
^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^9 - a*c^8*d - 3*a*c^7*d^2 + 3*
a*c^6*d^3 + 3*a*c^5*d^4 - 3*a*c^4*d^5 - a*c^3*d^6 + a*c^2*d^7)*f*cos(f*x +
 e)^3 + (a*c^9 + a*c^8*d - 5*a*c^7*d^2 - 3*a*c^6*d^3 + 9*a*c^5*d^4 + 3*a*c
^4*d^5 - 7*a*c^3*d^6 - a*c^2*d^7 + 2*a*c*d^8)*f*cos(f*x + e)^2 + (2*a*c^8*
d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*
a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e) + (a*c^7*d^2 - a*c^6*d^3 - 3*a
*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f),
-1/2*(3*(2*c^2*d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(f
*x + e)^3 + (2*c^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (
4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arct
an(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2
*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6 + (2*c^6 + 4*c^5*d +...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{\sec(e+fx)}{c^3\sec(e+fx)+c^3+3c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^3(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^4(e+fx)+d^3\sec^3(e+fx)}\,dx}{a}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

output

```
Integral(sec(e + f*x)/(c**3*sec(e + f*x) + c**3 + 3*c**2*d*sec(e + f*x)**2
 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e + f*x
)**2 + d**3*sec(e + f*x)**4 + d**3*sec(e + f*x)**3), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="max
ima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.75

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx =$$

$$-\frac{\frac{3\,(2\,c^2d+2\,cd^2+d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(ac^5-ac^4d-2\,ac^3d^2+2\,ac^2d^3+acd^4-ad^5)\sqrt{-c^2+d^2}}-\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{ac^3-3\,ac^2d+3\,acd^2-ad^3}+\frac{6\,c^2d^2}{}}{f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="gia
c")
```

output
```
-(3*(2*c^2*d + 2*c*d^2 + d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^
2 + d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5
)*sqrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^3 - 3*a*c^2*d + 3*a*c*d^2
- a*d^3) + (6*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*c*d^3*tan(1/2*f*x + 1/2*e
)^3 - 3*d^4*tan(1/2*f*x + 1/2*e)^3 - 6*c^2*d^2*tan(1/2*f*x + 1/2*e) - 7*c*
d^3*tan(1/2*f*x + 1/2*e) - d^4*tan(1/2*f*x + 1/2*e))/((a*c^5 - a*c^4*d - 2
*a*c^3*d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*
tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 11.73 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.83

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,{\left(c-d\right)}^3}$$

$$-\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(d^3+6\,c\,d^2\right)}{c+d}+\frac{3\tan}{}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,a\,c^5-6\,a\,c^4\,d+4\,a\,c^3\,d^2+4\,a\,c^2\,d^3-6\,a\,c\,d^4+2\,a\,d^5\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(a\,c^5-5\right.\right.}$$

$$+\frac{d\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^4-4\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,d+6\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d^2-4\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^3+1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^4}{\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}\right)\,\left(2\,c^2+2\,c\,d+d\right.}{a\,f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{7/2}}$$

input

```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))^3),x)
```

output

```
tan(e/2 + (f*x)/2)/(a*f*(c - d)^3) - ((tan(e/2 + (f*x)/2)*(6*c*d^2 + d^3))
/(c + d) + (3*tan(e/2 + (f*x)/2)^3*(c*d^3 + d^4 - 2*c^2*d^2))/(c + d)^2)/(
f*(tan(e/2 + (f*x)/2)^2*(2*a*c^5 + 2*a*d^5 + 4*a*c^2*d^3 + 4*a*c^3*d^2 - 6
*a*c*d^4 - 6*a*c^4*d) - tan(e/2 + (f*x)/2)^4*(a*c^5 - a*d^5 - 10*a*c^2*d^3
 + 10*a*c^3*d^2 + 5*a*c*d^4 - 5*a*c^4*d) - a*c^5 + a*d^5 - 2*a*c^2*d^3 + 2
*a*c^3*d^2 - a*c*d^4 + a*c^4*d)) + (d*atan((c^4*tan(e/2 + (f*x)/2)*1i + d^
4*tan(e/2 + (f*x)/2)*1i - c*d^3*tan(e/2 + (f*x)/2)*4i - c^3*d*tan(e/2 + (f
*x)/2)*4i + c^2*d^2*tan(e/2 + (f*x)/2)*6i)/((c + d)^(1/2)*(c - d)^(7/2)))*
(2*c*d + 2*c^2 + d^2)*3i)/(a*f*(c + d)^(5/2)*(c - d)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1559, normalized size of antiderivative = 7.53

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

output

```
( - 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*tan((e + f*x)/2)**4*c**4*d + 6*sqrt( - c**2 + d**2)*
atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((
e + f*x)/2)**4*c**3*d**2 + 3*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c
 - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**4*c**2*d**3
 - 3*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/s
qrt( - c**2 + d**2))*tan((e + f*x)/2)**4*d**5 + 12*sqrt( - c**2 + d**2)*at
an((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e
+ f*x)/2)**2*c**4*d + 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - t
an((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*c**3*d**2 - 6
*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt(
 - c**2 + d**2))*tan((e + f*x)/2)**2*c**2*d**3 - 12*sqrt( - c**2 + d**2)*a
tan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e
 + f*x)/2)**2*c*d**4 - 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - t
an((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*d**5 - 6*sqrt
( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c*
*2 + d**2))*c**4*d - 18*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - ta
n((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**3*d**2 - 21*sqrt( - c**2 + d**2
)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**
2*d**3 - 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*...
```

3.217 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 258

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx \\
&= \frac{5(2c-d)d^2(2c^2-3cd+2d^2)\operatorname{arctanh}(\sin(e+fx))}{2a^2f} \\
&\quad - \frac{d(c^2+10cd-12d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} \\
&\quad + \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\
&\quad - \frac{d(4(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)+d(2c^3+20c^2d-57cd^2+30d^3)\sec(e+fx))\tan(e+fx)}{6a^2f}
\end{aligned}$$

output

```
5/2*(2*c-d)*d^2*(2*c^2-3*c*d+2*d^2)*arctanh(sin(f*x+e))/a^2/f-1/3*d*(c^2+1
0*c*d-12*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/a^2/f+1/3*(c-d)*(c+10*d)*(c+d*
sec(f*x+e))^3*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+1/3*(c-d)*(c+d*sec(f*x+e))
^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-1/6*d*(4*c^4+40*c^3*d-176*c^2*d^2+160*c
*d^3-48*d^4+d*(2*c^3+20*c^2*d-57*c*d^2+30*d^3)*sec(f*x+e))*tan(f*x+e)/a^2/
f
```

Mathematica [A] (verified)

Time = 6.45 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.73

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{240d^2(-4c^3+8c^2d-7cd^2+2d^3)\cos^4\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\right.\right.}{}$$

input

```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

output

```
(240*d^2*(-4*c^3 + 8*c^2*d - 7*c*d^2 + 2*d^3)*Cos[(e + f*x)/2]^4*(Log[Cos[
(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
]) + 2*Cos[(e + f*x)/2]*(6*c^5 + 15*c^4*d - 120*c^3*d^2 + 420*c^2*d^3 - 30
0*c*d^4 + 104*d^5 + (6*c^5 + 60*c^4*d - 300*c^3*d^2 + 840*c^2*d^3 - 585*c*
d^4 + 190*d^5)*Cos[e + f*x] + 4*(2*c^5 + 5*c^4*d - 40*c^3*d^2 + 130*c^2*d^
3 - 95*c*d^4 + 30*d^5)*Cos[2*(e + f*x)] + 2*c^5*Cos[3*(e + f*x)] + 20*c^4*
d*Cos[3*(e + f*x)] - 100*c^3*d^2*Cos[3*(e + f*x)] + 280*c^2*d^3*Cos[3*(e +
 f*x)] - 215*c*d^4*Cos[3*(e + f*x)] + 66*d^5*Cos[3*(e + f*x)] + 2*c^5*Cos[
4*(e + f*x)] + 5*c^4*d*Cos[4*(e + f*x)] - 40*c^3*d^2*Cos[4*(e + f*x)] + 10
0*c^2*d^3*Cos[4*(e + f*x)] - 80*c*d^4*Cos[4*(e + f*x)] + 24*d^5*Cos[4*(e +
 f*x)])*Sec[e + f*x]^3*Sin[(e + f*x)/2])/(24*a^2*f*(1 + Cos[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.52, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4475, 109, 25, 27, 167, 27, 170, 25, 27, 164, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a\sec(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc \left(e+fx+\frac{\pi}{2}\right) \left(c+d \csc \left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a \csc \left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \ 4475$$

$$-\frac{a^2 \tan(e+fx) \int \frac{(c+d\sec(e+fx))^5}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 109$$

$$-\frac{a^2 \tan(e+fx) \left(-\frac{\int -\frac{a^2(c+d\sec(e+fx))^3\left(c^2+6dc-4d^2-3(c-2d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^4}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 25$$

$$-\frac{a^2 \tan(e+fx) \left(\frac{\int \frac{a^2(c+d\sec(e+fx))^3\left(c^2+6dc-4d^2-3(c-2d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^4}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2 \tan(e+fx) \left(\frac{\int \frac{(c+d\sec(e+fx))^3\left(c^2+6dc-4d^2-3(c-2d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^4}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 167$$

$$-\frac{a^2 \tan(e+fx) \left(\frac{\frac{\int \frac{3a^2d(c+d\sec(e+fx))^2\left((11c-10d)d-\left(c^2+10dc-12d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^3}-\frac{(c-d)(c+10d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d\int\frac{(c+d\sec(e+fx))^2\left((11c-10d)d-\left(c^2+10dc-12d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{(c-d)(c+10d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 170$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d\left(\frac{\left(c^2+10cd-12d^2\right)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}{3a^2}-\frac{\int-\frac{a^2(c+d\sec(e+fx))\left(d\left(31c^2-50dc+24d^2\right)-\left(2c^3+20d\right.\right.}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx}}}{3a^2}\right.}{a}}{3a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d\left(\frac{\int\frac{a^2(c+d\sec(e+fx))\left(d\left(31c^2-50dc+24d^2\right)-\left(2c^3+20dc^2-57d^2c+30d^3\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{3a^2}+\frac{\left(c^2+10cd-12d^2\right)\sqrt{a-a\sec(e}}{}\right.}{a}}{3a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d\left(\frac{1}{3}\int\frac{(c+d\sec(e+fx))\left(d\left(31c^2-50dc+24d^2\right)-\left(2c^3+20dc^2-57d^2c+30d^3\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)+\frac{\left(c^2+10cd-12d^2\right)\sqrt{a-a\sec(e-}}{}\right.}{a}}{3a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

$$\Big\downarrow \; 164$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d\left(\frac{1}{3}\left(\frac{15}{2}d(2c-d)\left(2c^2-3cd+2d^2\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(2c^3+20c^2\right.\right.}{}\right.\right.}{}}{}\right.}{}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d\left(\frac{1}{3}\left(15d(2c-d)\left(2c^2-3cd+2d^2\right)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(2c^3+20c^2d-\right.\right.}{}\right.\right.}{}}{}\right.}{}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d\left(\frac{1}{3}\left(\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(2c^3+20c^2d-57cd^2+30d^3\right)\sec(e+fx)+4\left(c^4+10c^3d-44c^2d^2+40cd^3-12d^4\right)\right)}{2a^2}-\frac{15d(2c-}{}\right.\right.}{a}}{}\right.}{}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

output
```
-((a^2*(-1/3*((c - d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4)/(a^
2*(a + a*Sec[e + f*x])^(3/2)) + (-(((c - d)*(c + 10*d)*Sqrt[a - a*Sec[e +
f*x]]*(c + d*Sec[e + f*x])^3)/(a^2*Sqrt[a + a*Sec[e + f*x]])) + (3*d*(((c^
2 + 10*c*d - 12*d^2)*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]*(c
+ d*Sec[e + f*x])^2)/(3*a^2) + ((-15*(2*c - d)*d*(2*c^2 - 3*c*d + 2*d^2)*A
rcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]]])/a + (Sqrt[a - a*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]*(4*(c^4 + 10*c^3*d - 44*c^2*d^2 + 4
0*c*d^3 - 12*d^4) + d*(2*c^3 + 20*c^2*d - 57*c*d^2 + 30*d^3)*Sec[e + f*x])
)/(2*a^2))/3))/a)/(3*a))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
+ a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.48

method	result
parallelrisch	$-360\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\left(c^2-\frac{3}{2}cd+d^2\right)d^2\left(c-\frac{d}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+360\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\left(c^2-\frac{3}{2}\right.$
derivativedivides	$-\frac{c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{5c^4d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{10c^3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{10c^2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{5c\,d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3}$
default	$-\frac{c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{5c^4d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{10c^3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{10c^2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{5c\,d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3}$
norman	$-\frac{\left(c^5-5c^4d+10c^3d^2-10c^2d^3+5c\,d^4-d^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{6fa}-\frac{\left(c^5+5c^4d-30c^3d^2+90c^2d^3-65c\,d^4+21d^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2fa}+10\left(2c^5+5\right.$
risch	Expression too large to display

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output

```
1/12*(-360*(cos(f*x+e)+1/3*cos(3*f*x+3*e))*(c^2-3/2*c*d+d^2)*d^2*(c-1/2*d)
*ln(tan(1/2*f*x+1/2*e)-1)+360*(cos(f*x+e)+1/3*cos(3*f*x+3*e))*(c^2-3/2*c*d
+d^2)*d^2*(c-1/2*d)*ln(tan(1/2*f*x+1/2*e)+1)+2*sec(1/2*f*x+1/2*e)^2*tan(1/
2*f*x+1/2*e)*((c^5+33*d^5+10*c^4*d-50*c^3*d^2+140*c^2*d^3-215/2*c*d^4)*cos
(3*f*x+3*e)+(4*c^5+10*c^4*d-80*c^3*d^2+260*c^2*d^3-190*c*d^4+60*d^5)*cos(2
*f*x+2*e)+(c^5+12*d^5+5/2*c^4*d-20*c^3*d^2+50*c^2*d^3-40*c*d^4)*cos(4*f*x+
4*e)+(95*d^5+3*c^5-150*c^3*d^2+30*c^4*d+420*c^2*d^3-585/2*c*d^4)*cos(f*x+e
)+52*d^5+3*c^5+15/2*c^4*d-60*c^3*d^2+210*c^2*d^3-150*c*d^4))/f/a^2/(cos(3*
f*x+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.77

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{15\left(\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\cos\left(fx+e\right)^5+2\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\cos\left(fx+e\right)^4+(4c^3}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
1/12*(15*((4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*
c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2
*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((4*c^3
*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*c^3*d^2 - 8*c^2*
d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 -
 2*d^5)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(2*d^5 + 2*(2*c^5 + 5*c
^4*d - 40*c^3*d^2 + 100*c^2*d^3 - 80*c*d^4 + 24*d^5)*cos(f*x + e)^4 + (2*c
^5 + 20*c^4*d - 100*c^3*d^2 + 280*c^2*d^3 - 215*c*d^4 + 66*d^5)*cos(f*x +
e)^3 + 6*(10*c^2*d^3 - 5*c*d^4 + 2*d^5)*cos(f*x + e)^2 + (15*c*d^4 - 2*d^5
)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)
^4 + a^2*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{c^5\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d^5\sec^6\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{5cd^4\sec^5\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{10c^2d^3\sec^4}{\sec^2\left(e+fx\right)+2\mathrm{se}}}{a^2}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

output
```
(Integral(c**5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(d**5*sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(5*c*d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
x) + Integral(10*c**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*
x) + 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*s
ec(e + f*x) + 1), x) + Integral(5*c**4*d*sec(e + f*x)**2/(sec(e + f*x)**2
+ 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(247) = 494.

Time = 0.05 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.99

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="m
axima")
```

output

```
1/6*(d^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/(
cos(f*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a
^2) - 5*c*d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1)
 + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x
 + e) + 1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 1
0*c^2*d^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log
(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 10*c^3*d^2*((9*s
in(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
- 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a^2) + 5*c^4*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 + c^5*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(247) = 494.

Time = 0.28 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.96

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/6*(15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*log(abs(tan(1/2*f*x + 1/
2*e) + 1))/a^2 - 15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*log(abs(tan(
1/2*f*x + 1/2*e) - 1))/a^2 - 2*(60*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 75*c*d
^4*tan(1/2*f*x + 1/2*e)^5 + 30*d^5*tan(1/2*f*x + 1/2*e)^5 - 120*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 + 120*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 40*d^5*tan(1/2*f
*x + 1/2*e)^3 + 60*c^2*d^3*tan(1/2*f*x + 1/2*e) - 45*c*d^4*tan(1/2*f*x + 1
/2*e) + 18*d^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a^2)
- (a^4*c^5*tan(1/2*f*x + 1/2*e)^3 - 5*a^4*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 1
0*a^4*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 10*a^4*c^2*d^3*tan(1/2*f*x + 1/2*e)
^3 + 5*a^4*c*d^4*tan(1/2*f*x + 1/2*e)^3 - a^4*d^5*tan(1/2*f*x + 1/2*e)^3 -
 3*a^4*c^5*tan(1/2*f*x + 1/2*e) - 15*a^4*c^4*d*tan(1/2*f*x + 1/2*e) + 90*a
^4*c^3*d^2*tan(1/2*f*x + 1/2*e) - 150*a^4*c^2*d^3*tan(1/2*f*x + 1/2*e) + 1
05*a^4*c*d^4*tan(1/2*f*x + 1/2*e) - 27*a^4*d^5*tan(1/2*f*x + 1/2*e))/a^6)/
f
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{5\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,c-d\right)\,\left(2\,c^2-3\,c\,d+2\,d^2\right)}{a^2\,f}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{2\,{\left(c-d\right)}^5}{a^2}-\frac{5\,\left(c+d\right)\,{\left(c-d\right)}^4}{2\,a^2}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,{\left(c-d\right)}^5}{6\,a^2\,f}$$
$$-\frac{\left(20\,c^2\,d^3-25\,c\,d^4+10\,d^5\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\left(-40\,c^2\,d^3+40\,c\,d^4-\frac{40\,d^5}{3}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+(20\,c^2\,d^3-1}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-a^2\right)}$$

input
```
int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output

```
(5*d^2*atanh(tan(e/2 + (f*x)/2))*(2*c - d)*(2*c^2 - 3*c*d + 2*d^2))/(a^2*f
) - (tan(e/2 + (f*x)/2)*((2*(c - d)^5)/a^2 - (5*(c + d)*(c - d)^4)/(2*a^2)
))/f - (tan(e/2 + (f*x)/2)^3*(c - d)^5)/(6*a^2*f) - (tan(e/2 + (f*x)/2)*(6
*d^5 - 15*c*d^4 + 20*c^2*d^3) + tan(e/2 + (f*x)/2)^5*(10*d^5 - 25*c*d^4 +
20*c^2*d^3) - tan(e/2 + (f*x)/2)^3*((40*d^5)/3 - 40*c*d^4 + 40*c^2*d^3))/(
f*(3*a^2*tan(e/2 + (f*x)/2)^2 - 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 +
 (f*x)/2)^6 - a^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1404, normalized size of antiderivative = 5.44

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x)
```

output

```
( - 60*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**6*c**3*d**2 + 120*log(t
an((e + f*x)/2) - 1)*tan((e + f*x)/2)**6*c**2*d**3 - 105*log(tan((e + f*x)
/2) - 1)*tan((e + f*x)/2)**6*c*d**4 + 30*log(tan((e + f*x)/2) - 1)*tan((e
+ f*x)/2)**6*d**5 + 180*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*c**3
*d**2 - 360*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*c**2*d**3 + 315*
log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*c*d**4 - 90*log(tan((e + f*x
)/2) - 1)*tan((e + f*x)/2)**4*d**5 - 180*log(tan((e + f*x)/2) - 1)*tan((e
+ f*x)/2)**2*c**3*d**2 + 360*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2
*c**2*d**3 - 315*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*c*d**4 + 90
*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*d**5 + 60*log(tan((e + f*x)
/2) - 1)*c**3*d**2 - 120*log(tan((e + f*x)/2) - 1)*c**2*d**3 + 105*log(tan
((e + f*x)/2) - 1)*c*d**4 - 30*log(tan((e + f*x)/2) - 1)*d**5 + 60*log(tan
((e + f*x)/2) + 1)*tan((e + f*x)/2)**6*c**3*d**2 - 120*log(tan((e + f*x)/2
) + 1)*tan((e + f*x)/2)**6*c**2*d**3 + 105*log(tan((e + f*x)/2) + 1)*tan((
e + f*x)/2)**6*c*d**4 - 30*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**6*d
**5 - 180*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*c**3*d**2 + 360*lo
g(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*c**2*d**3 - 315*log(tan((e + f
*x)/2) + 1)*tan((e + f*x)/2)**4*c*d**4 + 90*log(tan((e + f*x)/2) + 1)*tan(
(e + f*x)/2)**4*d**5 + 180*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2*c
**3*d**2 - 360*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2*c**2*d**3 ...
```

3.218 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 193

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx \\
&= \frac{d^2(12c^2-16cd+7d^2)\operatorname{arctanh}(\sin(e+fx))}{2a^2f} \\
&\quad + \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\
&\quad - \frac{d(4(c^3+8c^2d-20cd^2+8d^3)+d(2c^2+16cd-21d^2)\sec(e+fx))\tan(e+fx)}{6a^2f}
\end{aligned}$$

output

```
1/2*d^2*(12*c^2-16*c*d+7*d^2)*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)*(c+8*d)*
(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+1/3*(c-d)*(c+d*sec(f*
x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-1/6*d*(4*c^3+32*c^2*d-80*c*d^2+32*
d^3+d*(2*c^2+16*c*d-21*d^2)*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.61

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{-24d^2(12c^2-16cd+7d^2)\cos^4\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

output
```
(-24*d^2*(12*c^2 - 16*c*d + 7*d^2)*Cos[(e + f*x)/2]^4*(Log[Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Cos[
(e + f*x)/2]*(2*c^4 + 16*c^3*d - 60*c^2*d^2 + 112*c*d^3 - 37*d^4 + 6*(c^4
+ 2*c^3*d - 12*c^2*d^2 + 28*c*d^3 - 10*d^4)*Cos[e + f*x] + (2*c^4 + 16*c^3
*d - 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*Cos[2*(e + f*x)] + 2*c^4*Cos[3*(e +
f*x)] + 4*c^3*d*Cos[3*(e + f*x)] - 24*c^2*d^2*Cos[3*(e + f*x)] + 40*c*d^3*
Cos[3*(e + f*x)] - 16*d^4*Cos[3*(e + f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2
])/(12*a^2*f*(1 + Cos[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.56, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4475, 109, 25, 27, 167, 27, 164, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a\sec(e+fx)+a)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\Big\downarrow \text{ 4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(c+d\sec(e+fx))^4}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2(c+d\sec(e+fx))^2\left(c^2+5dc-3d^2-(2c-5d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(c+d\sec(e+fx))^2\left(c^2+5dc-3d^2-(2c-5d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{(c+d\sec(e+fx))^2\left(c^2+5dc-3d^2-(2c-5d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{167}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{a^2d(c+d\sec(e+fx))\left((19c-16d)d-\left(2c^2+16dc-21d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^3}-\frac{(c-d)(c+8d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{d\int \frac{(c+d\sec(e+fx))\left((19c-16d)d-\left(2c^2+16dc-21d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{(c-d)(c+8d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{164}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{d\left(\frac{3}{2}d\left(12c^2-16cd+7d^2\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(2c^2+16cd-21d^2\right)\sec}{2a^2}}{a}}{3a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\,\mathrm{s}}}$$

$$\Big\downarrow \color{blue}{45}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{d\left(3d\left(12c^2-16cd+7d^2\right)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(2c^2+16cd-21d^2\right)\sec(e+}{2a^2}}{a}}{3a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\,\mathrm{se}}}$$

$$\Big\downarrow \color{blue}{218}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{d\left(\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(2c^2+16cd-21d^2\right)\sec(e+fx)+4\left(c^3+8c^2d-20cd^2+8d^3\right)\right)}{2a^2}-\frac{3d\left(12c^2-16cd+7d^2\right)\arctan\left(\frac{\sqrt{a}}{\sqrt{a}}\right.}{a}}{a}}{3a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

output

```
-((a^2*(-1/3*((c - d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3)/(a^
2*(a + a*Sec[e + f*x])^(3/2)) + (-(((c - d)*(c + 8*d)*Sqrt[a - a*Sec[e + f
*x]]*(c + d*Sec[e + f*x])^2)/(a^2*Sqrt[a + a*Sec[e + f*x]])) + (d*((-3*d*(
12*c^2 - 16*c*d + 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e
+ f*x]]])/a + (Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]*(4*(c^3 +
 8*c^2*d - 20*c*d^2 + 8*d^3) + d*(2*c^2 + 16*c*d - 21*d^2)*Sec[e + f*x]))/
(2*a^2))/a)/(3*a))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.42

method	result
parallelrisch	$-12d^2(1+\cos(2fx+2e))(c^2-\frac{4}{3}cd+\frac{7}{12}d^2)\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)+12d^2(1+\cos(2fx+2e))(c^2-\frac{4}{3}cd+\frac{7}{12}d^2)\ln(\tan(\frac{fx}{2}+$
derivativedivides	$-\frac{c^4\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+\frac{4c^3d\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}-2c^2d^2\tan(\frac{fx}{2}+\frac{e}{2})^3+\frac{4cd^3\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}-\frac{d^4\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+c^4\tan(\frac{fx}{2}+\frac{e}{2})+4$
default	$-\frac{c^4\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+\frac{4c^3d\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}-2c^2d^2\tan(\frac{fx}{2}+\frac{e}{2})^3+\frac{4cd^3\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}-\frac{d^4\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+c^4\tan(\frac{fx}{2}+\frac{e}{2})+4$
norman	$-\frac{(c^4-4c^3d+6c^2d^2-4cd^3+d^4)\tan(\frac{fx}{2}+\frac{e}{2})^{11}}{6fa}+\frac{(c^4+4c^3d-18c^2d^2+36cd^3-13d^4)\tan(\frac{fx}{2}+\frac{e}{2})}{2fa}-\frac{(3c^4+4c^3d-30c^2d^2+44cd^3-}{fa}$
risch	$i(8c^3d-48c^2d^2+80cd^3-32d^4+4c^4-132c^2d^2e^{2i(fx+e)}+256cd^3e^{2i(fx+e)}+16c^3de^{2i(fx+e)}-108c^2d^2e^{5i(fx+e)}-216c^2$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/2*(-12*d^2*(1+cos(2*f*x+2*e))*(c^2-4/3*c*d+7/12*d^2)*ln(tan(1/2*f*x+1/2*
e)-1)+12*d^2*(1+cos(2*f*x+2*e))*(c^2-4/3*c*d+7/12*d^2)*ln(tan(1/2*f*x+1/2*
e)+1)+((1/3*c^4+8/3*c^3*d-10*c^2*d^2+56/3*c*d^3-43/6*d^4)*cos(2*f*x+2*e)+(
1/3*c^4+2/3*c^3*d-4*c^2*d^2+20/3*c*d^3-8/3*d^4)*cos(3*f*x+3*e)+(c^4+2*c^3*
d-12*c^2*d^2+28*c*d^3-10*d^4)*cos(f*x+e)+1/3*c^4+8/3*c^3*d-10*c^2*d^2+56/3
*c*d^3-37/6*d^4)*tan(1/2*f*x+1/2*e)*sec(1/2*f*x+1/2*e)^2)/f/a^2/(1+cos(2*f
*x+2*e))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.87

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{3\left((12c^2d^2-16cd^3+7d^4)\cos(fx+e)^4+2(12c^2d^2-16cd^3+7d^4)\cos(fx+e)^3+(12c^2d^2-16cd^3\right.}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
1/12*(3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^4 + 2*(12*c^2*d^2 -
16*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x
 + e)^2)*log(sin(f*x + e) + 1) - 3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*
x + e)^4 + 2*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2
- 16*c*d^3 + 7*d^4)*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(3*d^4 + 4*
(c^4 + 2*c^3*d - 12*c^2*d^2 + 20*c*d^3 - 8*d^4)*cos(f*x + e)^3 + (2*c^4 +
16*c^3*d - 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*cos(f*x + e)^2 + 6*(4*c*d^3 -
d^4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x +
 e)^3 + a^2*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\int \frac{c^4\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d^4\sec^5(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{4cd^3\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{6c^2d^2\sec^3(}{\sec^2(e+fx)+2\,\mathrm{se}}}{a^2}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

output

```
(Integral(c**4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(4*c*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
x) + Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x
) + 1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(184) = 368$.

Time = 0.05 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.78

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
-1/6*(d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + s
in(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 4*c*d
^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^2*d^2*((9*sin(f*x +
 e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(
sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e
) + 1) - 1)/a^2) - 4*c^3*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x +
e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) -
sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.86

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{3\,(12\,c^2d^2-16\,cd^3+7\,d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} - \frac{3\,(12\,c^2d^2-16\,cd^3+7\,d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} - 6\left(8\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-5\,d^4\,\mathrm{ta}\right.}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/6*(3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/
a^2 - 3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))
/a^2 - 6*(8*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 5*d^4*tan(1/2*f*x + 1/2*e)^3 -
8*c*d^3*tan(1/2*f*x + 1/2*e) + 3*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x +
 1/2*e)^2 - 1)^2*a^2) - (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c^3*d*tan(
1/2*f*x + 1/2*e)^3 + 6*a^4*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c*d^3*ta
n(1/2*f*x + 1/2*e)^3 + a^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^4*tan(1/2*
f*x + 1/2*e) - 12*a^4*c^3*d*tan(1/2*f*x + 1/2*e) + 54*a^4*c^2*d^2*tan(1/2*
f*x + 1/2*e) - 60*a^4*c*d^3*tan(1/2*f*x + 1/2*e) + 21*a^4*d^4*tan(1/2*f*x
+ 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(8\,c\,d^3-3\,d^4\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(8\,c\,d^3-5\,d^4\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-2\,a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a^2\right)} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(\frac{3\,(c-d)^4}{2\,a^2}-\frac{2\,(c+d)\,(c-d)^3}{a^2}\right)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(c-d)^4}{6\,a^2\,f} + \frac{d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\,\left(12\,c^2-16\,c\,d+7\,d^2\right)}{a^2\,f}$$

input
```
int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output
```
(tan(e/2 + (f*x)/2)*(8*c*d^3 - 3*d^4) - tan(e/2 + (f*x)/2)^3*(8*c*d^3 - 5*
d^4))/(f*(a^2*tan(e/2 + (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) -
(tan(e/2 + (f*x)/2)*((3*(c - d)^4)/(2*a^2) - (2*(c + d)*(c - d)^3)/a^2))/f
 - (tan(e/2 + (f*x)/2)^3*(c - d)^4)/(6*a^2*f) + (d^2*atanh(tan(e/2 + (f*x)
/2))*(12*c^2 - 16*c*d + 7*d^2))/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 829, normalized size of antiderivative = 4.30

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x)
```

output

```
( - 36*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*c**2*d**2 + 48*log(ta
n((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*c*d**3 - 21*log(tan((e + f*x)/2) -
 1)*tan((e + f*x)/2)**4*d**4 + 72*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/
2)**2*c**2*d**2 - 96*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*c*d**3
+ 42*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*d**4 - 36*log(tan((e +
f*x)/2) - 1)*c**2*d**2 + 48*log(tan((e + f*x)/2) - 1)*c*d**3 - 21*log(tan(
(e + f*x)/2) - 1)*d**4 + 36*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*
c**2*d**2 - 48*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*c*d**3 + 21*l
og(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*d**4 - 72*log(tan((e + f*x)/2
) + 1)*tan((e + f*x)/2)**2*c**2*d**2 + 96*log(tan((e + f*x)/2) + 1)*tan((e
 + f*x)/2)**2*c*d**3 - 42*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2*d*
*4 + 36*log(tan((e + f*x)/2) + 1)*c**2*d**2 - 48*log(tan((e + f*x)/2) + 1)
*c*d**3 + 21*log(tan((e + f*x)/2) + 1)*d**4 - tan((e + f*x)/2)**7*c**4 + 4
*tan((e + f*x)/2)**7*c**3*d - 6*tan((e + f*x)/2)**7*c**2*d**2 + 4*tan((e +
 f*x)/2)**7*c*d**3 - tan((e + f*x)/2)**7*d**4 + 5*tan((e + f*x)/2)**5*c**4
 + 4*tan((e + f*x)/2)**5*c**3*d - 42*tan((e + f*x)/2)**5*c**2*d**2 + 52*ta
n((e + f*x)/2)**5*c*d**3 - 19*tan((e + f*x)/2)**5*d**4 - 7*tan((e + f*x)/2
)**3*c**4 - 20*tan((e + f*x)/2)**3*c**3*d + 102*tan((e + f*x)/2)**3*c**2*d
**2 - 164*tan((e + f*x)/2)**3*c*d**3 + 71*tan((e + f*x)/2)**3*d**4 + 3*tan
((e + f*x)/2)*c**4 + 12*tan((e + f*x)/2)*c**3*d - 54*tan((e + f*x)/2)*c...
```

3.219 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 133

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{(3c-2d)d^2\text{arctanh}(\sin(e+fx))}{a^2f} + \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^3+4c^2d-12cd^2+10d^3-(c-4d)d^2\sec(e+fx))\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))}$$

output
```
(3*c-2*d)*d^2*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)*(c+d*sec(f*x+e))^2*tan(f
*x+e)/f/(a+a*sec(f*x+e))^2+1/3*(c^3+4*c^2*d-12*c*d^2+10*d^3-(c-4*d)*d^2*se
c(f*x+e))*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 294 vs. 2(133) = 266.

Time = 3.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.21

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{2\cos^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(6d^2(-3c+2d)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log(\cos(\frac{1}{2}(e\right.\right.\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

output
```
(2*Cos[(e + f*x)/2]^6*Sec[e + f*x]*(6*d^2*(-3*c + 2*d)*(Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 8*(c
- d)^3*Csc[e + f*x]^3*Sin[(e + f*x)/2]^4 + 32*(c - d)^3*Csc[e + f*x]^5*Sin
[(e + f*x)/2]^8 + 2*(2*c^3 + 3*c^2*d - 12*c*d^2 + 13*d^3)*Tan[(e + f*x)/2]
 + 6*(3*c - 2*d)*d^2*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(
e + f*x)/2] + Sin[(e + f*x)/2]])*Tan[(e + f*x)/2]^2 - 2*(c - d)^2*(2*c + 7
*d)*Tan[(e + f*x)/2]^3))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 4475, 109, 25, 27, 160, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2(c+d\sec(e+fx))\left(c^2+4dc-2d^2-(c-4d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(c+d\sec(e+fx))\left(c^2+4dc-2d^2-(c-4d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{(c+d\sec(e+fx))\left(c^2+4dc-2d^2-(c-4d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 160$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d^2(3c-2d)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{\sqrt{a-a\sec(e+fx)}\left(c^3+4c^2d-d^2(c-4d)\sec(e+fx)-12cd^2+10d^3\right)}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-(\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{6d^2(3c-2d)\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{a}-\frac{\sqrt{a-a\sec(e+fx)}\left(c^3+4c^2d-d^2(c-4d)\sec(e+fx)-12cd^2+10d^3\right)}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-(c-\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{6d^2(3c-2d)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)}{a^2}-\frac{\sqrt{a-a\sec(e+fx)}\left(c^3+4c^2d-d^2(c-4d)\sec(e+fx)-12cd^2+10d^3\right)}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-\frac{(c-d)\sqrt{a-a\sec(e}{3a^2(a\sec}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

output
```
-((a^2*(-1/3*((c - d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2)/(a^
2*(a + a*Sec[e + f*x])^(3/2)) + ((-6*(3*c - 2*d)*d^2*ArcTan[Sqrt[a - a*Sec
[e + f*x]]/Sqrt[a + a*Sec[e + f*x]]])/a^2 - (Sqrt[a - a*Sec[e + f*x]]*(c^3
 + 4*c^2*d - 12*c*d^2 + 10*d^3 - (c - 4*d)*d^2*Sec[e + f*x]))/(a^2*Sqrt[a
+ a*Sec[e + f*x]]))/(3*a))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[
a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 160
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d*(f*g
+ e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Simp[(a*d*f*h*m + b*(d*
(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d)   Int[(a + b*x)^(m + 1)*(c + d*x)^n,
x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0] &&
NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.32

method	result
parallelrisch	$\frac{-18d^2\cos(fx+e)\left(c-\frac{2d}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+18d^2\cos(fx+e)\left(c-\frac{2d}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\left(\frac{3}{2}c^2d\right.\right.}{6f\,a^2\cos(fx+}$
derivativedivides	$-\frac{c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-c\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+\frac{d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-9c\,d$
default	$-\frac{c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-c\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+\frac{d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-9c\,d$
norman	$\frac{\frac{\left(c^3-3c\,d^2+2d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{fa}-\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{6fa}-\frac{\left(c^3+3c^2d-9c\,d^2+9d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2fa}-\frac{\left(2c^3+3c^2d-12c\right.}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3a}$
risch	$\frac{2i\left(3c^3e^{4i(fx+e)}-9c\,d^2e^{4i(fx+e)}+6d^3e^{4i(fx+e)}+3c^3e^{3i(fx+e)}+9c^2d\,e^{3i(fx+e)}-27c\,d^2e^{3i(fx+e)}+18d^3e^{3i(fx+e)}+5c^3e\right.}{3f}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/6*(-18*d^2*cos(f*x+e)*(c-2/3*d)*ln(tan(1/2*f*x+1/2*e)-1)+18*d^2*cos(f*x+
e)*(c-2/3*d)*ln(tan(1/2*f*x+1/2*e)+1)+sec(1/2*f*x+1/2*e)^2*((3/2*c^2*d-6*c
*d^2+c^3+5*d^3)*cos(2*f*x+2*e)+(c^3+6*c^2*d-15*c*d^2+14*d^3)*cos(f*x+e)+c^
3+3/2*c^2*d-6*c*d^2+8*d^3)*tan(1/2*f*x+1/2*e))/f/a^2/cos(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(130) = 260$.

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.02

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$$

$$= 3\left(\left(3\,cd^{2}-2\,d^{3}\right)\cos\left(fx+e\right)^{3}+2\left(3\,cd^{2}-2\,d^{3}\right)\cos\left(fx+e\right)^{2}+\left(3\,cd^{2}-2\,d^{3}\right)\cos\left(fx+e\right)\right)\log\left(\sin\left(fx\right.\right.$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
1/6*(3*((3*c*d^2 - 2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e
)^2 + (3*c*d^2 - 2*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*((3*c*d^2
- 2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (3*c*d^2 -
2*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(3*d^3 + (2*c^3 + 3*c^2*d
- 12*c*d^2 + 10*d^3)*cos(f*x + e)^2 + (c^3 + 6*c^2*d - 15*c*d^2 + 14*d^3)*
cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2
 + a^2*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$$

$$= \frac{\int \frac{c^{3}\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d^{3}\sec^{4}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{3cd^{2}\sec^{3}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{3c^{2}d\sec^{2}\left(e\right.}{\sec^{2}\left(e+fx\right)+2\,\mathrm{se}}}{a^{2}}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

output
```
(Integral(c**3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(3*c*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
x) + Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) +
 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(130) = 260.

Time = 0.04 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.57

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{d^3\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)-}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
1/6*(d^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 3*c*d^2*((9*sin(f
*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a^2) + 3*c^2*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*
x + e)^3/(cos(f*x + e) + 1)^3)/a^2 + c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1
) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.88

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{\frac{12\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2} - \frac{6\left(3\,cd^2-2\,d^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} + \frac{6\left(3\,cd^2-2\,d^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} + \frac{a^4c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{}}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
-1/6*(12*d^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) - 6*(
3*c*d^2 - 2*d^3)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 + 6*(3*c*d^2 - 2*d
^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 + (a^4*c^3*tan(1/2*f*x + 1/2*e)
^3 - 3*a^4*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a^4*c*d^2*tan(1/2*f*x + 1/2*e)
^3 - a^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^3*tan(1/2*f*x + 1/2*e) - 9*a
^4*c^2*d*tan(1/2*f*x + 1/2*e) + 27*a^4*c*d^2*tan(1/2*f*x + 1/2*e) - 15*a^4
*d^3*tan(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(3\,c-2\,d\right)}{a^2\,f} - \frac{2\,d^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(c-d\right)^3}{6\,a^2\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{(c-d)^3}{a^2}-\frac{3\,(c+d)\,(c-d)^2}{2\,a^2}\right)}{f}$$

input
```
int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output
```
(2*d^2*atanh(tan(e/2 + (f*x)/2))*(3*c - 2*d))/(a^2*f) - (2*d^3*tan(e/2 + (
f*x)/2))/(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2)) - (tan(e/2 + (f*x)/2)^3*(c -
 d)^3)/(6*a^2*f) - (tan(e/2 + (f*x)/2)*((c - d)^3/a^2 - (3*(c + d)*(c - d)
^2)/(2*a^2)))/f
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.98

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{-18\log\big(\tan\big(\frac{fx}{2}+\frac{e}{2}\big)-1\big)\tan\big(\frac{fx}{2}+\frac{e}{2}\big)^2cd^2+12\log\big(\tan\big(\frac{fx}{2}+\frac{e}{2}\big)-1\big)\tan\big(\frac{fx}{2}+\frac{e}{2}\big)^2d^3+18\log\big(\tan\big(}{}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x)
```

output
```
( - 18*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*c*d**2 + 12*log(tan((
e + f*x)/2) - 1)*tan((e + f*x)/2)**2*d**3 + 18*log(tan((e + f*x)/2) - 1)*c
*d**2 - 12*log(tan((e + f*x)/2) - 1)*d**3 + 18*log(tan((e + f*x)/2) + 1)*t
an((e + f*x)/2)**2*c*d**2 - 12*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)*
*2*d**3 - 18*log(tan((e + f*x)/2) + 1)*c*d**2 + 12*log(tan((e + f*x)/2) +
1)*d**3 - tan((e + f*x)/2)**5*c**3 + 3*tan((e + f*x)/2)**5*c**2*d - 3*tan(
(e + f*x)/2)**5*c*d**2 + tan((e + f*x)/2)**5*d**3 + 4*tan((e + f*x)/2)**3*
c**3 + 6*tan((e + f*x)/2)**3*c**2*d - 24*tan((e + f*x)/2)**3*c*d**2 + 14*t
an((e + f*x)/2)**3*d**3 - 3*tan((e + f*x)/2)*c**3 - 9*tan((e + f*x)/2)*c**
2*d + 27*tan((e + f*x)/2)*c*d**2 - 27*tan((e + f*x)/2)*d**3)/(6*a**2*f*(ta
n((e + f*x)/2)**2 - 1))
```

3.220 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 89

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{d^2\operatorname{arctanh}(\sin(e+fx))}{a^2f} + \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

output
```
d^2*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^2+
1/3*(c-d)*(c+5*d)*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(89) = 178.

Time = 1.85 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.03

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx =$$

$$-\,2\cos\left(\tfrac{1}{2}(e+fx)\right)\left(6d^2\cos^3\left(\tfrac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\tfrac{1}{2}(e+f\right.\right.\right.\right.$$

input

```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

output

```
(-2*Cos[(e + f*x)/2]*(6*d^2*Cos[(e + f*x)/2]^3*(Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)^2*Sec
[e/2]*Sin[(f*x)/2] - 4*(c^2 + c*d - 2*d^2)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin
[(f*x)/2] + (c - d)^2*Cos[(e + f*x)/2]*Tan[e/2]))/(3*a^2*f*(1 + Cos[e + f*
x])^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 4475, 100, 27, 87, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^3\left(c^2+4dc-2d^2+3d^2\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^4}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{c^2+4dc-2d^2+3d^2\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 87$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d^2\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{(c-d)(c+5d)\sqrt{a-a\sec(e+fx)}}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{6d^2\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{a}-\frac{(c-d)(c+5d)\sqrt{a-a\sec(e+fx)}}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{6d^2\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)}{a^2}-\frac{(c-d)(c+5d)\sqrt{a-a\sec(e+fx)}}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

output

```
-((a^2*(-1/3*((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(a^2*(a + a*Sec[e + f*x]
)^(3/2)) + ((-6*d^2*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x
]]])/a^2 - ((c - d)*(c + 5*d)*Sqrt[a - a*Sec[e + f*x]])/(a^2*Sqrt[a + a*Se
c[e + f*x]]))/(3*a))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*
Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

method	result
parallelrisch	$\frac{-6d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+6d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\left((c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3c-9d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(c-d)}{6a^2f}$
derivativedivides	$\frac{-2d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2d^2\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{2fa^2}$
default	$\frac{-2d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2d^2\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{2fa^2}$
risch	$\frac{2i\left(3c^2e^{2i(fx+e)}-3d^2e^{2i(fx+e)}+3c^2e^{i(fx+e)}+6d\,e^{i(fx+e)}c-9d^2e^{i(fx+e)}+2c^2+2cd-4d^2\right)}{3fa^2\left(e^{i(fx+e)}+1\right)^3}+\frac{d^2\ln\left(e^{i(fx+e)}+i\right)}{a^2f}-$
norman	$\frac{-\frac{\left(c^2-2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{6fa}+\frac{\left(c^2+2cd-3d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2fa}+\frac{\left(5c^2+2cd-7d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{6fa}-\frac{\left(7c^2+10cd-17d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{6fa}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2a}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/6*(-6*d^2*ln(tan(1/2*f*x+1/2*e)-1)+6*d^2*ln(tan(1/2*f*x+1/2*e)+1)-((c-d)
*tan(1/2*f*x+1/2*e)^2-3*c-9*d)*tan(1/2*f*x+1/2*e)*(c-d))/a^2/f
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.74

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)+d^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+\right.\right.}{6\left(a^2f\cos\left(fx+e\right)^2+2\,a^2\right.}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(sin(f*x + e) +
1) - 3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(-sin(f*x + e) +
 1) + 2*(c^2 + 4*c*d - 5*d^2 + 2*(c^2 + c*d - 2*d^2)*cos(f*x + e))*sin(f*x
 + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\int \frac{c^2\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d^2\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{2cd\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

output
```
(Integral(c**2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(2*c*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))
/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(85) = 170$.

Time = 0.05 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{d^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-\frac{2cd\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-}{6f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
-1/6*(d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 2*c*d*(3*sin(f*x + e)/(cos(f*x +
 e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/
(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.78

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{6d^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2}-\frac{6d^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2}-\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^4cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+a^4d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a}{a^6}}{6f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/6*(6*d^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 6*d^2*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/a^2 - (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^4*c*d*tan(1
/2*f*x + 1/2*e)^3 + a^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^2*tan(1/2*f*x
 + 1/2*e) - 6*a^4*c*d*tan(1/2*f*x + 1/2*e) + 9*a^4*d^2*tan(1/2*f*x + 1/2*e
))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{(c-d)^2}{2\,a^2}-\frac{c^2-d^2}{a^2}\right)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(c-d)^2}{6\,a^2\,f}$$

input
```
int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output
```
(2*d^2*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (tan(e/2 + (f*x)/2)*((c - d)^2
/(2*a^2) - (c^2 - d^2)/a^2))/f - (tan(e/2 + (f*x)/2)^3*(c - d)^2)/(6*a^2*f
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{-6\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d^2+6\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^2+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3cd-\tan}{6a^2f}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x)
```

output
```
( - 6*log(tan((e + f*x)/2) - 1)*d**2 + 6*log(tan((e + f*x)/2) + 1)*d**2 -
tan((e + f*x)/2)**3*c**2 + 2*tan((e + f*x)/2)**3*c*d - tan((e + f*x)/2)**3
*d**2 + 3*tan((e + f*x)/2)*c**2 + 6*tan((e + f*x)/2)*c*d - 9*tan((e + f*x)
/2)*d**2)/(6*a**2*f)
```

3.221 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c+2d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

output
```
1/3*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2+1/3*(c+2*d)*tan(f*x+e)/f/(a^2+a^
2*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(3(c+d)\sin\left(\frac{fx}{2}\right)-3c\sin\left(e+\frac{fx}{2}\right)+(2c+d)\sin\left(e+\frac{3fx}{2}\right)\right)}{3a^2f(1+\cos(e+fx))^2}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

output
```
(Cos[(e + f*x)/2]*Sec[e/2]*(3*(c + d)*Sin[(f*x)/2] - 3*c*Sin[e + (f*x)/2]
+ (2*c + d)*Sin[e + (3*f*x)/2]))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4488}$$

$$\frac{(c+2d)\int \frac{\sec(e+fx)}{\sec(e+fx)a+a}dx}{3a}+\frac{(c-d)\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+2d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{(c-d)\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{(c+2d)\tan(e+fx)}{3af(a\sec(e+fx)+a)}+\frac{(c-d)\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

output
```
((c - d)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((c + 2*d)*Tan[e + f
*x])/(3*a*f*(a + a*Sec[e + f*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$\frac{\left(\left(c+\frac{d}{2}\right)\cos\left(fx+e\right)+\frac{c}{2}+d\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3a^{2}f}$	46
derivativedivides	$\frac{-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f\,a^{2}}$	60
default	$\frac{-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f\,a^{2}}$	60
risch	$\frac{2i\left(3c\,{\mathrm e}^{2i\left(fx+e\right)}+3\,{\mathrm e}^{i\left(fx+e\right)}c+3d\,{\mathrm e}^{i\left(fx+e\right)}+2c+d\right)}{3f\,a^{2}\left({\mathrm e}^{i\left(fx+e\right)}+1\right)^{3}}$	64
norman	$\frac{-\frac{\left(c-d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{6fa}-\frac{\left(c+d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2fa}+\frac{\left(2c+d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3fa}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$	89

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/3*((c+1/2*d)*cos(f*x+e)+1/2*c+d)*sec(1/2*f*x+1/2*e)^2*tan(1/2*f*x+1/2*e)
/a^2/f
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{((2\,c+d)\cos\left(fx+e\right)+c+2\,d)\sin\left(fx+e\right)}{3\,\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output
```
1/3*((2*c + d)*cos(f*x + e) + c + 2*d)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2
+ 2*a^2*f*cos(f*x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{c\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

output
```
(Integral(c*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
1/6*(d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{6\,a^2f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
-1/6*(c*tan(1/2*f*x + 1/2*e)^3 - d*tan(1/2*f*x + 1/2*e)^3 - 3*c*tan(1/2*f*
x + 1/2*e) - 3*d*tan(1/2*f*x + 1/2*e))/(a^2*f)
```

Mupad [B] (verification not implemented)

Time = 10.42 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(c+d)}{2\,a^2\,f} - \frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,(c-d)}{6\,a^2\,f}$$

input
```
int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

output
```
(tan(e/2 + (f*x)/2)*(c + d))/(2*a^2*f) - (tan(e/2 + (f*x)/2)^3*(c - d))/(6
*a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 d+3c+3d\right)}{6a^2f}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

output
```
(tan((e + f*x)/2)*( - tan((e + f*x)/2)**2*c + tan((e + f*x)/2)**2*d + 3*c
+ 3*d))/(6*a**2*f)
```

3.222 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 129

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx &= \frac{2d^2\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2(c-d)^{5/2}\sqrt{c+d}f} + \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} \\ &\quad + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\left(a^2+a^2\sec(e+fx)\right)}\end{aligned}$$

output

```
2*d^2*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^2/(c-d)^(5/2)/
(c+d)^(1/2)/f+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2+1/3*(c-4*d)*tan(f*
x+e)/(c-d)^2/f/(a^2+a^2*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.62

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(-\frac{24id^2\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\sec\left(\frac{e}{2}\right)(3(\right.\right.}{3a^2(c-d)^2f(1+\cos(e+fx))^2}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

output
```
(Cos[(e + f*x)/2]*(((-24*I)*d^2*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-
d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])
]*Cos[(e + f*x)/2]^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] -
I*Sin[e])^2]) + Sec[e/2]*(3*(c - 3*d)*Sin[(f*x)/2] - 3*(c - 2*d)*Sin[e + (
f*x)/2] + (2*c - 5*d)*Sin[e + (3*f*x)/2])))/(3*a^2*(c - d)^2*f*(1 + Cos[e
+ f*x])^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.70, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 4475, 115, 25, 27, 169, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 115$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2(c-3d+d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(c-3d+d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{c-3d+d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 169$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{-\frac{\int -\frac{3a^2d^2}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a^3(c-d)}-\frac{(c-4d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/}}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{3d^2\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a(c-d)}-\frac{(c-4d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/}}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{6d^2\int \frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{a(c-d)}-\frac{(c-4d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{6d^2\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2(c-d)^{3/2}\sqrt{c+d}}-\frac{(c-4d)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

output

```
-((a^2*(-1/3*Sqrt[a - a*Sec[e + f*x]]/(a^2*(c - d)*(a + a*Sec[e + f*x])^(3
/2)) + ((6*d^2*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*
Sqrt[a - a*Sec[e + f*x]])])/(a^2*(c - d)^(3/2)*Sqrt[c + d]) - ((c - 4*d)*S
qrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*Sqrt[a + a*Sec[e + f*x]]))/(3*a*(c -
 d)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c-d)^{2}}+\frac{4d^{2}\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c-d)^{2}\sqrt{(c-d)(c+d)}}}{2f\,a^{2}}$
default	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c-d)^{2}}+\frac{4d^{2}\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c-d)^{2}\sqrt{(c-d)(c+d)}}}{2f\,a^{2}}$
risch	$\frac{2i\left(3c\,{\mathrm e}^{2i(fx+e)}-6d\,{\mathrm e}^{2i(fx+e)}+3\,{\mathrm e}^{i(fx+e)}c-9d\,{\mathrm e}^{i(fx+e)}+2c-5d\right)}{3f\,a^{2}(c-d)^{2}\left({\mathrm e}^{i(fx+e)}+1\right)^{3}}+\frac{d^{2}\ln\left({\mathrm e}^{i(fx+e)}+\frac{ic^{2}-id^{2}+d\sqrt{c^{2}-d^{2}}}{\sqrt{c^{2}-d^{2}}\,c}\right)}{\sqrt{c^{2}-d^{2}}\,(c-d)^{2}f\,a^{2}}-\frac{d^{2}\ln\left(\right.}{}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
1/2/f/a^2*(-1/(c-d)^2*(1/3*c*tan(1/2*f*x+1/2*e)^3-1/3*d*tan(1/2*f*x+1/2*e)
^3-c*tan(1/2*f*x+1/2*e)+3*d*tan(1/2*f*x+1/2*e))+4*d^2/(c-d)^2/((c-d)*(c+d)
)^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(116) = 232.

Time = 0.15 (sec) , antiderivative size = 598, normalized size of antiderivative = 4.64

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$
$$= \left[\frac{3\left(d^{2}\cos\left(fx+e\right)^{2}+2\,d^{2}\cos\left(fx+e\right)+d^{2}\right)\sqrt{c^{2}-d^{2}}\log\left(\frac{2\,cd\cos(fx+e)-\left(c^{2}-2\,d^{2}\right)\cos(fx+e)^{2}+2\,\sqrt{c^{2}-d^{2}}(d\cos(fx}{c^{2}\cos(fx+e)^{2}+2\,cd\cos(fx+e)+c}\right.}{6\left(\left(a^{2}c^{4}-2\,a^{2}c^{3}d+2\,a^{2}cd^{3}-a^{2}d^{4}\right)f\cos\left(fx+e\right)^{2}+2\left(a^{2}c^{4}-2\,a^{2}c\right.\right.}\right.$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
[1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt(c^2 - d^2)*lo
g((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(
d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*
d*cos(f*x + e) + d^2)) + 2*(c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2
*d - 2*c*d^2 + 5*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d
+ 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a
^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3
- a^2*d^4)*f), 1/3*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt
(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*si
n(f*x + e))) + (c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2*d - 2*c*d^2
 + 5*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^
3 - a^2*d^4)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a
^2*d^4)*f*cos(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f
)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$
$$= \frac{\int \frac{\sec{(e+fx)}}{c\sec^2{(e+fx)}+2c\sec{(e+fx)}+c+d\sec^3{(e+fx)}+2d\sec^2{(e+fx)}+d\sec{(e+fx)}}\,dx}{a^2}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

output

```
Integral(sec(e + f*x)/(c*sec(e + f*x)**2 + 2*c*sec(e + f*x) + c + d*sec(e
+ f*x)**3 + 2*d*sec(e + f*x)**2 + d*sec(e + f*x)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(116) = 232$.

Time = 0.19 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.93

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = -\frac{\frac{12\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{(a^2c^2-2a^2cd+a^2d^2)\sqrt{-c^2+d^2}}+\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^4cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+a^4d^2}{}}{6f}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
-1/6*(12*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((a^2*c^
2 - 2*a^2*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + (a^4*c^2*tan(1/2*f*x + 1/2*e)
^3 - 2*a^4*c*d*tan(1/2*f*x + 1/2*e)^3 + a^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 3
*a^4*c^2*tan(1/2*f*x + 1/2*e) + 12*a^4*c*d*tan(1/2*f*x + 1/2*e) - 9*a^4*d^
2*tan(1/2*f*x + 1/2*e))/(a^6*c^3 - 3*a^6*c^2*d + 3*a^6*c*d^2 - a^6*d^3))/f
```

Mupad [B] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.30

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{1}{a^2\,(c-d)}-\frac{c+d}{2\,a^2\,{(c-d)}^2}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{6\,a^2\,f\,\left(c-d\right)}$$
$$-\frac{d^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3-3\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d+3\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^2-1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^3}{\sqrt{c+d}\,{(c-d)}^{5/2}}\right)\,2\mathrm{i}}{a^2\,f\,\sqrt{c+d}\,{(c-d)}^{5/2}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))),x)
```

output
```
(tan(e/2 + (f*x)/2)*(1/(a^2*(c - d)) - (c + d)/(2*a^2*(c - d)^2)))/f - tan
(e/2 + (f*x)/2)^3/(6*a^2*f*(c - d)) - (d^2*atan((c^3*tan(e/2 + (f*x)/2)*1i
 - d^3*tan(e/2 + (f*x)/2)*1i + c*d^2*tan(e/2 + (f*x)/2)*3i - c^2*d*tan(e/2
 + (f*x)/2)*3i)/((c + d)^(1/2)*(c - d)^(5/2)))*2i)/(a^2*f*(c + d)^(1/2)*(c
 - d)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.60

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$
$$= \frac{12\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)d^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^3+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^2d+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c}{6a^2f\left(c^4-2c^3\right.}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x)
```

output

```
(12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sq
rt( - c**2 + d**2))*d**2 - tan((e + f*x)/2)**3*c**3 + tan((e + f*x)/2)**3*
c**2*d + tan((e + f*x)/2)**3*c*d**2 - tan((e + f*x)/2)**3*d**3 + 3*tan((e
+ f*x)/2)*c**3 - 9*tan((e + f*x)/2)*c**2*d - 3*tan((e + f*x)/2)*c*d**2 + 9
*tan((e + f*x)/2)*d**3)/(6*a**2*f*(c**4 - 2*c**3*d + 2*c*d**3 - d**4))
```

3.223 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 211

$$\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx \\
&= \frac{2d^2(3c+2d)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2(c-d)^{7/2}(c+d)^{3/2}f}+\frac{d(c^2-6cd-10d^2)\tan(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sec(e+fx))} \\
&\quad+\frac{(c-6d)\tan(e+fx)}{3a^2(c-d)^2f(1+\sec(e+fx))(c+d\sec(e+fx))} \\
&\quad+\frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))}
\end{aligned}$$

output
```
2*d^2*(3*c+2*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^2/(c
-d)^(7/2)/(c+d)^(3/2)/f+1/3*d*(c^2-6*c*d-10*d^2)*tan(f*x+e)/a^2/(c-d)^3/(c
+d)/f/(c+d*sec(f*x+e))+1/3*(c-6*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))
/(c+d*sec(f*x+e))+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e
))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^4(e+fx)\left(\frac{12d^2(3c+2d)\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\,s}}\right.}{}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]
```

output
```
(2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^4*((12*d^2*(3*c + 2*
d)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/
(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]^3*(d + c*C
os[e + f*x])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] -
I*Sin[e])^2]) + (c - d)*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] - 4*(c
- 4*d)*Cos[(e + f*x)/2]^2*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (6*
d^3*Cos[(e + f*x)/2]^3*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] -
Sin[e/2])*(Cos[e/2] + Sin[e/2])) + (c - d)*Cos[(e + f*x)/2]*(d + c*Cos[e +
 f*x])*Tan[e/2]))/(3*a^2*(-c + d)^3*f*(1 + Sec[e + f*x])^2*(c + d*Sec[e +
f*x])^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4475, 114, 27, 169, 25, 27, 169, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(c+2d-2d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{a^2(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}}{a^2(c^2-d^2)(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{c+2d-2d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{c^2-d^2}+\frac{d\sqrt{a-a\sec(e+fx)}}{a^2(c^2-d^2)(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{\int -\frac{a^2((c-6d)(c+d)+d(c+4d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{(c+4d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{c^2-d^2}+\frac{d\sqrt{a-}}{a^2(c^2-d^2)(a\sec(e+f}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{a^2((c-6d)(c+d)+d(c+4d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{(c+4d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{c^2-d^2}+\frac{d\sqrt{a-a\,\mathrm{se}}}{a^2(c^2-d^2)(a\sec(e+fx)+}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{(c-6d)(c+d)+d(c+4d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a(c-d)}-\frac{(c+4d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{c^2-d^2}+\frac{d\sqrt{a-a\,\mathrm{se}}}{a^2(c^2-d^2)(a\sec(e+fx)+}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{\int -\frac{3a^2d^2(3c+2d)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a^3(c-d)}-\frac{\left(c^2-6cd-10d^2\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{(c+4d)\sqrt{a-a\sec(e+fx}}{3a^2(c-d)(a\sec(e+fx)+a)}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{3d^2(3c+2d)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a(c-d)}-\frac{\left(c^2-6cd-10d^2\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{(c+4d)\sqrt{a-a\,\mathrm{s}}}{3a^2(c-d)(a\sec(e-}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{6d^2(3c+2d)\int \frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{a(c-d)}-\frac{\left(c^2-6cd-10d^2\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{(c+4d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^3}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{6d^2(3c+2d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2(c-d)^{3/2}\sqrt{c+d}}-\frac{\left(c^2-6cd-10d^2\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{(c+4d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{c^2-d^2}+\frac{}{a^2(c^2-d^2)(}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]
```

output
```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c^2 - d^2)*(a + a*Sec[e + f*x])
^(3/2)*(c + d*Sec[e + f*x])) + (-1/3*((c + 4*d)*Sqrt[a - a*Sec[e + f*x]])/
(a^2*(c - d)*(a + a*Sec[e + f*x])^(3/2)) + ((6*d^2*(3*c + 2*d)*ArcTan[(Sqr
t[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]
)/(a^2*(c - d)^(3/2)*Sqrt[c + d]) - ((c^2 - 6*c*d - 10*d^2)*Sqrt[a - a*Sec
[e + f*x]])/(a^2*(c - d)*Sqrt[a + a*Sec[e + f*x]]))/(3*a*(c - d)))/(c^2 -
d^2))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+5d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-2cd+d^2\right)(c-d)}-\frac{4d^2\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\right.}{(c-d)^3}}{2f\,a^2}$
default	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+5d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-2cd+d^2\right)(c-d)}-\frac{4d^2\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\right.}{(c-d)^3}}{2f\,a^2}$
risch	$\underline{2i\left(-3c^4\mathrm{e}^{4i(fx+e)}+6c^3d\,\mathrm{e}^{4i(fx+e)}+9c^2d^2\mathrm{e}^{4i(fx+e)}+3d^4\mathrm{e}^{4i(fx+e)}-3c^4\mathrm{e}^{3i(fx+e)}+6c^3d\,\mathrm{e}^{3i(fx+e)}+27c^2d^2\mathrm{e}^{3i(fx+e)}+\right.}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/2/f/a^2*(-1/(c^2-2*c*d+d^2)/(c-d)*(1/3*c*tan(1/2*f*x+1/2*e)^3-1/3*d*tan(
1/2*f*x+1/2*e)^3-c*tan(1/2*f*x+1/2*e)+5*d*tan(1/2*f*x+1/2*e))-4*d^2/(c-d)^
3*(-d/(c+d)*tan(1/2*f*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^
2*d-c-d)-(3*c+2*d)/(c+d)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2
*e)/((c-d)*(c+d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(196) = 392$.

Time = 0.20 (sec) , antiderivative size = 1242, normalized size of antiderivative = 5.89

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
[-1/6*(3*(3*c*d^3 + 2*d^4 + (3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*
d^2 + 7*c*d^3 + 2*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(
f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x
+ e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2
)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(c^4*d - 6*c^3*d^2
- 11*c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5 - 6*c^4*d - 10*c^3*d^2 + 3*c^2*d^
3 + 8*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c^5 - 4*c^4*d - 14*c^3*d^2 - 10*c^2
*d^3 + 13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*c^7 - 2*a^2*c^
6*d - a^2*c^5*d^2 + 4*a^2*c^4*d^3 - a^2*c^3*d^4 - 2*a^2*c^2*d^5 + a^2*c*d^
6)*f*cos(f*x + e)^3 + (2*a^2*c^7 - 3*a^2*c^6*d - 4*a^2*c^5*d^2 + 7*a^2*c^4
*d^3 + 2*a^2*c^3*d^4 - 5*a^2*c^2*d^5 + a^2*d^7)*f*cos(f*x + e)^2 + (a^2*c^
7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*
c*d^6 + 2*a^2*d^7)*f*cos(f*x + e) + (a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d
^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f), 1/3*(3*(3*c*
d^3 + 2*d^4 + (3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*d^2 + 7*c*d^3
+ 2*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*x + e))*sqrt
(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*si
n(f*x + e))) + (c^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5
 - 6*c^4*d - 10*c^3*d^2 + 3*c^2*d^3 + 8*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c
^5 - 4*c^4*d - 14*c^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*cos(f*x + e...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{\int \frac{\sec{(e+fx)}}{c^2\sec^2{(e+fx)}+2c^2\sec{(e+fx)}+c^2+2cd\sec^3{(e+fx)}+4cd\sec^2{(e+fx)}+2cd\sec{(e+fx)}+d^2\sec^4{(e+fx)}+2d^2\sec^3{(e+fx)}+d^2\sec^2{(e+fx)}}}{a^2}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(sec(e + f*x)/(c**2*sec(e + f*x)**2 + 2*c**2*sec(e + f*x) + c**2 +
 2*c*d*sec(e + f*x)**3 + 4*c*d*sec(e + f*x)**2 + 2*c*d*sec(e + f*x) + d**2
*sec(e + f*x)**4 + 2*d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(196) = 392.

Time = 0.20 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.25

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = \frac{\frac{12\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)} + \frac{12\,(3\,cd^2+2\,d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\,\mathrm{ta}}{}\right.\right.}{(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4)\sqrt{}}}{}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
1/6*(12*d^3*tan(1/2*f*x + 1/2*e)/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a
^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) + 1
2*(3*c*d^2 + 2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + ar
ctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))
/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)) - (a^4
*c^4*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c
^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c*d^3*tan(1/2*f*x + 1/2*e)^3 + a^4*d
^4*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^4*tan(1/2*f*x + 1/2*e) + 24*a^4*c^3*d*
tan(1/2*f*x + 1/2*e) - 54*a^4*c^2*d^2*tan(1/2*f*x + 1/2*e) + 48*a^4*c*d^3*
tan(1/2*f*x + 1/2*e) - 15*a^4*d^4*tan(1/2*f*x + 1/2*e))/(a^6*c^6 - 6*a^6*c
^5*d + 15*a^6*c^4*d^2 - 20*a^6*c^3*d^3 + 15*a^6*c^2*d^4 - 6*a^6*c*d^5 + a^
6*d^6))/f
```

Mupad [B] (verification not implemented)

Time = 11.17 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.49

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3}{2\,a^2\,{\left(c-d\right)}^2}-\frac{c^2-d^2}{a^2\,{\left(c-d\right)}^4}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{6\,a^2\,f\,{\left(c-d\right)}^2}$$

$$+\frac{2\,d^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(c+d\right)\,\Big(a^2\,d^4-a^2\,c^4+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(a^2\,c^4-4\,a^2\,c^3\,d+6\,a^2\,c^2\,d^2-4\,a^2\,c\,d^3+a^2\,d^4\right)-2\,a^2\,c\,d^3+}$$

$$-\frac{d^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^4-4\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,d+6\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d^2-4\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^3+1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^4}{\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}\right)\,\left(3\,c+2\,d\right)\,2\mathrm{i}}{a^2\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{7/2}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^2),x)
```

output
```
(tan(e/2 + (f*x)/2)*(3/(2*a^2*(c - d)^2) - (c^2 - d^2)/(a^2*(c - d)^4)))/f
 - tan(e/2 + (f*x)/2)^3/(6*a^2*f*(c - d)^2) + (2*d^3*tan(e/2 + (f*x)/2))/(
f*(c + d)*(a^2*d^4 - a^2*c^4 + tan(e/2 + (f*x)/2)^2*(a^2*c^4 + a^2*d^4 - 4
*a^2*c*d^3 - 4*a^2*c^3*d + 6*a^2*c^2*d^2) - 2*a^2*c*d^3 + 2*a^2*c^3*d)) -
(d^2*atan((c^4*tan(e/2 + (f*x)/2)*1i + d^4*tan(e/2 + (f*x)/2)*1i - c*d^3*t
an(e/2 + (f*x)/2)*4i - c^3*d*tan(e/2 + (f*x)/2)*4i + c^2*d^2*tan(e/2 + (f*
x)/2)*6i)/((c + d)^(1/2)*(c - d)^(7/2)))*(3*c + 2*d)*2i)/(a^2*f*(c + d)^(3
/2)*(c - d)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 861, normalized size of antiderivative = 4.08

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x)
```

output
```
(36*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sq
rt( - c**2 + d**2))*tan((e + f*x)/2)**2*c**2*d**2 - 12*sqrt( - c**2 + d**2
)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan
((e + f*x)/2)**2*c*d**3 - 24*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c
 - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*d**4 - 36
*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt(
 - c**2 + d**2))*c**2*d**2 - 60*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2
)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c*d**3 - 24*sqrt( - c**2 +
 d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2)
)*d**4 - tan((e + f*x)/2)**5*c**5 + tan((e + f*x)/2)**5*c**4*d + 2*tan((e
+ f*x)/2)**5*c**3*d**2 - 2*tan((e + f*x)/2)**5*c**2*d**3 - tan((e + f*x)/2
)**5*c*d**4 + tan((e + f*x)/2)**5*d**5 + 4*tan((e + f*x)/2)**3*c**5 - 14*t
an((e + f*x)/2)**3*c**4*d - 8*tan((e + f*x)/2)**3*c**3*d**2 + 28*tan((e +
f*x)/2)**3*c**2*d**3 + 4*tan((e + f*x)/2)**3*c*d**4 - 14*tan((e + f*x)/2)*
*3*d**5 - 3*tan((e + f*x)/2)*c**5 + 9*tan((e + f*x)/2)*c**4*d + 30*tan((e
+ f*x)/2)*c**3*d**2 + 18*tan((e + f*x)/2)*c**2*d**3 - 27*tan((e + f*x)/2)*
c*d**4 - 27*tan((e + f*x)/2)*d**5)/(6*a**2*f*(tan((e + f*x)/2)**2*c**7 - 3
*tan((e + f*x)/2)**2*c**6*d + tan((e + f*x)/2)**2*c**5*d**2 + 5*tan((e + f
*x)/2)**2*c**4*d**3 - 5*tan((e + f*x)/2)**2*c**3*d**4 - tan((e + f*x)/2)**
2*c**2*d**5 + 3*tan((e + f*x)/2)**2*c*d**6 - tan((e + f*x)/2)**2*d**7 -...
```

3.224 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 284

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx &= \frac{d^2(12c^2+16cd+7d^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2(c-d)^{9/2}(c+d)^{5/2}f} \\ &\quad+\frac{d(2c^2-16cd-21d^2)\tan(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sec(e+fx))^2} \\ &\quad+\frac{(c-8d)\tan(e+fx)}{3a^2(c-d)^2f(1+\sec(e+fx))(c+d\sec(e+fx))^2} \\ &\quad+\frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} \\ &\quad+\frac{d(2c^3-16c^2d-59cd^2-32d^3)\tan(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sec(e+fx))}\end{aligned}$$

output

```
d^2*(12*c^2+16*c*d+7*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/
2))/a^2/(c-d)^(9/2)/(c+d)^(5/2)/f+1/6*d*(2*c^2-16*c*d-21*d^2)*tan(f*x+e)/a
^2/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))^2+1/3*(c-8*d)*tan(f*x+e)/a^2/(c-d)^2/f
/(1+sec(f*x+e))/(c+d*sec(f*x+e))^2+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))
^2/(c+d*sec(f*x+e))^2+1/6*d*(2*c^3-16*c^2*d-59*c*d^2-32*d^3)*tan(f*x+e)/a^
2/(c-d)^4/(c+d)^2/f/(c+d*sec(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.95 (sec) , antiderivative size = 2220, normalized size of antiderivative = 7.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
```

output

```
((12*c^2 + 16*c*d + 7*d^2)*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec
[e + f*x]^5*(((-4*I)*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt
[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*S
in[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2
 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (4*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e
]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d
^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)
/2])]*Sin[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^
4*(c + d)^2*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f
*x)/2]*(d + c*Cos[e + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-16*c^7*Sin[(f
*x)/2] + 14*c^6*d*Sin[(f*x)/2] + 220*c^5*d^2*Sin[(f*x)/2] + 334*c^4*d^3*Si
n[(f*x)/2] + 54*c^3*d^4*Sin[(f*x)/2] - 156*c^2*d^5*Sin[(f*x)/2] - 48*c*d^6
*Sin[(f*x)/2] + 18*d^7*Sin[(f*x)/2] + 14*c^7*Sin[(3*f*x)/2] - 16*c^6*d*Sin
[(3*f*x)/2] - 226*c^5*d^2*Sin[(3*f*x)/2] - 532*c^4*d^3*Sin[(3*f*x)/2] - 58
3*c^3*d^4*Sin[(3*f*x)/2] - 232*c^2*d^5*Sin[(3*f*x)/2] - 6*c*d^6*Sin[(3*f*x
)/2] + 6*d^7*Sin[(3*f*x)/2] - 12*c^7*Sin[e - (f*x)/2] + 20*c^6*d*Sin[e - (
f*x)/2] + 236*c^5*d^2*Sin[e - (f*x)/2] + 628*c^4*d^3*Sin[e - (f*x)/2] + 77
8*c^3*d^4*Sin[e - (f*x)/2] + 420*c^2*d^5*Sin[e - (f*x)/2] + 48*c*d^6*Sin[e
 - (f*x)/2] - 18*d^7*Sin[e - (f*x)/2] + 12*c^7*Sin[e + (f*x)/2] - 20*c^6*d
*Sin[e + (f*x)/2] - 236*c^5*d^2*Sin[e + (f*x)/2] - 460*c^4*d^3*Sin[e + ...
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4475, 114, 27, 168, 27, 169, 25, 27, 169, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(2(c+d)-3d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2a^2(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2a^2(c^2-d^2)(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{2(c+d)-3d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2a^2(c^2-d^2)(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{a^2\left(2c^2+12dc+7d^2-2d(5c+2d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{a^2\left(c^2-d^2\right)}+\frac{d(5c+2d)\sqrt{a-a\sec(e+fx)}}{a^2\left(c^2-d^2\right)(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{}{2a^2(c^2-}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{\int \frac{2c^2+12dc+7d^2-2d(5c+2d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{c^2-d^2}+\frac{d(5c+2d)\sqrt{a-a\sec(e+fx)}}{a^2\left(c^2-d^2\right)(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{}{2a^2(c^2-}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 169$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{-\frac{\int -\frac{a^2\left((c+d)\left(2c^2-16dc-21d^2\right)+d\left(2c^2+22dc+11d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{\left(2c^2+22cd+11d^2\right)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{c^2-d^2}+\frac{}{a^2\left(c^2-d^2\right)}}{2(c^2-d^2)}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{\frac{\int \frac{a^2\left((c+d)\left(2c^2-16dc-21d^2\right)+d\left(2c^2+22dc+11d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{\left(2c^2+22cd+11d^2\right)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{c^2-d^2}+\frac{d(5c}{a^2\left(c^2-d^2\right)(a\,\mathrm{s}}}{2(c^2-d^2)}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{\frac{\int \frac{(c+d)\left(2c^2-16dc-21d^2\right)+d\left(2c^2+22dc+11d^2\right)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a(c-d)}-\frac{\left(2c^2+22cd+11d^2\right)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{c^2-d^2}+\frac{d(5c+2d)}{a^2\left(c^2-d^2\right)(a\sec(e+}}{2(c^2-d^2)}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{-\frac{\int -\frac{3a^2d^2\left(12c^2+16dc+7d^2\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a^3(c-d)}-\frac{\left(2c^3-16c^2d-59cd^2-32d^3\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\left(2c^2+22cd+11\right.}{3a^2(c-d)(a}}{c^2-d^2}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec}}$$

$\Big\downarrow$ 27

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\frac{3d^2\left(12c^2+16cd+7d^2\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a(c-d)}-\frac{\left(2c^3-16c^2d-59cd^2-32d^3\right)\sqrt{a-a\sec(e+f}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}}{c^2-d^2}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}}$$

$\Big\downarrow$ 104

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\frac{6d^2\left(12c^2+16cd+7d^2\right)\int\frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{a(c-d)}-\frac{\left(2c^3-16c^2d-59cd^2-32d^3\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\left(2c^2\right.}{3a}}{c^2-d^2}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

$\Big\downarrow$ 218

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\frac{6d^2\left(12c^2+16cd+7d^2\right)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2(c-d)^{3/2}\sqrt{c+d}}-\frac{\left(2c^3-16c^2d-59cd^2-32d^3\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\left(2c^2+22cd+11d^2\right)\sqrt{a-a\,\mathrm{s}}}{3a^2(c-d)(a\sec(e+fx)+}}{c^2-d^2}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
```

output

```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]])/(2*a^2*(c^2 - d^2)*(a + a*Sec[e + f*x
])^(3/2)*(c + d*Sec[e + f*x])^2) + ((d*(5*c + 2*d)*Sqrt[a - a*Sec[e + f*x]
])/(a^2*(c^2 - d^2)*(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])) + (-1
/3*((2*c^2 + 22*c*d + 11*d^2)*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*(a +
a*Sec[e + f*x])^(3/2)) + ((6*d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTan[(Sqrt[c
+ d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])])/(a
^2*(c - d)^(3/2)*Sqrt[c + d]) - ((2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Sq
rt[a - a*Sec[e + f*x]])/(a^2*(c - d)*Sqrt[a + a*Sec[e + f*x]]))/(3*a*(c -
d)))/(c^2 - d^2))/(2*(c^2 - d^2)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x
]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+7d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)(c-d)}-\frac{8d^{2}\left(\frac{-\frac{d\left(8c^{2}-3cd-5d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{4\left(c^{2}+2cd+d^{2}\right)}+\frac{d(8c+3d)\tan(}{4c+4d}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}\right.}{(}}{2f\,a^{2}}$
default	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+7d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)(c-d)}-\frac{8d^{2}\left(\frac{-\frac{d\left(8c^{2}-3cd-5d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{4\left(c^{2}+2cd+d^{2}\right)}+\frac{d(8c+3d)\tan(}{4c+4d}}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}\right.}{(}}{2f\,a^{2}}$
risch	Expression too large to display

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/2/f/a^2*(-1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c-d)*(1/3*c*tan(1/2*f*x+1/2*e)^3-
1/3*d*tan(1/2*f*x+1/2*e)^3-c*tan(1/2*f*x+1/2*e)+7*d*tan(1/2*f*x+1/2*e))-8*
d^2/(c-d)^4*((-1/4*d*(8*c^2-3*c*d-5*d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e
)^3+1/4*d*(8*c+3*d)/(c+d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-tan(
1/2*f*x+1/2*e)^2*d-c-d)^2-1/4*(12*c^2+16*c*d+7*d^2)/(c^2+2*c*d+d^2)/((c-d)
*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. 2(267) = 534.

Time = 0.27 (sec) , antiderivative size = 2030, normalized size of antiderivative = 7.15

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
[1/12*(3*(12*c^2*d^4 + 16*c*d^5 + 7*d^6 + (12*c^4*d^2 + 16*c^3*d^3 + 7*c^2
*d^4)*cos(f*x + e)^4 + 2*(12*c^4*d^2 + 28*c^3*d^3 + 23*c^2*d^4 + 7*c*d^5)*
cos(f*x + e)^3 + (12*c^4*d^2 + 64*c^3*d^3 + 83*c^2*d^4 + 44*c*d^5 + 7*d^6)
*cos(f*x + e)^2 + 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x +
 e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^
2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^
2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d^2 - 16*c^4*d^3
- 61*c^3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7 + (4*c^7 - 14*c^6*d - 44*c^5
*d^2 - 32*c^4*d^3 + 28*c^3*d^4 + 49*c^2*d^5 + 12*c*d^6 - 3*d^7)*cos(f*x +
e)^3 + (2*c^7 - 8*c^6*d - 68*c^5*d^2 - 140*c^4*d^3 - 23*c^3*d^4 + 142*c^2*
d^5 + 89*c*d^6 + 6*d^7)*cos(f*x + e)^2 + (4*c^6*d - 28*c^5*d^2 - 118*c^4*d
^3 - 106*c^3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e))*sin(f*x
+ e))/((a^2*c^10 - 2*a^2*c^9*d - 2*a^2*c^8*d^2 + 6*a^2*c^7*d^3 - 6*a^2*c^5
*d^5 + 2*a^2*c^4*d^6 + 2*a^2*c^3*d^7 - a^2*c^2*d^8)*f*cos(f*x + e)^4 + 2*(
a^2*c^10 - a^2*c^9*d - 4*a^2*c^8*d^2 + 4*a^2*c^7*d^3 + 6*a^2*c^6*d^4 - 6*a
^2*c^5*d^5 - 4*a^2*c^4*d^6 + 4*a^2*c^3*d^7 + a^2*c^2*d^8 - a^2*c*d^9)*f*co
s(f*x + e)^3 + (a^2*c^10 + 2*a^2*c^9*d - 9*a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 2
2*a^2*c^6*d^4 - 22*a^2*c^4*d^6 + 4*a^2*c^3*d^7 + 9*a^2*c^2*d^8 - 2*a^2*c*d
^9 - a^2*d^10)*f*cos(f*x + e)^2 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d
^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{\sec(e+fx)}{c^3\sec^2(e+fx)+2c^3\sec(e+fx)+c^3+3c^2d\sec^3(e+fx)+6c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^4(e+fx)+6cd^2\sec^3(e+fx)+3cd^2\sec^2}}{a^2}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(sec(e + f*x)/(c**3*sec(e + f*x)**2 + 2*c**3*sec(e + f*x) + c**3 +
 3*c**2*d*sec(e + f*x)**3 + 6*c**2*d*sec(e + f*x)**2 + 3*c**2*d*sec(e + f*
x) + 3*c*d**2*sec(e + f*x)**4 + 6*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e
+ f*x)**2 + d**3*sec(e + f*x)**5 + 2*d**3*sec(e + f*x)**4 + d**3*sec(e + f
*x)**3), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(267) = 534$.

Time = 0.25 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.64

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="g
iac")
```

output

```
1/6*(6*(12*c^2*d^2 + 16*c*d^3 + 7*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*s
gn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))
/sqrt(-c^2 + d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3
- a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(-c^2 + d^2)) - (a^4*c^6*tan(1/
2*f*x + 1/2*e)^3 - 6*a^4*c^5*d*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^4*d^2*tan
(1/2*f*x + 1/2*e)^3 - 20*a^4*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^2*d
^4*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*c*d^5*tan(1/2*f*x + 1/2*e)^3 + a^4*d^6*t
an(1/2*f*x + 1/2*e)^3 - 3*a^4*c^6*tan(1/2*f*x + 1/2*e) + 36*a^4*c^5*d*tan(
1/2*f*x + 1/2*e) - 135*a^4*c^4*d^2*tan(1/2*f*x + 1/2*e) + 240*a^4*c^3*d^3*
tan(1/2*f*x + 1/2*e) - 225*a^4*c^2*d^4*tan(1/2*f*x + 1/2*e) + 108*a^4*c*d^
5*tan(1/2*f*x + 1/2*e) - 21*a^4*d^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9 - 9*a^6
*c^8*d + 36*a^6*c^7*d^2 - 84*a^6*c^6*d^3 + 126*a^6*c^5*d^4 - 126*a^6*c^4*d
^5 + 84*a^6*c^3*d^6 - 36*a^6*c^2*d^7 + 9*a^6*c*d^8 - a^6*d^9) + 6*(8*c^2*d
^3*tan(1/2*f*x + 1/2*e)^3 - 3*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 5*d^5*tan(1/2
*f*x + 1/2*e)^3 - 8*c^2*d^3*tan(1/2*f*x + 1/2*e) - 11*c*d^4*tan(1/2*f*x +
1/2*e) - 3*d^5*tan(1/2*f*x + 1/2*e))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2
 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*(c*tan(1/2*f*x + 1
/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,a^2\,c^6-8\,a^2\,c^5\,d+10\,a^2\,c^4\,d^2-10\,a^2\,c^2\,d^4+8\,a^2\,c\,d^5-2\,a^2\,d^6\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(a^2\right.\right.}$$

$$+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{2}{a^2\,(c-d)^3}-\frac{3\,(c+d)}{2\,a^2\,(c-d)^4}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{6\,a^2\,f\,(c-d)^3}$$

$$-\frac{d^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^5-5\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^4\,d+10\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,d^2-10\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d^3+5\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^4-1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d}{\sqrt{c+d}\,(c-d)^{9/2}}\right.}{a^2\,f\,(c+d)^{5/2}\,(c-d)^{9/2}}$$

input

```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^3),x)
```

output

```
((tan(e/2 + (f*x)/2)^3*(3*c*d^4 + 5*d^5 - 8*c^2*d^3))/(c + d)^2 + (tan(e/2
 + (f*x)/2)*(8*c*d^3 + 3*d^4))/(c + d))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^2*c^
6 - 2*a^2*d^6 + 8*a^2*c*d^5 - 8*a^2*c^5*d - 10*a^2*c^2*d^4 + 10*a^2*c^4*d^
2) - tan(e/2 + (f*x)/2)^4*(a^2*c^6 + a^2*d^6 - 6*a^2*c*d^5 - 6*a^2*c^5*d +
 15*a^2*c^2*d^4 - 20*a^2*c^3*d^3 + 15*a^2*c^4*d^2) - a^2*c^6 - a^2*d^6 + 2
*a^2*c*d^5 + 2*a^2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)) + (
tan(e/2 + (f*x)/2)*(2/(a^2*(c - d)^3) - (3*(c + d))/(2*a^2*(c - d)^4)))/f
- tan(e/2 + (f*x)/2)^3/(6*a^2*f*(c - d)^3) - (d^2*atan((c^5*tan(e/2 + (f*x
)/2)*1i - d^5*tan(e/2 + (f*x)/2)*1i + c*d^4*tan(e/2 + (f*x)/2)*5i - c^4*d*
tan(e/2 + (f*x)/2)*5i - c^2*d^3*tan(e/2 + (f*x)/2)*10i + c^3*d^2*tan(e/2 +
 (f*x)/2)*10i)/((c + d)^(1/2)*(c - d)^(9/2)))*(16*c*d + 12*c^2 + 7*d^2)*1i
)/(a^2*f*(c + d)^(5/2)*(c - d)^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1915, normalized size of antiderivative = 6.74

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx=\text{Too large to display}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x)
```

output

```
(72*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sq
rt( - c**2 + d**2))*tan((e + f*x)/2)**4*c**4*d**2 - 48*sqrt( - c**2 + d**2
)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan
((e + f*x)/2)**4*c**3*d**3 - 78*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2
)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**4*c**2*d
**4 + 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*
d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**4*c*d**5 + 42*sqrt( - c**2 + d*
*2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*t
an((e + f*x)/2)**4*d**6 - 144*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*
c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*c**4*d**
2 - 192*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*c**3*d**3 + 60*sqrt( - c**2 +
d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))
*tan((e + f*x)/2)**2*c**2*d**4 + 192*sqrt( - c**2 + d**2)*atan((tan((e + f
*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*c
*d**5 + 84*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2
)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*d**6 + 72*sqrt( - c**2 + d*
*2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c
**4*d**2 + 240*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*
x)/2)*d)/sqrt( - c**2 + d**2))*c**3*d**3 + 306*sqrt( - c**2 + d**2)*ata...
```

3.225 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 363

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx \\
&= \frac{d^3(40c^3-90c^2d+78cd^2-23d^3)\operatorname{arctanh}(\sin(e+fx))}{2a^3f} \\
&\quad - \frac{2d(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136d^5)\tan(e+fx)}{15a^3f} \\
&\quad - \frac{d^2(4c^4+36c^3d+216c^2d^2-626cd^3+345d^4)\sec(e+fx)\tan(e+fx)}{30a^3f} \\
&\quad - \frac{d(2c^3+18c^2d+111cd^2-136d^3)\,(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} \\
&\quad + \frac{(c-d)\,(2c^2+18cd+115d^2)\,(c+d\sec(e+fx))^3\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))} \\
&\quad + \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}
\end{aligned}$$

output
```
1/2*d^3*(40*c^3-90*c^2*d+78*c*d^2-23*d^3)*arctanh(sin(f*x+e))/a^3/f-2/15*d
*(2*c^5+18*c^4*d+107*c^3*d^2-472*c^2*d^3+456*c*d^4-136*d^5)*tan(f*x+e)/a^3
/f-1/30*d^2*(4*c^4+36*c^3*d+216*c^2*d^2-626*c*d^3+345*d^4)*sec(f*x+e)*tan(
f*x+e)/a^3/f-1/15*d*(2*c^3+18*c^2*d+111*c*d^2-136*d^3)*(c+d*sec(f*x+e))^2*
tan(f*x+e)/a^3/f+1/15*(c-d)*(2*c^2+18*c*d+115*d^2)*(c+d*sec(f*x+e))^3*tan(
f*x+e)/f/(a^3+a^3*sec(f*x+e))+1/15*(c-d)*(2*c+13*d)*(c+d*sec(f*x+e))^4*tan
(f*x+e)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*sec(f*x+e))^5*tan(f*x+e)/f/(
a+a*sec(f*x+e))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1338 vs. $2(363) = 726$.

Time = 9.67 (sec) , antiderivative size = 1338, normalized size of antiderivative = 3.69

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

output

```
(4*(-40*c^3*d^3 + 90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos
[e + f*x]^3*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*
x])^6)/(f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) - (4*(-40*c^3*d^3
 + 90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos[e + f*x]^3*Log
[Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^6)/(f*(d +
c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2]^2*Cos[e
+ f*x]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[e/2] - 6*c^5*d*Sin[e/2]
+ 15*c^4*d^2*Sin[e/2] - 20*c^3*d^3*Sin[e/2] + 15*c^2*d^4*Sin[e/2] - 6*c*d^
5*Sin[e/2] + d^6*Sin[e/2]))/(5*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x
])^3) + (8*Cos[e/2 + (f*x)/2]^4*Cos[e + f*x]^3*Sec[e/2]*(c + d*Sec[e + f*x
])^6*(-4*c^6*Sin[e/2] + 9*c^5*d*Sin[e/2] + 15*c^4*d^2*Sin[e/2] - 70*c^3*d^
3*Sin[e/2] + 90*c^2*d^4*Sin[e/2] - 51*c*d^5*Sin[e/2] + 11*d^6*Sin[e/2]))/(
15*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2
]*Cos[e + f*x]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[(f*x)/2] - 6*c^5
*d*Sin[(f*x)/2] + 15*c^4*d^2*Sin[(f*x)/2] - 20*c^3*d^3*Sin[(f*x)/2] + 15*c
^2*d^4*Sin[(f*x)/2] - 6*c*d^5*Sin[(f*x)/2] + d^6*Sin[(f*x)/2]))/(5*f*(d +
c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (8*Cos[e/2 + (f*x)/2]^3*Cos[e
+ f*x]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(-4*c^6*Sin[(f*x)/2] + 9*c^5*d*Si
n[(f*x)/2] + 15*c^4*d^2*Sin[(f*x)/2] - 70*c^3*d^3*Sin[(f*x)/2] + 90*c^2*d^
4*Sin[(f*x)/2] - 51*c*d^5*Sin[(f*x)/2] + 11*d^6*Sin[(f*x)/2]))/(15*f*(d...
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.38, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4475, 109, 25, 27, 167, 27, 167, 27, 170, 25, 27, 164, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int\frac{(c+d\sec(e+fx))^6}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 109$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int-\frac{a^2(c+d\sec(e+fx))^4\left(2c^2+8dc-5d^2-(3c-8d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^5}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int\frac{a^2(c+d\sec(e+fx))^4\left(2c^2+8dc-5d^2-(3c-8d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^5}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int\frac{(c+d\sec(e+fx))^4\left(2c^2+8dc-5d^2-(3c-8d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^5}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 167$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int\frac{a^2(c+d\sec(e+fx))^3\left(2c^3+10dc^2+55d^2c-52d^3-3d\left(2c^2+14dc-21d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^3}-\frac{(c-d)(2c+13d)\sqrt{a-a\sec(e+fx)}(c+}{3a^2(a\sec(e+fx)+a)^{3/}}}{5a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int\frac{(c+d\sec(e+fx))^3\left(2c^3+10dc^2+55d^2c-52d^3-3d\left(2c^2+14dc-21d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a}-\frac{(c-d)(2c+13d)\sqrt{a-a\sec(e+fx)}(c+d}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 167$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\int \frac{3a^2 d(c+d\sec(e+fx))^2\left(d\left(2c^2+118dc-115d^2\right)-\left(2c^3+18dc^2+111d^2c-136d^3\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^3}-\frac{(c-d)\left(2c^2+18cd+115d^2\right)}{a^2\sqrt{a}}}{3a}}{5a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\,\mathrm{s}}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{3d\int \frac{(c+d\sec(e+fx))^2\left(d\left(2c^2+118dc-115d^2\right)-\left(2c^3+18dc^2+111d^2c-136d^3\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{(c-d)\left(2c^2+18cd+115d^2\right)\sqrt{a}}{a^2\sqrt{a\,\mathrm{se}}}}{3a}}{5a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\,\mathrm{se}}}$$

$$\Big\downarrow 170$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d\left(\frac{\left(2c^3+18c^2d+111cd^2-136d^3\right)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}{3a^2}-\frac{\int -\frac{a^2(c+d\sec(e+fx))\left(d\left(2c^3+318dc^2-567\right.\right.}{\sqrt{a-}}}{}\right.}{a}}{}\right.}{}$$

$$\Big\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{3d\left(\frac{\int \frac{a^2(c+d\sec(e+fx))\left(d\left(2c^3+318dc^2-567d^2c+272d^3\right)-\left(4c^4+36dc^3+216d^2c^2-626d^3c+345d^4\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{3a^2}+\frac{\left(2c^3\right.}{}\right.}{a}}{}\right.}{}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{3d\left(\frac{1}{3}\int\frac{(c+d\sec(e+fx))\left(d\left(2c^3+318dc^2-567d^2c+272d^3\right)-\left(4c^4+36dc^3+216d^2c^2-626d^3c+345d^4\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)+\frac{\left(2c^3+\right.}{}\right.}{a}}{}}{}\right.}{}$$

$$\Big\downarrow 164$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{3d\left(\frac{1}{3}\left(\frac{15}{2}d^2\left(40c^3-90c^2d+78cd^2-23d^3\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(4c\right.\right.}{}\right.\right.}{}}{}}{}\right.}{}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{3d\left(\frac{1}{3}\left(15d^2\left(40c^3-90c^2d+78cd^2-23d^3\right)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(4c^4+3\right.\right.}{}\right.\right.}{}}{}}{}\right.}{}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{3d\left(\frac{1}{3}\left(\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(4c^4+36c^3d+216c^2d^2-626cd^3+345d^4\right)\sec(e+fx)+4\left(2c^5+18c^4d+107c^3d^2-472c^2d^3+4\right.\right.}{2a^2}\right.\right.}{}}{}}{}\right.}{}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

output

```
-((a^2*(-1/5*((c - d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^5)/(a^
2*(a + a*Sec[e + f*x])^(5/2)) + (-1/3*((c - d)*(2*c + 13*d)*Sqrt[a - a*Sec
[e + f*x]]*(c + d*Sec[e + f*x])^4)/(a^2*(a + a*Sec[e + f*x])^(3/2)) + (-((
(c - d)*(2*c^2 + 18*c*d + 115*d^2)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e +
 f*x])^3)/(a^2*Sqrt[a + a*Sec[e + f*x]])) + (3*d*(((2*c^3 + 18*c^2*d + 111
*c*d^2 - 136*d^3)*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]*(c + d
*Sec[e + f*x])^2)/(3*a^2) + ((-15*d^2*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d
^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]]])/a + (Sqrt[a
 - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]*(4*(2*c^5 + 18*c^4*d + 107*c^3
*d^2 - 472*c^2*d^3 + 456*c*d^4 - 136*d^5) + d*(4*c^4 + 36*c^3*d + 216*c^2*
d^2 - 626*c*d^3 + 345*d^4)*Sec[e + f*x]))/(2*a^2))/3))/a)/(3*a))/(5*a))*Ta
n[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 109

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.35

method	result
parallelrisch	$-14400\left(c^{3}-\frac{9}{4}c^{2}d+\frac{39}{20}c\,d^{2}-\frac{23}{40}d^{3}\right)\left(\cos\left(fx+e\right)+\frac{\cos\left(3fx+3e\right)}{3}\right)d^{3}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+14400\left(c^{3}-\frac{9}{4}c^{2}d+\frac{39}{20}c\,d^{2}-\frac{23}{40}d$
derivativedivides	$-\frac{4d^{6}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{4d^{6}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{c^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{d^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{2c^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{10d^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+$
default	$-\frac{4d^{6}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{4d^{6}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{c^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{d^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{2c^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{10d^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+$
risch	Expression too large to display

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/240*(-14400*(c^3-9/4*c^2*d+39/20*c*d^2-23/40*d^3)*(cos(f*x+e)+1/3*cos(3*
f*x+3*e))*d^3*ln(tan(1/2*f*x+1/2*e)-1)+14400*(c^3-9/4*c^2*d+39/20*c*d^2-23
/40*d^3)*(cos(f*x+e)+1/3*cos(3*f*x+3*e))*d^3*ln(tan(1/2*f*x+1/2*e)+1)+12*t
an(1/2*f*x+1/2*e)*((43/12*c^6+1549/6*d^6+27/2*c^5*d+95/2*c^4*d^2-1190/3*c^
3*d^3+1035*d^4*c^2-859*c*d^5)*cos(3*f*x+3*e)+(36*c^5*d+4*c^6+1382/3*d^6+60
*c^4*d^2-680*c^3*d^3+1860*d^4*c^2-1524*c*d^5)*cos(2*f*x+2*e)+(c^6+429/4*d^
6+9*c^5*d+15*c^4*d^2-170*c^3*d^3+855/2*d^4*c^2-717/2*c*d^5)*cos(4*f*x+4*e)
+(7/12*c^6+68/3*d^6+3/2*c^5*d+5/2*c^4*d^2-110/3*c^3*d^3+90*d^4*c^2-76*c*d^
5)*cos(5*f*x+5*e)+(3907/6*d^6+47/6*c^6+33*c^5*d+130*c^4*d^2+2655*d^4*c^2-2
137*c*d^5-3020/3*c^3*d^3)*cos(f*x+e)+4321/12*d^6+27*c^5*d+45*c^4*d^2-510*c
^3*d^3+2865/2*d^4*c^2-2331/2*c*d^5+3*c^6)*sec(1/2*f*x+1/2*e)^4)/f/a^3/(cos
(3*f*x+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.71

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/60*(15*((40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3
*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*
d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^
2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((40
*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3*(40*c^3*d^3
- 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*d^3 - 90*c^2*
d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^2*d^4 + 78*c*
d^5 - 23*d^6)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(10*d^6 + 2*(7*c^
6 + 18*c^5*d + 30*c^4*d^2 - 440*c^3*d^3 + 1080*c^2*d^4 - 912*c*d^5 + 272*d
^6)*cos(f*x + e)^5 + 3*(4*c^6 + 36*c^5*d + 60*c^4*d^2 - 680*c^3*d^3 + 1710
*c^2*d^4 - 1434*c*d^5 + 429*d^6)*cos(f*x + e)^4 + (4*c^6 + 36*c^5*d + 210*
c^4*d^2 - 1280*c^3*d^3 + 3510*c^2*d^4 - 2874*c*d^5 + 869*d^6)*cos(f*x + e)
^3 + 5*(90*c^2*d^4 - 54*c*d^5 + 19*d^6)*cos(f*x + e)^2 + 15*(6*c*d^5 - d^6
)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)
^5 + 3*a^3*f*cos(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$$

$$= \int \frac{c^6\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^6\sec^7(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{6cd^5\sec^6(e+}{\sec^3(e+fx)+3\sec^2(e+fx)}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

output

```
(Integral(c**6*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) + 1), x) + Integral(d**6*sec(e + f*x)**7/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*c*d**5*sec(e + f*x)**6/
(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(
15*c**2*d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(20*c**3*d**3*sec(e + f*x)**4/(sec(e + f*x)**3
 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*c**4*d**2*sec
(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x
) + Integral(6*c**5*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2
 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(349) = 698$.

Time = 0.07 (sec) , antiderivative size = 946, normalized size of antiderivative = 2.61

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/60*(d^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*
x + e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3
 + 690*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 6*c*d^5*(60*(5*sin(
f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3
- 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(
sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x +
 e) + 1) - 1)/a^3) + 45*c^2*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x +
e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 20*c^3*d^3*((105*sin(f*x +
e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
 + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^4*d...
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.85

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
1/60*(30*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*log(abs(tan(1/2*f*x
 + 1/2*e) + 1))/a^3 - 30*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*log
(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - 20*(90*c^2*d^4*tan(1/2*f*x + 1/2*e)^
5 - 126*c*d^5*tan(1/2*f*x + 1/2*e)^5 + 51*d^6*tan(1/2*f*x + 1/2*e)^5 - 180
*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 216*c*d^5*tan(1/2*f*x + 1/2*e)^3 - 76*d^
6*tan(1/2*f*x + 1/2*e)^3 + 90*c^2*d^4*tan(1/2*f*x + 1/2*e) - 90*c*d^5*tan(
1/2*f*x + 1/2*e) + 33*d^6*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 -
 1)^3*a^3) + (3*a^12*c^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^12*c^5*d*tan(1/2*f*
x + 1/2*e)^5 + 45*a^12*c^4*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^12*c^3*d^3*ta
n(1/2*f*x + 1/2*e)^5 + 45*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e)^5 - 18*a^12*c*
d^5*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^6*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c
^6*tan(1/2*f*x + 1/2*e)^3 + 150*a^12*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 400*
a^12*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 450*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e
)^3 - 240*a^12*c*d^5*tan(1/2*f*x + 1/2*e)^3 + 50*a^12*d^6*tan(1/2*f*x + 1/
2*e)^3 + 15*a^12*c^6*tan(1/2*f*x + 1/2*e) + 90*a^12*c^5*d*tan(1/2*f*x + 1/
2*e) + 225*a^12*c^4*d^2*tan(1/2*f*x + 1/2*e) - 2100*a^12*c^3*d^3*tan(1/2*f
*x + 1/2*e) + 3825*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e) - 2790*a^12*c*d^5*tan
(1/2*f*x + 1/2*e) + 735*a^12*d^6*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.90

$$
\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx \\
&= \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,{\left(c-d\right)}^6}{2\,a^3}-\frac{6\,\left(c+d\right)\,{\left(c-d\right)}^5}{a^3}+\frac{15\,{\left(c+d\right)}^2\,{\left(c-d\right)}^4}{4\,a^3}\right)}{f} \\
&\quad - \frac{\left(30\,c^2\,d^4-42\,c\,d^5+17\,d^6\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\left(-60\,c^2\,d^4+72\,c\,d^5-\frac{76\,d^6}{3}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(30\,c^2\,d^4-3\right.}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-3\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-a^3\right)} \\
&\quad + \frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{{\left(c-d\right)}^6}{3\,a^3}-\frac{\left(c+d\right)\,{\left(c-d\right)}^5}{2\,a^3}\right)}{f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,{\left(c-d\right)}^6}{20\,a^3\,f} \\
&\quad + \frac{d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(40\,c^3-90\,c^2\,d+78\,c\,d^2-23\,d^3\right)}{a^3\,f}
\end{aligned}
$$

input
```
int((c + d/cos(e + f*x))^6/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(tan(e/2 + (f*x)/2)*((5*(c - d)^6)/(2*a^3) - (6*(c + d)*(c - d)^5)/a^3 + (
15*(c + d)^2*(c - d)^4)/(4*a^3)))/f - (tan(e/2 + (f*x)/2)*(11*d^6 - 30*c*d
^5 + 30*c^2*d^4) + tan(e/2 + (f*x)/2)^5*(17*d^6 - 42*c*d^5 + 30*c^2*d^4) -
 tan(e/2 + (f*x)/2)^3*((76*d^6)/3 - 72*c*d^5 + 60*c^2*d^4))/(f*(3*a^3*tan(
e/2 + (f*x)/2)^2 - 3*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^6 -
 a^3)) + (tan(e/2 + (f*x)/2)^3*((c - d)^6/(3*a^3) - ((c + d)*(c - d)^5)/(2
*a^3)))/f + (tan(e/2 + (f*x)/2)^5*(c - d)^6)/(20*a^3*f) + (d^3*atanh(tan(e
/2 + (f*x)/2))*(78*c*d^2 - 90*c^2*d + 40*c^3 - 23*d^3))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1638, normalized size of antiderivative = 4.51

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x)
```

output
```
( - 1200*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**6*c**3*d**3 + 2700*lo
g(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**6*c**2*d**4 - 2340*log(tan((e +
f*x)/2) - 1)*tan((e + f*x)/2)**6*c*d**5 + 690*log(tan((e + f*x)/2) - 1)*ta
n((e + f*x)/2)**6*d**6 + 3600*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**
4*c**3*d**3 - 8100*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*c**2*d**4
 + 7020*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*c*d**5 - 2070*log(ta
n((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*d**6 - 3600*log(tan((e + f*x)/2) -
 1)*tan((e + f*x)/2)**2*c**3*d**3 + 8100*log(tan((e + f*x)/2) - 1)*tan((e
+ f*x)/2)**2*c**2*d**4 - 7020*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**
2*c*d**5 + 2070*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*d**6 + 1200*
log(tan((e + f*x)/2) - 1)*c**3*d**3 - 2700*log(tan((e + f*x)/2) - 1)*c**2*
d**4 + 2340*log(tan((e + f*x)/2) - 1)*c*d**5 - 690*log(tan((e + f*x)/2) -
1)*d**6 + 1200*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**6*c**3*d**3 - 2
700*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**6*c**2*d**4 + 2340*log(tan
((e + f*x)/2) + 1)*tan((e + f*x)/2)**6*c*d**5 - 690*log(tan((e + f*x)/2) +
 1)*tan((e + f*x)/2)**6*d**6 - 3600*log(tan((e + f*x)/2) + 1)*tan((e + f*x
)/2)**4*c**3*d**3 + 8100*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*c**
2*d**4 - 7020*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*c*d**5 + 2070*
log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*d**6 + 3600*log(tan((e + f*x
)/2) + 1)*tan((e + f*x)/2)**2*c**3*d**3 - 8100*log(tan((e + f*x)/2) + 1...
```

3.226 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 287

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx \\
&= \frac{d^3(20c^2-30cd+13d^2)\operatorname{arctanh}(\sin(e+fx))}{2a^3f} \\
&\quad - \frac{2d(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4)\tan(e+fx)}{15a^3f} \\
&\quad - \frac{d^2(4c^3+30c^2d+146cd^2-195d^3)\sec(e+fx)\tan(e+fx)}{30a^3f} \\
&\quad + \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} \\
&\quad + \frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}
\end{aligned}$$

output

```
1/2*d^3*(20*c^2-30*c*d+13*d^2)*arctanh(sin(f*x+e))/a^3/f-2/15*d*(2*c^4+15*
c^3*d+72*c^2*d^2-180*c*d^3+76*d^4)*tan(f*x+e)/a^3/f-1/30*d^2*(4*c^3+30*c^2
*d+146*c*d^2-195*d^3)*sec(f*x+e)*tan(f*x+e)/a^3/f+1/15*(c-d)*(2*c^2+15*c*d
+76*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+1/15*(c-d)*(
2*c+11*d)*(c+d*sec(f*x+e))^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(
c+d*sec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.53

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{-480d^3(20c^2-30cd+13d^2)\cos^6\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e\right.\right.\right.}{}$$

input

```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

output

```
(-480*d^3*(20*c^2 - 30*c*d + 13*d^2)*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Co
s[(e + f*x)/2]*(29*c^5 + 105*c^4*d + 340*c^3*d^2 - 1940*c^2*d^3 + 3420*c*d
^4 - 1354*d^5 + 3*(12*c^5 + 90*c^4*d + 120*c^3*d^2 - 1020*c^2*d^3 + 1910*c
*d^4 - 777*d^5)*Cos[e + f*x] + 6*(6*c^5 + 20*c^4*d + 60*c^3*d^2 - 360*c^2*
d^3 + 630*c*d^4 - 261*d^5)*Cos[2*(e + f*x)] + 12*c^5*Cos[3*(e + f*x)] + 90
*c^4*d*Cos[3*(e + f*x)] + 120*c^3*d^2*Cos[3*(e + f*x)] - 1020*c^2*d^3*Cos[
3*(e + f*x)] + 1710*c*d^4*Cos[3*(e + f*x)] - 717*d^5*Cos[3*(e + f*x)] + 7*
c^5*Cos[4*(e + f*x)] + 15*c^4*d*Cos[4*(e + f*x)] + 20*c^3*d^2*Cos[4*(e + f
*x)] - 220*c^2*d^3*Cos[4*(e + f*x)] + 360*c*d^4*Cos[4*(e + f*x)] - 152*d^5
*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2])/(120*a^3*f*(1 + Cos[e
+ f*x])^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4475, 109, 25, 27, 167, 27, 167, 27, 164, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(c+d\sec(e+fx))^5}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 109}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2(c+d\sec(e+fx))^3((2c-d)(c+4d)-(2c-7d)d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^4}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(c+d\sec(e+fx))^3((2c-d)(c+4d)-(2c-7d)d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^4}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{(c+d\sec(e+fx))^3((2c-d)(c+4d)-(2c-7d)d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^4}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 167}$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{\int\frac{a^2(c+d\sec(e+fx))^2\left(2c^3+9dc^2+37d^2c-33d^3-d\left(4c^2+24dc-43d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^3}-\frac{(c-d)(2c+11d)\sqrt{a-a\sec(e+fx)}(c+d}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{\int\frac{(c+d\sec(e+fx))^2\left(2c^3+9dc^2+37d^2c-33d^3-d\left(4c^2+24dc-43d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a}-\frac{(c-d)(2c+11d)\sqrt{a-a\sec(e+fx)}(c+d\sec}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 167$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{\frac{\int\frac{a^2d(c+d\sec(e+fx))\left(d\left(2c^2+165dc-152d^2\right)-\left(4c^3+30dc^2+146d^2c-195d^3\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^3}-\frac{(c-d)\left(2c^2+15cd+76d^2\right)\sqrt{a-}}{a^2\sqrt{a\sec}}}{3a}}{5a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{\frac{d\int\frac{(c+d\sec(e+fx))\left(d\left(2c^2+165dc-152d^2\right)-\left(4c^3+30dc^2+146d^2c-195d^3\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{(c-d)\left(2c^2+15cd+76d^2\right)\sqrt{a-a}}{a^2\sqrt{a\sec(e}}}{3a}}{5a}}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec}}$$

$$\Big\downarrow 164$$

$$-\frac{a^2\tan(e+fx)\Bigg(\frac{\frac{d\Bigg(\frac{15}{2}d^2\left(20c^2-30cd+13d^2\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(4c^3+30c^2d+146\right.\right.}{}}{a}}{3a}}{}$$

$$\Big\downarrow 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{d\left(15d^2\left(20c^2-30cd+13d^2\right)\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}+\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(4c^3+30c^2d+146cd^2\right.\right.}{}\right.}{a}}{3a}\right.}{}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{d\left(\frac{\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\left(d\left(4c^3+30c^2d+146cd^2-195d^3\right)\sec(e+fx)+4\left(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4\right)\right)}{2a^2}-\frac{15d^2\left(20\right.}{}\right.}{a}}{3a}\right.}{}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

output
```
-((a^2*(-1/5*((c - d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4)/(a^
2*(a + a*Sec[e + f*x])^(5/2)) + (-1/3*((c - d)*(2*c + 11*d)*Sqrt[a - a*Sec
[e + f*x]]*(c + d*Sec[e + f*x])^3)/(a^2*(a + a*Sec[e + f*x])^(3/2)) + (-((
(c - d)*(2*c^2 + 15*c*d + 76*d^2)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e +
f*x])^2)/(a^2*Sqrt[a + a*Sec[e + f*x]])) + (d*((-15*d^2*(20*c^2 - 30*c*d +
 13*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]]])/a + (S
qrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]*(4*(2*c^4 + 15*c^3*d + 72
*c^2*d^2 - 180*c*d^3 + 76*d^4) + d*(4*c^3 + 30*c^2*d + 146*c*d^2 - 195*d^3
)*Sec[e + f*x]))/(2*a^2)))/a)/(3*a))/(5*a))*Tan[e + f*x])/(f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.25

method	result
parallelrisch	$-2400d^3\left(c^2-\frac{3}{2}cd+\frac{13}{20}d^2\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+2400d^3\left(c^2-\frac{3}{2}cd+\frac{13}{20}d^2\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\right.\right.$
derivativedivides	$\frac{c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-c^4d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+2c^3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-2c^2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+c\,d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-\frac{d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}$
default	$\frac{c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-c^4d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+2c^3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-2c^2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+c\,d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-\frac{d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}$
norman	$\frac{\left(c^5-5c^4d+10c^3d^2-10c^2d^3+5c\,d^4-d^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{20fa}-\frac{5\left(c^5-3c^4d+2c^3d^2+2c^2d^3-3c\,d^4+d^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{12fa}-\frac{\left(c^5+5c^4d+10\right.}{}$
risch	Expression too large to display

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/240*(-2400*d^3*(c^2-3/2*c*d+13/20*d^2)*(1+cos(2*f*x+2*e))*ln(tan(1/2*f*x
+1/2*e)-1)+2400*d^3*(c^2-3/2*c*d+13/20*d^2)*(1+cos(2*f*x+2*e))*ln(tan(1/2*
f*x+1/2*e)+1)+7*sec(1/2*f*x+1/2*e)^4*tan(1/2*f*x+1/2*e)*(6*(20/7*c^4*d+60/
7*c^3*d^2+90*c*d^4+6/7*c^5-261/7*d^5-360/7*c^2*d^3)*cos(2*f*x+2*e)+3/7*(4*
c^5+30*c^4*d+40*c^3*d^2-340*c^2*d^3+570*c*d^4-239*d^5)*cos(3*f*x+3*e)+(c^5
-152/7*d^5+15/7*c^4*d+20/7*c^3*d^2-220/7*c^2*d^3+360/7*c*d^4)*cos(4*f*x+4*
e)+3*(-111*d^5+12/7*c^5+120/7*c^3*d^2+90/7*c^4*d-1020/7*c^2*d^3+1910/7*c*d
^4)*cos(f*x+e)-1354/7*d^5+29/7*c^5+15*c^4*d+340/7*c^3*d^2-1940/7*c^2*d^3+3
420/7*c*d^4))/f/a^3/(1+cos(2*f*x+2*e))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.75

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{15\left(\left(20\,c^2d^3-30\,cd^4+13\,d^5\right)\cos\left(fx+e\right)^5+3\left(20\,c^2d^3-30\,cd^4+13\,d^5\right)\cos\left(fx+e\right)^4+3\left(20\,c^2d^3-30\right.\right.}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/60*(15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20*c^2*d^3
- 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*c
os(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*log(sin(f
*x + e) + 1) - 15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20
*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 +
13*d^5)*cos(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*
log(-sin(f*x + e) + 1) + 2*(15*d^5 + 2*(7*c^5 + 15*c^4*d + 20*c^3*d^2 - 22
0*c^2*d^3 + 360*c*d^4 - 152*d^5)*cos(f*x + e)^4 + 3*(4*c^5 + 30*c^4*d + 40
*c^3*d^2 - 340*c^2*d^3 + 570*c*d^4 - 239*d^5)*cos(f*x + e)^3 + (4*c^5 + 30
*c^4*d + 140*c^3*d^2 - 640*c^2*d^3 + 1170*c*d^4 - 479*d^5)*cos(f*x + e)^2
+ 15*(10*c*d^4 - 3*d^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^5
+ 3*a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$$
$$= \int \frac{c^5\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^5\sec^6(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{5cd^4\sec^5(e+}{\sec^3(e+fx)+3\sec^2(e+fx)}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

output
```
(Integral(c**5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) + 1), x) + Integral(d**5*sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c*d**4*sec(e + f*x)**5/
(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(
10*c**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**3
 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c**4*d*sec(e +
 f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a
**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(275) = 550$.

Time = 0.06 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
-1/60*(d^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1)
+ 40*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(s
in(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 15*c*d^4*(40*sin(f*x + e)/((a^3
 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(
f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin
(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e)
+ 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^2*
d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e
)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1
)/a^3) - 10*c^3*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - c^5*
(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 15*c^4*d*(5*sin(f*x + e)
/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.76

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output

```
1/60*(30*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*log(abs(tan(1/2*f*x + 1/2*e) + 1
))/a^3 - 30*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*log(abs(tan(1/2*f*x + 1/2*e)
- 1))/a^3 - 60*(10*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 7*d^5*tan(1/2*f*x + 1/2*
e)^3 - 10*c*d^4*tan(1/2*f*x + 1/2*e) + 5*d^5*tan(1/2*f*x + 1/2*e))/((tan(1
/2*f*x + 1/2*e)^2 - 1)^2*a^3) + (3*a^12*c^5*tan(1/2*f*x + 1/2*e)^5 - 15*a^
12*c^4*d*tan(1/2*f*x + 1/2*e)^5 + 30*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 -
 30*a^12*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 15*a^12*c*d^4*tan(1/2*f*x + 1/2*
e)^5 - 3*a^12*d^5*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^5*tan(1/2*f*x + 1/2*e
)^3 + 100*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 200*a^12*c^2*d^3*tan(1/2*f
*x + 1/2*e)^3 + 150*a^12*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 40*a^12*d^5*tan(1/
2*f*x + 1/2*e)^3 + 15*a^12*c^5*tan(1/2*f*x + 1/2*e) + 75*a^12*c^4*d*tan(1/
2*f*x + 1/2*e) + 150*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e) - 1050*a^12*c^2*d^3
*tan(1/2*f*x + 1/2*e) + 1275*a^12*c*d^4*tan(1/2*f*x + 1/2*e) - 465*a^12*d^
5*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx
&= \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,{\left(c-d\right)}^5}{2\,a^3}-\frac{15\,\left(c+d\right)\,{\left(c-d\right)}^4}{4\,a^3}+\frac{5\,{\left(c+d\right)}^2\,{\left(c-d\right)}^3}{2\,a^3}\right)}{f} \\
&\quad+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(10\,c\,d^4-5\,d^5\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(10\,c\,d^4-7\,d^5\right)}{f\,\left(a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-2\,a^3\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a^3\right)} \\
&\quad+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{{\left(c-d\right)}^5}{4\,a^3}-\frac{5\,\left(c+d\right)\,{\left(c-d\right)}^4}{12\,a^3}\right)}{f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,{\left(c-d\right)}^5}{20\,a^3\,f} \\
&\quad+\frac{d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(20\,c^2-30\,c\,d+13\,d^2\right)}{a^3\,f}
\end{aligned}
$$

input

```
int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output

```
(tan(e/2 + (f*x)/2)*((3*(c - d)^5)/(2*a^3) - (15*(c + d)*(c - d)^4)/(4*a^3
) + (5*(c + d)^2*(c - d)^3)/(2*a^3)))/f + (tan(e/2 + (f*x)/2)*(10*c*d^4 -
5*d^5) - tan(e/2 + (f*x)/2)^3*(10*c*d^4 - 7*d^5))/(f*(a^3*tan(e/2 + (f*x)/
2)^4 - 2*a^3*tan(e/2 + (f*x)/2)^2 + a^3)) + (tan(e/2 + (f*x)/2)^3*((c - d)
^5/(4*a^3) - (5*(c + d)*(c - d)^4)/(12*a^3)))/f + (tan(e/2 + (f*x)/2)^5*(c
 - d)^5)/(20*a^3*f) + (d^3*atanh(tan(e/2 + (f*x)/2))*(20*c^2 - 30*c*d + 13
*d^2))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1007, normalized size of antiderivative = 3.51

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x)
```

output

```
( - 600*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*c**2*d**3 + 900*log(
tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4*c*d**4 - 390*log(tan((e + f*x)/2
) - 1)*tan((e + f*x)/2)**4*d**5 + 1200*log(tan((e + f*x)/2) - 1)*tan((e +
f*x)/2)**2*c**2*d**3 - 1800*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*
c*d**4 + 780*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*d**5 - 600*log(
tan((e + f*x)/2) - 1)*c**2*d**3 + 900*log(tan((e + f*x)/2) - 1)*c*d**4 - 3
90*log(tan((e + f*x)/2) - 1)*d**5 + 600*log(tan((e + f*x)/2) + 1)*tan((e +
 f*x)/2)**4*c**2*d**3 - 900*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*
c*d**4 + 390*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4*d**5 - 1200*log
(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2*c**2*d**3 + 1800*log(tan((e + f
*x)/2) + 1)*tan((e + f*x)/2)**2*c*d**4 - 780*log(tan((e + f*x)/2) + 1)*tan
((e + f*x)/2)**2*d**5 + 600*log(tan((e + f*x)/2) + 1)*c**2*d**3 - 900*log(
tan((e + f*x)/2) + 1)*c*d**4 + 390*log(tan((e + f*x)/2) + 1)*d**5 + 3*tan(
(e + f*x)/2)**9*c**5 - 15*tan((e + f*x)/2)**9*c**4*d + 30*tan((e + f*x)/2)
**9*c**3*d**2 - 30*tan((e + f*x)/2)**9*c**2*d**3 + 15*tan((e + f*x)/2)**9*
c*d**4 - 3*tan((e + f*x)/2)**9*d**5 - 16*tan((e + f*x)/2)**7*c**5 + 30*tan
((e + f*x)/2)**7*c**4*d + 40*tan((e + f*x)/2)**7*c**3*d**2 - 140*tan((e +
f*x)/2)**7*c**2*d**3 + 120*tan((e + f*x)/2)**7*c*d**4 - 34*tan((e + f*x)/2
)**7*d**5 + 38*tan((e + f*x)/2)**5*c**5 + 60*tan((e + f*x)/2)**5*c**4*d -
20*tan((e + f*x)/2)**5*c**3*d**2 - 680*tan((e + f*x)/2)**5*c**2*d**3 + ...
```

3.227 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 205

$$
\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx \\
&= \frac{(4c-3d)d^3\text{arctanh}(\sin(e+fx))}{a^3f} + \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} \\
&\quad + \frac{(2c^4+8c^3d+21c^2d^2-88cd^3+72d^4-d^2(2c^2+10cd-27d^2)\sec(e+fx))\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))}
\end{aligned}
$$

output

```
(4*c-3*d)*d^3*arctanh(sin(f*x+e))/a^3/f+1/15*(c-d)*(2*c+9*d)*(c+d*sec(f*x+
e))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*sec(f*x+e))^3*tan(f
*x+e)/f/(a+a*sec(f*x+e))^3+1/15*(2*c^4+8*c^3*d+21*c^2*d^2-88*c*d^3+72*d^4-
d^2*(2*c^2+10*c*d-27*d^2)*sec(f*x+e))*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.42

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(3(c-d)^4\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-8(c-d)^3(2c+3d)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+4(}{}$$

input

```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

output

```
(2*Cos[(e + f*x)/2]*(3*(c - d)^4*Sec[e/2]*Sin[(f*x)/2] - 8*(c - d)^3*(2*c
+ 3*d)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin[(f*x)/2] + 4*(c - d)^2*(7*c^2 + 26*
c*d + 57*d^2)*Cos[(e + f*x)/2]^4*Sec[e/2]*Sin[(f*x)/2] - 60*d^3*Cos[(e + f
*x)/2]^5*((4*c - 3*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]]) - d*Sec[e]*Sec[e + f*x]*Sin[f*x]) + 3*(c
 - d)^4*Cos[(e + f*x)/2]*Tan[e/2] - 8*(c - d)^3*(2*c + 3*d)*Cos[(e + f*x)/
2]^3*Tan[e/2]))/(15*a^3*f*(1 + Cos[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3042, 4475, 109, 25, 27, 167, 27, 160, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a\sec(e+fx)+a)^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \text{ 4475}$$

$$-\frac{a^2\tan(e+fx)\int\frac{(c+d\sec(e+fx))^4}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int-\frac{a^2(c+d\sec(e+fx))^2\left(2c^2+6dc-3d^2-(c-6d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int\frac{a^2(c+d\sec(e+fx))^2\left(2c^2+6dc-3d^2-(c-6d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int\frac{(c+d\sec(e+fx))^2\left(2c^2+6dc-3d^2-(c-6d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{167}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int\frac{a^2(c+d\sec(e+fx))\left(2c^3+8dc^2+23d^2c-18d^3-d\left(2c^2+10dc-27d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a^3}-\frac{(c-d)(2c+9d)\sqrt{a-a\sec(e+fx)}(c+d\,\mathrm{se}}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int\frac{(c+d\sec(e+fx))\left(2c^3+8dc^2+23d^2c-18d^3-d\left(2c^2+10dc-27d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a}-\frac{(c-d)(2c+9d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{160}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{15d^3(4c-3d)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a}-\frac{\sqrt{a-a\sec(e+fx)}\left(2c^4+8c^3d-d^2\left(2c^2+10cd-27d^2\right)\sec(e+fx)+21c^2d}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}}{5a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

$$\Big\downarrow \textbf{45}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{30d^3(4c-3d)\int\frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{a}-\frac{\sqrt{a-a\sec(e+fx)}\left(2c^4+8c^3d-d^2\left(2c^2+10cd-27d^2\right)\sec(e+fx)+21c^2d^2-\right.}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}}{5a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

$$\Big\downarrow \textbf{218}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{30d^3(4c-3d)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)}{a^2}-\frac{\sqrt{a-a\sec(e+fx)}\left(2c^4+8c^3d-d^2\left(2c^2+10cd-27d^2\right)\sec(e+fx)+21c^2d^2-88cd^3+72d^4\right)}{a^2\sqrt{a\sec(e+fx)+a}}}{3a}-}{5a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

output
```
-((a^2*(-1/5*((c - d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3)/(a^
2*(a + a*Sec[e + f*x])^(5/2)) + (-1/3*((c - d)*(2*c + 9*d)*Sqrt[a - a*Sec[
e + f*x]]*(c + d*Sec[e + f*x])^2)/(a^2*(a + a*Sec[e + f*x])^(3/2)) + ((-30
*(4*c - 3*d)*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]]]
)/a^2 - (Sqrt[a - a*Sec[e + f*x]]*(2*c^4 + 8*c^3*d + 21*c^2*d^2 - 88*c*d^3
 + 72*d^4 - d^2*(2*c^2 + 10*c*d - 27*d^2)*Sec[e + f*x]))/(a^2*Sqrt[a + a*S
ec[e + f*x]]))/(3*a))/(5*a))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 160
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d*(f*g
+ e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Simp[(a*d*f*h*m + b*(d*
(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d)   Int[(a + b*x)^(m + 1)*(c + d*x)^n,
x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0] &&
NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.20

method	result
parallelrisch	$-960\left(c-\frac{3d}{4}\right)d^3\cos(fx+e)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+960\left(c-\frac{3d}{4}\right)d^3\cos(fx+e)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\Bigg($
derivativedivides	$\frac{c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{4c^3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{6c^2d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{4c\,d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{2c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+4c$
default	$\frac{c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{4c^3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{6c^2d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{4c\,d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{2c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+4c$
norman	$\frac{\left(c^4-4c^3d+6c^2d^2-4c\,d^3+d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{20fa}+\frac{\left(c^4+4c^3d+6c^2d^2-28c\,d^3+25d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4fa}-\frac{\left(7c^4+24c^3d+30c^2d^2-160c\,d^3+\right.}{6fa}$
risch	$2i\left(12c^3d+12c^2d^2-88c\,d^3+72d^4+7c^4+132c^2d^2\mathrm{e}^{2i(fx+e)}-668c\,d^3\mathrm{e}^{2i(fx+e)}+72c^3d\,\mathrm{e}^{2i(fx+e)}+60c^2d^2\mathrm{e}^{3i(fx+e)}+60c^3\right.$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/240*(-960*(c-3/4*d)*d^3*cos(f*x+e)*ln(tan(1/2*f*x+1/2*e)-1)+960*(c-3/4*d
)*d^3*cos(f*x+e)*ln(tan(1/2*f*x+1/2*e)+1)+29*tan(1/2*f*x+1/2*e)*(6/29*(2*c
^4+12*c^3*d+12*c^2*d^2-68*c*d^3+57*d^4)*cos(2*f*x+2*e)+1/29*(7*c^4+12*c^3*
d+12*c^2*d^2-88*c*d^3+72*d^4)*cos(3*f*x+3*e)+(c^4+84/29*c^3*d+204/29*c^2*d
^2-776/29*c*d^3+684/29*d^4)*cos(f*x+e)+12/29*c^4+72/29*c^3*d+72/29*c^2*d^2
-408/29*c*d^3+402/29*d^4)*sec(1/2*f*x+1/2*e)^4)/f/a^3/cos(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.88

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{15\left(\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+3\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3+3\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^2+(4\,cd^3 -\right.}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
1/30*(15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d^3 - 3*d^4)*cos(f*x +
 e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3 - 3*d^4)*cos(f*x + e
))*log(sin(f*x + e) + 1) - 15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d
^3 - 3*d^4)*cos(f*x + e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3
 - 3*d^4)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(15*d^4 + (7*c^4 + 12*c
^3*d + 12*c^2*d^2 - 88*c*d^3 + 72*d^4)*cos(f*x + e)^3 + 3*(2*c^4 + 12*c^3*
d + 12*c^2*d^2 - 68*c*d^3 + 57*d^4)*cos(f*x + e)^2 + (2*c^4 + 12*c^3*d + 4
2*c^2*d^2 - 128*c*d^3 + 117*d^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*
x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x
 + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^4\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^4\sec^5(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{4cd^3\sec^4(e+}{\sec^3(e+fx)+3\sec^2(e+fx)}}{a^3}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

output
```
(Integral(c**4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) + 1), x) + Integral(d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/
(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(
6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) + 1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*
sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(199) = 398.

Time = 0.05 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.32

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output

```
1/60*(3*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 -
60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a^3) - 4*c*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) +
 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^2*d^2*(15*sin(f*x + e)/(cos(f*
x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/
a^3 + 12*c^3*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.82

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx =$$

$$-\frac{\frac{120\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3}-\frac{60\,\left(4\,cd^3-3\,d^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}+\frac{60\,\left(4\,cd^3-3\,d^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}-\frac{3\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\right.}{}}{}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output

```
-1/60*(120*d^4*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - 6
0*(4*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 + 60*(4*c*d^3 -
 3*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - (3*a^12*c^4*tan(1/2*f*x +
 1/2*e)^5 - 12*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 18*a^12*c^2*d^2*tan(1/2
*f*x + 1/2*e)^5 - 12*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^4*tan(1/
2*f*x + 1/2*e)^5 - 10*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 60*a^12*c^2*d^2*ta
n(1/2*f*x + 1/2*e)^3 - 80*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*d^4*
tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e) + 60*a^12*c^3*d*
tan(1/2*f*x + 1/2*e) + 90*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e) - 420*a^12*c*d
^3*tan(1/2*f*x + 1/2*e) + 255*a^12*d^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3(c-d)^4}{4a^3}+\frac{3(c^2-d^2)^2}{2a^3}-\frac{2(c+d)(c-d)^3}{a^3}\right)}{f}$$
$$+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{(c-d)^4}{6a^3}-\frac{(c+d)(c-d)^3}{3a^3}\right)}{f}-\frac{2d^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-a^3\right)}$$
$$+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(c-d)^4}{20a^3f}+\frac{2d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(4c-3d)}{a^3f}$$

input
```
int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(tan(e/2 + (f*x)/2)*((3*(c - d)^4)/(4*a^3) + (3*(c^2 - d^2)^2)/(2*a^3) - (
2*(c + d)*(c - d)^3)/a^3))/f + (tan(e/2 + (f*x)/2)^3*((c - d)^4/(6*a^3) -
((c + d)*(c - d)^3)/(3*a^3)))/f - (2*d^4*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e
/2 + (f*x)/2)^2 - a^3)) + (tan(e/2 + (f*x)/2)^5*(c - d)^4)/(20*a^3*f) + (2
*d^3*atanh(tan(e/2 + (f*x)/2))*(4*c - 3*d))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 538, normalized size of antiderivative = 2.62

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx=\text{Too large to display}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x)
```

output

```
( - 240*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*c*d**3 + 180*log(tan
((e + f*x)/2) - 1)*tan((e + f*x)/2)**2*d**4 + 240*log(tan((e + f*x)/2) - 1
)*c*d**3 - 180*log(tan((e + f*x)/2) - 1)*d**4 + 240*log(tan((e + f*x)/2) +
 1)*tan((e + f*x)/2)**2*c*d**3 - 180*log(tan((e + f*x)/2) + 1)*tan((e + f*
x)/2)**2*d**4 - 240*log(tan((e + f*x)/2) + 1)*c*d**3 + 180*log(tan((e + f*
x)/2) + 1)*d**4 + 3*tan((e + f*x)/2)**7*c**4 - 12*tan((e + f*x)/2)**7*c**3
*d + 18*tan((e + f*x)/2)**7*c**2*d**2 - 12*tan((e + f*x)/2)**7*c*d**3 + 3*
tan((e + f*x)/2)**7*d**4 - 13*tan((e + f*x)/2)**5*c**4 + 12*tan((e + f*x)/
2)**5*c**3*d + 42*tan((e + f*x)/2)**5*c**2*d**2 - 68*tan((e + f*x)/2)**5*c
*d**3 + 27*tan((e + f*x)/2)**5*d**4 + 25*tan((e + f*x)/2)**3*c**4 + 60*tan
((e + f*x)/2)**3*c**3*d + 30*tan((e + f*x)/2)**3*c**2*d**2 - 340*tan((e +
f*x)/2)**3*c*d**3 + 225*tan((e + f*x)/2)**3*d**4 - 15*tan((e + f*x)/2)*c**
4 - 60*tan((e + f*x)/2)*c**3*d - 90*tan((e + f*x)/2)*c**2*d**2 + 420*tan((
e + f*x)/2)*c*d**3 - 375*tan((e + f*x)/2)*d**4)/(60*a**3*f*(tan((e + f*x)/
2)**2 - 1))
```

3.228 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 133

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx &= \frac{d^3\text{arctanh}(\sin(e+fx))}{a^3f}+\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\ &\quad+\frac{(c-d)\left(2(2c^2+8cd+11d^2)+(2c^2+11cd+29d^2)\sec(e+fx)\right)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\end{aligned}$$

output

```
d^3*arctanh(sin(f*x+e))/a^3/f+1/5*(c-d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a
+a*sec(f*x+e))^3+1/15*(c-d)*(4*c^2+16*c*d+22*d^2+(2*c^2+11*c*d+29*d^2)*sec
(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 295 vs. 2(133) = 266.

Time = 2.44 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.22

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$

$$= -240d^3\cos^6(\tfrac{1}{2}(e+fx))(\log(\cos(\tfrac{1}{2}(e+fx))-\sin(\tfrac{1}{2}(e+fx)))-\log(\cos(\tfrac{1}{2}(e+fx))+\sin(\tfrac{1}{2}(e+$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

output
```
(-240*d^3*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - L
og[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Cos[(e + f*x)/2]*Sec[e/
2]*(5*(8*c^2 + 17*c*d + 29*d^2)*Sin[(f*x)/2] - 15*(2*c^2 + 5*c*d + 5*d^2)*
Sin[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] + 65*c*d*Sin[e + (3*f*x)/2] +
 95*d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] - 15*c*d*Sin[2*e
+ (3*f*x)/2] - 15*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[2*e + (5*f*x)/2] +
16*c*d*Sin[2*e + (5*f*x)/2] + 22*d^2*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(1 +
 Cos[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.69, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 4475, 109, 25, 27, 162, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{109}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2(c+d\sec(e+fx))\left(2c^2+5dc-2d^2+5d^2\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(c+d\sec(e+fx))\left(2c^2+5dc-2d^2+5d^2\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{(c+d\sec(e+fx))\left(2c^2+5dc-2d^2+5d^2\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 162$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{5d^3\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{a^2}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}\left(\left(2c^2+11cd+29d^2\right)\sec(e+fx)+2\left(2c^2+8cd+11d^2\right)\right)}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}-\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 45$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{10d^3\int \frac{1}{-\frac{(a-a\sec(e+fx))a}{\sec(e+fx)a+a}-a}d\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}}{a^2}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}\left(\left(2c^2+11cd+29d^2\right)\sec(e+fx)+2\left(2c^2+8cd+11d^2\right)\right)}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}-\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{10d^3\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}\right)}{a^3}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}\left(\left(2c^2+11cd+29d^2\right)\sec(e+fx)+2\left(2c^2+8cd+11d^2\right)\right)}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}-\frac{(c-d)\sqrt{a-a\,\mathrm{s}}}{5a^2(a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

output
```
-((a^2*(-1/5*((c - d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2)/(a^
2*(a + a*Sec[e + f*x])^(5/2)) + ((-10*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/
Sqrt[a + a*Sec[e + f*x]]])/a^3 - ((c - d)*Sqrt[a - a*Sec[e + f*x]]*(2*(2*c
^2 + 8*c*d + 11*d^2) + (2*c^2 + 11*c*d + 29*d^2)*Sec[e + f*x]))/(3*a^2*(a
+ a*Sec[e + f*x])^(3/2)))/(5*a))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 162

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{-60d^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+60d^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+3(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left((c-d)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-\frac{10(c+2d)(c-d)\,t}{3}\right.}{60a^3f}$
derivativedivides	$\frac{c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-4d^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-7d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-4$
default	$\frac{c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-4d^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-7d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-4$
risch	$\frac{2i\left(15c^3e^{4i(fx+e)}-15d^3e^{4i(fx+e)}+30c^3e^{3i(fx+e)}+45c^2d\,e^{3i(fx+e)}-75d^3e^{3i(fx+e)}+40c^3e^{2i(fx+e)}+45c^2d\,e^{2i(fx+e)}+\right.}{15f\,a^3\left(e\right.}$
norman	$\frac{\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{20fa}-\frac{\left(c^3+3c^2d+3c\,d^2-7d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4fa}+\frac{3\left(3c^3+c^2d-c\,d^2-3d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{10fa}+\frac{\left(11c^3+2\right.}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/60*(-60*d^3*ln(tan(1/2*f*x+1/2*e)-1)+60*d^3*ln(tan(1/2*f*x+1/2*e)+1)+3*(
c-d)*tan(1/2*f*x+1/2*e)*((c-d)^2*tan(1/2*f*x+1/2*e)^4-10/3*(c+2*d)*(c-d)*t
an(1/2*f*x+1/2*e)^2+5*c^2+20*c*d+35*d^2))/a^3/f
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.86

$$\int\frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{15\left(d^3\cos\left(fx+e\right)^3+3\,d^3\cos\left(fx+e\right)^2+3\,d^3\cos\left(fx+e\right)+d^3\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(d^3\cos\left(fx\right.\right.}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output

```
1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e) +
 d^3)*log(sin(f*x + e) + 1) - 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^
2 + 3*d^3*cos(f*x + e) + d^3)*log(-sin(f*x + e) + 1) + 2*(2*c^3 + 9*c^2*d
+ 21*c*d^2 - 32*d^3 + (7*c^3 + 9*c^2*d + 6*c*d^2 - 22*d^3)*cos(f*x + e)^2
+ 3*(2*c^3 + 9*c^2*d + 6*c*d^2 - 17*d^3)*cos(f*x + e))*sin(f*x + e))/(a^3*
f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^3\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^3\sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{3cd^2\sec^3(e+}{\sec^3(e+fx)+3\sec^2(e+fx)}}{a^3}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

output

```
(Integral(c**3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) + 1), x) + Integral(d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/
(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(
3*c**2*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(126) = 252.

Time = 0.04 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.31

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx =$$
$$-\frac{d^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3} + \frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right) - \frac{3cd^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\right.}{}}{60f}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output

```
-1/60*(d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(
f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) +
 1) - 1)/a^3) - 3*c*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 -
c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^2*d*(5*sin(f*x +
 e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(126) = 252$.

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.95

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{60\,d^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} - \frac{60\,d^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} + \frac{3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-9\,a^{12}c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+9\,a^{12}cd^2\tan\left(\frac{1}{2}\,fx\right.}{}}{}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output

```
1/60*(60*d^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 60*d^3*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^3 + (3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 9*a^12*c^
2*d*tan(1/2*f*x + 1/2*e)^5 + 9*a^12*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 3*a^12*
d^3*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^3*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*
c*d^2*tan(1/2*f*x + 1/2*e)^3 - 20*a^12*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^1
2*c^3*tan(1/2*f*x + 1/2*e) + 45*a^12*c^2*d*tan(1/2*f*x + 1/2*e) + 45*a^12*
c*d^2*tan(1/2*f*x + 1/2*e) - 105*a^12*d^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{(c-d)^3}{4\,a^3}-\frac{3\,(c+d)\,(c-d)^2}{4\,a^3}+\frac{3\,(c+d)^2\,(c-d)}{4\,a^3}\right)}{f} + \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{(c-d)^3}{12\,a^3}-\frac{(c+d)\,(c-d)^2}{4\,a^3}\right)}{f} + \frac{2\,d^3\,\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f} + \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(c-d)^3}{20\,a^3\,f}$$

input
```
int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(tan(e/2 + (f*x)/2)*((c - d)^3/(4*a^3) - (3*(c + d)*(c - d)^2)/(4*a^3) + (
3*(c + d)^2*(c - d))/(4*a^3)))/f + (tan(e/2 + (f*x)/2)^3*((c - d)^3/(12*a^
3) - ((c + d)*(c - d)^2)/(4*a^3)))/f + (2*d^3*atanh(tan(e/2 + (f*x)/2)))/(
a^3*f) + (tan(e/2 + (f*x)/2)^5*(c - d)^3)/(20*a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.62

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{-60\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d^3+60\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d^3+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^3-9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^2d+}{}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x)
```

output
```
( - 60*log(tan((e + f*x)/2) - 1)*d**3 + 60*log(tan((e + f*x)/2) + 1)*d**3
+ 3*tan((e + f*x)/2)**5*c**3 - 9*tan((e + f*x)/2)**5*c**2*d + 9*tan((e + f
*x)/2)**5*c*d**2 - 3*tan((e + f*x)/2)**5*d**3 - 10*tan((e + f*x)/2)**3*c**
3 + 30*tan((e + f*x)/2)**3*c*d**2 - 20*tan((e + f*x)/2)**3*d**3 + 15*tan((
e + f*x)/2)*c**3 + 45*tan((e + f*x)/2)*c**2*d + 45*tan((e + f*x)/2)*c*d**2
 - 105*tan((e + f*x)/2)*d**3)/(60*a**3*f)
```

3.229 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 115

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c^2+6cd+7d^2)\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))}$$

output
```
1/5*(c-d)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+2/15*(c-d)*(c+4*d)*tan(f*x+e)/
a/f/(a+a*sec(f*x+e))^2+1/15*(2*c^2+6*c*d+7*d^2)*tan(f*x+e)/f/(a^3+a^3*sec(
f*x+e))
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.73

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{(2c^2+6cd+7d^2+6(c^2+3cd+d^2)\cos(e+fx)+(7c^2+6cd+2d^2)\cos^2(e+fx))\sin(e+fx)}{15a^3f(1+\cos(e+fx))^3}$$

input

```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

output

```
((2*c^2 + 6*c*d + 7*d^2 + 6*(c^2 + 3*c*d + d^2)*Cos[e + f*x] + (7*c^2 + 6*
c*d + 2*d^2)*Cos[e + f*x]^2)*Sin[e + f*x])/(15*a^3*f*(1 + Cos[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.67, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 4475, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{100}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^3\left(2c^2+6dc-3d^2+5d^2\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a^4}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{2c^2+6dc-3d^2+5d^2\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{5a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\left(2c^2+6cd+7d^2\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{3a}-\frac{2(c-d)(c+4d)\sqrt{a-a\sec(e+fx)}}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx}}{5a^2(a\sec(e+fx)+a)^{5/2}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \color{blue}{48}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{\left(2c^2+6cd+7d^2\right)\sqrt{a-a\sec(e+fx)}}{3a^3\sqrt{a\sec(e+fx)+a}}-\frac{2(c-d)(c+4d)\sqrt{a-a\sec(e+fx)}}{3a^2(a\sec(e+fx)+a)^{3/2}}}{5a}-\frac{(c-d)^2\sqrt{a-a\sec(e+fx)}}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

output
```
-((a^2*(-1/5*((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(a^2*(a + a*Sec[e + f*x]
)^(5/2)) + ((-2*(c - d)*(c + 4*d)*Sqrt[a - a*Sec[e + f*x]])/(3*a^2*(a + a*
Sec[e + f*x])^(3/2)) - ((2*c^2 + 6*c*d + 7*d^2)*Sqrt[a - a*Sec[e + f*x]])/
(3*a^3*Sqrt[a + a*Sec[e + f*x]]))/(5*a))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e
 + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{(c-d)^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{2(-c-d)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(-c-d)^2}{4f\,a^3}$
default	$\frac{\frac{(c-d)^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{2(-c-d)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(-c-d)^2}{4f\,a^3}$
parallelrisch	$\frac{\left(\left(7c^2+6cd+2d^2\right)\cos(2fx+2e)+12\left(c^2+3cd+d^2\right)\cos(fx+e)+11c^2+18cd+16d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{120a^3 f}$
risch	$\frac{2i\left(15c^2 e^{4i(fx+e)}+30c^2 e^{3i(fx+e)}+30cd\,e^{3i(fx+e)}+40c^2 e^{2i(fx+e)}+30cd\,e^{2i(fx+e)}+20d^2 e^{2i(fx+e)}+20c^2 e^{i(fx+e)}+30d\right.}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}$
norman	$\frac{\frac{\left(c^2-2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{20fa}+\frac{\left(c^2+2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4fa}-\frac{\left(2c^2+3cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3fa}-\frac{\left(4c^2-3cd-d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{15fa}+\left(19\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2 a^2}$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output

```
1/4/f/a^3*(1/5*(c-d)^2*tan(1/2*f*x+1/2*e)^5+2/3*(-c-d)*(c-d)*tan(1/2*f*x+1
/2*e)^3+tan(1/2*f*x+1/2*e)*(-c-d)^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\left(\left(7\,c^{2}+6\,cd+2\,d^{2}\right)\cos\left(fx+e\right)^{2}+2\,c^{2}+6\,cd+7\,d^{2}+6\,\left(c^{2}+3\,cd+d^{2}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{15\,\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}+3\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output

```
1/15*((7*c^2 + 6*c*d + 2*d^2)*cos(f*x + e)^2 + 2*c^2 + 6*c*d + 7*d^2 + 6*(
c^2 + 3*c*d + d^2)*cos(f*x + e))*sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^
3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \frac{c^{2}\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{d^{2}\sec^{3}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{2cd\sec^{2}\left(e+\right.}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)}}{a^{3}}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

output
```
(Integral(c**2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) + 1), x) + Integral(d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.60

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\frac{d^2\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^2\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{6\,cd\left(\frac{5\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx}{(\cos(fx+}\right.}{a^3}}{60\,f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
1/60*(d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*
x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 6*c*d*(5*sin(f*x + e)/(cos(f*x + e
) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-6\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+10\,d^2}{60\,a^3f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
1/60*(3*c^2*tan(1/2*f*x + 1/2*e)^5 - 6*c*d*tan(1/2*f*x + 1/2*e)^5 + 3*d^2*
tan(1/2*f*x + 1/2*e)^5 - 10*c^2*tan(1/2*f*x + 1/2*e)^3 + 10*d^2*tan(1/2*f*
x + 1/2*e)^3 + 15*c^2*tan(1/2*f*x + 1/2*e) + 30*c*d*tan(1/2*f*x + 1/2*e) +
 15*d^2*tan(1/2*f*x + 1/2*e))/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c+d\right)^2}{4\,a^3\,f} - \frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,c^2-2\,d^2\right)}{12\,a^3\,f} + \frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,{\left(c-d\right)}^2}{20\,a^3\,f}$$

input
```
int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(tan(e/2 + (f*x)/2)*(c + d)^2)/(4*a^3*f) - (tan(e/2 + (f*x)/2)^3*(2*c^2 -
2*d^2))/(12*a^3*f) + (tan(e/2 + (f*x)/2)^5*(c - d)^2)/(20*a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4c^2-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4cd+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4d^2-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c^2+10\,\mathrm{ta}}{60a^3f}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x)
```

output
```
(tan((e + f*x)/2)*(3*tan((e + f*x)/2)**4*c**2 - 6*tan((e + f*x)/2)**4*c*d
+ 3*tan((e + f*x)/2)**4*d**2 - 10*tan((e + f*x)/2)**2*c**2 + 10*tan((e + f
*x)/2)**2*d**2 + 15*c**2 + 30*c*d + 15*d**2))/(60*a**3*f)
```

3.230 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 102

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c+3d)\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}$$

output
```
1/5*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+1/15*(2*c+3*d)*tan(f*x+e)/a/f/(a
+a*sec(f*x+e))^2+1/15*(2*c+3*d)*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(5(8c+3d)\sin\left(\frac{fx}{2}\right)-15(2c+d)\sin\left(e+\frac{fx}{2}\right)+20c\sin\left(e+\frac{3fx}{2}\right)+15d\sin(e+\right.}{30a^3f(1+\cos(e+fx))^3}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

output

```
(Cos[(e + f*x)/2]*Sec[e/2]*(5*(8*c + 3*d)*Sin[(f*x)/2] - 15*(2*c + d)*Sin[
e + (f*x)/2] + 20*c*Sin[e + (3*f*x)/2] + 15*d*Sin[e + (3*f*x)/2] - 15*c*Si
n[2*e + (3*f*x)/2] + 7*c*Sin[2*e + (5*f*x)/2] + 3*d*Sin[2*e + (5*f*x)/2]))
/(30*a^3*f*(1 + Cos[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3042, 4488, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4488}$$

$$\frac{(2c+3d)\int \frac{\sec(e+fx)}{(\sec(e+fx)a+a)^2}dx}{5a}+\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2c+3d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{4283}$$

$$\frac{(2c+3d)\left(\frac{\int \frac{\sec(e+fx)}{\sec(e+fx)a+a}dx}{3a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}\right)}{5a}+\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2c+3d)\left(\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}\right)}{5a}+\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

$$\Big\downarrow \text{ 4281}$$

$$\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}+\frac{(2c+3d)\left(\frac{\tan(e+fx)}{3af(a\sec(e+fx)+a)}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}\right)}{5a}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

output

```
((c - d)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((2*c + 3*d)*(Tan[e
+ f*x]/(3*f*(a + a*Sec[e + f*x])^2) + Tan[e + f*x]/(3*a*f*(a + a*Sec[e + f
*x]))))/(5*a)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4488

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result
parallelrisch	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left((c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-\frac{10c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3}+5c+5d\right)}{20a^{3}f}$
derivativedivides	$\frac{\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f\,a^{3}}$
default	$\frac{\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f\,a^{3}}$
risch	$\frac{2i\left(15c\,e^{4i(fx+e)}+30c\,e^{3i(fx+e)}+15d\,e^{3i(fx+e)}+40c\,e^{2i(fx+e)}+15d\,e^{2i(fx+e)}+20\,e^{i(fx+e)}c+15d\,e^{i(fx+e)}+7c+3d\right)}{15f\,a^{3}\left(e^{i(fx+e)}+1\right)^{5}}$
norman	$\frac{\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{20fa}-\frac{(c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4fa}+\frac{(5c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{12fa}-\frac{(13c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{60fa}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)a^{2}}$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output

```
1/20*tan(1/2*f*x+1/2*e)*((c-d)*tan(1/2*f*x+1/2*e)^4-10/3*c*tan(1/2*f*x+1/2
*e)^2+5*c+5*d)/a^3/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\left((7\,c+3\,d)\cos\left(fx+e\right)^2+3\,(2\,c+3\,d)\cos\left(fx+e\right)+2\,c+3\,d\right)\sin\left(fx+e\right)}{15\,\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
1/15*((7*c + 3*d)*cos(f*x + e)^2 + 3*(2*c + 3*d)*cos(f*x + e) + 2*c + 3*d)
*sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos
(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \frac{c\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{d\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx}{a^3}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

output
```
(Integral(c*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*
x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{3\,d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="max
ima")
```

output
```
1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 3*d*(5*sin(f*x +
 e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,d\tan(}{60\,a^3f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="gia
c")
```

output
```
1/60*(3*c*tan(1/2*f*x + 1/2*e)^5 - 3*d*tan(1/2*f*x + 1/2*e)^5 - 10*c*tan(1
/2*f*x + 1/2*e)^3 + 15*c*tan(1/2*f*x + 1/2*e) + 15*d*tan(1/2*f*x + 1/2*e))
/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 10.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(15\,c+15\,d-10\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\right)}{60\,a^3\,f}$$

input
```
int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

output
```
(tan(e/2 + (f*x)/2)*(15*c + 15*d - 10*c*tan(e/2 + (f*x)/2)^2 + 3*c*tan(e/2
 + (f*x)/2)^4 - 3*d*tan(e/2 + (f*x)/2)^4))/(60*a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4c-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4d-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+15c+15d\right)}{60a^3f}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

output
```
(tan((e + f*x)/2)*(3*tan((e + f*x)/2)**4*c - 3*tan((e + f*x)/2)**4*d - 10*
tan((e + f*x)/2)**2*c + 15*c + 15*d))/(60*a**3*f)
```

3.231 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 181

$$\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx \\
&= -\frac{2d^3\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3(c-d)^{7/2}\sqrt{c+d}f} + \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} \\
&\quad + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15(c-d)^3f\,(a^3+a^3\sec(e+fx))}
\end{aligned}$$

output

```
-2*d^3*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^3/(c-d)^(7/2)
/(c+d)^(1/2)/f+1/5*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^3+1/15*(2*c-7*d)*ta
n(f*x+e)/a/(c-d)^2/f/(a+a*sec(f*x+e))^2+1/15*(2*c^2-9*c*d+22*d^2)*tan(f*x+
e)/(c-d)^3/f/(a^3+a^3*sec(f*x+e))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.75 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx$$

$$= \frac{\cos\left(\frac{1}{2}(e+fx)\right)\Bigg(\frac{480d^3\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos^5\left(\frac{1}{2}(e+fx)\right)(i\cos(e)+\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\sec\left(\frac{e}{2}\right)\big(5(8}{}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

output
```
(Cos[(e + f*x)/2]*((480*d^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d +
c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Co
s[(e + f*x)/2]^5*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Si
n[e])^2]) + Sec[e/2]*(5*(8*c^2 - 27*c*d + 37*d^2)*Sin[(f*x)/2] - 15*(2*c^2
 - 7*c*d + 9*d^2)*Sin[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] - 75*c*d*Si
n[e + (3*f*x)/2] + 115*d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2
] + 45*c*d*Sin[2*e + (3*f*x)/2] - 45*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[
2*e + (5*f*x)/2] - 24*c*d*Sin[2*e + (5*f*x)/2] + 32*d^2*Sin[2*e + (5*f*x)/
2])))/(30*a^3*(c - d)^3*f*(1 + Cos[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.62, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4475, 115, 25, 27, 169, 25, 27, 169, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c+d\sec(e+fx))}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{115}$$

$$-\frac{a^2\tan(e+fx)\left(-\frac{\int -\frac{a^2(2c-5d+2d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{5a^3(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(2c-5d+2d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{5a^3(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{2c-5d+2d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{5a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+a)^{5/2}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{\int -\frac{a^2\left(2c^2-7dc+15d^2+(2c-7d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{(2c-7d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{5a(c-d)}-\frac{\sqrt{a-a\sec(e+f}}{5a^2(c-d)(a\sec(e+fx}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{a^2\left(2c^2-7dc+15d^2+(2c-7d)d\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{(2c-7d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{5a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{2c^2-7dc+15d^2+(2c-7d)d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a(c-d)}-\frac{(2c-7d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{5a(c-d)}-\frac{\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{\int \frac{15a^2d^3}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a^3(c-d)}-\frac{\left(2c^2-9cd+22d^2\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{(2c-7d)\sqrt{a-a\sec(e+fx}}{3a^2(c-d)(a\sec(e+fx)+a)}}{5a(c-d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{15d^3\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a(c-d)}-\frac{\left(2c^2-9cd+22d^2\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{(2c-7d)\sqrt{a-a\sec(}}{3a^2(c-d)(a\sec(e+fx}}{5a(c-d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{30d^3\int \frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{a(c-d)}-\frac{\left(2c^2-9cd+22d^2\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{(2c-7d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{5a(c-d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{30d^3\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2(c-d)^{3/2}\sqrt{c+d}}-\frac{\left(2c^2-9cd+22d^2\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{(2c-7d)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{5a(c-d)}-\frac{\sqrt{a-a\sec}}{5a^2(c-d)(a\sec(}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

output

```
-((a^2*(-1/5*Sqrt[a - a*Sec[e + f*x]]/(a^2*(c - d)*(a + a*Sec[e + f*x])^(5
/2)) + (-1/3*((2*c - 7*d)*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*(a + a*Se
c[e + f*x])^(3/2)) + ((-30*d^3*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]
])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])])/(a^2*(c - d)^(3/2)*Sqrt[c + d]
) - ((2*c^2 - 9*c*d + 22*d^2)*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*Sqrt[
a + a*Sec[e + f*x]]))/(3*a*(c - d)))/(5*a*(c - d)))*Tan[e + f*x])/(f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 115

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 169

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} c^{2}}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} d^{2}}{5}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-\frac{4d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c^{2}\tan(\frac{fx}{2}}{(c-d)^{3}}}{4f\,a^{3}}$
default	$\frac{\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} c^{2}}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} d^{2}}{5}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-\frac{4d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c^{2}\tan(\frac{fx}{2}}{(c-d)^{3}}}{4f\,a^{3}}$
risch	$\frac{2i(15c^{2}e^{4i(fx+e)}-45cd\,e^{4i(fx+e)}+45d^{2}e^{4i(fx+e)}+30c^{2}e^{3i(fx+e)}-105cd\,e^{3i(fx+e)}+135d^{2}e^{3i(fx+e)}+40c^{2}e^{2i(fx+e)}-}{15f\,a^{3}(c-d)^{3}(e^{i(fx+e)}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
1/4/f/a^3*(1/(c-d)^3*(1/5*tan(1/2*f*x+1/2*e)^5*c^2-2/5*tan(1/2*f*x+1/2*e)^
5*c*d+1/5*tan(1/2*f*x+1/2*e)^5*d^2-2/3*c^2*tan(1/2*f*x+1/2*e)^3+2*c*d*tan(
1/2*f*x+1/2*e)^3-4/3*d^2*tan(1/2*f*x+1/2*e)^3+c^2*tan(1/2*f*x+1/2*e)-4*tan
(1/2*f*x+1/2*e)*c*d+7*d^2*tan(1/2*f*x+1/2*e))-8*d^3/(c-d)^3/((c-d)*(c+d))^
(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(166) = 332$.

Time = 0.17 (sec) , antiderivative size = 1001, normalized size of antiderivative = 5.53

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
[-1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e)
 + d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x +
e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/
(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(2*c^4 - 9*c^3*d + 20
*c^2*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2*d^2 + 24*c*d^3 -
32*d^4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*d^
4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2
*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^
3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x
 + e)^2 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3
*c*d^4 + a^3*d^5)*f*cos(f*x + e) + (a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2
+ 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f), -1/15*(15*(d^3*cos(f*x + e)^3
 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e) + d^3)*sqrt(-c^2 + d^2)*arcta
n(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*
c^4 - 9*c^3*d + 20*c^2*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2
*d^2 + 24*c*d^3 - 32*d^4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2
 + 9*c*d^3 - 17*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d +
 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 +
 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 +
a^3*d^5)*f*cos(f*x + e)^2 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + ...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = \frac{\int \frac{\sec(e+fx)}{c\sec^3(e+fx)+3c\sec^2(e+fx)+3c\sec(e+fx)+c+d\sec^4(e+fx)+3d\sec^3(e+fx)+3d\sec^2(e+fx)+d\sec(e+fx)}\,dx}{a^3}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

output

```
Integral(sec(e + f*x)/(c*sec(e + f*x)**3 + 3*c*sec(e + f*x)**2 + 3*c*sec(e
 + f*x) + c + d*sec(e + f*x)**4 + 3*d*sec(e + f*x)**3 + 3*d*sec(e + f*x)**
2 + d*sec(e + f*x)), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(166) = 332$.

Time = 0.19 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.60

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx =$$

$$-\frac{\frac{120\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^3}{(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)\sqrt{-c^2+d^2}} - \frac{3a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12a^{12}c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e}{}}{}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
-1/60*(120*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*
tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^3/((a^
3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*sqrt(-c^2 + d^2)) - (3*a^12*c
^4*tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 18*a^12
*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 3
*a^12*d^4*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 50
*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 90*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e)^
3 + 70*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 20*a^12*d^4*tan(1/2*f*x + 1/2*e
)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e) - 90*a^12*c^3*d*tan(1/2*f*x + 1/2*e
) + 240*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e) - 270*a^12*c*d^3*tan(1/2*f*x + 1
/2*e) + 105*a^12*d^4*tan(1/2*f*x + 1/2*e))/(a^15*c^5 - 5*a^15*c^4*d + 10*a
^15*c^3*d^2 - 10*a^15*c^2*d^3 + 5*a^15*c*d^4 - a^15*d^5))/f
```

Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.26

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3}{4a^3(c-d)}-\frac{(c+d)\left(\frac{3}{4a^3(c-d)}-\frac{c+d}{4a^3(c-d)^2}\right)}{c-d}\right)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{1}{4a^3(c-d)}-\frac{c+d}{12a^3(c-d)^2}\right)}{f} + \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{20a^3f(c-d)} - \frac{2d^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2c-2d)\left(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3\right)}{2a^3\sqrt{c+d}\,(c-d)^{7/2}}\right)}{a^3f\sqrt{c+d}\,(c-d)^{7/2}}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))),x)
```

output
```
(tan(e/2 + (f*x)/2)*(3/(4*a^3*(c - d)) - ((c + d)*(3/(4*a^3*(c - d)) - (c
+ d)/(4*a^3*(c - d)^2)))/(c - d)))/f - (tan(e/2 + (f*x)/2)^3*(1/(4*a^3*(c
- d)) - (c + d)/(12*a^3*(c - d)^2)))/f + tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c
 - d)) - (2*d^3*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(a^3*c^3 - a^3*d^3 +
 3*a^3*c*d^2 - 3*a^3*c^2*d))/(2*a^3*(c + d)^(1/2)*(c - d)^(7/2))))/(a^3*f*
(c + d)^(1/2)*(c - d)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.80

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx$$

$$= \frac{-120\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)d^3+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^4-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^3d+6\tan(\frac{fx}{2}}{}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x)
```

output
```
( - 120*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*d**3 + 3*tan((e + f*x)/2)**5*c**4 - 6*tan((e + f*x
)/2)**5*c**3*d + 6*tan((e + f*x)/2)**5*c*d**3 - 3*tan((e + f*x)/2)**5*d**4
 - 10*tan((e + f*x)/2)**3*c**4 + 30*tan((e + f*x)/2)**3*c**3*d - 10*tan((e
 + f*x)/2)**3*c**2*d**2 - 30*tan((e + f*x)/2)**3*c*d**3 + 20*tan((e + f*x)
/2)**3*d**4 + 15*tan((e + f*x)/2)*c**4 - 60*tan((e + f*x)/2)*c**3*d + 90*t
an((e + f*x)/2)*c**2*d**2 + 60*tan((e + f*x)/2)*c*d**3 - 105*tan((e + f*x)
/2)*d**4)/(60*a**3*f*(c**5 - 3*c**4*d + 2*c**3*d**2 + 2*c**2*d**3 - 3*c*d*
*4 + d**5))
```

3.232 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 288

$$\begin{aligned}
\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx
&= -\frac{2d^3(4c+3d)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3(c-d)^{9/2}(c+d)^{3/2}f} \\
&\quad+\frac{d(2c^3-12c^2d+43cd^2+72d^3)\tan(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sec(e+fx))} \\
&\quad+\frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} \\
&\quad+\frac{(2c-9d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} \\
&\quad+\frac{(2c^2-12cd+45d^2)\tan(e+fx)}{15(c-d)^3f\left(a^3+a^3\sec(e+fx)\right)(c+d\sec(e+fx))}
\end{aligned}$$

output

```
-2*d^3*(4*c+3*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^3/(
c-d)^(9/2)/(c+d)^(3/2)/f+1/15*d*(2*c^3-12*c^2*d+43*c*d^2+72*d^3)*tan(f*x+e
)/a^3/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))+1/5*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x
+e))^3/(c+d*sec(f*x+e))+1/15*(2*c-9*d)*tan(f*x+e)/a/(c-d)^2/f/(a+a*sec(f*x
+e))^2/(c+d*sec(f*x+e))+1/15*(2*c^2-12*c*d+45*d^2)*tan(f*x+e)/(c-d)^3/f/(a
^3+a^3*sec(f*x+e))/(c+d*sec(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.67 (sec) , antiderivative size = 1772, normalized size of antiderivative = 6.15

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

output

```
((4*c + 3*d)*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^5*((
(16*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*
Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-
I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt
[Cos[2*e] - I*Sin[2*e]]) + (16*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 -
 d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[
2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])
/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^4*(c + d)*(a
+ a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2) + (Cos[e/2 + (f*x)/2]*(d + c*C
os[e + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-55*c^5*Sin[(f*x)/2] + 135*c^
4*d*Sin[(f*x)/2] - 20*c^3*d^2*Sin[(f*x)/2] - 810*c^2*d^3*Sin[(f*x)/2] - 45
0*c*d^4*Sin[(f*x)/2] + 150*d^5*Sin[(f*x)/2] + 47*c^5*Sin[(3*f*x)/2] - 137*
c^4*d*Sin[(3*f*x)/2] + 88*c^3*d^2*Sin[(3*f*x)/2] + 812*c^2*d^3*Sin[(3*f*x)
/2] + 690*c*d^4*Sin[(3*f*x)/2] + 75*d^5*Sin[(3*f*x)/2] - 50*c^5*Sin[e - (f
*x)/2] + 130*c^4*d*Sin[e - (f*x)/2] - 10*c^3*d^2*Sin[e - (f*x)/2] - 1030*c
^2*d^3*Sin[e - (f*x)/2] - 990*c*d^4*Sin[e - (f*x)/2] - 150*d^5*Sin[e - (f*
x)/2] + 50*c^5*Sin[e + (f*x)/2] - 130*c^4*d*Sin[e + (f*x)/2] + 10*c^3*d^2*
Sin[e + (f*x)/2] + 1030*c^2*d^3*Sin[e + (f*x)/2] + 765*c*d^4*Sin[e + (f*x)
/2] - 150*d^5*Sin[e + (f*x)/2] - 55*c^5*Sin[2*e + (f*x)/2] + 135*c^4*d*Sin
[2*e + (f*x)/2] - 20*c^3*d^2*Sin[2*e + (f*x)/2] - 810*c^2*d^3*Sin[2*e +...
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.36, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 4475, 114, 27, 169, 25, 27, 169, 25, 27, 169, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{114}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(c+3d-3d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}(c+d\sec(e+fx))}d\sec(e+fx)}{a^2(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}}{a^2(c^2-d^2)(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{c+3d-3d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}(c+d\sec(e+fx))}d\sec(e+fx)}{c^2-d^2}+\frac{d\sqrt{a-a\sec(e+fx)}}{a^2(c^2-d^2)(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{-\frac{\int -\frac{a^2\left(2c^2-8dc-15d^2+2d(c+6d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{5a^3(c-d)}-\frac{(c+6d)\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+a)^{5/2}}}{c^2-d^2}+\frac{d\sqrt{a-}}{a^2(c^2-d^2)(a\sec(e+f}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{a^2\left(2c^2-8dc-15d^2+2d(c+6d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{5a^3(c-d)}-\frac{(c+6d)\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+a)^{5/2}}}{c^2-d^2}+\frac{d\sqrt{a-a\,\mathrm{se}}}{a^2(c^2-d^2)(a\sec(e+fx)+}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{2c^2-8dc-15d^2+2d(c+6d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{5a(c-d)}-\frac{(c+6d)\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+a)^{5/2}}}{c^2-d^2}+\frac{d\sqrt{a-a\,\mathrm{se}}}{a^2(c^2-d^2)(a\sec(e+fx)+}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{\int -\frac{a^2\left((c+d)\left(2c^2-12dc+45d^2\right)+d\left(2c^2-10dc-27d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{\left(2c^2-10cd-27d^2\right)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{5a(c-d)}-\frac{(c+6d)\sqrt{}}{5a^2(c-d)(a}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\int \frac{a^2\left((c+d)\left(2c^2-12dc+45d^2\right)+d\left(2c^2-10dc-27d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{\left(2c^2-10cd-27d^2\right)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{5a(c-d)}-\frac{(c+6d)\sqrt{a-}}{5a^2(c-d)(a\,\mathrm{sec}}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\int \frac{(c+d)\left(2c^2-12dc+45d^2\right)+d\left(2c^2-10dc-27d^2\right)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a(c-d)}-\frac{\left(2c^2-10cd-27d^2\right)\sqrt{a-a\sec(e+fx)}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{5a(c-d)}-\frac{(c+6d)\sqrt{a-a\sec(}}{5a^2(c-d)(a\sec(e+f}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{-\frac{\int \frac{15a^2d^3(4c+3d)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a^3(c-d)}-\frac{\left(2c^3-12c^2d+43cd^2+72d^3\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\left(2c^2-10cd-27d\right.}{3a^2(c-d)(a\sec}}{5a(c-d)}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{-\frac{15d^3(4c+3d)\int \frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a(c-d)}-\frac{\left(2c^3-12c^2d+43cd^2+72d^3\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\left(2c\right.}{}}{5a(c-d)}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

$$\Big\downarrow 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{-\frac{30d^3(4c+3d)\int \frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{a(c-d)}-\frac{\left(2c^3-12c^2d+43cd^2+72d^3\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\left(2c^2-10cd-\right.}{3a^2(c-d)(}}{5a(c-d)}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+f}}$$

$$\Big\downarrow 218$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{-\frac{30d^3(4c+3d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2(c-d)^{3/2}\sqrt{c+d}}-\frac{\left(2c^3-12c^2d+43cd^2+72d^3\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\left(2c^2-10cd-27d^2\right)\sqrt{a-a\sec(e+fx}}{3a^2(c-d)(a\sec(e+fx)+a)^{3/2}}}{5a(c-d)}}{c^2-d^2}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

output
```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c^2 - d^2)*(a + a*Sec[e + f*x])
^(5/2)*(c + d*Sec[e + f*x])) + (-1/5*((c + 6*d)*Sqrt[a - a*Sec[e + f*x]])/
(a^2*(c - d)*(a + a*Sec[e + f*x])^(5/2)) + (-1/3*((2*c^2 - 10*c*d - 27*d^2
)*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*(a + a*Sec[e + f*x])^(3/2)) + ((-
30*d^3*(4*c + 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
 d]*Sqrt[a - a*Sec[e + f*x]])])/(a^2*(c - d)^(3/2)*Sqrt[c + d]) - ((2*c^3
- 12*c^2*d + 43*c*d^2 + 72*d^3)*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*Sqr
t[a + a*Sec[e + f*x]]))/(3*a*(c - d)))/(5*a*(c - d)))/(c^2 - d^2))*Tan[e +
 f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5c^2}{5}-\frac{2\tan(\frac{fx}{2}+\frac{e}{2})^5cd}{5}+\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5d^2}{5}-\frac{2c^2\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+\frac{8cd\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}-2d^2\tan(\frac{fx}{2}+\frac{e}{2})^3+c^2\tan(\frac{fx}{2}}{(c^2-2cd+d^2)(c-d)^2}$
default	$\frac{\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5c^2}{5}-\frac{2\tan(\frac{fx}{2}+\frac{e}{2})^5cd}{5}+\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5d^2}{5}-\frac{2c^2\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+\frac{8cd\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}-2d^2\tan(\frac{fx}{2}+\frac{e}{2})^3+c^2\tan(\frac{fx}{2}}{(c^2-2cd+d^2)(c-d)^2}$
risch	$2i(7c^5-27c^4d+38c^3d^2+72c^2d^3+15cd^4+90c^2d^3e^{6i(fx+e)}+75d^5e^{2i(fx+e)}+30c^5e^{5i(fx+e)}+55c^5e^{4i(fx+e)}+50c^5e^{3i(fx}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/4/f/a^3*(1/(c^2-2*c*d+d^2)/(c-d)^2*(1/5*tan(1/2*f*x+1/2*e)^5*c^2-2/5*tan
(1/2*f*x+1/2*e)^5*c*d+1/5*tan(1/2*f*x+1/2*e)^5*d^2-2/3*c^2*tan(1/2*f*x+1/2
*e)^3+8/3*c*d*tan(1/2*f*x+1/2*e)^3-2*d^2*tan(1/2*f*x+1/2*e)^3+c^2*tan(1/2*
f*x+1/2*e)-6*tan(1/2*f*x+1/2*e)*c*d+17*d^2*tan(1/2*f*x+1/2*e))+16*d^3/(c-d
)^4*(-1/2*d/(c+d)*tan(1/2*f*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1
/2*e)^2*d-c-d)-1/2*(4*c+3*d)/(c+d)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1
/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(271) = 542$.

Time = 0.22 (sec) , antiderivative size = 1693, normalized size of antiderivative = 5.88

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
[1/30*(15*(4*c*d^4 + 3*d^5 + (4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^
2*d^3 + 13*c*d^4 + 3*d^5)*cos(f*x + e)^3 + 3*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)
*cos(f*x + e)^2 + (4*c^2*d^3 + 15*c*d^4 + 9*d^5)*cos(f*x + e))*sqrt(c^2 -
d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 -
 d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2
 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*
c^2*d^4 - 43*c*d^5 - 72*d^6 + (7*c^6 - 27*c^5*d + 31*c^4*d^2 + 99*c^3*d^3
- 23*c^2*d^4 - 72*c*d^5 - 15*d^6)*cos(f*x + e)^3 + (6*c^6 - 29*c^5*d + 51*
c^4*d^2 + 193*c^3*d^3 + 60*c^2*d^4 - 164*c*d^5 - 117*d^6)*cos(f*x + e)^2 +
 (2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141*c*d^5 - 17
1*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^8 - 3*a^3*c^7*d + a^3*c^6*d^2 +
 5*a^3*c^5*d^3 - 5*a^3*c^4*d^4 - a^3*c^3*d^5 + 3*a^3*c^2*d^6 - a^3*c*d^7)*
f*cos(f*x + e)^4 + (3*a^3*c^8 - 8*a^3*c^7*d + 16*a^3*c^5*d^3 - 10*a^3*c^4*
d^4 - 8*a^3*c^3*d^5 + 8*a^3*c^2*d^6 - a^3*d^8)*f*cos(f*x + e)^3 + 3*(a^3*c
^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c
^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^2 + (a^3*c^8 - 8*a^3*c^6*d^
2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*
d^8)*f*cos(f*x + e) + (a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4
*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f), -1/15*(15*
(4*c*d^4 + 3*d^5 + (4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^2*d^3 +...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{\int \frac{\sec{\left(e+fx \right)}}{c^{2}\sec^{3}{\left(e+fx \right)}+3c^{2}\sec^{2}{\left(e+fx \right)}+3c^{2}\sec{\left(e+fx \right)}+c^{2}+2cd\sec^{4}{\left(e+fx \right)}+6cd\sec^{3}{\left(e+fx \right)}+6cd\sec^{2}{\left(e+fx \right)}+2cd\sec{\left(e+fx \right)}+d^{2}\sec^{5}{\left(e+fx \right)}+}}{a^{3}}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(sec(e + f*x)/(c**2*sec(e + f*x)**3 + 3*c**2*sec(e + f*x)**2 + 3*c
**2*sec(e + f*x) + c**2 + 2*c*d*sec(e + f*x)**4 + 6*c*d*sec(e + f*x)**3 +
6*c*d*sec(e + f*x)**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**5 + 3*d**2
*sec(e + f*x)**4 + 3*d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(271) = 542$.

Time = 0.22 (sec) , antiderivative size = 918, normalized size of antiderivative = 3.19

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="g
iac")
```

output

```
-1/60*(120*d^4*tan(1/2*f*x + 1/2*e)/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^
2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*t
an(1/2*f*x + 1/2*e)^2 - c - d)) + 120*(4*c*d^3 + 3*d^4)*(pi*floor(1/2*(f*x
 + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(
1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d
^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*sqrt(-c^2 + d^2)) - (3*a^12*c^
8*tan(1/2*f*x + 1/2*e)^5 - 24*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 84*a^12*
c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 +
 210*a^12*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^3*d^5*tan(1/2*f*x +
1/2*e)^5 + 84*a^12*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 24*a^12*c*d^7*tan(1/2*
f*x + 1/2*e)^5 + 3*a^12*d^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^8*tan(1/2*f
*x + 1/2*e)^3 + 100*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 420*a^12*c^6*d^2*t
an(1/2*f*x + 1/2*e)^3 + 980*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 1400*a^1
2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 + 1260*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e)^
3 - 700*a^12*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 220*a^12*c*d^7*tan(1/2*f*x +
 1/2*e)^3 - 30*a^12*d^8*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^8*tan(1/2*f*x +
 1/2*e) - 180*a^12*c^7*d*tan(1/2*f*x + 1/2*e) + 1020*a^12*c^6*d^2*tan(1/2*
f*x + 1/2*e) - 3180*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e) + 5850*a^12*c^4*d^4*
tan(1/2*f*x + 1/2*e) - 6540*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e) + 4380*a^12*
c^2*d^6*tan(1/2*f*x + 1/2*e) - 1620*a^12*c*d^7*tan(1/2*f*x + 1/2*e) + 2...
```

Mupad [B] (verification not implemented)

Time = 11.08 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.61

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{20\,a^3\,f\,(c-d)^2} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(\frac{2\,(c^2-d^2)\,\left(\frac{1}{a^3\,(c-d)^2}-\frac{c^2-d^2}{2\,a^3\,(c-d)^4}\right)}{(c-d)^2}-\frac{3}{2\,a^3\,(c-d)^2}+\frac{(c+d)^2}{4\,a^3\,(c-d)^4}\right)}{f}$$

$$- \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\,\left(\frac{1}{3\,a^3\,(c-d)^2}-\frac{c^2-d^2}{6\,a^3\,(c-d)^4}\right)}{f}$$

$$+ \frac{2\,d^4\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,(c+d)\,\Big(a^3\,c^5-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\,(a^3\,c^5-5\,a^3\,c^4\,d+10\,a^3\,c^3\,d^2-10\,a^3\,c^2\,d^3+5\,a^3\,c\,d^4-a^3\,d^5)+a^3}$$

$$+ \frac{d^3\,\mathrm{atan}\Big(\frac{1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^5-5\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^4\,d+10\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^3\,d^2-10\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2\,d^3+5\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c\,d^4-1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d}{\sqrt{c+d}\,(c-d)^{9/2}}}{a^3\,f\,(c+d)^{3/2}\,(c-d)^{9/2}}$$

input

```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^2),x)
```

output

```
tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c - d)^2) - (tan(e/2 + (f*x)/2)*((2*(c^2 -
 d^2)*(1/(a^3*(c - d)^2) - (c^2 - d^2)/(2*a^3*(c - d)^4)))/(c - d)^2 - 3/(
2*a^3*(c - d)^2) + (c + d)^2/(4*a^3*(c - d)^4)))/f - (tan(e/2 + (f*x)/2)^3
*(1/(3*a^3*(c - d)^2) - (c^2 - d^2)/(6*a^3*(c - d)^4)))/f + (2*d^4*tan(e/2
 + (f*x)/2))/(f*(c + d)*(a^3*c^5 - tan(e/2 + (f*x)/2)^2*(a^3*c^5 - a^3*d^5
 + 5*a^3*c*d^4 - 5*a^3*c^4*d - 10*a^3*c^2*d^3 + 10*a^3*c^3*d^2) + a^3*d^5
- 3*a^3*c*d^4 - 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2)) + (d^3*atan(
(c^5*tan(e/2 + (f*x)/2)*1i - d^5*tan(e/2 + (f*x)/2)*1i + c*d^4*tan(e/2 + (
f*x)/2)*5i - c^4*d*tan(e/2 + (f*x)/2)*5i - c^2*d^3*tan(e/2 + (f*x)/2)*10i
+ c^3*d^2*tan(e/2 + (f*x)/2)*10i)/((c + d)^(1/2)*(c - d)^(9/2)))*(4*c + 3*
d)*2i)/(a^3*f*(c + d)^(3/2)*(c - d)^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1060, normalized size of antiderivative = 3.68

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x)
```

output

```
( - 480*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*c**2*d**3 + 120*sqrt( - c**2 +
 d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2)
)*tan((e + f*x)/2)**2*c*d**4 + 360*sqrt( - c**2 + d**2)*atan((tan((e + f*x
)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*d**
5 + 480*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*c**2*d**3 + 840*sqrt( - c**2 + d**2)*atan((tan((e
+ f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c*d**4 + 360*sqrt(
 - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**
2 + d**2))*d**5 + 3*tan((e + f*x)/2)**7*c**6 - 6*tan((e + f*x)/2)**7*c**5*
d - 3*tan((e + f*x)/2)**7*c**4*d**2 + 12*tan((e + f*x)/2)**7*c**3*d**3 - 3
*tan((e + f*x)/2)**7*c**2*d**4 - 6*tan((e + f*x)/2)**7*c*d**5 + 3*tan((e +
 f*x)/2)**7*d**6 - 13*tan((e + f*x)/2)**5*c**6 + 40*tan((e + f*x)/2)**5*c*
*5*d - tan((e + f*x)/2)**5*c**4*d**2 - 80*tan((e + f*x)/2)**5*c**3*d**3 +
41*tan((e + f*x)/2)**5*c**2*d**4 + 40*tan((e + f*x)/2)**5*c*d**5 - 27*tan(
(e + f*x)/2)**5*d**6 + 25*tan((e + f*x)/2)**3*c**6 - 110*tan((e + f*x)/2)*
*3*c**5*d + 175*tan((e + f*x)/2)**3*c**4*d**2 + 220*tan((e + f*x)/2)**3*c*
*3*d**3 - 425*tan((e + f*x)/2)**3*c**2*d**4 - 110*tan((e + f*x)/2)**3*c*d*
*5 + 225*tan((e + f*x)/2)**3*d**6 - 15*tan((e + f*x)/2)*c**6 + 60*tan((e +
 f*x)/2)*c**5*d - 75*tan((e + f*x)/2)*c**4*d**2 - 480*tan((e + f*x)/2)*...
```

3.233 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 368

$$\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx \\
&= -\frac{d^3(20c^2+30cd+13d^2)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3(c-d)^{11/2}(c+d)^{5/2}f} \\
&\quad+\frac{d(4c^3-30c^2d+146cd^2+195d^3)\tan(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sec(e+fx))^2} \\
&\quad+\frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} \\
&\quad+\frac{(2c-11d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} \\
&\quad+\frac{(2c^2-15cd+76d^2)\tan(e+fx)}{15(c-d)^3f\,(a^3+a^3\sec(e+fx))\,(c+d\sec(e+fx))^2} \\
&\quad+\frac{d(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)\tan(e+fx)}{30a^3(c-d)^5(c+d)^2f(c+d\sec(e+fx))}
\end{aligned}$$

output
```
-d^3*(20*c^2+30*c*d+13*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(
1/2))/a^3/(c-d)^(11/2)/(c+d)^(5/2)/f+1/30*d*(4*c^3-30*c^2*d+146*c*d^2+195*
d^3)*tan(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))^2+1/5*tan(f*x+e)/(c-d
)/f/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2+1/15*(2*c-11*d)*tan(f*x+e)/a/(c-
d)^2/f/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2+1/15*(2*c^2-15*c*d+76*d^2)*ta
n(f*x+e)/(c-d)^3/f/(a^3+a^3*sec(f*x+e))/(c+d*sec(f*x+e))^2+1/30*d*(4*c^4-3
0*c^3*d+142*c^2*d^2+525*c*d^3+304*d^4)*tan(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c
+d*sec(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.45 (sec) , antiderivative size = 1096, normalized size of antiderivative = 2.98

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]
```

output

```
(4*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(-8
*c*Sin[e/2] + 23*d*Sin[e/2]))/(15*(-c + d)^4*f*(a + a*Sec[e + f*x])^3*(c +
 d*Sec[e + f*x])^3) + ((20*c^2 + 30*c*d + 13*d^2)*Cos[e/2 + (f*x)/2]^6*(d
+ c*Cos[e + f*x])^3*Sec[e + f*x]^6*(((-8*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e
]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d
^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)
/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (8*d^3*Arc
Tan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (
I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)
/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*
Sin[2*e]])))/((-c + d)^5*(c + d)^2*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f
*x])^3) - (2*Cos[e/2 + (f*x)/2]*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*
x]^6*Sin[(f*x)/2])/(5*(-c + d)^3*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f
*x])^3) + (4*Cos[e/2 + (f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e +
f*x]^6*(-8*c*Sin[(f*x)/2] + 23*d*Sin[(f*x)/2]))/(15*(-c + d)^4*f*(a + a*Se
c[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (8*Cos[e/2 + (f*x)/2]^5*(d + c*Cos
[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(7*c^2*Sin[(f*x)/2] - 44*c*d*Sin[(f*x
)/2] + 127*d^2*Sin[(f*x)/2]))/(15*(-c + d)^5*f*(a + a*Sec[e + f*x])^3*(c +
 d*Sec[e + f*x])^3) + (4*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])*Sec[e]*
Sec[e + f*x]^6*(d^6*Sin[e] - c*d^5*Sin[f*x]))/(c^2*(-c + d)^4*(c + d)*f...
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.38, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 4475, 114, 27, 168, 27, 169, 25, 27, 169, 25, 27, 169, 27, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a\sec(e+fx)+a)^3(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4475}$$

$$-\frac{a^2\tan(e+fx)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 114$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a^2(2c+3d-4d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2a^2(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2a^2(c^2-d^2)(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{2c+3d-4d\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2(c^2-d^2)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2a^2(c^2-d^2)(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 168$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{a^2\left(2c^2+21dc+13d^2-9d(2c+d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}(c+d\sec(e+fx))}d\sec(e+fx)}{a^2\left(c^2-d^2\right)}+\frac{3d(2c+d)\sqrt{a-a\sec(e+fx)}}{a^2\left(c^2-d^2\right)(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{}{2a^2(c^2-}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{2c^2+21dc+13d^2-9d(2c+d)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{7/2}(c+d\sec(e+fx))}d\sec(e+fx)}{c^2-d^2}+\frac{3d(2c+d)\sqrt{a-a\sec(e+fx)}}{a^2\left(c^2-d^2\right)(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{}{2a^2(c^2-}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{-\frac{\int -\frac{a^2\left((2c+5d)\left(2c^2-16dc-13d^2\right)+2d\left(2c^2+39dc+22d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{5a^3(c-d)}-\frac{\left(2c^2+39cd+22d^2\right)\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+a)^{5/2}}}{c^2-d^2}+\frac{}{a^2\left(c^2-\right.}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+}}$$

$$\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\int\frac{a^2\left((2c+5d)\left(2c^2-16dc-13d^2\right)+2d\left(2c^2+39dc+22d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{5a^3(c-d)}-\frac{\left(2c^2+39cd+22d^2\right)\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+a)^{5/2}}}{c^2-d^2}+\frac{3}{a^2\left(c^2-d^2\right)}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\int\frac{(2c+5d)\left(2c^2-16dc-13d^2\right)+2d\left(2c^2+39dc+22d^2\right)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{5/2}(c+d\sec(e+fx))}d\sec(e+fx)}{5a(c-d)}-\frac{\left(2c^2+39cd+22d^2\right)\sqrt{a-a\sec(e+fx)}}{5a^2(c-d)(a\sec(e+fx)+a)^{5/2}}}{c^2-d^2}+\frac{3d(2c}{a^2\left(c^2-d^2\right)(a\sec}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\downarrow 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{-\frac{\int-\frac{a^2\left((c+d)\left(4c^3-30dc^2+146d^2c+195d^3\right)+d\left(4c^3-26dc^2-184d^2c-109d^3\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{\left(4c^3-26c^2d-184cd^2-109d\right.}{3a^2(c-d)(a\sec(e+}}{5a(c-d)}}{c^2-d^2}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e}}$$

$$\downarrow 25$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\frac{\int\frac{a^2\left((c+d)\left(4c^3-30dc^2+146d^2c+195d^3\right)+d\left(4c^3-26dc^2-184d^2c-109d^3\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a^3(c-d)}-\frac{\left(4c^3-26c^2d-184cd^2-109d^3\right)\sqrt{}}{3a^2(c-d)(a\sec(e+fx)+}}{5a(c-d)}}{c^2-d^2}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+}}$$

$$\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\frac{\int\frac{(c+d)\left(4c^3-30dc^2+146d^2c+195d^3\right)+d\left(4c^3-26dc^2-184d^2c-109d^3\right)\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}(\sec(e+fx)a+a)^{3/2}(c+d\sec(e+fx))}d\sec(e+fx)}{3a(c-d)}-\frac{\left(4c^3-26c^2d-184cd^2-109d^3\right)\sqrt{a-a}}{3a^2(c-d)(a\sec(e+fx)+a)^{3}}}{5a(c-d)}}{c^2-d^2}}{2(c^2-d^2)}\right.}{f\sqrt{a-a\sec(e+}}$$

$$\Big\downarrow 169$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\frac{-\frac{\int\frac{15a^2d^3\left(20c^2+30dc+13d^2\right)}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a^3(c-d)}-\frac{\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\left(4c\right.}{}}{5a(c-d)}}{c^2-d^2}}{2(c^2}\right.}{f\sqrt{a-}}$$

$$\Big\downarrow 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{-\frac{15d^3\left(20c^2+30cd+13d^2\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}\sqrt{\sec(e+fx)a+a}(c+d\sec(e+fx))}d\sec(e+fx)}{a(c-d)}-\frac{\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right.}{a^2(c-d)\sqrt{a\sec(e+fx)}}}{3a(c-d)}}{5a(c-d)}}{c^2-d^2}\right.}{}$$

$$\Big\downarrow 104$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{-\frac{30d^3\left(20c^2+30cd+13d^2\right)\int\frac{1}{a(c-d)+\frac{a(c+d)(\sec(e+fx)a+a)}{a-a\sec(e+fx)}}d\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{a-a\sec(e+fx)}}}{a(c-d)}-\frac{\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)\sqrt{a-a\,\mathrm{se}}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}}{5a(c-d)}}{c^2-d^2}}{}\right.}{f}$$

$$\Big\downarrow \color{blue}{218}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{\frac{-\frac{30d^3\left(20c^2+30cd+13d^2\right)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2(c-d)^{3/2}\sqrt{c+d}}-\frac{\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)\sqrt{a-a\sec(e+fx)}}{a^2(c-d)\sqrt{a\sec(e+fx)+a}}}{3a(c-d)}-\frac{\left(4c^3-26c\right.}{3}}{5a(c-d)}}{c^2-d^2}}{2(c^2-d}\right.}{f\sqrt{a-a}}$$

input
```
Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]
```

output
```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]])/(2*a^2*(c^2 - d^2)*(a + a*Sec[e + f*x
])^(5/2)*(c + d*Sec[e + f*x])^2) + ((3*d*(2*c + d)*Sqrt[a - a*Sec[e + f*x]
])/(a^2*(c^2 - d^2)*(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])) + (-1
/5*((2*c^2 + 39*c*d + 22*d^2)*Sqrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*(a +
a*Sec[e + f*x])^(5/2)) + (-1/3*((4*c^3 - 26*c^2*d - 184*c*d^2 - 109*d^3)*S
qrt[a - a*Sec[e + f*x]])/(a^2*(c - d)*(a + a*Sec[e + f*x])^(3/2)) + ((-30*
d^3*(20*c^2 + 30*c*d + 13*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]
])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])])/(a^2*(c - d)^(3/2)*Sqrt[c + d]
) - ((4*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4)*Sqrt[a - a*Sec
[e + f*x]])/(a^2*(c - d)*Sqrt[a + a*Sec[e + f*x]]))/(3*a*(c - d)))/(5*a*(c
 - d)))/(c^2 - d^2))/(2*(c^2 - d^2)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e +
f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}c^{2}}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}d^{2}}{5}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{10cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\frac{8d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c^{2}\tan(}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)\left(c^{2}-2cd+d^{2}\right)}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}c^{2}}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}d^{2}}{5}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{10cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\frac{8d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c^{2}\tan(}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)\left(c^{2}-2cd+d^{2}\right)}$
risch	Expression too large to display

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/4/f/a^3*(1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c^2-2*c*d+d^2)*(1/5*tan(1/2*f*x+1/
2*e)^5*c^2-2/5*tan(1/2*f*x+1/2*e)^5*c*d+1/5*tan(1/2*f*x+1/2*e)^5*d^2-2/3*c
^2*tan(1/2*f*x+1/2*e)^3+10/3*c*d*tan(1/2*f*x+1/2*e)^3-8/3*d^2*tan(1/2*f*x+
1/2*e)^3+c^2*tan(1/2*f*x+1/2*e)-8*tan(1/2*f*x+1/2*e)*c*d+31*d^2*tan(1/2*f*
x+1/2*e))+16*d^3/(c-d)^5*((-1/4*d*(10*c^2-3*c*d-7*d^2)/(c^2+2*c*d+d^2)*tan
(1/2*f*x+1/2*e)^3+5/4*d*(2*c+d)/(c+d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1
/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-1/4*(20*c^2+30*c*d+13*d^2)/(c^2+2*c*
d+d^2)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^
(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(349) = 698$.

Time = 0.31 (sec) , antiderivative size = 2677, normalized size of antiderivative = 7.27

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="f
ricas")
```

output
```
[-1/60*(15*(20*c^2*d^5 + 30*c*d^6 + 13*d^7 + (20*c^4*d^3 + 30*c^3*d^4 + 13
*c^2*d^5)*cos(f*x + e)^5 + (60*c^4*d^3 + 130*c^3*d^4 + 99*c^2*d^5 + 26*c*d
^6)*cos(f*x + e)^4 + (60*c^4*d^3 + 210*c^3*d^4 + 239*c^2*d^5 + 108*c*d^6 +
 13*d^7)*cos(f*x + e)^3 + (20*c^4*d^3 + 150*c^3*d^4 + 253*c^2*d^5 + 168*c*
d^6 + 39*d^7)*cos(f*x + e)^2 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*
d^7)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)
*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*
c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(4*c^6*d^2
 - 30*c^5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*
d^8 + (14*c^8 - 60*c^7*d + 78*c^6*d^2 + 480*c^5*d^3 + 312*c^4*d^4 - 330*c^
3*d^5 - 419*c^2*d^6 - 90*c*d^7 + 15*d^8)*cos(f*x + e)^4 + (12*c^8 - 62*c^7
*d + 114*c^6*d^2 + 1056*c^5*d^3 + 1626*c^4*d^4 - 81*c^3*d^5 - 1707*c^2*d^6
 - 913*c*d^7 - 45*d^8)*cos(f*x + e)^3 + (4*c^8 - 6*c^7*d - 28*c^6*d^2 + 82
8*c^5*d^3 + 2400*c^4*d^4 + 1197*c^3*d^5 - 1897*c^2*d^6 - 2019*c*d^7 - 479*
d^8)*cos(f*x + e)^2 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 +
 1539*c^3*d^5 - 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e))*sin(f*x
+ e))/((a^3*c^11 - 3*a^3*c^10*d + 8*a^3*c^8*d^3 - 6*a^3*c^7*d^4 - 6*a^3*c^
6*d^5 + 8*a^3*c^5*d^6 - 3*a^3*c^3*d^8 + a^3*c^2*d^9)*f*cos(f*x + e)^5 + (3
*a^3*c^11 - 7*a^3*c^10*d - 6*a^3*c^9*d^2 + 24*a^3*c^8*d^3 - 2*a^3*c^7*d^4
- 30*a^3*c^6*d^5 + 12*a^3*c^5*d^6 + 16*a^3*c^4*d^7 - 9*a^3*c^3*d^8 - 3*...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{\int \frac{\sec(e+fx}{c^3\sec^3(e+fx)+3c^3\sec^2(e+fx)+3c^3\sec(e+fx)+c^3+3c^2d\sec^4(e+fx)+9c^2d\sec^3(e+fx)+9c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^5(e}}{a^3}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(sec(e + f*x)/(c**3*sec(e + f*x)**3 + 3*c**3*sec(e + f*x)**2 + 3*c
**3*sec(e + f*x) + c**3 + 3*c**2*d*sec(e + f*x)**4 + 9*c**2*d*sec(e + f*x)
**3 + 9*c**2*d*sec(e + f*x)**2 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e +
f*x)**5 + 9*c*d**2*sec(e + f*x)**4 + 9*c*d**2*sec(e + f*x)**3 + 3*c*d**2*s
ec(e + f*x)**2 + d**3*sec(e + f*x)**6 + 3*d**3*sec(e + f*x)**5 + 3*d**3*se
c(e + f*x)**4 + d**3*sec(e + f*x)**3), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. 2(349) = 698.

Time = 0.30 (sec) , antiderivative size = 1369, normalized size of antiderivative = 3.72

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="g
iac")
```

output
```
-1/60*(60*(20*c^2*d^3 + 30*c*d^4 + 13*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/
2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2
*e))/sqrt(-c^2 + d^2)))/((a^3*c^7 - 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4*
d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^3*d^7)*sqrt(-c^2 + d^2
)) - (3*a^12*c^12*tan(1/2*f*x + 1/2*e)^5 - 36*a^12*c^11*d*tan(1/2*f*x + 1/
2*e)^5 + 198*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^5 - 660*a^12*c^9*d^3*tan(1
/2*f*x + 1/2*e)^5 + 1485*a^12*c^8*d^4*tan(1/2*f*x + 1/2*e)^5 - 2376*a^12*c
^7*d^5*tan(1/2*f*x + 1/2*e)^5 + 2772*a^12*c^6*d^6*tan(1/2*f*x + 1/2*e)^5 -
 2376*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e)^5 + 1485*a^12*c^4*d^8*tan(1/2*f*x
+ 1/2*e)^5 - 660*a^12*c^3*d^9*tan(1/2*f*x + 1/2*e)^5 + 198*a^12*c^2*d^10*t
an(1/2*f*x + 1/2*e)^5 - 36*a^12*c*d^11*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^1
2*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 150*a^12*
c^11*d*tan(1/2*f*x + 1/2*e)^3 - 990*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^3 +
 3850*a^12*c^9*d^3*tan(1/2*f*x + 1/2*e)^3 - 9900*a^12*c^8*d^4*tan(1/2*f*x
+ 1/2*e)^3 + 17820*a^12*c^7*d^5*tan(1/2*f*x + 1/2*e)^3 - 23100*a^12*c^6*d^
6*tan(1/2*f*x + 1/2*e)^3 + 21780*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e)^3 - 148
50*a^12*c^4*d^8*tan(1/2*f*x + 1/2*e)^3 + 7150*a^12*c^3*d^9*tan(1/2*f*x + 1
/2*e)^3 - 2310*a^12*c^2*d^10*tan(1/2*f*x + 1/2*e)^3 + 450*a^12*c*d^11*tan(
1/2*f*x + 1/2*e)^3 - 40*a^12*d^12*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^12*ta
n(1/2*f*x + 1/2*e) - 270*a^12*c^11*d*tan(1/2*f*x + 1/2*e) + 2340*a^12*c...
```

Mupad [B] (verification not implemented)

Time = 11.24 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^3),x)
```

output

```
tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c - d)^3) - (tan(e/2 + (f*x)/2)*((3*(c + d
)^2)/(4*a^3*(c - d)^5) - 5/(2*a^3*(c - d)^3) + (3*(c + d)*(5/(4*a^3*(c - d
)^3) - (3*(c + d))/(4*a^3*(c - d)^4)))/(c - d)))/f - (tan(e/2 + (f*x)/2)^3
*(5/(12*a^3*(c - d)^3) - (c + d)/(4*a^3*(c - d)^4)))/f - ((tan(e/2 + (f*x)
/2)^3*(3*c*d^5 + 7*d^6 - 10*c^2*d^4))/(c + d)^2 + (5*tan(e/2 + (f*x)/2)*(2
*c*d^4 + d^5))/(c + d))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^3*c^7 + 2*a^3*d^7 -
10*a^3*c*d^6 - 10*a^3*c^6*d + 18*a^3*c^2*d^5 - 10*a^3*c^3*d^4 - 10*a^3*c^4
*d^3 + 18*a^3*c^5*d^2) - tan(e/2 + (f*x)/2)^4*(a^3*c^7 - a^3*d^7 + 7*a^3*c
*d^6 - 7*a^3*c^6*d - 21*a^3*c^2*d^5 + 35*a^3*c^3*d^4 - 35*a^3*c^4*d^3 + 21
*a^3*c^5*d^2) - a^3*c^7 + a^3*d^7 - 3*a^3*c*d^6 + 3*a^3*c^6*d + a^3*c^2*d^
5 + 5*a^3*c^3*d^4 - 5*a^3*c^4*d^3 - a^3*c^5*d^2)) + (d^3*atan((c^6*tan(e/2
 + (f*x)/2)*1i + d^6*tan(e/2 + (f*x)/2)*1i - c*d^5*tan(e/2 + (f*x)/2)*6i -
 c^5*d*tan(e/2 + (f*x)/2)*6i + c^2*d^4*tan(e/2 + (f*x)/2)*15i - c^3*d^3*ta
n(e/2 + (f*x)/2)*20i + c^4*d^2*tan(e/2 + (f*x)/2)*15i)/((c + d)^(1/2)*(c -
 d)^(11/2)))*(30*c*d + 20*c^2 + 13*d^2)*1i)/(a^3*f*(c + d)^(5/2)*(c - d)^(
11/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2231, normalized size of antiderivative = 6.06

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x)
```

output

```
( - 1200*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*
d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**4*c**4*d**3 + 600*sqrt( - c**2
+ d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2
))*tan((e + f*x)/2)**4*c**3*d**4 + 1620*sqrt( - c**2 + d**2)*atan((tan((e
+ f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**
4*c**2*d**5 - 240*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e +
 f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**4*c*d**6 - 780*sqrt( -
 c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2
+ d**2))*tan((e + f*x)/2)**4*d**7 + 2400*sqrt( - c**2 + d**2)*atan((tan((e
 + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)*
*2*c**4*d**3 + 3600*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e
 + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*c**3*d**4 - 840*sq
rt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( -
c**2 + d**2))*tan((e + f*x)/2)**2*c**2*d**5 - 3600*sqrt( - c**2 + d**2)*at
an((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e
+ f*x)/2)**2*c*d**6 - 1560*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
 tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*tan((e + f*x)/2)**2*d**7 - 1200
*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt(
 - c**2 + d**2))*c**4*d**3 - 4200*sqrt( - c**2 + d**2)*atan((tan((e + f*x)
/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*c**3*d**4 - 5580*sqrt...
```

3.234 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 61

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}$$

output
```
2*a^(1/2)*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*s
ec(f*x+e))^(1/2))/d^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{d+c\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{d}f\sqrt{c+d\sec(e+fx)}}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]]
,x]
```

output
```
(Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x
]]]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/
(Sqrt[d]*f*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3042, 4468, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a\sec(e+fx)+a}}{\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4468}$$

$$-\frac{2a\int \frac{1}{1-\frac{ad\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}$$

input
```
Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]
]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[d]*f)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4468
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*(b/f)   Subs
t[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
 + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(49) = 98$.

Time = 1.16 (sec) , antiderivative size = 311, normalized size of antiderivative = 5.10

method	result
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{-2\sqrt{2}\,\sqrt{-d}\,\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\,\sin(fx+e)+2c\cos(fx+e)-2\cos(fx+e)d+2\sin(fx+e)c+2\sin(fx+e)d-2c+2d}{\cos(fx+e)-1+\sin(fx+e)}\right)-\ln\left(\frac{2\sqrt{-\frac{2(d+}{\cos}}}{}\right.\right.}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/f*2^(1/2)/(-d)^(1/2)*(ln(2*(-2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-cos(f*x+e)*d+sin(f*x+e)*c+sin(
f*x+e)*d-c+d)/(cos(f*x+e)-1+sin(f*x+e)))-ln(2*((-2*(d+c*cos(f*x+e))/(cos(f
*x+e)+1))^(1/2)*(-d)^(1/2)*2^(1/2)*cos(f*x+e)+2^(1/2)*(-d)^(1/2)*(-2*(d+c*
cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-c*cos(f*x+e)-sin(f*x+e)*c-cos(f*x+e)*d+s
in(f*x+e)*d-c-d)/(sin(f*x+e)+cos(f*x+e)+1)))*(a*(1+sec(f*x+e)))^(1/2)*(c+d
*sec(f*x+e))^(1/2)*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x
+e)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(49) = 98$.

Time = 0.32 (sec) , antiderivative size = 307, normalized size of antiderivative = 5.03

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= \left[\frac{\sqrt{\frac{a}{d}}\log\left(-\frac{8\,acd\cos(fx+e)+(ac^2-6\,acd+ad^2)\cos(fx+e)^3+4\left(2\,d^2\cos(fx+e)+(cd-d^2)\cos(fx+e)^2\right)\sqrt{\frac{a}{d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e}{\cos(fx+}}}{\cos(fx+e)^3+\cos(fx+e)^2}\right.}{2\,f}\right.$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
[1/2*sqrt(a/d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(
f*x + e)^3 + 4*(2*d^2*cos(f*x + e) + (c*d - d^2)*cos(f*x + e)^2)*sqrt(a/d)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
 + e))*sin(f*x + e) + 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2
)/(cos(f*x + e)^3 + cos(f*x + e)^2))/f, sqrt(-a/d)*arctan(-2*d*sqrt(-a/d)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e)/((a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c
 + a*d)*cos(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)),
 x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c),
x)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),x
)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{\sec\left(fx+e\right)d+c}\,\sqrt{\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)d+c}dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x) + 1)*sec(e + f*x))
/(sec(e + f*x)*d + c),x)
```

3.235 $\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 140

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

output

```
2^(1/2)*(c-d)^(1/2)*arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a
*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/a^(1/2)/f+2*d^(1/2)*arctanh(a^(
1/2)*d^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/a^(
1/2)/f
```

Mathematica [A] (verified)

Time = 13.63 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$=\frac{\sqrt{c}\left(-\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+c\cos(e+fx)}}{\sqrt{c-d}\sqrt{c-c\cos(e+fx)}}\right)+2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+c\cos(e+fx)}}{\sqrt{d}\sqrt{c-c\cos(e+fx)}}\right)\right)\sqrt{c+d\sec(e+fx)}\sin(}{f\sqrt{c-c\cos(e+fx)}\sqrt{d+c\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

input

```
Integrate[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]]
,x]
```

output

```
(Sqrt[c]*(-(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + c*Cos[e +
 f*x]])/(Sqrt[c - d]*Sqrt[c - c*Cos[e + f*x]])]) + 2*Sqrt[d]*ArcTanh[(Sqrt
[c]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c - c*Cos[e + f*x]])])*Sqrt[c
+ d*Sec[e + f*x]]*Sin[e + f*x])/(f*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos
[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4469, 3042, 4468, 219, 4471, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4469}$$

$$(c-d)\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}dx+\frac{d\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d\sec(e+fx)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{ 4468}$$

$$(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2d\int \frac{1}{1-\frac{ad\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{ 219}$$

$$(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

$$\Big\downarrow \text{ 4471}$$

$$\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{2(c-d)\int \frac{1}{\frac{a(c-d)\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

input

```
Int[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

output

```
(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sq
rt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) + (2*Sqrt[d
]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 + d*Sec[e + f*x]])])/(Sqrt[a]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4468
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*(b/f)   Subs
t[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
 + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4469
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-(b*c - a*d)/d
  Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x],
 x] + Simp[b/d   Int[Csc[e + f*x]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

rule 4471
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*(a/(b*f))
Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e +
f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(113) = 226$.

Time = 1.08 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.92

method	result
default	$-\frac{\sqrt{2}\,\Bigg(d\ln\Bigg(-\frac{2\left(\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\,\sqrt{-d}\,\sqrt{2}\,\cos(fx+e)+\sqrt{2}\,\sqrt{-d}\,\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}-c\cos(fx+e)+\sin(fx+e)c-\cos(fx+e)d-\sin(fx+e}{-\sin(fx+e)+\cos(fx+e)+1}}{}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/f/a/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)*(d*ln(-2*((-2*(d+c*cos(f*x+e))/(cos(
f*x+e)+1))^(1/2)*(-d)^(1/2)*2^(1/2)*cos(f*x+e)+2^(1/2)*(-d)^(1/2)*(-2*(d+c
*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-c*cos(f*x+e)+sin(f*x+e)*c-cos(f*x+e)*d-
sin(f*x+e)*d-c-d)/(-sin(f*x+e)+cos(f*x+e)+1))*(c-d)^(1/2)-d*ln(2*((-2*(d+c
*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(-d)^(1/2)*2^(1/2)*cos(f*x+e)+2^(1/2)*(
-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-c*cos(f*x+e)-sin(f*x+
e)*c-cos(f*x+e)*d+sin(f*x+e)*d-c-d)/(sin(f*x+e)+cos(f*x+e)+1))*(c-d)^(1/2)
+ln(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c-d)^(1/2)-
c*cot(f*x+e)+d*cot(f*x+e)+c*csc(f*x+e)-d*csc(f*x+e)))*2^(1/2)*(-d)^(1/2)*c
-ln(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c-d)^(1/2)-
c*cot(f*x+e)+d*cot(f*x+e)+c*csc(f*x+e)-d*csc(f*x+e)))*2^(1/2)*(-d)^(1/2)*d
)*(a*(1+sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))^(1/2)*cos(f*x+e)/(cos(f*x+e)+1
)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1048, normalized size of antiderivative = 7.49

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output

```
[1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(-(2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d)*cos(f*x + e)
+ c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(d/a)*log(-((c^2 -
 6*c*d + d^2)*cos(f*x + e)^3 + 4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e
))*sqrt(d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
+ d)/cos(f*x + e))*sin(f*x + e) + 8*c*d*cos(f*x + e) + (c^2 + 2*c*d - 7*d^
2)*cos(f*x + e)^2 + 8*d^2)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/f, 1/2*(2*s
qrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/
((c - d)*sin(f*x + e))) + sqrt(d/a)*log(-((c^2 - 6*c*d + d^2)*cos(f*x + e)
^3 + 4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(d/a)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x
 + e) + 8*c*d*cos(f*x + e) + (c^2 + 2*c*d - 7*d^2)*cos(f*x + e)^2 + 8*d^2)
/(cos(f*x + e)^3 + cos(f*x + e)^2)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(
-(2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*c
os(f*x + e)^2 - 2*(c + d)*cos(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(
f*x + e) + 1)) + 2*sqrt(-d/a)*arctan(-2*sqrt(-d/a)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*s...
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{c+d\sec{(e+fx)}}\sec{(e+fx)}}{\sqrt{a\left(\sec{(e+fx)}+1\right)}}\,dx$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output

```
Integral(sqrt(c + d*sec(e + f*x))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1)),
 x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a),
x)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x
)
```

output
```
int((c + d/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)d+c}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x) + 1)*sec(e + f*x)
)/(sec(e + f*x) + 1),x))/a
```

3.236 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 78

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

output
```
2^(1/2)*arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))
^(1/2)/(c+d*sec(f*x+e))^(1/2))/a^(1/2)/(c-d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.37

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e+fx)}\sec(e+fx)}{\sqrt{c-d}f\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}}$$

input

```
Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
,x]
```

output

```
(2*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]]*Cos[(e
+ f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])/(Sqrt[c - d]*f*Sqrt[a*(1
+ Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3042, 4471, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4471}$$

$$-\frac{2\int \frac{1}{\frac{a(c-d)\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

input

```
Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec
[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4471
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*(a/(b*f))
Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e +
f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(63) = 126$.

Time = 0.74 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.73

method	result
default	$\frac{\sqrt{a(1+\sec(fx+e))}\left((-\cos(fx+e)+1)^2\csc(fx+e)^2-1\right)\sqrt{c+d\sec(fx+e)}\,\ln\left(\sqrt{c-d}\,(-\cot(fx+e)+\csc(fx+e))+\sqrt{-\frac{2(d+c\cos(fx+}{\cos(fx+e)+}}\right.}{fa\sqrt{c-d}\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}}$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/f/a/(c-d)^(1/2)*(a*(1+sec(f*x+e)))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2
-1)*(c+d*sec(f*x+e))^(1/2)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*ln((
c-d)^(1/2)*(-cot(f*x+e)+csc(f*x+e))+(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.15

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= \left[\frac{\sqrt{2}\sqrt{-\frac{1}{ac-ad}}\log\left(-\frac{2\sqrt{2}(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\sqrt{-\frac{1}{ac-ad}}\cos(fx+e)\sin(fx+e)-(3c-d)\cos(fx+e)^2-2(c+d)\cos(f}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{2f}\right.$$

$$\left.-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{ac-ad}\sin(fx+e)}\right)}{\sqrt{ac-ad}f}\right]$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/(a*c - a*d))*log(-(2*sqrt(2)*(c - d)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/(a
*c - a*d))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d
)*cos(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))/f, -sqrt(
2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
+ e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/(sqrt
(a*c - a*d)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x))
), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \int \frac{\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c))
, x)
```

Giac [F]

$$\begin{aligned}\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx\\ =\int \frac{\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx\end{aligned}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned}\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx\\ =\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx\end{aligned}$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)d+c}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2d+\sec(fx+e)c+\sec(fx+e)d+c}dx\right)}{a}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x) + 1)*sec(e + f*x)
)/(sec(e + f*x)**2*d + sec(e + f*x)*c + sec(e + f*x)*d + c),x))/a
```

3.237 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 141

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}+\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}f}$$

output
```
-2^(1/2)*arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e)
)^(1/2)/(c+d*sec(f*x+e))^(1/2))/a^(1/2)/(c-d)^(1/2)/f+2*arctanh(a^(1/2)*d^
(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/a^(1/2)/d^
(1/2)/f
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\left(-\sqrt{d}\arctan\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)+\sqrt{2}\sqrt{c-d}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e}}{\sqrt{c-d}\sqrt{d}f\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]
]),x]
```

output
```
(2*(-(Sqrt[d]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x
]]]) + Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt
[d + c*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f
*x])/(Sqrt[c - d]*Sqrt[d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e +
f*x]])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3042, 4473, 3042, 4468, 219, 4471, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4473}$$

$$\frac{\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d\sec(e+fx)}}dx}{a}-\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4468}$$

$$-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2\int \frac{1}{1-\frac{ad\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}f}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4471}$$

$$\frac{2\int \frac{1}{\frac{a(c-d)\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}f}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

input
```
Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
ec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)) + (2*Arc
Tanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*S
ec[e + f*x]])])/(Sqrt[a]*Sqrt[d]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4468
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*(b/f)   Subs
t[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
 + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4471
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*(a/(b*f))
Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e +
f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4473
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-a/b   Int[Cs
c[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Si
mp[1/b   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(114) = 228$.

Time = 1.12 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.84

method	result
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{-2\sqrt{2}\,\sqrt{-d}\,\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\,\sin(fx+e)+2c\cos(fx+e)-2\cos(fx+e)d+2\sin(fx+e)c+2\sin(fx+e)d-2c+2d}{\cos(fx+e)-1+\sin(fx+e)}\right)\sqrt{c-d}-\ln\right.}{}$

input

```
int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output

```
-1/f*2^(1/2)/a/(c-d)^(1/2)/(-d)^(1/2)*(ln(2*(-2^(1/2)*(-d)^(1/2)*(-2*(d+c*
cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-cos(f*x+e)*d+sin
(f*x+e)*c+sin(f*x+e)*d-c+d)/(cos(f*x+e)-1+sin(f*x+e)))*(c-d)^(1/2)-ln(2*((
-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(-d)^(1/2)*2^(1/2)*cos(f*x+e)+2^
(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-c*cos(f*x+e)-s
in(f*x+e)*c-cos(f*x+e)*d+sin(f*x+e)*d-c-d)/(sin(f*x+e)+cos(f*x+e)+1))*(c-d
)^(1/2)-ln((c-d)^(1/2)*csc(f*x+e)-(c-d)^(1/2)*cot(f*x+e)+(-2*(d+c*cos(f*x+
e))/(cos(f*x+e)+1))^(1/2))*2^(1/2)*(-d)^(1/2))*(a*(1+sec(f*x+e)))^(1/2)*(c
+d*sec(f*x+e))^(1/2)*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f
*x+e)+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 1100, normalized size of antiderivative = 7.80

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output

```
[1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-
1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c
 + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqr
t(a*d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)
^3 + 4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x
 + e) + 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e
)^3 + cos(f*x + e)^2)))/(a*d*f), 1/2*(2*sqrt(2)*a*d*arctan(sqrt(2)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*c
os(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(a*c - a*d) + sqrt(a*d)*lo
g(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)^3 + 4*((
c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) + 8
*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e)^3 + cos
(f*x + e)^2)))/(a*d*f), 1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(
2)*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) +
d)/cos(f*x + e))*sqrt(-1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d
)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*c
os(f*x + e) + 1)) + 2*sqrt(-a*d)*arctan(-2*sqrt(-a*d)*sqrt((a*cos(f*x +...
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt{a\left(\sec(e+fx)+1\right)}\sqrt{c+d\sec(e+fx)}}\,dx$$

input

```
integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output

```
Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*
x))), x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

input

```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output

```
integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c
)), x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

input

```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, al
gorithm="giac")
```

output

```
integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2
)),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)d+c}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2d+\sec(fx+e)c+\sec(fx+e)d+c}dx\right)}{a}$$

input
```
int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x) + 1)*sec(e + f*x)
**2)/(sec(e + f*x)**2*d + sec(e + f*x)*c + sec(e + f*x)*d + c),x))/a
```

3.238 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 61

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{d}\sqrt{c+d}f}$$

output

```
2*a^(1/2)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(
1/2))/d^(1/2)/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$
$$=\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)\sqrt{\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{d}\sqrt{c+d}f}$$

input

```
Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e
 + f*x]])]*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])
/(Sqrt[d]*Sqrt[c + d]*f)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 4455, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a\sec(e+fx)+a}}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4455}$$

$$-\frac{2a\int \frac{1}{\frac{a^2d\tan^2(e+fx)}{\sec(e+fx)a+a}+a(c+d)}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c+d}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*S
ec[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(47) = 94$.

Time = 7.51 (sec) , antiderivative size = 424, normalized size of antiderivative = 6.95

method	result
default	$-\frac{\sqrt{2}\,\Big(\ln\Big(\frac{2\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,\sqrt{(-\cos(fx+e)+1)^2\csc(fx+e)^2-1}\,c-2\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,\sqrt{(-\cos(fx+e)+1)^2\csc(fx+e)^2-1}\,d-2\sqrt{(c-d)(c+d)}\,(-\cot(fx+e)+}{c(-\cot(fx+e)+\csc(fx+e))-d(-\cot(fx+e)+\csc(fx+e))+\sqrt{(c-d)(c+d)}}}{}$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
-1/2/f*2^(1/2)/((c-d)*(c+d))^(1/2)/(d/(c-d))^(1/2)*(ln(2*(2^(1/2)*(d/(c-d)
)^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)
*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*d-((c-d)*(c+d))^(1/2)*(-cot(f*x+
e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-d*(-cot(f*x+e)+csc(f*x+e))
+((c-d)*(c+d))^(1/2)))-ln(-2*(2^(1/2)*(d/(c-d))^(1/2)*((-cos(f*x+e)+1)^2*c
sc(f*x+e)^2-1)^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+
e)^2-1)^(1/2)*d+((c-d)*(c+d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-co
t(f*x+e)+csc(f*x+e))+d*(-cot(f*x+e)+csc(f*x+e))+((c-d)*(c+d))^(1/2))))*((-
cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*(-2*a/((-cos(f*x+e)+1)^2*csc(f*x+e)^
2-1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.

Time = 0.34 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.62

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \left[\frac{\sqrt{-\frac{a}{cd+d^2}}\log\left(-\frac{(ac^2+8\,acd+8\,ad^2)\cos(fx+e)^3+ad^2+(ac^2+2\,acd)\cos(fx+e)^2-4\left((c^2d+3\,cd^2+2\,d^3)\cos(fx+e)^2-(cd^2+d^3)\cos(f\right.}{c^2\cos(fx+e)^3+(c^2+2\,cd)\cos(fx+e)^2+d^2+(2\,cd+d^2)\,c}\right.}{2\,f}\right.$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
[1/2*sqrt(-a/(c*d + d^2))*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3
 + a*d^2 + (a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*((c^2*d + 3*c*d^2 + 2*d^3)
*cos(f*x + e)^2 - (c*d^2 + d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*c*d + 7*a*d^2)*cos(f*
x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d
+ d^2)*cos(f*x + e)))/f, sqrt(a/(c*d + d^2))*arctan(2*(c*d + d^2)*sqrt(a/(
c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
 e)/((a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(47) = 94.

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$

$$=\frac{2\sqrt{-a}\arctan\Bigg(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2d+ac+}{4\sqrt{-cd-d^2}a}}{\sqrt{-cd-d^2}f}$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
2*sqrt(-a)*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))*sgn(
cos(f*x + e))/(sqrt(-c*d - d^2)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\sec\left(fx+e\right)+1}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)d+c}dx\right)$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)*d + c),x)
```

3.239 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 149

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\frac{2\sqrt{a}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df}-\frac{2\sqrt{a}\sqrt{c}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{d\sqrt{c+d}f}$$

output
```
2*a^(1/2)*g^(3/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/
(a+a*sec(f*x+e))^(1/2))/d/f-2*a^(1/2)*c^(1/2)*g^(3/2)*arctanh(a^(1/2)*c^(1
/2)*g^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(
1/2))/d/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.26

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=$$
$$-\frac{g^2\left(\sqrt{c+d}\log\left(\sqrt{2}-2\sin\left(\frac{1}{2}(e+fx)\right)\right)-\sqrt{c+d}\log\left(\sqrt{2}+2\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{c}\left(-\log\left(\sqrt{2}\sqrt{c+d}\right.\right.\right.}{\sqrt{2}d\sqrt{c+d}f}$$

input

```
Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e +
 f*x]),x]
```

output

```
-((g^2*(Sqrt[c + d]*Log[Sqrt[2] - 2*Sin[(e + f*x)/2]] - Sqrt[c + d]*Log[Sq
rt[2] + 2*Sin[(e + f*x)/2]] + Sqrt[c]*(-Log[Sqrt[2]*Sqrt[c + d] - 2*Sqrt[c
]*Sin[(e + f*x)/2]] + Log[Sqrt[2]*Sqrt[c + d] + 2*Sqrt[c]*Sin[(e + f*x)/2]
]))*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(Sqrt[2]*d*Sqrt[c + d]*f*
Sqrt[g*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 4458, 3042, 4289, 221, 4453, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}(g\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4458$$

$$\frac{g\int \sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}dx}{d}-\frac{cg\int \frac{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{g\int \sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{d}-\frac{cg\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{d}$$

$$\Big\downarrow \; 4289$$

$$-\frac{2ag^2\int \frac{1}{a-\frac{a^2\sin(e+fx)\tan(e+fx)}{\sec(e+fx)a+a}}d\left(-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}\right)}{df}-\frac{cg\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{2\sqrt{a}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df}-\frac{cg\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{d}$$

$$\Big\downarrow \ 4453$$

$$\frac{2acg^2\int \frac{1}{a(c+d)-\frac{a^2c\sin(e+fx)\tan(e+fx)}{\sec(e+fx)a+a}}d\left(-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}\right)}{df}+\frac{2\sqrt{a}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df}$$

$$\Big\downarrow \ 221$$

$$\frac{2\sqrt{a}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df}-\frac{2\sqrt{a}\sqrt{c}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df\sqrt{c+d}}$$

input

```
Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x])
,x]
```

output

```
(2*Sqrt[a]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e +
f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(d*f) - (2*Sqrt[a]*Sqrt[c]*g^(3/2)*ArcTa
nh[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]
]*Sqrt[a + a*Sec[e + f*x]])])/(d*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4289
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/f)   Subst[Int[1/(b - d*x^2), x], x, b*(
Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] &&  !GtQ[a*(d/b), 0]
```

rule 4453
```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*b*(g
/f)   Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, b*(Cot[e + f*x]/(Sqrt[g*Csc
[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4458
```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[g/d
 Int[Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[c*(g/d)
 Int[Sqrt[g*Csc[e + f*x]]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])),
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(117) = 234$.

Time = 1.91 (sec) , antiderivative size = 841, normalized size of antiderivative = 5.64

method	result	size
default	Expression too large to display	841

input
```
int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
-2*g/f/((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)/((2*2^(1/2)*(c*(c-d
))^(1/2)+3*c-d)*(c+d))^(1/2)/(c*(c-d))^(1/2)/d*(g/(2*cos(1/2*f*x+1/2*e)^2-
1))^(1/2)*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(2^(1/
2)*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)*(c*(c-d))^(1/2)*arctan(
(c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)-
3*c+d)*(c+d))^(1/2))*c-2^(1/2)*(c*(c-d))^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)
-3*c+d)*(c+d))^(1/2)*arctanh((c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))
/((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2))*c-((2*2^(1/2)*(c*(c-d))^
(1/2)+3*c-d)*(c+d))^(1/2)*arctan((c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2
*e))/((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2))*c^2+((2*2^(1/2)*(c*(
c-d))^(1/2)+3*c-d)*(c+d))^(1/2)*arctan((c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f
*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2))*c*d-((2*2^(1/2
)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)*arctanh((c+d)*(cot(1/2*f*x+1/2*e)-cs
c(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2))*c^2+((2
*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)*arctanh((c+d)*(cot(1/2*f*x+1/
2*e)-csc(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2))*
c*d+arctanh(1/2*2^(1/2)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e)-1))*(c*(c-d
))^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)*((2*2^(1/2)*(c*(c
-d))^(1/2)-3*c+d)*(c+d))^(1/2)+arctanh(1/2*2^(1/2)*(cot(1/2*f*x+1/2*e)-csc
(1/2*f*x+1/2*e)+1))*(c*(c-d))^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*...
```

Fricas [A] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 1108, normalized size of antiderivative = 7.44

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="fricas")
```

output

```
[1/2*(sqrt(a*c*g/(c + d))*g*log((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c
*d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^
2)*cos(f*x + e))*sqrt(a*c*g/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*a*c*d + a*d^2)*g*cos(f*x + e) +
(8*a*c^2 + 8*a*c*d + a*d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x
 + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + sqrt(a*g)*g*log((a*g*cos(f*
x + e)^3 - 7*a*g*cos(f*x + e)^2 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x
+ e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x
 + e) + 8*a*g)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/(d*f), -1/2*(2*sqrt(-a*
c*g/(c + d))*g*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt
(-a*c*g/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x +
e))/(a*c*g*sin(f*x + e))) - sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*co
s(f*x + e)^2 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos
(f*x + e)^3 + cos(f*x + e)^2)))/(d*f), 1/2*(2*sqrt(-a*g)*g*arctan(1/2*sqrt
(-a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(g/cos(f*x + e))/(a*g*sin(f*x + e))) + sqrt(a*c*g/(c + d))*g*lo
g((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c*d)*g*cos(f*x + e)^2 + 4*((c^2
 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(a*c*g/(c
 + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin...
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{c+d\sec\left(e+fx\right)}\,dx$$

input

```
integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x
)
```

output

```
Integral(sqrt(a*(sec(e + f*x) + 1))*(g*sec(e + f*x))**(3/2)/(c + d*sec(e +
 f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found %i
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="giac")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e)
+ c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,{\left(\frac{g}{\cos(e+f\,x)}\right)}^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x
)),x)
```

output
```
int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\sqrt{g}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)\,d+c}$$

input
```
int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
sqrt(g)*sqrt(a)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x) + 1)*sec(e + f*x
))/(sec(e + f*x)*d + c),x)*g
```

3.240 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 122

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}$$

output

```
2^(1/2)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1
/2)/(c-d)/f-2*d^(1/2)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*s
ec(f*x+e))^(1/2))/a^(1/2)/(c-d)/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\left(\sqrt{-c-d}\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{2}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{-c-d}\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right)}{\sqrt{-c-d}(c-d)f\sqrt{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
(2*(Sqrt[-c - d]*ArcSin[Tan[(e + f*x)/2]] + Sqrt[2]*Sqrt[d]*ArcTanh[(Sqrt[
d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])]
))/(Sqrt[-c - d]*(c - d)*f*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sqrt[a*(1
 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 4460, 3042, 4282, 216, 4455, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4460}$$

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{c-d}-\frac{d\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{a(c-d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{c-d}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a(c-d)}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{2\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f(c-d)}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a(c-d)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a(c-d)}$$

$$\Big\downarrow \text{4455}$$

$$\frac{2d\int \frac{1}{\frac{a^2d\tan^2(e+fx)}{\sec(e+fx)a+a}+a(c+d)}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f(c-d)}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}$$

input
```
Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/(Sqrt[a]*(c - d)*f) - (2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(
Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

rule 4460
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[b/(b*c - a*d)   Int
[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[d/(b*c - a*d)   Int[C
sc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[
c^2 - d^2, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(99) = 198$.

Time = 8.51 (sec) , antiderivative size = 505, normalized size of antiderivative = 4.14

method	result
default	$\left(2\sqrt{(c-d)(c+d)}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{(-\cos(fx+e)+1)^{2}\csc(fx+e)^{2}-1}\right)\sqrt{\frac{d}{c-d}}+d\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{d}{c-d}}\sqrt{(-\cos(fx+e)+1)}}{}\right.\right.$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output

```
1/2/f/(d/(c-d))^(1/2)/(c-d)/((c-d)*(c+d))^(1/2)/a*(2*((c-d)*(c+d))^(1/2)*l
n(csc(f*x+e)-cot(f*x+e)+((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2))*(d/(c-d)
)^(1/2)+d*2^(1/2)*ln(2*(2^(1/2)*(d/(c-d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x
+e)^2-1)^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1
)^(1/2)*d-((c-d)*(c+d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e
)+csc(f*x+e))-d*(-cot(f*x+e)+csc(f*x+e))+((c-d)*(c+d))^(1/2)))-d*2^(1/2)*l
n(-2*(2^(1/2)*(d/(c-d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*c-2
^(1/2)*(d/(c-d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*d+((c-d)*(
c+d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+d*(
-cot(f*x+e)+csc(f*x+e))+((c-d)*(c+d))^(1/2))))*((-cos(f*x+e)+1)^2*csc(f*x+
e)^2-1)^(1/2)*(-2*a/((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 963, normalized size of antiderivative = 7.89

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"fricas")
```

output

```
[-1/2*(sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
 + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(-d/(a*c + a*d))*l
og(-((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 -
 4*((c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d^2)*cos(f*x + e))*sqrt(
-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + d^2
 - (6*c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f
*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), -1/2*(sqrt(2)
*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1
/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(co
s(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(d/(a*c + a*d))*arctan(2*(c +
d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)*sin(f*x + e)/((c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d))/((c
 - d)*f), -1/2*(sqrt(-d/(a*c + a*d))*log(-((c^2 + 8*c*d + 8*d^2)*cos(f*x +
 e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 - 4*((c^2 + 3*c*d + 2*d^2)*cos(f*x +
e)^2 - (c*d + d^2)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e))/(c^
2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(
f*x + e))) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((c - d)*f), -(sqrt(d...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec{\left(e+fx \right)}}{\sqrt{a\left(\sec{\left(e+fx \right)}+1\right)}\left(c+d\sec{\left(e+fx \right)}\right)}\,dx$$

input

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output

```
Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x
)
```

Maxima [F]

$$\begin{aligned}&\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx\\&=\int \frac{\sec(fx+e)}{\sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx\end{aligned}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^{2}d+\sec(fx+e)c+\sec(fx+e)d+c}dx\right)}{a}$$

input
```
int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
(sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2*d + se
c(e + f*x)*c + sec(e + f*x)*d + c),x))/a
```

3.241 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 124

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f}+\frac{2c\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{d}\sqrt{c+d}f}$$

output
```
-2^(1/2)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(
1/2)/(c-d)/f+2*c*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*
x+e))^(1/2))/a^(1/2)/(c-d)/d^(1/2)/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{2\left(\sqrt{d}\sqrt{c+d}\arctan\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)-\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)}{(c-d)\sqrt{d}\sqrt{c+d}f\sqrt{\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x
]
```

output
```
(-2*(Sqrt[d]*Sqrt[c + d]*ArcTan[Sin[(e + f*x)/2]/Sqrt[Cos[e + f*x]]] - Sqr
t[2]*c*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e +
 f*x]])])*Cos[(e + f*x)/2])/((c - d)*Sqrt[d]*Sqrt[c + d]*f*Sqrt[Cos[e + f*
x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4464, 3042, 4282, 216, 4455, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4464}$$

$$\frac{c\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{a(c-d)}-\frac{\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{c-d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a(c-d)}-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{c-d}$$

$$\Big\downarrow \text{4282}$$

$$\frac{2\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f(c-d)}+\frac{c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a(c-d)}$$

$$\Big\downarrow \text{216}$$

$$\frac{c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a(c-d)}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

$$\Big\downarrow \text{4455}$$

$$-\frac{2c\int \frac{1}{\frac{a^2d\tan^2(e+fx)}{\sec(e+fx)a+a}+a(c+d)}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f(c-d)}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2c\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

input
```
Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]
)]])/(Sqrt[a]*(c - d)*f)) + (2*c*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[d]*Sqrt[c + d]*
f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

rule 4464
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-a/(b*c - a*d)
Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[c/(b*c - a*d)   In
t[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || E
qQ[c^2 - d^2, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(101) = 202$.

Time = 3.70 (sec) , antiderivative size = 505, normalized size of antiderivative = 4.07

method	result
default	$\frac{\sqrt{-\frac{2a}{(-\cos(fx+e)+1)^2\csc(fx+e)^2-1}}\,\sqrt{(-\cos(fx+e)+1)^2\csc(fx+e)^2-1}\,\left(c\sqrt{2}\,\ln\left(-\frac{2\left(\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,\sqrt{(-\cos(fx+e)+1)^2\csc(fx+e)^2-1}\,c}{-c(-\cot(fx}\right.\right.}{}$

input
```
int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output

```
1/2/f/a/((c-d)*(c+d))^(1/2)/(c-d)/(d/(c-d))^(1/2)*(-2*a/((-cos(f*x+e)+1)^2
*csc(f*x+e)^2-1))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*(c*2^(1/2
)*ln(-2*(2^(1/2)*(d/(c-d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*
c-2^(1/2)*(d/(c-d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*d+((c-d
)*(c+d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+
d*(-cot(f*x+e)+csc(f*x+e))+((c-d)*(c+d))^(1/2)))-c*2^(1/2)*ln(2*(2^(1/2)*(
d/(c-d))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*c-2^(1/2)*(d/(c-d)
)^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2)*d-((c-d)*(c+d))^(1/2)*(-c
ot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-d*(-cot(f*x+e)+csc(
f*x+e))+((c-d)*(c+d))^(1/2)))-2*((c-d)*(c+d))^(1/2)*ln(csc(f*x+e)-cot(f*x+
e)+((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)^(1/2))*(d/(c-d))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 1041, normalized size of antiderivative = 8.40

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorith
m="fricas")
```

output

```
[-1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x +
 e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(
-a*c*d - a*d^2)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2
 + (a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*co
s(f*x + e)^2 - d*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin
(f*x + e) - (6*a*c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 +
 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((a*c^2*d - a
*d^3)*f), -1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*
cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)
) - 2*sqrt(a*c*d + a*d^2)*c*arctan(2*sqrt(a*c*d + a*d^2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a*c + 2*a*d)*cos(f*x +
e)^2 - a*d + (a*c + a*d)*cos(f*x + e))))/((a*c^2*d - a*d^3)*f), 1/2*(sqrt(
-a*c*d - a*d^2)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2
 + (a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*co
s(f*x + e)^2 - d*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin
(f*x + e) - (6*a*c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 +
 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 2*sqrt(2)*(a
*c*d + a*d^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co...
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec^2{(e+fx)}}{\sqrt{a\left(\sec{(e+fx)}+1\right)}\left(c+d\sec{(e+fx)}\right)}\,dx$$

input

```
integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output

```
Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x)))
, x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{a\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)),
x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2d+\sec(fx+e)c+\sec(fx+e)d+c}dx\right)}{a}$$

input
```
int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
(sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**2*d +
 sec(e + f*x)*c + sec(e + f*x)*d + c),x))/a
```

3.242 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 167

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{\sqrt{2}g^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2\sqrt{c}g^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}$$

output

```
-2^(1/2)*g^(3/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2^(1/2)/(g*sec(f*x
+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/(c-d)/f+2*c^(1/2)*g^(3/2)*arcta
nh(a^(1/2)*c^(1/2)*g^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(g*sec(f*x+e))^(1/2)/(a+
a*sec(f*x+e))^(1/2))/a^(1/2)/(c-d)/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.19

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{g\cos\left(\frac{1}{2}(e+fx)\right)\left(2\sqrt{c+d}\log\left(\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e$$

input
```
Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e +
f*x])),x]
```

output
```
(g*Cos[(e + f*x)/2]*(2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] - Sin[(e + f*x)/4]
] - 2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] + Sin[(e + f*x)/4]] + Sqrt[2]*Sqrt[
c]*(-Log[Sqrt[2]*Sqrt[c + d] - 2*Sqrt[c]*Sin[(e + f*x)/2]] + Log[Sqrt[2]*S
qrt[c + d] + 2*Sqrt[c]*Sin[(e + f*x)/2]]))*Sqrt[g*Sec[e + f*x]])/((c - d)*
Sqrt[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 4462, 3042, 4295, 221, 4453, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4462}$$

$$\frac{cg\int \frac{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{a(c-d)}-\frac{g\int \frac{\sqrt{g\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{c-d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{cg\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a(c-d)}-\frac{g\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{c-d}$$

$$\Big\downarrow \text{4295}$$

$$\frac{2g^2\int \frac{1}{2a-\frac{a^2\sin(e+fx)\tan(e+fx)}{\sec(e+fx)a+a}}d\left(-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}\right)}{f(c-d)}+\frac{cg\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a(c-d)}$$

$$\Big\downarrow \text{221}$$

$$\frac{cg\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a(c-d)}-\frac{\sqrt{2}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)}$$

$$\Big\downarrow \text{4453}$$

$$-\frac{2cg^2\int \frac{1}{a(c+d)-\frac{a^2c\sin(e+fx)\tan(e+fx)}{\sec(e+fx)a+a}}d\left(-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}\right)}{f(c-d)}-\frac{\sqrt{2}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{c}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)}$$

input

```
Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
,x]
```

output

```
-((Sqrt[2]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[c
]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt
[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*
f)
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4453

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*b*(g
/f)   Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, b*(Cot[e + f*x]/(Sqrt[g*Csc
[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4462

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-a)*
(g/(b*c - a*d))   Int[Sqrt[g*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x]
 + Simp[c*(g/(b*c - a*d))   Int[Sqrt[g*Csc[e + f*x]]*(Sqrt[a + b*Csc[e + f*
x]]/(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(132) = 264$.

Time = 1.78 (sec) , antiderivative size = 832, normalized size of antiderivative = 4.98

method	result	size
default	Expression too large to display	832

input

```
int((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output

```
-2^(1/2)*g/f/((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)/((2*2^(1/2)*(
c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)/(c*(c-d))^(1/2)/(c-d)*(2^(1/2)*((2*2^(1
/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)*arctanh((c+d)*(cot(1/2*f*x+1/2*e)-
csc(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2))*c^2-2
^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)*arctanh((c+d)*(cot(
1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d
))^(1/2))*c*d+2^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)*arct
an((c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/
2)-3*c+d)*(c+d))^(1/2))*c^2-2^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+
d))^(1/2)*arctan((c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))/((2*2^(1/2)
*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2))*c*d-ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f
*x+1/2*e)-1)*(c*(c-d))^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/
2)*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)+ln(-cot(1/2*f*x+1/2*e)+
csc(1/2*f*x+1/2*e)+1)*(c*(c-d))^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(
c+d))^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)+2*((2*2^(1/2)*
(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)*arctanh((c+d)*(cot(1/2*f*x+1/2*e)-csc(
1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2))*(c*(c-d))
^(1/2)*c-2*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)*arctan((c+d)*(c
ot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(
c+d))^(1/2))*(c*(c-d))^(1/2)*c)*(g/(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*co...
```

Fricas [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 1103, normalized size of antiderivative = 6.60

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input

```
integrate((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="fricas")
```

output

```
[-1/2*(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(f
*x + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
 + sqrt(c*g/(a*c + a*d))*g*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d)*g*c
os(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(
f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2 +
8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 +
 (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), 1/2*(2*sqrt(2)*g*sqrt(-g/a)*ar
ctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos
(f*x + e))*cos(f*x + e)/(g*sin(f*x + e))) - sqrt(c*g/(a*c + a*d))*g*log((c
^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*co
s(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(c*g/(a*c + a*d))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) +
 (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2 + 8*c*d + d^2)*g)/(c^2*cos(f*x + e)
^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((
c - d)*f), -1/2*(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
 - g*cos(f*x + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x
+ e) + 1)) - 2*sqrt(-c*g/(a*c + a*d))*g*arctan(1/2*(c*cos(f*x + e)^2 - ...
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{a\,(\sec{(e+fx)}+1)}\,(c+d\sec{(e+fx)})}\,dx$$

input

```
integrate((g*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x
)
```

output

```
Integral((g*sec(e + f*x))**(3/2)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e
+ f*x))), x)
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{(g\sec{(fx+e)})^{\frac{3}{2}}}{\sqrt{a\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)}\,dx$$

input

```
integrate((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="maxima")
```

output
```
1/2*(sqrt(2)*c*f*g*integrate(((c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2*f*x + 2*
e)^2 - 2*(c*d - 2*d^2)*cos(f*x + e)^2 - (c^2 - 4*c*d)*sin(2*f*x + 2*e)*sin
(f*x + e) - 2*(c*d - 2*d^2)*sin(f*x + e)^2 + (c^2 - (c^2 - 4*c*d)*cos(f*x
+ e))*cos(2*f*x + 2*e) - (c^2 - 2*c*d)*cos(f*x + e))*cos(1/2*arctan2(sin(f
*x + e), cos(f*x + e))) - (c^2*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2*cos(f*
x + e) + c^2)*sin(2*f*x + 2*e) + (c^2 - 2*c*d)*sin(f*x + e))*sin(1/2*arcta
n2(sin(f*x + e), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x
+ e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*
d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^
2)*cos(2*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + (c^2
*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*
d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e
) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arctan2(s
in(f*x + e), cos(f*x + e)))^2), x) + sqrt(2)*c*f*g*integrate(((2*c*d*cos(f
*x + e)^2 + 2*c*d*sin(f*x + e)^2 - (c^2 - 2*c*d)*cos(2*f*x + 2*e)^2 + c^2*
cos(f*x + e) - (c^2 - 2*c*d)*sin(2*f*x + 2*e)^2 + (c^2 - 2*c*d + 4*d^2)*si
n(2*f*x + 2*e)*sin(f*x + e) - (c^2 - 2*c*d - (c^2 - 2*c*d + 4*d^2)*cos(f*x
 + e))*cos(2*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + (c
^2*sin(f*x + e) + (c^2 + 2*c*d - 4*d^2)*cos(2*f*x + 2*e)*sin(f*x + e) - (c
^2 - 2*c*d + (c^2 + 2*c*d - 4*d^2)*cos(f*x + e))*sin(2*f*x + 2*e))*sin(...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)
)),x)
```

output
```
int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\frac{\sqrt{g}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2d+\sec(fx+e)c+\sec(fx+e)d+c}dx\right)g}{a}$$

input
```
int((g*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
(sqrt(g)*sqrt(a)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x) + 1)*sec(e + f*
x))/(sec(e + f*x)**2*d + sec(e + f*x)*c + sec(e + f*x)*d + c),x)*g)/a
```

3.243 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 231

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}df} + \frac{\sqrt{2}g^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2c^{3/2}g^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)d\sqrt{c+d}f}$$

output

```
2*g^(5/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec
(f*x+e))^(1/2))/a^(1/2)/d/f+2^(1/2)*g^(5/2)*arctanh(1/2*a^(1/2)*g^(1/2)*ta
n(f*x+e)*2^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/(c-d
)/f-2*c^(3/2)*g^(5/2)*arctanh(a^(1/2)*c^(1/2)*g^(1/2)*tan(f*x+e)/(c+d)^(1/
2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/(c-d)/d/(c+d)^(1/2
)/f
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.67

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g^2\Big(d\sqrt{c+d}\operatorname{arctanh}\big(\sin\left(\frac{1}{2}(e+fx)\right)\big)+\sqrt{2}\Big((c-d)\sqrt{c+}}{}$$

input
```
Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e +
f*x])),x]
```

output
```
(2*g^2*(d*Sqrt[c + d]*ArcTanh[Sin[(e + f*x)/2]] + Sqrt[2]*((c - d)*Sqrt[c
+ d]*ArcTanh[Sqrt[2]*Sin[(e + f*x)/2]] - c^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[c]*
Sin[(e + f*x)/2])/Sqrt[c + d]]))*Cos[(e + f*x)/2]*Sqrt[g*Sec[e + f*x]])/((
c - d)*d*Sqrt[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {3042, 4466, 3042, 4453, 221, 4511, 3042, 4289, 221, 4295, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 4466$$

$$\frac{g^2\int \frac{\sqrt{g\sec(e+fx)}(ac+a(c-d)\sec(e+fx))}{\sqrt{\sec(e+fx)a+a}}dx}{ad(c-d)}-\frac{c^2g^2\int \frac{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{ad(c-d)}$$

$$\Big\downarrow \; 3042$$

$$\frac{g^2\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\left(ac+a(c-d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ad(c-d)}-\frac{c^2g^2\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ad(c-d)}$$

$$\Big\downarrow \ 4453$$

$$\frac{2c^2g^3\int \frac{1}{a(c+d)-\frac{a^2c\sin(e+fx)\tan(e+fx)}{\sec(e+fx)a+a}}d\left(-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}\right)}{df(c-d)}+\frac{g^2\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\left(ac+a(c-d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ad(c-d)}$$

$$\Big\downarrow \ 221$$

$$\frac{g^2\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\left(ac+a(c-d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ad(c-d)}-\frac{2c^{3/2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}}$$

$$\Big\downarrow \ 4511$$

$$\frac{g^2\left((c-d)\int \sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}dx+ad\int \frac{\sqrt{g\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx\right)}{ad(c-d)}-\frac{2c^{3/2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}}$$

$$\Big\downarrow \ 3042$$

$$\frac{g^2\left((c-d)\int \sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+ad\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)}{ad(c-d)}-\frac{2c^{3/2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}}$$

$$\Big\downarrow \ 4289$$

$$\frac{g^2\left(ad\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx-\frac{2ag(c-d)\int \frac{1}{a-\frac{a^2\sin(e+fx)\tan(e+fx)}{\sec(e+fx)a+a}}d\left(-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}\right)}{f}\right)}{ad(c-d)}-\frac{2c^{3/2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}}$$

$$\Big\downarrow \text{221}$$

$$\frac{g^2\left(ad\int \frac{\sqrt{g\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx+\frac{2\sqrt{a}\sqrt{g}(c-d)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{f}\right)}{ad(c-d)}-\frac{2c^{3/2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}}$$

$$\Big\downarrow \text{4295}$$

$$\frac{g^2\left(\frac{2\sqrt{a}\sqrt{g}(c-d)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{f}-\frac{2adg\int \frac{1}{2a-\frac{a^2\sin(e+fx)\tan(e+fx)}{\sec(e+fx)a+a}}d\left(-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{\sec(e+fx)a+a}}\right)}{f}\right)}{ad(c-d)}-\frac{2c^{3/2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}}$$

$$\Big\downarrow \text{221}$$

$$\frac{g^2\left(\frac{2\sqrt{a}\sqrt{g}(c-d)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{f}+\frac{\sqrt{2}\sqrt{a}d\sqrt{g}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{f}\right)}{ad(c-d)}-\frac{2c^{3/2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}}$$

input
```
Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
,x]
```

output

```
(g^2*((2*Sqrt[a]*(c - d)*Sqrt[g]*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(S
qrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/f + (Sqrt[2]*Sqrt[a]*d*Sqr
t[g]*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]])])/f))/(a*(c - d)*d) - (2*c^(3/2)*g^(5/2)*ArcTanh
[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*d*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4289

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/f)   Subst[Int[1/(b - d*x^2), x], x, b*(
Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] &&  !GtQ[a*(d/b), 0]
```

rule 4295

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4453

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*b*(g
/f)   Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, b*(Cot[e + f*x]/(Sqrt[g*Csc
[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4466
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-c^2
)*(g^2/(d*(b*c - a*d)))   Int[Sqrt[g*Csc[e + f*x]]*(Sqrt[a + b*Csc[e + f*x]
]/(c + d*Csc[e + f*x])), x], x] + Simp[g^2/(d*(b*c - a*d))   Int[Sqrt[g*Csc
[e + f*x]]*((a*c + (b*c - a*d)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
 x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^
2, 0]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3072 vs. $2(184) = 368$.

Time = 2.32 (sec) , antiderivative size = 3073, normalized size of antiderivative = 13.30

method	result	size
default	Expression too large to display	3073

input
```
int((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output

```
1/2*2^(1/2)*(-1/f/((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)/((2*2^(1
/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)/(c*(c-d))^(1/2)/(c-d)/d^2*((-8*cos
(1/2*f*x+1/2*e)^2+4)*((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)*(c*(c
-d))^(1/2)*c^3*arctanh((c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))/((2*2
^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2))+(-4*cos(1/2*f*x+1/2*e)^2+2)*2^
(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)*c^4*arctanh((c+d)*(c
ot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(
c+d))^(1/2))+(4*cos(1/2*f*x+1/2*e)^2-2)*2^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2
)-3*c+d)*(c+d))^(1/2)*c^3*d*arctanh((c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+
1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2))+(8*cos(1/2*f*x+1/
2*e)^2-4)*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)*(c*(c-d))^(1/2)*
c^3*arctan((c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c
-d))^(1/2)-3*c+d)*(c+d))^(1/2))+(-4*cos(1/2*f*x+1/2*e)^2+2)*2^(1/2)*((2*2^
(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)*c^4*arctan((c+d)*(cot(1/2*f*x+1/
2*e)-csc(1/2*f*x+1/2*e))/((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2))+
(4*cos(1/2*f*x+1/2*e)^2-2)*2^(1/2)*((2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d
))^(1/2)*c^3*d*arctan((c+d)*(cot(1/2*f*x+1/2*e)-csc(1/2*f*x+1/2*e))/((2*2^
(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2))+(4*cos(1/2*f*x+1/2*e)^2-2)*2^(1
/2)*(c*(c-d))^(1/2)*c^2*((2*2^(1/2)*(c*(c-d))^(1/2)-3*c+d)*(c+d))^(1/2)*((
2*2^(1/2)*(c*(c-d))^(1/2)+3*c-d)*(c+d))^(1/2)*arctanh(1/2*2^(1/2)*(cot(...
```

Fricas [A] (verification not implemented)

Time = 77.67 (sec) , antiderivative size = 1579, normalized size of antiderivative = 6.84

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input

```
integrate((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="fricas")
```

output

```
[-1/2*(sqrt(2)*d*g^2*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*
cos(f*x + e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e)
+ 1)) + c*sqrt(c*g/(a*c + a*d))*g^2*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6
*c*d)*g*cos(f*x + e)^2 - 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d +
d^2)*cos(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) +
(8*c^2 + 8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^
2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) - (c - d)*g^2*sqrt(g/a)*log((g*cos(
f*x + e)^3 - 4*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x +
 e)^2 + 8*g)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/((c*d - d^2)*f), -1/2*(sq
rt(2)*d*g^2*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*cos(f*x +
 e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2
*c*sqrt(-c*g/(a*c + a*d))*g^2*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos
(f*x + e))*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt(g/cos(f*x + e))/(c*g*sin(f*x + e))) - (c - d)*g^2*sqrt(g/a)*log((g*co
s(f*x + e)^3 - 4*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) - 7*g*cos(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\text{Timed out}$$

input

```
integrate((g*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x
)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

input

```
integrate((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="maxima")
```

output

```
-1/2*(sqrt(2)*c^2*f*g^2*integrate(((c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2*f*x
 + 2*e)^2 - 2*(c*d - 2*d^2)*cos(f*x + e)^2 - (c^2 - 4*c*d)*sin(2*f*x + 2*e
)*sin(f*x + e) - 2*(c*d - 2*d^2)*sin(f*x + e)^2 + (c^2 - (c^2 - 4*c*d)*cos
(f*x + e))*cos(2*f*x + 2*e) - (c^2 - 2*c*d)*cos(f*x + e))*cos(1/2*arctan2(
sin(f*x + e), cos(f*x + e))) - (c^2*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2*c
os(f*x + e) + c^2)*sin(2*f*x + 2*e) + (c^2 - 2*c*d)*sin(f*x + e))*sin(1/2*
arctan2(sin(f*x + e), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos
(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e)
 + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e)
 + c^2)*cos(2*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 +
 (c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 +
 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*
x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arct
an2(sin(f*x + e), cos(f*x + e)))^2), x) + sqrt(2)*c^2*f*g^2*integrate(((2*
c*d*cos(f*x + e)^2 + 2*c*d*sin(f*x + e)^2 - (c^2 - 2*c*d)*cos(2*f*x + 2*e)
^2 + c^2*cos(f*x + e) - (c^2 - 2*c*d)*sin(2*f*x + 2*e)^2 + (c^2 - 2*c*d +
4*d^2)*sin(2*f*x + 2*e)*sin(f*x + e) - (c^2 - 2*c*d - (c^2 - 2*c*d + 4*d^2
)*cos(f*x + e))*cos(2*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x +
e))) + (c^2*sin(f*x + e) + (c^2 + 2*c*d - 4*d^2)*cos(2*f*x + 2*e)*sin(f*x
+ e) - (c^2 - 2*c*d + (c^2 + 2*c*d - 4*d^2)*cos(f*x + e))*sin(2*f*x + 2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)
)),x)
```

output
```
int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\frac{\sqrt{g}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2d+\sec(fx+e)c+\sec(fx+e)d+c}dx\right)g^2}{a}$$

input
```
int((g*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
(sqrt(g)*sqrt(a)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x) + 1)*sec(e + f*
x)**2)/(sec(e + f*x)**2*d + sec(e + f*x)*c + sec(e + f*x)*d + c),x)*g**2)/
a
```

3.244 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 29, antiderivative size = 250

$$\begin{aligned}\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx &= \frac{(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4)\,\mathrm{arctanh}(\sin(e+fx))}{8f}\\ &+\frac{(12bc^4+95ac^3d+112bc^2d^2+80acd^3+16bd^4)\tan(e+fx)}{30f}\\ &+\frac{d(24bc^3+130ac^2d+116bcd^2+45ad^3)\sec(e+fx)\tan(e+fx)}{120f}\\ &+\frac{(12bc^2+35acd+16bd^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60f}\\ &+\frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f}\end{aligned}$$

output
```
1/8*(8*a*c^4+24*a*c^2*d^2+3*a*d^4+16*b*c^3*d+12*b*c*d^3)*arctanh(sin(f*x+e
))/f+1/30*(95*a*c^3*d+80*a*c*d^3+12*b*c^4+112*b*c^2*d^2+16*b*d^4)*tan(f*x+
e)/f+1/120*d*(130*a*c^2*d+45*a*d^3+24*b*c^3+116*b*c*d^2)*sec(f*x+e)*tan(f*
x+e)/f+1/60*(35*a*c*d+12*b*c^2+16*b*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1
/20*(5*a*d+4*b*c)*(c+d*sec(f*x+e))^3*tan(f*x+e)/f+1/5*b*(c+d*sec(f*x+e))^4
*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 4.37 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.82

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$
$$=\frac{120ac^4\coth^{-1}(\sin(e+fx))+15d(3ad(8c^2+d^2)+4b(4c^3+3cd^2))\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)}{}$$

input

```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

output

```
(120*a*c^4*ArcCoth[Sin[e + f*x]] + 15*d*(3*a*d*(8*c^2 + d^2) + 4*b*(4*c^3
+ 3*c*d^2))*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(15*d*(3*a*d*(8*c^2 + d^2
) + 4*b*(4*c^3 + 3*c*d^2))*Sec[e + f*x] + 30*d^3*(4*b*c + a*d)*Sec[e + f*x
]^3 + 8*(15*(4*a*c*d*(c^2 + d^2) + b*(c^4 + 6*c^2*d^2 + d^4)) + 10*d^2*(2*
a*c*d + b*(3*c^2 + d^2))*Tan[e + f*x]^2 + 3*b*d^4*Tan[e + f*x]^4)))/(120*f
)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {3042, 4490, 3042, 4490, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$
$$\Big\downarrow \text{4490}$$

$$\frac{1}{5}\int \sec(e+fx)(c+d\sec(e+fx))^3(5ac+4bd+(4bc+5ad)\sec(e+fx))dx+\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(5ac+4bd+(4bc+5ad)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{ 4490}$$

$$\frac{1}{5}\left(\frac{1}{4}\int \sec(e+fx)(c+d\sec(e+fx))^2\left(20ac^2+28bdc+15ad^2+\left(12bc^2+35adc+16bd^2\right)\sec(e+fx)\right)dx+\right.$$
$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(20ac^2+28bdc+15ad^2+\left(12bc^2+35adc+16bd^2\right)\csc\left(e\right.\right.\right.$$
$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{ 4490}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \sec(e+fx)(c+d\sec(e+fx))\left(60ac^3+108bdc^2+115ad^2c+32bd^3+\left(24bc^3+130adc^2+116bd^2c+\right.\right.\right.\right.$$
$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(60ac^3+108bdc^2+115ad^2c+32bd^3+\left(24bc^3+130ad\right.\right.\right.\right.$$
$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \text{ 4485}$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\int \sec(e+fx)\left(15\left(8ac^4+16bdc^3+24ad^2c^2+12bd^3c+3ad^4\right)+4\left(12bc^4+95adc^3+112bd^2c^2+80\right.\right.$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(15\left(8ac^4+16bdc^3+24ad^2c^2+12bd^3c+3ad^4\right)+4\left(12bc^4+95adc^3+112bd^2c\right.\right.$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \; 4274$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(4\left(95ac^3d+80acd^3+12bc^4+112bc^2d^2+16bd^4\right)\int \sec^2(e+fx)dx+15\left(8ac^4+24ac^2d^2+3ad^4+\right.$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(15\left(8ac^4+24ac^2d^2+3ad^4+16bc^3d+12bcd^3\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+4\left(95ac^3d+80acd^3+12b\right.$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \; 4254$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(15\left(8ac^4+24ac^2d^2+3ad^4+16bc^3d+12bcd^3\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{4\left(95ac^3d+80acd^3+12b\right.}{}$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(15\left(8ac^4+24ac^2d^2+3ad^4+16bc^3d+12bcd^3\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{4\left(95ac^3d+80acd^3+12b\right.}{}$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(\frac{15\left(8ac^4+24ac^2d^2+3ad^4+16bc^3d+12bcd^3\right)\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{4(95ac^3d+80acd^3+12bc}{}$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

output
```
(b*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f) + (((4*b*c + 5*a*d)*(c + d*S
ec[e + f*x])^3*Tan[e + f*x])/(4*f) + (((12*b*c^2 + 35*a*c*d + 16*b*d^2)*(c
 + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f) + ((d*(24*b*c^3 + 130*a*c^2*d + 1
16*b*c*d^2 + 45*a*d^3)*Sec[e + f*x]*Tan[e + f*x])/(2*f) + ((15*(8*a*c^4 +
16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*ArcTanh[Sin[e + f*x]])/f
 + (4*(12*b*c^4 + 95*a*c^3*d + 112*b*c^2*d^2 + 80*a*c*d^3 + 16*b*d^4)*Tan[
e + f*x])/f)/2)/3)/4)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.95

method	result
parts	$\frac{(a d^4+4bc d^3)\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}-\frac{(4ac d^3+6b c^2 d^2)\left(-\frac{2}{3}-\right.}{f}$
derivativedivides	$a c^4\ln(\sec(fx+e)+\tan(fx+e))+4a c^3 d\tan(fx+e)+6a c^2 d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4ac d^3\left(\right.$
default	$a c^4\ln(\sec(fx+e)+\tan(fx+e))+4a c^3 d\tan(fx+e)+6a c^2 d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4ac d^3\left(\right.$
parallelrisch	$-120\left(a c^4+3a c^2 d^2+\frac{3}{8}a d^4+2b c^3 d+\frac{3}{2}bc d^3\right)(\cos(5fx+5e)+5\cos(3fx+3e)+10\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+120$
norman	$-\frac{4\left(180a c^3 d+100ac d^3+45b c^4+150b c^2 d^2+29b d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}-\frac{\left(32a c^3 d-24a c^2 d^2+32ac d^3-5a d^4+8b c^4-16b c^3 d+48b c^2 d^2\right.}{4f}$
risch	$i\left(320ac d^3+480b c^2 d^2+480a c^3 d+120b c^4+64b d^4+1920a c^3 d\,\mathrm{e}^{6i(fx+e)}+960ac d^3\mathrm{e}^{6i(fx+e)}+1440b c^2 d^2\mathrm{e}^{6i(fx+e)}+288\right.$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE
)
```

output

```
(a*d^4+4*b*c*d^3)/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln
(sec(f*x+e)+tan(f*x+e)))-(4*a*c*d^3+6*b*c^2*d^2)/f*(-2/3-1/3*sec(f*x+e)^2)
*tan(f*x+e)+(6*a*c^2*d^2+4*b*c^3*d)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(se
c(f*x+e)+tan(f*x+e)))+(4*a*c^3*d+b*c^4)/f*tan(f*x+e)+a*c^4/f*ln(sec(f*x+e)
+tan(f*x+e))-b*d^4/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.12

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$

$$= \underline{15\left(8\,ac^4+16\,bc^3d+24\,ac^2d^2+12\,bcd^3+3\,ad^4\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(8\,ac^4+16\,bc^3\right.}$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fri
cas")
```

output

```
1/240*(15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos
(f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^
2 + 12*b*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*b*
d^4 + 8*(15*b*c^4 + 60*a*c^3*d + 60*b*c^2*d^2 + 40*a*c*d^3 + 8*b*d^4)*cos(
f*x + e)^4 + 15*(16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos(f*x
 + e)^3 + 16*(15*b*c^2*d^2 + 10*a*c*d^3 + 2*b*d^4)*cos(f*x + e)^2 + 30*(4*
b*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$
$$= \int \left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^4\sec\left(e+fx\right)\,dx$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

output

```
Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**4*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.52

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$

$$= \frac{480\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)bc^2d^2+320\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)acd^3+16\,(3\,\tan\,(fx}{}$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="max
ima")
```

output

```
1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c^2*d^2 + 320*(tan(f*x + e)
^3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 +
15*tan(f*x + e))*b*d^4 - 60*b*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))
/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log
(sin(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin
(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(
f*x + e) - 1)) - 240*b*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(si
n(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240
*a*c^4*log(sec(f*x + e) + tan(f*x + e)) + 240*b*c^4*tan(f*x + e) + 960*a*c
^3*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. 2(238) = 476.

Time = 0.22 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.40

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="gia
c")
```

output

```
1/120*(15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*log
(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2
+ 12*b*c*d^3 + 3*a*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(120*b*c^4*
tan(1/2*f*x + 1/2*e)^9 + 480*a*c^3*d*tan(1/2*f*x + 1/2*e)^9 - 240*b*c^3*d*
tan(1/2*f*x + 1/2*e)^9 - 360*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^9 + 720*b*c^2*
d^2*tan(1/2*f*x + 1/2*e)^9 + 480*a*c*d^3*tan(1/2*f*x + 1/2*e)^9 - 300*b*c*
d^3*tan(1/2*f*x + 1/2*e)^9 - 75*a*d^4*tan(1/2*f*x + 1/2*e)^9 + 120*b*d^4*t
an(1/2*f*x + 1/2*e)^9 - 480*b*c^4*tan(1/2*f*x + 1/2*e)^7 - 1920*a*c^3*d*ta
n(1/2*f*x + 1/2*e)^7 + 480*b*c^3*d*tan(1/2*f*x + 1/2*e)^7 + 720*a*c^2*d^2*
tan(1/2*f*x + 1/2*e)^7 - 1920*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 1280*a*c*
d^3*tan(1/2*f*x + 1/2*e)^7 + 120*b*c*d^3*tan(1/2*f*x + 1/2*e)^7 + 30*a*d^4
*tan(1/2*f*x + 1/2*e)^7 - 160*b*d^4*tan(1/2*f*x + 1/2*e)^7 + 720*b*c^4*tan
(1/2*f*x + 1/2*e)^5 + 2880*a*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 2400*b*c^2*d^2
*tan(1/2*f*x + 1/2*e)^5 + 1600*a*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 464*b*d^4*
tan(1/2*f*x + 1/2*e)^5 - 480*b*c^4*tan(1/2*f*x + 1/2*e)^3 - 1920*a*c^3*d*t
an(1/2*f*x + 1/2*e)^3 - 480*b*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 720*a*c^2*d^2
*tan(1/2*f*x + 1/2*e)^3 - 1920*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 1280*a*c
*d^3*tan(1/2*f*x + 1/2*e)^3 - 120*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 30*a*d^
4*tan(1/2*f*x + 1/2*e)^3 - 160*b*d^4*tan(1/2*f*x + 1/2*e)^3 + 120*b*c^4*ta
n(1/2*f*x + 1/2*e) + 480*a*c^3*d*tan(1/2*f*x + 1/2*e) + 240*b*c^3*d*tan...
```

Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.22

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$

$$=\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c^4+2\,b\,c^3\,d+3\,a\,c^2\,d^2+\frac{3\,b\,c\,d^3}{2}+\frac{3\,a\,d^4}{8}\right)}{4\,a\,c^4+8\,b\,c^3\,d+12\,a\,c^2\,d^2+6\,b\,c\,d^3+\frac{3\,a\,d^4}{2}}\right)\,\left(2\,a\,c^4+4\,b\,c^3\,d+6\,a\,c^2\,d^2+3\,b\,c\,d^3+\frac{3\,a\,d^4}{4}\right)}{f}$$

$$-\frac{\left(2\,b\,c^4-\frac{5\,a\,d^4}{4}+2\,b\,d^4-6\,a\,c^2\,d^2+12\,b\,c^2\,d^2+8\,a\,c\,d^3+8\,a\,c^3\,d-5\,b\,c\,d^3-4\,b\,c^3\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+}{}$$

input

```
int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

output
```
(atanh((4*tan(e/2 + (f*x)/2)*(a*c^4 + (3*a*d^4)/8 + 3*a*c^2*d^2 + (3*b*c*d
^3)/2 + 2*b*c^3*d))/(4*a*c^4 + (3*a*d^4)/2 + 12*a*c^2*d^2 + 6*b*c*d^3 + 8*
b*c^3*d))*(2*a*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*b*c*d^3 + 4*b*c^3*d))/f
 - (tan(e/2 + (f*x)/2)^5*(12*b*c^4 + (116*b*d^4)/15 + 40*b*c^2*d^2 + (80*a
*c*d^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*((5*a*d^4)/4 + 2*b*c^4 + 2*b*
d^4 + 6*a*c^2*d^2 + 12*b*c^2*d^2 + 8*a*c*d^3 + 8*a*c^3*d + 5*b*c*d^3 + 4*b
*c^3*d) + tan(e/2 + (f*x)/2)^9*(2*b*c^4 - (5*a*d^4)/4 + 2*b*d^4 - 6*a*c^2*
d^2 + 12*b*c^2*d^2 + 8*a*c*d^3 + 8*a*c^3*d - 5*b*c*d^3 - 4*b*c^3*d) - tan(
e/2 + (f*x)/2)^3*((a*d^4)/2 + 8*b*c^4 + (8*b*d^4)/3 + 12*a*c^2*d^2 + 32*b*
c^2*d^2 + (64*a*c*d^3)/3 + 32*a*c^3*d + 2*b*c*d^3 + 8*b*c^3*d) - tan(e/2 +
 (f*x)/2)^7*(8*b*c^4 - (a*d^4)/2 + (8*b*d^4)/3 - 12*a*c^2*d^2 + 32*b*c^2*d
^2 + (64*a*c*d^3)/3 + 32*a*c^3*d - 2*b*c*d^3 - 8*b*c^3*d))/(f*(5*tan(e/2 +
 (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/
2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1323, normalized size of antiderivative = 5.29

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x)
```

output

```
( - 120*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a*c**4 - 36
0*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a*c**2*d**2 - 45*
cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a*d**4 - 240*cos(e
+ f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*b*c**3*d - 180*cos(e + f*
x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*b*c*d**3 + 240*cos(e + f*x)*l
og(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*c**4 + 720*cos(e + f*x)*log(tan
((e + f*x)/2) - 1)*sin(e + f*x)**2*a*c**2*d**2 + 90*cos(e + f*x)*log(tan((
e + f*x)/2) - 1)*sin(e + f*x)**2*a*d**4 + 480*cos(e + f*x)*log(tan((e + f*
x)/2) - 1)*sin(e + f*x)**2*b*c**3*d + 360*cos(e + f*x)*log(tan((e + f*x)/2
) - 1)*sin(e + f*x)**2*b*c*d**3 - 120*cos(e + f*x)*log(tan((e + f*x)/2) -
1)*a*c**4 - 360*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*a*c**2*d**2 - 45*co
s(e + f*x)*log(tan((e + f*x)/2) - 1)*a*d**4 - 240*cos(e + f*x)*log(tan((e
+ f*x)/2) - 1)*b*c**3*d - 180*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*b*c*d
**3 + 120*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*a*c**4 +
360*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*a*c**2*d**2 + 4
5*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*a*d**4 + 240*cos(
e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*b*c**3*d + 180*cos(e +
f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*b*c*d**3 - 240*cos(e + f*x)
*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a*c**4 - 720*cos(e + f*x)*log(t
an((e + f*x)/2) + 1)*sin(e + f*x)**2*a*c**2*d**2 - 90*cos(e + f*x)*log(...
```

3.245 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 180

$$
\begin{aligned}
&\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx \\
&= \frac{(8ac^3+12bc^2d+12acd^2+3bd^3)\operatorname{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{(4ad(4c^2+d^2)+3b(c^3+4cd^2))\tan(e+fx)}{6f} \\
&\quad + \frac{d(6bc^2+20acd+9bd^2)\sec(e+fx)\tan(e+fx)}{24f} \\
&\quad + \frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f}
\end{aligned}
$$

output

```
1/8*(8*a*c^3+12*a*c*d^2+12*b*c^2*d+3*b*d^3)*arctanh(sin(f*x+e))/f+1/6*(4*a
*d*(4*c^2+d^2)+3*b*(c^3+4*c*d^2))*tan(f*x+e)/f+1/24*d*(20*a*c*d+6*b*c^2+9*
b*d^2)*sec(f*x+e)*tan(f*x+e)/f+1/12*(4*a*d+3*b*c)*(c+d*sec(f*x+e))^2*tan(f
*x+e)/f+1/4*b*(c+d*sec(f*x+e))^3*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$= \frac{24ac^3\coth^{-1}(\sin(e+fx))+9d(4acd+b(4c^2+d^2))\,\text{arctanh}(\sin(e+fx))+\tan(e+fx)\,(9d(4acd+b(4}{}$$

input

```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

output

```
(24*a*c^3*ArcCoth[Sin[e + f*x]] + 9*d*(4*a*c*d + b*(4*c^2 + d^2))*ArcTanh[
Sin[e + f*x]] + Tan[e + f*x]*(9*d*(4*a*c*d + b*(4*c^2 + d^2))*Sec[e + f*x]
 + 6*b*d^3*Sec[e + f*x]^3 + 8*(3*a*d*(3*c^2 + d^2) + 3*b*(c^3 + 3*c*d^2) +
 d^2*(3*b*c + a*d)*Tan[e + f*x]^2)))/(24*f)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {3042, 4490, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4490}$$

$$\frac{1}{4}\int \sec(e+fx)(c+d\sec(e+fx))^2(4ac+3bd+(3bc+4ad)\sec(e+fx))dx+\frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(4ac+3bd+(3bc+4ad)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \ 4490$$

$$\frac{1}{4}\left(\frac{1}{3}\int \sec(e+fx)(c+d\sec(e+fx))\left(12ac^2+15bdc+8ad^2+\left(6bc^2+20adc+9bd^2\right)\sec(e+fx)\right)dx+\frac{(4ad\ -}{} \right. \\ \frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\left(\frac{1}{3}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(12ac^2+15bdc+8ad^2+\left(6bc^2+20adc+9bd^2\right)\csc(e+f\right.\right. \\ \frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \ 4485$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(e+fx)\left(3\left(3bd\left(4c^2+d^2\right)+4a\left(2c^3+3d^2c\right)\right)+4\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4d^2c\right)\right)\sec(e+fx)\right)d\right.\right. \\ \frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(3\left(3bd\left(4c^2+d^2\right)+4a\left(2c^3+3d^2c\right)\right)+4\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4d^2c\right)\right)\csc(e+\right.\right.\right. \\ \frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \ 4274$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(4\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\int \sec^2(e+fx)dx+3\left(8ac^3+12acd^2+12bc^2d+3bd^3\right)\int \sec(e+f\right.\right.\right. \\ \frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(4\big(4ad\big(4c^2+d^2\big)+3b\big(c^3+4cd^2\big)\big)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx+3\big(8ac^3+12acd^2+12bc^2d+3bd^3\big)\int \text{cs}$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \; 4254$$

$$\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(3\big(8ac^3+12acd^2+12bc^2d+3bd^3\big)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{4\big(4ad\big(4c^2+d^2\big)+3b\big(c^3+4cd^2\big)\big)\int 1d(}{f}$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(3\big(8ac^3+12acd^2+12bc^2d+3bd^3\big)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{4\big(4ad\big(4c^2+d^2\big)+3b\big(c^3+4cd^2\big)\big)\tan(e}{f}$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{4}\Bigg(\frac{1}{3}\Bigg(\frac{1}{2}\Bigg(\frac{3\big(8ac^3+12acd^2+12bc^2d+3bd^3\big)\text{arctanh}(\sin(e+fx))}{f}+\frac{4\big(4ad\big(4c^2+d^2\big)+3b\big(c^3+4cd^2\big)\big)\tan(e+}{f}$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

input

```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

output

```
(b*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(4*f) + (((3*b*c + 4*a*d)*(c + d*S
ec[e + f*x])^2*Tan[e + f*x])/(3*f) + ((d*(6*b*c^2 + 20*a*c*d + 9*b*d^2)*Se
c[e + f*x]*Tan[e + f*x])/(2*f) + ((3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 +
3*b*d^3)*ArcTanh[Sin[e + f*x]])/f + (4*(4*a*d*(4*c^2 + d^2) + 3*b*(c^3 + 4
*c*d^2))*Tan[e + f*x])/f)/2)/3)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03

method	result
parts	$-\frac{\left(a\,d^{3}+3bc\,d^{2}\right)\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}+\frac{\left(3ac\,d^{2}+3b\,c^{2}d\right)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
derivativedivides	$\frac{a\,c^{3}\ln(\sec(fx+e)+\tan(fx+e))+3a\,c^{2}d\tan(fx+e)+3ac\,d^{2}\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,d^{3}\left(-\right.}{}$
default	$\frac{a\,c^{3}\ln(\sec(fx+e)+\tan(fx+e))+3a\,c^{2}d\tan(fx+e)+3ac\,d^{2}\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,d^{3}\left(-\right.}{}$
parallelrisch	$\frac{-96\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(a\,c^{3}+\frac{3}{2}ac\,d^{2}+\frac{3}{2}b\,c^{2}d+\frac{3}{8}b\,d^{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+96\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2\right.}{}$
norman	$\frac{-\frac{\left(24a\,c^{2}d-12ac\,d^{2}+8a\,d^{3}+8b\,c^{3}-12b\,c^{2}d+24bc\,d^{2}-5b\,d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{4f}+\frac{\left(24a\,c^{2}d+12ac\,d^{2}+8a\,d^{3}+8b\,c^{3}+12b\,c^{2}d+24bc\,d^{2}+5b\right.}{4f}}{}$
risch	$-\frac{i\left(-72a\,c^{2}d-48bc\,d^{2}-16a\,d^{3}-24b\,c^{3}-216a\,c^{2}d\,\mathrm{e}^{2i(fx+e)}-192bc\,d^{2}\mathrm{e}^{2i(fx+e)}-36ac\,d^{2}\mathrm{e}^{3i(fx+e)}-36b\,c^{2}d\,\mathrm{e}^{3i(fx+e)}+\right.}{}$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output
```
-(a*d^3+3*b*c*d^2)/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+(3*a*c*d^2+3*b*c^2
*d)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+(3*a*c^2*d
+b*c^3)/f*tan(f*x+e)+a*c^3/f*ln(sec(f*x+e)+tan(f*x+e))+b*d^3/f*(-(-1/4*sec
(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$

$$=\frac{3\left(8\,ac^{3}+12\,bc^{2}d+12\,acd^{2}+3\,bd^{3}\right)\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-3\left(8\,ac^{3}+12\,bc^{2}d+12\,acd^{2}+\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
1/48*(3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x + e)^4*log(s
in(f*x + e) + 1) - 3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x
 + e)^4*log(-sin(f*x + e) + 1) + 2*(6*b*d^3 + 8*(3*b*c^3 + 9*a*c^2*d + 6*b
*c*d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*b*c^2*d + 4*a*c*d^2 + b*d^3)*cos(f
*x + e)^2 + 8*(3*b*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
 e)^4)
```

Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$= \int (a+b\sec{(e+fx)})\,(c+d\sec{(e+fx)})^3\sec{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**3*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.48

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$= \frac{48\,(\tan{(fx+e)}^3+3\,\tan{(fx+e)})bcd^2+16\,(\tan{(fx+e)}^3+3\,\tan{(fx+e)})ad^3-3\,bd^3\Big(\frac{2\,(3\,\sin(fx+e)^3}{\sin(fx+e)^4-2}}{}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="max
ima")
```

output

```
1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c*d^2 + 16*(tan(f*x + e)^3 +
3*tan(f*x + e))*a*d^3 - 3*b*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(si
n(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 36*b*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(si
n(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin
(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c
^3*log(sec(f*x + e) + tan(f*x + e)) + 48*b*c^3*tan(f*x + e) + 144*a*c^2*d*
tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(170) = 340$.

Time = 0.21 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.26

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="gia
c")
```

output

```
1/24*(3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*log(abs(tan(1/2*f*x
+ 1/2*e) + 1)) - 3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*log(abs(t
an(1/2*f*x + 1/2*e) - 1)) - 2*(24*b*c^3*tan(1/2*f*x + 1/2*e)^7 + 72*a*c^2*
d*tan(1/2*f*x + 1/2*e)^7 - 36*b*c^2*d*tan(1/2*f*x + 1/2*e)^7 - 36*a*c*d^2*
tan(1/2*f*x + 1/2*e)^7 + 72*b*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 24*a*d^3*tan(
1/2*f*x + 1/2*e)^7 - 15*b*d^3*tan(1/2*f*x + 1/2*e)^7 - 72*b*c^3*tan(1/2*f*
x + 1/2*e)^5 - 216*a*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 36*b*c^2*d*tan(1/2*f*x
 + 1/2*e)^5 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 120*b*c*d^2*tan(1/2*f*x
+ 1/2*e)^5 - 40*a*d^3*tan(1/2*f*x + 1/2*e)^5 - 9*b*d^3*tan(1/2*f*x + 1/2*e
)^5 + 72*b*c^3*tan(1/2*f*x + 1/2*e)^3 + 216*a*c^2*d*tan(1/2*f*x + 1/2*e)^3
 + 36*b*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 +
 120*b*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 40*a*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*
b*d^3*tan(1/2*f*x + 1/2*e)^3 - 24*b*c^3*tan(1/2*f*x + 1/2*e) - 72*a*c^2*d*
tan(1/2*f*x + 1/2*e) - 36*b*c^2*d*tan(1/2*f*x + 1/2*e) - 36*a*c*d^2*tan(1/
2*f*x + 1/2*e) - 72*b*c*d^2*tan(1/2*f*x + 1/2*e) - 24*a*d^3*tan(1/2*f*x +
1/2*e) - 15*b*d^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.19

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$

$$=\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c^3+\frac{3\,b\,c^2\,d}{2}+\frac{3\,a\,c\,d^2}{2}+\frac{3\,b\,d^3}{8}\right)}{4\,a\,c^3+6\,b\,c^2\,d+6\,a\,c\,d^2+\frac{3\,b\,d^3}{2}}\right)\,\left(2\,a\,c^3+3\,b\,c^2\,d+3\,a\,c\,d^2+\frac{3\,b\,d^3}{4}\right)}{f}$$

$$-\frac{\left(2\,a\,d^3+2\,b\,c^3-\frac{5\,b\,d^3}{4}-3\,a\,c\,d^2+6\,a\,c^2\,d+6\,b\,c\,d^2-3\,b\,c^2\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(3\,a\,c\,d^2-6\,b\,c^3-\right.}{}$$

input
```
int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

output
```
(atanh((4*tan(e/2 + (f*x)/2)*(a*c^3 + (3*b*d^3)/8 + (3*a*c*d^2)/2 + (3*b*c
^2*d)/2))/(4*a*c^3 + (3*b*d^3)/2 + 6*a*c*d^2 + 6*b*c^2*d))*(2*a*c^3 + (3*b
*d^3)/4 + 3*a*c*d^2 + 3*b*c^2*d))/f - (tan(e/2 + (f*x)/2)^7*(2*a*d^3 + 2*b
*c^3 - (5*b*d^3)/4 - 3*a*c*d^2 + 6*a*c^2*d + 6*b*c*d^2 - 3*b*c^2*d) + tan(
e/2 + (f*x)/2)^3*((10*a*d^3)/3 + 6*b*c^3 - (3*b*d^3)/4 + 3*a*c*d^2 + 18*a*
c^2*d + 10*b*c*d^2 + 3*b*c^2*d) - tan(e/2 + (f*x)/2)^5*((10*a*d^3)/3 + 6*b
*c^3 + (3*b*d^3)/4 - 3*a*c*d^2 + 18*a*c^2*d + 10*b*c*d^2 - 3*b*c^2*d) - ta
n(e/2 + (f*x)/2)*(2*a*d^3 + 2*b*c^3 + (5*b*d^3)/4 + 3*a*c*d^2 + 6*a*c^2*d
+ 6*b*c*d^2 + 3*b*c^2*d))/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2
)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 838, normalized size of antiderivative = 4.66

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x)
```

output

```
( - 72*cos(e + f*x)*sin(e + f*x)**3*a*c**2*d - 16*cos(e + f*x)*sin(e + f*x
)**3*a*d**3 - 24*cos(e + f*x)*sin(e + f*x)**3*b*c**3 - 48*cos(e + f*x)*sin
(e + f*x)**3*b*c*d**2 + 72*cos(e + f*x)*sin(e + f*x)*a*c**2*d + 24*cos(e +
 f*x)*sin(e + f*x)*a*d**3 + 24*cos(e + f*x)*sin(e + f*x)*b*c**3 + 72*cos(e
 + f*x)*sin(e + f*x)*b*c*d**2 - 24*log(tan((e + f*x)/2) - 1)*sin(e + f*x)*
*4*a*c**3 - 36*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a*c*d**2 - 36*log
(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*b*c**2*d - 9*log(tan((e + f*x)/2) -
 1)*sin(e + f*x)**4*b*d**3 + 48*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*
a*c**3 + 72*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*c*d**2 + 72*log(ta
n((e + f*x)/2) - 1)*sin(e + f*x)**2*b*c**2*d + 18*log(tan((e + f*x)/2) - 1
)*sin(e + f*x)**2*b*d**3 - 24*log(tan((e + f*x)/2) - 1)*a*c**3 - 36*log(ta
n((e + f*x)/2) - 1)*a*c*d**2 - 36*log(tan((e + f*x)/2) - 1)*b*c**2*d - 9*l
og(tan((e + f*x)/2) - 1)*b*d**3 + 24*log(tan((e + f*x)/2) + 1)*sin(e + f*x
)**4*a*c**3 + 36*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*a*c*d**2 + 36*l
og(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*b*c**2*d + 9*log(tan((e + f*x)/2)
 + 1)*sin(e + f*x)**4*b*d**3 - 48*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**
2*a*c**3 - 72*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a*c*d**2 - 72*log(
tan((e + f*x)/2) + 1)*sin(e + f*x)**2*b*c**2*d - 18*log(tan((e + f*x)/2) +
 1)*sin(e + f*x)**2*b*d**3 + 24*log(tan((e + f*x)/2) + 1)*a*c**3 + 36*log(
tan((e + f*x)/2) + 1)*a*c*d**2 + 36*log(tan((e + f*x)/2) + 1)*b*c**2*d ...
```

3.246 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 115

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx = \frac{(2bcd+a(2c^2+d^2))\operatorname{arctanh}(\sin(e+fx))}{2f}+\frac{2(3acd+b(c^2+d^2))\tan(e+fx)}{3f}+\frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f}$$

output

```
1/2*(2*b*c*d+a*(2*c^2+d^2))*arctanh(sin(f*x+e))/f+2/3*(3*a*c*d+b*(c^2+d^2)
)*tan(f*x+e)/f+1/6*d*(3*a*d+2*b*c)*sec(f*x+e)*tan(f*x+e)/f+1/3*b*(c+d*sec(
f*x+e))^2*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.81

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{6ac^2\coth^{-1}(\sin(e+fx))+3d(2bc+ad)\text{arctanh}(\sin(e+fx))+\tan(e+fx)\left(12acd+6b(c^2+d^2)+3d\right.}{6f}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

output
```
(6*a*c^2*ArcCoth[Sin[e + f*x]] + 3*d*(2*b*c + a*d)*ArcTanh[Sin[e + f*x]] +
 Tan[e + f*x]*(12*a*c*d + 6*b*(c^2 + d^2) + 3*d*(2*b*c + a*d)*Sec[e + f*x]
 + 2*b*d^2*Tan[e + f*x]^2))/(6*f)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4490}$$

$$\frac{1}{3}\int \sec(e+fx)(c+d\sec(e+fx))(3ac+2bd+(2bc+3ad)\sec(e+fx))dx+\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(3ac+2bd+(2bc+3ad)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 4485$$

$$\frac{1}{3}\bigg(\frac{1}{2}\int \sec(e+fx)\left(3\left(2bcd+a\left(2c^2+d^2\right)\right)+4\left(3acd+b\left(c^2+d^2\right)\right)\sec(e+fx)\right)dx+\frac{d(3ad+2bc)\tan(e+fx)}{2f}\quad\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\bigg(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(3\left(2bcd+a\left(2c^2+d^2\right)\right)+4\left(3acd+b\left(c^2+d^2\right)\right)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{d(3ad+2bc}{}\quad\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 4274$$

$$\frac{1}{3}\bigg(\frac{1}{2}\left(4\left(3acd+b\left(c^2+d^2\right)\right)\int \sec^2(e+fx)dx+3\left(a\left(2c^2+d^2\right)+2bcd\right)\int \sec(e+fx)dx\right)+\frac{d(3ad+2bc)\tan(e}{2}\quad\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\bigg(\frac{1}{2}\left(3\left(a\left(2c^2+d^2\right)+2bcd\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+4\left(3acd+b\left(c^2+d^2\right)\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)+\frac{d(3ac}{}\quad\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 4254$$

$$\frac{1}{3}\bigg(\frac{1}{2}\left(3\left(a\left(2c^2+d^2\right)+2bcd\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{4\left(3acd+b\left(c^2+d^2\right)\right)\int 1d(-\tan(e+fx))}{f}\right)+\frac{d(3ad+}{}\quad\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3\left(a\left(2c^2+d^2\right)+2bcd\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{4\left(3acd+b\left(c^2+d^2\right)\right)\tan(e+fx)}{f}\right)+\frac{d(3ad+2bc)\tan}{}\right.$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

$$\Big\downarrow \color{blue}{4257}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3\left(a\left(2c^2+d^2\right)+2bcd\right)\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{4\left(3acd+b\left(c^2+d^2\right)\right)\tan(e+fx)}{f}\right)+\frac{d(3ad+2bc)\tan(e}{2}\right.$$

$$\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

output
```
(b*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f) + ((d*(2*b*c + 3*a*d)*Sec[e
+ f*x]*Tan[e + f*x])/(2*f) + ((3*(2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e
 + f*x]])/f + (4*(3*a*c*d + b*(c^2 + d^2))*Tan[e + f*x])/f)/2)/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

method	result
parts	$\frac{(a\,d^2+2bcd)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{(2acd+b\,c^2)\tan(fx+e)}{f}+\frac{a\,c^2\ln(\sec(fx+e)+\tan(}{f}$
derivativedivides	$\frac{a\,c^2\ln(\sec(fx+e)+\tan(fx+e))+2acd\tan(fx+e)+a\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\,c^2\tan(fx+}{f}$
default	$\frac{a\,c^2\ln(\sec(fx+e)+\tan(fx+e))+2acd\tan(fx+e)+a\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\,c^2\tan(fx+}{f}$
parallelrisch	$\frac{-9\left(a\,c^2+\frac{1}{2}a\,d^2+bcd\right)\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+9\left(a\,c^2+\frac{1}{2}a\,d^2+bcd\right)\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right.}{3f(\cos(3fx+3e}$
norman	$\frac{\frac{4\left(6acd+3b\,c^2+b\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{\left(4acd-a\,d^2+2b\,c^2-2bcd+2b\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{\left(4acd+a\,d^2+2b\,c^2+2bcd+2b\,d^2\right)\tan\left(\frac{fx}{2}+\right.}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
risch	$-\frac{i\left(3a\,d^2e^{5i(fx+e)}+6bcd\,e^{5i(fx+e)}-12acd\,e^{4i(fx+e)}-6b\,c^2e^{4i(fx+e)}-24acd\,e^{2i(fx+e)}-12b\,c^2e^{2i(fx+e)}-12b\,d^2e^{2i(fx\right.}{3f\left(e^{2i(fx+e)}+1\right)^3}$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
(a*d^2+2*b*c*d)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))
)+(2*a*c*d+b*c^2)/f*tan(f*x+e)+a*c^2/f*ln(sec(f*x+e)+tan(f*x+e))-b*d^2/f*(
-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.30

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= \frac{3\left(2\,ac^2+2\,bcd+ad^2\right)\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\left(2\,ac^2+2\,bcd+ad^2\right)\cos\left(fx+e\right)^3\log\left(-\right.}{12\,f\,c}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output
```
1/12*(3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
 3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(
2*b*d^2 + 2*(3*b*c^2 + 6*a*c*d + 2*b*d^2)*cos(f*x + e)^2 + 3*(2*b*c*d + a*
d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= \int \left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^2\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**2*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.43

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{4\,(\tan(fx+e)^3+3\,\tan(fx+e))bd^2-6\,bcd\Bigl(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-}{}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*d^2 - 6*b*c*d*(2*sin(f*x + e)/
(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*
a*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(s
in(f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*b*c^2*t
an(f*x + e) + 24*a*c*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(107) = 214.

Time = 0.16 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.56

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{3\,(2\,ac^2+2\,bcd+ad^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,(2\,ac^2+2\,bcd+ad^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
1/6*(3*(2*a*c^2 + 2*b*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*
(2*a*c^2 + 2*b*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(6*b*c^
2*tan(1/2*f*x + 1/2*e)^5 + 12*a*c*d*tan(1/2*f*x + 1/2*e)^5 - 6*b*c*d*tan(1
/2*f*x + 1/2*e)^5 - 3*a*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*b*d^2*tan(1/2*f*x +
 1/2*e)^5 - 12*b*c^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*c*d*tan(1/2*f*x + 1/2*e
)^3 - 4*b*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*b*c^2*tan(1/2*f*x + 1/2*e) + 12*a
*c*d*tan(1/2*f*x + 1/2*e) + 6*b*c*d*tan(1/2*f*x + 1/2*e) + 3*a*d^2*tan(1/2
*f*x + 1/2*e) + 6*b*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)
^3)/f
```

Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.97

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$

$$= \frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a\,c^2+b\,c\,d+\frac{a\,d^2}{2}\right)}{4\,a\,c^2+4\,b\,c\,d+2\,a\,d^2}\right)\,(2\,a\,c^2+2\,b\,c\,d+a\,d^2)}{f} - \frac{(2\,b\,c^2-a\,d^2+2\,b\,d^2+4\,a\,c\,d-2\,b\,c\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-4\,b\,c^2-8\,a\,c\,d-\frac{4\,b\,d^2}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+(a}{f\,\Big(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}$$

input
```
int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

output
```
(atanh((4*tan(e/2 + (f*x)/2)*(a*c^2 + (a*d^2)/2 + b*c*d))/(4*a*c^2 + 2*a*d
^2 + 4*b*c*d))*(2*a*c^2 + a*d^2 + 2*b*c*d))/f - (tan(e/2 + (f*x)/2)*(a*d^2
 + 2*b*c^2 + 2*b*d^2 + 4*a*c*d + 2*b*c*d) - tan(e/2 + (f*x)/2)^3*(4*b*c^2
+ (4*b*d^2)/3 + 8*a*c*d) + tan(e/2 + (f*x)/2)^5*(2*b*c^2 - a*d^2 + 2*b*d^2
 + 4*a*c*d - 2*b*c*d))/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4
 + tan(e/2 + (f*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 469, normalized size of antiderivative = 4.08

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$

$$= \frac{-6\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^2\,a\,c^2-3\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+}{}$$

input

```
int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x)
```

output

```
( - 6*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*c**2 - 3*co
s(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*d**2 - 6*cos(e + f*
x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*b*c*d + 6*cos(e + f*x)*log(ta
n((e + f*x)/2) - 1)*a*c**2 + 3*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*a*d*
*2 + 6*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*b*c*d + 6*cos(e + f*x)*log(t
an((e + f*x)/2) + 1)*sin(e + f*x)**2*a*c**2 + 3*cos(e + f*x)*log(tan((e +
f*x)/2) + 1)*sin(e + f*x)**2*a*d**2 + 6*cos(e + f*x)*log(tan((e + f*x)/2)
+ 1)*sin(e + f*x)**2*b*c*d - 6*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*a*c*
*2 - 3*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*a*d**2 - 6*cos(e + f*x)*log(
tan((e + f*x)/2) + 1)*b*c*d - 3*cos(e + f*x)*sin(e + f*x)*a*d**2 - 6*cos(e
 + f*x)*sin(e + f*x)*b*c*d + 12*sin(e + f*x)**3*a*c*d + 6*sin(e + f*x)**3*
b*c**2 + 4*sin(e + f*x)**3*b*d**2 - 12*sin(e + f*x)*a*c*d - 6*sin(e + f*x)
*b*c**2 - 6*sin(e + f*x)*b*d**2)/(6*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.247 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 61

$$\begin{aligned}\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx\\ =\frac{(2ac+bd)\text{arctanh}(\sin(e+fx))}{2f}\\ +\frac{(bc+ad)\tan(e+fx)}{f}+\frac{bd\sec(e+fx)\tan(e+fx)}{2f}\end{aligned}$$

output

```
1/2*(2*a*c+b*d)*arctanh(sin(f*x+e))/f+(a*d+b*c)*tan(f*x+e)/f+1/2*b*d*sec(f
*x+e)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\begin{aligned}\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx\\ =\frac{ac\coth^{-1}(\sin(e+fx))}{f}+\frac{bd\text{arctanh}(\sin(e+fx))}{2f}\\ +\frac{bc\tan(e+fx)}{f}+\frac{ad\tan(e+fx)}{f}+\frac{bd\sec(e+fx)\tan(e+fx)}{2f}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

output
```
(a*c*ArcCoth[Sin[e + f*x]])/f + (b*d*ArcTanh[Sin[e + f*x]])/(2*f) + (b*c*T
an[e + f*x])/f + (a*d*Tan[e + f*x])/f + (b*d*Sec[e + f*x]*Tan[e + f*x])/(2
*f)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{2}\int \sec(e+fx)(2ac+bd+2(bc+ad)\sec(e+fx))dx+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(2ac+bd+2(bc+ad)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{2}\left(2(ad+bc)\int \sec^2(e+fx)dx+(2ac+bd)\int \sec(e+fx)dx\right)+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left((2ac+bd)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+2(ad+bc)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx\right)+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{2}\left((2ac+bd)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{2(ad+bc)\int 1d(-\tan(e+fx))}{f}\right)+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left((2ac+bd)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{2(ad+bc)\tan(e+fx)}{f}\right)+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(\frac{(2ac+bd)\text{arctanh}(\sin(e+fx))}{f}+\frac{2(ad+bc)\tan(e+fx)}{f}\right)+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

output
```
(b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f) + (((2*a*c + b*d)*ArcTanh[Sin[e + f*
x]])/f + (2*(b*c + a*d)*Tan[e + f*x])/f)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)+bc\tan(fx+e)+bd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
default	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)+bc\tan(fx+e)+bd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parts	$\frac{(ad+bc)\tan(fx+e)}{f}+\frac{ac\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{bd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parallelrisch	$\frac{-\left(ac+\frac{bd}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(ac+\frac{bd}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+(ad+bc)\sin(2f}{f(1+\cos(2fx+2e))}$
norman	$\frac{\frac{(2ad+2bc+bd)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(2ad+2bc-bd)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}}-\frac{(2ac+bd)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{(2ac+bd)\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{2f}$
risch	$-\frac{i\left(bd\,e^{3i(fx+e)}-2ad\,e^{2i(fx+e)}-2bc\,e^{2i(fx+e)}-bd\,e^{i(fx+e)}-2ad-2bc\right)}{f\left(e^{2i(fx+e)}+1\right)^{2}}+\frac{ac\ln\left(e^{i(fx+e)}+i\right)}{f}+\frac{\ln\left(e^{i(fx+e)}+i\right)bd}{2f}$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output

```
1/f*(a*c*ln(sec(f*x+e)+tan(f*x+e))+a*d*tan(f*x+e)+b*c*tan(f*x+e)+b*d*(1/2*
sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$= \frac{(2\,ac+bd)\cos\left(fx+e\right)^{2}\log\left(\sin\left(fx+e\right)+1\right)-(2\,ac+bd)\cos\left(fx+e\right)^{2}\log\left(-\sin\left(fx+e\right)+1\right)+2\,(b}{4\,f\cos\left(fx+e\right)^{2}}$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="frica
s")
```

output

```
1/4*((2*a*c + b*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + b*d)*co
s(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(b*d + 2*(b*c + a*d)*cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$= \int (a+b\sec\left(e+fx\right))\left(c+d\sec\left(e+fx\right)\right)\sec\left(e+fx\right)\,dx$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

output

```
Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx =$$
$$-\frac{bd\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+\right.\right.}{4\,f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
-1/4*(b*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + l
og(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*b*c*tan
(f*x + e) - 4*a*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(57) = 114$.

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.51

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$=\frac{(2\,ac+bd)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-(2\,ac+bd)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(2\,bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\right.}{}}{2\,f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
1/2*((2*a*c + b*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - (2*a*c + b*d)*log(
abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(2*b*c*tan(1/2*f*x + 1/2*e)^3 + 2*a*d*t
an(1/2*f*x + 1/2*e)^3 - b*d*tan(1/2*f*x + 1/2*e)^3 - 2*b*c*tan(1/2*f*x + 1
/2*e) - 2*a*d*tan(1/2*f*x + 1/2*e) - b*d*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*
x + 1/2*e)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$=\frac{\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,a\,c+b\,d\right)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,a\,d+2\,b\,c+b\,d\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,a\,d+2\,b\,c-b\,d\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

input
```
int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

output
```
(atanh(tan(e/2 + (f*x)/2))*(2*a*c + b*d))/f + (tan(e/2 + (f*x)/2)*(2*a*d +
 2*b*c + b*d) - tan(e/2 + (f*x)/2)^3*(2*a*d + 2*b*c - b*d))/(f*(tan(e/2 +
(f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.57

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$=\frac{-2\cos\left(fx+e\right)\sin\left(fx+e\right)ad-2\cos\left(fx+e\right)\sin\left(fx+e\right)bc-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^2a}{}$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

output
```
( - 2*cos(e + f*x)*sin(e + f*x)*a*d - 2*cos(e + f*x)*sin(e + f*x)*b*c - 2*
log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*c - log(tan((e + f*x)/2) - 1)*
sin(e + f*x)**2*b*d + 2*log(tan((e + f*x)/2) - 1)*a*c + log(tan((e + f*x)/
2) - 1)*b*d + 2*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a*c + log(tan((e
 + f*x)/2) + 1)*sin(e + f*x)**2*b*d - 2*log(tan((e + f*x)/2) + 1)*a*c - lo
g(tan((e + f*x)/2) + 1)*b*d - sin(e + f*x)*b*d)/(2*f*(sin(e + f*x)**2 - 1)
)
```

3.248 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{b\operatorname{arctanh}(\sin(e+fx))}{df} - \frac{2(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,d\sqrt{c+d}\,f}$$

output
```
b*arctanh(sin(f*x+e))/d/f-2*(-a*d+b*c)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2
*e)/(c+d)^(1/2))/(c-d)^(1/2)/d/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{\frac{2(bc-ad)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + b\big(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}{df}$$

input
```
Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

output
```
((2*(b*c - a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt
[c^2 - d^2] + b*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]))/(d*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4486}$$

$$\frac{b\int \sec(e+fx)dx}{d}-\frac{(bc-ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{d}-\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\text{arctanh}(\sin(e+fx))}{df}-\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{d}$$

$$\Big\downarrow \text{4318}$$

$$\frac{b\text{arctanh}(\sin(e+fx))}{df}-\frac{(bc-ad)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\text{arctanh}(\sin(e+fx))}{df}-\frac{(bc-ad)\int\frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{d^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{b\text{arctanh}(\sin(e+fx))}{df}-\frac{2(bc-ad)\int\frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{d^2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\text{arctanh}(\sin(e+fx))}{df}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c-d}\sqrt{c+d}}$$

input
```
Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

output
```
(b*ArcTanh[Sin[e + f*x]])/(d*f) - (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[
(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*d*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2(-ad+bc)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{d\sqrt{(c-d)(c+d)}}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{d}+\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{d}}{f}$
default	$\frac{-\frac{2(-ad+bc)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{d\sqrt{(c-d)(c+d)}}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{d}+\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{d}}{f}$
risch	$\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+d\sqrt{c^2-d^2}}{\sqrt{c^2-d^2}\,c}\right)a}{\sqrt{c^2-d^2}\,f}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+d\sqrt{c^2-d^2}}{\sqrt{c^2-d^2}\,c}\right)bc}{\sqrt{c^2-d^2}\,fd}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-d\sqrt{c^2-d^2}}{\sqrt{c^2-d^2}\,c}\right)a}{\sqrt{c^2-d^2}\,f}$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/d*(-a*d+b*c)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/
((c-d)*(c+d))^(1/2))-b/d*ln(tan(1/2*f*x+1/2*e)-1)+b/d*ln(tan(1/2*f*x+1/2*e
)+1))
```

Fricas [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.16

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$$

$$= \Bigg[-\frac{(bc-ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)-(bc^2-b}{2\,(c^2d-d^3)f}$$

$$-\frac{2\,(bc-ad)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)-(bc^2-bd^2)\log\left(\sin\left(fx+e\right)+1\right)+(bc^2-b}{2\,(c^2d-d^3)f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="frica
s")
```

output
```
[-1/2*((b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)
*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*
c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (b*c^2 - b*d
^2)*log(sin(f*x + e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*
d - d^3)*f), -1/2*(2*(b*c - a*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)
*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (b*c^2 - b*d^2)*log(si
n(f*x + e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*d - d^3)*f
)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \int \frac{(a+b\sec\left(e+fx\right))\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output
```
Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$$
$$= \frac{\frac{b\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d} - \frac{b\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d} + \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(bc-}{\sqrt{-c^2+d^2}\,d}}{f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
(b*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d - b*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/d + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*ta
n(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(b*c - a*d
)/(sqrt(-c^2 + d^2)*d))/f
```

Mupad [B] (verification not implemented)

Time = 11.94 (sec) , antiderivative size = 573, normalized size of antiderivative = 7.54

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

output
```
(a*c^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/
2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (a*d^2*
log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2
 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (2*b*d*atanh(s
in(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)) - (a*log((c*cos(e/2
 + (f*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))
/cos(e/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(f*(c^2 - d^2)) + (b*c*d*log
((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 -
d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) + (2*b*c^2*atanh(si
n(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(c^2 - d^2)) - (b*c^3*log((c*si
n(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(
1/2))/cos(e/2 + (f*x)/2)))/(d*f*(c^2 - d^2)^(3/2)) + (b*c*log((c*cos(e/2 +
 (f*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/c
os(e/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(d*f*(c^2 - d^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.54

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$$
$$= \frac{2\sqrt{-c^2+d^2}\, atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right) ad - 2\sqrt{-c^2+d^2}\, atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right) bc - \log\left(\tan\right.}{df\,(c}$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output
```
(2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqr
t( - c**2 + d**2))*a*d - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
 tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*b*c - log(tan((e + f*x)/2) - 1)
*b*c**2 + log(tan((e + f*x)/2) - 1)*b*d**2 + log(tan((e + f*x)/2) + 1)*b*c
**2 - log(tan((e + f*x)/2) + 1)*b*d**2)/(d*f*(c**2 - d**2))
```

3.249 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 99

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}$$

output

```
2*(a*c-b*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3/2
)/(c+d)^(3/2)/f+(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{-\frac{2(ac-bd)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}} + \frac{(bc-ad)\sin(e+fx)}{(c-d)(c+d)(d+c\cos(e+fx))}}{f}$$

input

```
Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

output
```
((-2*(a*c - b*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^
2 - d^2)^(3/2) + ((b*c - a*d)*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e
+ f*x])))/f
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = $\{3042, 4491, 25, 27, 3042, 4318, 3042, 3138, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4491}$$

$$\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}-\frac{\int -\frac{(ac-bd)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ac-bd)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}+\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ac-bd)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}+\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(ac-bd)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c^2-d^2}+\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{4318}$$

$$\frac{(ac-bd)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{d\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(ac-bd)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{d\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{2(ac-bd)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{df\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

input

```
Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

output

```
(2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[
c - d]*Sqrt[c + d]*(c^2 - d^2)*f) + ((b*c - a*d)*Tan[e + f*x])/((c^2 - d^2
)*f*(c + d*Sec[e + f*x]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^{2}-d^{2}\right)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}+\frac{2(ac-bd)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c-d)(c+d)\sqrt{(c-d)(c+d)}}}{f}$
default	$\frac{\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^{2}-d^{2}\right)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}+\frac{2(ac-bd)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c-d)(c+d)\sqrt{(c-d)(c+d)}}}{f}$
risch	$\frac{2i(-ad+bc)\left(d\,\mathrm{e}^{i(fx+e)}+c\right)}{c(c^{2}-d^{2})f\left(c\,\mathrm{e}^{2i(fx+e)}+2d\,\mathrm{e}^{i(fx+e)}+c\right)}+\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^{2}-id^{2}+d\sqrt{c^{2}-d^{2}}}{\sqrt{c^{2}-d^{2}}\,c}\right)ac}{\sqrt{c^{2}-d^{2}}\,(c+d)(c-d)f}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^{2}-id^{2}+d\sqrt{c^{2}-d^{2}}}{\sqrt{c^{2}-d^{2}}\,c}\right.}{\sqrt{c^{2}-d^{2}}\,(c+d)(c-d)f}$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/f*(2*(a*d-b*c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-tan(
1/2*f*x+1/2*e)^2*d-c-d)+2*(a*c-b*d)/(c-d)/(c+d)/((c-d)*(c+d))^(1/2)*arctan
h((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.93

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$$
$$= \Bigg[\frac{(acd-bd^2+(ac^2-bcd)\cos(fx+e))\sqrt{c^2-d^2}\log\bigg(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}}{2\,((c^5-2\,c^3d^2+cd^4)f\cos(fx+e)+(c^4d-2\,c^2d^3}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output
```
[1/2*((a*c*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(c^2 - d^2)*log((
2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*c
os(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(b*c^3 - a*c^2*d - b*c*d^2 + a*d^3)*sin(f*x + e))/
((c^5 - 2*c^3*d^2 + c*d^4)*f*cos(f*x + e) + (c^4*d - 2*c^2*d^3 + d^5)*f),
((a*c*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-s
qrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^3
- a*c^2*d - b*c*d^2 + a*d^3)*sin(f*x + e))/((c^5 - 2*c^3*d^2 + c*d^4)*f*co
s(f*x + e) + (c^4*d - 2*c^2*d^3 + d^5)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a+b\sec{(e+fx)})\sec{(e+fx)}}{(c+d\sec{(e+fx)})^2}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx =$$

$$-\frac{2\Bigg(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(ac-bd)}{(c^2-d^2)\sqrt{-c^2+d^2}}+\frac{bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)}}{f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
-2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f
*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(a*c - b*d)/((c^2
 - d^2)*sqrt(-c^2 + d^2)) + (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x +
1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)*(c^
2 - d^2)))/f
```

Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)\,\left(a\,c-b\,d\right)}{f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{3/2}} - \frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,d-b\,c\right)}{f\,\left(c+d\right)\,\left(c-d\right)\,\left(\left(d-c\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+c+d\right)}$$

input
```
int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

output
```
(2*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(a*c - b*d))/(f
*(c + d)^(3/2)*(c - d)^(3/2)) - (2*tan(e/2 + (f*x)/2)*(a*d - b*c))/(f*(c +
 d)*(c - d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.36

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{2\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)\cos\left(fx+e\right)a\,c^2-2\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right.}{}$$

input

```
int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

output

```
(2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqr
t( - c**2 + d**2))*cos(e + f*x)*a*c**2 - 2*sqrt( - c**2 + d**2)*atan((tan(
(e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*b*
c*d + 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*a*c*d - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)
/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*b*d**2 - sin(e + f*x)*a*
c**2*d + sin(e + f*x)*a*d**3 + sin(e + f*x)*b*c**3 - sin(e + f*x)*b*c*d**2
)/(f*(cos(e + f*x)*c**5 - 2*cos(e + f*x)*c**3*d**2 + cos(e + f*x)*c*d**4 +
 c**4*d - 2*c**2*d**3 + d**5))
```

3.250 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 166

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = -\frac{(3bcd-a(2c^2+d^2))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))}$$

output

```
-(3*b*c*d-a*(2*c^2+d^2))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2
))/(c-d)^(5/2)/(c+d)^(5/2)/f+1/2*(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/f/(c+d*se
c(f*x+e))^2-1/2*(3*a*c*d-b*(c^2+2*d^2))*tan(f*x+e)/(c^2-d^2)^2/f/(c+d*sec(
f*x+e))
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{-\frac{2(-3bcd+a(2c^2+d^2))\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}}+\frac{d(-bc+ad)\sin(e+fx)}{c(c-d)(c+d)(d+c\cos(e+fx))^2}+\frac{(ad(-4c^2+d^2)+bc(2c^2+d^2))\sin(e+fx)}{c(c-d)^2(c+d)^2(d+c\cos(e+fx))}}{2f}$$

input
```
Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

output
```
((-2*(-3*b*c*d + a*(2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt
[c^2 - d^2]])/(c^2 - d^2)^(5/2) + (d*(-(b*c) + a*d)*Sin[e + f*x])/(c*(c -
d)*(c + d)*(d + c*Cos[e + f*x])^2) + ((a*d*(-4*c^2 + d^2) + b*c*(2*c^2 + d
^2))*Sin[e + f*x])/(c*(c - d)^2*(c + d)^2*(d + c*Cos[e + f*x])))/(2*f)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {3042, 4491, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4491}$$

$$\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}-\frac{\int -\frac{\sec(e+fx)(2(ac-bd)+(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^2}dx}{2\left(c^2-d^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(e+fx)(2(ac-bd)+(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^2}dx}{2\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(2(ac-bd)+(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{4491}$$

$$\frac{-\frac{\int \frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c^2-d^2}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{4318}$$

$$\frac{-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{d\left(c^2-d^2\right)}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{d\left(c^2-d^2\right)}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{-\frac{2\left(3bcd-a\left(2c^2+d^2\right)\right)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{df\left(c^2-d^2\right)}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{f\left(c^2-d^2\right)\left(c+d\sec(e+fx)\right)}}{2\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)\left(c+d\sec(e+fx)\right)^2}$$

$$\Big\downarrow 221$$

$$\frac{-\frac{2\left(3bcd-a\left(2c^2+d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}\left(c^2-d^2\right)}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{f\left(c^2-d^2\right)\left(c+d\sec(e+fx)\right)}}{2\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)\left(c+d\sec(e+fx)\right)^2}$$

input
```
Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

output
```
((b*c - a*d)*Tan[e + f*x])/(2*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^2) + ((-2
*(3*b*c*d - a*(2*c^2 + d^2))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c
 + d]])/(Sqrt[c - d]*Sqrt[c + d]*(c^2 - d^2)*f) - ((3*a*c*d - b*(c^2 + 2*d
^2))*Tan[e + f*x])/((c^2 - d^2)*f*(c + d*Sec[e + f*x])))/(2*(c^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(4acd+a d^2-2b c^2-bcd-2b d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{\left(4acd-a d^2-2b c^2+bcd-2b d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c^2-2cd+d^2\right)}\right)}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 d-c-d\right)^2}+\frac{\left(2a c^2+a d^2-3bcd\right)\mathrm{arcta}}{\left(c^4-2c^2d^2+d\right.}}{f}$
default	$\frac{-\frac{2\left(-\frac{\left(4acd+a d^2-2b c^2-bcd-2b d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{\left(4acd-a d^2-2b c^2+bcd-2b d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c^2-2cd+d^2\right)}\right)}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 d-c-d\right)^2}+\frac{\left(2a c^2+a d^2-3bcd\right)\mathrm{arcta}}{\left(c^4-2c^2d^2+d\right.}}{f}$
risch	$\frac{i\left(-5a c^3d^2\mathrm{e}^{3i(fx+e)}+2ac d^4\mathrm{e}^{3i(fx+e)}+3b c^4d\,\mathrm{e}^{3i(fx+e)}-4a c^4d\,\mathrm{e}^{2i(fx+e)}-7a c^2d^3\mathrm{e}^{2i(fx+e)}+2a d^5\mathrm{e}^{2i(fx+e)}+2b c^5\mathrm{e}^{2}\right.}{c^2\left(-c^2+\right.}$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/f*(-2*(-1/2*(4*a*c*d+a*d^2-2*b*c^2-b*c*d-2*b*d^2)/(c-d)/(c^2+2*c*d+d^2)*
tan(1/2*f*x+1/2*e)^3+1/2*(4*a*c*d-a*d^2-2*b*c^2+b*c*d-2*b*d^2)/(c+d)/(c^2-
2*c*d+d^2)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^
2*d-c-d)^2+(2*a*c^2+a*d^2-3*b*c*d)/(c^4-2*c^2*d^2+d^4)/((c-d)*(c+d))^(1/2)
*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(153) = 306$.

Time = 0.20 (sec) , antiderivative size = 752, normalized size of antiderivative = 4.53

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fri
cas")
```

output
```
[1/4*((2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)
*cos(f*x + e)^2 + 2*(2*a*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt
(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqr
t(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x
 + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^
3 + 3*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a*c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 -
 b*c*d^4 - a*d^5)*cos(f*x + e))*sin(f*x + e))/((c^8 - 3*c^6*d^2 + 3*c^4*d^
4 - c^2*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*
f*cos(f*x + e) + (c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f), 1/2*((2*a*c^2
*d^2 - 3*b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)*cos(f*x + e)^
2 + 2*(2*a*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*a
rctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) +
 (b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a
*c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d^4 - a*d^5)*cos(f*x + e))*sin(f*x
+ e))/((c^8 - 3*c^6*d^2 + 3*c^4*d^4 - c^2*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d
 - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*cos(f*x + e) + (c^6*d^2 - 3*c^4*d^4 +
3*c^2*d^6 - d^8)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a+b\sec{(e+fx)})\sec{(e+fx)}}{(c+d\sec{(e+fx)})^3}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(153) = 306.

Time = 0.20 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{\frac{(2\,ac^2-3\,bcd+ad^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4-2\,c^2d^2+d^4)\sqrt{-c^2+d^2}} - \frac{2\,bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,ac^2d\tan\left(\frac{1}{2}\,fx}{}}{}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="gia
c")
```

output
```
((2*a*c^2 - 3*b*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2
 + d^2)))/((c^4 - 2*c^2*d^2 + d^4)*sqrt(-c^2 + d^2)) - (2*b*c^3*tan(1/2*f*
x + 1/2*e)^3 - 4*a*c^2*d*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d*tan(1/2*f*x + 1/
2*e)^3 + 3*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + b*c*d^2*tan(1/2*f*x + 1/2*e)^3
 + a*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*c^3
*tan(1/2*f*x + 1/2*e) + 4*a*c^2*d*tan(1/2*f*x + 1/2*e) - b*c^2*d*tan(1/2*f
*x + 1/2*e) + 3*a*c*d^2*tan(1/2*f*x + 1/2*e) - b*c*d^2*tan(1/2*f*x + 1/2*e
) - a*d^3*tan(1/2*f*x + 1/2*e) - 2*b*d^3*tan(1/2*f*x + 1/2*e))/((c^4 - 2*c
^2*d^2 + d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d
)^2))/f
```

Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$
$$=\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,d^2+2\,b\,c^2+2\,b\,d^2-4\,a\,c\,d-b\,c\,d\right)}{\left(c+d\right)\,\left(c^2-2\,c\,d+d^2\right)}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,b\,c^2-a\,d^2+2\,b\,d^2-4\,a\,c\,d+b\,c\,d\right)}{{\left(c+d\right)}^2\,\left(c-d\right)}}{f\,\left(2\,c\,d-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2-2\,d^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(c^2-2\,c\,d+d^2\right)+c^2+d^2\right)}$$
$$+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-2\,d\right)\,\left(c^2-2\,c\,d+d^2\right)}{2\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}}\right)\,\left(2\,a\,c^2-3\,b\,c\,d+a\,d^2\right)}{f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{5/2}}$$

input
```
int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

output

```
((tan(e/2 + (f*x)/2)*(a*d^2 + 2*b*c^2 + 2*b*d^2 - 4*a*c*d - b*c*d))/((c +
d)*(c^2 - 2*c*d + d^2)) - (tan(e/2 + (f*x)/2)^3*(2*b*c^2 - a*d^2 + 2*b*d^2
 - 4*a*c*d + b*c*d))/((c + d)^2*(c - d)))/(f*(2*c*d - tan(e/2 + (f*x)/2)^2
*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2))
+ (atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(c^2 - 2*c*d + d^2))/(2*(c + d)^(
1/2)*(c - d)^(5/2)))*(2*a*c^2 + a*d^2 - 3*b*c*d))/(f*(c + d)^(5/2)*(c - d)
^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 977, normalized size of antiderivative = 5.89

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

output

```
(8*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqr
t( - c**2 + d**2))*cos(e + f*x)*a*c**3*d + 4*sqrt( - c**2 + d**2)*atan((ta
n((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*
a*c*d**3 - 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x
)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*b*c**2*d**2 - 4*sqrt( - c**2 +
d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))
*sin(e + f*x)**2*a*c**4 - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c
- tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*a*c**2*d**2 +
6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt
( - c**2 + d**2))*sin(e + f*x)**2*b*c**3*d + 4*sqrt( - c**2 + d**2)*atan((
tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*a*c**4 + 6*
sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt(
- c**2 + d**2))*a*c**2*d**2 + 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2
)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*a*d**4 - 6*sqrt( - c**2 +
d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))
*b*c**3*d - 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x
)/2)*d)/sqrt( - c**2 + d**2))*b*c*d**3 - 4*cos(e + f*x)*sin(e + f*x)*a*c**
4*d + 5*cos(e + f*x)*sin(e + f*x)*a*c**2*d**3 - cos(e + f*x)*sin(e + f*x)*
a*d**5 + 2*cos(e + f*x)*sin(e + f*x)*b*c**5 - cos(e + f*x)*sin(e + f*x)*b*
c**3*d**2 - cos(e + f*x)*sin(e + f*x)*b*c*d**4 - 3*sin(e + f*x)*a*c**3*...
```

3.251 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 237

$$\begin{aligned}\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx &= \frac{(2ac^3-4bc^2d+3acd^2-bd^3)\,\mathrm{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{7/2}(c+d)^{7/2}f}\\ &\quad+\frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3}+\frac{(2bc^2-5acd+3bd^2)\tan(e+fx)}{6\left(c^2-d^2\right)^2f(c+d\sec(e+fx))^2}\\ &\quad+\frac{(2bc^3-11ac^2d+13bcd^2-4ad^3)\tan(e+fx)}{6\left(c^2-d^2\right)^3f(c+d\sec(e+fx))}\end{aligned}$$

output
```
(2*a*c^3+3*a*c*d^2-4*b*c^2*d-b*d^3)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)
/(c+d)^(1/2))/(c-d)^(7/2)/(c+d)^(7/2)/f+1/3*(-a*d+b*c)*tan(f*x+e)/(c^2-d^2
)/f/(c+d*sec(f*x+e))^3+1/6*(-5*a*c*d+2*b*c^2+3*b*d^2)*tan(f*x+e)/(c^2-d^2)
^2/f/(c+d*sec(f*x+e))^2+1/6*(-11*a*c^2*d-4*a*d^3+2*b*c^3+13*b*c*d^2)*tan(f
*x+e)/(c^2-d^2)^3/f/(c+d*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.71

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec^3(e+fx)(a+b\sec(e+fx))\left(\frac{24(-bd(4c^2+d^2)+a(2c^3+3cd^2))\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}\right.}{}$$

input

```
Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

output

```
((d + c*Cos[e + f*x])*Sec[e + f*x]^3*(a + b*Sec[e + f*x])*((24*(-(b*d*(4*c
^2 + d^2)) + a*(2*c^3 + 3*c*d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt
[c^2 - d^2]]*(d + c*Cos[e + f*x])^3)/Sqrt[c^2 - d^2] - 6*b*c^5*Sin[e + f*x
] + 18*a*c^4*d*Sin[e + f*x] - 18*b*c^3*d^2*Sin[e + f*x] + 39*a*c^2*d^3*Sin
[e + f*x] - 51*b*c*d^4*Sin[e + f*x] + 18*a*d^5*Sin[e + f*x] - 12*b*c^4*d*S
in[2*(e + f*x)] + 54*a*c^3*d^2*Sin[2*(e + f*x)] - 54*b*c^2*d^3*Sin[2*(e +
f*x)] + 6*a*c*d^4*Sin[2*(e + f*x)] + 6*b*d^5*Sin[2*(e + f*x)] - 6*b*c^5*Si
n[3*(e + f*x)] + 18*a*c^4*d*Sin[3*(e + f*x)] - 10*b*c^3*d^2*Sin[3*(e + f*x
)] - 5*a*c^2*d^3*Sin[3*(e + f*x)] + b*c*d^4*Sin[3*(e + f*x)] + 2*a*d^5*Sin
[3*(e + f*x)]))/(24*(-c^2 + d^2)^3*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f
*x])^4)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.17, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {3042, 4491, 25, 3042, 4491, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4491}$$

$$\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}-\frac{\int -\frac{\sec(e+fx)(3(ac-bd)+2(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^3}dx}{3\left(c^2-d^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(e+fx)(3(ac-bd)+2(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^3}dx}{3\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(3(ac-bd)+2(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{3\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \text{4491}$$

$$\frac{\frac{\left(-5acd+2bc^2+3bd^2\right)\tan(e+fx)}{2f(c^2-d^2)(c+d\sec(e+fx))^2}-\frac{\int -\frac{\sec(e+fx)\left(2\left(3ac^2-5bdc+2ad^2\right)+\left(2bc^2-5adc+3bd^2\right)\sec(e+fx)\right)}{(c+d\sec(e+fx))^2}dx}{2(c^2-d^2)}}{3\left(c^2-d^2\right)}+$$
$$\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\sec(e+fx)\left(2\left(3ac^2-5bdc+2ad^2\right)+\left(2bc^2-5adc+3bd^2\right)\sec(e+fx)\right)}{(c+d\sec(e+fx))^2}dx}{2(c^2-d^2)}+\frac{\left(-5acd+2bc^2+3bd^2\right)\tan(e+fx)}{2f(c^2-d^2)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+$$
$$\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(2\left(3ac^2-5bdc+2ad^2\right)+\left(2bc^2-5adc+3bd^2\right)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2(c^2-d^2)}+\frac{\left(-5acd+2bc^2+3bd^2\right)\tan(e+fx)}{2f(c^2-d^2)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+$$
$$\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \text{4491}$$

$$\frac{\frac{\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}-\frac{\int -\frac{3\left(2ac^3-4bdc^2+3ad^2c-bd^3\right)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}}{2(c^2-d^2)}+\frac{(-5acd+2bc^2+3bd^2)\tan(e+fx)}{2f(c^2-d^2)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c^2-d^2}+\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{(-5acd+2bc^2+3bd^2)\tan(e+fx)}{2f(c^2-d^2)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c^2-d^2}+\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{(-5acd+2bc^2+3bd^2)\tan(e+fx)}{2f(c^2-d^2)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{3\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{d\left(c^2-d^2\right)}+\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{(-5acd+2bc^2+3bd^2)\tan(e+fx)}{2f(c^2-d^2)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{d\left(c^2-d^2\right)}+\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{(-5acd+2bc^2+3bd^2)\tan(e+fx)}{2f(c^2-d^2)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{6\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{df\left(c^2-d^2\right)}+\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{\left(-5acd+2bc^2+3bd^2\right)\tan(e}{2f(c^2-d^2)(c+d\sec(e+f}}{3\left(c^2-d^2\right)}$$

$$\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

$$\Big\downarrow 221$$

$$\frac{\frac{\frac{6\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}\left(c^2-d^2\right)}+\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}}{2(c^2-d^2)}+\frac{\left(-5acd+2bc^2+3bd^2\right)\tan(e+fx)}{2f(c^2-d^2)(c+d\sec(e+fx))^2}}{3\left(c^2-d^2\right)}+$$

$$\frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

input
```
Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

output
```
((b*c - a*d)*Tan[e + f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + (((2
*b*c^2 - 5*a*c*d + 3*b*d^2)*Tan[e + f*x])/(2*(c^2 - d^2)*f*(c + d*Sec[e +
f*x])^2) + ((6*(2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*ArcTanh[(Sqrt[c -
 d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*(c^2 - d^2)*f
) + ((2*b*c^3 - 11*a*c^2*d + 13*b*c*d^2 - 4*a*d^3)*Tan[e + f*x])/((c^2 - d
^2)*f*(c + d*Sec[e + f*x])))/(2*(c^2 - d^2)))/(3*(c^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.59

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{\left(6a\,c^{2}d+3ac\,d^{2}+2a\,d^{3}-2b\,c^{3}-2b\,c^{2}d-6bc\,d^{2}-b\,d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{2(c-d)\left(c^{3}+3c^{2}d+3c\,d^{2}+d^{3}\right)}+\frac{2\left(9a\,c^{2}d+a\,d^{3}-3b\,c^{3}-7bc\,d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3\left(c^{2}+2cd+d^{2}\right)\left(c^{2}-2cd+d^{2}\right)}-\frac{\left(6a\,c^{2}\right.}{}\right.}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{3}}}{f}$
default	$-\frac{\frac{2\left(-\frac{\left(6a\,c^{2}d+3ac\,d^{2}+2a\,d^{3}-2b\,c^{3}-2b\,c^{2}d-6bc\,d^{2}-b\,d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{2(c-d)\left(c^{3}+3c^{2}d+3c\,d^{2}+d^{3}\right)}+\frac{2\left(9a\,c^{2}d+a\,d^{3}-3b\,c^{3}-7bc\,d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3\left(c^{2}+2cd+d^{2}\right)\left(c^{2}-2cd+d^{2}\right)}-\frac{\left(6a\,c^{2}\right.}{}\right.}{\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{3}}}{f}$
risch	Expression too large to display

input

```
int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE
)
```

output

```
1/f*(-2*(-1/2*(6*a*c^2*d+3*a*c*d^2+2*a*d^3-2*b*c^3-2*b*c^2*d-6*b*c*d^2-b*d
^3)/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(9*a*c^2*d+a*
d^3-3*b*c^3-7*b*c*d^2)/(c^2+2*c*d+d^2)/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e)^
3-1/2*(6*a*c^2*d-3*a*c*d^2+2*a*d^3-2*b*c^3+2*b*c^2*d-6*b*c*d^2+b*d^3)/(c+d
)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-ta
n(1/2*f*x+1/2*e)^2*d-c-d)^3+(2*a*c^3+3*a*c*d^2-4*b*c^2*d-b*d^3)/(c^6-3*c^4
*d^2+3*c^2*d^4-d^6)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/(
(c-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(222) = 444$.

Time = 0.23 (sec) , antiderivative size = 1238, normalized size of antiderivative = 5.22

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fri
cas")
```

output

```
[1/12*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2*a*c^6 - 4*b*c
^5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*d - 4*b*c^4*d^
2 + 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2 - 4*b*c^3*d^3
 + 3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x
 + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) +
 c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) +
 d^2)) + 2*(2*b*c^5*d^2 - 11*a*c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b
*c*d^6 + 4*a*d^7 + (6*b*c^7 - 18*a*c^6*d + 4*b*c^5*d^2 + 23*a*c^4*d^3 - 11
*b*c^3*d^4 - 7*a*c^2*d^5 + b*c*d^6 + 2*a*d^7)*cos(f*x + e)^2 + 3*(2*b*c^6*
d - 9*a*c^5*d^2 + 7*b*c^4*d^3 + 8*a*c^3*d^4 - 10*b*c^2*d^5 + a*c*d^6 + b*d
^7)*cos(f*x + e))*sin(f*x + e))/((c^11 - 4*c^9*d^2 + 6*c^7*d^4 - 4*c^5*d^6
 + c^3*d^8)*f*cos(f*x + e)^3 + 3*(c^10*d - 4*c^8*d^3 + 6*c^6*d^5 - 4*c^4*d
^7 + c^2*d^9)*f*cos(f*x + e)^2 + 3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^
3*d^8 + c*d^10)*f*cos(f*x + e) + (c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*
d^9 + d^11)*f), 1/6*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2
*a*c^6 - 4*b*c^5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*
d - 4*b*c^4*d^2 + 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2
 - 4*b*c^3*d^3 + 3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arc
tan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (
2*b*c^5*d^2 - 11*a*c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b*c*d^6 + ...
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx=\int \frac{(a+b\sec{(e+fx)})\sec{(e+fx)}}{(c+d\sec{(e+fx)})^4}\,dx$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
```

output

```
Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(222) = 444.

Time = 0.23 (sec) , antiderivative size = 693, normalized size of antiderivative = 2.92

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="gia
c")
```

output
```
-1/3*(3*(2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/
pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x
+ 1/2*e))/sqrt(-c^2 + d^2)))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(-c^
2 + d^2)) + (6*b*c^5*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^4*d*tan(1/2*f*x + 1/2
*e)^5 - 6*b*c^4*d*tan(1/2*f*x + 1/2*e)^5 + 27*a*c^3*d^2*tan(1/2*f*x + 1/2*
e)^5 + 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 6*a*c^2*d^3*tan(1/2*f*x + 1/2
*e)^5 - 27*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a*c*d^4*tan(1/2*f*x + 1/2*
e)^5 + 12*b*c*d^4*tan(1/2*f*x + 1/2*e)^5 - 6*a*d^5*tan(1/2*f*x + 1/2*e)^5
+ 3*b*d^5*tan(1/2*f*x + 1/2*e)^5 - 12*b*c^5*tan(1/2*f*x + 1/2*e)^3 + 36*a*
c^4*d*tan(1/2*f*x + 1/2*e)^3 - 16*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 32*a*
c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 28*b*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*a*d
^5*tan(1/2*f*x + 1/2*e)^3 + 6*b*c^5*tan(1/2*f*x + 1/2*e) - 18*a*c^4*d*tan(
1/2*f*x + 1/2*e) + 6*b*c^4*d*tan(1/2*f*x + 1/2*e) - 27*a*c^3*d^2*tan(1/2*f
*x + 1/2*e) + 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 6*a*c^2*d^3*tan(1/2*f*x
+ 1/2*e) + 27*b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 3*a*c*d^4*tan(1/2*f*x + 1/2
*e) + 12*b*c*d^4*tan(1/2*f*x + 1/2*e) - 6*a*d^5*tan(1/2*f*x + 1/2*e) - 3*b
*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*(c*tan(1/2
*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

Mupad [B] (verification not implemented)

Time = 15.60 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.85

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(2\,b\,c^3-2\,a\,d^3+b\,d^3-3\,a\,c\,d^2-6\,a\,c^2\,d+6\,b\,c\,d^2+2\,b\,c^2\,d\right)}{{\left(c+d\right)}^3\,\left(c-d\right)}+\frac{4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(-3\,b\,c^3+9\,a\,c^2\,d-7\,b\,c\,d^2+a\,d^3\right)}{3\,{\left(c+d\right)}^2\,\left(c^2-2\,c\,d+d^2\right)}-}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3\right)+3\,c\,d\right.}$$

$$+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-2\,d\right)\,\left(c^3-3\,c^2\,d+3\,c\,d^2-d^3\right)}{2\,\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}\right)\,\left(2\,a\,c^3-4\,b\,c^2\,d+3\,a\,c\,d^2-b\,d^3\right)}{f\,{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{7/2}}$$

input
```
int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

output

```
((tan(e/2 + (f*x)/2)^5*(2*b*c^3 - 2*a*d^3 + b*d^3 - 3*a*c*d^2 - 6*a*c^2*d
+ 6*b*c*d^2 + 2*b*c^2*d))/((c + d)^3*(c - d)) + (4*tan(e/2 + (f*x)/2)^3*(a
*d^3 - 3*b*c^3 + 9*a*c^2*d - 7*b*c*d^2))/(3*(c + d)^2*(c^2 - 2*c*d + d^2))
 - (tan(e/2 + (f*x)/2)*(2*a*d^3 - 2*b*c^3 + b*d^3 - 3*a*c*d^2 + 6*a*c^2*d
- 6*b*c*d^2 + 2*b*c^2*d))/((c + d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(t
an(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/
2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 -
 tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) + (atanh((tan(e/2
+ (f*x)/2)*(2*c - 2*d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3))/(2*(c + d)^(1/2)*(
c - d)^(7/2)))*(2*a*c^3 - b*d^3 + 3*a*c*d^2 - 4*b*c^2*d))/(f*(c + d)^(7/2)
*(c - d)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1900, normalized size of antiderivative = 8.02

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x)
```

output

```
(12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sq
rt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a*c**6 + 18*sqrt( - c**2
+ d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2
))*cos(e + f*x)*sin(e + f*x)**2*a*c**4*d**2 - 24*sqrt( - c**2 + d**2)*atan
((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f
*x)*sin(e + f*x)**2*b*c**5*d - 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/
2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)
**2*b*c**3*d**3 - 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((
e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a*c**6 - 54*sqrt( - c**2
 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**
2))*cos(e + f*x)*a*c**4*d**2 - 54*sqrt( - c**2 + d**2)*atan((tan((e + f*x)
/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a*c**2*d**4
 + 24*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*cos(e + f*x)*b*c**5*d + 78*sqrt( - c**2 + d**2)*atan
((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f
*x)*b*c**3*d**3 + 18*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((
e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*b*c*d**5 + 36*sqrt( - c*
*2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d
**2))*sin(e + f*x)**2*a*c**5*d + 54*sqrt( - c**2 + d**2)*atan((tan((e + f*
x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*a*c...
```

3.252 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 247

$$\begin{aligned}
\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx
&= \frac{d^3(4bc-ad)\operatorname{arctanh}(\sin(e+fx))}{2b^2f} \\
&\quad + \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\operatorname{arctanh}(\sin(e+fx))}{b^4f} \\
&\quad + \frac{2(bc-ad)^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}f} \\
&\quad + \frac{d^4\tan(e+fx)}{bf} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)\tan(e+fx)}{b^3f} \\
&\quad + \frac{d^3(4bc-ad)\sec(e+fx)\tan(e+fx)}{2b^2f} + \frac{d^4\tan^3(e+fx)}{3bf}
\end{aligned}$$

output

```
1/2*d^3*(-a*d+4*b*c)*arctanh(sin(f*x+e))/b^2/f+d*(-a*d+2*b*c)*(a^2*d^2-2*a
*b*c*d+2*b^2*c^2)*arctanh(sin(f*x+e))/b^4/f+2*(-a*d+b*c)^4*arctanh((a-b)^(
1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/(a-b)^(1/2)/b^4/(a+b)^(1/2)/f+d^4*tan
(f*x+e)/b/f+d^2*(a^2*d^2-4*a*b*c*d+6*b^2*c^2)*tan(f*x+e)/b^3/f+1/2*d^3*(-a
*d+4*b*c)*sec(f*x+e)*tan(f*x+e)/b^2/f+1/3*d^4*tan(f*x+e)^3/b/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 580 vs. $2(247) = 494$.

Time = 5.24 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.35

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx$$

$$= \frac{\cos^3(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^4\left(-\frac{24(bc-ad)^4\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-6d(8a^2bc\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```

output
```
(Cos[e + f*x]^3*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((-24*(b*c - a
*d)^4*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2
] - 6*d*(8*a^2*b*c*d^2 - 2*a^3*d^3 + 4*b^3*c*(2*c^2 + d^2) - a*b^2*d*(12*c
^2 + d^2))*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 6*d*(-8*a^2*b*c*d^2
+ 2*a^3*d^3 - 4*b^3*c*(2*c^2 + d^2) + a*b^2*d*(12*c^2 + d^2))*Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^3*(-3*a*d + b*(12*c + d)))/(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^2 + (2*b^3*d^4*Sin[(e + f*x)/2])/(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^3 + (4*b*d^2*(-12*a*b*c*d + 3*a^2*d^2 + 2*b^2*(9
*c^2 + d^2))*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (2*
b^3*d^4*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (b^2*d
^3*(-3*a*d + b*(12*c + d)))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*b
*d^2*(-12*a*b*c*d + 3*a^2*d^2 + 2*b^2*(9*c^2 + d^2))*Sin[(e + f*x)/2])/(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])))/(12*b^4*f*(d + c*Cos[e + f*x])^4*(a +
 b*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4476, 3042, 3431, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4476$$

$$\int \frac{\sec^4(e+fx)(c\cos(e+fx)+d)^4}{a\cos(e+fx)+b}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^4}{\sin\left(e+fx+\frac{\pi}{2}\right)^4\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\Big\downarrow \; 3431$$

$$\int \left(\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)\sec(e+fx)}{b^4}+\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\sec^2(e+fx)}{b^3}+\frac{(bc-ad)}{b^4(a\cos(e+f}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)\text{arctanh}(\sin(e+fx))}{b^4f}+\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\tan(e+fx)}{b^3f}+\frac{2(bc-ad)^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^4f\sqrt{a-b}\sqrt{a+b}}+\frac{d^3(4bc-ad)\text{arctanh}(\sin(e+fx))}{2b^2f}+\frac{d^3(4bc-ad)\tan(e+fx)\sec(e+fx)}{2b^2f}+\frac{d^4\tan^3(e+fx)}{3bf}+\frac{d^4\tan(e+fx)}{bf}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```

output
```
(d^3*(4*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d*(2*b*c - a*d)*(2*
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c -
a*d)^4*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b
^4*Sqrt[a + b]*f) + (d^4*Tan[e + f*x])/(b*f) + (d^2*(6*b^2*c^2 - 4*a*b*c*d
 + a^2*d^2)*Tan[e + f*x])/(b^3*f) + (d^3*(4*b*c - a*d)*Sec[e + f*x]*Tan[e
+ f*x])/(2*b^2*f) + (d^4*Tan[e + f*x]^3)/(3*b*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3431
```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

rule 4476
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(232) = 464.

Time = 0.89 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.94

method	result
derivativedivides	$-\frac{d^{4}}{3b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{d\left(2a^{3}d^{3}-8a^{2}bc\,d^{2}+12a\,b^{2}c^{2}d+a\,b^{2}d^{3}-8c^{3}b^{3}-4c\,d^{2}b^{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^{4}}-\frac{d^{2}(2a^{2}d^{2}-8abcd+b\,d^{2}}{2b^{3}(\tan(}$
default	$-\frac{d^{4}}{3b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{d\left(2a^{3}d^{3}-8a^{2}bc\,d^{2}+12a\,b^{2}c^{2}d+a\,b^{2}d^{3}-8c^{3}b^{3}-4c\,d^{2}b^{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^{4}}-\frac{d^{2}(2a^{2}d^{2}-8abcd+b\,d^{2}}{2b^{3}(\tan(}$
risch	Expression too large to display

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output

```
1/f*(-1/3*d^4/b/(tan(1/2*f*x+1/2*e)+1)^3-1/2*d*(2*a^3*d^3-8*a^2*b*c*d^2+12
*a*b^2*c^2*d+a*b^2*d^3-8*b^3*c^3-4*b^3*c*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)+1)
-1/2*d^2*(2*a^2*d^2-8*a*b*c*d+a*b*d^2+12*b^2*c^2-4*b^2*c*d+2*b^2*d^2)/b^3/
(tan(1/2*f*x+1/2*e)+1)+1/2*d^3*(a*d-4*b*c+b*d)/b^2/(tan(1/2*f*x+1/2*e)+1)^
2-2/b^4*(-a^4*d^4+4*a^3*b*c*d^3-6*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d-b^4*c^4)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))-1
/3*d^4/b/(tan(1/2*f*x+1/2*e)-1)^3+1/2*d*(2*a^3*d^3-8*a^2*b*c*d^2+12*a*b^2*
c^2*d+a*b^2*d^3-8*b^3*c^3-4*b^3*c*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)-1)-1/2*d^
2*(2*a^2*d^2-8*a*b*c*d+a*b*d^2+12*b^2*c^2-4*b^2*c*d+2*b^2*d^2)/b^3/(tan(1/
2*f*x+1/2*e)-1)-1/2*d^3*(a*d-4*b*c+b*d)/b^2/(tan(1/2*f*x+1/2*e)-1)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(232) = 464$.

Time = 150.81 (sec) , antiderivative size = 1093, normalized size of antiderivative = 4.43

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
[1/12*(6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*sqrt(a^2 - b^2)*cos(f*x + e)^3*log((2*a*b*cos(f*x + e) - (a^2 - 2*b
^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) +
 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + 3*(8*(a^2
*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 -
b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)^3*log(sin(f*x + e
) + 1) - 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*
a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)
^3*log(-sin(f*x + e) + 1) + 2*(2*(a^2*b^3 - b^5)*d^4 + 2*(18*(a^2*b^3 - b^
5)*c^2*d^2 - 12*(a^3*b^2 - a*b^4)*c*d^3 + (3*a^4*b - a^2*b^3 - 2*b^5)*d^4)
*cos(f*x + e)^2 + 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*d^4)*cos(
f*x + e))*sin(f*x + e))/((a^2*b^4 - b^6)*f*cos(f*x + e)^3), 1/12*(12*(b^4*
c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a
^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f
*x + e)))*cos(f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^
4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)
*d^4)*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*(8*(a^2*b^3 - b^5)*c^3*d -
12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5
- a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*(a^2*
b^3 - b^5)*d^4 + 2*(18*(a^2*b^3 - b^5)*c^2*d^2 - 12*(a^3*b^2 - a*b^4)*c...
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \int \frac{(c+d\sec{(e+fx)})^4\sec{(e+fx)}}{a+b\sec{(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e)),x)
```

output
```
Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(232) = 464$.

Time = 0.22 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.45

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="gia
c")
```

output

```
1/6*(3*(8*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a
^3*d^4 - a*b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^4 - 3*(8*b^3*c^3*
d - 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^3*d^4 - a*b^2*d^4
)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/b^4 - 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1
/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*
e))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^4) - 2*(36*b^2*c^2*d^2*tan(1/2*
f*x + 1/2*e)^5 - 24*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^5 - 12*b^2*c*d^3*tan(1/
2*f*x + 1/2*e)^5 + 6*a^2*d^4*tan(1/2*f*x + 1/2*e)^5 + 3*a*b*d^4*tan(1/2*f*
x + 1/2*e)^5 + 6*b^2*d^4*tan(1/2*f*x + 1/2*e)^5 - 72*b^2*c^2*d^2*tan(1/2*f
*x + 1/2*e)^3 + 48*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*d^4*tan(1/2*f
*x + 1/2*e)^3 - 4*b^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 36*b^2*c^2*d^2*tan(1/2*
f*x + 1/2*e) - 24*a*b*c*d^3*tan(1/2*f*x + 1/2*e) + 12*b^2*c*d^3*tan(1/2*f*
x + 1/2*e) + 6*a^2*d^4*tan(1/2*f*x + 1/2*e) - 3*a*b*d^4*tan(1/2*f*x + 1/2*
e) + 6*b^2*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*b^3))
/f
```

Mupad [B] (verification not implemented)

Time = 20.05 (sec) , antiderivative size = 9987, normalized size of antiderivative = 40.43

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

input

```
int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

output

```
(atan(((((((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 1
6*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^
9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11
*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a
^3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 - (8*tan(e/2 + (f
*x)/2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2)
- b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/
2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4
 + (8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d
^8 - a^2*b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6
*b^3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d
^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*
c^5*d^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3
*b^6*c*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 1
28*a^7*b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7
*c^4*d^4 - 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6
 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a
^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*
c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5
+ 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2357, normalized size of antiderivative = 9.54

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x)
```

output

```
(12*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sq
rt( - a**2 + b**2))*cos(e + f*x)*sin(e + f*x)**2*a**4*d**4 - 48*sqrt( - a*
*2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b
**2))*cos(e + f*x)*sin(e + f*x)**2*a**3*b*c*d**3 + 72*sqrt( - a**2 + b**2)
*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(
e + f*x)*sin(e + f*x)**2*a**2*b**2*c**2*d**2 - 48*sqrt( - a**2 + b**2)*ata
n((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e +
f*x)*sin(e + f*x)**2*a*b**3*c**3*d + 12*sqrt( - a**2 + b**2)*atan((tan((e
+ f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*sin(e
 + f*x)**2*b**4*c**4 - 12*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a -
tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a**4*d**4 + 48*sqrt
( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a*
*2 + b**2))*cos(e + f*x)*a**3*b*c*d**3 - 72*sqrt( - a**2 + b**2)*atan((tan
((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a
**2*b**2*c**2*d**2 + 48*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - ta
n((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a*b**3*c**3*d - 12*sq
rt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( -
a**2 + b**2))*cos(e + f*x)*b**4*c**4 + 6*cos(e + f*x)*log(tan((e + f*x)/2)
 - 1)*sin(e + f*x)**2*a**5*d**4 - 24*cos(e + f*x)*log(tan((e + f*x)/2) - 1
)*sin(e + f*x)**2*a**4*b*c*d**3 + 36*cos(e + f*x)*log(tan((e + f*x)/2) ...
```

3.253 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 170

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx &= \frac{d^3\operatorname{arctanh}(\sin(e+fx))}{2bf} \\ &\quad+ \frac{d(3b^2c^2-3abcd+a^2d^2)\operatorname{arctanh}(\sin(e+fx))}{b^3f} \\ &\quad+ \frac{2(bc-ad)^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,f} \\ &\quad+ \frac{d^2(3bc-ad)\tan(e+fx)}{b^2f} \\ &\quad+ \frac{d^3\sec(e+fx)\tan(e+fx)}{2bf}\end{aligned}$$

output

```
1/2*d^3*arctanh(sin(f*x+e))/b/f+d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*arctanh(si
n(f*x+e))/b^3/f+2*(-a*d+b*c)^3*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b
)^(1/2))/(a-b)^(1/2)/b^3/(a+b)^(1/2)/f+d^2*(-a*d+3*b*c)*tan(f*x+e)/b^2/f+1
/2*d^3*sec(f*x+e)*tan(f*x+e)/b/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 389 vs. $2(170) = 340$.

Time = 2.98 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.29

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$$

$$= \frac{\cos^2(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^3\left(\frac{8(-bc+ad)^3\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-2d(-6abcd\right.}{}$$

input

```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

output

```
(Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((8*(-(b*c) +
a*d)^3*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^
2] - 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] + 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^3)/(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + (4*b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]) - (b^2*d^3)/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2 + (4*b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])))/(4*b^3*f*(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4476, 3042, 3431, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc \left(e+fx+\frac{\pi}{2}\right) \left(c+d \csc \left(e+fx+\frac{\pi}{2}\right)\right)^3}{a+b \csc \left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \textbf{4476}$$

$$\int \frac{\sec^3(e+fx)(c \cos(e+fx)+d)^3}{a \cos(e+fx)+b}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\left(c \sin \left(e+fx+\frac{\pi}{2}\right)+d\right)^3}{\sin \left(e+fx+\frac{\pi}{2}\right)^3 \left(a \sin \left(e+fx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\Big\downarrow \textbf{3431}$$

$$\int \left(\frac{d\left(a^2d^2-3abcd+3b^2c^2\right) \sec(e+fx)}{b^3}+\frac{(bc-ad)^3}{b^3(a \cos(e+fx)+b)}+\frac{d^2(3bc-ad) \sec^2(e+fx)}{b^2}+\frac{d^3 \sec^3(e+f}{b}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right) \text{arctanh}(\sin(e+fx))}{b^3 f}+\frac{2(bc-ad)^3\text{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^3 f \sqrt{a-b}\sqrt{a+b}}+\frac{d^2(3bc-ad) \tan(e+fx)}{b^2 f}+\frac{d^3\text{arctanh}(\sin(e+fx))}{2bf}+\frac{d^3 \tan(e+fx) \sec(e+fx)}{2bf}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

output
```
(d^3*ArcTanh[Sin[e + f*x]])/(2*b*f) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)
*ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcTanh[(Sqrt[a - b]*Ta
n[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*f) + (d^2*(3*b*
c - a*d)*Tan[e + f*x])/(b^2*f) + (d^3*Sec[e + f*x]*Tan[e + f*x])/(2*b*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3431
```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

rule 4476
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{2\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-c^{3}b^{3}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{3}\sqrt{(a+b)(a-b)}}+\frac{d^{3}}{2b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{d\left(2a^{2}d^{2}-6abcd+6b^{2}c^{2}+b^{2}d^{2}\right)\ln}{2b^{3}}$
default	$-\frac{2\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-c^{3}b^{3}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{3}\sqrt{(a+b)(a-b)}}+\frac{d^{3}}{2b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{d\left(2a^{2}d^{2}-6abcd+6b^{2}c^{2}+b^{2}d^{2}\right)\ln}{2b^{3}}$
risch	$-\frac{id^{2}\left(bd\,e^{3i(fx+e)}+2ad\,e^{2i(fx+e)}-6bc\,e^{2i(fx+e)}-bd\,e^{i(fx+e)}+2ad-6bc\right)}{f\,b^{2}\left(e^{2i(fx+e)}+1\right)^{2}}+\frac{d^{3}\ln\left(e^{i(fx+e)}+i\right)a^{2}}{f\,b^{3}}-\frac{3d^{2}\ln\left(e^{i(fx+}\right.}{f\,b^{2}}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output

```
1/f*(-2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^3/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))+1/2*d^3/b/(tan(1
/2*f*x+1/2*e)-1)^2-1/2*d*(2*a^2*d^2-6*a*b*c*d+6*b^2*c^2+b^2*d^2)/b^3*ln(ta
n(1/2*f*x+1/2*e)-1)+1/2*d^2*(2*a*d-6*b*c+b*d)/b^2/(tan(1/2*f*x+1/2*e)-1)-1
/2*d^3/b/(tan(1/2*f*x+1/2*e)+1)^2+1/2*d*(2*a^2*d^2-6*a*b*c*d+6*b^2*c^2+b^2
*d^2)/b^3*ln(tan(1/2*f*x+1/2*e)+1)+1/2*d^2*(2*a*d-6*b*c+b*d)/b^2/(tan(1/2*
f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(157) = 314$.

Time = 32.15 (sec) , antiderivative size = 779, normalized size of antiderivative = 4.58

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
[-1/4*(2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a^2 - b^
2)*cos(f*x + e)^2*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*c
os(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - (6*(a^2*b^2 - b^4)*c^2*d - 6*
(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(si
n(f*x + e) + 1) + (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*
a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*((a^2*
b^2 - b^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)*d^3)*cos(f*x
 + e))*sin(f*x + e))/((a^2*b^3 - b^5)*f*cos(f*x + e)^2), 1/4*(4*(b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e)^2
+ (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 -
b^4)*d^3)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (6*(a^2*b^2 - b^4)*c^2*d
- 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*lo
g(-sin(f*x + e) + 1) + 2*((a^2*b^2 - b^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2
 - (a^3*b - a*b^3)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*b^3 - b^5)*f*cos
(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \int \frac{(c+d\sec{(e+fx)})^3\sec{(e+fx)}}{a+b\sec{(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e)),x)
```

output
```
Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(157) = 314.

Time = 0.22 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.99

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$$

$$= \frac{\frac{(6\,b^2c^2d-6\,abcd^2+2\,a^2d^3+b^2d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^3} - \frac{(6\,b^2c^2d-6\,abcd^2+2\,a^2d^3+b^2d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b^3} - \frac{4\,\bigl(b^3c^3-3\,ab^2c^2}{}}{}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
1/2*((6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*log(abs(tan(1/2*f*x
 + 1/2*e) + 1))/b^3 - (6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*lo
g(abs(tan(1/2*f*x + 1/2*e) - 1))/b^3 - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arct
an((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/(s
qrt(-a^2 + b^2)*b^3) - 2*(6*b*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*d^3*tan(1
/2*f*x + 1/2*e)^3 - b*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*b*c*d^2*tan(1/2*f*x +
 1/2*e) + 2*a*d^3*tan(1/2*f*x + 1/2*e) - b*d^3*tan(1/2*f*x + 1/2*e))/((tan
(1/2*f*x + 1/2*e)^2 - 1)^2*b^2))/f
```

Mupad [B] (verification not implemented)

Time = 18.32 (sec) , antiderivative size = 6730, normalized size of antiderivative = 39.59

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

output

```
((tan(e/2 + (f*x)/2)*(b*d^3 - 2*a*d^3 + 6*b*c*d^2))/b^2 + (tan(e/2 + (f*x)
/2)^3*(2*a*d^3 + b*d^3 - 6*b*c*d^2))/b^2)/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan
(e/2 + (f*x)/2)^2 + 1)) - (atan(((((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^
7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6
 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*
b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36
*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 +
96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*
c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 -
 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^
6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 + (((8*(4*b^10*c^3 + 2*b^1
0*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*
b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^
2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/
2 + (f*x)/2)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2) + a
^2*d^3 - 3*a*b*c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2
))/b^3)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2)*1i)/b^3 + (((8*tan
(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d
^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a
^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1366, normalized size of antiderivative = 8.04

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x)
```

output

```
( - 4*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/
sqrt( - a**2 + b**2))*sin(e + f*x)**2*a**3*d**3 + 12*sqrt( - a**2 + b**2)*
atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*sin(e
 + f*x)**2*a**2*b*c*d**2 - 12*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*
a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*sin(e + f*x)**2*a*b**2*c**2*
d + 4*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/
sqrt( - a**2 + b**2))*sin(e + f*x)**2*b**3*c**3 + 4*sqrt( - a**2 + b**2)*a
tan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*a**3*d
**3 - 12*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*
b)/sqrt( - a**2 + b**2))*a**2*b*c*d**2 + 12*sqrt( - a**2 + b**2)*atan((tan
((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*a*b**2*c**2*d
- 4*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sq
rt( - a**2 + b**2))*b**3*c**3 + 2*cos(e + f*x)*sin(e + f*x)*a**3*b*d**3 -
6*cos(e + f*x)*sin(e + f*x)*a**2*b**2*c*d**2 - 2*cos(e + f*x)*sin(e + f*x)
*a*b**3*d**3 + 6*cos(e + f*x)*sin(e + f*x)*b**4*c*d**2 - 2*log(tan((e + f*
x)/2) - 1)*sin(e + f*x)**2*a**4*d**3 + 6*log(tan((e + f*x)/2) - 1)*sin(e +
 f*x)**2*a**3*b*c*d**2 - 6*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a**2*
b**2*c**2*d + log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a**2*b**2*d**3 - 6
*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*b**3*c*d**2 + 6*log(tan((e +
f*x)/2) - 1)*sin(e + f*x)**2*b**4*c**2*d + log(tan((e + f*x)/2) - 1)*si...
```

3.254 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 103

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = {} & \frac{d(2bc-ad)\text{arctanh}(\sin(e+fx))}{b^2f} \\ & + \frac{2(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}f} \\ & + \frac{d^2\tan(e+fx)}{bf}\end{aligned}$$

output

```
d*(-a*d+2*b*c)*arctanh(sin(f*x+e))/b^2/f+2*(-a*d+b*c)^2*arctanh((a-b)^(1/2
)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/(a-b)^(1/2)/b^2/(a+b)^(1/2)/f+d^2*tan(f*
x+e)/b/f
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx$$

$$= \frac{-\frac{2(bc-ad)^2\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + d\big(-\big((2bc-ad)\,\big(\log\big(\cos\big(\tfrac{1}{2}(e+fx)\big)-\sin\big(\tfrac{1}{2}(e+fx)\big)\big)-\log\big(}{b^2f}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

output
```
((-2*(b*c - a*d)^2*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + d*(-((2*b*c - a*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + b*d*Tan[e + f*x]))/(b^2
*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4476, 3042, 3431, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4476}$$

$$\int \frac{\sec^2(e+fx)(c\cos(e+fx)+d)^2}{a\cos(e+fx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^2}{\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\Big\downarrow \text{3431}$$

$$\int \left(\frac{(bc-ad)^2}{b^2(a\cos(e+fx)+b)}+\frac{d(2bc-ad)\sec(e+fx)}{b^2}+\frac{d^2\sec^2(e+fx)}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(2bc-ad)\operatorname{arctanh}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a-b}\sqrt{a+b}}+\frac{d^2\tan(e+fx)}{bf}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

output
```
(d*(2*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh
[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]
*f) + (d^2*Tan[e + f*x])/(b*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3431
```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

rule 4476
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-\frac{2\left(-a^2d^2+2abcd-b^2c^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d(ad-2bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}}{f}$
default	$\frac{-\frac{2\left(-a^2d^2+2abcd-b^2c^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d(ad-2bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}}{f}$
risch	$\frac{2id^2}{fb\left(e^{2i(fx+e)}+1\right)}+\frac{\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+\sqrt{a^2-b^2}\,b}{\sqrt{a^2-b^2}\,a}\right)a^2d^2}{\sqrt{a^2-b^2}\,f\,b^2}-\frac{2\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+\sqrt{a^2-b^2}\,b}{\sqrt{a^2-b^2}\,a}\right)acd}{\sqrt{a^2-b^2}\,fb}+\frac{\ln\left(e^{i(f\right.}}{}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
1/f*(-2/b^2*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))-d^2/b/(tan(1/2*f*x+1/2*e)+1)-d*(a
*d-2*b*c)/b^2*ln(tan(1/2*f*x+1/2*e)+1)-d^2/b/(tan(1/2*f*x+1/2*e)-1)+d*(a*d
-2*b*c)/b^2*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(94) = 188.

Time = 4.69 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.03

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx$$
$$=\left[\frac{2\,(a^2b-b^3)d^2\sin(fx+e)+(b^2c^2-2\,abcd+a^2d^2)\sqrt{a^2-b^2}\cos(fx+e)\log\left(\frac{2\,ab\cos(fx+e)-(a^2-2\,b^2)\cos(f}{a^2\cos}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="fri
cas")
```

output
```
[1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*s
qrt(a^2 - b^2)*cos(f*x + e)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*
x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b
^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + (2*(a^2*b - b^3)*c*
d - (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b - b^
3)*c*d - (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b^2
 - b^4)*f*cos(f*x + e)), 1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + 2*(b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(
f*x + e) + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e) + (2*(a^2*b - b^3)*
c*d - (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b -
b^3)*c*d - (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b
^2 - b^4)*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \int \frac{(c+d\sec{(e+fx)})^2\sec{(e+fx)}}{a+b\sec{(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e)),x)
```

output
```
Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(94) = 188$.

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.89

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx =$$

$$-\frac{\frac{2\,d^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)b}-\frac{\left(2\,bcd-ad^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^{2}}+\frac{\left(2\,bcd-ad^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b^{2}}+\frac{2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(\pi\right.}{}}{f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
-(2*d^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*b) - (2*b*c*d -
 a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^2 + (2*b*c*d - a*d^2)*log(abs
(tan(1/2*f*x + 1/2*e) - 1))/b^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*fl
oor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e
) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^2))/f
```

Mupad [B] (verification not implemented)

Time = 16.49 (sec) , antiderivative size = 3559, normalized size of antiderivative = 34.55

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

output
```
- (2*d^2*tan(e/2 + (f*x)/2))/(b*f*(tan(e/2 + (f*x)/2)^2 - 1)) - (atan(((((
a + b)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4
*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^
4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2
*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*
b*c*d^3))/b^2 + (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b
^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c
*d + 2*a^2*b^5*c*d))/b^3 - (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*
(a*d - b*c)^2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a
*d - b*c)^2)/(b^4 - a^2*b^2))*(a*d - b*c)^2*1i)/(b^4 - a^2*b^2) + (((a + b
)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4
- 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2
*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*
c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d
^3))/b^2 - (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^
2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d +
2*a^2*b^5*c*d))/b^3 + (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d
- b*c)^2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d -
b*c)^2)/(b^4 - a^2*b^2))*(a*d - b*c)^2*1i)/(b^4 - a^2*b^2))/((64*(a^4*b*d^
6 - a^5*d^6 - 2*b^5*c^5*d + 4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + a*b^4*c^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 445, normalized size of antiderivative = 4.32

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx$$

$$= \frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{\sqrt{-a^2+b^2}}\right)\cos\left(fx+e\right)a^2d^2-4\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{\sqrt{-a^2+b^2}}\right.}{}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x)
```

output

```
(2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(e + f*x)*a**2*d**2 - 4*sqrt( - a**2 + b**2)*atan((t
an((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)
*a*b*c*d + 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)
/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*b**2*c**2 + cos(e + f*x)*log(tan
((e + f*x)/2) - 1)*a**3*d**2 - 2*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*a*
*2*b*c*d - cos(e + f*x)*log(tan((e + f*x)/2) - 1)*a*b**2*d**2 + 2*cos(e +
f*x)*log(tan((e + f*x)/2) - 1)*b**3*c*d - cos(e + f*x)*log(tan((e + f*x)/2
) + 1)*a**3*d**2 + 2*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*a**2*b*c*d + c
os(e + f*x)*log(tan((e + f*x)/2) + 1)*a*b**2*d**2 - 2*cos(e + f*x)*log(tan
((e + f*x)/2) + 1)*b**3*c*d + sin(e + f*x)*a**2*b*d**2 - sin(e + f*x)*b**3
*d**2)/(cos(e + f*x)*b**2*f*(a**2 - b**2))
```

3.255 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \frac{d\text{arctanh}(\sin(e+fx))}{bf} + \frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}f}$$

output
```
d*arctanh(sin(f*x+e))/b/f+2*(-a*d+b*c)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2
*e)/(a+b)^(1/2))/(a-b)^(1/2)/b/(a+b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$$
$$= \frac{\frac{2(-bc+ad)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + d\bigl(-\log\bigl(\cos\bigl(\frac{1}{2}(e+fx)\bigr) - \sin\bigl(\frac{1}{2}(e+fx)\bigr)\bigr) + \log(\cos(\frac{1}{2}(e+fx)}{bf}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

output
```
((2*(-(b*c) + a*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + d*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]))/(b*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4486}$$

$$\frac{(bc-ad)\int \frac{\sec(e+fx)}{a+b\sec(e+fx)}dx}{b}+\frac{d\int \sec(e+fx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{b}+\frac{d\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{b}+\frac{d\text{arctanh}(\sin(e+fx))}{bf}$$

$$\Big\downarrow \text{4318}$$

$$\frac{(bc-ad)\int \frac{1}{\frac{a\cos(e+fx)}{b}+1}dx}{b^2}+\frac{d\text{arctanh}(\sin(e+fx))}{bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bc-ad)\int \frac{1}{\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}+\frac{d\text{arctanh}(\sin(e+fx))}{bf}$$

$$\Big\downarrow \; \color{blue}{3138}$$

$$\frac{2(bc-ad)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(e+fx)\right)}{b^2 f}+\frac{d\text{arctanh}(\sin(e+fx))}{bf}$$

$$\Big\downarrow \; \color{blue}{221}$$

$$\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{bf\sqrt{a-b}\sqrt{a+b}}+\frac{d\text{arctanh}(\sin(e+fx))}{bf}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

output

```
(d*ArcTanh[Sin[e + f*x]])/(b*f) + (2*(b*c - a*d)*ArcTanh[(Sqrt[a - b]*Tan[
(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*f)
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2(ad-bc)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b}+\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b}}{f}$
default	$\frac{-\frac{2(ad-bc)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b}+\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b}}{f}$
risch	$\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ia^2+ib^2+\sqrt{a^2-b^2}\,b}{\sqrt{a^2-b^2}\,a}\right)ad}{\sqrt{a^2-b^2}\,fb}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ia^2+ib^2+\sqrt{a^2-b^2}\,b}{\sqrt{a^2-b^2}\,a}\right)c}{\sqrt{a^2-b^2}\,f}-\frac{\ln\Big(\mathrm{e}^{i(fx+e)}+\frac{ia^2-ib^2+\sqrt{a^2-b^2}\,b}{\sqrt{a^2-b^2}\,a}}{\sqrt{a^2-b^2}\,fb}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2*(a*d-b*c)/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/(
(a+b)*(a-b))^(1/2))-d/b*ln(tan(1/2*f*x+1/2*e)-1)+d/b*ln(tan(1/2*f*x+1/2*e)
+1))
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.07

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$$

$$= \left[\frac{(a^2-b^2)d\log\left(\sin\left(fx+e\right)+1\right)-(a^2-b^2)d\log\left(-\sin\left(fx+e\right)+1\right)-\sqrt{a^2-b^2}(bc-ad)\log\left(\frac{2\,ab\cos(}{}\right.}{2\left(a^2b-b^3\right)f}\right.$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="frica
s")
```

output
```
[1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(f*x + e
) + 1) - sqrt(a^2 - b^2)*(b*c - a*d)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^
2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) +
2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/((a^2*b - b
^3)*f), 1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(
f*x + e) + 1) + 2*sqrt(-a^2 + b^2)*(b*c - a*d)*arctan(-sqrt(-a^2 + b^2)*(b
*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))))/((a^2*b - b^3)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \int \frac{(c+d\sec\left(e+fx\right))\sec\left(e+fx\right)}{a+b\sec\left(e+fx\right)}\,dx$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x)
```

output
```
Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$$
$$= \frac{\frac{d\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b} - \frac{d\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-a^2+b^2}}\right)\right)(bc}{\sqrt{-a^2+b^2}b}}{f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
(d*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b - d*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/b - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*ta
n(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*(b*c - a*d
)/(sqrt(-a^2 + b^2)*b))/f
```

Mupad [B] (verification not implemented)

Time = 12.32 (sec) , antiderivative size = 571, normalized size of antiderivative = 7.51

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

output
```
(b^2*c*log((b*sin(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/
2)*(a^2 - b^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) - (a^2*c*
log((b*sin(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2
 - b^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) - (2*b*d*atanh(s
in(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)) + (c*log((a*cos(e/2
 + (f*x)/2) + b*cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))
/cos(e/2 + (f*x)/2))*((a + b)*(a - b))^(1/2))/(f*(a^2 - b^2)) - (a*b*d*log
((b*sin(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 -
b^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) + (2*a^2*d*atanh(si
n(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(b*f*(a^2 - b^2)) + (a^3*d*log((b*si
n(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 - b^2)^(
1/2))/cos(e/2 + (f*x)/2)))/(b*f*(a^2 - b^2)^(3/2)) - (a*d*log((a*cos(e/2 +
 (f*x)/2) + b*cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/c
os(e/2 + (f*x)/2))*((a + b)*(a - b))^(1/2))/(b*f*(a^2 - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.54

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{\sqrt{-a^2+b^2}}\right)ad+2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{\sqrt{-a^2+b^2}}\right)bc-\log(t}{bf\,(a}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x)
```

output

```
( - 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/
sqrt( - a**2 + b**2))*a*d + 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*
a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*b*c - log(tan((e + f*x)/2) -
 1)*a**2*d + log(tan((e + f*x)/2) - 1)*b**2*d + log(tan((e + f*x)/2) + 1)*
a**2*d - log(tan((e + f*x)/2) + 1)*b**2*d)/(b*f*(a**2 - b**2))
```

3.256 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)f} - \frac{2d\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}(bc-ad)f}$$

output

```
2*b*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^
(1/2)/(-a*d+b*c)/f-2*d*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))
/(c-d)^(1/2)/(c+d)^(1/2)/(-a*d+b*c)/f
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = -\frac{2b\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}(bc-ad)f} - \frac{2d\,\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(-bc+ad)\sqrt{c^2-d^2}f}$$

input

```
Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

output
```
(-2*b*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^
2]*(b*c - a*d)*f) - (2*d*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^
2]])/((-(b*c) + a*d)*Sqrt[c^2 - d^2]*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 4476, 3042, 3480, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4476}$$

$$\int \frac{\cos(e+fx)}{(a\cos(e+fx)+b)(c\cos(e+fx)+d)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)}dx$$

$$\Big\downarrow \text{3480}$$

$$\frac{b\int \frac{1}{b+a\cos(e+fx)}dx}{bc-ad}-\frac{d\int \frac{1}{d+c\cos(e+fx)}dx}{bc-ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{1}{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{bc-ad}-\frac{d\int \frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{bc-ad}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2b\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(bc-ad)}-\frac{2d\int \frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+c+d}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(bc-ad)}$$

$$\Big\downarrow 221$$

$$\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)}-\frac{2d\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}(bc-ad)}$$

input
```
Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

output
```
(2*b*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]*(b*c - a*d)*f) - (2*d*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt
[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a*d)*f)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4476

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{2d\ \operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(ad-bc)\sqrt{(c-d)(c+d)}}-\frac{2b\ \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(ad-bc)\sqrt{(a+b)(a-b)}}}{f}$
default	$\frac{\frac{2d\ \operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(ad-bc)\sqrt{(c-d)(c+d)}}-\frac{2b\ \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(ad-bc)\sqrt{(a+b)(a-b)}}}{f}$
risch	$\frac{b\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ia^2+ib^2+\sqrt{a^2-b^2}\,b}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(ad-bc)f}-\frac{b\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ia^2-ib^2+\sqrt{a^2-b^2}\,b}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(ad-bc)f}+\frac{d\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+d\sqrt{c^2-d^2}}{\sqrt{c^2-d^2}\,c}\right.}{\sqrt{c^2-d^2}\,(ad-bc)f}$

input

```
int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output

```
1/f*(2*d/(a*d-b*c)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((
c-d)*(c+d))^(1/2))-2*b/(a*d-b*c)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2
*f*x+1/2*e)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 1040, normalized size of antiderivative = 8.60

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx=\text{Too large to display}$$

input

```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="frica
s")
```

output

```
[-1/2*((a^2 - b^2)*sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^
2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) +
2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (b*c^2 - b
*d^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)
^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a
^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3 - (a^3
 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -1/2*((a^2
- b^2)*sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x +
 e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)
/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(b*c^2 - b*d^2)*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*si
n(f*x + e))))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*
d^2 + (a^3 - a*b^2)*d^3)*f), -1/2*(2*(a^2 - b^2)*sqrt(-c^2 + d^2)*d*arctan
(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c
^2 - b*d^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*
x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b
^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3
- (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -((a^
2 - b^2)*sqrt(-c^2 + d^2)*d*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/
((c^2 - d^2)*sin(f*x + e)) - (b*c^2 - b*d^2)*sqrt(-a^2 + b^2)*arctan(-...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \int \frac{\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

input

```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output

```
Integral(sec(e + f*x)/((a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(103) = 206$.

Time = 0.26 (sec) , antiderivative size = 522, normalized size of antiderivative = 4.31

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx$$
$$= \frac{\left(\sqrt{-c^2+d^2}b(c-2d)|c-d|+\sqrt{-c^2+d^2}ad|c-d|+\sqrt{-c^2+d^2}|-bc+ad||c-d|\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-\frac{2ac-2bd+\sqrt{-4(ac+bc+ad+bd)(ac-bc-ad}}{ac-bc-ad+bd}}}\right.\right.}{(bc-ad)^2(c^2-2cd+d^2)+(c^3-2c^2d+cd^2)a|-bc+ad|-(c^2d-2cd^2+d^3)b|-bc+ad|}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac"
)
```

output
```
((sqrt(-c^2 + d^2)*b*(c - 2*d)*abs(c - d) + sqrt(-c^2 + d^2)*a*d*abs(c - d
) + sqrt(-c^2 + d^2)*abs(-b*c + a*d)*abs(c - d))*(pi*floor(1/2*(f*x + e)/p
i + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*b*d +
sqrt(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b*d)^2)
)/(a*c - b*c - a*d + b*d))))/((b*c - a*d)^2*(c^2 - 2*c*d + d^2) + (c^3 - 2
*c^2*d + c*d^2)*a*abs(-b*c + a*d) - (c^2*d - 2*c*d^2 + d^3)*b*abs(-b*c + a
*d)) + (sqrt(-a^2 + b^2)*b*c*abs(a - b) + sqrt(-a^2 + b^2)*(a - 2*b)*d*abs
(a - b) - sqrt(-a^2 + b^2)*abs(-b*c + a*d)*abs(a - b))*(pi*floor(1/2*(f*x
+ e)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*
b*d - sqrt(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b
*d)^2))/(a*c - b*c - a*d + b*d))))/((a^2 - 2*a*b + b^2)*(b*c - a*d)^2 - (a
^3 - 2*a^2*b + a*b^2)*c*abs(-b*c + a*d) + (a^2*b - 2*a*b^2 + b^3)*d*abs(-b
*c + a*d)))/f
```

Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 2665, normalized size of antiderivative = 22.02

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x))*(c + d/cos(e + f*x))),x)
```

output
```
(b*c^2*atan((b^5*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^5*d^2*tan
(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i + b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b
^2)^(3/2)*2i + b^5*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a^2*b^3*c
^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^3*b^2*c^2*tan(e/2 + (f*x)/2
)*(a^2 - b^2)^(1/2)*1i - a^2*b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*
3i + a^3*b^2*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - b^3*c*d*tan(e/2
 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i - b^5*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^
(1/2)*2i + a*b^2*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i + a*b^4*c^2*t
an(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i + a^4*b*d^2*tan(e/2 + (f*x)/2)*(a^2
 - b^2)^(1/2)*1i + a^2*b^3*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a
^3*b^2*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a*b^2*c*d*tan(e/2 + (
f*x)/2)*(a^2 - b^2)^(3/2)*2i - a*b^4*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1
/2)*2i)/(a^6*d^2 - b^6*c^2 + 2*a^2*b^4*c^2 - a^4*b^2*c^2 + a^2*b^4*d^2 - 2
*a^4*b^2*d^2))*(a^2 - b^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a
*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) - (b*d^2*at
an((b^5*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^5*d^2*tan(e/2 + (f
*x)/2)*(a^2 - b^2)^(1/2)*1i + b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)
*2i + b^5*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a^2*b^3*c^2*tan(e/
2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^3*b^2*c^2*tan(e/2 + (f*x)/2)*(a^2 -
b^2)^(1/2)*1i - a^2*b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*3i + a...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx$$
$$= \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{\sqrt{-a^2+b^2}}\right)b\,c^2+2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{\sqrt{-a^2+b^2}}\right)b\,d^2+2\surd}{f\,(a^3c^2d-a^3d^3-a^2b\,c^3+a^2bc\,d^2-a\,b}$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output

```
(2*( - sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)
/sqrt( - a**2 + b**2))*b*c**2 + sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2
)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*b*d**2 + sqrt( - c**2 + d*
*2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*a
**2*d - sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*b**2*d))/(f*(a**3*c**2*d - a**3*d**3 - a**2*b*c**3
 + a**2*b*c*d**2 - a*b**2*c**2*d + a*b**2*d**3 + b**3*c**3 - b**3*c*d**2))
```

3.257 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 188

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx &= \frac{2b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)^2f} + \frac{2d(acd-b(2c^2-d^2))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2f}\\ &\quad + \frac{d^2\sin(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(d+c\cos(e+fx))}\end{aligned}$$

output
```
2*b^2*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b
)^(1/2)/(-a*d+b*c)^2/f+2*d*(a*c*d-b*(2*c^2-d^2))*arctanh((c-d)^(1/2)*tan(1
/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3/2)/(c+d)^(3/2)/(-a*d+b*c)^2/f+d^2*sin(
f*x+e)/(-a*d+b*c)/(c^2-d^2)/f/(d+c*cos(f*x+e))
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{-2b^2(c^2-d^2)^{3/2}\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(d+c\cos(e+fx)) - \sqrt{a^2-b^2}\,d\Big(-2(2bc^2-acd-bd^2)\,\text{ar}}{\sqrt{a^2-b^2}(c-d)(c+d)(bc-ad)^2\sqrt{c^2-d^2}}$$

input
```
Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

output
```
(-2*b^2*(c^2 - d^2)^(3/2)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b
^2]]*(d + c*Cos[e + f*x]) - Sqrt[a^2 - b^2]*d*(-2*(2*b*c^2 - a*c*d - b*d^2
)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x]
) + d*(-(b*c) + a*d)*Sqrt[c^2 - d^2]*Sin[e + f*x]))/(Sqrt[a^2 - b^2]*(c -
d)*(c + d)*(b*c - a*d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x]))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4476, 3042, 3535, 25, 3042, 3480, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4476}$$
$$\int \frac{\cos^2(e+fx)}{(a\cos(e+fx)+b)(c\cos(e+fx)+d)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^2}dx$$

$$\Big\downarrow \; 3535$$

$$\frac{\int -\frac{bcd+\left(acd-b\left(c^2-d^2\right)\right)\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}dx}{\left(c^2-d^2\right)(bc-ad)}+\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{\int \frac{bcd+\left(acd-b\left(c^2-d^2\right)\right)\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}dx}{\left(c^2-d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{\int \frac{bcd+\left(acd-b\left(c^2-d^2\right)\right)\sin\left(e+fx+\frac{\pi}{2}\right)}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{\left(c^2-d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 3480$$

$$\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{-\frac{b^2\left(c^2-d^2\right)\int \frac{1}{b+a\cos(e+fx)}dx}{bc-ad}-\frac{d\left(acd-b\left(2c^2-d^2\right)\right)\int \frac{1}{d+c\cos(e+fx)}dx}{bc-ad}}{\left(c^2-d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{-\frac{b^2\left(c^2-d^2\right)\int \frac{1}{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{bc-ad}-\frac{d\left(acd-b\left(2c^2-d^2\right)\right)\int \frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{bc-ad}}{\left(c^2-d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 3138$$

$$\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{-\frac{2b^2\left(c^2-d^2\right)\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(bc-ad)}-\frac{2d\left(acd-b\left(2c^2-d^2\right)\right)\int \frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+c+d}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(bc-ad)}}{\left(c^2-d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{-\frac{2b^2\left(c^2-d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)}-\frac{2d\left(acd-b\left(2c^2-d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}(bc-ad)}}{\left(c^2-d^2\right)(bc-ad)}$$

input

```
Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

output

```
-(((-2*b^2*(c^2 - d^2)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*Sqrt[a + b]*(b*c - a*d)*f) - (2*d*(a*c*d - b*(2*c^2 - d^2))
*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c
+ d]*(b*c - a*d)*f))/((b*c - a*d)*(c^2 - d^2))) + (d^2*Sin[e + f*x])/((b*c
 - a*d)*(c^2 - d^2)*f*(d + c*Cos[e + f*x]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4476
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.11

method	result
derivativedivides	$-\frac{\frac{2d\left(-\frac{d(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^{2}-d^{2}\right)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}-\frac{\left(acd-2b\,c^{2}+b\,d^{2}\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c-d)(c+d)\sqrt{(c-d)(c+d)}}\right)}{(ad-bc)^{2}}+\frac{2b^{2}\operatorname{arctanh}(}{(ad-bc)}}{f}$
default	$-\frac{\frac{2d\left(-\frac{d(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^{2}-d^{2}\right)\left(c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}-\frac{\left(acd-2b\,c^{2}+b\,d^{2}\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c-d)(c+d)}}\right)}{(c-d)(c+d)\sqrt{(c-d)(c+d)}}\right)}{(ad-bc)^{2}}+\frac{2b^{2}\operatorname{arctanh}(}{(ad-bc)}}{f}$
risch	$\frac{2id^{2}\left(d\,e^{i(fx+e)}+c\right)}{c(c^{2}-d^{2})(-ad+bc)f\left(c\,e^{2i(fx+e)}+2d\,e^{i(fx+e)}+c\right)}+\frac{b^{2}\ln\left(e^{i(fx+e)}+\frac{ia^{2}-ib^{2}+\sqrt{a^{2}-b^{2}}\,b}{\sqrt{a^{2}-b^{2}}\,a}\right)}{\sqrt{a^{2}-b^{2}}\,(ad-bc)^{2}f}-\frac{b^{2}\ln(e^{i(fx+e)}-ia^{2}-}{\sqrt{a^{2}-b^{2}}\,(ad}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/f*(-2*d/(a*d-b*c)^2*(-d*(a*d-b*c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(c*tan(1/
2*f*x+1/2*e)^2-tan(1/2*f*x+1/2*e)^2*d-c-d)-(a*c*d-2*b*c^2+b*d^2)/(c-d)/(c+
d)/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c-d)*(c+d))^(1/2
)))+2*b^2/(a*d-b*c)^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)
/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(170) = 340$.

Time = 121.84 (sec) , antiderivative size = 2863, normalized size of antiderivative = 15.23

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^{2}}\,dx=\text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output

```
[1/2*((b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^
2*c*d^4)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*
b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e)
+ 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + (2*(a^2*
b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a^2*b - b
^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x + e))*sqr
t(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sq
rt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*
x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^3 -
a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))/((
(a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*
c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 -
2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*
b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c
^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2
*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), 1/2*(2*(b^2*c^4*d - 2*b^
2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4)*cos(f*x + e))*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2
)*sin(f*x + e))) + (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b
 - b^3)*d^4 + (2*(a^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b -...
```

Sympy [F]

$$
\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx \\
&= \int \frac{\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^2}\,dx
\end{aligned}
$$

input

```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

output

```
Integral(sec(e + f*x)/((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.76

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{-a^2+b^2}}+\frac{d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(bc^3-ac^2d-bcd^2+ad^3)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}\right.\right.}\right.}{f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
-2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f
*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*b^2/((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)) + d^2*tan(1/2*f*x + 1/2*e)/((b*c^3 -
 a*c^2*d - b*c*d^2 + a*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/
2*e)^2 - c - d)) - (2*b*c^2*d - a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
 1/2*e))/sqrt(-c^2 + d^2)))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 - b^2*c^
2*d^2 + 2*a*b*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)))/f
```

Mupad [B] (verification not implemented)

Time = 24.71 (sec) , antiderivative size = 20827, normalized size of antiderivative = 110.78

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x))*(c + d/cos(e + f*x))^2),x)
```

output
```
(2*d^2*tan(e/2 + (f*x)/2))/(f*(c + d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d
))*(a*d^2 + b*c^2 - a*c*d - b*c*d)) - (d*atan(((d*((32*tan(e/2 + (f*x)/2)*
(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d
 + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d
^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2
 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 -
 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^
2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c
^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^
2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*
d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) + (d*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*
b*d^9 + a^7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^
9 - a^7*c^2*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3
+ b^7*c^7*d^2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b
^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*
b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*
c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^
5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 +
 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*
c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 ...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1678, normalized size of antiderivative = 8.93

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

output

```
(2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(e + f*x)*b**2*c**5 - 4*sqrt( - a**2 + b**2)*atan((t
an((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)
*b**2*c**3*d**2 + 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e
 + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*b**2*c*d**4 + 2*sqrt( - a
**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 +
b**2))*b**2*c**4*d - 4*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan
((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*b**2*c**2*d**3 + 2*sqrt( - a**2 + b
**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*
b**2*d**5 + 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x
)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a**3*c**2*d**2 - 4*sqrt( - c**2
 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**
2))*cos(e + f*x)*a**2*b*c**3*d + 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x
)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a**2*b*c*d
**3 - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*cos(e + f*x)*a*b**2*c**2*d**2 + 4*sqrt( - c**2 + d
**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*
cos(e + f*x)*b**3*c**3*d - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c
 - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*b**3*c*d**3 + 2*
sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqr...
```

3.258 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 379

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx \\
&= \frac{d^4(5bc-2ad)\operatorname{arctanh}(\sin(e+fx))}{2b^3f} \\
&\quad + \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\operatorname{arctanh}(\sin(e+fx))}{b^5f} \\
&\quad + \frac{2(bc-ad)^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b^3(a+b)^{3/2}f} \\
&\quad + \frac{2(bc-ad)^4(bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^5\sqrt{a+b}f} \\
&\quad - \frac{(bc-ad)^5\sin(e+fx)}{b^4(a^2-b^2)f(b+a\cos(e+fx))} + \frac{d^5\tan(e+fx)}{b^2f} \\
&\quad + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\tan(e+fx)}{b^4f} \\
&\quad + \frac{d^4(5bc-2ad)\sec(e+fx)\tan(e+fx)}{2b^3f} + \frac{d^5\tan^3(e+fx)}{3b^2f}
\end{aligned}$$

output
```
1/2*d^4*(-2*a*d+5*b*c)*arctanh(sin(f*x+e))/b^3/f+d^2*(-4*a^3*d^3+15*a^2*b*
c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*arctanh(sin(f*x+e))/b^5/f+2*(-a*d+b*c)^5*
arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(3/2)/b^3/(a+b
)^(3/2)/f+2*(-a*d+b*c)^4*(4*a*d+b*c)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e
)/(a+b)^(1/2))/a/(a-b)^(1/2)/b^5/(a+b)^(1/2)/f-(-a*d+b*c)^5*sin(f*x+e)/b^4
/(a^2-b^2)/f/(b+a*cos(f*x+e))+d^5*tan(f*x+e)/b^2/f+d^3*(3*a^2*d^2-10*a*b*c
*d+10*b^2*c^2)*tan(f*x+e)/b^4/f+1/2*d^4*(-2*a*d+5*b*c)*sec(f*x+e)*tan(f*x+
e)/b^3/f+1/3*d^5*tan(f*x+e)^3/b^2/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 784 vs. 2(379) = 758.

Time = 9.49 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.07

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx$$

$$= \frac{(b+a\cos(e+fx))(c+d\sec(e+fx))^5 \Bigg(-\frac{24(bc-ad)^4(abc+4a^2d-5b^2d)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)\cos^3(e+fx)(b+}{(a^2-b^2)^{3/2}}}{}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```

output

```
((b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^5*((-24*(b*c - a*d)^4*(a*b*c +
4*a^2*d - 5*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]]*Co
s[e + f*x]^3*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) + 6*d^2*(-30*a^2*b*c*
d^2 + 8*a^3*d^3 - 5*b^3*c*(4*c^2 + d^2) + 2*a*b^2*d*(20*c^2 + d^2))*Cos[e
+ f*x]^3*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 6
*d^2*(30*a^2*b*c*d^2 - 8*a^3*d^3 + 5*b^3*c*(4*c^2 + d^2) - 2*a*b^2*d*(20*c
^2 + d^2))*Cos[e + f*x]^3*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]] + (b*(-60*a^2*b^3*c^2*d^3 + 60*b^5*c^2*d^3 + 45*a^3*b^2*c*d^
4 - 45*a*b^4*c*d^4 - 12*a^4*b*d^5 + 4*a^2*b^3*d^5 + 8*b^5*d^5 + (135*a^4*b
*c*d^4 - 36*a^5*d^5 + 30*a^2*b^3*c*d^2*(3*c^2 - 4*d^2) + a^3*b^2*d^3*(-180
*c^2 + 29*d^2) + a*b^4*d*(-45*c^4 + 90*c^2*d^2 - 2*d^4) + b^5*(9*c^5 + 30*
c*d^4))*Cos[e + f*x] + b*(-a^2 + b^2)*d^3*(-45*a*b*c*d + 12*a^2*d^2 + 4*b^
2*(15*c^2 + d^2))*Cos[2*(e + f*x)] + 3*b^5*c^5*Cos[3*(e + f*x)] - 15*a*b^4
*c^4*d*Cos[3*(e + f*x)] + 30*a^2*b^3*c^3*d^2*Cos[3*(e + f*x)] - 60*a^3*b^2
*c^2*d^3*Cos[3*(e + f*x)] + 30*a*b^4*c^2*d^3*Cos[3*(e + f*x)] + 45*a^4*b*c
*d^4*Cos[3*(e + f*x)] - 30*a^2*b^3*c*d^4*Cos[3*(e + f*x)] - 12*a^5*d^5*Cos
[3*(e + f*x)] + 7*a^3*b^2*d^5*Cos[3*(e + f*x)] + 2*a*b^4*d^5*Cos[3*(e + f*
x)])*Sin[e + f*x])/(-a^2 + b^2)))/(12*b^5*f*(d + c*Cos[e + f*x])^5*(a + b*
Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4476, 3042, 3431, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4476}$$

$$\int \frac{\sec^4(e+fx)(c\cos(e+fx)+d)^5}{(a\cos(e+fx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^5}{\sin\left(e+fx+\frac{\pi}{2}\right)^4\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{3431}$$

$$\int \left(\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\sec^2(e+fx)}{b^4}+\frac{d^2\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)\sec(e+fx)}{b^5}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\tan(e+fx)}{b^4f}-\frac{(bc-ad)^5\sin(e+fx)}{b^4f\left(a^2-b^2\right)\left(a\cos(e+fx)+b\right)}+\frac{d^2\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)\operatorname{arctanh}(\sin(e+fx))}{b^5f}+\frac{2(bc-ad)^4(4ad+bc)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^5f\sqrt{a-b}\sqrt{a+b}}+\frac{d^4(5bc-2ad)\operatorname{arctanh}(\sin(e+fx))}{2b^3f}+\frac{2(bc-ad)^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^3f(a-b)^{3/2}(a+b)^{3/2}}+\frac{d^4(5bc-2ad)\tan(e+fx)\sec(e+fx)}{2b^3f}+\frac{d^5\tan^3(e+fx)}{3b^2f}+\frac{d^5\tan(e+fx)}{b^2f}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```

output

```
(d^4*(5*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(2*b^3*f) + (d^2*(10*b^3*c^3 -
 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*ArcTanh[Sin[e + f*x]])/(b^5*
f) + (2*(b*c - a*d)^5*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])
/(a*(a - b)^(3/2)*b^3*(a + b)^(3/2)*f) + (2*(b*c - a*d)^4*(b*c + 4*a*d)*Ar
cTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^5*Sqrt
[a + b]*f) - ((b*c - a*d)^5*Sin[e + f*x])/(b^4*(a^2 - b^2)*f*(b + a*Cos[e
+ f*x])) + (d^5*Tan[e + f*x])/(b^2*f) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*
a^2*d^2)*Tan[e + f*x])/(b^4*f) + (d^4*(5*b*c - 2*a*d)*Sec[e + f*x]*Tan[e +
 f*x])/(2*b^3*f) + (d^5*Tan[e + f*x]^3)/(3*b^2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3431
```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

rule 4476
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.82

method	result
derivativedivides	$-\frac{d^{5}}{3b^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{d^{2}\left(8a^{3}d^{3}-30a^{2}bc\,d^{2}+40a\,b^{2}c^{2}d+2a\,b^{2}d^{3}-20c^{3}b^{3}-5c\,d^{2}b^{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^{5}}-\frac{d^{3}\left(6a^{2}d^{2}-20abc}{2b}$
default	$-\frac{d^{5}}{3b^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{d^{2}\left(8a^{3}d^{3}-30a^{2}bc\,d^{2}+40a\,b^{2}c^{2}d+2a\,b^{2}d^{3}-20c^{3}b^{3}-5c\,d^{2}b^{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^{5}}-\frac{d^{3}\left(6a^{2}d^{2}-20abc}{2b}$
risch	Expression too large to display

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output

```
1/f*(-1/3*d^5/b^2/(tan(1/2*f*x+1/2*e)+1)^3-1/2*d^2*(8*a^3*d^3-30*a^2*b*c*d
^2+40*a*b^2*c^2*d+2*a*b^2*d^3-20*b^3*c^3-5*b^3*c*d^2)/b^5*ln(tan(1/2*f*x+1
/2*e)+1)-1/2*d^3*(6*a^2*d^2-20*a*b*c*d+2*a*b*d^2+20*b^2*c^2-5*b^2*c*d+2*b^
2*d^2)/b^4/(tan(1/2*f*x+1/2*e)+1)+1/2*d^4*(2*a*d-5*b*c+b*d)/b^3/(tan(1/2*f
*x+1/2*e)+1)^2-1/3*d^5/b^2/(tan(1/2*f*x+1/2*e)-1)^3+1/2*d^2*(8*a^3*d^3-30*
a^2*b*c*d^2+40*a*b^2*c^2*d+2*a*b^2*d^3-20*b^3*c^3-5*b^3*c*d^2)/b^5*ln(tan(
1/2*f*x+1/2*e)-1)-1/2*d^3*(6*a^2*d^2-20*a*b*c*d+2*a*b*d^2+20*b^2*c^2-5*b^2
*c*d+2*b^2*d^2)/b^4/(tan(1/2*f*x+1/2*e)-1)-1/2*d^4*(2*a*d-5*b*c+b*d)/b^3/(
tan(1/2*f*x+1/2*e)-1)^2-2/b^5*(b*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3
-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(t
an(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(4*a^6*d^5-15*a^5*b*c*d^
4+20*a^4*b^2*c^2*d^3-5*a^4*b^2*d^5-10*a^3*b^3*c^3*d^2+20*a^3*b^3*c*d^4-30*
a^2*b^4*c^2*d^3+a*b^5*c^5+20*a*b^5*c^3*d^2-5*b^6*c^4*d)/(a+b)/(a-b)/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx=\text{Timed out}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c+d\sec(e+fx))^5\sec(e+fx)}{(a+b\sec(e+fx))^2}\,dx$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((c + d*sec(e + f*x))**5*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. 2(355) = 710.

Time = 0.31 (sec) , antiderivative size = 857, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="g
iac")
```

output

```
-1/6*(12*(a*b^5*c^5 - 5*b^6*c^4*d - 10*a^3*b^3*c^3*d^2 + 20*a*b^5*c^3*d^2
+ 20*a^4*b^2*c^2*d^3 - 30*a^2*b^4*c^2*d^3 - 15*a^5*b*c*d^4 + 20*a^3*b^3*c*
d^4 + 4*a^6*d^5 - 5*a^4*b^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a
 - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a
^2 + b^2)))/((a^2*b^5 - b^7)*sqrt(-a^2 + b^2)) - 12*(b^5*c^5*tan(1/2*f*x +
 1/2*e) - 5*a*b^4*c^4*d*tan(1/2*f*x + 1/2*e) + 10*a^2*b^3*c^3*d^2*tan(1/2*
f*x + 1/2*e) - 10*a^3*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 5*a^4*b*c*d^4*tan
(1/2*f*x + 1/2*e) - a^5*d^5*tan(1/2*f*x + 1/2*e))/((a^2*b^4 - b^6)*(a*tan(
1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)) - 3*(20*b^3*c^3*d^
2 - 40*a*b^2*c^2*d^3 + 30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*
d^5)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^5 + 3*(20*b^3*c^3*d^2 - 40*a*b^2
*c^2*d^3 + 30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*log(abs
(tan(1/2*f*x + 1/2*e) - 1))/b^5 + 2*(60*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^5
 - 60*a*b*c*d^4*tan(1/2*f*x + 1/2*e)^5 - 15*b^2*c*d^4*tan(1/2*f*x + 1/2*e)
^5 + 18*a^2*d^5*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*d^5*tan(1/2*f*x + 1/2*e)^5
+ 6*b^2*d^5*tan(1/2*f*x + 1/2*e)^5 - 120*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^
3 + 120*a*b*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 36*a^2*d^5*tan(1/2*f*x + 1/2*e)
^3 - 4*b^2*d^5*tan(1/2*f*x + 1/2*e)^3 + 60*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e
) - 60*a*b*c*d^4*tan(1/2*f*x + 1/2*e) + 15*b^2*c*d^4*tan(1/2*f*x + 1/2*e)
+ 18*a^2*d^5*tan(1/2*f*x + 1/2*e) - 6*a*b*d^5*tan(1/2*f*x + 1/2*e) + 6*...
```

Mupad [B] (verification not implemented)

Time = 26.35 (sec) , antiderivative size = 17256, normalized size of antiderivative = 45.53

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

output

```
(atan(((((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^
10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*
d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^1
0 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*
d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^
11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3
 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7
*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520
*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*
c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^
6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7
- 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160
*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*
b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8
*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6
 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*
a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*
b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c
^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^
8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 ...
```

Reduce [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 9042, normalized size of antiderivative = 23.86

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x)
```

output

```
(48*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sq
rt( - a**2 + b**2))*cos(e + f*x)*sin(e + f*x)**2*a**6*b*d**5 - 180*sqrt( -
 a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2
+ b**2))*cos(e + f*x)*sin(e + f*x)**2*a**5*b**2*c*d**4 + 240*sqrt( - a**2
+ b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2
))*cos(e + f*x)*sin(e + f*x)**2*a**4*b**3*c**2*d**3 - 60*sqrt( - a**2 + b*
*2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*c
os(e + f*x)*sin(e + f*x)**2*a**4*b**3*d**5 - 120*sqrt( - a**2 + b**2)*atan
((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f
*x)*sin(e + f*x)**2*a**3*b**4*c**3*d**2 + 240*sqrt( - a**2 + b**2)*atan((t
an((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)
*sin(e + f*x)**2*a**3*b**4*c*d**4 - 360*sqrt( - a**2 + b**2)*atan((tan((e
+ f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*sin(e
 + f*x)**2*a**2*b**5*c**2*d**3 + 12*sqrt( - a**2 + b**2)*atan((tan((e + f*
x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*sin(e + f
*x)**2*a*b**6*c**5 + 240*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - t
an((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*sin(e + f*x)**2*a*b*
*6*c**3*d**2 - 60*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e +
 f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*sin(e + f*x)**2*b**7*c**4*d
 - 48*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*...
```

3.259 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 297

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx \\
&= \frac{d^4\operatorname{arctanh}(\sin(e+fx))}{2b^2f} + \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\operatorname{arctanh}(\sin(e+fx))}{b^4f} \\
&\quad + \frac{2(bc-ad)^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b^2(a+b)^{3/2}f} \\
&\quad + \frac{2(bc-ad)^3(bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^4\sqrt{a+b}f} \\
&\quad - \frac{(bc-ad)^4\sin(e+fx)}{b^3\left(a^2-b^2\right)f(b+a\cos(e+fx))} \\
&\quad + \frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f} + \frac{d^4\sec(e+fx)\tan(e+fx)}{2b^2f}
\end{aligned}$$

output

```
1/2*d^4*arctanh(sin(f*x+e))/b^2/f+d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*arct
anh(sin(f*x+e))/b^4/f+2*(-a*d+b*c)^4*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e
)/(a+b)^(1/2))/a/(a-b)^(3/2)/b^2/(a+b)^(3/2)/f+2*(-a*d+b*c)^3*(3*a*d+b*c)*
arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(1/2)/b^4/(a+b
)^(1/2)/f-(-a*d+b*c)^4*sin(f*x+e)/b^3/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*d^3*(
-a*d+2*b*c)*tan(f*x+e)/b^3/f+1/2*d^4*sec(f*x+e)*tan(f*x+e)/b^2/f
```

Mathematica [A] (verified)

Time = 5.03 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.72

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx$$

$$= \frac{\cos^2(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^4\left(\frac{8(-bc+ad)^3(abc+3a^2d-4b^2d)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(}{(a^2-b^2)^{3/2}}\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

output
```
(Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((8*(-(b*c) +
a*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqr
t[a^2 - b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) - 2*d^2*(-16*a*b*c*d
 + 6*a^2*d^2 + b^2*(12*c^2 + d^2))*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]] + 2*d^2*(-16*a*b*c*d + 6*a^2*d^2 + b^2*(12*c^2 + d^
2))*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d
^4*(b + a*Cos[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (8*b*d^
3*(2*b*c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - (b^2*d^4*(b + a*Cos[e + f*x]))/(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2 + (8*b*d^3*(2*b*c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f
*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (4*b*(b*c - a*d)^4*Sin[e +
 f*x])/((-a + b)*(a + b))))/(4*b^4*f*(d + c*Cos[e + f*x])^4*(a + b*Sec[e +
 f*x])^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4476, 3042, 3431, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4476}$$

$$\int \frac{\sec^3(e+fx)(c\cos(e+fx)+d)^4}{(a\cos(e+fx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^4}{\sin\left(e+fx+\frac{\pi}{2}\right)^3\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{3431}$$

$$\int \left(\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\sec(e+fx)}{b^4}-\frac{(ad-bc)^3(3ad+bc)}{ab^4(a\cos(e+fx)+b)}+\frac{2d^3(2bc-ad)\sec^2(e+fx)}{b^3}-\frac{(a}{ab^3(a\cos}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\text{arctanh}(\sin(e+fx))}{b^4f}-\frac{(bc-ad)^4\sin(e+fx)}{b^3f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2(bc-ad)^3(3ad+bc)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^4f\sqrt{a-b}\sqrt{a+b}}+\frac{2(bc-ad)^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^2f(a-b)^{3/2}(a+b)^{3/2}}+\frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f}+\frac{d^4\text{arctanh}(\sin(e+fx))}{2b^2f}+\frac{d^4\tan(e+fx)\sec(e+fx)}{2b^2f}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

output
```
(d^4*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^
2*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*d)^4*ArcTanh[(Sqrt[a -
 b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b^2*(a + b)^(3/2)*f)
+ (2*(b*c - a*d)^3*(b*c + 3*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sq
rt[a + b]])/(a*Sqrt[a - b]*b^4*Sqrt[a + b]*f) - ((b*c - a*d)^4*Sin[e + f*x
])/(b^3*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (2*d^3*(2*b*c - a*d)*Tan[e +
 f*x])/(b^3*f) + (d^4*Sec[e + f*x]*Tan[e + f*x])/(2*b^2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3431
```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

rule 4476
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.57

method	result
derivativedivides	$-\frac{d^4}{2b^2(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}+\frac{d^2(6a^2d^2-16abcd+12b^2c^2+b^2d^2)\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{2b^4}+\frac{d^3(4ad-8bc+bd)}{2b^3(\tan(\frac{fx}{2}+\frac{e}{2})+1)}+\frac{2b(a^4d^4-4a^3bc\,d^3}{(a^2-b^2)(\tan}$
default	$-\frac{d^4}{2b^2(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}+\frac{d^2(6a^2d^2-16abcd+12b^2c^2+b^2d^2)\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{2b^4}+\frac{d^3(4ad-8bc+bd)}{2b^3(\tan(\frac{fx}{2}+\frac{e}{2})+1)}+\frac{2b(a^4d^4-4a^3bc\,d^3}{(a^2-b^2)(\tan}$
risch	Expression too large to display

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output

```
1/f*(-1/2*d^4/b^2/(tan(1/2*f*x+1/2*e)+1)^2+1/2*d^2*(6*a^2*d^2-16*a*b*c*d+1
2*b^2*c^2+b^2*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)+1)+1/2*d^3*(4*a*d-8*b*c+b*d)/
b^3/(tan(1/2*f*x+1/2*e)+1)+2/b^4*(b*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)
^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(3*a^5*d^4-8*a^4*b*c*d^3+6*a^3*b^2*c^2*d^
2-4*a^3*b^2*d^4+12*a^2*b^3*c*d^3-a*b^4*c^4-12*a*b^4*c^2*d^2+4*b^5*c^3*d)/(
a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-
b))^(1/2)))+1/2*d^4/b^2/(tan(1/2*f*x+1/2*e)-1)^2-1/2*d^2*(6*a^2*d^2-16*a*b
*c*d+12*b^2*c^2+b^2*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)-1)+1/2*d^3*(4*a*d-8*b*c
+b*d)/b^3/(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(275) = 550$.

Time = 172.17 (sec) , antiderivative size = 1925, normalized size of antiderivative = 6.48

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
[-1/4*(2*((a^2*b^4*c^4 - 4*a*b^5*c^3*d - 6*(a^4*b^2 - 2*a^2*b^4)*c^2*d^2 +
 4*(2*a^5*b - 3*a^3*b^3)*c*d^3 - (3*a^6 - 4*a^4*b^2)*d^4)*cos(f*x + e)^3 +
 (a*b^5*c^4 - 4*b^6*c^3*d - 6*(a^3*b^3 - 2*a*b^5)*c^2*d^2 + 4*(2*a^4*b^2 -
 3*a^2*b^4)*c*d^3 - (3*a^5*b - 4*a^3*b^3)*d^4)*cos(f*x + e)^2)*sqrt(a^2 -
b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 -
 b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2
 + 2*a*b*cos(f*x + e) + b^2)) - ((12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^2*d^2
 - 16*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^
4 + a*b^6)*d^4)*cos(f*x + e)^3 + (12*(a^4*b^3 - 2*a^2*b^5 + b^7)*c^2*d^2 -
 16*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^
5 + b^7)*d^4)*cos(f*x + e)^2)*log(sin(f*x + e) + 1) + ((12*(a^5*b^2 - 2*a^
3*b^4 + a*b^6)*c^2*d^2 - 16*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 -
 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d^4)*cos(f*x + e)^3 + (12*(a^4*b^3 - 2*a^
2*b^5 + b^7)*c^2*d^2 - 16*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b -
 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d^4)*cos(f*x + e)^2)*log(-sin(f*x + e) + 1)
 - 2*((a^4*b^3 - 2*a^2*b^5 + b^7)*d^4 - 2*((a^2*b^5 - b^7)*c^4 - 4*(a^3*b^
4 - a*b^6)*c^3*d + 6*(a^4*b^3 - a^2*b^5)*c^2*d^2 - 4*(2*a^5*b^2 - 3*a^3*b^
4 + a*b^6)*c*d^3 + (3*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*d^4)*cos(f*x + e)^2 +
 (8*(a^4*b^3 - 2*a^2*b^5 + b^7)*c*d^3 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^
4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^4 - 2*a^3*b^6 + a*b^8)*f*cos(f*x...
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx=\int \frac{(c+d\sec{(e+fx)})^4\sec{(e+fx)}}{(a+b\sec{(e+fx)})^2}\,dx$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e))**2,x)
```

output

```
Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(275) = 550$.

Time = 0.25 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.86

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="g
iac")
```

output

```
-1/2*(4*(a*b^4*c^4 - 4*b^5*c^3*d - 6*a^3*b^2*c^2*d^2 + 12*a*b^4*c^2*d^2 +
8*a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 - 3*a^5*d^4 + 4*a^3*b^2*d^4)*(pi*floor(1/
2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*
tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2)
) - 4*(b^4*c^4*tan(1/2*f*x + 1/2*e) - 4*a*b^3*c^3*d*tan(1/2*f*x + 1/2*e) +
 6*a^2*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) - 4*a^3*b*c*d^3*tan(1/2*f*x + 1/2*
e) + a^4*d^4*tan(1/2*f*x + 1/2*e))/((a^2*b^3 - b^5)*(a*tan(1/2*f*x + 1/2*e
)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)) - (12*b^2*c^2*d^2 - 16*a*b*c*d^3
+ 6*a^2*d^4 + b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^4 + (12*b^2*c^
2*d^2 - 16*a*b*c*d^3 + 6*a^2*d^4 + b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/b^4 + 2*(8*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 4*a*d^4*tan(1/2*f*x + 1/2
*e)^3 - b*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*b*c*d^3*tan(1/2*f*x + 1/2*e) + 4*
a*d^4*tan(1/2*f*x + 1/2*e) - b*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1
/2*e)^2 - 1)^2*b^3))/f
```

Mupad [B] (verification not implemented)

Time = 23.77 (sec) , antiderivative size = 12483, normalized size of antiderivative = 42.03

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

output
```
(atan(((((((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4
+ 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*
a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*
c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a
^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3
+ 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^
10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10
 - a^3*b^9) - (8*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 -
 8*a*b*c*d^3)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b
^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*(d^4/2 + 6*
c^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x)/2)*(72*a^10*
d^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d
^8 - 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 1
20*a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64
*b^10*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 -
 384*a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*
d^7 - 160*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*
b^2*c*d^7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^
4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a
^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5286, normalized size of antiderivative = 17.80

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x)
```

output

```
( - 12*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)
/sqrt( - a**2 + b**2))*cos(e + f*x)*sin(e + f*x)**2*a**6*d**4 + 32*sqrt( -
 a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2
+ b**2))*cos(e + f*x)*sin(e + f*x)**2*a**5*b*c*d**3 - 24*sqrt( - a**2 + b*
*2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*c
os(e + f*x)*sin(e + f*x)**2*a**4*b**2*c**2*d**2 + 16*sqrt( - a**2 + b**2)*
atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e
 + f*x)*sin(e + f*x)**2*a**4*b**2*d**4 - 48*sqrt( - a**2 + b**2)*atan((tan
((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*s
in(e + f*x)**2*a**3*b**3*c*d**3 + 4*sqrt( - a**2 + b**2)*atan((tan((e + f*
x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*sin(e + f
*x)**2*a**2*b**4*c**4 + 48*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a -
 tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*sin(e + f*x)**2*a*
*2*b**4*c**2*d**2 - 16*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan
((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*sin(e + f*x)**2*a*b**5
*c**3*d + 12*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)
/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a**6*d**4 - 32*sqrt( - a**2 + b*
*2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*c
os(e + f*x)*a**5*b*c*d**3 + 24*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)
*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a**4*b**2*c...
```

3.260 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 228

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx \\
&= \frac{d^2(3bc-2ad)\operatorname{arctanh}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b(a+b)^{3/2}f} \\
&\quad + \frac{2(bc-ad)^2(bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,b^3\sqrt{a+b}\,f} \\
&\quad - \frac{(bc-ad)^3\sin(e+fx)}{b^2\left(a^2-b^2\right)f(b+a\cos(e+fx))} + \frac{d^3\tan(e+fx)}{b^2f}
\end{aligned}$$

output

```
d^2*(-2*a*d+3*b*c)*arctanh(sin(f*x+e))/b^3/f+2*(-a*d+b*c)^3*arctanh((a-b)^
(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(3/2)/b/(a+b)^(3/2)/f+2*(-a*
d+b*c)^2*(2*a*d+b*c)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a
/(a-b)^(1/2)/b^3/(a+b)^(1/2)/f-(-a*d+b*c)^3*sin(f*x+e)/b^2/(a^2-b^2)/f/(b+
a*cos(f*x+e))+d^3*tan(f*x+e)/b^2/f
```

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.59

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx$$

$$= \frac{\cos(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^3\left(-\frac{2(bc-ad)^2(abc+2a^2d-3b^2d)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(}{(a^2-b^2)^{3/2}}\right.}{}$$

input

```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]
```

output

```
(Cos[e + f*x]*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((-2*(b*c - a*d)
^2*(a*b*c + 2*a^2*d - 3*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) + d^2*(-3*b*c + 2*a*d)*(
b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + d^2*(3*b*c
- 2*a*d)*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (
b*d^3*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]) + (b*d^3*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]) + (b*(b*c - a*d)^3*Sin[e + f*x])/((-a + b)*(a + b))))
/(b^3*f*(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4476, 3042, 3431, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4476}$$

$$\int \frac{\sec^2(e+fx)(c\cos(e+fx)+d)^3}{(a\cos(e+fx)+b)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^3}{\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{ 3431}$$

$$\int \left(\frac{d^2(3bc-2ad)\sec(e+fx)}{b^3}+\frac{(ad-bc)^2(2ad+bc)}{ab^3(a\cos(e+fx)+b)}+\frac{(ad-bc)^3}{ab^2(a\cos(e+fx)+b)^2}+\frac{d^3\sec^2(e+fx)}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(bc-ad)^3\sin(e+fx)}{b^2f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{d^2(3bc-2ad)\text{arctanh}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^2(2ad+bc)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^3f\sqrt{a-b}\sqrt{a+b}}+\frac{2(bc-ad)^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{abf(a-b)^{3/2}(a+b)^{3/2}}+\frac{d^3\tan(e+fx)}{b^2f}$$

input

```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]
```

output

```
(d^2*(3*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*Arc
Tanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b*(a +
b)^(3/2)*f) + (2*(b*c - a*d)^2*(b*c + 2*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e +
 f*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^3*Sqrt[a + b]*f) - ((b*c - a*d)^3
*Sin[e + f*x])/(b^2*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (d^3*Tan[e + f*x
])/(b^2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3431
```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

rule 4476
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.38

method	result
derivativedivides	$-\frac{d^3}{b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d^2(2ad-3bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^3}-2\left(\frac{b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-c^3b^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\left(2a^4d^3-3a^3\right.$
default	$-\frac{d^3}{b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d^2(2ad-3bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^3}-2\left(\frac{b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-c^3b^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\left(2a^4d^3-3a^3\right.$
risch	Expression too large to display

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output

```
1/f*(-d^3/b^2/(tan(1/2*f*x+1/2*e)+1)-d^2*(2*a*d-3*b*c)/b^3*ln(tan(1/2*f*x+
1/2*e)+1)-2/b^3*(b*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a^2-b^2)
*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(2
*a^4*d^3-3*a^3*b*c*d^2-3*a^2*b^2*d^3+a*b^3*c^3+6*a*b^3*c*d^2-3*b^4*c^2*d)/
(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a
-b))^(1/2)))-d^3/b^2/(tan(1/2*f*x+1/2*e)-1)+d^2*(2*a*d-3*b*c)/b^3*ln(tan(1
/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(210) = 420$.

Time = 37.83 (sec) , antiderivative size = 1326, normalized size of antiderivative = 5.82

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
[1/2*(((a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5
 - 3*a^3*b^2)*d^3)*cos(f*x + e)^2 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2
- 2*a*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*cos(f*x + e))*sqrt(a^2 - b^2
)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^
2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 +
2*a*b*cos(f*x + e) + b^2)) + ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^
6 - 2*a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b
^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(sin(f*x +
 e) + 1) - ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 + a^
2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5
*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*((a^
4*b^2 - 2*a^2*b^4 + b^6)*d^3 - ((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^5)*
c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)*c
os(f*x + e))*sin(f*x + e))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*f*cos(f*x + e)^2
 + (a^4*b^4 - 2*a^2*b^6 + b^8)*f*cos(f*x + e)), 1/2*(2*((a^2*b^3*c^3 - 3*a
*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5 - 3*a^3*b^2)*d^3)*cos(f*
x + e)^2 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a*b^4)*c*d^2 + (2*a^4
*b - 3*a^2*b^3)*d^3)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) + ((3*(a^5*b - 2*a^3*b
^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 +...
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx=\int \frac{(c+d\sec{(e+fx)})^3\sec{(e+fx)}}{(a+b\sec{(e+fx)})^2}\,dx$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e))**2,x)
```

output

```
Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(210) = 420$.

Time = 0.21 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.36

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="g
iac")
```

output

```
-(2*(a*b^3*c^3 - 3*b^4*c^2*d - 3*a^3*b*c*d^2 + 6*a*b^3*c*d^2 + 2*a^4*d^3 -
 3*a^2*b^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan(
(a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2
*b^3 - b^5)*sqrt(-a^2 + b^2)) - 2*(b^3*c^3*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^
2*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*
a^3*d^3*tan(1/2*f*x + 1/2*e)^3 + a^2*b*d^3*tan(1/2*f*x + 1/2*e)^3 + a*b^2*
d^3*tan(1/2*f*x + 1/2*e)^3 - b^3*d^3*tan(1/2*f*x + 1/2*e)^3 - b^3*c^3*tan(
1/2*f*x + 1/2*e) + 3*a*b^2*c^2*d*tan(1/2*f*x + 1/2*e) - 3*a^2*b*c*d^2*tan(
1/2*f*x + 1/2*e) + 2*a^3*d^3*tan(1/2*f*x + 1/2*e) + a^2*b*d^3*tan(1/2*f*x
+ 1/2*e) - a*b^2*d^3*tan(1/2*f*x + 1/2*e) - b^3*d^3*tan(1/2*f*x + 1/2*e))/
((a*tan(1/2*f*x + 1/2*e)^4 - b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + a + b)*(a^2*b^2 - b^4)) - (3*b*c*d^2 - 2*a*d^3)*log(abs(tan(1/2
*f*x + 1/2*e) + 1))/b^3 + (3*b*c*d^2 - 2*a*d^3)*log(abs(tan(1/2*f*x + 1/2*
e) - 1))/b^3)/f
```

Mupad [B] (verification not implemented)

Time = 20.65 (sec) , antiderivative size = 7958, normalized size of antiderivative = 34.90

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

output

```
(d^2*atan(((d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6
*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16
*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7
*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5
*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 +
36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c
^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*
a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^
3*b^4) + (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*
d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*
d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a
^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*
b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*d^2*
tan(e/2 + (f*x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*
a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))
*(2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c)*1i)/b^3 + (d^2*((32*tan(e/2 + (f*x)
/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6
 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8
*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3
*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 4...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2455, normalized size of antiderivative = 10.77

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x)
```

output

```
(4*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(e + f*x)*a**4*b*d**3 - 6*sqrt( - a**2 + b**2)*atan(
(tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*
x)*a**3*b**2*c*d**2 - 6*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - ta
n((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a**2*b**3*d**3 + 2*sq
rt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( -
a**2 + b**2))*cos(e + f*x)*a*b**4*c**3 + 12*sqrt( - a**2 + b**2)*atan((tan
((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a
*b**4*c*d**2 - 6*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e +
f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*b**5*c**2*d - 4*sqrt( - a**2
 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**
2))*sin(e + f*x)**2*a**5*d**3 + 6*sqrt( - a**2 + b**2)*atan((tan((e + f*x)
/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*sin(e + f*x)**2*a**4*b*c
*d**2 + 6*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)
*b)/sqrt( - a**2 + b**2))*sin(e + f*x)**2*a**3*b**2*d**3 - 2*sqrt( - a**2
+ b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2
))*sin(e + f*x)**2*a**2*b**3*c**3 - 12*sqrt( - a**2 + b**2)*atan((tan((e +
 f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*sin(e + f*x)**2*a**
2*b**3*c*d**2 + 6*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e +
 f*x)/2)*b)/sqrt( - a**2 + b**2))*sin(e + f*x)**2*a*b**4*c**2*d + 4*sqr...
```

3.261 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 198

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = {} & \frac{d^2\operatorname{arctanh}(\sin(e+fx))}{b^2f} \\ & + \frac{2(bc-ad)^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}f} \\ & + \frac{2(b^2c^2-a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^2\sqrt{a+b}f} \\ & - \frac{(bc-ad)^2\sin(e+fx)}{b(a^2-b^2)f(b+a\cos(e+fx))}\end{aligned}$$

output

```
d^2*arctanh(sin(f*x+e))/b^2/f+2*(-a*d+b*c)^2*arctanh((a-b)^(1/2)*tan(1/2*f
*x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(3/2)/(a+b)^(3/2)/f+2*(-a^2*d^2+b^2*c^2)*ar
ctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(1/2)/b^2/(a+b)^
(1/2)/f-(-a*d+b*c)^2*sin(f*x+e)/b/(a^2-b^2)/f/(b+a*cos(f*x+e))
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx$$
$$= \frac{\frac{2(2b^3cd+a^3d^2-ab^2(c^2+2d^2))\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-d^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+d^2\log\left(\right.}{b^2f}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```

output
```
((2*(2*b^3*c*d + a^3*d^2 - a*b^2*(c^2 + 2*d^2))*ArcTanh[((-a + b)*Tan[(e +
 f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - d^2*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] + d^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b*(b*
c - a*d)^2*Sin[e + f*x])/((-a + b)*(a + b)*(b + a*Cos[e + f*x])))/(b^2*f)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 4476, 3042, 3431, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4476}$$
$$\int \frac{\sec(e+fx)(c\cos(e+fx)+d)^2}{(a\cos(e+fx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^2}{\sin\left(e+fx+\frac{\pi}{2}\right)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}dx$$

$$\Big\downarrow \text{3431}$$

$$\int \left(\frac{b^2c^2-a^2d^2}{ab^2(a\cos(e+fx)+b)}-\frac{(ad-bc)^2}{ab(a\cos(e+fx)+b)^2}+\frac{d^2\sec(e+fx)}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(b^2c^2-a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^2f\sqrt{a-b}\sqrt{a+b}}-\frac{(bc-ad)^2\sin(e+fx)}{bf\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2(bc-ad)^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{af(a-b)^{3/2}(a+b)^{3/2}}+\frac{d^2\operatorname{arctanh}(\sin(e+fx))}{b^2f}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```

output
```
(d^2*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[(Sqrt[a - b
]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*f) + (2*(
b^2*c^2 - a^2*d^2)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a
*Sqrt[a - b]*b^2*Sqrt[a + b]*f) - ((b*c - a*d)^2*Sin[e + f*x])/(b*(a^2 - b
^2)*f*(b + a*Cos[e + f*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3431
```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

rule 4476
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\frac{2b\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}b-a-b\right)}-\frac{2\left(a^{3}d^{2}-b^{2}c^{2}a-2b^{2}d^{2}a+2cd\,b^{3}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{b^{2}}+\frac{d^{2}\ln\left(\tan\right.}{}}{f}$
default	$\frac{\frac{\frac{2b\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}b-a-b\right)}-\frac{2\left(a^{3}d^{2}-b^{2}c^{2}a-2b^{2}d^{2}a+2cd\,b^{3}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{b^{2}}+\frac{d^{2}\ln\left(\tan\right.}{}}{f}$
risch	$-\frac{2i\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\left(b\,e^{i(fx+e)}+a\right)}{\left(a^{2}-b^{2}\right)fba\left(a\,e^{2i(fx+e)}+2b\,e^{i(fx+e)}+a\right)}+\frac{\ln\left(e^{i(fx+e)}-\frac{ia^{2}-ib^{2}-\sqrt{a^{2}-b^{2}}\,b}{\sqrt{a^{2}-b^{2}}\,a}\right)a^{3}d^{2}}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)f\,b^{2}}-\frac{\ln\left(e^{i(fx+e)}-\frac{ia^{2}-ib^{2}-\sqrt{}}{\sqrt{a^{2}-}}\right.}{\sqrt{a^{2}-b^{2}}\,(a+b)(a}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/f*(2/b^2*(b*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(ta
n(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(a^3*d^2-a*b^2*c^2-2*a*b^
2*d^2+2*b^3*c*d)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x
+1/2*e)/((a+b)*(a-b))^(1/2)))+d^2/b^2*ln(tan(1/2*f*x+1/2*e)+1)-d^2/b^2*ln(
tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(180) = 360$.

Time = 5.33 (sec) , antiderivative size = 798, normalized size of antiderivative = 4.03

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="f
ricas")
```

output

```
[-1/2*((a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 + (a^2*b^2*c^2 - 2*a
*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b
*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*
x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*
x + e) + b^2)) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*
a^2*b^3 + b^5)*d^2)*log(sin(f*x + e) + 1) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2
*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) + 2*
((a^2*b^3 - b^5)*c^2 - 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*si
n(f*x + e))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a
^2*b^5 + b^7)*f), 1/2*(2*(a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 +
(a^2*b^2*c^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*
x + e))) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^
3 + b^5)*d^2)*log(sin(f*x + e) + 1) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f
*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) - 2*((a^2*
b^3 - b^5)*c^2 - 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*sin(f*x
+ e))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a^2*b^5
 + b^7)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \int \frac{\left(c+d\sec\left(e+fx\right)\right)^2\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^2}\,dx$$

input

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.35

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx$$
$$= \frac{\frac{d^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{b^2} - \frac{d^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{b^2} - \frac{2\left(ab^2c^2-2b^3cd-a^3d^2+2ab^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\right.}{}\right.\right.}{\left(a^2b^2-b^4\right)\sqrt{-a^2+b^2}}}{f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="g
iac")
```

output
```
(d^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^2 - d^2*log(abs(tan(1/2*f*x + 1/
2*e) - 1))/b^2 - 2*(a*b^2*c^2 - 2*b^3*c*d - a^3*d^2 + 2*a*b^2*d^2)*(pi*flo
or(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e)
 - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 +
 b^2)) + 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1/2*e)
+ a^2*d^2*tan(1/2*f*x + 1/2*e))/((a^2*b - b^3)*(a*tan(1/2*f*x + 1/2*e)^2 -
 b*tan(1/2*f*x + 1/2*e)^2 - a - b)))/f
```

Mupad [B] (verification not implemented)

Time = 19.25 (sec) , antiderivative size = 4926, normalized size of antiderivative = 24.88

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

output
```
- (d^2*atan(((d^2*((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d
^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2
*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2
*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) +
 (d^2*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2
+ a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*
a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b
^3) + (32*d^2*tan(e/2 + (f*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*
b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^
2)*1i)/b^2 + (d^2*((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d
^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2
*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2
*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) -
 (d^2*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2
+ a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*
a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b
^3) - (32*d^2*tan(e/2 + (f*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*
b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^
2)*1i)/b^2)/((64*(a^5*d^6 + 2*a*b^4*d^6 - a^4*b*d^6 - 2*b^5*c*d^5 + 2*a^2*
b^3*d^6 - 3*a^3*b^2*d^6 + 4*b^5*c^2*d^4 + a*b^4*c^2*d^4 - 4*a*b^4*c^3*d...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 921, normalized size of antiderivative = 4.65

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x)
```

output

```
( - 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/
sqrt( - a**2 + b**2))*cos(e + f*x)*a**4*d**2 + 2*sqrt( - a**2 + b**2)*atan
((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f
*x)*a**2*b**2*c**2 + 4*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan
((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a**2*b**2*d**2 - 4*sqr
t( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a
**2 + b**2))*cos(e + f*x)*a*b**3*c*d - 2*sqrt( - a**2 + b**2)*atan((tan((e
 + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*a**3*b*d**2 + 2*s
qrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( -
 a**2 + b**2))*a*b**3*c**2 + 4*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)
*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*a*b**3*d**2 - 4*sqrt( - a**
2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b*
*2))*b**4*c*d - cos(e + f*x)*log(tan((e + f*x)/2) - 1)*a**5*d**2 + 2*cos(e
 + f*x)*log(tan((e + f*x)/2) - 1)*a**3*b**2*d**2 - cos(e + f*x)*log(tan((e
 + f*x)/2) - 1)*a*b**4*d**2 + cos(e + f*x)*log(tan((e + f*x)/2) + 1)*a**5*
d**2 - 2*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*a**3*b**2*d**2 + cos(e + f
*x)*log(tan((e + f*x)/2) + 1)*a*b**4*d**2 - log(tan((e + f*x)/2) - 1)*a**4
*b*d**2 + 2*log(tan((e + f*x)/2) - 1)*a**2*b**3*d**2 - log(tan((e + f*x)/2
) - 1)*b**5*d**2 + log(tan((e + f*x)/2) + 1)*a**4*b*d**2 - 2*log(tan((e +
f*x)/2) + 1)*a**2*b**3*d**2 + log(tan((e + f*x)/2) + 1)*b**5*d**2 - sin...
```

3.262 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 100

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}f} - \frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}$$

output
```
2*(a*c-b*d)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/(a-b)^(3/2
)/(a+b)^(3/2)/f-(-a*d+b*c)*tan(f*x+e)/(a^2-b^2)/f/(a+b*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \frac{-\frac{2(ac-bd)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{(-bc+ad)\sin(e+fx)}{(a-b)(a+b)(b+a\cos(e+fx))}}{f}$$

input
```
Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

output

```
((-2*(a*c - b*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^
2 - b^2)^(3/2) + ((-(b*c) + a*d)*Sin[e + f*x])/((a - b)*(a + b)*(b + a*Cos
[e + f*x])))/f
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {3042, 4491, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4491}$$

$$-\frac{\int -\frac{(ac-bd)\sec(e+fx)}{a+b\sec(e+fx)}dx}{a^2-b^2}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ac-bd)\sec(e+fx)}{a+b\sec(e+fx)}dx}{a^2-b^2}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ac-bd)\int \frac{\sec(e+fx)}{a+b\sec(e+fx)}dx}{a^2-b^2}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(ac-bd)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

$$\Big\downarrow \text{4318}$$

$$\frac{(ac-bd)\int \frac{1}{\frac{a\cos(e+fx)}{b}+1}dx}{b\left(a^2-b^2\right)}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(ac-bd)\int \frac{1}{\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{b}+1}dx}{b\left(a^2-b^2\right)}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2(ac-bd)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(e+fx)\right)}{bf\left(a^2-b^2\right)}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

input
```
Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

output
```
(2*(a*c - b*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[
a - b]*Sqrt[a + b]*(a^2 - b^2)*f) - ((b*c - a*d)*Tan[e + f*x])/((a^2 - b^2
)*f*(a + b*Sec[e + f*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{-\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}+\frac{2(ac-bd)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{f}$
default	$\frac{-\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}+\frac{2(ac-bd)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{f}$
risch	$\frac{2i(ad-bc)\left(b\,e^{i(fx+e)}+a\right)}{a(a^2-b^2)f\left(a\,e^{2i(fx+e)}+2b\,e^{i(fx+e)}+a\right)}+\frac{\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+\sqrt{a^2-b^2}\,b}{\sqrt{a^2-b^2}\,a}\right)ac}{\sqrt{a^2-b^2}\,(a+b)(a-b)f}-\frac{\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)f}$

input
```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/f*(-2*(a*d-b*c)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan
(1/2*f*x+1/2*e)^2*b-a-b)+2*(a*c-b*d)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.94

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx$$
$$= \left[\frac{(abc - b^2d + (a^2c - abd)\cos(fx+e))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(fx+e) - (a^2-2\,b^2)\cos(fx+e)^2 + 2\,\sqrt{a^2-b^2}(b\cos(fx+e)+a)}{a^2\cos(fx+e)^2 + 2\,ab\cos(fx+e) + b^2}\right.}{2\left((a^5 - 2\,a^3b^2 + ab^4)f\cos(fx+e) + (a^4b - 2\,a^2b\right.}\right.$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="fri
cas")
```

output
```
[1/2*((a*b*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(a^2 - b^2)*log((
2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*c
os(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*c
os(f*x + e) + b^2)) - 2*((a^2*b - b^3)*c - (a^3 - a*b^2)*d)*sin(f*x + e))/
((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f),
((a*b*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-s
qrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) - ((a^2*b
 - b^3)*c - (a^3 - a*b^2)*d)*sin(f*x + e))/((a^5 - 2*a^3*b^2 + a*b^4)*f*co
s(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c+d\sec{(e+fx)})\sec{(e+fx)}}{(a+b\sec{(e+fx)})^2}\,dx$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.73

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx =$$

$$-\frac{2\Bigg(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)(ac-bd)}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a-b\right)}}{f}$$

input
```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="gia
c")
```

output
```
-2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f
*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*(a*c - b*d)/((a^2
 - b^2)*sqrt(-a^2 + b^2)) - (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x +
1/2*e))/((a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)*(a^
2 - b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 11.75 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a-b}}{\sqrt{a+b}}\right)\,\left(a\,c-b\,d\right)}{f\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,d-b\,c\right)}{f\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(b-a\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a+b\right)}$$

input
```
int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

output
```
(2*atanh((tan(e/2 + (f*x)/2)*(a - b)^(1/2))/(a + b)^(1/2))*(a*c - b*d))/(f
*(a + b)^(3/2)*(a - b)^(3/2)) + (2*tan(e/2 + (f*x)/2)*(a*d - b*c))/(f*(a +
 b)*(a - b)*(a + b - tan(e/2 + (f*x)/2)^2*(a - b)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.33

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx$$

$$= \frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{\sqrt{-a^2+b^2}}\right)\cos\left(fx+e\right)a^2c-2\sqrt{-a^2+b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{\sqrt{-a^2+b^2}}\right.}{}$$

input

```
int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x)
```

output

```
(2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqr
t( - a**2 + b**2))*cos(e + f*x)*a**2*c - 2*sqrt( - a**2 + b**2)*atan((tan(
(e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a*
b*d + 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b
)/sqrt( - a**2 + b**2))*a*b*c - 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)
/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*b**2*d + sin(e + f*x)*a*
*3*d - sin(e + f*x)*a**2*b*c - sin(e + f*x)*a*b**2*d + sin(e + f*x)*b**3*c
)/(f*(cos(e + f*x)*a**5 - 2*cos(e + f*x)*a**3*b**2 + cos(e + f*x)*a*b**4 +
 a**4*b - 2*a**2*b**3 + b**5))
```

3.263 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 186

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx &= \frac{2b(abc-2a^2d+b^2d)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2f}\\ &\quad+\frac{2d^2\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}(bc-ad)^2f}-\frac{b^2\sin(e+fx)}{(a^2-b^2)(bc-ad)f(b+a\cos(e+fx))}\end{aligned}$$

output

```
2*b*(-2*a^2*d+a*b*c+b^2*d)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1
/2))/(a-b)^(3/2)/(a+b)^(3/2)/(-a*d+b*c)^2/f+2*d^2*arctanh((c-d)^(1/2)*tan(
1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(1/2)/(c+d)^(1/2)/(-a*d+b*c)^2/f-b^2*sin
(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \frac{\frac{2b(abc-2a^2d+b^2d)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{2(a^2-b^2)d^2\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{b^2(bc-ad)\sin(e+fx)}{b+a\cos(e+fx)}}{(-a+b)(a+b)(bc-ad)^2f}$$

input

```
Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

output

```
((2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a
^2 - b^2]])/Sqrt[a^2 - b^2] + (2*(a^2 - b^2)*d^2*ArcTanh[((-c + d)*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + (b^2*(b*c - a*d)*Sin[e + f*
x])/(b + a*Cos[e + f*x]))/((-a + b)*(a + b)*(b*c - a*d)^2*f)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4476, 3042, 3535, 25, 3042, 3480, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4476}$$

$$\int \frac{\cos^2(e+fx)}{(a\cos(e+fx)+b)^2(c\cos(e+fx)+d)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin \left(e+fx+\frac{\pi}{2}\right)^2}{\left(a \sin \left(e+fx+\frac{\pi}{2}\right)+b\right)^2 \left(c \sin \left(e+fx+\frac{\pi}{2}\right)+d\right)}dx$$

$$\Big\downarrow \text{3535}$$

$$-\frac{\int -\frac{abd+\left(-da^2+bca+b^2d\right)\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}dx}{\left(a^2-b^2\right)(bc-ad)}-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{abd+\left(-da^2+bca+b^2d\right)\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}dx}{\left(a^2-b^2\right)(bc-ad)}-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{abd+\left(-da^2+bca+b^2d\right)\sin\left(e+fx+\frac{\pi}{2}\right)}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{\left(a^2-b^2\right)(bc-ad)}-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{d^2\left(a^2-b^2\right)\int \frac{1}{d+c\cos(e+fx)}dx}{bc-ad}+\frac{b\left(-2a^2d+abc+b^2d\right)\int \frac{1}{b+a\cos(e+fx)}dx}{bc-ad}}{\left(a^2-b^2\right)(bc-ad)}-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{d^2\left(a^2-b^2\right)\int \frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{bc-ad}+\frac{b\left(-2a^2d+abc+b^2d\right)\int \frac{1}{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{bc-ad}}{\left(a^2-b^2\right)(bc-ad)}-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{2d^2\left(a^2-b^2\right)\int \frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+c+d}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(bc-ad)}+\frac{2b\left(-2a^2d+abc+b^2d\right)\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(bc-ad)}}{\left(a^2-b^2\right)(bc-ad)}-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2d^2(a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}(bc-ad)}+\frac{2b(-2a^2d+abc+b^2d)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)}}{(a^2-b^2)\,(bc-ad)}-\frac{b^2\sin(e+fx)}{f\,(a^2-b^2)\,(bc-ad)(a\cos(e+fx)+b)}$$

input

```
Int[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

output

```
((2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqr
t[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(b*c - a*d)*f) + (2*(a^2 - b^2)*d^2*Ar
cTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d
]*(b*c - a*d)*f))/((a^2 - b^2)*(b*c - a*d)) - (b^2*Sin[e + f*x])/((a^2 - b
^2)*(b*c - a*d)*f*(b + a*Cos[e + f*x]))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4476
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[1
/g^(m + n)   Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{2b\left(-\frac{b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2d-abc-d\,b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{(ad-bc)^2}+\frac{2d^2\operatorname{arctanh}\left(\frac{(c-}{}\right.}{(ad-bc)^2}}{f}$
default	$\frac{\frac{2b\left(-\frac{b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2d-abc-d\,b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{(ad-bc)^2}+\frac{2d^2\operatorname{arctanh}\left(\frac{(c-}{}\right.}{(ad-bc)^2}}{f}$
risch	$\frac{2ib^2\left(b\,\mathrm{e}^{i(fx+e)}+a\right)}{a(a^2-b^2)(ad-bc)f\left(a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{i(fx+e)}+a\right)}+\frac{d^2\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+d\sqrt{c^2-d^2}}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(ad-bc)^2f}-\frac{d^2\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ic^2+}{}\right.}{\sqrt{c^2-d^2}\,(ad-}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE
)
```

output
```
1/f*(2*b/(a*d-b*c)^2*(-b*(a*d-b*c)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f
*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(2*a^2*d-a*b*c-b^2*d)/(a+b)/(a-b
)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2)
))+2*d^2/(a*d-b*c)^2/((c-d)*(c+d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/
((c-d)*(c+d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(168) = 336.

Time = 58.93 (sec) , antiderivative size = 2852, normalized size of antiderivative = 15.33

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fri
cas")
```

output

```
[-1/2*((a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b
^4)*d^3 + (a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*
b - a*b^3)*d^3)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a
^2 - 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*
x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - (
(a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2
)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 +
 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*c
os(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b^3 - b^5)*c^3 - (a^3
*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*sin(f
*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*
b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*
a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f*cos(f*x + e)
 + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*
d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4
 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f), 1/2*(2*((a^5 - 2*
a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(-c
^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) - (a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^
2 - b^4)*d^3 + (a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d +...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$
$$= \int \frac{\sec{(e+fx)}}{\left(a+b\sec{(e+fx)}\right)^2\left(c+d\sec{(e+fx)}\right)}\,dx$$

input

```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

output

```
Integral(sec(e + f*x)/((a + b*sec(e + f*x))**2*(c + d*sec(e + f*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.77

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$
$$= \frac{2 \Bigg(\frac{\left(\pi \left\lfloor \frac{fx+e}{2\pi}+\frac{1}{2} \right\rfloor \operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{-c^2+d^2}} + \frac{b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(a^2bc-b^3c-a^3d+ab^2d)\Big(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan(\frac{1}{2}}}{f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="gia
c")
```

output
```
2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*
f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + b^2*tan(1/2*f*x + 1/2*e)/((a^2*b*
c - b^3*c - a^3*d + a*b^2*d)*(a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1
/2*e)^2 - a - b)) - (a*b^2*c - 2*a^2*b*d + b^3*d)*(pi*floor(1/2*(f*x + e)/
pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x
+ 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^2*c^2 - b^4*c^2 - 2*a^3*b*c*d + 2*a*b
^3*c*d + a^4*d^2 - a^2*b^2*d^2)*sqrt(-a^2 + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 24.92 (sec) , antiderivative size = 20827, normalized size of antiderivative = 111.97

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^2*(c + d/cos(e + f*x))),x)
```

output
```
(d^2*atan(((d^2*(c^2 - d^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6
*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 -
 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d
^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*
d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*
d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b
^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c
*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*
c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*
d - 2*a^3*b^2*c*d) + (d^2*(c^2 - d^2)^(1/2)*((32*(a*b^8*c^7 - a^9*d^7 + 2*
a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*
c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^
3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 -
5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*
a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^
7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a
^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5
+ 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5
*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 1
6*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1678, normalized size of antiderivative = 9.02

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x)
```

output

```
( - 4*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/
sqrt( - a**2 + b**2))*cos(e + f*x)*a**3*b*c**2*d + 4*sqrt( - a**2 + b**2)*
atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e
 + f*x)*a**3*b*d**3 + 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - ta
n((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a**2*b**2*c**3 - 2*sq
rt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( -
a**2 + b**2))*cos(e + f*x)*a**2*b**2*c*d**2 + 2*sqrt( - a**2 + b**2)*atan(
(tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*
x)*a*b**3*c**2*d - 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((
e + f*x)/2)*b)/sqrt( - a**2 + b**2))*cos(e + f*x)*a*b**3*d**3 - 4*sqrt( -
a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 +
 b**2))*a**2*b**2*c**2*d + 4*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a
 - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*a**2*b**2*d**3 + 2*sqrt( - a*
*2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b
**2))*a*b**3*c**3 - 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan(
(e + f*x)/2)*b)/sqrt( - a**2 + b**2))*a*b**3*c*d**2 + 2*sqrt( - a**2 + b**
2)*atan((tan((e + f*x)/2)*a - tan((e + f*x)/2)*b)/sqrt( - a**2 + b**2))*b*
*4*c**2*d - 2*sqrt( - a**2 + b**2)*atan((tan((e + f*x)/2)*a - tan((e + f*x
)/2)*b)/sqrt( - a**2 + b**2))*b**4*d**3 + 2*sqrt( - c**2 + d**2)*atan((tan
((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x...
```

3.264 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 213

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{df} - \frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

output

```
2*(a+b)^(1/2)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^
(1/2)/d/f-2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+
d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c
+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$

$$= \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\,((a-b)(c+d)\operatorname{EllipticF}(\arcsin(\tan(\frac{1}{2}(e+fx))),\frac{a-}{a-}}{(c-d)(c+d)f(b+a\cos(e}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

output
```
(4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSi
n[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(b*c - a*d)*EllipticPi[(c - d)/(
c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x
]])/((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 4457, 3042, 4319, 4461}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4457}$$

$$\frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{d}-\frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{d}-\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{4319}$$

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}-\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{4461}$$

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{df(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

input

```
Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

output

```
(2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c
+ d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*S
ec[e + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*
Sqrt[-Tan[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4457
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[b/d   Int[Csc[e + f
*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[(b*c - a*d)/d   Int[Csc[e + f*x
]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 6.81 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

method	result
default	$-\frac{2\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)ac+\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)ad-\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e\right.\right.}{}$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
-2/f/(c-d)/(c+d)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*c
+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*d-EllipticF(cot(f*
x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*c-EllipticF(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*b*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a
-b)/(a+b))^(1/2))*a*d+2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b
)/(a+b))^(1/2))*b*c)*(cos(f*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(
a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*co
s(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+b\sec{(e+fx)}}\sec{(e+fx)}}{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)b+a}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)d+c}dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
int((sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(e + f*x)*d + c),x)
```

3.265 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 196

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(}{(c-d)(a}}}{d\sqrt{\frac{a+b}{c+d}}f}$$

output

```
2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*se
c(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(-(-a*d+b
*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e)
)/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))/d/((a+b)/(c+d))^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.37 (sec) , antiderivative size = 44664, normalized size of antiderivative = 227.88

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]]
,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3042, 4470}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4470}$$

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c}}{\sqrt{a+b}}\right.\right.}{df\sqrt{\frac{a+b}{c+d}}}$$

input

```
Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

output

```
(2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(
c + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c +
 d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(
a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a +
 b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(d*Sqrt[(a + b)/(c + d)]*f)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4470

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 10.64 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.64

method	result
default	$\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)a-\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+\right.\right.}{f}$

input

```
int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(Ell
ipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+
d))^(1/2))*a-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)
*(c-d)/(a-b)/(c+d))^(1/2))*b+2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)
+csc(f*x+e)),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b)*(1/(c
+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(
f*x+e)+1))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

input

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output

```
Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)), x
)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{b\sec(fx+e)+a}\sec(fx+e)}{\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c),
x)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{b\sec(fx+e)+a}\sec(fx+e)}{\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos(e+f\,x)\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),x
)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$
$$=\int \frac{\sqrt{\sec\left(fx+e\right)d+c}\,\sqrt{\sec\left(fx+e\right)b+a}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)d+c}dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(
e + f*x)*d + c),x)
```

3.266 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 192

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{a+b}\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(1+}{(a-b)(c+d}}}{\sqrt{c+d}(bc-ad)f}$$

output
```
2*(a+b)^(1/2)*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b
)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((-a*d+b*c
)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))
/(a-b)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/(c+d)^(1/2)/(-a*d+b*c)/f
```

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{4\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}\csc(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2(}{-bc+ad}}}{\sqrt{2}}\right.\right.}{(a+b)f\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{bc-ad}}\sqrt{c+d\sec(}}$$

input

```
Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
,x]
```

output

```
(4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e +
 f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*EllipticF[ArcSin[S
qrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]/Sqrt
[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sqrt[a + b*Sec[e + f*x]]*Sin[(e +
 f*x)/2]^2)/((a + b)*f*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)/(b*c - a*d)]*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = $\{3042, 4472\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4472}$$

$$\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b}}{\sqrt{a+b}\sqrt{c+d}}\right.\right.}{f\sqrt{c+d}(bc-ad)}$$

input

```
Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e
 + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e
+ f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e +
f*x])))]*(c + d*Sec[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4472
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 7.84 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99

method	result
default	$\frac{2\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\ \text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)\sqrt{\frac{d+c\,\mathrm{co}}{(c+d)(\mathrm{cos}}}}{f\sqrt{\frac{a-b}{a+b}}\,(d+c\cos(fx+e))(b+a\cos(fx+e))}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/f/((a-b)/(a+b))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a
+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*
(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(1/(c+d)*(d+c*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x
+e)^2+cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/(b
*d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))),
 x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c))
, x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)d+c}\,\sqrt{\sec\left(fx+e\right)b+a}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)^{2}bd+\sec\left(fx+e\right)ad+\sec\left(fx+e\right)bc+ac}dx$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(
e + f*x)**2*b*d + sec(e + f*x)*a*d + sec(e + f*x)*b*c + a*c),x)
```

3.267 $\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 110

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2+3\sec(e+fx)}}{\sqrt{5}\sqrt{-4+5\sec(e+fx)}}\right),45\right)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{1+\sec(e+fx)}{4-5\sec(e+fx)}}}{f}$$

output
```
2*cot(f*x+e)*EllipticF(1/5*(2+3*sec(f*x+e))^(1/2)*5^(1/2)/(-4+5*sec(f*x+e)
)^(1/2),3*5^(1/2))*(4-5*sec(f*x+e))*((1-sec(f*x+e))/(4-5*sec(f*x+e)))^(1/2
)*((1+sec(f*x+e))/(4-5*sec(f*x+e)))^(1/2)/f
```

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.60

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = -\frac{4\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{-\left((3+2\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\sqrt{-\left((-5+4\cos(e+fx))\csc^2\left(\frac{1}{2}(\right.\right.}}{3\sqrt{5}f\sqrt{2+3\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]]
),x]
```

output
```
(-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Ellipt
icF[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44
/45]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[2 + 3*Sec[e + f*x]
]*Sqrt[-4 + 5*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3042, 4472}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{3\sec(e+fx)+2}\sqrt{5\sec(e+fx)-4}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{3\csc\left(e+fx+\frac{\pi}{2}\right)+2}\sqrt{5\csc\left(e+fx+\frac{\pi}{2}\right)-4}}dx$$

$$\Big\downarrow \text{4472}$$

$$\frac{2\cot(e+fx)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{\sec(e+fx)+1}{4-5\sec(e+fx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3\sec(e+fx)+2}}{\sqrt{5}\sqrt{5\sec(e+fx)-4}}\right),45\right)}{f}$$

input
```
Int[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]]),x]
```

output
```
(2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[2 + 3*Sec[e + f*x]]/(Sqrt[5]*Sqrt[-4
 + 5*Sec[e + f*x]])], 45]*(4 - 5*Sec[e + f*x])*Sqrt[(1 - Sec[e + f*x])/(4
- 5*Sec[e + f*x])]*Sqrt[(1 + Sec[e + f*x])/(4 - 5*Sec[e + f*x])])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4472
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

method	result
default	$\frac{i\sqrt{5}\,\sqrt{2+3\sec(fx+e)}\,\sqrt{-4+5\sec(fx+e)}\,\sqrt{10}\,\sqrt{\frac{2\cos(fx+e)+3}{\cos(fx+e)+1}}\ \mathrm{EllipticF}\left(\frac{i\sqrt{5}\,(-\cot(fx+e)+\csc(fx+e))}{5},3\sqrt{5}\right)\sqrt{-\frac{2(4\cos(fx+e)-5)}{\cos(fx+e)+1}}}{5f\left(8\cos(fx+e)^2+2\cos(fx+e)-15\right)}$

input
```
int(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
1/5*I/f*5^(1/2)*(2+3*sec(f*x+e))^(1/2)*(-4+5*sec(f*x+e))^(1/2)*10^(1/2)*((
2*cos(f*x+e)+3)/(cos(f*x+e)+1))^(1/2)*EllipticF(1/5*I*5^(1/2)*(-cot(f*x+e)
+csc(f*x+e)),3*5^(1/2))*(-2*(4*cos(f*x+e)-5)/(cos(f*x+e)+1))^(1/2)/(8*cos(
f*x+e)^2+2*cos(f*x+e)-15)*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{5\,\sec\left(fx+e\right)-4}\sqrt{3\,\sec\left(fx+e\right)+2}}\,dx$$

input
```
integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)*sec(f*x + e)/(1
5*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{3\sec\left(e+fx\right)+2}\sqrt{5\sec\left(e+fx\right)-4}}\,dx$$

input
```
integrate(sec(f*x+e)/(2+3*sec(f*x+e))**(1/2)/(-4+5*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(3*sec(e + f*x) + 2)*sqrt(5*sec(e + f*x) - 4)),
 x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{5\,\sec\left(fx+e\right)-4}\sqrt{3\,\sec\left(fx+e\right)+2}}\,dx$$

input
```
integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2))
, x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{5\,\sec\left(fx+e\right)-4}\sqrt{3\,\sec\left(fx+e\right)+2}}\,dx$$

input
```
integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{\frac{3}{\cos(e+f\,x)}+2}\,\sqrt{\frac{5}{\cos(e+f\,x)}-4}}\,dx$$

input

```
int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(5/cos(e + f*x) - 4)^(1/2))
,x)
```

output

```
int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(5/cos(e + f*x) - 4)^(1/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{\sqrt{3\sec\left(fx+e\right)+2}\,\sqrt{5\sec\left(fx+e\right)-4}\,\sec\left(fx+e\right)}{15\sec\left(fx+e\right)^{2}-2\sec\left(fx+e\right)-8}dx$$

input

```
int(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x)
```

output

```
int((sqrt(3*sec(e + f*x) + 2)*sqrt(5*sec(e + f*x) - 4)*sec(e + f*x))/(15*s
ec(e + f*x)**2 - 2*sec(e + f*x) - 8),x)
```

3.268 $\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 125

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \frac{2i\cot(e+fx)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{2+3\sec(e+fx)}}\right),\frac{1}{45}\right)\sqrt{\frac{1-\sec(e+fx)}{2+3\sec(e+fx)}}\sqrt{\frac{1+\sec(e+fx)}{2+3\sec(e+fx)}}(2+3\sec(e+fx))}{3\sqrt{5}f}$$

output
```
2/15*I*cot(f*x+e)*InverseJacobiAM(I*arcsinh(5^(1/2)*(4-5*sec(f*x+e))^(1/2)
/(2+3*sec(f*x+e))^(1/2)),1/15*5^(1/2))*((1-sec(f*x+e))/(2+3*sec(f*x+e)))^(
1/2)*((1+sec(f*x+e))/(2+3*sec(f*x+e)))^(1/2)*(2+3*sec(f*x+e))*5^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = -\frac{4\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{-\left((3+2\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\sqrt{-((-5+4\cos(e+fx))\csc^2(\frac{1}{2}(}}{3\sqrt{5}f\sqrt{4-5\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]])
,x]
```

output
```
(-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]
^2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Ellipt
icF[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44
/45]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[4 - 5*Sec[e + f*x]
]*Sqrt[2 + 3*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3042, 4472}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{3\sec(e+fx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{4-5\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{3\csc\left(e+fx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{4472}$$

$$\frac{2i\cot(e+fx)\sqrt{\frac{1-\sec(e+fx)}{3\sec(e+fx)+2}}\sqrt{\frac{\sec(e+fx)+1}{3\sec(e+fx)+2}}(3\sec(e+fx)+2)\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{3\sec(e+fx)+2}}\right),\frac{1}{45}\right)}{3\sqrt{5}f}$$

input
```
Int[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]]),x]
```

output
```
(((2*I)/3)*Cot[e + f*x]*EllipticF[I*ArcSinh[(Sqrt[5]*Sqrt[4 - 5*Sec[e + f*
x]])/Sqrt[2 + 3*Sec[e + f*x]]], 1/45]*Sqrt[(1 - Sec[e + f*x])/(2 + 3*Sec[e
 + f*x])]*Sqrt[(1 + Sec[e + f*x])/(2 + 3*Sec[e + f*x])]*(2 + 3*Sec[e + f*x
]))/(Sqrt[5]*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4472
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

method	result
default	$\frac{i \operatorname{EllipticF}\left(3 i (\cot(fx+e)-\csc(fx+e)), \frac{\sqrt{5}}{15}\right) \sqrt{2+3\sec(fx+e)}\, \sqrt{4-5\sec(fx+e)}\, \sqrt{-\frac{2(4\cos(fx+e)-5)}{\cos(fx+e)+1}}\, \sqrt{10}\, \sqrt{\frac{2\cos(fx+e)+3}{\cos(fx+e)+1}}\, \Big(\cos(fx}{15 f \left(8\cos(fx+e)^2+2\cos(fx+e)-15\right)}$

input
```
int(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/15*I/f*EllipticF(3*I*(cot(f*x+e)-csc(f*x+e)),1/15*5^(1/2))*(2+3*sec(f*x+
e))^(1/2)*(4-5*sec(f*x+e))^(1/2)*(-2*(4*cos(f*x+e)-5)/(cos(f*x+e)+1))^(1/2
)*10^(1/2)*((2*cos(f*x+e)+3)/(cos(f*x+e)+1))^(1/2)/(8*cos(f*x+e)^2+2*cos(f
*x+e)-15)*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{3\,\sec\left(fx+e\right)+2}\sqrt{-5\,\sec\left(fx+e\right)+4}}\,dx$$

input
```
integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4)*sec(f*x + e)/
(15*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{4-5\sec\left(e+fx\right)}\sqrt{3\sec\left(e+fx\right)+2}}\,dx$$

input
```
integrate(sec(f*x+e)/(4-5*sec(f*x+e))**(1/2)/(2+3*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(4 - 5*sec(e + f*x))*sqrt(3*sec(e + f*x) + 2)),
 x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{3\sec(fx+e)+2}\sqrt{-5\sec(fx+e)+4}}\,dx$$

input
```
integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4)
), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{3\sec(fx+e)+2}\sqrt{-5\sec(fx+e)+4}}\,dx$$

input
```
integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{\frac{3}{\cos\left(e+f\,x\right)}+2}\,\sqrt{4-\frac{5}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(4 - 5/cos(e + f*x))^(1/2))
,x)
```

output
```
int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(4 - 5/cos(e + f*x))^(1/2))
, x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = -\left(\int \frac{\sqrt{3\sec\left(fx+e\right)+2}\,\sqrt{-5\sec\left(fx+e\right)+4}\,\sec\left(fx+e\right)}{15\sec\left(fx+e\right)^{2}-2\sec\left(fx+e\right)-8}dx\right)$$

input
```
int(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x)
```

output
```
 - int((sqrt(3*sec(e + f*x) + 2)*sqrt( - 5*sec(e + f*x) + 4)*sec(e + f*x))
/(15*sec(e + f*x)**2 - 2*sec(e + f*x) - 8),x)
```

3.269 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 396

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}}{bd\sqrt{\frac{a+b}{c+d}}f} - \frac{2a\sqrt{a+b}\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}}{b\sqrt{c+d}(bc-ad)f}$$

output

```
2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*se
c(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(-(-a*d+b
*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e)
)/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))/b/d/((a+b)/(c+d))^(1/2)/f
-2*a*(a+b)^(1/2)*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(
a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((-a*d+
b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+
e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/b/(c+d)^(1/2)/(-a*d+b*c
)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.24 (sec) , antiderivative size = 39359, normalized size of antiderivative = 99.39

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]
]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3042, 4473, 3042, 4470, 4472}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4473}$$

$$\frac{\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx}{b}-\frac{a\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{4470}$$

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c}}{\sqrt{a+b}}\right.\right.}{bdf\sqrt{\frac{a+b}{c+d}}}$$

$$\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{4472}$$

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c}}{\sqrt{a+b}}\right.\right.}{bdf\sqrt{\frac{a+b}{c+d}}}$$

$$\frac{2a\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a-}}{\sqrt{a+b}\sqrt{c+}}\right.\right.}{bf\sqrt{c+d}(bc-ad)}$$

input

```
Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output

```
(2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(
c + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c +
 d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(
a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a +
 b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(b*d*Sqrt[(a + b)/(c + d)]*f) - (
2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[
e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e
 + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e +
 f*x])))]*(c + d*Sec[e + f*x]))/(b*Sqrt[c + d]*(b*c - a*d)*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4470
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4472
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4473
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-a/b   Int[Cs
c[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Si
mp[1/b   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 12.95 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\,\Bigg(\mathrm{EllipticF}\Big(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e))}{f\sqrt{\frac{a-b}{a+b}}\,(d+c\cos(fx+e))(b+a}$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/
(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(co
s(f*x+e)+1))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e))
,((a+b)*(c-d)/(a-b)/(c+d))^(1/2))-2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f
*x+e)+csc(f*x+e)),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)))/(d
+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$=\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x)
)), x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$=\int \frac{\sec(fx+e)^2}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c
)), x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{{\cos\left(e+f\,x\right)}^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2
)),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$=\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)^2}{\sec(fx+e)^2\,bd+\sec(fx+e)\,ad+\sec(fx+e)\,bc+ac}dx$$

input

```
int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output

```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2)/(s
ec(e + f*x)**2*b*d + sec(e + f*x)*a*d + sec(e + f*x)*b*c + a*c),x)
```

3.270 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 170

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \frac{2dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+}}{bf\sqrt{c+d\sec(e+fx)}}$$

$$+\frac{2(bc-ad)g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{b(a+b)f\sqrt{c+d\sec(e+fx)}}$$

output

```
2*d*g*((d+c*cos(f*x+e))/(c+d))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^(1/
2)*(c/(c+d))^(1/2))*(g*sec(f*x+e))^(1/2)/b/f/(c+d*sec(f*x+e))^(1/2)+2*(-a*
d+b*c)*g*((d+c*cos(f*x+e))/(c+d))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2*a/
(a+b),2^(1/2)*(c/(c+d))^(1/2))*(g*sec(f*x+e))^(1/2)/b/(a+b)/f/(c+d*sec(f*x
+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.31

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx =$$
$$-\frac{2ig\sqrt{-\frac{c(-1+\cos(e+fx))}{c+d}}\sqrt{\frac{c(1+\cos(e+fx))}{c-d}}\cot(e+fx)\Bigl(\text{EllipticPi}\Bigl(1-\frac{c}{d}, i\text{arcsinh}\Bigl(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(e+fx)}}{b\sqrt{\frac{1}{c-d}}f\sqrt{}}$$

input
```
Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e +
 f*x]),x]
```

output
```
((-2*I)*g*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*
x]))/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1
)]*Sqrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(a*(-c + d))/
(-(b*c) + a*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-
c + d)/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(b*Sqrt[(c
 - d)^(-1)]*f*Sqrt[d + c*Cos[e + f*x]])
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4459, 3042, 4346, 3042, 3286, 3042, 3284, 4463, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4459}$$

$$\frac{(bc-ad)\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}dx}{b}+\frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}+\frac{d\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{4346}$$

$$\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}+\frac{dg\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+fx)}}dx}{b\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}+\frac{dg\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{b\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}+\frac{dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{\sec(e+fx)}{\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}dx}{b\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}+\frac{dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{\frac{d}{c+d}+\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{c+d}}}dx}{b\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{b}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4463}$$

$$\frac{g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{1}{(b+a\cos(e+fx))\sqrt{d+c\cos(e+fx)}}dx}{b\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{1}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{b\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3286}$$

$$\frac{g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{1}{(b+a\cos(e+fx))\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}dx}{b\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{1}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{\frac{d}{c+d}+\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{c+d}}}dx}{b\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3284}$$

$$\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{bf(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

input
```
Int[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e + f*x])
,x]
```

output
```
(2*d*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)
/(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c -
 a*d)*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e +
f*x)/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*(a + b)*f*Sqrt[c + d*Sec[e
 + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4459
```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[b/d
 Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[(b*c -
a*d)/d   Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e
+ f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0]
```

rule 4463
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])   Int
[1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.27 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.62

method	result
default	$2ig\left(\text{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{c-d}{c+d}}\right)abc-\text{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{c-d}{c+d}}\right)abd+\text{EllipticF}\left(i(-\cot\right.\right.$

input
```
int((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output

```
2*I*g/f/b/(a+b)/(a-b)*(EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(c-d)/(c+d))
^(1/2))*a*b*c-EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*a
*b*d+EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*b^2*c-Elli
pticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*b^2*d-2*EllipticPi(
I*(-cot(f*x+e)+csc(f*x+e)),-1,I*((c-d)/(c+d))^(1/2))*a^2*d+2*EllipticPi(I*
(-cot(f*x+e)+csc(f*x+e)),-1,I*((c-d)/(c+d))^(1/2))*b^2*d+2*EllipticPi(I*(-
cot(f*x+e)+csc(f*x+e)),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*a^2*d-2*Ellipti
cPi(I*(-cot(f*x+e)+csc(f*x+e)),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*a*b*c)*
(g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos
(f*x+e)+1))^(1/2)*cos(f*x+e)/(d+c*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \text{Timed out}$$

input

```
integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x,
algorithm="fricas")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}\sqrt{c+d\sec{(e+fx)}}}{a+b\sec{(e+fx)}}\,dx$$

input

```
integrate((g*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e)),x
)
```

output

```
Integral((g*sec(e + f*x))**(3/2)*sqrt(c + d*sec(e + f*x))/(a + b*sec(e + f
*x)), x)
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}(g\sec\left(fx+e\right))^{\frac{3}{2}}}{b\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x,
algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e)
+ a), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}(g\sec\left(fx+e\right))^{\frac{3}{2}}}{b\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x,
algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e)
+ a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx=\int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}\,{\left(\frac{g}{\cos(e+f\,x)}\right)}^{3/2}}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(a + b/cos(e + f*x
)),x)
```

output

```
int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(a + b/cos(e + f*x
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx=\sqrt{g}\left(\int \frac{\sqrt{\sec\left(fx+e\right)}\,\sqrt{\sec\left(fx+e\right)d+c}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)b+a}dx\right)$$

input

```
int((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x)
```

output

```
sqrt(g)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x)*d + c)*sec(e + f*x))/(se
c(e + f*x)*b + a),x)*g
```

3.271 $\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 83

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e}{(a+b)f\sqrt{c+d\sec(e+fx)}}$$

output
```
2*g*((d+c*cos(f*x+e))/(c+d))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2*a/(a+b)
,2^(1/2)*(c/(c+d))^(1/2))*(g*sec(f*x+e))^(1/2)/(a+b)/f/(c+d*sec(f*x+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e}{(a+b)f\sqrt{c+d\sec(e+fx)}}$$

input
```
Integrate[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e +
f*x]]),x]
```

output

```
(2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x
)/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*
x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 4463, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4463}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{1}{(b+a\cos(e+fx))\sqrt{d+c\cos(e+fx)}}dx}{\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{1}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{1}{(b+a\cos(e+fx))\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}dx}{\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{1}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{\frac{d}{c+d}+\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{c+d}}}dx}{\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{f(a+b)\sqrt{c+d\sec(e+fx)}}$$

input
```
Int[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]])
,x]
```

output
```
(2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x
)/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*
x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4463
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])   Int
[1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.78 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.69

method	result
default	$\frac{2ig\left(2a\,\mathrm{EllipticPi}\left(i(\cot(fx+e)-\csc(fx+e)),-\frac{a-b}{a+b},i\sqrt{\frac{c-d}{c+d}}\right)-a\,\mathrm{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{c-d}{c+d}}\right)-b\,\mathrm{EllipticF}\left(i(\cot(\right.\right.}{f(a-b)(a+b)(d+c\cos(fx+e))\sqrt{\frac{1}{\cos(fx}}}$

input
```
int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I*g/f/(a-b)/(a+b)*(2*a*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-(a-b)/(a+b)
,I*((c-d)/(c+d))^(1/2))-a*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(c+d
))^(1/2))-b*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(c+d))^(1/2)))*(g*
sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*
x+e)+1))^(1/2)*cos(f*x+e)/(d+c*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$$

input

```
integrate((g*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x
)
```

output

```
Integral((g*sec(e + f*x))**(3/2)/((a + b*sec(e + f*x))*sqrt(c + d*sec(e +
f*x))), x)
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e)+a)\sqrt{d\sec(fx+e)+c}}\,dx$$

input

```
integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,
algorithm="maxima")
```

output

```
integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
 + c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e)+a)\sqrt{d\sec(fx+e)+c}}\,dx$$

input

```
integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,
algorithm="giac")
```

output
```
integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(e+f\,x)}\right)\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2
)),x)
```

output
```
int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx=\sqrt{g}\,\Bigg(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)\,d+c}\,\sec(fx+}{\sec(fx+e)^{2}\,bd+\sec(fx+e)\,ad+\sec(fx+e)\,b}$$

input
```
int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(g)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x)*d + c)*sec(e + f*x))/(se
c(e + f*x)**2*b*d + sec(e + f*x)*a*d + sec(e + f*x)*b*c + a*c),x)*g
```

3.272 $\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 168

$$\begin{aligned}\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx &= \frac{2d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{af\sqrt{c+d\sec(e+fx)}}\\ &\quad+\frac{2(ac-bd)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{a(a+b)f\sqrt{c+d\sec(e+fx)}}\end{aligned}$$

output

```
2*d*((d+c*cos(f*x+e))/(c+d))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^(1/2)
*(c/(c+d))^(1/2))*(g*sec(f*x+e))^(1/2)/a/f/(c+d*sec(f*x+e))^(1/2)+2*(a*c-b
*d)*((d+c*cos(f*x+e))/(c+d))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(a+b)
,2^(1/2)*(c/(c+d))^(1/2))*(g*sec(f*x+e))^(1/2)/a/(a+b)/f/(c+d*sec(f*x+e))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.90 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = -\frac{2i\sqrt{-\frac{c(-1+\cos(e+fx))}{c+d}}\sqrt{\frac{c(1+\cos(e+fx))}{c-d}}\cot(e+fx)\Bigg(\text{EllipticPi}\Bigg(1-\frac{c}{d},i\text{arcsinh}\Bigg(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(e+fx}}{a\sqrt{\frac{1}{c-d}}}$$

input

```
Integrate[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f
*x]),x]
```

output

```
((-2*I)*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*x]
))/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1)]
*Sqrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(b*(-c + d))/(-
(a*c) + b*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c
+ d)/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a*Sqrt[(c -
 d)^(-1)]*f*Sqrt[d + c*Cos[e + f*x]])
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3441, 3042, 4459, 3042, 4346, 3042, 3286, 3042, 3284, 4463, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{g \csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{a+b\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3441$$

$$\frac{\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{b+a\sec(e+fx)}dx}{g}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{b+a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{g}$$

$$\Big\downarrow \ 4459$$

$$\frac{\frac{(ac-bd)\int \frac{(g\sec(e+fx))^{3/2}}{(b+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}dx}{a}+\frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}dx}{a}}{g}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(ac-bd)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{d\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}}{g}$$

$$\Big\downarrow \ 4346$$

$$\frac{\frac{(ac-bd)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{dg\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+fx)}}dx}{a\sqrt{c+d\sec(e+fx)}}}{g}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(ac-bd)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{dg\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a\sqrt{c+d\sec(e+fx)}}}{g}$$

$$\Big\downarrow \ 3286$$

$$\frac{\frac{(ac-bd)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{\sec(e+fx)}{\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}dx}{a\sqrt{c+d\sec(e+fx)}}}{g}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(ac-bd)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{\frac{d}{c+d}+\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{c+d}}}dx}{a\sqrt{c+d\sec(e+fx)}}}{g}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{(ac-bd)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(b+a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{af\sqrt{c+d\sec(e+fx)}}}{g}$$

$$\Big\downarrow \text{4463}$$

$$\frac{\frac{g(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{1}{(a+b\cos(e+fx))\sqrt{d+c\cos(e+fx)}}dx}{a\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{af\sqrt{c+d\sec(e+fx)}}}{g}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{g(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{1}{\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+f\right.}{af\sqrt{c+d\sec(e+fx)}}}{g}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\frac{g(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{1}{(a+b\cos(e+fx))\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}dx}{a\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+}\right.}{af\sqrt{c+d\sec(e+fx)}}}{g}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{g(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\int \frac{1}{\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)\sqrt{\frac{d}{c+d}+\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{c+d}}}dx}{a\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e\right.}{af\sqrt{c+d\sec(e+fx)}}}{g}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{2g(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{af(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{af\sqrt{c+d\sec(e+fx)}}}{g}$$

input
```
Int[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f*x]),x
]
```

output
```
((2*d*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c
)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(a*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(a*c
- b*d)*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*b)/(a + b), (e +
 f*x)/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(a*(a + b)*f*Sqrt[c + d*Sec[
e + f*x]]))/g
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3441
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
(c_))^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp
[g^m   Int[(g*Csc[e + f*x])^(p - m)*(b + a*Csc[e + f*x])^m*(c + d*Csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[m]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4459
```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[b/d
 Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[(b*c -
a*d)/d   Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e
+ f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0]
```

rule 4463
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])   Int
[1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.12 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.63

method	result
default	$\frac{2i\Big(\text{EllipticF}\Big(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{c-d}{c+d}}\Big)a^2c-\text{EllipticF}\Big(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{c-d}{c+d}}\Big)a^2d+\text{EllipticF}\Big(i(\cot(fx+e)}{}$

input
```
int((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f/a/(a+b)/(a-b)*(EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(c+d))^(1
/2))*a^2*c-EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*a^2*d
+EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*a*b*c-EllipticF
(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*a*b*d+2*EllipticPi(I*(cot
(f*x+e)-csc(f*x+e)),-1,I*((c-d)/(c+d))^(1/2))*a^2*d-2*EllipticPi(I*(cot(f*
x+e)-csc(f*x+e)),-1,I*((c-d)/(c+d))^(1/2))*b^2*d-2*EllipticPi(I*(cot(f*x+e
)-csc(f*x+e)),(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*a*b*c+2*EllipticPi(I*(cot
(f*x+e)-csc(f*x+e)),(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*b^2*d)*(g*sec(f*x+e
))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^
(1/2)*cos(f*x+e)/(d+c*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \int \frac{\sqrt{g\sec\left(e+fx\right)}\sqrt{c+d\sec\left(e+fx\right)}}{a+b\cos\left(e+fx\right)}\,dx$$

input
```
integrate((g*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*cos(f*x+e)),x
)
```

output
```
Integral(sqrt(g*sec(e + f*x))*sqrt(c + d*sec(e + f*x))/(a + b*cos(e + f*x)
), x)
```

Maxima [F]

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sqrt{g\sec\left(fx+e\right)}}{b\cos\left(fx+e\right)+a}\,dx$$

input
```
integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x,
algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) +
a), x)
```

Giac [F]

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sqrt{g\sec\left(fx+e\right)}}{b\cos\left(fx+e\right)+a}\,dx$$

input
```
integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x,
algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) +
a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}\,\sqrt{\frac{g}{\cos(e+f\,x)}}}{a+b\,\cos\left(e+f\,x\right)}\,dx$$

input
```
int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(1/2))/(a + b*cos(e + f*x
)),x)
```

output
```
int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(1/2))/(a + b*cos(e + f*x
)), x)
```

Reduce [F]

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx$$
$$=\sqrt{g}\left(\int \frac{\sqrt{\sec\left(fx+e\right)}\,\sqrt{\sec\left(fx+e\right)d+c}}{\cos\left(fx+e\right)b+a}dx\right)$$

input
```
int((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x)
```

output
```
sqrt(g)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x)*d + c))/(cos(e + f*x)*b
+ a),x)
```

3.273 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

output
```
EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*(1/(1+sec(f*x+e))
)^(1/2)*(a+b*sec(f*x+e))^(1/2)/c/f/((a+b*sec(f*x+e))/(a+b)/(1+sec(f*x+e)))
^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 264 vs. 2(95) = 190.

Time = 5.72 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.78

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|\frac{a-b}{a+b}\right)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{1+}}{\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}}\right.}{4cf(1}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

output
```
(Cos[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((2*Sqrt[C
os[e + f*x]/(1 + Cos[e + f*x])]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b
)/(a + b)]*Sec[(e + f*x)/2]^4*Sqrt[1 + Sec[e + f*x]])/(((1 + Cos[e + f*x])
^(-1))^(3/2)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]) + (S
ec[(e + f*x)/2]^5*Sqrt[1 + Sec[e + f*x]]*(-Sin[(e + f*x)/2] + Sin[(3*(e +
f*x))/2]))/((1 + Cos[e + f*x])^(-1))^(3/2) - 8*Sqrt[Sec[e + f*x]]*(Sin[e +
 f*x] - Tan[(e + f*x)/2])))/(4*c*f*(1 + Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3042, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c\sec(e+fx)+c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{c\csc\left(e+fx+\frac{\pi}{2}\right)+c}dx$$

$$\Big\downarrow \text{4456}$$

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

output
```
(EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[
(1 + Sec[e + f*x])^(-1)]*Sqrt[a + b*Sec[e + f*x]])/(c*f*Sqrt[(a + b*Sec[e
+ f*x])/((a + b)*(1 + Sec[e + f*x]))])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{(-a-b)(\cos(fx+e)+1)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\operatorname{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{a+b\sec(fx+e)}}{cf(b+a\cos(fx+e))}$	123

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
1/c/f*(-a-b)*(cos(f*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+
a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)
/(a+b))^(1/2))*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{\int \frac{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{c}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x)/c
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + c/cos(e + f*x))),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + c/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\frac{\int \frac{\sqrt{\sec(fx+e)b+a}\,\sec(fx+e)}{\sec(fx+e)+1}dx}{c}$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
int((sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(e + f*x) + 1),x)/c
```

3.274 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 295

$$\begin{aligned}\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx &= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)\middle|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}}\\
&+\frac{(a-b)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}\\
&+\frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\mathrm{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}\\
&-\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}\end{aligned}$$

output
```
g*(b+a*cos(f*x+e))*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*(
g*sec(f*x+e))^(1/2)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e))^(1
/2)+(a-b)*g*((b+a*cos(f*x+e))/(a+b))^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2
^(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)+2*
b*g*((b+a*cos(f*x+e))/(a+b))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^(1/2)
*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)-g*(b+a*c
os(f*x+e))*(g*sec(f*x+e))^(1/2)*sin(f*x+e)/f/(c+c*cos(f*x+e))/(a+b*sec(f*x
+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$$

input

```
Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
 f*x]),x]
```

output

```
Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
 f*x]), x]
```

Rubi [A] (verified)

Time = 2.69 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 4459, 3042, 4346, 3042, 3286, 3042, 3284, 4463, 3042, 3247, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c\sec(e+fx)+c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{c\csc\left(e+fx+\frac{\pi}{2}\right)+c}dx$$

$$\Big\downarrow \text{4459}$$

$$\frac{b\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}dx}{c}+(a-b)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(\sec(e+fx)c+c)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}+(a-b)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx$$

$$\Big\downarrow \textbf{4346}$$

$$(a-b)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx+\frac{bg\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}dx}{c\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$(a-b)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx+\frac{bg\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \textbf{3286}$$

$$(a-b)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx+\frac{bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}dx}{c\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$(a-b)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx+\frac{bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}}dx}{c\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \textbf{3284}$$

$$(a-b)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx+\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \; 4463$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sqrt{b+a\cos(e+fx)}(\cos(e+fx)c+c)}dx}{\sqrt{a+b\sec(e+fx)}}+\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(\sin\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx}{\sqrt{a+b\sec(e+fx)}}+\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \; 3247$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(-\frac{a\int -\frac{\cos(e+fx)c+c}{2\sqrt{b+a\cos(e+fx)}}dx}{c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}+\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\int \frac{\cos(e+fx)c+c}{\sqrt{b+a\cos(e+fx)}}dx}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}+\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)c+c}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}+\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \ \color{blue}{3231}$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\Bigg(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\cos(e+fx)}}dx}{a}+\frac{c\int\sqrt{b+a\cos(e+fx)}dx}{a}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+}}{f(a-b)(c\cos(e+fx)+c)}}{\sqrt{a+b\sec(e+fx)}}$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\Bigg(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{c\int\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{a}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(}}{f(a-b)(c\cos(e+}}{\sqrt{a+b\sec(e+fx)}}$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \ \color{blue}{3134}$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\Bigg(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{c\sqrt{a\cos(e+fx)+b}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}dx}{a\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\frac{\sin(}{f}}{\sqrt{a+b\sec(e+fx)}}$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{c\sqrt{a\cos(e+fx)+b}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow\ 3132$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\frac{\sin(e+f}{f(a-b}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow\ 3142$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}dx}{a\sqrt{a\cos(e+fx)+b}}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right.}{2c^2(a-b)}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow\ 3042$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a\cos(e+fx)+b}}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\right.}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right.}{2c^2(a-b)}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{2c(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{af\sqrt{a\cos(e+fx)+b}}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right.}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right.}{2c^2(a-b)}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

input

```
Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x])
,x]
```

output

```
(2*b*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)
/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + ((a - b)*
g*Sqrt[b + a*Cos[e + f*x]]*Sqrt[g*Sec[e + f*x]]*((a*((2*c*Sqrt[b + a*Cos[e
 + f*x]]*EllipticE[(e + f*x)/2, (2*a)/(a + b)])/(a*f*Sqrt[(b + a*Cos[e + f
*x])/(a + b)]) + (2*(a - b)*c*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF
[(e + f*x)/2, (2*a)/(a + b)])/(a*f*Sqrt[b + a*Cos[e + f*x]])))/(2*(a - b)*
c^2) - (Sqrt[b + a*Cos[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e + f
*x]))))/Sqrt[a + b*Sec[e + f*x]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4459
```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[b/d
 Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[(b*c -
a*d)/d   Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e
+ f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0]
```

rule 4463
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])   Int
[1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.93

method	result
default	$\frac{ig\left(2a\,\mathrm{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)-2b\,\mathrm{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)-a\,\mathrm{EllipticE}\left(i(\cot(fx+e)\right.\right.}{}$

input
```
int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
I*g/c/f*(2*a*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-2*b
*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-a*EllipticE(I*(
cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(cot(f*x+e)-csc
(f*x+e)),(-(a-b)/(a+b))^(1/2))+4*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1,I
*((a-b)/(a+b))^(1/2))*b)*(g*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(1/(a
+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)/(b+a*cos(f*x+e))/(1/
(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}\sqrt{a+b\sec(e+fx)}}{\sec(e+fx)+1}\,dx}{c}$$

input
```
integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x
)
```

output
```
Integral((g*sec(e + f*x))**(3/2)*sqrt(a + b*sec(e + f*x))/(sec(e + f*x) +
1), x)/c
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}\,(g\sec(fx+e))^{\frac{3}{2}}}{c\sec(fx+e)+c}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,
algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e)
+ c), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}(g\sec\left(fx+e\right))^{\frac{3}{2}}}{c\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,
algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e)
+ c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,{\left(\frac{g}{\cos(e+f\,x)}\right)}^{3/2}}{c+\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + c/cos(e + f*x
)),x)
```

output
```
int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + c/cos(e + f*x
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\frac{\sqrt{g}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)b+a}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)g}{c}$$

input
```
int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

output

```
(sqrt(g)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(s
ec(e + f*x) + 1),x)*g)/c
```

3.275 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 209

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)cf} + \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{(a-b)cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

output
```
-2*(a+b)^(1/2)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a
+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))
^(1/2)/(a-b)/c/f+EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*
(1/(1+sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a-b)/c/f/((a+b*sec(f*x+e)
)/(a+b)/(1+sec(f*x+e)))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 12.51 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.79

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$
$$= \frac{\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(b+a\cos(e+fx))\sec^2(e+fx)\left(\frac{2\sin(e+fx)}{-a+b}-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{-a+b}\right)}{f\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}$$
$$- \frac{2\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^{\frac{3}{2}}(e+fx)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}\Bigg((a-b)E\Bigg(\arcsin\Bigg(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)}{\left(\frac{a-b}{a+b}\right)^{3/2}(a+b)f\sqrt{\cos(e+fx)s}}$$

input
```
Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

output
```
(Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*Sec[e + f*x]^2*((2*Sin[e + f*x]
)/(-a + b) - (2*Tan[(e + f*x)/2])/(-a + b)))/(f*Sqrt[a + b*Sec[e + f*x]]*(
c + c*Sec[e + f*x])) - (2*Cos[e/2 + (f*x)/2]^2*Sec[e + f*x]^(3/2)*Sqrt[Cos
[(e + f*x)/2]^2*Sec[e + f*x]]*((a - b)*EllipticE[ArcSin[Sqrt[(a - b)/(a +
b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[e + f*x])*Sec[(e
+ f*x)/2]^2)/(a + b)] + Sqrt[2]*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[e + f*x]/(1
 + Cos[e + f*x])]*(b + a*Cos[e + f*x])*Tan[(e + f*x)/2])*(-1 + Tan[(e + f*
x)/2]^2))/(((a - b)/(a + b))^(3/2)*(a + b)*f*Sqrt[Cos[e + f*x]*Sec[(e + f*
x)/2]^4]*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 4460, 3042, 4319, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(c\sec(e+fx)+c)\sqrt{a+b\sec(e+fx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c\csc\left(e+fx+\frac{\pi}{2}\right)+c\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4460$$

$$\frac{\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sec(e+fx)c+c}dx}{a-b}-\frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{c(a-b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)c+c}dx}{a-b}-\frac{b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c(a-b)}$$

$$\Big\downarrow \ 4319$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)c+c}dx}{a-b}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{cf(a-b)}$$

$$\Big\downarrow \ 4456$$

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{cf(a-b)}$$

input

```
Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

output

```
(-2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[e + f*x]))/(a - b))])/((a - b)*c*f) + (EllipticE[ArcSin[Tan[e +
f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*S
qrt[a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(
1 + Sec[e + f*x]))])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))]]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

rule 4460
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[b/(b*c - a*d)   Int
[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[d/(b*c - a*d)   Int[C
sc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[
c^2 - d^2, 0])
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\big(a\,\mathrm{EllipticE}\big(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\big)+b\,\mathrm{EllipticE}\big(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\big)-2\,\mathrm{EllipticF}\big(\cot(fx+e)-\csc(fx+e)}{cf(a-b)(b+a\cos(fx+e))}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
-1/c/f/(a-b)*(a*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))+b*Ell
ipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))-2*EllipticF(cot(f*x+e)-c
sc(f*x+e),((a-b)/(a+b))^(1/2))*b)*(cos(f*x+e)+1)*(a+b*sec(f*x+e))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^
(1/2)/(b+a*cos(f*x+e))
```

Fricas [F]

$$\begin{aligned}\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx\\ =\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)+c)}\,dx\end{aligned}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(b*c*sec(f*x + e)^2 + (a +
b)*c*sec(f*x + e) + a*c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx=\frac{\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)+\sqrt{a+b\sec\left(e+fx\right)}}\,dx}{c}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a + b*
sec(e + f*x))), x)/c
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))),x)
```

output
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx = \frac{\int \frac{\sqrt{\sec(fx+e)b+a}\,\sec(fx+e)}{\sec(fx+e)^2b+\sec(fx+e)a+\sec(fx+e)b+a}dx}{c}$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
int((sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(e + f*x)**2*b + sec(e + f
*x)*a + sec(e + f*x)*b + a),x)/c
```

3.276 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 214

$$\begin{aligned}\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx &= \frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)bcf} \\ &\quad - \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{(a-b)cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}\end{aligned}$$

output

```
2*a*(a+b)^(1/2)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b)
)^(1/2)/(a-b)/b/c/f-EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2
))*(1/(1+sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a-b)/c/f/((a+b*sec(f*x
+e))/(a+b)/(1+sec(f*x+e)))^(1/2)
```

Mathematica [A] (verified)

Time = 4.88 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$
$$= \frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\Big((a+b)E\Big(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\Big)-2a\,\text{EllipticF}\,\big(\arcsin\,(\text{ta}}{(-a+b)cf\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}(1+\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x
]
```

output
```
(4*Cos[(e + f*x)/2]^4*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]
))]*((a + b)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*a*El
lipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/((-a + b)*c*f*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])]*(1 + Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x
]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4464, 3042, 4319, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(c\sec(e+fx)+c)\sqrt{a+b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\left(c\csc\left(e+fx+\frac{\pi}{2}\right)+c\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4464}$$

$$\frac{a\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{c(a-b)}-\frac{\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sec(e+fx)c+c}dx}{a-b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c(a-b)}-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)c+c}dx}{a-b}$$

$$\Big\downarrow \text{4319}$$

$$\frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bcf(a-b)}-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)c+c}dx}{a-b}$$

$$\Big\downarrow \text{4456}$$

$$\frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bcf(a-b)}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

input

```
Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

output

```
(2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[e + f*x]))/(a - b))])/((a - b)*b*c*f) - (EllipticE[ArcSin[Tan[e
 + f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)
]*Sqrt[a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b
)*(1 + Sec[e + f*x]))])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))]]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

rule 4464
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-a/(b*c - a*d)
Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[c/(b*c - a*d)   In
t[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || E
qQ[c^2 - d^2, 0])
```

Maple [A] (verified)

Time = 5.75 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\left(2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a-a\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)-b\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e\right.\right.}{cf(a-b)(b+a\cos(fx+e))}$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output

```
-1/c/f/(a-b)*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a-a*E
llipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))-b*EllipticE(cot(f*x+e)
-csc(f*x+e),((a-b)/(a+b))^(1/2)))*(cos(f*x+e)+1)*(a+b*sec(f*x+e))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^
(1/2)/(b+a*cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$
$$= \int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)+c)}\,dx$$

input

```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorith
m="fricas")
```

output

```
integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)^2/(b*c*sec(f*x + e)^2 + (a
+ b)*c*sec(f*x + e) + a*c), x)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)+\sqrt{a+b\sec\left(e+fx\right)}}\,dx}{c}$$

input

```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

output

```
Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a +
 b*sec(e + f*x))), x)/c
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)+c)}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)),
x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)+c)}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorith
m="giac")
```

output
```
integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$
$$=\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{\int \frac{\sqrt{\sec(fx+e)b+a}\,\sec(fx+e)^2}{\sec(fx+e)^2b+\sec(fx+e)a+\sec(fx+e)b+a}dx}{c}$$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
int((sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2)/(sec(e + f*x)**2*b + sec(e
+ f*x)*a + sec(e + f*x)*b + a),x)/c
```

3.277 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 229

$$\begin{aligned}\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx &= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)\middle|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}}\\ &+\frac{g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}\\ &-\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}\end{aligned}$$

output

```
g*(b+a*cos(f*x+e))*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*(
g*sec(f*x+e))^(1/2)/(a-b)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+
e))^(1/2)+g*((b+a*cos(f*x+e))/(a+b))^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2
^(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)-g*
(b+a*cos(f*x+e))*(g*sec(f*x+e))^(1/2)*sin(f*x+e)/(a-b)/f/(c+c*cos(f*x+e))/
(a+b*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.29 (sec) , antiderivative size = 1019, normalized size of antiderivative = 4.45

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \text{Too large to display}$$

input

```
Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e +
f*x])),x]
```

output

```
(Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*(g*Sec[e + f*x])^(3/2)*((2*Csc[
e])/((-a + b)*f) + (2*Sec[e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/((-a + b)*
f)))/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])) + (AppellF1[1/2, 1/2,
 1/2, 3/2, (Csc[e]*(b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e
]]]))/(a*Sqrt[1 + Cot[e]^2]*(1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2]))), (Csc
[e]*(b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]]))/(a*Sqrt[1
 + Cot[e]^2]*(-1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2])))]*Cos[e/2 + (f*x)/2]
^2*Sqrt[b + a*Cos[e + f*x]]*Csc[e/2]*Sec[e/2]*(g*Sec[e + f*x])^(3/2)*Sec[f
*x - ArcTan[Cot[e]]]*Sqrt[(a*Sqrt[1 + Cot[e]^2] - a*Sqrt[1 + Cot[e]^2]*Sin
[f*x - ArcTan[Cot[e]]])/(a*Sqrt[1 + Cot[e]^2] - b*Csc[e])]*Sqrt[(a*Sqrt[1
+ Cot[e]^2] + a*Sqrt[1 + Cot[e]^2]*Sin[f*x - ArcTan[Cot[e]]])/(a*Sqrt[1 +
Cot[e]^2] + b*Csc[e])]*Sqrt[b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcT
an[Cot[e]]]])/((-a + b)*f*Sqrt[1 + Cot[e]^2]*Sqrt[a + b*Sec[e + f*x]]*(c +
 c*Sec[e + f*x])) + (a*Cos[e/2 + (f*x)/2]^2*Sqrt[b + a*Cos[e + f*x]]*Csc[e
/2]*Sec[e/2]*(g*Sec[e + f*x])^(3/2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((S
ec[e]*(b + a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt
[1 + Tan[e]^2]*(1 - (b*Sec[e])/(a*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(b + a
*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt[1 + Tan[e]^
2]*(-1 - (b*Sec[e])/(a*Sqrt[1 + Tan[e]^2]))))]*Sin[f*x + ArcTan[Tan[e]]]*T
an[e])/(Sqrt[1 + Tan[e]^2]*Sqrt[(a*Sqrt[1 + Tan[e]^2] - a*Cos[f*x + Arc...
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.359$, Rules used = {3042, 4463, 3042, 3247, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{3/2}}{(c\sec(e+fx)+c)\sqrt{a+b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(c\csc\left(e+fx+\frac{\pi}{2}\right)+c\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4463}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sqrt{b+a\cos(e+fx)}(\cos(e+fx)c+c)}dx}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(\sin\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3247}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(-\frac{a\int -\frac{\cos(e+fx)c+c}{2\sqrt{b+a\cos(e+fx)}}dx}{c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\int \frac{\cos(e+fx)c+c}{\sqrt{b+a\cos(e+fx)}}dx}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)c+c}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 3231

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\cos(e+fx)}}dx}{a}+\frac{c\int\sqrt{b+a\cos(e+fx)}dx}{a}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{c\int\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{a}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+}}{f(a-b)(c\cos(e+fx)+c)}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 3134

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{c\sqrt{a\cos(e+fx)+b}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}dx}{a\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{}}{f(a-b)(c\,c}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{c\sqrt{a\cos(e+fx)+b}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\frac{\sin(e+f}{f(a-b}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \color{blue}{3132}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos}}{f(a-b)(c\cos(e}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \color{blue}{3142}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}dx}{a\sqrt{a\cos(e+fx)+b}}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\mathrm{si}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a\cos(e+fx)+b}}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \color{blue}{3140}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{2c(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{af\sqrt{a\cos(e+fx)+b}}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\mathrm{si}\right.}{\sqrt{a+b\sec(e+fx)}}$$

input
```
Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))
,x]
```

output
```
(g*Sqrt[b + a*Cos[e + f*x]]*Sqrt[g*Sec[e + f*x]]*((a*((2*c*Sqrt[b + a*Cos[
e + f*x]]*EllipticE[(e + f*x)/2, (2*a)/(a + b)])/(a*f*Sqrt[(b + a*Cos[e +
f*x])/(a + b)]) + (2*(a - b)*c*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Elliptic
F[(e + f*x)/2, (2*a)/(a + b)])/(a*f*Sqrt[b + a*Cos[e + f*x]])))/(2*(a - b)
*c^2) - (Sqrt[b + a*Cos[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e +
f*x]))))/Sqrt[a + b*Sec[e + f*x]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4463
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])   Int
[1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90

method	result
default	$\frac{ig\left(2a\,\text{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)-a\,\text{EllipticE}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)-b\,\text{EllipticE}\left(i(\cot(fx+e)-\right.\right.}{cf(a-b)(b+a\cos(fx+e))\sqrt{\frac{1}{\cos(fx+e)+1}}}$

input
```
int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
I*g/c/f/(a-b)*(2*a*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2
))-a*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-b*EllipticE
(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2)))*(g*sec(f*x+e))^(1/2)*(a+
b*sec(f*x+e))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*
x+e)/(b+a*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.26

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,
algorithm="fricas")
```

output
```
-1/6*(6*a*g*sqrt((a*cos(f*x + e) + b)/cos(f*x + e))*sqrt(g/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + sqrt(2)*(I*(3*a - 2*b)*g*cos(f*x + e) + I*(3*a
- 2*b)*g)*sqrt(a*g)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(f*x + e) + 3*I*a*sin(f*x + e) + 2*b)/a) +
 sqrt(2)*(-I*(3*a - 2*b)*g*cos(f*x + e) - I*(3*a - 2*b)*g)*sqrt(a*g)*weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(
3*a*cos(f*x + e) - 3*I*a*sin(f*x + e) + 2*b)/a) - 3*sqrt(2)*(I*a*g*cos(f*x
 + e) + I*a*g)*sqrt(a*g)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(f*x + e) + 3*I*a*sin(f*x + e) + 2*b)/a))
 - 3*sqrt(2)*(-I*a*g*cos(f*x + e) - I*a*g)*sqrt(a*g)*weierstrassZeta(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(f*x + e) - 3
*I*a*sin(f*x + e) + 2*b)/a)))/((a^2 - a*b)*c*f*cos(f*x + e) + (a^2 - a*b)*
c*f)
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{\sqrt{a+b\sec(e+fx)}\sec(e+fx)+\sqrt{a+b\sec(e+fx)}}\,dx}{c}$$

input
```
integrate((g*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x
)
```

output
```
Integral((g*sec(e + f*x))**(3/2)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) +
sqrt(a + b*sec(e + f*x))), x)/c
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,
algorithm="maxima")
```

output
```
integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
 + c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,
algorithm="giac")
```

output
```
integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx=\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x)
)),x)
```

output
```
int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x)
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx=\frac{\sqrt{g}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)b+a}\,\sec(fx+e)}{\sec(fx+e)^{2}b+\sec(fx+e)a+\sec(fx+e)b+a}dx\right)g}{c}$$

input
```
int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
(sqrt(g)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(s
ec(e + f*x)**2*b + sec(e + f*x)*a + sec(e + f*x)*b + a),x)*g)/c
```

3.278 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 312

$$\begin{aligned}
\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx =
&-\frac{g^2(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} \\
&-\frac{g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} \\
&+\frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} \\
&+\frac{g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}
\end{aligned}$$

output

```
-g^2*(b+a*cos(f*x+e))*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2)
)*(g*sec(f*x+e))^(1/2)/(a-b)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f
*x+e))^(1/2)-g^2*((b+a*cos(f*x+e))/(a+b))^(1/2)*InverseJacobiAM(1/2*f*x+1/
2*e,2^(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/
2)+2*g^2*((b+a*cos(f*x+e))/(a+b))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^
(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)+g^2
*(b+a*cos(f*x+e))*(g*sec(f*x+e))^(1/2)*sin(f*x+e)/(a-b)/f/(c+c*cos(f*x+e))
/(a+b*sec(f*x+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx=\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$

input

```
Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e +
f*x])),x]
```

output

```
Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e +
f*x])), x]
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.99, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 4467, 3042, 4346, 3042, 3286, 3042, 3284, 4463, 3042, 3247, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{5/2}}{(c\sec(e+fx)+c)\sqrt{a+b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(c\csc\left(e+fx+\frac{\pi}{2}\right)+c\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4467}$$

$$\frac{g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}dx}{c}-g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(\sec(e+fx)c+c)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{g\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-g\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx$$

$$\Big\downarrow \text{4346}$$

$$\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}dx}{c\sqrt{a+b\sec(e+fx)}}-g\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c\sqrt{a+b\sec(e+fx)}}-g\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx$$

$$\Big\downarrow \text{3286}$$

$$\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}dx}{c\sqrt{a+b\sec(e+fx)}}-g\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}}dx}{c\sqrt{a+b\sec(e+fx)}}-g\int\frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx$$

$$\Big\downarrow \text{3284}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-g\int\frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx$$

$$\Big\downarrow \text{4463}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int\frac{1}{\sqrt{b+a\cos(e+fx)}(\cos(e+fx)c+c)}dx}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(\sin\left(e+fx+\frac{\pi}{2}\right)c+c\right)}dx}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3247}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(-\frac{a\int-\frac{\cos(e+fx)c+c}{2\sqrt{b+a\cos(e+fx)}}dx}{c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\int\frac{\cos(e+fx)c+c}{\sqrt{b+a\cos(e+fx)}}dx}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)c+c}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3231}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\cos(e+fx)}}dx}{a}+\frac{c\int\sqrt{b+a\cos(e+fx)}dx}{a}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{f(a-b)(c\cos(e+fx)+c)}\right)}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{c\int\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{a}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\cos(e+fx)+}}{f(a-b)(c\cos(e+fx)+c)}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{c\sqrt{a\cos(e+fx)+b}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}dx}{a\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)}{f(a-b)(c}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{c\sqrt{a\cos(e+fx)+b}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\frac{\sin(e+}{f(a-}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \; 3132$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\frac{\sin(e+fx)\sqrt{a\,co}}{f(a-b)(c\cos(e}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \; 3142$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}dx}{a\sqrt{a\cos(e+fx)+b}}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{c(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a\cos(e+fx)+b}}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}\right.}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\left(\frac{a\left(\frac{2c(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{af\sqrt{a\cos(e+fx)+b}}+\frac{2c\sqrt{a\cos(e+fx)+b}E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{af\sqrt{\frac{a\cos(e+fx)+b}{a+b}}}\right)}{2c^2(a-b)}-\right.}{\sqrt{a+b\sec(e+fx)}}$$

input

```
Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))
,x]
```

output

```
(2*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)
/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g^2*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[g*Sec[e + f*x]]*((a*((2*c*Sqrt[b + a*Cos[e + f*x
]]*EllipticE[(e + f*x)/2, (2*a)/(a + b)])/(a*f*Sqrt[(b + a*Cos[e + f*x])/(
a + b)]) + (2*(a - b)*c*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e +
f*x)/2, (2*a)/(a + b)])/(a*f*Sqrt[b + a*Cos[e + f*x]])))/(2*(a - b)*c^2) -
 (Sqrt[b + a*Cos[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e + f*x]))
)/Sqrt[a + b*Sec[e + f*x]]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4463
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])   Int
[1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4467
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g/d
 Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[c*(g/d)
   Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.04

method	result
default	$i\Big(4\,\mathrm{EllipticF}\Big(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\Big)a-2\,\mathrm{EllipticF}\Big(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\Big)b-a\,\mathrm{EllipticE}\Big(i(-\cot(f$

input
```
int((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
I/c/f/(a-b)*(4*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*
a-2*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*b-a*Ellipti
cE(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(-cot(f*
x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-4*EllipticPi(I*(-cot(f*x+e)+csc(f*x
+e)),-1,I*((a-b)/(a+b))^(1/2))*a+4*b*EllipticPi(I*(-cot(f*x+e)+csc(f*x+e))
,-1,I*((a-b)/(a+b))^(1/2)))*(a+b*sec(f*x+e))^(1/2)*(g*sec(f*x+e))^(1/2)*(1
/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*g^2/(b+a*cos(f*x+
e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx=\text{Timed out}$$

input
```
integrate((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx=\text{Timed out}$$

input
```
integrate((g*sec(f*x+e))**(5/2)/(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx=\int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

input
```
integrate((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,
algorithm="maxima")
```

output
```
integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
 + c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

input
```
integrate((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,
algorithm="giac")
```

output
```
integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x)
)),x)
```

output
```
int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x)
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{\sqrt{g}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)b+a}\,\sec(fx+e)^2}{\sec(fx+e)^2b+\sec(fx+e)a+\sec(fx+e)b+a}dx\right)g^2}{c}$$

input
```
int((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
(sqrt(g)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2)
/(sec(e + f*x)**2*b + sec(e + f*x)*a + sec(e + f*x)*b + a),x)*g**2)/c
```

3.279 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 213

$$\begin{aligned}\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx &= \frac{2\sqrt{a+b}\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{df}\\ &\quad-\frac{2(bc-ad)\operatorname{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}\end{aligned}$$

output

```
2*(a+b)^(1/2)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^
(1/2)/d/f-2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+
d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c
+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$
$$= \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a-b)(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-}{a-}\right.\right.}{(c-d)(c+d)f(b+a\cos(e}$$

input
```
Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

output
```
(4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSi
n[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(b*c - a*d)*EllipticPi[(c - d)/(
c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x
]])/((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 4457, 3042, 4319, 4461}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 4457}$$
$$\frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{d}-\frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{\csc \left(e+fx+\frac{\pi }{2}\right)}{\sqrt{a+b \csc \left(e+fx+\frac{\pi }{2}\right)}}dx}{d}-\frac{(bc-ad) \int \frac{\csc \left(e+fx+\frac{\pi }{2}\right)}{\sqrt{a+b \csc \left(e+fx+\frac{\pi }{2}\right)} \left(c+d \csc \left(e+fx+\frac{\pi }{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{4319}$$

$$\frac{2 \sqrt{a+b} \cot (e+fx) \sqrt{\frac{b (1-\sec (e+fx))}{a+b}} \sqrt{-\frac{b (\sec (e+fx)+1)}{a-b}} \text{EllipticF}\left(\arcsin \left(\frac{\sqrt{a+b \sec (e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}-\frac{(bc-ad) \int \frac{\csc \left(e+fx+\frac{\pi }{2}\right)}{\sqrt{a+b \csc \left(e+fx+\frac{\pi }{2}\right)} \left(c+d \csc \left(e+fx+\frac{\pi }{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{4461}$$

$$\frac{2 \sqrt{a+b} \cot (e+fx) \sqrt{\frac{b (1-\sec (e+fx))}{a+b}} \sqrt{-\frac{b (\sec (e+fx)+1)}{a-b}} \text{EllipticF}\left(\arcsin \left(\frac{\sqrt{a+b \sec (e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}-\frac{2 (bc-ad) \tan (e+fx) \sqrt{\frac{a+b \sec (e+fx)}{a+b}} \text{EllipticPi}\left(\frac{2 d}{c+d},\arcsin \left(\frac{\sqrt{1-\sec (e+fx)}}{\sqrt{2}}\right),\frac{2 b}{a+b}\right)}{df (c+d) \sqrt{-\tan ^2(e+fx)} \sqrt{a+b \sec (e+fx)}}$$

input
```
Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

output
```
(2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c
+ d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*S
ec[e + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*
Sqrt[-Tan[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4457
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[b/d   Int[Csc[e + f
*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[(b*c - a*d)/d   Int[Csc[e + f*x
]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 6.72 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

method	result
default	$2\left(2\,\mathrm{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),\frac{c-d}{c+d},\sqrt{\frac{a-b}{a+b}}\right)ad-2\,\mathrm{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),\frac{c-d}{c+d},\sqrt{\frac{a-b}{a+b}}\right)bc-\mathrm{EllipticF}\left(\cot(fx+e)\right.\right.$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f/(c-d)/(c+d)*(2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+
b))^(1/2))*a*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b)
)^(1/2))*b*c-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*c-Elli
pticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*d+EllipticF(cot(f*x+e)-
csc(f*x+e),((a-b)/(a+b))^(1/2))*b*c+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)
/(a+b))^(1/2))*b*d)*(cos(f*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a
+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos
(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+b\sec{(e+fx)}}\sec{(e+fx)}}{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)\,d+c}dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
int((sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(e + f*x)*d + c),x)
```

3.280 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 170

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+}}{df\sqrt{a+b\sec(e+fx)}}$$

$$-\frac{2(bc-ad)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{d(c+d)f\sqrt{a+b\sec(e+fx)}}$$

output

```
2*b*g*((b+a*cos(f*x+e))/(a+b))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^(1/
2)*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/d/f/(a+b*sec(f*x+e))^(1/2)-2*(-a*
d+b*c)*g*((b+a*cos(f*x+e))/(a+b))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2*c/
(c+d),2^(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/d/(c+d)/f/(a+b*sec(f*x
+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.18 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.31

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx =$$

$$-\frac{2ig\sqrt{-\frac{a(-1+\cos(e+fx))}{a+b}}\sqrt{\frac{a(1+\cos(e+fx))}{a-b}}\cot(e+fx)\Big(\text{EllipticPi}\Big(1-\frac{a}{b}, i\text{arcsinh}\Big(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(e+fx)}}{\sqrt{\frac{1}{a-b}}df\sqrt{}}$$

input
```
Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e +
 f*x]),x]
```

output
```
((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*
x]))/(a - b)]*Cot[e + f*x]*(EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1
)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)] - EllipticPi[((a - b)*c)/(
-(b*c) + a*d), I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a
 + b)/(a + b)])*Sqrt[g*Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[(a -
b)^(-1)]*d*f*Sqrt[b + a*Cos[e + f*x]])
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4459, 3042, 4346, 3042, 3286, 3042, 3284, 4463, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4459}$$

$$\frac{b\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}dx}{d}-\frac{(bc-ad)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{d}-\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{4346}$$

$$\frac{bg\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}dx}{d\sqrt{a+b\sec(e+fx)}}-\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{bg\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{d\sqrt{a+b\sec(e+fx)}}-\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{3286}$$

$$\frac{bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}dx}{d\sqrt{a+b\sec(e+fx)}}-\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}}dx}{d\sqrt{a+b\sec(e+fx)}}-\frac{(bc-ad)\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{(bc-ad)\int\frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow 4463$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int\frac{1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))}dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow 3042$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int\frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow 3286$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}(d+c\cos(e+fx))}dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow 3042$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow 3284$$

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

input
```
Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x])
,x]
```

output
```
(2*b*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)
/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*(b*c -
 a*d)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e +
f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e
 + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4459
```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[b/d
 Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[(b*c -
a*d)/d   Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e
+ f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0]
```

rule 4463
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])   Int
[1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.07 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.62

method	result
default	$2ig\left(\mathrm{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)acd+\mathrm{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)a\,d^2-\mathrm{EllipticF}\left(i(-\mathrm{co}\right.\right.$

input
```
int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
2*I*g/f/d/(c-d)/(c+d)*(EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))
^(1/2))*a*c*d+EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*a
*d^2-EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*b*c*d-Elli
pticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*b*d^2-2*EllipticPi(
I*(-cot(f*x+e)+csc(f*x+e)),-1,I*((a-b)/(a+b))^(1/2))*b*c^2+2*EllipticPi(I*
(-cot(f*x+e)+csc(f*x+e)),-1,I*((a-b)/(a+b))^(1/2))*b*d^2-2*EllipticPi(I*(-
cot(f*x+e)+csc(f*x+e)),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2))*a*c*d+2*Ellipti
cPi(I*(-cot(f*x+e)+csc(f*x+e)),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2))*b*c^2)*
(g*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos
(f*x+e)+1))^(1/2)*cos(f*x+e)/(b+a*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}\sqrt{a+b\sec{(e+fx)}}}{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x
)
```

output
```
Integral((g*sec(e + f*x))**(3/2)*sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f
*x)), x)
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}(g\sec\left(fx+e\right))^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e)
+ c), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}(g\sec\left(fx+e\right))^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e)
+ c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,{\left(\frac{g}{\cos(e+f\,x)}\right)}^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x
)),x)
```

output
```
int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\sqrt{g}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)\,d+c}dx\right)$$

input
```
int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
sqrt(g)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(se
c(e + f*x)*d + c),x)*g
```

3.281 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

output
```
2*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),2^(1/2)*(b/(a+b))^
(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/(c+d)/f/(a+b*sec(f*x+e))^
(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 8.38 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.83

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)-2d\,\text{EllipticPi}\bigl(\frac{c-d}{c+d},\arcsin\bigl(\tan\bigl(\right.}{(c-d)(c+d)f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{}}$$

input

```
Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output

```
(2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*Ellipt
icF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*d*EllipticPi[(c - d)/(c
 + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[Cos[e + f*x]*Sec[(
e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])/((c - d)*(c + d)
*f*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = $\{3042, 4461\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4461}$$

$$\frac{2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\operatorname{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{f(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output

```
(2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)
/(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/((c + d)*f*Sqrt
[a + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(97) = 194$.

Time = 6.57 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.01

method	result
default	$\frac{2\left(2\,\mathrm{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),\frac{c-d}{c+d},\sqrt{\frac{a-b}{a+b}}\right)d-\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)c-\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx\right.\right.}{f(c-d)(c+d)(b+a\cos(fx+e))}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f/(c-d)/(c+d)*(2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+
b))^(1/2))*d-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*c-Ellipt
icF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*d)*(cos(f*x+e)+1)*(a+b*sec(
f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/
(cos(f*x+e)+1))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\begin{aligned}&\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx\\&=\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx\end{aligned}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\begin{aligned}&\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx\\&=\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)}\,dx\end{aligned}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm=
"giac")
```

output
```
integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$$
$$=\int \frac{\sqrt{\sec\left(fx+e\right)b+a}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)^{2}bd+\sec\left(fx+e\right)ad+\sec\left(fx+e\right)bc+ac}dx$$

input

```
int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output

```
int((sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(e + f*x)**2*b*d + sec(e +
 f*x)*a*d + sec(e + f*x)*b*c + a*c),x)
```

3.282 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 209

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bdf} - \frac{2c\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

output
```
2*(a+b)^(1/2)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^
(1/2)/b/d/f-2*c*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),2^(1
/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/f/(
a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 3.99 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.79

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx =$$
$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((c+d)\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)-\right.}{(c-d)(c+d)f\sqrt{a+b\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x
]
```

output
```
(-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*C
os[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*EllipticF[ArcSin[Tan[(
e + f*x)/2]], (a - b)/(a + b)] - 2*c*EllipticPi[(c - d)/(c + d), ArcSin[Ta
n[(e + f*x)/2]], (a - b)/(a + b)])*Sec[e + f*x])/((c - d)*(c + d)*f*Sqrt[a
 + b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4465, 3042, 4319, 4461}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \text{4465}$$

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{d}-\frac{c\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{d}-\frac{c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \textbf{4319}$$

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bdf}-\frac{c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \textbf{4461}$$

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bdf}-\frac{2c\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{df(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

input

```
Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output

```
(2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/(b*d*f) - (2*c*EllipticPi[(2*d)/(c + d), Ar
cSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f
*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Ta
n[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4465
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[1/d   Int[Csc[e +
 f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[c/d   Int[Csc[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 7.15 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.98

method	result
default	$-\frac{2\big(2c\,\mathrm{EllipticPi}\big(\cot(fx+e)-\csc(fx+e),\frac{c-d}{c+d},\sqrt{\frac{a-b}{a+b}}\big)-\mathrm{EllipticF}\big(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\big)c-\mathrm{EllipticF}\big(\cot(fx+e)-\csc(f}{f(c-d)(c+d)(b+a\cos(fx+e))}$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
-2/f/(c-d)/(c+d)*(2*c*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/
(a+b))^(1/2))-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*c-Ellip
ticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*d)*(cos(f*x+e)+1)*(a+b*sec
(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))
/(cos(f*x+e)+1))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec^2{(e+fx)}}{\sqrt{a+b\sec{(e+fx)}}\,(c+d\sec{(e+fx)})}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))),
x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)),
x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorith
m="giac")
```

output
```
integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)b+a}\,\sec\left(fx+e\right)^2}{\sec\left(fx+e\right)^2bd+\sec\left(fx+e\right)ad+\sec\left(fx+e\right)bc+ac}dx$$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
int((sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2)/(sec(e + f*x)**2*b*d + sec(
e + f*x)*a*d + sec(e + f*x)*b*c + a*c),x)
```

3.283 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 83

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e}}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

output
```
2*g*((b+a*cos(f*x+e))/(a+b))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2*c/(c+d)
,2^(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/(c+d)/f/(a+b*sec(f*x+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e}}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

input
```
Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e +
f*x])),x]
```

output
```
(2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x
)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*
x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 4463, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4463}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))}dx}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}(d+c\cos(e+fx))}dx}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow 3284$$

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{f(c+d)\sqrt{a+b\sec(e+fx)}}$$

input

```
Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
,x]
```

output

```
(2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x
)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*
x]])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4463

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])   Int
[1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.74 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.66

method	result
default	$\frac{2ig\left(c\,\mathrm{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)+d\,\mathrm{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)-2c\,\mathrm{EllipticPi}\left(i(-\cot\right.\right.}{f(c-d)(c+d)(b+a\cos(fx+e))\sqrt{\cos}}$

input
```
int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
2*I*g/f/(c-d)/(c+d)*(c*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))
^(1/2))+d*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-2*c*E
llipticPi(I*(-cot(f*x+e)+csc(f*x+e)),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2)))*
(g*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos
(f*x+e)+1))^(1/2)*cos(f*x+e)/(b+a*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\text{Timed out}$$

input
```
integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{a+b\sec{(e+fx)}}\,(c+d\sec{(e+fx)})}\,dx$$

input
```
integrate((g*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x
)
```

output
```
Integral((g*sec(e + f*x))**(3/2)/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e +
f*x))), x)
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{(g\sec{(fx+e)})^{\frac{3}{2}}}{\sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="maxima")
```

output
```
integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
 + c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{(g\sec{(fx+e)})^{\frac{3}{2}}}{\sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)}\,dx$$

input
```
integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="giac")
```

output
```
integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)
)),x)
```

output
```
int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\sqrt{g}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+}{\sec(fx+e)^2\,bd+\sec(fx+e)\,ad+\sec(fx+e)\,b}\right.$$

input
```
int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
sqrt(g)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(se
c(e + f*x)**2*b*d + sec(e + f*x)*a*d + sec(e + f*x)*b*c + a*c),x)*g
```

3.284 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 166

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+}}{df\sqrt{a+b\sec(e+fx)}}$$

$$-\frac{2cg^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\operatorname{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{d(c+d)f\sqrt{a+b\sec(e+fx)}}$$

output
```
2*g^2*((b+a*cos(f*x+e))/(a+b))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^(1/
2)*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/d/f/(a+b*sec(f*x+e))^(1/2)-2*c*g^
2*((b+a*cos(f*x+e))/(a+b))^(1/2)*EllipticPi(sin(1/2*f*x+1/2*e),2*c/(c+d),2
^(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x+e))^(1/2)/d/(c+d)/f/(a+b*sec(f*x+e))^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.25 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.48

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx =$$
$$-\frac{2ig\sqrt{-\frac{a(-1+\cos(e+fx))}{a+b}}\sqrt{\frac{a(1+\cos(e+fx))}{a-b}}\sqrt{b+a\cos(e+fx)}\cot(e+fx)\left((-bc+ad)\,\mathrm{EllipticPi}\left(1-\frac{a}{b},i\,\mathrm{arc}\right.\right.}{\sqrt{\frac{1}{a-b}}b}$$

input
```
Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e +
f*x])),x]
```

output
```
((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*
x]))/(a - b)]*Sqrt[b + a*Cos[e + f*x]]*Cot[e + f*x]*((-(b*c) + a*d)*Ellipt
icPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a
+ b)/(a + b)] + b*c*EllipticPi[((a - b)*c)/(-(b*c) + a*d), I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)])*(g*Sec[e + f*x
])^(3/2))/(Sqrt[(a - b)^(-1)]*b*d*(-(b*c) + a*d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4467, 3042, 4346, 3042, 3286, 3042, 3284, 4463, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(g \csc \left(e+f x+\frac{\pi }{2}\right)\right)^{5/2}}{\sqrt{a+b \csc \left(e+f x+\frac{\pi }{2}\right)} \left(c+d \csc \left(e+f x+\frac{\pi }{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4467}$$

$$\frac{g \int \frac{(g \sec (e+f x))^{3/2}}{\sqrt{a+b \sec (e+f x)}}dx}{d}-\frac{c g \int \frac{(g \sec (e+f x))^{3/2}}{\sqrt{a+b \sec (e+f x)} (c+d \sec (e+f x))}dx}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{g \int \frac{\left(g \csc \left(e+f x+\frac{\pi }{2}\right)\right)^{3/2}}{\sqrt{a+b \csc \left(e+f x+\frac{\pi }{2}\right)}}dx}{d}-\frac{c g \int \frac{\left(g \csc \left(e+f x+\frac{\pi }{2}\right)\right)^{3/2}}{\sqrt{a+b \csc \left(e+f x+\frac{\pi }{2}\right)} \left(c+d \csc \left(e+f x+\frac{\pi }{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{ 4346}$$

$$\frac{g^2 \sqrt{g \sec (e+f x)} \sqrt{a \cos (e+f x)+b} \int \frac{\sec (e+f x)}{\sqrt{b+a \cos (e+f x)}}dx}{d \sqrt{a+b \sec (e+f x)}}-\frac{c g \int \frac{\left(g \csc \left(e+f x+\frac{\pi }{2}\right)\right)^{3/2}}{\sqrt{a+b \csc \left(e+f x+\frac{\pi }{2}\right)} \left(c+d \csc \left(e+f x+\frac{\pi }{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{g^2 \sqrt{g \sec (e+f x)} \sqrt{a \cos (e+f x)+b} \int \frac{1}{\sin \left(e+f x+\frac{\pi }{2}\right) \sqrt{b+a \sin \left(e+f x+\frac{\pi }{2}\right)}}dx}{d \sqrt{a+b \sec (e+f x)}}-\frac{c g \int \frac{\left(g \csc \left(e+f x+\frac{\pi }{2}\right)\right)^{3/2}}{\sqrt{a+b \csc \left(e+f x+\frac{\pi }{2}\right)} \left(c+d \csc \left(e+f x+\frac{\pi }{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{ 3286}$$

$$\frac{g^2 \sqrt{g \sec (e+f x)} \sqrt{\frac{a \cos (e+f x)+b}{a+b}} \int \frac{\sec (e+f x)}{\sqrt{\frac{b}{a+b}+\frac{a \cos (e+f x)}{a+b}}}dx}{d \sqrt{a+b \sec (e+f x)}}-\frac{c g \int \frac{\left(g \csc \left(e+f x+\frac{\pi }{2}\right)\right)^{3/2}}{\sqrt{a+b \csc \left(e+f x+\frac{\pi }{2}\right)} \left(c+d \csc \left(e+f x+\frac{\pi }{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int \frac{1}{\sin\left(e+fx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}}dx}{d\sqrt{a+b\sec(e+fx)}}-\frac{cg\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{cg\int \frac{\left(g\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d}$$

$$\Big\downarrow \text{4463}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{cg^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))}dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{cg^2\sqrt{g\sec(e+fx)}\sqrt{a\cos(e+fx)+b}\int \frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{cg^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}(d+c\cos(e+fx))}dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{cg^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(e+fx+\frac{\pi}{2}\right)}{a+b}}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \color{blue}{3284}$$

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2cg^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

input
```
Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]))
,x]
```

output
```
(2*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)
/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*c*g^2*
Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)/2,
(2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4463
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])   Int
[1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4467
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[g/d
 Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[c*(g/d)
   Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]
)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.54 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.89

method	result
default	$-\frac{2i\left(\operatorname{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)dc+\operatorname{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)d^{2}+2c^{2}\operatorname{EllipticPi}\left(i(-\right.\right.}{}$

input
```
int((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
-2*I/f/d/(c-d)/(c+d)*(EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^
(1/2))*d*c+EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*d^2+
2*c^2*EllipticPi(I*(-cot(f*x+e)+csc(f*x+e)),-1,I*((a-b)/(a+b))^(1/2))-2*El
lipticPi(I*(-cot(f*x+e)+csc(f*x+e)),-1,I*((a-b)/(a+b))^(1/2))*d^2-2*c^2*El
lipticPi(I*(-cot(f*x+e)+csc(f*x+e)),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2)))*(
a+b*sec(f*x+e))^(1/2)*(g*sec(f*x+e))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(
f*x+e)+1))^(1/2)*cos(f*x+e)*g^2/(b+a*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate((g*sec(f*x+e))**(5/2)/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

input

```
integrate((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="maxima")
```

output

```
integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
 + c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

input

```
integrate((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,
algorithm="giac")
```

output

```
integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input

```
int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)
)),x)
```

output
```
int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)
)), x)
```

Reduce [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \sqrt{g}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e}{\sec(fx+e)^2\,bd+\sec(fx+e)\,ad+\sec(fx+e)\,b}\right.$$

input
```
int((g*sec(f*x+e))^(5/2)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
sqrt(g)*int((sqrt(sec(e + f*x))*sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2)/
(sec(e + f*x)**2*b*d + sec(e + f*x)*a*d + sec(e + f*x)*b*c + a*c),x)*g**2
```

3.285 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx=\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}-\frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}$$

output
```
1/20*cot(1/2*f*x+1/2*e)^5/c^7/f-1/14*cot(1/2*f*x+1/2*e)^7/c^7/f+1/36*cot(1
/2*f*x+1/2*e)^9/c^7/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(67) = 134.

Time = 3.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx$$
$$=\frac{\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(-971082\sin\left(\frac{fx}{2}\right)-718830\sin\left(e+\frac{fx}{2}\right)+467208\sin\left(e+\frac{3fx}{2}\right)+659400\sin\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```

output
```
(Csc[e/2]*Csc[(e + f*x)/2]^9*(-971082*Sin[(f*x)/2] - 718830*Sin[e + (f*x)/
2] + 467208*Sin[e + (3*f*x)/2] + 659400*Sin[2*e + (3*f*x)/2] - 303192*Sin[
2*e + (5*f*x)/2] - 179640*Sin[3*e + (5*f*x)/2] + 30753*Sin[3*e + (7*f*x)/2
] + 89955*Sin[4*e + (7*f*x)/2] - 13427*Sin[4*e + (9*f*x)/2] + 15*Sin[5*e +
 (9*f*x)/2]))/(23063040*c^7*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 4902, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)\sec(e+fx)}{(c-c\sec(e+fx))^7}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4\sec(e+fx)}{(c-c\sec(e+fx))^7}dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int -\frac{\cot^{10}\left(\frac{1}{2}(e+fx)\right)\left(1-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}{8c^7}d\tan\left(\frac{1}{2}(e+fx)\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \cot^{10}\left(\frac{1}{2}(e+fx)\right)\left(1-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2d\tan\left(\frac{1}{2}(e+fx)\right)}{4c^7f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cot^{10}\left(\frac{1}{2}(e+fx)\right)-2\cot^8\left(\frac{1}{2}(e+fx)\right)+\cot^6\left(\frac{1}{2}(e+fx)\right)\right)d\tan\left(\frac{1}{2}(e+fx)\right)}{4c^7f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{9}\cot^9\left(\frac{1}{2}(e+fx)\right)+\frac{2}{7}\cot^7\left(\frac{1}{2}(e+fx)\right)-\frac{1}{5}\cot^5\left(\frac{1}{2}(e+fx)\right)}{4c^7f}$$

input
```
Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```

output
```
-1/4*(-1/5*Cot[(e + f*x)/2]^5 + (2*Cot[(e + f*x)/2]^7)/7 - Cot[(e + f*x)/2
]^9/9)/(c^7*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

Maple [A] (verified)

Time = 15.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{4f\,c^{7}}$
default	$\frac{-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{4f\,c^{7}}$
risch	$\frac{2i\left(315\,\mathrm{e}^{8i(fx+e)}-630\,\mathrm{e}^{7i(fx+e)}+2310\,\mathrm{e}^{6i(fx+e)}-2520\,\mathrm{e}^{5i(fx+e)}+3402\,\mathrm{e}^{4i(fx+e)}-1638\,\mathrm{e}^{3i(fx+e)}+1062\,\mathrm{e}^{2i(fx+e)}-10\right.}{315f\,c^{7}\left(\mathrm{e}^{i(fx+e)}-1\right)^{9}}$

input
```
int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x,method=_RETURNVERBOSE)
```

output
```
1/4/f/c^7*(-2/7/tan(1/2*f*x+1/2*e)^7+1/9/tan(1/2*f*x+1/2*e)^9+1/5/tan(1/2*
f*x+1/2*e)^5)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(58) = 116.

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{47\cos(fx+e)^5+127\cos(fx+e)^4+101\cos(fx+e)^3+11\cos(fx+e)^2-8\cos(fx+e)+2}{315\left(c^7f\cos(fx+e)^4-4\,c^7f\cos(fx+e)^3+6\,c^7f\cos(fx+e)^2-4\,c^7f\cos(fx+e)+c^7f\right)\sin(fx+e}$$

input
```
integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="fricas"
)
```

output
```
1/315*(47*cos(f*x + e)^5 + 127*cos(f*x + e)^4 + 101*cos(f*x + e)^3 + 11*co
s(f*x + e)^2 - 8*cos(f*x + e) + 2)/((c^7*f*cos(f*x + e)^4 - 4*c^7*f*cos(f*
x + e)^3 + 6*c^7*f*cos(f*x + e)^2 - 4*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x
+ e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = -\frac{\int \frac{\tan^4(e+fx)\sec(e+fx)}{\sec^7(e+fx)-7\sec^6(e+fx)+21\sec^5(e+fx)-35\sec^4(e+fx)+35\sec^3(e+fx)-21\sec^2(e+fx)+7\sec(e+fx)-1}\,dx}{c^7}$$

input
```
integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**7,x)
```

output
```
-Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**7 - 7*sec(e + f*x)**
6 + 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(
e + f*x)**2 + 7*sec(e + f*x) - 1), x)/c**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = -\frac{\left(\frac{90\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{63\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-35\right)(\cos(fx+e)+1)^9}{1260\,c^7 f\sin(fx+e)^9}$$

input
```
integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="maxima"
)
```

output
```
-1/1260*(90*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 63*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 35)*(cos(f*x + e) + 1)^9/(c^7*f*sin(f*x + e)^9)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{63\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-90\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35}{1260\,c^7f\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

input
```
integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="giac")
```

output
```
1/1260*(63*tan(1/2*f*x + 1/2*e)^4 - 90*tan(1/2*f*x + 1/2*e)^2 + 35)/(c^7*f
*tan(1/2*f*x + 1/2*e)^9)
```

Mupad [B] (verification not implemented)

Time = 11.91 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{63\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-90\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+35}{1260\,c^7\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

input
```
int(tan(e + f*x)^4/(cos(e + f*x)*(c - c/cos(e + f*x))^7),x)
```

output
```
(63*tan(e/2 + (f*x)/2)^4 - 90*tan(e/2 + (f*x)/2)^2 + 35)/(1260*c^7*f*tan(e
/2 + (f*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{63\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+35}{1260\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9c^7f}$$

input
```
int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x)
```

output
```
(63*tan((e + f*x)/2)**4 - 90*tan((e + f*x)/2)**2 + 35)/(1260*tan((e + f*x)
/2)**9*c**7*f)
```

3.286 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 89

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f} - \frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f} + \frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f} - \frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}$$

output
```
1/40*cot(1/2*f*x+1/2*e)^5/c^8/f-3/56*cot(1/2*f*x+1/2*e)^7/c^8/f+1/24*cot(1
/2*f*x+1/2*e)^9/c^8/f-1/88*cot(1/2*f*x+1/2*e)^11/c^8/f
```

Mathematica [A] (verified)

Time = 4.91 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.97

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx =$$
$$-\frac{\csc\left(\frac{e}{2}\right)\csc^{11}\left(\frac{1}{2}(e+fx)\right)\left(425964\sin\left(\frac{fx}{2}\right)+486024\sin\left(e+\frac{fx}{2}\right)-351450\sin\left(e+\frac{3fx}{2}\right)-299970\sin\right.}{}$$

input
```
Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

output
```
-1/15375360*(Csc[e/2]*Csc[(e + f*x)/2]^11*(425964*Sin[(f*x)/2] + 486024*Si
n[e + (f*x)/2] - 351450*Sin[e + (3*f*x)/2] - 299970*Sin[2*e + (3*f*x)/2] +
 145695*Sin[2*e + (5*f*x)/2] + 180015*Sin[3*e + (5*f*x)/2] - 63580*Sin[3*e
 + (7*f*x)/2] - 44990*Sin[4*e + (7*f*x)/2] + 6710*Sin[4*e + (9*f*x)/2] + 1
5004*Sin[5*e + (9*f*x)/2] - 1975*Sin[5*e + (11*f*x)/2] + Sin[6*e + (11*f*x
)/2]))/(c^8*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 4902, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)\sec(e+fx)}{(c-c\sec(e+fx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4\sec(e+fx)}{(c-c\sec(e+fx))^8}dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int \frac{\cot^{12}\left(\frac{1}{2}(e+fx)\right)\left(1-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^3}{16c^8}d\tan\left(\frac{1}{2}(e+fx)\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cot^{12}\left(\frac{1}{2}(e+fx)\right)\left(1-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^3d\tan\left(\frac{1}{2}(e+fx)\right)}{8c^8f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^{12}\left(\frac{1}{2}(e+fx)\right)-3\cot^{10}\left(\frac{1}{2}(e+fx)\right)+3\cot^8\left(\frac{1}{2}(e+fx)\right)-\cot^6\left(\frac{1}{2}(e+fx)\right)\right)d\tan\left(\frac{1}{2}(e+fx)\right)}{8c^8f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{11}\cot^{11}\left(\frac{1}{2}(e+fx)\right)+\frac{1}{3}\cot^9\left(\frac{1}{2}(e+fx)\right)-\frac{3}{7}\cot^7\left(\frac{1}{2}(e+fx)\right)+\frac{1}{5}\cot^5\left(\frac{1}{2}(e+fx)\right)}{8c^8f}$$

input
```
Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

output
```
(Cot[(e + f*x)/2]^5/5 - (3*Cot[(e + f*x)/2]^7)/7 + Cot[(e + f*x)/2]^9/3 -
Cot[(e + f*x)/2]^11/11)/(8*c^8*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

Maple [A] (verified)

Time = 25.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}}{8f\,c^{8}}$
default	$\frac{\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}}{8f\,c^{8}}$
risch	$\frac{2i\left(1155\,\mathrm{e}^{10i(fx+e)}-3465\,\mathrm{e}^{9i(fx+e)}+13860\,\mathrm{e}^{8i(fx+e)}-23100\,\mathrm{e}^{7i(fx+e)}+37422\,\mathrm{e}^{6i(fx+e)}-32802\,\mathrm{e}^{5i(fx+e)}+27060\,\mathrm{e}^{4i(}\right.}{1155f\,c^{8}\left(\mathrm{e}^{i(fx+e)}-1\right)^{11}}$

input
```
int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x,method=_RETURNVERBOSE)
```

output
```
1/8/f/c^8*(1/5/tan(1/2*f*x+1/2*e)^5-1/11/tan(1/2*f*x+1/2*e)^11+1/3/tan(1/2
*f*x+1/2*e)^9-3/7/tan(1/2*f*x+1/2*e)^7)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.64

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx$$

$$=\frac{152\,\cos\left(fx+e\right)^{6}+395\,\cos\left(fx+e\right)^{5}+289\,\cos\left(fx+e\right)^{4}+15\,\cos\left(fx+e\right)^{3}-19\,\cos\left(fx+e\right)^{2}+}{1155\,\left(c^{8}f\cos\left(fx+e\right)^{5}-5\,c^{8}f\cos\left(fx+e\right)^{4}+10\,c^{8}f\cos\left(fx+e\right)^{3}-10\,c^{8}f\cos\left(fx+e\right)^{2}+5\,c^{8}f\cos\left(f\right.\right.}$$

input
```
integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="fricas"
)
```

output
```
1/1155*(152*cos(f*x + e)^6 + 395*cos(f*x + e)^5 + 289*cos(f*x + e)^4 + 15*
cos(f*x + e)^3 - 19*cos(f*x + e)^2 + 10*cos(f*x + e) - 2)/((c^8*f*cos(f*x
+ e)^5 - 5*c^8*f*cos(f*x + e)^4 + 10*c^8*f*cos(f*x + e)^3 - 10*c^8*f*cos(f
*x + e)^2 + 5*c^8*f*cos(f*x + e) - c^8*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{\int \frac{\tan^4(e+fx)\sec(e+fx)}{\sec^8(e+fx)-8\sec^7(e+fx)+28\sec^6(e+fx)-56\sec^5(e+fx)+70\sec^4(e+fx)-56\sec^3(e+fx)+28\sec^2(e+fx)-8\sec(e+fx)+1}\,dx}{c^8}$$

input
```
integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**8,x)
```

output
```
Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**8 - 8*sec(e + f*x)**7
 + 28*sec(e + f*x)**6 - 56*sec(e + f*x)**5 + 70*sec(e + f*x)**4 - 56*sec(e
 + f*x)**3 + 28*sec(e + f*x)**2 - 8*sec(e + f*x) + 1), x)/c**8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{495\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{231\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-105\right)(\cos(fx+e)+1)^{11}}{9240\,c^8 f\sin(fx+e)^{11}}$$

input
```
integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="maxima"
)
```

output
```
1/9240*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 495*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 231*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 105)*(cos(f*x
+ e) + 1)^11/(c^8*f*sin(f*x + e)^11)
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{231\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-495\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+385\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-105}{9240\,c^8f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

input
```
integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="giac")
```

output
```
1/9240*(231*tan(1/2*f*x + 1/2*e)^6 - 495*tan(1/2*f*x + 1/2*e)^4 + 385*tan(
1/2*f*x + 1/2*e)^2 - 105)/(c^8*f*tan(1/2*f*x + 1/2*e)^11)
```

Mupad [B] (verification not implemented)

Time = 12.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{5}-\frac{3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{7}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}-\frac{1}{11}}{8\,c^8\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}$$

input
```
int(tan(e + f*x)^4/(cos(e + f*x)*(c - c/cos(e + f*x))^8),x)
```

output
```
(tan(e/2 + (f*x)/2)^2/3 - (3*tan(e/2 + (f*x)/2)^4)/7 + tan(e/2 + (f*x)/2)^
6/5 - 1/11)/(8*c^8*f*tan(e/2 + (f*x)/2)^11)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{231\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-495\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+385\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-105}{9240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}c^8f}$$

input
```
int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x)
```

output
```
(231*tan((e + f*x)/2)**6 - 495*tan((e + f*x)/2)**4 + 385*tan((e + f*x)/2)*
*2 - 105)/(9240*tan((e + f*x)/2)**11*c**8*f)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
      else:
         return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file