Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.5-Secant/240-4.5.4.7

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**470**]. This is test number [240].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.57 (468)	0.43 (2)
Mathematica	93.19 (438)	6.81 (32)
Maple	91.49 (430)	8.51 (40)
Fricas	88.51 (416)	11.49 (54)
Maxima	61.70 (290)	38.30 (180)
Giac	61.06 (287)	38.94 (183)
Mupad	51.70 (243)	48.30 (227)
Reduce	46.60 (219)	53.40 (251)
Sympy	4.68 (22)	95.32 (448)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

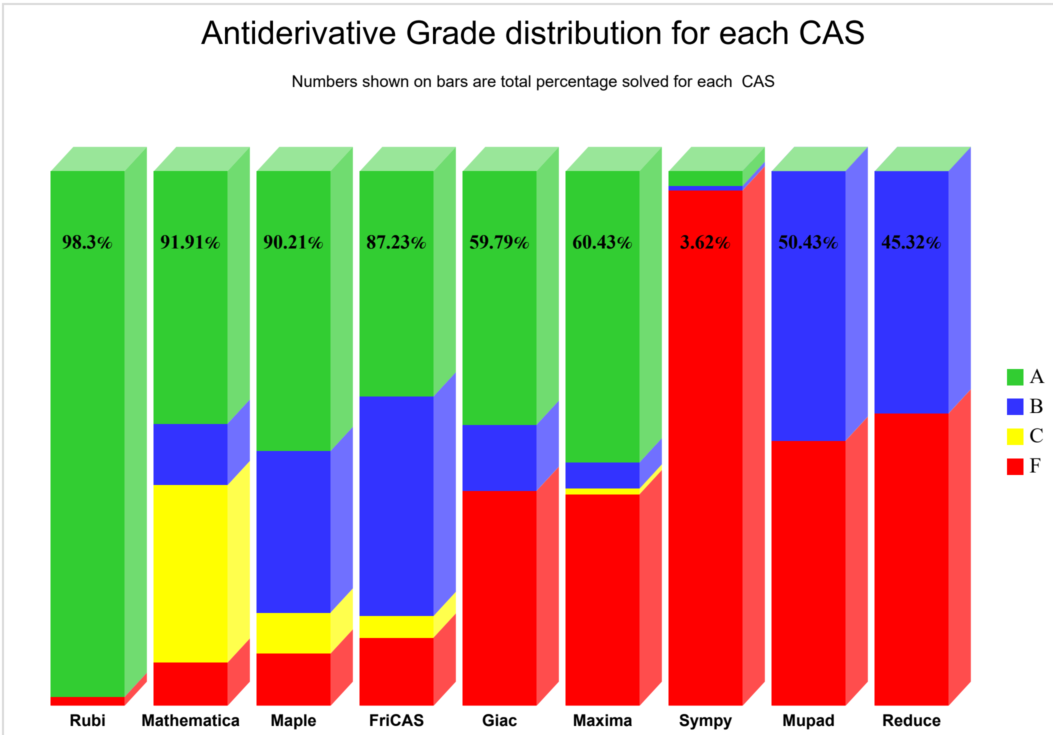
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

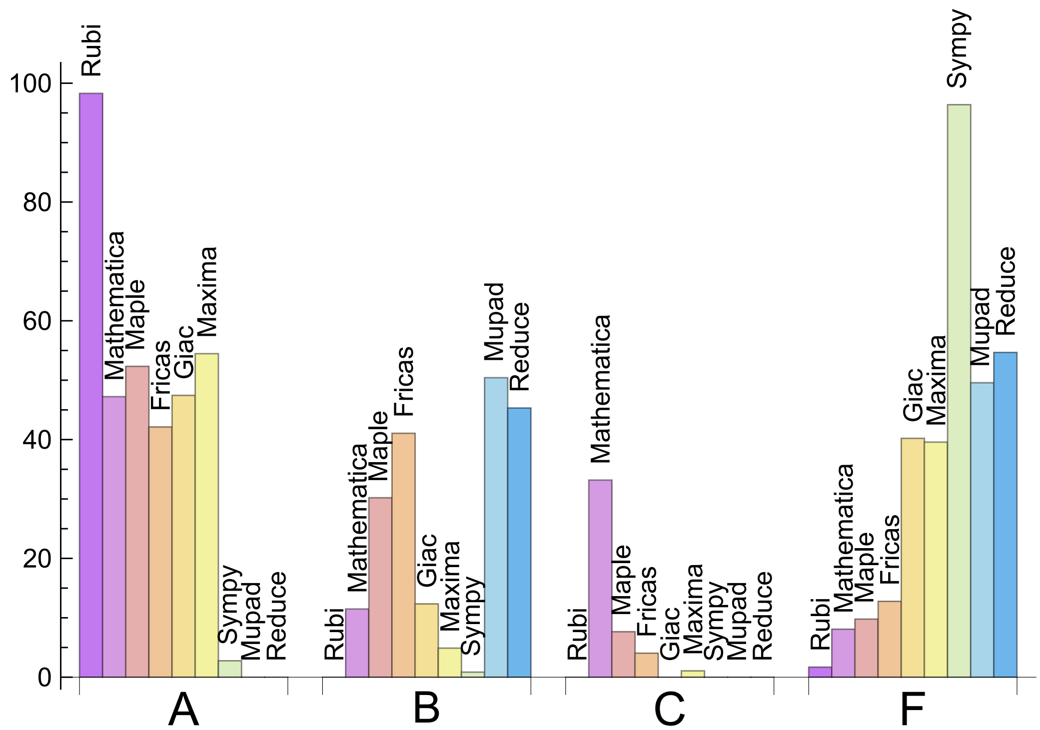
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.298	0.000	0.000	1.702
Maxima	54.468	4.894	1.064	39.574
Maple	52.340	30.213	7.660	9.787
Giac	47.447	12.340	0.000	40.213
Mathematica	47.234	11.489	33.191	8.085
Fricas	42.128	41.064	4.043	12.766
Sympy	2.766	0.851	0.000	96.383
Mupad	0.000	50.426	0.000	49.574
Reduce	0.000	45.319	0.000	54.681

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	32	100.00	0.00	0.00
Maple	40	97.50	2.50	0.00
Fricas	54	100.00	0.00	0.00
Maxima	180	87.22	12.78	0.00
Giac	183	91.26	2.73	6.01
Mupad	227	0.00	100.00	0.00
Reduce	251	100.00	0.00	0.00
Sympy	448	71.21	28.79	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.21
Giac	0.35
Rubi	0.35
Fricas	0.72
Reduce	1.21
Mathematica	3.30
Maple	9.18
Sympy	10.23
Mupad	14.49

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	73.41	1.68	63.50	1.31
Rubi	127.99	1.03	107.00	1.00
Maxima	223.76	2.82	101.50	1.19
Giac	254.11	2.24	117.00	1.29
Mathematica	414.37	3.55	166.00	1.52
Reduce	521.68	4.32	216.00	2.89
Fricas	608.75	4.44	401.00	3.62
Maple	653.60	3.92	147.00	1.29
Mupad	1563.74	11.06	105.00	1.18

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

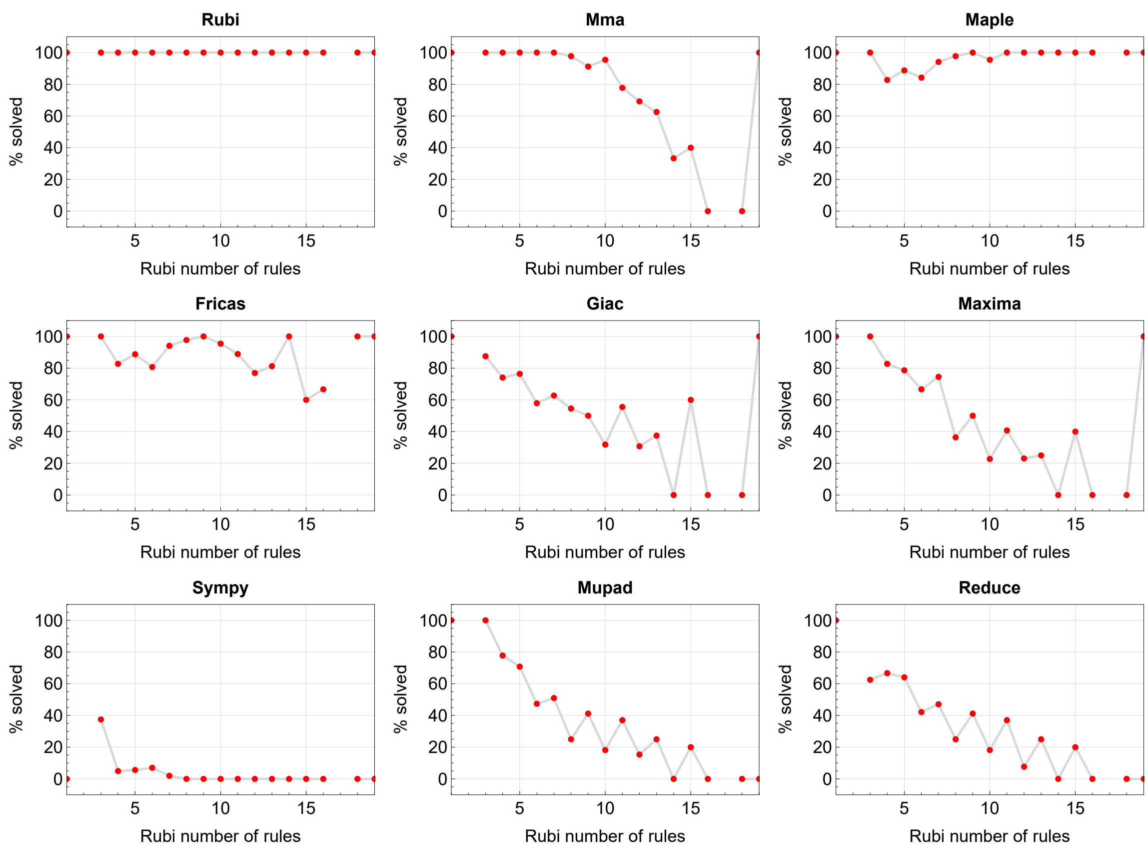


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

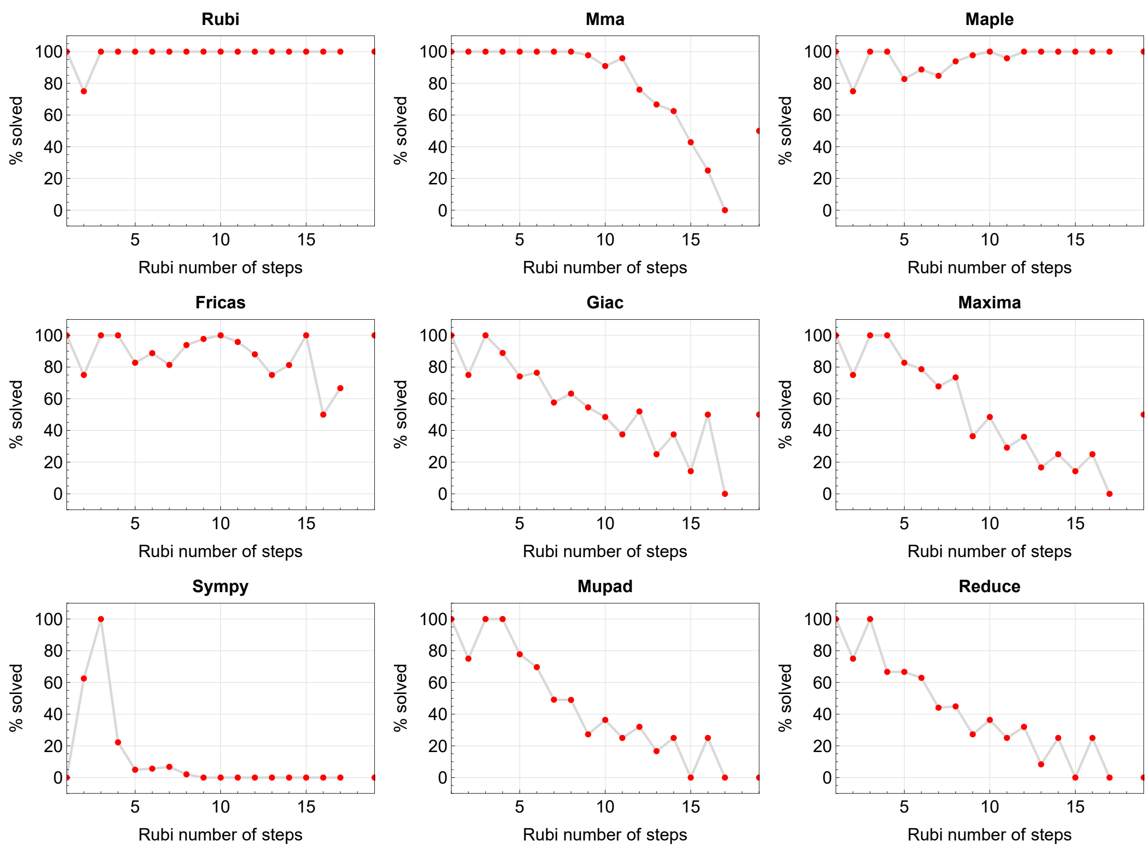


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

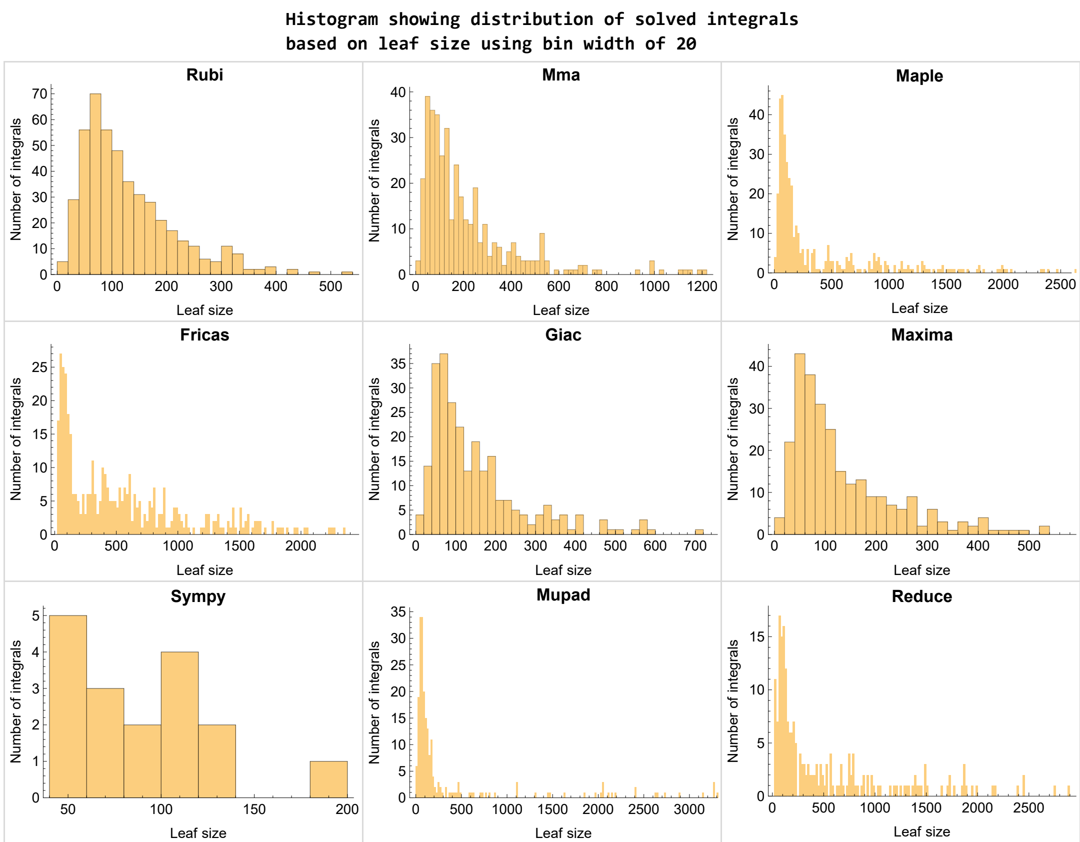


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

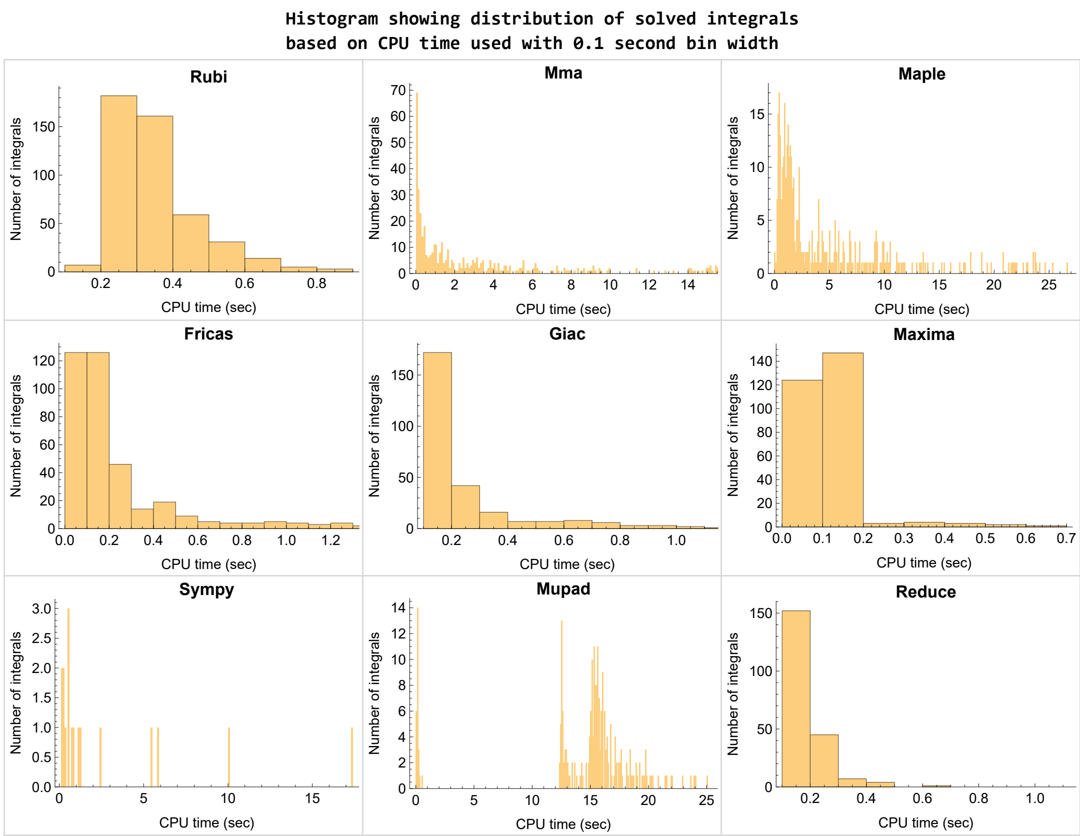


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

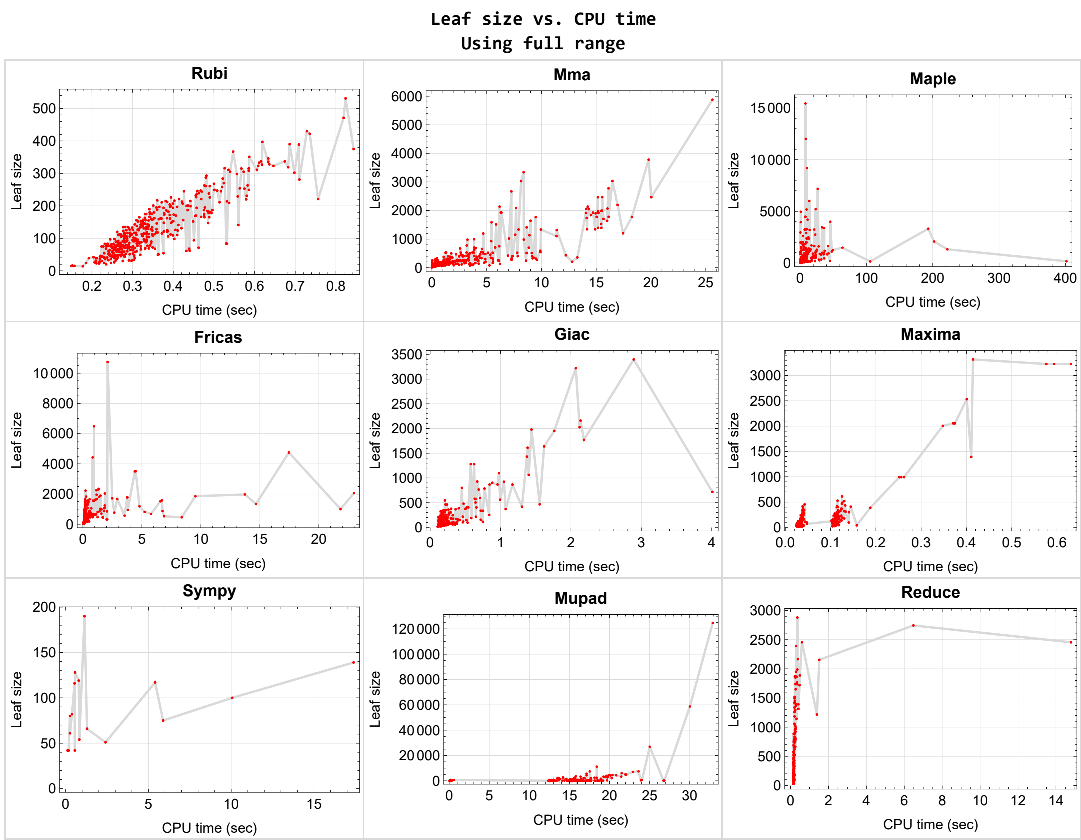


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{462, 466, 467, 468, 469, 470}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {143, 304, 310, 311, 312, 324, 325, 326, 327, 337}

Mathematica {28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 78, 88, 89, 91, 125, 126, 127, 128, 132, 133, 134, 136, 137, 138, 139, 140, 180, 186, 187, 189, 193, 199, 201, 202, 205, 208, 211, 213, 214, 215, 217, 218, 219, 240, 250, 254, 264, 268, 269, 276, 277, 280, 281, 282, 289, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 384, 392, 393, 394, 397, 398, 399, 428, 429, 430, 437, 441, 447, 448, 449, 450, 451, 463, 464}

Maple {73, 84, 86, 87, 88, 89, 123, 124, 241, 247, 250, 251, 254, 255, 261, 262, 270, 271, 272, 273, 275, 283, 284, 285, 286, 287, 288, 326, 380, 381, 382, 383, 392, 393, 394, 395, 396, 397, 398, 399, 406, 407, 408, 409, 418, 419, 420, 421, 431, 432, 433}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

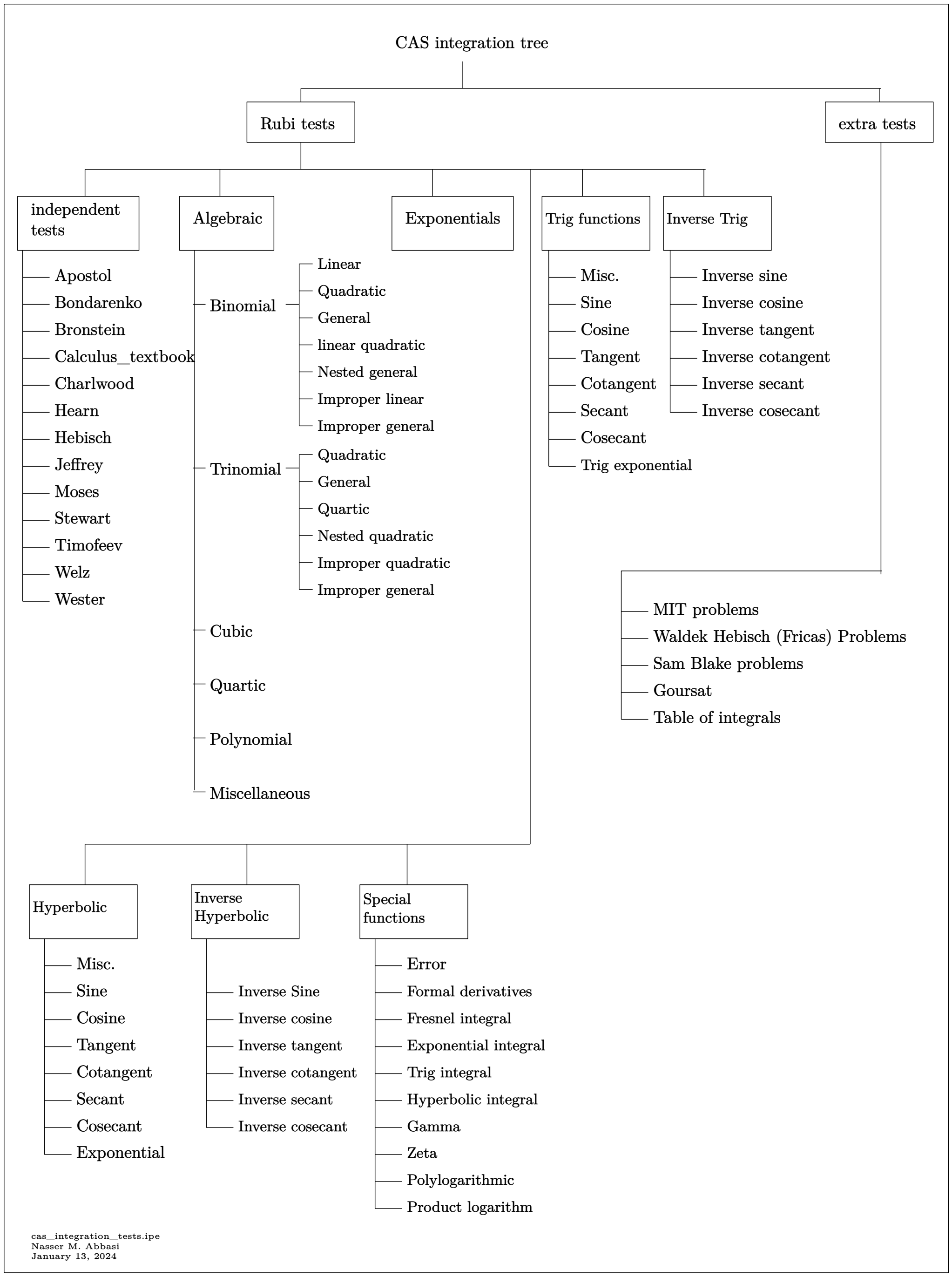
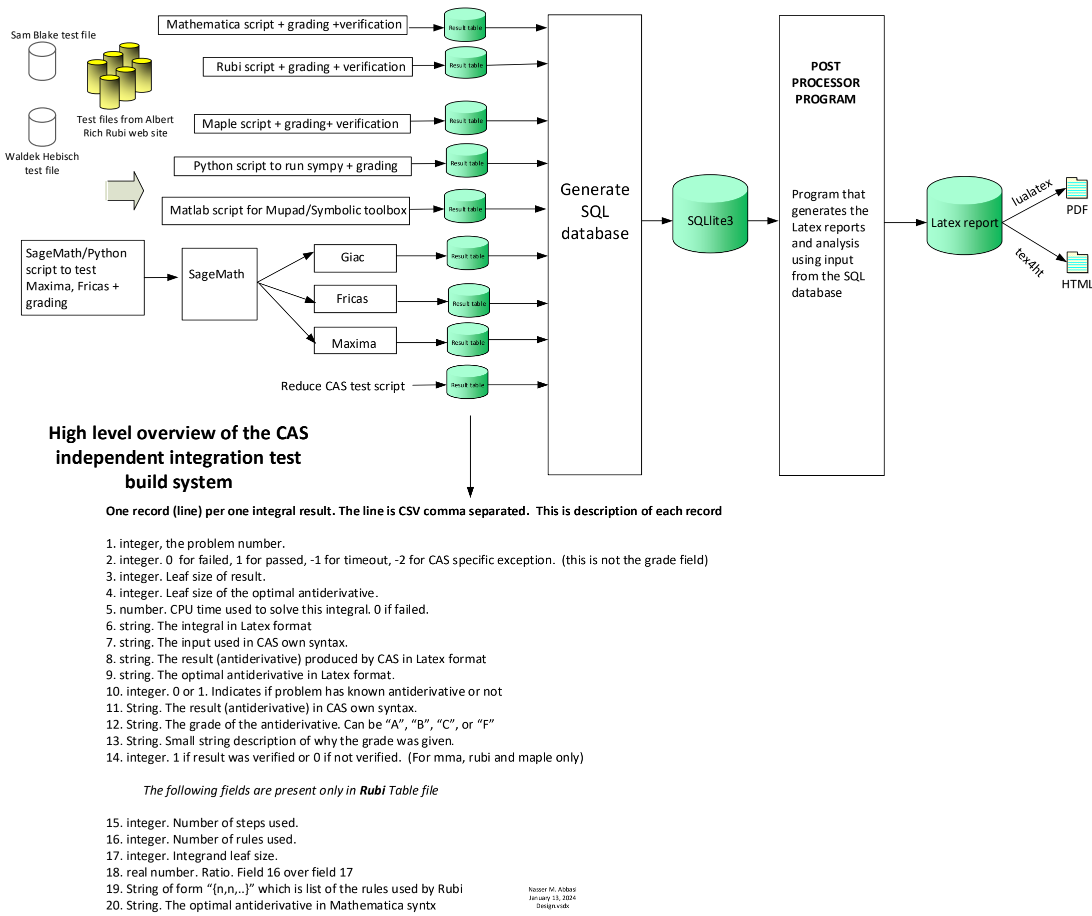


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465

}

B grade { }

C grade { }

F normal fail { 132, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 23, 24, 26, 67, 68, 69, 70, 71, 72, 80, 81, 83, 84, 85, 87, 93, 94, 95, 96, 97, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 116, 117, 118, 119, 120, 121, 129, 130, 131, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 188, 190, 191, 192, 194, 195, 196, 197, 198, 200, 203, 204, 206, 207, 209, 210, 212, 216, 220, 221, 222, 223, 224, 225, 226, 227, 231, 238, 239, 245, 252, 253, 259, 267, 271, 278, 283, 290, 291, 304, 305, 306, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 333, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 376, 377, 379, 382, 383, 386, 387, 388, 389, 390, 395, 396, 402, 403, 404, 408, 409, 412, 413, 414, 415, 416, 421, 422, 425, 426, 427, 438, 439, 440, 442, 443, 444, 445, 446, 452, 453, 454, 455, 456, 459, 463, 464, 465 }

B grade { 5, 6, 7, 18, 19, 21, 25, 27, 102, 115, 125, 126, 127, 132, 136, 137, 138, 139, 140, 236, 256, 265, 266, 279, 297, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 330, 331, 332, 334, 335, 336, 378, 411, 423, 424, 434, 435, 436, 441, 447, 448, 449, 450, 451 }

C grade { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 76, 77, 78, 79, 82, 88, 89, 90, 91, 92, 98, 110, 111, 122, 123, 124, 128, 148, 149, 150, 151, 180, 186, 187, 189, 193, 199, 201, 202, 205, 208, 211, 213, 214, 215, 217, 218, 219, 232, 234, 235, 237, 240, 246, 247, 248, 249, 250, 251, 254, 255, 260, 263, 264, 268, 269, 272, 276, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 321, 322, 323, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 384, 385, 391, 392, 393, 394, 397, 398, 399, 400, 401, 405, 410, 417, 428, 429, 430, 437, 457, 458, 460, 461 }

F normal fail { 73, 74, 86, 228, 229, 230, 233, 241, 242, 243, 244, 257, 258, 261, 262, 270, 273, 274, 275, 285, 286, 287, 288, 381, 406, 407, 418, 419, 420, 431, 432, 433 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 82, 93, 94, 95, 103, 104, 105, 106, 107, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 378, 391, 404, 417, 430, 452, 453, 454, 455, 456, 459, 461 }

B grade { 19, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98, 99, 100, 101, 102, 109, 110, 111, 113, 114, 115, 122, 123, 124, 125, 126, 127, 128, 234, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 263, 264, 265, 266, 267, 268, 269, 276, 277, 279, 280, 281, 282, 289, 292, 293, 294, 295, 296, 297, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade { 26, 144, 145, 228, 229, 230, 231, 232, 233, 241, 242, 243, 244, 245, 246, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 457, 458, 460 }

F normal fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { 112 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 8, 9, 10, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 41, 42, 43, 47, 48, 49, 54, 55, 56, 60, 61, 67, 68, 69, 70, 73, 74, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 99, 100, 103, 104, 105, 106, 107, 108, 112, 113, 116, 117, 119, 120, 121, 125, 126, 129, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 198, 203, 204, 205, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 239, 240, 247, 248, 249, 253, 263, 268, 269, 281, 282, 290, 295, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 337, 338, 339, 340, 341, 345, 346, 350, 351, 352, 353, 363, 364, 365, 382, 395, 396, 452, 453, 454, 455, 456, 459 }

B grade { 5, 6, 7, 11, 18, 19, 20, 33, 39, 40, 44, 45, 46, 50, 51, 52, 53, 57, 58, 59, 62, 63, 64, 65, 66, 71, 72, 75, 76, 77, 78, 79, 88, 89, 97, 98, 101, 102, 109, 110, 111, 114, 115, 118, 122, 123, 124, 127, 128, 130, 131, 149, 150, 151, 161, 186, 187, 188, 197, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 236, 237, 238, 250, 251, 252, 254, 255, 256, 264, 265, 266, 267, 276, 277, 278, 279, 280, 289, 291, 292, 293, 294, 296, 297, 320, 322, 323, 335, 336, 342, 343, 344, 347, 348, 349, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade { 228, 229, 230, 241, 242, 243, 257, 258, 259, 270, 271, 272, 283, 284, 285, 457, 458, 460, 461 }

F normal fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 231, 232, 233, 244, 245, 246, 260, 261, 262, 273, 274, 275, 286, 287, 288, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 60, 61, 62, 63, 67, 68, 69, 77, 78, 79, 80, 81, 82, 90, 91, 92, 93, 94, 95, 103, 104, 105, 106, 107, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 235, 236, 247, 248, 249, 263, 264, 265, 276, 277, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461 }

B grade { 33, 46, 53, 58, 59, 64, 65, 66, 102, 115, 219, 234, 266, 279, 289, 297, 355, 367, 368, 411, 416, 423, 424 }

C grade { 76, 237, 256, 379, 385 }

F normal fail { 70, 71, 72, 73, 74, 75, 83, 84, 85, 86, 87, 88, 89, 96, 97, 98, 99, 100, 101, 109, 112, 113, 114, 122, 125, 126, 127, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 228, 229, 230, 231, 232, 233, 238, 239, 240, 241, 242, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 285, 286, 287, 288, 293, 294, 295, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 378, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 415, 417, 418, 422, 425, 430, 431, 434, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { 110, 111, 123, 124, 128, 292, 296, 414, 419, 420, 421, 426, 427, 428, 429, 432, 433, 435, 436, 437, 438, 439, 440 }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 81, 82, 94, 95, 96, 107, 108, 120, 121, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 226, 227, 238, 239, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461 }

B grade { 33, 67, 70, 71, 72, 80, 93, 97, 98, 109, 110, 111, 119, 122, 123, 124, 148, 149, 150, 151, 222, 223, 278, 290, 291, 321, 322, 323, 335, 336, 355, 367, 376, 377, 378, 379, 380, 381, 389, 390, 391, 392, 402, 403, 404, 405, 406, 407, 415, 416, 417, 419, 420, 428, 429, 430, 432, 433 }

C grade { }

F normal fail { 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 101, 102, 104, 105, 112, 113, 114, 115, 116, 117, 118, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 224, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 382, 383, 384, 385, 386, 387, 388, 395, 396, 397, 398, 399, 400, 401, 408, 409, 410, 411, 413, 414, 421, 422, 423, 424, 425, 426, 427, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { 99, 100, 103, 106, 412 }

F(-2) exception fail { 83, 84, 85, 393, 394, 418, 431, 445, 446, 447, 448 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 82, 95, 103, 104, 105, 108, 116, 117, 121, 129, 135, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 378, 391, 404, 417, 430, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461 }

C grade { }

F normal fail { }

F(-1) timedout fail { 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 107, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-2) exception fail { }

Sympy

A grade { 162, 312, 313, 317, 318, 319, 325, 326, 417, 430, 452, 453, 454 }

B grade { 311, 324, 339, 459 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 16, 17, 18, 19, 23, 24, 25, 30, 31, 32, 33, 35, 36, 37, 38, 39, 43, 44, 45, 48, 49, 50, 51, 52, 63, 69, 70, 71, 72, 74, 75, 76, 77, 78, 82, 83, 89, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 122, 123, 124, 126, 127, 128, 129, 130, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 206, 207, 208, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 247, 248, 249, 250, 254, 255, 256, 257, 258, 259, 260, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 296, 297, 298, 300, 306, 307, 314, 315, 316, 320, 321, 322, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 372, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 395, 396, 397, 398, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 443, 444, 445, 448, 449, 455, 456, 457, 458, 460, 461, 464, 465 }

F(-1) timedout fail { 1, 8, 14, 15, 20, 21, 22, 26, 27, 28, 29, 34, 40, 41, 42, 46, 47, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 73, 79, 80, 81, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 106, 107, 119, 120, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 164, 170, 177, 184, 185, 192, 197, 198, 205, 209, 210, 211, 216, 217, 218, 219, 220, 232, 233, 240, 245, 246, 251, 252, 253, 261, 262, 274, 275, 287, 288, 295, 299, 301, 302, 303, 304, 305, 308, 309, 310, 336, 351, 352, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 393, 394, 400, 401, 441, 442, 446, 447, 450, 451, 463, 467 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 452, 453, 454, 455, 456, 459 }

C grade { }

F normal fail { 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 457, 458, 460, 461, 463, 464, 465 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	74	120	98	73	75	0	90	136	70
N.S.	1	0.89	1.45	1.18	0.88	0.90	0.00	1.08	1.64	0.84
time (sec)	N/A	0.252	0.281	1.259	0.026	0.077	0.000	0.163	0.149	0.084

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	59	88	81	58	60	0	68	106	55
N.S.	1	0.89	1.33	1.23	0.88	0.91	0.00	1.03	1.61	0.83
time (sec)	N/A	0.249	0.172	1.050	0.029	0.074	0.000	0.174	0.153	12.506

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	40	53	62	40	42	0	45	125	39
N.S.	1	0.91	1.20	1.41	0.91	0.95	0.00	1.02	2.84	0.89
time (sec)	N/A	0.230	0.127	0.914	0.027	0.076	0.000	0.132	0.151	0.069

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	23	35	25	25	27	0	25	34	25
N.S.	1	0.96	1.46	1.04	1.04	1.12	0.00	1.04	1.42	1.04
time (sec)	N/A	0.211	0.028	0.337	0.025	0.077	0.000	0.156	0.150	0.044

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	58	38	44	60	0	50	59	29
N.S.	1	0.96	2.15	1.41	1.63	2.22	0.00	1.85	2.19	1.07
time (sec)	N/A	0.206	0.149	0.299	0.028	0.080	0.000	0.135	0.148	0.092

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	60	236	90	76	124	0	90	138	62
N.S.	1	1.13	4.45	1.70	1.43	2.34	0.00	1.70	2.60	1.17
time (sec)	N/A	0.242	0.450	0.810	0.034	0.084	0.000	0.133	0.153	12.534

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	97	198	120	101	178	0	112	160	86
N.S.	1	1.20	2.44	1.48	1.25	2.20	0.00	1.38	1.98	1.06
time (sec)	N/A	0.279	1.471	0.925	0.029	0.084	0.000	0.182	0.186	12.518

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	124	78	89	111	86	0	104	132	105
N.S.	1	1.27	0.80	0.91	1.13	0.88	0.00	1.06	1.35	1.07
time (sec)	N/A	0.349	0.424	1.180	0.105	0.082	0.000	0.191	0.159	13.063

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	90	54	71	82	68	0	82	102	79
N.S.	1	1.29	0.77	1.01	1.17	0.97	0.00	1.17	1.46	1.13
time (sec)	N/A	0.259	0.397	0.975	0.107	0.079	0.000	0.134	0.158	12.625

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	55	54	46	47	50	0	47	61	35
N.S.	1	1.31	1.29	1.10	1.12	1.19	0.00	1.12	1.45	0.83
time (sec)	N/A	0.230	0.107	0.451	0.110	0.072	0.000	0.148	0.153	12.539

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	31	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	2.07	1.13
time (sec)	N/A	0.150	0.003	0.719	0.027	0.073	0.000	0.108	0.154	12.513

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	24	36	43	26	39	0	26	48	28
N.S.	1	0.92	1.38	1.65	1.00	1.50	0.00	1.00	1.85	1.08
time (sec)	N/A	0.230	0.228	0.800	0.032	0.066	0.000	0.142	0.151	12.540

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	84	73	43	66	0	50	72	46
N.S.	1	0.89	1.83	1.59	0.93	1.43	0.00	1.09	1.57	1.00
time (sec)	N/A	0.238	0.194	1.357	0.034	0.065	0.000	0.154	0.155	12.603

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	60	128	89	64	91	0	76	94	60
N.S.	1	0.88	1.88	1.31	0.94	1.34	0.00	1.12	1.38	0.88
time (sec)	N/A	0.252	0.209	1.500	0.037	0.074	0.000	0.172	0.206	12.717

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	86	118	155	89	90	0	115	261	87
N.S.	1	0.89	1.22	1.60	0.92	0.93	0.00	1.19	2.69	0.90
time (sec)	N/A	0.292	1.108	1.397	0.031	0.084	0.000	0.153	0.166	12.425

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	65	83	69	67	67	0	77	474	66
N.S.	1	0.90	1.15	0.96	0.93	0.93	0.00	1.07	6.58	0.92
time (sec)	N/A	0.264	0.833	1.123	0.032	0.077	0.000	0.181	0.162	12.506

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	75	42	42	44	0	43	138	45
N.S.	1	0.89	1.63	0.91	0.91	0.96	0.00	0.93	3.00	0.98
time (sec)	N/A	0.231	0.080	0.474	0.035	0.076	0.000	0.129	0.160	0.058

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	48	108	94	82	101	0	95	266	53
N.S.	1	0.92	2.08	1.81	1.58	1.94	0.00	1.83	5.12	1.02
time (sec)	N/A	0.257	0.839	0.521	0.032	0.082	0.000	0.124	0.160	0.118

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	117	1021	163	126	193	0	147	408	96
N.S.	1	1.46	12.76	2.04	1.58	2.41	0.00	1.84	5.10	1.20
time (sec)	N/A	0.308	7.037	1.213	0.028	0.083	0.000	0.124	0.159	12.535

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	162	218	211	165	286	0	191	446	135
N.S.	1	1.34	1.80	1.74	1.36	2.36	0.00	1.58	3.69	1.12
time (sec)	N/A	0.354	1.962	1.499	0.038	0.089	0.000	0.185	0.161	12.602

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	168	499	165	164	131	0	183	353	163
N.S.	1	1.19	3.54	1.17	1.16	0.93	0.00	1.30	2.50	1.16
time (sec)	N/A	0.476	1.766	1.770	0.112	0.094	0.000	0.295	0.184	12.944

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	129	153	100	120	107	0	123	275	116
N.S.	1	1.19	1.42	0.93	1.11	0.99	0.00	1.14	2.55	1.07
time (sec)	N/A	0.337	1.670	1.469	0.102	0.082	0.000	0.191	0.170	12.524

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	76	126	71	67	81	0	67	167	94
N.S.	1	1.17	1.94	1.09	1.03	1.25	0.00	1.03	2.57	1.45
time (sec)	N/A	0.266	1.351	0.746	0.112	0.079	0.000	0.150	0.159	12.652

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	41	46	44	58	0	49	106	42
N.S.	1	1.10	1.02	1.15	1.10	1.45	0.00	1.22	2.65	1.05
time (sec)	N/A	0.220	0.316	1.294	0.026	0.082	0.000	0.164	0.160	12.436

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	45	109	96	54	71	0	60	124	56
N.S.	1	0.90	2.18	1.92	1.08	1.42	0.00	1.20	2.48	1.12
time (sec)	N/A	0.257	1.354	0.959	0.030	0.072	0.000	0.191	0.156	12.479

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	68	151	130	80	101	0	98	145	85
N.S.	1	0.89	1.99	1.71	1.05	1.33	0.00	1.29	1.91	1.12
time (sec)	N/A	0.274	3.480	1.539	0.032	0.073	0.000	0.242	0.155	12.545

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	92	353	154	107	139	0	141	200	108
N.S.	1	0.89	3.43	1.50	1.04	1.35	0.00	1.37	1.94	1.05
time (sec)	N/A	0.301	1.868	1.839	0.030	0.073	0.000	0.193	0.156	12.817

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	91	425	115	102	229	0	146	315	123
N.S.	1	0.93	4.34	1.17	1.04	2.34	0.00	1.49	3.21	1.26
time (sec)	N/A	0.301	2.825	1.688	0.116	0.105	0.000	0.138	0.183	12.458

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	376	67	63	154	0	85	182	76
N.S.	1	0.97	5.30	0.94	0.89	2.17	0.00	1.20	2.56	1.07
time (sec)	N/A	0.253	1.077	0.915	0.112	0.094	0.000	0.137	0.221	0.109

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	46	329	44	40	118	0	42	76	39
N.S.	1	0.98	7.00	0.94	0.85	2.51	0.00	0.89	1.62	0.83
time (sec)	N/A	0.222	0.459	0.418	0.106	0.093	0.000	0.140	0.156	12.398

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	54	239	71	64	156	0	73	84	123
N.S.	1	0.98	4.35	1.29	1.16	2.84	0.00	1.33	1.53	2.24
time (sec)	N/A	0.241	1.013	0.421	0.111	0.102	0.000	0.167	0.158	0.199

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	99	371	115	128	327	0	140	162	392
N.S.	1	1.15	4.31	1.34	1.49	3.80	0.00	1.63	1.88	4.56
time (sec)	N/A	0.279	1.792	0.557	0.112	0.123	0.000	0.149	0.161	12.987

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	155	549	181	231	693	0	233	268	870
N.S.	1	1.20	4.26	1.40	1.79	5.37	0.00	1.81	2.08	6.74
time (sec)	N/A	0.339	4.677	0.728	0.117	0.148	0.000	0.138	0.168	15.937

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	199	357	170	209	428	0	237	384	1448
N.S.	1	1.20	2.15	1.02	1.26	2.58	0.00	1.43	2.31	8.72
time (sec)	N/A	0.442	3.374	2.303	0.110	0.120	0.000	0.174	0.196	13.844

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	141	303	119	137	332	0	152	230	494
N.S.	1	1.21	2.59	1.02	1.17	2.84	0.00	1.30	1.97	4.22
time (sec)	N/A	0.337	1.341	1.210	0.124	0.107	0.000	0.159	0.176	12.753

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	91	245	78	77	257	0	92	104	111
N.S.	1	1.20	3.22	1.03	1.01	3.38	0.00	1.21	1.37	1.46
time (sec)	N/A	0.276	0.549	0.697	0.116	0.110	0.000	0.172	0.204	12.666

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	44	231	0	65	85	460
N.S.	1	1.00	4.04	1.02	0.98	5.13	0.00	1.44	1.89	10.22
time (sec)	N/A	0.258	1.384	0.332	0.110	0.098	0.000	0.149	0.157	12.657

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	52	189	52	50	271	0	71	120	46
N.S.	1	0.96	3.50	0.96	0.93	5.02	0.00	1.31	2.22	0.85
time (sec)	N/A	0.241	0.920	0.449	0.111	0.105	0.000	0.191	0.158	12.507

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	77	226	69	82	397	0	103	204	80
N.S.	1	1.01	2.97	0.91	1.08	5.22	0.00	1.36	2.68	1.05
time (sec)	N/A	0.261	1.974	0.587	0.115	0.110	0.000	0.150	0.164	12.418

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	97	318	93	137	587	0	173	329	112
N.S.	1	0.92	3.03	0.89	1.30	5.59	0.00	1.65	3.13	1.07
time (sec)	N/A	0.328	1.671	0.764	0.102	0.112	0.000	0.244	0.189	12.944

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	136	454	159	148	405	0	206	782	195
N.S.	1	0.95	3.17	1.11	1.03	2.83	0.00	1.44	5.47	1.36
time (sec)	N/A	0.339	5.330	3.409	0.106	0.114	0.000	0.117	0.254	0.152

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	107	403	102	104	297	0	133	523	130
N.S.	1	0.94	3.54	0.89	0.91	2.61	0.00	1.17	4.59	1.14
time (sec)	N/A	0.325	2.911	1.915	0.115	0.114	0.000	0.114	0.234	0.137

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	393	69	70	201	0	72	525	72
N.S.	1	0.99	4.68	0.82	0.83	2.39	0.00	0.86	6.25	0.86
time (sec)	N/A	0.239	2.482	0.869	0.110	0.099	0.000	0.147	0.170	12.363

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	106	384	103	138	390	0	150	530	2188
N.S.	1	1.07	3.88	1.04	1.39	3.94	0.00	1.52	5.35	22.10
time (sec)	N/A	0.285	1.636	0.643	0.107	0.141	0.000	0.176	0.171	13.660

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	157	468	148	231	698	0	235	770	1845
N.S.	1	1.07	3.18	1.01	1.57	4.75	0.00	1.60	5.24	12.55
time (sec)	N/A	0.348	2.203	0.793	0.107	0.161	0.000	0.195	0.207	13.478

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	221	450	213	369	1202	0	325	895	4338
N.S.	1	1.12	2.28	1.08	1.87	6.10	0.00	1.65	4.54	22.02
time (sec)	N/A	0.414	2.659	1.059	0.125	0.200	0.000	0.201	0.231	17.136

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	298	2468	204	302	674	0	294	967	1461
N.S.	1	1.12	9.24	0.76	1.13	2.52	0.00	1.10	3.62	5.47
time (sec)	N/A	0.556	20.020	4.392	0.117	0.145	0.000	0.171	0.280	14.623

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	213	1105	147	205	522	0	193	664	435
N.S.	1	1.12	5.79	0.77	1.07	2.73	0.00	1.01	3.48	2.28
time (sec)	N/A	0.421	11.371	2.537	0.111	0.125	0.000	0.179	0.252	13.653

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	145	699	109	126	441	0	149	663	816
N.S.	1	1.12	5.38	0.84	0.97	3.39	0.00	1.15	5.10	6.28
time (sec)	N/A	0.331	8.846	1.408	0.112	0.120	0.000	0.194	0.186	13.015

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	111	240	90	106	435	0	114	562	2056
N.S.	1	1.21	2.61	0.98	1.15	4.73	0.00	1.24	6.11	22.35
time (sec)	N/A	0.265	2.707	0.494	0.114	0.125	0.000	0.168	0.169	14.229

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	242	78	117	407	0	127	427	91
N.S.	1	1.01	2.66	0.86	1.29	4.47	0.00	1.40	4.69	1.00
time (sec)	N/A	0.265	2.246	0.650	0.115	0.114	0.000	0.162	0.192	12.501

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	125	303	106	193	663	0	185	735	141
N.S.	1	1.02	2.46	0.86	1.57	5.39	0.00	1.50	5.98	1.15
time (sec)	N/A	0.385	4.873	0.882	0.115	0.147	0.000	0.225	0.215	13.421

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	195	777	124	268	987	0	254	927	198
N.S.	1	1.32	5.25	0.84	1.81	6.67	0.00	1.72	6.26	1.34
time (sec)	N/A	0.478	3.178	1.129	0.115	0.147	0.000	0.261	0.250	14.375

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	214	1641	190	204	579	0	254	1391	255
N.S.	1	1.07	8.20	0.95	1.02	2.90	0.00	1.27	6.96	1.28
time (sec)	N/A	0.532	9.004	7.334	0.111	0.142	0.000	0.221	0.412	12.510

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	152	1153	125	149	439	0	165	1390	172
N.S.	1	0.99	7.49	0.81	0.97	2.85	0.00	1.07	9.03	1.12
time (sec)	N/A	0.449	7.606	4.056	0.122	0.122	0.000	0.122	0.290	0.151

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	120	656	83	103	299	0	89	1410	105
N.S.	1	1.03	5.66	0.72	0.89	2.58	0.00	0.77	12.16	0.91
time (sec)	N/A	0.262	4.566	2.148	0.110	0.111	0.000	0.124	0.221	0.138

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	173	447	156	261	779	0	245	1447	3557
N.S.	1	1.12	2.90	1.01	1.69	5.06	0.00	1.59	9.40	23.10
time (sec)	N/A	0.378	2.672	1.178	0.114	0.203	0.000	0.161	0.227	15.792

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	237	532	198	399	1332	0	329	1863	2728
N.S.	1	1.11	2.50	0.93	1.87	6.25	0.00	1.54	8.75	12.81
time (sec)	N/A	0.458	3.705	1.339	0.114	0.244	0.000	0.194	0.285	14.206

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	284	549	259	528	1833	0	462	1988	5613
N.S.	1	1.11	2.14	1.01	2.05	7.13	0.00	1.80	7.74	21.84
time (sec)	N/A	0.522	4.511	1.724	0.115	0.275	0.000	0.239	0.359	17.132

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	346	1639	247	418	930	0	351	1719	2117
N.S.	1	1.10	5.22	0.79	1.33	2.96	0.00	1.12	5.47	6.74
time (sec)	N/A	0.634	15.999	8.401	0.115	0.180	0.000	0.227	0.477	15.567

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	261	2469	193	299	803	0	306	1736	1317
N.S.	1	1.10	10.37	0.81	1.26	3.37	0.00	1.29	7.29	5.53
time (sec)	N/A	0.480	20.024	4.609	0.141	0.163	0.000	0.204	0.352	15.051

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	216	1915	155	272	815	0	208	1738	2628
N.S.	1	1.17	10.41	0.84	1.48	4.43	0.00	1.13	9.45	14.28
time (sec)	N/A	0.403	14.057	3.026	0.114	0.156	0.000	0.246	0.257	15.713

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	175	332	147	231	819	0	196	1496	3271
N.S.	1	1.22	2.31	1.02	1.60	5.69	0.00	1.36	10.39	22.72
time (sec)	N/A	0.333	5.519	1.152	0.115	0.141	0.000	0.215	0.216	16.389

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	132	749	97	219	615	0	177	922	146
N.S.	1	1.06	6.04	0.78	1.77	4.96	0.00	1.43	7.44	1.18
time (sec)	N/A	0.291	6.148	1.185	0.116	0.130	0.000	0.326	0.262	13.177

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	166	994	140	323	1009	0	264	1367	207
N.S.	1	1.01	6.06	0.85	1.97	6.15	0.00	1.61	8.34	1.26
time (sec)	N/A	0.468	3.848	1.311	0.122	0.163	0.000	0.263	0.286	14.656

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	249	479	180	434	1423	0	367	1865	267
N.S.	1	1.20	2.30	0.87	2.09	6.84	0.00	1.76	8.97	1.28
time (sec)	N/A	0.586	4.881	1.816	0.132	0.175	0.000	0.361	0.321	15.135

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	143	152	326	171	306	0	1281	24	0
N.S.	1	1.03	1.09	2.35	1.23	2.20	0.00	9.22	0.17	0.00
time (sec)	N/A	0.322	0.620	5.246	0.116	0.234	0.000	0.577	0.172	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	95	120	236	116	243	0	85	24	0
N.S.	1	0.95	1.20	2.36	1.16	2.43	0.00	0.85	0.24	0.00
time (sec)	N/A	0.287	0.255	4.056	0.113	0.218	0.000	0.210	0.158	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	99	93	88	193	0	55	22	87
N.S.	1	0.97	1.50	1.41	1.33	2.92	0.00	0.83	0.33	1.32
time (sec)	N/A	0.244	0.094	0.240	0.114	0.208	0.000	0.176	0.162	12.862

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	80	119	554	0	536	0	405	22	0
N.S.	1	0.98	1.45	6.76	0.00	6.54	0.00	4.94	0.27	0.00
time (sec)	N/A	0.290	0.091	3.984	0.000	0.161	0.000	0.543	0.167	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	131	163	1244	0	907	0	578	24	0
N.S.	1	1.06	1.31	10.03	0.00	7.31	0.00	4.66	0.19	0.00
time (sec)	N/A	0.327	0.299	4.069	0.000	0.227	0.000	0.633	0.163	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	207	209	2007	0	1516	0	867	24	0
N.S.	1	1.13	1.14	10.97	0.00	8.28	0.00	4.74	0.13	0.00
time (sec)	N/A	0.423	1.074	4.122	0.000	0.453	0.000	0.958	0.165	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	240	270	0	831	0	1715	0	0	24	0
N.S.	1	1.12	0.00	3.46	0.00	7.15	0.00	0.00	0.10	0.00
time (sec)	N/A	0.524	0.000	35.727	0.000	2.485	0.000	0.000	0.160	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	197	0	645	0	1565	0	0	24	0
N.S.	1	1.09	0.00	3.56	0.00	8.65	0.00	0.00	0.13	0.00
time (sec)	N/A	0.392	0.000	21.786	0.000	0.863	0.000	0.000	0.174	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	127	432	481	0	1417	0	0	24	0
N.S.	1	1.03	3.51	3.91	0.00	11.52	0.00	0.00	0.20	0.00
time (sec)	N/A	0.314	3.327	18.800	0.000	0.402	0.000	0.000	0.160	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	284	351	3227	1227	0	0	15	0
N.S.	1	0.97	3.59	4.44	40.85	15.53	0.00	0.00	0.19	0.00
time (sec)	N/A	0.236	1.373	17.822	0.631	0.298	0.000	0.000	0.161	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	66	61	324	50	306	0	0	24	0
N.S.	1	0.97	0.90	4.76	0.74	4.50	0.00	0.00	0.35	0.00
time (sec)	N/A	0.254	0.176	17.876	0.035	0.135	0.000	0.000	0.158	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	100	285	589	82	436	0	0	24	0
N.S.	1	0.95	2.71	5.61	0.78	4.15	0.00	0.00	0.23	0.00
time (sec)	N/A	0.281	5.574	18.878	0.032	0.272	0.000	0.000	0.165	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	157	422	903	143	656	0	0	24	0
N.S.	1	1.05	2.83	6.06	0.96	4.40	0.00	0.00	0.16	0.00
time (sec)	N/A	0.325	6.943	19.313	0.038	1.117	0.000	0.000	0.163	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	175	188	498	277	362	0	1952	61	0
N.S.	1	0.89	0.96	2.54	1.41	1.85	0.00	9.96	0.31	0.00
time (sec)	N/A	0.330	0.984	16.865	0.115	0.334	0.000	1.763	0.225	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	144	164	399	250	289	0	146	61	0
N.S.	1	0.89	1.01	2.46	1.54	1.78	0.00	0.90	0.38	0.00
time (sec)	N/A	0.323	0.524	9.302	0.113	0.313	0.000	0.396	0.236	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	98	73	121	142	242	0	87	57	61
N.S.	1	0.98	0.73	1.21	1.42	2.42	0.00	0.87	0.57	0.61
time (sec)	N/A	0.255	0.475	0.219	0.109	0.238	0.000	0.196	0.226	14.111

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	119	171	1466	0	755	0	0	57	0
N.S.	1	0.98	1.40	12.02	0.00	6.19	0.00	0.00	0.47	0.00
time (sec)	N/A	0.335	0.424	8.936	0.000	0.224	0.000	0.000	0.220	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	161	162	202	2037	0	1024	0	0	61	0
N.S.	1	1.01	1.25	12.65	0.00	6.36	0.00	0.00	0.38	0.00
time (sec)	N/A	0.384	1.062	9.216	0.000	0.240	0.000	0.000	0.229	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	226	262	2890	0	1551	0	0	61	0
N.S.	1	1.04	1.20	13.26	0.00	7.11	0.00	0.00	0.28	0.00
time (sec)	N/A	0.490	2.553	9.266	0.000	0.278	0.000	0.000	0.230	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	298	311	0	1172	0	1855	0	0	61	0
N.S.	1	1.04	0.00	3.93	0.00	6.22	0.00	0.00	0.20	0.00
time (sec)	N/A	0.606	0.000	37.056	0.000	9.540	0.000	0.000	0.232	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	217	221	225	961	0	1667	0	0	61	0
N.S.	1	1.02	1.04	4.43	0.00	7.68	0.00	0.00	0.28	0.00
time (sec)	N/A	0.469	3.126	36.352	0.000	2.919	0.000	0.000	0.221	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	161	159	493	825	0	1535	0	0	61	0
N.S.	1	0.99	3.06	5.12	0.00	9.53	0.00	0.00	0.38	0.00
time (sec)	N/A	0.364	3.439	23.526	0.000	0.937	0.000	0.000	0.231	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	116	527	673	0	1457	0	0	44	0
N.S.	1	0.98	4.47	5.70	0.00	12.35	0.00	0.00	0.37	0.00
time (sec)	N/A	0.280	3.167	20.885	0.000	0.439	0.000	0.000	0.183	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	103	64	616	98	370	0	0	61	0
N.S.	1	0.98	0.61	5.87	0.93	3.52	0.00	0.00	0.58	0.00
time (sec)	N/A	0.279	0.184	20.866	0.032	0.287	0.000	0.000	0.229	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	154	163	654	243	472	0	0	61	0
N.S.	1	0.90	0.95	3.80	1.41	2.74	0.00	0.00	0.35	0.00
time (sec)	N/A	0.324	5.555	21.831	0.040	1.243	0.000	0.000	0.235	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	188	512	999	273	682	0	0	61	0
N.S.	1	0.90	2.45	4.78	1.31	3.26	0.00	0.00	0.29	0.00
time (sec)	N/A	0.361	8.643	23.002	0.037	5.777	0.000	0.000	0.235	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	126	93	89	162	87	0	928	38	0
N.S.	1	1.02	0.76	0.72	1.32	0.71	0.00	7.54	0.31	0.00
time (sec)	N/A	0.312	0.775	2.409	0.035	0.101	0.000	0.669	0.170	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	64	58	83	57	0	59	38	0
N.S.	1	0.97	0.86	0.78	1.12	0.77	0.00	0.80	0.51	0.00
time (sec)	N/A	0.272	0.156	2.125	0.035	0.089	0.000	0.319	0.169	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	48	31	28	37	0	31	36	46
N.S.	1	1.00	1.60	1.03	0.93	1.23	0.00	1.03	1.20	1.53
time (sec)	N/A	0.224	0.086	0.398	0.026	0.082	0.000	0.179	0.172	13.296

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	86	247	0	149	0	49	36	0
N.S.	1	1.00	2.00	5.74	0.00	3.47	0.00	1.14	0.84	0.00
time (sec)	N/A	0.250	0.082	1.326	0.000	0.137	0.000	0.192	0.170	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	93	140	567	0	314	0	480	38	0
N.S.	1	1.07	1.61	6.52	0.00	3.61	0.00	5.52	0.44	0.00
time (sec)	N/A	0.287	0.631	1.409	0.000	0.149	0.000	0.474	0.173	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	157	78	975	0	500	0	785	38	0
N.S.	1	1.14	0.57	7.07	0.00	3.62	0.00	5.69	0.28	0.00
time (sec)	N/A	0.348	0.120	1.443	0.000	0.179	0.000	0.771	0.163	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	215	163	633	0	639	0	0	38	0
N.S.	1	1.11	0.84	3.28	0.00	3.31	0.00	0.00	0.20	0.00
time (sec)	N/A	0.442	1.000	19.707	0.000	1.077	0.000	0.000	0.172	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	148	145	463	0	565	0	0	38	0
N.S.	1	1.10	1.07	3.43	0.00	4.19	0.00	0.00	0.28	0.00
time (sec)	N/A	0.326	0.278	9.355	0.000	0.368	0.000	0.000	0.164	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	125	297	0	497	0	0	38	0
N.S.	1	1.05	1.47	3.49	0.00	5.85	0.00	0.00	0.45	0.00
time (sec)	N/A	0.279	0.123	5.857	0.000	0.228	0.000	0.000	0.173	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	138	992	408	0	0	30	0
N.S.	1	1.00	2.23	3.54	25.44	10.46	0.00	0.00	0.77	0.00
time (sec)	N/A	0.192	0.050	3.317	0.252	0.211	0.000	0.000	0.167	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	55	48	33	47	0	0	38	74
N.S.	1	1.00	1.67	1.45	1.00	1.42	0.00	0.00	1.15	2.24
time (sec)	N/A	0.240	0.078	2.289	0.040	0.089	0.000	0.000	0.177	12.811

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	76	74	77	96	96	0	0	38	123
N.S.	1	0.97	0.95	0.99	1.23	1.23	0.00	0.00	0.49	1.58
time (sec)	N/A	0.273	0.143	4.346	0.034	0.133	0.000	0.000	0.174	18.136

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	137	100	120	191	171	0	0	38	723
N.S.	1	1.04	0.76	0.91	1.45	1.30	0.00	0.00	0.29	5.48
time (sec)	N/A	0.325	0.248	6.227	0.037	0.393	0.000	0.000	0.163	24.053

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	160	130	163	250	136	0	0	54	0
N.S.	1	1.01	0.82	1.03	1.57	0.86	0.00	0.00	0.34	0.00
time (sec)	N/A	0.346	6.209	401.533	0.037	0.159	0.000	0.000	0.177	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	106	93	113	141	98	0	117	54	0
N.S.	1	0.98	0.86	1.05	1.31	0.91	0.00	1.08	0.50	0.00
time (sec)	N/A	0.293	2.480	1.727	0.038	0.123	0.000	0.469	0.163	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	60	64	59	57	67	0	48	52	155
N.S.	1	0.97	1.03	0.95	0.92	1.08	0.00	0.77	0.84	2.50
time (sec)	N/A	0.241	0.918	0.233	0.035	0.101	0.000	0.249	0.169	18.705

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	78	113	907	0	353	0	503	52	0
N.S.	1	0.98	1.41	11.34	0.00	4.41	0.00	6.29	0.65	0.00
time (sec)	N/A	0.277	0.569	1.395	0.000	0.158	0.000	0.644	0.174	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	134	97	1190	0	556	0	903	54	0
N.S.	1	1.06	0.77	9.44	0.00	4.41	0.00	7.17	0.43	0.00
time (sec)	N/A	0.333	0.263	1.510	0.000	0.215	0.000	0.882	0.174	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	200	100	1986	0	882	0	1612	54	0
N.S.	1	1.13	0.56	11.22	0.00	4.98	0.00	9.11	0.31	0.00
time (sec)	N/A	0.393	0.355	1.499	0.000	0.264	0.000	1.384	0.164	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	268	256	0	0	813	0	0	54	0
N.S.	1	1.11	1.06	0.00	0.00	3.36	0.00	0.00	0.22	0.00
time (sec)	N/A	0.500	6.030	180.000	0.000	5.236	0.000	0.000	0.177	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	194	229	682	0	703	0	0	54	0
N.S.	1	1.11	1.31	3.90	0.00	4.02	0.00	0.00	0.31	0.00
time (sec)	N/A	0.376	2.488	10.296	0.000	1.568	0.000	0.000	0.170	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	125	190	479	0	607	0	0	54	0
N.S.	1	1.03	1.57	3.96	0.00	5.02	0.00	0.00	0.45	0.00
time (sec)	N/A	0.315	0.776	6.737	0.000	0.490	0.000	0.000	0.167	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	168	465	2055	601	0	0	46	0
N.S.	1	0.97	2.18	6.04	26.69	7.81	0.00	0.00	0.60	0.00
time (sec)	N/A	0.224	1.001	4.322	0.371	0.262	0.000	0.000	0.158	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	66	76	84	64	102	0	0	54	2151
N.S.	1	0.97	1.12	1.24	0.94	1.50	0.00	0.00	0.79	31.63
time (sec)	N/A	0.263	1.203	4.565	0.034	0.166	0.000	0.000	0.166	20.267

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	102	139	156	189	0	0	54	124682
N.S.	1	1.02	0.89	1.21	1.36	1.64	0.00	0.00	0.47	1084.19
time (sec)	N/A	0.304	0.491	5.017	0.036	0.515	0.000	0.000	0.174	32.873

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	176	126	203	282	314	0	0	54	0
N.S.	1	1.05	0.75	1.22	1.69	1.88	0.00	0.00	0.32	0.00
time (sec)	N/A	0.348	0.709	9.132	0.040	2.014	0.000	0.000	0.166	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	195	182	203	334	189	0	1638	70	0
N.S.	1	0.96	0.89	1.00	1.64	0.93	0.00	8.03	0.34	0.00
time (sec)	N/A	0.358	3.063	1.578	0.037	0.295	0.000	1.619	0.172	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	138	129	143	195	138	0	154	70	0
N.S.	1	0.92	0.86	0.95	1.30	0.92	0.00	1.03	0.47	0.00
time (sec)	N/A	0.310	2.179	105.738	0.035	0.203	0.000	0.718	0.169	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	90	86	101	0	70	68	26927
N.S.	1	1.00	0.91	0.93	0.89	1.04	0.00	0.72	0.70	277.60
time (sec)	N/A	0.245	0.862	0.227	0.033	0.145	0.000	0.320	0.164	25.015

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	134	108	1320	0	601	0	1097	68	0
N.S.	1	1.06	0.85	10.39	0.00	4.73	0.00	8.64	0.54	0.00
time (sec)	N/A	0.319	3.774	222.005	0.000	0.209	0.000	0.977	0.168	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	171	186	151	2074	0	950	0	1980	70	0
N.S.	1	1.09	0.88	12.13	0.00	5.56	0.00	11.58	0.41	0.00
time (sec)	N/A	0.383	1.025	202.141	0.000	0.289	0.000	1.440	0.166	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	234	263	129	3313	0	1316	0	3221	70	0
N.S.	1	1.12	0.55	14.16	0.00	5.62	0.00	13.76	0.30	0.00
time (sec)	N/A	0.477	1.308	193.193	0.000	0.387	0.000	2.069	0.171	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	315	1705	1127	0	1003	0	0	70	0
N.S.	1	1.09	5.92	3.91	0.00	3.48	0.00	0.00	0.24	0.00
time (sec)	N/A	0.568	15.663	23.701	0.000	21.843	0.000	0.000	0.175	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	246	1315	894	0	873	0	0	70	0
N.S.	1	1.08	5.79	3.94	0.00	3.85	0.00	0.00	0.31	0.00
time (sec)	N/A	0.459	11.395	11.608	0.000	6.752	0.000	0.000	0.165	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	177	983	888	0	879	0	0	70	0
N.S.	1	1.06	5.89	5.32	0.00	5.26	0.00	0.00	0.42	0.00
time (sec)	N/A	0.378	8.390	7.733	0.000	1.800	0.000	0.000	0.170	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	135	1927	861	0	881	0	0	62	0
N.S.	1	1.08	15.42	6.89	0.00	7.05	0.00	0.00	0.50	0.00
time (sec)	N/A	0.281	14.708	6.835	0.000	0.557	0.000	0.000	0.165	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	108	108	131	94	192	0	0	70	336
N.S.	1	1.02	1.02	1.24	0.89	1.81	0.00	0.00	0.66	3.17
time (sec)	N/A	0.273	1.554	8.292	0.035	0.533	0.000	0.000	0.168	23.915

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	154	138	207	216	320	0	0	70	0
N.S.	1	0.96	0.86	1.29	1.35	2.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.325	4.268	11.165	0.037	2.066	0.000	0.000	0.164	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	211	173	291	373	460	0	0	70	0
N.S.	1	0.98	0.80	1.35	1.73	2.14	0.00	0.00	0.33	0.00
time (sec)	N/A	0.372	4.783	9.226	0.039	8.382	0.000	0.000	0.164	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	0	286	0	0	0	0	0	29	0
N.S.	1	0.00	2.33	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	3.931	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	180	253	0	0	0	0	0	25	0
N.S.	1	1.01	1.42	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.353	3.355	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	115	178	0	0	0	0	0	25	0
N.S.	1	0.97	1.51	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.279	1.839	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	68	0	0	0	0	0	23	79
N.S.	1	0.99	0.99	0.00	0.00	0.00	0.00	0.00	0.33	1.14
time (sec)	N/A	0.234	0.671	0.000	0.000	0.000	0.000	0.000	0.169	14.836

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	77	1532	0	0	0	0	0	23	0
N.S.	1	0.99	19.64	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.256	15.450	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	81	266	0	0	0	0	0	25	0
N.S.	1	0.99	3.24	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.271	3.172	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	5878	0	0	0	0	0	25	0
N.S.	1	0.98	65.31	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.282	25.607	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	3781	0	0	0	0	0	25	0
N.S.	1	0.98	42.01	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.279	19.788	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	2137	0	0	0	0	0	16	0
N.S.	1	0.98	25.14	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.253	14.229	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	73	72	0	0	0	0	0	25	0
N.S.	1	0.97	0.96	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.269	0.897	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	132	0	0	0	0	0	25	0
N.S.	1	0.97	1.02	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.301	1.580	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	197	149	0	0	0	0	0	25	0
N.S.	1	1.02	0.77	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.366	1.279	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	62	72	97	129	82	0	66	151	61
N.S.	1	0.84	0.97	1.31	1.74	1.11	0.00	0.89	2.04	0.82
time (sec)	N/A	0.440	0.031	2.790	0.032	0.087	0.000	0.137	0.166	14.924

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	49	58	75	81	69	0	53	113	49
N.S.	1	0.88	1.04	1.34	1.45	1.23	0.00	0.95	2.02	0.88
time (sec)	N/A	0.362	0.027	2.047	0.033	0.075	0.000	0.125	0.169	15.309

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	32	42	47	45	56	0	39	75	33
N.S.	1	0.84	1.11	1.24	1.18	1.47	0.00	1.03	1.97	0.87
time (sec)	N/A	0.291	0.018	1.451	0.033	0.082	0.000	0.123	0.164	14.902

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	17	16	32	0	16	31	16
N.S.	1	1.00	1.62	1.06	1.00	2.00	0.00	1.00	1.94	1.00
time (sec)	N/A	0.152	0.007	0.769	0.037	0.076	0.000	0.106	0.165	15.144

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	20	31	24	26	31	0	45	31	19
N.S.	1	1.05	1.63	1.26	1.37	1.63	0.00	2.37	1.63	1.00
time (sec)	N/A	0.235	0.024	0.337	0.105	0.072	0.000	0.115	0.163	15.384

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	31	36	34	40	75	0	80	53	31
N.S.	1	0.84	0.97	0.92	1.08	2.03	0.00	2.16	1.43	0.84
time (sec)	N/A	0.297	0.023	0.378	0.105	0.078	0.000	0.126	0.169	15.308

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	36	44	50	109	0	111	69	41
N.S.	1	0.91	0.65	0.80	0.91	1.98	0.00	2.02	1.25	0.75
time (sec)	N/A	0.363	0.040	0.415	0.112	0.078	0.000	0.133	0.173	15.589

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	61	36	54	60	147	0	139	85	51
N.S.	1	0.84	0.49	0.74	0.82	2.01	0.00	1.90	1.16	0.70
time (sec)	N/A	0.432	0.016	0.490	0.108	0.083	0.000	0.148	0.165	15.762

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	94	137	108	126	114	0	121	436	102
N.S.	1	0.96	1.40	1.10	1.29	1.16	0.00	1.23	4.45	1.04
time (sec)	N/A	0.451	0.024	1.777	0.028	0.085	0.000	0.140	0.171	15.882

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	93	85	97	95	0	98	312	78
N.S.	1	0.97	1.33	1.21	1.39	1.36	0.00	1.40	4.46	1.11
time (sec)	N/A	0.355	0.017	1.482	0.031	0.081	0.000	0.132	0.166	0.154

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	55	58	72	0	60	177	41
N.S.	1	1.00	1.20	1.38	1.45	1.80	0.00	1.50	4.42	1.02
time (sec)	N/A	0.255	0.014	0.816	0.032	0.079	0.000	0.121	0.167	15.526

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	30	38	40	0	40	42	22
N.S.	1	1.00	1.46	1.25	1.58	1.67	0.00	1.67	1.75	0.92
time (sec)	N/A	0.247	0.032	0.302	0.024	0.083	0.000	0.116	0.159	15.297

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	50	31	27	28	0	34	29	28
N.S.	1	1.00	1.67	1.03	0.90	0.93	0.00	1.13	0.97	0.93
time (sec)	N/A	0.276	0.036	0.911	0.027	0.073	0.000	0.117	0.166	0.049

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	47	71	49	43	45	0	57	51	43
N.S.	1	0.94	1.42	0.98	0.86	0.90	0.00	1.14	1.02	0.86
time (sec)	N/A	0.304	0.019	1.652	0.026	0.081	0.000	0.126	0.166	15.431

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	70	81	78	60	74	0	79	124	56
N.S.	1	0.80	0.93	0.90	0.69	0.85	0.00	0.91	1.43	0.64
time (sec)	N/A	0.305	0.230	1.630	0.026	0.073	0.000	0.131	0.166	15.694

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	58	61	58	43	56	0	57	92	42
N.S.	1	0.89	0.94	0.89	0.66	0.86	0.00	0.88	1.42	0.65
time (sec)	N/A	0.306	0.158	1.515	0.027	0.070	0.000	0.126	0.170	15.739

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	34	37	0	34	60	28
N.S.	1	1.00	0.84	0.81	0.79	0.86	0.00	0.79	1.40	0.65
time (sec)	N/A	0.280	0.070	1.382	0.025	0.068	0.000	0.126	0.164	15.800

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	31	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	2.07	1.13
time (sec)	N/A	0.153	0.001	0.774	0.030	0.075	0.000	0.112	0.165	15.961

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	24	37	28	51	37	29	25
N.S.	1	1.00	1.06	0.77	1.19	0.90	1.65	1.19	0.94	0.81
time (sec)	N/A	0.210	0.057	0.326	0.107	0.074	2.410	0.116	0.165	16.417

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	59	45	44	73	49	0	73	63	67
N.S.	1	0.97	0.74	0.72	1.20	0.80	0.00	1.20	1.03	1.10
time (sec)	N/A	0.275	0.118	0.942	0.109	0.081	0.000	0.140	0.162	15.973

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	85	68	61	103	68	0	96	97	91
N.S.	1	0.96	0.76	0.69	1.16	0.76	0.00	1.08	1.09	1.02
time (sec)	N/A	0.346	0.132	1.720	0.106	0.078	0.000	0.141	0.161	16.777

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	258	180	200	168	0	221	886	170
N.S.	1	1.08	1.69	1.18	1.31	1.10	0.00	1.44	5.79	1.11
time (sec)	N/A	0.317	0.043	2.608	0.038	0.092	0.000	0.168	0.166	15.903

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	136	187	147	166	143	0	183	690	134
N.S.	1	1.16	1.60	1.26	1.42	1.22	0.00	1.56	5.90	1.15
time (sec)	N/A	0.321	0.028	2.280	0.035	0.087	0.000	0.165	0.161	16.021

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	105	109	107	119	116	0	120	470	86
N.S.	1	1.30	1.35	1.32	1.47	1.43	0.00	1.48	5.80	1.06
time (sec)	N/A	0.271	0.025	1.513	0.031	0.093	0.000	0.142	0.163	16.095

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	59	80	69	87	94	0	79	215	55
N.S.	1	1.05	1.43	1.23	1.55	1.68	0.00	1.41	3.84	0.98
time (sec)	N/A	0.266	0.045	0.981	0.031	0.087	0.000	0.136	0.152	0.112

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	44	72	55	63	66	0	70	74	48
N.S.	1	0.90	1.47	1.12	1.29	1.35	0.00	1.43	1.51	0.98
time (sec)	N/A	0.256	0.043	1.013	0.032	0.087	0.000	0.137	0.153	14.881

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	48	106	67	55	59	0	76	64	44
N.S.	1	0.91	2.00	1.26	1.04	1.11	0.00	1.43	1.21	0.83
time (sec)	N/A	0.272	0.040	1.724	0.026	0.084	0.000	0.133	0.154	14.986

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	95	96	134	103	120	0	152	228	94
N.S.	1	0.90	0.91	1.26	0.97	1.13	0.00	1.43	2.15	0.89
time (sec)	N/A	0.307	0.337	2.407	0.031	0.077	0.000	0.148	0.157	15.127

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	72	75	104	81	94	0	114	180	70
N.S.	1	0.90	0.94	1.30	1.01	1.18	0.00	1.42	2.25	0.88
time (sec)	N/A	0.281	0.306	2.215	0.026	0.082	0.000	0.159	0.154	14.995

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	48	48	71	71	69	0	76	132	44
N.S.	1	0.91	0.91	1.34	1.34	1.30	0.00	1.43	2.49	0.83
time (sec)	N/A	0.263	0.221	2.065	0.027	0.074	0.000	0.135	0.159	15.493

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	41	46	44	58	0	49	106	42
N.S.	1	1.10	1.02	1.15	1.10	1.45	0.00	1.22	2.65	1.05
time (sec)	N/A	0.225	0.084	1.393	0.040	0.079	0.000	0.130	0.154	15.401

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	57	52	41	53	56	0	53	94	66
N.S.	1	1.21	1.11	0.87	1.13	1.19	0.00	1.13	2.00	1.40
time (sec)	N/A	0.274	0.163	0.980	0.107	0.079	0.000	0.180	0.151	16.058

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	100	58	54	87	62	0	87	78	76
N.S.	1	1.39	0.81	0.75	1.21	0.86	0.00	1.21	1.08	1.06
time (sec)	N/A	0.286	0.208	0.950	0.109	0.076	0.000	0.140	0.154	15.834

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	129	99	82	135	89	0	150	132	123
N.S.	1	1.19	0.92	0.76	1.25	0.82	0.00	1.39	1.22	1.14
time (sec)	N/A	0.315	0.265	1.730	0.109	0.080	0.000	0.178	0.158	16.582

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	70	84	83	90	0	91	208	73
N.S.	1	1.01	0.96	1.15	1.14	1.23	0.00	1.25	2.85	1.00
time (sec)	N/A	0.248	0.947	2.154	0.027	0.079	0.000	0.162	0.159	16.769

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	109	106	130	134	130	0	148	342	119
N.S.	1	0.98	0.95	1.17	1.21	1.17	0.00	1.33	3.08	1.07
time (sec)	N/A	0.281	3.636	2.381	0.036	0.088	0.000	0.144	0.157	15.460

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	95	1195	106	124	272	0	113	493	591
N.S.	1	1.10	13.90	1.23	1.44	3.16	0.00	1.31	5.73	6.87
time (sec)	N/A	0.300	4.687	0.718	0.104	0.106	0.000	0.137	0.183	0.589

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	49	63	83	157	0	74	159	456
N.S.	1	0.96	0.89	1.15	1.51	2.85	0.00	1.35	2.89	8.29
time (sec)	N/A	0.252	0.077	0.444	0.104	0.108	0.000	0.142	0.169	15.530

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	50	117	0	38	92	28
N.S.	1	1.00	1.00	0.78	1.39	3.25	0.00	1.06	2.56	0.78
time (sec)	N/A	0.220	0.054	0.347	0.109	0.104	0.000	0.123	0.158	0.119

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	52	45	67	164	0	53	122	44
N.S.	1	0.96	1.00	0.87	1.29	3.15	0.00	1.02	2.35	0.85
time (sec)	N/A	0.242	0.082	0.575	0.108	0.094	0.000	0.143	0.162	16.266

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	71	105	70	88	230	0	85	156	72
N.S.	1	0.93	1.38	0.92	1.16	3.03	0.00	1.12	2.05	0.95
time (sec)	N/A	0.280	0.237	0.923	0.112	0.094	0.000	0.129	0.172	16.323

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	100	136	110	117	305	0	129	199	111
N.S.	1	0.93	1.26	1.02	1.08	2.82	0.00	1.19	1.84	1.03
time (sec)	N/A	0.307	0.567	1.668	0.106	0.103	0.000	0.150	0.188	0.152

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	72	224	70	65	354	0	96	290	72
N.S.	1	0.94	2.91	0.91	0.84	4.60	0.00	1.25	3.77	0.94
time (sec)	N/A	0.282	2.979	0.810	0.109	0.104	0.000	0.139	0.192	17.174

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	192	45	45	286	0	66	118	44
N.S.	1	0.96	3.69	0.87	0.87	5.50	0.00	1.27	2.27	0.85
time (sec)	N/A	0.245	1.612	0.544	0.113	0.095	0.000	0.165	0.165	17.477

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	27	209	0	48	67	31
N.S.	1	1.00	1.00	0.78	0.75	5.81	0.00	1.33	1.86	0.86
time (sec)	N/A	0.232	0.034	0.339	0.110	0.098	0.000	0.149	0.159	18.215

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	44	231	0	65	85	460
N.S.	1	1.00	4.04	1.02	0.98	5.13	0.00	1.44	1.89	10.22
time (sec)	N/A	0.260	0.281	0.310	0.119	0.105	0.000	0.130	0.157	16.505

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	91	67	76	72	272	0	94	146	373
N.S.	1	1.21	0.89	1.01	0.96	3.63	0.00	1.25	1.95	4.97
time (sec)	N/A	0.281	0.384	0.703	0.116	0.107	0.000	0.137	0.172	17.429

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	141	95	116	126	343	0	141	231	1114
N.S.	1	1.21	0.81	0.99	1.08	2.93	0.00	1.21	1.97	9.52
time (sec)	N/A	0.340	0.465	1.220	0.107	0.120	0.000	0.140	0.185	16.447

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	197	133	165	189	424	0	216	332	1979
N.S.	1	1.21	0.82	1.01	1.16	2.60	0.00	1.33	2.04	12.14
time (sec)	N/A	0.409	0.893	2.262	0.113	0.124	0.000	0.163	0.203	17.643

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	112	980	110	146	392	0	126	879	2039
N.S.	1	1.10	9.61	1.08	1.43	3.84	0.00	1.24	8.62	19.99
time (sec)	N/A	0.298	3.242	0.883	0.105	0.131	0.000	0.154	0.202	16.888

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	71	68	98	262	0	76	371	62
N.S.	1	0.97	0.96	0.92	1.32	3.54	0.00	1.03	5.01	0.84
time (sec)	N/A	0.252	0.272	0.548	0.112	0.095	0.000	0.145	0.173	0.144

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	82	80	111	301	0	91	583	71
N.S.	1	0.98	0.99	0.96	1.34	3.63	0.00	1.10	7.02	0.86
time (sec)	N/A	0.248	0.258	0.552	0.111	0.099	0.000	0.144	0.174	15.674

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	96	89	92	133	391	0	113	665	94
N.S.	1	0.95	0.88	0.91	1.32	3.87	0.00	1.12	6.58	0.93
time (sec)	N/A	0.312	0.402	1.081	0.106	0.113	0.000	0.136	0.179	0.192

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	118	139	120	154	490	0	146	746	124
N.S.	1	0.94	1.10	0.95	1.22	3.89	0.00	1.16	5.92	0.98
time (sec)	N/A	0.335	0.817	2.259	0.106	0.128	0.000	0.159	0.215	15.789

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	146	171	158	183	583	0	188	805	173
N.S.	1	0.93	1.09	1.01	1.17	3.71	0.00	1.20	5.13	1.10
time (sec)	N/A	0.359	1.609	3.438	0.110	0.131	0.000	0.148	0.267	0.193

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	95	248	89	110	516	0	120	622	113
N.S.	1	0.95	2.48	0.89	1.10	5.16	0.00	1.20	6.22	1.13
time (sec)	N/A	0.308	3.018	0.964	0.110	0.117	0.000	0.142	0.214	15.687

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	80	84	76	88	406	0	89	483	70
N.S.	1	0.98	1.02	0.93	1.07	4.95	0.00	1.09	5.89	0.85
time (sec)	N/A	0.262	0.204	0.593	0.114	0.107	0.000	0.141	0.173	15.614

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	71	211	64	71	368	0	83	318	69
N.S.	1	0.97	2.89	0.88	0.97	5.04	0.00	1.14	4.36	0.95
time (sec)	N/A	0.250	1.879	0.516	0.107	0.099	0.000	0.164	0.170	15.472

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	111	240	90	106	435	0	114	562	2056
N.S.	1	1.21	2.61	0.98	1.15	4.73	0.00	1.24	6.11	22.35
time (sec)	N/A	0.268	1.581	0.430	0.119	0.115	0.000	0.175	0.177	17.678

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	161	103	120	175	544	0	192	792	2401
N.S.	1	1.13	0.73	0.85	1.23	3.83	0.00	1.35	5.58	16.91
time (sec)	N/A	0.368	1.091	1.454	0.130	0.125	0.000	0.163	0.201	18.403

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	232	138	160	268	656	0	195	963	2880
N.S.	1	1.14	0.68	0.79	1.32	3.23	0.00	0.96	4.74	14.19
time (sec)	N/A	0.455	1.497	2.551	0.111	0.139	0.000	0.192	0.238	19.493

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	316	499	210	369	789	0	270	1142	3310
N.S.	1	1.14	1.79	0.76	1.33	2.84	0.00	0.97	4.11	11.91
time (sec)	N/A	0.526	4.680	4.435	0.123	0.149	0.000	0.162	0.313	21.460

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	113	90	108	179	472	0	117	834	113
N.S.	1	1.05	0.83	1.00	1.66	4.37	0.00	1.08	7.72	1.05
time (sec)	N/A	0.270	0.729	1.075	0.115	0.116	0.000	0.185	0.212	0.225

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	163	124	212	544	0	155	1184	129
N.S.	1	1.00	1.30	0.99	1.70	4.35	0.00	1.24	9.47	1.03
time (sec)	N/A	0.285	0.670	1.266	0.112	0.112	0.000	0.229	0.211	16.750

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	156	927	142	233	613	0	178	1518	149
N.S.	1	1.11	6.62	1.01	1.66	4.38	0.00	1.27	10.84	1.06
time (sec)	N/A	0.309	5.413	1.218	0.114	0.121	0.000	0.211	0.213	0.278

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	148	176	149	253	727	0	197	1649	175
N.S.	1	0.95	1.13	0.96	1.62	4.66	0.00	1.26	10.57	1.12
time (sec)	N/A	0.377	1.061	2.219	0.115	0.126	0.000	0.243	0.271	17.038

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	170	194	177	272	856	0	229	1766	256
N.S.	1	0.94	1.07	0.98	1.50	4.73	0.00	1.27	9.76	1.41
time (sec)	N/A	0.406	3.111	3.974	0.109	0.147	0.000	0.188	0.340	0.363

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	200	2670	214	303	995	0	271	1887	257
N.S.	1	0.93	12.48	1.00	1.42	4.65	0.00	1.27	8.82	1.20
time (sec)	N/A	0.433	7.256	6.946	0.115	0.176	0.000	0.185	0.493	0.261

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	152	125	137	210	722	0	185	1296	149
N.S.	1	1.10	0.91	0.99	1.52	5.23	0.00	1.34	9.39	1.08
time (sec)	N/A	0.320	0.718	1.118	0.119	0.121	0.000	0.176	0.250	17.280

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	283	118	187	654	0	163	1026	125
N.S.	1	1.00	2.30	0.96	1.52	5.32	0.00	1.33	8.34	1.02
time (sec)	N/A	0.277	4.174	1.115	0.114	0.130	0.000	0.182	0.244	17.657

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	111	265	100	156	580	0	124	752	112
N.S.	1	1.05	2.50	0.94	1.47	5.47	0.00	1.17	7.09	1.06
time (sec)	N/A	0.267	3.333	1.191	0.105	0.122	0.000	0.174	0.241	15.730

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	175	332	147	231	819	0	196	1496	3271
N.S.	1	1.22	2.31	1.02	1.60	5.69	0.00	1.36	10.39	22.72
time (sec)	N/A	0.343	4.527	1.069	0.117	0.142	0.000	0.139	0.234	19.251

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	239	156	177	338	970	0	228	1949	3708
N.S.	1	1.19	0.78	0.88	1.68	4.83	0.00	1.13	9.70	18.45
time (sec)	N/A	0.463	2.988	3.142	0.117	0.159	0.000	0.175	0.286	19.593

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	305	1134	215	464	1129	0	464	2166	4158
N.S.	1	1.13	4.22	0.80	1.72	4.20	0.00	1.72	8.05	15.46
time (sec)	N/A	0.540	8.906	4.667	0.125	0.177	0.000	0.218	0.387	20.128

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	397	1770	267	611	1296	0	350	2455	4594
N.S.	1	1.13	5.03	0.76	1.74	3.68	0.00	0.99	6.97	13.05
time (sec)	N/A	0.619	9.476	8.789	0.125	0.226	0.000	0.190	0.601	20.376

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	245	1411	229	401	1323	0	324	2880	4506
N.S.	1	1.20	6.92	1.12	1.97	6.49	0.00	1.59	14.12	22.09
time (sec)	N/A	0.426	8.648	2.227	0.117	0.179	0.000	0.149	0.358	19.895

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	83	70	143	81	100	0	217	100	0
N.S.	1	0.62	0.52	1.07	0.60	0.75	0.00	1.62	0.75	0.00
time (sec)	N/A	0.532	1.051	4.088	0.112	0.090	0.000	0.463	0.228	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	68	83	133	62	87	0	182	72	0
N.S.	1	0.67	0.82	1.32	0.61	0.86	0.00	1.80	0.71	0.00
time (sec)	N/A	0.439	0.284	4.026	0.111	0.083	0.000	0.298	0.208	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	51	48	100	40	68	0	137	46	0
N.S.	1	0.80	0.75	1.56	0.62	1.06	0.00	2.14	0.72	0.00
time (sec)	N/A	0.363	0.100	1.237	0.158	0.082	0.000	0.232	0.200	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	38	21	53	0	141	18	0
N.S.	1	1.00	1.00	1.15	0.64	1.61	0.00	4.27	0.55	0.00
time (sec)	N/A	0.300	0.028	0.519	0.104	0.085	0.000	0.242	0.182	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	32	48	37	56	0	0	35	0
N.S.	1	1.06	1.00	1.50	1.16	1.75	0.00	0.00	1.09	0.00
time (sec)	N/A	0.304	0.040	0.519	0.112	0.080	0.000	0.000	0.183	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	54	48	87	60	94	0	108	44	0
N.S.	1	0.81	0.72	1.30	0.90	1.40	0.00	1.61	0.66	0.00
time (sec)	N/A	0.376	0.115	1.218	0.111	0.084	0.000	0.258	0.194	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	71	59	105	79	125	0	147	55	0
N.S.	1	0.71	0.59	1.05	0.79	1.25	0.00	1.47	0.55	0.00
time (sec)	N/A	0.462	0.216	1.290	0.105	0.087	0.000	0.424	0.192	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	84	71	115	94	162	0	171	64	0
N.S.	1	0.63	0.53	0.86	0.71	1.22	0.00	1.29	0.48	0.00
time (sec)	N/A	0.530	0.112	1.318	0.107	0.090	0.000	0.367	0.200	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	390	0	3376	0	887	0	0	24	0
N.S.	1	1.04	0.00	8.98	0.00	2.36	0.00	0.00	0.06	0.00
time (sec)	N/A	0.686	0.000	36.366	0.000	0.152	0.000	0.000	0.176	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	305	0	2475	0	782	0	0	24	0
N.S.	1	1.04	0.00	8.48	0.00	2.68	0.00	0.00	0.08	0.00
time (sec)	N/A	0.573	0.000	12.586	0.000	0.122	0.000	0.000	0.171	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	238	0	1831	0	603	0	0	22	0
N.S.	1	1.07	0.00	8.25	0.00	2.72	0.00	0.00	0.10	0.00
time (sec)	N/A	0.488	0.000	7.983	0.000	0.113	0.000	0.000	0.167	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	90	69	1770	0	0	0	0	22	0
N.S.	1	1.10	0.84	21.59	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.313	0.390	3.954	0.000	0.000	0.000	0.000	0.181	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	249	402	2386	0	0	0	0	24	0
N.S.	1	1.00	1.61	9.54	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.501	9.131	9.783	0.000	0.000	0.000	0.000	0.177	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	328	0	3236	0	0	0	0	24	0
N.S.	1	0.96	0.00	9.46	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.636	0.000	21.351	0.000	0.000	0.000	0.000	0.187	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	180	433	1106	317	468	0	0	24	0
N.S.	1	1.01	2.43	6.21	1.78	2.63	0.00	0.00	0.13	0.00
time (sec)	N/A	0.342	12.226	47.892	0.040	0.842	0.000	0.000	0.172	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	117	349	797	173	390	0	0	24	0
N.S.	1	0.96	2.86	6.53	1.42	3.20	0.00	0.00	0.20	0.00
time (sec)	N/A	0.294	9.291	31.992	0.033	0.239	0.000	0.000	0.160	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	74	210	509	69	320	0	0	24	0
N.S.	1	0.97	2.76	6.70	0.91	4.21	0.00	0.00	0.32	0.00
time (sec)	N/A	0.259	1.217	23.766	0.048	0.138	0.000	0.000	0.169	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	284	353	3227	1227	0	0	15	0
N.S.	1	0.97	3.59	4.47	40.85	15.53	0.00	0.00	0.19	0.00
time (sec)	N/A	0.247	0.321	4.978	0.594	0.296	0.000	0.000	0.149	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	136	278	0	499	0	78	24	0
N.S.	1	1.05	1.66	3.39	0.00	6.09	0.00	0.95	0.29	0.00
time (sec)	N/A	0.270	0.488	5.560	0.000	0.235	0.000	0.247	0.158	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	143	152	441	0	567	0	117	24	0
N.S.	1	1.02	1.09	3.15	0.00	4.05	0.00	0.84	0.17	0.00
time (sec)	N/A	0.305	0.897	9.616	0.000	0.337	0.000	0.239	0.158	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	215	1902	625	0	641	0	0	24	0
N.S.	1	1.10	9.70	3.19	0.00	3.27	0.00	0.00	0.12	0.00
time (sec)	N/A	0.381	14.950	21.964	0.000	0.950	0.000	0.000	0.157	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	454	471	0	3995	0	981	0	0	53	0
N.S.	1	1.04	0.00	8.80	0.00	2.16	0.00	0.00	0.12	0.00
time (sec)	N/A	0.819	0.000	45.559	0.000	0.177	0.000	0.000	0.189	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	389	0	3442	0	890	0	0	53	0
N.S.	1	1.04	0.00	9.18	0.00	2.37	0.00	0.00	0.14	0.00
time (sec)	N/A	0.709	0.000	33.151	0.000	0.148	0.000	0.000	0.203	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	308	0	2743	0	798	0	0	51	0
N.S.	1	1.05	0.00	9.33	0.00	2.71	0.00	0.00	0.17	0.00
time (sec)	N/A	0.586	0.000	22.911	0.000	0.135	0.000	0.000	0.184	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	244	0	1987	0	0	0	0	57	0
N.S.	1	1.07	0.00	8.71	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.470	0.000	9.904	0.000	0.000	0.000	0.000	0.228	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	179	2625	0	0	0	0	61	0
N.S.	1	1.00	0.73	10.71	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.517	1.836	4.511	0.000	0.000	0.000	0.000	0.233	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	356	3284	0	0	0	0	61	0
N.S.	1	1.00	1.10	10.17	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.646	13.277	10.433	0.000	0.000	0.000	0.000	0.238	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	237	217	512	1474	415	566	0	0	53	0
N.S.	1	0.92	2.16	6.22	1.75	2.39	0.00	0.00	0.22	0.00
time (sec)	N/A	0.361	9.117	63.987	0.038	3.536	0.000	0.000	0.195	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	152	400	1151	243	470	0	0	53	0
N.S.	1	0.92	2.42	6.98	1.47	2.85	0.00	0.00	0.32	0.00
time (sec)	N/A	0.313	7.837	48.296	0.034	0.825	0.000	0.000	0.193	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	109	84	829	104	390	0	0	53	0
N.S.	1	0.98	0.76	7.47	0.94	3.51	0.00	0.00	0.48	0.00
time (sec)	N/A	0.280	0.268	40.878	0.035	0.242	0.000	0.000	0.196	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	116	527	677	0	1457	0	0	44	0
N.S.	1	0.98	4.47	5.74	0.00	12.35	0.00	0.00	0.37	0.00
time (sec)	N/A	0.287	1.294	10.138	0.000	0.433	0.000	0.000	0.182	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	124	128	466	507	0	1403	0	0	61	0
N.S.	1	1.03	3.76	4.09	0.00	11.31	0.00	0.00	0.49	0.00
time (sec)	N/A	0.345	4.410	21.639	0.000	0.447	0.000	0.000	0.228	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	191	464	0	563	0	0	61	0
N.S.	1	1.06	1.53	3.71	0.00	4.50	0.00	0.00	0.49	0.00
time (sec)	N/A	0.299	0.739	5.826	0.000	0.352	0.000	0.000	0.233	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	190	165	649	0	647	0	0	61	0
N.S.	1	0.98	0.85	3.36	0.00	3.35	0.00	0.00	0.32	0.00
time (sec)	N/A	0.341	1.332	10.572	0.000	0.975	0.000	0.000	0.233	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	706	988	0	1611	0	0	76	0
N.S.	1	1.00	4.25	5.95	0.00	9.70	0.00	0.00	0.46	0.00
time (sec)	N/A	0.342	5.777	33.528	0.000	1.055	0.000	0.000	0.205	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	109	181	0	160	0	0	24	0
N.S.	1	1.00	2.60	4.31	0.00	3.81	0.00	0.00	0.57	0.00
time (sec)	N/A	0.204	0.134	9.306	0.000	0.099	0.000	0.000	0.170	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	57	135	1391	131	0	0	9	0
N.S.	1	1.00	2.38	5.62	57.96	5.46	0.00	0.00	0.38	0.00
time (sec)	N/A	0.185	0.046	7.092	0.411	0.090	0.000	0.000	0.165	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	315	0	2359	0	784	0	0	38	0
N.S.	1	0.94	0.00	7.06	0.00	2.35	0.00	0.00	0.11	0.00
time (sec)	N/A	0.585	0.000	13.509	0.000	0.137	0.000	0.000	0.174	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	201	0	1518	0	594	0	0	38	0
N.S.	1	1.17	0.00	8.83	0.00	3.45	0.00	0.00	0.22	0.00
time (sec)	N/A	0.436	0.000	6.355	0.000	0.109	0.000	0.000	0.163	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	90	69	230	0	303	0	0	36	0
N.S.	1	1.10	0.84	2.80	0.00	3.70	0.00	0.00	0.44	0.00
time (sec)	N/A	0.346	0.539	2.918	0.000	0.100	0.000	0.000	0.158	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	115	274	1526	0	0	0	0	36	0
N.S.	1	1.07	2.56	14.26	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.327	7.270	5.539	0.000	0.000	0.000	0.000	0.169	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	259	258	0	2331	0	0	0	0	38	0
N.S.	1	1.00	0.00	9.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.520	0.000	10.205	0.000	0.000	0.000	0.000	0.160	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	349	337	0	3198	0	0	0	0	38	0
N.S.	1	0.97	0.00	9.16	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.616	0.000	14.443	0.000	0.000	0.000	0.000	0.164	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	142	326	879	160	396	0	0	38	0
N.S.	1	1.07	2.45	6.61	1.20	2.98	0.00	0.00	0.29	0.00
time (sec)	N/A	0.320	7.282	29.715	0.041	0.252	0.000	0.000	0.156	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	79	326	572	74	324	0	0	38	0
N.S.	1	0.98	4.02	7.06	0.91	4.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.277	9.193	23.800	0.033	0.164	0.000	0.000	0.162	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	241	23	215	0	0	38	0
N.S.	1	1.00	2.23	6.18	0.59	5.51	0.00	0.00	0.97	0.00
time (sec)	N/A	0.256	0.079	9.138	0.034	0.129	0.000	0.000	0.160	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	138	992	408	0	0	30	0
N.S.	1	1.00	2.23	3.54	25.44	10.46	0.00	0.00	0.77	0.00
time (sec)	N/A	0.213	0.046	3.508	0.262	0.195	0.000	0.000	0.160	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	91	126	295	0	502	0	0	38	0
N.S.	1	1.05	1.45	3.39	0.00	5.77	0.00	0.00	0.44	0.00
time (sec)	N/A	0.285	0.183	6.211	0.000	0.228	0.000	0.000	0.164	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	156	1840	463	0	567	0	0	38	0
N.S.	1	1.09	12.87	3.24	0.00	3.97	0.00	0.00	0.27	0.00
time (sec)	N/A	0.332	14.513	9.977	0.000	0.372	0.000	0.000	0.163	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	226	1739	636	0	643	0	0	38	0
N.S.	1	1.11	8.52	3.12	0.00	3.15	0.00	0.00	0.19	0.00
time (sec)	N/A	0.394	14.130	22.079	0.000	0.923	0.000	0.000	0.161	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	293	316	0	3242	0	960	0	0	54	0
N.S.	1	1.08	0.00	11.06	0.00	3.28	0.00	0.00	0.18	0.00
time (sec)	N/A	0.586	0.000	9.245	0.000	0.141	0.000	0.000	0.162	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	152	187	113	1783	0	894	0	0	54	0
N.S.	1	1.23	0.74	11.73	0.00	5.88	0.00	0.00	0.36	0.00
time (sec)	N/A	0.435	2.720	5.399	0.000	0.137	0.000	0.000	0.160	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	233	247	206	1783	0	762	0	0	52	0
N.S.	1	1.06	0.88	7.65	0.00	3.27	0.00	0.00	0.22	0.00
time (sec)	N/A	0.508	12.802	5.636	0.000	0.128	0.000	0.000	0.165	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	244	261	0	3227	0	0	0	0	52	0
N.S.	1	1.07	0.00	13.23	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.490	0.000	7.560	0.000	0.000	0.000	0.000	0.162	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	336	0	4192	0	0	0	0	54	0
N.S.	1	0.99	0.00	12.37	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.633	0.000	11.398	0.000	0.000	0.000	0.000	0.157	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	440	422	0	5206	0	0	0	0	54	0
N.S.	1	0.96	0.00	11.83	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.736	0.000	23.532	0.000	0.000	0.000	0.000	0.155	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	143	259	1285	161	524	0	0	54	0
N.S.	1	1.18	2.14	10.62	1.33	4.33	0.00	0.00	0.45	0.00
time (sec)	N/A	0.337	6.053	25.352	0.036	0.283	0.000	0.000	0.153	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	405	890	78	410	0	0	54	0
N.S.	1	0.97	5.26	11.56	1.01	5.32	0.00	0.00	0.70	0.00
time (sec)	N/A	0.282	7.359	9.945	0.036	0.152	0.000	0.000	0.158	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	57	49	30	65	0	103	54	199
N.S.	1	1.00	1.78	1.53	0.94	2.03	0.00	3.22	1.69	6.22
time (sec)	N/A	0.259	0.439	3.611	0.035	0.096	0.000	0.465	0.158	17.729

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	168	450	2055	601	0	0	46	0
N.S.	1	0.97	2.18	5.84	26.69	7.81	0.00	0.00	0.60	0.00
time (sec)	N/A	0.236	1.000	4.488	0.375	0.261	0.000	0.000	0.154	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	134	2059	666	0	699	0	0	54	0
N.S.	1	1.02	15.72	5.08	0.00	5.34	0.00	0.00	0.41	0.00
time (sec)	N/A	0.348	14.090	7.393	0.000	0.445	0.000	0.000	0.165	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	207	2046	858	0	811	0	0	54	0
N.S.	1	1.07	10.55	4.42	0.00	4.18	0.00	0.00	0.28	0.00
time (sec)	N/A	0.399	14.329	11.561	0.000	1.248	0.000	0.000	0.160	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	293	2068	1099	0	941	0	0	54	0
N.S.	1	1.08	7.63	4.06	0.00	3.47	0.00	0.00	0.20	0.00
time (sec)	N/A	0.482	16.034	24.627	0.000	3.808	0.000	0.000	0.157	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	325	327	167	4473	0	1374	0	0	70	0
N.S.	1	1.01	0.51	13.76	0.00	4.23	0.00	0.00	0.22	0.00
time (sec)	N/A	0.618	3.088	7.739	0.000	0.210	0.000	0.000	0.163	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	1204	3241	0	1244	0	0	70	0
N.S.	1	1.00	3.73	10.03	0.00	3.85	0.00	0.00	0.22	0.00
time (sec)	N/A	0.609	17.453	6.990	0.000	0.205	0.000	0.000	0.154	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	331	333	0	4473	0	1293	0	0	68	0
N.S.	1	1.01	0.00	13.51	0.00	3.91	0.00	0.00	0.21	0.00
time (sec)	N/A	0.611	0.000	8.365	0.000	0.213	0.000	0.000	0.160	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	353	351	0	5176	0	0	0	0	68	0
N.S.	1	0.99	0.00	14.66	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.587	0.000	9.842	0.000	0.000	0.000	0.000	0.155	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	445	430	0	6007	0	0	0	0	70	0
N.S.	1	0.97	0.00	13.50	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.729	0.000	13.989	0.000	0.000	0.000	0.000	0.159	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	563	531	0	7179	0	0	0	0	70	0
N.S.	1	0.94	0.00	12.75	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.824	0.000	26.633	0.000	0.000	0.000	0.000	0.165	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	145	592	1637	275	688	0	0	70	0
N.S.	1	1.14	4.66	12.89	2.17	5.42	0.00	0.00	0.55	0.00
time (sec)	N/A	0.328	9.886	21.561	0.040	0.316	0.000	0.000	0.161	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	79	74	75	117	134	0	336	70	153
N.S.	1	0.95	0.89	0.90	1.41	1.61	0.00	4.05	0.84	1.84
time (sec)	N/A	0.261	3.469	5.505	0.034	0.158	0.000	0.750	0.156	26.837

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	78	84	61	134	0	337	70	172
N.S.	1	0.97	1.10	1.18	0.86	1.89	0.00	4.75	0.99	2.42
time (sec)	N/A	0.256	5.251	5.869	0.035	0.182	0.000	0.718	0.153	26.723

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	135	1927	861	0	881	0	0	62	0
N.S.	1	1.08	15.42	6.89	0.00	7.05	0.00	0.00	0.50	0.00
time (sec)	N/A	0.288	6.346	6.880	0.000	0.532	0.000	0.000	0.153	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	204	1775	1107	0	1023	0	0	70	0
N.S.	1	1.09	9.49	5.92	0.00	5.47	0.00	0.00	0.37	0.00
time (sec)	N/A	0.391	15.069	9.105	0.000	1.484	0.000	0.000	0.156	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	287	1777	1342	0	1187	0	0	70	0
N.S.	1	1.10	6.81	5.14	0.00	4.55	0.00	0.00	0.27	0.00
time (sec)	N/A	0.480	16.133	13.797	0.000	4.798	0.000	0.000	0.162	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	367	1776	1602	0	1337	0	0	70	0
N.S.	1	1.11	5.35	4.83	0.00	4.03	0.00	0.00	0.21	0.00
time (sec)	N/A	0.547	18.257	27.990	0.000	14.675	0.000	0.000	0.160	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	201	1777	1292	0	1241	0	0	78	0
N.S.	1	1.12	9.93	7.22	0.00	6.93	0.00	0.00	0.44	0.00
time (sec)	N/A	0.351	15.037	11.174	0.000	1.855	0.000	0.000	0.160	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	37	76	388	53	0	0	18	0
N.S.	1	1.00	2.64	5.43	27.71	3.79	0.00	0.00	1.29	0.00
time (sec)	N/A	0.178	0.019	1.388	0.188	0.082	0.000	0.000	0.148	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	0	2195	0	0	0	0	0	29	0
N.S.	1	0.00	19.60	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.000	16.948	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	1392	0	0	0	0	0	25	0
N.S.	1	1.08	13.26	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.348	15.469	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	1394	0	0	0	0	0	23	0
N.S.	1	1.08	13.28	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.326	15.163	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	111	1983	0	0	0	0	0	23	0
N.S.	1	1.08	19.25	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.309	15.160	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	1987	0	0	0	0	0	25	0
N.S.	1	1.08	18.92	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.334	15.438	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	1997	0	0	0	0	0	25	0
N.S.	1	1.08	19.02	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.354	15.521	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	212	149	0	0	0	0	0	25	0
N.S.	1	0.99	0.70	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.385	1.392	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	127	126	0	0	0	0	0	25	0
N.S.	1	0.97	0.96	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.293	1.378	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	71	0	0	0	0	0	25	0
N.S.	1	0.97	0.96	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.249	0.695	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	2137	0	0	0	0	0	16	0
N.S.	1	0.98	25.14	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.238	6.178	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	1343	0	0	0	0	0	25	0
N.S.	1	0.98	15.80	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.270	14.162	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	1342	0	0	0	0	0	25	0
N.S.	1	0.98	15.79	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.267	14.904	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	1914	0	0	0	0	0	25	0
N.S.	1	0.98	22.52	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.267	15.205	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	63	60	55	95	69	116	64	91	52
N.S.	1	0.88	0.83	0.76	1.32	0.96	1.61	0.89	1.26	0.72
time (sec)	N/A	0.258	0.032	1.154	0.033	0.094	0.540	0.251	0.155	15.547

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	46	43	45	64	50	80	44	60	46
N.S.	1	0.94	0.88	0.92	1.31	1.02	1.63	0.90	1.22	0.94
time (sec)	N/A	0.245	0.077	0.628	0.029	0.091	0.260	0.185	0.165	16.225

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	28	30	26	33	37	42	29	29	32
N.S.	1	0.93	1.00	0.87	1.10	1.23	1.40	0.97	0.97	1.07
time (sec)	N/A	0.218	0.014	0.384	0.032	0.083	0.120	0.200	0.156	16.025

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	37	38	24	33	35	0	35	76	32
N.S.	1	1.32	1.36	0.86	1.18	1.25	0.00	1.25	2.71	1.14
time (sec)	N/A	0.247	0.016	0.377	0.028	0.086	0.000	0.159	0.155	15.108

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	41	46	39	29	50	0	31	86	51
N.S.	1	1.28	1.44	1.22	0.91	1.56	0.00	0.97	2.69	1.59
time (sec)	N/A	0.256	0.019	0.507	0.027	0.085	0.000	0.171	0.150	15.033

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	66	58	55	49	83	0	58	110	61
N.S.	1	1.29	1.14	1.08	0.96	1.63	0.00	1.14	2.16	1.20
time (sec)	N/A	0.266	0.022	0.958	0.027	0.087	0.000	0.135	0.150	15.642

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	60	73	57	56	89	66	72	393	51
N.S.	1	0.94	1.14	0.89	0.88	1.39	1.03	1.12	6.14	0.80
time (sec)	N/A	0.287	0.026	1.541	0.106	0.083	1.290	0.380	0.153	15.305

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	47	45	72	54	57	224	40
N.S.	1	1.00	1.19	0.98	0.94	1.50	1.12	1.19	4.67	0.83
time (sec)	N/A	0.280	0.016	0.839	0.109	0.095	0.828	0.355	0.151	15.571

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	36	41	37	33	53	42	43	105	29
N.S.	1	1.12	1.28	1.16	1.03	1.66	1.31	1.34	3.28	0.91
time (sec)	N/A	0.261	0.017	0.536	0.111	0.081	0.556	0.226	0.155	15.491

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	31	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	2.07	1.13
time (sec)	N/A	0.157	0.001	0.850	0.032	0.072	0.000	0.118	0.146	15.315

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	26	43	33	25	34	0	53	42	19
N.S.	1	1.37	2.26	1.74	1.32	1.79	0.00	2.79	2.21	1.00
time (sec)	N/A	0.255	0.069	0.991	0.108	0.076	0.000	0.140	0.150	15.084

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	41	51	48	41	76	0	111	78	35
N.S.	1	1.24	1.55	1.45	1.24	2.30	0.00	3.36	2.36	1.06
time (sec)	N/A	0.264	0.020	0.682	0.110	0.080	0.000	0.147	0.150	15.674

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	49	51	63	52	110	0	170	113	46
N.S.	1	0.96	1.00	1.24	1.02	2.16	0.00	3.33	2.22	0.90
time (sec)	N/A	0.267	0.026	1.280	0.105	0.079	0.000	0.163	0.158	15.963

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	91	126	86	147	99	190	94	154	124
N.S.	1	0.91	1.26	0.86	1.47	0.99	1.90	0.94	1.54	1.24
time (sec)	N/A	0.304	0.332	5.581	0.034	0.094	1.139	0.347	0.151	15.184

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	71	107	76	114	79	128	73	101	92
N.S.	1	0.92	1.39	0.99	1.48	1.03	1.66	0.95	1.31	1.19
time (sec)	N/A	0.281	0.183	2.971	0.032	0.090	0.567	0.289	0.150	15.627

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	48	46	83	41	67	53	61	46	46	61
N.S.	1	0.96	1.73	0.85	1.40	1.10	1.27	0.96	0.96	1.27
time (sec)	N/A	0.248	0.075	1.550	0.028	0.090	0.265	0.156	0.144	15.367

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	55	84	50	64	79	0	66	353	58
N.S.	1	1.04	1.58	0.94	1.21	1.49	0.00	1.25	6.66	1.09
time (sec)	N/A	0.274	0.157	1.298	0.033	0.094	0.000	0.159	0.150	15.529

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	64	81	64	60	100	0	71	187	68
N.S.	1	1.12	1.42	1.12	1.05	1.75	0.00	1.25	3.28	1.19
time (sec)	N/A	0.293	0.124	1.759	0.035	0.100	0.000	0.122	0.146	15.459

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	69	77	71	61	97	0	68	141	83
N.S.	1	1.35	1.51	1.39	1.20	1.90	0.00	1.33	2.76	1.63
time (sec)	N/A	0.288	0.191	3.460	0.034	0.088	0.000	0.156	0.143	15.256

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	90	275	88	84	137	0	113	574	126
N.S.	1	0.95	2.89	0.93	0.88	1.44	0.00	1.19	6.04	1.33
time (sec)	N/A	0.334	2.309	7.359	0.108	0.093	0.000	0.748	0.152	15.373

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	395	78	71	113	0	96	364	97
N.S.	1	0.97	5.13	1.01	0.92	1.47	0.00	1.25	4.73	1.26
time (sec)	N/A	0.320	1.468	4.052	0.109	0.083	0.000	0.532	0.148	15.198

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	60	281	68	58	86	0	81	208	69
N.S.	1	1.02	4.76	1.15	0.98	1.46	0.00	1.37	3.53	1.17
time (sec)	N/A	0.310	1.100	2.127	0.107	0.085	0.000	0.378	0.147	15.081

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	41	46	44	58	0	49	106	42
N.S.	1	1.10	1.02	1.15	1.10	1.45	0.00	1.22	2.65	1.05
time (sec)	N/A	0.226	0.088	1.502	0.039	0.077	0.000	0.127	0.146	15.174

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	82	66	46	67	0	46	91	44
N.S.	1	1.11	2.28	1.83	1.28	1.86	0.00	1.28	2.53	1.22
time (sec)	N/A	0.305	1.999	1.839	0.105	0.080	0.000	0.161	0.149	15.061

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	160	73	59	98	0	176	117	53
N.S.	1	1.09	3.56	1.62	1.31	2.18	0.00	3.91	2.60	1.18
time (sec)	N/A	0.319	0.983	3.043	0.106	0.078	0.000	0.152	0.152	15.201

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	66	256	107	72	136	0	273	172	68
N.S.	1	1.02	3.94	1.65	1.11	2.09	0.00	4.20	2.65	1.05
time (sec)	N/A	0.320	2.464	4.767	0.108	0.080	0.000	0.177	0.149	16.436

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	63	99	67	81	84	0	73	766	103
N.S.	1	0.91	1.43	0.97	1.17	1.22	0.00	1.06	11.10	1.49
time (sec)	N/A	0.301	0.196	1.243	0.033	0.130	0.000	0.183	0.175	16.230

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	42	50	41	0	45	207	64
N.S.	1	1.00	0.91	0.93	1.11	0.91	0.00	1.00	4.60	1.42
time (sec)	N/A	0.280	0.079	0.721	0.034	0.105	0.000	0.151	0.156	16.079

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	35	26	21	117	22	35	63
N.S.	1	1.00	1.13	1.52	1.13	0.91	5.09	0.96	1.52	2.74
time (sec)	N/A	0.218	0.131	0.382	0.025	0.086	5.403	0.143	0.146	16.082

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	52	43	68	50	42	0	53	135	65
N.S.	1	1.13	0.93	1.48	1.09	0.91	0.00	1.15	2.93	1.41
time (sec)	N/A	0.283	0.075	0.637	0.027	0.119	0.000	0.118	0.153	16.289

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	100	119	87	126	0	102	283	98
N.S.	1	1.04	1.35	1.61	1.18	1.70	0.00	1.38	3.82	1.32
time (sec)	N/A	0.312	0.157	0.986	0.033	0.173	0.000	0.193	0.166	16.069

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	118	138	180	145	265	0	162	414	160
N.S.	1	1.09	1.28	1.67	1.34	2.45	0.00	1.50	3.83	1.48
time (sec)	N/A	0.358	0.449	1.723	0.036	0.291	0.000	0.144	0.170	15.754

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	98	229	101	95	373	0	121	654	1109
N.S.	1	1.18	2.76	1.22	1.14	4.49	0.00	1.46	7.88	13.36
time (sec)	N/A	0.403	2.452	1.616	0.140	0.120	0.000	0.262	0.183	15.343

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	70	206	72	66	297	0	75	196	410
N.S.	1	1.19	3.49	1.22	1.12	5.03	0.00	1.27	3.32	6.95
time (sec)	N/A	0.331	1.177	1.003	0.112	0.114	0.000	0.304	0.160	15.141

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	53	184	48	45	226	0	51	78	126
N.S.	1	1.15	4.00	1.04	0.98	4.91	0.00	1.11	1.70	2.74
time (sec)	N/A	0.301	0.409	0.694	0.113	0.104	0.000	0.250	0.150	15.639

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	44	231	0	65	85	460
N.S.	1	1.00	4.04	1.02	0.98	5.13	0.00	1.44	1.89	10.22
time (sec)	N/A	0.271	0.281	0.333	0.110	0.109	0.000	0.152	0.158	16.382

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	77	204	68	66	310	0	87	164	637
N.S.	1	1.24	3.29	1.10	1.06	5.00	0.00	1.40	2.65	10.27
time (sec)	N/A	0.337	1.479	0.816	0.117	0.114	0.000	0.150	0.160	17.283

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	109	390	90	106	533	0	134	280	2644
N.S.	1	1.27	4.53	1.05	1.23	6.20	0.00	1.56	3.26	30.74
time (sec)	N/A	0.399	2.811	1.315	0.114	0.117	0.000	0.195	0.165	19.749

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	147	671	118	161	833	0	212	423	4324
N.S.	1	1.22	5.59	0.98	1.34	6.94	0.00	1.77	3.52	36.03
time (sec)	N/A	0.486	2.574	2.359	0.118	0.129	0.000	0.183	0.191	20.831

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	70	109	72	98	118	0	91	756	170
N.S.	1	0.91	1.42	0.94	1.27	1.53	0.00	1.18	9.82	2.21
time (sec)	N/A	0.305	0.319	3.762	0.047	0.144	0.000	0.194	0.175	15.323

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	48	81	50	59	53	0	48	107	97
N.S.	1	0.94	1.59	0.98	1.16	1.04	0.00	0.94	2.10	1.90
time (sec)	N/A	0.277	0.638	2.098	0.031	0.095	0.000	0.152	0.146	15.199

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	44	79	59	57	52	0	46	108	90
N.S.	1	0.90	1.61	1.20	1.16	1.06	0.00	0.94	2.20	1.84
time (sec)	N/A	0.254	0.312	1.467	0.029	0.085	0.000	0.138	0.148	15.006

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	84	112	91	117	138	0	118	740	106
N.S.	1	1.01	1.35	1.10	1.41	1.66	0.00	1.42	8.92	1.28
time (sec)	N/A	0.310	0.261	2.283	0.037	0.209	0.000	0.147	0.174	15.281

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	108	130	141	192	312	0	173	1057	160
N.S.	1	0.97	1.17	1.27	1.73	2.81	0.00	1.56	9.52	1.44
time (sec)	N/A	0.349	0.951	4.213	0.037	0.338	0.000	0.147	0.204	15.420

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	148	162	201	279	557	0	308	1312	206
N.S.	1	1.06	1.16	1.44	1.99	3.98	0.00	2.20	9.37	1.47
time (sec)	N/A	0.395	1.377	7.773	0.041	0.628	0.000	0.184	0.415	16.261

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	131	286	125	127	514	0	145	799	765
N.S.	1	1.10	2.40	1.05	1.07	4.32	0.00	1.22	6.71	6.43
time (sec)	N/A	0.420	4.065	5.147	0.108	0.120	0.000	0.396	0.206	16.081

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	104	249	85	96	393	0	107	470	285
N.S.	1	1.16	2.77	0.94	1.07	4.37	0.00	1.19	5.22	3.17
time (sec)	N/A	0.340	2.235	2.882	0.110	0.112	0.000	0.319	0.168	15.602

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	346	77	75	458	0	83	519	711
N.S.	1	1.12	4.07	0.91	0.88	5.39	0.00	0.98	6.11	8.36
time (sec)	N/A	0.332	5.379	1.675	0.113	0.111	0.000	0.269	0.163	15.572

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	111	240	90	106	435	0	114	562	2056
N.S.	1	1.21	2.61	0.98	1.15	4.73	0.00	1.24	6.11	22.35
time (sec)	N/A	0.271	1.386	0.447	0.106	0.119	0.000	0.155	0.172	16.760

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	141	288	102	163	604	0	175	784	3146
N.S.	1	1.17	2.38	0.84	1.35	4.99	0.00	1.45	6.48	26.00
time (sec)	N/A	0.413	3.293	3.185	0.114	0.129	0.000	0.222	0.187	19.782

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	183	1588	124	235	979	0	212	993	4987
N.S.	1	1.14	9.92	0.78	1.47	6.12	0.00	1.32	6.21	31.17
time (sec)	N/A	0.486	5.444	5.809	0.114	0.145	0.000	0.221	0.281	21.963

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	231	3028	152	319	1505	0	298	1216	6017
N.S.	1	1.12	14.63	0.73	1.54	7.27	0.00	1.44	5.87	29.07
time (sec)	N/A	0.581	8.141	10.394	0.130	0.173	0.000	0.269	1.391	21.513

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	73	136	80	112	116	0	77	216	166
N.S.	1	0.94	1.74	1.03	1.44	1.49	0.00	0.99	2.77	2.13
time (sec)	N/A	0.297	1.942	11.794	0.044	0.101	0.000	0.197	0.160	16.500

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	75	131	73	113	111	0	74	226	153
N.S.	1	0.93	1.62	0.90	1.40	1.37	0.00	0.91	2.79	1.89
time (sec)	N/A	0.301	0.709	6.824	0.034	0.106	0.000	0.213	0.153	16.765

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	69	129	81	102	102	0	65	196	142
N.S.	1	0.93	1.74	1.09	1.38	1.38	0.00	0.88	2.65	1.92
time (sec)	N/A	0.270	1.245	6.691	0.034	0.100	0.000	0.182	0.144	17.108

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	128	158	144	243	307	0	190	1871	190
N.S.	1	0.98	1.22	1.11	1.87	2.36	0.00	1.46	14.39	1.46
time (sec)	N/A	0.359	0.829	9.878	0.039	0.409	0.000	0.156	0.229	18.950

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	151	176	193	344	584	0	305	2392	272
N.S.	1	0.98	1.14	1.25	2.23	3.79	0.00	1.98	15.53	1.77
time (sec)	N/A	0.397	1.277	17.365	0.042	0.756	0.000	0.175	0.286	19.693

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	194	208	254	454	859	0	359	2744	327
N.S.	1	1.01	1.08	1.32	2.36	4.47	0.00	1.87	14.29	1.70
time (sec)	N/A	0.458	4.899	29.097	0.043	1.423	0.000	0.260	6.480	19.040

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	170	523	134	193	664	0	188	1259	615
N.S.	1	1.16	3.56	0.91	1.31	4.52	0.00	1.28	8.56	4.18
time (sec)	N/A	0.456	5.121	15.479	0.113	0.133	0.000	0.584	0.211	15.686

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	153	1333	124	163	763	0	147	1337	1117
N.S.	1	1.12	9.73	0.91	1.19	5.57	0.00	1.07	9.76	8.15
time (sec)	N/A	0.400	9.949	9.061	0.111	0.132	0.000	0.515	0.209	15.694

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	161	1334	125	191	860	0	161	1422	2405
N.S.	1	1.17	9.67	0.91	1.38	6.23	0.00	1.17	10.30	17.43
time (sec)	N/A	0.392	7.909	6.913	0.126	0.142	0.000	0.393	0.211	18.132

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	175	332	147	231	819	0	196	1496	3271
N.S.	1	1.22	2.31	1.02	1.60	5.69	0.00	1.36	10.39	22.72
time (sec)	N/A	0.344	4.070	1.049	0.124	0.138	0.000	0.138	0.224	19.722

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	211	2089	149	311	1060	0	247	1851	4890
N.S.	1	1.17	11.54	0.82	1.72	5.86	0.00	1.36	10.23	27.02
time (sec)	N/A	0.514	7.683	13.257	0.133	0.165	0.000	0.321	0.260	22.078

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	263	3340	171	409	1649	0	302	2154	7057
N.S.	1	1.14	14.52	0.74	1.78	7.17	0.00	1.31	9.37	30.68
time (sec)	N/A	0.583	8.374	22.671	0.145	0.193	0.000	0.292	1.516	22.914

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	319	547	199	520	2229	0	388	2455	7460
N.S.	1	1.12	1.92	0.70	1.82	7.82	0.00	1.36	8.61	26.18
time (sec)	N/A	0.683	9.912	45.069	0.130	0.226	0.000	0.407	14.794	23.614

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	107	139	322	0	456	0	1280	199	0
N.S.	1	0.96	1.25	2.90	0.00	4.11	0.00	11.53	1.79	0.00
time (sec)	N/A	0.324	1.176	28.317	0.000	1.675	0.000	0.625	0.198	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	74	234	0	386	0	800	133	0
N.S.	1	0.99	0.92	2.92	0.00	4.82	0.00	10.00	1.66	0.00
time (sec)	N/A	0.284	0.496	10.569	0.000	0.413	0.000	0.451	0.187	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	56	119	58	0	312	0	377	57	46
N.S.	1	1.04	2.20	1.07	0.00	5.78	0.00	6.98	1.06	0.85
time (sec)	N/A	0.253	0.328	0.272	0.000	0.178	0.000	0.298	0.170	16.267

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	70	540	3317	963	0	402	22	0
N.S.	1	1.03	1.00	7.71	47.39	13.76	0.00	5.74	0.31	0.00
time (sec)	N/A	0.302	0.168	1.497	0.414	0.246	0.000	0.519	0.163	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	527	1200	0	1342	0	574	24	0
N.S.	1	1.04	4.83	11.01	0.00	12.31	0.00	5.27	0.22	0.00
time (sec)	N/A	0.322	3.818	1.594	0.000	0.366	0.000	0.644	0.165	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	161	182	0	1944	0	1953	0	866	24	0
N.S.	1	1.13	0.00	12.07	0.00	12.13	0.00	5.38	0.15	0.00
time (sec)	N/A	0.388	0.000	1.618	0.000	1.162	0.000	1.169	0.171	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	219	229	263	1257	0	1775	0	0	24	0
N.S.	1	1.05	1.20	5.74	0.00	8.11	0.00	0.00	0.11	0.00
time (sec)	N/A	0.559	2.461	49.500	0.000	3.746	0.000	0.000	0.170	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	165	169	208	933	0	1621	0	0	24	0
N.S.	1	1.02	1.26	5.65	0.00	9.82	0.00	0.00	0.15	0.00
time (sec)	N/A	0.454	1.858	42.960	0.000	1.049	0.000	0.000	0.168	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	115	526	636	0	1471	0	0	24	0
N.S.	1	0.97	4.46	5.39	0.00	12.47	0.00	0.00	0.20	0.00
time (sec)	N/A	0.372	2.635	24.107	0.000	0.429	0.000	0.000	0.159	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	284	353	3227	1227	0	0	15	0
N.S.	1	0.97	3.59	4.47	40.85	15.53	0.00	0.00	0.19	0.00
time (sec)	N/A	0.241	0.304	5.033	0.577	0.301	0.000	0.000	0.150	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	67	130	195	0	499	0	0	24	0
N.S.	1	0.97	1.88	2.83	0.00	7.23	0.00	0.00	0.35	0.00
time (sec)	N/A	0.330	0.431	4.003	0.000	0.242	0.000	0.000	0.163	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	112	176	359	0	629	0	0	24	0
N.S.	1	0.98	1.54	3.15	0.00	5.52	0.00	0.00	0.21	0.00
time (sec)	N/A	0.393	0.526	5.081	0.000	0.567	0.000	0.000	0.169	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	175	178	548	0	849	0	0	24	0
N.S.	1	1.05	1.07	3.28	0.00	5.08	0.00	0.00	0.14	0.00
time (sec)	N/A	0.476	1.679	6.350	0.000	2.049	0.000	0.000	0.173	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	128	149	438	0	527	0	2026	295	0
N.S.	1	0.95	1.10	3.24	0.00	3.90	0.00	15.01	2.19	0.00
time (sec)	N/A	0.332	1.382	36.375	0.000	6.883	0.000	2.123	0.246	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	102	99	337	0	443	0	1431	195	0
N.S.	1	0.98	0.95	3.24	0.00	4.26	0.00	13.76	1.88	0.00
time (sec)	N/A	0.302	1.023	32.079	0.000	1.702	0.000	1.374	0.217	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	84	81	0	373	0	867	87	66
N.S.	1	1.01	1.08	1.04	0.00	4.78	0.00	11.12	1.12	0.85
time (sec)	N/A	0.269	0.182	0.233	0.000	0.406	0.000	0.844	0.176	18.614

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	506	1301	0	1075	0	718	57	0
N.S.	1	1.00	5.56	14.30	0.00	11.81	0.00	7.89	0.63	0.00
time (sec)	N/A	0.329	3.470	6.801	0.000	0.399	0.000	4.010	0.221	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	118	622	901	0	1300	0	0	61	0
N.S.	1	1.04	5.46	7.90	0.00	11.40	0.00	0.00	0.54	0.00
time (sec)	N/A	0.362	3.864	1.578	0.000	0.417	0.000	0.000	0.235	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	159	172	684	1451	0	1801	0	0	61	0
N.S.	1	1.08	4.30	9.13	0.00	11.33	0.00	0.00	0.38	0.00
time (sec)	N/A	0.391	4.001	1.610	0.000	1.376	0.000	0.000	0.944	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	290	302	353	1644	0	1973	0	0	61	0
N.S.	1	1.04	1.22	5.67	0.00	6.80	0.00	0.00	0.21	0.00
time (sec)	N/A	0.698	4.374	42.323	0.000	13.736	0.000	0.000	0.223	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	214	220	258	1306	0	1777	0	0	61	0
N.S.	1	1.03	1.21	6.10	0.00	8.30	0.00	0.00	0.29	0.00
time (sec)	N/A	0.579	2.835	16.064	0.000	3.775	0.000	0.000	0.216	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	166	165	703	971	0	1627	0	0	61	0
N.S.	1	0.99	4.23	5.85	0.00	9.80	0.00	0.00	0.37	0.00
time (sec)	N/A	0.488	3.862	11.237	0.000	1.064	0.000	0.000	0.212	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	116	527	673	0	1457	0	0	44	0
N.S.	1	0.98	4.47	5.70	0.00	12.35	0.00	0.00	0.37	0.00
time (sec)	N/A	0.294	1.244	6.285	0.000	0.427	0.000	0.000	0.189	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	111	106	410	477	0	1446	0	0	61	0
N.S.	1	0.95	3.69	4.30	0.00	13.03	0.00	0.00	0.55	0.00
time (sec)	N/A	0.387	3.868	3.138	0.000	0.459	0.000	0.000	0.226	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	110	100	250	0	597	0	0	61	0
N.S.	1	0.98	0.89	2.23	0.00	5.33	0.00	0.00	0.54	0.00
time (sec)	N/A	0.410	0.251	3.657	0.000	0.648	0.000	0.000	0.418	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	139	450	0	767	0	0	61	0
N.S.	1	1.00	0.84	2.73	0.00	4.65	0.00	0.00	0.37	0.00
time (sec)	N/A	0.477	0.974	4.702	0.000	2.661	0.000	0.000	2.256	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	87	107	193	0	410	0	765	38	0
N.S.	1	0.98	1.20	2.17	0.00	4.61	0.00	8.60	0.43	0.00
time (sec)	N/A	0.311	1.029	11.875	0.000	0.467	0.000	0.689	0.151	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	59	56	154	0	328	0	373	38	0
N.S.	1	1.05	1.00	2.75	0.00	5.86	0.00	6.66	0.68	0.00
time (sec)	N/A	0.279	0.378	4.445	0.000	0.191	0.000	0.501	0.153	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	261	0	111	36	27
N.S.	1	1.00	1.00	1.27	0.00	7.91	0.00	3.36	1.09	0.82
time (sec)	N/A	0.240	0.078	0.145	0.000	0.139	0.000	0.380	0.164	16.023

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	294	343	0	1015	0	403	36	0
N.S.	1	1.03	4.20	4.90	0.00	14.50	0.00	5.76	0.51	0.00
time (sec)	N/A	0.293	1.637	0.301	0.000	0.240	0.000	0.657	0.164	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	128	0	1048	0	1550	0	595	38	0
N.S.	1	1.10	0.00	9.03	0.00	13.36	0.00	5.13	0.33	0.00
time (sec)	N/A	0.346	0.000	0.434	0.000	0.417	0.000	0.709	0.169	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	166	194	0	1978	0	2257	0	924	38	0
N.S.	1	1.17	0.00	11.92	0.00	13.60	0.00	5.57	0.23	0.00
time (sec)	N/A	0.399	0.000	0.487	0.000	1.145	0.000	1.048	0.212	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	173	181	230	1054	0	1673	0	0	38	0
N.S.	1	1.05	1.33	6.09	0.00	9.67	0.00	0.00	0.22	0.00
time (sec)	N/A	0.491	2.248	12.996	0.000	1.201	0.000	0.000	0.165	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	196	732	0	1507	0	0	38	0
N.S.	1	1.02	1.63	6.10	0.00	12.56	0.00	0.00	0.32	0.00
time (sec)	N/A	0.377	1.559	4.362	0.000	0.487	0.000	0.000	0.163	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	78	296	342	0	1259	0	0	38	0
N.S.	1	0.98	3.70	4.28	0.00	15.74	0.00	0.00	0.48	0.00
time (sec)	N/A	0.343	1.505	2.188	0.000	0.333	0.000	0.000	0.160	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	138	992	408	0	0	30	0
N.S.	1	1.00	2.23	3.54	25.44	10.46	0.00	0.00	0.77	0.00
time (sec)	N/A	0.210	0.050	1.107	0.256	0.196	0.000	0.000	0.153	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	127	298	0	525	0	0	38	0
N.S.	1	0.97	1.72	4.03	0.00	7.09	0.00	0.00	0.51	0.00
time (sec)	N/A	0.349	0.147	2.016	0.000	0.236	0.000	0.000	0.152	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	126	168	505	0	723	0	0	38	0
N.S.	1	1.06	1.41	4.24	0.00	6.08	0.00	0.00	0.32	0.00
time (sec)	N/A	0.405	0.935	5.053	0.000	0.479	0.000	0.000	0.185	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	187	199	741	0	987	0	0	38	0
N.S.	1	1.09	1.16	4.31	0.00	5.74	0.00	0.00	0.22	0.00
time (sec)	N/A	0.475	2.089	7.647	0.000	1.581	0.000	0.000	0.356	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	121	337	0	458	0	560	54	0
N.S.	1	1.00	1.38	3.83	0.00	5.20	0.00	6.36	0.61	0.00
time (sec)	N/A	0.333	3.419	7.118	0.000	0.627	0.000	0.994	0.164	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	62	874	2532	417	0	251	54	0
N.S.	1	1.03	0.98	13.87	40.19	6.62	0.00	3.98	0.86	0.00
time (sec)	N/A	0.298	0.473	0.443	0.401	0.206	0.000	0.839	0.157	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	59	382	62	0	392	75	216	52	49
N.S.	1	1.04	6.70	1.09	0.00	6.88	1.32	3.79	0.91	0.86
time (sec)	N/A	0.261	4.091	0.072	0.000	0.194	5.887	0.650	0.159	17.535

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	100	113	0	2949	0	1569	0	0	52	0
N.S.	1	1.13	0.00	29.49	0.00	15.69	0.00	0.00	0.52	0.00
time (sec)	N/A	0.339	0.000	0.496	0.000	0.429	0.000	0.000	0.156	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	153	173	0	3946	0	2347	0	1062	54	0
N.S.	1	1.13	0.00	25.79	0.00	15.34	0.00	6.94	0.35	0.00
time (sec)	N/A	0.401	0.000	1.132	0.000	1.340	0.000	1.400	0.165	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	213	247	0	4962	0	3501	0	1771	54	0
N.S.	1	1.16	0.00	23.30	0.00	16.44	0.00	8.31	0.25	0.00
time (sec)	N/A	0.467	0.000	1.209	0.000	4.402	0.000	2.185	0.522	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	172	178	247	1296	0	1895	0	0	54	0
N.S.	1	1.03	1.44	7.53	0.00	11.02	0.00	0.00	0.31	0.00
time (sec)	N/A	0.494	6.118	13.582	0.000	1.494	0.000	0.000	0.179	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	121	201	921	0	1655	0	0	54	0
N.S.	1	1.04	1.73	7.94	0.00	14.27	0.00	0.00	0.47	0.00
time (sec)	N/A	0.377	3.076	6.139	0.000	0.568	0.000	0.000	0.173	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	169	282	2005	548	0	0	54	0
N.S.	1	0.97	2.38	3.97	28.24	7.72	0.00	0.00	0.76	0.00
time (sec)	N/A	0.340	1.694	2.287	0.348	0.260	0.000	0.000	0.165	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	168	465	2055	601	0	0	46	0
N.S.	1	0.97	2.18	6.04	26.69	7.81	0.00	0.00	0.60	0.00
time (sec)	N/A	0.228	0.937	3.418	0.373	0.258	0.000	0.000	0.172	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	124	182	661	0	741	0	0	54	0
N.S.	1	1.04	1.53	5.55	0.00	6.23	0.00	0.00	0.45	0.00
time (sec)	N/A	0.411	2.976	5.640	0.000	0.557	0.000	0.000	0.181	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	184	224	918	0	1061	0	0	54	0
N.S.	1	1.06	1.29	5.28	0.00	6.10	0.00	0.00	0.31	0.00
time (sec)	N/A	0.484	2.663	10.023	0.000	1.815	0.000	0.000	0.247	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	254	237	1161	0	1517	0	0	54	0
N.S.	1	1.05	0.98	4.82	0.00	6.29	0.00	0.00	0.22	0.00
time (sec)	N/A	0.564	4.241	15.240	0.000	6.568	0.000	0.000	2.276	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	96	187	1570	0	564	0	466	70	0
N.S.	1	0.99	1.93	16.19	0.00	5.81	0.00	4.80	0.72	0.00
time (sec)	N/A	0.335	5.572	5.462	0.000	0.787	0.000	1.556	0.175	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	522	1541	0	522	0	418	70	0
N.S.	1	1.04	5.87	17.31	0.00	5.87	0.00	4.70	0.79	0.00
time (sec)	N/A	0.306	7.788	1.410	0.000	0.645	0.000	1.304	0.176	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	A	B	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	521	86	0	494	100	370	68	68
N.S.	1	1.05	6.28	1.04	0.00	5.95	1.20	4.46	0.82	0.82
time (sec)	N/A	0.264	5.545	0.083	0.000	0.526	10.058	1.073	0.169	19.215

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	137	161	0	9176	0	2279	0	0	68	0
N.S.	1	1.18	0.00	66.98	0.00	16.64	0.00	0.00	0.50	0.00
time (sec)	N/A	0.371	0.000	10.529	0.000	1.245	0.000	0.000	0.164	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	200	227	0	12014	0	3507	0	2160	70	0
N.S.	1	1.14	0.00	60.07	0.00	17.54	0.00	10.80	0.35	0.00
time (sec)	N/A	0.456	0.000	8.632	0.000	4.476	0.000	2.137	0.181	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	268	311	0	15443	0	4751	0	3397	70	0
N.S.	1	1.16	0.00	57.62	0.00	17.73	0.00	12.68	0.26	0.00
time (sec)	N/A	0.536	0.000	8.175	0.000	17.468	0.000	2.892	8.843	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	178	316	1423	0	2035	0	0	70	0
N.S.	1	1.13	2.01	9.06	0.00	12.96	0.00	0.00	0.45	0.00
time (sec)	N/A	0.487	9.484	5.409	0.000	1.865	0.000	0.000	0.164	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	409	489	0	661	0	0	70	0
N.S.	1	1.02	3.41	4.08	0.00	5.51	0.00	0.00	0.58	0.00
time (sec)	N/A	0.387	4.296	2.838	0.000	0.777	0.000	0.000	0.165	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	120	410	692	0	773	0	0	70	0
N.S.	1	1.01	3.45	5.82	0.00	6.50	0.00	0.00	0.59	0.00
time (sec)	N/A	0.385	3.165	3.877	0.000	0.650	0.000	0.000	0.167	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	135	1927	861	0	881	0	0	62	0
N.S.	1	1.08	15.42	6.89	0.00	7.05	0.00	0.00	0.50	0.00
time (sec)	N/A	0.277	6.292	6.072	0.000	0.552	0.000	0.000	0.166	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	187	247	1109	0	1097	0	0	70	0
N.S.	1	1.07	1.42	6.37	0.00	6.30	0.00	0.00	0.40	0.00
time (sec)	N/A	0.496	4.200	9.801	0.000	1.844	0.000	0.000	0.167	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	253	234	1397	0	1579	0	0	70	0
N.S.	1	1.07	0.99	5.92	0.00	6.69	0.00	0.00	0.30	0.00
time (sec)	N/A	0.575	6.127	13.352	0.000	6.705	0.000	0.000	1.948	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	337	272	1698	0	2059	0	0	70	0
N.S.	1	1.07	0.86	5.39	0.00	6.54	0.00	0.00	0.22	0.00
time (sec)	N/A	0.674	9.588	17.253	0.000	22.982	0.000	0.000	30.941	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	105	259	0	0	0	0	0	29	0
N.S.	1	0.98	2.42	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.335	2.972	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	113	94	0	0	0	0	0	932	0
N.S.	1	0.92	0.76	0.00	0.00	0.00	0.00	0.00	7.58	0.00
time (sec)	N/A	0.327	0.436	0.000	0.000	0.000	0.000	0.000	0.509	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	83	61	0	0	0	0	0	288	0
N.S.	1	0.95	0.70	0.00	0.00	0.00	0.00	0.00	3.31	0.00
time (sec)	N/A	0.266	0.123	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	54	54	0	0	0	0	0	65	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.230	0.037	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	110	115	0	0	0	0	0	23	0
N.S.	1	0.96	1.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.294	1.226	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	163	139	0	0	0	0	0	25	0
N.S.	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.330	2.330	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	2777	0	0	0	0	0	25	0
N.S.	1	0.98	30.86	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.322	16.126	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	2465	0	0	0	0	0	25	0
N.S.	1	0.98	27.39	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.314	15.155	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	1512	0	0	0	0	0	16	0
N.S.	1	0.98	17.79	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.232	5.855	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	84	2469	0	0	0	0	0	25	0
N.S.	1	0.98	28.71	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.317	15.548	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	3033	0	0	0	0	0	25	0
N.S.	1	0.98	33.70	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.315	16.483	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	77	92	70	73	81	119	74	91	227
N.S.	1	0.84	1.00	0.76	0.79	0.88	1.29	0.80	0.99	2.47
time (sec)	N/A	0.289	0.199	1.914	0.031	0.089	0.791	0.288	0.180	18.513

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	55	59	49	51	59	82	52	61	167
N.S.	1	0.90	0.97	0.80	0.84	0.97	1.34	0.85	1.00	2.74
time (sec)	N/A	0.273	0.163	0.938	0.027	0.085	0.383	0.254	0.181	18.447

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	28	30	26	28	37	42	39	31	83
N.S.	1	0.93	1.00	0.87	0.93	1.23	1.40	1.30	1.03	2.77
time (sec)	N/A	0.219	0.009	0.434	0.027	0.081	0.181	0.182	0.187	15.982

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	51	57	42	45	61	0	52	82	72
N.S.	1	0.94	1.06	0.78	0.83	1.13	0.00	0.96	1.52	1.33
time (sec)	N/A	0.293	0.008	0.414	0.032	0.093	0.000	0.163	0.185	15.392

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	63	108	63	62	99	0	78	103	86
N.S.	1	0.97	1.66	0.97	0.95	1.52	0.00	1.20	1.58	1.32
time (sec)	N/A	0.265	0.053	0.804	0.034	0.088	0.000	0.168	0.183	15.172

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	C	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	209	251	233	218	4427	0	229	0	7402
N.S.	1	0.95	1.15	1.06	1.00	20.21	0.00	1.05	0.00	33.80
time (sec)	N/A	0.537	0.536	2.288	0.109	0.828	0.000	0.216	0.523	17.536

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	C	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	156	242	113	159	1052	0	163	25	1620
N.S.	1	0.94	1.46	0.68	0.96	6.34	0.00	0.98	0.15	9.76
time (sec)	N/A	0.424	0.412	2.698	0.112	0.715	0.000	0.178	0.185	18.913

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	35	21	21	139	22	72	114
N.S.	1	1.00	1.00	1.52	0.91	0.91	6.04	0.96	3.13	4.96
time (sec)	N/A	0.217	0.014	1.039	0.031	0.093	17.392	0.156	0.185	15.388

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	C	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	281	215	288	306	6482	0	329	0	11182
N.S.	1	0.95	0.73	0.98	1.04	21.97	0.00	1.12	0.00	37.91
time (sec)	N/A	0.710	0.433	2.096	0.116	0.939	0.000	0.170	0.232	18.409

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	C	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	375	336	374	488	10746	0	545	0	58699
N.S.	1	0.95	0.85	0.95	1.24	27.34	0.00	1.39	0.00	149.36
time (sec)	N/A	0.844	1.647	5.512	0.126	2.094	0.000	0.209	0.294	30.025

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	26	29	32	31
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.96	1.07	1.19	1.15
time (sec)	N/A	0.239	3.503	0.724	5.802	0.091	98.501	0.735	0.212	14.681

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	221	245	0	0	0	0	0	1184	0
N.S.	1	0.97	1.07	0.00	0.00	0.00	0.00	0.00	5.17	0.00
time (sec)	N/A	0.756	8.592	0.000	0.000	0.000	0.000	0.000	0.337	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	141	162	0	0	0	0	0	364	0
N.S.	1	0.97	1.12	0.00	0.00	0.00	0.00	0.00	2.51	0.00
time (sec)	N/A	0.560	3.973	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	59	59	0	0	0	0	0	76	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.272	0.078	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	26	27
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.13	1.17
time (sec)	N/A	0.226	2.668	0.309	3.642	0.085	9.710	0.461	0.194	15.057

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08	1.16
time (sec)	N/A	0.235	19.688	0.331	9.315	0.087	0.000	0.856	200.028	15.982

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	24	27	28	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.96	1.08	1.12	1.16
time (sec)	N/A	0.239	2.113	0.262	6.092	0.090	18.350	0.740	0.200	14.945

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	19	20
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.19	1.25
time (sec)	N/A	0.189	1.096	0.229	3.849	0.082	1.715	0.366	0.197	14.855

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	24	27	28	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.96	1.08	1.12	1.16
time (sec)	N/A	0.229	1.763	0.319	6.227	0.087	48.819	0.920	0.219	15.579

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [227] had the largest ratio of [1.11765000000000003]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	0.89	21	0.190
2	A	5	4	0.89	21	0.190
3	A	5	4	0.91	21	0.190
4	A	5	4	0.96	19	0.211
5	A	5	4	0.96	19	0.211
6	A	7	6	1.13	21	0.286
7	A	9	8	1.20	21	0.381
8	A	10	9	1.27	21	0.429
9	A	7	6	1.29	21	0.286
10	A	7	6	1.31	21	0.286
11	A	1	1	1.00	12	0.083
12	A	5	4	0.92	21	0.190
13	A	5	4	0.89	21	0.190
14	A	5	4	0.88	21	0.190
15	A	5	4	0.89	23	0.174
16	A	5	4	0.90	23	0.174
17	A	5	4	0.89	21	0.190
18	A	5	4	0.92	21	0.190
19	A	8	7	1.46	23	0.304
20	A	10	9	1.34	23	0.391
21	A	9	8	1.19	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	7	6	1.19	23	0.261
23	A	7	6	1.17	23	0.261
24	A	5	4	1.10	14	0.286
25	A	5	4	0.90	23	0.174
26	A	5	4	0.89	23	0.174
27	A	5	4	0.89	23	0.174
28	A	5	4	0.93	23	0.174
29	A	6	5	0.97	23	0.217
30	A	5	4	0.98	21	0.190
31	A	6	5	0.98	21	0.238
32	A	7	6	1.15	23	0.261
33	A	8	7	1.20	23	0.304
34	A	10	9	1.20	23	0.391
35	A	8	7	1.21	23	0.304
36	A	7	6	1.20	23	0.261
37	A	6	5	1.00	14	0.357
38	A	5	4	0.96	23	0.174
39	A	6	5	1.01	23	0.217
40	A	5	4	0.92	23	0.174
41	A	8	7	0.95	23	0.304
42	A	6	5	0.94	23	0.217
43	A	6	5	0.99	21	0.238
44	A	7	6	1.07	21	0.286
45	A	9	8	1.07	23	0.348
46	A	11	10	1.12	23	0.435
47	A	12	11	1.12	23	0.478
48	A	11	10	1.12	23	0.435
49	A	9	8	1.12	23	0.348
50	A	7	6	1.21	14	0.429
51	A	6	5	1.01	23	0.217
52	A	7	6	1.02	23	0.261
53	A	8	7	1.32	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	8	7	1.07	23	0.304
55	A	8	7	0.99	23	0.304
56	A	7	6	1.03	21	0.286
57	A	8	7	1.12	21	0.333
58	A	12	11	1.11	23	0.478
59	A	12	11	1.11	23	0.478
60	A	14	13	1.10	23	0.565
61	A	12	11	1.10	23	0.478
62	A	10	9	1.17	23	0.391
63	A	8	7	1.22	14	0.500
64	A	7	6	1.06	23	0.261
65	A	8	7	1.01	23	0.304
66	A	9	8	1.20	23	0.348
67	A	9	8	1.03	25	0.320
68	A	8	7	0.95	25	0.280
69	A	6	5	0.97	23	0.217
70	A	9	8	0.98	23	0.348
71	A	9	8	1.06	25	0.320
72	A	12	11	1.13	25	0.440
73	A	12	11	1.12	25	0.440
74	A	10	9	1.09	25	0.360
75	A	9	8	1.03	25	0.320
76	A	8	7	0.97	16	0.438
77	A	6	5	0.97	25	0.200
78	A	7	6	0.95	25	0.240
79	A	8	7	1.05	25	0.280
80	A	10	9	0.89	25	0.360
81	A	9	8	0.89	25	0.320
82	A	7	6	0.98	23	0.261
83	A	11	10	0.98	23	0.435
84	A	11	10	1.01	25	0.400
85	A	15	14	1.04	25	0.560

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	14	13	1.04	25	0.520
87	A	13	12	1.02	25	0.480
88	A	11	10	0.99	25	0.400
89	A	9	8	0.98	16	0.500
90	A	7	6	0.98	25	0.240
91	A	8	7	0.90	25	0.280
92	A	10	9	0.90	25	0.360
93	A	7	6	1.02	25	0.240
94	A	6	5	0.97	25	0.200
95	A	4	3	1.00	23	0.130
96	A	6	5	1.00	23	0.217
97	A	7	6	1.07	25	0.240
98	A	9	8	1.14	25	0.320
99	A	9	8	1.11	25	0.320
100	A	8	7	1.10	25	0.280
101	A	7	6	1.05	25	0.240
102	A	5	4	1.00	16	0.250
103	A	4	3	1.00	25	0.120
104	A	5	4	0.97	25	0.160
105	A	6	5	1.04	25	0.200
106	A	8	7	1.01	25	0.280
107	A	7	6	0.98	25	0.240
108	A	5	4	0.97	23	0.174
109	A	7	6	0.98	23	0.261
110	A	9	8	1.06	25	0.320
111	A	12	11	1.13	25	0.440
112	A	11	10	1.11	25	0.400
113	A	10	9	1.11	25	0.360
114	A	8	7	1.03	25	0.280
115	A	6	5	0.97	16	0.312
116	A	5	4	0.97	25	0.160
117	A	6	5	1.02	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	6	1.05	25	0.240
119	A	9	8	0.96	25	0.320
120	A	8	7	0.92	25	0.280
121	A	6	5	1.00	23	0.217
122	A	9	8	1.06	23	0.348
123	A	11	10	1.09	25	0.400
124	A	14	13	1.12	25	0.520
125	A	14	13	1.09	25	0.520
126	A	11	10	1.08	25	0.400
127	A	10	9	1.06	25	0.360
128	A	8	7	1.08	16	0.438
129	A	6	5	1.02	25	0.200
130	A	7	6	0.96	25	0.240
131	A	8	7	0.98	25	0.280
132	F	0	0	N/A	0.000	N/A
133	A	8	7	1.01	23	0.304
134	A	7	6	0.97	23	0.261
135	A	5	4	0.99	21	0.190
136	A	6	5	0.99	21	0.238
137	A	5	4	0.99	23	0.174
138	A	5	4	0.98	23	0.174
139	A	5	4	0.98	23	0.174
140	A	5	4	0.98	14	0.286
141	A	5	4	0.97	23	0.174
142	A	6	5	0.97	23	0.217
143	A	7	6	1.02	23	0.261
144	A	11	11	0.84	15	0.733
145	A	9	9	0.88	15	0.600
146	A	7	7	0.84	15	0.467
147	A	1	1	1.00	13	0.077
148	A	5	5	1.05	15	0.333
149	A	7	7	0.84	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	9	9	0.91	15	0.600
151	A	11	11	0.84	15	0.733
152	A	8	8	0.96	21	0.381
153	A	6	6	0.97	21	0.286
154	A	4	4	1.00	19	0.211
155	A	4	4	1.00	19	0.211
156	A	6	5	1.00	21	0.238
157	A	7	6	0.94	21	0.286
158	A	6	5	0.80	21	0.238
159	A	6	5	0.89	21	0.238
160	A	6	5	1.00	21	0.238
161	A	1	1	1.00	12	0.083
162	A	3	3	1.00	21	0.143
163	A	5	5	0.97	21	0.238
164	A	7	7	0.96	21	0.333
165	A	9	8	1.08	23	0.348
166	A	8	7	1.16	23	0.304
167	A	7	6	1.30	21	0.286
168	A	5	4	1.05	21	0.190
169	A	5	4	0.90	23	0.174
170	A	5	4	0.91	23	0.174
171	A	5	4	0.90	23	0.174
172	A	5	4	0.90	23	0.174
173	A	5	4	0.91	23	0.174
174	A	5	4	1.10	14	0.286
175	A	5	4	1.21	23	0.174
176	A	6	5	1.39	23	0.217
177	A	7	6	1.19	23	0.261
178	A	5	4	1.01	14	0.286
179	A	5	4	0.98	14	0.286
180	A	8	7	1.10	23	0.304
181	A	6	5	0.96	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	4	3	1.00	21	0.143
183	A	5	4	0.96	21	0.190
184	A	5	4	0.93	23	0.174
185	A	5	4	0.93	23	0.174
186	A	5	4	0.94	23	0.174
187	A	5	4	0.96	23	0.174
188	A	4	3	1.00	23	0.130
189	A	6	5	1.00	14	0.357
190	A	8	7	1.21	23	0.304
191	A	10	9	1.21	23	0.391
192	A	12	11	1.21	23	0.478
193	A	8	7	1.10	23	0.304
194	A	5	4	0.97	23	0.174
195	A	5	4	0.98	21	0.190
196	A	5	4	0.95	21	0.190
197	A	5	4	0.94	23	0.174
198	A	5	4	0.93	23	0.174
199	A	5	4	0.95	23	0.174
200	A	5	4	0.98	23	0.174
201	A	5	4	0.97	23	0.174
202	A	7	6	1.21	14	0.429
203	A	10	9	1.13	23	0.391
204	A	12	11	1.14	23	0.478
205	A	14	13	1.14	23	0.565
206	A	6	5	1.05	23	0.217
207	A	6	5	1.00	23	0.217
208	A	7	6	1.11	21	0.286
209	A	5	4	0.95	21	0.190
210	A	5	4	0.94	23	0.174
211	A	5	4	0.93	23	0.174
212	A	6	5	1.10	23	0.217
213	A	6	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	6	5	1.05	23	0.217
215	A	8	7	1.22	14	0.500
216	A	11	10	1.19	23	0.435
217	A	14	13	1.13	23	0.565
218	A	16	15	1.13	23	0.652
219	A	10	9	1.20	14	0.643
220	A	12	12	0.62	17	0.706
221	A	10	10	0.67	17	0.588
222	A	8	8	0.80	17	0.471
223	A	6	6	1.00	17	0.353
224	A	7	7	1.06	17	0.412
225	A	11	11	0.81	17	0.647
226	A	15	15	0.71	17	0.882
227	A	19	19	0.63	17	1.118
228	A	17	16	1.04	25	0.640
229	A	15	14	1.04	25	0.560
230	A	13	12	1.07	23	0.522
231	A	7	6	1.10	23	0.261
232	A	12	11	1.00	25	0.440
233	A	14	13	0.96	25	0.520
234	A	9	8	1.01	25	0.320
235	A	7	6	0.96	25	0.240
236	A	6	5	0.97	25	0.200
237	A	8	7	0.97	16	0.438
238	A	6	5	1.05	25	0.200
239	A	7	6	1.02	25	0.240
240	A	11	10	1.10	25	0.400
241	A	19	18	1.04	25	0.720
242	A	17	16	1.04	25	0.640
243	A	15	14	1.05	23	0.609
244	A	13	12	1.07	23	0.522
245	A	11	10	1.00	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	13	12	1.00	25	0.480
247	A	10	9	0.92	25	0.360
248	A	8	7	0.92	25	0.280
249	A	7	6	0.98	25	0.240
250	A	9	8	0.98	16	0.500
251	A	9	8	1.03	25	0.320
252	A	7	6	1.06	25	0.240
253	A	8	7	0.98	25	0.280
254	A	10	9	1.00	16	0.562
255	A	9	8	1.00	10	0.800
256	A	7	6	1.00	10	0.600
257	A	15	14	0.94	25	0.560
258	A	10	9	1.17	25	0.360
259	A	7	6	1.10	23	0.261
260	A	7	6	1.07	23	0.261
261	A	12	11	1.00	25	0.440
262	A	14	13	0.97	25	0.520
263	A	8	7	1.07	25	0.280
264	A	6	5	0.98	25	0.200
265	A	5	4	1.00	25	0.160
266	A	5	4	1.00	16	0.250
267	A	6	5	1.05	25	0.200
268	A	10	9	1.09	25	0.360
269	A	11	10	1.11	25	0.400
270	A	15	14	1.08	25	0.560
271	A	9	8	1.23	25	0.320
272	A	12	11	1.06	23	0.478
273	A	12	11	1.07	23	0.478
274	A	14	13	0.99	25	0.520
275	A	16	15	0.96	25	0.600
276	A	7	6	1.18	25	0.240
277	A	6	5	0.97	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	4	3	1.00	25	0.120
279	A	6	5	0.97	16	0.312
280	A	9	8	1.02	25	0.320
281	A	11	10	1.07	25	0.400
282	A	13	12	1.08	25	0.480
283	A	14	13	1.01	25	0.520
284	A	14	13	1.00	25	0.520
285	A	14	13	1.01	23	0.565
286	A	13	12	0.99	23	0.522
287	A	16	15	0.97	25	0.600
288	A	17	16	0.94	25	0.640
289	A	7	6	1.14	25	0.240
290	A	5	4	0.95	25	0.160
291	A	5	4	0.97	25	0.160
292	A	8	7	1.08	16	0.438
293	A	10	9	1.09	25	0.360
294	A	12	11	1.10	25	0.440
295	A	14	13	1.11	25	0.520
296	A	9	8	1.12	16	0.500
297	A	5	4	1.00	10	0.400
298	F	0	0	N/A	0.000	N/A
299	A	7	6	1.08	23	0.261
300	A	7	6	1.08	21	0.286
301	A	7	6	1.08	21	0.286
302	A	7	6	1.08	23	0.261
303	A	7	6	1.08	23	0.261
304	A	8	7	0.99	23	0.304
305	A	6	5	0.97	23	0.217
306	A	5	4	0.97	23	0.174
307	A	5	4	0.98	14	0.286
308	A	5	4	0.98	23	0.174
309	A	5	4	0.98	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	5	4	0.98	23	0.174
311	A	6	5	0.88	21	0.238
312	A	6	5	0.94	21	0.238
313	A	5	4	0.93	19	0.211
314	A	6	5	1.32	19	0.263
315	A	6	5	1.28	21	0.238
316	A	6	5	1.29	21	0.238
317	A	7	6	0.94	21	0.286
318	A	7	6	1.00	21	0.286
319	A	7	6	1.12	21	0.286
320	A	1	1	1.00	12	0.083
321	A	6	5	1.37	21	0.238
322	A	7	6	1.24	21	0.286
323	A	8	7	0.96	21	0.333
324	A	6	5	0.91	23	0.217
325	A	6	5	0.92	23	0.217
326	A	6	5	0.96	21	0.238
327	A	6	5	1.04	21	0.238
328	A	6	5	1.12	23	0.217
329	A	6	5	1.35	23	0.217
330	A	6	5	0.95	23	0.217
331	A	6	5	0.97	23	0.217
332	A	6	5	1.02	23	0.217
333	A	5	4	1.10	14	0.286
334	A	6	5	1.11	23	0.217
335	A	6	5	1.09	23	0.217
336	A	6	5	1.02	23	0.217
337	A	6	5	0.91	23	0.217
338	A	6	5	1.00	23	0.217
339	A	4	3	1.00	21	0.143
340	A	6	5	1.13	21	0.238
341	A	6	5	1.04	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	6	5	1.09	23	0.217
343	A	10	9	1.18	23	0.391
344	A	8	7	1.19	23	0.304
345	A	7	6	1.15	23	0.261
346	A	6	5	1.00	14	0.357
347	A	9	8	1.24	23	0.348
348	A	10	9	1.27	23	0.391
349	A	12	11	1.22	23	0.478
350	A	6	5	0.91	23	0.217
351	A	6	5	0.94	23	0.217
352	A	6	5	0.90	21	0.238
353	A	6	5	1.01	21	0.238
354	A	6	5	0.97	23	0.217
355	A	6	5	1.06	23	0.217
356	A	9	8	1.10	23	0.348
357	A	8	7	1.16	23	0.304
358	A	8	7	1.12	23	0.304
359	A	7	6	1.21	14	0.429
360	A	9	8	1.17	23	0.348
361	A	11	10	1.14	23	0.435
362	A	13	12	1.12	23	0.522
363	A	6	5	0.94	23	0.217
364	A	6	5	0.93	23	0.217
365	A	6	5	0.93	21	0.238
366	A	6	5	0.98	21	0.238
367	A	6	5	0.98	23	0.217
368	A	6	5	1.01	23	0.217
369	A	10	9	1.16	23	0.391
370	A	10	9	1.12	23	0.391
371	A	9	8	1.17	23	0.348
372	A	8	7	1.22	14	0.500
373	A	10	9	1.17	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	12	11	1.14	23	0.478
375	A	14	13	1.12	23	0.565
376	A	6	5	0.96	25	0.200
377	A	9	8	0.99	25	0.320
378	A	7	6	1.04	23	0.261
379	A	8	7	1.03	23	0.304
380	A	9	8	1.04	25	0.320
381	A	12	11	1.13	25	0.440
382	A	13	12	1.05	25	0.480
383	A	12	11	1.02	25	0.440
384	A	10	9	0.97	25	0.360
385	A	8	7	0.97	16	0.438
386	A	9	8	0.97	25	0.320
387	A	9	8	0.98	25	0.320
388	A	11	10	1.05	25	0.400
389	A	6	5	0.95	25	0.200
390	A	10	9	0.98	25	0.360
391	A	8	7	1.01	23	0.304
392	A	10	9	1.00	23	0.391
393	A	9	8	1.04	25	0.320
394	A	12	11	1.08	25	0.440
395	A	15	14	1.04	25	0.560
396	A	14	13	1.03	25	0.520
397	A	12	11	0.99	25	0.440
398	A	9	8	0.98	16	0.500
399	A	11	10	0.95	25	0.400
400	A	10	9	0.98	25	0.360
401	A	12	11	1.00	25	0.440
402	A	6	5	0.98	25	0.200
403	A	8	7	1.05	25	0.280
404	A	6	5	1.00	23	0.217
405	A	8	7	1.03	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
406	A	9	8	1.10	25	0.320
407	A	12	11	1.17	25	0.440
408	A	11	10	1.05	25	0.400
409	A	10	9	1.02	25	0.360
410	A	9	8	0.98	25	0.320
411	A	5	4	1.00	16	0.250
412	A	9	8	0.97	25	0.320
413	A	10	9	1.06	25	0.360
414	A	11	10	1.09	25	0.400
415	A	6	5	1.00	25	0.200
416	A	8	7	1.03	25	0.280
417	A	7	6	1.04	23	0.261
418	A	10	9	1.13	23	0.391
419	A	11	10	1.13	25	0.400
420	A	14	13	1.16	25	0.520
421	A	11	10	1.03	25	0.400
422	A	10	9	1.04	25	0.360
423	A	7	6	0.97	25	0.240
424	A	6	5	0.97	16	0.312
425	A	9	8	1.04	25	0.320
426	A	10	9	1.06	25	0.360
427	A	11	10	1.05	25	0.400
428	A	6	5	0.99	25	0.200
429	A	9	8	1.04	25	0.320
430	A	8	7	1.05	23	0.304
431	A	12	11	1.18	23	0.478
432	A	13	12	1.14	25	0.480
433	A	16	15	1.16	25	0.600
434	A	13	12	1.13	25	0.480
435	A	9	8	1.02	25	0.320
436	A	9	8	1.01	25	0.320
437	A	8	7	1.08	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	10	9	1.07	25	0.360
439	A	12	11	1.07	25	0.440
440	A	13	12	1.07	25	0.480
441	A	6	5	0.98	25	0.200
442	A	6	5	0.92	23	0.217
443	A	7	6	0.95	23	0.261
444	A	5	4	0.98	21	0.190
445	A	8	7	0.96	21	0.333
446	A	9	8	1.03	23	0.348
447	A	6	5	0.98	23	0.217
448	A	6	5	0.98	23	0.217
449	A	5	4	0.98	14	0.286
450	A	6	5	0.98	23	0.217
451	A	6	5	0.98	23	0.217
452	A	5	4	0.84	21	0.190
453	A	5	4	0.90	21	0.190
454	A	5	4	0.93	19	0.211
455	A	5	4	0.94	19	0.211
456	A	8	7	0.97	21	0.333
457	A	5	4	0.95	23	0.174
458	A	13	12	0.94	23	0.522
459	A	4	3	1.00	21	0.143
460	A	5	4	0.95	21	0.190
461	A	5	4	0.95	23	0.174
462	N/A	2	0	1.00	27	0.000
463	A	5	4	0.97	25	0.160
464	A	6	5	0.97	25	0.200
465	A	7	6	0.98	23	0.261
466	N/A	2	0	1.00	23	0.000
467	N/A	2	0	1.00	25	0.000
468	N/A	2	0	1.00	25	0.000
469	N/A	2	0	1.00	16	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	N/A	2	0	1.00	25	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+b\sec^2(e+fx))\sin^7(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (a+b\sec^2(e+fx))\sin^7(e+fx)\,dx = -\frac{(a-3b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(3a-b)\cos^5(e+fx)}{5f}+\frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

output
```
-(a-3*b)*cos(f*x+e)/f+(a-b)*cos(f*x+e)^3/f-1/5*(3*a-b)*cos(f*x+e)^5/f+1/7*
a*cos(f*x+e)^7/f+b*sec(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.45

$$\int (a+b\sec^2(e+fx))\sin^7(e+fx)\,dx = -\frac{35a\cos(e+fx)}{64f}+\frac{19b\cos(e+fx)}{8f}+\frac{7a\cos(3(e+fx))}{64f}-\frac{3b\cos(3(e+fx))}{16f}-\frac{7a\cos(5(e+fx))}{320f}+\frac{b\cos(5(e+fx))}{80f}+\frac{a\cos(7(e+fx))}{448f}+\frac{b\sec(e+fx)}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

output
```
(-35*a*Cos[e + f*x])/(64*f) + (19*b*Cos[e + f*x])/(8*f) + (7*a*Cos[3*(e +
f*x)])/(64*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (7*a*Cos[5*(e + f*x)])/(32
0*f) + (b*Cos[5*(e + f*x)])/(80*f) + (a*Cos[7*(e + f*x)])/(448*f) + (b*Sec
[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^7\left(a+b\sec(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^3\left(a\cos^2(e+fx)+b\right)\sec^2(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(-a\cos^6(e+fx)+(3a-b)\cos^4(e+fx)-3(a-b)\cos^2(e+fx)+b\sec^2(e+fx)+a\left(1-\frac{3b}{a}\right)\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{5}(3a-b)\cos^5(e+fx)-(a-b)\cos^3(e+fx)+(a-3b)\cos(e+fx)-\frac{1}{7}a\cos^7(e+fx)-b\sec(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

output
```
-(((a - 3*b)*Cos[e + f*x] - (a - b)*Cos[e + f*x]^3 + ((3*a - b)*Cos[e + f*
x]^5)/5 - (a*Cos[e + f*x]^7)/7 - b*Sec[e + f*x])/f)
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{(-980a+4900b)\cos(2fx+2e)+(196a-392b)\cos(4fx+4e)+(-44a+28b)\cos(6fx+6e)+5a\cos(8fx+8e)+(-2048a+14}{4480f\cos(fx+e)}$
derivativedivides	$\frac{-\frac{a\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{7}+b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx-}{5}\right.\right.}{f}$
default	$\frac{-\frac{a\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{7}+b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx-}{5}\right.\right.}{f}$
parts	$-\frac{a\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{7f}+\frac{b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8}{}\right.\right.}{f}$
norman	$\frac{\frac{32a-224b}{35f}-\frac{32(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{f}+\frac{6(32a-224b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{35f}+\frac{2(32a-224b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{5f}+\frac{2(32a-224b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{5f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^7}$
risch	$\frac{19\,e^{i(fx+e)}b}{16f}-\frac{35\,e^{i(fx+e)}a}{128f}+\frac{19\,e^{-i(fx+e)}b}{16f}-\frac{35\,e^{-i(fx+e)}a}{128f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{a\cos(7fx+7e)}{448f}-\frac{7\cos(}{3}$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x,method=_RETURNVERBOSE)
```

output
```
1/4480*((-980*a+4900*b)*cos(2*f*x+2*e)+(196*a-392*b)*cos(4*f*x+4*e)+(-44*a
+28*b)*cos(6*f*x+6*e)+5*a*cos(8*f*x+8*e)+(-2048*a+14336*b)*cos(f*x+e)-1225
*a+9800*b)/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx$$
$$=\frac{5\,a\cos\left(fx+e\right)^8-7\left(3\,a-b\right)\cos\left(fx+e\right)^6+35\left(a-b\right)\cos\left(fx+e\right)^4-35\left(a-3\,b\right)\cos\left(fx+e\right)^2+35\,b}{35\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="fricas")
```

output
```
1/35*(5*a*cos(f*x + e)^8 - 7*(3*a - b)*cos(f*x + e)^6 + 35*(a - b)*cos(f*x
 + e)^4 - 35*(a - 3*b)*cos(f*x + e)^2 + 35*b)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx$$
$$=\frac{5\,a\cos\left(fx+e\right)^7-7\,(3\,a-b)\cos\left(fx+e\right)^5+35\,(a-b)\cos\left(fx+e\right)^3-35\,(a-3\,b)\cos\left(fx+e\right)+\frac{35}{\cos(f}}{35\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="maxima")
```

output
```
1/35*(5*a*cos(f*x + e)^7 - 7*(3*a - b)*cos(f*x + e)^5 + 35*(a - b)*cos(f*x
 + e)^3 - 35*(a - 3*b)*cos(f*x + e) + 35*b/cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx$$
$$=\frac{5\,a\cos\left(fx+e\right)^7-21\,a\cos\left(fx+e\right)^5+7\,b\cos\left(fx+e\right)^5+35\,a\cos\left(fx+e\right)^3-35\,b\cos\left(fx+e\right)^3-35\,a}{35\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="giac")
```

output
```
1/35*(5*a*cos(f*x + e)^7 - 21*a*cos(f*x + e)^5 + 7*b*cos(f*x + e)^5 + 35*a
*cos(f*x + e)^3 - 35*b*cos(f*x + e)^3 - 35*a*cos(f*x + e) + 105*b*cos(f*x
+ e) + 35*b/cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int (a + b\sec^2(e + fx))\sin^7(e + fx)\,dx = \frac{\frac{a\cos(e+f\,x)^7}{7} - \cos(e + f\,x)\,(a - 3\,b) - \cos(e + f\,x)^5\left(\frac{3\,a}{5} - \frac{b}{5}\right) + \frac{b}{\cos(e+f\,x)} + \cos(e + f\,x)^3\,(a - b)}{f}$$

input
```
int(sin(e + f*x)^7*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*cos(e + f*x)^7)/7 - cos(e + f*x)*(a - 3*b) - cos(e + f*x)^5*((3*a)/5 -
 b/5) + b/cos(e + f*x) + cos(e + f*x)^3*(a - b))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.64

$$\int (a + b\sec^2(e + fx))\sin^7(e + fx)\,dx = \frac{-5\cos(fx+e)^2\sin(fx+e)^6 a - 6\cos(fx+e)^2\sin(fx+e)^4 a - 8\cos(fx+e)^2\sin(fx+e)^2 a - 16\,\text{c}}{3}$$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x)
```

output
```
( - 5*cos(e + f*x)**2*sin(e + f*x)**6*a - 6*cos(e + f*x)**2*sin(e + f*x)**
4*a - 8*cos(e + f*x)**2*sin(e + f*x)**2*a - 16*cos(e + f*x)**2*a + 16*cos(
e + f*x)*a - 112*cos(e + f*x)*b - 7*sin(e + f*x)**6*b - 14*sin(e + f*x)**4
*b - 56*sin(e + f*x)**2*b + 112*b)/(35*cos(e + f*x)*f)
```

3.2 $\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx = -\frac{(a-2b)\cos(e+fx)}{f}+\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

output
```
-(a-2*b)*cos(f*x+e)/f+1/3*(2*a-b)*cos(f*x+e)^3/f-1/5*a*cos(f*x+e)^5/f+b*se
c(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx = -\frac{5a\cos(e+fx)}{8f}+\frac{7b\cos(e+fx)}{4f}+\frac{5a\cos(3(e+fx))}{48f}-\frac{b\cos(3(e+fx))}{12f}-\frac{a\cos(5(e+fx))}{80f}+\frac{b\sec(e+fx)}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

output
```
(-5*a*Cos[e + f*x])/(8*f) + (7*b*Cos[e + f*x])/(4*f) + (5*a*Cos[3*(e + f*x
)])/(48*f) - (b*Cos[3*(e + f*x)])/(12*f) - (a*Cos[5*(e + f*x)])/(80*f) + (
b*Sec[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(e+fx)^5 \left(a+b\sec(e+fx)^2\right) dx$$

$$\Big\downarrow \; 4621$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2 \left(a\cos^2(e+fx)+b\right)\sec^2(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 355$$

$$-\frac{\int \left(a\cos^4(e+fx)-(2a-b)\cos^2(e+fx)+b\sec^2(e+fx)+a\left(1-\frac{2b}{a}\right)\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{1}{3}(2a-b)\cos^3(e+fx)+(a-2b)\cos(e+fx)+\frac{1}{5}a\cos^5(e+fx)-b\sec(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

output
```
-(((a - 2*b)*Cos[e + f*x] - ((2*a - b)*Cos[e + f*x]^3)/3 + (a*Cos[e + f*x]
^5)/5 - b*Sec[e + f*x])/f)
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{(-125a+400b)\cos(2fx+2e)+(22a-20b)\cos(4fx+4e)-3\cos(6fx+6e)a+(-256a+1280b)\cos(fx+e)-150a+900b}{480f\cos(fx+e)}$
derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)}{f}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)}{f}$
parts	$-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5f}+\frac{b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)}{f}$
norman	$\frac{\frac{16a-80b}{15f}-\frac{32(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3f}+\frac{4(16a-80b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15f}+\frac{(16a-80b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^5}$
risch	$-\frac{5\,e^{i(fx+e)}a}{16f}+\frac{7\,e^{i(fx+e)}b}{8f}-\frac{5\,e^{-i(fx+e)}a}{16f}+\frac{7\,e^{-i(fx+e)}b}{8f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}-\frac{\cos(5fx+5e)a}{80f}+\frac{5\cos(3f}{48}$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/480*((-125*a+400*b)*cos(2*f*x+2*e)+(22*a-20*b)*cos(4*f*x+4*e)-3*cos(6*f*
x+6*e)*a+(-256*a+1280*b)*cos(f*x+e)-150*a+900*b)/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx = -\frac{3\,a\cos(fx+e)^6-5\,(2\,a-b)\cos(fx+e)^4+15\,(a-2\,b)\cos(fx+e)^2-15\,b}{15\,f\cos(fx+e)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/15*(3*a*cos(f*x + e)^6 - 5*(2*a - b)*cos(f*x + e)^4 + 15*(a - 2*b)*cos(
f*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\sin^5{(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**5,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx = -\frac{3\,a\cos\left(fx+e\right)^5-5\,(2\,a-b)\cos\left(fx+e\right)^3+15\,(a-2\,b)\cos\left(fx+e\right)-\frac{15\,b}{\cos(fx+e)}}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/15*(3*a*cos(f*x + e)^5 - 5*(2*a - b)*cos(f*x + e)^3 + 15*(a - 2*b)*cos(
f*x + e) - 15*b/cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx = -\frac{3\,a\cos\left(fx+e\right)^5-10\,a\cos\left(fx+e\right)^3+5\,b\cos\left(fx+e\right)^3+15\,a\cos\left(fx+e\right)-30\,b\cos\left(fx+e\right)-\frac{1}{\cos(}}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
-1/15*(3*a*cos(f*x + e)^5 - 10*a*cos(f*x + e)^3 + 5*b*cos(f*x + e)^3 + 15*
a*cos(f*x + e) - 30*b*cos(f*x + e) - 15*b/cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx = \frac{\cos(e+f\,x)^3\,\left(\frac{2\,a}{3}-\frac{b}{3}\right)-\cos(e+f\,x)\,\left(a-2\,b\right)-\frac{a\cos(e+f\,x)^5}{5}+\frac{b}{\cos(e+f\,x)}}{f}$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

output
```
(cos(e + f*x)^3*((2*a)/3 - b/3) - cos(e + f*x)*(a - 2*b) - (a*cos(e + f*x)
^5)/5 + b/cos(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.61

$$\int (a+b\sec^2(e+fx))\sin^5(e+fx)\,dx = \frac{-3\cos(fx+e)^2\sin(fx+e)^4a-4\cos(fx+e)^2\sin(fx+e)^2a-8\cos(fx+e)^2a+8\cos(fx+e)a-}{15\cos(fx+e)f}$$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x)
```

output
```
( - 3*cos(e + f*x)**2*sin(e + f*x)**4*a - 4*cos(e + f*x)**2*sin(e + f*x)**
2*a - 8*cos(e + f*x)**2*a + 8*cos(e + f*x)*a - 40*cos(e + f*x)*b - 5*sin(e
 + f*x)**4*b - 20*sin(e + f*x)**2*b + 40*b)/(15*cos(e + f*x)*f)
```

3.3 $\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx=-\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

output
```
-(a-b)*cos(f*x+e)/f+1/3*a*cos(f*x+e)^3/f+b*sec(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx=-\frac{3a\cos(e+fx)}{4f}+\frac{b\cos(e+fx)}{f}+\frac{a\cos(3(e+fx))}{12f}+\frac{b\sec(e+fx)}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
```

output
```
(-3*a*Cos[e + f*x])/(4*f) + (b*Cos[e + f*x])/f + (a*Cos[3*(e + f*x)])/(12*
f) + (b*Sec[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3 \left(a+b\sec(e+fx)^2\right) dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right) \left(a\cos^2(e+fx)+b\right) \sec^2(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(-a\cos^2(e+fx)+b\sec^2(e+fx)+a\left(1-\frac{b}{a}\right)\right) d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a-b)\cos(e+fx)-\frac{1}{3}a\cos^3(e+fx)-b\sec(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
```

output
```
-(((a - b)*Cos[e + f*x] - (a*Cos[e + f*x]^3)/3 - b*Sec[e + f*x])/f)
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{-\frac{a\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)}{3}+b\left(\frac{\sin(fx+e)^{4}}{\cos(fx+e)}+\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)\right)}{f}$	62
default	$\frac{-\frac{a\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)}{3}+b\left(\frac{\sin(fx+e)^{4}}{\cos(fx+e)}+\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)\right)}{f}$	62
parallelrisch	$\frac{(-8a+12b)\cos(2fx+2e)+\cos(4fx+4e)a+(-16a+48b)\cos(fx+e)-9a+36b}{24f\cos(fx+e)}$	63
parts	$-\frac{a\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)}{3f}+\frac{b\left(\frac{\sin(fx+e)^{4}}{\cos(fx+e)}+\left(2+\sin(fx+e)^{2}\right)\cos(fx+e)\right)}{f}$	64
norman	$\frac{\frac{4a-12b}{3f}-\frac{4(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{f}+\frac{2(4a-12b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)^{3}}$	87
risch	$-\frac{3\,e^{i(fx+e)}a}{8f}+\frac{e^{i(fx+e)}b}{2f}-\frac{3\,e^{-i(fx+e)}a}{8f}+\frac{e^{-i(fx+e)}b}{2f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\cos(3fx+3e)a}{12f}$	105

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(
f*x+e)^2)*cos(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx=\frac{a\cos\left(fx+e\right)^4-3\,(a-b)\cos\left(fx+e\right)^2+3\,b}{3\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/3*(a*cos(f*x + e)^4 - 3*(a - b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\sin^3\left(e+fx\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx = \frac{a\cos(fx+e)^3 - 3(a-b)\cos(fx+e) + \frac{3b}{\cos(fx+e)}}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/3*(a*cos(f*x + e)^3 - 3*(a - b)*cos(f*x + e) + 3*b/cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx = \frac{a\cos(fx+e)^3 - 3a\cos(fx+e) + 3b\cos(fx+e) + \frac{3b}{\cos(fx+e)}}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
1/3*(a*cos(f*x + e)^3 - 3*a*cos(f*x + e) + 3*b*cos(f*x + e) + 3*b/cos(f*x
+ e))/f
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int (a+b\sec^2(e+fx))\sin^3(e+fx)\,dx = \frac{\frac{a\cos(e+fx)^3}{3} - \cos(e+fx)\,(a-b) + \frac{b}{\cos(e+fx)}}{f}$$

input
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*cos(e + f*x)^3)/3 - cos(e + f*x)*(a - b) + b/cos(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.84

$$\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx$$
$$=\frac{-\cos(fx+e)\sin(fx+e)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4a+\cos(fx+e)\sin(fx+e)^2a-2\cos(fx+e)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-1\right)}$$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x)
```

output
```
( - cos(e + f*x)*sin(e + f*x)**2*tan((e + f*x)/2)**4*a + cos(e + f*x)*sin(
e + f*x)**2*a - 2*cos(e + f*x)*tan((e + f*x)/2)**4*a + 2*cos(e + f*x)*a +
2*tan((e + f*x)/2)**4*a - 12*tan((e + f*x)/2)**4*b - 2*a)/(3*f*(tan((e + f
*x)/2)**4 - 1))
```

3.4 $\int (a+b\sec^2(e+fx))\sin(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (a+b\sec^2(e+fx))\sin(e+fx)\,dx=-\frac{a\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

output
```
-a*cos(f*x+e)/f+b*sec(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int (a+b\sec^2(e+fx))\sin(e+fx)\,dx=-\frac{a\cos(e)\cos(fx)}{f}+\frac{b\sec(e+fx)}{f}+\frac{a\sin(e)\sin(fx)}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

output
```
-((a*Cos[e]*Cos[f*x])/f) + (b*Sec[e + f*x])/f + (a*Sin[e]*Sin[f*x])/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4621, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx) \left(a+b\sec(e+fx)^2\right) dx$$

$$\Big\downarrow \text{ 4621}$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)\sec^2(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{\int \left(b\sec^2(e+fx)+a\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\cos(e+fx)-b\sec(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

output
```
-((a*Cos[e + f*x] - b*Sec[e + f*x])/f)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\sec(fx+e)b-\frac{a}{\sec(fx+e)}}{f}$	25
default	$\frac{\sec(fx+e)b-\frac{a}{\sec(fx+e)}}{f}$	25
parts	$-\frac{a\cos(fx+e)}{f}+\frac{b\sec(fx+e)}{f}$	25
parallelrisch	$\frac{-a\cos(2fx+2e)+(-2a+2b)\cos(fx+e)-a+2b}{2f\cos(fx+e)}$	47
risch	$-\frac{a\,e^{3i(fx+e)}+(3a-4b)\cos(fx+e)+i(a-4b)\sin(fx+e)}{2f\left(e^{2i(fx+e)}+1\right)}$	59
norman	$\frac{\frac{2a-2b}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}$	63

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(sec(f*x+e)*b-a/sec(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx=-\frac{a\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
-(a*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\sin\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx=-\frac{a\cos\left(fx+e\right)-\frac{b}{\cos(fx+e)}}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
-(a*cos(f*x + e) - b/cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (a + b\sec^2(e + fx))\sin(e + fx)\,dx = -\frac{a\cos(fx + e) - \frac{b}{\cos(fx+e)}}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="giac")
```

output
```
-(a*cos(f*x + e) - b/cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (a + b\sec^2(e + fx))\sin(e + fx)\,dx = -\frac{a\,\cos(e + f\,x) - \frac{b}{\cos(e+f\,x)}}{f}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

output
```
-(a*cos(e + f*x) - b/cos(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int (a + b\sec^2(e + fx))\sin(e + fx)\,dx = \frac{-\cos(fx + e)^2 a - \cos(fx + e)\,b + b}{\cos(fx + e)\,f}$$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e),x)
```

output
```
( - cos(e + f*x)**2*a - cos(e + f*x)*b + b)/(cos(e + f*x)*f)
```

3.5 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\text{arctanh}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}$$

output
```
-(a+b)*arctanh(cos(f*x+e))/f+b*sec(f*x+e)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{a\text{arctanh}(\cos(e+fx))}{f}-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{b\sec(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((a*ArcTanh[Cos[e + f*x]])/f) - (b*Log[Cos[(e + f*x)/2]])/f + (b*Log[Sin[
(e + f*x)/2]])/f + (b*Sec[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4621, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$-\frac{(a+b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-b\sec(e+fx)}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(a+b)\text{arctanh}(\cos(e+fx))-b\sec(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
-(((a + b)*ArcTanh[Cos[e + f*x]] - b*Sec[e + f*x])/f)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

method	result	size
parallelrisch	$\frac{\left(b+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)(a+b)\right)\cos(fx+e)+b}{f\cos(fx+e)}$	38
norman	$-\frac{2b}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{(a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$	40
derivativedivides	$\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$	51
default	$\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$	51
risch	$\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right)b}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)b}{f}$	100

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((b+ln(tan(1/2*f*x+1/2*e))*(a+b))*cos(f*x+e)+b)/cos(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+b)\cos(fx+e)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)-(a+b)\cos(fx+e)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)-2b}{2f\cos(fx+e)}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*((a + b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - (a + b)*cos(f*x +
 e)*log(-1/2*cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\csc(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+b)\log\left(\cos\left(fx+e\right)+1\right)-(a+b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,b}{\cos(fx+e)}}{2\,f}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*((a + b)*log(cos(f*x + e) + 1) - (a + b)*log(cos(f*x + e) - 1) - 2*b/
cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+b)\log\left(|\cos\left(fx+e\right)+1|\right)}{2\,f} + \frac{(a+b)\log\left(|\cos\left(fx+e\right)-1|\right)}{2\,f} + \frac{b}{f\cos\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(a + b)*log(abs(cos(f*x + e) + 1))/f + 1/2*(a + b)*log(abs(cos(f*x +
e) - 1))/f + b/(f*cos(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b}{f\,\cos\left(e+f\,x\right)}-\frac{\mathrm{atanh}(\cos\left(e+f\,x\right))\,\left(a+b\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x),x)
```

output
```
b/(f*cos(e + f*x)) - (atanh(cos(e + f*x))*(a + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)b-\cos\left(fx+e\right)b+b}{\cos\left(fx+e\right)f}$$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

output
```
(cos(e + f*x)*log(tan((e + f*x)/2))*a + cos(e + f*x)*log(tan((e + f*x)/2))
*b - cos(e + f*x)*b + b)/(cos(e + f*x)*f)
```

3.6 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+3b)\text{arctanh}(\cos(e+fx))}{2f} - \frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(e+fx)}{f}$$

output
```
-1/2*(a+3*b)*arctanh(cos(f*x+e))/f-1/2*(a+b)*cot(f*x+e)*csc(f*x+e)/f+b*sec
(f*x+e)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 236 vs. $2(53) = 106$.

Time = 0.45 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.45

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}-\frac{3b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{3b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{b\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}-\frac{b\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/8*(a*Csc[(e + f*x)/2]^2)/f - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[
(e + f*x)/2]])/(2*f) - (3*b*Log[Cos[(e + f*x)/2]])/(2*f) + (a*Log[Sin[(e +
 f*x)/2]])/(2*f) + (3*b*Log[Sin[(e + f*x)/2]])/(2*f) + (a*Sec[(e + f*x)/2]
^2)/(8*f) + (b*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sin[(e + f*x)/2])/(f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])) - (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4621, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{ 4621}$$

$$-\frac{\int \frac{(a\cos^2(e+fx)+b)\sec^2(e+fx)}{(1-\cos^2(e+fx))^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 361}$$

$$-\frac{\frac{(a+b)\cos(e+fx)}{2(1-\cos^2(e+fx))}-\frac{1}{2}\int -\frac{((a+b)\cos^2(e+fx)+2b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{1}{2}\int \frac{((a+b)\cos^2(e+fx)+2b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{(a+b)\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{\frac{1}{2}\left((a+3b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-2b\sec(e+fx)\right)+\frac{(a+b)\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{1}{2}((a+3b)\text{arctanh}(\cos(e+fx))-2b\sec(e+fx))+\frac{(a+b)\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((((a + b)*Cos[e + f*x])/(2*(1 - Cos[e + f*x]^2)) + ((a + 3*b)*ArcTanh[Co
s[e + f*x]] - 2*b*Sec[e + f*x])/2)/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e}{2}\right.}{f}$
default	$\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e}{2}\right.}{f}$
norman	$\frac{\frac{a+b}{8f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8f}-\frac{(a+9b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{(a+3b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}$
parallelrisch	$\frac{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)(a+3b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(a+b)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^2-2a-18b}{8f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-8f}$
risch	$\frac{e^{i(fx+e)}\left(a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}+a+3b\right)}{f\left(e^{2i(fx+e)}+1\right)\left(e^{2i(fx+e)}-1\right)^2}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{2f}+\frac{3\ln\left(e^{i(fx+e)}-1\right)b}{2f}$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))+b*(-1/2/
sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(49) = 98.

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.34

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{2\,(a+3\,b)\cos\left(fx+e\right)^2-\left((a+3\,b)\cos\left(fx+e\right)^3-(a+3\,b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+(}{4\left(f\cos\left(fx+e\right)^3-f\cos\left(fx\right.\right.}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*(2*(a + 3*b)*cos(f*x + e)^2 - ((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*co
s(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 3*b)*cos(f*x + e)^3 - (a +
 3*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3
- f*cos(f*x + e))
```

Sympy [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)\csc^3\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=$$
$$-\frac{(a+3\,b)\log\left(\cos\left(fx+e\right)+1\right)-(a+3\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left((a+3\,b)\cos(fx+e)^2-2\,b\right)}{\cos(fx+e)^3-\cos(fx+e)}}{4\,f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/4*((a + 3*b)*log(cos(f*x + e) + 1) - (a + 3*b)*log(cos(f*x + e) - 1) -
2*((a + 3*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+3b)\log\left(|\cos\left(fx+e\right)+1|\right)}{4f} + \frac{(a+3b)\log\left(|\cos\left(fx+e\right)-1|\right)}{4f} + \frac{a\cos\left(fx+e\right)^2+3b\cos\left(fx+e\right)^2-2b}{2\left(\cos\left(fx+e\right)^3-\cos\left(fx+e\right)\right)f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/4*(a + 3*b)*log(abs(cos(f*x + e) + 1))/f + 1/4*(a + 3*b)*log(abs(cos(f*
x + e) - 1))/f + 1/2*(a*cos(f*x + e)^2 + 3*b*cos(f*x + e)^2 - 2*b)/((cos(f
*x + e)^3 - cos(f*x + e))*f)
```

Mupad [B] (verification not implemented)

Time = 12.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b-\cos\left(e+f\,x\right)^2\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f\,\left(\cos\left(e+f\,x\right)-\cos\left(e+f\,x\right)^3\right)} - \frac{\mathrm{atanh}(\cos\left(e+f\,x\right))\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^3,x)
```

output
```
(b - cos(e + f*x)^2*(a/2 + (3*b)/2))/(f*(cos(e + f*x) - cos(e + f*x)^3)) -
 (atanh(cos(e + f*x))*(a/2 + (3*b)/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.60

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{4\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^2 a+12\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^2 b-\cos}{8\cos(fx+e)}$$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

output
```
(4*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**2*a + 12*cos(e + f*x)*
log(tan((e + f*x)/2))*sin(e + f*x)**2*b - cos(e + f*x)*sin(e + f*x)**2*a -
 9*cos(e + f*x)*sin(e + f*x)**2*b + 4*sin(e + f*x)**2*a + 12*sin(e + f*x)*
*2*b - 4*a - 4*b)/(8*cos(e + f*x)*sin(e + f*x)**2*f)
```

3.7 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{3(a+5b)\text{arctanh}(\cos(e+fx))}{8f} - \frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{b\sec(e+fx)}{f}$$

output
```
-3/8*(a+5*b)*arctanh(cos(f*x+e))/f-1/8*(3*a+7*b)*cot(f*x+e)*csc(f*x+e)/f-1
/4*(a+b)*cot(f*x+e)*csc(f*x+e)^3/f+b*sec(f*x+e)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(81) = 162.

Time = 1.47 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.44

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{-2(3a+7b)\csc^2\left(\frac{1}{2}(e+fx)\right)-(a+b)\csc^4\left(\frac{1}{2}(e+fx)\right)+\frac{2\left(-3(a+13b)+4\cos(e+fx)\left(8b+3(a+5b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{}}{64f}$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-2*(3*a + 7*b)*Csc[(e + f*x)/2]^2 - (a + b)*Csc[(e + f*x)/2]^4 + (2*(-3*(
a + 13*b) + 4*Cos[e + f*x]*(8*b + 3*(a + 5*b)*Log[Cos[(e + f*x)/2]] - 3*(a
 + 5*b)*Log[Sin[(e + f*x)/2]]))*Sec[(e + f*x)/2]^2 - (a + b)*Sec[(e + f*x)
/2]^4 + (4*(a + 2*b) + (3*a + 7*b)*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Tan[(e
 + f*x)/2]^2)/(-1 + Tan[(e + f*x)/2]^2))/(64*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4621, 361, 25, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \text{ 4621}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)\sec^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 361}$$

$$-\frac{\frac{(a+b)\cos(e+fx)}{4\left(1-\cos^2(e+fx)\right)^2}-\frac{1}{4}\int -\frac{\left(3(a+b)\cos^2(e+fx)+4b\right)\sec^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{1}{4}\int \frac{\left(3(a+b)\cos^2(e+fx)+4b\right)\sec^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)+\frac{(a+b)\cos(e+fx)}{4\left(1-\cos^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{ 361}$$

$$-\frac{\frac{1}{4}\left(\frac{(3a+7b)\cos(e+fx)}{2(1-\cos^2(e+fx))}-\frac{1}{2}\int -\frac{((3a+7b)\cos^2(e+fx)+8b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)+\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{((3a+7b)\cos^2(e+fx)+8b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{(3a+7b)\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)+\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 359$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(3(a+5b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-8b\sec(e+fx)\right)+\frac{(3a+7b)\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)+\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}(3(a+5b)\text{arctanh}(\cos(e+fx))-8b\sec(e+fx))+\frac{(3a+7b)\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)+\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((((a + b)*Cos[e + f*x])/(4*(1 - Cos[e + f*x]^2)^2) + (((3*a + 7*b)*Cos[e
 + f*x])/(2*(1 - Cos[e + f*x]^2)) + (3*(a + 5*b)*ArcTanh[Cos[e + f*x]] - 8
*b*Sec[e + f*x])/2)/4)/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos(}\right.}{f}$
default	$\frac{a\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos(}\right.}{f}$
parallelrisch	$\frac{24\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)(a+5b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+(7a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(a-}{64f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-64f}$
norman	$\frac{\frac{a+b}{64f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{64f}+\frac{(7a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{64f}+\frac{(7a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{64f}-\frac{(a+10b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{3(a+5b)\ln\left(\tan\right.}{8f}$
risch	$\frac{e^{i(fx+e)}\left(3a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-8a\,e^{6i(fx+e)}-40b\,e^{6i(fx+e)}-22a\,e^{4i(fx+e)}+18b\,e^{4i(fx+e)}-8a\,e^{2i(fx+e)}-40b\,e\right.}{4f\left(e^{2i(fx+e)}-1\right)^4\left(e^{2i(fx+e)}+1\right)}$

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-co
t(f*x+e)))+b*(-1/4/sin(f*x+e)^4/cos(f*x+e)-5/8/sin(f*x+e)^2/cos(f*x+e)+15/
8/cos(f*x+e)+15/8*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(75) = 150$.

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.20

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{6\,(a+5\,b)\cos{(fx+e)}^4-10\,(a+5\,b)\cos{(fx+e)}^2-3\,\left((a+5\,b)\cos{(fx+e)}^5-2\,(a+5\,b)\cos{(fx+e}\right.}{}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/16*(6*(a + 5*b)*cos(f*x + e)^4 - 10*(a + 5*b)*cos(f*x + e)^2 - 3*((a + 5
*b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f*x + e)^3 + (a + 5*b)*cos(f*x + e))*
log(1/2*cos(f*x + e) + 1/2) + 3*((a + 5*b)*cos(f*x + e)^5 - 2*(a + 5*b)*co
s(f*x + e)^3 + (a + 5*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b
)/(f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

Sympy [F]

$$\int \csc^5(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \int \left(a+b\sec^2{(e+fx)}\right)\csc^5{(e+fx)}\, dx$$

input
```
integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \csc^5(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx =$$

$$-\frac{3\,(a+5\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+5\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+5\,b)\cos(fx+e)^4-5\,(a+5\,b)\cos(fx}{\cos(fx+e)^5-2\,\cos(fx+e)^3+\cos(fx}}{16\,f}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/16*(3*(a + 5*b)*log(cos(f*x + e) + 1) - 3*(a + 5*b)*log(cos(f*x + e) -
1) - 2*(3*(a + 5*b)*cos(f*x + e)^4 - 5*(a + 5*b)*cos(f*x + e)^2 + 8*b)/(co
s(f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= -\frac{3\,(a+5\,b)\log\left(\left|\cos\left(fx+e\right)+1\right|\right)}{16\,f} + \frac{3\,(a+5\,b)\log\left(\left|\cos\left(fx+e\right)-1\right|\right)}{16\,f} + \frac{b}{f\cos\left(fx+e\right)} + \frac{3\,a\cos\left(fx+e\right)^3+7\,b\cos\left(fx+e\right)^3-5\,a\cos\left(fx+e\right)-9\,b\cos\left(fx+e\right)}{8\left(\cos\left(fx+e\right)^2-1\right)^2 f}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-3/16*(a + 5*b)*log(abs(cos(f*x + e) + 1))/f + 3/16*(a + 5*b)*log(abs(cos(
f*x + e) - 1))/f + b/(f*cos(f*x + e)) + 1/8*(3*a*cos(f*x + e)^3 + 7*b*cos(
f*x + e)^3 - 5*a*cos(f*x + e) - 9*b*cos(f*x + e))/((cos(f*x + e)^2 - 1)^2*
f)
```

Mupad [B] (verification not implemented)

Time = 12.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{\left(\frac{3\,a}{8}+\frac{15\,b}{8}\right)\,\cos\left(e+f\,x\right)^4+\left(-\frac{5\,a}{8}-\frac{25\,b}{8}\right)\,\cos\left(e+f\,x\right)^2+b}{f\,\left(\cos\left(e+f\,x\right)^5-2\cos\left(e+f\,x\right)^3+\cos\left(e+f\,x\right)\right)} - \frac{\mathrm{atanh}(\cos\left(e+f\,x\right))\,\left(\frac{3\,a}{8}+\frac{15\,b}{8}\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^5,x)
```

output
```
(b + cos(e + f*x)^4*((3*a)/8 + (15*b)/8) - cos(e + f*x)^2*((5*a)/8 + (25*b
)/8))/(f*(cos(e + f*x) - 2*cos(e + f*x)^3 + cos(e + f*x)^5)) - (atanh(cos(
e + f*x))*((3*a)/8 + (15*b)/8))/f
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.98

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{3\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^4 a+15\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^4 b-\cos}{}$$

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

output
```
(3*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**4*a + 15*cos(e + f*x)*
log(tan((e + f*x)/2))*sin(e + f*x)**4*b - cos(e + f*x)*sin(e + f*x)**4*a -
 10*cos(e + f*x)*sin(e + f*x)**4*b + 3*sin(e + f*x)**4*a + 15*sin(e + f*x)
**4*b - sin(e + f*x)**2*a - 5*sin(e + f*x)**2*b - 2*a - 2*b)/(8*cos(e + f*
x)*sin(e + f*x)**4*f)
```

3.8 $\int (a+b\sec^2(e+fx))\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a+b\sec^2(e+fx))\sin^6(e+fx)\,dx=\frac{5}{16}(a-6b)x-\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{b\tan(e+fx)}{f}$$

output
```
5/16*(a-6*b)*x-1/16*(11*a-18*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(13*a-6*b)*co
s(f*x+e)^3*sin(f*x+e)/f-1/6*a*cos(f*x+e)^5*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int (a+b\sec^2(e+fx))\sin^6(e+fx)\,dx=\frac{60ae-360be+60afx-360bfx+(-45a+96b)\sin(2(e+fx))+(9a-6b)\sin(4(e+fx))-a\sin(6(e}{192f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

output
```
(60*a*e - 360*b*e + 60*a*f*x - 360*b*f*x + (-45*a + 96*b)*Sin[2*(e + f*x)]
 + (9*a - 6*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + 192*b*Tan[e + f*x])
/(192*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4620, 360, 25, 2345, 27, 1471, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 360$$

$$\frac{-\frac{1}{6}\int -\frac{6b\tan^6(e+fx)+6a\tan^4(e+fx)-6a\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{a\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{6}\int \frac{6b\tan^6(e+fx)+6a\tan^4(e+fx)-6a\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{a\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \ 2345$$

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{1}{4}\int\frac{3(-8b\tan^4(e+fx)-8(a-b)\tan^2(e+fx)+3a-2b)}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)\right)-\frac{a\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{3}{4}\int\frac{-8b\tan^4(e+fx)-8(a-b)\tan^2(e+fx)+3a-2b}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)\right)-\frac{a\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}}{f}$$

$\Big\downarrow$ 1471

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{3}{4}\left(\frac{(11a-18b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}-\frac{1}{2}\int\frac{16b\tan^2(e+fx)+5a-14b}{\tan^2(e+fx)+1}d\tan(e+fx)\right)\right)-\frac{a\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}}{f}$$

$\Big\downarrow$ 299

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{3}{4}\left(\frac{1}{2}\left(-5(a-6b)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-16b\tan(e+fx)\right)+\frac{(11a-18b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)\right)-}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{3}{4}\left(\frac{1}{2}(-5(a-6b)\arctan(\tan(e+fx))-16b\tan(e+fx))+\frac{(11a-18b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)\right)-\frac{a\tan}{6(\tan^2(e}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

output
```
(-1/6*(a*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^3 + (((13*a - 6*b)*Tan[e + f*x
])/(4*(1 + Tan[e + f*x]^2)^2) - (3*((-5*(a - 6*b)*ArcTan[Tan[e + f*x]] - 1
6*b*Tan[e + f*x])/2 + ((11*a - 18*b)*Tan[e + f*x])/(2*(1 + Tan[e + f*x]^2)
)))/4)/6)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{(-36a+90b)\sin(3fx+3e)+(8a-6b)\sin(5fx+5e)-\sin(7fx+7e)a+120fx(a-6b)\cos(fx+e)-45\left(a-\frac{32b}{3}\right)\sin(fx+e)}{384\cos(fx+e)f}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15}{}\right.\right.}{f}$
default	$\frac{a\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15}{}\right.\right.}{f}$
parts	$\frac{a\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)}{f}+\frac{b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\right.\right.}{f}$
risch	$\frac{5ax}{16}-\frac{15xb}{8}+\frac{15i e^{2i(fx+e)}a}{128f}-\frac{i e^{2i(fx+e)}b}{4f}-\frac{15i e^{-2i(fx+e)}a}{128f}+\frac{i e^{-2i(fx+e)}b}{4f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}-\frac{a\sin(6}{19}$
norman	$\left(-\frac{5a}{16}+\frac{15b}{8}\right)x+\left(-\frac{45a}{16}+\frac{135b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{25a}{16}+\frac{75b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{25a}{16}+\frac{75b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{5a}{16}-\right.$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
1/384*((-36*a+90*b)*sin(3*f*x+3*e)+(8*a-6*b)*sin(5*f*x+5*e)-sin(7*f*x+7*e)
*a+120*f*x*(a-6*b)*cos(f*x+e)-45*(a-32/3*b)*sin(f*x+e))/cos(f*x+e)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$=\frac{15\,(a-6\,b)fx\cos\left(fx+e\right)-\left(8\,a\cos\left(fx+e\right)^6-2\,(13\,a-6\,b)\cos\left(fx+e\right)^4+3\,(11\,a-18\,b)\cos\left(fx+\right.\right.}{48\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
1/48*(15*(a - 6*b)*f*x*cos(f*x + e) - (8*a*cos(f*x + e)^6 - 2*(13*a - 6*b)
*cos(f*x + e)^4 + 3*(11*a - 18*b)*cos(f*x + e)^2 - 48*b)*sin(f*x + e))/(f*
cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$=\frac{15\,(fx+e)(a-6\,b)+48\,b\tan\left(fx+e\right)-\frac{3\,(11\,a-18\,b)\tan(fx+e)^5+8\,(5\,a-12\,b)\tan(fx+e)^3+3\,(5\,a-14\,b)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (3*(11*a - 18*b)*tan(f*
x + e)^5 + 8*(5*a - 12*b)*tan(f*x + e)^3 + 3*(5*a - 14*b)*tan(f*x + e))/(t
an(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$= \frac{15\,(fx+e)(a-6\,b)+48\,b\tan\left(fx+e\right)-\frac{33\,a\tan(fx+e)^5-54\,b\tan(fx+e)^5+40\,a\tan(fx+e)^3-96\,b\tan(fx+e)^3+15\,a\tan(fx}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (33*a*tan(f*x + e)^5 -
54*b*tan(f*x + e)^5 + 40*a*tan(f*x + e)^3 - 96*b*tan(f*x + e)^3 + 15*a*tan
(f*x + e) - 42*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$= x\left(\frac{5\,a}{16}-\frac{15\,b}{8}\right)$$
$$-\frac{\left(\frac{11\,a}{16}-\frac{9\,b}{8}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a}{6}-2\,b\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a}{16}-\frac{7\,b}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$
$$+\frac{b\tan(e+f\,x)}{f}$$

input
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

output
```
x*((5*a)/16 - (15*b)/8) - (tan(e + f*x)^3*((5*a)/6 - 2*b) + tan(e + f*x)^5
*((11*a)/16 - (9*b)/8) + tan(e + f*x)*((5*a)/16 - (7*b)/8))/(f*(3*tan(e +
f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (b*tan(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.35

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$=\frac{-8\cos(fx+e)^2\sin(fx+e)^5a-10\cos(fx+e)^2\sin(fx+e)^3a-15\cos(fx+e)^2\sin(fx+e)a+15}{4}$$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x)
```

output
```
( - 8*cos(e + f*x)**2*sin(e + f*x)**5*a - 10*cos(e + f*x)**2*sin(e + f*x)*
*3*a - 15*cos(e + f*x)**2*sin(e + f*x)*a + 15*cos(e + f*x)*a*f*x - 90*cos(
e + f*x)*b*e - 90*cos(e + f*x)*b*f*x - 12*sin(e + f*x)**5*b - 30*sin(e + f
*x)**3*b + 90*sin(e + f*x)*b)/(48*cos(e + f*x)*f)
```

3.9 $\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx = \frac{3}{8}(a-4b)x - \frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{b\tan(e+fx)}{f}$$

output
```
3/8*(a-4*b)*x-1/8*(5*a-4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*sin
(f*x+e)/f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx = \frac{12(a-4b)(e+fx)-8(a-b)\sin(2(e+fx))+a\sin(4(e+fx))+32b\tan(e+fx)}{32f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

output
```
(12*(a - 4*b)*(e + f*x) - 8*(a - b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)]
+ 32*b*Tan[e + f*x])/(32*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4620, 360, 1471, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$\frac{\frac{a\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{1}{4}\int \frac{-4b\tan^4(e+fx)-4a\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b\tan^2(e+fx)+3a-4b}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{(5a-4b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{a\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3(a-4b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+8b\tan(e+fx)\right)-\frac{(5a-4b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{a\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(3(a-4b)\arctan(\tan(e+fx))+8b\tan(e+fx))-\frac{(5a-4b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{a\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

output
```
((a*Tan[e + f*x])/(4*(1 + Tan[e + f*x]^2)^2) + ((3*(a - 4*b)*ArcTan[Tan[e
+ f*x]] + 8*b*Tan[e + f*x])/2 - ((5*a - 4*b)*Tan[e + f*x])/(2*(1 + Tan[e +
 f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{(-7a+8b)\sin(3fx+3e)+\sin(5fx+5e)a+24fx(a-4b)\cos(fx+e)-8\sin(fx+e)(a-9b)}{64\cos(fx+e)f}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\right.}{f}$
default	$\frac{a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\right.}{f}$
parts	$\frac{a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+\frac{b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}\right.}{f}$
risch	$\frac{3ax}{8}-\frac{3xb}{2}+\frac{i e^{2i(fx+e)}a}{8f}-\frac{i e^{2i(fx+e)}b}{8f}-\frac{i e^{-2i(fx+e)}a}{8f}+\frac{i e^{-2i(fx+e)}b}{8f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\sin(4fx+4e)}{32f}$
norman	$\frac{\left(-\frac{3a}{8}+\frac{3b}{2}\right)x+\left(-\frac{9a}{8}+\frac{9b}{2}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{3a}{4}+3b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{3a}{4}-3b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{3a}{8}-\frac{3b}{2}\right)x\tan\left(\right.}{\left(\tan\right.}$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/64*((-7*a+8*b)*sin(3*f*x+3*e)+sin(5*f*x+5*e)*a+24*f*x*(a-4*b)*cos(f*x+e)
-8*sin(f*x+e)*(a-9*b))/cos(f*x+e)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx = \frac{3\,(a-4\,b)fx\cos(fx+e)+\left(2\,a\cos(fx+e)^4-(5\,a-4\,b)\cos(fx+e)^2+8\,b\right)\sin(fx+e)}{8\,f\cos(fx+e)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/8*(3*(a - 4*b)*f*x*cos(f*x + e) + (2*a*cos(f*x + e)^4 - (5*a - 4*b)*cos(
f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx = \int (a+b\sec^2(e+fx))\sin^4(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**4,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx = \frac{3\,(fx+e)(a-4\,b)+8\,b\tan\left(fx+e\right)-\frac{(5\,a-4\,b)\tan(fx+e)^3+(3\,a-4\,b)\tan(fx+e)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{8\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - ((5*a - 4*b)*tan(f*x + e)^
3 + (3*a - 4*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx = \frac{3\,(fx+e)(a-4\,b)+8\,b\tan\left(fx+e\right)-\frac{5\,a\tan(fx+e)^3-4\,b\tan(fx+e)^3+3\,a\tan(fx+e)-4\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - (5*a*tan(f*x + e)^3 - 4*b*
tan(f*x + e)^3 + 3*a*tan(f*x + e) - 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)
^2)/f
```

Mupad [B] (verification not implemented)

Time = 12.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int (a + b\sec^2(e + fx))\sin^4(e + fx)\,dx$$
$$= x\left(\frac{3a}{8} - \frac{3b}{2}\right) - \frac{\left(\frac{5a}{8} - \frac{b}{2}\right)\tan(e + f\,x)^3 + \left(\frac{3a}{8} - \frac{b}{2}\right)\tan(e + f\,x)}{f\,\left(\tan(e + f\,x)^4 + 2\tan(e + f\,x)^2 + 1\right)} + \frac{b\tan(e + f\,x)}{f}$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

output
```
x*((3*a)/8 - (3*b)/2) - (tan(e + f*x)^3*((5*a)/8 - b/2) + tan(e + f*x)*((3
*a)/8 - b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b*tan(e + f*x
))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.46

$$\int (a + b\sec^2(e + fx))\sin^4(e + fx)\,dx$$
$$= \frac{-2\cos(fx + e)^2\sin(fx + e)^3 a - 3\cos(fx + e)^2\sin(fx + e)\,a + 3\cos(fx + e)\,afx - 12\cos(fx + e)\,b}{8\cos(fx + e)\,f}$$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x)
```

output
```
( - 2*cos(e + f*x)**2*sin(e + f*x)**3*a - 3*cos(e + f*x)**2*sin(e + f*x)*a
 + 3*cos(e + f*x)*a*f*x - 12*cos(e + f*x)*b*e - 12*cos(e + f*x)*b*f*x - 4*
sin(e + f*x)**3*b + 12*sin(e + f*x)*b)/(8*cos(e + f*x)*f)
```

3.10 $\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx=\frac{1}{2}(a-2b)x-\frac{a\cos(e+fx)\sin(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

output
```
1/2*(a-2*b)*x-1/2*a*cos(f*x+e)*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx=\frac{a(e+fx)}{2f}-\frac{b\arctan(\tan(e+fx))}{f}-\frac{a\sin(2(e+fx))}{4f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

output
```
(a*(e + f*x))/(2*f) - (b*ArcTan[Tan[e + f*x]])/f - (a*Sin[2*(e + f*x)])/(4
*f) + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4620, 360, 25, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^2 \left(a+b\sec(e+fx)^2\right) dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$\frac{-\frac{1}{2}\int -\frac{2b\tan^2(e+fx)+a}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{a\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{2b\tan^2(e+fx)+a}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{a\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{2}\left((a-2b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+2b\tan(e+fx)\right)-\frac{a\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}((a-2b)\arctan(\tan(e+fx))+2b\tan(e+fx))-\frac{a\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

output
```
(((a - 2*b)*ArcTan[Tan[e + f*x]] + 2*b*Tan[e + f*x])/2 - (a*Tan[e + f*x])/
(2*(1 + Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(\tan(fx+e)-fx-e)}{f}$
default	$\frac{a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(\tan(fx+e)-fx-e)}{f}$
parts	$\frac{a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b(\tan(fx+e)-fx-e)}{f}$
parallelrisch	$\frac{-\sin(3fx+3e)a+4fx(a-2b)\cos(fx+e)-\sin(fx+e)(a-8b)}{8\cos(fx+e)f}$
risch	$\frac{ax}{2}-xb+\frac{i\mathrm{e}^{2i(fx+e)}a}{8f}-\frac{i\mathrm{e}^{-2i(fx+e)}a}{8f}+\frac{2ib}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}$
norman	$\frac{\left(-\frac{a}{2}+b\right)x+\frac{(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\left(-\frac{a}{2}+b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(\frac{a}{2}-b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{a}{2}-b\right)x\tan(}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+b*(tan(f*x+e)-f*x-e))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx = \frac{(a-2\,b)fx\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)^2-2\,b\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/2*((a - 2*b)*f*x*cos(f*x + e) - (a*cos(f*x + e)^2 - 2*b)*sin(f*x + e))/(
f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a + b\sec^2(e + fx)\right)\sin^2(e + fx)\,dx = \int \left(a + b\sec^2{(e + fx)}\right)\sin^2{(e + fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx = \frac{(fx+e)(a-2\,b)+2\,b\tan\left(fx+e\right)-\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)
^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx = \frac{(fx+e)(a-2\,b)+2\,b\tan(fx+e)-\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)
^2 + 1))/f
```

Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx = \frac{b\tan(e+f\,x)-\frac{a\,\sin(2\,e+2\,f\,x)}{4}+f\,x\,\left(\frac{a}{2}-b\right)}{f}$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

output
```
(b*tan(e + f*x) - (a*sin(2*e + 2*f*x))/4 + f*x*(a/2 - b))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int (a+b\sec^2(e+fx))\sin^2(e+fx)\,dx$$
$$= \frac{-\cos(fx+e)^2\sin(fx+e)\,a+\cos(fx+e)\,afx-2\cos(fx+e)\,bfx+2\sin(fx+e)\,b}{2\cos(fx+e)\,f}$$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x)
```

output
```
( - cos(e + f*x)**2*sin(e + f*x)*a + cos(e + f*x)*a*f*x - 2*cos(e + f*x)*b
*f*x + 2*sin(e + f*x)*b)/(2*cos(e + f*x)*f)
```

3.11 $\int (a+b\sec^2(e+fx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int (a+b\sec^2(e+fx))\,dx = ax+\frac{b\tan(e+fx)}{f}$$

output
```
a*x+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a+b\sec^2(e+fx))\,dx = ax+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b\tan(e+fx)}{f}$$

input
```
Int[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\tan(fx+e)}{f}$	16
parts	$ax+\frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
parallelrisch	$\frac{b\sin(fx+e)}{\cos(fx+e)f}+ax$	24
risch	$ax+\frac{2ib}{f(e^{2i(fx+e)}+1)}$	25
norman	$\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-ax-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}$	51

input
```
int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*tan(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

output
```
(a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\,dx$$

input
```
integrate(a+b*sec(f*x+e)**2,x)
```

output
```
Integral(a + b*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan{(fx+e)}}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

output
```
a*x + b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan{(fx+e)}}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

output
```
a*x + b*tan(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(a+b\sec^2(e+fx)\right) \, dx = \frac{b\tan(e+f\,x)+a\,f\,x}{f}$$

input
```
int(a + b/cos(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x) + a*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(a+b\sec^2(e+fx)\right) \, dx = \frac{\cos\left(fx+e\right)afx+\sin\left(fx+e\right)b}{\cos\left(fx+e\right)f}$$

input
```
int(a+b*sec(f*x+e)^2,x)
```

output
```
(cos(e + f*x)*a*f*x + sin(e + f*x)*b)/(cos(e + f*x)*f)
```

3.12 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

output
```
-(a+b)*cot(f*x+e)/f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{a\cot(e+fx)}{f}-\frac{b\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((a*Cot[e + f*x])/f) - (b*Cot[e + f*x])/f + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4620, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 244$$

$$\frac{\int \left((a+b)\cot^2(e+fx)+b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 2009$$

$$\frac{b\tan(e+fx)-(a+b)\cot(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a + b)*Cot[e + f*x]) + b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
derivativedivides	$\frac{-\cot(fx+e)a+b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	43
default	$\frac{-\cot(fx+e)a+b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	43
risch	$-\frac{2i\left(a\,e^{2i(fx+e)}+a+2b\right)}{f\left(e^{2i(fx+e)}+1\right)\left(e^{2i(fx+e)}-1\right)}$	49
parallelrisch	$\frac{\left((a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+(-2a-6b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+a+b\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-2f}$	68
norman	$\frac{\frac{a+b}{2f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{2f}-\frac{(a+3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$	77

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-cot(f*x+e)*a+b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+2b)\cos{(fx+e)}^2-b}{f\cos{(fx+e)}\sin{(fx+e)}}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-((a + 2*b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2{(e+fx)}\right)\csc^2{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\tan{(fx+e)}-\frac{a+b}{\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
(b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan\left(fx+e\right)-\frac{a+b}{\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)}{f}-\frac{a+b}{f\tan\left(e+f\,x\right)}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x))/f - (a + b)/(f*tan(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\sin\left(fx+e\right)^2a+2\sin\left(fx+e\right)^2b-a-b}{\cos\left(fx+e\right)\sin\left(fx+e\right)f}$$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
```

output
```
(sin(e + f*x)**2*a + 2*sin(e + f*x)**2*b - a - b)/(cos(e + f*x)*sin(e + f*
x)*f)
```

3.13 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+2b)\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

output
```
-(a+2*b)*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{2a\cot(e+fx)}{3f}-\frac{5b\cot(e+fx)}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{b\cot(e+fx)\csc^2(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-2*a*Cot[e + f*x])/(3*f) - (5*b*Cot[e + f*x])/(3*f) - (a*Cot[e + f*x]*Csc
[e + f*x]^2)/(3*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) + (b*Tan[e + f*
x])/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4620, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 355}$$

$$\frac{\int \left((a+b)\cot^4(e+fx)+(a+2b)\cot^2(e+fx)+b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{3}(a+b)\cot^3(e+fx)-(a+2b)\cot(e+fx)+b\tan(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a + 2*b)*Cot[e + f*x]) - ((a + b)*Cot[e + f*x]^3)/3 + b*Tan[e + f*x])/
f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}-\frac{\csc(fx+e)^{2}}{3}\right)\cot(fx+e)+b\left(-\frac{1}{3\sin(fx+e)^{3}\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)}{f}$
default	$\frac{a\left(-\frac{2}{3}-\frac{\csc(fx+e)^{2}}{3}\right)\cot(fx+e)+b\left(-\frac{1}{3\sin(fx+e)^{3}\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)}{f}$
risch	$\frac{4i\left(3a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+8b\,e^{2i(fx+e)}-a-4b\right)}{3f\left(e^{2i(fx+e)}-1\right)^{3}\left(e^{2i(fx+e)}+1\right)}$
parallelrisch	$\frac{\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left((a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+(8a+20b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+(-18a-90b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+(8a+20b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+\right.}{24f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-24f}$
norman	$\frac{\frac{a+b}{24f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{24f}-\frac{3(a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{4f}+\frac{(2a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{6f}+\frac{(2a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{6f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+
4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2\,(a+4\,b)\cos\left(fx+e\right)^4-3\,(a+4\,b)\cos\left(fx+e\right)^2+3\,b}{3\,\left(f\cos\left(fx+e\right)^3-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a + 4*b)*cos(f*x + e)^4 - 3*(a + 4*b)*cos(f*x + e)^2 + 3*b)/((f*c
os(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\csc^4\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{3\,b\tan\left(fx+e\right)-\frac{3\,(a+2\,b)\tan(fx+e)^2+a+b}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*b*tan(f*x + e) - (3*(a + 2*b)*tan(f*x + e)^2 + a + b)/tan(f*x + e)^
3)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{3\,b\tan\left(fx+e\right)-\frac{3\,a\tan(fx+e)^2+6\,b\tan(fx+e)^2+a+b}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 + a + b)/
tan(f*x + e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 12.60 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\tan(e+f\,x)}{f}-\frac{(a+2\,b)\,\tan(e+f\,x)^2+\frac{a}{3}+\frac{b}{3}}{f\tan\left(e+f\,x\right)^3}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^4,x)
```

output
```
(b*tan(e + f*x))/f - (a/3 + b/3 + tan(e + f*x)^2*(a + 2*b))/(f*tan(e + f*x
)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{2\sin(fx+e)^4a+8\sin(fx+e)^4b-\sin(fx+e)^2a-4\sin(fx+e)^2b-a-b}{3\cos(fx+e)\sin(fx+e)^3f}$$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

output
```
(2*sin(e + f*x)**4*a + 8*sin(e + f*x)**4*b - sin(e + f*x)**2*a - 4*sin(e +
 f*x)**2*b - a - b)/(3*cos(e + f*x)*sin(e + f*x)**3*f)
```

3.14 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(a+3b)\cot(e+fx)}{f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

output
```
-(a+3*b)*cot(f*x+e)/f-1/3*(2*a+3*b)*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/
f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.88

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{8a\cot(e+fx)}{15f}-\frac{11b\cot(e+fx)}{5f}-\frac{4a\cot(e+fx)\csc^2(e+fx)}{15f}-\frac{3b\cot(e+fx)\csc^2(e+fx)}{5f}-\frac{a\cot(e+fx)\csc^4(e+fx)}{5f}-\frac{b\cot(e+fx)\csc^4(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-8*a*Cot[e + f*x])/(15*f) - (11*b*Cot[e + f*x])/(5*f) - (4*a*Cot[e + f*x]
*Csc[e + f*x]^2)/(15*f) - (3*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*f) - (a*Cot
[e + f*x]*Csc[e + f*x]^4)/(5*f) - (b*Cot[e + f*x]*Csc[e + f*x]^4)/(5*f) +
(b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4620, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left((a+b)\cot^6(e+fx)+(2a+3b)\cot^4(e+fx)+(a+3b)\cot^2(e+fx)+b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{5}(a+b)\cot^5(e+fx)-\frac{1}{3}(2a+3b)\cot^3(e+fx)-(a+3b)\cot(e+fx)+b\tan(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a + 3*b)*Cot[e + f*x]) - ((2*a + 3*b)*Cot[e + f*x]^3)/3 - ((a + b)*Cot
[e + f*x]^5)/5 + b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31

method	result
parallelrisch	$-\frac{\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\left((a+6b)\cos(2fx+2e)+\frac{4(-a-6b)\cos(4fx+4e)}{5}+\frac{(a+6b)\cos(6fx+6e)}{5}+2a\right)}{384f\cos(fx+e)}$
risch	$-\frac{16i\left(10a\,e^{6i(fx+e)}+5a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-4a\,e^{2i(fx+e)}-24b\,e^{2i(fx+e)}+a+6b\right)}{15f\left(e^{2i(fx+e)}-1\right)^{5}\left(e^{2i(fx+e)}+1\right)}$
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\csc(fx+e)^{4}}{5}-\frac{4\csc(fx+e)^{2}}{15}\right)\cot(fx+e)+b\left(-\frac{1}{5\sin(fx+e)^{5}\cos(fx+e)}-\frac{2}{5\sin(fx+e)^{3}\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(fx}\right.}{f}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\csc(fx+e)^{4}}{5}-\frac{4\csc(fx+e)^{2}}{15}\right)\cot(fx+e)+b\left(-\frac{1}{5\sin(fx+e)^{5}\cos(fx+e)}-\frac{2}{5\sin(fx+e)^{3}\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(fx}\right.}{f}$
norman	$\frac{\frac{a+b}{160f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{160f}-\frac{5(a+7b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{8f}+\frac{5(5a+21b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{96f}+\frac{5(5a+21b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{96f}+\frac{(11a+21b)\tan\left(\frac{fx}{2}\right.}{240f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*sec(1/2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)^5*((a+6*b)*cos(2*f*x+2*e)+4
/5*(-a-6*b)*cos(4*f*x+4*e)+1/5*(a+6*b)*cos(6*f*x+6*e)+2*a)/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.34

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{8\,(a+6\,b)\cos\left(fx+e\right)^{6}-20\,(a+6\,b)\cos\left(fx+e\right)^{4}+15\,(a+6\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\left(f\cos\left(fx+e\right)^{5}-2\,f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/15*(8*(a + 6*b)*cos(f*x + e)^6 - 20*(a + 6*b)*cos(f*x + e)^4 + 15*(a +
6*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*co
s(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{15\,b\tan\left(fx+e\right)-\frac{15\,(a+3\,b)\tan(fx+e)^4+5\,(2\,a+3\,b)\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(15*b*tan(f*x + e) - (15*(a + 3*b)*tan(f*x + e)^4 + 5*(2*a + 3*b)*tan
(f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{15\,b\tan\left(fx+e\right)-\frac{15\,a\tan(fx+e)^4+45\,b\tan(fx+e)^4+10\,a\tan(fx+e)^2+15\,b\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 45*b*tan(f*x + e)^4 + 10*
a*tan(f*x + e)^2 + 15*b*tan(f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 12.72 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)}{f}-\frac{(a+3\,b)\,\tan(e+f\,x)^4+\left(\frac{2\,a}{3}+b\right)\,\tan(e+f\,x)^2+\frac{a}{5}+\frac{b}{5}}{f\tan\left(e+f\,x\right)^5}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^6,x)
```

output
```
(b*tan(e + f*x))/f - (a/5 + b/5 + tan(e + f*x)^2*((2*a)/3 + b) + tan(e + f
*x)^4*(a + 3*b))/(f*tan(e + f*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{8\sin\left(fx+e\right)^6a+48\sin\left(fx+e\right)^6b-4\sin\left(fx+e\right)^4a-24\sin\left(fx+e\right)^4b-\sin\left(fx+e\right)^2a-6\sin\left(fx\right.}{15\cos\left(fx+e\right)\sin\left(fx+e\right)^5f}$$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

output
```
(8*sin(e + f*x)**6*a + 48*sin(e + f*x)**6*b - 4*sin(e + f*x)**4*a - 24*sin
(e + f*x)**4*b - sin(e + f*x)**2*a - 6*sin(e + f*x)**2*b - 3*a - 3*b)/(15*
cos(e + f*x)*sin(e + f*x)**5*f)
```

3.15 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx = -\frac{(a^2-4ab+b^2)\cos(e+fx)}{f} + \frac{2a(a-b)\cos^3(e+fx)}{3f} - \frac{a^2\cos^5(e+fx)}{5f} + \frac{2(a-b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-(a^2-4*a*b+b^2)*cos(f*x+e)/f+2/3*a*(a-b)*cos(f*x+e)^3/f-1/5*a^2*cos(f*x+e
)^5/f+2*(a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx = -\frac{(425a^2-4400ab+2000b^2+24(22a^2-215ab+120b^2)\cos(2(e+fx))+12(7a^2-60ab+20b^2)\cos(4}{1920f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

output
```
-1/1920*((425*a^2 - 4400*a*b + 2000*b^2 + 24*(22*a^2 - 215*a*b + 120*b^2)*
Cos[2*(e + f*x)] + 12*(7*a^2 - 60*a*b + 20*b^2)*Cos[4*(e + f*x)] - 16*a^2*
Cos[6*(e + f*x)] + 40*a*b*Cos[6*(e + f*x)] + 3*a^2*Cos[8*(e + f*x)])*Sec[e
 + f*x]^3)/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^5 \left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2 \left(a\cos^2(e+fx)+b\right)^2 \sec^4(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(a^2\cos^4(e+fx)-2a(a-b)\cos^2(e+fx)+b^2\sec^4(e+fx)+2(a-b)b\sec^2(e+fx)+a^2\left(\frac{b(b-4a)}{a^2}+1\right)\right) d c}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a^2-4ab+b^2\right)\cos(e+fx)+\frac{1}{5}a^2\cos^5(e+fx)-\frac{2}{3}a(a-b)\cos^3(e+fx)-2b(a-b)\sec(e+fx)-\frac{1}{3}b^2\sec^3(e}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

output
```
-(((a^2 - 4*a*b + b^2)*Cos[e + f*x] - (2*a*(a - b)*Cos[e + f*x]^3)/3 + (a^
2*Cos[e + f*x]^5)/5 - 2*(a - b)*b*Sec[e + f*x] - (b^2*Sec[e + f*x]^3)/3)/f
)
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)+b^2\left(\frac{s}{3}\right.}{f}$
default	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)+b^2\left(\frac{s}{3}\right.}{f}$
parts	$-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5f}+\frac{b^2\left(\frac{\sin(fx+e)^6}{3\cos(fx+e)^3}-\frac{\sin(fx+e)^6}{\cos(fx+e)}-\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\right.}{f}$
parallelrisch	$\frac{(-256a^2+2560ab-1280b^2)\cos(3fx+3e)+(-528a^2+5160ab-2880b^2)\cos(2fx+2e)+(-84a^2+720ab-240b^2)\cos(4fx}{480f(\cos(3fx+3}$
norman	$\frac{\frac{16a^2-160ab+80b^2}{15f}+\frac{16(5a^2-2ab-7b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{3f}+\frac{2(16a^2-160ab+80b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15f}-\frac{2(16a^2-160ab+80b^2)\tan\left(\frac{fx}{2}+\right.}{15f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)}$
risch	$-\frac{a^2e^{5i(fx+e)}}{160f}+\frac{5e^{3i(fx+e)}a^2}{96f}-\frac{e^{3i(fx+e)}ab}{12f}-\frac{5e^{i(fx+e)}a^2}{16f}+\frac{7e^{i(fx+e)}ab}{4f}-\frac{e^{i(fx+e)}b^2}{2f}-\frac{5e^{-i(fx+e)}a^2}{16f}$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*
x+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3
*sin(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f*x+e)^4+4/3*s
in(f*x+e)^2)*cos(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int (a+b\sec^2(e+fx))^2\sin^5(e+fx)\,dx =$$
$$-\frac{3a^2\cos(fx+e)^8-10(a^2-ab)\cos(fx+e)^6+15(a^2-4ab+b^2)\cos(fx+e)^4-30(ab-b^2)\cos(f}{15f\cos(fx+e)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/15*(3*a^2*cos(f*x + e)^8 - 10*(a^2 - a*b)*cos(f*x + e)^6 + 15*(a^2 - 4*
a*b + b^2)*cos(f*x + e)^4 - 30*(a*b - b^2)*cos(f*x + e)^2 - 5*b^2)/(f*cos(
f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx=$$
$$-\frac{3\,a^2\cos\left(fx+e\right)^5-10\left(a^2-ab\right)\cos\left(fx+e\right)^3+15\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)-\frac{5\left(6\left(ab-b^2\right)\cos(fx+e)^2\right.}{\cos(fx+e)^3}}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/15*(3*a^2*cos(f*x + e)^5 - 10*(a^2 - a*b)*cos(f*x + e)^3 + 15*(a^2 - 4*
a*b + b^2)*cos(f*x + e) - 5*(6*(a*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x +
 e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx = -\frac{3\,a^2\cos\left(fx+e\right)^5-10\,a^2\cos\left(fx+e\right)^3+10\,ab\cos\left(fx+e\right)^3+15\,a^2\cos\left(fx+e\right)-60\,ab\cos\left(fx+e\right)}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
-1/15*(3*a^2*cos(f*x + e)^5 - 10*a^2*cos(f*x + e)^3 + 10*a*b*cos(f*x + e)^
3 + 15*a^2*cos(f*x + e) - 60*a*b*cos(f*x + e) + 15*b^2*cos(f*x + e) - 5*(6
*a*b*cos(f*x + e)^2 - 6*b^2*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx = \frac{\frac{\frac{b^2}{3}+\cos(e+f\,x)^2\,\left(2\,a\,b-2\,b^2\right)}{\cos(e+f\,x)^3}-\cos\left(e+f\,x\right)\,\left(a^2-4\,a\,b+b^2\right)-\frac{a^2\cos(e+f\,x)^5}{5}+\frac{2\,a\cos(e+f\,x)^3\,(a-b)}{3}}{f}$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2/3 + cos(e + f*x)^2*(2*a*b - 2*b^2))/cos(e + f*x)^3 - cos(e + f*x)*(a
^2 - 4*a*b + b^2) - (a^2*cos(e + f*x)^5)/5 + (2*a*cos(e + f*x)^3*(a - b))/
3)/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.69

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^5(e+fx)\,dx$$
$$= \frac{-3\cos\left(fx+e\right)^2\sin\left(fx+e\right)^6a^2-\cos\left(fx+e\right)^2\sin\left(fx+e\right)^4a^2-4\cos\left(fx+e\right)^2\sin\left(fx+e\right)^2a^2+8\,\mathrm{c}}{}$$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x)
```

output
```
( - 3*cos(e + f*x)**2*sin(e + f*x)**6*a**2 - cos(e + f*x)**2*sin(e + f*x)*
*4*a**2 - 4*cos(e + f*x)**2*sin(e + f*x)**2*a**2 + 8*cos(e + f*x)**2*a**2
+ 8*cos(e + f*x)*sin(e + f*x)**2*a**2 - 80*cos(e + f*x)*sin(e + f*x)**2*a*
b + 40*cos(e + f*x)*sin(e + f*x)**2*b**2 - 8*cos(e + f*x)*a**2 + 80*cos(e
+ f*x)*a*b - 40*cos(e + f*x)*b**2 - 10*sin(e + f*x)**6*a*b - 30*sin(e + f*
x)**4*a*b + 15*sin(e + f*x)**4*b**2 + 120*sin(e + f*x)**2*a*b - 60*sin(e +
 f*x)**2*b**2 - 80*a*b + 40*b**2)/(15*cos(e + f*x)*f*(sin(e + f*x)**2 - 1)
)
```

3.16 $\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx = -\frac{a(a-2b)\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{3f}+\frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-a*(a-2*b)*cos(f*x+e)/f+1/3*a^2*cos(f*x+e)^3/f+(2*a-b)*b*sec(f*x+e)/f+1/3*
b^2*sec(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx = \frac{(-26a^2+168ab-16b^2-3(11a^2-64ab+16b^2)\cos(2(e+fx))-6a(a-4b)\cos(4(e+fx))+a^2\cos(6$$

$$96f$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

output
```
((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cos[2*(e + f*x
)] - 6*a*(a - 4*b)*Cos[4*(e + f*x)] + a^2*Cos[6*(e + f*x)])*Sec[e + f*x]^3
)/(96*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3 \left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right) \left(a\cos^2(e+fx)+b\right)^2 \sec^4(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left(b^2\sec^4(e+fx)+(2a-b)b\sec^2(e+fx)-a^2\cos^2(e+fx)+a(a-2b)\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}a^2\cos^3(e+fx)+a(a-2b)\cos(e+fx)-b(2a-b)\sec(e+fx)-\frac{1}{3}b^2\sec^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

output
```
-((a*(a - 2*b)*Cos[e + f*x] - (a^2*Cos[e + f*x]^3)/3 - (2*a - b)*b*Sec[e +
 f*x] - (b^2*Sec[e + f*x]^3)/3)/f)
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{b^2 \sec(fx+e)^3}{3}+2\sec(fx+e)ab-\sec(fx+e)b^2+\frac{a^2}{3\sec(fx+e)^3}-\frac{a(a-2b)}{\sec(fx+e)}}{f}$
default	$\frac{\frac{b^2 \sec(fx+e)^3}{3}+2\sec(fx+e)ab-\sec(fx+e)b^2+\frac{a^2}{3\sec(fx+e)^3}-\frac{a(a-2b)}{\sec(fx+e)}}{f}$
parts	$-\frac{a^2\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3f}+\frac{b^2\left(\frac{\sec(fx+e)^3}{3}-\sec(fx+e)\right)}{f}+\frac{2ab\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos(fx+e\right.}{f}$
norman	$\frac{\frac{4a^2-24ab+4b^2}{3f}+\frac{32\left(a^2-b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3f}-\frac{\left(4a^2+8ab+4b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{f}-\frac{\left(8a^2-16ab+8b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}$
parallelrisch	$\frac{\left(-16a^2+96ab-16b^2\right)\cos(3fx+3e)+\left(-33a^2+192ab-48b^2\right)\cos(2fx+2e)-6a(a-4b)\cos(4fx+4e)+a^2\cos(6fx+6e)+}{24f(\cos(3fx+3e)+3\cos(fx+e))}$
risch	$\frac{e^{3i(fx+e)}a^2}{24f}-\frac{3\,e^{i(fx+e)}a^2}{8f}+\frac{e^{i(fx+e)}ab}{f}-\frac{3\,e^{-i(fx+e)}a^2}{8f}+\frac{e^{-i(fx+e)}ab}{f}+\frac{e^{-3i(fx+e)}a^2}{24f}-\frac{2b\,e^{i(fx+e)}(-}{}$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*b^2*sec(f*x+e)^3+2*sec(f*x+e)*a*b-sec(f*x+e)*b^2+1/3*a^2/sec(f*x+
e)^3-a*(a-2*b)/sec(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = \frac{a^2\cos\left(fx+e\right)^6-3\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4+3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/3*(a^2*cos(f*x + e)^6 - 3*(a^2 - 2*a*b)*cos(f*x + e)^4 + 3*(2*a*b - b^2)
*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\sin^3\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = \frac{a^2\cos(fx+e)^3-3\,(a^2-2\,ab)\cos(fx+e)+\frac{3\,(2\,ab-b^2)\cos(fx+e)^2+b^2}{\cos(fx+e)^3}}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/3*(a^2*cos(f*x + e)^3 - 3*(a^2 - 2*a*b)*cos(f*x + e) + (3*(2*a*b - b^2)*
cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = \frac{a^2\cos(fx+e)^3-3\,a^2\cos(fx+e)+6\,ab\cos(fx+e)+\frac{6\,ab\cos(fx+e)^2-3\,b^2\cos(fx+e)^2+b^2}{\cos(fx+e)^3}}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
1/3*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e) + 6*a*b*cos(f*x + e) + (6*a*b
*cos(f*x + e)^2 - 3*b^2*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = \frac{\frac{\frac{b^2}{3}+\cos(e+f\,x)^2\,\left(2\,a\,b-b^2\right)}{\cos(e+f\,x)^3}+\frac{a^2\cos(e+f\,x)^3}{3}-a\,\cos\left(e+f\,x\right)\,\left(a-2\,b\right)}{f}$$

input
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2/3 + cos(e + f*x)^2*(2*a*b - b^2))/cos(e + f*x)^3 + (a^2*cos(e + f*x)
^3)/3 - a*cos(e + f*x)*(a - 2*b))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 474, normalized size of antiderivative = 6.58

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = \frac{-\cos\left(fx+e\right)^2\sin\left(fx+e\right)^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4a^2+\cos\left(fx+e\right)^2\sin\left(fx+e\right)^4a^2-\cos\left(fx+e\right)^2\sin\left(fx+\right.}{}$$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x)
```

output

```
( - cos(e + f*x)**2*sin(e + f*x)**4*tan((e + f*x)/2)**4*a**2 + cos(e + f*x
)**2*sin(e + f*x)**4*a**2 - cos(e + f*x)**2*sin(e + f*x)**2*tan((e + f*x)/
2)**4*a**2 + cos(e + f*x)**2*sin(e + f*x)**2*a**2 + 2*cos(e + f*x)**2*tan(
(e + f*x)/2)**4*a**2 - 2*cos(e + f*x)**2*a**2 + 2*cos(e + f*x)*sin(e + f*x
)**2*tan((e + f*x)/2)**4*a**2 - 24*cos(e + f*x)*sin(e + f*x)**2*tan((e + f
*x)/2)**4*a*b + 2*cos(e + f*x)*sin(e + f*x)**2*tan((e + f*x)/2)**4*b**2 -
2*cos(e + f*x)*sin(e + f*x)**2*a**2 - 2*cos(e + f*x)*sin(e + f*x)**2*b**2
- 2*cos(e + f*x)*tan((e + f*x)/2)**4*a**2 + 24*cos(e + f*x)*tan((e + f*x)/
2)**4*a*b - 2*cos(e + f*x)*tan((e + f*x)/2)**4*b**2 + 2*cos(e + f*x)*a**2
+ 2*cos(e + f*x)*b**2 - 3*sin(e + f*x)**2*tan((e + f*x)/2)**4*b**2 + 3*sin
(e + f*x)**2*b**2 + 2*tan((e + f*x)/2)**4*b**2 - 2*b**2)/(3*cos(e + f*x)*f
*(sin(e + f*x)**2*tan((e + f*x)/2)**4 - sin(e + f*x)**2 - tan((e + f*x)/2)
**4 + 1))
```

3.17 $\int (a+b\sec^2(e+fx))^2\sin(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=-\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-a^2*cos(f*x+e)/f+2*a*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=\frac{4(b+a\cos^2(e+fx))^2\left(b^2+6ab\cos^2(e+fx)-3a^2\cos^4(e+fx)\right)\sec^3(e+fx)}{3f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

output
```
(4*(b + a*Cos[e + f*x]^2)^2*(b^2 + 6*a*b*Cos[e + f*x]^2 - 3*a^2*Cos[e + f*
x]^4)*Sec[e + f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4621, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(b^2\sec^4(e+fx)+2ab\sec^2(e+fx)+a^2\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\cos(e+fx)-2ab\sec(e+fx)-\frac{1}{3}b^2\sec^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

output
```
-((a^2*Cos[e + f*x] - 2*a*b*Sec[e + f*x] - (b^2*Sec[e + f*x]^3)/3)/f)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{b^2 \sec(fx+e)^3}{3}+2\sec(fx+e)ab-\frac{a^2}{\sec(fx+e)}}{f}$
default	$\frac{\frac{b^2 \sec(fx+e)^3}{3}+2\sec(fx+e)ab-\frac{a^2}{\sec(fx+e)}}{f}$
parts	$-\frac{a^2\cos(fx+e)}{f}+\frac{2ab\sec(fx+e)}{f}+\frac{b^2\sec(fx+e)^3}{3f}$
risch	$-\frac{e^{i(fx+e)}a^2}{2f}-\frac{e^{-i(fx+e)}a^2}{2f}+\frac{4b\left(3a\,e^{5i(fx+e)}+6a\,e^{3i(fx+e)}+2b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)}\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
parallelrisch	$\frac{2(-3a^2+6ab+b^2)\cos(3fx+3e)-12a(a-2b)\cos(2fx+2e)-3a^2\cos(4fx+4e)+6(-3a^2+6ab+b^2)\cos(fx+e)-9a^2+24a}{6f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{\frac{6a^2-12ab-2b^2}{3f}+\frac{2(3a^2+2ab-b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}-\frac{2(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{f}-\frac{2(9a^2-6ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)}$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*b^2*sec(f*x+e)^3+2*sec(f*x+e)*a*b-a^2/sec(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=-\frac{3\,a^2\cos\left(fx+e\right)^4-6\,ab\cos\left(fx+e\right)^2-b^2}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*cos(f*x + e)^4 - 6*a*b*cos(f*x + e)^2 - b^2)/(f*cos(f*x + e)^3
)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sin\left(e+fx\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=-\frac{3\,a^2\cos\left(fx+e\right)-\frac{6\,ab}{\cos(fx+e)}-\frac{b^2}{\cos(fx+e)^3}}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="maxima")
```

output
```
-1/3*(3*a^2*cos(f*x + e) - 6*a*b/cos(f*x + e) - b^2/cos(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=-\frac{3\,a^2\cos\left(fx+e\right)-\frac{6\,ab\cos(fx+e)^2+b^2}{\cos(fx+e)^3}}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="giac")
```

output
```
-1/3*(3*a^2*cos(f*x + e) - (6*a*b*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx=\frac{\frac{b^2}{3}+2\,a\,b\cos\left(e+f\,x\right)^2}{f\cos\left(e+f\,x\right)^3}-\frac{a^2\,\cos\left(e+f\,x\right)}{f}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2/3 + 2*a*b*cos(e + f*x)^2)/(f*cos(e + f*x)^3) - (a^2*cos(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.00

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx$$
$$=\frac{-3\cos\left(fx+e\right)^2\sin\left(fx+e\right)^2a^2+3\cos\left(fx+e\right)^2a^2-6\cos\left(fx+e\right)\sin\left(fx+e\right)^2ab-\cos\left(fx+e\right)\sin}{3\cos\left(fx+e\right)f\left(\sin\left(fx+e\right)\right.}$$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x)
```

output
```
( - 3*cos(e + f*x)**2*sin(e + f*x)**2*a**2 + 3*cos(e + f*x)**2*a**2 - 6*co
s(e + f*x)*sin(e + f*x)**2*a*b - cos(e + f*x)*sin(e + f*x)**2*b**2 + 6*cos
(e + f*x)*a*b + cos(e + f*x)*b**2 + 6*sin(e + f*x)**2*a*b - 6*a*b - b**2)/
(3*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.18 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a+b)^2\text{arctanh}(\cos(e+fx))}{f}+\frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-(a+b)^2*arctanh(cos(f*x+e))/f+b*(2*a+b)*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3
/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(52) = 104.

Time = 0.84 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.08

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{4\left(b+a\cos^2(e+fx)\right)^2\left(-b^2-3b(2a+b)\cos^2(e+fx)+3(a+b)^2\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{3f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-4*(b + a*Cos[e + f*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cos[e + f*x]^2 + 3*(a +
 b)^2*Cos[e + f*x]^3*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[
e + f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4621, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{ 4621}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 364}$$

$$-\frac{\int \left(b^2\sec^4(e+fx)+b(2a+b)\sec^2(e+fx)-\frac{(a+b)^2}{\cos^2(e+fx)-1}\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(a+b)^2\text{arctanh}(\cos(e+fx))-b(2a+b)\sec(e+fx)-\frac{1}{3}b^2\sec^3(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-(((a + b)^2*ArcTanh[Cos[e + f*x]] - b*(2*a + b)*Sec[e + f*x] - (b^2*Sec[e
 + f*x]^3)/3)/f)
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{a^2 \ln(\csc(fx+e)-\cot(fx+e))+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx\right.}{f}$
default	$\frac{a^2 \ln(\csc(fx+e)-\cot(fx+e))+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx\right.}{f}$
norman	$\frac{\frac{(8ab+4b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}-\frac{12ab+8b^2}{3f}-\frac{(4ab+4b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}+\frac{(a^2+2ab+b^2)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$
parallelrisch	$\frac{3(a+b)^2\left(\frac{\cos(3fx+3e)}{3}+\cos(fx+e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+6\left(\frac{\left(a+\frac{2b}{3}\right)\cos(3fx+3e)}{3}+\frac{(2a+b)\cos(2fx+2e)}{3}+\left(a+\frac{2b}{3}\right)\cos(fx+\right.}{f(\cos(3fx+3e)+3\cos(fx+e))}$
risch	$\frac{2b\,e^{i(fx+e)}\left(6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}+12a\,e^{2i(fx+e)}+10b\,e^{2i(fx+e)}+6a+3b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}-\frac{\ln\left(e^{i(fx+e)}+1\right)a^2}{f}-\frac{2\ln\left(e^{i(fx+e\right.}{f}$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*ln(csc(f*x+e)-cot(f*x+e))+2*a*b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f
*x+e)))+b^2*(1/3/cos(f*x+e)^3+1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(50) = 100$.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.94

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{3\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^3\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-3\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^3\log\left(-\frac{1}{2}\cos\right.}{6\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3
*(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*
b + b^2)*cos(f*x + e)^2 - 2*b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\csc\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.58

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{3\,(a^2+2\,ab+b^2)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a^2+2\,ab+b^2)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,(3\,(2\,ab+b^2)\cos(fx+}{\cos(fx+e)^3}}{6\,f}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(a^2 + 2*a*b + b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 2*a*b + b^2)*
log(cos(f*x + e) - 1) - 2*(3*(2*a*b + b^2)*cos(f*x + e)^2 + b^2)/cos(f*x +
 e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.83

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a^2+2\,ab+b^2)\log\left(\left|\cos\left(fx+e\right)+1\right|\right)}{2\,f} + \frac{(a^2+2\,ab+b^2)\log\left(\left|\cos\left(fx+e\right)-1\right|\right)}{2\,f} + \frac{6\,ab\cos\left(fx+e\right)^2+3\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(a^2 + 2*a*b + b^2)*log(abs(cos(f*x + e) + 1))/f + 1/2*(a^2 + 2*a*b +
 b^2)*log(abs(cos(f*x + e) - 1))/f + 1/3*(6*a*b*cos(f*x + e)^2 + 3*b^2*cos
(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\cos\left(e+f\,x\right)^2\,\left(b^2+2\,a\,b\right)+\frac{b^2}{3}}{f\cos\left(e+f\,x\right)^3}-\frac{\mathrm{atanh}(\cos\left(e+f\,x\right))\,\left(a+b\right)^2}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x),x)
```

output
```
(cos(e + f*x)^2*(2*a*b + b^2) + b^2/3)/(f*cos(e + f*x)^3) - (atanh(cos(e +
 f*x))*(a + b)^2)/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 266, normalized size of antiderivative = 5.12

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{3\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(fx+e\right)^2a^2+6\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(fx+e\right)^2ab+3}{}$$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(3*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**2 + 6*cos(e + f*x
)*log(tan((e + f*x)/2))*sin(e + f*x)**2*a*b + 3*cos(e + f*x)*log(tan((e +
f*x)/2))*sin(e + f*x)**2*b**2 - 3*cos(e + f*x)*log(tan((e + f*x)/2))*a**2
- 6*cos(e + f*x)*log(tan((e + f*x)/2))*a*b - 3*cos(e + f*x)*log(tan((e + f
*x)/2))*b**2 - 6*cos(e + f*x)*sin(e + f*x)**2*a*b - 4*cos(e + f*x)*sin(e +
 f*x)**2*b**2 + 6*cos(e + f*x)*a*b + 4*cos(e + f*x)*b**2 + 6*sin(e + f*x)*
*2*a*b + 3*sin(e + f*x)**2*b**2 - 6*a*b - 4*b**2)/(3*cos(e + f*x)*f*(sin(e
 + f*x)**2 - 1))
```

3.19 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(a+b)(a+5b)\operatorname{arctanh}(\cos(e+fx))}{2f}-\frac{(a+b)^2\cot(e+fx)\csc(e+fx)}{2f}+\frac{2b(a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-1/2*(a+b)*(a+5*b)*arctanh(cos(f*x+e))/f-1/2*(a+b)^2*cot(f*x+e)*csc(f*x+e)
/f+2*b*(a+b)*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1021 vs. 2(80) = 160.

Time = 7.04 (sec) , antiderivative size = 1021, normalized size of antiderivative = 12.76

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((-a^2 - 2*a*b - b^2)*Cos[e + f*x]^4*Csc[e/2 + (f*x)/2]^2*(a + b*Sec[e + f
*x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) - (2*(a^2 + 6*a*b + 5*b^2
)*Cos[e + f*x]^4*Log[Cos[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a +
 2*b + a*Cos[2*e + 2*f*x])^2) + (2*(a^2 + 6*a*b + 5*b^2)*Cos[e + f*x]^4*Lo
g[Sin[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*b + a*Cos[2*e +
2*f*x])^2) + (2*b*(12*a + 13*b)*Cos[e + f*x]^4*Sec[e]*(a + b*Sec[e + f*x]^
2)^2)/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + ((a^2 + 2*a*b + b^2)*Cos[e
+ f*x]^4*Sec[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*x]^2)^2)/(2*f*(a + 2*b + a*
Cos[2*e + 2*f*x])^2) + (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*Sin[
(f*x)/2])/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos
[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[e +
f*x]^2)^2*(b^2*Cos[e/2] + b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x
])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) +
(2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*S
in[(f*x)/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*
(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) - (2*b^2*Cos[e + f*x]^4*(a + b*
Sec[e + f*x]^2)^2*Sin[(f*x)/2])/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos
[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^3) + (Cos[e +
f*x]^4*(a + b*Sec[e + f*x]^2)^2*(b^2*Cos[e/2] - b^2*Sin[e/2]))/(3*f*(a + 2
*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + ...
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4621, 365, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{365}$$

$$-\frac{\frac{1}{3}\int \frac{\left(3a^2\cos^2(e+fx)+b(6a+5b)\right)\sec^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)-\frac{b^2\sec^3(e+fx)}{3\left(1-\cos^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{361}$$

$$-\frac{\frac{1}{3}\left(\frac{\left(3a^2+6ab+5b^2\right)\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}-\frac{1}{2}\int -\frac{\left(\left(3a^2+6ba+5b^2\right)\cos^2(e+fx)+2b(6a+5b)\right)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)-\frac{b^2\sec^3(e+fx)}{3\left(1-\cos^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{3}\left(\frac{1}{2}\int \frac{\left(\left(3a^2+6ba+5b^2\right)\cos^2(e+fx)+2b(6a+5b)\right)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{\left(3a^2+6ab+5b^2\right)\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}\right)-\frac{b^2\sec^3(e+fx)}{3\left(1-\cos^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{359}$$

$$-\frac{\frac{1}{3}\left(\frac{1}{2}\left(3(a+b)(a+5b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-2b(6a+5b)\sec(e+fx)\right)+\frac{\left(3a^2+6ab+5b^2\right)\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}\right)-\frac{b^2 s}{3(1-}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{1}{3}\left(\frac{\left(3a^2+6ab+5b^2\right)\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}+\frac{1}{2}\left(3(a+b)(a+5b)\text{arctanh}(\cos(e+fx))-2b(6a+5b)\sec(e+fx)\right)\right)-\frac{b^2\sec^3(e+f}{3(1-\cos^2(e+f}}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((-1/3*(b^2*Sec[e + f*x]^3)/(1 - Cos[e + f*x]^2) + (((3*a^2 + 6*a*b + 5*b
^2)*Cos[e + f*x])/(2*(1 - Cos[e + f*x]^2)) + (3*(a + b)*(a + 5*b)*ArcTanh[
Cos[e + f*x]] - 2*b*(6*a + 5*b)*Sec[e + f*x])/2)/3)/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(74) = 148$.

Time = 1.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.04

method	result
derivativedivides	$\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(f \cdots}{2} \cdots}{f}$
default	$\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(f \cdots}{2} \cdots}{f}$
norman	$\frac{\frac{a^2+2ab+b^2}{8f}+\frac{(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{8f}-\frac{(7a^2+46ab+55b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{8f}-\frac{(9a^2+66ab+65b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{12f}+\frac{(11a^2+86a \cdots}{\cdots}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
parallelrisch	$\frac{\frac{3(a+b)\left(\frac{\cos(3fx+3e)}{3}+\cos(fx+e)\right)(a+5b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{3\left(\left(a^2+\frac{22}{3}ab+\frac{65}{9}b^2\right)\cos(3fx+3e)+\frac{32\left(a^2+2ab+\frac{5}{3}b^2\right)\cos(2fx+2e \cdots}{3} \cdots\right)}{\cdots}}{f(\cos(3 \cdots}$
risch	$\frac{e^{i(fx+e)}\left(3a^2e^{8i(fx+e)}+18ab\,e^{8i(fx+e)}+15b^2e^{8i(fx+e)}+12a^2e^{6i(fx+e)}+24ab\,e^{6i(fx+e)}+20b^2e^{6i(fx+e)}+18a^2e^{4i(fx+ \cdots}\right)}{3f\left(e^{2i(fx+e)}+1\right)^3\left(e^{2i(fx \cdots}\right)}$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))+2*a*b*
(-1/2/sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e))
)+b^2*(1/3/sin(f*x+e)^2/cos(f*x+e)^3-5/6/sin(f*x+e)^2/cos(f*x+e)+5/2/cos(f
*x+e)+5/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(74) = 148$.

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.41

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{6\,(a^2+6\,ab+5\,b^2)\cos{(fx+e)}^4-4\,(6\,ab+5\,b^2)\cos{(fx+e)}^2-4\,b^2-3\,\left((a^2+6\,ab+5\,b^2)\cos{(fx+e}\right.}{}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/12*(6*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 4*(6*a*b + 5*b^2)*cos(f*x +
 e)^2 - 4*b^2 - 3*((a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^5 - (a^2 + 6*a*b + 5
*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^2 + 6*a*b + 5*b^
2)*cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x
 + e) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2{(e+fx)}\right)^2\csc^3{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{3\,(a^2+6\,ab+5\,b^2)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a^2+6\,ab+5\,b^2)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a^2+6\,ab+5\,b\right.}{}}{12\,f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/12*(3*(a^2 + 6*a*b + 5*b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 6*a*b + 5*
b^2)*log(cos(f*x + e) - 1) - 2*(3*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 2
*(6*a*b + 5*b^2)*cos(f*x + e)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3)
)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.84

$$\begin{aligned}\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= -\frac{(a^2+6\,ab+5\,b^2)\log\left(\left|\cos\left(fx+e\right)+1\right|\right)}{4\,f} \\ &\quad+\frac{(a^2+6\,ab+5\,b^2)\log\left(\left|\cos\left(fx+e\right)-1\right|\right)}{4\,f} \\ &\quad+\frac{a^2\cos\left(fx+e\right)+2\,ab\cos\left(fx+e\right)+b^2\cos\left(fx+e\right)}{2\left(\cos\left(fx+e\right)^2-1\right)f} \\ &\quad+\frac{6\,ab\cos\left(fx+e\right)^2+6\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}\end{aligned}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/4*(a^2 + 6*a*b + 5*b^2)*log(abs(cos(f*x + e) + 1))/f + 1/4*(a^2 + 6*a*b
 + 5*b^2)*log(abs(cos(f*x + e) - 1))/f + 1/2*(a^2*cos(f*x + e) + 2*a*b*cos
(f*x + e) + b^2*cos(f*x + e))/((cos(f*x + e)^2 - 1)*f) + 1/3*(6*a*b*cos(f*
x + e)^2 + 6*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \csc^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx = \frac{\frac{b^2}{3}+\cos\left(e+f\,x\right)^2\,\left(\frac{5\,b^2}{3}+2\,a\,b\right)-\cos\left(e+f\,x\right)^4\,\left(\frac{a^2}{2}+3\,a\,b+\frac{5\,b^2}{2}\right)}{f\,\left({\cos\left(e+f\,x\right)}^3-{\cos\left(e+f\,x\right)}^5\right)} - \frac{\mathrm{atanh}\left(\cos\left(e+f\,x\right)\right)\,\left(a+b\right)\,\left(a+5\,b\right)}{2\,f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^3,x)
```

output
```
(b^2/3 + cos(e + f*x)^2*(2*a*b + (5*b^2)/3) - cos(e + f*x)^4*(3*a*b + a^2/
2 + (5*b^2)/2))/(f*(cos(e + f*x)^3 - cos(e + f*x)^5)) - (atanh(cos(e + f*x
))*(a + b)*(a + 5*b))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 408, normalized size of antiderivative = 5.10

$$\int \csc^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx = \frac{12\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(fx+e\right)^4 a^2+72\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(fx+e\right)^4 ab+}{}$$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(12*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**4*a**2 + 72*cos(e + f
*x)*log(tan((e + f*x)/2))*sin(e + f*x)**4*a*b + 60*cos(e + f*x)*log(tan((e
 + f*x)/2))*sin(e + f*x)**4*b**2 - 12*cos(e + f*x)*log(tan((e + f*x)/2))*s
in(e + f*x)**2*a**2 - 72*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**
2*a*b - 60*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**2*b**2 - 9*cos
(e + f*x)*sin(e + f*x)**4*a**2 - 66*cos(e + f*x)*sin(e + f*x)**4*a*b - 65*
cos(e + f*x)*sin(e + f*x)**4*b**2 + 9*cos(e + f*x)*sin(e + f*x)**2*a**2 +
66*cos(e + f*x)*sin(e + f*x)**2*a*b + 65*cos(e + f*x)*sin(e + f*x)**2*b**2
 + 12*sin(e + f*x)**4*a**2 + 72*sin(e + f*x)**4*a*b + 60*sin(e + f*x)**4*b
**2 - 24*sin(e + f*x)**2*a**2 - 96*sin(e + f*x)**2*a*b - 80*sin(e + f*x)**
2*b**2 + 12*a**2 + 24*a*b + 12*b**2)/(24*cos(e + f*x)*sin(e + f*x)**2*f*(s
in(e + f*x)**2 - 1))
```

3.20 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(3a^2+30ab+35b^2)\operatorname{arctanh}(\cos(e+fx))}{8f} - \frac{(a+b)(3a+11b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)^2\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{b(2a+3b)\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-1/8*(3*a^2+30*a*b+35*b^2)*arctanh(cos(f*x+e))/f-1/8*(a+b)*(3*a+11*b)*cot(
f*x+e)*csc(f*x+e)/f-1/4*(a+b)^2*cot(f*x+e)*csc(f*x+e)^3/f+b*(2*a+3*b)*sec(
f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.80

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{\left(b+a\cos^2(e+fx)\right)^2\left((90a^2+132ab-102b^2+(6a^2+60ab+70b^2)\cos(4(e+fx))-3(3a^2+30ab+\right.}{}$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/192*((b + a*Cos[e + f*x]^2)^2*((90*a^2 + 132*a*b - 102*b^2 + (6*a^2 + 6
0*a*b + 70*b^2)*Cos[4*(e + f*x)] - 3*(3*a^2 + 30*a*b + 35*b^2)*Cos[6*(e +
f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 + ((105*a^2 + 282*a*b + 329*b^2)*(Cos[e
 + f*x] + Cos[3*(e + f*x)])*Csc[e + f*x]^4)/2 + 96*(3*a^2 + 30*a*b + 35*b^
2)*Cos[e + f*x]^4*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e +
 f*x]^4)/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 4621, 365, 361, 25, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^5}dx$$
$$\Big\downarrow \text{4621}$$
$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)}{\left(1-\cos^2(e+fx)\right)^3}d\cos(e+fx)}{f}$$

$$\downarrow 365$$

$$-\frac{\frac{1}{3}\int \frac{(3a^2\cos^2(e+fx)+b(6a+7b))\sec^2(e+fx)}{(1-\cos^2(e+fx))^3}d\cos(e+fx)-\frac{b^2\sec^3(e+fx)}{3(1-\cos^2(e+fx))^2}}{f}$$

$$\downarrow 361$$

$$-\frac{\frac{1}{3}\left(\frac{(3a^2+6ab+7b^2)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}-\frac{1}{4}\int -\frac{(3(3a^2+6ba+7b^2)\cos^2(e+fx)+4b(6a+7b))\sec^2(e+fx)}{(1-\cos^2(e+fx))^2}d\cos(e+fx)\right)-\frac{b^2\sec^3(e+fx)}{3(1-\cos^2(e+fx))^2}}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\int \frac{(3(3a^2+6ba+7b^2)\cos^2(e+fx)+4b(6a+7b))\sec^2(e+fx)}{(1-\cos^2(e+fx))^2}d\cos(e+fx)+\frac{(3a^2+6ab+7b^2)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}\right)-\frac{b^2\sec^3(e+fx)}{3(1-\cos^2(e+fx))^2}}{f}$$

$$\downarrow 361$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\left(\frac{(3a+7b)^2\cos(e+fx)}{2(1-\cos^2(e+fx))}-\frac{1}{2}\int -\frac{((3a+7b)^2\cos^2(e+fx)+8b(6a+7b))\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)+\frac{(3a^2+6ab+7b^2)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}\right)-}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{((3a+7b)^2\cos^2(e+fx)+8b(6a+7b))\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{(3a+7b)^2\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)+\frac{(3a^2+6ab+7b^2)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}\right)-}{f}$$

$$\downarrow 359$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\left(\frac{1}{2}\left(3(3a^2+30ab+35b^2)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-8b(6a+7b)\sec(e+fx)\right)+\frac{(3a+7b)^2\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)+\right.}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\left(\frac{1}{2}(3(3a^2+30ab+35b^2)\operatorname{arctanh}(\cos(e+fx))-8b(6a+7b)\sec(e+fx))+\frac{(3a+7b)^2\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)+\frac{(3a^2+6ab}{4(1-}\right.}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((-1/3*(b^2*Sec[e + f*x]^3)/(1 - Cos[e + f*x]^2)^2 + (((3*a^2 + 6*a*b + 7
*b^2)*Cos[e + f*x])/(4*(1 - Cos[e + f*x]^2)^2) + (((3*a + 7*b)^2*Cos[e + f
*x])/(2*(1 - Cos[e + f*x]^2)) + (3*(3*a^2 + 30*a*b + 35*b^2)*ArcTanh[Cos[e
 + f*x]] - 8*b*(6*a + 7*b)*Sec[e + f*x])/2)/4)/3)/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.74

method	result
derivativedivides	$a^{2}\left(\left(-\frac{\csc(fx+e)^{3}}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{1}{4\sin(fx+e)^{4}\cos(fx+e)}-\frac{5}{8\sin(fx+e)^{2}}$
default	$a^{2}\left(\left(-\frac{\csc(fx+e)^{3}}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{1}{4\sin(fx+e)^{4}\cos(fx+e)}-\frac{5}{8\sin(fx+e)^{2}}$
norman	$\frac{a^{2}+2ab+b^{2}}{64f}+\frac{(a^{2}+2ab+b^{2})\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}}{64f}+\frac{(5a^{2}+26ab+21b^{2})\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{64f}+\frac{(5a^{2}+26ab+21b^{2})\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{64f}-(3a^{2}+29a$ / $\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right.$
parallelrisch	$55296\left(\frac{\cos(3fx+3e)}{3}+\cos(fx+e)\right)\left(a^{2}+10ab+\frac{35}{3}b^{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-63\left(\left(3a^{2}+\frac{218}{7}ab+\frac{121}{3}b^{2}\right)\cos(3fx+3e)+\left(\frac{80}{3}a^{2}\right.\right.$
risch	$e^{i(fx+e)}\left(9a^{2}e^{12i(fx+e)}+90ab\,e^{12i(fx+e)}+105b^{2}e^{12i(fx+e)}-6a^{2}e^{10i(fx+e)}-60ab\,e^{10i(fx+e)}-70b^{2}e^{10i(fx+e)}-105a^{2}e\right.$

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-
cot(f*x+e)))+2*a*b*(-1/4/sin(f*x+e)^4/cos(f*x+e)-5/8/sin(f*x+e)^2/cos(f*x+
e)+15/8/cos(f*x+e)+15/8*ln(csc(f*x+e)-cot(f*x+e)))+b^2*(-1/4/sin(f*x+e)^4/
cos(f*x+e)^3+7/12/sin(f*x+e)^2/cos(f*x+e)^3-35/24/sin(f*x+e)^2/cos(f*x+e)+
35/8/cos(f*x+e)+35/8*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(113) = 226$.

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.36

$$\int \csc^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$$
$$= \frac{6\,(3\,a^2+30\,ab+35\,b^2)\cos{(fx+e)}^6-10\,(3\,a^2+30\,ab+35\,b^2)\cos{(fx+e)}^4+16\,(6\,ab+7\,b^2)\cos{(fx+\cdots}}{\cdots}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/48*(6*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 30*a*b + 35
*b^2)*cos(f*x + e)^4 + 16*(6*a*b + 7*b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*
a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b^2)*cos(f*
x + e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e)
+ 1/2) + 3*((3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b +
 35*b^2)*cos(f*x + e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(-1
/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x
 + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.36

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{3\,(3\,a^2+30\,ab+35\,b^2)\log\left(\cos\left(fx+e\right)+1\right)-3\,(3\,a^2+30\,ab+35\,b^2)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(3\,\cdots\right.}{\cdots}}{48\,f}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/48*(3*(3*a^2 + 30*a*b + 35*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 30*a
*b + 35*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 30*a*b + 35*b^2)*cos(f*
x + e)^6 - 5*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^4 + 8*(6*a*b + 7*b^2)*
cos(f*x + e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^
3))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.58

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(3\,a^2+30\,ab+35\,b^2)\log\left(|\cos\left(fx+e\right)+1|\right)}{16\,f}$$
$$+\frac{(3\,a^2+30\,ab+35\,b^2)\log\left(|\cos\left(fx+e\right)-1|\right)}{16\,f}$$
$$+\frac{3\,a^2\cos\left(fx+e\right)^3+14\,ab\cos\left(fx+e\right)^3+11\,b^2\cos\left(fx+e\right)^3-5\,a^2\cos\left(fx+e\right)-18\,ab\cos\left(fx+e\right)-\cdots}{8\left(\cos\left(fx+e\right)^2-1\right)^2 f}$$
$$+\frac{6\,ab\cos\left(fx+e\right)^2+9\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/16*(3*a^2 + 30*a*b + 35*b^2)*log(abs(cos(f*x + e) + 1))/f + 1/16*(3*a^2
 + 30*a*b + 35*b^2)*log(abs(cos(f*x + e) - 1))/f + 1/8*(3*a^2*cos(f*x + e)
^3 + 14*a*b*cos(f*x + e)^3 + 11*b^2*cos(f*x + e)^3 - 5*a^2*cos(f*x + e) -
18*a*b*cos(f*x + e) - 13*b^2*cos(f*x + e))/((cos(f*x + e)^2 - 1)^2*f) + 1/
3*(6*a*b*cos(f*x + e)^2 + 9*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 12.60 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\frac{b^2}{3}+\cos(e+f\,x)^2\left(\frac{7\,b^2}{3}+2\,a\,b\right)+\cos(e+f\,x)^6\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)-\cos(e+f\,x)^4\left(\frac{5\,a^2}{8}+\frac{25\,a\,b}{4}+\frac{175}{2}\right.}{f\,\left(\cos(e+f\,x)^7-2\cos(e+f\,x)^5+\cos(e+f\,x)^3\right)} - \frac{\operatorname{atanh}(\cos(e+f\,x))\,\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^5,x)
```

output
```
(b^2/3 + cos(e + f*x)^2*(2*a*b + (7*b^2)/3) + cos(e + f*x)^6*((15*a*b)/4 +
 (3*a^2)/8 + (35*b^2)/8) - cos(e + f*x)^4*((25*a*b)/4 + (5*a^2)/8 + (175*b
^2)/24))/(f*(cos(e + f*x)^3 - 2*cos(e + f*x)^5 + cos(e + f*x)^7)) - (atanh
(cos(e + f*x))*((15*a*b)/4 + (3*a^2)/8 + (35*b^2)/8))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 446, normalized size of antiderivative = 3.69

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{72\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^6a^2+720\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^6ab}{}$$

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(72*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**6*a**2 + 720*cos(e +
f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**6*a*b + 840*cos(e + f*x)*log(tan(
(e + f*x)/2))*sin(e + f*x)**6*b**2 - 72*cos(e + f*x)*log(tan((e + f*x)/2))
*sin(e + f*x)**4*a**2 - 720*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x
)**4*a*b - 840*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**4*b**2 - 6
3*cos(e + f*x)*sin(e + f*x)**6*a**2 - 654*cos(e + f*x)*sin(e + f*x)**6*a*b
 - 847*cos(e + f*x)*sin(e + f*x)**6*b**2 + 63*cos(e + f*x)*sin(e + f*x)**4
*a**2 + 654*cos(e + f*x)*sin(e + f*x)**4*a*b + 847*cos(e + f*x)*sin(e + f*
x)**4*b**2 + 72*sin(e + f*x)**6*a**2 + 720*sin(e + f*x)**6*a*b + 840*sin(e
 + f*x)**6*b**2 - 96*sin(e + f*x)**4*a**2 - 960*sin(e + f*x)**4*a*b - 1120
*sin(e + f*x)**4*b**2 - 24*sin(e + f*x)**2*a**2 + 144*sin(e + f*x)**2*a*b
+ 168*sin(e + f*x)**2*b**2 + 48*a**2 + 96*a*b + 48*b**2)/(192*cos(e + f*x)
*sin(e + f*x)**4*f*(sin(e + f*x)**2 - 1))
```

3.21 $\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx = \frac{5}{16}\left(a^2-12ab+8b^2\right)x-\frac{(11a^2-36ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f}+\frac{a(13a-12b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{a^2\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{2(a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
5/16*(a^2-12*a*b+8*b^2)*x-1/16*(11*a^2-36*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)
/f+1/24*a*(13*a-12*b)*cos(f*x+e)^3*sin(f*x+e)/f-1/6*a^2*cos(f*x+e)^5*sin(f
*x+e)/f+2*(a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 499 vs. $2(141) = 282$.

Time = 1.77 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.54

$$\int (a+b\sec^2(e+fx))^2\sin^6(e+fx)\,dx$$
$$= \frac{(b+a\cos^2(e+fx))^2\sec(e)\sec^3(e+fx)\,(360(a^2-12ab+8b^2)\,fx\cos(fx)+360(a^2-12ab+8b^2)\,fx\cos$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

output
```
((b + a*Cos[e + f*x]^2)^2*Sec[e]*Sec[e + f*x]^3*(360*(a^2 - 12*a*b + 8*b^2
)*f*x*Cos[f*x] + 360*(a^2 - 12*a*b + 8*b^2)*f*x*Cos[2*e + f*x] + 120*a^2*f
*x*Cos[2*e + 3*f*x] - 1440*a*b*f*x*Cos[2*e + 3*f*x] + 960*b^2*f*x*Cos[2*e
+ 3*f*x] + 120*a^2*f*x*Cos[4*e + 3*f*x] - 1440*a*b*f*x*Cos[4*e + 3*f*x] +
960*b^2*f*x*Cos[4*e + 3*f*x] - 81*a^2*Sin[f*x] + 3444*a*b*Sin[f*x] - 3168*
b^2*Sin[f*x] - 81*a^2*Sin[2*e + f*x] - 1164*a*b*Sin[2*e + f*x] + 2208*b^2*
Sin[2*e + f*x] - 109*a^2*Sin[2*e + 3*f*x] + 2076*a*b*Sin[2*e + 3*f*x] - 19
36*b^2*Sin[2*e + 3*f*x] - 109*a^2*Sin[4*e + 3*f*x] + 540*a*b*Sin[4*e + 3*f
*x] - 144*b^2*Sin[4*e + 3*f*x] - 21*a^2*Sin[4*e + 5*f*x] + 156*a*b*Sin[4*e
 + 5*f*x] - 48*b^2*Sin[4*e + 5*f*x] - 21*a^2*Sin[6*e + 5*f*x] + 156*a*b*Si
n[6*e + 5*f*x] - 48*b^2*Sin[6*e + 5*f*x] + 6*a^2*Sin[6*e + 7*f*x] - 12*a*b
*Sin[6*e + 7*f*x] + 6*a^2*Sin[8*e + 7*f*x] - 12*a*b*Sin[8*e + 7*f*x] - a^2
*Sin[8*e + 9*f*x] - a^2*Sin[10*e + 9*f*x]))/(768*f*(a + 2*b + a*Cos[2*(e +
 f*x)])^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4620, 366, 360, 25, 2345, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$\downarrow$ 3042

$$\int \sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2dx$$

$\downarrow$ 4620

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$\downarrow$ 366

$$\frac{\frac{a^2\tan^7(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}-\frac{1}{6}\int \frac{\tan^6(e+fx)\left(7a^2-6(a+b)^2-6b^2\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$\downarrow$ 360

$$\frac{\frac{1}{6}\left(\frac{1}{4}\int -\frac{-24b^2\tan^6(e+fx)+4a(a-12b)\tan^4(e+fx)-4a(a-12b)\tan^2(e+fx)+a(a-12b)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{a(a-12b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)+\frac{a^2}{6(\mathrm{ta}}}{f}$$

$\downarrow$ 25

$$\frac{\frac{1}{6}\left(\frac{a(a-12b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{1}{4}\int \frac{-24b^2\tan^6(e+fx)+4a(a-12b)\tan^4(e+fx)-4a(a-12b)\tan^2(e+fx)+a(a-12b)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)\right)+\frac{a^2\,\mathrm{ta}}{6(\tan^2}}{f}$$

$\downarrow$ 2345

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{48b^2\tan^4(e+fx)-8\left(a^2-12ba+6b^2\right)\tan^2(e+fx)+7a^2+24b^2-84ab}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{3\left(3a^2-36ab+8b^2\right)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{a(a-12}{4(\tan^2}}{f}$$

$\downarrow$ 1467

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\int \left(48b^2\tan^2(e+fx)-8\left(a^2-12ba+12b^2\right)+\frac{15\left(a^2-12ba+8b^2\right)}{\tan^2(e+fx)+1}\right)d\tan(e+fx)-\frac{3\left(3a^2-36ab+8b^2\right)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right.}{f}$$

$\downarrow$ 2009

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\left(15\left(a^2-12ab+8b^2\right)\arctan(\tan(e+fx))-8\left(a^2-12ab+12b^2\right)\tan(e+fx)+16b^2\tan^3(e+fx)\right)-\frac{3(3}{}\right.\right.}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

output
```
((a^2*Tan[e + f*x]^7)/(6*(1 + Tan[e + f*x]^2)^3) + ((a*(a - 12*b)*Tan[e +
f*x])/(4*(1 + Tan[e + f*x]^2)^2) + ((-3*(3*a^2 - 36*a*b + 8*b^2)*Tan[e + f
*x])/(2*(1 + Tan[e + f*x]^2)) + (15*(a^2 - 12*a*b + 8*b^2)*ArcTan[Tan[e +
f*x]] - 8*(a^2 - 12*a*b + 12*b^2)*Tan[e + f*x] + 16*b^2*Tan[e + f*x]^3)/2)
/4)/6)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.17

method	result
parallelrisch	$-\frac{109\left(-\frac{120fx\left(a^2-12ab+8b^2\right)\cos(3fx+3e)}{109}+\left(a^2-12ab+\frac{1040}{109}b^2\right)\sin(3fx+3e)+\frac{3\left(7a^2-52ab+16b^2\right)\sin(5fx+5e)}{109}-\frac{6a(a-}{}\right.}{1536f}$
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}\right.\right.}{}$
default	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}\right.\right.}{}$
parts	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)}{f}+\frac{b^2\left(\frac{\sin(fx+e)^7}{3\cos(fx+e)^3}-\frac{4\sin(fx+e)^7}{3\cos(fx+e)}-\frac{4\left(\sin(fx+\right.}{}\right.}{}$
risch	$\frac{5a^2x}{16}-\frac{15xab}{4}+\frac{5x\,b^2}{2}-\frac{3ie^{4i(fx+e)}a^2}{128f}+\frac{ie^{4i(fx+e)}ab}{32f}+\frac{15ie^{2i(fx+e)}a^2}{128f}-\frac{ie^{2i(fx+e)}ab}{2f}+\frac{ie^{2i(fx+e)}b^2}{8f}-$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
-109/1536*(-120/109*f*x*(a^2-12*a*b+8*b^2)*cos(3*f*x+3*e)+(a^2-12*a*b+1040
/109*b^2)*sin(3*f*x+3*e)+3/109*(7*a^2-52*a*b+16*b^2)*sin(5*f*x+5*e)-6/109*
a*(a-2*b)*sin(7*f*x+7*e)+1/109*a^2*sin(9*f*x+9*e)-360/109*f*x*(a^2-12*a*b+
8*b^2)*cos(f*x+e)+81/109*sin(f*x+e)*(a^2-380/27*a*b+160/27*b^2))*sec(f*x+e
)^3/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx = \frac{15\left(a^2-12\,ab+8\,b^2\right)fx\cos\left(fx+e\right)^3-\left(8\,a^2\cos\left(fx+e\right)^8-2\left(13\,a^2-12\,ab\right)\cos\left(fx+e\right)^6+3\left(11\,a^2-\right.\right.}{48\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
1/48*(15*(a^2 - 12*a*b + 8*b^2)*f*x*cos(f*x + e)^3 - (8*a^2*cos(f*x + e)^8
 - 2*(13*a^2 - 12*a*b)*cos(f*x + e)^6 + 3*(11*a^2 - 36*a*b + 8*b^2)*cos(f*
x + e)^4 - 16*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 16*b^2)*sin(f*x + e))/(f*co
s(f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.16

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$$
$$=\frac{16\,b^2\tan\left(fx+e\right)^3+15\,(a^2-12\,ab+8\,b^2)(fx+e)+96\,(ab-b^2)\tan\left(fx+e\right)-\frac{3\,(11\,a^2-36\,ab+8\,b^2)\tan(fx+\cdots}{\tan\cdots}}{48\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
1/48*(16*b^2*tan(f*x + e)^3 + 15*(a^2 - 12*a*b + 8*b^2)*(f*x + e) + 96*(a*
b - b^2)*tan(f*x + e) - (3*(11*a^2 - 36*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5
*a^2 - 24*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(5*a^2 - 28*a*b + 8*b^2)*tan(f*x
 + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.30

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$$
$$=\frac{16\,b^2\tan\left(fx+e\right)^3+96\,ab\tan\left(fx+e\right)-96\,b^2\tan\left(fx+e\right)+15\,(a^2-12\,ab+8\,b^2)(fx+e)-\frac{33\,a^2\tan(f\cdots}{\cdots}}{\cdots}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
1/48*(16*b^2*tan(f*x + e)^3 + 96*a*b*tan(f*x + e) - 96*b^2*tan(f*x + e) +
15*(a^2 - 12*a*b + 8*b^2)*(f*x + e) - (33*a^2*tan(f*x + e)^5 - 108*a*b*tan
(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 192*a*b*tan(
f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) - 84*a*b*tan(f*x
+ e) + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx = x\left(\frac{5\,a^2}{16}-\frac{15\,a\,b}{4}+\frac{5\,b^2}{2}\right) - \frac{\left(\frac{11\,a^2}{16}-\frac{9\,a\,b}{4}+\frac{b^2}{2}\right)\tan(e+f\,x)^5+\left(\frac{5\,a^2}{6}-4\,a\,b+b^2\right)\tan(e+f\,x)^3+\left(\frac{5\,a^2}{16}-\frac{7\,a\,b}{4}+\frac{b^2}{2}\right)\tan(e+f\,x}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)} + \frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\tan(e+f\,x)\,\left(4\,b^2-2\,b\,(a+b)\right)}{f}$$

input
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
x*((5*a^2)/16 - (15*a*b)/4 + (5*b^2)/2) - (tan(e + f*x)*((5*a^2)/16 - (7*a
*b)/4 + b^2/2) + tan(e + f*x)^3*((5*a^2)/6 - 4*a*b + b^2) + tan(e + f*x)^5
*((11*a^2)/16 - (9*a*b)/4 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^
4 + tan(e + f*x)^6 + 1)) + (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*(4*b
^2 - 2*b*(a + b)))/f
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.50

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx = \frac{-8\cos\left(fx+e\right)^2\sin\left(fx+e\right)^7a^2-2\cos\left(fx+e\right)^2\sin\left(fx+e\right)^5a^2-5\cos\left(fx+e\right)^2\sin\left(fx+e\right)^3a^2+1}{}$$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x)
```

output
```
( - 8*cos(e + f*x)**2*sin(e + f*x)**7*a**2 - 2*cos(e + f*x)**2*sin(e + f*x
)**5*a**2 - 5*cos(e + f*x)**2*sin(e + f*x)**3*a**2 + 15*cos(e + f*x)**2*si
n(e + f*x)*a**2 + 15*cos(e + f*x)*sin(e + f*x)**2*a**2*f*x - 180*cos(e + f
*x)*sin(e + f*x)**2*a*b*e - 180*cos(e + f*x)*sin(e + f*x)**2*a*b*f*x + 120
*cos(e + f*x)*sin(e + f*x)**2*b**2*e + 120*cos(e + f*x)*sin(e + f*x)**2*b*
*2*f*x - 15*cos(e + f*x)*a**2*f*x + 180*cos(e + f*x)*a*b*e + 180*cos(e + f
*x)*a*b*f*x - 120*cos(e + f*x)*b**2*e - 120*cos(e + f*x)*b**2*f*x - 24*sin
(e + f*x)**7*a*b - 36*sin(e + f*x)**5*a*b + 24*sin(e + f*x)**5*b**2 + 240*
sin(e + f*x)**3*a*b - 160*sin(e + f*x)**3*b**2 - 180*sin(e + f*x)*a*b + 12
0*sin(e + f*x)*b**2)/(48*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.22 $\int (a+b\sec^2(e+fx))^2\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int (a+b\sec^2(e+fx))^2\sin^4(e+fx)\,dx=\frac{1}{8}\left(3a^2-24ab+8b^2\right)x-\frac{a(5a-8b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
1/8*(3*a^2-24*a*b+8*b^2)*x-1/8*a*(5*a-8*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a^2
*cos(f*x+e)^3*sin(f*x+e)/f+(2*a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int (a+b\sec^2(e+fx))^2\sin^4(e+fx)\,dx$$
$$=\frac{(b+a\cos^2(e+fx))^2\sec^3(e+fx)\left(32b^2\sec(e)\sin(fx)+64(3a-2b)b\cos^2(e+fx)\sec(e)\sin(fx)+3\,\mathrm{co}\right.}{24f(a+2b}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

output
```
((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(32*b^2*Sec[e]*Sin[f*x] + 64*(3*a
 - 2*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*Cos[e + f*x]^3*(4*(3*a^2 - 24
*a*b + 8*b^2)*f*x - 8*a*(a - 2*b)*Sin[2*(e + f*x)] + a^2*Sin[4*(e + f*x)])
 + 32*b^2*Cos[e + f*x]*Tan[e]))/(24*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 366, 360, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{a^2\tan^5(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{1}{4}\int \frac{\tan^4(e+fx)\left(5a^2-4(a+b)^2-4b^2\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b^2\tan^4(e+fx)-2a(a-8b)\tan^2(e+fx)+a(a-8b)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{a(a-8b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{a^2\tan^5(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int\left(8b^2\tan^2(e+fx)-2(a^2-8ba+4b^2)+\frac{3a^2-24ba+8b^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)-\frac{a(a-8b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{a^2\tan^5(e+fx)}{4(\tan^2(e+fx)+1}}{f}$$

$\big\downarrow$ 2009

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left((3a^2-24ab+8b^2)\arctan(\tan(e+fx))-2(a^2-8ab+4b^2)\tan(e+fx)+\frac{8}{3}b^2\tan^3(e+fx)\right)-\frac{a(a-8b)\tan}{2(\tan^2(e+f}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

output
```
((a^2*Tan[e + f*x]^5)/(4*(1 + Tan[e + f*x]^2)^2) + (-1/2*(a*(a - 8*b)*Tan[
e + f*x])/(1 + Tan[e + f*x]^2) + ((3*a^2 - 24*a*b + 8*b^2)*ArcTan[Tan[e +
f*x]] - 2*(a^2 - 8*a*b + 4*b^2)*Tan[e + f*x] + (8*b^2*Tan[e + f*x]^3)/3)/2
)/4)/f
```

Defintions of rubi rules used

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
parallelrisch	$\frac{48ab\tan(fx+e)\cos(2fx+2e)-24a^2\sin(2fx+2e)+3a^2\sin(4fx+4e)+\left(32b^2\sec(fx+e)^2+240\left(a-\frac{8b}{15}\right)b\right)\tan(fx+e)+3}{96f}$
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3f}{2}\right.}{f}$
default	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3f}{2}\right.}{f}$
parts	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)}{f}+\frac{2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}\right.}{}$
risch	$\frac{3a^2x}{8}-3xab+x\,b^2-\frac{i e^{4i(fx+e)}a^2}{64f}+\frac{i e^{2i(fx+e)}a^2}{8f}-\frac{i e^{2i(fx+e)}ab}{4f}-\frac{i e^{-2i(fx+e)}a^2}{8f}+\frac{i e^{-2i(fx+e)}ab}{4f}+$
norman	$\frac{(-\frac{3}{8}a^2+3ab-b^2)x+(-\frac{9}{8}a^2+9ab-3b^2)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(-\frac{9}{8}a^2+9ab-3b^2)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-\frac{3}{8}a^2+3ab-b^2)x\tan\left(\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*(48*a*b*tan(f*x+e)*cos(2*f*x+2*e)-24*a^2*sin(2*f*x+2*e)+3*a^2*sin(4*f
*x+4*e)+(32*b^2*sec(f*x+e)^2+240*(a-8/15*b)*b)*tan(f*x+e)+36*x*f*(a^2-8*a*
b+8/3*b^2))/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^4(e+fx)\,dx$$
$$=\frac{3\left(3a^2-24ab+8b^2\right)fx\cos(fx+e)^3+\left(6a^2\cos(fx+e)^6-3\left(5a^2-8ab\right)\cos(fx+e)^4+16\left(3ab-2\right.\right.}{24f\cos(fx+e)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/24*(3*(3*a^2 - 24*a*b + 8*b^2)*f*x*cos(f*x + e)^3 + (6*a^2*cos(f*x + e)^
6 - 3*(5*a^2 - 8*a*b)*cos(f*x + e)^4 + 16*(3*a*b - 2*b^2)*cos(f*x + e)^2 +
 8*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a + b\sec^2(e + fx)\right)^2 \sin^4(e + fx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11

$$\int \left(a + b\sec^2(e + fx)\right)^2 \sin^4(e + fx)\, dx$$
$$= \frac{8\,b^2\tan\left(fx+e\right)^3 + 3\,(3\,a^2 - 24\,ab + 8\,b^2)(fx+e) + 24\,(2\,ab - b^2)\tan\left(fx+e\right) - \frac{3\left(\left(5\,a^2 - 8\,ab\right)\tan(fx+e)^3 + \right.}{\tan(fx+e)^4 + 2\,\mathrm{ta}}}{24\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) + 24*(2*
a*b - b^2)*tan(f*x + e) - 3*((5*a^2 - 8*a*b)*tan(f*x + e)^3 + (3*a^2 - 8*a
*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14

$$\int (a+b\sec^2(e+fx))^2\sin^4(e+fx)\,dx$$
$$=\frac{8\,b^2\tan(fx+e)^3+48\,ab\tan(fx+e)-24\,b^2\tan(fx+e)+3\,(3\,a^2-24\,ab+8\,b^2)(fx+e)-\frac{3\left(5\,a^2\tan(\right.}{}}{24\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
1/24*(8*b^2*tan(f*x + e)^3 + 48*a*b*tan(f*x + e) - 24*b^2*tan(f*x + e) + 3
*(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) - 3*(5*a^2*tan(f*x + e)^3 - 8*a*b*tan(
f*x + e)^3 + 3*a^2*tan(f*x + e) - 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)
^2)/f
```

Mupad [B] (verification not implemented)

Time = 12.52 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\int (a+b\sec^2(e+fx))^2\sin^4(e+fx)\,dx$$
$$=x\left(\frac{3\,a^2}{8}-3\,a\,b+b^2\right)+\frac{\left(a\,b-\frac{5\,a^2}{8}\right)\tan(e+f\,x)^3+\left(a\,b-\frac{3\,a^2}{8}\right)\tan(e+f\,x)}{f\,\left(\tan(e+f\,x)^4+2\tan(e+f\,x)^2+1\right)}$$
$$+\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\tan(e+f\,x)\,\left(3\,b^2-2\,b\,(a+b)\right)}{f}$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
x*((3*a^2)/8 - 3*a*b + b^2) + (tan(e + f*x)*(a*b - (3*a^2)/8) + tan(e + f*
x)^3*(a*b - (5*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b^2
*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*(3*b^2 - 2*b*(a + b)))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.55

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^4(e+fx)\,dx$$
$$= \frac{-6\cos(fx+e)^2\sin(fx+e)^5a^2-3\cos(fx+e)^2\sin(fx+e)^3a^2+9\cos(fx+e)^2\sin(fx+e)a^2+9\,\mathrm{c}}{}$$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x)
```

output
```
( - 6*cos(e + f*x)**2*sin(e + f*x)**5*a**2 - 3*cos(e + f*x)**2*sin(e + f*x
)**3*a**2 + 9*cos(e + f*x)**2*sin(e + f*x)*a**2 + 9*cos(e + f*x)*sin(e + f
*x)**2*a**2*f*x - 72*cos(e + f*x)*sin(e + f*x)**2*a*b*e - 72*cos(e + f*x)*
sin(e + f*x)**2*a*b*f*x + 24*cos(e + f*x)*sin(e + f*x)**2*b**2*f*x - 9*cos
(e + f*x)*a**2*f*x + 72*cos(e + f*x)*a*b*e + 72*cos(e + f*x)*a*b*f*x - 24*
cos(e + f*x)*b**2*f*x - 24*sin(e + f*x)**5*a*b + 96*sin(e + f*x)**3*a*b -
32*sin(e + f*x)**3*b**2 - 72*sin(e + f*x)*a*b + 24*sin(e + f*x)*b**2)/(24*
cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.23 $\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx=\frac{1}{2}a(a-4b)x-\frac{a^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
1/2*a*(a-4*b)*x-1/2*a^2*cos(f*x+e)*sin(f*x+e)/f+2*a*b*tan(f*x+e)/f+1/3*b^2
*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.94

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx = -\frac{\left(b+a\cos^2(e+fx)\right)^2\sec^3(e+fx)\left(-4b^2\sec(e)\sin(fx)-4(6a-b)b\cos^2(e+fx)\sec(e)\sin(fx)+3a\right.}{3f(a+2b+a\cos(2(e+fx))}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

output
```
-1/3*((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(-4*b^2*Sec[e]*Sin[f*x] - 4*
(6*a - b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*a*Cos[e + f*x]^3*(-2*(a - 4
*b)*f*x + a*Sin[2*(e + f*x)]) - 4*b^2*Cos[e + f*x]*Tan[e]))/(f*(a + 2*b +
a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4620, 366, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{a^2\tan^3(e+fx)}{2(\tan^2(e+fx)+1)}-\frac{1}{2}\int \frac{\tan^2(e+fx)\left(3a^2-2(a+b)^2-2b^2\tan^2(e+fx)\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tan^3(e+fx)-a(a-4b)\int \frac{\tan^2(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)\right)+\frac{a^2\tan^3(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tan^3(e+fx)-a(a-4b)\left(\tan(e+fx)-\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)\right)\right)+\frac{a^2\tan^3(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{a^2\tan^3(e+fx)}{2(\tan^2(e+fx)+1)}+\frac{1}{2}\left(\frac{2}{3}b^2\tan^3(e+fx)-a(a-4b)(\tan(e+fx)-\arctan(\tan(e+fx)))\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

output
```
((a^2*Tan[e + f*x]^3)/(2*(1 + Tan[e + f*x]^2)) + ((2*b^2*Tan[e + f*x]^3)/3
 - a*(a - 4*b)*(-ArcTan[Tan[e + f*x]] + Tan[e + f*x]))/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(\tan(fx+e)-fx-e)+\frac{b^2\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$
default	$\frac{a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(\tan(fx+e)-fx-e)+\frac{b^2\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$
parts	$\frac{a^2\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b^2\sin(fx+e)^3}{3f\cos(fx+e)^3}+\frac{2ab(\tan(fx+e)-fx-e)}{f}$
risch	$\frac{a^2x}{2}-2xab+\frac{i\mathrm{e}^{2i(fx+e)}a^2}{8f}-\frac{i\mathrm{e}^{-2i(fx+e)}a^2}{8f}-\frac{2ib\left(-6a\,\mathrm{e}^{4i(fx+e)}+3b\,\mathrm{e}^{4i(fx+e)}-12a\,\mathrm{e}^{2i(fx+e)}-6a+b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$
parallelrisch	$\frac{12afx(a-4b)\cos(3fx+3e)+(-9a^2+48ab-8b^2)\sin(3fx+3e)-3\sin(5fx+5e)a^2+36afx(a-4b)\cos(fx+e)-6\sin(fx+}{24f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{(-\frac{1}{2}a^2+2ab)x+(-\frac{1}{2}a^2+2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(\frac{1}{2}a^2-2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+(\frac{1}{2}a^2-2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-a^2+4ab}{}$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(tan(f*x+e)-f*x-
e)+1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx = \frac{3\,(a^2-4\,ab)fx\cos\left(fx+e\right)^3-\left(3\,a^2\cos\left(fx+e\right)^4-2\,(6\,ab-b^2)\cos\left(fx+e\right)^2-2\,b^2\right)\sin\left(fx+e\right)}{6\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/6*(3*(a^2 - 4*a*b)*f*x*cos(f*x + e)^3 - (3*a^2*cos(f*x + e)^4 - 2*(6*a*b
 - b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\sin^2\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx = \frac{2\,b^2\tan\left(fx+e\right)^3+12\,ab\tan\left(fx+e\right)+3\,(a^2-4\,ab)(fx+e)-\frac{3\,a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e
) - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx = \frac{2\,b^2\tan\left(fx+e\right)^3+12\,ab\tan\left(fx+e\right)+3\left(a^2-4\,ab\right)(fx+e)-\frac{3\,a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e
) - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Mupad [B] (verification not implemented)

Time = 12.65 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx = \frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{a^2\sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{\tan(e+f\,x)\,\left(2\,b^2-2\,b\,(a+b)\right)}{f}-\frac{a\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)\,(a-4\,b)}{2\,\left(2\,a\,b-\frac{a^2}{2}\right)}\right)\,(a-4\,b)}{2\,f}$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - (a^2*sin(2*e + 2*f*x))/(4*f) - (tan(e + f*x)*
(2*b^2 - 2*b*(a + b)))/f - (a*atan((a*tan(e + f*x)*(a - 4*b))/(2*(2*a*b -
a^2/2)))*(a - 4*b))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.57

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx$$
$$= \frac{-3\cos(fx+e)^2\sin(fx+e)^3a^2+3\cos(fx+e)^2\sin(fx+e)\,a^2+3\cos(fx+e)\sin(fx+e)^2a^2fx-}{}$$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x)
```

output
```
( - 3*cos(e + f*x)**2*sin(e + f*x)**3*a**2 + 3*cos(e + f*x)**2*sin(e + f*x
)*a**2 + 3*cos(e + f*x)*sin(e + f*x)**2*a**2*f*x - 12*cos(e + f*x)*sin(e +
 f*x)**2*a*b*f*x - 3*cos(e + f*x)*a**2*f*x + 12*cos(e + f*x)*a*b*f*x + 12*
sin(e + f*x)**3*a*b - 2*sin(e + f*x)**3*b**2 - 12*sin(e + f*x)*a*b)/(6*cos
(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.24 $\int \left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{3a^2fx+3b(2a+b)\tan(e+fx)+b^2\tan^3(e+fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(3*a^2*f*x + 3*b*(2*a + b)*Tan[e + f*x] + b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{a^2}{\tan^2(e+fx)+1}+b^2\tan^2(e+fx)+b(2a+b)\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\arctan(\tan(e+fx))+b(2a+b)\tan(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] + b*(2*a + b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^3
)/3)/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result
parts	$a^{2} x -\frac{b^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x +e \right)^{2}}{3}\right) \tan \left(f x +e \right)}{f}+\frac{2 a b \tan \left(f x +e \right)}{f}$
derivativedivides	$\frac{a^{2} \left(f x +e \right)+2 a b \tan \left(f x +e \right)-b^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x +e \right)^{2}}{3}\right) \tan \left(f x +e \right)}{f}$
default	$\frac{a^{2} \left(f x +e \right)+2 a b \tan \left(f x +e \right)-b^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x +e \right)^{2}}{3}\right) \tan \left(f x +e \right)}{f}$
risch	$a^{2} x +\frac{4 i b \left(3 a \,{\mathrm e}^{4 i \left(f x +e \right)}+6 a \,{\mathrm e}^{2 i \left(f x +e \right)}+3 b \,{\mathrm e}^{2 i \left(f x +e \right)}+3 a +b \right)}{3 f \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)^{3}}$
norman	$\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-a^{2} x +3 a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-3 a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-\frac{2 b \left(2 a +b \right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{f}-\frac{2 b \left(2 a +b \right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{f}+\frac{4 b (\cdots}{\cdots}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$
parallelrisch	$\frac{3 x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6} a^{2} f +\left(-12 a b -6 b^{2}\right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-9 x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4} a^{2} f +\left(24 a b +4 b^{2}\right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+9 x \tan \left(\frac{f x}{2}+\frac{e}{\cdots}\right.}{3 f \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{3} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}$

input
```
int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*tan(f*x+e)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^2fx\cos\left(fx+e\right)^3+\left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin
(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2 \, dx = a^2x + \frac{\left(\tan\left(fx+e\right)^3 + 3\,\tan\left(fx+e\right)\right)b^2}{3\,f} + \frac{2\,ab\tan\left(fx+e\right)}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{b^2\tan\left(fx+e\right)^3 + 3\left(fx+e\right)a^2 + 6\,ab\tan\left(fx+e\right) + 3\,b^2\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan
(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{\frac{b^2\tan(e+f\,x)^3}{3} - \tan(e+f\,x)\,\left(b^2 - 2\,b\,(a+b)\right) + a^2\,f\,x}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.65

$$\int \left(a+b\sec^2(e+fx)\right)^2 dx$$
$$= \frac{3\cos(fx+e)\sin(fx+e)^2 a^2 fx - 3\cos(fx+e)\,a^2 fx + 6\sin(fx+e)^3 ab + 2\sin(fx+e)^3 b^2 - 6\sin(\ldots}{3\cos(fx+e)\,f\left(\sin(fx+e)^2-1\right)}$$

input
```
int((a+b*sec(f*x+e)^2)^2,x)
```

output
```
(3*cos(e + f*x)*sin(e + f*x)**2*a**2*f*x - 3*cos(e + f*x)*a**2*f*x + 6*sin
(e + f*x)**3*a*b + 2*sin(e + f*x)**3*b**2 - 6*sin(e + f*x)*a*b - 3*sin(e +
 f*x)*b**2)/(3*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.25 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a+b)^2\cot(e+fx)}{f} + \frac{2b(a+b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

output
```
-(a+b)^2*cot(f*x+e)/f+2*b*(a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(50) = 100.

Time = 1.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.18

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{4\left(b+a\cos^2(e+fx)\right)^2\sec^3(e+fx)\left(b^2\sec(e)\sin(fx)+\cos^2(e+fx)\left(3(a+b)^2\cot(e+fx)\csc(e)+b(6\right.\right.}{3f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(b^2*Sec[e]*Sin[f*x] + Cos[e +
f*x]^2*(3*(a + b)^2*Cot[e + f*x]*Csc[e] + b*(6*a + 5*b)*Sec[e])*Sin[f*x] +
 b^2*Cos[e + f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2 d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left((a+b)^2\cot^2(e+fx)+b^2\tan^2(e+fx)+2b(a+b)\right) d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b(a+b)\tan(e+fx)-(a+b)^2\cot(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-((a + b)^2*Cot[e + f*x]) + 2*b*(a + b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^
3)/3)/f
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{-a^2\cot(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+}{3}\right.}{f}$
default	$\frac{-a^2\cot(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+}{3}\right.}{f}$
risch	$-\frac{2i\left(3a^2e^{6i(fx+e)}+9a^2e^{4i(fx+e)}+12ab\,e^{4i(fx+e)}+9a^2e^{2i(fx+e)}+24ab\,e^{2i(fx+e)}+16b^2e^{2i(fx+e)}+3a^2+12ab+8b^2\right)}{3f\left(e^{2i(fx+e)}+1\right)^3\left(e^{2i(fx+e)}-1\right)}$
parallelrisch	$\frac{\left((a+b)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-4(a+3b)(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(6a^2+28ab+\frac{50}{3}b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-4(a+3b)(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
norman	$\frac{\frac{a^2+2ab+b^2}{2f}+\frac{\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{2f}-\frac{2\left(a^2+4ab+3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}-\frac{2\left(a^2+4ab+3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{f}+\frac{(9a^2+42ab+25b}{3}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*cot(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e))+b^2*(1/3
/sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=-\frac{(3a^2+12ab+8b^2)\cos(fx+e)^4-2(3ab+2b^2)\cos(fx+e)^2-b^2}{3f\cos(fx+e)^3\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 2*b^2)*cos(f*x
+ e)^2 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2(e+fx)\right)^2\csc^2(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \csc^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{b^2\tan\left(fx+e\right)^3+6\left(ab+b^2\right)\tan\left(fx+e\right)-\frac{3\left(a^2+2\,ab+b^2\right)}{\tan(fx+e)}}{3\,f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 6*(a*b + b^2)*tan(f*x + e) - 3*(a^2 + 2*a*b + b^
2)/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \csc^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{b^2\tan\left(fx+e\right)^3+6\,ab\tan\left(fx+e\right)+6\,b^2\tan\left(fx+e\right)-\frac{3\left(a^2+2\,ab+b^2\right)}{\tan(fx+e)}}{3\,f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 6*b^2*tan(f*x + e) - 3*(a^2
 + 2*a*b + b^2)/tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{a^2+2\,a\,b+b^2}{f\tan\left(e+f\,x\right)}+\frac{2\,b\tan(e+f\,x)\,\left(a+b\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^2,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - (2*a*b + a^2 + b^2)/(f*tan(e + f*x)) + (2*b*t
an(e + f*x)*(a + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.48

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\sin\left(fx+e\right)^4a^2+12\sin\left(fx+e\right)^4ab+8\sin\left(fx+e\right)^4b^2-6\sin\left(fx+e\right)^2a^2-18\sin\left(fx+e\right)^2ab-1}{3\cos\left(fx+e\right)\sin\left(fx+e\right)f\left(\sin\left(fx+e\right)^2-1\right)}$$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(3*sin(e + f*x)**4*a**2 + 12*sin(e + f*x)**4*a*b + 8*sin(e + f*x)**4*b**2
- 6*sin(e + f*x)**2*a**2 - 18*sin(e + f*x)**2*a*b - 12*sin(e + f*x)**2*b**
2 + 3*a**2 + 6*a*b + 3*b**2)/(3*cos(e + f*x)*sin(e + f*x)*f*(sin(e + f*x)*
*2 - 1))
```

3.26 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a+b)(a+3b)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f} + \frac{b(2a+3b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

output
```
-(a+b)*(a+3*b)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f+b*(2*a+3*b)*tan(f*x
+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 3.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.99

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\csc(2e)\csc^3(2(e+fx))\left(8a(a+2b)\sin(2e)-6(a+2b)^2\sin(2fx)-3a^2\sin(2(e+fx))-6ab\sin(2(e\cdots\right.}{}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/6*(Csc[2*e]*Csc[2*(e + f*x)]^3*(8*a*(a + 2*b)*Sin[2*e] - 6*(a + 2*b)^2*
Sin[2*f*x] - 3*a^2*Sin[2*(e + f*x)] - 6*a*b*Sin[2*(e + f*x)] + a^2*Sin[6*(
e + f*x)] + 2*a*b*Sin[6*(e + f*x)] + 3*a^2*Sin[4*e + 2*f*x] + a^2*Sin[4*e
+ 6*f*x] + 8*a*b*Sin[4*e + 6*f*x] + 8*b^2*Sin[4*e + 6*f*x]))/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left((a+b)^2\cot^4(e+fx)+(a+b)(a+3b)\cot^2(e+fx)+b^2\tan^2(e+fx)+b(2a+3b)\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(2a+3b)\tan(e+fx)-\frac{1}{3}(a+b)^2\cot^3(e+fx)-(a+b)(a+3b)\cot(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-((a + b)*(a + 3*b)*Cot[e + f*x]) - ((a + b)^2*Cot[e + f*x]^3)/3 + b*(2*a
 + 3*b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

method	result
risch	$\frac{4i\left(3a^2 e^{8i(fx+e)}+8a^2 e^{6i(fx+e)}+16ab\, e^{6i(fx+e)}+6a^2 e^{4i(fx+e)}+24ab\, e^{4i(fx+e)}+24b^2 e^{4i(fx+e)}-a^2-8ab-8b^2\right)}{3f\left(e^{2i(fx+e)}+1\right)^3\left(e^{2i(fx+e)}-1\right)^3}$
parallelrisch	$\frac{(-8a^2-16ab-24\cos(2fx+2e)ab-9\cos(2fx+2e)a^2+8b^2\cos(6fx+6e)-24\cos(2fx+2e)b^2+a^2\cos(6fx+6e)+8ab\cos(}{96f(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+b^2\left(\frac{}{3\sin(fx+e)^3}\right.}{f}$
default	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+b^2\left(\frac{}{3\sin(fx+e)^3}\right.}{f}$
norman	$\frac{\frac{a^2+2ab+b^2}{24f}+\frac{(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{24f}+\frac{(a^2+6ab+5b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{4f}+\frac{(a^2+6ab+5b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{4f}-\frac{(11a^2+86ab+91}{}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right.}$

input

```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
4/3*I*(3*a^2*exp(8*I*(f*x+e))+8*a^2*exp(6*I*(f*x+e))+16*a*b*exp(6*I*(f*x+e
))+6*a^2*exp(4*I*(f*x+e))+24*a*b*exp(4*I*(f*x+e))+24*b^2*exp(4*I*(f*x+e))-
a^2-8*a*b-8*b^2)/f/(exp(2*I*(f*x+e))+1)^3/(exp(2*I*(f*x+e))-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{2\,(a^2+8\,ab+8\,b^2)\cos(fx+e)^6-3\,(a^2+8\,ab+8\,b^2)\cos(fx+e)^4+6\,(ab+b^2)\cos(fx+e)^2+b^2}{3\left(f\cos(fx+e)^5-f\cos(fx+e)^3\right)\sin(fx+e)}$$

input

```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output

```
-1/3*(2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^6 - 3*(a^2 + 8*a*b + 8*b^2)*cos
(f*x + e)^4 + 6*(a*b + b^2)*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*c
os(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\tan\left(fx+e\right)^3+3\left(2\,ab+3\,b^2\right)\tan\left(fx+e\right)-\frac{3\left(a^2+4\,ab+3\,b^2\right)\tan(fx+e)^2+a^2+2\,ab+b^2}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + 3*b^2)*tan(f*x + e) - (3*(a^2 + 4*a*b
 + 3*b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2)/tan(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\tan\left(fx+e\right)^3+6\,ab\tan\left(fx+e\right)+9\,b^2\tan\left(fx+e\right)-\frac{3\,a^2\tan(fx+e)^2+12\,ab\tan(fx+e)^2+9\,b^2\tan(fx+e)^2+a^2+2\,ab}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e) - (3*a^2
*tan(f*x + e)^2 + 12*a*b*tan(f*x + e)^2 + 9*b^2*tan(f*x + e)^2 + a^2 + 2*a
*b + b^2)/tan(f*x + e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\frac{2\,a\,b}{3}+\tan(e+f\,x)^2\,\left(a^2+4\,a\,b+3\,b^2\right)+\frac{a^2}{3}+\frac{b^2}{3}}{f\,\tan\left(e+f\,x\right)^3}+\frac{b\,\tan(e+f\,x)\,\left(2\,a+3\,b\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^4,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/3 + tan(e + f*x)^2*(4*a*b + a^2 + 3*
b^2) + a^2/3 + b^2/3)/(f*tan(e + f*x)^3) + (b*tan(e + f*x)*(2*a + 3*b))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.91

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2\sin\left(fx+e\right)^6a^2+16\sin\left(fx+e\right)^6ab+16\sin\left(fx+e\right)^6b^2-3\sin\left(fx+e\right)^4a^2-24\sin\left(fx+e\right)^4ab-}{3\cos\left(fx+e\right)\sin\left(fx+e\right)^3f\left(\sin\left(fx+\right.\right.}$$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(2*sin(e + f*x)**6*a**2 + 16*sin(e + f*x)**6*a*b + 16*sin(e + f*x)**6*b**2
 - 3*sin(e + f*x)**4*a**2 - 24*sin(e + f*x)**4*a*b - 24*sin(e + f*x)**4*b*
*2 + 6*sin(e + f*x)**2*a*b + 6*sin(e + f*x)**2*b**2 + a**2 + 2*a*b + b**2)
/(3*cos(e + f*x)*sin(e + f*x)**3*f*(sin(e + f*x)**2 - 1))
```

3.27 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a^2+6ab+6b^2)\cot(e+fx)}{f} - \frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f} - \frac{(a+b)^2\cot^5(e+fx)}{5f} + \frac{2b(a+2b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

output
```
-(a^2+6*a*b+6*b^2)*cot(f*x+e)/f-2/3*(a+b)*(a+2*b)*cot(f*x+e)^3/f-1/5*(a+b)
^2*cot(f*x+e)^5/f+2*b*(a+2*b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 353 vs. $2(103) = 206$.

Time = 1.87 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.43

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx =$$
$$-\frac{\csc(e)\csc^5(e+fx)\sec(e)\sec^3(e+fx)\left(20a(5a+12b)\sin(2e)-32(2a^2+9ab+12b^2)\sin(2fx)-24a^2\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/1920*(Csc[e]*Csc[e + f*x]^5*Sec[e]*Sec[e + f*x]^3*(20*a*(5*a + 12*b)*Si
n[2*e] - 32*(2*a^2 + 9*a*b + 12*b^2)*Sin[2*f*x] - 24*a^2*Sin[2*(e + f*x)]
- 108*a*b*Sin[2*(e + f*x)] - 54*b^2*Sin[2*(e + f*x)] + 8*a^2*Sin[4*(e + f*
x)] + 36*a*b*Sin[4*(e + f*x)] + 18*b^2*Sin[4*(e + f*x)] + 8*a^2*Sin[6*(e +
 f*x)] + 36*a*b*Sin[6*(e + f*x)] + 18*b^2*Sin[6*(e + f*x)] - 4*a^2*Sin[8*(
e + f*x)] - 18*a*b*Sin[8*(e + f*x)] - 9*b^2*Sin[8*(e + f*x)] + 8*a^2*Sin[2
*(e + 2*f*x)] + 96*a*b*Sin[2*(e + 2*f*x)] + 128*b^2*Sin[2*(e + 2*f*x)] + 4
0*a^2*Sin[4*e + 2*f*x] + 8*a^2*Sin[4*e + 6*f*x] + 96*a*b*Sin[4*e + 6*f*x]
+ 128*b^2*Sin[4*e + 6*f*x] - 4*a^2*Sin[6*e + 8*f*x] - 48*a*b*Sin[6*e + 8*f
*x] - 64*b^2*Sin[6*e + 8*f*x]))/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^6}dx$$

$\downarrow$ 4620

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2d\tan(e+fx)}{f}$$

$\downarrow$ 355

$$\frac{\int \left((a+b)^2\cot^6(e+fx)+2(a+b)(a+2b)\cot^4(e+fx)+\left(a^2+6ba+6b^2\right)\cot^2(e+fx)+b^2\tan^2(e+fx)+2b\right.}{f}$$

$\downarrow$ 2009

$$\frac{-\left(a^2+6ab+6b^2\right)\cot(e+fx)+2b(a+2b)\tan(e+fx)-\frac{1}{5}(a+b)^2\cot^5(e+fx)-\frac{2}{3}(a+b)(a+2b)\cot^3(e+f}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-((a^2 + 6*a*b + 6*b^2)*Cot[e + f*x]) - (2*(a + b)*(a + 2*b)*Cot[e + f*x]
^3)/3 - ((a + b)^2*Cot[e + f*x]^5)/5 + 2*b*(a + 2*b)*Tan[e + f*x] + (b^2*T
an[e + f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

method	result
parallelrisch	$\frac{\left(\left(-13a^2-36ab-48b^2\right)\cos(2fx+2e)+\left(a^2+12ab+16b^2\right)\cos(4fx+4e)+\left(a^2+12ab+16b^2\right)\cos(6fx+6e)+\left(-\frac{1}{2}a^2-6ab-\right.\right.}{960f(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{a^2\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)+2ab\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos}\right.}{}$
default	$\frac{a^2\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)+2ab\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos}\right.}{}$
risch	$-\frac{16i\left(10a^2e^{10i(fx+e)}+25a^2e^{8i(fx+e)}+60ab\,e^{8i(fx+e)}+16a^2e^{6i(fx+e)}+72ab\,e^{6i(fx+e)}+96b^2e^{6i(fx+e)}-2a^2e^{4i(fx+e)}\right.}{15f\left(e^{2i(fx+e)}+1\right)^3\left(e^{2i(fx\right.}}$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/960*((-13*a^2-36*a*b-48*b^2)*cos(2*f*x+2*e)+(a^2+12*a*b+16*b^2)*cos(4*f*
x+4*e)+(a^2+12*a*b+16*b^2)*cos(6*f*x+6*e)+(-1/2*a^2-6*a*b-8*b^2)*cos(8*f*x
+8*e)-25/2*a^2-30*a*b)*csc(1/2*f*x+1/2*e)^5*sec(1/2*f*x+1/2*e)^5/f/(cos(3*
f*x+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{8\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^8-20\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2+12\,ab+16\,b^2\right)\cos}{15\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)\sin}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*(8*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^8 - 20*(a^2 + 12*a*b + 16*b^
2)*cos(f*x + e)^6 + 15*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^4 - 10*(3*a*b
+ 4*b^2)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 +
 f*cos(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{5\,b^2\tan\left(fx+e\right)^3 + 30\,(ab+2\,b^2)\tan\left(fx+e\right) - \frac{15\,(a^2+6\,ab+6\,b^2)\tan(fx+e)^4+10\,(a^2+3\,ab+2\,b^2)\tan(fx+e)^2+3\,a^2+6\,a}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + 2*b^2)*tan(f*x + e) - (15*(a^2 + 6*
a*b + 6*b^2)*tan(f*x + e)^4 + 10*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*
a^2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.37

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{5\,b^2\tan\left(fx+e\right)^3+30\,ab\tan\left(fx+e\right)+60\,b^2\tan\left(fx+e\right)-\frac{15\,a^2\tan(fx+e)^4+90\,ab\tan(fx+e)^4+90\,b^2\tan(fx+e)^4+}{}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 60*b^2*tan(f*x + e) - (
15*a^2*tan(f*x + e)^4 + 90*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 + 10
*a^2*tan(f*x + e)^2 + 30*a*b*tan(f*x + e)^2 + 20*b^2*tan(f*x + e)^2 + 3*a^
2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 12.82 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\tan(e+f\,x)^3}{3\,f}$$
$$-\frac{\frac{2\,a\,b}{5}+\tan(e+f\,x)^4\,\left(a^2+6\,a\,b+6\,b^2\right)+\frac{a^2}{5}+\frac{b^2}{5}+\tan(e+f\,x)^2\,\left(\frac{2\,a^2}{3}+2\,a\,b+\frac{4\,b^2}{3}\right)}{f\tan\left(e+f\,x\right)^5}$$
$$+\frac{2\,b\tan(e+f\,x)\,\left(a+2\,b\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^6,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/5 + tan(e + f*x)^4*(6*a*b + a^2 + 6*
b^2) + a^2/5 + b^2/5 + tan(e + f*x)^2*(2*a*b + (2*a^2)/3 + (4*b^2)/3))/(f*
tan(e + f*x)^5) + (2*b*tan(e + f*x)*(a + 2*b))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.94

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{8\sin(fx+e)^8a^2+96\sin(fx+e)^8ab+128\sin(fx+e)^8b^2-12\sin(fx+e)^6a^2-144\sin(fx+e)^6ab}{}$$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(8*sin(e + f*x)**8*a**2 + 96*sin(e + f*x)**8*a*b + 128*sin(e + f*x)**8*b**
2 - 12*sin(e + f*x)**6*a**2 - 144*sin(e + f*x)**6*a*b - 192*sin(e + f*x)**
6*b**2 + 3*sin(e + f*x)**4*a**2 + 36*sin(e + f*x)**4*a*b + 48*sin(e + f*x)
**4*b**2 - 2*sin(e + f*x)**2*a**2 + 6*sin(e + f*x)**2*a*b + 8*sin(e + f*x)
**2*b**2 + 3*a**2 + 6*a*b + 3*b**2)/(15*cos(e + f*x)*sin(e + f*x)**5*f*(si
n(e + f*x)**2 - 1))
```

3.28 $\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}(a+b)^2\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f} - \frac{(a+b)^2\cos(e+fx)}{a^3 f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2 f} - \frac{\cos^5(e+fx)}{5af}$$

output
```
b^(1/2)*(a+b)^2*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(7/2)/f-(a+b)^2*cos(f
*x+e)/a^3/f+1/3*(2*a+b)*cos(f*x+e)^3/a^2/f-1/5*cos(f*x+e)^5/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 425, normalized size of antiderivative = 4.34

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\left(15(5a^3+64a^2b+128ab^2+64b^3)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)}{\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*(15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3
)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Ta
n[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan
[(f*x)/2]))/Sqrt[b]] + 15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)*ArcTan[(
(-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2])
)/Sqrt[b]] - 75*a^3*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b
]] - 75*a^3*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 8*S
qrt[a]*Sqrt[b]*Cos[e + f*x]*(89*a^2 + 220*a*b + 120*b^2 - 4*a*(7*a + 5*b)*
Cos[2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)]))*Sec[e + f*x]^2)/(1920*a^(7/2)*
Sqrt[b]*f*(a + b*Sec[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4621, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^5}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cos^2(e+fx)\left(1-\cos^2(e+fx)\right)^2}{a\cos^2(e+fx)+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$-\frac{\int \left(\frac{\cos^4(e+fx)}{a}-\frac{(2a+b)\cos^2(e+fx)}{a^2}+\frac{(a+b)^2}{a^3}+\frac{-b^3-2ab^2-a^2b}{a^3\left(a\cos^2(e+fx)+b\right)}\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\sqrt{b}(a+b)^2\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}}+\frac{(a+b)^2\cos(e+fx)}{a^3}-\frac{(2a+b)\cos^3(e+fx)}{3a^2}+\frac{\cos^5(e+fx)}{5a}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((Sqrt[b]*(a + b)^2*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/a^(7/2)) +
 ((a + b)^2*Cos[e + f*x])/a^3 - ((2*a + b)*Cos[e + f*x]^3)/(3*a^2) + Cos[e
 + f*x]^5/(5*a))/f)
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{a\cos(fx+e)^3b}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)+b^2\cos(fx+e)}{a^3}+\frac{b(a^2+2ab+b^2)\arctan\left(\frac{a\cos(}{\sqrt{}}\right.}{a^3\sqrt{ab}}}{f}$
default	$\frac{-\frac{\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{a\cos(fx+e)^3b}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)+b^2\cos(fx+e)}{a^3}+\frac{b(a^2+2ab+b^2)\arctan\left(\frac{a\cos(}{\sqrt{}}\right.}{a^3\sqrt{ab}}}{f}$
risch	$-\frac{5\,e^{i(fx+e)}}{16af}-\frac{7\,e^{i(fx+e)}b}{8a^2f}-\frac{e^{i(fx+e)}b^2}{2a^3f}-\frac{5\,e^{-i(fx+e)}}{16af}-\frac{7\,e^{-i(fx+e)}b}{8a^2f}-\frac{e^{-i(fx+e)}b^2}{2a^3f}-\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e}\right.}{2}$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-1/3*a*cos(f*x+e)^3*
b+a^2*cos(f*x+e)+2*a*b*cos(f*x+e)+b^2*cos(f*x+e))+b*(a^2+2*a*b+b^2)/a^3/(a
*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.34

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[-\frac{6\,a^2\cos(fx+e)^5-10\,(2\,a^2+ab)\cos(fx+e)^3-15\,(a^2+2\,ab+b^2)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\,c}{a\cos(fx+e)^2-}\right.}{30\,a^3f}\right.$$
$$-\frac{3\,a^2\cos(fx+e)^5-5\,(2\,a^2+ab)\cos(fx+e)^3-15\,(a^2+2\,ab+b^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)+15\,(}{15\,a^3f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/30*(6*a^2*cos(f*x + e)^5 - 10*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2 +
 2*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x +
 e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(a^2 + 2*a*b + b^2)*cos(f*x + e))/(a
^3*f), -1/15*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(
a^2 + 2*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(a^2
+ 2*a*b + b^2)*cos(f*x + e))/(a^3*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\,(a^2b+2\,ab^2+b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{3\,a^2\cos(fx+e)^5-5\,(2\,a^2+ab)\cos(fx+e)^3+15\,(a^2+2\,ab+b^2)\cos(fx+e)}{a^3}}{15\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*
b)*a^3) - (3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 + 15*(a^2
 + 2*a*b + b^2)*cos(f*x + e))/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a^2b+2\,ab^2+b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3f} - \frac{3\,a^4f^4\cos\left(fx+e\right)^5 - 10\,a^4f^4\cos\left(fx+e\right)^3 - 5\,a^3bf^4\cos\left(fx+e\right)^3 + 15\,a^4f^4\cos\left(fx+e\right) + 30\,a^3bf^4}{15\,a^5f^5}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(a^2*b + 2*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f)
 - 1/15*(3*a^4*f^4*cos(f*x + e)^5 - 10*a^4*f^4*cos(f*x + e)^3 - 5*a^3*b*f^
4*cos(f*x + e)^3 + 15*a^4*f^4*cos(f*x + e) + 30*a^3*b*f^4*cos(f*x + e) + 1
5*a^2*b^2*f^4*cos(f*x + e))/(a^5*f^5)
```

Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\cos\left(e+f\,x\right)^3\left(\frac{b}{3\,a^2}+\frac{2}{3\,a}\right)}{f} - \frac{\cos\left(e+f\,x\right)^5}{5\,a\,f} - \frac{\cos\left(e+f\,x\right)\,\left(\frac{1}{a}+\frac{b\left(\frac{b}{a^2}+\frac{2}{a}\right)}{a}\right)}{f} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(a+b)^2}{a^2\,b+2\,a\,b^2+b^3}\right)\,(a+b)^2}{a^{7/2}\,f}$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

output
```
(cos(e + f*x)^3*(b/(3*a^2) + 2/(3*a)))/f - cos(e + f*x)^5/(5*a*f) - (cos(e
 + f*x)*(1/a + (b*(b/a^2 + 2/a))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*co
s(e + f*x)*(a + b)^2)/(2*a*b^2 + a^2*b + b^3))*(a + b)^2)/(a^(7/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.21

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)a^2-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)ab-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a}}{}\right.}{}$$

input

```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

output

```
( - 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*a**2 - 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*a*b - 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - s
qrt(a))/sqrt(b))*b**2 + 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)
/2) + sqrt(a))/sqrt(b))*a**2 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e
 + f*x)/2) + sqrt(a))/sqrt(b))*a*b + 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*
tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**2 - 3*cos(e + f*x)*sin(e + f*x)**4
*a**3 - 4*cos(e + f*x)*sin(e + f*x)**2*a**3 - 5*cos(e + f*x)*sin(e + f*x)*
*2*a**2*b - 8*cos(e + f*x)*a**3 - 25*cos(e + f*x)*a**2*b - 15*cos(e + f*x)
*a*b**2 + 8*a**3 + 25*a**2*b + 15*a*b**2)/(15*a**4*f)
```

3.29 $\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f} - \frac{(a+b)\cos(e+fx)}{a^2f} + \frac{\cos^3(e+fx)}{3af}$$

output
```
b^(1/2)*(a+b)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(5/2)/f-(a+b)*cos(f*x+e
)/a^2/f+1/3*cos(f*x+e)^3/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.30

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\left(3(a^2+8ab+8b^2)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}\right.}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]
*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]
] + 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e]
 - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] - S
qrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] + Sqrt[a + b
]*Tan[(e + f*x)/2])/Sqrt[b]] + 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(-5*a - 6*b
+ a*Cos[2*(e + f*x)]))*Sec[e + f*x]^2)/(48*a^(5/2)*Sqrt[b]*f*(a + b*Sec[e
+ f*x]^2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4621, 363, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^3}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cos^2(e+fx)\left(1-\cos^2(e+fx)\right)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{363}$$

$$-\frac{\frac{(a+b)\int \frac{\cos^2(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a}-\frac{\cos^3(e+fx)}{3a}}{f}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{(a+b)\left(\frac{\cos(e+fx)}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a}\right)}{a}-\frac{\cos^3(e+fx)}{3a}}{f}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{(a+b)\left(\frac{\cos(e+fx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}-\frac{\cos^3(e+fx)}{3a}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-1/3*Cos[e + f*x]^3/a + ((a + b)*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*
x])/Sqrt[b]])/a^(3/2)) + Cos[e + f*x]/a))/a)/f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-\cos(fx+e)b}{a^2}+\frac{b(a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
default	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-\cos(fx+e)b}{a^2}+\frac{b(a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
risch	$-\frac{3\,\mathrm{e}^{i(fx+e)}}{8af}-\frac{\mathrm{e}^{i(fx+e)}b}{2a^2f}-\frac{3\,\mathrm{e}^{-i(fx+e)}}{8af}-\frac{\mathrm{e}^{-i(fx+e)}b}{2a^2f}+\frac{i\sqrt{ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{ab}\,\mathrm{e}^{i(fx+e)}}{a}+1\right)}{2a^2f}+\frac{i\sqrt{ab}\,\ln(}{}$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^2*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-cos(f*x+e)*b)+b*(a+b)/a^2/(a*b
)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.17

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[\frac{2\,a\cos\left(fx+e\right)^3+3\,(a+b)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-6\,(a+b)\cos\left(fx+e\right)}{6\,a^2f},\ \frac{a\cos(}{}\right.$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/6*(2*a*cos(f*x + e)^3 + 3*(a + b)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2
*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(a + b)*cos(f*
x + e))/(a^2*f), 1/3*(a*cos(f*x + e)^3 + 3*(a + b)*sqrt(b/a)*arctan(a*sqrt
(b/a)*cos(f*x + e)/b) - 3*(a + b)*cos(f*x + e))/(a^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{3\,(ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{a\cos(fx+e)^3-3\,(a+b)\cos(fx+e)}{a^2}}{3\,f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*(a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2) + (a*c
os(f*x + e)^3 - 3*(a + b)*cos(f*x + e))/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2f} + \frac{a^2f^2\cos{(fx+e)}^3 - 3\,a^2f^2\cos{(fx+e)} - 3\,abf^2\cos{(fx+e)}}{3\,a^3f^3}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) + 1/3*(a^2*
f^2*cos(f*x + e)^3 - 3*a^2*f^2*cos(f*x + e) - 3*a*b*f^2*cos(f*x + e))/(a^3
*f^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\cos{(e+f\,x)}^3}{3\,a\,f} - \frac{\cos{(e+f\,x)}\,\left(\frac{b}{a^2}+\frac{1}{a}\right)}{f} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(a+b)}{b^2+a\,b}\right)\,(a+b)}{a^{5/2}\,f}$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

output
```
cos(e + f*x)^3/(3*a*f) - (cos(e + f*x)*(b/a^2 + 1/a))/f + (b^(1/2)*atan((a
^(1/2)*b^(1/2)*cos(e + f*x)*(a + b))/(a*b + b^2))*(a + b))/(a^(5/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.56

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)a-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)b+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\tan\left(\right.}{\right.}\right.}{}$$

input

```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

output

```
( - 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*a - 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqr
t(b))*b + 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/
sqrt(b))*a + 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a
))/sqrt(b))*b - cos(e + f*x)*sin(e + f*x)**2*a**2 - 2*cos(e + f*x)*a**2 -
3*cos(e + f*x)*a*b + 2*a**2 + 3*a*b)/(3*a**3*f)
```

3.30 $\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)} \, dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f} - \frac{\cos(e+fx)}{af}$$

output
```
b^(1/2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(3/2)/f-cos(f*x+e)/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 329, normalized size of antiderivative = 7.00

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)} \, dx = \frac{\left((a+4b)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right) + (a +$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
(((a + 4*b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])
*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (a + 4*b)*ArcTan[((-Sqrt[a] + I*Sqrt[a +
b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sq
rt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - a*ArcTan[(
Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - a*ArcTan[(Sqrt[a] + Sqr
t[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x])*(a +
 2*b + a*Cos[2*(e + f*x)]))/(8*a^(3/2)*Sqrt[b]*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4621, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cos^2(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{\cos(e+fx)}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a}}{f}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\cos(e+fx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/a^(3/2)) + Cos[e + f
*x]/a)/f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{b\sec(fx+e)}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{a\sec(fx+e)}}{f}$
default	$\frac{-\frac{b\arctan\left(\frac{b\sec(fx+e)}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{a\sec(fx+e)}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2af}-\frac{e^{-i(fx+e)}}{2af}-\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a^2f}+\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a^2f}$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a/(a*b)^(1/2)*arctan(b*sec(f*x+e)/(a*b)^(1/2))-1/a/sec(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.51

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-2\,\cos(fx+e)}{2\,af}, \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)-\cos(fx+e)}{af}\right]$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)
/(a*cos(f*x + e)^2 + b)) - 2*cos(f*x + e))/(a*f), (sqrt(b/a)*arctan(a*sqrt
(b/a)*cos(f*x + e)/b) - cos(f*x + e))/(a*f)]
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{\cos(fx+e)}{a}}{f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
(b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a) - cos(f*x + e)/a)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}af} - \frac{\cos{(fx+e)}}{af}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a*f) - cos(f*x + e)/(a*f)
```

Mupad [B] (verification not implemented)

Time = 12.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{a^{3/2}\,f} - \frac{\cos\left(e+f\,x\right)}{a\,f}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

output
```
(b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(a^(3/2)*f) - cos(e + f*x)/
(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)-\cos\left(fx+e\right)a}{a^2 f}$$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))
 + sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))
- cos(e + f*x)*a)/(a**2*f)
```

3.31 $\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)f}$$

output
```
b^(1/2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(1/2)/(a+b)/f-arctanh(cos(f*x
+e))/(a+b)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 239, normalized size of antiderivative = 4.35

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos \ldots}{\ldots}\right.}{\ldots}}{(a+\ldots}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
((Sqrt[b]*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqr
t[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a
] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[
a] - Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]])/((a + b)*f)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4621, 383, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \ 4621$$

$$-\frac{\int \frac{\cos^2(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \ 383$$

$$-\frac{\frac{\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}}{f}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}}{f}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{\operatorname{arctanh}(\cos(e+fx))}{a+b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a + b))) +
 ArcTanh[Cos[e + f*x]]/(a + b))/f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{\ln(1+\cos(fx+e))}{2a+2b}+\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}}{f}$
default	$\frac{-\frac{\ln(1+\cos(fx+e))}{2a+2b}+\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}}{f}$
risch	$\frac{\ln(e^{i(fx+e)}-1)}{f(a+b)}-\frac{\ln(e^{i(fx+e)}+1)}{f(a+b)}+\frac{i\sqrt{ab}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a(a+b)f}-\frac{i\sqrt{ab}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e^{i(fx}}{a}\right.}{2a(a+b)f}$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(2*a+2*b)*ln(1+cos(f*x+e))+1/(a+b)*b/(a*b)^(1/2)*arctan(a*cos(f*x+
e)/(a*b)^(1/2))+1/(2*a+2*b)*ln(-1+cos(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.84

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{2(a+b)f},\ 2\ldots\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)
/(a*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x
+ e) + 1/2))/((a + b)*f), 1/2*(2*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)
/b) - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/((a + b)
*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{\log(\cos(fx+e)+1)}{a+b} + \frac{\log(\cos(fx+e)-1)}{a+b}}{2\,f}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - log(cos(f*
x + e) + 1)/(a + b) + log(cos(f*x + e) - 1)/(a + b))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(af+bf)} - \frac{\log(|\cos(fx+e)+1|)}{2\,(af+bf)} + \frac{\log(|\cos(fx+e)-1|)}{2\,(af+bf)}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a*f + b*f)) - 1/2*log(abs(c
os(f*x + e) + 1))/(a*f + b*f) + 1/2*log(abs(cos(f*x + e) - 1))/(a*f + b*f)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.24

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\operatorname{atanh}\left(\frac{\cos(e+f\,x)\,\left(2\,a^3+2\,a\,b^2\right)-\frac{\cos(e+f\,x)\,\left(8\,a^5+8\,a^4\,b-8\,a^3\,b^2-8\,a^2\,b^3\right)}{4\,(a+b)^2}}{2\,a\,b\,(a+b)}\right)}{f\,(a+b)} - \frac{\operatorname{atanh}\left(\frac{\cos(e+f\,x)\,\sqrt{-a\,b}}{b}\right)\,\sqrt{-a\,b}}{f\,(a^2+b\,a)}$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)),x)
```

output
```
- atanh((cos(e + f*x)*(2*a*b^2 + 2*a^3) - (cos(e + f*x)*(8*a^4*b + 8*a^5 -
 8*a^2*b^3 - 8*a^3*b^2))/(4*(a + b)^2))/(2*a*b*(a + b)))/(f*(a + b)) - (at
anh((cos(e + f*x)*(-a*b)^(1/2))/b)*(-a*b)^(1/2))/(f*(a*b + a^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)+\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a}{af\,(a+b)}$$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))
 + sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))
+ log(tan((e + f*x)/2))*a)/(a*f*(a + b))
```

3.32 $\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^2 f} - \frac{(a-b)\text{arctanh}(\cos(e+fx))}{2(a+b)^2 f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f}$$

output
```
a^(1/2)*b^(1/2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/(a+b)^2/f-1/2*(a-b)*arc
tanh(cos(f*x+e))/(a+b)^2/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.31

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(-8\sqrt{a}\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\right.}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/16*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a
] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e
]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b
]] - 8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + a*Csc[(e + f*x)/2]^2 + b*Csc[(
e + f*x)/2]^2 + 4*a*Log[Cos[(e + f*x)/2]] - 4*b*Log[Cos[(e + f*x)/2]] - 4*
a*Log[Sin[(e + f*x)/2]] + 4*b*Log[Sin[(e + f*x)/2]] - a*Sec[(e + f*x)/2]^2
 - b*Sec[(e + f*x)/2]^2)*Sec[e + f*x]^2)/((a + b)^2*f*(a + b*Sec[e + f*x]^
2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4621, 373, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)}-\frac{\int \frac{b-a\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2(a+b)}}{f}$$

$$\Big\downarrow \; 397$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))}-\frac{\frac{2ab\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{(a-b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}}{f}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))}-\frac{\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{(a-b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}}{f}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))}-\frac{\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{(a-b)\text{arctanh}(\cos(e+fx))}{a+b}}{2(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-1/2*((2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a + b
) - ((a - b)*ArcTanh[Cos[e + f*x]])/(a + b))/(a + b) + Cos[e + f*x]/(2*(a
+ b)*(1 - Cos[e + f*x]^2)))/f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a+b)\ln(1+\cos(fx+e))}{4(a+b)^2}+\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(a-b)\ln(-1+\cos(fx+e))}{4(a+b)^2}+\frac{ab\arctan\left(\frac{a\cos(fx+e}{\sqrt{ab}}\right.}{(a+b)^2\sqrt{ab}}}{f}$
default	$\frac{\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a+b)\ln(1+\cos(fx+e))}{4(a+b)^2}+\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(a-b)\ln(-1+\cos(fx+e))}{4(a+b)^2}+\frac{ab\arctan\left(\frac{a\cos(fx+e}{\sqrt{ab}}\right.}{(a+b)^2\sqrt{ab}}}{f}$
risch	$\frac{e^{3i(fx+e)}+e^{i(fx+e)}}{f(a+b)\left(e^{2i(fx+e)}-1\right)^2}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{2f(a^2+2ab+b^2)}-\frac{\ln\left(e^{i(fx+e)}-1\right)b}{2f(a^2+2ab+b^2)}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{2f(a^2+2ab+b^2)}+\frac{\ln\left(e^{i(fx+e)}+1\right)b}{2f(a^2+2ab+b^2)}-\frac{i\sqrt{a}}{}$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(4*a+4*b)/(1+cos(f*x+e))+1/4/(a+b)^2*(-a+b)*ln(1+cos(f*x+e))+1/(4*a
+4*b)/(-1+cos(f*x+e))+1/4*(a-b)/(a+b)^2*ln(-1+cos(f*x+e))+a*b/(a+b)^2/(a*b
)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.80

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{2\sqrt{-ab}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{-ab}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)+2\left(a+b\right)\cos\left(fx+e\right)-\left(\left(a-b\right)\cos\right.}{4\left(\left(a^2+2\,ab+b^2\right)f\cos\left(f\right.\right.}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a
*b)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(a + b)*cos(f*x + e) - (
(a - b)*cos(f*x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos
(f*x + e)^2 - a + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*
cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f), 1/4*(4*sqrt(a*b)*(cos(f*x + e)^2
- 1)*arctan(sqrt(a*b)*cos(f*x + e)/b) + 2*(a + b)*cos(f*x + e) - ((a - b)*
cos(f*x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*x + e
)^2 - a + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(f*x
+ e)^2 - (a^2 + 2*a*b + b^2)*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{4\,ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} - \frac{(a-b)\log(\cos(fx+e)+1)}{a^2+2\,ab+b^2} + \frac{(a-b)\log(\cos(fx+e)-1)}{a^2+2\,ab+b^2} + \frac{2\cos(fx+e)}{(a+b)\cos(fx+e)^2-a-b}}{4\,f}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*(4*a*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)
) - (a - b)*log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) + (a - b)*log(cos(f*
x + e) - 1)/(a^2 + 2*a*b + b^2) + 2*cos(f*x + e)/((a + b)*cos(f*x + e)^2 -
 a - b))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.63

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^2f+2\,abf+b^2f)\sqrt{ab}} - \frac{(a-b)\log\left(|\cos(fx+e)+1|\right)}{4\,(a^2f+2\,abf+b^2f)} + \frac{(a-b)\log\left(|\cos(fx+e)-1|\right)}{4\,(a^2f+2\,abf+b^2f)} + \frac{\cos(fx+e)}{2\,(af+bf)\left(\cos(fx+e)^2-1\right)}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
a*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^2*f + 2*a*b*f + b^2*f)*sqrt(a*b))
 - 1/4*(a - b)*log(abs(cos(f*x + e) + 1))/(a^2*f + 2*a*b*f + b^2*f) + 1/4*
(a - b)*log(abs(cos(f*x + e) - 1))/(a^2*f + 2*a*b*f + b^2*f) + 1/2*cos(f*x
 + e)/((a*f + b*f)*(cos(f*x + e)^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.56

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{2\,a\,\cos\left(e+f\,x\right)+2\,b\,\cos\left(e+f\,x\right)-a\,\ln\left(\cos\left(e+f\,x\right)-1\right)+a\,\ln\left(\cos\left(e+f\,x\right)+1\right)+b\,\ln\left(\cos\left(e\right.\right.}{}$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

output
```
-(2*a*cos(e + f*x) + 2*b*cos(e + f*x) - atan((a^3*cos(e + f*x)*(-a*b)^(1/2
)*1i + a*b^2*cos(e + f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2
)*2i)/(a*b^3 + a^3*b + 2*a^2*b^2))*(-a*b)^(1/2)*4i - a*log(cos(e + f*x) -
1) + a*log(cos(e + f*x) + 1) + b*log(cos(e + f*x) - 1) - b*log(cos(e + f*x
) + 1) + cos(e + f*x)^2*atan((a^3*cos(e + f*x)*(-a*b)^(1/2)*1i + a*b^2*cos
(e + f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2)*2i)/(a*b^3 + a
^3*b + 2*a^2*b^2))*(-a*b)^(1/2)*4i + a*log(cos(e + f*x) - 1)*cos(e + f*x)^
2 - a*log(cos(e + f*x) + 1)*cos(e + f*x)^2 - b*log(cos(e + f*x) - 1)*cos(e
 + f*x)^2 + b*log(cos(e + f*x) + 1)*cos(e + f*x)^2)/(4*a^2*f + 4*b^2*f - 4
*a^2*f*cos(e + f*x)^2 - 4*b^2*f*cos(e + f*x)^2 + 8*a*b*f - 8*a*b*f*cos(e +
 f*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.88

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^2+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^2-}{2\sin\left(fx+e\right)^2 f}$$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**2 + 2*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2)
+ sqrt(a))/sqrt(b))*sin(e + f*x)**2 - cos(e + f*x)*a - cos(e + f*x)*b + lo
g(tan((e + f*x)/2))*sin(e + f*x)**2*a - log(tan((e + f*x)/2))*sin(e + f*x)
**2*b)/(2*sin(e + f*x)**2*f*(a**2 + 2*a*b + b**2))
```

3.33 $\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^3f} - \frac{(3a^2-6ab-b^2)\operatorname{arctanh}(\cos(e+fx))}{8(a+b)^3f} - \frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f}$$

output
```
a^(3/2)*b^(1/2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/(a+b)^3/f-1/8*(3*a^2-6*
a*b-b^2)*arctanh(cos(f*x+e))/(a+b)^3/f-1/8*(3*a-b)*cot(f*x+e)*csc(f*x+e)/(
a+b)^2/f-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 549, normalized size of antiderivative = 4.26

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{(a+2b+a\cos(2(e+fx)))\left(-64a^{3/2}\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a}\right.}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/128*((a + 2*b + a*Cos[2*(e + f*x)])*(-64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt
[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos
[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]] - 64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] -
 I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + 6*a^2*Csc[(e + f*x)/2]^2 +
4*a*b*Csc[(e + f*x)/2]^2 - 2*b^2*Csc[(e + f*x)/2]^2 + a^2*Csc[(e + f*x)/2]
^4 + 2*a*b*Csc[(e + f*x)/2]^4 + b^2*Csc[(e + f*x)/2]^4 + 24*a^2*Log[Cos[(e
 + f*x)/2]] - 48*a*b*Log[Cos[(e + f*x)/2]] - 8*b^2*Log[Cos[(e + f*x)/2]] -
 24*a^2*Log[Sin[(e + f*x)/2]] + 48*a*b*Log[Sin[(e + f*x)/2]] + 8*b^2*Log[S
in[(e + f*x)/2]] - 6*a^2*Sec[(e + f*x)/2]^2 - 4*a*b*Sec[(e + f*x)/2]^2 + 2
*b^2*Sec[(e + f*x)/2]^2 - a^2*Sec[(e + f*x)/2]^4 - 2*a*b*Sec[(e + f*x)/2]^
4 - b^2*Sec[(e + f*x)/2]^4)*Sec[e + f*x]^2)/((a + b)^3*f*(a + b*Sec[e + f*
x]^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4621, 373, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)^5\,(a+b\sec(e+fx)^2)}\,dx$$

$$\Big\downarrow \; 4621$$

$$-\frac{\int \frac{\cos^2(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)}\,d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 373$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\int \frac{b-3a\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)}\,d\cos(e+fx)}{4(a+b)}}{f}$$

$$\Big\downarrow \; 402$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\frac{\int \frac{b(5a+b)-a(3a-b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}\,d\cos(e+fx)}{2(a+b)}-\frac{(3a-b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)}}{4(a+b)}}{f}$$

$$\Big\downarrow \; 397$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\frac{\frac{8a^2b\int \frac{1}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{1-\cos^2(e+fx)}\,d\cos(e+fx)}{a+b}}{2(a+b)}-\frac{(3a-b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)}}{4(a+b)}}{f}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\frac{\frac{8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{1-\cos^2(e+fx)}\,d\cos(e+fx)}{a+b}}{2(a+b)}-\frac{(3a-b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)}}{4(a+b)}}{f}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\frac{\frac{8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\text{arctanh}(\cos(e+fx))}{a+b}}{2(a+b)}-\frac{(3a-b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)}}{4(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((Cos[e + f*x]/(4*(a + b)*(1 - Cos[e + f*x]^2)^2) - (((8*a^(3/2)*Sqrt[b]*
ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a + b) - ((3*a^2 - 6*a*b - b^2)*A
rcTanh[Cos[e + f*x]])/(a + b))/(2*(a + b)) - ((3*a - b)*Cos[e + f*x])/(2*(
a + b)*(1 - Cos[e + f*x]^2)))/(4*(a + b)))/f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{-3a+b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{(3a^2-6ab-b^2)\ln(-1+\cos(fx+e))}{16(a+b)^3}+\frac{1}{2(8a+8b)(1+\cos(fx+e))^2}-\ldots}{f}$
default	$\frac{-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{-3a+b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{(3a^2-6ab-b^2)\ln(-1+\cos(fx+e))}{16(a+b)^3}+\frac{1}{2(8a+8b)(1+\cos(fx+e))^2}-\ldots}{f}$
risch	$\frac{3a\,e^{7i(fx+e)}-b\,e^{7i(fx+e)}-11a\,e^{5i(fx+e)}-7b\,e^{5i(fx+e)}-11a\,e^{3i(fx+e)}-7b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)}-b\,e^{i(fx+e)}}{4f(a+b)^2(e^{2i(fx+e)}-1)^4}-\ldots$

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/(8*a+8*b)/(-1+cos(f*x+e))^2-1/16*(-3*a+b)/(a+b)^2/(-1+cos(f*x+e)
)+1/16*(3*a^2-6*a*b-b^2)/(a+b)^3*ln(-1+cos(f*x+e))+1/2/(8*a+8*b)/(1+cos(f*
x+e))^2-1/16*(-3*a+b)/(a+b)^2/(1+cos(f*x+e))+1/16/(a+b)^3*(-3*a^2+6*a*b+b^
2)*ln(1+cos(f*x+e))+a^2*b/(a+b)^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(115) = 230$.

Time = 0.15 (sec) , antiderivative size = 693, normalized size of antiderivative = 5.37

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3 + 8*(a*cos(f*x + e)^4 - 2*a*
cos(f*x + e)^2 + a)*sqrt(-a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*b)*cos(f
*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e
) - ((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*
x + e)^2 + 3*a^2 - 6*a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*
a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2
 - 6*a*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2
 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f), 1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f
*x + e)^3 + 16*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 + a)*sqrt(a*b)*arcta
n(sqrt(a*b)*cos(f*x + e)/b) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) - ((3*a
^2 - 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2
+ 3*a^2 - 6*a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b - b^2
)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a*b
- b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*co
s(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^5{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(115) = 230.

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.79

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{16\,a^2b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}} - \frac{(3\,a^2-6\,ab-b^2)\log(\cos(fx+e)+1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{(3\,a^2-6\,ab-b^2)\log(\cos(fx+e)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left((3\,a-b)\cos(fx+\right.}{(a^2+2\,ab+b^2)\cos(fx+e)^4-2\,(a}}{16\,f}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/16*(16*a^2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*sqrt(a*b)) - (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) + 1)/(a^3 + 3*a
^2*b + 3*a*b^2 + b^3) + (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) - 1)/(a^3 +
 3*a^2*b + 3*a*b^2 + b^3) + 2*((3*a - b)*cos(f*x + e)^3 - (5*a + b)*cos(f*
x + e))/((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 + 2*a*b + b^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(115) = 230$.

Time = 0.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.81

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{a^2b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3f+3\,a^2bf+3\,ab^2f+b^3f)\sqrt{ab}} - \frac{(3\,a^2-6\,ab-b^2)\log\left(|\cos\left(fx+e\right)+1|\right)}{16\,(a^3f+3\,a^2bf+3\,ab^2f+b^3f)} + \frac{(3\,a^2-6\,ab-b^2)\log\left(|\cos\left(fx+e\right)-1|\right)}{16\,(a^3f+3\,a^2bf+3\,ab^2f+b^3f)} + \frac{3\,a\cos\left(fx+e\right)^3-b\cos\left(fx+e\right)^3-5\,a\cos\left(fx+e\right)-b\cos\left(fx+e\right)}{8\,(a^2f+2\,abf+b^2f)\left(\cos\left(fx+e\right)^2-1\right)^2}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
a^2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3*f + 3*a^2*b*f + 3*a*b^2*f + b
^3*f)*sqrt(a*b)) - 1/16*(3*a^2 - 6*a*b - b^2)*log(abs(cos(f*x + e) + 1))/(
a^3*f + 3*a^2*b*f + 3*a*b^2*f + b^3*f) + 1/16*(3*a^2 - 6*a*b - b^2)*log(ab
s(cos(f*x + e) - 1))/(a^3*f + 3*a^2*b*f + 3*a*b^2*f + b^3*f) + 1/8*(3*a*co
s(f*x + e)^3 - b*cos(f*x + e)^3 - 5*a*cos(f*x + e) - b*cos(f*x + e))/((a^2
*f + 2*a*b*f + b^2*f)*(cos(f*x + e)^2 - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 870, normalized size of antiderivative = 6.74

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

output
```
(atan((a^5*cos(e + f*x)*(-a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^
(1/2)*12i + a^3*b^2*cos(e + f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(
-a^3*b)^(1/2)*1i + a^4*b*cos(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b
^5 + 12*a^3*b^4 + 30*a^4*b^3 + 28*a^5*b^2))*(-a^3*b)^(1/2)*8i - 5*a^2*cos(
e + f*x) - b^2*cos(e + f*x) + 3*a^2*cos(e + f*x)^3 - b^2*cos(e + f*x)^3 -
3*a^2*atanh(cos(e + f*x)) + b^2*atanh(cos(e + f*x)) - atan((a^5*cos(e + f*
x)*(-a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*b^2*c
os(e + f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i + a^
4*b*cos(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4 + 30*
a^4*b^3 + 28*a^5*b^2))*cos(e + f*x)^2*(-a^3*b)^(1/2)*16i + atan((a^5*cos(e
 + f*x)*(-a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*
b^2*cos(e + f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i
 + a^4*b*cos(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4
+ 30*a^4*b^3 + 28*a^5*b^2))*cos(e + f*x)^4*(-a^3*b)^(1/2)*8i - 6*a*b*cos(e
 + f*x) + 6*a^2*cos(e + f*x)^2*atanh(cos(e + f*x)) - 3*a^2*cos(e + f*x)^4*
atanh(cos(e + f*x)) - 2*b^2*cos(e + f*x)^2*atanh(cos(e + f*x)) + b^2*cos(e
 + f*x)^4*atanh(cos(e + f*x)) + 2*a*b*cos(e + f*x)^3 + 6*a*b*atanh(cos(e +
 f*x)) - 12*a*b*cos(e + f*x)^2*atanh(cos(e + f*x)) + 6*a*b*cos(e + f*x)^4*
atanh(cos(e + f*x)))/(8*a^3*f + 8*b^3*f - 16*a^3*f*cos(e + f*x)^2 + 8*a^3*
f*cos(e + f*x)^4 - 16*b^3*f*cos(e + f*x)^2 + 8*b^3*f*cos(e + f*x)^4 + 2...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.08

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{-8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^4 a+8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^4 a}{}$$

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 8*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**4*a + 8*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2
) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a - 3*cos(e + f*x)*sin(e + f*x)**2*a
**2 - 2*cos(e + f*x)*sin(e + f*x)**2*a*b + cos(e + f*x)*sin(e + f*x)**2*b*
*2 - 2*cos(e + f*x)*a**2 - 4*cos(e + f*x)*a*b - 2*cos(e + f*x)*b**2 + 3*lo
g(tan((e + f*x)/2))*sin(e + f*x)**4*a**2 - 6*log(tan((e + f*x)/2))*sin(e +
 f*x)**4*a*b - log(tan((e + f*x)/2))*sin(e + f*x)**4*b**2)/(8*sin(e + f*x)
**4*f*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

3.34 $\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(5a^3+30a^2b+40ab^2+16b^3)\,x}{16a^4} - \frac{\sqrt{b}(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f} - \frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af}$$

output
```
1/16*(5*a^3+30*a^2*b+40*a*b^2+16*b^3)*x/a^4-b^(1/2)*(a+b)^(5/2)*arctan(b^(
1/2)*tan(f*x+e)/(a+b)^(1/2))/a^4/f-1/16*(11*a^2+18*a*b+8*b^2)*cos(f*x+e)*s
in(f*x+e)/a^3/f+1/8*(3*a+2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*cos(f*x+e)
^3*sin(f*x+e)^3/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.15

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3\sqrt{b}(9a^4+136a^3b+384a^2b^2+384ab^3+128b^4)\arctan\left(\frac{\sec(fx}{}\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*Sqrt[b]*(9*a^4 + 136*a^3
*b + 384*a^2*b^2 + 384*a*b^3 + 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(
Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e
])^4]*(3*a^3*(9*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + 2*Sq
rt[b]*Sqrt[a + b]*(-12*a^3*e + 60*a^3*f*x + 360*a^2*b*f*x + 480*a*b^2*f*x
+ 192*b^3*f*x - 3*a*(15*a^2 + 32*a*b + 16*b^2)*Sin[2*(e + f*x)] + 3*a^2*(3
*a + 2*b)*Sin[4*(e + f*x)] - a^3*Sin[6*(e + f*x)]))))/(768*a^4*Sqrt[b]*Sqr
t[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4620, 372, 27, 440, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(e+fx)^6}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\int \frac{3\tan^2(e+fx)\left(-\left((2a+b)\tan^2(e+fx)\right)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{6a}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\int \frac{\tan^2(e+fx)\left(-\left((2a+b)\tan^2(e+fx)\right)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{f}$$

$$\Big\downarrow \; 440$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\int \frac{(a+b)(3a+2b)-\left(8a^2+13ba+6b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}-\frac{(3a+2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{2a}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{\left(11a^2+18ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{(a+b)(a+2b)(5a+4b)-b\left(11a^2+18ba+8b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{4a}-\frac{(3a+2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{2a}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{\left(11a^2+18ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(5a^3+30a^2b+40ab^2+16b^3\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{16b(a+b)^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e}{a}}{2a}}{4a}}{2a}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{(11a^2+18ab+8b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)}-\frac{\frac{(5a^3+30a^2b+40ab^2+16b^3)\arctan(\tan(e+fx))}{a}-\frac{16b(a+b)^3\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}}{4a}-\frac{(3}{4a}}{2a}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{(11a^2+18ab+8b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)}-\frac{\frac{(5a^3+30a^2b+40ab^2+16b^3)\arctan(\tan(e+fx))}{a}-\frac{16\sqrt{b}(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a}}{2a}}{4a}-\frac{(3a+2b}{4a(\tan^2}}{2a}}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3) - (-1/4*((3*a + 2*b)*Tan[e +
f*x])/(a*(1 + Tan[e + f*x]^2)^2) + (-1/2*(((5*a^3 + 30*a^2*b + 40*a*b^2 +
16*b^3)*ArcTan[Tan[e + f*x]])/a - (16*Sqrt[b]*(a + b)^(5/2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/a)/a + ((11*a^2 + 18*a*b + 8*b^2)*Tan[e + f*
x])/(2*a*(1 + Tan[e + f*x]^2)))/(4*a))/(2*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{9}{8}a^2b-\frac{1}{2}a\,b^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-2a^2b-a\,b^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{7}{8}a^2b-\frac{1}{2}a\,b^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{(5a^3+30a^2b+40a}{}}{a^4}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{9}{8}a^2b-\frac{1}{2}a\,b^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-2a^2b-a\,b^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{7}{8}a^2b-\frac{1}{2}a\,b^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{(5a^3+30a^2b+40a}{}}{a^4}}{f}$
risch	$\frac{5x}{16a}+\frac{15xb}{8a^2}+\frac{5x\,b^2}{2a^3}+\frac{x\,b^3}{a^4}+\frac{15i\mathrm{e}^{2i(fx+e)}}{128af}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{4a^2f}-\frac{i\mathrm{e}^{-2i(fx+e)}b^2}{8a^3f}+\frac{i\mathrm{e}^{2i(fx+e)}b}{4a^2f}-\frac{15i\mathrm{e}^{-2i(fx+}}{128af}$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(((-9/8*a^2*b-1/2*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-2*a^2*b-a*b^2
-5/6*a^3)*tan(f*x+e)^3+(-5/16*a^3-7/8*a^2*b-1/2*a*b^2)*tan(f*x+e))/(1+tan(
f*x+e)^2)^3+1/16*(5*a^3+30*a^2*b+40*a*b^2+16*b^3)*arctan(tan(f*x+e)))-b/a^
4*(a+b)^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.58

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[\frac{3\,(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3)fx+12\,(a^2+2\,ab+b^2)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4}{}\right.}{}\right.$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 12*(a^2 + 2*a*b + b^
2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b +
 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqr
t(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2)) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x + e)^3 +
 3*(11*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), 1/48
*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 24*(a^2 + 2*a*b + b^2)*sq
rt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*c
os(f*x + e)*sin(f*x + e))) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*
cos(f*x + e)^3 + 3*(11*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e
))/(a^4*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.26

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\frac{3\,(11\,a^2+18\,ab+8\,b^2)\tan(fx+e)^5+8\,(5\,a^2+12\,ab+6\,b^2)\tan(fx+e)^3+3\,(5\,a^2+14\,ab+8\,b^2)\tan(fx+e)}{a^3\tan(fx+e)^6+3\,a^3\tan(fx+e)^4+3\,a^3\tan(fx+e)^2+a^3}-\frac{3\,(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3)(f}{a^4}}{48\,f}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/48*((3*(11*a^2 + 18*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6
*b^2)*tan(f*x + e)^3 + 3*(5*a^2 + 14*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f
*x + e)^6 + 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) - 3*(5*a^3
+ 30*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 + 48*(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))
/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.43

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\frac{3\left(5a^3+30a^2b+40ab^2+16b^3\right)(fx+e)}{a^4} - \frac{48\left(a^3b+3a^2b^2+3ab^3+b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^4} - \frac{33a^2\tan(fx+e)^5+54}{}}{4}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 - 48*(a^3*b +
 3*a^2*b^2 + 3*a*b^3 + b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(
b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^4) - (33*a^2*tan(f*x +
 e)^5 + 54*a*b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e
)^3 + 96*a*b*tan(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e)
+ 42*a*b*tan(f*x + e) + 24*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^3))
/f
```

Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 1448, normalized size of antiderivative = 8.72

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

output
```
(atanh((25*b^3*tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3
*b^3 - 5*a^4*b^2)^(1/2))/(128*((227*a*b^5)/128 + (217*b^6)/128 + (119*a^2*
b^4)/128 + (25*a^3*b^3)/128 + (13*b^7)/(16*a) + (5*b^8)/(32*a^2))) + (11*b
^4*tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4
*b^2)^(1/2))/(32*((217*a*b^6)/128 + (13*b^7)/16 + (227*a^2*b^5)/128 + (119
*a^3*b^4)/128 + (25*a^4*b^3)/128 + (5*b^8)/(32*a))) + (5*b^5*tan(e + f*x)*
(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4*b^2)^(1/2))/(32
*((13*a*b^7)/16 + (5*b^8)/32 + (217*a^2*b^6)/128 + (227*a^3*b^5)/128 + (11
9*a^4*b^4)/128 + (25*a^5*b^3)/128)))*(-b*(a + b)^5)^(1/2))/(a^4*f) - (atan
((((((512*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9
) - (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a
^3*5i + b^3*16i))/(4096*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))
/(32*a^4) - (tan(e + f*x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*
b^6 + 5140*a^4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i +
a^2*b*30i + a^3*5i + b^3*16i)*1i)/(32*a^4) - (((((512*a^8*b^5 + 1408*a^9*b
^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9) + (tan(e + f*x)*(2048*a^8*b^3
 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(4096*a^10))*
(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4) + (tan(e + f*x)*(2816
*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^4*b^5 + 1836*a^5*b
^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.31

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{-48\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)a^2-96\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)ab-48\sqrt{b}\,\sqrt{a+}}{}$$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 48*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*a**2 - 96*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) -
 sqrt(a))/sqrt(b))*a*b - 48*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
 f*x)/2) - sqrt(a))/sqrt(b))*b**2 - 48*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 - 96*sqrt(b)*sqrt(a + b)*atan
((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b - 48*sqrt(b)*sqrt(a
 + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**2 - 8*cos(
e + f*x)*sin(e + f*x)**5*a**3 - 10*cos(e + f*x)*sin(e + f*x)**3*a**3 - 12*
cos(e + f*x)*sin(e + f*x)**3*a**2*b - 15*cos(e + f*x)*sin(e + f*x)*a**3 -
42*cos(e + f*x)*sin(e + f*x)*a**2*b - 24*cos(e + f*x)*sin(e + f*x)*a*b**2
+ 15*a**3*e + 15*a**3*f*x + 90*a**2*b*e + 90*a**2*b*f*x + 120*a*b**2*e + 1
20*a*b**2*f*x + 48*b**3*e + 48*b**3*f*x)/(48*a**4*f)
```

3.35 $\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(3a^2+12ab+8b^2)\,x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f} - \frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}$$

output
```
1/8*(3*a^2+12*a*b+8*b^2)*x/a^3-b^(1/2)*(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+
e)/(a+b)^(1/2))/a^3/f-1/8*(5*a+4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1/4*cos(f*
x+e)^3*sin(f*x+e)/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.59

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{b}(3a^3+34a^2b+64ab^2+32b^3)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(}{2\sqrt{a+b}\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[b]*(3*a^3 + 34*a^2*b
+ 64*a*b^2 + 32*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)
*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4]*(a^2*(3*a + 2
*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a + b]*(-2*a
^2*e + 12*a^2*f*x + 48*a*b*f*x + 32*b^2*f*x - 8*a*(a + b)*Sin[2*(e + f*x)]
 + a^2*Sin[4*(e + f*x)]))))/(64*a^3*Sqrt[b]*Sqrt[a + b]*f*(a + b*Sec[e + f
*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4620, 372, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)^4}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}-\frac{\int \frac{(b-4(a+b))\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{(a+b)(3a+4b)-b(5a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{8b(a+b)^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{8b(a+b)^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{8\sqrt{b}(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a}}{2a}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2) - (-1/2*(((3*a^2 + 12*a*b + 8*b
^2)*ArcTan[Tan[e + f*x]])/a - (8*Sqrt[b]*(a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan
[e + f*x])/Sqrt[a + b]])/a)/a + ((5*a + 4*b)*Tan[e + f*x])/(2*a*(1 + Tan[e
 + f*x]^2)))/(4*a))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{b(a+b)^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\frac{\left(-\frac{1}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
default	$\frac{-\frac{b(a+b)^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\frac{\left(-\frac{1}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
risch	$\frac{3x}{8a}+\frac{3xb}{2a^2}+\frac{x\,b^2}{a^3}+\frac{ie^{2i(fx+e)}}{8af}+\frac{ie^{2i(fx+e)}b}{8a^2f}-\frac{ie^{-2i(fx+e)}}{8af}-\frac{ie^{-2i(fx+e)}b}{8a^2f}+\frac{\sqrt{-ab-b^2}\,\ln\left(e^{2i(fx+e)}+\frac{2i}{}\right.}{2f\,a^2}$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^3*(a+b)^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1
/a^3*(((-1/2*a*b-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-1/2*a*b)*tan(f*x+e))/(1+t
an(f*x+e)^2)^2+1/8*(3*a^2+12*a*b+8*b^2)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.84

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[\frac{(3\,a^2+12\,ab+8\,b^2)fx+2\sqrt{-ab-b^2}(a+b)\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a+2\,b)\cos\right.}{a^2\cos(fx+e)^4+2\,ab\cos(fx+}\right.}{8\,a^3}\right.$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/8*((3*a^2 + 12*a*b + 8*b^2)*f*x + 2*sqrt(-a*b - b^2)*(a + b)*log(((a^2
+ 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a
 + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b
^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^2*cos(f*x +
e)^3 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/8*((3*a^2 +
12*a*b + 8*b^2)*f*x + 4*sqrt(a*b + b^2)*(a + b)*arctan(1/2*((a + 2*b)*cos(
f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + (2*a^2*cos(
f*x + e)^3 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{(5a+4b)\tan(fx+e)^3+(3a+4b)\tan(fx+e)}{a^2\tan(fx+e)^4+2a^2\tan(fx+e)^2+a^2} - \frac{(3a^2+12ab+8b^2)(fx+e)}{a^3} + \frac{8(a^2b+2ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3}}{8f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/8*(((5*a + 4*b)*tan(f*x + e)^3 + (3*a + 4*b)*tan(f*x + e))/(a^2*tan(f*x
 + e)^4 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 + 12*a*b + 8*b^2)*(f*x + e)
/a^3 + 8*(a^2*b + 2*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(s
qrt((a + b)*b)*a^3))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(3\,a^2+12\,ab+8\,b^2)(fx+e)}{a^3} - \frac{8\,(a^2b+2\,ab^2+b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^3} - \frac{5\,a\tan(fx+e)^3+4\,b\tan(fx+e)^3+3\,a\tan(f\cdots}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/8*((3*a^2 + 12*a*b + 8*b^2)*(f*x + e)/a^3 - 8*(a^2*b + 2*a*b^2 + b^3)*(p
i*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)
))/(sqrt(a*b + b^2)*a^3) - (5*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 3*a*
tan(f*x + e) + 4*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 12.75 (sec) , antiderivative size = 494, normalized size of antiderivative = 4.22

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}\left(\frac{9\,b^3\tan(e+f\,x)\sqrt{-a^3\,b-3\,a^2\,b^2-3\,a\,b^3-b^4}}{32\left(\frac{13\,a\,b^4}{16}+\frac{25\,b^5}{32}+\frac{9\,a^2\,b^3}{32}+\frac{b^6}{4\,a}\right)}+\frac{b^4\tan(e+f\,x)\sqrt{-a^3\,b-3\,a^2\,b^2-3\,a\,b^3-b^4}}{4\left(\frac{9\,a^3\,b^3}{32}+\frac{13\,a^2\,b^4}{16}+\frac{25\,a\,b^5}{32}+\frac{b^6}{4}\right)}\right)\sqrt{-b\,(a+b)^3}}{a^3\,f} - \frac{\frac{\tan(e+f\,x)\,(3\,a+4\,b)}{8\,a^2}+\frac{\tan(e+f\,x)^3\,(5\,a+4\,b)}{8\,a^2}}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)} - \frac{\operatorname{atan}\left(\frac{159\,b^3\tan(e+f\,x)}{256\left(\frac{27\,a\,b^2}{256}+\frac{159\,b^3}{256}+\frac{75\,b^4}{64\,a}+\frac{29\,b^5}{32\,a^2}+\frac{b^6}{4\,a^3}\right)}+\frac{75\,b^4\tan(e+f\,x)}{64\left(\frac{159\,a\,b^3}{256}+\frac{75\,b^4}{64}+\frac{27\,a^2\,b^2}{256}+\frac{29\,b^5}{32\,a}+\frac{b^6}{4\,a^2}\right)}+\frac{29\,b^5\tan(e+f\,x)}{32\left(\frac{75\,a\,b^4}{64}+\frac{29\,b^5}{32}+\frac{159\,a^2\,b^3}{256}+\frac{27\,a^3\,b^2}{256}\cdots\right)}\cdots\right)\cdots}{8\,a\cdots}$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

output
```
(atanh((9*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(3
2*((13*a*b^4)/16 + (25*b^5)/32 + (9*a^2*b^3)/32 + b^6/(4*a))) + (b^4*tan(e
 + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(4*((25*a*b^5)/32 + b
^6/4 + (13*a^2*b^4)/16 + (9*a^3*b^3)/32)))*(-b*(a + b)^3)^(1/2))/(a^3*f) -
 ((tan(e + f*x)*(3*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(5*a + 4*b))/(8*a^2
))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((159*b^3*tan(e + f*
x))/(256*((27*a*b^2)/256 + (159*b^3)/256 + (75*b^4)/(64*a) + (29*b^5)/(32*
a^2) + b^6/(4*a^3))) + (75*b^4*tan(e + f*x))/(64*((159*a*b^3)/256 + (75*b^
4)/64 + (27*a^2*b^2)/256 + (29*b^5)/(32*a) + b^6/(4*a^2))) + (29*b^5*tan(e
 + f*x))/(32*((75*a*b^4)/64 + (29*b^5)/32 + (159*a^2*b^3)/256 + (27*a^3*b^
2)/256 + b^6/(4*a))) + (b^6*tan(e + f*x))/(4*((29*a*b^5)/32 + b^6/4 + (75*
a^2*b^4)/64 + (159*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (27*b^2*tan(e + f*x
))/(256*((27*b^2)/256 + (159*b^3)/(256*a) + (75*b^4)/(64*a^2) + (29*b^5)/(
32*a^3) + b^6/(4*a^4))))*(a*b*12i + a^2*3i + b^2*8i)*1i)/(8*a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.97

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-8\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)a-8\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)b-8\sqrt{b}\,\sqrt{a+b}\,ata}{}$$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*a - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt
(a))/sqrt(b))*b - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2)
 + sqrt(a))/sqrt(b))*a - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) + sqrt(a))/sqrt(b))*b - 2*cos(e + f*x)*sin(e + f*x)**3*a**2 - 3*co
s(e + f*x)*sin(e + f*x)*a**2 - 4*cos(e + f*x)*sin(e + f*x)*a*b + 3*a**2*e
+ 3*a**2*f*x + 12*a*b*e + 12*a*b*f*x + 8*b**2*e + 8*b**2*f*x)/(8*a**3*f)
```

3.36 $\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b)x}{2a^2} - \frac{\sqrt{b}\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f} - \frac{\cos(e+fx)\sin(e+fx)}{2af}$$

output
```
1/2*(a+2*b)*x/a^2-b^(1/2)*(a+b)^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2
))/a^2/f-1/2*cos(f*x+e)*sin(f*x+e)/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.22

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f} - \frac{-4(a+2b)x-\frac{(a^2+8ab+8b^2)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2\ldots}{2\sqrt{a+b}\ldots}\right)}{\sqrt{a+b}f\sqrt{\ldots}}}{\ldots}\right.}{16\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f) - (-4*(a + 2*b)*x - ((a^2 + 8*a*b
 + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x])
+ a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2
*e] - I*Sin[2*e]))/(Sqrt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*a*Co
s[2*f*x]*Sin[2*e])/f + (2*a*Cos[2*e]*Sin[2*f*x])/f)/a^2))/(16*(a + b*Sec[e
 + f*x]^2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 373, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{a+b\sec(e+fx)^2}dx$$

$\Big\downarrow$ 4620

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 373

$$\frac{\frac{\int \frac{-b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{(a+2b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{2b(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{(a+2b)\arctan(\tan(e+fx))}{a}-\frac{2b(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{(a+2b)\arctan(\tan(e+fx))}{a}-\frac{2\sqrt{b}\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
((((a + 2*b)*ArcTan[Tan[e + f*x]])/a - (2*Sqrt[b]*Sqrt[a + b]*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a + b]])/a)/(2*a) - Tan[e + f*x]/(2*a*(1 + Tan[e +
f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{(a+b)b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
default	$\frac{-\frac{(a+b)b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
risch	$\frac{x}{2a}+\frac{xb}{a^2}+\frac{i\mathrm{e}^{2i(fx+e)}}{8af}-\frac{i\mathrm{e}^{-2i(fx+e)}}{8af}-\frac{\sqrt{-ab-b^2}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-ab-b^2}-a-2b}{a}\right)}{2f\,a^2}+\frac{\sqrt{-ab-b^2}\,\ln\left(\mathrm{e}^{2i(fx-\dots\right.}}{2\dots}$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-(a+b)*b/a^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a
^2*(-1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+2*b)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.38

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[\frac{2\,(a+2\,b)fx-2\,a\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e\dots}{a^2\cos(fx+\dots}\right.}{4\,a^2f}\right.$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a + 2*b)*f*x - 2*a*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b - b^2)*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x
 + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^2*f),
1/2*((a + 2*b)*f*x - a*cos(f*x + e)*sin(f*x + e) + sqrt(a*b + b^2)*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x +
 e))))/(a^2*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin^2{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(fx+e)(a+2b)}{a^2} - \frac{\tan(fx+e)}{a\tan(fx+e)^2+a} - \frac{2(ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2}}{2f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*((f*x + e)*(a + 2*b)/a^2 - tan(f*x + e)/(a*tan(f*x + e)^2 + a) - 2*(a*
b + b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(fx+e)(a+2b)}{a^2} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)\sqrt{ab+b^2}}{a^2} - \frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*((f*x + e)*(a + 2*b)/a^2 - 2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + ar
ctan(b*tan(f*x + e)/sqrt(a*b + b^2)))*sqrt(a*b + b^2)/a^2 - tan(f*x + e)/(
(tan(f*x + e)^2 + 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}\left(\frac{\sin(e+f\,x)\,\sqrt{-b^2-a\,b}}{a\,\cos(e+f\,x)+b\,\cos(e+f\,x)}\right)\,\sqrt{-b^2-a\,b}-a\,\left(\frac{\sin(2\,e+2\,f\,x)}{4}-\frac{\operatorname{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)}{2}\right)+b\,\operatorname{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)}{a^2\,f}$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

output
```
(atanh((sin(e + f*x)*(- a*b - b^2)^(1/2))/(a*cos(e + f*x) + b*cos(e + f*x)
))*(- a*b - b^2)^(1/2) - a*(sin(2*e + 2*f*x)/4 - atan(sin(e + f*x)/cos(e +
 f*x))/2) + b*atan(sin(e + f*x)/cos(e + f*x)))/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{-2\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)-2\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)-\cos\left(fx+e\right)\sin\left(f\right.}{2a^2f}$$

input

```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

output

```
( - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b)) - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a
))/sqrt(b)) - cos(e + f*x)*sin(e + f*x)*a + a*e + a*f*x + 2*b*e + 2*b*f*x)
/(2*a**2*f)
```

3.37 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{x}{a}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}$$

output
```
x/a+b^(1/2)*arctan((a+b)^(1/2)*cot(f*x+e)/b^(1/2))/a/(a+b)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx =\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(}{2\sqrt{a+}}\right.\right.}{2a\sqrt{a+b}f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(e)-i\sin(e)}}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Co
s[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e
])^4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2
)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4615, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4615}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b}dx}{a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{b\int \frac{1}{(a+b)\cot^2(e+fx)+b}d\cot(e+fx)}{af}+\frac{x}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*
f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 4615
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Simp[b/a   Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f},
x] && NeQ[a + b, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
default	$\frac{-\frac{b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)fa}$	114

input
```
int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a*arctan(
tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{\cdots}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,af}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a
 + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*
x + e)*sin(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \int \frac{1}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(1/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)
/a)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{\operatorname{atan}\left(\frac{\frac{\left(2\,b^3\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\frac{\left(2\,b^3\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)}{\cdots}\right.}{\cdots}\right.}{\cdots}}\right.\cdots}{f\,\left(a^2+b\,a\right)\cdots}$$

input
```
int(1/(a + b/cos(e + f*x)^2),x)
```

output
```
x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^
3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2
*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) +
 ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/
(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1
i)/(a*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^
2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2)
)/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x)
+ ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))
/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))
/(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.89

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)-\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)+afx+bfx}{af\,(a+b)}$$

input
```
int(1/(a+b*sec(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b)) - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/s
qrt(b)) + a*f*x + b*f*x)/(a*f*(a + b))
```

3.38 $\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{\cot(e+fx)}{(a+b)f}$$

output
```
-b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(3/2)/f-cot(f*x+e)/(
a+b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.50

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos\right.}{2(a+b)^{3/2}f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(}}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b*ArcTan[(Sec[f*x]*(Cos[2*
e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b
]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Cs
c[e]*Csc[e + f*x]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*Sin[f*x]))/(2*(a + b)^(3/2
)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{b\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{a+b}}{f}$$

$$\Big\downarrow \ 218$$

$$\frac{-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cot(e+fx)}{a+b}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)) - C
ot[e + f*x]/(a + b))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)\sqrt{(a+b)b}}}{f}$
risch	$-\frac{2i}{f(a+b)\left(e^{2i(fx+e)}-1\right)}-\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)^2f}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+}{a}\right)}{2(a+b)^2f}$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a+b)/tan(f*x+e)-1/(a+b)*b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a
+b)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 5.02

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a^2+3ab+2b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right.}{4(a+b)f\sin(fx+e)}\right.$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*
b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b
 + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x +
e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/((a + b
)*f*sin(f*x + e)), 1/2*(sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)
^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 2*co
s(f*x + e))/((a + b)*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(a+b)} + \frac{1}{(a+b)\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)) + 1/(
(a + b)*tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}(a+b)} + \frac{1}{(a+b)\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*b/(sqrt(a*b + b^2)*(a + b)) + 1/((a + b)*tan(f*x + e)))/f
```

Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\cot(e+f\,x)}{f\,(a+b)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)}{f\,(a+b)^{3/2}}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
```

output
```
- cot(e + f*x)/(f*(a + b)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x))/(a + b)^
(1/2)))/(f*(a + b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.22

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin(fx+e) - \sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\sin(fx+e}{\sin(fx+e)\,f\,(a^2+2ab+b^2)}$$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

output
```
( - (sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqr
t(b))*sin(e + f*x) + sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2
) + sqrt(a))/sqrt(b))*sin(e + f*x) + cos(e + f*x)*a + cos(e + f*x)*b))/(si
n(e + f*x)*f*(a**2 + 2*a*b + b**2))
```

3.39 $\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{a\cot(e+fx)}{(a+b)^2f} - \frac{\cot^3(e+fx)}{3(a+b)f}$$

output
```
-a*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(5/2)/f-a*cot(f*x+
e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.97

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3ab\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(c\ldots}{6(a+b)^{5/2}f\,(a\ldots}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*a*b*ArcTan[(Sec[f*x]*(Co
s[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a +
b]*Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(6*a*Sin[f*x] - 3*b
*Sin[2*e + f*x] + (-2*a + b)*Sin[2*e + 3*f*x]))/4))/(6*(a + b)^(5/2)*f*(a
+ b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4620, 359, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(\tan^2(e+fx)+1\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{a\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{a\left(-\frac{b\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{a+b}\right)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{a\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cot(e+fx)}{a+b}\right)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/(a + b) + (a*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b]])/(a + b)^(3/2)) - Cot[e + f*x]/(a + b)))/(a + b))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{a}{(a+b)^2\tan(fx+e)}-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{a}{(a+b)^2\tan(fx+e)}-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2\sqrt{(a+b)b}}}{f}$
risch	$\frac{2i\left(3b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-2a+b\right)}{3f(a+b)^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-(a+b)b}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)^3f}-\frac{\sqrt{-(a+b)b}\,a\ln\left(e^{2i(fx\right.}}{2(a}$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3/(a+b)/tan(f*x+e)^3-a/(a+b)^2/tan(f*x+e)-a*b/(a+b)^2/((a+b)*b)^(1
/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(66) = 132$.

Time = 0.11 (sec) , antiderivative size = 397, normalized size of antiderivative = 5.22

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,(2\,a-b)\cos\left(fx+e\right)^3 - 3\,\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\,\left(3\,ab+4\,b^2\right)\cos(fx+}{a^2}\right.}{12\,\left(\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^2-\right.}\right.$$

$$-\frac{2\,(2\,a-b)\cos\left(fx+e\right)^3-3\,\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(fx+e\right.}{6\,\left(\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^2-\left(a^2+2\,ab+b^2\right)f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/12*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(-b/(a +
 b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
 + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x +
e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f
*x + e)^2 + b^2))*sin(f*x + e) - 12*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*
f*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), -1/6*(2*(2*a - b)
*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(b/(a + b))*arctan(1/2*((a
+ 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*
sin(f*x + e) - 6*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 -
(a^2 + 2*a*b + b^2)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}+\frac{3\,a\tan(fx+e)^2+a+b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*a*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sq
rt((a + b)*b)) + (3*a*tan(f*x + e)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x
 + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)ab}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}}+\frac{3\,a\tan(fx+e)^2+a+b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*a*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*a*tan(f*x + e)
^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{1}{3\,(a+b)}+\frac{a\tan(e+f\,x)^2}{(a+b)^2}}{f\tan\left(e+f\,x\right)^3}-\frac{a\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^2+2\,a\,b+b^2\right)}{(a+b)^{5/2}}\right)}{f\,{\left(a+b\right)}^{5/2}}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
```

output
```
- (1/(3*(a + b)) + (a*tan(e + f*x)^2)/(a + b)^2)/(f*tan(e + f*x)^3) - (a*b
^(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^2 + b^2))/(a + b)^(5/2)))/(f*
(a + b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.68

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-3\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^3 a-3\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\sin\left(f\right.}{}$$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*sin(e + f*x)**3*a - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**3*a - 2*cos(e + f*x)*sin(e + f
*x)**2*a**2 - cos(e + f*x)*sin(e + f*x)**2*a*b + cos(e + f*x)*sin(e + f*x)
**2*b**2 - cos(e + f*x)*a**2 - 2*cos(e + f*x)*a*b - cos(e + f*x)*b**2)/(3*
sin(e + f*x)**3*f*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

3.40 $\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}f} - \frac{a^2\cot(e+fx)}{(a+b)^3f} - \frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f}$$

output
```
-a^2*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(7/2)/f-a^2*cot(
f*x+e)/(a+b)^3/f-1/3*(2*a+b)*cot(f*x+e)^3/(a+b)^2/f-1/5*cot(f*x+e)^5/(a+b)
/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.03

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(240a^2b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(240*a^2*b*ArcTan[(Sec[f*x]
*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sq
rt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a
 + b]*Csc[e]*Csc[e + f*x]^5*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(10*(8*a^2 + b^2
)*Sin[f*x] - 30*b*(3*a + b)*Sin[2*e + f*x] - 40*a^2*Sin[2*e + 3*f*x] + 30*
a*b*Sin[2*e + 3*f*x] + 10*b^2*Sin[2*e + 3*f*x] + 15*a*b*Sin[4*e + 3*f*x] +
 8*a^2*Sin[4*e + 5*f*x] - 9*a*b*Sin[4*e + 5*f*x] - 2*b^2*Sin[4*e + 5*f*x])
))/(480*(a + b)^(7/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^
4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(\frac{\cot^6(e+fx)}{a+b}+\frac{(2a+b)\cot^4(e+fx)}{(a+b)^2}+\frac{a^2\cot^2(e+fx)}{(a+b)^3}-\frac{a^2b}{(a+b)^3\left(b\tan^2(e+fx)+a+b\right)}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}}-\frac{a^2\cot(e+fx)}{(a+b)^3}-\frac{\cot^5(e+fx)}{5(a+b)}-\frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(7/2))
 - (a^2*Cot[e + f*x])/(a + b)^3 - ((2*a + b)*Cot[e + f*x]^3)/(3*(a + b)^2)
 - Cot[e + f*x]^5/(5*(a + b)))/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{a^2}{(a+b)^3\tan(fx+e)}-\frac{2a+b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2 b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{a^2}{(a+b)^3\tan(fx+e)}-\frac{2a+b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2 b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3\sqrt{(a+b)b}}}{f}$
risch	$\frac{2i(15ab\,e^{8i(fx+e)}-90ab\,e^{6i(fx+e)}-30b^2e^{6i(fx+e)}-80a^2e^{4i(fx+e)}-10b^2e^{4i(fx+e)}+40a^2e^{2i(fx+e)}-30ab\,e^{2i(fx+e)}-1}{15f(a+b)^3\left(e^{2i(fx+e)}-1\right)^5}$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/5/(a+b)/tan(f*x+e)^5-a^2/(a+b)^3/tan(f*x+e)-1/3*(2*a+b)/(a+b)^2/ta
n(f*x+e)^3-a^2*b/(a+b)^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(93) = 186.

Time = 0.11 (sec) , antiderivative size = 587, normalized size of antiderivative = 5.59

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\left(8a^2-9ab-2b^2\right)\cos\left(fx+e\right)^5-20\left(4a^2-3ab-b^2\right)\cos\left(fx+e\right)^3-15\left(a^2\cos\left(fx+e\right)^4-2a^2\right.}{60\left(\left(a^3+3a^2b+3ab^2+b^3\right)f\cos\right.}\right.$$

$$-\frac{2\left(8a^2-9ab-2b^2\right)\cos\left(fx+e\right)^5-10\left(4a^2-3ab-b^2\right)\cos\left(fx+e\right)^3-15\left(a^2\cos\left(fx+e\right)^4-2a^2\right.}{30\left(\left(a^3+3a^2b+3ab^2+b^3\right)f\cos\left(fx+e\right)^4-2\left(a^3+3a^2b+3ab^2+\right.\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/60*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 3*a*b - b^2
)*cos(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sq
rt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^
2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*
cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*a^2*cos(f*x + e))/(((a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)), -1
/30*(2*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 10*(4*a^2 - 3*a*b - b^2)*c
os(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(
b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*a^2*cos(f*x + e))/(((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{15\,a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}}+\frac{15\,a^2\tan(fx+e)^4+5\,(2\,a^2+3\,ab+b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*a^2*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*sqrt((a + b)*b)) + (15*a^2*tan(f*x + e)^4 + 5*(2*a^2 + 3*a*b
 + b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*tan(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.65

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2b}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab+b^2}}+\frac{15a^2\tan(fx+e)^4+10a^2\tan(fx+e)^2+15ab\tan(fx+e)^2+5b^2\tan(fx+e)^2+3a^2+6\cdots}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)^5}}{15f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqr
t(a*b + b^2)))*a^2*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (
15*a^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 15*a*b*tan(f*x + e)^2 + 5*
b^2*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*tan(f*x + e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{1}{5(a+b)}+\frac{\tan(e+fx)^2(2a+b)}{3(a+b)^2}+\frac{a^2\tan(e+fx)^4}{(a+b)^3}}{f\tan(e+fx)^5} - \frac{a^2\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+fx)(a^3+3a^2b+3ab^2+b^3)}{(a+b)^{7/2}}\right)}{f(a+b)^{7/2}}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

output
```
- (1/(5*(a + b)) + (tan(e + f*x)^2*(2*a + b))/(3*(a + b)^2) + (a^2*tan(e +
 f*x)^4)/(a + b)^3)/(f*tan(e + f*x)^5) - (a^2*b^(1/2)*atan((b^(1/2)*tan(e
+ f*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(f*(a + b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.13

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-15\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^5 a^2-15\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\sin}{}$$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**5*a**2 - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**5*a**2 - 8*cos(e + f*x)*s
in(e + f*x)**4*a**3 + cos(e + f*x)*sin(e + f*x)**4*a**2*b + 11*cos(e + f*x
)*sin(e + f*x)**4*a*b**2 + 2*cos(e + f*x)*sin(e + f*x)**4*b**3 - 4*cos(e +
 f*x)*sin(e + f*x)**2*a**3 - 7*cos(e + f*x)*sin(e + f*x)**2*a**2*b - 2*cos
(e + f*x)*sin(e + f*x)**2*a*b**2 + cos(e + f*x)*sin(e + f*x)**2*b**3 - 3*c
os(e + f*x)*a**3 - 9*cos(e + f*x)*a**2*b - 9*cos(e + f*x)*a*b**2 - 3*cos(e
 + f*x)*b**3)/(15*sin(e + f*x)**5*f*(a**4 + 4*a**3*b + 6*a**2*b**2 + 4*a*b
**3 + b**4))
```

3.41 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\sqrt{b}(a+b)(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f} - \frac{(a+b)(a+3b)\cos(e+fx)}{a^4f} + \frac{2(a+b)\cos^3(e+fx)}{3a^3f} - \frac{\cos^5(e+fx)}{5a^2f} - \frac{b(a+b)^2\cos(e+fx)}{2a^4f\left(b+a\cos^2(e+fx)\right)}$$

output
```
1/2*b^(1/2)*(a+b)*(3*a+7*b)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(9/2)/f-(
a+b)*(a+3*b)*cos(f*x+e)/a^4/f+2/3*(a+b)*cos(f*x+e)^3/a^3/f-1/5*cos(f*x+e)^
5/a^2/f-1/2*b*(a+b)^2*cos(f*x+e)/a^4/f/(b+a*cos(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.17

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{15(3a^4+384a^2b^2+1280ab^3+896b^4)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}}+}{}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] - I*Sq
rt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[
a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3
/2) + (15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] +
 I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(
Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])
/b^(3/2) - (45*a^4*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]
])/b^(3/2) - (45*a^4*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[
b]])/b^(3/2) - (16*Sqrt[a]*Cos[e + f*x]*(150*a^3 + 1436*a^2*b + 2960*a*b^2
 + 1680*b^3 + a*(125*a^2 + 688*a*b + 560*b^2)*Cos[2*(e + f*x)] - 2*a^2*(11
*a + 14*b)*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)]))/(a + 2*b + a*Cos[2*
(e + f*x)]))/(3840*a^(9/2)*f)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4621, 366, 25, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^5}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \; 4621$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\cos^2(e+fx))^2}{(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 366$$

$$-\frac{\frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)}-\frac{\int -\frac{\cos^4(e+fx)\left(2a^2+2b\cos^2(e+fx)a-5(a+b)^2\right)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a^2b}}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int \frac{\cos^4(e+fx)\left(2a^2+2b\cos^2(e+fx)a-5(a+b)^2\right)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a^2b}+\frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 363$$

$$-\frac{\frac{\frac{2}{5}b\cos^5(e+fx)-(a+b)(3a+7b)\int \frac{\cos^4(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a^2b}+\frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 254$$

$$-\frac{\frac{\frac{2}{5}b\cos^5(e+fx)-(a+b)(3a+7b)\int \left(\frac{b^2}{a^2\left(a\cos^2(e+fx)+b\right)}-\frac{b}{a^2}+\frac{\cos^2(e+fx)}{a}\right)d\cos(e+fx)}{2a^2b}+\frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)}+\frac{\frac{2}{5}b\cos^5(e+fx)-(a+b)(3a+7b)\left(\frac{b^{3/2}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}}-\frac{b\cos(e+fx)}{a^2}+\frac{\cos^3(e+fx)}{3a}\right)}{2a^2b}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((((a + b)^2*Cos[e + f*x]^5)/(2*a^2*b*(b + a*Cos[e + f*x]^2)) + ((2*b*Cos
[e + f*x]^5)/5 - (a + b)*(3*a + 7*b)*((b^(3/2)*ArcTan[(Sqrt[a]*Cos[e + f*x
])/Sqrt[b]])/a^(5/2) - (b*Cos[e + f*x])/a^2 + Cos[e + f*x]^3/(3*a)))/(2*a^
2*b))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{\frac{\cos(fx+e)^5 a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{2a\cos(fx+e)^3 b}{3}+a^2\cos(fx+e)+4ab\cos(fx+e)+3b^2\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\cos(f}{b+a\cos(fx+e)^2}\right.}{}}{f}$
default	$\frac{-\frac{\frac{\cos(fx+e)^5 a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{2a\cos(fx+e)^3 b}{3}+a^2\cos(fx+e)+4ab\cos(fx+e)+3b^2\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\cos(f}{b+a\cos(fx+e)^2}\right.}{}}{f}$
risch	$\frac{5\,\mathrm{e}^{3i(fx+e)}}{96a^2f}+\frac{\mathrm{e}^{3i(fx+e)}b}{12a^3f}-\frac{5\,\mathrm{e}^{i(fx+e)}}{16f\,a^2}-\frac{7\,\mathrm{e}^{i(fx+e)}b}{4f\,a^3}-\frac{3\,\mathrm{e}^{i(fx+e)}b^2}{2f\,a^4}-\frac{5\,\mathrm{e}^{-i(fx+e)}}{16f\,a^2}-\frac{7\,\mathrm{e}^{-i(fx+e)}b}{4f\,a^3}-\frac{3\,\mathrm{e}^{-i}}{2}$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^4*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-2/3*a*cos(f*x+e)^3*
b+a^2*cos(f*x+e)+4*a*b*cos(f*x+e)+3*b^2*cos(f*x+e))+b/a^4*((-1/2*a^2-a*b-1
/2*b^2)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a^2+10*a*b+7*b^2)/(a*b)^(1/2)
*arctan(a*cos(f*x+e)/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.83

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[-\frac{12\,a^3\cos{(fx+e)}^7 - 4\,(10\,a^3+7\,a^2b)\cos{(fx+e)}^5 + 20\,(3\,a^3+10\,a^2b+7\,ab^2)\cos{(fx+e)}^3 - 15\,(3\right.$$
$$-\frac{6\,a^3\cos{(fx+e)}^7 - 2\,(10\,a^3+7\,a^2b)\cos{(fx+e)}^5 + 10\,(3\,a^3+10\,a^2b+7\,ab^2)\cos{(fx+e)}^3 - 15\,(3\,a}{30}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/60*(12*a^3*cos(f*x + e)^7 - 4*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 20*(
3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^
3 + (3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*
x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(
3*a^2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f)
, -1/30*(6*a^3*cos(f*x + e)^7 - 2*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 10*(
3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^
3 + (3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b
/a)*cos(f*x + e)/b) + 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f
*cos(f*x + e)^2 + a^4*b*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{15\,(a^2b+2\,ab^2+b^3)\cos(fx+e)}{a^5\cos(fx+e)^2+a^4b} - \frac{15\,(3\,a^2b+10\,ab^2+7\,b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{2\left(3\,a^2\cos(fx+e)^5-10\,(a^2+ab)\cos(fx+e)^3+15\,(a^2+4\right.}{a^4}}{30\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/30*(15*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)/(a^5*cos(f*x + e)^2 + a^4*b
) - 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt
(a*b)*a^4) + 2*(3*a^2*cos(f*x + e)^5 - 10*(a^2 + a*b)*cos(f*x + e)^3 + 15*
(a^2 + 4*a*b + 3*b^2)*cos(f*x + e))/a^4)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.44

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(3\,a^2b+10\,ab^2+7\,b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4f} - \frac{a^2b\cos(fx+e)+2\,ab^2\cos(fx+e)+b^3\cos(fx+e)}{2\left(a\cos(fx+e)^2+b\right)a^4f} - \frac{3\,a^8f^4\cos(fx+e)^5-10\,a^8f^4\cos(fx+e)^3-10\,a^7bf^4\cos(fx+e)^3+15\,a^8f^4\cos(fx+e)+60\,a^7bf}{15\,a^{10}f^5}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*
b)*a^4*f) - 1/2*(a^2*b*cos(f*x + e) + 2*a*b^2*cos(f*x + e) + b^3*cos(f*x +
 e))/((a*cos(f*x + e)^2 + b)*a^4*f) - 1/15*(3*a^8*f^4*cos(f*x + e)^5 - 10*
a^8*f^4*cos(f*x + e)^3 - 10*a^7*b*f^4*cos(f*x + e)^3 + 15*a^8*f^4*cos(f*x
+ e) + 60*a^7*b*f^4*cos(f*x + e) + 45*a^6*b^2*f^4*cos(f*x + e))/(a^10*f^5)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.36

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\cos(e+f\,x)^3\,\left(\frac{2\,b}{3\,a^3}+\frac{2}{3\,a^2}\right)}{f}-\frac{\cos(e+f\,x)^5}{5\,a^2\,f}-\frac{\cos(e+f\,x)\,\left(\frac{1}{a^2}-\frac{b^2}{a^4}+\frac{2\,b\,\left(\frac{2\,b}{a^3}+\frac{2}{a^2}\right)}{a}\right)}{f}-\frac{\cos(e+f\,x)\,\left(\frac{a^2\,b}{2}+a\,b^2+\frac{b^3}{2}\right)}{f\,\left(a^5\cos(e+f\,x)^2+b\,a^4\right)}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(a+b)\,(3\,a+7\,b)}{3\,a^2\,b+10\,a\,b^2+7\,b^3}\right)\,(a+b)\,(3\,a+7\,b)}{2\,a^{9/2}\,f}$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(cos(e + f*x)^3*((2*b)/(3*a^3) + 2/(3*a^2)))/f - cos(e + f*x)^5/(5*a^2*f)
- (cos(e + f*x)*(1/a^2 - b^2/a^4 + (2*b*((2*b)/a^3 + 2/a^2))/a))/f - (cos(
e + f*x)*(a*b^2 + (a^2*b)/2 + b^3/2))/(f*(a^4*b + a^5*cos(e + f*x)^2)) + (
b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(a + b)*(3*a + 7*b))/(10*a*b^2
+ 3*a^2*b + 7*b^3))*(a + b)*(3*a + 7*b))/(2*a^(9/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 782, normalized size of antiderivative = 5.47

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 45*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**2*a**3 - 150*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*b - 105*sqrt(b)*sqrt(a)*a
tan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b*
*2 + 45*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*a**3 + 195*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(
a))/sqrt(b))*a**2*b + 255*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/
2) - sqrt(a))/sqrt(b))*a*b**2 + 105*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*b**3 + 45*sqrt(b)*sqrt(a)*atan((sqrt(a +
b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3 + 150*sqrt(b)
*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*
x)**2*a**2*b + 105*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sq
rt(a))/sqrt(b))*sin(e + f*x)**2*a*b**2 - 45*sqrt(b)*sqrt(a)*atan((sqrt(a +
 b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**3 - 195*sqrt(b)*sqrt(a)*atan((
sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2*b - 255*sqrt(b)*sqrt
(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b**2 - 105*sq
rt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**3
- 6*cos(e + f*x)*sin(e + f*x)**6*a**4 - 2*cos(e + f*x)*sin(e + f*x)**4*a**
4 - 14*cos(e + f*x)*sin(e + f*x)**4*a**3*b - 8*cos(e + f*x)*sin(e + f*x)**
2*a**4 - 72*cos(e + f*x)*sin(e + f*x)**2*a**3*b - 70*cos(e + f*x)*sin(e...
```

3.42 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f} - \frac{(a+2b)\cos(e+fx)}{a^3 f} + \frac{\cos^3(e+fx)}{3a^2 f} - \frac{b(a+b)\cos(e+fx)}{2a^3 f\left(b+a\cos^2(e+fx)\right)}$$

output

```
1/2*b^(1/2)*(3*a+5*b)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(7/2)/f-(a+2*b)
*cos(f*x+e)/a^3/f+1/3*cos(f*x+e)^3/a^2/f-1/2*b*(a+b)*cos(f*x+e)/a^3/f/(b+a
*cos(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.54

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{3\left(3a^3+192ab^2+320b^3\right)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}} + \frac{3\left(3a^3+192\ldots\right.}{}}{}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((3*(3*a^3 + 192*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]
*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) + (3*(3*a^3
+ 192*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a
] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[
a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (32*Sqrt[a]*Cos[e +
 f*x]*(9*a^2 + 56*a*b + 60*b^2 + 4*a*(2*a + 5*b)*Cos[2*(e + f*x)] - a^2*Co
s[4*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))/(384*a^(7/2)*f)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4621, 360, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^3}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\cos^2(e+fx))}{(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$-\frac{\frac{b(a+b)\cos(e+fx)}{2a^3(a\cos^2(e+fx)+b)}-\frac{\int \frac{2a^2\cos^4(e+fx)-2a(a+b)\cos^2(e+fx)+b(a+b)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a^3}}{f}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{\frac{b(a+b)\cos(e+fx)}{2a^3(a\cos^2(e+fx)+b)}-\frac{\int\left(2a\cos^2(e+fx)-2(a+2b)+\frac{5b^2+3ab}{a\cos^2(e+fx)+b}\right)d\cos(e+fx)}{2a^3}}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b(a+b)\cos(e+fx)}{2a^3(a\cos^2(e+fx)+b)}-\frac{\frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}}-2(a+2b)\cos(e+fx)+\frac{2}{3}a\cos^3(e+fx)}{2a^3}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-(((b*(a + b)*Cos[e + f*x])/(2*a^3*(b + a*Cos[e + f*x]^2)) - ((Sqrt[b]*(3*
a + 5*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/Sqrt[a] - 2*(a + 2*b)*Cos
[e + f*x] + (2*a*Cos[e + f*x]^3)/3)/(2*a^3))/f
```

Defintions of rubi rules used

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-2\cos(fx+e)b}{a^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a+5b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
default	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-2\cos(fx+e)b}{a^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a+5b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24a^2f}-\frac{3\,e^{i(fx+e)}}{8f\,a^2}-\frac{e^{i(fx+e)}b}{f\,a^3}-\frac{3\,e^{-i(fx+e)}}{8f\,a^2}-\frac{e^{-i(fx+e)}b}{f\,a^3}+\frac{e^{-3i(fx+e)}}{24a^2f}-\frac{(a+b)b(e^{3i(fx+e}}{a^3f(a\,e^{4i(fx+e)}+2a\,e^{2i(fx}}$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-2*cos(f*x+e)*b)+b/a^3*((-1/2*a
-1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a+5*b)/(a*b)^(1/2)*arctan(a*c
os(f*x+e)/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.61

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[\frac{4a^2\cos(fx+e)^5 - 4(3a^2+5ab)\cos(fx+e)^3 + 3\left((3a^2+5ab)\cos(fx+e)^2 + 3ab + 5b^2\right)\sqrt{-\frac{b}{a}}\log\cdots}{12\left(a^4f\cos(fx+e)^2 + a^3bf\right)}\right.$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*a^2*cos(f*x + e)^5 - 4*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2
 + 5*a*b)*cos(f*x + e)^2 + 3*a*b + 5*b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^
2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(3*a*b +
5*b^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), 1/6*(2*a^2*cos(f*x
+ e)^5 - 2*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2 + 5*a*b)*cos(f*x + e
)^2 + 3*a*b + 5*b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 3*(3*a
*b + 5*b^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,(ab+b^2)\cos(fx+e)}{a^4\cos(fx+e)^2+a^3b} - \frac{3\,(3\,ab+5\,b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\left(a\cos(fx+e)^3-3\,(a+2\,b)\cos(fx+e)\right)}{a^3}}{6\,f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(a*b + b^2)*cos(f*x + e)/(a^4*cos(f*x + e)^2 + a^3*b) - 3*(3*a*b +
 5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2*(a*cos(f*x +
e)^3 - 3*(a + 2*b)*cos(f*x + e))/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(3\,ab+5\,b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3f} - \frac{ab\cos(fx+e)+b^2\cos(fx+e)}{2\left(a\cos(fx+e)^2+b\right)a^3f} + \frac{a^4f^2\cos(fx+e)^3-3\,a^4f^2\cos(fx+e)-6\,a^3bf^2\cos(fx+e)}{3\,a^6f^3}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(3*a*b + 5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - 1
/2*(a*b*cos(f*x + e) + b^2*cos(f*x + e))/((a*cos(f*x + e)^2 + b)*a^3*f) +
1/3*(a^4*f^2*cos(f*x + e)^3 - 3*a^4*f^2*cos(f*x + e) - 6*a^3*b*f^2*cos(f*x
 + e))/(a^6*f^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.14

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\cos(e+f\,x)^3}{3\,a^2\,f} - \frac{\cos(e+f\,x)\,\left(\frac{2\,b}{a^3}+\frac{1}{a^2}\right)}{f} - \frac{\cos(e+f\,x)\,\left(\frac{b^2}{2}+\frac{a\,b}{2}\right)}{f\,\left(a^4\cos(e+f\,x)^2+b\,a^3\right)} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(3\,a+5\,b)}{5\,b^2+3\,a\,b}\right)\,(3\,a+5\,b)}{2\,a^{7/2}\,f}$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
cos(e + f*x)^3/(3*a^2*f) - (cos(e + f*x)*((2*b)/a^3 + 1/a^2))/f - (cos(e +
 f*x)*((a*b)/2 + b^2/2))/(f*(a^3*b + a^4*cos(e + f*x)^2)) + (b^(1/2)*atan(
(a^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 5*b))/(3*a*b + 5*b^2))*(3*a + 5*b))/(
2*a^(7/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 523, normalized size of antiderivative = 4.59

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 9*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**2*a**2 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*
x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 9*sqrt(b)*sqrt(a)*atan((sq
rt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 + 24*sqrt(b)*sqrt(a)*a
tan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b + 15*sqrt(b)*sqr
t(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**2 + 9*sqrt(
b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e +
f*x)**2*a**2 + 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqr
t(a))/sqrt(b))*sin(e + f*x)**2*a*b - 9*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*t
an((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 - 24*sqrt(b)*sqrt(a)*atan((sqrt(a
 + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b - 15*sqrt(b)*sqrt(a)*atan((
sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**2 - 2*cos(e + f*x)*sin
(e + f*x)**4*a**3 - 2*cos(e + f*x)*sin(e + f*x)**2*a**3 - 10*cos(e + f*x)*
sin(e + f*x)**2*a**2*b + 4*cos(e + f*x)*a**3 + 19*cos(e + f*x)*a**2*b + 15
*cos(e + f*x)*a*b**2 + 4*sin(e + f*x)**2*a**3 + 15*sin(e + f*x)**2*a**2*b
- 4*a**3 - 19*a**2*b - 15*a*b**2)/(6*a**4*f*(sin(e + f*x)**2*a - a - b))
```

3.43 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f} - \frac{3\cos(e+fx)}{2a^2f} + \frac{\cos^3(e+fx)}{2af\left(b+a\cos^2(e+fx)\right)}$$

output
```
3/2*b^(1/2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(5/2)/f-3/2*cos(f*x+e)/a^
2/f+1/2*cos(f*x+e)^3/a/f/(b+a*cos(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.68

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^2 \left(\frac{(a^2+24b^2)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e}{\sqrt{b}}\right.}{b^{3/2}}\right.}{}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*(((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] - I*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt
[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(
3/2) + ((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a]
- Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] +
 Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (16*Sqrt[a]*Cos[e + f*x
]*(a + 3*b + a*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))*Sec[e +
f*x]^4)/(64*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4621, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{3\int \frac{\cos^2(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a}-\frac{\cos^3(e+fx)}{2a(a\cos^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \ 262$$

$$-\frac{\frac{3\left(\frac{\cos(e+fx)}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a}\right)}{2a}-\frac{\cos^3(e+fx)}{2a(a\cos^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{3\left(\frac{\cos(e+fx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{\cos^3(e+fx)}{2a(a\cos^2(e+fx)+b)}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-(((3*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/a^(3/2)) + Cos[e
 + f*x]/a))/(2*a) - Cos[e + f*x]^3/(2*a*(b + a*Cos[e + f*x]^2)))/f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\sec(fx+e)}{2a+2b\sec(fx+e)^2}+\frac{3\arctan\left(\frac{b\sec(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2\sec(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\sec(fx+e)}{2a+2b\sec(fx+e)^2}+\frac{3\arctan\left(\frac{b\sec(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2\sec(fx+e)}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2f\,a^2}-\frac{e^{-i(fx+e)}}{2f\,a^2}-\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{3i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e^{i(fx+}}{a}\right.}{4a^3f}$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^2*(1/2*sec(f*x+e)/(a+b*sec(f*x+e)^2)+3/2/(a*b)^(1/2)*arctan(b*se
c(f*x+e)/(a*b)^(1/2)))-1/a^2/sec(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.39

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[-\frac{4\,a\cos\left(fx+e\right)^3 - 3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right) + 6\,b\cos\left(fx+\right.}{4\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}, -\frac{2\,a\cos\left(fx+e\right)^3 - 3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right) + 3\,b\cos\left(fx+e\right)}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)} \right]$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/a)*log(-(a*co
s(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) +
6*b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), -1/2*(2*a*cos(f*x + e)
^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b)
 + 3*b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sin\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{b\cos(fx+e)}{a^3\cos(fx+e)^2+a^2b} - \frac{3\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\cos(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*cos(f*x + e)/(a^3*cos(f*x + e)^2 + a^2*b) - 3*b*arctan(a*cos(f*x +
 e)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2*cos(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{3\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2f} - \frac{\cos\left(fx+e\right)}{a^2f} - \frac{b\cos\left(fx+e\right)}{2\left(a\cos\left(fx+e\right)^2+b\right)a^2f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
3/2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) - cos(f*x + e)/(a
^2*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 + b)*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 12.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{2\,a^{5/2}\,f} - \frac{b\,\cos\left(e+f\,x\right)}{2\,f\,\left(a^3\cos\left(e+f\,x\right)^2+b\,a^2\right)} - \frac{\cos\left(e+f\,x\right)}{a^2\,f}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(3*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(2*a^(5/2)*f) - (b*cos(e
+ f*x))/(2*f*(a^2*b + a^3*cos(e + f*x)^2)) - cos(e + f*x)/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 525, normalized size of antiderivative = 6.25

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^2 a^2-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^2}{}$$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**2*a**2 - 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x
)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqrt(a)*atan((sqr
t(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 + 6*sqrt(b)*sqrt(a)*ata
n((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b + 3*sqrt(b)*sqrt(a
)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**2 + 3*sqrt(b)*
sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x
)**2*a**2 + 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a)
)/sqrt(b))*sin(e + f*x)**2*a*b - 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((
e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 - 6*sqrt(b)*sqrt(a)*atan((sqrt(a + b)
*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b - 3*sqrt(b)*sqrt(a)*atan((sqrt(a
 + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**2 - 2*cos(e + f*x)*sin(e + f
*x)**2*a**3 - 2*cos(e + f*x)*sin(e + f*x)**2*a**2*b + 2*cos(e + f*x)*a**3
+ 5*cos(e + f*x)*a**2*b + 3*cos(e + f*x)*a*b**2 + 2*sin(e + f*x)**2*a**3 +
 3*sin(e + f*x)**2*a**2*b - 2*a**3 - 5*a**2*b - 3*a*b**2)/(2*a**3*f*(sin(e
 + f*x)**2*a**2 + sin(e + f*x)**2*a*b - a**2 - 2*a*b - b**2))
```

3.44 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)^2f} - \frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)}$$

output
```
1/2*b^(1/2)*(3*a+b)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(3/2)/(a+b)^2/f-a
rctanh(cos(f*x+e))/(a+b)^2/f-1/2*b*cos(f*x+e)/a/(a+b)/f/(b+a*cos(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.88

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(-\frac{2b(a+b)}{a}+\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cdots}{\sqrt{b}}\right)}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*b*(a + b))/a + (Sqrt[b
]*(3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])
*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]
)/a^(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos
[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sq
rt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sec[e + f*x])/a^(3/2) - 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[(e
 + f*x)/2]]*Sec[e + f*x] + 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Sin[(e + f
*x)/2]]*Sec[e + f*x]))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4621, 372, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$-\frac{\frac{b\cos(e+fx)}{2a(a+b)(a\cos^2(e+fx)+b)}-\frac{\int \frac{b-(2a+b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2a(a+b)}}{f}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{b\cos(e+fx)}{2a(a+b)(a\cos^2(e+fx)+b)}-\frac{\frac{b(3a+b)\int\frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{2a\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2a(a+b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{b\cos(e+fx)}{2a(a+b)(a\cos^2(e+fx)+b)}-\frac{\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{2a\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2a(a+b)}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{b\cos(e+fx)}{2a(a+b)(a\cos^2(e+fx)+b)}-\frac{\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{2a\text{arctanh}(\cos(e+fx))}{a+b}}{2a(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((-1/2*((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[
a]*(a + b)) - (2*a*ArcTanh[Cos[e + f*x]])/(a + b))/(a*(a + b)) + (b*Cos[e
+ f*x])/(2*a*(a + b)*(b + a*Cos[e + f*x]^2)))/f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{b\left(-\frac{(a+b)\cos(fx+e)}{2a\left(b+a\cos(fx+e)^{2}\right)}+\frac{(3a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^{2}}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^{2}}-\frac{\ln(1+\cos(fx+e))}{2(a+b)^{2}}}{f}$
default	$\frac{\frac{b\left(-\frac{(a+b)\cos(fx+e)}{2a\left(b+a\cos(fx+e)^{2}\right)}+\frac{(3a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^{2}}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^{2}}-\frac{\ln(1+\cos(fx+e))}{2(a+b)^{2}}}{f}$
risch	$-\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{af(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{i(fx+e)}+1\right)}{f(a^{2}+2ab+b^{2})}+\frac{\ln\left(e^{i(fx+e)}-1\right)}{f(a^{2}+2ab+b^{2})}+\frac{3i\sqrt{ab}\,\ln\left(e^{2i(fx\right.}{4}$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b/(a+b)^2*(-1/2*(a+b)/a*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a+b)/a/(
a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/2/(a+b)^2*ln(-1+cos(f*x+e))
-1/2/(a+b)^2*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(87) = 174$.

Time = 0.14 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.94

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \left[\frac{\left((3a^2+ab)\cos(fx+e)^2+3ab+b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-2(ab+b^2)\cos(fx}{4\left((a^4+2a^3b+a^2b^2)f\cos\right.}\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(
f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*
(a*b + b^2)*cos(f*x + e) - 2*(a^2*cos(f*x + e)^2 + a*b)*log(1/2*cos(f*x +
e) + 1/2) + 2*(a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e) + 1/2))/((a
^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f),
 1/2*(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt
(b/a)*cos(f*x + e)/b) - (a*b + b^2)*cos(f*x + e) - (a^2*cos(f*x + e)^2 + a
*b)*log(1/2*cos(f*x + e) + 1/2) + (a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(
f*x + e) + 1/2))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*
a^2*b^2 + a*b^3)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.39

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{b\cos(fx+e)}{a^2b+ab^2+(a^3+a^2b)\cos(fx+e)^2} - \frac{(3\,ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{ab}} + \frac{\log(\cos(fx+e)+1)}{a^2+2\,ab+b^2} - \frac{\log(\cos(fx+e)-1)}{a^2+2\,ab+b^2}}{2\,f}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*cos(f*x + e)/(a^2*b + a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2) - (3*a
*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a
*b)) + log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) - log(cos(f*x + e) - 1)/(
a^2 + 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.52

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(3\,ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\,(a^3f+2\,a^2bf+ab^2f)\sqrt{ab}} - \frac{b\cos(fx+e)}{2\,(a^2f+abf)\left(a\cos(fx+e)^2+b\right)} + \frac{\log(|-\cos(fx+e)+1|)}{2\,(a^2f+2\,abf+b^2f)} - \frac{\log(|-\cos(fx+e)-1|)}{2\,(a^2f+2\,abf+b^2f)}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(3*a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3*f + 2*a^2*b*f + a
*b^2*f)*sqrt(a*b)) - 1/2*b*cos(f*x + e)/((a^2*f + a*b*f)*(a*cos(f*x + e)^2
 + b)) + 1/2*log(abs(-cos(f*x + e) + 1))/(a^2*f + 2*a*b*f + b^2*f) - 1/2*l
og(abs(-cos(f*x + e) - 1))/(a^2*f + 2*a*b*f + b^2*f)
```

Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 2188, normalized size of antiderivative = 22.10

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(atan((((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9
*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) + ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6
*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^
4 + 3*a^2*b^2) - (cos(e + f*x)*(3*a + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8
 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b
 + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4
*(2*a^4*b + a^5 + a^3*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*
a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) - ((3*a + b)
*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2
)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) + (cos(e + f*x)*(3*a + b)*(-a^3*b)^(
1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^
2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b +
a^5 + a^3*b^2)))*1i)/(4*(2*a^4*b + a^5 + a^3*b^2)))/(((5*a*b^2)/2 + 3*a^2*
b + b^3/2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - ((3*a + b)*(-a^3*b)^(1/2)
*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b +
 a^3)) + ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*
a^4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - (cos(e + f*x)*(
3*a + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*
a^5*b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2
))))/(4*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)) + ((...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 530, normalized size of antiderivative = 5.35

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**2*a**2 - sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/
2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqrt(a)*atan((sqrt(
a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 + 4*sqrt(b)*sqrt(a)*atan(
(sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b + sqrt(b)*sqrt(a)*at
an((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**2 + 3*sqrt(b)*sqrt
(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2
*a**2 + sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt
(b))*sin(e + f*x)**2*a*b - 3*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*
x)/2) + sqrt(a))/sqrt(b))*a**2 - 4*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((
e + f*x)/2) + sqrt(a))/sqrt(b))*a*b - sqrt(b)*sqrt(a)*atan((sqrt(a + b)*ta
n((e + f*x)/2) + sqrt(a))/sqrt(b))*b**2 + cos(e + f*x)*a**2*b + cos(e + f*
x)*a*b**2 + 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**3 - 2*log(tan((e +
f*x)/2))*a**3 - 2*log(tan((e + f*x)/2))*a**2*b + sin(e + f*x)**2*a**2*b -
a**2*b - a*b**2)/(2*a**2*f*(sin(e + f*x)**2*a**3 + 2*sin(e + f*x)**2*a**2*
b + sin(e + f*x)**2*a*b**2 - a**3 - 3*a**2*b - 3*a*b**2 - b**3))
```

3.45 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}(a+b)^3f}-\frac{(a-3b)\text{arctanh}(\cos(e+fx))}{2(a+b)^3f}+\frac{(a-b)\cos(e+fx)}{2(a+b)^2f\left(b+a\cos^2(e+fx)\right)}-\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)}$$

output
```
1/2*(3*a-b)*b^(1/2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(1/2)/(a+b)^3/f-1
/2*(a-3*b)*arctanh(cos(f*x+e))/(a+b)^3/f+1/2*(a-b)*cos(f*x+e)/(a+b)^2/f/(b
+a*cos(f*x+e)^2)-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(b+a*cos(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.18

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(-8b(a+b)-\frac{4\sqrt{b}(-3a+b)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan}{}\right.}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(-8*b*(a + b) - (4*Sqrt[b]*
(-3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*
Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])
/Sqrt[a] - (4*Sqrt[b]*(-3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*
Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e
+ f*x)])*Sec[e + f*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Cs
c[(e + f*x)/2]^2*Sec[e + f*x] - 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])
*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e +
f*x)])*Log[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b + a*Cos[2*(e
+ f*x)])*Sec[(e + f*x)/2]^2*Sec[e + f*x]))/(32*(a + b)^3*f*(a + b*Sec[e +
f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4621, 372, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^2}dx$$

$\Big\downarrow$ 4621

$$-\frac{\int \frac{\cos^4(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{f}$$

$\Big\downarrow$ 372

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\int \frac{b-(a-2b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{2(a+b)}}{f}$$

$\Big\downarrow$ 402

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\int -\frac{2b\left(2b-(a-b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}}{2(a+b)}}{f}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{\int \frac{2b-(a-b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{a+b}+\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}}{2(a+b)}}{f}$$

$\Big\downarrow$ 397

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{\frac{b(3a-b)\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{(a-3b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{a+b}+\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}}{2(a+b)}}{f}$$

$\Big\downarrow$ 218

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{(a-3b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{a+b}+\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}}{2(a+b)}}{f}$$

$\downarrow$ 219

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{(a-3b)\text{arctanh}(\cos(e+fx))}{a+b}}{a+b}+\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}}{2(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((Cos[e + f*x]/(2*(a + b)*(1 - Cos[e + f*x]^2)*(b + a*Cos[e + f*x]^2)) -
((((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a +
 b)) - ((a - 3*b)*ArcTanh[Cos[e + f*x]])/(a + b))/(a + b) + ((a - b)*Cos[e
 + f*x])/((a + b)*(b + a*Cos[e + f*x]^2)))/(2*(a + b)))/f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^3}+\frac{1}{4(a+b)^2(1+\cos(fx+e))}+\frac{(-a+3b)\ln(1+\cos(fx+e))}{4(a+b)^3}+\frac{1}{4(a+b)^2(-1+c}}{f}$
default	$\frac{\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^3}+\frac{1}{4(a+b)^2(1+\cos(fx+e))}+\frac{(-a+3b)\ln(1+\cos(fx+e))}{4(a+b)^3}+\frac{1}{4(a+b)^2(-1+c}}{f}$
risch	$\frac{a\,e^{7i(fx+e)}-b\,e^{7i(fx+e)}+3a\,e^{5i(fx+e)}+5b\,e^{5i(fx+e)}+3a\,e^{3i(fx+e)}+5b\,e^{3i(fx+e)}+a\,e^{i(fx+e)}-b\,e^{i(fx+e)}}{f(a+b)^2\left(e^{2i(fx+e)}-1\right)^2\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln(e^{i(}}{2f(a^3+3a}$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b/(a+b)^3*((-1/2*a-1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a-b)/(
a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/4/(a+b)^2/(1+cos(f*x+e))+1/
4/(a+b)^3*(-a+3*b)*ln(1+cos(f*x+e))+1/4/(a+b)^2/(-1+cos(f*x+e))+1/4*(a-3*b
)/(a+b)^3*ln(-1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(131) = 262$.

Time = 0.16 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.75

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 - ((3*a^2 - a*b)*cos(f*x + e)^4 - (3*a^
2 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(-b/a)*log((a*cos(f*x +
 e)^2 - 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 4*(a*b
+ b^2)*cos(f*x + e) - ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2
)*cos(f*x + e)^2 - a*b + 3*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*
b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*lo
g(-1/2*cos(f*x + e) + 1/2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x
 + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4)*f), 1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 + 2*((3*a^2 -
 a*b)*cos(f*x + e)^4 - (3*a^2 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)
*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 4*(a*b + b^2)*cos(f*x + e)
 - ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 -
a*b + 3*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*cos(f*x + e)^4 -
 (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2*cos(f*x + e)
 + 1/2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*
a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b
^4)*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.57

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{(a-3b)\log(\cos(fx+e)+1)}{a^3+3a^2b+3ab^2+b^3} - \frac{(a-3b)\log(\cos(fx+e)-1)}{a^3+3a^2b+3ab^2+b^3} - \frac{2(3ab-b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}} - \frac{2\left((a-b)\cos(fx+e)\right.}{(a^3+2a^2b+ab^2)\cos(fx+e)^4-a^2b-2ab}}{4f}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*((a - 3*b)*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a
 - 3*b)*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(3*a*b -
 b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sq
rt(a*b)) - 2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))/((a^3 + 2*a^2*b +
 a*b^2)*cos(f*x + e)^4 - a^2*b - 2*a*b^2 - b^3 - (a^3 + a^2*b - a*b^2 - b^
3)*cos(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.60

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{(a-3\,b)\log(|\cos(fx+e)+1|)}{4\,(a^3f+3\,a^2bf+3\,ab^2f+b^3f)} + \frac{(a-3\,b)\log(|\cos(fx+e)-1|)}{4\,(a^3f+3\,a^2bf+3\,ab^2f+b^3f)} + \frac{(3\,ab-b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\,(a^3f+3\,a^2bf+3\,ab^2f+b^3f)\sqrt{ab}} + \frac{a\cos(fx+e)^3-b\cos(fx+e)^3+2\,b\cos(fx+e)}{2\,(a\cos(fx+e)^4-a\cos(fx+e)^2+b\cos(fx+e)^2-b)(a^2f+2\,abf+b^2f)}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/4*(a - 3*b)*log(abs(cos(f*x + e) + 1))/(a^3*f + 3*a^2*b*f + 3*a*b^2*f +
 b^3*f) + 1/4*(a - 3*b)*log(abs(cos(f*x + e) - 1))/(a^3*f + 3*a^2*b*f + 3*
a*b^2*f + b^3*f) + 1/2*(3*a*b - b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^
3*f + 3*a^2*b*f + 3*a*b^2*f + b^3*f)*sqrt(a*b)) + 1/2*(a*cos(f*x + e)^3 -
b*cos(f*x + e)^3 + 2*b*cos(f*x + e))/((a*cos(f*x + e)^4 - a*cos(f*x + e)^2
 + b*cos(f*x + e)^2 - b)*(a^2*f + 2*a*b*f + b^2*f))
```

Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 1845, normalized size of antiderivative = 12.55

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
- ((cos(e + f*x)^3*(a - b))/(2*(2*a*b + a^2 + b^2)) + (b*cos(e + f*x))/(2*
a*b + a^2 + b^2))/(f*(b - a*cos(e + f*x)^4 + cos(e + f*x)^2*(a - b))) - (l
og(cos(e + f*x) - 1)*(b/(a + b)^3 - 1/(4*(a + b)^2)))/f - (log(cos(e + f*x
) + 1)*(a - 3*b))/(4*f*(a + b)^3) - (atan((((-a*b)^(1/2)*((cos(e + f*x)*(a
*b^4 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^
4 + b^4 + 6*a^2*b^2)) + ((-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 +
 60*a^4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a
^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2) - (cos(e + f*x)*(-a*b)^(1
/2)*(3*a - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 -
 80*a^5*b^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2
*b^2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 +
 3*a^3*b + a^4 + 3*a^2*b^2)))*(3*a - b)*1i)/(4*(a*b^3 + 3*a^3*b + a^4 + 3*
a^2*b^2)) + ((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4 - 6*a^4*b + a^5 - 6*a^2*b^
3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) - ((-a*b)
^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^4*b^5 + 80*a^5*b^4 + 60*a
^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*
b^3 + 15*a^4*b^2) + (cos(e + f*x)*(-a*b)^(1/2)*(3*a - b)*(80*a^8*b + 16*a^
9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b^4 + 80*a^6*b^3 + 144*
a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4
+ b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2))...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 770, normalized size of antiderivative = 5.24

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 6*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**4*a**2 + 2*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x
)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a*b + 6*sqrt(b)*sqrt(a)*atan((sqr
t(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2 + 4*sqr
t(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e
+ f*x)**2*a*b - 2*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqr
t(a))/sqrt(b))*sin(e + f*x)**2*b**2 + 6*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*
tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2 - 2*sqrt(b)*sqrt
(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4
*a*b - 6*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqr
t(b))*sin(e + f*x)**2*a**2 - 4*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e +
f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 2*sqrt(b)*sqrt(a)*atan((
sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*b**2 - 2*
cos(e + f*x)*sin(e + f*x)**2*a**3 + 2*cos(e + f*x)*sin(e + f*x)**2*a*b**2
+ 2*cos(e + f*x)*a**3 + 4*cos(e + f*x)*a**2*b + 2*cos(e + f*x)*a*b**2 + 2*
log(tan((e + f*x)/2))*sin(e + f*x)**4*a**3 - 6*log(tan((e + f*x)/2))*sin(e
 + f*x)**4*a**2*b - 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**3 + 4*log(t
an((e + f*x)/2))*sin(e + f*x)**2*a**2*b + 6*log(tan((e + f*x)/2))*sin(e +
f*x)**2*a*b**2 - sin(e + f*x)**4*a**3 + 3*sin(e + f*x)**4*a**2*b + sin(e +
 f*x)**2*a**3 - 2*sin(e + f*x)**2*a**2*b - 3*sin(e + f*x)**2*a*b**2)/(4...
```

3.46 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 197

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2(a+b)^4f} - \frac{3(a^2-6ab+b^2)\,\mathrm{arctanh}(\cos(e+fx))}{8(a+b)^4f} + \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\,(b+a\cos^2(e+fx))} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\,(b+a\cos^2(e+fx))} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\,(b+a\cos^2(e+fx))}$$

output
```
3/2*a^(1/2)*(a-b)*b^(1/2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/(a+b)^4/f-3/8
*(a^2-6*a*b+b^2)*arctanh(cos(f*x+e))/(a+b)^4/f+3/8*a*(a-3*b)*cos(f*x+e)/(a
+b)^3/f/(b+a*cos(f*x+e)^2)-1/8*(a-5*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(b+
a*cos(f*x+e)^2)-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f/(b+a*cos(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.28

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\left(96\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\right.}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*(96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqrt
[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos
[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]]*(a + 2*b + a*Cos[2*(e + f*x)]) + 96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((
-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
+ Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a + b)*(11*a^2 + 43*a*b - 4*
b^2 + 4*(2*a^2 - 5*a*b + 5*b^2)*Cos[2*(e + f*x)] - 3*a*(a - 3*b)*Cos[4*(e
+ f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 - 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a
*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/2]] + 24*(a^2 - 6*a*b + b^2)*(a + 2*b
 + a*Cos[2*(e + f*x)])*Log[Sin[(e + f*x)/2]])*Sec[e + f*x]^4)/(256*(a + b)
^4*f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4621, 372, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\sin(e+fx)^5\left(a+b\sec(e+fx)^2\right)^2}dx$$

$\Big\downarrow$ 4621

$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$\Big\downarrow$ 372

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\int \frac{b-(a-4b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4(a+b)}}{f}$$

$\Big\downarrow$ 402

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\frac{\int \frac{3\left((a-b)b-a(a-5b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{2(a+b)}-\frac{(a-5b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}}{4(a+b)}}{f}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\frac{3\int \frac{(a-b)b-a(a-5b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{2(a+b)}-\frac{(a-5b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}}{4(a+b)}}{f}$$

$\Big\downarrow$ 402

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\frac{3\left(\frac{a(a-3b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\int -\frac{2b\left((3a-b)b-a(a-3b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}\right)}{2(a+b)}-\frac{(a-5}{2(a+b)\left(1-\cos^2(e\right.}}{4(a+b)}}{f}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\frac{3\left(\frac{\int\frac{(3a-b)b-a(a-3b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{a+b}+\frac{a(a-3b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}\right)}{2(a+b)}-\frac{(a-5b)\cdots}{2(a+b)\left(1-\cos^2(e+\cdots\right.}}{4(a+b)}}{f}$$

$\downarrow$ 397

$$-\frac{-\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\frac{3\left(\frac{\frac{4ab(a-b)\int\frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{\left(a^2-6ab+b^2\right)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{a+b}+\frac{a(a-3b)\cdots}{(a+b)\left(a\cos\cdots\right.}\right.}{2(a+b)}\cdots}{4(a+b)}}{f}$$

$\downarrow$ 218

$$-\frac{-\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\frac{3\left(\frac{\frac{4\sqrt{a}\sqrt{b}(a-b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{\left(a^2-6ab+b^2\right)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{a+b}+\frac{a(a-3b)\cos\cdots}{(a+b)\left(a\cos^2(e\cdots\right.}\right.}{2(a+b)}\cdots}{4(a+b)}}{f}$$

$\downarrow$ 219

$$-\frac{-\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\frac{3\left(\frac{\frac{4\sqrt{a}\sqrt{b}(a-b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{\left(a^2-6ab+b^2\right)\text{arctanh}(\cos(e+fx))}{a+b}}{a+b}+\frac{a(a-3b)\cos(e+fx\cdots}{(a+b)\left(a\cos^2(e+fx)\cdots\right.}\right.}{2(a+b)}\cdots}{4(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((Cos[e + f*x]/(4*(a + b)*(1 - Cos[e + f*x]^2)^2*(b + a*Cos[e + f*x]^2))
- (-1/2*((a - 5*b)*Cos[e + f*x])/((a + b)*(1 - Cos[e + f*x]^2)*(b + a*Cos[
e + f*x]^2)) + (3*(((4*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x
])/Sqrt[b]])/(a + b) - ((a^2 - 6*a*b + b^2)*ArcTanh[Cos[e + f*x]])/(a + b)
)/(a + b) + (a*(a - 3*b)*Cos[e + f*x])/((a + b)*(b + a*Cos[e + f*x]^2))))/
(2*(a + b)))/(4*(a + b)))/f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{ab\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{3(a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^4}+\frac{1}{16(a+b)^2(1+\cos(fx+e))^2}-\frac{-3a+5b}{16(a+b)^3(1+\cos(fx+e))}+\frac{\left(-3a^2+18ab\cdots}{f}$
default	$\frac{\frac{ab\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{3(a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^4}+\frac{1}{16(a+b)^2(1+\cos(fx+e))^2}-\frac{-3a+5b}{16(a+b)^3(1+\cos(fx+e))}+\frac{\left(-3a^2+18ab\cdots}{f}$
risch	$-\frac{-3a^2e^{11i(fx+e)}+9ab\,e^{11i(fx+e)}+5a^2e^{9i(fx+e)}-11ab\,e^{9i(fx+e)}+20b^2e^{9i(fx+e)}+30a^2e^{7i(fx+e)}+66ab\,e^{7i(fx+e)}+1\cdots}{4f(a+b)^3\left(e^{2i(fx+e)}-1\right)\cdots}$

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*b/(a+b)^4*((-1/2*a-1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2*(a-b)/(
a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/16/(a+b)^2/(1+cos(f*x+e))^2
-1/16*(-3*a+5*b)/(a+b)^3/(1+cos(f*x+e))+1/16/(a+b)^4*(-3*a^2+18*a*b-3*b^2)
*ln(1+cos(f*x+e))-1/16/(a+b)^2/(-1+cos(f*x+e))^2-1/16*(-3*a+5*b)/(a+b)^3/(
-1+cos(f*x+e))+1/16/(a+b)^4*(3*a^2-18*a*b+3*b^2)*ln(-1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(179) = 358$.

Time = 0.20 (sec) , antiderivative size = 1202, normalized size of antiderivative = 6.10

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/16*(6*(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9
*a*b^2 + 5*b^3)*cos(f*x + e)^3 - 12*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 -
 3*a*b + b^2)*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b
- b^2)*sqrt(-a*b)*log((a*cos(f*x + e)^2 - 2*sqrt(-a*b)*cos(f*x + e) - b)/(
a*cos(f*x + e)^2 + b)) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e) - 3*((a^
3 - 6*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*c
os(f*x + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)
*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b + a*b^2)*
cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4 + a^2*b
 - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^2)*log(
-1/2*cos(f*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)
*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b
^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 -
2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*
f), 1/16*(6*(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b
- 9*a*b^2 + 5*b^3)*cos(f*x + e)^3 + 24*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^
2 - 3*a*b + b^2)*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a
*b - b^2)*sqrt(a*b)*arctan(sqrt(a*b)*cos(f*x + e)/b) - 6*(3*a^2*b + 2*a*b^
2 - b^3)*cos(f*x + e) - 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3
 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + b^3 + (...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(179) = 358.

Time = 0.12 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.87

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx =$$

$$-\frac{\frac{3\,(a^2-6\,ab+b^2)\log(\cos(fx+e)+1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{3\,(a^2-6\,ab+b^2)\log(\cos(fx+e)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{24\,(a^2b-ab^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab}} - \frac{\cdots}{(a^4+3\,a^3b+3\,a^2b^2+a\cdots}}{16\,f}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/16*(3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4) - 3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) - 1)/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 24*(a^2*b - a*b^2)*arctan(a*cos(f*x
 + e)/sqrt(a*b))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) -
 2*(3*(a^2 - 3*a*b)*cos(f*x + e)^5 - (5*a^2 - 14*a*b + 5*b^2)*cos(f*x + e)
^3 - 3*(3*a*b - b^2)*cos(f*x + e))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*co
s(f*x + e)^6 - (2*a^4 + 5*a^3*b + 3*a^2*b^2 - a*b^3 - b^4)*cos(f*x + e)^4
+ a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4 + (a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 -
 2*b^4)*cos(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.65

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= -\frac{ab\cos(fx+e)}{2(a^3f+3a^2bf+3ab^2f+b^3f)(a\cos(fx+e)^2+b)} + \frac{3(a^2-6ab+b^2)\log(|-\cos(fx+e)+1|)}{16(a^4f+4a^3bf+6a^2b^2f+4ab^3f+b^4f)} - \frac{3(a^2-6ab+b^2)\log(|-\cos(fx+e)-1|)}{16(a^4f+4a^3bf+6a^2b^2f+4ab^3f+b^4f)} + \frac{3(a^2b-ab^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2(a^4f+4a^3bf+6a^2b^2f+4ab^3f+b^4f)\sqrt{ab}} + \frac{3a\cos(fx+e)^3-5b\cos(fx+e)^3-5a\cos(fx+e)+3b\cos(fx+e)}{8(a^3f+3a^2bf+3ab^2f+b^3f)(\cos(fx+e)^2-1)^2}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*a*b*cos(f*x + e)/((a^3*f + 3*a^2*b*f + 3*a*b^2*f + b^3*f)*(a*cos(f*x
+ e)^2 + b)) + 3/16*(a^2 - 6*a*b + b^2)*log(abs(-cos(f*x + e) + 1))/(a^4*f
 + 4*a^3*b*f + 6*a^2*b^2*f + 4*a*b^3*f + b^4*f) - 3/16*(a^2 - 6*a*b + b^2)
*log(abs(-cos(f*x + e) - 1))/(a^4*f + 4*a^3*b*f + 6*a^2*b^2*f + 4*a*b^3*f
+ b^4*f) + 3/2*(a^2*b - a*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^4*f +
4*a^3*b*f + 6*a^2*b^2*f + 4*a*b^3*f + b^4*f)*sqrt(a*b)) + 1/8*(3*a*cos(f*x
 + e)^3 - 5*b*cos(f*x + e)^3 - 5*a*cos(f*x + e) + 3*b*cos(f*x + e))/((a^3*
f + 3*a^2*b*f + 3*a*b^2*f + b^3*f)*(cos(f*x + e)^2 - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 17.14 (sec) , antiderivative size = 4338, normalized size of antiderivative = 22.02

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(atan((((-a*b)^(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396
*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 +
 20*a^3*b^3 + 15*a^4*b^2)) + (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a
^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a
^7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b +
 a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*
b^3 + 36*a^7*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 25
6*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a
^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^
3*b + a^4 + b^4 + 6*a^2*b^2)*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 +
 20*a^3*b^3 + 15*a^4*b^2))))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2
)))*(a - b)*3i)/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + ((-a*b)^
(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*
a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 1
5*a^4*b^2)) - (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*b^10)/2 - (1
5*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^
8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36
*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^
2) + (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11 - 256*a^
2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 358...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 895, normalized size of antiderivative = 4.54

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 96*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**6*a**2 + 96*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f
*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**6*a*b + 96*sqrt(b)*sqrt(a)*atan((
sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2 - 96
*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*si
n(e + f*x)**4*b**2 + 96*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2)
 + sqrt(a))/sqrt(b))*sin(e + f*x)**6*a**2 - 96*sqrt(b)*sqrt(a)*atan((sqrt(
a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**6*a*b - 96*sqrt(
b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e +
f*x)**4*a**2 + 96*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqr
t(a))/sqrt(b))*sin(e + f*x)**4*b**2 - 24*cos(e + f*x)*sin(e + f*x)**4*a**3
 + 48*cos(e + f*x)*sin(e + f*x)**4*a**2*b + 72*cos(e + f*x)*sin(e + f*x)**
4*a*b**2 + 8*cos(e + f*x)*sin(e + f*x)**2*a**3 - 24*cos(e + f*x)*sin(e + f
*x)**2*a**2*b - 72*cos(e + f*x)*sin(e + f*x)**2*a*b**2 - 40*cos(e + f*x)*s
in(e + f*x)**2*b**3 + 16*cos(e + f*x)*a**3 + 48*cos(e + f*x)*a**2*b + 48*c
os(e + f*x)*a*b**2 + 16*cos(e + f*x)*b**3 + 24*log(tan((e + f*x)/2))*sin(e
 + f*x)**6*a**3 - 144*log(tan((e + f*x)/2))*sin(e + f*x)**6*a**2*b + 24*lo
g(tan((e + f*x)/2))*sin(e + f*x)**6*a*b**2 - 24*log(tan((e + f*x)/2))*sin(
e + f*x)**4*a**3 + 120*log(tan((e + f*x)/2))*sin(e + f*x)**4*a**2*b + 120*
log(tan((e + f*x)/2))*sin(e + f*x)**4*a*b**2 - 24*log(tan((e + f*x)/2))...
```

3.47 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 267

$$\begin{aligned}\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{(5a^3+60a^2b+120ab^2+64b^3)\,x}{16a^5} \\ &\quad - \frac{\sqrt{b}(a+b)^{3/2}(3a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f} \\ &\quad - \frac{(33a^2+82ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} \\ &\quad + \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} \\ &\quad + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} \\ &\quad - \frac{b(19a^2+52ab+32b^2)\tan(e+fx)}{16a^4f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output
```
1/16*(5*a^3+60*a^2*b+120*a*b^2+64*b^3)*x/a^5-1/2*b^(1/2)*(a+b)^(3/2)*(3*a+
8*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/f-1/48*(33*a^2+82*a*b+48*b
^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(9*a+8*b)*cos(f*
x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3
/a/f/(a+b+b*tan(f*x+e)^2)-1/16*b*(19*a^2+52*a*b+32*b^2)*tan(f*x+e)/a^4/f/(
a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.02 (sec) , antiderivative size = 2468, normalized size of antiderivative = 9.24

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/512*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6
*a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a
 + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I
*Sin[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))
/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + S
in[e]))))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x
])^2*Sec[e + f*x]^4*(-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 25
6*a*b^3 - 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]
)]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^
4]) + (16*a*Cos[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3
+ 18*a^2*b + 48*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a
 + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(4096*a^3*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x]
)^2*Sec[e + f*x]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(a + b)^(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*
(e + f*x)]))))/(2048*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*C
os[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b]])/(a + b)^(3/2)) + (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b...
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4620, 372, 440, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^6}{\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\int \frac{\tan^2(e+fx)\left((b-6(a+b))\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{6a}}{f}$$

$$\Big\downarrow \text{440}$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\frac{\int \frac{(a+b)(9a+8b)-\left(24a^2+65ba+40b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a}-\frac{(9a+8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)}}{6a}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{\int \frac{3\left((a+b)\left(5a^2+22ba+16b^2\right)-b\left(33a^2+82ba+48b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}}{2a}}{4a}}{6a}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\frac{\frac{(33a^2+82ab+48b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}-\frac{3\int\frac{(a+b)(5a^2+22ba+16b^2)-b(33a^2+82ba+48b^2)\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d}{2a}}{4a}}{6a}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\frac{\frac{(33a^2+82ab+48b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}-\frac{3\left(\frac{\int\frac{2(a+b)((a+b)(5a^2+36ba+32b^2)-b(19a^2+52ba+32b^2)\tan}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}}{2a(a+b)}\right.}{2a}}{4a}}{6a}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\frac{\frac{(33a^2+82ab+48b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}-\frac{3\left(\frac{\int\frac{(a+b)(5a^2+36ba+32b^2)-b(19a^2+52ba+32b^2)\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}}{a}\right.}{2a}}{4a}}{6a}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\frac{(33a^2+82ab+48b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}-\frac{3\left(\frac{\frac{(5a^3+60a^2b+120ab^2+64b^3)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-}{a}\right.}{4a}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3\left(\frac{\frac{\left(5a^3+60a^2b+120ab^2+64b^3\right)\arctan(\tan(e+fx))}{a}-\frac{8b(a+b)^2(3}{a}}{}\right.}{4a}}{}}{f}$$

$\big\downarrow$ 218

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3\left(\frac{\frac{\left(5a^3+60a^2b+120ab^2+64b^3\right)\arctan(\tan(e+fx))}{a}-\frac{8\sqrt{b}(a+b)^3}{a}}{}\right.}{4a}}{}}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)) -
(-1/4*((9*a + 8*b)*Tan[e + f*x])/(a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[
e + f*x]^2)) + (((33*a^2 + 82*a*b + 48*b^2)*Tan[e + f*x])/(2*a*(1 + Tan[e
+ f*x]^2)*(a + b + b*Tan[e + f*x]^2)) - (3*((((5*a^3 + 60*a^2*b + 120*a*b^
2 + 64*b^3)*ArcTan[Tan[e + f*x]])/a - (8*Sqrt[b]*(a + b)^(3/2)*(3*a + 8*b)
*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/a)/a - (b*(19*a^2 + 52*a*b +
32*b^2)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2))))/(2*a))/(4*a))/(6*a)
)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{-\frac{b(a+b)^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(-\frac{9}{4}a^2b-\frac{3}{2}ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-4a^2b-3ab^2-\frac{5}{6}\right.}{\left(1+\tan(fx\right.}}{f}$
default	$\frac{-\frac{b(a+b)^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(-\frac{9}{4}a^2b-\frac{3}{2}ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-4a^2b-3ab^2-\frac{5}{6}\right.}{\left(1+\tan(fx\right.}}{f}$
risch	$\frac{5x}{16a^2}+\frac{15xb}{4a^3}+\frac{15x\,b^2}{2a^4}+\frac{4x\,b^3}{a^5}-\frac{ie^{4i(fx+e)}b}{32a^3f}+\frac{3ie^{-4i(fx+e)}}{128a^2f}-\frac{3ie^{4i(fx+e)}}{128a^2f}-\frac{ie^{-2i(fx+e)}b}{2a^3f}+\frac{3ie^{2i(fx+e}}{8a^4f}$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b*(a+b)^2/a^5*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+8*b)/(
(a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^5*(((-9/4*a^2*b-3
/2*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-4*a^2*b-3*a*b^2-5/6*a^3)*tan(f*x+e)^3+(
-5/16*a^3-7/4*a^2*b-3/2*a*b^2)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*(5*a^3+
60*a^2*b+120*a*b^2+64*b^3)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.52

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/48*(3*(5*a^4 + 60*a^3*b + 120*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 +
3*(5*a^3*b + 60*a^2*b^2 + 120*a*b^3 + 64*b^4)*f*x + 6*(3*a^2*b + 11*a*b^2
+ 8*b^3 + (3*a^3 + 11*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*lo
g(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x
+ e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - (8*a^4*co
s(f*x + e)^7 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b +
48*a^2*b^2)*cos(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x
+ e))*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), 1/48*(3*(5*a^4 + 60*
a^3*b + 120*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3*(5*a^3*b + 60*a^2*b
^2 + 120*a*b^3 + 64*b^4)*f*x + 12*(3*a^2*b + 11*a*b^2 + 8*b^3 + (3*a^3 + 1
1*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*c
os(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - (8*a^4*c
os(f*x + e)^7 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b +
 48*a^2*b^2)*cos(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x
 + e))*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.13

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{\frac{3\,(19\,a^2b+52\,ab^2+32\,b^3)\,\tan(fx+e)^7+(33\,a^3+253\,a^2b+516\,ab^2+288\,b^3)\,\tan(fx+e)^5+(40\,a^3+319\,a^2b+564\,ab^2+288\,b^3)\,\tan(fx+e)^3+3\,(5\,a}{a^4b\tan(fx+e)^8+(a^5+4\,a^4b)\tan(fx+e)^6+a^5+a^4b+3\,(a^5+2\,a^4b)\tan(fx+e)^4+(3\,a^5+4\,a^4b)\tan(fx+e)}}{48}$$

input

```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output

```
-1/48*((3*(19*a^2*b + 52*a*b^2 + 32*b^3)*tan(f*x + e)^7 + (33*a^3 + 253*a^
2*b + 516*a*b^2 + 288*b^3)*tan(f*x + e)^5 + (40*a^3 + 319*a^2*b + 564*a*b^
2 + 288*b^3)*tan(f*x + e)^3 + 3*(5*a^3 + 41*a^2*b + 68*a*b^2 + 32*b^3)*tan
(f*x + e))/(a^4*b*tan(f*x + e)^8 + (a^5 + 4*a^4*b)*tan(f*x + e)^6 + a^5 +
a^4*b + 3*(a^5 + 2*a^4*b)*tan(f*x + e)^4 + (3*a^5 + 4*a^4*b)*tan(f*x + e)^
2) - 3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 + 24*(3*a^3*b
 + 14*a^2*b^2 + 19*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(
sqrt((a + b)*b)*a^5))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.10

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{3\,(5\,a^3+60\,a^2b+120\,ab^2+64\,b^3)(fx+e)}{a^5}-\frac{24\,(3\,a^3b+14\,a^2b^2+19\,ab^3+8\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^5}-\frac{24\,(a^2b\tan(fx-}{(}}{}$$

input

```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/48*(3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 - 24*(3*a^3*
b + 14*a^2*b^2 + 19*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^5) - 24*(a^2*b*
tan(f*x + e) + 2*a*b^2*tan(f*x + e) + b^3*tan(f*x + e))/((b*tan(f*x + e)^2
 + a + b)*a^4) - (33*a^2*tan(f*x + e)^5 + 108*a*b*tan(f*x + e)^5 + 72*b^2*
tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 192*a*b*tan(f*x + e)^3 + 144*b^2*
tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 84*a*b*tan(f*x + e) + 72*b^2*tan(f*
x + e))/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 1461, normalized size of antiderivative = 5.47

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(atanh((75*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(
256*((211*a*b^4)/128 + (811*b^5)/256 + (75*a^2*b^3)/256 + (41*b^6)/(16*a)
+ (3*b^7)/(4*a^2))) + (17*b^4*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^
2*b^2)^(1/2))/(16*((811*a*b^5)/256 + (41*b^6)/16 + (211*a^2*b^4)/128 + (75
*a^3*b^3)/256 + (3*b^7)/(4*a))) + (3*b^5*tan(e + f*x)*(- 3*a*b^3 - a^3*b -
 b^4 - 3*a^2*b^2)^(1/2))/(4*((41*a*b^6)/16 + (3*b^7)/4 + (811*a^2*b^5)/256
 + (211*a^3*b^4)/128 + (75*a^4*b^3)/256)))*(-b*(a + b)^3)^(1/2)*(3*a + 8*b
))/(2*a^5*f) - (atan((((((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5
*a^13*b^2)/4)/a^12 - (tan(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*
120i + a^2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i
 + a^3*5i + b^3*64i))/(32*a^5) - (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 5
9520*a^2*b^7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3)
)/(128*a^8))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i)*1i)/(32*a^5) - ((
(((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^2)/4)/a^12 + (ta
n(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^2*b*60i + a^3*5
i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(3
2*a^5) + (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^7 + 52160*a^3
*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))*(a*b^2*120i
 + a^2*b*60i + a^3*5i + b^3*64i)*1i)/(32*a^5))/((376*a*b^10 + 64*b^11 + 93
7*a^2*b^9 + (10285*a^3*b^8)/8 + (33701*a^4*b^7)/32 + (8333*a^5*b^6)/16 ...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 967, normalized size of antiderivative = 3.62

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 72*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**2*a**3 - 264*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*t
an((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*b - 192*sqrt(b)*s
qrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e +
f*x)**2*a*b**2 + 72*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2)
 - sqrt(a))/sqrt(b))*a**3 + 336*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*b + 456*sqrt(b)*sqrt(a + b)*atan((sq
rt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b**2 + 192*sqrt(b)*sqrt(a
 + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**3 - 72*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*si
n(e + f*x)**2*a**3 - 264*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*
x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*b - 192*sqrt(b)*sqrt(a + b)
*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*
b**2 + 72*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a)
)/sqrt(b))*a**3 + 336*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/
2) + sqrt(a))/sqrt(b))*a**2*b + 456*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*
tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b**2 + 192*sqrt(b)*sqrt(a + b)*atan
((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**3 - 8*cos(e + f*x)*s
in(e + f*x)**7*a**4 - 2*cos(e + f*x)*sin(e + f*x)**5*a**4 - 16*cos(e + f*x
)*sin(e + f*x)**5*a**3*b - 5*cos(e + f*x)*sin(e + f*x)**3*a**4 - 50*cos...
```

3.48 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3(a^2+8ab+8b^2)\,x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f} - \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))} - \frac{3b(3a+4b)\tan(e+fx)}{8a^3f\,(a+b+b\tan^2(e+fx))}$$

output
```
3/8*(a^2+8*a*b+8*b^2)*x/a^4-3/2*b^(1/2)*(a+b)^(1/2)*(a+2*b)*arctan(b^(1/2)
*tan(f*x+e)/(a+b)^(1/2))/a^4/f-1/8*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(
a+b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-3
/8*b*(3*a+4*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.37 (sec) , antiderivative size = 1105, normalized size of antiderivative = 5.79

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6
*a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a
 + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I
*Sin[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))
/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + S
in[e]))))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x
])^2*Sec[e + f*x]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]
)/(a + b)^(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2
*(e + f*x)]))))/(1024*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*
Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 160
0*a^2*b^3 - 1920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])
*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
 - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4])) + (Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^
3 + 48*b^4)*f*x*Cos[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f
*x*Cos[2*f*x] + 80*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e +
2*f*x] + 768*a^2*b^3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x]
 + a^5*Sin[2*e] + 34*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*S
in[2*e] + 640*a*b^4*Sin[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a...
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4620, 372, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)^4}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}-\frac{\int \frac{-\left((4a+5b)\tan^2(e+fx)\right)+a+b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}-\frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{\int \frac{3\left((a+b)(a+2b)-b(5a+6b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}-\frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3\int \frac{(a+b)(a+2b)-b(5a+6b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}-\frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3\left(\frac{\int \frac{2(a+b)\left((a+b)(a+4b)-b(3a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\right.}{2a}}{4a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}-\frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3\left(\frac{\int \frac{(a+b)(a+4b)-b(3a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a}-\frac{b(3a}{a\left(a+\right.}\right.}{2a}}{4a}}{f}$$

$\downarrow$ 397

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}-\frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3\left(\frac{\left(a^2+8ab+8b^2\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{4b(a+b)(a+2b}{a}\right.}{2a}}{4a}}{f}$$

$\downarrow$ 216

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}-\frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3\left(\frac{\left(a^2+8ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{4b(a+b)(a+2b)\int \frac{}{b\tan^2(}}{a}\right.}{2a}}{4a}}{f}$$

$\downarrow$ 218

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}-\frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3\left(\frac{\left(a^2+8ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{\frac{4\sqrt{b}\sqrt{a+b}(a+2b)\arctan\left(\right.}{a}}{a}\right.}{2a}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)) - ((
(5*a + 6*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x
]^2)) - (3*((((a^2 + 8*a*b + 8*b^2)*ArcTan[Tan[e + f*x]])/a - (4*Sqrt[b]*S
qrt[a + b]*(a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/a)/a - (b
*(3*a + 4*b)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2))))/(2*a))/(4*a))/
f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{\frac{\left(-ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2+8ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{(a+b)b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3(a+}{}\right.}{a^4}}{f}$
default	$\frac{\frac{\frac{\left(-ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2+8ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{(a+b)b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3(a+}{}\right.}{a^4}}{f}$
risch	$\frac{3x}{8a^2}+\frac{3xb}{a^3}+\frac{3x\,b^2}{a^4}-\frac{ie^{4i(fx+e)}}{64a^2f}+\frac{ie^{2i(fx+e)}}{8a^2f}+\frac{ie^{2i(fx+e)}b}{4a^3f}-\frac{ie^{-2i(fx+e)}}{8a^2f}-\frac{ie^{-2i(fx+e)}b}{4a^3f}+\frac{ie^{-4i(fx+e}}{64a^2f}$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(((-a*b-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-a*b)*tan(f*x+e))/(1+tan
(f*x+e)^2)^2+3/8*(a^2+8*a*b+8*b^2)*arctan(tan(f*x+e)))-(a+b)*b/a^4*(1/2*a*
tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+3/2*(a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(f
*x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.73

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[\frac{3\,(a^3+8\,a^2b+8\,ab^2)fx\cos{(fx+e)}^2+3\,(a^2b+8\,ab^2+8\,b^3)fx+3\left((a^2+2\,ab)\cos{(fx+e)}^2+ab+2\right.}{}\right.$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(3*(a^3 + 8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2
+ 8*b^3)*f*x + 3*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a*b -
b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
 + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*s
in(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2
*a^3*cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*a*
b^2)*cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), 1/8*(3*
(a^3 + 8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 + 8*b^3)
*f*x + 6*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arct
an(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*
x + e))) + (2*a^3*cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3
*a^2*b + 4*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*
b*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\int \frac{\sin^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^2}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.07

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx =$$

$$-\frac{\frac{3\,(3\,ab+4\,b^2)\tan(fx+e)^5+(5\,a^2+24\,ab+24\,b^2)\tan(fx+e)^3+3\,(a^2+5\,ab+4\,b^2)\tan(fx+e)}{a^3b\tan(fx+e)^6+(a^4+3\,a^3b)\tan(fx+e)^4+a^4+a^3b+(2\,a^4+3\,a^3b)\tan(fx+e)^2}-\frac{3\,(a^2+8\,ab+8\,b^2)(fx+e)}{a^4}+\frac{12\,(a^2b+3\,ab^2-}{}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*((3*(3*a*b + 4*b^2)*tan(f*x + e)^5 + (5*a^2 + 24*a*b + 24*b^2)*tan(f*
x + e)^3 + 3*(a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(a^3*b*tan(f*x + e)^6 + (
a^4 + 3*a^3*b)*tan(f*x + e)^4 + a^4 + a^3*b + (2*a^4 + 3*a^3*b)*tan(f*x +
e)^2) - 3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 + 12*(a^2*b + 3*a*b^2 + 2*b^
3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{3\,(a^2+8\,ab+8\,b^2)(fx+e)}{a^4} - \frac{12\,(a^2b+3\,ab^2+2\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^4} - \frac{4\,(ab\tan(fx+e)+b^2\tan(fx+e))}{\left(b\tan(fx+e)^2+a+b\right)a^3} - \frac{5\,a\,\mathrm{t}}{}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/8*(3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 - 12*(a^2*b + 3*a*b^2 + 2*b^3)*
(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))/(sqrt(a*b + b^2)*a^4) - 4*(a*b*tan(f*x + e) + b^2*tan(f*x + e))/((b*t
an(f*x + e)^2 + a + b)*a^3) - (5*a*tan(f*x + e)^3 + 8*b*tan(f*x + e)^3 + 3
*a*tan(f*x + e) + 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.28

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{3\,\mathrm{atanh}\left(\frac{27\,b^3\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{64\left(\frac{27\,a\,b^3}{64}+\frac{81\,b^4}{64}+\frac{27\,b^5}{32\,a}\right)}+\frac{27\,b^4\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{32\left(\frac{27\,a^2\,b^3}{64}+\frac{81\,a\,b^4}{64}+\frac{27\,b^5}{32}\right)}\right)(a+2\,b)\,\sqrt{-b\,(a+b)}}{2\,a^4\,f}$$
$$-\frac{\frac{3\tan(e+f\,x)^5\,(4\,b^2+3\,a\,b)}{8\,a^3}+\frac{\tan(e+f\,x)^3\,(5\,a^2+24\,a\,b+24\,b^2)}{8\,a^3}+\frac{3\tan(e+f\,x)\,(a^2+5\,a\,b+4\,b^2)}{8\,a^3}}{f\,\left(b\tan(e+f\,x)^6+(a+3\,b)\,\tan(e+f\,x)^4+(2\,a+3\,b)\,\tan(e+f\,x)^2+a+b\right)}$$
$$-\frac{\mathrm{atan}\left(\frac{27\,b^2\tan(e+f\,x)}{256\left(\frac{27\,b^2}{256}+\frac{243\,b^3}{256\,a}+\frac{27\,b^4}{16\,a^2}+\frac{27\,b^5}{32\,a^3}\right)}+\frac{243\,b^3\tan(e+f\,x)}{256\left(\frac{27\,a\,b^2}{256}+\frac{243\,b^3}{256}+\frac{27\,b^4}{16\,a}+\frac{27\,b^5}{32\,a^2}\right)}+\frac{27\,b^4\tan(e+f\,x)}{16\left(\frac{243\,a\,b^3}{256}+\frac{27\,b^4}{16}+\frac{27\,a^2\,b^2}{256}+\frac{27\,b^5}{32\,a}\right)}+\frac{}{32\left(\frac{27\,a^3\,b}{256}\right.}\right.}{8\,a^4\,f}$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(3*atanh((27*b^3*tan(e + f*x)*(- a*b - b^2)^(1/2))/(64*((27*a*b^3)/64 + (8
1*b^4)/64 + (27*b^5)/(32*a))) + (27*b^4*tan(e + f*x)*(- a*b - b^2)^(1/2))/
(32*((81*a*b^4)/64 + (27*b^5)/32 + (27*a^2*b^3)/64)))*(a + 2*b)*(-b*(a + b
))^(1/2))/(2*a^4*f) - (atan((27*b^2*tan(e + f*x))/(256*((27*b^2)/256 + (24
3*b^3)/(256*a) + (27*b^4)/(16*a^2) + (27*b^5)/(32*a^3))) + (243*b^3*tan(e
+ f*x))/(256*((27*a*b^2)/256 + (243*b^3)/256 + (27*b^4)/(16*a) + (27*b^5)/
(32*a^2))) + (27*b^4*tan(e + f*x))/(16*((243*a*b^3)/256 + (27*b^4)/16 + (2
7*a^2*b^2)/256 + (27*b^5)/(32*a))) + (27*b^5*tan(e + f*x))/(32*((27*a*b^4)
/16 + (27*b^5)/32 + (243*a^2*b^3)/256 + (27*a^3*b^2)/256)))*(a*b*8i + a^2*
1i + b^2*8i)*3i)/(8*a^4*f) - ((3*tan(e + f*x)^5*(3*a*b + 4*b^2))/(8*a^3) +
 (tan(e + f*x)^3*(24*a*b + 5*a^2 + 24*b^2))/(8*a^3) + (3*tan(e + f*x)*(5*a
*b + a^2 + 4*b^2))/(8*a^3))/(f*(a + b + tan(e + f*x)^2*(2*a + 3*b) + b*tan
(e + f*x)^6 + tan(e + f*x)^4*(a + 3*b)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 664, normalized size of antiderivative = 3.48

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 12*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**2*a**2 - 24*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 12*sqrt(b)*sqrt(a
 + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 + 36*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*
b + 24*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*b**2 - 12*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) +
 sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2 - 24*sqrt(b)*sqrt(a + b)*atan((sqr
t(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 12*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*
*2 + 36*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/
sqrt(b))*a*b + 24*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) +
 sqrt(a))/sqrt(b))*b**2 - 2*cos(e + f*x)*sin(e + f*x)**5*a**3 - cos(e + f*
x)*sin(e + f*x)**3*a**3 - 6*cos(e + f*x)*sin(e + f*x)**3*a**2*b + 3*cos(e
+ f*x)*sin(e + f*x)*a**3 + 15*cos(e + f*x)*sin(e + f*x)*a**2*b + 12*cos(e
+ f*x)*sin(e + f*x)*a*b**2 + 3*sin(e + f*x)**2*a**3*e + 3*sin(e + f*x)**2*
a**3*f*x + 24*sin(e + f*x)**2*a**2*b*e + 24*sin(e + f*x)**2*a**2*b*f*x + 2
4*sin(e + f*x)**2*a*b**2*e + 24*sin(e + f*x)**2*a*b**2*f*x - 3*a**3*e - 3*
a**3*f*x - 27*a**2*b*e - 27*a**2*b*f*x - 48*a*b**2*e - 48*a*b**2*f*x - 24*
b**3*e - 24*b**3*f*x)/(8*a**4*f*(sin(e + f*x)**2*a - a - b))
```

3.49 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx = \frac{(a+4b)x}{2a^3} - \frac{\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/2*(a+4*b)*x/a^3-1/2*b^(1/2)*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(1/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-
b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.85 (sec) , antiderivative size = 699, normalized size of antiderivative = 5.38

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(-\frac{2\left(16x+\frac{(-a^3+6a^2b+24ab^2+16b^3)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(f\ldots}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e)\ldots}}\right)\ldots}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}\ldots\right)}{\ldots}\ldots\right)}{\ldots}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*((-2*(16*x + ((-a^3 + 6*a
^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin
[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(
b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin
[e]))))/a^2 - (-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^
3 - 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x
]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Co
s[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) +
(16*a*Cos[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a
^2*b + 48*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*
f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))/a^3
 + (2*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2
) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))
)/(b^(3/2)*f) + (-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^
(3/2)) + (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*
(e + f*x)])))/(b^(3/2)*f)))/(256*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4620, 373, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\int \frac{-3b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{2(a+b)\left(-2b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{-2b\tan^2(e+fx)+a+2b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a}-\frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{(a+4b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b(3a+4b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a}-\frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{(a+4b)\arctan(\tan(e+fx))}{a}-\frac{b(3a+4b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a}-\frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{(a+4b)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a}-\frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/2*Tan[e + f*x]/(a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)) + (
(((a + 4*b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/a - (2*b*Tan[e + f*x])/(a*(a
 + b + b*Tan[e + f*x]^2)))/(2*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+4b)\arctan(\tan(fx+e))}{2}}{a^3}-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^3}}{f}$
default	$\frac{\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+4b)\arctan(\tan(fx+e))}{2}}{a^3}-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^3}}{f}$
risch	$\frac{x}{2a^2}+\frac{2xb}{a^3}+\frac{ie^{2i(fx+e)}}{8a^2f}-\frac{ie^{-2i(fx+e)}}{8a^2f}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^3f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{3\sqrt{-(a+b)b}\,\ln\Big(}{}$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*(-1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+4*b)*arctan(tan(f*x+
e)))-1/a^3*b*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+4*b)/((a+b)*b
)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.39

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$=\left[\frac{4\,(a^2+4\,ab)fx\cos{(fx+e)}^2+4\,(ab+4\,b^2)fx+\left((3\,a^2+4\,ab)\cos{(fx+e)}^2+3\,ab+4\,b^2\right)\sqrt{-\frac{b}{a+b}}\log}{}\right.$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 4*(a*b + 4*b^2)*f*x + ((3*a^2 +
 4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b
 + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*
a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*s
in(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*
(a^2*cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f*x + e
)^2 + a^3*b*f), 1/4*(2*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 2*(a*b + 4*b^2)*
f*x + ((3*a^2 + 4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(b/(a + b))*arc
tan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin
(f*x + e))) - 2*(a^2*cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a
^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{2\,b\tan(fx+e)^3+(a+2\,b)\tan(fx+e)}{a^2b\tan(fx+e)^4+a^3+a^2b+(a^3+2\,a^2b)\tan(fx+e)^2}-\frac{(fx+e)(a+4\,b)}{a^3}+\frac{(3\,ab+4\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3}}{2\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*b*tan(f*x + e)^3 + (a + 2*b)*tan(f*x + e))/(a^2*b*tan(f*x + e)^4
+ a^3 + a^2*b + (a^3 + 2*a^2*b)*tan(f*x + e)^2) - (f*x + e)*(a + 4*b)/a^3
+ (3*a*b + 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*
a^3))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{(fx+e)(a+4\,b)}{a^3} - \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+4\,b^2)}{\sqrt{ab+b^2}a^3} - \frac{2\,b\tan(fx+e)^3+a\tan(fx+e)+2\,b\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2\,b\tan(fx+e)^2+a+b\right)a^2}}{2\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((f*x + e)*(a + 4*b)/a^3 - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arct
an(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a*b + 4*b^2)/(sqrt(a*b + b^2)*a^3)
- (2*b*tan(f*x + e)^3 + a*tan(f*x + e) + 2*b*tan(f*x + e))/((b*tan(f*x + e
)^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 816, normalized size of antiderivative = 6.28

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

output

```
(atan(((((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 + (((4*a^6*b^3
 + 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(
-b*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/
(a^3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4) + (((tan
(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 + 2*a^7*b^2)
/a^6 - (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b*(a + b))^(
1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4)
)*((3*a)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4))/((8*a*b^4 + 8*b^5 +
(3*a^2*b^3)/2)/a^6 + (((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4
+ (((4*a^6*b^3 + 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*(
(3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a
 + b))^(1/2))/(a^3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^
4) - (((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 +
 2*a^7*b^2)/a^6 - (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b
*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a
^3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4)))*((3*a)/4 +
b)*(-b*(a + b))^(1/2)*2i)/(f*(a^3*b + a^4)) - (atan((b^2*tan(e + f*x))/(4*
(b^2/4 + b^3/a)) + (b^3*tan(e + f*x))/((a*b^2)/4 + b^3))*(a*1i + b*4i)*1i)
/(2*a^3*f) - ((tan(e + f*x)*(a + 2*b))/(2*a^2) + (b*tan(e + f*x)^3)/a^2)/(
f*(a + b + b*tan(e + f*x)^4 + tan(e + f*x)^2*(a + 2*b)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 663, normalized size of antiderivative = 5.10

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input

```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*sin(e + f*x)**2*a**2 - 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 + 7*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b +
4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*b**2 - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(
a))/sqrt(b))*sin(e + f*x)**2*a**2 - 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 + 7*s
qrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*
a*b + 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/
sqrt(b))*b**2 - cos(e + f*x)*sin(e + f*x)**3*a**3 - cos(e + f*x)*sin(e + f
*x)**3*a**2*b + cos(e + f*x)*sin(e + f*x)*a**3 + 3*cos(e + f*x)*sin(e + f*
x)*a**2*b + 2*cos(e + f*x)*sin(e + f*x)*a*b**2 + sin(e + f*x)**2*a**3*e +
sin(e + f*x)**2*a**3*f*x + 5*sin(e + f*x)**2*a**2*b*e + 5*sin(e + f*x)**2*
a**2*b*f*x + 4*sin(e + f*x)**2*a*b**2*e + 4*sin(e + f*x)**2*a*b**2*f*x - a
**3*e - a**3*f*x - 6*a**2*b*e - 6*a**2*b*f*x - 9*a*b**2*e - 9*a*b**2*f*x -
 4*b**3*e - 4*b**3*f*x)/(2*a**3*f*(sin(e + f*x)**2*a**2 + sin(e + f*x)**2*
a*b - a**2 - 2*a*b - b**2))
```

3.50 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{x}{a^2}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^2-1/2*b^(1/2)*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.61

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx))) + \frac{b(3a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(\cdots}{2\sqrt{a+\cdots}}\right.}{\cdots}\right.}{8a^2\left(a+b\sec^2(e+\cdots\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e
+ f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin
[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)
^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2
*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Se
c[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4616, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-b\tan^2(e+fx)+2a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2(a+b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b(3a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{b(3a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
(((2*(a + b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*Tan[e +
f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{4(a+b)^2fa}+$

input
```
int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^2*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(
f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time = 0.12 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.73

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \left[\frac{8\,(a^2+ab)fx\cos{(fx+e)}^2 - 4\,ab\cos{(fx+e)}\sin{(fx+e)} + 8\,(ab+b^2)fx + \left((3\,a^2+2\,ab)\cos{(fx+\right.}{8\,\left((a^4+\right.}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
 8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*
b^2)*f), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*
x + e) + 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b
^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3
*b + a^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^2}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{b\tan(fx+e)}{a^3+2\,a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2} + \frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)
^2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b
)*sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{\left(\pi\left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}} + \frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e
)/((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 2056, normalized size of antiderivative = 22.35

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b
^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/
(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b
^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a
^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*
a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a
^2*b^2)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a
^3*b + a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/
(2*(2*a^4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 6
4*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) +
(tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*
b^2)))/a^2 + (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5
 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^
7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20
*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*
a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan(((((tan(e + f*
x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(
a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*
b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5
*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.11

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*sin(e + f*x)**2*a**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 + 5*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b +
2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*b**2 - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(
a))/sqrt(b))*sin(e + f*x)**2*a**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 + 5*s
qrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*
a*b + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/
sqrt(b))*b**2 + cos(e + f*x)*sin(e + f*x)*a**2*b + cos(e + f*x)*sin(e + f*
x)*a*b**2 + 2*sin(e + f*x)**2*a**3*f*x + 4*sin(e + f*x)**2*a**2*b*f*x + 2*
sin(e + f*x)**2*a*b**2*f*x - 2*a**3*f*x - 6*a**2*b*f*x - 6*a*b**2*f*x - 2*
b**3*f*x)/(2*a**2*f*(sin(e + f*x)**2*a**3 + 2*sin(e + f*x)**2*a**2*b + sin
(e + f*x)**2*a*b**2 - a**3 - 3*a**2*b - 3*a*b**2 - b**3))
```

3.51 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f}-\frac{3\cot(e+fx)}{2(a+b)^2f}+\frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-3/2*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(5/2)/f-3/2*cot(
f*x+e)/(a+b)^2/f+1/2*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.66

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$=\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{3b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(\cdots}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{8(a+b)^2f\,(a\cdots}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((3*b*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2
*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2*(a + 2*
b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + (b*((a + 2*b)*Sin[2
*e] - a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a + b)^
2*f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4620, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{3\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2(a+b)}+\frac{\cot(e+fx)}{2(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{3\left(-\frac{b\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{a+b}\right)}{2(a+b)}+\frac{\cot(e+fx)}{2(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cot(e+fx)}{a+b}\right)}{2(a+b)}+\frac{\cot(e+fx)}{2(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((3*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2))
 - Cot[e + f*x]/(a + b)))/(2*(a + b)) + Cot[e + f*x]/(2*(a + b)*(a + b + b
*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)^2\tan(fx+e)}-\frac{b\left(\frac{\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^2}}{f}$
default	$\frac{-\frac{1}{(a+b)^2\tan(fx+e)}-\frac{b\left(\frac{\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^2}}{f}$
risch	$-\frac{i(2a^2e^{4i(fx+e)}+ab\,e^{4i(fx+e)}+2b^2e^{4i(fx+e)}+4a^2e^{2i(fx+e)}+8ab\,e^{2i(fx+e)}-2b^2e^{2i(fx+e)}+2a^2-ab)}{a(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}-1\right)}-\frac{3\sqrt{-(a+b)}}{}$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a+b)^2/tan(f*x+e)-1/(a+b)^2*b*(1/2*tan(f*x+e)/(a+b+b*tan(f*x+e)^2
)+3/2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(77) = 154$.

Time = 0.11 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.47

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[-\frac{4\,(2\,a-b)\cos\left(fx+e\right)^3-3\,\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+\right.}{\left.a^2\right.}}{8\,\left((a^3+2\,a^2b+ab^2)f\cos\left(fx+e\right)^2+\right.}\right.$$

$$-\frac{2\,(2\,a-b)\cos\left(fx+e\right)^3-3\,\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(fx+e\right.}{4\,\left((a^3+2\,a^2b+ab^2)f\cos\left(fx+e\right)^2+(a^2b+2\,ab^2+b^3)f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/(a +
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
+ e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e
))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*
x + e)^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^
2)*f*cos(f*x + e)^2 + (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e)), -1/4*(2*(2
*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(b/(a + b))*arctan(1
/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x
+ e)))*sin(f*x + e) + 6*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*
x + e)^2 + (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}} + \frac{3\,b\tan(fx+e)^2+2\,a+2\,b}{(a^2b+2\,ab^2+b^3)\tan(fx+e)^3+(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)}}{2\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt
((a + b)*b)) + (3*b*tan(f*x + e)^2 + 2*a + 2*b)/((a^2*b + 2*a*b^2 + b^3)*t
an(f*x + e)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.40

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}} + \frac{3\,b\tan(fx+e)^2+2\,a+2\,b}{\left(b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)\right)(a^2+2\,ab+b^2)}}{2\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^2
 + 2*a + 2*b)/((b*tan(f*x + e)^3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^2 +
 2*a*b + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{1}{a+b}+\frac{3\,b\tan(e+f\,x)^2}{2\,(a+b)^2}}{f\,\left(b\tan\left(e+f\,x\right)^3+\left(a+b\right)\,\tan\left(e+f\,x\right)\right)} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^2+2\,a\,b+b^2\right)}{(a+b)^{5/2}}\right)}{2\,f\,(a+b)^{5/2}}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
- (1/(a + b) + (3*b*tan(e + f*x)^2)/(2*(a + b)^2))/(f*(b*tan(e + f*x)^3 +
tan(e + f*x)*(a + b))) - (3*b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^
2 + b^2))/(a + b)^(5/2)))/(2*f*(a + b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.69

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{-3\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^3 a+3\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(f\right.}{}$$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*sin(e + f*x)**3*a + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*a + 3*sqrt(b)*sqrt(a + b)*atan(
(sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*b - 3*sqrt(
b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(
e + f*x)**3*a + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) +
 sqrt(a))/sqrt(b))*sin(e + f*x)*a + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)*b - 2*cos(e + f*x)*sin
(e + f*x)**2*a**2 - cos(e + f*x)*sin(e + f*x)**2*a*b + cos(e + f*x)*sin(e
+ f*x)**2*b**2 + 2*cos(e + f*x)*a**2 + 4*cos(e + f*x)*a*b + 2*cos(e + f*x)
*b**2)/(2*sin(e + f*x)*f*(sin(e + f*x)**2*a**4 + 3*sin(e + f*x)**2*a**3*b
+ 3*sin(e + f*x)**2*a**2*b**2 + sin(e + f*x)**2*a*b**3 - a**4 - 4*a**3*b -
 6*a**2*b**2 - 4*a*b**3 - b**4))
```

3.52 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{(3a-2b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f}-\frac{(a-b)\cot(e+fx)}{(a+b)^3f}-\frac{\cot^3(e+fx)}{3(a+b)^2f}-\frac{ab\tan(e+fx)}{2(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-1/2*(3*a-2*b)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(7/2)/
f-(a-b)*cot(f*x+e)/(a+b)^3/f-1/3*cot(f*x+e)^3/(a+b)^2/f-1/2*a*b*tan(f*x+e)
/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.87 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.46

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-2(a+b)(a+2b+a\cos(2(e+fx)))\cot(e)\csc^2(e+fx)+\right.}{}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-2*(a + b)*(a + 2*b + a*Co
s[2*(e + f*x)])*Cot[e]*Csc[e + f*x]^2 + (3*(3*a - 2*b)*b*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqr
t[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(C
os[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 4*(a
- 2*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + 2*(a
+ b)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]^3*Sin[f*x] - 3*a*b
*Sec[2*e]*Sin[2*f*x] + 3*b*(a + 2*b)*Tan[2*e]))/(24*(a + b)^3*f*(a + b*Sec
[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^4\,(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 361$$

$$\frac{-\frac{1}{2}b\int -\frac{\cot^4(e+fx)\left(-\frac{a\tan^4(e+fx)}{(a+b)^3}+\frac{2a\tan^2(e+fx)}{b(a+b)^2}+\frac{2}{b(a+b)}\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{ab\tan(e+fx)}{2(a+b)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}b\int \frac{\cot^4(e+fx)\left(-\frac{a\tan^4(e+fx)}{(a+b)^3}+\frac{2a\tan^2(e+fx)}{b(a+b)^2}+\frac{2}{b(a+b)}\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{ab\tan(e+fx)}{2(a+b)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 1584$$

$$\frac{\frac{1}{2}b\int \left(\frac{2\cot^4(e+fx)}{b(a+b)^2}+\frac{2(a-b)\cot^2(e+fx)}{b(a+b)^3}+\frac{2b-3a}{(a+b)^3(b\tan^2(e+fx)+a+b)}\right)d\tan(e+fx)-\frac{ab\tan(e+fx)}{2(a+b)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{2}b\left(-\frac{(3a-2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}(a+b)^{7/2}}-\frac{2\cot^3(e+fx)}{3b(a+b)^2}-\frac{2(a-b)\cot(e+fx)}{b(a+b)^3}\right)-\frac{ab\tan(e+fx)}{2(a+b)^3(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b*(-(((3*a - 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(Sqrt[b]*(
a + b)^(7/2))) - (2*(a - b)*Cot[e + f*x])/(b*(a + b)^3) - (2*Cot[e + f*x]^
3)/(3*b*(a + b)^2)))/2 - (a*b*Tan[e + f*x])/(2*(a + b)^3*(a + b + b*Tan[e
+ f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{a-b}{(a+b)^3\tan(fx+e)}-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^3}}{f}$
default	$\frac{-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{a-b}{(a+b)^3\tan(fx+e)}-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^3}}{f}$
risch	$\frac{i(9\,e^{8i(fx+e)}ab-6\,e^{8i(fx+e)}b^2+12a^2\,e^{6i(fx+e)}+18ab\,e^{6i(fx+e)}+66b^2\,e^{6i(fx+e)}+20a^2\,e^{4i(fx+e)}+44ab\,e^{4i(fx+e)}-66b^2}{3f(a+b)^3\left(e^{2i(fx+e)}-1\right)^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i}\right.}$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3/(a+b)^2/tan(f*x+e)^3-(a-b)/(a+b)^3/tan(f*x+e)-1/(a+b)^3*b*(1/2*a
*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a-2*b)/((a+b)*b)^(1/2)*arctan(b*ta
n(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(109) = 218.

Time = 0.15 (sec) , antiderivative size = 663, normalized size of antiderivative = 5.39

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/24*(4*(4*a^2 - 11*a*b)*cos(f*x + e)^5 - 8*(3*a^2 - 8*a*b + 4*b^2)*cos(
f*x + e)^3 + 3*((3*a^2 - 2*a*b)*cos(f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*c
os(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b
^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e)
 - 12*(3*a*b - 2*b^2)*cos(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f
*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*
b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x + e)), -1/12*(2*(4*a^2 - 11*a*b)
*cos(f*x + e)^5 - 4*(3*a^2 - 8*a*b + 4*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - 2
*a*b)*cos(f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 - 3*a*b + 2*
b^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
 b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(3*a*b - 2*b^2)*cos(f
*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2
*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 +
b^4)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.57

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,(3\,ab-2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}}+\frac{3\,(3\,ab-2\,b^2)\tan(fx+e)^4+2\,(3\,a^2+ab-2\,b^2)\tan(fx+e)^2+2\,a^2+4\,ab+2\,b^2}{(a^3b+3\,a^2b^2+3\,ab^3+b^4)\tan(fx+e)^5+(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\tan(fx+e)^3}}{6\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(3*a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*sqrt((a + b)*b)) + (3*(3*a*b - 2*b^2)*tan(f*x + e)^4
 + 2*(3*a^2 + a*b - 2*b^2)*tan(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)/((a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*tan(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.50

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,ab\tan(fx+e)}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)} + \frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab-2\,b^2)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab+b^2}} + \frac{2\left(3\,a\tan(fx+e)^2-3\,b\tan\cdots\right.}{(a^3+3\,a^2b+3\,ab^2+b^3)\,\cdots}}{6\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*a*b*tan(f*x + e)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*tan(f*x + e)^
2 + a + b)) + 3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x +
e)/sqrt(a*b + b^2)))*(3*a*b - 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt
(a*b + b^2)) + 2*(3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^2 + a + b)/((a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{1}{3\,(a+b)}+\frac{\tan(e+f\,x)^2\,(3\,a-2\,b)}{3\,(a+b)^2}+\frac{b\tan(e+f\,x)^4\,(3\,a-2\,b)}{2\,(a+b)^3}}{f\,\left(b\tan\left(e+f\,x\right)^5+(a+b)\,\tan\left(e+f\,x\right)^3\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{(a+b)^{7/2}}\right)\,(3\,a-2\,b)}{2\,f\,(a+b)^{7/2}}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
- (1/(3*(a + b)) + (tan(e + f*x)^2*(3*a - 2*b))/(3*(a + b)^2) + (b*tan(e +
 f*x)^4*(3*a - 2*b))/(2*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b) + b*tan(e +
 f*x)^5)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(3*a*b^2 + 3*a^2*b + a^3 +
 b^3))/(a + b)^(7/2))*(3*a - 2*b))/(2*f*(a + b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 735, normalized size of antiderivative = 5.98

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 9*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*sin(e + f*x)**5*a**2 + 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a*b + 9*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*
a**2 + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))
/sqrt(b))*sin(e + f*x)**3*a*b - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*b**2 - 9*sqrt(b)*sqrt(a
 + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)*
*5*a**2 + 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(
a))/sqrt(b))*sin(e + f*x)**5*a*b + 9*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**3*a**2 + 3*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*
x)**3*a*b - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqr
t(a))/sqrt(b))*sin(e + f*x)**3*b**2 - 4*cos(e + f*x)*sin(e + f*x)**4*a**3
+ 7*cos(e + f*x)*sin(e + f*x)**4*a**2*b + 11*cos(e + f*x)*sin(e + f*x)**4*
a*b**2 + 2*cos(e + f*x)*sin(e + f*x)**2*a**3 - 4*cos(e + f*x)*sin(e + f*x)
**2*a**2*b - 14*cos(e + f*x)*sin(e + f*x)**2*a*b**2 - 8*cos(e + f*x)*sin(e
 + f*x)**2*b**3 + 2*cos(e + f*x)*a**3 + 6*cos(e + f*x)*a**2*b + 6*cos(e +
f*x)*a*b**2 + 2*cos(e + f*x)*b**3)/(6*sin(e + f*x)**3*f*(sin(e + f*x)**2*a
**5 + 4*sin(e + f*x)**2*a**4*b + 6*sin(e + f*x)**2*a**3*b**2 + 4*sin(e ...
```

3.53 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{a(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{9/2}f} - \frac{a(a-2b)\cot(e+fx)}{(a+b)^4f} - \frac{2a\cot^3(e+fx)}{3(a+b)^3f} - \frac{\cot^5(e+fx)}{5(a+b)^2f} - \frac{a^2b\tan(e+fx)}{2(a+b)^4f\,(a+b+b\tan^2(e+fx))}$$

output
```
-1/2*a*(3*a-4*b)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(9/2
)/f-a*(a-2*b)*cot(f*x+e)/(a+b)^4/f-2/3*a*cot(f*x+e)^3/(a+b)^3/f-1/5*cot(f*
x+e)^5/(a+b)^2/f-1/2*a^2*b*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.18 (sec) , antiderivative size = 777, normalized size of antiderivative = 5.25

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{960a(3a-4b)b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

input

```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output

```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((960*a*(3*a - 4*b)*b*ArcTa
n[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f
*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e
 + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])
^4]) - Csc[e]*Csc[e + f*x]^5*Sec[2*e]*(10*a*(16*a^2 + 34*a*b + 123*b^2)*Si
n[f*x] - a*(16*a^2 - 223*a*b + 1336*b^2)*Sin[3*f*x] + 240*a^3*Sin[2*e - f*
x] + 640*a^2*b*Sin[2*e - f*x] - 1460*a*b^2*Sin[2*e - f*x] + 240*b^3*Sin[2*
e - f*x] - 240*a^3*Sin[2*e + f*x] - 715*a^2*b*Sin[2*e + f*x] + 860*a*b^2*S
in[2*e + f*x] - 240*b^3*Sin[2*e + f*x] + 160*a^3*Sin[4*e + f*x] + 415*a^2*
b*Sin[4*e + f*x] + 1830*a*b^2*Sin[4*e + f*x] + 165*a^2*b*Sin[2*e + 3*f*x]
 - 30*a*b^2*Sin[2*e + 3*f*x] + 120*b^3*Sin[2*e + 3*f*x] - 16*a^3*Sin[4*e +
3*f*x] + 208*a^2*b*Sin[4*e + 3*f*x] - 1036*a*b^2*Sin[4*e + 3*f*x] + 180*a^
2*b*Sin[6*e + 3*f*x] - 330*a*b^2*Sin[6*e + 3*f*x] + 120*b^3*Sin[6*e + 3*f*
x] + 48*a^3*Sin[2*e + 5*f*x] - 268*a^2*b*Sin[2*e + 5*f*x] + 290*a*b^2*Sin[
2*e + 5*f*x] - 24*b^3*Sin[2*e + 5*f*x] + 48*a^3*Sin[6*e + 5*f*x] - 223*a^2
*b*Sin[6*e + 5*f*x] + 230*a*b^2*Sin[6*e + 5*f*x] - 24*b^3*Sin[6*e + 5*f*x]
 - 45*a^2*b*Sin[8*e + 5*f*x] + 60*a*b^2*Sin[8*e + 5*f*x] - 16*a^3*Sin[4*e
+ 7*f*x] + 83*a^2*b*Sin[4*e + 7*f*x] - 6*a*b^2*Sin[4*e + 7*f*x] - 15*a^2*b
*Sin[6*e + 7*f*x] - 16*a^3*Sin[8*e + 7*f*x] + 68*a^2*b*Sin[8*e + 7*f*x] -
6*a*b^2*Sin[8*e + 7*f*x])))/(7680*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4620, 365, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 365}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+10a+3b\right)}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 361}$$

$$\frac{\frac{-\frac{1}{2}b\int -\frac{\cot^4(e+fx)\left(-\frac{\left(5a^2+2b^2\right)\tan^4(e+fx)}{(a+b)^3}+\frac{2\left(5a^2+2b^2\right)\tan^2(e+fx)}{b(a+b)^2}+\frac{2(10a+3b)}{b(a+b)}\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{2(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}}{5(a+b)}-\frac{\cot^5(e+}{5(a+b)(a+b\tan}}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{1}{2}b\int \frac{\cot^4(e+fx)\left(-\frac{\left(5a^2+2b^2\right)\tan^4(e+fx)}{(a+b)^3}+\frac{2\left(5a^2+2b^2\right)\tan^2(e+fx)}{b(a+b)^2}+\frac{2(10a+3b)}{b(a+b)}\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{2(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}}{5(a+b)}-\frac{\cot^5(e+fx}{5(a+b)(a+b\tan^2(e}}{f}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{\frac{\frac{1}{2}b\int\left(\frac{2(10a+3b)\cot^4(e+fx)}{b(a+b)^2}-\frac{2\left(-5a^2+10ba+b^2\right)\cot^2(e+fx)}{b(a+b)^3}-\frac{5a(3a-4b)}{(a+b)^3\left(b\tan^2(e+fx)+a+b\right)}\right)d\tan(e+fx)-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{2(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}}{5(a+b)}-\frac{\text{co}}{5(a+b)(a\text{-}}}{f}$$

$\downarrow$ 2009

$$\frac{\frac{\frac{1}{2}b\left(-\frac{2\left(5a^2-10ab-b^2\right)\cot(e+fx)}{b(a+b)^3}-\frac{5a(3a-4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}(a+b)^{7/2}}-\frac{2(10a+3b)\cot^3(e+fx)}{3b(a+b)^2}\right)-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{2(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}}{5(a+b)}-\frac{\cot^5(e+fx}{5(a+b)(a+b\tan^2(e}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/5*Cot[e + f*x]^5/((a + b)*(a + b + b*Tan[e + f*x]^2)) + ((b*((-5*a*(3*
a - 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(Sqrt[b]*(a + b)^(7/2
)) - (2*(5*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(b*(a + b)^3) - (2*(10*a + 3*
b)*Cot[e + f*x]^3)/(3*b*(a + b)^2)))/2 - (b*(5*a^2 + 2*b^2)*Tan[e + f*x])/
(2*(a + b)^3*(a + b + b*Tan[e + f*x]^2)))/(5*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
 - a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{ba\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{a(a-2b)}{(a+b)^4\tan(fx+e)}-\frac{2a}{3(a+b)^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{ba\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{a(a-2b)}{(a+b)^4\tan(fx+e)}-\frac{2a}{3(a+b)^3\tan(fx+e)}}{f}$
risch	$-\frac{i(-45a^2b\,e^{12i(fx+e)}+60a\,b^2e^{12i(fx+e)}+180a^2b\,e^{10i(fx+e)}-330a\,b^2e^{10i(fx+e)}+120b^3e^{10i(fx+e)}+160a^3e^{8i(fx+e)}+}{}$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b*a/(a+b)^4*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a-4*b)/((a
+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/5/(a+b)^2/tan(f*x+e)^
5-a*(a-2*b)/(a+b)^4/tan(f*x+e)-2/3*a/(a+b)^3/tan(f*x+e)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(132) = 264$.

Time = 0.15 (sec) , antiderivative size = 987, normalized size of antiderivative = 6.67

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/120*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(40*a^3 - 201*
a^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 29*a^2*b + 28*a*b^2
)*cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11*a^2*
b + 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b + 8*a*
b^2)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*
x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/
(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^
2*b - 4*a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*
b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4
 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b
^5)*f)*sin(f*x + e)), -1/60*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^
7 - 2*(40*a^3 - 201*a^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 10*(6*a^3 -
 29*a^2*b + 28*a*b^2)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^
6 - (6*a^3 - 11*a^2*b + 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a
^3 - 10*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a +
2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*si
n(f*x + e) + 30*(3*a^2*b - 4*a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3
*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(132) = 264$.

Time = 0.12 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.81

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{15\,(3\,a^2b-4\,ab^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)b}} + \frac{15\,(3\,a^2b-4\,ab^2)\tan(fx+e)^6+10\,(3\,a^3-a^2b-4\,ab^2)\tan(fx+e)^4+6\,a^3+18\,a^2b+18\,ab^2+6\,b^3+2}{(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5)\tan(fx+e)^7+(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+}}{30\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/30*(15*(3*a^2*b - 4*a*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a^2*b -
4*a*b^2)*tan(f*x + e)^6 + 10*(3*a^3 - a^2*b - 4*a*b^2)*tan(f*x + e)^4 + 6*
a^3 + 18*a^2*b + 18*a*b^2 + 6*b^3 + 2*(10*a^3 + 23*a^2*b + 16*a*b^2 + 3*b^
3)*tan(f*x + e)^2)/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*
x + e)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f
*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.72

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{15\,a^2b\tan(fx+e)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)} + \frac{15\,(3\,a^2b-4\,ab^2)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab+b^2}} + \frac{2\left(15\,a^2\tan(fx\right.}{}}{30\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/30*(15*a^2*b*tan(f*x + e)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*
(b*tan(f*x + e)^2 + a + b)) + 15*(3*a^2*b - 4*a*b^2)*(pi*floor((f*x + e)/p
i + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b + b^2)) + 2*(15*a^2*tan(f*x + e)^4 -
 30*a*b*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3
*a^2 + 6*a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x
 + e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.34

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{a\sqrt{b}\,\mathrm{atan}\left(\frac{a\sqrt{b}\tan(e+f\,x)\,(3\,a-4\,b)\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}{(a+b)^{9/2}\,(4\,a\,b-3\,a^2)}\right)\,(3\,a-4\,b)}{2\,f\,(a+b)^{9/2}} - \frac{\frac{1}{5\,(a+b)}-\frac{\tan(e+f\,x)^4\,(4\,a\,b-3\,a^2)}{3\,(a+b)^3}+\frac{\tan(e+f\,x)^2\,(10\,a+3\,b)}{15\,(a+b)^2}-\frac{b\tan(e+f\,x)^6\,(4\,a\,b-3\,a^2)}{2\,(a+b)^4}}{f\,\left(b\tan\left(e+f\,x\right)^7+\left(a+b\right)\,\tan\left(e+f\,x\right)^5\right)}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(a*b^(1/2)*atan((a*b^(1/2)*tan(e + f*x)*(3*a - 4*b)*(4*a*b^3 + 4*a^3*b + a
^4 + b^4 + 6*a^2*b^2))/((a + b)^(9/2)*(4*a*b - 3*a^2)))*(3*a - 4*b))/(2*f*
(a + b)^(9/2)) - (1/(5*(a + b)) - (tan(e + f*x)^4*(4*a*b - 3*a^2))/(3*(a +
 b)^3) + (tan(e + f*x)^2*(10*a + 3*b))/(15*(a + b)^2) - (b*tan(e + f*x)^6*
(4*a*b - 3*a^2))/(2*(a + b)^4))/(f*(tan(e + f*x)^5*(a + b) + b*tan(e + f*x
)^7))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 927, normalized size of antiderivative = 6.26

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 45*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**7*a**3 + 60*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**2*b + 45*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*
x)**5*a**3 - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - s
qrt(a))/sqrt(b))*sin(e + f*x)**5*a**2*b - 60*sqrt(b)*sqrt(a + b)*atan((sqr
t(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a*b**2 - 45*
sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))
*sin(e + f*x)**7*a**3 + 60*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**2*b + 45*sqrt(b)*sqrt(a + b
)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**5*a
**3 - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))
/sqrt(b))*sin(e + f*x)**5*a**2*b - 60*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**5*a*b**2 - 16*cos(e +
 f*x)*sin(e + f*x)**6*a**4 + 67*cos(e + f*x)*sin(e + f*x)**6*a**3*b + 77*c
os(e + f*x)*sin(e + f*x)**6*a**2*b**2 - 6*cos(e + f*x)*sin(e + f*x)**6*a*b
**3 + 8*cos(e + f*x)*sin(e + f*x)**4*a**4 - 40*cos(e + f*x)*sin(e + f*x)**
4*a**3*b - 98*cos(e + f*x)*sin(e + f*x)**4*a**2*b**2 - 44*cos(e + f*x)*sin
(e + f*x)**4*a*b**3 + 6*cos(e + f*x)*sin(e + f*x)**4*b**4 + 2*cos(e + f*x)
*sin(e + f*x)**2*a**4 - 6*cos(e + f*x)*sin(e + f*x)**2*a**3*b - 30*cos(...
```

3.54 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 200

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} \, dx = \frac{\sqrt{b}(15a^2+70ab+63b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f} - \frac{(a^2+6ab+6b^2)\cos(e+fx)}{a^5f} + \frac{(2a+3b)\cos^3(e+fx)}{3a^4f} - \frac{\cos^5(e+fx)}{5a^3f} + \frac{b^2(a+b)^2\cos(e+fx)}{4a^5f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(a+b)(9a+17b)\cos(e+fx)}{8a^5f\left(b+a\cos^2(e+fx)\right)}$$

output
```
1/8*b^(1/2)*(15*a^2+70*a*b+63*b^2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(1
1/2)/f-(a^2+6*a*b+6*b^2)*cos(f*x+e)/a^5/f+1/3*(2*a+3*b)*cos(f*x+e)^3/a^4/f
-1/5*cos(f*x+e)^5/a^3/f+1/4*b^2*(a+b)^2*cos(f*x+e)/a^5/f/(b+a*cos(f*x+e)^2
)^2-1/8*b*(a+b)*(9*a+17*b)*cos(f*x+e)/a^5/f/(b+a*cos(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.00 (sec) , antiderivative size = 1641, normalized size of antiderivative = 8.20

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-900*a^(11/2)*b^(3/2)*Cos[
e + f*x] - 109000*a^(9/2)*b^(5/2)*Cos[e + f*x] - 936000*a^(7/2)*b^(7/2)*Co
s[e + f*x] - 2803072*a^(5/2)*b^(9/2)*Cos[e + f*x] - 3763200*a^(3/2)*b^(11/
2)*Cos[e + f*x] - 1935360*Sqrt[a]*b^(13/2)*Cos[e + f*x] - 900*a^(11/2)*b^(
3/2)*Cos[e + f*x]*Cos[2*(e + f*x)] + 900*a^(9/2)*b^(3/2)*Cos[e + f*x]*(a +
 2*b + a*Cos[2*(e + f*x)]) + 24000*a^(7/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b +
 a*Cos[2*(e + f*x)]) + 43200*a^(5/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos
[2*(e + f*x)]) + 225*a^5*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])
^2 + 115200*a^2*b^3*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin
[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e]
- I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 +
537600*a*b^4*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
)*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin
[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 483840*
b^5*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*
Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*T
an[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 225*a^5*ArcTan[(
(-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)...
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4621, 366, 25, 360, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)^5}{\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 4621}$$

$$-\frac{\int \frac{\cos^6(e+fx)\left(1-\cos^2(e+fx)\right)^2}{\left(a\cos^2(e+fx)+b\right)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 366}$$

$$-\frac{\frac{(a+b)^2\cos^7(e+fx)}{4a^2b\left(a\cos^2(e+fx)+b\right)^2}-\frac{\int -\frac{\cos^6(e+fx)\left(4a^2+4b\cos^2(e+fx)a-7(a+b)^2\right)}{\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4a^2b}}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int \frac{\cos^6(e+fx)\left(4a^2+4b\cos^2(e+fx)a-7(a+b)^2\right)}{\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4a^2b}+\frac{(a+b)^2\cos^7(e+fx)}{4a^2b\left(a\cos^2(e+fx)+b\right)^2}}{f}$$

$$\Big\downarrow \text{ 360}$$

$$-\frac{\frac{\frac{b^2(a+b)(3a+11b)\cos(e+fx)}{2a^3\left(a\cos^2(e+fx)+b\right)}-\frac{\int \frac{-8a^4b\cos^6(e+fx)+2a^3(a+b)(3a+11b)\cos^4(e+fx)-2a^2b(a+b)(3a+11b)\cos^2(e+fx)+ab^2(a+b)(3a+11b)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a^4}}{4a^2b}+\frac{\quad}{4a}}{f}$$

$$\Big\downarrow \text{ 2341}$$

$$-\frac{\frac{\frac{b^2(a+b)(3a+11b)\cos(e+fx)}{2a^3\left(a\cos^2(e+fx)+b\right)}-\frac{\int \left(-8a^3b\cos^4(e+fx)+2a^2(a+3b)(3a+5b)\cos^2(e+fx)-4ab\left(3a^2+14ba+13b^2\right)+\frac{63ab^4+70a^2b^3+15a^3b^2}{a\cos^2(e+fx)+b}\right)d\cos(e+fx)}{2a^4}}{4a^2b}+\frac{(a}{4a^2}}{f}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{(a+b)^2\cos^7(e+fx)}{4a^2b(a\cos^2(e+fx)+b)^2}+\frac{\frac{b^2(a+b)(3a+11b)\cos(e+fx)}{2a^3\left(a\cos^2(e+fx)+b\right)}-\frac{-\frac{8}{5}a^3b\cos^5(e+fx)+\sqrt{a}b^{3/2}\left(15a^2+70ab+63b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)-4ab\left(3a^2+14ab+13b^2\right.}{2a^4}}{4a^2b}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-((((a + b)^2*Cos[e + f*x]^7)/(4*a^2*b*(b + a*Cos[e + f*x]^2)^2) + ((b^2*(
a + b)*(3*a + 11*b)*Cos[e + f*x])/(2*a^3*(b + a*Cos[e + f*x]^2)) - (Sqrt[a
]*b^(3/2)*(15*a^2 + 70*a*b + 63*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]
] - 4*a*b*(3*a^2 + 14*a*b + 13*b^2)*Cos[e + f*x] + (2*a^2*(a + 3*b)*(3*a +
 5*b)*Cos[e + f*x]^3)/3 - (8*a^3*b*Cos[e + f*x]^5)/5)/(2*a^4))/(4*a^2*b))/
f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 7.33 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-a\cos(fx+e)^3b+a^2\cos(fx+e)+6ab\cos(fx+e)+6b^2\cos(fx+e)}{a^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{13}{4}a^2b-\frac{17}{8}ab^2\right)}{(}\right.}{}}{f}$
default	$\frac{-\frac{\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-a\cos(fx+e)^3b+a^2\cos(fx+e)+6ab\cos(fx+e)+6b^2\cos(fx+e)}{a^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{13}{4}a^2b-\frac{17}{8}ab^2\right)}{(}\right.}{}}{f}$
risch	$-\frac{e^{5i(fx+e)}}{160a^3f}+\frac{5e^{3i(fx+e)}}{96a^3f}+\frac{e^{3i(fx+e)}b}{8a^4f}-\frac{5e^{i(fx+e)}}{16a^3f}-\frac{21e^{i(fx+e)}b}{8a^4f}-\frac{3e^{i(fx+e)}b^2}{a^5f}-\frac{5e^{-i(fx+e)}}{16a^3f}-\frac{21e^{-}}{8}$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^5*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-a*cos(f*x+e)^3*b+a^
2*cos(f*x+e)+6*a*b*cos(f*x+e)+6*b^2*cos(f*x+e))+b/a^5*(((-9/8*a^3-13/4*a^2
*b-17/8*a*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+22*a*b+15*b^2)*cos(f*x+e))/(b+a*c
os(f*x+e)^2)^2+1/8*(15*a^2+70*a*b+63*b^2)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/
(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.90

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[-\frac{48\,a^4\cos(fx+e)^9-16\,(10\,a^4+9\,a^3b)\cos(fx+e)^7+16\,(15\,a^4+70\,a^3b+63\,a^2b^2)\cos(fx+e)^5+5}{}\right.$$

$$-\frac{24\,a^4\cos(fx+e)^9-8\,(10\,a^4+9\,a^3b)\cos(fx+e)^7+8\,(15\,a^4+70\,a^3b+63\,a^2b^2)\cos(fx+e)^5+25\,(}{}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/240*(48*a^4*cos(f*x + e)^9 - 16*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 16
*(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 50*(15*a^3*b + 70*a^2*b
^2 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f
*x + e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63
*a*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)
*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 70*a*b^3 + 6
3*b^4)*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 + a^
5*b^2*f), -1/120*(24*a^4*cos(f*x + e)^9 - 8*(10*a^4 + 9*a^3*b)*cos(f*x + e
)^7 + 8*(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 25*(15*a^3*b + 7
0*a^2*b^2 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2
)*cos(f*x + e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b
^2 + 63*a*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b
) + 15*(15*a^2*b^2 + 70*a*b^3 + 63*b^4)*cos(f*x + e))/(a^7*f*cos(f*x + e)^
4 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.02

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{15\left(\left(9\,a^3b+26\,a^2b^2+17\,ab^3\right)\cos(fx+e)^3+\left(7\,a^2b^2+22\,ab^3+15\,b^4\right)\cos(fx+e)\right)}{a^7\cos(fx+e)^4+2\,a^6b\cos(fx+e)^2+a^5b^2} - \frac{15\left(15\,a^2b+70\,ab^2+63\,b^3\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} + \frac{8\left(3\,\cdots\right.}{\cdots}}{120\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/120*(15*((9*a^3*b + 26*a^2*b^2 + 17*a*b^3)*cos(f*x + e)^3 + (7*a^2*b^2
+ 22*a*b^3 + 15*b^4)*cos(f*x + e))/(a^7*cos(f*x + e)^4 + 2*a^6*b*cos(f*x +
 e)^2 + a^5*b^2) - 15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(a*cos(f*x + e)
/sqrt(a*b))/(sqrt(a*b)*a^5) + 8*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + 3*a*b)*
cos(f*x + e)^3 + 15*(a^2 + 6*a*b + 6*b^2)*cos(f*x + e))/a^5)/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.27

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(15\,a^2b+70\,ab^2+63\,b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5f} - \frac{9\,a^3b\cos(fx+e)^3+26\,a^2b^2\cos(fx+e)^3+17\,ab^3\cos(fx+e)^3+7\,a^2b^2\cos(fx+e)+22\,ab^3\cos(fx}{8\,\left(a\cos(fx+e)^2+b\right)^2a^5f} - \frac{3\,a^{12}f^4\cos(fx+e)^5-10\,a^{12}f^4\cos(fx+e)^3-15\,a^{11}bf^4\cos(fx+e)^3+15\,a^{12}f^4\cos(fx+e)+90\,a}{15\,a^{15}f^5}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(
a*b)*a^5*f) - 1/8*(9*a^3*b*cos(f*x + e)^3 + 26*a^2*b^2*cos(f*x + e)^3 + 17
*a*b^3*cos(f*x + e)^3 + 7*a^2*b^2*cos(f*x + e) + 22*a*b^3*cos(f*x + e) + 1
5*b^4*cos(f*x + e))/((a*cos(f*x + e)^2 + b)^2*a^5*f) - 1/15*(3*a^12*f^4*co
s(f*x + e)^5 - 10*a^12*f^4*cos(f*x + e)^3 - 15*a^11*b*f^4*cos(f*x + e)^3 +
 15*a^12*f^4*cos(f*x + e) + 90*a^11*b*f^4*cos(f*x + e) + 90*a^10*b^2*f^4*c
os(f*x + e))/(a^15*f^5)
```

Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.28

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\cos(e+f\,x)^3\,\left(\frac{b}{a^4}+\frac{2}{3\,a^3}\right)}{f} - \frac{\left(\frac{9\,a^3\,b}{8}+\frac{13\,a^2\,b^2}{4}+\frac{17\,a\,b^3}{8}\right)\cos(e+f\,x)^3+\left(\frac{7\,a^2\,b^2}{8}+\frac{11\,a\,b^3}{4}+\frac{15\,b^4}{8}\right)\cos(e+f\,x)}{f\,\left(a^7\cos(e+f\,x)^4+2\,a^6\,b\cos(e+f\,x)^2+a^5\,b^2\right)} - \frac{\cos(e+f\,x)^5}{5\,a^3\,f} - \frac{\cos(e+f\,x)\,\left(\frac{1}{a^3}-\frac{3\,b^2}{a^5}+\frac{3\,b\,\left(\frac{3\,b}{a^4}+\frac{2}{a^3}\right)}{a}\right)}{f} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(15\,a^2+70\,a\,b+63\,b^2)}{15\,a^2\,b+70\,a\,b^2+63\,b^3}\right)\,(15\,a^2+70\,a\,b+63\,b^2)}{8\,a^{11/2}\,f}$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(cos(e + f*x)^3*(b/a^4 + 2/(3*a^3)))/f - (cos(e + f*x)^3*((17*a*b^3)/8 + (
9*a^3*b)/8 + (13*a^2*b^2)/4) + cos(e + f*x)*((11*a*b^3)/4 + (15*b^4)/8 + (
7*a^2*b^2)/8))/(f*(a^5*b^2 + a^7*cos(e + f*x)^4 + 2*a^6*b*cos(e + f*x)^2))
 - cos(e + f*x)^5/(5*a^3*f) - (cos(e + f*x)*(1/a^3 - (3*b^2)/a^5 + (3*b*((
3*b)/a^4 + 2/a^3))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(70
*a*b + 15*a^2 + 63*b^2))/(70*a*b^2 + 15*a^2*b + 63*b^3))*(70*a*b + 15*a^2
+ 63*b^2))/(8*a^(11/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1391, normalized size of antiderivative = 6.96

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 225*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*sin(e + f*x)**4*a**4 - 1050*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 945*sqrt(b)*sqrt(a)
*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a*
*2*b**2 + 450*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*sin(e + f*x)**2*a**4 + 2550*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*t
an((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b + 3990*sqrt(b)*
sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x
)**2*a**2*b**2 + 1890*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) -
 sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 - 225*sqrt(b)*sqrt(a)*atan((sqrt
(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 - 1500*sqrt(b)*sqrt(a)*a
tan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3*b - 3270*sqrt(b
)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*b**2
 - 2940*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*a*b**3 - 945*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqr
t(a))/sqrt(b))*b**4 + 225*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/
2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4 + 1050*sqrt(b)*sqrt(a)*atan((s
qrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b + 9
45*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*
sin(e + f*x)**4*a**2*b**2 - 450*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((...
```

3.55 $\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f} - \frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f} + \frac{b^2(a+b)\cos(e+fx)}{4a^4f\,(b+a\cos^2(e+fx))^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\,(b+a\cos^2(e+fx))}$$

output
```
5/8*b^(1/2)*(3*a+7*b)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(9/2)/f-(a+3*b)
*cos(f*x+e)/a^4/f+1/3*cos(f*x+e)^3/a^3/f+1/4*b^2*(a+b)*cos(f*x+e)/a^4/f/(b
+a*cos(f*x+e)^2)^2-1/8*b*(9*a+13*b)*cos(f*x+e)/a^4/f/(b+a*cos(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.61 (sec) , antiderivative size = 1153, normalized size of antiderivative = 7.49

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(3*(9*a^4 + 1920*a*b^3 +
4480*b^4)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Tan[(f*x)/2]))/Sqrt[b]] - (-27*a^(11/2)*Sqrt[b]*Cos[e + f*x] + 162*a^
(9/2)*b^(3/2)*Cos[e + f*x] + 10816*a^(7/2)*b^(5/2)*Cos[e + f*x] + 51552*a^
(5/2)*b^(7/2)*Cos[e + f*x] + 87424*a^(3/2)*b^(9/2)*Cos[e + f*x] + 53760*Sq
rt[a]*b^(11/2)*Cos[e + f*x] - 27*a^(11/2)*Sqrt[b]*Cos[e + f*x]*Cos[2*(e +
f*x)] + 47936*a^(5/2)*b^(7/2)*Cos[e + f*x]*Cos[2*(e + f*x)] + 44800*a^(3/2
)*b^(9/2)*Cos[e + f*x]*Cos[2*(e + f*x)] + 27*a^(9/2)*Sqrt[b]*Cos[e + f*x]*
(a + 2*b + a*Cos[2*(e + f*x)]) - 216*a^(7/2)*b^(3/2)*Cos[e + f*x]*(a + 2*b
 + a*Cos[2*(e + f*x)]) - 3600*a^(5/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b + a*Co
s[2*(e + f*x)]) - 5184*a^(3/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e
+ f*x)]) - 27*a^4*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 57
60*a*b^3*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])
^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 13440*b^4*A
rcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(
f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan...
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4621, 360, 25, 2345, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^3}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cos^6(e+fx)(1-\cos^2(e+fx))}{(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$-\frac{-\frac{\int -\frac{-4a^3\cos^6(e+fx)+4a^2(a+b)\cos^4(e+fx)-4ab(a+b)\cos^2(e+fx)+b^2(a+b)}{\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4a^4}-\frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{-4a^3\cos^6(e+fx)+4a^2(a+b)\cos^4(e+fx)-4ab(a+b)\cos^2(e+fx)+b^2(a+b)}{\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4a^4}-\frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\frac{\frac{b(9a+13b)\cos(e+fx)}{2\left(a\cos^2(e+fx)+b\right)}-\frac{\int \frac{8a^2b\cos^4(e+fx)-8ab(a+2b)\cos^2(e+fx)+b^2(7a+11b)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2b}}{4a^4}-\frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{\frac{\frac{b(9a+13b)\cos(e+fx)}{2\left(a\cos^2(e+fx)+b\right)}-\frac{\int \left(8ab\cos^2(e+fx)-8b(a+3b)+\frac{5\left(7b^3+3ab^2\right)}{a\cos^2(e+fx)+b}\right)d\cos(e+fx)}{2b}}{4a^4}-\frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{\frac{b(9a+13b)\cos(e+fx)}{2\left(a\cos^2(e+fx)+b\right)}-\frac{\frac{5b^{3/2}(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}}+\frac{8}{3}ab\cos^3(e+fx)-8b(a+3b)\cos(e+fx)}{2b}}{4a^4}-\frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-((-1/4*(b^2*(a + b)*Cos[e + f*x])/(a^4*(b + a*Cos[e + f*x]^2)^2) + ((b*(9
*a + 13*b)*Cos[e + f*x])/(2*(b + a*Cos[e + f*x]^2)) - ((5*b^(3/2)*(3*a + 7
*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/Sqrt[a] - 8*b*(a + 3*b)*Cos[e
+ f*x] + (8*a*b*Cos[e + f*x]^3)/3)/(2*b))/(4*a^4))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-3\cos(fx+e)b}{a^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{13}{8}ab\right)\cos(fx+e)^3-\frac{b(7a+11b)\cos(fx+e)}{8}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{5(3a+7b)\arctan\left(\frac{a\cos(fx+e}{\sqrt{ab}}\right.}{8\sqrt{ab}}\right.}{a^4}}{f}$
default	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-3\cos(fx+e)b}{a^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{13}{8}ab\right)\cos(fx+e)^3-\frac{b(7a+11b)\cos(fx+e)}{8}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{5(3a+7b)\arctan\left(\frac{a\cos(fx+e}{\sqrt{ab}}\right.}{8\sqrt{ab}}\right.}{a^4}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24a^3f}-\frac{3\,e^{i(fx+e)}}{8a^3f}-\frac{3\,e^{i(fx+e)}b}{2a^4f}-\frac{3\,e^{-i(fx+e)}}{8a^3f}-\frac{3\,e^{-i(fx+e)}b}{2a^4f}+\frac{e^{-3i(fx+e)}}{24a^3f}-\frac{b\left(9a^2e^{7i(fx+e)}+13ab\,e\right.}{}$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-3*cos(f*x+e)*b)+b/a^4*(((-9/8*
a^2-13/8*a*b)*cos(f*x+e)^3-1/8*b*(7*a+11*b)*cos(f*x+e))/(b+a*cos(f*x+e)^2)
^2+5/8*(3*a+7*b)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.85

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{16\,a^3\cos\left(fx+e\right)^7-16\left(3\,a^3+7\,a^2b\right)\cos\left(fx+e\right)^5-50\left(3\,a^2b+7\,ab^2\right)\cos\left(fx+e\right)^3+15\left(\left(3\,a^3+7\,a^2\right.\right.}{48\left(a^6f\,c\right.}\right.$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*a^3*cos(f*x + e)^7 - 16*(3*a^3 + 7*a^2*b)*cos(f*x + e)^5 - 50*(3
*a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*cos(f*x + e)^4 +
3*a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a
*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b))
 - 30*(3*a*b^2 + 7*b^3)*cos(f*x + e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*co
s(f*x + e)^2 + a^4*b^2*f), 1/24*(8*a^3*cos(f*x + e)^7 - 8*(3*a^3 + 7*a^2*b
)*cos(f*x + e)^5 - 25*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*
a^2*b)*cos(f*x + e)^4 + 3*a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x +
e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 15*(3*a*b^2 + 7*b^3)*
cos(f*x + e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 + a^4*b^2*f
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$
$$-\frac{\frac{3\left((9\,a^2b+13\,ab^2)\cos(fx+e)^3+(7\,ab^2+11\,b^3)\cos(fx+e)\right)}{a^6\cos(fx+e)^4+2\,a^5b\cos(fx+e)^2+a^4b^2}-\frac{15\,(3\,ab+7\,b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^4}-\frac{8\left(a\cos(fx+e)^3-3\,(a+3\,b)\cos(fx+\right.}{a^4}}{24\,f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*(3*((9*a^2*b + 13*a*b^2)*cos(f*x + e)^3 + (7*a*b^2 + 11*b^3)*cos(f*x
 + e))/(a^6*cos(f*x + e)^4 + 2*a^5*b*cos(f*x + e)^2 + a^4*b^2) - 15*(3*a*b
 + 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4) - 8*(a*cos(f*x
+ e)^3 - 3*(a + 3*b)*cos(f*x + e))/a^4)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{5\,(3\,ab+7\,b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4f} - \frac{9\,a^2b\cos\left(fx+e\right)^3+13\,ab^2\cos\left(fx+e\right)^3+7\,ab^2\cos\left(fx+e\right)+11\,b^3\cos\left(fx+e\right)}{8\,\left(a\cos\left(fx+e\right)^2+b\right)^2a^4f} + \frac{a^6f^2\cos\left(fx+e\right)^3-3\,a^6f^2\cos\left(fx+e\right)-9\,a^5bf^2\cos\left(fx+e\right)}{3\,a^9f^3}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
5/8*(3*a*b + 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4*f) - 1
/8*(9*a^2*b*cos(f*x + e)^3 + 13*a*b^2*cos(f*x + e)^3 + 7*a*b^2*cos(f*x + e
) + 11*b^3*cos(f*x + e))/((a*cos(f*x + e)^2 + b)^2*a^4*f) + 1/3*(a^6*f^2*c
os(f*x + e)^3 - 3*a^6*f^2*cos(f*x + e) - 9*a^5*b*f^2*cos(f*x + e))/(a^9*f^
3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\cos\left(e+f\,x\right)^3}{3\,a^3\,f} - \frac{\left(\frac{9\,a^2\,b}{8}+\frac{13\,a\,b^2}{8}\right)\,\cos\left(e+f\,x\right)^3+\left(\frac{11\,b^3}{8}+\frac{7\,a\,b^2}{8}\right)\,\cos\left(e+f\,x\right)}{f\,\left(a^6\cos\left(e+f\,x\right)^4+2\,a^5\,b\cos\left(e+f\,x\right)^2+a^4\,b^2\right)} - \frac{\cos\left(e+f\,x\right)\,\left(\frac{3\,b}{a^4}+\frac{1}{a^3}\right)}{f} + \frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(3\,a+7\,b)}{7\,b^2+3\,a\,b}\right)\,(3\,a+7\,b)}{8\,a^{9/2}\,f}$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
cos(e + f*x)^3/(3*a^3*f) - (cos(e + f*x)^3*((13*a*b^2)/8 + (9*a^2*b)/8) +
cos(e + f*x)*((7*a*b^2)/8 + (11*b^3)/8))/(f*(a^4*b^2 + a^6*cos(e + f*x)^4
+ 2*a^5*b*cos(e + f*x)^2)) - (cos(e + f*x)*((3*b)/a^4 + 1/a^3))/f + (5*b^(
1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 7*b))/(3*a*b + 7*b^2))*(3*a
 + 7*b))/(8*a^(9/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1390, normalized size of antiderivative = 9.03

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 45*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**4*a**4 - 150*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 105*sqrt(b)*sqrt(a)*a
tan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2
*b**2 + 90*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**2*a**4 + 390*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((
e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b + 510*sqrt(b)*sqrt(
a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*
a**2*b**2 + 210*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(
a))/sqrt(b))*sin(e + f*x)**2*a*b**3 - 45*sqrt(b)*sqrt(a)*atan((sqrt(a + b)
*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 - 240*sqrt(b)*sqrt(a)*atan((sqr
t(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3*b - 450*sqrt(b)*sqrt(a)
*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*b**2 - 360*sq
rt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b**
3 - 105*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*b**4 + 45*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a
))/sqrt(b))*sin(e + f*x)**4*a**4 + 150*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*t
an((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b + 105*sqrt(b)*s
qrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)
**4*a**2*b**2 - 90*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) +...
```

3.56 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f} - \frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2f\left(b+a\cos^2(e+fx)\right)}$$

output
```
15/8*b^(1/2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(7/2)/f-15/8*cos(f*x+e)/
a^3/f+1/4*cos(f*x+e)^5/a/f/(b+a*cos(f*x+e)^2)^2+5/8*cos(f*x+e)^3/a^2/f/(b+
a*cos(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.57 (sec) , antiderivative size = 656, normalized size of antiderivative = 5.66

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(15(a^3+64b^3)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right.}{}\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(15*(a^3 + 64*b^3)*ArcTan
[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/
2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2
]))/Sqrt[b]] + 15*(a^3 + 64*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*
Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (Sqrt[a]*(24*a^4*Sqr
t[b]*Cos[e + f*x] - 24*a^3*b^(3/2)*Cos[e + f*x] - 144*a^2*b^(5/2)*Cos[e +
f*x] + 512*b^(9/2)*Cos[e + f*x] - 72*a^3*b^(3/2)*Cos[e + f*x]*Cos[2*(e + f
*x)] - 24*a^3*Sqrt[b]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) + 72*a^2
*b^(3/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 1152*b^(7/2)*Cos[e
+ f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 15*a^(5/2)*ArcTan[(Sqrt[a] - Sqrt[
a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 15*a^
(5/2)*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b +
a*Cos[2*(e + f*x)])^2 - 512*b^(5/2)*Cos[e]*Cos[f*x]*(a + 2*b + a*Cos[2*(e
+ f*x)])^2 + 512*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x]
+ 6*a^4*Sqrt[b]*Csc[e + f*x]*Sin[4*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*
x)])^2))/(4096*a^(7/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4621, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{\sin(e+fx)}{(a+b\sec(e+fx)^2)^3}dx$$

$\Big\downarrow$ 4621

$$-\frac{\int \frac{\cos^6(e+fx)}{(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$\Big\downarrow$ 252

$$-\frac{\frac{5\int \frac{\cos^4(e+fx)}{\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4a}-\frac{\cos^5(e+fx)}{4a(a\cos^2(e+fx)+b)^2}}{f}$$

$\Big\downarrow$ 252

$$-\frac{\frac{5\left(\frac{3\int \frac{\cos^2(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a}-\frac{\cos^3(e+fx)}{2a\left(a\cos^2(e+fx)+b\right)}\right)}{4a}-\frac{\cos^5(e+fx)}{4a(a\cos^2(e+fx)+b)^2}}{f}$$

$\Big\downarrow$ 262

$$-\frac{\frac{5\left(\frac{3\left(\frac{\cos(e+fx)}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a}\right)}{2a}-\frac{\cos^3(e+fx)}{2a\left(a\cos^2(e+fx)+b\right)}\right)}{4a}-\frac{\cos^5(e+fx)}{4a(a\cos^2(e+fx)+b)^2}}{f}$$

$\Big\downarrow$ 218

$$-\frac{\frac{5\left(\frac{3\left(\frac{\cos(e+fx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{\cos^3(e+fx)}{2a\left(a\cos^2(e+fx)+b\right)}\right)}{4a}-\frac{\cos^5(e+fx)}{4a(a\cos^2(e+fx)+b)^2}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-((-1/4*Cos[e + f*x]^5/(a*(b + a*Cos[e + f*x]^2)^2) + (5*((3*(-((Sqrt[b]*A
rcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/a^(3/2)) + Cos[e + f*x]/a))/(2*a) -
 Cos[e + f*x]^3/(2*a*(b + a*Cos[e + f*x]^2))))/(4*a))/f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{-\frac{1}{a^3 \sec(fx+e)}-\frac{b\left(\frac{\frac{7b\sec(fx+e)^3}{8}+\frac{9a\sec(fx+e)}{8}}{\left(a+b\sec(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\sec(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}}{f}$
default	$\frac{-\frac{1}{a^3 \sec(fx+e)}-\frac{b\left(\frac{\frac{7b\sec(fx+e)^3}{8}+\frac{9a\sec(fx+e)}{8}}{\left(a+b\sec(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\sec(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2a^3 f}-\frac{e^{-i(fx+e)}}{2a^3 f}-\frac{b\left(9a\,e^{7i(fx+e)}+27a\,e^{5i(fx+e)}+28b\,e^{5i(fx+e)}+27a\,e^{3i(fx+e)}+28b\,e^{3i(fx+e)}+9a\,e^{i(fx+e}\right.}{4a^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2 f}$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3/sec(f*x+e)-1/a^3*b*((7/8*b*sec(f*x+e)^3+9/8*a*sec(f*x+e))/(a+b
*sec(f*x+e)^2)^2+15/8/(a*b)^(1/2)*arctan(b*sec(f*x+e)/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.58

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[-\frac{16\,a^2\cos(fx+e)^5+50\,ab\cos(fx+e)^3+30\,b^2\cos(fx+e)-15\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+\right.}{16\left(a^5f\cos(fx+e)^4+2\,a^4bf\cos(fx+e)^2+a^3b^2\right.}\right.$$

$$-\frac{8\,a^2\cos(fx+e)^5+25\,ab\cos(fx+e)^3+15\,b^2\cos(fx+e)-15\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)\right.}{8\left(a^5f\cos(fx+e)^4+2\,a^4bf\cos(fx+e)^2+a^3b^2f\right)}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*a^2*cos(f*x + e)^5 + 50*a*b*cos(f*x + e)^3 + 30*b^2*cos(f*x + e
) - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/a)*log(-(
a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)
))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), -1/8*(8*
a^2*cos(f*x + e)^5 + 25*a*b*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) - 15*(a^2
*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)
*cos(f*x + e)/b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b
^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{9\,ab\cos(fx+e)^3+7\,b^2\cos(fx+e)}{a^5\cos(fx+e)^4+2\,a^4b\cos(fx+e)^2+a^3b^2} - \frac{15\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{8\,\cos(fx+e)}{a^3}}{8\,f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((9*a*b*cos(f*x + e)^3 + 7*b^2*cos(f*x + e))/(a^5*cos(f*x + e)^4 + 2*
a^4*b*cos(f*x + e)^2 + a^3*b^2) - 15*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(s
qrt(a*b)*a^3) + 8*cos(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{15\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3f} - \frac{\cos(fx+e)}{a^3f} - \frac{9\,ab\cos(fx+e)^3 + 7\,b^2\cos(fx+e)}{8\left(a\cos(fx+e)^2+b\right)^2a^3f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
15/8*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - cos(f*x + e)/(
a^3*f) - 1/8*(9*a*b*cos(f*x + e)^3 + 7*b^2*cos(f*x + e))/((a*cos(f*x + e)^
2 + b)^2*a^3*f)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{8\,a^{7/2}\,f} - \frac{\frac{7\,b^2\,\cos(e+f\,x)}{8}+\frac{9\,a\,b\cos(e+f\,x)^3}{8}}{f\,\left(a^5\cos\left(e+f\,x\right)^4+2\,a^4\,b\cos\left(e+f\,x\right)^2+a^3\,b^2\right)} - \frac{\cos\left(e+f\,x\right)}{a^3\,f}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(15*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(8*a^(7/2)*f) - ((7*b^2*
cos(e + f*x))/8 + (9*a*b*cos(e + f*x)^3)/8)/(f*(a^3*b^2 + a^5*cos(e + f*x)
^4 + 2*a^4*b*cos(e + f*x)^2)) - cos(e + f*x)/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1410, normalized size of antiderivative = 12.16

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**4*a**4 - 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f
*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 15*sqrt(b)*sqrt(a)*ata
n((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b
**2 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqr
t(b))*sin(e + f*x)**2*a**4 + 90*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e +
 f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b + 90*sqrt(b)*sqrt(a)*a
tan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2
*b**2 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**2*a*b**3 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 - 60*sqrt(b)*sqrt(a)*atan((sqrt(a +
b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3*b - 90*sqrt(b)*sqrt(a)*atan((
sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*b**2 - 60*sqrt(b)*sq
rt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b**3 - 15*s
qrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4
 + 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b
))*sin(e + f*x)**4*a**4 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*
x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b + 15*sqrt(b)*sqrt(a)*atan
((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b*
*2 - 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/s...
```

3.57 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)^3f} - \frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)}$$

output
```
1/8*b^(1/2)*(15*a^2+10*a*b+3*b^2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(5/
2)/(a+b)^3/f-arctanh(cos(f*x+e))/(a+b)^3/f-1/4*b*cos(f*x+e)^3/a/(a+b)/f/(b
+a*cos(f*x+e)^2)^2-1/8*b*(7*a+3*b)*cos(f*x+e)/a^2/(a+b)^2/f/(b+a*cos(f*x+e
)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.90

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\frac{8b^2(a+b)^2}{a^2} - \frac{2b(a+b)(9a+5b)(a+2b+a\cos(2(e+fx)))}{a^2} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\,a}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a^2 - (2
*b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a^2 + (Sqrt[b]*(15*
a^2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*
Sec[e + f*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[
a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[
e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[
b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(5/2) - 8*(a + 2*b +
a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 8*(a + 2*b + a*
Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x]))/(64*(a + b)^3*f*(
a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4621, 372, 440, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\sin(e+fx)\left(a+b\sec(e+fx)^2\right)^3}dx$$

$\Big\downarrow$ 4621

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$\Big\downarrow$ 372

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\int \frac{\cos^2(e+fx)\left(3b-(4a+3b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4a(a+b)}}{f}$$

$\Big\downarrow$ 440

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\frac{\int \frac{b(7a+3b)-\left(8a^2+7ba+3b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2a(a+b)}-\frac{b(7a+3b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)}}{4a(a+b)}}{f}$$

$\Big\downarrow$ 397

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{b\left(15a^2+10ab+3b^2\right)\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{8a^2\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)}}{4a(a+b)}}{f}$$

$\Big\downarrow$ 218

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{8a^2\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)}}{4a(a+b)}}{f}$$

$\Big\downarrow$ 219

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{8a^2\text{arctanh}(\cos(e+fx))}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)}}{4a(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-(((b*Cos[e + f*x]^3)/(4*a*(a + b)*(b + a*Cos[e + f*x]^2)^2) - (((Sqrt[b]*
(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]
*(a + b)) - (8*a^2*ArcTanh[Cos[e + f*x]])/(a + b))/(2*a*(a + b)) - (b*(7*a
 + 3*b)*Cos[e + f*x])/(2*a*(a + b)*(b + a*Cos[e + f*x]^2)))/(4*a*(a + b)))
/f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}+\frac{b\left(\frac{-\frac{\left(9a^2+14ab+5b^2\right)\cos(fx+e)^3}{8a}-\frac{b\left(7a^2+10ab+3b^2\right)\cos(fx+e)}{8a^2}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2+10ab+3b^2\right.}{8}\right.}{(a+b)^3}}{f}$
default	$\frac{-\frac{\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}+\frac{b\left(\frac{-\frac{\left(9a^2+14ab+5b^2\right)\cos(fx+e)^3}{8a}-\frac{b\left(7a^2+10ab+3b^2\right)\cos(fx+e)}{8a^2}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2+10ab+3b^2\right.}{8}\right.}{(a+b)^3}}{f}$
risch	$-\frac{b\left(9a^2e^{7i(fx+e)}+5ab\,e^{7i(fx+e)}+27a^2e^{5i(fx+e)}+43ab\,e^{5i(fx+e)}+12b^2e^{5i(fx+e)}+27a^2e^{3i(fx+e)}+43ab\,e^{3i(fx+e)}+12\right.}{4a^2f(a+b)^2\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/(a+b)^3*ln(1+cos(f*x+e))+1/2/(a+b)^3*ln(-1+cos(f*x+e))+b/(a+b)^3
*((-1/8*(9*a^2+14*a*b+5*b^2)/a*cos(f*x+e)^3-1/8*b*(7*a^2+10*a*b+3*b^2)/a^2
*cos(f*x+e))/(b+a*cos(f*x+e)^2)^2+1/8*(15*a^2+10*a*b+3*b^2)/a^2/(a*b)^(1/2
)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(140) = 280$.

Time = 0.20 (sec) , antiderivative size = 779, normalized size of antiderivative = 5.06

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 10*
a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15*
a^3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x +
 e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(7*a^
2*b^2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 8*(a^4*cos(f*x + e)^4 + 2*a^3*b*c
os(f*x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2) - 8*(a^4*cos(f*x + e)
^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7
 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2
+ 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
 + a^2*b^5)*f), -1/8*((9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((
15*a^4 + 10*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*
b^4 + 2*(15*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan
(a*sqrt(b/a)*cos(f*x + e)/b) + (7*a^2*b^2 + 10*a*b^3 + 3*b^4)*cos(f*x + e)
 + 4*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(1/2*cos(f
*x + e) + 1/2) - 4*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)
*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*co
s(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^
2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.69

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\frac{(15\,a^2b+10\,ab^2+3\,b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sqrt{ab}} - \frac{(9\,a^2b+5\,ab^2)\cos(fx+e)^3+(7\,ab^2+3\,b^3)\cos(fx+e)}{a^4b^2+2\,a^3b^3+a^2b^4+(a^6+2\,a^5b+a^4b^2)\cos(fx+e)^4+2\,(a^5b+2\,a^4b^2+a^3b^3)\cos(fx+e)^2} - \frac{4\log}{a^3+}}{8\,f}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)) - ((9*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 + (7*a*b^2 + 3*b^3)*cos(f*x + e))/(a^4*b^2 + 2*a^3*b^3 + a^2*b^4 +
 (a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^4 + 2*(a^5*b + 2*a^4*b^2 + a^3*b^3
)*cos(f*x + e)^2) - 4*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3
) + 4*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.59

$$\begin{aligned}\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx &= \frac{(15\,a^2b+10\,ab^2+3\,b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\,(a^5f+3\,a^4bf+3\,a^3b^2f+a^2b^3f)\sqrt{ab}} \\ &\quad + \frac{\log\left(\left|-\cos\left(fx+e\right)+1\right|\right)}{2\,(a^3f+3\,a^2bf+3\,ab^2f+b^3f)} - \frac{\log\left(\left|-\cos\left(fx+e\right)-1\right|\right)}{2\,(a^3f+3\,a^2bf+3\,ab^2f+b^3f)} \\ &\quad - \frac{9\,a^2b\cos\left(fx+e\right)^3+5\,ab^2\cos\left(fx+e\right)^3+7\,ab^2\cos\left(fx+e\right)+3\,b^3\cos\left(fx+e\right)}{8\,(a^4f+2\,a^3bf+a^2b^2f)\left(a\cos\left(fx+e\right)^2+b\right)^2}\end{aligned}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5*f
 + 3*a^4*b*f + 3*a^3*b^2*f + a^2*b^3*f)*sqrt(a*b)) + 1/2*log(abs(-cos(f*x
+ e) + 1))/(a^3*f + 3*a^2*b*f + 3*a*b^2*f + b^3*f) - 1/2*log(abs(-cos(f*x
+ e) - 1))/(a^3*f + 3*a^2*b*f + 3*a*b^2*f + b^3*f) - 1/8*(9*a^2*b*cos(f*x
+ e)^3 + 5*a*b^2*cos(f*x + e)^3 + 7*a*b^2*cos(f*x + e) + 3*b^3*cos(f*x + e
))/((a^4*f + 2*a^3*b*f + a^2*b^2*f)*(a*cos(f*x + e)^2 + b)^2)
```

Mupad [B] (verification not implemented)

Time = 15.79 (sec) , antiderivative size = 3557, normalized size of antiderivative = 23.10

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
(atan((((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2
*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^
3 + 6*a^5*b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6
+ 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b
+ a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (co
s(e + f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^1
2 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^
3 + 2304*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b +
a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 +
 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*
b^2)*1i)/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((co
s(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^
4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((224*a^
10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b
^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5
 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) + (cos(e + f*x)*(-a^5*b)^(1/2)*(
10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*
b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(512*(3
*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 +
 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + ...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1447, normalized size of antiderivative = 9.40

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**4*a**4 - 10*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f
*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 3*sqrt(b)*sqrt(a)*atan
((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b*
*2 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*sin(e + f*x)**2*a**4 + 50*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b + 26*sqrt(b)*sqrt(a)*at
an((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*
b**2 + 6*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqr
t(b))*sin(e + f*x)**2*a*b**3 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e
 + f*x)/2) - sqrt(a))/sqrt(b))*a**4 - 40*sqrt(b)*sqrt(a)*atan((sqrt(a + b)
*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3*b - 38*sqrt(b)*sqrt(a)*atan((sq
rt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*b**2 - 16*sqrt(b)*sqrt
(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b**3 - 3*sqrt
(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4 +
15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*
sin(e + f*x)**4*a**4 + 10*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/
2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b + 3*sqrt(b)*sqrt(a)*atan((sq
rt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 -
 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(...
```

3.58 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 213

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4f} - \frac{(a-5b)\text{arctanh}(\cos(e+fx))}{2(a+b)^4f} - \frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\,(b+a\cos^2(e+fx))^2} + \frac{(4a^2-9ab-b^2)\cos(e+fx)}{8a(a+b)^3f\,(b+a\cos^2(e+fx))} - \frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\,(b+a\cos^2(e+fx))^2}$$

output

```
1/8*b^(1/2)*(15*a^2-10*a*b-b^2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(3/2)
/(a+b)^4/f-1/2*(a-5*b)*arctanh(cos(f*x+e))/(a+b)^4/f-1/4*(2*a-b)*b*cos(f*x
+e)/a/(a+b)^2/f/(b+a*cos(f*x+e)^2)^2+1/8*(4*a^2-9*a*b-b^2)*cos(f*x+e)/a/(a
+b)^3/f/(b+a*cos(f*x+e)^2)-1/2*cos(f*x+e)*cot(f*x+e)^2/(a+b)/f/(b+a*cos(f*
x+e)^2)^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.70 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.50

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\frac{8b^2(a+b)^2}{a}-\frac{2b(a+b)(9a+b)(a+2b+a\cos(2(e+fx)))}{a}-\frac{\sqrt{b}(-15a^2+10ab+b^2)\,\mathrm{a}}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a - (2*b
*(a + b)*(9*a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a - (Sqrt[b]*(-15*a^2 +
 10*a*b + b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e +
 f*x])/a^(3/2) - (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt
[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a +
 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(3/2) - (a + b)*(a + 2*b + a*
Cos[2*(e + f*x)])^2*Csc[(e + f*x)/2]^2*Sec[e + f*x] - 4*(a - 5*b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 5*b)*
(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a +
 b)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[(e + f*x)/2]^2*Sec[e + f*x]))/(64
*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4621, 372, 440, 27, 402, 25, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4621}$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\int \frac{\cos^2(e+fx)\left(3b-(a-2b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^3}d\cos(e+fx)}{2(a+b)}}{f}$$

$$\Big\downarrow \text{440}$$

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\int \frac{2\left((2a-b)b-\left(2a^2-8ba-b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4a(a+b)}-\frac{b(2a-b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)^2}}{2(a+b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\int \frac{(2a-b)b-\left(2a^2-8ba-b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{2a(a+b)}-\frac{b(2a-b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)^2}}{2(a+b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{2(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\int -\frac{b\left((11a-b)b-\left(4a^2-9ba-b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}}{2a(a+b)}-\frac{b(2a-b)\,c}{2a(a+b)\left(a\cos\right.}}{2(a+b)}}{f}$$

$\downarrow$ 25

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{\int \frac{b\left((11a-b)b-\left(4a^2-9ba-b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}+\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{2(a+b)\left(a\cos^2(e+fx)+b\right)}}{2a(a+b)}-\frac{b(2a-b)\cos}{2a(a+b)\left(a\cos^2(\right.}}{2(a+b)}}{f}$$

$\downarrow$ 27

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{\int \frac{(11a-b)b-\left(4a^2-9ba-b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2(a+b)}+\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{2(a+b)\left(a\cos^2(e+fx)+b\right)}}{2a(a+b)}-\frac{b(2a-b)\cos(e+}{2a(a+b)\left(a\cos^2(e+\right.}}{2(a+b)}}{f}$$

$\downarrow$ 397

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{\frac{b\left(15a^2-10ab-b^2\right)\int\frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{4a(a-5b)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}+\frac{\left(4a^2-9ab\right.}{2(a+b)\left(a\right.}}{2a(a+b)}}{2(a+b)}}{f}$$

$\downarrow$ 218

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{4a(a-5b)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}+\frac{\left(4a^2-9ab-b^2\right)}{2(a+b)\left(a\cos^2(\right.}}{2a(a+b)}}{2(a+b)}}{f}$$

$\downarrow$ 219

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{4a(a-5b)\text{arctanh}(\cos(e+fx))}{a+b}}{2(a+b)}+\frac{\left(4a^2-9ab-b^2\right)\cos(e}{2(a+b)\left(a\cos^2(e+fx\right.}}{2a(a+b)}}{2(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-((Cos[e + f*x]^3/(2*(a + b)*(1 - Cos[e + f*x]^2)*(b + a*Cos[e + f*x]^2)^2
) - (-1/2*((2*a - b)*b*Cos[e + f*x])/(a*(a + b)*(b + a*Cos[e + f*x]^2)^2)
+ (((Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]
])/(Sqrt[a]*(a + b)) - (4*a*(a - 5*b)*ArcTanh[Cos[e + f*x]])/(a + b))/(2*(
a + b)) + ((4*a^2 - 9*a*b - b^2)*Cos[e + f*x])/(2*(a + b)*(b + a*Cos[e + f
*x]^2)))/(2*a*(a + b)))/(2*(a + b)))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\cos(fx+e)^3-\frac{b\left(7a^2+6ab-b^2\right)\cos(fx+e)}{8a}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2-10ab-b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{(a+b)^4}+\frac{1}{4(a+b)^3(-1+\cos}}{f}$
default	$\frac{\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\cos(fx+e)^3-\frac{b\left(7a^2+6ab-b^2\right)\cos(fx+e)}{8a}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2-10ab-b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{(a+b)^4}+\frac{1}{4(a+b)^3(-1+\cos}}{f}$
risch	$-\frac{-4a^3e^{11i(fx+e)}+9a^2b\,e^{11i(fx+e)}+a\,b^2e^{11i(fx+e)}-20a^3e^{9i(fx+e)}-23a^2b\,e^{9i(fx+e)}+29a\,b^2e^{9i(fx+e)}-4b^3e^{9i(fx+e)}}{}$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b/(a+b)^4*(((-9/8*a^2-5/4*a*b-1/8*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+6*a*
b-b^2)/a*cos(f*x+e))/(b+a*cos(f*x+e)^2)^2+1/8*(15*a^2-10*a*b-b^2)/a/(a*b)^
(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/4/(a+b)^3/(-1+cos(f*x+e))+1/4*(a
-5*b)/(a+b)^4*ln(-1+cos(f*x+e))+1/4/(a+b)^3/(1+cos(f*x+e))+1/4/(a+b)^4*(-a
+5*b)*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(195) = 390$.

Time = 0.24 (sec) , antiderivative size = 1332, normalized size of antiderivative = 6.25

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + 2*(17*a^3
*b + 11*a^2*b^2 - 5*a*b^3 + b^4)*cos(f*x + e)^3 - ((15*a^4 - 10*a^3*b - a^
2*b^2)*cos(f*x + e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x
 + e)^4 - 15*a^2*b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 +
 b^4)*cos(f*x + e)^2)*sqrt(-b/a)*log((a*cos(f*x + e)^2 - 2*a*sqrt(-b/a)*co
s(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 + 10*a*b^3 - b^4)*
cos(f*x + e) - 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2
*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3
)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 5*a^3*b)*cos(f*x
 + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3
- (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) +
 1/2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f
*x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b
^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^
6)*f), 1/8*((4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + (17*a^
3*b + 11*a^2*b^2 - 5*a*b^3 + b^4)*cos(f*x + e)^3 + ((15*a^4 - 10*a^3*b - a
^2*b^2)*cos(f*x + e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*
x + e)^4 - 15*a^2*b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3
+ b^4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + (...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(195) = 390.

Time = 0.11 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.87

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\,(a-5\,b)\log(\cos(fx+e)+1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{2\,(a-5\,b)\log(\cos(fx+e)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{(15\,a^2b-10\,ab^2-b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\sqrt{ab}} - \frac{\cdots}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\,\mathrm{co}\cdots}}{8\,f}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*(a - 5*b)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4) - 2*(a - 5*b)*log(cos(f*x + e) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4) - (15*a^2*b - 10*a*b^2 - b^3)*arctan(a*cos(f*x + e)/sqrt(a
*b))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - ((4*a^3
 - 9*a^2*b - a*b^2)*cos(f*x + e)^5 + (17*a^2*b - 6*a*b^2 + b^3)*cos(f*x +
e)^3 + (11*a*b^2 - b^3)*cos(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*cos(f*x + e)^6 - a^4*b^2 - 3*a^3*b^3 - 3*a^2*b^4 - a*b^5 - (a^6 + a^5*b
 - 3*a^4*b^2 - 5*a^3*b^3 - 2*a^2*b^4)*cos(f*x + e)^4 - (2*a^5*b + 5*a^4*b^
2 + 3*a^3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.54

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{(a-5\,b)\log\left(|\cos\left(fx+e\right)+1|\right)}{4\,(a^4f+4\,a^3bf+6\,a^2b^2f+4\,ab^3f+b^4f)} + \frac{(a-5\,b)\log\left(|\cos\left(fx+e\right)-1|\right)}{4\,(a^4f+4\,a^3bf+6\,a^2b^2f+4\,ab^3f+b^4f)} + \frac{(15\,a^2b-10\,ab^2-b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\,(a^5f+4\,a^4bf+6\,a^3b^2f+4\,a^2b^3f+ab^4f)\sqrt{ab}} + \frac{\cos\left(fx+e\right)}{2\,(a^3f+3\,a^2bf+3\,ab^2f+b^3f)\left(\cos\left(fx+e\right)^2-1\right)} - \frac{9\,a^2b\cos\left(fx+e\right)^3+ab^2\cos\left(fx+e\right)^3+7\,ab^2\cos\left(fx+e\right)-b^3\cos\left(fx+e\right)}{8\,(a^4f+3\,a^3bf+3\,a^2b^2f+ab^3f)\left(a\cos\left(fx+e\right)^2+b\right)^2}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/4*(a - 5*b)*log(abs(cos(f*x + e) + 1))/(a^4*f + 4*a^3*b*f + 6*a^2*b^2*f
 + 4*a*b^3*f + b^4*f) + 1/4*(a - 5*b)*log(abs(cos(f*x + e) - 1))/(a^4*f +
4*a^3*b*f + 6*a^2*b^2*f + 4*a*b^3*f + b^4*f) + 1/8*(15*a^2*b - 10*a*b^2 -
b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5*f + 4*a^4*b*f + 6*a^3*b^2*f +
4*a^2*b^3*f + a*b^4*f)*sqrt(a*b)) + 1/2*cos(f*x + e)/((a^3*f + 3*a^2*b*f +
 3*a*b^2*f + b^3*f)*(cos(f*x + e)^2 - 1)) - 1/8*(9*a^2*b*cos(f*x + e)^3 +
a*b^2*cos(f*x + e)^3 + 7*a*b^2*cos(f*x + e) - b^3*cos(f*x + e))/((a^4*f +
3*a^3*b*f + 3*a^2*b^2*f + a*b^3*f)*(a*cos(f*x + e)^2 + b)^2)
```

Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 2728, normalized size of antiderivative = 12.81

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
- ((cos(e + f*x)^3*(17*a^2*b - 6*a*b^2 + b^3))/(8*(a*b^3 + 3*a^3*b + a^4 +
 3*a^2*b^2)) - (cos(e + f*x)^5*(9*a*b - 4*a^2 + b^2))/(8*(3*a*b^2 + 3*a^2*
b + a^3 + b^3)) + (b^2*cos(e + f*x)*(11*a - b))/(8*(a*b^3 + 3*a^3*b + a^4
+ 3*a^2*b^2)))/(f*(b^2 - cos(e + f*x)^4*(2*a*b - a^2) + cos(e + f*x)^2*(2*
a*b - b^2) - a^2*cos(e + f*x)^6)) - (log(cos(e + f*x) - 1)*((3*b)/(2*(a +
b)^4) - 1/(4*(a + b)^3)))/f - (log(cos(e + f*x) + 1)*(a - 5*b))/(4*f*(a +
b)^4) - (atan(((((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a
^2*b^4 - 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^
5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) + ((-a^3*b)^(1/2)*(((11*a^11*b)
/2 - (a^2*b^10)/2 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6
 + 357*a^7*b^5 + 294*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a
^9*b + a^10 + 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*
b^4 + 84*a^7*b^3 + 36*a^8*b^2) - (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15
*a^2 + b^2)*(1792*a^11*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^
5*b^7 - 7168*a^6*b^6 - 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a
^10*b^2))/(512*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 +
6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*
b - 15*a^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))
*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*1i)/(16*(4*a^6*b + a^7 + a^3*b^4 +
 4*a^4*b^3 + 6*a^5*b^2)) + (((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1863, normalized size of antiderivative = 8.75

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
( - 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**6*a**4 + 10*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f
*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**6*a**3*b + sqrt(b)*sqrt(a)*atan((
sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**6*a**2*b**2
 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**4*a**4 + 10*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*
x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 22*sqrt(b)*sqrt(a)*atan
((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b*
*2 - 2*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**4*a*b**3 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e +
 f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4 - 20*sqrt(b)*sqrt(a)*ata
n((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b
 + 6*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
)*sin(e + f*x)**2*a**2*b**2 + 12*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 + sqrt(b)*sqrt(a)*ata
n((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*b**4 +
 15*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))
*sin(e + f*x)**6*a**4 - 10*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)
/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**6*a**3*b - sqrt(b)*sqrt(a)*atan((sqr
t(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**6*a**2*b**2...
```

3.59 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 257

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}(a+b)^5 f} - \frac{3(a^2-10ab+5b^2)\,\mathrm{arctanh}(\cos(e+fx))}{8(a+b)^5 f} + \frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3 f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3(a^2-6ab+b^2)\cos(e+fx)}{8(a+b)^4 f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2}$$

output
```
3/8*b^(1/2)*(5*a^2-10*a*b+b^2)*arctan(a^(1/2)*cos(f*x+e)/b^(1/2))/a^(1/2)/
(a+b)^5/f-3/8*(a^2-10*a*b+5*b^2)*arctanh(cos(f*x+e))/(a+b)^5/f+1/8*(a^2-9*
a*b+2*b^2)*cos(f*x+e)/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+3/8*(a^2-6*a*b+b^2)*c
os(f*x+e)/(a+b)^4/f/(b+a*cos(f*x+e)^2)-1/8*(a-7*b)*cot(f*x+e)*csc(f*x+e)/(
a+b)^2/f/(b+a*cos(f*x+e)^2)^2-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(b+a*cos
(f*x+e)^2)^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.14

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\left(\frac{48\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos}\right.}{\sqrt{b}}\right.}{\sqrt{a}}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*((48*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan
[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/
2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2
]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] + (48*Sqrt[b]*(5*a^
2 - 10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqr
t[a] - 2*(a + b)*(30*a^3 + 112*a^2*b + 182*a*b^2 - 140*b^3 + (35*a^3 + 78*
a^2*b - 93*a*b^2 + 224*b^3)*Cos[2*(e + f*x)] + 2*(a^3 - 8*a^2*b + 53*a*b^2
 - 10*b^3)*Cos[4*(e + f*x)] - 3*a^3*Cos[6*(e + f*x)] + 18*a^2*b*Cos[6*(e +
 f*x)] - 3*a*b^2*Cos[6*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 - 48*(a^2 -
 10*a*b + 5*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]] +
48*(a^2 - 10*a*b + 5*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*
x)/2]])*Sec[e + f*x]^6)/(1024*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4621, 372, 440, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(e+fx)^5\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \ 4621$$

$$-\frac{\int \frac{\cos^6(e+fx)}{\left(1-\cos^2(e+fx)\right)^3\left(a\cos^2(e+fx)+b\right)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \ 372$$

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{\int \frac{\cos^2(e+fx)\left(3b-(a-4b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^3}d\cos(e+fx)}{4(a+b)}}{f}$$

$$\Big\downarrow \ 440$$

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{\frac{\int \frac{(a-7b)b-\left(3a^2-29ba+8b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^3}d\cos(e+fx)}{2(a+b)}-\frac{(a-7b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}}{4(a+b)}}{f}$$

$$\Big\downarrow \ 402$$

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{\frac{\frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)^2}-\frac{\int -\frac{12b\left((a-3b)b-\left(a^2-9ba+2b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4b(a+b)}}{2(a+b)}-\frac{\cdots}{2(a+b)\left(1-\mathrm{c}\cdots\right.}}{4(a+b)}}{f}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{3\int \frac{(a-3b)b-\left(a^2-9ba+2b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{a+b}+\frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)^2}}{2(a+b)}-\frac{(a-7\ldots}{2(a+b)\left(1-\cos^2(e\ldots\right.}}{4(a+b)}}{f}$$

$\Big\downarrow$ 402

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{3\left(\frac{\left(a^2-6ab+b^2\right)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\int -\frac{2b\left(4(a-b)b-\left(a^2-6ba+b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}\right)}{a+b}+\frac{\left(a^2-9ab+\ldots\right.}{(a+b)\left(a\cos\ldots\right.}}{2(a+b)}\ldots}{4(a+b)}}{f}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{3\left(\frac{\int \frac{4(a-b)b-\left(a^2-6ba+b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{a+b}+\frac{\left(a^2-6ab+b^2\right)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}\right)}{a+b}+\frac{\left(a^2-9ab+2b^2\right)c\ldots}{(a+b)\left(a\cos^2(e+\ldots\right.}}{2(a+b)}\ldots}{4(a+b)}}{f}$$

$\Big\downarrow$ 397

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\frac{\frac{3\left(\frac{\frac{b\left(5a^2-10ab+b^2\right)\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{\left(a^2-10ab+5b^2\right)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{a+b}+\ldots\right.}{a+b}\ldots}{2(a+b)}\ldots}{4(a\ldots}}{f}$$

$\Big\downarrow$ 218

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\frac{3\left(\frac{\frac{\sqrt{b}\left(5a^2-10ab+b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{\left(a^2-10ab+5b^2\right)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{a+b}+\frac{\left(a^2 \ldots\right.}{(a+\ldots}\right)}{a+b}}{2(a+b)}}{4(a+\ldots}}{f}$$

$\Big\downarrow$ 219

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\frac{3\left(\frac{\frac{\sqrt{b}\left(5a^2-10ab+b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{\left(a^2-10ab+5b^2\right)\text{arctanh}(\cos(e+fx))}{a+b}}{a+b}+\frac{\left(a^2-6ab+\ldots\right.}{(a+b)\left(a\,c\ldots\right.}\right)}{a+b}}{2(a+b)}}{4(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-((Cos[e + f*x]^3/(4*(a + b)*(1 - Cos[e + f*x]^2)^2*(b + a*Cos[e + f*x]^2)
^2) - (-1/2*((a - 7*b)*Cos[e + f*x])/((a + b)*(1 - Cos[e + f*x]^2)*(b + a*
Cos[e + f*x]^2)^2) + (((a^2 - 9*a*b + 2*b^2)*Cos[e + f*x])/((a + b)*(b + a
*Cos[e + f*x]^2)^2) + (3*(((Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[a]
*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a + b)) - ((a^2 - 10*a*b + 5*b^2)*ArcTa
nh[Cos[e + f*x]])/(a + b))/(a + b) + ((a^2 - 6*a*b + b^2)*Cos[e + f*x])/((
a + b)*(b + a*Cos[e + f*x]^2))))/(a + b))/(2*(a + b)))/(4*(a + b)))/f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{3}{4}a^2b+\frac{3}{8}a\,b^2\right)\cos(fx+e)^3-\frac{b\left(7a^2+2ab-5b^2\right)\cos(fx+e)}{8}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{3\left(5a^2-10ab+b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{(a+b)^5}+\frac{1}{16(a+b)^3(1+}}{}$
default	$\frac{\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{3}{4}a^2b+\frac{3}{8}a\,b^2\right)\cos(fx+e)^3-\frac{b\left(7a^2+2ab-5b^2\right)\cos(fx+e)}{8}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{3\left(5a^2-10ab+b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{(a+b)^5}+\frac{1}{16(a+b)^3(1+}}{}$
risch	Expression too large to display

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b/(a+b)^5*(((-9/8*a^3-3/4*a^2*b+3/8*a*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+
2*a*b-5*b^2)*cos(f*x+e))/(b+a*cos(f*x+e)^2)^2+3/8*(5*a^2-10*a*b+b^2)/(a*b)
^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/16/(a+b)^3/(1+cos(f*x+e))^2-1/1
6*(-3*a+9*b)/(a+b)^4/(1+cos(f*x+e))+1/16/(a+b)^5*(-3*a^2+30*a*b-15*b^2)*ln
(1+cos(f*x+e))-1/16/(a+b)^3/(-1+cos(f*x+e))^2-1/16*(-3*a+9*b)/(a+b)^4/(-1+
cos(f*x+e))+1/16/(a+b)^5*(3*a^2-30*a*b+15*b^2)*ln(-1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(237) = 474$.

Time = 0.27 (sec) , antiderivative size = 1833, normalized size of antiderivative = 7.13

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*(a^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 2
6*a^3*b + 26*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15
*a*b^3 + 19*b^4)*cos(f*x + e)^3 + 3*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x
+ e)^8 - 2*(5*a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4
 - 30*a^3*b + 46*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10
*a*b^3 + b^4 + 2*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*s
qrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos
(f*x + e)^2 + b)) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos
(f*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e
)^4 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^
4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b + 5*a^
2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x
+ e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 +
 a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*co
s(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5
*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b
- 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x
 + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(237) = 474.

Time = 0.12 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.05

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\,(a^2-10\,ab+5\,b^2)\log(\cos(fx+e)+1)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5} - \frac{3\,(a^2-10\,ab+5\,b^2)\log(\cos(fx+e)-1)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5} - \frac{6\,(5\,a^2b-10\,ab^2+b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\sqrt{ab}} - \frac{\cdots}{(a^6 \cdots}}{\cdots}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(3*(a^2 - 10*a*b + 5*b^2)*log(cos(f*x + e) + 1)/(a^5 + 5*a^4*b + 10*
a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 3*(a^2 - 10*a*b + 5*b^2)*log(cos(f
*x + e) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 6
*(5*a^2*b - 10*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b)) - 2*(3*(a^3 - 6*a
^2*b + a*b^2)*cos(f*x + e)^7 - (5*a^3 - 31*a^2*b + 31*a*b^2 - 5*b^3)*cos(f
*x + e)^5 - (19*a^2*b - 34*a*b^2 + 19*b^3)*cos(f*x + e)^3 - 12*(a*b^2 - b^
3)*cos(f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*
x + e)^8 - 2*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*c
os(f*x + e)^6 + a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6 + (a^6 - 9
*a^4*b^2 - 16*a^3*b^3 - 9*a^2*b^4 + b^6)*cos(f*x + e)^4 + 2*(a^5*b + 3*a^4
*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cos(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.80

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{3\,(a^2-10\,ab+5\,b^2)\log(|-\cos(fx+e)+1|)}{16\,(a^5f+5\,a^4bf+10\,a^3b^2f+10\,a^2b^3f+5\,ab^4f+b^5f)} - \frac{3\,(a^2-10\,ab+5\,b^2)\log(|-\cos(fx+e)-1|)}{16\,(a^5f+5\,a^4bf+10\,a^3b^2f+10\,a^2b^3f+5\,ab^4f+b^5f)} + \frac{3\,(5\,a^2b-10\,ab^2+b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\,(a^5f+5\,a^4bf+10\,a^3b^2f+10\,a^2b^3f+5\,ab^4f+b^5f)\sqrt{ab}} + \frac{3\,a^3\cos(fx+e)^7-18\,a^2b\cos(fx+e)^7+3\,ab^2\cos(fx+e)^7-5\,a^3\cos(fx+e)^5+31\,a^2b\cos(fx+e}{8\,(a^4f+4\,a^3bf+6\,a^2b^2}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
3/16*(a^2 - 10*a*b + 5*b^2)*log(abs(-cos(f*x + e) + 1))/(a^5*f + 5*a^4*b*f
 + 10*a^3*b^2*f + 10*a^2*b^3*f + 5*a*b^4*f + b^5*f) - 3/16*(a^2 - 10*a*b +
 5*b^2)*log(abs(-cos(f*x + e) - 1))/(a^5*f + 5*a^4*b*f + 10*a^3*b^2*f + 10
*a^2*b^3*f + 5*a*b^4*f + b^5*f) + 3/8*(5*a^2*b - 10*a*b^2 + b^3)*arctan(a*
cos(f*x + e)/sqrt(a*b))/((a^5*f + 5*a^4*b*f + 10*a^3*b^2*f + 10*a^2*b^3*f
+ 5*a*b^4*f + b^5*f)*sqrt(a*b)) + 1/8*(3*a^3*cos(f*x + e)^7 - 18*a^2*b*cos
(f*x + e)^7 + 3*a*b^2*cos(f*x + e)^7 - 5*a^3*cos(f*x + e)^5 + 31*a^2*b*cos
(f*x + e)^5 - 31*a*b^2*cos(f*x + e)^5 + 5*b^3*cos(f*x + e)^5 - 19*a^2*b*co
s(f*x + e)^3 + 34*a*b^2*cos(f*x + e)^3 - 19*b^3*cos(f*x + e)^3 - 12*a*b^2*
cos(f*x + e) + 12*b^3*cos(f*x + e))/((a^4*f + 4*a^3*b*f + 6*a^2*b^2*f + 4*
a*b^3*f + b^4*f)*(a*cos(f*x + e)^4 - a*cos(f*x + e)^2 + b*cos(f*x + e)^2 -
 b)^2)
```

Mupad [B] (verification not implemented)

Time = 17.13 (sec) , antiderivative size = 5613, normalized size of antiderivative = 21.84

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
(atan(((((cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a
^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8
+ 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) + (3*(-
a*b)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*
b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4
 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a
^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*
b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (3*cos(e + f*x)*(-a*b)^(1
/2)*(5*a^2 - 10*a*b + b^2)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a
^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 1
0752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^
5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*
b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6
*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 1
0*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*3i)/(16*(a*b
^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) + (((cos(e + f*
x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^
3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^
3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) - (3*(-a*b)^(1/2)*((6*a^13*
b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8...
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 1988, normalized size of antiderivative = 7.74

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 60*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**8*a**4 + 120*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**8*a**3*b - 12*sqrt(b)*sqrt(a)*at
an((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**8*a**2*
b**2 + 120*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**6*a**4 - 120*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((
e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**6*a**3*b - 216*sqrt(b)*sqrt(
a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**6*
a**2*b**2 + 24*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a
))/sqrt(b))*sin(e + f*x)**6*a*b**3 - 60*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*
tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4 + 168*sqrt(b)*sq
rt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*
*4*a**2*b**2 + 96*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqr
t(a))/sqrt(b))*sin(e + f*x)**4*a*b**3 - 12*sqrt(b)*sqrt(a)*atan((sqrt(a +
b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*b**4 + 60*sqrt(b)*
sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x
)**8*a**4 - 120*sqrt(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(
a))/sqrt(b))*sin(e + f*x)**8*a**3*b + 12*sqrt(b)*sqrt(a)*atan((sqrt(a + b)
*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**8*a**2*b**2 - 120*sqrt
(b)*sqrt(a)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(...
```

3.60 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 314

$$\begin{aligned}\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx &= \frac{5(a+2b)\left(a^2+16ab+16b^2\right)x}{16a^6} \\ &\quad - \frac{5\sqrt{b}\sqrt{a+b}(a+4b)(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f} \\ &\quad - \frac{(33a^2+110ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &\quad + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &\quad + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} \\ &\quad - \frac{5b(9a^2+32ab+24b^2)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &\quad - \frac{5b(5a^2+20ab+16b^2)\tan(e+fx)}{16a^5f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output
```
5/16*(a+2*b)*(a^2+16*a*b+16*b^2)*x/a^6-5/8*b^(1/2)*(a+b)^(1/2)*(a+4*b)*(3*
a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^6/f-1/48*(33*a^2+110*a*b+8
0*b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+1/24*(9*a+10*b)*
cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e)^3*sin(
f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^2-5/48*b*(9*a^2+32*a*b+24*b^2)*tan(f*x+e
)/a^4/f/(a+b+b*tan(f*x+e)^2)^2-5/16*b*(5*a^2+20*a*b+16*b^2)*tan(f*x+e)/a^5
/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.00 (sec) , antiderivative size = 1639, normalized size of antiderivative = 5.22

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*
(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)
])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(65536*b^(5/2)*f*(a + b*
Sec[e + f*x]^2)^3) - (15*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(
(-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) +
a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e
] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (a*Sec[2*e]
*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sin[2*f*x] + a*(-3
*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x)] + (3*a^4 - 64*a^2*b
^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^
2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*(a + 2*b + a*Cos[2*(
e + f*x)])^2)))/(262144*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + (3*(a
+ 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2*b)*x - (3*(a^6
- 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 3072*a*b^5 + 1024*
b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*S
in[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] -
 I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*(a^
4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a + 2*b)*Sin[
2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])^2) + ...
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4620, 372, 440, 402, 27, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^6}{\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$\downarrow$ 372

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\int \frac{\tan^2(e+fx)\left(3(a+b)-(6a+7b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{6a}}{f}$$

$\downarrow$ 440

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\int \frac{(a+b)(9a+10b)-\left(24a^2+91ba+70b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a}-\frac{(9a+10b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)}}{6a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\int \frac{5\left((a+b)\left(3a^2+18ba+16b^2\right)-b\left(33a^2+110ba+80b^2\right)\tan^2(e+\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}}{2a}}{4a}}{6a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{5\int \frac{(a+b)\left(3a^2+18ba+16b^2\right)-b\left(33a^2+110ba+80b^2\right)\tan^2(e+f}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}}{2a}}{4a}}{6a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{5\left(\frac{\int \frac{12(a+b)\left((a+b)\left(a^2+8ba+8b^2\right)-b\left(9a^2+32ba+24b^2\right)\tan^2\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}}{4a(a+b)}\right)}{2a}}{4a}}{6a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\frac{(33a^2+110ab+80b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}-\frac{5\left(\frac{3\int\frac{(a+b)(a^2+8ba+8b^2)-b(9a^2+32ba+24b^2)\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}}{a}\right.}{2a}}{4a}}{6a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(33a^2+110ab+80b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}-\frac{5\left(3\left(\frac{\int\frac{2(a+b)((a+b)(a^2+12ba+16b^2)-b(5a^2+20ba+16b^2)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}}{2a(a+b)}\right.\right.}{}}{4a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(33a^2+110ab+80b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}-\frac{5\left(\frac{3\left(\frac{\int\frac{(a+b)(a^2+12ba+16b^2)-b(5a^2+20ba+16b^2)\tan^2(e+}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}}{a}\right.}{a}\right.}{}}{4a}}{f}$$

$\downarrow$ 397

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{5\left(\frac{3\left(\frac{(a+2b)\left(a^2+16ab+16b^2\right)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}\right.}{}\right.}{}}{}$$

$\downarrow$ 216

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{5\left(\frac{3\left(\frac{(a+2b)\left(a^2+16ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{2b(a+b)(}{a}\right.}{}\right.}{}}{}$$

$\downarrow$ 218

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{5\left(\frac{3\left(\frac{(a+2b)\left(a^2+16ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{2\sqrt{b}\sqrt{a+}}{a}\right.}{}\right.}{}}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)^2)
- (-1/4*((9*a + 10*b)*Tan[e + f*x])/(a*(1 + Tan[e + f*x]^2)^2*(a + b + b*T
an[e + f*x]^2)^2) + (((33*a^2 + 110*a*b + 80*b^2)*Tan[e + f*x])/(2*a*(1 +
Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^2) - (5*(-((b*(9*a^2 + 32*a*b +
 24*b^2)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2)^2)) + (3*((((a + 2*b)
*(a^2 + 16*a*b + 16*b^2)*ArcTan[Tan[e + f*x]])/a - (2*Sqrt[b]*Sqrt[a + b]*
(a + 4*b)*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/a)/a - (
b*(5*a^2 + 20*a*b + 16*b^2)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2))))
/a))/(2*a))/(4*a))/(6*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 8.40 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{\left(-\frac{27}{8}a^2b-3a\,b^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-6a^2b-6a\,b^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{21}{8}a^2b-3a\,b^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{5\left(a^3+18a^2b+48\right.}{}}{a^6}$
default	$\frac{\frac{\left(-\frac{27}{8}a^2b-3a\,b^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-6a^2b-6a\,b^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{21}{8}a^2b-3a\,b^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{5\left(a^3+18a^2b+48\right.}{}}{a^6}$
risch	$\frac{5x}{16a^3}+\frac{45xb}{8a^4}+\frac{15x\,b^2}{a^5}+\frac{10x\,b^3}{a^6}-\frac{3i\mathrm{e}^{4i(fx+e)}b}{64a^4f}+\frac{3i\mathrm{e}^{-4i(fx+e)}}{128a^3f}-\frac{i\mathrm{e}^{-6i(fx+e)}}{384a^3f}+\frac{3i\mathrm{e}^{2i(fx+e)}b}{4a^4f}-\frac{15i\mathrm{e}^{-2i}}{128a}$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^6*(((-27/8*a^2*b-3*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-6*a^2*b-6*a*b^
2-5/6*a^3)*tan(f*x+e)^3+(-5/16*a^3-21/8*a^2*b-3*a*b^2)*tan(f*x+e))/(1+tan(
f*x+e)^2)^3+5/16*(a^3+18*a^2*b+48*a*b^2+32*b^3)*arctan(tan(f*x+e)))-(a+b)*
b/a^6*(((7/8*a^2*b+2*a*b^2)*tan(f*x+e)^3+1/8*a*(9*a^2+25*a*b+16*b^2)*tan(f
*x+e))/(a+b+b*tan(f*x+e)^2)^2+5/8*(3*a^2+16*a*b+16*b^2)/((a+b)*b)^(1/2)*ar
ctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 930, normalized size of antiderivative = 2.96

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/96*(30*(a^5 + 18*a^4*b + 48*a^3*b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 +
60*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3 + 32*a*b^4)*f*x*cos(f*x + e)^2 + 30*(a
^3*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*
a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*
a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8
*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos
(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 2*(8*a^5*cos(f*x + e)^9 - 2*
(13*a^5 + 10*a^4*b)*cos(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos
(f*x + e)^5 + 20*(6*a^4*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(
5*a^3*b^2 + 20*a^2*b^3 + 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(
f*x + e)^4 + 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*f), 1/48*(15*(a^5 + 18*a^4
*b + 48*a^3*b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 30*(a^4*b + 18*a^3*b^2
+ 48*a^2*b^3 + 32*a*b^4)*f*x*cos(f*x + e)^2 + 15*(a^3*b^2 + 18*a^2*b^3 + 4
8*a*b^4 + 32*b^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4
 + 3*a^2*b^2 + 16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*b^2 + 16*a*b^3)*cos
(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sq
rt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - (8*a^5*cos(f*x + e)^9 - 2*(13*
a^5 + 10*a^4*b)*cos(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x
 + e)^5 + 20*(6*a^4*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.33

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{15\left(5a^2b^2+20ab^3+16b^4\right)\tan(fx+e)^9+40\left(3a^3b+19a^2b^2+39ab^3+24b^4\right)\tan(fx+e)^7+\left(33a^4+470a^3b+1910a^2b^2+2880ab^3+1440b^4\right)\tan}{a^5b^2\tan(fx+e)^{10}+\left(2a^6b+5a^5b^2\right)\tan(fx+e)^8+a^7+2a^6b+a^5b^2+\left(a^7+8a^6b+10a^5b^2\right)\tan(fx+e)}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/48*((15*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*tan(f*x + e)^9 + 40*(3*a^3*b +
19*a^2*b^2 + 39*a*b^3 + 24*b^4)*tan(f*x + e)^7 + (33*a^4 + 470*a^3*b + 191
0*a^2*b^2 + 2880*a*b^3 + 1440*b^4)*tan(f*x + e)^5 + 40*(a^4 + 14*a^3*b + 4
6*a^2*b^2 + 57*a*b^3 + 24*b^4)*tan(f*x + e)^3 + 15*(a^4 + 14*a^3*b + 41*a^
2*b^2 + 44*a*b^3 + 16*b^4)*tan(f*x + e))/(a^5*b^2*tan(f*x + e)^10 + (2*a^6
*b + 5*a^5*b^2)*tan(f*x + e)^8 + a^7 + 2*a^6*b + a^5*b^2 + (a^7 + 8*a^6*b
+ 10*a^5*b^2)*tan(f*x + e)^6 + (3*a^7 + 12*a^6*b + 10*a^5*b^2)*tan(f*x + e
)^4 + (3*a^7 + 8*a^6*b + 5*a^5*b^2)*tan(f*x + e)^2) - 15*(a^3 + 18*a^2*b +
 48*a*b^2 + 32*b^3)*(f*x + e)/a^6 + 30*(3*a^3*b + 19*a^2*b^2 + 32*a*b^3 +
16*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^6))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{\frac{15\left(a^3+18a^2b+48ab^2+32b^3\right)(fx+e)}{a^6}-\frac{30\left(3a^3b+19a^2b^2+32ab^3+16b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^6}-\frac{6\left(7a^2b^2\tan(f\right.}{}}{}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/48*(15*(a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*(f*x + e)/a^6 - 30*(3*a^3*b
+ 19*a^2*b^2 + 32*a*b^3 + 16*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + a
rctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^6) - 6*(7*a^2*b^
2*tan(f*x + e)^3 + 23*a*b^3*tan(f*x + e)^3 + 16*b^4*tan(f*x + e)^3 + 9*a^3
*b*tan(f*x + e) + 34*a^2*b^2*tan(f*x + e) + 41*a*b^3*tan(f*x + e) + 16*b^4
*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^5) - (33*a^2*tan(f*x + e)^5
 + 162*a*b*tan(f*x + e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3
 + 288*a*b*tan(f*x + e)^3 + 288*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) +
 126*a*b*tan(f*x + e) + 144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5)
)/f
```

Mupad [B] (verification not implemented)

Time = 15.57 (sec) , antiderivative size = 2117, normalized size of antiderivative = 6.74

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(5*atan(((5*((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 17
6000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) -
(((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 - (t
an(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*b)*(16*a*b + a^2 + 16*b
^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6))*(a +
2*b)*(16*a*b + a^2 + 16*b^2))/(32*a^6) + (5*((tan(e + f*x)*(179200*a*b^8 +
 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b
^4 + 925*a^6*b^3))/(128*a^10) + (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^
3)/4 + (5*a^15*b^2)/4)/a^15 + (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2
)*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a
^2 + 16*b^2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2))/(32*a^6))/((
4750*a*b^10 + 1000*b^11 + (18875*a^2*b^9)/2 + (40625*a^3*b^8)/4 + (204875*
a^4*b^7)/32 + (305125*a^5*b^6)/128 + (256125*a^6*b^5)/512 + (53125*a^7*b^4
)/1024 + (1875*a^8*b^3)/1024)/a^15 - (((tan(e + f*x)*(179200*a*b^8 + 51200
*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 9
25*a^6*b^3))/(128*a^10) - (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 +
 (5*a^15*b^2)/4)/a^15 - (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a +
 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 1
6*b^2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6) + (((t
an(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6...
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1719, normalized size of antiderivative = 5.47

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
( - 90*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**4*a**4 - 480*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*t
an((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 480*sqrt(b)*s
qrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e +
f*x)**4*a**2*b**2 + 180*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x
)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4 + 1140*sqrt(b)*sqrt(a + b)*a
tan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3
*b + 1920*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*sin(e + f*x)**2*a**2*b**2 + 960*sqrt(b)*sqrt(a + b)*atan((sqrt(
a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 - 90*sq
rt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a
**4 - 660*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*a**3*b - 1530*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*
x)/2) - sqrt(a))/sqrt(b))*a**2*b**2 - 1440*sqrt(b)*sqrt(a + b)*atan((sqrt(
a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b**3 - 480*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4 - 90*sqrt(b
)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e
 + f*x)**4*a**4 - 480*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/
2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 480*sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a*...
```

3.61 $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 238

$$\begin{aligned}\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= \frac{3(a^2+12ab+16b^2)\,x}{8a^5} \\ &\quad - \frac{3\sqrt{b}(5a^2+20ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}f} \\ &\quad - \frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^2} \\ &\quad + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^2} \\ &\quad - \frac{b(7a+12b)\tan(e+fx)}{8a^3f\,(a+b+b\tan^2(e+fx))^2} \\ &\quad - \frac{3b(a+2b)\tan(e+fx)}{2a^4f\,(a+b+b\tan^2(e+fx))}\end{aligned}$$

output
```
3/8*(a^2+12*a*b+16*b^2)*x/a^5-3/8*b^(1/2)*(5*a^2+20*a*b+16*b^2)*arctan(b^(
1/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(1/2)/f-1/8*(5*a+8*b)*cos(f*x+e)*si
n(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b
+b*tan(f*x+e)^2)^2-1/8*b*(7*a+12*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^
2-3/2*b*(a+2*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.02 (sec) , antiderivative size = 2469, normalized size of antiderivative = 10.37

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(3*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*
(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)
])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(16384*b^(5/2)*f*(a + b*
Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3
*a*(a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (
Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*
Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x
)])^2)))/(16384*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - (3*(a + 2*b + a*Cos[
2*e + 2*f*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^
2*b^3 + 640*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((
a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*
Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4]) + (Sec[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e
] + 512*a*b^2*(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(
e + 2*f*x)] + 256*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(
e + 2*f*x)] + 512*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e
+ 2*f*x] + 2560*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*
f*x] + 128*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x]
 + 128*a^2*b^4*f*x*Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e...
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4620, 372, 402, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^4}{\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}-\frac{\int \frac{-\left((4a+7b)\tan^2(e+fx)\right)+a+b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\int \frac{(a+b)(3a+8b)-5b(5a+8b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{\int \frac{12(a+b)\left((a+b)(a+4b)-b(7a+12b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{3\int\frac{(a+b)(a+4b)-b(7a+12b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{a}-\frac{b(\cdots}{a(\cdots}}{2a}}{4a}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\frac{\int\frac{2(a+b)\left(a^2+8ba+8b^2-4b(a+2b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+f\cdots}{2a(a+b)}\cdots\right)}{a}\cdots}{2a}}{4a}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\frac{\int\frac{a^2+8ba+8b^2-4b(a+2b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a}-\frac{4\cdots}{a(\cdots}\right)}{a}\cdots}{2a}}{4a}}{f}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\frac{\frac{\left(a^2+12ab+16b^2\right)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b\left(5a^2+2\cdots\right.}{a}}{\cdots}\right)}{\cdots}}{4a}}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\frac{\frac{\left(a^2+12ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{b\left(5a^2+20ab+16b^2\right)\cdots}{a}}{\cdots}\right)}{\cdots}}{4a}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\frac{\frac{\left(a^2+12ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}\left(5a^2+20ab+16b\right.}{a}}{a}\right.}{}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)^2) -
(((5*a + 8*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f
*x]^2)^2) - (-((b*(7*a + 12*b)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2)
^2)) + (3*((((a^2 + 12*a*b + 16*b^2)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(5
*a^2 + 20*a*b + 16*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqr
t[a + b]))/a - (4*b*(a + 2*b)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2))
))/a)/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397

```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}a^2b+\frac{3}{2}a\,b^2\right)\tan(fx+e)^3+\frac{3a\left(3a^2+7ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{3\left(5a^2+20ab+16b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(-\frac{3}{2}ab-\frac{5}{8}a^2\right)\cdots}{\cdots}}{f}$
default	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}a^2b+\frac{3}{2}a\,b^2\right)\tan(fx+e)^3+\frac{3a\left(3a^2+7ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{3\left(5a^2+20ab+16b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(-\frac{3}{2}ab-\frac{5}{8}a^2\right)\cdots}{\cdots}}{f}$
risch	$\frac{3x}{8a^3}+\frac{9xb}{2a^4}+\frac{6x\,b^2}{a^5}+\frac{i\mathrm{e}^{-4i(fx+e)}}{64a^3f}-\frac{i\mathrm{e}^{4i(fx+e)}}{64a^3f}-\frac{i\mathrm{e}^{-2i(fx+e)}}{8a^3f}+\frac{i\mathrm{e}^{2i(fx+e)}}{8a^3f}-\frac{ib\left(9a^3\mathrm{e}^{6i(fx+e)}+36a^2b\,\mathrm{e}^{6i(\cdots}{\cdots}$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^5*(((7/8*a^2*b+3/2*a*b^2)*tan(f*x+e)^3+3/8*a*(3*a^2+7*a*b+4*b^2)
*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+3/8*(5*a^2+20*a*b+16*b^2)/((a+b)*b)^(1
/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^5*(((-3/2*a*b-5/8*a^2)*tan(f
*x+e)^3+(-3/8*a^2-3/2*a*b)*tan(f*x+e))/(1+tan(f*x+e)^2)^2+3/8*(a^2+12*a*b+
16*b^2)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 803, normalized size of antiderivative = 3.37

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(12*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 24*(a^3*b + 1
2*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 12*a*b^3 + 16*b^4
)*f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^2 + 20
*a*b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)) + 4*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b)*co
s(f*x + e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2*a*
b^3)*cos(f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x
 + e)^2 + a^5*b^2*f), 1/16*(6*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x +
e)^4 + 12*(a^3*b + 12*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 6*(a^2*b^2
+ 12*a*b^3 + 16*b^4)*f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)
^4 + 5*a^2*b^2 + 20*a*b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*s
qrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*a^4*cos(f*x + e)^7 -
(5*a^4 + 8*a^3*b)*cos(f*x + e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3
- 12*(a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4
 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.26

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{12\,(ab^2+2\,b^3)\tan(fx+e)^7+(19\,a^2b+72\,ab^2+72\,b^3)\tan(fx+e)^5+(5\,a^3+46\,a^2b+108\,ab^2+72\,b^3)\tan(fx+e)^3+3\,(a^3+9\,a^2b+16\,ab^2+8\,b^3)\tan}{a^4b^2\tan(fx+e)^8+2\,(a^5b+2\,a^4b^2)\tan(fx+e)^6+a^6+2\,a^5b+a^4b^2+(a^6+6\,a^5b+6\,a^4b^2)\tan(fx+e)^4+2\,(a^6+3\,a^5b+2\,a^4b^2)\tan(fx+e)^2}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((12*(a*b^2 + 2*b^3)*tan(f*x + e)^7 + (19*a^2*b + 72*a*b^2 + 72*b^3)*
tan(f*x + e)^5 + (5*a^3 + 46*a^2*b + 108*a*b^2 + 72*b^3)*tan(f*x + e)^3 +
3*(a^3 + 9*a^2*b + 16*a*b^2 + 8*b^3)*tan(f*x + e))/(a^4*b^2*tan(f*x + e)^8
 + 2*(a^5*b + 2*a^4*b^2)*tan(f*x + e)^6 + a^6 + 2*a^5*b + a^4*b^2 + (a^6 +
 6*a^5*b + 6*a^4*b^2)*tan(f*x + e)^4 + 2*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f
*x + e)^2) - 3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 + 3*(5*a^2*b + 20*a*b
^2 + 16*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^5))
/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.29

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{\frac{3\,(a^2+12\,ab+16\,b^2)(fx+e)}{a^5}-\frac{3\,(5\,a^2b+20\,ab^2+16\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^5}-\frac{12\,ab^2\tan(fx+e)^7+24\,b^3\tan(fx+e)}{}}{}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 - 3*(5*a^2*b + 20*a*b^2 + 16*
b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))/(sqrt(a*b + b^2)*a^5) - (12*a*b^2*tan(f*x + e)^7 + 24*b^3*tan(f*
x + e)^7 + 19*a^2*b*tan(f*x + e)^5 + 72*a*b^2*tan(f*x + e)^5 + 72*b^3*tan(
f*x + e)^5 + 5*a^3*tan(f*x + e)^3 + 46*a^2*b*tan(f*x + e)^3 + 108*a*b^2*ta
n(f*x + e)^3 + 72*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) + 27*a^2*b*tan(f
*x + e) + 48*a*b^2*tan(f*x + e) + 24*b^3*tan(f*x + e))/((b*tan(f*x + e)^4
+ a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2*a^4))/f
```

Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 1317, normalized size of antiderivative = 5.53

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(atan(((((tan(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^
4 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)
/2)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128*a^11*b^2)*(-b*(a + b))^(1/2
)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(
20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b +
 5*a^2 + 16*b^2)*3i)/(16*(a^5*b + a^6)) + (((tan(e + f*x)*(4608*a*b^6 + 23
04*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) + (3*((12*a^
10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 + (3*tan(e + f*x)*(256*a^10*b^
3 + 128*a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(
a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b +
a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2)*3i)/(16*(a^5*b + a^6))
)/((540*a*b^7 + 216*b^8 + (999*a^2*b^6)/2 + (837*a^3*b^5)/4 + (1215*a^4*b^
4)/32 + (135*a^5*b^3)/64)/a^12 - (3*((tan(e + f*x)*(4608*a*b^6 + 2304*b^7
+ 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4
+ 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128
*a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b +
 a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*
(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)) + (3*((ta
n(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*
b^3))/(16*a^8) + (3*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 ...
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1736, normalized size of antiderivative = 7.29

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**4*a**4 - 60*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 48*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*
x)**4*a**2*b**2 + 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2
) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4 + 150*sqrt(b)*sqrt(a + b)*atan(
(sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b +
 216*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqr
t(b))*sin(e + f*x)**2*a**2*b**2 + 96*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 - 15*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 -
90*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*a**3*b - 183*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) -
sqrt(a))/sqrt(b))*a**2*b**2 - 156*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b**3 - 48*sqrt(b)*sqrt(a + b)*atan((s
qrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4 - 15*sqrt(b)*sqrt(a +
 b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4
*a**4 - 60*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a
))/sqrt(b))*sin(e + f*x)**4*a**3*b - 48*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
 b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 + 30...
```

3.62 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+6b)x}{2a^4} - \frac{\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+12b)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/2*(a+6*b)*x/a^4-1/8*b^(1/2)*(15*a^2+40*a*b+24*b^2)*arctan(b^(1/2)*tan(f*
x+e)/(a+b)^(1/2))/a^4/(a+b)^(3/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*t
an(f*x+e)^2)^2-3/4*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+1
2*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.06 (sec) , antiderivative size = 1915, normalized size of antiderivative = 10.41

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*
(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)
])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^(5/2)*f*(a + b*S
ec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*
a*(a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (S
qrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*C
os[2*(e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)
])^2)))/(2048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e
+ 2*f*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^
3 + 640*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4]) + (Sec[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] +
512*a*b^2*(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e +
2*f*x)] + 256*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e +
2*f*x)] + 512*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*
f*x] + 2560*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x]
 + 128*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 1
28*a^2*b^4*f*x*Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + ...
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4620, 373, 402, 27, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(e+fx)^2}{\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 373$$

$$\frac{\frac{\int \frac{-5b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{\int \frac{2(a+b)\left(-9b\tan^2(e+fx)+2a+3b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}-\frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{-9b\tan^2(e+fx)+2a+3b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}-\frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{\frac{\int \frac{4a^2+17ba+12b^2-b(11a+12b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(11a+12b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}-\frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{4(a+b)(a+6b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b\left(15a^2+40ab+24b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(11a+12b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}-\frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2}}{2a} \cdots}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{4(a+b)(a+6b)\arctan(\tan(e+fx))}{a}-\frac{b\left(15a^2+40ab+24b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(11a+12b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}-\frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}-\frac{\cdots}{2a\cdots}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{4(a+b)(a+6b)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(11a+12b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}-\frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}-\frac{\cdots}{2a(\tan^2(e\cdots}}{f}$$

input

```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output

```
(-1/2*Tan[e + f*x]/(a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^2) +
 ((-3*b*Tan[e + f*x])/(2*a*(a + b + b*Tan[e + f*x]^2)^2) + (((4*(a + b)*(a
 + 6*b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcT
an[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (
b*(11*a + 12*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(2
*a))/(2*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{ab(7a+8b)\tan(fx+e)^3}{8a+8b}+\frac{a(9a+8b)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+40ab+24b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}\right)}{a^4}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+6b}{}}{a^4}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{ab(7a+8b)\tan(fx+e)^3}{8a+8b}+\frac{a(9a+8b)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+40ab+24b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}\right)}{a^4}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+6b}{}}{a^4}}{f}$
risch	$\frac{x}{2a^3}+\frac{3xb}{a^4}+\frac{ie^{2i(fx+e)}}{8a^3f}-\frac{ie^{-2i(fx+e)}}{8a^3f}-\frac{ib\left(9a^3e^{6i(fx+e)}+32a^2b\,e^{6i(fx+e)}+24a\,b^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+\right.}{4a^4(}$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^4*b*((1/8*a*b*(7*a+8*b)/(a+b)*tan(f*x+e)^3+1/8*a*(9*a+8*b)*tan(f
*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+40*a*b+24*b^2)/(a+b)/((a+b)*b)^(
1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^4*(-1/2*a*tan(f*x+e)/(1+tan
(f*x+e)^2)+1/2*(a+6*b)*arctan(tan(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(166) = 332$.

Time = 0.16 (sec) , antiderivative size = 815, normalized size of antiderivative = 4.43

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(16*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b + 7*a
^2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 7*a*b^3 + 6*b^4)*f*x
+ ((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3
 + 24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(
a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(
f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x
 + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)) - 4*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^3*b + 18*
a^2*b^2)*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*sin(f*x +
e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2
 + (a^5*b^2 + a^4*b^3)*f), 1/16*(8*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x
 + e)^4 + 16*(a^3*b + 7*a^2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2
 + 7*a*b^3 + 6*b^4)*f*x + ((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4
 + 15*a^2*b^2 + 40*a*b^3 + 24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*s
qrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) - 2*(4*(a^4 + a^3*b)*cos(f*x
 + e)^5 + (17*a^3*b + 18*a^2*b^2)*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)
*cos(f*x + e))*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b +
a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^2 + a^4*b^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.48

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{(15\,a^2b+40\,ab^2+24\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+a^4b)\sqrt{(a+b)b}}+\frac{(11\,ab^2+12\,b^3)\tan(fx+e)^5+(17\,a^2b+40\,ab^2+24\,b^3)\tan(fx+e)^3+(4\,a^3+21\,a^2}{(a^4b^2+a^3b^3)\tan(fx+e)^6+a^6+3\,a^5b+3\,a^4b^2+a^3b^3+(2\,a^5b+5\,a^4b^2+3\,a^3b^3)\tan(fx+e)^4+}}{8\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b)
)/((a^5 + a^4*b)*sqrt((a + b)*b)) + ((11*a*b^2 + 12*b^3)*tan(f*x + e)^5 +
(17*a^2*b + 40*a*b^2 + 24*b^3)*tan(f*x + e)^3 + (4*a^3 + 21*a^2*b + 29*a*b
^2 + 12*b^3)*tan(f*x + e))/((a^4*b^2 + a^3*b^3)*tan(f*x + e)^6 + a^6 + 3*a
^5*b + 3*a^4*b^2 + a^3*b^3 + (2*a^5*b + 5*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e
)^4 + (a^6 + 5*a^5*b + 7*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e)^2) - 4*(f*x + e
)*(a + 6*b)/a^4)/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.13

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{(15\,a^2b+40\,ab^2+24\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+a^4b)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan(fx+e)^3+8\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+17\,ab^2\tan}{(a^4+a^3b)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
+ arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2))
+ (7*a*b^2*tan(f*x + e)^3 + 8*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) +
17*a*b^2*tan(f*x + e) + 8*b^3*tan(f*x + e))/((a^4 + a^3*b)*(b*tan(f*x + e)
^2 + a + b)^2) - 4*(f*x + e)*(a + 6*b)/a^4 + 4*tan(f*x + e)/((tan(f*x + e)
^2 + 1)*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 15.71 (sec) , antiderivative size = 2628, normalized size of antiderivative = 14.28

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(atan((((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^
4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*
b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(
e + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a
^11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*
1i + b*6i)*1i)/(4*a^4) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2
*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*
a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11
+ a^9*b^2) + (tan(e + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 102
4*a^10*b^3 + 256*a^11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b
*6i))/(4*a^4))*(a*1i + b*6i)*1i)/(4*a^4))/(((297*a*b^6)/4 + 27*b^7 + (279*
a^2*b^5)/4 + (805*a^3*b^4)/32 + (165*a^4*b^3)/64)/(2*a^10*b + a^11 + a^9*b
^2) - (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4
 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*b
^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e
 + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^
11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1
i + b*6i))/(4*a^4) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5
 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*
b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 +...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1738, normalized size of antiderivative = 9.45

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**4*a**4 - 40*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 24*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*
x)**4*a**2*b**2 + 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2
) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4 + 110*sqrt(b)*sqrt(a + b)*atan(
(sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b +
 128*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqr
t(b))*sin(e + f*x)**2*a**2*b**2 + 48*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 - 15*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 -
70*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*a**3*b - 119*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) -
sqrt(a))/sqrt(b))*a**2*b**2 - 88*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan
((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b**3 - 24*sqrt(b)*sqrt(a + b)*atan((sq
rt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4 - 15*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*
a**4 - 40*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a)
)/sqrt(b))*sin(e + f*x)**4*a**3*b - 24*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 + 30*...
```

3.63 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{x}{a^3}-\frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^3-1/8*b^(1/2)*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1
/8*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.52 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.31

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2+\frac{b(15a^2+20ab+8b^2)\arctan\left(\frac{\sec(fx)(\cdots}{\cdots}\right)}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e
+ f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*S
in[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*S
in[2*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2
*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e
])) + (b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e
] - 3*a*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] +
Sin[e]))))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4616, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \; 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\int \frac{-3b\tan^2(e+fx)+4a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{8a^2+9ba+4b^2-b(7a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
(-1/4*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (((8*(a
+ b)^2*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*
(7*a + 4*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*(
a + b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3 e^{6i(fx+e)}+28a^2 b\, e^{6i(fx+e)}+16a\, b^2 e^{6i(fx+e)}+27a^3 e^{4i(fx+e)}+90a^2 b\, e^{4i(fx+e)}+120a\, b^2 e^{4i(fx+e)}+48b^3 e\right.}{4a^3(a+b)^2 f\left(a\, e^{4i(fx+e)}+2a\, e^{2i(fx+e)}+4b\, e^{2i(f\right.}$

input
```
int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(9*a+4*b)
*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+20*a*b+8*b^2)/(a^2
+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*ar
ctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(130) = 260$.

Time = 0.14 (sec) , antiderivative size = 819, normalized size of antiderivative = 5.69

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2
*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15
*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^
4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sq
rt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e
)^2 + b^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a
*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e
)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*
b^3 + a^3*b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 +
 32*(a^3*b + 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3
 + b^4)*f*x + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 20*a*b^3 + 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*
sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3
 + (7*a^2*b^2 + 4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5
*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2
+ (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^3}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(15a^2b+20ab^2+8b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+2a^4b+a^3b^2)\sqrt{(a+b)b}}+\frac{(7ab^2+4b^3)\tan(fx+e)^3+(9a^2b+13ab^2+4b^3)\tan(fx+e)}{a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4+(a^4b^2+2a^3b^3+a^2b^4)\tan(fx+e)^4+2(a^5b+3a^4b^2+3a^3b^3+\dots}}{8f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x
+ e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^
4 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x +
 e)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(15a^2b+20ab^2+8b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+2a^4b+a^3b^2)\sqrt{ab+b^2}}+\frac{7ab^2\tan(fx+e)^3+4b^3\tan(fx+e)^3+9a^2b\tan(fx+e)+13ab^2\tan(\dots}{(a^4+2a^3b+a^2b^2)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a
*b + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(
f*x + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a
^2*b^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 16.39 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int \frac{1}{(a + b\sec^2(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^1
0*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e
 + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a
^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 +
856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a
^6*b^2)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)
/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)
) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a
^5*b^3 + 6*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*
a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5
*b^3 + 6*a^6*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*
b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b
^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*
a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*
b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^1
1*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(
2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 +
 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1496, normalized size of antiderivative = 10.39

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input

```
int(1/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
( - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**4*a**4 - 20*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 8*sqrt(b)*sqrt
(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x
)**4*a**2*b**2 + 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2)
 - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4 + 70*sqrt(b)*sqrt(a + b)*atan((s
qrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b + 5
6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**2*a**2*b**2 + 16*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 - 15*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 - 50*
sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))
*a**3*b - 63*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt
(a))/sqrt(b))*a**2*b**2 - 36*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*a*b**3 - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a
+ b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4 - 15*sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4
- 20*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqr
t(b))*sin(e + f*x)**4*a**3*b - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan
((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 + 30*sqrt(b...
```

3.64 $\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{15\cot(e+fx)}{8(a+b)^3f} + \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-15/8*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(7/2)/f-15/8*co
t(f*x+e)/(a+b)^3/f+1/4*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+5/8*cot(f
*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.15 (sec) , antiderivative size = 749, normalized size of antiderivative = 6.04

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(\frac{120b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((120*b*ArcTan[(Sec[f*x]*(C
os[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(C
os[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Csc[
e]*Csc[e + f*x]*Sec[2*e]*((-32*a^4 - 64*a^3*b + 22*a^2*b^2 + 80*a*b^3 + 16
*b^4)*Sin[f*x] + 2*a*(16*a^3 + 23*a^2*b - 27*a*b^2 - 4*b^3)*Sin[3*f*x] - 4
8*a^4*Sin[2*e - f*x] - 128*a^3*b*Sin[2*e - f*x] - 106*a^2*b^2*Sin[2*e - f*
x] + 80*a*b^3*Sin[2*e - f*x] + 16*b^4*Sin[2*e - f*x] + 48*a^4*Sin[2*e + f*
x] + 146*a^3*b*Sin[2*e + f*x] + 182*a^2*b^2*Sin[2*e + f*x] + 80*a*b^3*Sin[
2*e + f*x] + 16*b^4*Sin[2*e + f*x] - 32*a^4*Sin[4*e + f*x] - 82*a^3*b*Sin[
4*e + f*x] - 54*a^2*b^2*Sin[4*e + f*x] - 80*a*b^3*Sin[4*e + f*x] - 16*b^4*
Sin[4*e + f*x] - 8*a^4*Sin[2*e + 3*f*x] + 18*a^3*b*Sin[2*e + 3*f*x] + 54*a
^2*b^2*Sin[2*e + 3*f*x] + 8*a*b^3*Sin[2*e + 3*f*x] + 32*a^4*Sin[4*e + 3*f*
x] + 73*a^3*b*Sin[4*e + 3*f*x] + 24*a^2*b^2*Sin[4*e + 3*f*x] + 8*a*b^3*Sin
[4*e + 3*f*x] - 8*a^4*Sin[6*e + 3*f*x] - 9*a^3*b*Sin[6*e + 3*f*x] - 24*a^2
*b^2*Sin[6*e + 3*f*x] - 8*a*b^3*Sin[6*e + 3*f*x] + 8*a^4*Sin[2*e + 5*f*x]
- 9*a^3*b*Sin[2*e + 5*f*x] - 2*a^2*b^2*Sin[2*e + 5*f*x] + 9*a^3*b*Sin[4*e
+ 5*f*x] + 2*a^2*b^2*Sin[4*e + 5*f*x] + 8*a^4*Sin[6*e + 5*f*x]))/a^2))/(51
2*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{5\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4(a+b)}+\frac{\cot(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{5\left(\frac{3\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2(a+b)}+\frac{\cot(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)}+\frac{\cot(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{5\left(\frac{3\left(-\frac{b\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{a+b}\right)}{2(a+b)}+\frac{\cot(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)}+\frac{\cot(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cot(e+fx)}{a+b}\right)}{2(a+b)}+\frac{\cot(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)}+\frac{\cot(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Cot[e + f*x]/(4*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (5*((3*(-((Sqrt[b
]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)) - Cot[e + f*x
]/(a + b)))/(2*(a + b)) + Cot[e + f*x]/(2*(a + b)*(a + b + b*Tan[e + f*x]^
2))))/(4*(a + b)))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{7b\tan(fx+e)^3}{8}+\left(\frac{9a}{8}+\frac{9b}{8}\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{(a+b)^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{7b\tan(fx+e)^3}{8}+\left(\frac{9a}{8}+\frac{9b}{8}\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{(a+b)^3\tan(fx+e)}}{f}$
risch	$-\frac{i(8a^4\mathrm{e}^{8i(fx+e)}+9a^3b\,\mathrm{e}^{8i(fx+e)}+24a^2b^2\mathrm{e}^{8i(fx+e)}+8a\,b^3\mathrm{e}^{8i(fx+e)}+32a^4\mathrm{e}^{6i(fx+e)}+82a^3b\,\mathrm{e}^{6i(fx+e)}+54a^2b^2\mathrm{e}^{6i(fx-}}{}$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a+b)^3*b*((7/8*b*tan(f*x+e)^3+(9/8*a+9/8*b)*tan(f*x+e))/(a+b+b*ta
n(f*x+e)^2)^2+15/8/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1
/(a+b)^3/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(108) = 216.

Time = 0.13 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.96

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 20*(5*a*b - b^2)*cos(f*
x + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/(a
 + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f
*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x
+ e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos
(f*x + e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b +
 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3
+ a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f
*x + e)), -1/16*(2*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 10*(5*a*b - b^
2)*cos(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*s
qrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(
b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*b^2*cos(f*x + e))/(((a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 +
3*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(108) = 216$.

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.77

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{15\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}} + \frac{15\,b^2\tan(fx+e)^4+25\,(ab+b^2)\tan(fx+e)^2+8\,a^2+16\,ab+8\,b^2}{(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)\tan(fx+e)^5+2\,(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5)\tan(fx+e)^3+(a^5+5\,a^4b+10\,a}}{8\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(15*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*sqrt((a + b)*b)) + (15*b^2*tan(f*x + e)^4 + 25*(a*b + b^2)*tan(f*
x + e)^2 + 8*a^2 + 16*a*b + 8*b^2)/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*
tan(f*x + e)^5 + 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*x
 + e)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*
x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.43

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab+b^2}} + \frac{7\,b^2\tan(fx+e)^3+9\,ab\tan(fx+e)+9\,b^2\tan(fx+e)}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2} + \frac{8}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+}}{8\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (7*b^2
*tan(f*x + e)^3 + 9*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e))/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2) + 8/((a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*tan(f*x + e)))/f
```

Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.18

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= -\frac{\frac{1}{a+b}+\frac{25\,b\tan(e+f\,x)^2}{8\,(a+b)^2}+\frac{15\,b^2\tan(e+f\,x)^4}{8\,(a+b)^3}}{f\,\left(\tan\left(e+f\,x\right)^3\,\left(2\,b^2+2\,a\,b\right)+\tan\left(e+f\,x\right)\,\left(a^2+2\,a\,b+b^2\right)+b^2\tan\left(e+f\,x\right)^5\right)} - \frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{(a+b)^{7/2}}\right)}{8\,f\,(a+b)^{7/2}}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
- (1/(a + b) + (25*b*tan(e + f*x)^2)/(8*(a + b)^2) + (15*b^2*tan(e + f*x)^
4)/(8*(a + b)^3))/(f*(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b
 + a^2 + b^2) + b^2*tan(e + f*x)^5)) - (15*b^(1/2)*atan((b^(1/2)*tan(e + f
*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(8*f*(a + b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 922, normalized size of antiderivative = 7.44

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**5*a**2 + 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*a**2 + 30*sqrt(b)*sqrt(
a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)
**3*a*b - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt
(a))/sqrt(b))*sin(e + f*x)*a**2 - 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*a*b - 15*sqrt(b)*sqrt(a
 + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*
b**2 - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a)
)/sqrt(b))*sin(e + f*x)**5*a**2 + 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**3*a**2 + 30*sqrt(b)*sq
rt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f
*x)**3*a*b - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + s
qrt(a))/sqrt(b))*sin(e + f*x)*a**2 - 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
 b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)*a*b - 15*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*
x)*b**2 - 8*cos(e + f*x)*sin(e + f*x)**4*a**3 + cos(e + f*x)*sin(e + f*x)*
*4*a**2*b + 11*cos(e + f*x)*sin(e + f*x)**4*a*b**2 + 2*cos(e + f*x)*sin(e
+ f*x)**4*b**3 + 16*cos(e + f*x)*sin(e + f*x)**2*a**3 + 23*cos(e + f*x)*si
n(e + f*x)**2*a**2*b - 2*cos(e + f*x)*sin(e + f*x)**2*a*b**2 - 9*cos(e ...
```

3.65 $\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{5(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} - \frac{(a-2b)\cot(e+fx)}{(a+b)^4 f} - \frac{\cot^3(e+fx)}{3(a+b)^3 f} - \frac{ab\tan(e+fx)}{4(a+b)^3 f\,(a+b+b\tan^2(e+fx))^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4 f\,(a+b+b\tan^2(e+fx))}$$

output
```
-5/8*(3*a-4*b)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(9/2)/
f-(a-2*b)*cot(f*x+e)/(a+b)^4/f-1/3*cot(f*x+e)^3/(a+b)^3/f-1/4*a*b*tan(f*x+
e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^2-1/8*(7*a-4*b)*b*tan(f*x+e)/(a+b)^4/f/(
a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.06

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(\frac{480(3a-4b)b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((480*(3*a - 4*b)*b*ArcTan[
(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x
]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])
^4]) - (Csc[e]*Csc[e + f*x]^3*Sec[2*e]*(4*(44*a^4 + 122*a^3*b + 63*a^2*b^2
 + 126*a*b^3 + 36*b^4)*Sin[f*x] + (-96*a^4 - 71*a^3*b + 344*a^2*b^2 - 1208
*a*b^3 + 48*b^4)*Sin[3*f*x] + 224*a^4*Sin[2*e - f*x] + 576*a^3*b*Sin[2*e -
 f*x] + 124*a^2*b^2*Sin[2*e - f*x] - 2184*a*b^3*Sin[2*e - f*x] + 144*b^4*S
in[2*e - f*x] - 224*a^4*Sin[2*e + f*x] - 657*a^3*b*Sin[2*e + f*x] - 538*a^
2*b^2*Sin[2*e + f*x] + 984*a*b^3*Sin[2*e + f*x] + 144*b^4*Sin[2*e + f*x] +
 176*a^4*Sin[4*e + f*x] + 569*a^3*b*Sin[4*e + f*x] + 666*a^2*b^2*Sin[4*e +
 f*x] + 1704*a*b^3*Sin[4*e + f*x] - 144*b^4*Sin[4*e + f*x] + 48*a^4*Sin[2*
e + 3*f*x] + 111*a^3*b*Sin[2*e + 3*f*x] + 360*a^2*b^2*Sin[2*e + 3*f*x] + 3
12*a*b^3*Sin[2*e + 3*f*x] - 48*b^4*Sin[2*e + 3*f*x] - 96*a^4*Sin[4*e + 3*f
*x] - 152*a^3*b*Sin[4*e + 3*f*x] + 146*a^2*b^2*Sin[4*e + 3*f*x] - 728*a*b^
3*Sin[4*e + 3*f*x] - 48*b^4*Sin[4*e + 3*f*x] + 48*a^4*Sin[6*e + 3*f*x] + 1
92*a^3*b*Sin[6*e + 3*f*x] + 558*a^2*b^2*Sin[6*e + 3*f*x] - 168*a*b^3*Sin[6
*e + 3*f*x] + 48*b^4*Sin[6*e + 3*f*x] + 16*a^4*Sin[2*e + 5*f*x] - 598*a^2*
b^2*Sin[2*e + 5*f*x] + 48*a*b^3*Sin[2*e + 5*f*x] + 72*a^3*b*Sin[4*e + 5*f*
x] + 150*a^2*b^2*Sin[4*e + 5*f*x] - 48*a*b^3*Sin[4*e + 5*f*x] + 16*a^4*...
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4620, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \frac{\cot^4(e+fx)\left(\tan^2(e+fx)+1\right)}{\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 361$$

$$\frac{-\frac{1}{4}b\int -\frac{\cot^4(e+fx)\left(-\frac{3a\tan^4(e+fx)}{(a+b)^3}+\frac{4a\tan^2(e+fx)}{b(a+b)^2}+\frac{4}{b(a+b)}\right)}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)-\frac{ab\tan(e+fx)}{4(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{4}b\int \frac{\cot^4(e+fx)\left(-\frac{3a\tan^4(e+fx)}{(a+b)^3}+\frac{4a\tan^2(e+fx)}{b(a+b)^2}+\frac{4}{b(a+b)}\right)}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)-\frac{ab\tan(e+fx)}{4(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}}{f}$$

$$\Big\downarrow \ 1582$$

$$\frac{\frac{1}{4}b\left(\frac{\int \frac{\cot^4(e+fx)\left(-\frac{(7a-4b)b^2\tan^4(e+fx)}{a+b}+8(a-b)b\tan^2(e+fx)+8b(a+b)\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2b^2(a+b)^3}-\frac{(7a-4b)\tan(e+fx)}{2(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}\right)-\frac{ab\tan(e+}{4(a+b)^3(a+b\tan^2}}{f}$$

$$\Big\downarrow \ 1584$$

$$\frac{\frac{1}{4}b\left(\frac{\int \left(8b\cot^4(e+fx)+\frac{8(a-2b)b\cot^2(e+fx)}{a+b}+\frac{5b^2(4b-3a)}{(a+b)\left(b\tan^2(e+fx)+a+b\right)}\right)d\tan(e+fx)}{2b^2(a+b)^3}-\frac{(7a-4b)\tan(e+fx)}{2(a+b)^4(a+b\tan^2(e+fx)+b)}\right)-\frac{ab\tan(e}{4(a+b)^3(a+b\tan}}{f}$$

$\Big\downarrow$ 2009

$$\frac{\frac{1}{4}b\left(\frac{-\frac{5b^{3/2}(3a-4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{8b(a-2b)\cot(e+fx)}{a+b}-\frac{8}{3}b\cot^3(e+fx)}{2b^2(a+b)^3}-\frac{(7a-4b)\tan(e+fx)}{2(a+b)^4(a+b\tan^2(e+fx)+b)}\right)-\frac{ab\tan(e+fx)}{4(a+b)^3(a+b\tan^2(e+f}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(a*b*Tan[e + f*x])/((a + b)^3*(a + b + b*Tan[e + f*x]^2)^2) + (b*(((
-5*(3*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)
^(3/2) - (8*(a - 2*b)*b*Cot[e + f*x])/(a + b) - (8*b*Cot[e + f*x]^3)/3)/(2
*b^2*(a + b)^3) - ((7*a - 4*b)*Tan[e + f*x])/(2*(a + b)^4*(a + b + b*Tan[e
 + f*x]^2))))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{-\frac{1}{3(a+b)^3 \tan(fx+e)^3}-\frac{a-2b}{(a+b)^4 \tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)^3+\left(\frac{9}{8}a^2+\frac{5}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{5(3a-4b)\arctan\left(\frac{b\tan}{\sqrt{(}}\right.}{8\sqrt{(a+b)b}}\right.}{(a+b)^4}}{f}$
default	$\frac{-\frac{1}{3(a+b)^3 \tan(fx+e)^3}-\frac{a-2b}{(a+b)^4 \tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)^3+\left(\frac{9}{8}a^2+\frac{5}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{5(3a-4b)\arctan\left(\frac{b\tan}{\sqrt{(}}\right.}{8\sqrt{(a+b)b}}\right.}{(a+b)^4}}{f}$
risch	$-\frac{i\left(16a^4+6a^2b^2-83b\,a^3-576a^3b\,e^{6i(fx+e)}-124a^2b^2e^{6i(fx+e)}+2184a\,b^3e^{6i(fx+e)}-71a^3b\,e^{4i(fx+e)}+344a^2b^2e^{4i(fx+\right.}{}$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3/(a+b)^3/tan(f*x+e)^3-(a-2*b)/(a+b)^4/tan(f*x+e)-1/(a+b)^4*b*(((7
/8*a*b-1/2*b^2)*tan(f*x+e)^3+(9/8*a^2+5/8*a*b-1/2*b^2)*tan(f*x+e))/(a+b+b*
tan(f*x+e)^2)^2+5/8*(3*a-4*b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b
)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(148) = 296.

Time = 0.16 (sec) , antiderivative size = 1009, normalized size of antiderivative = 6.15

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/96*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(24*a^3 - 134*a
^2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 32*a*b^2 + 16*b
^3)*cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (3*a^3 - 10*a^
2*b + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b - 11*a*b^2 + 4*
b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*
x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/
(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 60*(3*a*
b^2 - 4*b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*
b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4
 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^
4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4
*a*b^5 + b^6)*f)*sin(f*x + e)), -1/48*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos
(f*x + e)^7 - 2*(24*a^3 - 134*a^2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 -
 10*(15*a^2*b - 32*a*b^2 + 16*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*
cos(f*x + e)^6 - (3*a^3 - 10*a^2*b + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4
*b^3 - (6*a^2*b - 11*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan
(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*
x + e)))*sin(f*x + e) - 30*(3*a*b^2 - 4*b^3)*cos(f*x + e))/(((a^6 + 4*a^5*
b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^5*b ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(148) = 296$.

Time = 0.12 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.97

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\,(3\,ab-4\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)b}} + \frac{15\,(3\,ab^2-4\,b^3)\tan(fx+e)^6+25\,(3\,a^2b-ab^2-4\,b^3)\tan(fx+e)^4+8\,a^3+24\,a^2b-}{(a^4b^2+4\,a^3b^3+6\,a^2b^4+4\,ab^5+b^6)\tan(fx+e)^7+2\,(a^5b+5\,a^4b^2+10\,a^3b^3+10\,a^2b^4+5\,ab^5+b^6)\tan}}{24\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*(15*(3*a*b - 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a*b^2 - 4*b^
3)*tan(f*x + e)^6 + 25*(3*a^2*b - a*b^2 - 4*b^3)*tan(f*x + e)^4 + 8*a^3 +
24*a^2*b + 24*a*b^2 + 8*b^3 + 8*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*tan(f*
x + e)^2)/((a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*tan(f*x + e)^
7 + 2*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*tan(f*
x + e)^5 + (a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5
 + b^6)*tan(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.61

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab-4\,b^2)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab+b^2}} + \frac{3\left(7\,ab^2\tan(fx+e)^3-4\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+5\,ab^2\tan(fx+e)\right.}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)^2}}{24\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/24*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqr
t(a*b + b^2)))*(3*a*b - 4*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
)*sqrt(a*b + b^2)) + 3*(7*a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f*x + e)^3 + 9*
a^2*b*tan(f*x + e) + 5*a*b^2*tan(f*x + e) - 4*b^3*tan(f*x + e))/((a^4 + 4*
a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(3*a*
tan(f*x + e)^2 - 6*b*tan(f*x + e)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*tan(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.26

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{1}{3\,(a+b)}+\frac{25\tan(e+f\,x)^4\,(3\,a\,b-4\,b^2)}{24\,(a+b)^3}+\frac{\tan(e+f\,x)^2\,(3\,a-4\,b)}{3\,(a+b)^2}+\frac{5\tan(e+f\,x)^6\,(3\,a\,b^2-4\,b^3)}{8\,(a+b)^4}}{f\,\left(\tan\left(e+f\,x\right)^3\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^5\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^7\right)} - \frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}{(a+b)^{9/2}}\right)\,(3\,a-4\,b)}{8\,f\,(a+b)^{9/2}}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
- (1/(3*(a + b)) + (25*tan(e + f*x)^4*(3*a*b - 4*b^2))/(24*(a + b)^3) + (t
an(e + f*x)^2*(3*a - 4*b))/(3*(a + b)^2) + (5*tan(e + f*x)^6*(3*a*b^2 - 4*
b^3))/(8*(a + b)^4))/(f*(tan(e + f*x)^3*(2*a*b + a^2 + b^2) + tan(e + f*x)
^5*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^7)) - (5*b^(1/2)*atan((b^(1/2)*tan(e
 + f*x)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))/(a + b)^(9/2))*(3*a -
 4*b))/(8*f*(a + b)^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1367, normalized size of antiderivative = 8.34

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 45*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**7*a**3 + 60*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**2*b + 90*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*
x)**5*a**3 - 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - s
qrt(a))/sqrt(b))*sin(e + f*x)**5*a**2*b - 120*sqrt(b)*sqrt(a + b)*atan((sq
rt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a*b**2 - 45
*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
)*sin(e + f*x)**3*a**3 - 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
 f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*a**2*b + 75*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*
a*b**2 + 60*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(
a))/sqrt(b))*sin(e + f*x)**3*b**3 - 45*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**3 + 60*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e +
 f*x)**7*a**2*b + 90*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2
) + sqrt(a))/sqrt(b))*sin(e + f*x)**5*a**3 - 30*sqrt(b)*sqrt(a + b)*atan((
sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**5*a**2*b -
120*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt
(b))*sin(e + f*x)**5*a*b**2 - 45*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*...
```

3.66 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 208

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3} dx = -\frac{\sqrt{b}(15a^2-40ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{11/2}f} - \frac{(a^2-4ab+b^2)\cot(e+fx)}{(a+b)^5f} - \frac{(2a-b)\cot^3(e+fx)}{3(a+b)^4f} - \frac{\cot^5(e+fx)}{5(a+b)^3f} - \frac{a^2b\tan(e+fx)}{4(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{a(7a-8b)b\tan(e+fx)}{8(a+b)^5f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-1/8*b^(1/2)*(15*a^2-40*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/
(a+b)^(11/2)/f-(a^2-4*a*b+b^2)*cot(f*x+e)/(a+b)^5/f-1/3*(2*a-b)*cot(f*x+e)
^3/(a+b)^4/f-1/5*cot(f*x+e)^5/(a+b)^3/f-1/4*a^2*b*tan(f*x+e)/(a+b)^4/f/(a+
b+b*tan(f*x+e)^2)^2-1/8*a*(7*a-8*b)*b*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+
e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.88 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.30

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-8(4a-11b)(a+b)(a+2b+a\cos(2(e+fx)))^2\cot(e)\csc^2(\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-8*(4*a - 11*b)*(a + b)*(a
 + 2*b + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^2 - 24*(a + b)^2*(a + 2
*b + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^4 + (15*b*(15*a^2 - 40*a*b
+ 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) +
 a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*
b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Co
s[e] - I*Sin[e])^4]) + 8*(8*a^2 - 59*a*b + 23*b^2)*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*Csc[e]*Csc[e + f*x]*Sin[f*x] + 8*(4*a - 11*b)*(a + b)*(a + 2*b +
 a*Cos[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^3*Sin[f*x] + 24*(a + b)^2*(a +
2*b + a*Cos[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^5*Sin[f*x] - 60*b^2*(a + b
)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]) + 15*b*(a + 2*b + a*Cos[2*(
e + f*x)])*Sec[2*e]*((9*a^2 + 16*a*b - 8*b^2)*Sin[2*e] + 3*a*(-3*a + 2*b)*
Sin[2*f*x])))/(960*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4620, 365, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)^3}dx$$

$\Big\downarrow$ 4620

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 365

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+10a+b\right)}{\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\Big\downarrow$ 361

$$\frac{\frac{-\frac{1}{4}b\int -\frac{\cot^4(e+fx)\left(-\frac{3\left(5a^2+4b^2\right)\tan^4(e+fx)}{(a+b)^3}+\frac{4\left(5a^2+4b^2\right)\tan^2(e+fx)}{b(a+b)^2}+\frac{4(10a+b)}{b(a+b)}\right)}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)-\frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{4(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}}{5(a+b)}-\frac{\cot^5(\cdots}{5(a+b)(a+b\,\mathrm{ta}\cdots}}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{1}{4}b\int \frac{\cot^4(e+fx)\left(-\frac{3\left(5a^2+4b^2\right)\tan^4(e+fx)}{(a+b)^3}+\frac{4\left(5a^2+4b^2\right)\tan^2(e+fx)}{b(a+b)^2}+\frac{4(10a+b)}{b(a+b)}\right)}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)-\frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{4(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}}{5(a+b)}-\frac{\cot^5(e+f\cdots}{5(a+b)(a+b\tan^2(\cdots}}{f}$$

$\Big\downarrow$ 1582

$$\frac{\frac{\frac{1}{4}b\left(\frac{\int \frac{\cot^4(e+fx)\left(-\frac{b^2\left(35a^2-40ba+24b^2\right)\tan^4(e+fx)}{a+b}+8b\left(5a^2-10ba+3b^2\right)\tan^2(e+fx)+8b(a+b)(10a+b)\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2b^2(a+b)^3}-\frac{\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{2(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}\right)}{5(a+b)}\cdots}{f}$$

$\Big\downarrow$ 1584

$$\frac{\frac{\frac{1}{4}b\left(\frac{\int\left(8b(10a+b)\cot^4(e+fx)+\frac{8b\left(5a^2-20ba+2b^2\right)\cot^2(e+fx)}{a+b}-\frac{5b^2\left(15a^2-40ba+8b^2\right)}{(a+b)\left(b\tan^2(e+fx)+a+b\right)}\right)d\tan(e+fx)}{2b^2(a+b)^3}-\frac{\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{2(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}\right)-\frac{b\left(5a^2+4\ldots\right.}{4(a+b)^3\left(a+\ldots\right.}}{5(a+b)}}{f}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\frac{1}{4}b\left(\frac{-\frac{5b^{3/2}\left(15a^2-40ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{8b\left(5a^2-20ab+2b^2\right)\cot(e+fx)}{a+b}-\frac{8}{3}b(10a+b)\cot^3(e+fx)}{2b^2(a+b)^3}-\frac{\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{2(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}\right)-\frac{b\left(5\ldots\right.}{4(a+b)\ldots}}{5(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/5*Cot[e + f*x]^5/((a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (-1/4*(b*(5*
a^2 + 4*b^2)*Tan[e + f*x])/((a + b)^3*(a + b + b*Tan[e + f*x]^2)^2) + (b*(
((-5*b^(3/2)*(15*a^2 - 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a + b]])/(a + b)^(3/2) - (8*b*(5*a^2 - 20*a*b + 2*b^2)*Cot[e + f*x])/(a +
b) - (8*b*(10*a + b)*Cot[e + f*x]^3)/3)/(2*b^2*(a + b)^3) - ((35*a^2 - 40*
a*b + 24*b^2)*Tan[e + f*x])/(2*(a + b)^4*(a + b + b*Tan[e + f*x]^2))))/4)/
(5*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{1}{5(a+b)^3\tan(fx+e)^5}-\frac{2a-b}{3(a+b)^4\tan(fx+e)^3}-\frac{a^2-4ab+b^2}{(a+b)^5\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b-a\,b^2\right)\tan(fx+e)^3+\frac{a\left(9a^2+ab-8b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+}{(a+b)^5}}{f}$
default	$\frac{-\frac{1}{5(a+b)^3\tan(fx+e)^5}-\frac{2a-b}{3(a+b)^4\tan(fx+e)^3}-\frac{a^2-4ab+b^2}{(a+b)^5\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b-a\,b^2\right)\tan(fx+e)^3+\frac{a\left(9a^2+ab-8b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+}{(a+b)^5}}{f}$
risch	Expression too large to display

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/5/(a+b)^3/tan(f*x+e)^5-1/3*(2*a-b)/(a+b)^4/tan(f*x+e)^3-(a^2-4*a*b
+b^2)/(a+b)^5/tan(f*x+e)-b/(a+b)^5*(((7/8*a^2*b-a*b^2)*tan(f*x+e)^3+1/8*a*
(9*a^2+a*b-8*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2-40*a*b+8*
b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(190) = 380.

Time = 0.18 (sec) , antiderivative size = 1423, normalized size of antiderivative = 6.84

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/480*(4*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*x + e)^9 - 4*(160*a^4
- 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)^7 + 4*(120*a^4 - 120
5*a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x + e)^5 + 20*(75*a^3
*b - 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 40*a
^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b + 48*a^2*b^2 - 8*a
*b^3)*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2 - 72*a*b^3 + 8*b^
4)*cos(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*a^3*b - 55*a^2*b
^2 + 48*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b
+ 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a
*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*si
n(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f
*x + e) + 60*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(((a^7 + 5*a^6*
b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a
^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6
 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6
+ b^7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4
*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^
2*b^5 + 5*a*b^6 + b^7)*f)*sin(f*x + e)), -1/240*(2*(64*a^4 - 607*a^3*b + 2
74*a^2*b^2)*cos(f*x + e)^9 - 2*(160*a^4 - 1533*a^3*b + 1599*a^2*b^2 - 488*
a*b^3)*cos(f*x + e)^7 + 2*(120*a^4 - 1205*a^3*b + 2769*a^2*b^2 - 1392*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(190) = 380$.

Time = 0.13 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.09

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\,(15\,a^2b-40\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\sqrt{(a+b)b}}+\frac{15\,(15\,a^2b^2-40\,ab^3+8\,b^4)\tan(fx+e)^8+25\,(15\,a^3b-25\,a^2b^2-32\,ab^3+8\,b^4)\tan(fx+e)}{(a^5b^2+5\,a^4b^3+10\,a^3b^4+10\,a^2b^5+5\,ab^6+b^7)\tan(fx+e)^9+2\,(a^6b+6}}{}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt((a +
 b)*b)) + (15*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*tan(f*x + e)^8 + 25*(15*a^3*
b - 25*a^2*b^2 - 32*a*b^3 + 8*b^4)*tan(f*x + e)^6 + 8*(15*a^4 - 10*a^3*b -
 57*a^2*b^2 - 24*a*b^3 + 8*b^4)*tan(f*x + e)^4 + 24*a^4 + 96*a^3*b + 144*a
^2*b^2 + 96*a*b^3 + 24*b^4 + 8*(10*a^4 + 31*a^3*b + 33*a^2*b^2 + 13*a*b^3
+ b^4)*tan(f*x + e)^2)/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5
*a*b^6 + b^7)*tan(f*x + e)^9 + 2*(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*
b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*tan(f*x + e)^7 + (a^7 + 7*a^6*b + 21*a^5
*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*tan(f*x + e)^
5))/f
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.76

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\,(15\,a^2b-40\,ab^2+8\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\sqrt{ab+b^2}}+\frac{15\left(7\,a^2b^2\tan(fx+e)^3-8\,ab^3\tan(fx+e)^3+9\,a^3b\tan(fx+e)+a^2}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5)\left(b\tan(\right.}}{}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2
 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b + b^2)) + 15*(7*a^2*b^2*tan(f*x +
e)^3 - 8*a*b^3*tan(f*x + e)^3 + 9*a^3*b*tan(f*x + e) + a^2*b^2*tan(f*x + e
) - 8*a*b^3*tan(f*x + e))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*
b^4 + b^5)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(15*a^2*tan(f*x + e)^4 - 60*a
*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 5*a*b*
tan(f*x + e)^2 - 5*b^2*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 15.13 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.28

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{1}{5\,(a+b)}+\frac{\tan(e+f\,x)^2\,(10\,a+b)}{15\,(a+b)^2}+\frac{5\tan(e+f\,x)^6\,(15\,a^2\,b-40\,a\,b^2+8\,b^3)}{24\,(a+b)^4}+\frac{\tan(e+f\,x)^4\,(15\,a^2-40\,a\,b+8\,b^2)}{15\,(a+b)^3}+\frac{\tan(e+f\,x)^8\,(15\,a^2\,b^2}{8\,(a+b)}}{f\,\left(\tan\left(e+f\,x\right)^5\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^7\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^9\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5)}{(a+b)^{11/2}}\right)\,\left(15\,a^2-40\,a\,b+8\,b^2\right)}{8\,f\,{(a+b)}^{11/2}}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
- (1/(5*(a + b)) + (tan(e + f*x)^2*(10*a + b))/(15*(a + b)^2) + (5*tan(e +
 f*x)^6*(15*a^2*b - 40*a*b^2 + 8*b^3))/(24*(a + b)^4) + (tan(e + f*x)^4*(1
5*a^2 - 40*a*b + 8*b^2))/(15*(a + b)^3) + (tan(e + f*x)^8*(8*b^4 - 40*a*b^
3 + 15*a^2*b^2))/(8*(a + b)^5))/(f*(tan(e + f*x)^5*(2*a*b + a^2 + b^2) + t
an(e + f*x)^7*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^9)) - (b^(1/2)*atan((b^(1
/2)*tan(e + f*x)*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)
)/(a + b)^(11/2))*(15*a^2 - 40*a*b + 8*b^2))/(8*f*(a + b)^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1865, normalized size of antiderivative = 8.97

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 225*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/
sqrt(b))*sin(e + f*x)**9*a**4 + 600*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*
tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**9*a**3*b - 120*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e +
 f*x)**9*a**2*b**2 + 450*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*
x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**4 - 750*sqrt(b)*sqrt(a + b)*a
tan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**3
*b - 960*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))
/sqrt(b))*sin(e + f*x)**7*a**2*b**2 + 240*sqrt(b)*sqrt(a + b)*atan((sqrt(a
 + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**7*a*b**3 - 225*sq
rt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*s
in(e + f*x)**5*a**4 + 150*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f
*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a**3*b + 855*sqrt(b)*sqrt(a + b
)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a
**2*b**2 + 360*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sq
rt(a))/sqrt(b))*sin(e + f*x)**5*a*b**3 - 120*sqrt(b)*sqrt(a + b)*atan((sqr
t(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*b**4 - 225*s
qrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*
sin(e + f*x)**9*a**4 + 600*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**9*a**3*b - 120*sqrt(b)*sqrt(a...
```

3.67 $\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\begin{aligned}\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx &= \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} \\ &\quad + \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f} \\ &\quad - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af}\end{aligned}$$

output
```
b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f-cos(f*x+e)*
(a+b*sec(f*x+e)^2)^(1/2)/f+2/15*(5*a+b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3
/2)/a^2/f-1/5*cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2)/a/f
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx =$$
$$-\frac{\cos(e+fx)\left(-2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+2\sqrt{b+a\cos^2(e+fx)}-\frac{2(2a+b)(b+a\cos^2(e+fx))^{3/2}}{3a^2}+\frac{2(b+\cdots}{\cdots}\right.}{\sqrt{2}f\sqrt{a+2b+a\cos(2e+2fx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

output
```
-((Cos[e + f*x]*(-2*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[b]] +
2*Sqrt[b + a*Cos[e + f*x]^2] - (2*(2*a + b)*(b + a*Cos[e + f*x]^2)^(3/2))/
(3*a^2) + (2*(b + a*Cos[e + f*x]^2)^(5/2))/(5*a^2))*Sqrt[a + b*Sec[e + f*x
]^2])/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 365, 25, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx)^5\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4622}$$

$$\frac{\int \cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 365}$$

$$\frac{\frac{\int -\cos^4(e+fx)\left(2(5a+b)-5a\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5a}}{f}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \cos^4(e+fx)\left(2(5a+b)-5a\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5a}}{f}$$

$\Big\downarrow$ 358

$$\frac{-\frac{-5a\int \cos^2(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)-\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5a}}{f}$$

$\Big\downarrow$ 247

$$\frac{-\frac{-5a\left(b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5a}}{f}$$

$\Big\downarrow$ 224

$$\frac{-\frac{-5a\left(b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5a}}{f}$$

$\Big\downarrow$ 219

$$\frac{-\frac{-5a\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5a}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

output
```
(-1/5*(Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2))/a - ((-2*(5*a + b)*Cos
[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a) - 5*a*(Sqrt[b]*ArcTanh[(Sq
rt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - Cos[e + f*x]*Sqrt[a + b*
Sec[e + f*x]^2]))/(5*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(123) = 246$.

Time = 5.25 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.35

method	result
default	$\frac{\left(15\sqrt{b}\,\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sec(fx+e)-4b\sec(fx+e)\right)a^2+\left(-3\cos(fx+e)^5-3\cos(fx+e)^4+10\,\text{co}\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/15/f/a^2*(15*b^(1/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*
b*sec(f*x+e))*a^2+(-3*cos(f*x+e)^5-3*cos(f*x+e)^4+10*cos(f*x+e)^3+10*cos(f
*x+e)^2-15*cos(f*x+e)-15)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+
(-cos(f*x+e)^3-cos(f*x+e)^2+10*cos(f*x+e)+10)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a*b+(2*cos(f*x+e)+2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^2)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.20

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$$
$$= \left[\frac{15\,a^2\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\left(3\,a^2\cos\left(fx+e\right)^5-\left(10\,a^2-ab\right)\cos\left(fx+\right.\right.}{30\,a^2f}\right.$$
$$-\frac{15\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a\cos(fx+e)^2+b}\right)+\left(3\,a^2\cos\left(fx+e\right)^5-\left(10\,a^2-ab\right)\cos\left(fx+e\right)^3+(1\right.}{15\,a^2f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
[1/30*(15*a^2*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(3*a^2*c
os(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15*a^2 - 10*a*b - 2*b^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f), -1/15*
(15*a^2*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b)) + (3*a^2*cos(f*x + e)^5 - (10*a^2
- a*b)*cos(f*x + e)^3 + (15*a^2 - 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*sin(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.23

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$$
$$= \frac{\frac{20\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a} - 30\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e) - 15\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right) - \frac{2\,(\cdots}{\cdots}}{30\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
1/30*(20*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 30*sqrt(a + b/cos
(f*x + e)^2)*cos(f*x + e) - 15*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos
(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)))
- 2*(3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5*(a + b/cos(f*x + e)
^2)^(3/2)*b*cos(f*x + e)^3)/a^2)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. 2(123) = 246.

Time = 0.58 (sec) , antiderivative size = 1281, normalized size of antiderivative = 9.22

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
-2/15*(15*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) -
2*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
+ 1/2*e)^2 + a + b))^9*b + 165*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*b - 20*(sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))^7*(16*a^2 + 5*a*b - 27*b^2) + 20*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(32*a^2 - 83*a*b
 + 33*b^2)*sqrt(a + b) + 2*(416*a^3 + 625*a^2*b - 1230*a*b^2 - 15*b^3)*(sq
rt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^
2 + a + b))^5 - 10*(256*a^3 - 391*a^2*b + 90*a*b^2 + 81*b^3)*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)^4*sqrt(a + b) + 20*(16*a^4 - 161*a^3*b + 157*a^2*b^2 + 45*a*b^3 - 33*...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx=\int \sin\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx=\int \sqrt{\sec(fx+e)^2b+a}\,\sin(fx+e)^5\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^5,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**5,x)
```

3.68 $\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx=\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}$$

output
```
b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f-cos(f*x+e)*
(a+b*sec(f*x+e)^2)^(1/2)/f+1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/a/f
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$$
$$=\frac{\sqrt{2}\cos(e+fx)\left(3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+\sqrt{b+a\cos^2(e+fx)}(-3a+b+a\cos^2(e+fx))\right)\sqrt{a\cdots}}{3af\sqrt{a+2b+a\cos(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

output
```
(Sqrt[2]*Cos[e + f*x]*(3*a*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt
[b]] + Sqrt[b + a*Cos[e + f*x]^2]*(-3*a + b + a*Cos[e + f*x]^2))*Sqrt[a +
b*Sec[e + f*x]^2])/(3*a*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4622, 25, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{358}$$

$$\frac{\int \cos^2(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{f}$$

$$\Big\downarrow \text{247}$$

$$\frac{b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$\Big\downarrow$ 224

$$\frac{b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - Cos[
e + f*x]*Sqrt[a + b*Sec[e + f*x]^2] + (Cos[e + f*x]^3*(a + b*Sec[e + f*x]^
2)^(3/2))/(3*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(88) = 176$.

Time = 4.06 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.36

method	result
default	$\frac{\left(3\sqrt{b}\,\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sec(fx+e)-4b\sec(fx+e)\right)a+\left(\cos(fx+e)^3+\cos(fx+e)^2-3\cos(fx+e)\right.}{3fa(1+\cos(fx+e))\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}}$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/f/a*(3*b^(1/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*b*se
c(f*x+e))*a+(cos(f*x+e)^3+cos(f*x+e)^2-3*cos(f*x+e)-3)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a+(1+cos(f*x+e))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.43

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$$
$$=\left[\frac{3\,a\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)+2\left(a\cos\left(fx+e\right)^3-\left(3\,a-b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{6\,af},\right.$$
$$\left.-\frac{3\,a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a\cos(fx+e)^2+b}\right)-\left(a\cos\left(fx+e\right)^3-\left(3\,a-b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\,af}\right]$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
[1/6*(3*a*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(a*cos(f*x +
 e)^3 - (3*a - b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
))/(a*f), -1/3*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b)) - (a*cos(f*x + e)^3 -
(3*a - b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f)
]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*sin(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$$
$$=\frac{\frac{2\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a}-6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-3\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{6f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/6*(2*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 6*sqrt(a + b/cos(f*
x + e)^2)*cos(f*x + e) - 3*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x
 + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$$
$$=-\frac{\left(\frac{3\,b\arctan\left(\frac{\sqrt{a\cos(fx+e)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}a^2-3\sqrt{a\cos(fx+e)^2+b}a^3}{a^3}\right)\operatorname{sgn}(\cos(fx+e))}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/3*(3*b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) - ((a*cos(f
*x + e)^2 + b)^(3/2)*a^2 - 3*sqrt(a*cos(f*x + e)^2 + b)*a^3)/a^3)*sgn(cos(
f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx=\int \sin\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx=\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^3 dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^3,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**3,x)
```

3.69 $\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx=\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

output
```
b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f-cos(f*x+e)*
(a+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{\sqrt{2}\cos(e+fx)\left(-\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+\sqrt{b+a\cos^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

output
```
-((Sqrt[2]*Cos[e + f*x]*(-(Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt
[b]]) + Sqrt[b + a*Cos[e + f*x]^2])*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a
+ 2*b + a*Cos[2*e + 2*f*x]]))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4622, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx)\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4622}$$

$$\frac{\int \cos^2(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 247}$$

$$\frac{b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - Cos[
e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{fa}+\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\sec(fx+e)+\sqrt{a+b\sec(fx+e)^2}\right)}{f}$	93
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{fa}+\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\sec(fx+e)+\sqrt{a+b\sec(fx+e)^2}\right)}{f}$	93

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)+1/f*b/a*sec(f*x+e)*(a+b*sec(f*x
+e)^2)^(1/2)+1/f*b^(1/2)*ln(b^(1/2)*sec(f*x+e)+(a+b*sec(f*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.92

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$$

$$=\left[-\frac{2\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2f},\right.$$
$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a\cos(fx+e)^2+b}\right)+\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{f}\right]$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
[-1/2*(2*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b
)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -(sqrt(-b)*arctan(sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2
+ b)) + sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/f]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\sin{(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*sin(f*x+e),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{2\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{2f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)*log((sqrt(a + b/
cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*
x + e) + sqrt(b))))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{\left(\frac{b\arctan\left(\frac{\sqrt{a\cos(fx+e)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{a\cos\left(fx+e\right)^2+b}\right)\operatorname{sgn}(\cos\left(fx+e\right))}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e),x, algorithm="giac")
```

output
```
-(b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(a*cos(f*x
+ e)^2 + b))*sgn(cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 12.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{f} - \frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{a}\,\cos(e+f\,x)}\right)\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,1\mathrm{i}}{\sqrt{a}\,f\,\sqrt{\frac{b}{a\cos(e+f\,x)^2}+1}}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
- (cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2))/f - (b^(1/2)*asin((b^(1/2)*1
i)/(a^(1/2)*cos(e + f*x)))*(a + b/cos(e + f*x)^2)^(1/2)*1i)/(a^(1/2)*f*(b/
(a*cos(e + f*x)^2) + 1)^(1/2))
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx=\int \sqrt{\sec\left(fx+e\right)^{2}b+a}\,\sin\left(fx+e\right)dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x),x)
```

3.70 $\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

output
```
b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f-(a+b)^(1/2)
*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\sqrt{2}\left(\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right) - \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\right)\cos(e+fx)\sqrt{a+b\sec^2(e+f}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$

input
```
Integrate[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[2]*(Sqrt[b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a
 + b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]])*Cos[e + f*x]*Sq
rt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4622, 25, 301, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{ 4622}$$

$$\frac{\int -\frac{\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 301}$$

$$\frac{b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-(a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-(a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-(a+b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \textbf{291}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-(a+b)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

input
```
Int[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - Sqrt
[a + b]*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(70) = 140$.

Time = 3.98 (sec) , antiderivative size = 554, normalized size of antiderivative = 6.76

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)^2}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)\left(2\sqrt{b}\ln\left(\frac{4b(1-\cos(fx+e))^2\csc(fx+e)^2+8\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4b}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)\sqrt{a+b}-}{}$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(a+b)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)*(2*b^(1/2)*ln(4*(b*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))*(a+
b)^(1/2)-ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a-b*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))-ln(-4*((a+b
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e
)))*a-ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a
+b)/(-1+cos(f*x+e)))*b)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 536, normalized size of antiderivative = 6.54

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \left[\frac{\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)+\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\mathrm{c}}{\cos(fx+e)^2}\right.}{2f}\right.$$
$$\left. -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a\cos(fx+e)^2+b}\right)-\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a\right.}{\cos(fx+e)^2-1}\right.}{2f}\right.$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1))
+ sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, 1/2*(2*sqrt(-a - b)*
arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/(a*cos(f*x + e)^2 + b)) + sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^
2))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b)) - sqrt(a + b)*log(2*(a*cos(
f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/f, (sqrt(-a - b)*arctan(sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x
 + e)^2 + b)) - sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b)))/f]
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\csc{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(70) = 140$.

Time = 0.54 (sec) , antiderivative size = 405, normalized size of antiderivative = 4.94

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\left(\frac{4\,b\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}-\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{a}\right.}{}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(4*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) + sq
rt(a + b)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))) - sqrt(a + b)*log(abs(-sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b) - sqrt(a + b))) - sqrt(a + b)*log(abs((sqrt(a + b)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a
*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - s
qrt(a + b)*(a - b))))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec(fx+e)^2\,b+a}\,\csc(fx+e)\,dx$$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x),x)
```

3.71 $\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{a+b}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

output
```
b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f-1/2*(a+2*b)
*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(1/2)/f-1/
2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)-\sqrt{a+b}(a+2b)\text{arctanh}\left(\frac{\sqrt{a+}}{}\right.\right.}{\sqrt{2}(a+b)f\sqrt{a+2b+a\cos(2(e+fx))}}$$

input
```
Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[b]*(a + b)*ArcTanh[Sqrt[a
 + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a + b]*(a + 2*b)*ArcTanh[Sqrt[a +
 b - a*Sin[e + f*x]^2]/Sqrt[a + b]] - (a + b)*Csc[e + f*x]^2*Sqrt[a + b -
a*Sin[e + f*x]^2]))/(Sqrt[2]*(a + b)*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 369, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \frac{\sec^2(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{369}$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))}-\frac{1}{2}\int \frac{2b\sec^2(e+fx)+a}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$\downarrow$ 398

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-(a+2b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(e}}{2(1-\sec^2(e+fx)}}{f}$$

$\downarrow$ 224

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-(a+2b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\sqrt{a+b\,\mathrm{se}}}{2(1-\sec^2(e+}}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-(a+2b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+f}}{2(1-\sec^2(e+fx))}}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{2}\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-(a+2b)\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))}}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}}\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))}}{f}$$

input
```
Int[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - (
(a + 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/
Sqrt[a + b])/2 + (Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*(1 - Sec[e +
 f*x]^2)))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. $2(106) = 212$.

Time = 4.07 (sec) , antiderivative size = 1244, normalized size of antiderivative = 10.03

method	result	size
default	Expression too large to display	1244

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(a+b)^(5/2)*((4*cos(f*x+e)-4)*b^(1/2)*(a+b)^(3/2)*ln(4*(b*cot(f*x+e
)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+
e)^2-1))*a+(-5*cos(f*x+e)+5)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^2+(-5*cos(f*x+e)
+5)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b
)/(-1+cos(f*x+e)))*a*b^2+(-4*cos(f*x+e)+4)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b+
(-4*cos(f*x+e)+4)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+
cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b+(4*cos(f*x+e)-4)*b^(3/2)*(a+b)^(3/2
)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*
cot(f*x+e)+csc(f*x+e)^2-1))+(1-cos(f*x+e))*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3+(-
2*cos(f*x+e)+2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(106) = 212$.

Time = 0.23 (sec) , antiderivative size = 907, normalized size of antiderivative = 7.31

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 ((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2
- 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(
b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^2 - (a
 + b)*f), 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos
(f*x + e)^2 + b)) + (a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e) + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*log((a*cos(f*x + e)^
2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2
*b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f), -1/4*(4*((a +
 b)*cos(f*x + e)^2 - a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b)) - 2*(a + b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - ((a + 2*b)*cos(f*
x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x
 + e)^2 - 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/2*(((a + 2*b)*cos
(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b)) - 2*((a ...
```

Sympy [F]

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\csc^3(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

Maxima [F]

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(106) = 212$.

Time $= 0.63$ (sec) , antiderivative size $= 578$, normalized size of antiderivative $= 4.66$

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(4*(a + 2*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) - 16*b*
arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/
2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) + 2*(a + 2*b)*
log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f
*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/sqrt(a + b) - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*((sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b)
- (a + b)^(3/2))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)^3 dx$$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**3,x)
```

3.72 $\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\begin{aligned}\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{(3a^2+12ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{3/2}f}\\ &\quad-\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f}\\ &\quad-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}\end{aligned}$$

output
```
b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f-1/8*(3*a^2+
12*a*b+8*b^2)*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a+
b)^(3/2)/f-1/8*(3*a+4*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a
+b)/f-1/4*cot(f*x+e)*csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$=\frac{\cos(e+fx)\left(8\sqrt{b}(a+b)^2\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)-\sqrt{a+b}(3a^2+12ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e-}}{\sqrt{a+b}}\right.\right.}{4\sqrt{2}(a+b)^2f\sqrt{a+2b+a\cos}}$$

input
```
Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*(8*Sqrt[b]*(a + b)^2*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/
Sqrt[b]] - Sqrt[a + b]*(3*a^2 + 12*a*b + 8*b^2)*ArcTanh[Sqrt[a + b - a*Sin
[e + f*x]^2]/Sqrt[a + b]] + ((a + b)*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(-
7*a - 8*b + (3*a + 4*b)*Cos[2*(e + f*x)])*Csc[e + f*x]^4)/(2*Sqrt[2]))*Sqr
t[a + b*Sec[e + f*x]^2])/(4*Sqrt[2]*(a + b)^2*f*Sqrt[a + 2*b + a*Cos[2*(e
+ f*x)]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4622, 25, 369, 440, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^5}dx$$
$$\Big\downarrow \text{4622}$$
$$\frac{\int -\frac{\sec^4(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$

$\downarrow$ 25

$$-\frac{\int \frac{\sec^4(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$

$\downarrow$ 369

$$\frac{\frac{1}{4}\int \frac{\sec^2(e+fx)\left(4b\sec^2(e+fx)+3a\right)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-\frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(1-\sec^2(e+fx))^2}}{f}$$

$\downarrow$ 440

$$\frac{\frac{1}{4}\left(\frac{\int -\frac{8b(a+b)\sec^2(e+fx)+a(3a+4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}+\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}\right)-\frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(1-\sec^2(e+fx))^2}}{f}$$

$\downarrow$ 25

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\int \frac{8b(a+b)\sec^2(e+fx)+a(3a+4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}\right)-\frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(1-\sec^2(e+fx))^2}}{f}$$

$\downarrow$ 398

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-8b(a+b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec}{2(a+b)}\right.}{f}$$

$\downarrow$ 224

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-8b(a+b)\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d}{2(a+b)}\right.}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{(3a^2+12ab+8b^2)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-8\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e}{\sqrt{a+b\sec^2}}\right.}{2(a+b)}\right.}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{(3a^2+12ab+8b^2)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-8\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right.}{2(a+b)}\right.}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\frac{(3a^2+12ab+8b^2)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}}-8\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)}\right)-\frac{\text{s}}{}}{f}$$

input
```
Int[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/4*(Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(1 - Sec[e + f*x]^2)^2 +
 (-1/2*(-8*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e
 + f*x]^2]] + ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])
/Sqrt[a + b*Sec[e + f*x]^2]])/Sqrt[a + b])/(a + b) + ((3*a + 4*b)*Sec[e +
f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a + b)*(1 - Sec[e + f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. $2(161) = 322$.

Time = 4.12 (sec) , antiderivative size = 2007, normalized size of antiderivative = 10.97

method	result	size
default	Expression too large to display	2007

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/(a+b)^(9/2)*((-16*cos(f*x+e)+16)*sin(f*x+e)^2*(a+b)^(5/2)*b^(5/2)*l
n(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot
(f*x+e)+csc(f*x+e)^2-1))+(-32*cos(f*x+e)+32)*sin(f*x+e)^2*(a+b)^(5/2)*b^(3
/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e
)*cot(f*x+e)+csc(f*x+e)^2-1))*a+(-16*cos(f*x+e)+16)*sin(f*x+e)^2*(a+b)^(5/
2)*b^(1/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2
*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*cs
c(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*a^2+(6*cos(f*x+e)^2-10)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*(a+b)^(5/2)+(14*cos(f*x+e)^2-22)*(a+b)^
(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+(8*cos(f*x+e)^2-12)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*(a+b)^(5/2)+(3*cos(f*x+e)-
3)*sin(f*x+e)^2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^5+(21*cos(f*x+e)-21)*sin(f*x+e)
^2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(
f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b+(53*cos(f*x+e)-53)*sin(f*x+e)^2*ln(2/(a+
b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(161) = 322$.

Time = 0.45 (sec) , antiderivative size = 1516, normalized size of antiderivative = 8.28

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^
2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x +
 e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 8*((a^2 + 2*a*b + b^2)*cos(f*x + e
)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*lo
g((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x
 + e)^3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b
 + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b +
 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2
 + 12*a*b + 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b)) + 4*((a^2 + 2*a
*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*
a*b + b^2)*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + ((3*a^2 + 7*a
*b + 4*b^2)*cos(f*x + e)^3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)
^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/1
6*(16*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f...
```

Sympy [F]

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\csc^5{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**5, x)
```

Maxima [F]

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec(fx+e)^2+a}\csc(fx+e)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(161) = 322$.

Time = 0.96 (sec) , antiderivative size = 867, normalized size of antiderivative = 4.74

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*(tan(1/2*f*x + 1/
2*e)^2 + (9*a + 11*b)/(a + b)) + 128*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a
+ b))/sqrt(-b))/sqrt(-b) - 8*(3*a^2 + 12*a*b + 8*b^2)*arctan(-(sqrt(a + b)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) - 4*(3*a^2 + 12*a*b + 8*b^2)*log(a
bs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/(a + b)^(3/2) + 4*(2*(s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))^3*(2*a^2 - 3*b^2) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(3*a^2 + 10*a*b + 7*b^2)
*sqrt(a + b) - 2*(3*a^3 + 3*a^2*b - 2*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + 5...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

Reduce [F]

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^5\,dx$$

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**5,x)
```

3.73 $\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 240

$$\begin{aligned}\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx &= \frac{(5a^3-15a^2b-5ab^2-b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f}\\ &+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\ &-\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f}\\ &-\frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af}\\ &-\frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}\end{aligned}$$

output
```
1/16*(5*a^3-15*a^2*b-5*a*b^2-b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)^(1/2))/a^(5/2)/f+b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+
e)^2)^(1/2))/f-1/16*(a-b)*(5*a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^
2)^(1/2)/a^2/f-1/24*(5*a-b)*cos(f*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(
1/2)/a/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx=\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
```

output
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6, x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4620, 369, 440, 27, 440, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^6\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^6(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{369}$$

$$\frac{\frac{1}{6}\int \frac{\tan^4(e+fx)\left(6b\tan^2(e+fx)+5(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \text{440}$$

$$\frac{\frac{1}{6}\left(\frac{\int \frac{3\tan^2(e+fx)\left(8ab\tan^2(e+fx)+(5a-b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}-\frac{(5a-b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right)-\frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(\frac{3\int \frac{\tan^2(e+fx)\left(8ab\tan^2(e+fx)+(5a-b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}-\frac{(5a-b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right)-\frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$\downarrow$ 440

$$\frac{\frac{1}{6}\left(\frac{3\left(\frac{\int \frac{16a^2b\tan^2(e+fx)+(5a+b)\left(a^2-b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(a-b)(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}\right)}{4a}-\frac{(5a-b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4a(\tan^2(e+fx)+1)^2}\right.}{f}$$

$\downarrow$ 398

$$\frac{\frac{1}{6}\left(\frac{3\left(\frac{16a^2b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(a-b)(5a+b)\tan(e+fx)\sqrt{}}{2a\left(\tan^2(e+f\right.}\right.}{4a}\right.}{f}$$

$\downarrow$ 224

$$\frac{\frac{1}{6}\left(\frac{3\left(\frac{16a^2b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(a-b)(5a+b)}{}\right.}{4a}\right.}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{6}\left(\frac{3\left(\frac{16a^2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(a-b)(5a+b)\tan(e+fx}{2a\left(\tan^2(e\right.}}{4a}\right.}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{6}\left(\frac{3\left(\frac{16a^2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}-\frac{(a-b)(5a+b)\tan(e+fx)\sqrt{a-}}{2a\left(\tan^2(e+fx)\right.}}{4a}\right.}{f}$$

$\downarrow$ 216

$$\frac{\frac{1}{6}\left(\frac{3\left(\frac{16a^2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\frac{\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{2a}-\frac{(a-b)(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}\right.}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
```

output
```
(-1/6*(Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(1 + Tan[e + f*x]^2)
^3 + (-1/4*((5*a - b)*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a*(1
 + Tan[e + f*x]^2)^2) + (3*((-(((16*a^2*b - (5*a + b)*(a^2 - b^2))*ArcTan[
(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a]) + 16*a^2*
Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2
*a) - ((a - b)*(5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a
*(1 + Tan[e + f*x]^2))))/(4*a))/6)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(214) = 428$.

Time = 35.73 (sec) , antiderivative size = 831, normalized size of antiderivative = 3.46

method	result
default	$-\frac{\left(-24\ln\left(\frac{4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)\sqrt{b}\,\sqrt{-a}\,a^2-24\ln\left(-\frac{4\left(\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+}{(1+\cos(fx+}}\right.}{}\right.\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/48/f/a^2/(-a)^(1/2)*(-24*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(1/2)*(-a)^(1/2)*a^2-24*ln(-4*(b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(1/
2)*(-a)^(1/2)*a^2-15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)*a^3+45*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*a^2*b+15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3+(8*cos(f*x+e)^5+8*cos(f*x+e)^4-26*cos(f*x
+e)^3-26*cos(f*x+e)^2+33*cos(f*x+e)+33)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a^2+2*(cos(f*x+e)^3+cos(f*x+e)^2-7*cos(f*x
+e)-7)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a
*b+3*(-1-cos(f*x+e))*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(-a)^(1/2)*b^2)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 1715, normalized size of antiderivative = 7.15

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
[1/384*(96*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32
*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*co
s(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x +
 e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a
^3*f), 1/384*(192*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*c
os(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*co
s(f*x + e)^2 + b^2)*sin(f*x + e))) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*
sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 3
2*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*
b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)
^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)...
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*sin(f*x+e)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\sin\left(fx+e\right)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\sin\left(fx+e\right)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx=\int \sin\left(e+f\,x\right)^6\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \int \sqrt{\sec(fx+e)^2\,b+a}\,\sin(fx+e)^6\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^6,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**6,x)
```

3.74 $\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 181

$$\begin{aligned}\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx &= \frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\ &\quad - \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} \\ &\quad - \frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}\end{aligned}$$

output
```
1/8*(3*a^2-6*a*b-b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2)
)/a^(3/2)/f+b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/f-1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f-1/4*co
s(f*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

output
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4, x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4620, 369, 440, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^4\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{369}$$

$$\frac{\frac{1}{4}\int \frac{\tan^2(e+fx)(4b\tan^2(e+fx)+3(a+b))}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \text{440}$$

$$\frac{\frac{1}{4}\left(\frac{\int \frac{8ab\tan^2(e+fx)+(3a-b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}\right)-\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$\Big\downarrow$ 398

$$\frac{\frac{1}{4}\left(\frac{(3a^2-6ab-b^2)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+8ab\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(}}{2a(\tan^2(e+fx)+1)}\right.}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{4}\left(\frac{(3a^2-6ab-b^2)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+8ab\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{}}{2a(\tan^2(e}\right.}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{(3a^2-6ab-b^2)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+8a\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2}}{2a(\tan^2(e+fx)+1)}\right.}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{1}{4}\left(\frac{(3a^2-6ab-b^2)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+8a\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx}}{2a(\tan^2(e+fx)+1)}\right.}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{4}\left(\frac{\frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}+8a\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}\right)-\tan^3}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

output
```
(-1/4*(Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(1 + Tan[e + f*x]^2)
^2 + ((((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/Sqrt[a] + 8*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b + b*Tan[e + f*x]^2]])/(2*a) - ((3*a - b)*Tan[e + f*x]*Sqrt[a + b
+ b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(159) = 318$.

Time = 21.79 (sec) , antiderivative size = 645, normalized size of antiderivative = 3.56

method	result
default	$\left(4\sqrt{-a}\,\sqrt{b}\,\ln\left(\frac{4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)a+4\sqrt{-a}\,\sqrt{b}\,\ln\left(-\frac{4\left(\sqrt{b}\,\sqrt{\frac{b+a\cos(fx}{(1+\cos(fx}}\right.}{}\right.\right.$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/f/a/(-a)^(1/2)*(4*(-a)^(1/2)*b^(1/2)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a+4*(-a)^(1/2)*b^(1/2)*ln(
-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)
-1))*a+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*a^2-6*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x
+e)*a)*a*b-ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x
+e)*a)*b^2+(2*cos(f*x+e)^3+2*cos(f*x+e)^2-5*cos(f*x+e)-5)*sin(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+(1+cos(f*x+e))*sin(f*x
+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b)*cos(f*x+e)*(
a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 1565, normalized size of antiderivative = 8.65

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
[1/64*(16*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b
^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b
+ 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
 a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (
a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 -
 a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e))/(a^2*f), 1/64*(32*a^2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*
b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*lo
g(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*
b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*
a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b
 + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8...
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\sin^4{(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*sin(f*x+e)**4,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\sin{(fx+e)}^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\sin{(fx+e)}^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sin{(e+f\,x)}^4\,\sqrt{a+\frac{b}{\cos{(e+f\,x)}^2}}\,dx$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^4\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^4,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**4,x)
```

3.75 $\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f+
b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/2*cos(f
*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.51

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx$$
$$= \frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(i\left(-1+e^{2i(e+fx)}\right)+\frac{2e^{2i(e+fx)}\left(2afx-2bfx-i(a-b)\log\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

output
```
(Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f
*x]*(I*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a*f*x - 2*b*
f*x - I*(a - b)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^(
(2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + I*(a - b)*Log[a + a*E
^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*Sqrt[a]*Sqrt[b]*Log[((-(Sq
rt[b]*(-1 + E^((2*I)*(e + f*x)))) + I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
+ E^((2*I)*(e + f*x)))^2])*f)/(2*b*(1 + E^((2*I)*(e + f*x))))]))/(Sqrt[a]*
Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b
*Sec[e + f*x]^2])/(4*Sqrt[2]*E^(I*(e + f*x))*f*Sqrt[a + 2*b + a*Cos[2*e +
2*f*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4620, 369, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \sin(e+fx)^2\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 369$$

$$\frac{\frac{1}{2}\int \frac{2b\tan^2(e+fx)+a+b}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)-\frac{\tan(e+fx)\sqrt{a+\cdots}}{2(\tan^2(e-\cdots}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)-\frac{\tan(e+fx\cdots}{2(t\cdots}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)-\frac{\tan(e+fx)\sqrt{a+b\tan\cdots}}{2(\tan^2(e+fx\cdots}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+\cdots}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{2}\left(\frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}+2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

output
```
((((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/
Sqrt[a] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 - (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*(1 + Tan[e
+ f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(105) = 210$.

Time = 18.80 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.91

method	result
default	$\frac{\left(\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)\sqrt{b}\sqrt{-a}+\ln\left(-\frac{4\left(\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+\right.}{\ldots}\right.\right.}{\ldots}$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e
)*a-a-b)/(sin(f*x+e)+1))*b^(1/2)*(-a)^(1/2)+ln(-4*(b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(1/2)*(-a)^(1/2)+ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-ln(4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b+(-1-cos
(f*x+e))*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(105) = 210$.

Time = 0.40 (sec) , antiderivative size = 1417, normalized size of antiderivative = 11.52

$$\int \sqrt{a + b\sec^2(e + fx)}\sin^2(e + fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
[-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) - sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
 + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) - 4*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)
^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
 8*b^2)/cos(f*x + e)^4))/(a*f), -1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - 8*a*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*(a -
b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*
a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x
+ e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)...
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\sin^2{(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*sin(f*x+e)**2,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^2\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*sin(f*x+e)^2,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**2,x)
```

3.76 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

output
```
a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+b^(1/2)*ar
ctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.59

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{i\left(1+e^{2i(e+fx)}\right)\left(2\sqrt{b}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a}\operatorname{arctanh}\left(\frac{a+2b+ae^{2i(e+fx)}}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\right.}{2\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\,f}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-1/2*I)*(1 + E^((2*I)*(e + f*x)))*(2*Sqrt[b]*ArcTan[(Sqrt[b]*(-1 + E^((2
*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + Sqrt[a]*ArcTanh[(a + 2*b + a*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])] - Sqrt[a]*ArcTanh
[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])])*Sqrt[a + b*Sec[e +
f*x]^2])/(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 4616, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{a\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + S
qrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(67) = 134$.

Time = 17.82 (sec) , antiderivative size = 351, normalized size of antiderivative = 4.44

method	result
default	$\left(\ln\left(-\frac{4\left(\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-\sin(fx+e)a+a+b\right)}{\sin(fx+e)-1}\right)\sqrt{b}\sqrt{-a}+\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+\ldots}{\mathrm{si}\ldots}\right.\right.$

input
```
int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a+a+b)/(sin(f*x+e)-1))*b^(1/2)*(-a)^(1/2)+ln(4*(b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(1/2)*(-a)^(1/2)+2*l
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a)*(a
+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time = 0.30 (sec) , antiderivative size = 1227, normalized size of antiderivative = 15.53

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4))/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos
(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 -
256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3
*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
- (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a
^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*...
```

Sympy [F]

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = \int \sqrt{a + b\sec^2{(e + fx)}}\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 3227, normalized size of antiderivative = 40.85

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(a)*b^(3/2)*arctan2(a*sin(2*f*x + 2*e) + (a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^
2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*
arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), a*cos(2*f*x + 2*e) + (a^2*cos(4*
f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x
+ 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*
a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a
)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos
(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + a + 2*b) + a^(3/2)*sq
rt(b)*arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2
+ 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2
+ 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*...
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec(fx+e)^2+a}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec(fx+e)^2b+a}dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a),x)
```

3.77 $\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

output
```
b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=-\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f}$$

input
```
Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, (b*Sin[e + f*x]^2)/(a + b
- a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4620, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{247}$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] -
Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(60) = 120$.

Time = 17.88 (sec) , antiderivative size = 324, normalized size of antiderivative = 4.76

method	result
default	$\frac{\left(\ln\left(-\frac{4\left(\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-\sin(fx+e)a+a+b\right)}{\sin(fx+e)-1}\right)\sqrt{b}\sin(fx+e)+\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(f}{}\right.}{2f(1+\text{co}}$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(
sin(f*x+e)-1))*b^(1/2)*sin(f*x+e)+ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(1/2)*sin(f*x+e)-2*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)-2*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2))*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(60) = 120$.

Time = 0.13 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.50

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{b}\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)}{4f\sin(fx+e)}\right.$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(
f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
*sin(f*x + e) - 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
)/(f*sin(f*x + e)), 1/2*(sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*
b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b
*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

Sympy [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\csc^2{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\frac{\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)-\frac{\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
(sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - sqrt(b*tan(f*x + e)^2 +
 a + b)/tan(f*x + e))/f
```

Giac [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)^2dx$$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**2,x)
```

3.78 $\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}$$

output
```
b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(3/2)
/(a+b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.57 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.71

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\sqrt{2}\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(\frac{4b\,\text{Hypergeometric2F1}\left(2,2,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{\cdots}\right.}{3f\sqrt{a+2b+a\cdots}}$$

input
```
Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/3*(Sqrt[2]*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*(1 -
(a*Sin[e + f*x]^2)/(a + b))*((4*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*(a
 + b - a*Sin[e + f*x]^2)^2*Tan[e + f*x]^2)/(a + b)^2 + (a + b + 2*a*Sin[e
+ f*x]^2)*(Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)] + ArcSin[Sqrt[-((b*Tan[e +
 f*x]^2)/(a + b))]]*Sqrt[-((b*Tan[e + f*x]^2)/(a + b))])))/(f*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a
*Sin[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4620, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 358

$$\frac{\int \cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{f}$$

$\Big\downarrow$ 247

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\Big\downarrow$ 224

$$\frac{b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] -
Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2] - (Cot[e + f*x]^3*(a + b + b*T
an[e + f*x]^2)^(3/2))/(3*(a + b)))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(93) = 186$.

Time = 18.88 (sec) , antiderivative size = 589, normalized size of antiderivative = 5.61

method	result
default	$\frac{\left((-3\cos(fx+e)+3)\sin(fx+e)b^{\frac{3}{2}}\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)+(-3\cos(fx+ \right.}{}$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f/(a+b)*((-3*cos(f*x+e)+3)*sin(f*x+e)*b^(3/2)*ln(4*(b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+(-3*cos(f*x+e)+3)
*sin(f*x+e)*b^(1/2)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x
+e)*a-a-b)/(sin(f*x+e)+1))*a+(-3*cos(f*x+e)+3)*sin(f*x+e)*b^(3/2)*ln(-4*(b
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))+
(-3*cos(f*x+e)+3)*sin(f*x+e)*b^(1/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a+(4*cos(f*x+e)^2-6)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+(6*cos(f*x+e)^2-8)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(93) = 186$.

Time = 0.27 (sec) , antiderivative size = 436, normalized size of antiderivative = 4.15

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{3\left((a+b)\cos\left(fx+e\right)^2-a-b\right)\sqrt{b}\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(f\right.}{\cos(fx+e)^4}\right.}{12\left((a+b)f\cos\left(f\right.\right.}\right.$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)
*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3
 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x
 + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), 1/6*(3*((a +
 b)*cos(f*x + e)^2 - a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 +
 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*((2*a + 3*b)*cos
(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\csc^4{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{3\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) - \frac{3\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)} - \frac{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan(fx+e)^3}}{3f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 3*sqrt(b*tan(f*x
+ e)^2 + a + b)/tan(f*x + e) - (b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*t
an(f*x + e)^3))/f
```

Giac [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin(e+f\,x)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^4\,dx$$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**4,x)
```

3.79 $\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 149

$$\begin{aligned}\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}\\&\quad-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2f}\\&\quad-\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f}\end{aligned}$$

output
```
b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-2/15*(5*a+4*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+
e)^2)^(3/2)/(a+b)^2/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.94 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.83

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(-\frac{i\left(8a^2\left(1-6e^{2i(e+fx)}+16e^{4i(e+fx)}-6e^{6i(e+fx)}+e^{8i(e+fx)}\right)\cdots\right.}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]*(((-I)*(8*a^2*(1 - 6*E^((2*I)*(e + f*x)) + 16*
E^((4*I)*(e + f*x)) - 6*E^((6*I)*(e + f*x)) + E^((8*I)*(e + f*x))) + b^2*(
15 - 80*E^((2*I)*(e + f*x)) + 178*E^((4*I)*(e + f*x)) - 80*E^((6*I)*(e + f
*x)) + 15*E^((8*I)*(e + f*x))) + a*b*(25 - 136*E^((2*I)*(e + f*x)) + 318*E
^((4*I)*(e + f*x)) - 136*E^((6*I)*(e + f*x)) + 25*E^((8*I)*(e + f*x)))))/(
(a + b)^2*(-1 + E^((2*I)*(e + f*x)))^5) - (15*Sqrt[b]*Log[(-4*Sqrt[b]*(-1
+ E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e +
 f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sqrt[a + b*Sec[e + f*x]^2])/(15*f
*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4620, 365, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int \cot^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}(5(a+b)\tan^2(e+fx)+2(5a+4b))d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{5(a+b)}}{f}$$

$$\Big\downarrow \; 358$$

$$\frac{\frac{5(a+b)\int \cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{5(a+b)}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{5(a+b)}}{f}$$

$$\Big\downarrow \; 247$$

$$\frac{\frac{5(a+b)\left(b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{5(a+b)}-\frac{\cot^5(e+fx)(a+}{5}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{5(a+b)\left(b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{5(a+b)}-\cot^5(}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{5(a+b)\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{5(a+b)}-\cot^5(e+fx)(a}{f}$$

input
```
Int[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(3/2))/(a + b) + ((-2*(5*
a + 4*b)*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(3/2))/(3*(a + b)) + 5*
(a + b)*(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]] - Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]))/(5*(a + b)))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(133) = 266$.

Time = 19.31 (sec) , antiderivative size = 903, normalized size of antiderivative = 6.06

method	result	size
default	Expression too large to display	903

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30/f/(a+b)^2*(sin(f*x+e)^3*(15*cos(f*x+e)-15)*b^(5/2)*ln(4*(b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+sin(f*x+e)
^3*(30*cos(f*x+e)-30)*b^(3/2)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a+sin(f*x+e)^3*(15*cos(f*x+e)-15)*b^(
1/2)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin
(f*x+e)+1))*a^2+sin(f*x+e)^3*(15*cos(f*x+e)-15)*b^(5/2)*ln(-4*(b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))+sin(f*x+e)
^3*(30*cos(f*x+e)-30)*b^(3/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a+sin(f*x+e)^3*(15*cos(f*x+e)-15)*b^
(1/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(s
in(f*x+e)-1))*a^2+(16*cos(f*x+e)^4-40*cos(f*x+e)^2+30)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^2+(50*cos(f*x+e)^4-118*cos(f*x+e)^2+80)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+(30*cos(f*x+e)^4-70*cos(f*x+e)^2+
46)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)*(a+b*sec(f*x+e)^2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(133) = 266$.

Time = 1.12 (sec) , antiderivative size = 656, normalized size of antiderivative = 4.40

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/60*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(
f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x +
 e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((8*a^2 + 25*a*b + 15*b^2)*cos
(f*x + e)^5 - (20*a^2 + 59*a*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b
 + 23*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a
^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^
2 + (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/30*(15*((a^2 + 2*a*b + b^2)*co
s(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*s
qrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin
(f*x + e)))*sin(f*x + e) - 2*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x + e)^5 - (
20*a^2 + 59*a*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*b^2)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b + b^
2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*
b + b^2)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{15\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) - \frac{15\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)} - \frac{10\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan(fx+e)^3} + \frac{2\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}{(a+b)^2\tan(fx+e)^3} - \frac{3\left(b\tan(fx\cdots\right.}{(a+b)\tan\cdots}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(15*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 15*sqrt(b*tan(f
*x + e)^2 + a + b)/tan(f*x + e) - 10*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a
+ b)*tan(f*x + e)^3) + 2*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)^2*tan
(f*x + e)^3) - 3*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*tan(f*x + e)^5)
)/f
```

Giac [F]

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sin\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6, x)
```

Reduce [F]

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)^6dx$$

input

```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output

```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**6,x)
```

3.80 $\int (a+b\sec^2(e+fx))^{3/2}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 196

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \frac{(3a-4b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}$$

output
```
1/2*(3*a-4*b)*b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))
/f+1/2*(3*a-4*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f-1/3*(3*a-4*b)*c
os(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+2/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)
^(5/2)/a/f-1/5*cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(5/2)/a/f
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^5(e+fx)\,dx = \frac{\sqrt{2}\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}\left(-\frac{6b(a+b-a\sin^2(e+fx))^{5/2}}{a}+15\sec^2(e+fx)\,(a+b-a\sin\right.}{15bf(}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

output
```
(Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-6*b*(a + b - a*Sin
[e + f*x]^2)^(5/2))/a + 15*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)^(5/2)
 - 5*(3*a - 4*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b
]] + Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*Sin[e + f*x]^2))))/(15*b*
f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4622, 365, 27, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx)^5\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4622}$$

$$\frac{\int \cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int -5a\cos^4(e+fx)\left(2-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5a}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\int \cos^4(e+fx)\left(2-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5a}}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{(3a-4b)\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{3a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5a}+\frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$$\Big\downarrow \; 247$$

$$\frac{\frac{(3a-4b)\left(3b\int \sqrt{b\sec^2(e+fx)+a}\,d\sec(e+fx)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5a}+\frac{2\cos^3(e+fx)(a+b\sec}{3a}}{f}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{(3a-4b)\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cos^5(e+fx)(a+b\sec}{5a}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{(3a-4b)\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cos^5(e+fx)(}{}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(3a-4b)\left(3b\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cos^5(e+fx)(a+b\sec^2(}{5a}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

output
```
((2*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a) - (Cos[e + f*x]^5*(
a + b*Sec[e + f*x]^2)^(5/2))/(5*a) + ((3*a - 4*b)*(-(Cos[e + f*x]*(a + b*S
ec[e + f*x]^2)^(3/2)) + 3*b*((a*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*
Sec[e + f*x]^2]])/(2*Sqrt[b]) + (Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/
2)))/(3*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(172) = 344$.

Time = 16.86 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.54

method	result
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(60\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sec(fx+e)-4b\sec(fx+e)\right)b^{\frac{5}{2}}a\cos(fx+e)^3-45\ln\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/30/f/a/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(60*ln(-4*b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*b^(5/2)*a*cos(f*x+e)^3-45
*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*b^(3/2)*
a^2*cos(f*x+e)^3+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b*(6*cos(
f*x+e)^8+6*cos(f*x+e)^7-20*cos(f*x+e)^6-20*cos(f*x+e)^5+30*cos(f*x+e)^4+30
*cos(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2*(12*cos(f
*x+e)^6+12*cos(f*x+e)^5-80*cos(f*x+e)^4-80*cos(f*x+e)^3-15*cos(f*x+e)^2-15
*cos(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*(6*cos(f*x+e)
^4+6*cos(f*x+e)^3))
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.85

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \left[-\frac{15\,(3\,a^2-4\,ab)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos(fx+e)^2-2\,\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)+2\,(6\,a^2\cos(}{60\,a}\right.$$

$$-\frac{15\,(3\,a^2-4\,ab)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a\cos(fx+e)^2+b}\right)\cos\left(fx+e\right)+\left(6\,a^2\cos\left(fx+e\right)^6-4\,(5\,a^2-3}{30\,af\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
[-1/60*(15*(3*a^2 - 4*a*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/co
s(f*x + e)^2) + 2*(6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4
 + 2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)), -1/30*(15*(3*a^2 - 4*a*b)*s
qrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)/(a*cos(f*x + e)^2 + b))*cos(f*x + e) + (6*a^2*cos(f*x + e)^6 - 4*(5
*a^2 - 3*a*b)*cos(f*x + e)^4 + 2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2
- 15*a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx =$$

$$-\frac{\frac{12\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5}{a}-40\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3+60\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a\cos(fx+e)-120}{}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/60*(12*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5/a - 40*(a + b/cos(f*
x + e)^2)^(3/2)*cos(f*x + e)^3 + 60*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x +
 e) - 120*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) - 30*sqrt(a + b/cos(f*
x + e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 4
5*a*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(
a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 60*b^(3/2)*log((sqrt(a +
b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(
f*x + e) + sqrt(b))))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1952 vs. $2(172) = 344$.

Time = 1.76 (sec) , antiderivative size = 1952, normalized size of antiderivative = 9.96

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
-1/15*(15*(3*a*b - 4*b^2)*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b
))*sgn(cos(f*x + e))/sqrt(-b) + 30*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(a*b + 2*b^2)*sgn(cos
(f*x + e)) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b))^2*(3*a*b - 2*b^2)*sqrt(a + b)*sgn(cos(f*x + e)
) + (3*a^2*b - 3*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) - (a^2*b
- a*b^2 + 2*b^3)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*
a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))*sqrt(a + b) + a - 3*b)^2 - 4*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan
(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*(2*a*b - b...
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^5(e+fx)\,dx=\int \sin(e+f\,x)^5\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\sin(fx+e)^5\,dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^5\,dx\right)a$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*sin(e + f*x)**5,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**5,x)*a
```

3.81 $\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{(3a-2b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{3af} + \frac{\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{5/2}}{3af}$$

output
```
1/2*(3*a-2*b)*b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))
/f+1/2*(3*a-2*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f-1/3*(3*a-2*b)*c
os(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)
^(5/2)/a/f
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx=\frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(3\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)^{5/2}-(3a-2b)\right.}{3bf(a+2b+a\cos(}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

output
```
(Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*(3*Sec[e + f*x]^2*(a
+ b - a*Sin[e + f*x]^2)^(5/2) - (3*a - 2*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b
 - a*Sin[e + f*x]^2]/Sqrt[b]] + Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b -
a*Sin[e + f*x]^2))))/(3*b*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4622, 25, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \; 4622$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 359

$$\frac{\frac{(3a-2b)\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{3a}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$\Big\downarrow$ 247

$$\frac{\frac{(3a-2b)\left(3b\int \sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$\Big\downarrow$ 211

$$\frac{\frac{(3a-2b)\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}+\frac{\cos^3(e+fx)\left(a+b\sec^2\right.}{3a}}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{(3a-2b)\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}+\frac{\cos^3(e+fx)\left(\right.}{}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{(3a-2b)\left(3b\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}+\frac{\cos^3(e+fx)\left(a+b\sec^2\right.}{3a}}{f}$$

input

```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

output

```
((Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a) + ((3*a - 2*b)*(-(Cos
[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2)) + 3*b*((a*ArcTanh[(Sqrt[b]*Sec[e +
 f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2*Sqrt[b]) + (Sec[e + f*x]*Sqrt[a + b
*Sec[e + f*x]^2])/2)))/(3*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(142) = 284$.

Time = 9.30 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.46

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(-6b^{\frac{5}{2}}\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sec(fx+e)-4b\sec(fx+e)\right)\cos(fx+e)^3+9b^{\frac{3}{2}}\ln\left(\right.\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6/f/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(-6*b^(5/2)*ln(-4*b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*cos(f*x+e)^3+9*b^(3/2
)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*a*cos(f
*x+e)^3+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*(2*cos(f*x+e)^6+2*
cos(f*x+e)^5-6*cos(f*x+e)^4-6*cos(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^2*(8*cos(f*x+e)^4+8*cos(f*x+e)^3+3*cos(f*x+e)^2+3*cos(f*x+
e)))
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.78

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \left[-\frac{3\,(3\,a-2\,b)\sqrt{b}\cos(fx+e)\log\left(\frac{a\cos(fx+e)^2-2\,\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\,(2\,a\cos(fx+\dots}{12\,f\cos(fx+e)}\right.$$
$$-\frac{3\,(3\,a-2\,b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a\cos(fx+e)^2+b}\right)\cos(fx+e)-(2\,a\cos(fx+e)^4-2\,(3\,a-4\,b)\cos\dots}{6\,f\cos(fx+e)}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
[-1/12*(3*(3*a - 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*sqrt(
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x
 + e)^2) - 2*(2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)^2 + 3*b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/6*(3*(3*a -
 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)/(a*cos(f*x + e)^2 + b))*cos(f*x + e) - (2*a*cos(f*x + e)^4 -
 2*(3*a - 4*b)*cos(f*x + e)^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.54

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx = \frac{4\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-12\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a\cos(fx+e)+12\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(\ldots}{\ldots}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/12*(4*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 12*sqrt(a + b/cos(f*
x + e)^2)*a*cos(f*x + e) + 12*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) +
6*sqrt(a + b/cos(f*x + e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(
f*x + e)^2 - b) - 9*a*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)
 - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) + 6*b^(3/
2)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos
(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx =$$

$$-\frac{a\left(\frac{3\,(3\,ab-2\,b^2)\arctan\left(\frac{\sqrt{a\cos(fx+e)^2+b}}{\sqrt{-b}}\right)}{a\sqrt{-b}}-\frac{3\,\sqrt{a\cos(fx+e)^2+b}\,b}{a\cos(fx+e)^2}-\frac{2\left(\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}a^2-3\,\sqrt{a\cos(fx+e)^2+b}\,a^3+3\,\sqrt{a\cos(fx+\ldots}{a^3}\ldots\right)}{6\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/6*a*(3*(3*a*b - 2*b^2)*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/(a*s
qrt(-b)) - 3*sqrt(a*cos(f*x + e)^2 + b)*b/(a*cos(f*x + e)^2) - 2*((a*cos(f
*x + e)^2 + b)^(3/2)*a^2 - 3*sqrt(a*cos(f*x + e)^2 + b)*a^3 + 3*sqrt(a*cos
(f*x + e)^2 + b)*a^2*b)/a^3)*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx=\int \sin(e+f\,x)^3\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx=\left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\sin(fx+e)^3\,dx\right)b+\left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^3\,dx\right)a$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*sin(e + f*x)**3,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**3,x)*a
```

3.82 $\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}$$

output
```
3/2*a*b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f+3/2*b
*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)
/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx = -\frac{a\cos(e+fx)(a+2b+a\cos(2(e+fx)))^2\,\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1+\frac{a\cos^2(e+fx)}{b}\right)\sqrt{a+b\sec^2(e+}}{20b^2f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

output
```
-1/20*(a*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Hypergeometric2F1[2
, 5/2, 7/2, 1 + (a*Cos[e + f*x]^2)/b]*Sqrt[a + b*Sec[e + f*x]^2])/(b^2*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4622, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{247}$$

$$\frac{3b\int \sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

output
```
(-(Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2)) + 3*b*((a*ArcTanh[(Sqrt[b]*S
ec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2*Sqrt[b]) + (Sec[e + f*x]*Sqrt
[a + b*Sec[e + f*x]^2])/2))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.21

method	result
derivativedivides	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa}+\frac{3b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\,\mathrm{se}\right.}{}$
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa}+\frac{3b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\,\mathrm{se}\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(5/2)+1/f*b/a*sec(f*x+e)*(a+b*sec(f*x
+e)^2)^(3/2)+3/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f+3/2/f*b^(1/2)*a*l
n(b^(1/2)*sec(f*x+e)+(a+b*sec(f*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.42

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = \left[\frac{3\,a\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos(fx+e)^2+2\,\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\left(2\,a\cos\left(fx+e\right)^2-b\right)\sqrt{\cdots}}{4\,f\cos\left(fx+e\right)}, -\frac{3\,a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a\cos(fx+e)^2+b}\right)\cos\left(fx+e\right)+\left(2\,a\cos\left(fx+e\right)^2-b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\,f\cos\left(fx+e\right)}\right]$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
[1/4*(3*a*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2
*(2*a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*
cos(f*x + e)), -1/2*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b))*cos(f*x + e) + (2
*a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos
(f*x + e))]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sin\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.42

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = -\frac{4\sqrt{a+\frac{b}{\cos(fx+e)^2}}a\cos(fx+e)-\frac{2\sqrt{a+\frac{b}{\cos(fx+e)^2}}ab\cos(fx+e)}{\left(a+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2-b}+3a\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{4f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x + e) - 2*sqrt(a + b/cos(f*x +
 e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 3*a*
sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a +
b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = -\frac{\left(\frac{3b\arctan\left(\frac{\sqrt{a\cos(fx+e)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\sqrt{a\cos(fx+e)^2+b}-\frac{\sqrt{a\cos(fx+e)^2+b}b}{a\cos(fx+e)^2}\right)a\operatorname{sgn}(\cos(fx+e))}{2f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="giac")
```

output
```
-1/2*(3*b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(a*
cos(f*x + e)^2 + b) - sqrt(a*cos(f*x + e)^2 + b)*b/(a*cos(f*x + e)^2))*a*s
gn(cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.61

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = -\frac{\cos\left(e+f\,x\right)\,\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b}{a\,{\cos\left(e+f\,x\right)}^2}\right)}{f\,{\left(\frac{b}{a\,{\cos\left(e+f\,x\right)}^2}+1\right)}^{3/2}}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
-(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -
b/(a*cos(e + f*x)^2)))/(f*(b/(a*cos(e + f*x)^2) + 1)^(3/2))
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = \left(\int \sqrt{\sec\left(fx+e\right)^{2} b+a}\,\sec\left(fx+e\right)^{2}\sin\left(fx+e\right)dx\right)b + \left(\int \sqrt{\sec\left(fx+e\right)^{2} b+a}\,\sin\left(fx+e\right)dx\right)a$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*sin(e + f*x),x)*b + int(sq
rt(sec(e + f*x)**2*b + a)*sin(e + f*x),x)*a
```

3.83 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

output
```
1/2*b^(1/2)*(3*a+2*b)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))
/f-(a+b)^(3/2)*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f+
1/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.40

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\left(2\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)\cos^2(e+fx) - 4(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+}}{\sqrt{a+b}}\right.\right.}{2\sqrt{2}f\sqrt{a+2b+a}}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((2*Sqrt[b]*(3*a + 2*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Co
s[e + f*x]^2 - 4*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt
[a + b]]*Cos[e + f*x]^2 + Sqrt[2]*b*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])*Se
c[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2
*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4622, 25, 318, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\frac{\left(b\sec^2(e+fx)+a\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(b\sec^2(e+fx)+a\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{1}{2}\int -\frac{b(3a+2b)\sec^2(e+fx)+a(2a+b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{1}{2}b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}-\frac{1}{2}\int\frac{b(3a+2b)\sec^2(e+fx)+a(2a+b)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{2}\left(b(3a+2b)\int\frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-2(a+b)^2\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{1}{2}b\sec(e}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left(b(3a+2b)\int\frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-2(a+b)^2\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{1}{2}b\sec}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-2(a+b)^2\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{1}{2}b\sec(e+f}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-2(a+b)^2\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\right)+\frac{1}{2}b\sec(e+fx)\sqrt{a+}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)\right)+\frac{1}{2}b\sec(e+fx)\sqrt{a+b\sec^2(e}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*
x]^2]] - 2*(a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec
[e + f*x]^2]])/2 + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1465 vs. $2(104) = 208$.

Time = 8.94 (sec) , antiderivative size = 1466, normalized size of antiderivative = 12.02

method	result	size
default	Expression too large to display	1466

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/b/(a+b)^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(2*b^(5/2)*(
a+b)^(5/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2
*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*cs
c(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*cos(f*x+e)^3+3*b^(3/2)*(a+b)^(5/2)*ln
(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(
f*x+e)+csc(f*x+e)^2-1))*a*cos(f*x+e)^3+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(5/2)*b^2*(cos(f*x+e)^2+cos(f*x+e))-ln(-4*((a+b)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^4*b*co
s(f*x+e)^3-4*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f
*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*b^2*cos(f*x+e)^3-6*ln(-4*((a+b)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b^3*
cos(f*x+e)^3-4*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos
(f*x+e)*a+b)/(-1+cos(f*x+e)))*a*b^4*cos(f*x+e)^3-ln(-4*((a+b)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(...
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 755, normalized size of antiderivative = 6.19

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(
f*x + e)^2 - 1)) + (3*a + 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2
+ 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b
)/cos(f*x + e)^2) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*co
s(f*x + e)), 1/4*(4*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b))*cos(f*
x + e) + (3*a + 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x
+ e)^2) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)
), -1/2*((3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b))*cos(f*x + e) - (a + b
)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1
)) - b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), 1/2*
(2*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b))*cos(f*x + e) - (3*a + 2
*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*c
os(f*x + e)/(a*cos(f*x + e)^2 + b))*cos(f*x + e) + b*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\csc\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int\sqrt{\sec\left(fx+e\right)^{2}b+a}\,\csc\left(fx+e\right)\sec\left(fx+e\right)^{2}dx\right)b+\left(\int\sqrt{\sec\left(fx+e\right)^{2}b+a}\,\csc\left(fx+e\right)dx\right)a$$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)*sec(e + f*x)**2,x)*b + int(sq
rt(sec(e + f*x)**2*b + a)*csc(e + f*x),x)*a
```

3.84 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{b}(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}(a+4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

output
```
1/2*b^(1/2)*(3*a+4*b)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))
/f-1/2*(a+b)^(1/2)*(a+4*b)*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^
2)^(1/2))/f+b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f-1/2*cot(f*x+e)*csc(f*x
+e)*(a+b*sec(f*x+e)^2)^(3/2)/f
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{a+2b+a\cos(2(e+fx))}(a+(a+2b)\cos(2(e+fx$$

$$4\sqrt{2}f\sqrt{}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*(Csc[e + f*x]^2*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*Sqrt
[a + 2*b + a*Cos[2*(e + f*x)]]*(a + (a + 2*b)*Cos[2*(e + f*x)]) - Sqrt[b]*
(3*a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Sin[2*(e + f*x
)]^2 + Sqrt[a + b]*(a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a
 + b]]*Sin[2*(e + f*x)]^2))/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4622, 369, 403, 27, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \frac{\sec^2(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 369

$$\frac{\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2(1-\sec^2(e+fx))}-\frac{1}{2}\int\frac{\sqrt{b\sec^2(e+fx)+a}\,(4b\sec^2(e+fx)+a)}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 403

$$\frac{\frac{1}{2}\left(\frac{1}{2}\int-\frac{2\left(b(3a+4b)\sec^2(e+fx)+a(a+2b)\right)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+2b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)+\frac{\sec(e+fx)\left(a+b\sec^2(e+fx\right.}{2(1-\sec^2(e+fx))}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{2}\left(2b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}-\int\frac{b(3a+4b)\sec^2(e+fx)+a(a+2b)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/}}{2(1-\sec^2(e+fx))}}{f}$$

$\Big\downarrow$ 398

$$\frac{\frac{1}{2}\left(b(3a+4b)\int\frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-(a+b)(a+4b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+2b\,\text{se}\right.}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{2}\left(-(a+b)(a+4b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+b(3a+4b)\int\frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\right.}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{2}\left(-(a+b)(a+4b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\sqrt{b}(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+2b\sec\right.}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{1}{2}\left(-(a+b)(a+4b)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\sqrt{b}(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+2b\sec(e+fx)\right.}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-\sqrt{a+b}(a+4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+2b\sec(e+fx)\sqrt{a+b\sec}\right.}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(2*(1 - Sec[e + f*x]^2)) + (S
qrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]] - Sqrt[a + b]*(a + 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*
Sec[e + f*x]^2]] + 2*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2036 vs. $2(139) = 278$.

Time = 9.22 (sec) , antiderivative size = 2037, normalized size of antiderivative = 12.65

method	result	size
default	Expression too large to display	2037

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(a+b)^(7/2)/b*(b^(7/2)*(a+b)^(5/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x
+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*(8*cos
(f*x+e)^3-8*cos(f*x+e)^2)+b^(5/2)*(a+b)^(5/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot
(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*a*
(14*cos(f*x+e)^3-14*cos(f*x+e)^2)+b^(3/2)*(a+b)^(5/2)*ln(4*(b*cot(f*x+e)^2
-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^
2-1))*a^2*(6*cos(f*x+e)^3-6*cos(f*x+e)^2)+2*cos(f*x+e)^2*(a+b)^(5/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+(6*cos(f*x+e)^2-2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(5/2)*a*b^2+(4*cos(f*x+e)^2-2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(5/2)*b^3+cos(f*x+e)^2*(1-c
os(f*x+e))*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^5*b+cos(f*x+e)^2*(-8*cos(f*x+e)+8)*l
n(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+
e)*a+b)/(1+cos(f*x+e)))*a^4*b^2+cos(f*x+e)^2*(-22*cos(f*x+e)+22)*ln(2/(a+b
)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*...
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1024, normalized size of antiderivative = 6.36

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + 4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(
2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + ((3*a + 4*b)*cos(f*x
 + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt
(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*
x + e)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/4*(2*((a + 4*b)*cos
(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 +
b)) + ((3*a + 4*b)*cos(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log(
(a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x +
e)), -1/4*(2*((3*a + 4*b)*cos(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(
-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/(a*cos(f*x + e)^2 + b)) - ((a + 4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x
 + e))*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1))
 - 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/2*(((a + 4*b)*cos(f*x + e...
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{16384,[5,4]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0
,1]%%%}]%
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^3\sec(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^3\,dx\right)a$$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**3*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**3,x)*a
```

3.85 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{3\sqrt{b}(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{3(a^2+8ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8\sqrt{a+b}f} + \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f}$$

output
```
3/2*b^(1/2)*(a+2*b)*arctanh(b^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/f
-3/8*(a^2+8*a*b+8*b^2)*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(
1/2))/(a+b)^(1/2)/f+3/8*(a+4*b)*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f-3/8*
(a+2*b)*csc(f*x+e)^2*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f-1/4*cot(f*x+e)*
csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/f
```

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.20

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{(b+a\cos^2(e+fx))\left(-12b^{3/2}(a^2+3ab+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)\cos^2(e+fx)+3b\sqrt{a+b}(a^2+\right.}{}$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*((b + a*Cos[e + f*x]^2)*(-12*b^(3/2)*(a^2 + 3*a*b + 2*b^2)*ArcTanh[Sq
rt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos[e + f*x]^2 + 3*b*Sqrt[a + b]*(a^
2 + 8*a*b + 8*b^2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Cos
[e + f*x]^2 + (b*(a + b)*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(11*a + 4*b +
8*(a + 3*b)*Cos[2*(e + f*x)] - 3*(a + 4*b)*Cos[4*(e + f*x)])*Csc[e + f*x]^
4)/(8*Sqrt[2]))*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b*(a + b
)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4622, 25, 369, 27, 439, 25, 444, 27, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 369$$

$$\frac{\frac{1}{4}\int \frac{3\sec^2(e+fx)\sqrt{b\sec^2(e+fx)+a}\left(2b\sec^2(e+fx)+a\right)}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)-\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{4}\int \frac{\sec^2(e+fx)\sqrt{b\sec^2(e+fx)+a}\left(2b\sec^2(e+fx)+a\right)}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)-\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \; 439$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int -\frac{\sec^2(e+fx)\left(2b(a+4b)\sec^2(e+fx)+a(a+6b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{(a+2b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)}\right)-\frac{\sec^3(e+fx)\left(a+b\sec^2(\ldots\right.}{4\left(1-\sec^2(e+f\ldots\right.}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3}{4}\left(\frac{(a+2b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)}-\frac{1}{2}\int \frac{\sec^2(e+fx)\left(2b(a+4b)\sec^2(e+fx)+a(a+6b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)-\frac{\sec^3(e+fx)\left(a+b\sec^2(e+\ldots\right.}{4\left(1-\sec^2(e+fx)\right)\ldots}}{f}$$

$$\Big\downarrow \; 444$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left((a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}-\frac{\int \frac{2b\left(4b(a+2b)\sec^2(e+fx)+a(a+4b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2b}\right)+\frac{(a+2b)\sec^3(e+fx)\sqrt{a+b\ldots}}{2\left(1-\sec^2(e+fx\ldots\right.}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left((a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}-\int \frac{4b(a+2b)\sec^2(e+fx)+a(a+4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{(a+2b)\sec^3(e+fx)\ldots}{2\left(1-\sec^2(\ldots\right.}\right.}{f}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\left(a^2+8ab+8b^2\right)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+4b(a+2b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right.\right.}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\left(a^2+8ab+8b^2\right)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+4b(a+2b)\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)}}\right.\right.}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\left(a^2+8ab+8b^2\right)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+4\sqrt{b}(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+(\right.\right.}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\left(a^2+8ab+8b^2\right)\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+4\sqrt{b}(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+(a+4b)\,s\right.\right.}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{(a^2+8ab+8b^2)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}}+4\sqrt{b}(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+(a+4b)\sec(e+fx)\sqrt{a}\right.\right.}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/4*(Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/(1 - Sec[e + f*x]^2)^2
 + (3*(((a + 2*b)*Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(2*(1 - Sec[e
 + f*x]^2)) + (4*Sqrt[b]*(a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a +
 b*Sec[e + f*x]^2]] - ((a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e +
f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/Sqrt[a + b] + (a + 4*b)*Sec[e + f*x]*Sq
rt[a + b*Sec[e + f*x]^2])/2))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2889 vs. $2(190) = 380$.

Time = 9.27 (sec) , antiderivative size = 2890, normalized size of antiderivative = 13.26

method	result	size
default	Expression too large to display	2890

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/b/(a+b)^(11/2)*(cos(f*x+e)^2*(-48*cos(f*x+e)+48)*sin(f*x+e)^2*b^(9/
2)*(a+b)^(7/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)
^2+2*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-
2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))+cos(f*x+e)^2*(-120*cos(f*x+e)+120
)*sin(f*x+e)^2*b^(7/2)*(a+b)^(7/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc
(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*a+cos(f*x+e)^
2*(-96*cos(f*x+e)+96)*sin(f*x+e)^2*b^(5/2)*(a+b)^(7/2)*ln(4*(b*cot(f*x+e)^
2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)
^2-1))*a^2+cos(f*x+e)^2*(-24*cos(f*x+e)+24)*sin(f*x+e)^2*b^(3/2)*(a+b)^(7/
2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)
*cot(f*x+e)+csc(f*x+e)^2-1))*a^3+cos(f*x+e)^2*(6*cos(f*x+e)^2-10)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^3*b+(36*cos(f*x+e)^4-56*
cos(f*x+e)^2+8)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^
2*b^2+(54*cos(f*x+e)^4-82*cos(f*x+e)^2+16)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(7/2)*a*b^3+(24*cos(f*x+e)^4-36*cos(f*x+e)^2+8)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^4+cos(f*x+e)^2*(3*cos(f
*x+e)-3)*sin(f*x+e)^2*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x...
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1551, normalized size of antiderivative = 7.11

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*c
os(f*x + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*c
os(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + 3*a*b + 2*b^2)*
cos(f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2
*b^2)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3
*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x
+ e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)
), 1/8*(3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*
cos(f*x + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(-a - b)*arctan(s
qrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*co
s(f*x + e)^2 + b)) + 6*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*
a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(b)*
log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x
 + e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a +
b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)), -1/16*(24*((a^2 + 3*a*b ...
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{262144,[6,6]%%%},[12]%%%}+%%%{%%{[%%%{1572864,[6,6]%%%
},0]:[1,0
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

Reduce [F]

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^5\sec(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^5\,dx\right)a$$

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**5*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**5,x)*a
```

3.86 $\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 298

$$\begin{aligned}
&\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e \\
&+fx)\,dx=\frac{(5a^3-45a^2b+15ab^2+b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} \\
&+\frac{(3a-5b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} \\
&-\frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} \\
&+\frac{(5a^2-40ab+3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af} \\
&+\frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} \\
&-\frac{\cos(e+fx)\sin^5(e+fx)\,(a+b+b\tan^2(e+fx))^{3/2}}{6f}
\end{aligned}$$

output
```
1/16*(5*a^3-45*a^2*b+15*a*b^2+b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*
x+e)^2)^(1/2))/a^(3/2)/f+1/2*(3*a-5*b)*b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/
(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/16*(5*a^2-26*a*b+b^2)*tan(f*x+e)*(a+b+b*ta
n(f*x+e)^2)^(1/2)/a/f+1/48*(5*a^2-40*a*b+3*b^2)*sin(f*x+e)^2*tan(f*x+e)*(a
+b+b*tan(f*x+e)^2)^(1/2)/a/f+1/24*(5*a-3*b)*sin(f*x+e)^4*tan(f*x+e)*(a+b+b
*tan(f*x+e)^2)^(1/2)/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3
/2)/f
```

Mathematica [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

output
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6, x]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4620, 369, 439, 440, 27, 444, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$\downarrow$ 369

$$\frac{\frac{1}{6}\int \frac{\tan^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}\left(8b\tan^2(e+fx)+5(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$\downarrow$ 439

$$\frac{\frac{1}{6}\left(\frac{(5a-3b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{1}{4}\int \frac{\tan^4(e+fx)\left(2(5a-19b)b\tan^2(e+fx)+5(a-7b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)-\frac{\tan^5(e+fx)}{6(}}{f}$$

$\downarrow$ 440

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{3\tan^2(e+fx)\left(2b\left(5a^2-26ba+b^2\right)\tan^2(e+fx)+(a+b)\left(5a^2-40ba+3b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx}{2a}\right.\right.}{f}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3\int \frac{\tan^2(e+fx)\left(2b\left(5a^2-26ba+b^2\right)\tan^2(e+fx)+(a+b)\left(5a^2-40ba+3b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx}{2a}\right.\right.}{f}$$

$\downarrow$ 444

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3\left(\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{\int \frac{2b\left(8a(3a-5b)b\tan^2(e+f}{\left(\tan^2(e+fx)+1\right)\sqrt{}}}{}\right.}{2a}\right.\right.}{f}$$

$\downarrow$ 27

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(5a^2-40ab+3b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}-\frac{3\left((5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\int\frac{8a(3a-5b)b\tan^2(e+fx)+}{\left(\tan^2(e+fx)+1\right)\sqrt{b}}\right.}{2a}\right.\right.$$
$$f$$

$\downarrow$ 398

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(5a^2-40ab+3b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}-\frac{3\left(-(5a^3-45a^2b+15ab^2+b^3)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right.}{}\right.\right.$$

$\downarrow$ 224

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(5a^2-40ab+3b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}-\frac{3\left(-(5a^3-45a^2b+15ab^2+b^3)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right.}{}\right.\right.$$

$\downarrow$ 219

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(5a^2-40ab+3b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}-\frac{3\left(-(5a^3-45a^2b+15ab^2+b^3)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right.}{}\right.\right.$$

$\downarrow$ 291

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(5a^2-40ab+3b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}-\frac{3\left(-(5a^3-45a^2b+15ab^2+b^3)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+(5a\right.}{}\right.\right.$$

$\downarrow$ 216

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{(5a^2-40ab+3b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}-\frac{3\left((5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{(5a^3-45a^2b+15ab^2+b^3)\,\mathrm{a}}{}\right.}{2\mathrm{c}}\right.\right.$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

output
```
(-1/6*(Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(3/2))/(1 + Tan[e + f*x]^
2)^3 + (((5*a - 3*b)*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*(1
+ Tan[e + f*x]^2)^2) + (((5*a^2 - 40*a*b + 3*b^2)*Tan[e + f*x]^3*Sqrt[a +
b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)) - (3*(-(((5*a^3 - 45*a^2
*b + 15*a*b^2 + b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/Sqrt[a]) - 8*a*(3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b + b*Tan[e + f*x]^2]] + (5*a^2 - 26*a*b + b^2)*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2]))/(2*a))/4)/6)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1171 vs. $2(266) = 532$.

Time = 37.06 (sec) , antiderivative size = 1172, normalized size of antiderivative = 3.93

method	result	size
default	Expression too large to display	1172

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/48/f/(-a)^(1/2)/b/a*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(60*ln(4*
(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e
)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1
))*b^(5/2)*(-a)^(1/2)*a*cos(f*x+e)^3-36*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*b^(3/2)*(-a)^(1/2)*a^2*cos
(f*x+e)^3+60*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
+a+b)/(sin(f*x+e)+1))*b^(5/2)*(-a)^(1/2)*a*cos(f*x+e)^3-36*ln(-4*(-b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*b^(3/2
)*(-a)^(1/2)*a^2*cos(f*x+e)^3-15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b*cos(f*x+e)^3+135*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2*cos(f*x+e)^3-45*ln
(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3*
cos(f*x+e)^3-3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1...
```

Fricas [A] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 1855, normalized size of antiderivative = 6.22

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
[-1/384*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(-a)*cos(f*x + e)*log(1
28*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*
a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3
 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3
*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 48*(3*a
^3 - 5*a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8
*b^2)/cos(f*x + e)^4) + 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos
(f*x + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)
), 1/384*(96*(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 3*(5*a^3 - 45*a
^2*b + 15*a*b^2 + b^3)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 -
256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3
*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*...
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\int \sin\left(e+f\,x\right)^6\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^2\sin\left(fx+e\right)^6 dx\right)b+\left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^6 dx\right)a$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*sin(e + f*x)**6,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**6,x)*a
```

3.87 $\int (a+b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{3(a^2-6ab+b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a-b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} - \frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

output
```
3/8*(a^2-6*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
a^(1/2)/f+3/2*(a-b)*b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2
)^(1/2))/f-3/8*(a-3*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f+3/8*(a-b)*s
in(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/4*cos(f*x+e)*sin(f*x
+e)^3*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.04

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\frac{3\left(\left(a^2-6ab+b^2\right)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)+4\sqrt{a}(a-b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\right)\cos(e}{2\sqrt{2}\sqrt{a}f(a+2b+a\cos(2(e+fx)))^{3/2}}$$
$$+\frac{(-7a+26b+(-6a+10b)\cos(2(e+fx))+a\cos(4(e+fx)))\sqrt{a+b\sec^2(e+fx)}\tan(e+fx)}{32f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

output
```
(3*((a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
 + f*x]^2]] + 4*Sqrt[a]*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqr
t[a + b - a*Sin[e + f*x]^2]])*Cos[e + f*x]*(b + a*Cos[e + f*x]^2)*Sqrt[a +
 b*Sec[e + f*x]^2])/(2*Sqrt[2]*Sqrt[a]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3
/2)) + ((-7*a + 26*b + (-6*a + 10*b)*Cos[2*(e + f*x)] + a*Cos[4*(e + f*x)]
)*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(32*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 4620, 369, 27, 439, 444, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$
$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 369

$$\frac{\frac{1}{4}\int \frac{3\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}\left(2b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)-\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{3}{4}\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}\left(2b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)-\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$\Big\downarrow$ 439

$$\frac{\frac{3}{4}\left(\frac{(a-b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}-\frac{1}{2}\int \frac{\tan^2(e+fx)\left(2(a-3b)b\tan^2(e+fx)+(a-5b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)-\frac{\tan^3(e+fx)(a+b}{4(\tan^2(}}{f}$$

$\Big\downarrow$ 444

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int \frac{2b\left(4(a-b)b\tan^2(e+fx)+(a-3b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}-(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{(a-b)\tan^3(e+}{2(\tan}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\int \frac{4(a-b)b\tan^2(e+fx)+(a-3b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{(a-b)\tan}{}}{f}$$

$\Big\downarrow$ 398

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-6ab+b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+4b(a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx\right.\right.}{}$$

$\Big\downarrow$ 224

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-6ab+b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+4b(a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+f}{\sqrt{b\tan^2(e+f}}\right.\right.}{}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-6ab+b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+4\sqrt{b}(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\right.\right.$$

$$\Big\downarrow \text{291}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-6ab+b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+4\sqrt{b}(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a-3b\right.\right.$$

$$f$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\left(a^2-6ab+b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}+4\sqrt{b}(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a-3b)\tan(e+fx)\sqrt{a-}\right.\right.$$

$$f$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

output
```
(-1/4*(Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(3/2))/(1 + Tan[e + f*x]^
2)^2 + (3*(((a - b)*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*(1 +
 Tan[e + f*x]^2)) + (((a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sq
rt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a] + 4*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] - (a - 3*b)*Tan[e + f*x]*S
qrt[a + b + b*Tan[e + f*x]^2])/2))/4)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(189) = 378$.

Time = 36.35 (sec) , antiderivative size = 961, normalized size of antiderivative = 4.43

method	result	size
default	Expression too large to display	961

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/f/b/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(-6*ln(4*(b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^
(5/2)*(-a)^(1/2)*cos(f*x+e)^3+6*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(3/2)*(-a)^(1/2)*a*cos(f*x+e)^3-6
*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*
x+e)-1))*b^(5/2)*(-a)^(1/2)*cos(f*x+e)^3+6*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(3/2)*(-a)^(1/2)*a*co
s(f*x+e)^3+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*a^2*b*cos(f*x+e)^3-18*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2*cos(f*x+e)^3+3*ln(4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3*cos(f*x+e)^3+sin(f*x+e)*co
s(f*x+e)^3*(2*cos(f*x+e)^3+2*cos(f*x+e)^2-5*cos(f*x+e)-5)*((b+a*cos(f*x...
```

Fricas [A] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 1667, normalized size of antiderivative = 7.68

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2
)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 -
 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
+ e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 24*(a^2 - a*b)*sqrt(b)*c
os(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*
x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) -
 8*(2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), 1/64*
(48*(a^2 - a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
 + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 3*(a^2 - 6*a*b + b^2)*sqrt(-a)
*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -...
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\int \sin\left(e+f\,x\right)^4\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx=\left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\sin(fx+e)^4\,dx\right)b+\left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^4\,dx\right)a$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*sin(e + f*x)**4,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**4,x)*a
```

3.88 $\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3a-b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

output
```
1/2*a^(1/2)*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
f+1/2*(3*a-b)*b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/f+b*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/2*cos(f*x+e)*sin(f*x+e)*(
a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.06

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{i\left(-1+e^{2i(e+fx)}\right)\left(-4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2\right)}{\left(1+e^{2i(e+fx)}\right)^2}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

output
```
(Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f
*x]^3*((I*(-1 + E^((2*I)*(e + f*x)))*(-4*b*E^((2*I)*(e + f*x)) + a*(1 + E^
((2*I)*(e + f*x)))^2))/(1 + E^((2*I)*(e + f*x)))^2 + (2*E^((2*I)*(e + f*x)
)*(2*Sqrt[a]*(a - 3*b)*f*x - I*Sqrt[a]*(a - 3*b)*Log[a + 2*b + a*E^((2*I)*
(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f
*x)))^2]] + I*Sqrt[a]*(a - 3*b)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
+ f*x)))^2]] + 2*Sqrt[b]*(-3*a + b)*Log[((Sqrt[b]*(-1 + E^((2*I)*(e + f*x)
)) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/(
b*(-3*a + b)*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*I)*(e + f*x)) +
a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*E
^(I*(e + f*x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4620, 369, 403, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 369}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{b\tan^2(e+fx)+a+b}\left(4b\tan^2(e+fx)+a+b\right)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{2\left(a^2-b^2+(3a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{\tan(e+fx)\left(a+b\tan\ldots\right.}{2\left(\tan^2(e+\ldots\right.}}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{2}\left(\int \frac{a^2-b^2+(3a-b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{\tan(e+fx)\left(a+b\tan^2\ldots\right.}{2\left(\tan^2(e+f\ldots\right.}}{f}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{1}{2}\left(b(3a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a(a-3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2b\tan\ldots\right.}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{2}\left(a(a-3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\ldots\right.}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}\left(a(a-3b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+2b\tan(e\right.}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}\left(a(a-3b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+2b\tan(e+fx)\sqrt{\right.}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}\left(\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+2b\tan(e+fx)\sqrt{a+b\tan\right.}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

output
```
(-1/2*(Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(1 + Tan[e + f*x]^2)
 + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]] + (3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]] + 2*b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(139) = 278$.

Time = 23.53 (sec) , antiderivative size = 825, normalized size of antiderivative = 5.12

method	result	size
default	Expression too large to display	825

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/b/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(ln(4*(b^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(5
/2)*(-a)^(1/2)*cos(f*x+e)^3-3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(3/2)*(-a)^(1/2)*a*cos(f*x+e)^3+ln(
-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)
-1))*b^(5/2)*(-a)^(1/2)*cos(f*x+e)^3-3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(3/2)*(-a)^(1/2)*a*cos(f*
x+e)^3-2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*a^2*b*cos(f*x+e)^3+6*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-4*sin(f*x+e)*a)*a*b^2*cos(f*x+e)^3+sin(f*x+e)*cos(f*x+e)^3*(2*cos(f*
x+e)+2)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+(-2*cos
(f*x+e)-2)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b^2*cos(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(139) = 278$.

Time = 0.94 (sec) , antiderivative size = 1535, normalized size of antiderivative = 9.53

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
[-1/16*(sqrt(-a)*(a - 3*b)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^
3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 2*(3*a - b)*sqrt(b)*cos(f*x + e)*log
(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*(a*cos(f*x +
 e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*co
s(f*x + e)), 1/16*(4*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - sqrt(-a)*(a - 3
*b)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b +
 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(
f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -
 b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2...
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\sin(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^2\,dx\right)a$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*sin(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**2,x)*a
```

3.89 $\int (a+b\sec^2(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*b^(1/2
)*(3*a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*b*t
an(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.17 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.47

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\mathrm{lo}\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^
((2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)
*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f
*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2]] - 3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqr
t[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)
*(1 + E^((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*f + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]*f)/(b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(
a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$\big\downarrow$ 3042

$$\int \left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$\big\downarrow$ 4616

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 318

$$\frac{\frac{1}{2}\int \frac{b(3a+b)\tan^2(e+fx)+(a+b)(2a+b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\big\downarrow$ 398

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}b\tan(e-}{f}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}b\,\mathrm{ta}}{f}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+f}{f}$$

$\big\downarrow$ 291

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+}}{f}$$

$\big\downarrow$ 216

$$\frac{\frac{1}{2}\left(2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
+ Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(100) = 200$.

Time = 20.88 (sec) , antiderivative size = 673, normalized size of antiderivative = 5.70

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)b^{\frac{5}{2}}\sqrt{-a}\cos(fx+e)^3+3\ln\cdots}{\cdots}$

input
```
int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(ln(4*(b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(5/
2)*(-a)^(1/2)*cos(f*x+e)^3+3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(3/2)*(-a)^(1/2)*a*cos(f*x+e)^3+ln(-
4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-
1))*b^(5/2)*(-a)^(1/2)*cos(f*x+e)^3+3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(3/2)*(-a)^(1/2)*a*cos(f*x
+e)^3+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)
*a)*a^2*b*cos(f*x+e)^3+(2*cos(f*x+e)+2)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(100) = 200$.

Time = 0.44 (sec) , antiderivative size = 1457, normalized size of antiderivative = 12.35

$$\int \left(a + b\sec^2(e + fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*
b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4
 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*
b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqr
t(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^
8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e
)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)...
```

Sympy [F]

$$\int \left(a + b\sec^2(e + fx)\right)^{3/2} dx = \int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2} dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}dx\right)a + \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^2 dx\right)b$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a),x)*a + int(sqrt(sec(e + f*x)**2*b + a)*sec
(e + f*x)**2,x)*b
```

3.90 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{3\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}-\frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f}$$

output
```
3/2*b^(1/2)*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f
+3/2*b*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-cot(f*x+e)*(a+b+b*tan(f*x+e
)^2)^(3/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.61

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=-\frac{(a+b)\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-(((a + b)*Cot[e + f*x]*Hypergeometric2F1[-1/2, 2, 1/2, (b*Sin[e + f*x]^2)
/(a + b - a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4620, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{247}$$

$$\frac{3b\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{3b\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\,\mathrm{ta}\right.}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{3b\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a-\right.}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx\right.}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2)) + 3*b*(((a + b)*ArcTanh[
(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (Tan
[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(91) = 182$.

Time = 20.87 (sec) , antiderivative size = 616, normalized size of antiderivative = 5.87

method	result
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(-3\ln\left(\frac{4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)\cos(fx+e)^3 b^{\frac{5}{2}}-3\ln\right.}{}$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(-3*ln(4*(b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)^3
*b^(5/2)-3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b
)/(sin(f*x+e)+1))*cos(f*x+e)^3*b^(3/2)*a-3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)^3*b^(5/2)-3*
ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x
+e)-1))*cos(f*x+e)^3*b^(3/2)*a+(4*cos(f*x+e)+4)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^2*cot(f*x+e)+(6*cos(f*x+e)^3+6*cos(f*x+e)
^2-2*cos(f*x+e)-2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cot(f*x
+e))
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.52

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\,(a+b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\,\left(ab-b^2\right)\cos(fx+e)^2+4\,\left((a-b)\cos(fx+e)^3+2\,b\cos\right.}{\cos(fx+e)^4}\right.}{8\,f\cos\left(fx\right.}\right.$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)
^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e)^2 - b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x + e))
, 1/4*(3*(a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
 + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - 2*((2*a + 3*b)*c
os(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x
 + e)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{3\,a\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+3\,b^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+3\sqrt{b\tan\left(fx+e\right)^2+a+b}b\tan\left(fx\right.}{2\,f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(3*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*b^(3/2)*arcsi
nh(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*b*ta
n(f*x + e) - 2*(b*tan(f*x + e)^2 + a + b)^(3/2)/tan(f*x + e))/f
```

Giac [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^2\sec(fx+e)^2\,dx\right)b+\left(\int \sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^2\,dx\right)a$$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**2*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**2,x)*a
```

3.91 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}$$

output
```
1/2*b^(1/2)*(3*a+5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/f+1/2*b*(3*a+5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/3*(3*
a+5*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b
+b*tan(f*x+e)^2)^(5/2)/(a+b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.55 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$
$$-\frac{2(a+b\sec^2(e+fx))^{3/2}\left(-\left((a+b)(-2a-b+a\cos(2(e+fx)))\csc^2(e+fx)\,\text{Hypergeometric2F1}\left(1,2\right.\right.\right.}{3}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-2*(a + b*Sec[e + f*x]^2)^(3/2)*(-((a + b)*(-2*a - b + a*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2*Hypergeometric2F1[1, 2, 1/2, -((b*Tan[e + f*x]^2)/(a + b)
)]) + 8*b*Hypergeometric2F1[2, 3, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a +
 b*Sec[e + f*x]^2))*((a + b)*Csc[e + f*x] - a*Sin[e + f*x])^2*Tan[e + f*x]
)/(3*(a + b)^3*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4620, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^4}dx$$
$$\Big\downarrow \text{4620}$$
$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 359

$$\frac{\frac{(3a+5b)\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{3(a+b)}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{3(a+b)}}{f}$$

$\Big\downarrow$ 247

$$\frac{\frac{(3a+5b)\left(3b\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{3(a+b)}}{f}$$

$\Big\downarrow$ 211

$$\frac{\frac{(3a+5b)\left(3b\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)}-\frac{\cot^3(e}{}}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{(3a+5b)\left(3b\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{(3a+5b)\left(3b\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)}-\frac{\cot^3(e+}{}}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/3*(Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(a + b) + ((3*a +
5*b)*(-(Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2)) + 3*b*(((a + b)*Arc
Tanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) +
 (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)))/(3*(a + b)))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(152) = 304$.

Time = 21.83 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.80

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left((-15\cos(fx+e)+15)b^{\frac{5}{2}}\ln\left(\frac{4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)\text{co}\right.}{}$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/f/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)/(b+a*cos(f*x+e)^2)*((-15*cos(f*x+e)+15)*b^(5/2)*ln(4*(b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)*cot(
f*x+e)^2+(-9*cos(f*x+e)+9)*b^(3/2)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*cos(f*x+e)*cot(f*x+e)^2+(-15*c
os(f*x+e)+15)*b^(5/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(
f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)*cot(f*x+e)^2+(-9*cos(f*x+e)+9)*b^
(3/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(s
in(f*x+e)-1))*a*cos(f*x+e)*cot(f*x+e)^2+(8*cos(f*x+e)^2-12)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cot(f*x+e)^3+(30*cos(f*x+e)^4-40*cos(f*x
+e)^2+6)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cot(f*x+e)*csc(f*
x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.74

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left((3a+5b)\cos(fx+e)^3-(3a+5b)\cos(fx+e)\right)\sqrt{b}\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2}{}\right.}{}\right.$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e))*sqrt(b)*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) -
 4*((4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b)*cos(f*x + e)^2 + 3*b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x +
 e))*sin(f*x + e)), 1/12*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(
f*x + e))*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*((4*a + 15*b)*cos(f*x + e)^4 - 2*(
3*a + 10*b)*cos(f*x + e)^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/((f*cos(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.41

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{9\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+\frac{6\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a+b}+9\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+\frac{6\,b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{b\tan(\cdots}{\sqrt{(a\cdots}}\right.}{a+b}\cdots}{\cdots}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(9*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 6*a*b^(3/2)*arc
sinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 9*b^(3/2)*arcsinh(b*tan(f*x
 + e)/sqrt((a + b)*b)) + 6*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))
/(a + b) + 9*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 6*sqrt(b*tan(
f*x + e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 6*(b*tan(f*x + e)^2 + a + b
)^(3/2)/tan(f*x + e) - 4*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)*tan(f
*x + e)) - 2*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^3))/f
```

Giac [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^4} \,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)^4 \sec\left(fx+e\right)^2 dx\right) b + \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)^4 dx\right) a$$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**4*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**4,x)*a
```

3.92 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{b}(3a+7b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f}-\frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}-\frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}-\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f}$$

output

```
1/2*b^(1/2)*(3*a+7*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/f+1/2*b*(3*a+7*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/3*(3*
a+7*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-2/3*cot(f*x+e)^3*(a+b
+b*tan(f*x+e)^2)^(5/2)/(a+b)/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(5/2)
/(a+b)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.64 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.45

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(16a^2\left(1+e^{2i(e+fx)}\right)^2\left(1-6e^{2i(e+fx)}+1\right.\right.}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(16*a^2*(1 + E^((2*I)*(e + f*x)))^2*(
1 - 6*E^((2*I)*(e + f*x)) + 16*E^((4*I)*(e + f*x)) - 6*E^((6*I)*(e + f*x))
 + E^((8*I)*(e + f*x))) + b^2*(105 - 350*E^((2*I)*(e + f*x)) + 231*E^((4*I
)*(e + f*x)) + 412*E^((6*I)*(e + f*x)) + 231*E^((8*I)*(e + f*x)) - 350*E^(
(10*I)*(e + f*x)) + 105*E^((12*I)*(e + f*x))) + a*b*(115 - 402*E^((2*I)*(e
 + f*x)) + 317*E^((4*I)*(e + f*x)) + 708*E^((6*I)*(e + f*x)) + 317*E^((8*I
)*(e + f*x)) - 402*E^((10*I)*(e + f*x)) + 115*E^((12*I)*(e + f*x)))))/((a
+ b)*(-1 + E^((2*I)*(e + f*x)))^5*(1 + E^((2*I)*(e + f*x)))^2) - (15*Sqrt[
b]*(3*a + 7*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(
e + f*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]
)*(a + b*Sec[e + f*x]^2)^(3/2))/(15*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)
)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4620, 365, 27, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{365}$$

$$\frac{\frac{\int 5(a+b)\cot^4(e+fx)\left(\tan^2(e+fx)+2\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{5(a+b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+2\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{5(a+b)}}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a+7b)\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{3(a+b)}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{5(a+b)}-\frac{2\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{3(a+b)}}{f}$$

$$\Big\downarrow \text{247}$$

$$\frac{\frac{(3a+7b)\left(3b\int \sqrt{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{5(a+b)}-\frac{2\cot^3(e+f\ldots}{}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{(3a+7b)\left(3b\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)}-\frac{\cot^5(e}{}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{(3a+7b)\left(3b\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(3a+7b)\left(3b\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)}-\frac{\cot^5(e+}{}}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((-2*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*(a + b)) - (Cot[e
 + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(5/2))/(5*(a + b)) + ((3*a + 7*b)*(-(
Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2)) + 3*b*(((a + b)*ArcTanh[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (Tan[e
+ f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)))/(3*(a + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(185) = 370$.

Time = 23.00 (sec) , antiderivative size = 999, normalized size of antiderivative = 4.78

method	result	size
default	Expression too large to display	999

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60/f/(a+b)/b*(sin(f*x+e)^3*cos(f*x+e)^2*(105*cos(f*x+e)-105)*ln(4*(b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(
7/2)+sin(f*x+e)^3*cos(f*x+e)^2*(150*cos(f*x+e)-150)*ln(4*(b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(5/2)*a+sin(f
*x+e)^3*cos(f*x+e)^2*(45*cos(f*x+e)-45)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(3/2)*a^2+sin(f*x+e)^3*co
s(f*x+e)^2*(105*cos(f*x+e)-105)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(7/2)+sin(f*x+e)^3*cos(f*x+e)^2*
(150*cos(f*x+e)-150)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f
*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(5/2)*a+sin(f*x+e)^3*cos(f*x+e)^2*(45*cos(f
*x+e)-45)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b
)/(sin(f*x+e)-1))*b^(3/2)*a^2+cos(f*x+e)^2*(32*cos(f*x+e)^4-80*cos(f*x+e)^
2+60)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+(230*cos(f*x+e)^6-
546*cos(f*x+e)^4+370*cos(f*x+e)^2-30)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)...
```

Fricas [A] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.26

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/120*(15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b +
7*b^2)*cos(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(b)*log
(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4
*((16*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 245*b^
2)*cos(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a*b -
 15*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x
+ e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x + e)
), 1/60*(15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b +
 7*b^2)*cos(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(-b)*a
rctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)
))*sin(f*x + e) - 2*((16*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2
 + 273*a*b + 245*b^2)*cos(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*
x + e)^2 - 15*a*b - 15*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(
((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x
 + e))*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.31

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{45\,a\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+\frac{60\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a+b}+45\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+\frac{60\,b^{\frac{5}{2}}\operatorname{arsinh}(}{a+}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/30*(45*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 60*a*b^(3/2)*
arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 45*b^(3/2)*arcsinh(b*tan
(f*x + e)/sqrt((a + b)*b)) + 60*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b
)*b))/(a + b) + 45*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 60*sqrt
(b*tan(f*x + e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 30*(b*tan(f*x + e)^2
 + a + b)^(3/2)/tan(f*x + e) - 40*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a +
 b)*tan(f*x + e)) - 20*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x +
 e)^3) - 6*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6, x)
```

Reduce [F]

$$\int \csc^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)^6 \sec\left(fx+e\right)^2 dx\right) b + \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)^6 dx\right) a$$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**6*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**6,x)*a
```

3.93 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(15a^2+20ab+8b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f} + \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af}$$

output
```
-1/15*(15*a^2+20*a*b+8*b^2)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^3/f+2/15
*(5*a+2*b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f-1/5*cos(f*x+e)^5*(a
+b*sec(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(89a^2+144ab+64b^2-4a(7a+4b)\cos(2(e+fx))+3a^2\cos(4(e+fx))\right)}{240a^3f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/240*((a + 2*b + a*Cos[2*(e + f*x)])*(89*a^2 + 144*a*b + 64*b^2 - 4*a*(7
*a + 4*b)*Cos[2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x])/(a^3*f*
Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4622, 365, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^5}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{365}$$

$$\frac{\frac{\int -\frac{\cos^4(e+fx)\left(2(5a+2b)-5a\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a}}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(2(5a+2b)-5a\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a}}{f}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{-\frac{-\frac{\left(15a^2+20ab+8b^2\right)\int \frac{\cos^2(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{3a}-\frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a}}{5a}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a}}{f}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{-\frac{\frac{\left(15a^2+20ab+8b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2}-\frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a}}{5a}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a}}{f}$$

input
```
Int[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2])/a - (((15*a^2 + 20*a*b +
 8*b^2)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(3*a^2) - (2*(5*a + 2*b)*
Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(3*a))/(5*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{\left(b+a\cos\left(fx+e\right)^{2}\right)\left(-4\cos\left(fx+e\right)^{2}ab+20ab+\left(3\cos\left(fx+e\right)^{4}-10\cos\left(fx+e\right)^{2}+15\right)a^{2}+8b^{2}\right)\sec\left(fx+e\right)}{15f\,a^{3}\sqrt{a+b\sec\left(fx+e\right)^{2}}}$	89

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/a^3*(b+a*cos(f*x+e)^2)*(-4*cos(f*x+e)^2*a*b+20*a*b+(3*cos(f*x+e)^4
-10*cos(f*x+e)^2+15)*a^2+8*b^2)/(a+b*sec(f*x+e)^2)^(1/2)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(3\,a^2\cos\left(fx+e\right)^5-2\left(5\,a^2+2\,ab\right)\cos\left(fx+e\right)^3+\left(15\,a^2+20\,ab+8\,b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{15\,a^3f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(3*a^2*cos(f*x + e)^5 - 2*(5*a^2 + 2*a*b)*cos(f*x + e)^3 + (15*a^2 +
 20*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/(a^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a}-\frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a+\frac{b}{\cos(fx+e)^2}}b\cos(fx+e)\right)}{a^2}+\frac{3\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e}{}}{15\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - 10*((a + b/cos(f*x +
 e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^2 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a + b/cos(f*x
+ e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x
 + e))/a^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(111) = 222$.

Time = 0.67 (sec) , antiderivative size = 928, normalized size of antiderivative = 7.54

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-256/15*(5*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2
*f*x + 1/2*e)^2 + a + b))^7*(a + b) - 5*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2
*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(2*a - b)*sqrt(a
+ b) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
 + 1/2*e)^2 + a + b))^5*(13*a^2 + 40*a*b + 15*b^2) + 5*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(8
*a^2 + 5*a*b - 3*b^2)*sqrt(a + b) - 5*(a^3 - 9*a^2*b - 13*a*b^2 - 3*b^3)*(
sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e
)^2 + a + b))^3 - 5*(10*a^3 + 17*a^2*b + 4*a*b^2 - 3*b^3)*(sqrt(a + b)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2
*sqrt(a + b) + 5*(9*a^4 + 14*a^3*b - 6*a*b^3 - b^4)*(sqrt(a + b)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) - (12*a
^4 + 9*a^3*b - 13*a^2*b^2 - 5*a*b^3 + 5*b^4)*sqrt(a + b))/(((sqrt(a + b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin\left(e+f\,x\right)^5}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec (fx+e)^2 b+a}\,\sin (fx+e)^5}{\sec (fx+e)^2 b+a}dx$$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**5)/(sec(e + f*x)**2*b + a),
x)
```

3.94 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f} + \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}$$

output
```
-1/3*(3*a+2*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f+1/3*cos(f*x+e)^3*
(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(-5a-4b+a\cos(2(e+fx)))(a+2b+a\cos(2(e+fx)))\sec(e+fx)}{12a^2f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-5*a - 4*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e +
f*x])/(12*a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4622, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(e+fx)^3}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \ 4622$$

$$\frac{\int -\frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \ 359$$

$$\frac{\frac{(3a+2b)\int \frac{\cos^2(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{3a}+\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a}}{f}$$

$$\Big\downarrow \ 242$$

$$\frac{\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a}-\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2}}{f}$$

input
```
Int[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/3*((3*a + 2*b)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/a^2 + (Cos[e +
 f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(3*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(a\cos(fx+e)^2-3a-2b\right)\sec(fx+e)}{3f\,a^2\sqrt{a+b\sec(fx+e)^2}}$	58

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/a^2*(b+a*cos(f*x+e)^2)*(a*cos(f*x+e)^2-3*a-2*b)/(a+b*sec(f*x+e)^2)^(
1/2)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\left(a\cos\left(fx+e\right)^3-\left(3\,a+2\,b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\,a^2f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/(a^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a+\frac{b}{\cos(fx+e)^2}}b\cos(fx+e)}{a^2}}{3f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - ((a + b/cos(f*x + e)^2
)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^2)
/f
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{f\left(\frac{\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}}{a^2f^2}-\frac{3\sqrt{a\cos(fx+e)^2+b}(a+b)}{a^2f^2}\right)}{3\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*f*((a*cos(f*x + e)^2 + b)^(3/2)/(a^2*f^2) - 3*sqrt(a*cos(f*x + e)^2 +
b)*(a + b)/(a^2*f^2))/sgn(cos(f*x + e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^3}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**3)/(sec(e + f*x)**2*b + a),
x)
```

3.95 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

output
```
-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2e+2fx))\sec(e+fx)}{2af\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x])/(a*f*Sqrt[a + b*Sec[e +
 f*x]^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4622, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \frac{\cos^2(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{242}$$

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

input
```
Int[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(a*f))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-\frac{\sqrt{a+b\sec(fx+e)^{2}}}{fa\sec(fx+e)}$	31
default	$-\frac{\sqrt{a+b\sec(fx+e)^{2}}}{fa\sec(fx+e)}$	31

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{af}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{af}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a\cos(fx+e)^2+b}}{af\,\mathrm{sgn}(\cos(fx+e))}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(a*cos(f*x + e)^2 + b)/(a*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\cos(e+fx)\,\sqrt{\frac{a+2\,b+a\,\cos(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{a\,f}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
-(cos(e + f*x)*((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/
2))/(a*f)
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)}{\sec(fx+e)^2 b+a}dx$$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x))/(sec(e + f*x)**2*b + a),x)
```

3.96 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}f}$$

output
```
-arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a+b}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + 2*b + a*Co
s[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a + b]*f*Sqrt[a + b*Sec[e + f*
x]^2]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4622, 25, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

input
```
Int[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/(Sqrt[a +
 b]*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(37) = 74$.

Time = 1.33 (sec) , antiderivative size = 247, normalized size of antiderivative = 5.74

method	result
default	$-\frac{\left(\ln\left(\frac{2\sqrt{a+b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+2\sqrt{a+b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-2\cos(fx+e)a+2b}{\sqrt{a+b}\,(1+\cos(fx+e))}\right)+\ln\left(-\frac{4\left(\sqrt{a+b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)\right.}{-1+}\right.}{2f\sqrt{a+b}\,\sqrt{a+b\sec(fx+e)^2}}$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(a+b)^(1/2)*(ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))+ln(-4*((a+b)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e))))*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.47

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \left[\frac{\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)}{2\sqrt{a+b}\,f}, \frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a\cos(fx+e)^2+b}\right)}{(a+b)f}\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1))/(sqrt(a + b)*
f), sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b))/((a + b)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc{(fx+e)}}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{-a\sin(fx+e)^2+a+b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}f\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(-a*sin(f*x + e)^2 + a + b)/sqrt(-a - b))/(sqrt(-a - b)*f*sgn(c
os(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x))/(sec(e + f*x)**2*b + a),x)
```

3.97 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}$$

output
```
-1/2*a*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2
)/f-1/2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.61

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{a\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\left(\frac{(a+b)\csc^2(e+fx)}{a}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{a\sin^2}{a}}\right.}{\sqrt{1-\frac{a\sin^2(e+fx}{a+b}}}\right.}{2\sqrt{2}(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(a*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]*Sqrt[a + b - a*Sin
[e + f*x]^2]*(((a + b)*Csc[e + f*x]^2)/a + ArcTanh[Sqrt[1 - (a*Sin[e + f*x
]^2)/(a + b)]]/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))/(Sqrt[2]*(a + b)^2*f
*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4622, 373, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(e+fx)^3\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 4622}$$

$$\frac{\int \frac{\sec^2(e+fx)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 373}$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\int \frac{a}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{a\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{a\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{2(a+b)}}{f}$$

$\big\downarrow$ 219

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{a\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/2*(a*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(
a + b)^(3/2) + (Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a + b)*(1 - S
ec[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(75) = 150$.

Time = 1.41 (sec) , antiderivative size = 567, normalized size of antiderivative = 6.52

method	result
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\left(\ln\left(\frac{2\sqrt{a+b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+2\sqrt{a+b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-2\cos(fx+e)a+2b}{\sqrt{a+b}\,(1+\cos(fx+e))}\right)a^2(1-\cos(fx+e))^2+\ln\left(\frac{2\sqrt{}}{}\right.\right.}{}$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(a+b)^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x
+e)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)/(1-cos(f*x+e))^2*(ln(2/(a+b
)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/
(1+cos(f*x+e)))*a^2*(1-cos(f*x+e))^2+ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*(1-cos(f*x+e
))^2*a*b+ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e
)*a+b)/(-1+cos(f*x+e)))*a^2*(1-cos(f*x+e))^2+ln(-4*((a+b)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*(1-cos(f*x+e))
^2*a*b+(-2*cos(f*x+e)+2)*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(75) = 150$.

Time = 0.15 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.61

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{2\,(a+b)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) + \left(a\cos(fx+e)^2 - a\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2 - 2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)}{\cos(fx+e)}}\right.}{\cos(fx+e)^2-1}\right.}{4\left((a^2+2\,ab+b^2)f\cos(fx+e)^2 - (a^2+2\,ab+b^2)f\right)}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 (a*cos(f*x + e)^2 - a)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos
(f*x + e)^2 - 1)))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b +
b^2)*f), 1/2*((a*cos(f*x + e)^2 - a)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b
)) + (a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/((a
^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc^3{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc{(fx+e)}^3}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(75) = 150$.

Time = 0.47 (sec) , antiderivative size = 480, normalized size of antiderivative = 5.52

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(4*a*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) + 2*a
*log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*
f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a - b))/(a + b)^(3/2) - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)/(a + b) - 2*((sqrt(a + b)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b)
 - (a + b)^(3/2))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b)*(a + b)))/(f*sgn(cos(f*x + e
)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sin(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec (fx+e)^2 b+a}\,\csc (fx+e)^3}{\sec (fx+e)^2 b+a}dx$$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**3)/(sec(e + f*x)**2*b + a),
x)
```

3.98 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{3a^2\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{5/2}f} - \frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f}$$

output
```
-3/8*a^2*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5
/2)/f-1/8*(5*a+2*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2
/f-1/4*cot(f*x+e)^3*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{a^2(a+2b+a\cos(2(e+fx)))\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right)\sec(e+fx)}{2(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(a^2*(a + 2*b + a*Cos[2*(e + f*x)])*Hypergeometric2F1[1/2, 3, 3/2, 1
- (a*Sin[e + f*x]^2)/(a + b)]*Sec[e + f*x])/((a + b)^3*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 25, 372, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^5\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \ 372$$

$$\frac{\frac{\int \frac{2(2a+b)\sec^2(e+fx)+a}{\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{4(a+b)}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{\int -\frac{3a^2}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}+\frac{(5a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{(5a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)\left(1-\sec^2(e+fx)\right)}-\frac{3a^2\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}}{4(a+b)}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{\frac{(5a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)\left(1-\sec^2(e+fx)\right)}-\frac{3a^2\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{2(a+b)}}{4(a+b)}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{(5a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)\left(1-\sec^2(e+fx)\right)}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}}}{4(a+b)}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/4*(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/((a + b)*(1 - Sec[e + f*x]
^2)^2) + ((-3*a^2*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*
x]^2]])/(2*(a + b)^(3/2)) + ((5*a + 2*b)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f
*x]^2])/(2*(a + b)*(1 - Sec[e + f*x]^2)))/(4*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(122) = 244$.

Time = 1.44 (sec) , antiderivative size = 975, normalized size of antiderivative = 7.07

method	result	size
default	Expression too large to display	975

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f/(a+b)^(9/2)*(cos(f*x+e)^2*(-6*cos(f*x+e)^2+10)*(a+b)^(5/2)*a^2+(-2
*cos(f*x+e)^2+10)*(a+b)^(5/2)*a*b+4*(a+b)^(5/2)*b^2+(3*cos(f*x+e)+3)*sin(f
*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)
))*a^4+(6*cos(f*x+e)+6)*sin(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b+(3*cos(f*x+e)+3)*sin(f*x+e)^4*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^2+(3*
cos(f*x+e)+3)*sin(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+
b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+c
os(f*x+e)))*a^4+(6*cos(f*x+e)+6)*sin(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+co
s(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*b+(3*cos(f*x+e)+3)*sin(f*x+e)^4*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(122) = 244$.

Time = 0.18 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.62

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \left[\frac{3\left(a^2\cos(fx+e)^4-2a^2\cos(fx+e)^2+a^2\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2\cdots\right.}{\cos(fx+e)^2-1}\cdots\right.}{16\left((a^3+3a^2b+3ab^2+b^3)f\cos(fx+e)^4-2(a^3+3a^2b+3ab^2\cdots\right.}\cdots\right.$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(a + b)*log
(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 + a*b)*co
s(f*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4
- 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*f), 1/8*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*
sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b)) + (3*(a^2 + a*b)*cos(f*x + e)^3 -
(5*a^2 + 7*a*b + 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)
]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc^5{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\csc(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(122) = 244$.

Time = 0.77 (sec) , antiderivative size = 785, normalized size of antiderivative = 5.69

$$\int \frac{\csc^5(e + fx)}{\sqrt{a + b\sec^2(e + fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*((a + b)*tan(1/2*
f*x + 1/2*e)^2/(a^2 + 2*a*b + b^2) + 3*(3*a + b)/(a^2 + 2*a*b + b^2)) - 24
*a^2*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a - b
)) - 12*a^2*log(abs((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) - a + b))/((a^2 + 2*a*b + b
^2)*sqrt(a + b)) + 4*(2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(2*a^2 - 2*a*b - b^2) - 3*(sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))^2*(a^2 + 2*a*b + b^2)*sqrt(a + b) - 2*(3*a^3 + a^2*b - 2*a*b^2)*(s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b)) + (5*a^3 + 11*a^2*b + 7*a*b^2 + b^3)*sqrt(a + b))/(((sqrt(a +
 b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x
 + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)^5}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**5)/(sec(e + f*x)**2*b + a),
x)
```

3.99 $\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{5(a+b)^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} - \frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}$$

output
```
5/16*(a+b)^3*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)
/f-1/48*(33*a^2+40*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^
(1/2)/a^3/f+1/24*(9*a+5*b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1
/2)/a^2/f+1/6*cos(f*x+e)^3*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.84

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left(15(a+b)^3\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\sin(e+fx)\sqrt{a+b}\right.}{48\sqrt{2}a^{7/2}f\sqrt{a+b\sec^2(e+f}}$$

input
```
Integrate[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(15*(a + b)^3*ArcTan[(Sqr
t[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*
Sqrt[a + b - a*Sin[e + f*x]^2]*(15*(a + b)^2 + 10*a*(a + b)*Sin[e + f*x]^2
 + 8*a^2*Sin[e + f*x]^4)))/(48*Sqrt[2]*a^(7/2)*f*Sqrt[a + b*Sec[e + f*x]^2
])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4620, 372, 440, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)^6}{\sqrt{a+b\sec(e+fx)^2}}dx$$
$$\Big\downarrow \text{4620}$$
$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 372

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\int \frac{\tan^2(e+fx)\left(3(a+b)-2(3a+b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{6a}}{f}$$

$\Big\downarrow$ 440

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\int \frac{(a+b)(9a+5b)-2\left(12a^2+13ba+5b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}-\frac{(9a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{\left(33a^2+40ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{15(a+b)^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}-\frac{(9a+5b\cdots}{\cdots}}{6a}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{\left(33a^2+40ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{15(a+b)^3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}\cdots}{6a}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{\left(33a^2+40ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{15(a+b)^3\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}}{4a}-\frac{(9\cdots}{\cdots}}{6a}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{\left(33a^2+40ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{15(a+b)^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}}{4a}-\frac{(9a+5b)\tan(e+fx}{4a\left(\tan^2\right.}}{6a}}{f}$$

input
```
Int[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*(1 + Tan[e + f*x]^2)
^3) - (-1/4*((9*a + 5*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a*(
1 + Tan[e + f*x]^2)^2) + ((-15*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqr
t[a + b + b*Tan[e + f*x]^2]])/(2*a^(3/2)) + ((33*a^2 + 40*a*b + 15*b^2)*Ta
n[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)))/(4*
a))/(6*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(173) = 346$.

Time = 19.71 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.28

method	result
default	$15\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^3\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)+45\sqrt{\frac{b+}{(1-}}$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a^3/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(15*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-4*sin(f*x+e)*a)*(sec(f*x+e)+1)+45*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^2*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin
(f*x+e)*a)*(sec(f*x+e)+1)+45*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)
*(sec(f*x+e)+1)+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*ln(4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(sec(f*x+e)
+1)+sin(f*x+e)*cos(f*x+e)*(-8*cos(f*x+e)^4+26*cos(f*x+e)^2-33)*a^3*(-a)^(1
/2)+(2*cos(f*x+e)^4-14*cos(f*x+e)^2-33)*(-a)^(1/2)*a^2*b*tan(f*x+e)+5*(-8-
cos(f*x+e)^2)*(-a)^(1/2)*a*b^2*tan(f*x+e)-15*(-a)^(1/2)*b^3*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 639, normalized size of antiderivative = 3.31

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
 e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7
- 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5
 - 2*(13*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 15*a*b^2)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4
*f), -1/192*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*
cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*
x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*
x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
+ 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3
+ 40*a^2*b + 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(a^4*f)]
```

Sympy [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin^6{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^6}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin\left(e+f\,x\right)^6}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2b+a}\,\sin\left(fx+e\right)^6}{\sec\left(fx+e\right)^2b+a}dx$$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**6)/(sec(e + f*x)**2*b + a),
x)
```

3.100 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}$$

output
```
3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/
f-1/8*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/f+1/4
*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left(3(a+b)^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\sin(e+fx)\sqrt{a+b}\right.}{8\sqrt{2}a^{5/2}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(3*(a + b)^2*ArcTan[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*S
qrt[a + b - a*Sin[e + f*x]^2]*(3*(a + b) + 2*a*Sin[e + f*x]^2)))/(8*Sqrt[2
]*a^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4620, 372, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^4}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\int \frac{-2(2a+b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3(a+b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3(a+b)^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*a*(1 + Tan[e + f*x]^2)^2
) - ((-3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*
x]^2]])/(2*a^(3/2)) + ((5*a + 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(2*a*(1 + Tan[e + f*x]^2)))/(4*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(119) = 238.

Time = 9.36 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.43

method	result
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a^2 \ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(3+3\sec(fx+e))+\sqrt{\frac{b+a\cos}{(1+\cos}}}{}$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)*(3+3*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)
*a)*(6+6*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*ln(4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(3+3*sec(
f*x+e))+sin(f*x+e)*cos(f*x+e)*(2*cos(f*x+e)^2-5)*a^2*(-a)^(1/2)+(-cos(f*x+
e)^2-5)*(-a)^(1/2)*a*b*tan(f*x+e)-3*(-a)^(1/2)*b^2*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.19

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{3\,(a^2+2\,ab+b^2)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-14\,a^3b+\right.}{}\right.$$

$$-\frac{3\,(a^2+2\,ab+b^2)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)-}{32\,a^3f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7
*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 + 3*
a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^3*f), -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))
*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^
4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*
a^2*cos(f*x + e)^3 - (5*a^2 + 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^4}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**4)/(sec(e + f*x)**2*b + a),
x)
```

3.101 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

output
```
1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-
1/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.47

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left((a+b)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\sin(e+fx)\sqrt{a+b-\right.}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]*
Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqrt[
a + b - a*Sin[e + f*x]^2]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2
])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4620, 373, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^2}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 373$$

$$\frac{\frac{\int \frac{a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(3/2)) - (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e
 + f*x]^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(73) = 146$.

Time = 5.86 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.49

method	result
default	$-\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(-1-\sec(fx+e))+\sqrt{\frac{b+a}{(1+c}}}{}$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/a/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*(-1-sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-
1-sec(f*x+e))+sin(f*x+e)*cos(f*x+e)*(-a)^(1/2)*a+(-a)^(1/2)*b*tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(73) = 146$.

Time = 0.23 (sec) , antiderivative size = 497, normalized size of antiderivative = 5.85

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+\sqrt{-a}(a+b)\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\right.}{}\right.$$

$$-\frac{4\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+(a+b)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-\right.}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-\right.}\right.}{8\,a^2f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
 + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)))/(a^2*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a + b)*sqrt(a)*arctan(1/4*(8*a^2
*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f
*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f
*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
)/(a^2*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin^2{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^2}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**2)/(sec(e + f*x)**2*b + a),
x)
```

3.102 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4616, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

input
```
Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 3.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

method	result	size
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)}{f\sqrt{-a}\,\sqrt{a+b\sec(fx+e)^2}}$	138

input
```
int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)/(a+b*sec(f*x+e)^2)^
(1/2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(33) = 66$.

Time = 0.21 (sec) , antiderivative size = 408, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos\left(fx+\right.\right.}{}\right.$$

$$\left.-\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\,\sqrt{a}f}\right]$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + b\sec^2(e + fx)}}\,dx = \int \frac{1}{\sqrt{a + b\sec^2{(e + fx)}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(33) = 66$.

Time = 0.25 (sec) , antiderivative size = 992, normalized size of antiderivative = 25.44

$$\int \frac{1}{\sqrt{a + b\sec^2(e + fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*
f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b
)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x +
2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
 + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4
*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*
b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*co
s(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(
4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt
(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*...
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(1/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)/(sec(e + f*x)**2*b + a),x)
```

3.103 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

output
```
-cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\csc(e+fx)\sec(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/((a + b)*f
*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4620, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(e+fx)^2\sqrt{a+b\sec(e+fx)^2}}dx$$
$$\Big\downarrow \text{4620}$$
$$\frac{\int \frac{\cot^2(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{242}$$
$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

input
```
Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\cot(fx+e)a+b\sec(fx+e)\csc(fx+e)}{f(a+b)\sqrt{a+b\sec(fx+e)^{2}}}$	48

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a+b)/(a+b*sec(f*x+e)^2)^(1/2)*(cot(f*x+e)*a+b*sec(f*x+e)*csc(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=-\frac{\sqrt{\frac{a\cos(fx+e)^{2}+b}{\cos(fx+e)^{2}}}\cos(fx+e)}{(a+b)f\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a + b)*f*sin(f
*x + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)f\tan(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*f*tan(f*x + e))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 12.81 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(2\,\sin(2\,e+2\,f\,x)+\sin(4\,e+4\,f\,x))\,\sqrt{\frac{a+2\,b+a\,\cos(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{2\,f\,\sin(2\,e+2\,f\,x)^2\,(a+b)}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
-((2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*((a + 2*b + a*cos(2*e + 2*f*x))/
(cos(2*e + 2*f*x) + 1))^(1/2))/(2*f*sin(2*e + 2*f*x)^2*(a + b))
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^2}{\sec(fx+e)^2 b+a}dx$$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**2)/(sec(e + f*x)**2*b + a),
x)
```

3.104 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

output
```
-1/3*(3*a+b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e
)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(-2a-b+a\cos(2(e+fx)))(a+2b+a\cos(2(e+fx)))\csc^3(e+fx)\sec(e+fx)}{6(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-2*a - b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*
x]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4620, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(e+fx)^4\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{\frac{(3a+b)\int \frac{\cot^2(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}-\frac{(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)^2}}{f}$$

input
```
Int[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/3*((3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b)^2 -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)))/f
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\left(b+a\cos\left(fx+e\right)^{2}\right)\left(2a\cos\left(fx+e\right)^{2}-3a-b\right)\sec\left(fx+e\right)\csc\left(fx+e\right)^{3}}{3f\left(a^{2}+2ab+b^{2}\right)\sqrt{a+b\sec\left(fx+e\right)^{2}}}$	77

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^2+2*a*b+b^2)*(b+a*cos(f*x+e)^2)*(2*a*cos(f*x+e)^2-3*a-b)/(a+b*sec
(f*x+e)^2)^(1/2)*sec(f*x+e)*csc(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(2\,a\cos\left(fx+e\right)^3-\left(3\,a+b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^2-\left(a^2+2\,ab+b^2\right)f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*a*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b
 + b^2)*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{3\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)}-\frac{2\sqrt{b\tan(fx+e)^2+a+b}b}{(a+b)^2\tan(fx+e)}+\frac{\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 2*sqrt(b*t
an(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + sqrt(b*tan(f*x + e)^2
+ a + b)/((a + b)*tan(f*x + e)^3))/f
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 18.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.58

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{2\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(a\,1i-a\,e^{e\,2i+f\,x\,2i}\,4i+a\,e^{e\,4i+f\,x\,4i}\,1i-b\,e^{e\,2i+f\,x\,2i}\,2i\right)}{3\,f\,(a+b)^2\left(e^{e\,2i+f\,x\,2i}-1\right)^3}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
-(2*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2)^2)^(1/2)*(a*1i - a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*
1i - b*exp(e*2i + f*x*2i)*2i))/(3*f*(a + b)^2*(exp(e*2i + f*x*2i) - 1)^3)
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^4}{\sec(fx+e)^2 b+a}dx$$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**4)/(sec(e + f*x)**2*b + a),
x)
```

3.105 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} - \frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f}$$

output
```
-1/15*(15*a^2+10*a*b+3*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/
f-2/15*(5*a+3*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot
(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(8a^2+8ab+3b^2-2a(3a+b)\cos(2(e+fx))+a^2\cos(4(e+fx))\right)\csc^5(e}{30(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(8*a^2 + 8*a*b + 3*b^2 - 2*a*(3*a +
b)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/(
(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4620, 365, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^6\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{365}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+2(5a+3b)\right)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

$\Big\downarrow$ 359

$$\frac{\frac{\frac{\left(15a^2+10ab+3b^2\right)\int \frac{\cot^2(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

$\Big\downarrow$ 242

$$\frac{\frac{-\frac{\left(15a^2+10ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)^2}-\frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) + (-1/3*((15
*a^2 + 10*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b
)^2 - (2*(5*a + 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a
+ b)))/(5*(a + b)))/f
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 6.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\left(b+a\cos\left(fx+e\right)^{2}\right)\left(8\cos\left(fx+e\right)^{4}a^{2}-20a^{2}\cos\left(fx+e\right)^{2}-4\cos\left(fx+e\right)^{2}ab+15a^{2}+10ab+3b^{2}\right)\sec\left(fx+e\right)\csc\left(fx+e\right)^{5}}{15f\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)\sqrt{a+b\sec\left(fx+e\right)^{2}}}$	120

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a^3+3*a^2*b+3*a*b^2+b^3)*(b+a*cos(f*x+e)^2)*(8*cos(f*x+e)^4*a^2-2
0*a^2*cos(f*x+e)^2-4*cos(f*x+e)^2*a*b+15*a^2+10*a*b+3*b^2)/(a+b*sec(f*x+e)
^2)^(1/2)*sec(f*x+e)*csc(f*x+e)^5
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$
$$-\frac{\left(8\,a^2\cos\left(fx+e\right)^5-4\left(5\,a^2+ab\right)\cos\left(fx+e\right)^3+\left(15\,a^2+10\,ab+3\,b^2\right)\cos\left(fx+e\right)\right)\sqrt{}}{15\left(\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\left(fx+e\right)^4-2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\left(fx+e\right)^2+\left(a^3+3\,a^2b\right.\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(8*a^2*cos(f*x + e)^5 - 4*(5*a^2 + a*b)*cos(f*x + e)^3 + (15*a^2 + 1
0*a*b + 3*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x
+ e))
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^6\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{15\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)} - \frac{20\sqrt{b\tan(fx+e)^2+a+b}b}{(a+b)^2\tan(fx+e)} + \frac{8\sqrt{b\tan(fx+e)^2+a+b}b^2}{(a+b)^3\tan(fx+e)} + \frac{10\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)^3} - \frac{4\sqrt{b\tan(fx+e)^2+a}}{(a+b)^2\tan(fx+e)}}{15f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 20*sqrt(
b*tan(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + 8*sqrt(b*tan(f*x +
e)^2 + a + b)*b^2/((a + b)^3*tan(f*x + e)) + 10*sqrt(b*tan(f*x + e)^2 + a
+ b)/((a + b)*tan(f*x + e)^3) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*b/((a + b
)^2*tan(f*x + e)^3) + 3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x +
e)^5))/f
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 24.05 (sec) , antiderivative size = 723, normalized size of antiderivative = 5.48

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
(((32*a + 16*b)/(5*f*(6*a + 6*b)*(a*1i + b*1i)) + (32*a + 80*b)/(5*f*(6*a
+ 6*b)*(a*1i + b*1i)))*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)
/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i +
 f*x*2i) - 1)^3*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e*1i - f*x*1i)/
2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*((a*(2*a + b)*32i)/(15*f*(a*1i + b*1i)^
2*(8*a + 8*b)) + (a*(2*a + 3*b)*32i)/(15*f*(a*1i + b*1i)^2*(8*a + 8*b)))*(
2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^
2*(exp(e*2i + f*x*2i) + 1)) + ((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
 f*x*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*((96*
a + 32*b)/(5*f*(a*1i + b*1i)*(16*a + 16*b)) + (160*a + 160*b)/(5*f*(a*1i +
 b*1i)*(16*a + 16*b)) + (256*a + 320*b)/(5*f*(a*1i + b*1i)*(16*a + 16*b)))
)/((exp(e*2i + f*x*2i) - 1)^4*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e
*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + e
xp(e*4i + f*x*4i) + 1)*32i)/(f*(exp(e*2i + f*x*2i) - 1)^5*(exp(e*2i + f*x*
2i) + 1)*(10*a + 10*b)) - (8*a^2*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i
 + f*x*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/(1
5*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)*(a*1i + b*1i)^3)
```

Reduce [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^6}{\sec(fx+e)^2 b+a}dx$$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**6)/(sec(e + f*x)**2*b + a),
x)
```

3.106 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{b(a+b)^2\sec(e+fx)}{a^4f\sqrt{a+b\sec^2(e+fx)}} - \frac{(15a^2+50ab+33b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^4f} + \frac{(10a+9b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a^2f}$$

output

```
-b*(a+b)^2*sec(f*x+e)/a^4/f/(a+b*sec(f*x+e)^2)^(1/2)-1/15*(15*a^2+50*a*b+3
3*b^2)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^4/f+1/15*(10*a+9*b)*cos(f*x+e
)^3*(a+b*sec(f*x+e)^2)^(1/2)/a^3/f-1/5*cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/
2)/a^2/f
```

Mathematica [A] (verified)

Time = 6.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.82

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(150a^3+1528a^2b+2944ab^2+1536b^3+a(125a^2+544ab+384b^2)\cos(2(e+\right.}{960a^4f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/960*((a + 2*b + a*Cos[2*(e + f*x)])*(150*a^3 + 1528*a^2*b + 2944*a*b^2
+ 1536*b^3 + a*(125*a^2 + 544*a*b + 384*b^2)*Cos[2*(e + f*x)] - 2*a^2*(11*
a + 12*b)*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)])*Sec[e + f*x]^3)/(a^4*
f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4622, 365, 25, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^5}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \; 4622$$

$$\frac{\int \frac{\cos^6(e+fx)(1-\sec^2(e+fx))^2}{(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int -\frac{\cos^4(e+fx)\left(2(5a+3b)-5a\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(2(5a+3b)-5a\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$\Big\downarrow$ 359

$$\frac{-\frac{-\frac{\left(15a^2+40ab+24b^2\right)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a}-\frac{2(5a+3b)\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{5a}-\frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$\Big\downarrow$ 245

$$\frac{-\frac{-\frac{\left(15a^2+40ab+24b^2\right)\left(-\frac{2b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{a}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}\right)}{3a}-\frac{2(5a+3b)\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{5a}-\frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$\Big\downarrow$ 208

$$\frac{-\frac{-\frac{\left(15a^2+40ab+24b^2\right)\left(-\frac{2b\sec(e+fx)}{a^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}\right)}{3a}-\frac{2(5a+3b)\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{5a}-\frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/5*Cos[e + f*x]^5/(a*Sqrt[a + b*Sec[e + f*x]^2]) - ((-2*(5*a + 3*b)*Cos
[e + f*x]^3)/(3*a*Sqrt[a + b*Sec[e + f*x]^2]) - ((15*a^2 + 40*a*b + 24*b^2
)*(-(Cos[e + f*x]/(a*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^
2*Sqrt[a + b*Sec[e + f*x]^2])))/(3*a))/(5*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 401.53 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

method	result
default	$-\frac{(a+b)^6 a^2 \left(24a\cos(fx+e)^2 b^2+\left(-6\cos(fx+e)^4+40\cos(fx+e)^2\right) b\, a^2+80a\, b^2+30a^2 b+\left(3\cos(fx+e)^6-10\cos(fx+e)^4+15\cos(f\right.\right.}{15 f\left(\sqrt{-ab}+a\right)^6\left(\sqrt{-ab}-a\right)^6\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f*(a+b)^6*a^2/((-a*b)^(1/2)+a)^6/((-a*b)^(1/2)-a)^6*(24*a*cos(f*x+e)
^2*b^2+(-6*cos(f*x+e)^4+40*cos(f*x+e)^2)*b*a^2+80*a*b^2+30*a^2*b+(3*cos(f*
x+e)^6-10*cos(f*x+e)^4+15*cos(f*x+e)^2)*a^3+48*b^3)*(b+a*cos(f*x+e)^2)/(a+
b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.86

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{\left(3\,a^3\cos\left(fx+e\right)^7-2\left(5\,a^3+3\,a^2b\right)\cos\left(fx+e\right)^5+\left(15\,a^3+40\,a^2b+24\,ab^2\right)\cos\left(fx+e\right)^3+2\left(15\,a^2b+\right.\right.}{15\left(a^5f\cos\left(fx+e\right)^2+a^4bf\right)}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(3*a^3*cos(f*x + e)^7 - 2*(5*a^3 + 3*a^2*b)*cos(f*x + e)^5 + (15*a^3
 + 40*a^2*b + 24*a*b^2)*cos(f*x + e)^3 + 2*(15*a^2*b + 40*a*b^2 + 24*b^3)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x +
 e)^2 + a^4*b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.57

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2} - \frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^3} + \frac{15\,b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^2\cos(fx+e)} + \cdots}{\cdots}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - 10*((a + b/cos(f*x
 + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e
))/a^3 + 15*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 30*b^2/(sqrt
(a + b/cos(f*x + e)^2)*a^3*cos(f*x + e)) + 15*b^3/(sqrt(a + b/cos(f*x + e)
^2)*a^4*cos(f*x + e)) + 3*((a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5
*(a + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e
)^2)*b^2*cos(f*x + e))/a^4)/f
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^5}{\sec\left(fx+e\right)^4 b^2+2\sec\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**5)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.107 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{b(a+b)\sec(e+fx)}{a^3f\sqrt{a+b\sec^2(e+fx)}} - \frac{(3a+5b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^3f} + \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f}$$

output
```
-b*(a+b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)-1/3*(3*a+5*b)*cos(f*x+e
)*(a+b*sec(f*x+e)^2)^(1/2)/a^3/f+1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)
/a^2/f
```

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(9a^2+64ab+64b^2+8a(a+2b)\cos(2(e+fx))-a^2\cos(4(e+fx))\right)\sec^3(e}{48a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/48*((a + 2*b + a*Cos[2*(e + f*x)])*(9*a^2 + 64*a*b + 64*b^2 + 8*a*(a +
2*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^3)/(a^3*f*(a +
b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4622, 25, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^3}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a+4b)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a}+\frac{\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{(3a+4b)\left(-\frac{2b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{a}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}\right)}{3a}+\frac{\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{(3a+4b)\left(-\frac{2b\sec(e+fx)}{a^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}\right)}{3a}+\frac{\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]^3/(3*a*Sqrt[a + b*Sec[e + f*x]^2]) + ((3*a + 4*b)*(-(Cos[e +
 f*x]/(a*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^2*Sqrt[a + b
*Sec[e + f*x]^2])))/(3*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{a(a+b)^4\left(b+a\cos(fx+e)^2\right)\left(-4\cos(fx+e)^2ab-6ab+\left(\cos(fx+e)^4-3\cos(fx+e)^2\right)a^2-8b^2\right)\sec(fx+e)^3}{3f\left(\sqrt{-ab}-a\right)^4\left(\sqrt{-ab}+a\right)^4\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$	113

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*a/((-a*b)^(1/2)-a)^4/((-a*b)^(1/2)+a)^4*(a+b)^4*(b+a*cos(f*x+e)^2)*(
-4*cos(f*x+e)^2*a*b-6*a*b+(cos(f*x+e)^4-3*cos(f*x+e)^2)*a^2-8*b^2)/(a+b*se
c(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(a^2\cos\left(fx+e\right)^5-\left(3\,a^2+4\,ab\right)\cos\left(fx+e\right)^3-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right.}{3\left(a^4f\cos\left(fx+e\right)^2+a^3bf\right)}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a^2*cos(f*x + e)^5 - (3*a^2 + 4*a*b)*cos(f*x + e)^3 - 2*(3*a*b + 4*b^
2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^4*f*cos(f*
x + e)^2 + a^3*b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.31

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=$$

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^3}+\frac{3b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^2\cos(fx+e)}+\frac{\cdots}{\sqrt{a+\cdots}}}{3f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - ((a + b/cos(f*x + e)
^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^
3 + 3*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 3*b^2/(sqrt(a + b/
cos(f*x + e)^2)*a^3*cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{f\left(\frac{3\,(ab+b^2)}{\sqrt{a\cos(fx+e)^2+b}a^3f^2} - \frac{\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}a^6f^4-3\sqrt{a\cos(fx+e)^2+b}a^7f^4-6\sqrt{a\cos(fx+e)^2+b}a^6bf^4}{a^9f^6}\right)}{3\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3*f*(3*(a*b + b^2)/(sqrt(a*cos(f*x + e)^2 + b)*a^3*f^2) - ((a*cos(f*x +
 e)^2 + b)^(3/2)*a^6*f^4 - 3*sqrt(a*cos(f*x + e)^2 + b)*a^7*f^4 - 6*sqrt(a
*cos(f*x + e)^2 + b)*a^6*b*f^4)/(a^9*f^6))/sgn(cos(f*x + e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^3}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**3)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.108 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(1/2)-2*b*sec(f*x+e)/a^2/f/(a+b*sec(f*x
+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))(a+4b+a\cos(2(e+fx)))\sec^3(e+fx)}{4a^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 4*b + a*Cos[2*(e + f*x)])*Sec[e
+ f*x]^3)/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4622, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \frac{\cos^2(e+fx)}{(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{-\frac{2b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{a}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{-\frac{2b\sec(e+fx)}{a^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cos[e + f*x]/(a*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^2*
Sqrt[a + b*Sec[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}-\frac{2b\sec(fx+e)}{a^2\sqrt{a+b\sec(fx+e)^2}}}{f}$	59
default	$\frac{-\frac{1}{a\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}-\frac{2b\sec(fx+e)}{a^2\sqrt{a+b\sec(fx+e)^2}}}{f}$	59

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)-2*b/a^2*sec(f*x+e)/(a+b*sec(
f*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\left(a\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{a^3f\cos\left(fx+e\right)^2+a^2bf}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-(a*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/(a^3*f*cos(f*x + e)^2 + a^2*b*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2}+\frac{b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}a^2\cos(fx+e)}}{f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 + b/(sqrt(a + b/cos(f*x + e)
^2)*a^2*cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a\cos\left(fx+e\right)^2+b}+\frac{b}{\sqrt{a\cos(fx+e)^2+b}}}{a^2f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-(sqrt(a*cos(f*x + e)^2 + b) + b/sqrt(a*cos(f*x + e)^2 + b))/(a^2*f*sgn(co
s(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.50

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{e^{-e\,1i-f\,x\,1i}\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+8\,b\,e^{e\,2i+f\,x\,2i}\right)}{2\,a^2\,f\,\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+4\,b\,e^{e\,2i+f\,x\,2i}\right)}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
-(exp(- e*1i - f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1
i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(
e*4i + f*x*4i) + 8*b*exp(e*2i + f*x*2i)))/(2*a^2*f*(a + 2*a*exp(e*2i + f*x
*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i)))
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input

```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output

```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x))/(sec(e + f*x)**4*b**2 + 2*s
ec(e + f*x)**2*a*b + a**2),x)
```

3.109 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}f}-\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2)/f-b*
sec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(b\sqrt{a+b}+a\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\sqrt{a+b-a\sin^2(e+f}{2a(a+b)^{3/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(b*Sqrt[a + b] + a*Arc
Tanh[Sqrt[a + b - a*Sin[e + f*x]^2/Sqrt[a + b]]*Sqrt[a + b - a*Sin[e + f*
x]^2]))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4622, 25, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(e+fx)\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 4622}$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 296}$$

$$\frac{-\frac{\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}-\frac{b\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{-\frac{\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}-\frac{b\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}-\frac{b\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/(a + b)^
(3/2)) - (b*Sec[e + f*x])/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(72) = 144$.

Time = 1.40 (sec) , antiderivative size = 907, normalized size of antiderivative = 11.34

method	result	size
default	Expression too large to display	907

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^3*(sec(f*x+e)+1)-1/2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a^2*b*(1+sec(f*x+e)+sec(f*x+e)^2+sec(f*x+e)^3)-1/2*
ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x
+e)*a+b)/(1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2
*(sec(f*x+e)^2+sec(f*x+e)^3)-1/2*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^3*(sec(f*x+e)+1)-1/2*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^2*b*(1+sec(f*x+e)+sec(f*x+e)^2+sec(f*x+e)^3)-1
/2*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)
/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2*(se...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(72) = 144.

Time = 0.16 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.41

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{2\,(ab+b^2)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)-\left(a^2\cos\left(fx+e\right)^2+ab\right)\sqrt{a+b}}{2\left((a^4+2\,a^3b+a^2b^2)f\cos\left(fx+e\right)^2+\right.}\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*(a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e) - (a^2*cos(f*x + e)^2 + a*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*s
qrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a +
2*b)/(cos(f*x + e)^2 - 1)))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 +
(a^3*b + 2*a^2*b^2 + a*b^3)*f), ((a^2*cos(f*x + e)^2 + a*b)*sqrt(-a - b)*a
rctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)/(a*cos(f*x + e)^2 + b)) - (a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3
*b + 2*a^2*b^2 + a*b^3)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(72) = 144$.

Time = 0.64 (sec) , antiderivative size = 503, normalized size of antiderivative = 6.29

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(2*((a*b^2 + b^3)*tan(1/2*f*x + 1/2*e)^2/(a^3*b*sgn(cos(f*x + e)) + 2
*a^2*b^2*sgn(cos(f*x + e)) + a*b^3*sgn(cos(f*x + e))) + (a*b^2 + b^3)/(a^3
*b*sgn(cos(f*x + e)) + 2*a^2*b^2*sgn(cos(f*x + e)) + a*b^3*sgn(cos(f*x + e
))))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - log(abs(-sqrt(a +
 b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x
 + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
+ b) - sqrt(a + b)))/((a + b)^(3/2)*sgn(cos(f*x + e))) - log(abs((sqrt(a +
 b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x
 + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
+ b))*sqrt(a + b) - a + b))/((a + b)^(3/2)*sgn(cos(f*x + e))) + log(abs((s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))*sqrt(a + b) - a - b))/((a + b)^(3/2)*sgn(cos(f*x + e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x))/(sec(e + f*x)**4*b**2 + 2*s
ec(e + f*x)**2*a*b + a**2),x)
```

3.110 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-1/2*(a-2*b)*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b
)^(5/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-3/2*b
*sec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.77

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\csc^2(e+fx)-(a-2b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{a\sin^2(e+f}{a+b}\right.\right.}{4(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Csc[e + f*x]^2 - (a - 2*b)*H
ypergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*
x]^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4622, 373, 402, 25, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \frac{\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int \frac{a-2b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{2(a+b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{a(a-2b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}}{2(a+b)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{\int \frac{a(a-2b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}+\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{(a-2b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}+\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{(a-2b)\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}+\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}+\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sec[e + f*x]/(2*(a + b)*(1 - Sec[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2])
- (((a - 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2
]])/(a + b)^(3/2) + (3*b*Sec[e + f*x])/((a + b)*Sqrt[a + b*Sec[e + f*x]^2]
))/(2*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1189 vs. $2(110) = 220$.

Time = 1.51 (sec) , antiderivative size = 1190, normalized size of antiderivative = 9.44

method	result	size
default	Expression too large to display	1190

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(a+b)^(9/2)*((a+b)^(5/2)*b+(a+b)^(5/2)*a+((1-cos(f*x+e))^6*csc(f*x+e
)^6-(1-cos(f*x+e))^4*csc(f*x+e)^4-(1-cos(f*x+e))^2*csc(f*x+e)^2)*a*(a+b)^(
5/2)+((1-cos(f*x+e))^6*csc(f*x+e)^6+11*(1-cos(f*x+e))^4*csc(f*x+e)^4+11*(1
-cos(f*x+e))^2*csc(f*x+e)^2)*b*(a+b)^(5/2)+4*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos
(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*(1-cos(f*x+e))^2*csc(f*x+e)^2-12*b^2*ln(
-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+
b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+c
os(f*x+e)))*a*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(1-cos(f*x+e))^2
*csc(f*x+e)^2-8*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*((a+b)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))
*b^3*(1-cos(f*x+e))^2*csc(f*x+e)^2+4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*(1-cos(f*x+e))^2*csc(f*x+e)^2-12*b^2*l
n(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+
e)*a+b)/(1+cos(f*x+e)))*a*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(110) = 220$.

Time = 0.22 (sec) , antiderivative size = 556, normalized size of antiderivative = 4.41

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{\left((a^2-2\,ab)\cos\left(fx+e\right)^4-(a^2-3\,ab+2\,b^2)\cos\left(fx+e\right)^2-ab+2\,b^2\right.}{4\left((a^4+3\,a^3b+3\,a^2b^2+\right.}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((a^2 - 2*a*b)*cos(f*x + e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^
2 - a*b + 2*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x +
e)^2 - 1)) - 2*((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x
 + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*
x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f), 1/2*(((a^2 - 2*a*b)*cos
(f*x + e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-a
- b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/(a*cos(f*x + e)^2 + b)) + ((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*
(a*b + b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*
b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(110) = 220$.

Time = 0.88 (sec) , antiderivative size = 903, normalized size of antiderivative = 7.17

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*((((a^5*b*sgn(cos(f*x + e)) + 4*a^4*b^2*sgn(cos(f*x + e)) + 6*a^3*b^3
*sgn(cos(f*x + e)) + 4*a^2*b^4*sgn(cos(f*x + e)) + a*b^5*sgn(cos(f*x + e))
)*tan(1/2*f*x + 1/2*e)^2/(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*
a^2*b^5 + a*b^6) - 2*(a^5*b*sgn(cos(f*x + e)) - 2*a^4*b^2*sgn(cos(f*x + e)
) - 12*a^3*b^3*sgn(cos(f*x + e)) - 14*a^2*b^4*sgn(cos(f*x + e)) - 5*a*b^5*
sgn(cos(f*x + e)))/(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a^3*b^4 + 5*a^2*b^
5 + a*b^6))*tan(1/2*f*x + 1/2*e)^2 + (a^5*b*sgn(cos(f*x + e)) + 12*a^4*b^2
*sgn(cos(f*x + e)) + 30*a^3*b^3*sgn(cos(f*x + e)) + 28*a^2*b^4*sgn(cos(f*x
 + e)) + 9*a*b^5*sgn(cos(f*x + e)))/(a^6*b + 5*a^5*b^2 + 10*a^4*b^3 + 10*a
^3*b^4 + 5*a^2*b^5 + a*b^6))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x
 + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
+ b) - 4*(a - 2*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)
^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a^2 + 2*a*b + b^
2)*sqrt(-a - b)*sgn(cos(f*x + e))) - 2*(a - 2*b)*log(abs((sqrt(a + b)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e
)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sq
rt(a + b) - a + b))/((a^2 + 2*a*b + b^2)*sqrt(a + b)*sgn(cos(f*x + e))) +
2*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^3}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**3)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.111 $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{3a(a-4b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{7/2}f} - \frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e+fx)}{8(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-3/8*a*(a-4*b)*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a
+b)^(7/2)/f-5/8*a*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-1/8*(13*a-2*
b)*b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)^2\csc^4(e+fx)-a(a-4b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},1-\frac{a\sin^2(e}{a+}\right.\right.}{8(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/8*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)^2*Csc[e + f*x]^4 - a*(a - 4*
b)*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e
+ f*x]^3)/((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4622, 25, 372, 27, 402, 25, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^5\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\int \frac{a\left(4\sec^2(e+fx)+1\right)}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a\int \frac{4\sec^2(e+fx)+1}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{a\left(\frac{\int -\frac{-10b\sec^2(e+fx)+3a-2b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{2(a+b)}+\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}\right)}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int \frac{-10b\sec^2(e+fx)+3a-2b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{2(a+b)}\right)}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{3a(a-4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}}{2(a+b)}\right)}{4(a+b)}-\frac{\sec(e+}{4(a+b)(1-\sec^2(e+fx)}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{3(a-4b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}+\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}\right)}{4(a+b)}-\frac{\cdots}{4(a+b)(1-\sec^2(\cdots}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{3(a-4b)\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}+\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}\right)}{4(a+b)}-\frac{\sec(e\cdots}{4(a+b)(1-\sec^2(e+fx\cdots}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{3(a-4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}+\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}\right)}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\cdots}}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/4*Sec[e + f*x]/((a + b)*(1 - Sec[e + f*x]^2)^2*Sqrt[a + b*Sec[e + f*x]
^2]) + (a*((5*Sec[e + f*x])/(2*(a + b)*(1 - Sec[e + f*x]^2)*Sqrt[a + b*Sec
[e + f*x]^2]) - ((3*(a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a +
b*Sec[e + f*x]^2]])/(a + b)^(3/2) + ((13*a - 2*b)*b*Sec[e + f*x])/(a*(a +
b)*Sqrt[a + b*Sec[e + f*x]^2]))/(2*(a + b))))/(4*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1985 vs. $2(157) = 314$.

Time = 1.50 (sec) , antiderivative size = 1986, normalized size of antiderivative = 11.22

method	result	size
default	Expression too large to display	1986

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f/(a+b)^(13/2)/(a+b*sec(f*x+e)^2)^(3/2)*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^6*(3+3*sec(f*x+e))+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^5*b*(3*tan(f*x
+e)^2+3*tan(f*x+e)^2*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(
f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b^2*(-27-27*sec(f*x+e)-3*sec(f*x+e)^2-3*se
c(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(
(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*
x+e)))*a^3*b^3*(-33-33*sec(f*x+e)-27*sec(f*x+e)^2-27*sec(f*x+e)^3)+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^4*(-12
-12*sec(f*x+e)-33*sec(f*x+e)^2-33*sec(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(157) = 314$.

Time = 0.26 (sec) , antiderivative size = 882, normalized size of antiderivative = 4.98

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^2)*co
s(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e)^2)
*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(
3*(a^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5 - (5*a^3 - 16*a^2*b - 17*a*b^2
+ 4*b^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a
^2*b^3 + a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^
3 - 2*a*b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b
^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
4*a*b^4 + b^5)*f), 1/8*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2
*b + 4*a*b^2)*cos(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)
*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e)/(a*cos(f*x + e)^2 + b)) + (3*(a^3 - 3*a^2
*b - 4*a*b^2)*cos(f*x + e)^5 - (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b^3)*cos(f
*x + e)^3 - (13*a^2*b + 11*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4
)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 -
b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 -
 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b...
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. $2(157) = 314$.

Time = 1.38 (sec) , antiderivative size = 1612, normalized size of antiderivative = 9.11

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/64*(((((a^8*b + 7*a^7*b^2 + 21*a^6*b^3 + 35*a^5*b^4 + 35*a^4*b^5 + 21*a
^3*b^6 + 7*a^2*b^7 + a*b^8)*tan(1/2*f*x + 1/2*e)^2/(a^9*b*sgn(cos(f*x + e)
) + 8*a^8*b^2*sgn(cos(f*x + e)) + 28*a^7*b^3*sgn(cos(f*x + e)) + 56*a^6*b^
4*sgn(cos(f*x + e)) + 70*a^5*b^5*sgn(cos(f*x + e)) + 56*a^4*b^6*sgn(cos(f*
x + e)) + 28*a^3*b^7*sgn(cos(f*x + e)) + 8*a^2*b^8*sgn(cos(f*x + e)) + a*b
^9*sgn(cos(f*x + e))) + (7*a^8*b + 39*a^7*b^2 + 87*a^6*b^3 + 95*a^5*b^4 +
45*a^4*b^5 - 3*a^3*b^6 - 11*a^2*b^7 - 3*a*b^8)/(a^9*b*sgn(cos(f*x + e)) +
8*a^8*b^2*sgn(cos(f*x + e)) + 28*a^7*b^3*sgn(cos(f*x + e)) + 56*a^6*b^4*sg
n(cos(f*x + e)) + 70*a^5*b^5*sgn(cos(f*x + e)) + 56*a^4*b^6*sgn(cos(f*x +
e)) + 28*a^3*b^7*sgn(cos(f*x + e)) + 8*a^2*b^8*sgn(cos(f*x + e)) + a*b^9*s
gn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 - (17*a^8*b - 9*a^7*b^2 - 291*a^
6*b^3 - 725*a^5*b^4 - 765*a^4*b^5 - 363*a^3*b^6 - 49*a^2*b^7 + 9*a*b^8)/(a
^9*b*sgn(cos(f*x + e)) + 8*a^8*b^2*sgn(cos(f*x + e)) + 28*a^7*b^3*sgn(cos(
f*x + e)) + 56*a^6*b^4*sgn(cos(f*x + e)) + 70*a^5*b^5*sgn(cos(f*x + e)) +
56*a^4*b^6*sgn(cos(f*x + e)) + 28*a^3*b^7*sgn(cos(f*x + e)) + 8*a^2*b^8*sg
n(cos(f*x + e)) + a*b^9*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 + (9*a^
8*b + 113*a^7*b^2 + 425*a^6*b^3 + 745*a^5*b^4 + 675*a^4*b^5 + 299*a^3*b^6
+ 43*a^2*b^7 - 5*a*b^8)/(a^9*b*sgn(cos(f*x + e)) + 8*a^8*b^2*sgn(cos(f*x +
 e)) + 28*a^7*b^3*sgn(cos(f*x + e)) + 56*a^6*b^4*sgn(cos(f*x + e)) + 70*a^
5*b^5*sgn(cos(f*x + e)) + 56*a^4*b^6*sgn(cos(f*x + e)) + 28*a^3*b^7*sgn...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{1}{\sin(e+f\,x)^5\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^5}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**5)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.112 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 242

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{5(a+b)^2(a+7b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} - \frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(81a^2+190ab+105b^2)\tan(e+fx)}{48a^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
5/16*(a+b)^2*(a+7*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(9/2)/f-1/48*(a+b)*(33*a+35*b)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f
*x+e)^2)^(1/2)+1/24*(9*a+7*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x
+e)^2)^(1/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-
1/48*b*(81*a^2+190*a*b+105*b^2)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(1/2
)
```

Mathematica [A] (verified)

Time = 6.03 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.06

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(120(a+b)^2(a+7b)\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\right.}{}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(120*(a + b)^2*(a + 7*b)*Ar
cSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) -
2*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]
*(37*a^3 + 439*a^2*b + 830*a*b^2 + 420*b^3 + a*(29*a^2 + 108*a*b + 70*b^2)
*Cos[2*(e + f*x)] - 7*a^2*(a + b)*Cos[4*(e + f*x)] + a^3*Cos[6*(e + f*x)])
*Sin[e + f*x]))/(1536*a^(9/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*S
qrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4620, 372, 440, 27, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^6}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$\downarrow$ 372

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int\frac{\tan^2(e+fx)\left(2(b-3(a+b))\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{6a}}{f}$$

$\downarrow$ 440

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{\int\frac{(a+b)\left(-4(6a+7b)\tan^2(e+fx)+9a+7b\right)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{4a}-\frac{(9a+7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}}}{6a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{(a+b)\int\frac{-4(6a+7b)\tan^2(e+fx)+9a+7b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{4a}-\frac{(9a+7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+}}}{6a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{(a+b)\left(\frac{(33a+35b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int\frac{15a^2+54ba+35b^2-2b(33a+35b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx}{2a}\right)}{4a}}{6a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{(a+b)\left(\frac{(33a+35b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{\int\frac{15(a+b)^2(a+7b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-b}{2a}\right)}{4a}}{6a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{(a+b)\left(\frac{(33a+35b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{15(a+b)(a+7b)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d}{a}}{2a}\right.}{4a}}{6a}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{(a+b)\left(\frac{(33a+35b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{15(a+b)(a+7b)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)}}}{a}}{2a}\right.}{4a}}{6a}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{(a+b)\left(\frac{(33a+35b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{15(a+b)(a+7b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b(81a^2}{a(a}}{2a}\right.}{4a}}{6a}}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3*Sqrt[a + b + b*Tan[e + f*x]^2]
) - (-1/4*((9*a + 7*b)*Tan[e + f*x])/(a*(1 + Tan[e + f*x]^2)^2*Sqrt[a + b
+ b*Tan[e + f*x]^2]) + ((a + b)*(((33*a + 35*b)*Tan[e + f*x])/(2*a*(1 + Ta
n[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((15*(a + b)*(a + 7*b)*Arc
Tan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) - (b*(
81*a^2 + 190*a*b + 105*b^2)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e
+ f*x]^2]))/(2*a)))/(4*a))/(6*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 813, normalized size of antiderivative = 3.36

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^
2*b^2 + 7*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b
+ 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
 a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (
a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^4*cos(f*x + e)^7 - 2*(13*a^
4 + 7*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x + e
)^3 + (81*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f),
 -1/192*(15*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^
2*b^2 + 7*a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*
b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^4*cos(
f*x + e)^7 - 2*(13*a^4 + 7*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35
*a^2*b^2)*cos(f*x + e)^3 + (81*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f
*x + e)^2 + a^5*b*f)]
```

Sympy [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\sin^6{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^6}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**6)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.113 $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{3(a+b)(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
3/8*(a+b)*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^
(7/2)/f-5/8*(a+b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1
/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/8*b*(13*a+15*b
)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.31

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(24(a^2+6ab+5b^2)\arcsin\left(\frac{\sqrt{a}\sin}{\sqrt{}}\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(24*(a^2 + 6*a*b + 5*b^2)*A
rcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) -
 2*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)
]*(7*a^2 + 62*a*b + 60*b^2 + 2*a*(3*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*
(e + f*x)])*Sin[e + f*x]))/(256*a^(7/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^
2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4620, 372, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)^4}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 372$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int\frac{-4(a+b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int\frac{(a+b)\left(-10b\tan^2(e+fx)+3a+5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\int\frac{-10b\tan^2(e+fx)+3a+5b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\left(\frac{\int\frac{3(a+b)(a+5b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{}{a}\right.}{2a}}{4a}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\left(\frac{3(a+5b)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e-}{a}\right.}{2a}}{4a}}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\left(\frac{3(a+5b)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}\right.}{2a}}{4a}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\left(\frac{3(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b(13a+15b)}{a(a+b)\sqrt{a+b}}\right.}{2a}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*Sqrt[a + b + b*Tan[e + f*x]^2])
- ((5*(a + b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan[e
 + f*x]^2]) - ((a + b)*((3*(a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a
+ b + b*Tan[e + f*x]^2]])/a^(3/2) - (b*(13*a + 15*b)*Tan[e + f*x])/(a*(a +
 b)*Sqrt[a + b + b*Tan[e + f*x]^2])))/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(155) = 310$.

Time = 10.30 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.90

method	result
default	$\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a^3 \ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(3+3\sec(fx+e))+\sqrt{\frac{b+a\cos}{(1+\cos}}$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/a^3/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(3/2)*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)*(3+3*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a^2*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+
e)*a)*(18+18*sec(f*x+e)+3*sec(f*x+e)^2+3*sec(f*x+e)^3)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*(15+15*sec(f*x+e)+18*sec(f*x+e)^2+18*sec(f*x+e)
^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(15*sec(f*x+e)^2+15*sec(f*
x+e)^3)+sin(f*x+e)*cos(f*x+e)*(2*cos(f*x+e)^2-5)*a^3*(-a)^(1/2)+(-3*cos(f*
x+e)^2-18)*(-a)^(1/2)*a^2*b*tan(f*x+e)+(-a)^(1/2)*a*b^2*(-20*tan(f*x+e)-13
*tan(f*x+e)*sec(f*x+e)^2)-15*(-a)^(1/2)*b^3*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.02

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^2*b + 6*a*b^2 + 5*b^3 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e
)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^
3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e)) - 8*(2*a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13
*a^2*b + 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/32*(3*(a^2*b + 6*a*b
^2 + 5*b^3 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)
*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3
*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e))) - 4*(2*a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^
2*b + 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin{(fx+e)}^4}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^4}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2b+a}\,\sin\left(fx+e\right)^4}{\sec\left(fx+e\right)^4b^2+2\sec\left(fx+e\right)^2ab+a^2}dx$$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**4)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.114 $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
1/2*(a+3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/
f-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-3/2*b*tan(f*x+e
)/a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.57

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(4(a+3b)\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(\cdots}{32a^{5/2}\sqrt{a+\cdots}}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(4*(a + 3*b)*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*Sqrt[2]*Sq
rt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(a + 6*b +
a*Cos[2*(e + f*x)])*Sin[e + f*x]))/(32*a^(5/2)*Sqrt[a + b]*f*(a + b*Sec[e
+ f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4620, 373, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\int \frac{-2b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}-\frac{\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{(a+b)(a+3b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{3b\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{2a}-\frac{\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{(a+3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{3b\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{(a+3b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{3b\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{3b\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*Tan[e + f*x]/(a*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2])
 + (((a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2
]])/a^(3/2) - (3*b*Tan[e + f*x])/(a*Sqrt[a + b + b*Tan[e + f*x]^2]))/(2*a)
)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(105) = 210$.

Time = 6.74 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.96

method	result
default	$-\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a^2 \ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(-1-\sec(fx+e))+\sqrt{\frac{b+}{(1+}}}{}$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(3/2)*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*(-1-sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)
*a)*(-3-3*sec(f*x+e)-sec(f*x+e)^2-sec(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*(-3*sec(f*x+e)^2-3*sec(f*x+e)^3)+(-a)^(1/2)*a^2*cos(f*x+
e)*sin(f*x+e)+4*b*a*(-a)^(1/2)*tan(f*x+e)+3*(-a)^(1/2)*b^2*tan(f*x+e)*sec(
f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(105) = 210$.

Time = 0.49 (sec) , antiderivative size = 607, normalized size of antiderivative = 5.02

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(((a^2 + 3*a*b)*cos(f*x + e)^2 + a*b + 3*b^2)*sqrt(-a)*log(128*a^4*
cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4
- 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*
x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2
)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(a^2*cos(f*x
 + e)^3 + 3*a*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), -1/8*(((a^2 + 3*a*b)*cos(f
*x + e)^2 + a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2
 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^
2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(a^2*cos(f*x + e)^3
 + 3*a*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin{(fx+e)}^2}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+fx)^2}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^2}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**2)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.115 $\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+
e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.

Time = 1.00 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+\right.}{4a^{3/2}(a+b)f\,(a+b\sec^2(e+f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt
[a]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/
2)*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/
(a + b)])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4616, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/a^(3/2) - (
b*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(69) = 138$.

Time = 4.32 (sec) , antiderivative size = 465, normalized size of antiderivative = 6.04

method	result
default	$\frac{-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(-1-\sec(fx+e))-\sqrt{\frac{b+a}{(1+}}}{}$

input
```
int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-1-sec(f*x+e))-((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-1-sec(f*x+e)-sec(f*x+e)^2-sec(f*x+e)
^3)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-sec(f*x+e)^2-sec(f*x+e)^
3)-b*a*(-a)^(1/2)*tan(f*x+e)-(-a)^(1/2)*b^2*tan(f*x+e)*sec(f*x+e)^2)/(a+b)
/a/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(69) = 138$.

Time = 0.26 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.81

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f
*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e
)^2 + a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^
2 + (a^3*b + a^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. $2(69) = 138$.

Time = 0.37 (sec) , antiderivative size = 2055, normalized size of antiderivative = 26.69

$$\int \frac{1}{(a + b\sec^2(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2
*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2
*e) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2
*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2
*(a*b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) +
2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x +
 2*e) + a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b +
4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 +
 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))^(1/4)*(((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(
2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 +
(a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*si
n(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a
^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a
^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*...
```

Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}}{\sec\left(fx+e\right)^4 b^2+2\sec\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(1/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)/(sec(e + f*x)**4*b**2 + 2*sec(e + f*x)**2*
a*b + a**2),x)
```

3.116 $\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2*b*tan(f*x+e)/(a+b)^2/f/(a
+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))(a+3b+(a-b)\cos(2(e+fx)))\csc(e+fx)\sec^3(e+fx)}{4(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 3*b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]*Sec[e + f*x]^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4620, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{-\frac{2b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{-\frac{2b\tan(e+fx)}{(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cot[e + f*x]/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f
*x])/((a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\left(b+a\cos\left(fx+e\right)^{2}\right)\left(a\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)^{2}b+2b\right)\sec\left(fx+e\right)^{3}\csc\left(fx+e\right)}{f\left(a^{2}+2ab+b^{2}\right)\left(a+b\sec\left(fx+e\right)^{2}\right)^{\frac{3}{2}}}$	84

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a^2+2*a*b+b^2)*(b+a*cos(f*x+e)^2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+2*b
)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3*csc(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.50

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{\left((a^3+2\,a^2b+ab^2)f\cos(fx+e)^2+(a^2b+2\,ab^2+b^3)f\right)\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 + (a^2*b + 2*a*b
^2 + b^3)*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\frac{2\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2}+\frac{1}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(2*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + 1/(sqrt(b*
tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)))/f
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\csc(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 20.27 (sec) , antiderivative size = 2151, normalized size of antiderivative = 31.63

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
-((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e
*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(exp(e*4i + f*x*4i)*(((a + 4*b)*((
(a + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a
 + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*
b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^
2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a - (((a + 3
*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/
(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (((a^2*(a + 3*b)*(a*(a
- b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*
b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (a^3*(a + 3*b)*3i)
/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*
b + a^2)*(a*b^2 + a^2*b))))/a + ((a + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) -
(a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a
*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(
a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2
)*(a*b^2 + a^2*b))))/a + (((a + 3*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)
^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a
+ 3*b)) + (((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a
+ 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b
)*(a + 3*b)) - (a^3*(a + 3*b)*1i)/(4*f*(a*b + a^2)*(a*b^2 + a^2*b))) + ...
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^2}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**2)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.117 $\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{ab\tan(e+fx)}{(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(3a-2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^3f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f}$$

output
```
-a*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(3*a-2*b)*cot(f*x
+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f-1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e
)^2)^(1/2)/(a+b)^2/f
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\left(-2a(a-3b)+(a^2-2ab-3b^2)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)\right)\sec^2(}{6(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(-2*a*(a - 3*b) + (a^2 - 2*a*b - 3*b^
2)*Csc[e + f*x]^2 + (a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x])
/((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4620, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(e+fx)^4\,(a+b\sec(e+fx)^2)^{3/2}}dx$$
$$\Big\downarrow \text{4620}$$
$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a-b)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)}-\frac{\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{(3a-b)\left(-\frac{2b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)}-\frac{\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{(3a-b)\left(-\frac{2b\tan(e+fx)}{(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)}-\frac{\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((3*a - b)
*(-(Cot[e + f*x]/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) - (2*b*Tan[e +
f*x])/((a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(3*(a + b)))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

method	result
default	$\frac{\left(b+a\cos(fx+e)^{2}\right)\left(\left(-6\cos(fx+e)^{4}+10\cos(fx+e)^{2}\right)ab-6ab+\left(2\cos(fx+e)^{4}-3\cos(fx+e)^{2}\right)a^{2}+\left(-3\cos(fx+e)^{2}+2\right)b^{2}\right)\sec(f}{3f\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)\left(a+b\sec(fx+e)^{2}\right)^{\frac{3}{2}}}$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^3+3*a^2*b+3*a*b^2+b^3)*(b+a*cos(f*x+e)^2)*((-6*cos(f*x+e)^4+10*co
s(f*x+e)^2)*a*b-6*a*b+(2*cos(f*x+e)^4-3*cos(f*x+e)^2)*a^2+(-3*cos(f*x+e)^2
+2)*b^2)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3*csc(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.64

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{(2\,(a^2-3\,ab)\cos{(fx+e)}^5-(3\,a^2-10\,ab+3\,b^2)\cos{(fx+e)}^3-2\,(3\,ab-b^2)\cos{(fx+e)})}{3\,((a^4+3\,a^3b+3\,a^2b^2+ab^3)f\cos{(fx+e)}^4-(a^4+2\,a^3b-2\,ab^3-b^4)f\cos{(fx+e)}^2-(a^3b+3\,a^2b^2+}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a^2 - 3*a*b)*cos(f*x + e)^5 - (3*a^2 - 10*a*b + 3*b^2)*cos(f*x +
e)^3 - 2*(3*a*b - b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a
^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4)*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.36

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{6\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2} - \frac{8\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3} + \frac{3}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)\tan(fx+e)} - \frac{4\,b}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2\tan}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(6*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 8*b^2*
tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 3/(sqrt(b*tan(f*
x + e)^2 + a + b)*(a + b)*tan(f*x + e)) - 4*b/(sqrt(b*tan(f*x + e)^2 + a +
 b)*(a + b)^2*tan(f*x + e)) + 1/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*ta
n(f*x + e)^3))/f
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 32.87 (sec) , antiderivative size = 124682, normalized size of antiderivative = 1084.19

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
(a^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + (
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*128
i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 3
2*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(
cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)
*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) -
 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a
*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*
f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) - (5*a*(a + b/(((cos(2*f*x) -
 sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b
^2*f*3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + s...
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^4}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output

```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**4)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.118 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\begin{aligned}\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = &-\frac{a^2b\tan(e+fx)}{(a+b)^4f\sqrt{a+b+b\tan^2(e+fx)}}\\ &-\frac{(15a^2-20ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^4f}\\ &-\frac{(10a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f}\\ &-\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)^2f}\end{aligned}$$

output
```
-a^2*b*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^2-20*a*b
-2*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^4/f-1/15*(10*a+b)*cot(
f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f-1/5*cot(f*x+e)^5*(a+b+b*tan(
f*x+e)^2)^(1/2)/(a+b)^2/f
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.75

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\left(-8a^2(a-5b)+4a(a^2-4ab-5b^2)\csc^2(e+fx)+(a-5b)(a+b)^2\csc^4(e-\right.}{30(a+b)^4f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*a^2*(a - 5*b) + 4*a*(a^2 - 4*a*b
 - 5*b^2)*Csc[e + f*x]^2 + (a - 5*b)*(a + b)^2*Csc[e + f*x]^4 + 3*(a + b)^
3*Csc[e + f*x]^6)*Sec[e + f*x]^2*Tan[e + f*x])/((a + b)^4*f*(a + b*Sec[e +
 f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4620, 365, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$
$$\Big\downarrow \text{4620}$$
$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{365}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+2(5a+2b)\right)}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{\frac{\left(15a^2-10ab-b^2\right)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)}-\frac{2(5a+2b)\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{\frac{\left(15a^2-10ab-b^2\right)\left(-\frac{2b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)}-\frac{2(5a+2b)\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\frac{\left(15a^2-10ab-b^2\right)\left(-\frac{2b\tan(e+fx)}{(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)}-\frac{2(5a+2b)\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/5*Cot[e + f*x]^5/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((-2*(5*a
+ 2*b)*Cot[e + f*x]^3)/(3*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((15*a
^2 - 10*a*b - b^2)*(-(Cot[e + f*x]/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]
)) - (2*b*Tan[e + f*x])/((a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(3*(a
 + b)))/(5*(a + b)))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 9.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(\left(-20\cos(fx+e)^4+49\cos(fx+e)^2\right)a\,b^2+\left(-40\cos(fx+e)^6+104\cos(fx+e)^4-85\cos(fx+e)^2\right)b\,a^2-20a\,b^2+}{15f(a^4+4b\,a^3+6a^2b^2+4a\,b^3+b^4)\left(a-\right.}$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(b+a*cos(f*x+e)^2)*((-20*cos(f
*x+e)^4+49*cos(f*x+e)^2)*a*b^2+(-40*cos(f*x+e)^6+104*cos(f*x+e)^4-85*cos(f
*x+e)^2)*b*a^2-20*a*b^2+30*a^2*b+(8*cos(f*x+e)^6-20*cos(f*x+e)^4+15*cos(f*
x+e)^2)*a^3+(5*cos(f*x+e)^2-2)*b^3)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3*
csc(f*x+e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(153) = 306.

Time = 2.01 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.88

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(8\,(a^3-5\,a^2b)\cos\left(fx+e\right)^7-4\,(5\,a^3-26\,a^2b+5\,ab^2)\cos\left(fx+e\right)^5+(15\right.}{15\,\left((a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)f\cos\left(fx+e\right)^6-(2\,a^5+7\,a^4b+8\,a^3b^2+2\,a^2b^3-2\,ab^4-b^5)f\cos\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(8*(a^3 - 5*a^2*b)*cos(f*x + e)^7 - 4*(5*a^3 - 26*a^2*b + 5*a*b^2)*c
os(f*x + e)^5 + (15*a^3 - 85*a^2*b + 49*a*b^2 + 5*b^3)*cos(f*x + e)^3 + 2*
(15*a^2*b - 10*a*b^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x +
e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x
 + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(
f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f*x +
e))
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^6{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.69

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\frac{30\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2} - \frac{80\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3} + \frac{48\,b^3\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^4} + \frac{15}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)\tan(fx+e)} -}{}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(30*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 80*b
^2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 48*b^3*tan(f*
x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) + 15/(sqrt(b*tan(f*x + e
)^2 + a + b)*(a + b)*tan(f*x + e)) - 40*b/(sqrt(b*tan(f*x + e)^2 + a + b)*
(a + b)^2*tan(f*x + e)) + 24*b^2/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3
*tan(f*x + e)) + 10/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^3
) - 6*b/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)^3) + 3/(sqr
t(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^6}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**6)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.119 $\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{b(a+b)^2\sec(e+fx)}{3a^4f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(a+b)(5a+11b)\sec(e+fx)}{3a^5f\sqrt{a+b\sec^2(e+fx)}} - \frac{(15a^2+80ab+73b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^5f} + \frac{2(5a+7b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^4f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a^3f}$$

output
```
-1/3*b*(a+b)^2*sec(f*x+e)/a^4/f/(a+b*sec(f*x+e)^2)^(3/2)-1/3*b*(a+b)*(5*a+
11*b)*sec(f*x+e)/a^5/f/(a+b*sec(f*x+e)^2)^(1/2)-1/15*(15*a^2+80*a*b+73*b^2
)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^5/f+2/15*(5*a+7*b)*cos(f*x+e)^3*(a
+b*sec(f*x+e)^2)^(1/2)/a^4/f-1/5*cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2)/a^3
/f
```

Mathematica [A] (verified)

Time = 3.06 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.89

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\,(425a^4+6400a^3b+22784a^2b^2+32768ab^3+16384b^4+48a(11a^3+150a^2b\cdots}{\cdots}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/3840*((a + 2*b + a*Cos[2*(e + f*x)])*(425*a^4 + 6400*a^3*b + 22784*a^2*
b^2 + 32768*a*b^3 + 16384*b^4 + 48*a*(11*a^3 + 150*a^2*b + 384*a*b^2 + 256
*b^3)*Cos[2*(e + f*x)] + 12*a^2*(7*a^2 + 64*a*b + 64*b^2)*Cos[4*(e + f*x)]
 - 16*a^4*Cos[6*(e + f*x)] - 32*a^3*b*Cos[6*(e + f*x)] + 3*a^4*Cos[8*(e +
f*x)])*Sec[e + f*x]^5)/(a^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 365, 25, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)^5}{(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\Big\downarrow \text{4622}$$
$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int -\frac{\cos^4(e+fx)\left(2(5a+4b)-5a\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)}{5a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(2(5a+4b)-5a\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)}{5a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{-\frac{-\frac{\left(5a^2+20ab+16b^2\right)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{a}-\frac{2(5a+4b)\cos^3(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cos^5(e+fx)}{5a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 245$$

$$\frac{-\frac{-\frac{\left(5a^2+20ab+16b^2\right)\left(-\frac{4b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{2(5a+4b)\cos^3(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cos^5(e+fx)}{5a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 209$$

$$\frac{-\frac{-\frac{\left(5a^2+20ab+16b^2\right)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{2(5a+4b)\cos^3(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{5a}-}{f}$$

$$\Big\downarrow \; 208$$

$$-\frac{-\frac{-\frac{\left(5a^2+20ab+16b^2\right)\left(-\frac{4b\left(\frac{2\sec(e+fx)}{3a^2\sqrt{a+b\sec^2(e+fx)}}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{2(5a+4b)\cos^3(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cos^5(e+f}{5a(a+b\sec^2(e+}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/5*Cos[e + f*x]^5/(a*(a + b*Sec[e + f*x]^2)^(3/2)) - ((-2*(5*a + 4*b)*C
os[e + f*x]^3)/(3*a*(a + b*Sec[e + f*x]^2)^(3/2)) - ((5*a^2 + 20*a*b + 16*
b^2)*(-(Cos[e + f*x]/(a*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*(Sec[e + f*x
]/(3*a*(a + b*Sec[e + f*x]^2)^(3/2)) + (2*Sec[e + f*x])/(3*a^2*Sqrt[a + b*
Sec[e + f*x]^2])))/a))/a)/(5*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\frac{a^2\left(192a\,b^3\cos\left(fx+e\right)^2+\left(-8\cos\left(fx+e\right)^6+60\cos\left(fx+e\right)^4+60\cos\left(fx+e\right)^2\right)b\,a^3+160a\,b^3+40a^2b}{}$$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
1/15/f*a^2/((-a*b)^(1/2)+a)^7/((-a*b)^(1/2)-a)^7*(192*a*b^3*cos(f*x+e)^2+(
-8*cos(f*x+e)^6+60*cos(f*x+e)^4+60*cos(f*x+e)^2)*b*a^3+160*a*b^3+40*a^2*b^
2+(48*cos(f*x+e)^4+240*cos(f*x+e)^2)*a^2*b^2+128*b^4+(3*cos(f*x+e)^8-10*co
s(f*x+e)^6+15*cos(f*x+e)^4)*a^4)*(a+b)^7*(b+a*cos(f*x+e)^2)/(a+b*sec(f*x+e
)^2)^(5/2)*sec(f*x+e)^5
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(3\,a^4\cos\left(fx+e\right)^9-2\left(5\,a^4+4\,a^3b\right)\cos\left(fx+e\right)^7+3\left(5\,a^4+20\,a^3b+16\,a^2b^2\right)\cos\left(fx+e\right)^5+12\left(5\,a^3b\right.}{15\left(a^7f\cos\left(fx+e\right)^4+2\,a^6bf\cos\right.}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(3*a^4*cos(f*x + e)^9 - 2*(5*a^4 + 4*a^3*b)*cos(f*x + e)^7 + 3*(5*a^
4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^5 + 12*(5*a^3*b + 20*a^2*b^2 + 16*
a*b^3)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*
f*cos(f*x + e)^2 + a^5*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.64

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3} - \frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 9\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^4} + \frac{3\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5 -}{}}{}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - 10*((a + b/cos(f*x
 + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e
))/a^4 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a + b/cos(f*
x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f
*x + e))/a^5 + 5*(6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b
/cos(f*x + e)^2)^(3/2)*a^3*cos(f*x + e)^3) + 10*(9*(a + b/cos(f*x + e)^2)*
b^2*cos(f*x + e)^2 - b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3
) + 5*(12*(a + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((a + b/cos(f*x
 + e)^2)^(3/2)*a^5*cos(f*x + e)^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. 2(184) = 368.

Time = 1.62 (sec) , antiderivative size = 1638, normalized size of antiderivative = 8.03

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/15*(5*((((6*a^16*b^3*sgn(cos(f*x + e)) + 23*a^15*b^4*sgn(cos(f*x + e))
+ 28*a^14*b^5*sgn(cos(f*x + e)) + 11*a^13*b^6*sgn(cos(f*x + e)))*tan(1/2*f
*x + 1/2*e)^2/(a^18*b^2) - 3*(2*a^16*b^3*sgn(cos(f*x + e)) + a^15*b^4*sgn(
cos(f*x + e)) - 12*a^14*b^5*sgn(cos(f*x + e)) - 11*a^13*b^6*sgn(cos(f*x +
e)))/(a^18*b^2))*tan(1/2*f*x + 1/2*e)^2 - 3*(2*a^16*b^3*sgn(cos(f*x + e))
+ a^15*b^4*sgn(cos(f*x + e)) - 12*a^14*b^5*sgn(cos(f*x + e)) - 11*a^13*b^6
*sgn(cos(f*x + e)))/(a^18*b^2))*tan(1/2*f*x + 1/2*e)^2 + (6*a^16*b^3*sgn(c
os(f*x + e)) + 23*a^15*b^4*sgn(cos(f*x + e)) + 28*a^14*b^5*sgn(cos(f*x + e
)) + 11*a^13*b^6*sgn(cos(f*x + e)))/(a^18*b^2))/(a*tan(1/2*f*x + 1/2*e)^4
+ b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
+ 1/2*e)^2 + a + b)^(3/2) + 4*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*(2*a*b + 3*b^2) + 15*(s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))^8*(22*a*b + 21*b^2)*sqrt(a + b) + 20*(16*a^3 + 38*a^2*b + 63*
a*b^2 + 45*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*ta
n(1/2*f*x + 1/2*e)^2 + a + b))^7 - 20*(32*a^3 + 118*a^2*b + 51*a*b^2 - 63*
b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sin\left(e+f\,x\right)^5}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^5}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a b^2+3\sec(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**5)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.120 $\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{b(a+b)\sec(e+fx)}{3a^3f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(5a+8b)\sec(e+fx)}{3a^4f\sqrt{a+b\sec^2(e+fx)}} - \frac{(3a+8b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^4f} + \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^3f}$$

output
```
-1/3*b*(a+b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(3/2)-1/3*b*(5*a+8*b)*sec
(f*x+e)/a^4/f/(a+b*sec(f*x+e)^2)^(1/2)-1/3*(3*a+8*b)*cos(f*x+e)*(a+b*sec(f
*x+e)^2)^(1/2)/a^4/f+1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/a^3/f
```

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\left(26a^3+264a^2b+640ab^2+512b^3+3a(11a^2+96ab+128b^2)\cos(2(e+fx))\right.}{192a^4f\,(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/192*((a + 2*b + a*Cos[2*(e + f*x)])*(26*a^3 + 264*a^2*b + 640*a*b^2 + 5
12*b^3 + 3*a*(11*a^2 + 96*a*b + 128*b^2)*Cos[2*(e + f*x)] + 6*a^2*(a + 4*b
)*Cos[4*(e + f*x)] - a^3*Cos[6*(e + f*x)])*Sec[e + f*x]^5)/(a^4*f*(a + b*S
ec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4622, 25, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(e+fx)^3}{(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\Big\downarrow \; 4622$$
$$\frac{\int -\frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$
$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \textbf{359}$$

$$\frac{\frac{(a+2b)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{a}+\frac{\cos^3(e+fx)}{3a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \textbf{245}$$

$$\frac{\frac{(a+2b)\left(-\frac{4b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}+\frac{\cos^3(e+fx)}{3a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \textbf{209}$$

$$\frac{\frac{(a+2b)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}+\frac{\cos^3(e+fx)}{3a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \textbf{208}$$

$$\frac{\frac{(a+2b)\left(-\frac{4b\left(\frac{2\sec(e+fx)}{3a^2\sqrt{a+b\sec^2(e+fx)}}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}+\frac{\cos^3(e+fx)}{3a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Cos[e + f*x]^3/(3*a*(a + b*Sec[e + f*x]^2)^(3/2)) + ((a + 2*b)*(-(Cos[e +
 f*x]/(a*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*(Sec[e + f*x]/(3*a*(a + b*S
ec[e + f*x]^2)^(3/2)) + (2*Sec[e + f*x])/(3*a^2*Sqrt[a + b*Sec[e + f*x]^2]
)))/a))/a)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 105.74 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95

method	result
default	$-\frac{a\left(-24a\cos(fx+e)^{2}b^{2}+\left(-6\cos(fx+e)^{4}-12\cos(fx+e)^{2}\right)b\,a^{2}-8a\,b^{2}+\left(\cos(fx+e)^{6}-3\cos(fx+e)^{4}\right)a^{3}-16b^{3}\right)(a+b)^{5}\left(b+a\cos\right.}{3f\left(\sqrt{-ab}+a\right)^{5}\left(\sqrt{-ab}-a\right)^{5}\left(a+b\sec(fx+e)^{2}\right)^{\frac{5}{2}}}$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*a/((-a*b)^(1/2)+a)^5/((-a*b)^(1/2)-a)^5*(-24*a*cos(f*x+e)^2*b^2+(-6
*cos(f*x+e)^4-12*cos(f*x+e)^2)*b*a^2-8*a*b^2+(cos(f*x+e)^6-3*cos(f*x+e)^4)
*a^3-16*b^3)*(a+b)^5*(b+a*cos(f*x+e)^2)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e
)^5
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\left(a^3\cos\left(fx+e\right)^7-3\left(a^3+2\,a^2b\right)\cos\left(fx+e\right)^5-12\left(a^2b+2\,ab^2\right)\cos\left(fx+\right.\right.}{3\left(a^6f\cos\left(fx+e\right)^4+2\,a^5bf\cos\left(fx+\right.\right.}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(a^3*cos(f*x + e)^7 - 3*(a^3 + 2*a^2*b)*cos(f*x + e)^5 - 12*(a^2*b + 2
*a*b^2)*cos(f*x + e)^3 - 8*(a*b^2 + 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^
2 + a^4*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.30

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-9\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^4}+\frac{6\left(a+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^3\cos(fx+e)^3}+\frac{9\ldots}{\ldots}}{3f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - ((a + b/cos(f*x + e)
^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^
4 + (6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)
^2)^(3/2)*a^3*cos(f*x + e)^3) + (9*(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e)
^2 - b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{f\left(\frac{6\left(a\cos(fx+e)^2+b\right)ab+9\left(a\cos(fx+e)^2+b\right)b^2-ab^2-b^3}{\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}a^4f^2}-\frac{\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}a^8f^4-3\sqrt{a\cos(fx+e)^2+b}\,a^9f^4-9\sqrt{a\cos(fx+e)^2+b}\,a^8\cdots}{a^{12}f^6}\cdots\right.}{3\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*f*((6*(a*cos(f*x + e)^2 + b)*a*b + 9*(a*cos(f*x + e)^2 + b)*b^2 - a*b
^2 - b^3)/((a*cos(f*x + e)^2 + b)^(3/2)*a^4*f^2) - ((a*cos(f*x + e)^2 + b)
^(3/2)*a^8*f^4 - 3*sqrt(a*cos(f*x + e)^2 + b)*a^9*f^4 - 9*sqrt(a*cos(f*x +
 e)^2 + b)*a^8*b*f^4)/(a^12*f^6))/sgn(cos(f*x + e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^3}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**3)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.121 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(3/2)-4/3*b*sec(f*x+e)/a^2/f/(a+b*sec(f
*x+e)^2)^(3/2)-8/3*b*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((3a+8b)^2+12a(a+4b)\cos(2(e+fx))+3a^2\cos(4(e+fx))\right)\sec^5(e+fx}{48a^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/48*((a + 2*b + a*Cos[2*(e + f*x)])*((3*a + 8*b)^2 + 12*a*(a + 4*b)*Cos[
2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^5)/(a^3*f*(a + b*Sec[e
 + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4622, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{-\frac{4b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{209}$$

$$\frac{-\frac{4b\left(\frac{2\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{-\frac{4b\left(\frac{2\sec(e+fx)}{3a^2\sqrt{a+b\sec^2(e+fx)}}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-(Cos[e + f*x]/(a*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*(Sec[e + f*x]/(3*
a*(a + b*Sec[e + f*x]^2)^(3/2)) + (2*Sec[e + f*x])/(3*a^2*Sqrt[a + b*Sec[e
 + f*x]^2]))))/a)/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\sec(fx+e)}{3a\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sec(fx+e)}{3a^2\sqrt{a+b\sec(fx+e)^2}}\right)}{a}}{f}$	90
default	$\frac{-\frac{1}{a\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\sec(fx+e)}{3a\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sec(fx+e)}{3a^2\sqrt{a+b\sec(fx+e)^2}}\right)}{a}}{f}$	90

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2)-4*b/a*(1/3*sec(f*x+e)/a/(a+b
*sec(f*x+e)^2)^(3/2)+2/3/a^2*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\left(3\,a^2\cos\left(fx+e\right)^5+12\,ab\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\,\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*cos(f*x + e)^5 + 12*a*b*cos(f*x + e)^3 + 8*b^2*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)^4 + 2*a^4*b
*f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3} + \frac{6\left(a+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^3\cos(fx+e)^3}}{3f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 + (6*(a + b/cos(f*x +
e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)^(3/2)*a^3*cos(f*x +
e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{3\sqrt{a\cos{(fx+e)}^2+b} + \frac{6\left(a\cos(fx+e)^2+b\right)b-b^2}{\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}}}{3\,a^3f\operatorname{sgn}\left(\cos{(fx+e)}\right)}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(a*cos(f*x + e)^2 + b) + (6*(a*cos(f*x + e)^2 + b)*b - b^2)/(a
*cos(f*x + e)^2 + b)^(3/2))/(a^3*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 25.01 (sec) , antiderivative size = 26927, normalized size of antiderivative = 277.60

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*3i
 + f*x*3i)*(((2*a + 4*b)*(((2*a + 4*b)*((((32*a*b^2 + 30*a^2*b + 3*a^3)/(4
8*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(
a + b)))*(2*a + 4*b))/a + (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3*b*f*(a + b)) -
 (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b))))/a - (a + 6*b)/(24*a*b*f*(a
 + b)) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) + ((2*a + 4*b)
*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))/a - (((32*a*b^2 + 30*a^
2*b + 3*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3)
)/(48*a^4*b*f*(a + b)))*(2*a + 4*b))/a - (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3
*b*f*(a + b)) + (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b)) + (32*a*b^2 +
 40*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b))) + exp(e*1i + f*x*1i)*(((2*a + 4*b
)*((((32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*
a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b)))*(2*a + 4*b))/a + (8*a*b^2 +
8*a^2*b + a^3)/(48*a^3*b*f*(a + b)) - (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*
(a + b))))/a + (48*a*b^2 + 18*a^2*b + a^3 + 32*b^3)/(48*a^3*b*f*(a + b)) -
 (a + 6*b)/(24*a*b*f*(a + b)) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*
(a + b)) + ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))*
(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) + 1)
*(a + exp(e*2i + f*x*2i)*(2*a + 4*b) + a*exp(e*4i + f*x*4i))) - ((a + b/(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*1i + f*x*...
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x))/(sec(e + f*x)**6*b**3 + 3*s
ec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.122 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{5/2}f} - \frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5/2)/f-1/
3*b*sec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-1/3*b*(5*a+2*b)*sec(f*x+
e)/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.77 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.85

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\right.\right.}{6a^2(a+b)f\left(a+b\sec^2(\right.}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(a^2*Hypergeometric2F1[-3/2
, 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*(-2*(2*a + b) + 3*a*S
in[e + f*x]^2)))/(6*a^2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4622, 25, 316, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{2b\sec^2(e+fx)+b-3(a+b)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a(a+b)}-\frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{-\frac{\int \frac{3a^2}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}-\frac{b(5a+2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3a(a+b)}-\frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-\frac{3a\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}-\frac{b(5a+2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3a(a+b)}-\frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{-\frac{3a\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}-\frac{b(5a+2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3a(a+b)}-\frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{-\frac{3a\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}-\frac{b(5a+2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3a(a+b)}-\frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

input

```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output

```
(-1/3*(b*Sec[e + f*x])/(a*(a + b)*(a + b*Sec[e + f*x]^2)^(3/2)) + ((-3*a*A
rcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(a + b)^(3/
2) - (b*(5*a + 2*b)*Sec[e + f*x])/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/
(3*a*(a + b)))/f
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1319 vs. 2(113) = 226.

Time = 222.00 (sec) , antiderivative size = 1320, normalized size of antiderivative = 10.39

method	result	size
default	Expression too large to display	1320

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f/a^2/(2*(-a*b)^(1/2)-a+b)^2/(2*(-a*b)^(1/2)+a-b)^2/(a+b)^(9/2)*(a^4+
4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^2*(1+cos(f
*x+e))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*((a+b)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^5+3*(2
*cos(f*x+e)^3+2*cos(f*x+e)^2+cos(f*x+e)+1)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f
*x+e)*a+b)/(-1+cos(f*x+e)))*a^4*b+3*(cos(f*x+e)^3+cos(f*x+e)^2+2*cos(f*x+e
)+2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*((a+b)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*b^2+3*
(1+cos(f*x+e))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*((a+b)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*
a^2*b^3+3*cos(f*x+e)^2*(1+cos(f*x+e))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^5+3*(2*cos(f*x+e)^3+2*cos(f*x+e)^2+cos(
f*x+e)+1)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(113) = 226$.

Time = 0.21 (sec) , antiderivative size = 601, normalized size of antiderivative = 4.73

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(a^4\cos(fx+e)^4+2a^3b\cos(fx+e)^2+a^2b^2\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2\right.}{}\right.}{6\left((a^7+3a^6b+3a^5b^2+a^4b^3)f\cos\right.}\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(a + b
)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(2*a^3*b
+ 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*
x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 3*a^6*b + 3*a
^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^
3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f),
1/3*(3*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(-a - b
)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e)/(a*cos(f*x + e)^2 + b)) - (3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x +
e)^3 + (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e
)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*
b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(113) = 226$.

Time = 0.98 (sec) , antiderivative size = 1097, normalized size of antiderivative = 8.64

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(4*((((3*a^6*b^3 + 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7
+ 8*a*b^8 + b^9)*tan(1/2*f*x + 1/2*e)^2/(a^8*b^2*sgn(cos(f*x + e)) + 6*a^7
*b^3*sgn(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a^5*b^5*sgn(cos
(f*x + e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos(f*x + e)) +
a^2*b^8*sgn(cos(f*x + e))) - 3*(a^6*b^3 + 2*a^5*b^4 - 3*a^4*b^5 - 12*a^3*b
^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2*sgn(cos(f*x + e)) + 6*a^7*b^3*sg
n(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a^5*b^5*sgn(cos(f*x +
e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos(f*x + e)) + a^2*b^8
*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 - 3*(a^6*b^3 + 2*a^5*b^4 - 3*a
^4*b^5 - 12*a^3*b^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2*sgn(cos(f*x + e
)) + 6*a^7*b^3*sgn(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a^5*b
^5*sgn(cos(f*x + e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos(f*
x + e)) + a^2*b^8*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 + (3*a^6*b^3
+ 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*a*b^8 + b^9)/(a^8*
b^2*sgn(cos(f*x + e)) + 6*a^7*b^3*sgn(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f
*x + e)) + 20*a^5*b^5*sgn(cos(f*x + e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6
*a^3*b^7*sgn(cos(f*x + e)) + a^2*b^8*sgn(cos(f*x + e))))/(a*tan(1/2*f*x +
1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b)^(3/2) - 3*log(abs(-sqrt(a + b)*tan(1/2*f*x +
1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x))/(sec(e + f*x)**6*b**3 + 3*s
ec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.123 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a-4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(13a-2b)b\sec(e+fx)}{6a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-1/2*(a-4*b)*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b
)^(7/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-5/6*b
*sec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/6*(13*a-2*b)*b*sec(f*x+e)
/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\left(3a^2+6ab-2b^2+(3a^2+2b^2)\cos(2(e+fx))\right)\csc^2(e+fx)-3a(a\right.}{24a(a+b)^3f\,(a+b\sec}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/24*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*(3*a^2 + 6*a*b - 2*b^2 + (3
*a^2 + 2*b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - 3*a*(a - 4*b)*(a + 2*b +
a*Cos[2*(e + f*x)])*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)
/(a + b)])*Sec[e + f*x]^5)/(a*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4622, 373, 402, 25, 27, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin(e+fx)^3\,(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\Big\downarrow \; 4622$$
$$\frac{\int \frac{\sec^2(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$\downarrow$ 373

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}-\frac{\int \frac{a-4b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{2(a+b)}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}-\frac{\frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{a\left(-10b\sec^2(e+fx)+3a-2b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a(a+b)}}{2(a+b)}}{f}$$

$\downarrow$ 25

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}-\frac{\frac{\int \frac{a\left(-10b\sec^2(e+fx)+3a-2b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a(a+b)}+\frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}-\frac{\frac{\int \frac{-10b\sec^2(e+fx)+3a-2b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3(a+b)}+\frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}-\frac{\frac{\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{3a(a-4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}}{3(a+b)}+\frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}-\frac{\frac{\frac{3(a-4b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}+\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)}+\frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{f}$$

$\downarrow$ 291

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}-\frac{\frac{\frac{3(a-4b)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}+\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)}+\frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{f}$$

$\downarrow$ 219

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}-\frac{\frac{\frac{3(a-4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}+\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)}+\frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sec[e + f*x]/(2*(a + b)*(1 - Sec[e + f*x]^2)*(a + b*Sec[e + f*x]^2)^(3/2)
) - ((5*b*Sec[e + f*x])/(3*(a + b)*(a + b*Sec[e + f*x]^2)^(3/2)) + ((3*(a
- 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(a
+ b)^(3/2) + ((13*a - 2*b)*b*Sec[e + f*x])/(a*(a + b)*Sqrt[a + b*Sec[e + f
*x]^2]))/(3*(a + b)))/(2*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2073 vs. $2(151) = 302$.

Time = 202.14 (sec) , antiderivative size = 2074, normalized size of antiderivative = 12.13

method	result	size
default	Expression too large to display	2074

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/f/a/(2*(-a*b)^(1/2)+a-b)^3/(2*(-a*b)^(1/2)-a+b)^3/(a+b)^(13/2)*(a^6+
6*a^5*b+15*a^4*b^2+20*a^3*b^3+15*a^2*b^4+6*a*b^5+b^6)*(b+a*cos(f*x+e)^2)*(
-6*a^3*(a+b)^(7/2)*cos(f*x+e)^4+12*cos(f*x+e)^2*(2*cos(f*x+e)^2-3)*a^2*(a+
b)^(7/2)*b+2*(-13+10*cos(f*x+e)^2)*b^2*(a+b)^(7/2)*a+4*sin(f*x+e)^2*b^3*(a
+b)^(7/2)+3*cos(f*x+e)^2*(-1-cos(f*x+e))*sin(f*x+e)^2*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^6+3*(-1-cos(f*x+e))*sin
(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a
+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+
e)))*a^5*b+3*(9*cos(f*x+e)^3+9*cos(f*x+e)^2+cos(f*x+e)+1)*sin(f*x+e)^2*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b^2+
3*(11*cos(f*x+e)^3+11*cos(f*x+e)^2+9*cos(f*x+e)+9)*sin(f*x+e)^2*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b^3+3*(11+4
*cos(f*x+e)^3+4*cos(f*x+e)^2+11*cos(f*x+e))*sin(f*x+e)^2*((b+a*cos(f*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(151) = 302$.

Time = 0.29 (sec) , antiderivative size = 950, normalized size of antiderivative = 5.56

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((a^4 - 4*a^3*b)*cos(f*x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*co
s(f*x + e)^4 - a^2*b^2 + 4*a*b^3 - (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x
 + e)^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1
)) - 2*(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6
*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*
b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a
^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^
5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/6*(3*((a^4 - 4*a^
3*b)*cos(f*x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2
 + 4*a*b^3 - (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*
arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/(a*cos(f*x + e)^2 + b)) + (3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5
 + 2*(9*a^3*b + 4*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 +
11*a*b^3 - 2*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (
a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x +
 e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^...
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1980 vs. $2(151) = 302$.

Time = 1.44 (sec) , antiderivative size = 1980, normalized size of antiderivative = 11.58

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/24*(((((3*(a^12*b^2*sgn(cos(f*x + e)) + 10*a^11*b^3*sgn(cos(f*x + e)) +
 45*a^10*b^4*sgn(cos(f*x + e)) + 120*a^9*b^5*sgn(cos(f*x + e)) + 210*a^8*b
^6*sgn(cos(f*x + e)) + 252*a^7*b^7*sgn(cos(f*x + e)) + 210*a^6*b^8*sgn(cos
(f*x + e)) + 120*a^5*b^9*sgn(cos(f*x + e)) + 45*a^4*b^10*sgn(cos(f*x + e))
 + 10*a^3*b^11*sgn(cos(f*x + e)) + a^2*b^12*sgn(cos(f*x + e)))*tan(1/2*f*x
 + 1/2*e)^2/(a^13*b^2 + 11*a^12*b^3 + 55*a^11*b^4 + 165*a^10*b^5 + 330*a^9
*b^6 + 462*a^8*b^7 + 462*a^7*b^8 + 330*a^6*b^9 + 165*a^5*b^10 + 55*a^4*b^1
1 + 11*a^3*b^12 + a^2*b^13) - 4*(3*a^12*b^2*sgn(cos(f*x + e)) + 12*a^11*b^
3*sgn(cos(f*x + e)) - 25*a^10*b^4*sgn(cos(f*x + e)) - 270*a^9*b^5*sgn(cos(
f*x + e)) - 810*a^8*b^6*sgn(cos(f*x + e)) - 1344*a^7*b^7*sgn(cos(f*x + e))
 - 1386*a^6*b^8*sgn(cos(f*x + e)) - 900*a^5*b^9*sgn(cos(f*x + e)) - 345*a^
4*b^10*sgn(cos(f*x + e)) - 60*a^3*b^11*sgn(cos(f*x + e)) + 3*a^2*b^12*sgn(
cos(f*x + e)) + 2*a*b^13*sgn(cos(f*x + e)))/(a^13*b^2 + 11*a^12*b^3 + 55*a
^11*b^4 + 165*a^10*b^5 + 330*a^9*b^6 + 462*a^8*b^7 + 462*a^7*b^8 + 330*a^6
*b^9 + 165*a^5*b^10 + 55*a^4*b^11 + 11*a^3*b^12 + a^2*b^13))*tan(1/2*f*x +
 1/2*e)^2 + 6*(3*a^12*b^2*sgn(cos(f*x + e)) + 14*a^11*b^3*sgn(cos(f*x + e)
) + 27*a^10*b^4*sgn(cos(f*x + e)) + 68*a^9*b^5*sgn(cos(f*x + e)) + 262*a^8
*b^6*sgn(cos(f*x + e)) + 644*a^7*b^7*sgn(cos(f*x + e)) + 910*a^6*b^8*sgn(c
os(f*x + e)) + 752*a^5*b^9*sgn(cos(f*x + e)) + 343*a^4*b^10*sgn(cos(f*x +
e)) + 62*a^3*b^11*sgn(cos(f*x + e)) - 9*a^2*b^12*sgn(cos(f*x + e)) - 4*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^3\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^3}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**3)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.124 $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 234

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(3a^2-24ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{9/2}f} - \frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(23a-12b)b\sec(e+fx)}{24(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{5(11a-10b)b\sec(e+fx)}{24(a+b)^4f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-1/8*(3*a^2-24*a*b+8*b^2)*arctanh((a+b)^(1/2)*sec(f*x+e)/(a+b*sec(f*x+e)^2
)^(1/2))/(a+b)^(9/2)/f-1/8*(5*a-2*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*
sec(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)
^(3/2)-1/24*(23*a-12*b)*b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)-5/
24*(11*a-10*b)*b*sec(f*x+e)/(a+b)^4/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.55

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\left(3(a+b)(3a-4b+(a+8b)\cos(2(e+fx)))\csc^4(e+fx)-2(3a^2-24ab+\right.}{96(a+b)^3f\,(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/96*((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a + b)*(3*a - 4*b + (a + 8*b)*Co
s[2*(e + f*x)])*Csc[e + f*x]^4 - 2*(3*a^2 - 24*a*b + 8*b^2)*Hypergeometric
2F1[-3/2, 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]^5)/((a +
b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4622, 25, 372, 402, 25, 402, 25, 27, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(e+fx)^5\,(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\Big\downarrow \text{4622}$$
$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\int \frac{2(2a-b)\sec^2(e+fx)+a}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int -\frac{a(3a-4b)-4(5a-2b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{2(a+b)}+\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\int \frac{a(3a-4b)-4(5a-2b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{2(a+b)}}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{a\left(a(9a-26b)-2(23a-12b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a(a+b)}}{2(a+b)}}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\int \frac{a\left(a(9a-26b)-2(23a-12b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a(a+b)}+\frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\int\frac{a(9a-26b)-2(23a-12b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3(a+b)}+\frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{4(a+b)}-\frac{\text{s}\cdots}{4(a+b)(1-\sec^2(e+\cdots}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{5b(11a-10b)\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int-\frac{3a\left(3a^2-24ba+8b^2\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}}{3(a+b)}+\frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{4(a+b)}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2-24ab+8b^2\right)\int\frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}+\frac{5b(11a-10b)\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)}+\frac{b(23a-12\cdots}{3(a+b)\left(a+b\,\text{s}\cdots\right.}}{2(a+b)}}{4(a+b)}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2-24ab+8b^2\right)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}+\frac{5b(11a-10b)\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)}+\frac{b(23a-12b)\sec(\cdots}{3(a+b)\left(a+b\sec^2(e\cdots\right.}}{2(a+b)}}{4(a+b)}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2-24ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}+\frac{5b(11a-10b)\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)}+\frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{4(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/4*Sec[e + f*x]/((a + b)*(1 - Sec[e + f*x]^2)^2*(a + b*Sec[e + f*x]^2)^
(3/2)) + (((5*a - 2*b)*Sec[e + f*x])/(2*(a + b)*(1 - Sec[e + f*x]^2)*(a +
b*Sec[e + f*x]^2)^(3/2)) - (((23*a - 12*b)*b*Sec[e + f*x])/(3*(a + b)*(a +
 b*Sec[e + f*x]^2)^(3/2)) + ((3*(3*a^2 - 24*a*b + 8*b^2)*ArcTanh[(Sqrt[a +
 b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(a + b)^(3/2) + (5*(11*a -
10*b)*b*Sec[e + f*x])/((a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(3*(a + b)))/(
2*(a + b)))/(4*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3312 vs. $2(210) = 420$.

Time = 193.19 (sec) , antiderivative size = 3313, normalized size of antiderivative = 14.16

method	result	size
default	Expression too large to display	3313

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/(2*(-a*b)^(1/2)+a-b)^4/(2*(-a*b)^(1/2)-a+b)^4/(a+b)^(17/2)*(a^8+8*a
^7*b+28*a^6*b^2+56*a^5*b^3+70*a^4*b^4+56*a^3*b^5+28*a^2*b^6+8*a*b^7+b^8)/(
a+b*sec(f*x+e)^2)^(5/2)*((18*cos(f*x+e)^2-30)*(a+b)^(9/2)*a^4*cot(f*x+e)*c
sc(f*x+e)^3+(-144*cos(f*x+e)^4+282*cos(f*x+e)^2-186)*(a+b)^(9/2)*a^3*b*sec
(f*x+e)*csc(f*x+e)^4+(48*cos(f*x+e)^6-416*cos(f*x+e)^4+562*cos(f*x+e)^2-26
6)*(a+b)^(9/2)*a^2*b^2*sec(f*x+e)^3*csc(f*x+e)^4+(112*cos(f*x+e)^6-448*cos
(f*x+e)^4+398*cos(f*x+e)^2-110)*(a+b)^(9/2)*a*b^3*sec(f*x+e)^5*csc(f*x+e)^
4+(64*cos(f*x+e)^4-176*cos(f*x+e)^2+100)*(a+b)^(9/2)*b^4*sec(f*x+e)^5*csc(
f*x+e)^4+ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e
)*a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^8*(-
9-9*sec(f*x+e))+ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+co
s(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^7*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*b*(36+36*sec(f*x+e)-18*sec(f*x+e)^2-18*sec(f*x+e)^3)+ln(-4*((a+b)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a
^6*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(210+210*sec(f*x+e)+72*
sec(f*x+e)^2+72*sec(f*x+e)^3-9*sec(f*x+e)^4-9*sec(f*x+e)^5)+ln(-4*((a+b)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(210) = 420$.

Time = 0.39 (sec) , antiderivative size = 1316, normalized size of antiderivative = 5.62

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^
3*b + 32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b
^2 - 56*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(
3*a^3*b - 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2
*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(3*a^4 - 21*a^3*
b - 16*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117*a^3*b + 4*a^2*b^2
 + 104*a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^2*b^2 - 61*a*b^3
+ 88*b^4)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5
*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b
- 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x
 + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*
cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7)*f), 1/24*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a
^4 - 27*a^3*b + 32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b +
 107*a^2*b^2 - 56*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8
*b^4 + 2*(3*a^3*b - 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-a
 - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*c...
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3221 vs. $2(210) = 420$.

Time = 2.07 (sec) , antiderivative size = 3221, normalized size of antiderivative = 13.76

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/192*(((((3*((a^17*b^2 + 15*a^16*b^3 + 105*a^15*b^4 + 455*a^14*b^5 + 136
5*a^13*b^6 + 3003*a^12*b^7 + 5005*a^11*b^8 + 6435*a^10*b^9 + 6435*a^9*b^10
 + 5005*a^8*b^11 + 3003*a^7*b^12 + 1365*a^6*b^13 + 455*a^5*b^14 + 105*a^4*
b^15 + 15*a^3*b^16 + a^2*b^17)*tan(1/2*f*x + 1/2*e)^2/(a^18*b^2*sgn(cos(f*
x + e)) + 16*a^17*b^3*sgn(cos(f*x + e)) + 120*a^16*b^4*sgn(cos(f*x + e)) +
 560*a^15*b^5*sgn(cos(f*x + e)) + 1820*a^14*b^6*sgn(cos(f*x + e)) + 4368*a
^13*b^7*sgn(cos(f*x + e)) + 8008*a^12*b^8*sgn(cos(f*x + e)) + 11440*a^11*b
^9*sgn(cos(f*x + e)) + 12870*a^10*b^10*sgn(cos(f*x + e)) + 11440*a^9*b^11*
sgn(cos(f*x + e)) + 8008*a^8*b^12*sgn(cos(f*x + e)) + 4368*a^7*b^13*sgn(co
s(f*x + e)) + 1820*a^6*b^14*sgn(cos(f*x + e)) + 560*a^5*b^15*sgn(cos(f*x +
 e)) + 120*a^4*b^16*sgn(cos(f*x + e)) + 16*a^3*b^17*sgn(cos(f*x + e)) + a^
2*b^18*sgn(cos(f*x + e))) + (5*a^17*b^2 + 61*a^16*b^3 + 329*a^15*b^4 + 100
1*a^14*b^5 + 1729*a^13*b^6 + 1001*a^12*b^7 - 3003*a^11*b^8 - 9867*a^10*b^9
 - 15873*a^9*b^10 - 17017*a^8*b^11 - 13013*a^7*b^12 - 7189*a^6*b^13 - 2821
*a^5*b^14 - 749*a^4*b^15 - 121*a^3*b^16 - 9*a^2*b^17)/(a^18*b^2*sgn(cos(f*
x + e)) + 16*a^17*b^3*sgn(cos(f*x + e)) + 120*a^16*b^4*sgn(cos(f*x + e)) +
 560*a^15*b^5*sgn(cos(f*x + e)) + 1820*a^14*b^6*sgn(cos(f*x + e)) + 4368*a
^13*b^7*sgn(cos(f*x + e)) + 8008*a^12*b^8*sgn(cos(f*x + e)) + 11440*a^11*b
^9*sgn(cos(f*x + e)) + 12870*a^10*b^10*sgn(cos(f*x + e)) + 11440*a^9*b^11*
sgn(cos(f*x + e)) + 8008*a^8*b^12*sgn(cos(f*x + e)) + 4368*a^7*b^13*sgn...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\text{Hanged}$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^5}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**5)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.125 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 288

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{5(a+b)\left(a^2+14ab+21b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} - \frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b\left(113a^2+420ab+315b^2\right)\tan(e+fx)}{48a^5f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
5/16*(a+b)*(a^2+14*a*b+21*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)
^2)^(1/2))/a^(11/2)/f-1/16*(a+b)*(11*a+21*b)*cos(f*x+e)*sin(f*x+e)/a^3/f/(
a+b+b*tan(f*x+e)^2)^(3/2)+3/8*(a+b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*t
an(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^
(3/2)-7/48*b*(a+b)*(7*a+15*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(3/2)-
1/48*b*(113*a^2+420*a*b+315*b^2)*tan(f*x+e)/a^5/f/(a+b+b*tan(f*x+e)^2)^(1/
2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1705 vs. $2(288) = 576$.

Time = 15.66 (sec) , antiderivative size = 1705, normalized size of antiderivative = 5.92

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/3072*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2
*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*
a*b^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Co
s[2*(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(
a + b)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^
4 + 1120*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 22
40*b^4)*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a +
 b)^2*Sin[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^
(7/2)*(a + b*Sec[e + f*x]^2)^(5/2)) + (((a + 2*b + a*Cos[2*(e + f*x)])/(a
+ b))^(3/2)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(420*Sqrt[
a + b]*(a^4 + 9*a^3*b + 26*a^2*b^2 + 30*a*b^3 + 12*b^4)*ArcSin[(Sqrt[a]*Si
n[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - Sqrt[a]*Sin[e
+ f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]*(3*(561*a^6 + 6161*a^5*b +
 25200*a^4*b^2 + 50960*a^3*b^3 + 54880*a^2*b^4 + 30240*a*b^5 + 6720*b^6) -
 2*a*(1151*a^5 + 11230*a^4*b + 39200*a^3*b^2 + 62720*a^2*b^3 + 47040*a*b^4
 + 13440*b^5)*Sin[e + f*x]^2 + 672*a^2*(a + b)^2*(a^2 + 3*a*b + 6*b^2)*Sin
[e + f*x]^4 - 576*a^3*(a - 2*b)*(a + b)^2*Sin[e + f*x]^6 + 512*a^4*(a + b)
^2*Sin[e + f*x]^8)))/(3072*Sqrt[2]*a^(11/2)*f*(a + 2*b + a*Cos[2*(e + f*x)
])^(7/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (5*(a + 2*b + a*Cos[2*e + 2*f*x])
^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b ...
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4620, 372, 27, 440, 27, 402, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^6}{\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \frac{\tan^6(e+fx)}{\left(\tan^2(e+fx)+1\right)^4\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int \frac{3\tan^2(e+fx)\left(-2(a+b)\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{6a}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int \frac{\tan^2(e+fx)\left(-2(a+b)\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}}{f}$$

$$\Big\downarrow \; 440$$

$$\frac{\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\int \frac{(a+b)\left(3(a+b)-2(4a+9b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a}-\frac{3(a+b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/}}}{2a}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\int\frac{3(a+b)-2(4a+9b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a}-\frac{3(a+b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)\cdots\right.}\cdots}{2a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int\frac{(a+b)(5a+21b)-4b(11a+21b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan\cdots}{2a}\cdots\right.}{4a}\cdots}{2a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\int\frac{(a+b)\left(15a^2+112ba+105b^2-14b(7a+15b)\tan^2(\cdots\right.}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/\cdots}}}{3a(a+b)}\cdots}{2\cdots}\right.}{4a}\cdots}{2a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\int\frac{15a^2+112ba+105b^2-14b(7a+15b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\cdots}{3a}\cdots}{2a}\right.}{4a}\cdots}{2a}}{f}$$

$\downarrow$ 402

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\frac{\int\frac{15(a+b)\left(a^2+14ba+21b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+\cdots}{a(a+b)}\cdots}{3a}\cdots}{\cdots}\right.}{4a}\cdots}{f}$$

$\Big\downarrow$ 27

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\frac{15\left(a^2+14ab+21b^2\right)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(\cdots}}\cdots}{a}\cdots}{\cdots}}{\cdots}\right.}{4a}\cdots}{f}$$

$\Big\downarrow$ 291

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\frac{15\left(a^2+14ab+21b^2\right)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\cdots}{\sqrt{\cdots}}}{a}\cdots}{\cdots}}{\cdots}\right.}{4a}\cdots}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{15\left(a^2+14ab+21b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+\cdots}}\right)}{a^{3/2}}\cdots}{3a}\cdots\right)}{4a}\cdots}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)^(3/
2)) - ((-3*(a + b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Ta
n[e + f*x]^2)^(3/2)) + ((a + b)*(((11*a + 21*b)*Tan[e + f*x])/(2*a*(1 + Ta
n[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2)) - ((-7*b*(7*a + 15*b)*Tan[
e + f*x])/(3*a*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((15*(a^2 + 14*a*b + 21
*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/
2) - (b*(113*a^2 + 420*a*b + 315*b^2)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b
+ b*Tan[e + f*x]^2]))/(3*a))/(2*a)))/(4*a))/(2*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(260) = 520$.

Time = 23.70 (sec) , antiderivative size = 1127, normalized size of antiderivative = 3.91

method	result	size
default	Expression too large to display	1127

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a^5/(-a)^(1/2)*(15*cos(f*x+e)^4*(1+cos(f*x+e))*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)+15*cos(f*x+e)^2*(15*cos(f*x+e)^3+15*cos(f*x+e)^2+2
*cos(f*x+e)+2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b*ln(4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+15*(35*cos(f*
x+e)^5+35*cos(f*x+e)^4+30*cos(f*x+e)^3+30*cos(f*x+e)^2+cos(f*x+e)+1)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+15*(21*cos(f*x+e)^5+21*cos(f*x+
e)^4+70*cos(f*x+e)^3+70*cos(f*x+e)^2+15*cos(f*x+e)+15)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^2*b^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)+105*(6*cos(f*x+e)^3+6*cos(f*x+e)^2+5*cos(f*x+
e)+5)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^4*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+315*(1+cos(f*x+e))*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)...
```

Fricas [A] (verification not implemented)

Time = 21.84 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.48

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
[-1/384*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b +
35*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*b
^3 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^
3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5
+ 9*a^4*b)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3*b^2)*cos(f*x + e
)^5 + 2*(81*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e)^3 + (113*a^3*b
^2 + 420*a^2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*cos(f*x + e)
^2 + a^6*b^2*f), -1/192*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a
^5 + 15*a^4*b + 35*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^
3*b^2 + 35*a^2*b^3 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
 + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) +
 4*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 9*a^4*b)*cos(f*x + e)^7 + 3*(11*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^6}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**6)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.126 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 227

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(3a^2+30ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
1/8*(3*a^2+30*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(
1/2))/a^(9/2)/f-1/8*(5*a+7*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e
)^2)^(3/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/24
*b*(23*a+35*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)-5/24*b*(11*a+21
*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1315 vs. $2(227) = 454$.

Time = 11.39 (sec) , antiderivative size = 1315, normalized size of antiderivative = 5.79

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/768*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*
e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a
*b^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos
[2*(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a
 + b)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4
 + 1120*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 224
0*b^4)*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a +
b)^2*Sin[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(
7/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)
*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2
*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16
*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b
)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*
x]^4)/(a + b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)
/(a + b)]))/a^3))/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*S
in[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]
*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*
Sin[e + f*x]^2)/(a + b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]
^6)/a + (80*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))...
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4620, 372, 402, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)^4}{\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\tan^4(e+fx)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int \frac{-2(2a+3b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int \frac{(a+b)(3a+7b)-4b(5a+7b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\int \frac{(a+b)\left((a+b)(9a+35b)-2b(23a+35b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan}{3a(a+b)}}{2a}}{4a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\int\frac{(a+b)(9a+35b)-2b(23a+35b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a}}{2a}}{4a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\frac{\int\frac{3(a+b)\left(3a^2+30ba+35b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-5\ldots}{3a}}{2a}}{4a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{3\left(3a^2+30ab+35b^2\right)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+\ldots}}}{a}\ldots}{3a}\ldots}{4a}}{f}$$

$\downarrow$ 291

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{3\left(3a^2+30ab+35b^2\right)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(\ldots}{\sqrt{b\tan^2(\ldots}}}{a}\ldots}{3a}\ldots}{4a}}{f}$$

$\downarrow$ 216

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\frac{\frac{3\left(3a^2+30ab+35b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{5b}{a}}{3a}}{2a}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)^(3/2)
) - (((5*a + 7*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e
 + f*x]^2)^(3/2)) - (-1/3*(b*(23*a + 35*b)*Tan[e + f*x])/(a*(a + b + b*Tan
[e + f*x]^2)^(3/2)) + ((3*(3*a^2 + 30*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e
+ f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) - (5*b*(11*a + 21*b)*Tan[
e + f*x])/(a*Sqrt[a + b + b*Tan[e + f*x]^2]))/(3*a))/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(203) = 406$.

Time = 11.61 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.94

method	result	size
default	Expression too large to display	894

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/f/a^4/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(5/2)*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*(9+9*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^3*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x
+e)*a)*(90+90*sec(f*x+e)+18*sec(f*x+e)^2+18*sec(f*x+e)^3)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(105+105*sec(f*x+e)+180*sec(f*x+e)^2+180*s
ec(f*x+e)^3+9*sec(f*x+e)^4+9*sec(f*x+e)^5)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a*b^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4
*sin(f*x+e)*a)*(210*sec(f*x+e)^2+210*sec(f*x+e)^3+90*sec(f*x+e)^4+90*sec(f
*x+e)^5)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^4*ln(4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(105*sec(f*x+e)^4+105
*sec(f*x+e)^5)+sin(f*x+e)*cos(f*x+e)*(6*cos(f*x+e)^2-15)*a^4*(-a)^(1/2)+(-
15*cos(f*x+e)^2-93)*(-a)^(1/2)*a^3*b*tan(f*x+e)+(-a)^(1/2)*a^2*b^2*(-161*t
an(f*x+e)-133*tan(f*x+e)*sec(f*x+e)^2)+(-245*cos(f*x+e)^2-55)*(-a)^(1/2...
```

Fricas [A] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.85

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*((3*a^4 + 30*a^3*b + 35*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 3
0*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^2)*sqr
t(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(
5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
+ 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f
*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
) - 8*(6*a^4*cos(f*x + e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a
^3*b + 70*a^2*b^2)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x
 + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f), -1/96*(3*((3*a^4 + 30*a^3
*b + 35*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30*a*b^3 + 35*b^4 + 2*(3*a^3
*b + 30*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*
(6*a^4*cos(f*x + e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b +
 70*a^2*b^2)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + ...
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^4}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sin(fx+e)^4}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**4)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.127 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)\tan(e+fx)}{6a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
1/2*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/
f-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-5/6*b*tan(f*x+e
)/a^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/6*b*(13*a+15*b)*tan(f*x+e)/a^3/(a+b)/
f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 983 vs. $2(167) = 334$.

Time = 8.39 (sec) , antiderivative size = 983, normalized size of antiderivative = 5.89

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(
Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(
a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e + f*x]^2)*(1
 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a
*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^
2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*S
in[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(Sqrt[2]*f*(
a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)) - ((a + 2*b
+ a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a
 + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (24*Si
n[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b - a*Sin[e + f*x
]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a +
2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e
+ f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a
+ b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3 - (160*
(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)
^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a + b)^(3/
2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b -
a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e + f*x]^2)
/(a + b))^2))/a^4))/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b ...
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4620, 373, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\int \frac{-4b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{(a+b)\left(-10b\tan^2(e+fx)+3a+5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}-\frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{-10b\tan^2(e+fx)+3a+5b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a}-\frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\frac{\int \frac{3(a+b)(a+5b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b(13a+15b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}-\frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{3(a+5b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{b(13a+15b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}-\frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{\frac{3(a+5b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{b(13a+15b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}-\frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{3(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b(13a+15b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}-\frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/2*Tan[e + f*x]/(a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2
)) + ((-5*b*Tan[e + f*x])/(3*a*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*(a
+ 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3
/2) - (b*(13*a + 15*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x
]^2]))/(3*a))/(2*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(147) = 294$.

Time = 7.73 (sec) , antiderivative size = 888, normalized size of antiderivative = 5.32

method	result	size
default	Expression too large to display	888

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f/(a+b)/a^3/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(5/2)*(((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a^4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)*(-3-3*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a^3*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*(-18-18*sec(f*x+e)-6*sec(f*x+e)^2-6*sec(f*x+e)^3)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-15-15*sec(f*x+e)-36*sec(f*x+e)^2-3
6*sec(f*x+e)^3-3*sec(f*x+e)^4-3*sec(f*x+e)^5)+((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a*b^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*(-30*sec(f*x+e)^2-30*sec(f*x+e)^3-18*sec(f*x+e)^4-18*sec
(f*x+e)^5)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^4*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-15*sec(f*x+e)^4-1
5*sec(f*x+e)^5)+3*(-a)^(1/2)*a^4*cos(f*x+e)*sin(f*x+e)+(3*cos(f*x+e)^2+21)
*(-a)^(1/2)*a^3*b*tan(f*x+e)+(-a)^(1/2)*a^2*b^2*(23*tan(f*x+e)+31*tan(f*x+
e)*sec(f*x+e)^2)+(35*cos(f*x+e)^2+13)*(-a)^(1/2)*a*b^3*tan(f*x+e)*sec(f...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(147) = 294$.

Time = 1.80 (sec) , antiderivative size = 879, normalized size of antiderivative = 5.26

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 6*a*b^3
+ 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128
*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
 b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*c
os(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*
a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt
(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(3*(a^4
 + a^3*b)*cos(f*x + e)^5 + 2*(9*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a
^2*b^2 + 15*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*c
os(f*x + e)^2 + (a^5*b^2 + a^4*b^3)*f), -1/24*(3*((a^4 + 6*a^3*b + 5*a^2*b
^2)*cos(f*x + e)^4 + a^2*b^2 + 6*a*b^3 + 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*
a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 -
 a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2
- (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(3*(a^4 + a^3*b)*cos(
f*x + e)^5 + 2*(9*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 + 15*a*
b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)...
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin{(fx+e)}^2}{(b\sec{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin{(fx+e)}^2}{(b\sec{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sin\left(e+f\,x\right)^2}{{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^{2}b+a}\,\sin\left(fx+e\right)^{2}}{\sec\left(fx+e\right)^{6}b^{3}+3\sec\left(fx+e\right)^{4}a\,b^{2}+3\sec\left(fx+e\right)^{2}a^{2}b+a^{3}}dx$$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sin(e + f*x)**2)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.128 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\tan(e+fx)}{3a(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*tan(
f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*b*(5*a+3*b)*tan(f*x+e)/a^2
/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 14.71 (sec) , antiderivative size = 1927, normalized size of antiderivative = 15.42

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^
(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7
/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[
3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*
x]^2)*((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^5)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*Appe
llF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a
*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt
[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, ...
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4616, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 4616}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$\downarrow$ 316

$$\frac{\frac{\int \frac{-2b\tan^2(e+fx)+3a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\downarrow$ 291

$$\frac{\frac{\frac{3(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\downarrow$ 216

$$\frac{\frac{\frac{3(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output
```
(-1/3*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*
(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(
3/2) - (b*(5*a + 3*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]
^2]))/(3*a*(a + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(111) = 222$.

Time = 6.84 (sec) , antiderivative size = 861, normalized size of antiderivative = 6.89

method	result	size
default	Expression too large to display	861

input
```
int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*ln(4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-3-3*sec(f*x+e))-1/
3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-6-6*sec(f*x+e)-6*sec(f*x
+e)^2-6*sec(f*x+e)^3)-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*
b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*
(-3-3*sec(f*x+e)-12*sec(f*x+e)^2-12*sec(f*x+e)^3-3*sec(f*x+e)^4-3*sec(f*x+
e)^5)-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-6*sec(f*x+e)^2-6
*sec(f*x+e)^3-6*sec(f*x+e)^4-6*sec(f*x+e)^5)-1/3*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*b^4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-4*sin(f*x+e)*a)*(-3*sec(f*x+e)^4-3*sec(f*x+e)^5)-2*(-a)^(1/2)*a^3*b*tan
(f*x+e)-1/3*(-a)^(1/2)*a^2*b^2*(4*tan(f*x+e)+11*tan(f*x+e)*sec(f*x+e)^2)-1
/3*(7*cos(f*x+e)^2+5)*(-a)^(1/2)*a*b^3*tan(f*x+e)*sec(f*x+e)^4-(-a)^(1/2)*
b^4*tan(f*x+e)*sec(f*x+e)^4)/(a+b)^2/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(111) = 222$.

Time = 0.56 (sec) , antiderivative size = 881, normalized size of antiderivative = 7.05

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 +
b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b +
 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2
)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a
^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(
f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*
x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a
^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x
+ e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 +
 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^...
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(1/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)/(sec(e + f*x)**6*b**3 + 3*sec(e + f*x)**4*
a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.129 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b\tan(e+fx)}{3(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-4/3*b*tan(f*x+e)/(a+b)^2/f/
(a+b+b*tan(f*x+e)^2)^(3/2)-8/3*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(3a^2-6ab-b^2-6(a^2-b^2)\csc^2(e+fx)+3(a+b)^2\csc^4(e+fx)\right)\sec^2(e+}{6(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(3*a^2 - 6*a*b - b^2 - 6*(a^2 - b^2)*
Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x]^3
)/((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4620, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{-\frac{4b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{209}$$

$$\frac{-\frac{4b\left(\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{-\frac{4b\left(\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-(Cot[e + f*x]/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2))) - (4*b*(Tan[e
+ f*x]/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*x])/(3*
(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(a + b))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 8.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(\left(-6\cos(fx+e)^4+12\cos(fx+e)^2\right)ab+3\cos(fx+e)^4a^2+\left(-\cos(fx+e)^4-4\cos(fx+e)^2+8\right)b^2\right)\sec(fx+e)^5}{3f(a^3+3a^2b+3a\,b^2+b^3)\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a^3+3*a^2*b+3*a*b^2+b^3)*(b+a*cos(f*x+e)^2)*((-6*cos(f*x+e)^4+12*c
os(f*x+e)^2)*a*b+3*cos(f*x+e)^4*a^2+(-cos(f*x+e)^4-4*cos(f*x+e)^2+8)*b^2)/
(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5*csc(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.81

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\left((3\,a^2-6\,ab-b^2)\cos\left(fx+e\right)^5+4\,(3\,ab-b^2)\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{}}{3\,\left((a^5+3\,a^4b+3\,a^3b^2+a^2b^3)f\cos\left(fx+e\right)^4+2\,(a^4b+3\,a^3b^2+3\,a^2b^3+ab^4)f\cos\left(fx+e\right)^2+(a^3b^2+\right.}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^5 + 4*(3*a*b - b^2)*cos(f*x + e)^
3 + 8*b^2*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^5
 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2
+ 3*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b
^5)*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3}+\frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}+\frac{3}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)}}{3\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*ta
n(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) + 3/((b*tan(f*x +
e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)))/f
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 23.91 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.17

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(-a\,b\,6i+a^2\,3i-b^2\,1i+a^2\,e^{e\,2i+f\,x\,2i}\,12i+a^2\,e^{e\,4i+f\,x\,4i}\,18i+\dots\right)}{3\,f\dots}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
-((exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2)^2)^(1/2)*(a^2*3i - a*b*6i - b^2*1i + a^2*exp(e*2i + f*x*2i)*12i +
a^2*exp(e*4i + f*x*4i)*18i + a^2*exp(e*6i + f*x*6i)*12i + a^2*exp(e*8i + f
*x*8i)*3i - b^2*exp(e*2i + f*x*2i)*20i + b^2*exp(e*4i + f*x*4i)*90i - b^2*
exp(e*6i + f*x*6i)*20i - b^2*exp(e*8i + f*x*8i)*1i + a*b*exp(e*2i + f*x*2i
)*24i + a*b*exp(e*4i + f*x*4i)*60i + a*b*exp(e*6i + f*x*6i)*24i - a*b*exp(
e*8i + f*x*8i)*6i))/(3*f*(a + b)^3*(exp(e*2i + f*x*2i) - 1)*(a + 2*a*exp(e
*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^2}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a b^2+3\sec(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**2)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.130 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\begin{aligned}\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx &= -\frac{ab\tan(e+fx)}{3(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}\\ &\quad-\frac{(5a-3b)b\tan(e+fx)}{3(a+b)^4f\sqrt{a+b+b\tan^2(e+fx)}}\\ &\quad-\frac{(3a-5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^4f}\\ &\quad-\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^3f}\end{aligned}$$

output
```
-1/3*a*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*(5*a-3*b)*b*t
an(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(3*a-5*b)*cot(f*x+e)*(a
+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^4/f-1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(
1/2)/(a+b)^3/f
```

Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^3\left(\frac{4b^2(a+b)}{(a+2b+a\cos(2(e+fx)))^2}+\frac{4b(-3a+b)}{a+2b+a\cos(2(e+fx))}-2(a\right.}{24(a+b)^4f\,(a+b\,\mathrm{s}}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*((4*b^2*(a + b))/(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (4*b*(-3*a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a - 3*b)*
Csc[e + f*x]^2 - (a + b)*Csc[e + f*x]^4)*Sec[e + f*x]^4*Tan[e + f*x])/(24*
(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4620, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)^4\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 4620$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{(a-b)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{(a-b)\left(-\frac{4b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{(a-b)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{(a-b)\left(-\frac{4b\left(\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((a - b)
*(-(Cot[e + f*x]/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2))) - (4*b*(Tan[e
 + f*x]/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*x])/(3
*(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(a + b)))/(a + b))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 11.16 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.29

method	result
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(\left(2\cos(fx+e)^6-21\cos(fx+e)^4+24\cos(fx+e)^2\right)a\,b^2+\left(-12\cos(fx+e)^6+21\cos(fx+e)^4-12\cos(fx+e)^2\right)b\,a^2}{3f(a^4+4b\,a^3+6a^2b^2+4a\,b^3+b^4)\left(a+b\right.}$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(b+a*cos(f*x+e)^2)*((2*cos(f*x+e
)^6-21*cos(f*x+e)^4+24*cos(f*x+e)^2)*a*b^2+(-12*cos(f*x+e)^6+21*cos(f*x+e)
^4-12*cos(f*x+e)^2)*b*a^2-8*a*b^2+(2*cos(f*x+e)^6-3*cos(f*x+e)^4)*a^3+(3*c
os(f*x+e)^4-12*cos(f*x+e)^2+8)*b^3)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5*
csc(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(144) = 288$.

Time = 2.07 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.00

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{\left(2\left(a^3-6a^2b+ab^2\right)\cos\left(fx+e\right)^7-3\left(a^3-7a^2b+7ab^2-b^3\right)\cos}{3\left(\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^6-\left(a^6+2a^5b-2a^4b^2-8a^3b^3-7a^2b^4-2ab^5\right)f\right.}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^7 - 3*(a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e)^5 - 12*(a^2*b - 2*a*b^2 + b^3)*cos(f*x + e)^3 - 8*(a
*b^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a
^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 +
2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 -
(2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x +
e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.35

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3} + \frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2} - \frac{16\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^4} - \frac{8\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^3} + \frac{\cdots}{(b\,\mathrm{ta}\cdots}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*ta
n(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 16*b^2*tan(f*x +
 e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 8*b^2*tan(f*x + e)/((b*ta
n(f*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 3/((b*tan(f*x + e)^2 + a + b)^(3/
2)*(a + b)*tan(f*x + e)) - 6*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2
*tan(f*x + e)) + 1/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^
3))/f
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^4}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**4)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.131 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 215

$$\begin{aligned}\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = &-\frac{a^2b\tan(e+fx)}{3(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}\\&-\frac{a(5a-6b)b\tan(e+fx)}{3(a+b)^5f\sqrt{a+b+b\tan^2(e+fx)}}\\&-\frac{(15a^2-50ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^5f}\\&-\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^4f}\\&-\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)^3f}\end{aligned}$$

output
```
-1/3*a^2*b*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*a*(5*a-6*b)
*b*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^2-50*a*b+8*b
^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^5/f-2/15*(5*a-2*b)*cot(f*x
+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^4/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x
+e)^2)^(1/2)/(a+b)^3/f
```

Mathematica [A] (verified)

Time = 4.78 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.80

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^3\left(\frac{20ab^2(a+b)}{(a+2b+a\cos(2(e+fx)))^2}+\frac{10ab(-6a+5b)}{a+2b+a\cos(2(e+fx))}+(-\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*((20*a*b^2*(a + b))/(a + 2*b + a*Cos[2*(
e + f*x)])^2 + (10*a*b*(-6*a + 5*b))/(a + 2*b + a*Cos[2*(e + f*x)]) + (-8*
a^2 + 50*a*b - 15*b^2)*Csc[e + f*x]^2 + 2*(a + b)*(-2*a + 5*b)*Csc[e + f*x
]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x]^4*Tan[e + f*x])/(120*(a + b
)^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4620, 365, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 365}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+2(5a+b)\right)}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{\frac{\left(5a^2-10ab+b^2\right)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{a+b}-\frac{2(5a+b)\cot^3(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 245$$

$$\frac{\frac{\left(5a^2-10ab+b^2\right)\left(-\frac{4b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{2(5a+b)\cot^3(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 209$$

$$\frac{\frac{\left(5a^2-10ab+b^2\right)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{2(5a+b)\cot^3(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{\left(5a^2-10ab+b^2\right)\left(-\frac{4b\left(\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{2(5a+b)\cot^3(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input

```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/5*Cot[e + f*x]^5/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((-2*(5*
a + b)*Cot[e + f*x]^3)/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((5*
a^2 - 10*a*b + b^2)*(-(Cot[e + f*x]/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3
/2))) - (4*b*(Tan[e + f*x]/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) +
(2*Tan[e + f*x])/(3*(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(a + b)))/
(a + b))/(5*(a + b)))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 9.23 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(\left(60\cos(fx+e)^6-180\cos(fx+e)^4+212\cos(fx+e)^2\right)a\,b^3+\left(-80\cos(fx+e)^8+212\cos(fx+e)^6-180\cos(fx+\right.\right.}{}$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3+5*a*b^4+b^5)*(b+a*cos(f*x+e)^2)
*((60*cos(f*x+e)^6-180*cos(f*x+e)^4+212*cos(f*x+e)^2)*a*b^3+(-80*cos(f*x+e
)^8+212*cos(f*x+e)^6-180*cos(f*x+e)^4+60*cos(f*x+e)^2)*b*a^3-80*a*b^3+40*a
^2*b^2+(40*cos(f*x+e)^8-220*cos(f*x+e)^6+378*cos(f*x+e)^4-220*cos(f*x+e)^2
)*a^2*b^2+(15*cos(f*x+e)^4-20*cos(f*x+e)^2+8)*b^4+(8*cos(f*x+e)^8-20*cos(f
*x+e)^6+15*cos(f*x+e)^4)*a^4)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5*csc(f*
x+e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(195) = 390$.

Time = 8.38 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.14

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(8\,(a^4-10\,a^3b+5\,a^2b^2)\cos\left(fx+e\right)^9-4\,(5\,a^4-53\,a^3b+55\,a^2\cdots\right.}{15\,\left((a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5)f\cos\left(fx+e\right)^8-2\,(a^7+4\,a^6b+5\,a^5b^2-5\,a^3b^4-4\,a\cdots\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(8*(a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f*x + e)^9 - 4*(5*a^4 - 53*a^3*b
 + 55*a^2*b^2 - 15*a*b^3)*cos(f*x + e)^7 + 3*(5*a^4 - 60*a^3*b + 126*a^2*b
^2 - 60*a*b^3 + 5*b^4)*cos(f*x + e)^5 + 4*(15*a^3*b - 55*a^2*b^2 + 53*a*b^
3 - 5*b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 - 10*a*b^3 + b^4)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5
*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b
- 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x
 + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*
cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7)*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.73

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{40\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3}+\frac{20\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}-\frac{160\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^4}-\frac{80\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^3}+\frac{\ldots}{\sqrt{b\,\mathrm{t}\ldots}}}{\ldots}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/15*(40*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 20*b
*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 160*b^2*tan(f
*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 80*b^2*tan(f*x + e)/(
(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 128*b^3*tan(f*x + e)/(sqrt(b
*tan(f*x + e)^2 + a + b)*(a + b)^5) + 64*b^3*tan(f*x + e)/((b*tan(f*x + e)
^2 + a + b)^(3/2)*(a + b)^4) + 15/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b
)*tan(f*x + e)) - 60*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x
 + e)) + 48*b^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3*tan(f*x + e))
+ 10/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^3) - 8*b/((b*t
an(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e)^3) + 3/((b*tan(f*x + e
)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\csc(fx+e)^6}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*csc(e + f*x)**6)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.132 $\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx = \frac{\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{1}{2}+p,-p,\frac{3+m}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^{\frac{1}{2}+p}(a+b\sec^2(e+fx))^p(d\sin(e$$

$$f(1+m)$$

output
```
AppellF1(1/2+1/2*m,1/2+p,-p,3/2+1/2*m,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(
cos(f*x+e)^2)^(1/2+p)*(a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m*tan(f*x+e)/f/(
1+m)/(((a+b-a*sin(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(123) = 246.

Time = 3.93 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.33

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx = \frac{\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{2+m}{2},-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cos(e}{f(1+m)\left(\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{2+m}{2},-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)-\frac{\left(-2bp\operatorname{AppellF1}\left(\frac{3+m}{2},\frac{2+m}{2},1-p,\frac{5+m}{2}\right.\right.}{}\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

output
```
(AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d
*Sin[e + f*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - ((-2*b*p*AppellF1[(3 +
 m)/2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] + (a + b)*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)/((a + b)*
(3 + m)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sin(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sin(e+fx))^m \left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4623}$$

$$\int (d\sin(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4623
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Sin[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^{2}\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p(d\sin(e+fx))^m\,dx=\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*(d*sin(f*x+e))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p (d\sin\left(fx+e\right))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p (d\sin\left(fx+e\right))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx = \int (d\,\sin(e+f\,x))^m\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx = d^m\left(\int \sin(fx+e)^m\,\left(\sec(fx+e)^2\,b+a\right)^p dx\right)$$

input
```
int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

output
```
d**m*int(sin(e + f*x)**m*(sec(e + f*x)**2*b + a)**p,x)
```

3.133 $\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\begin{aligned}
&\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx \\
&= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} \\
&\quad - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{5af} \\
&\quad - \frac{(15a^2+b(10a+b(3-2p))(1-2p))\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b}{15a^2f}
\end{aligned}$$

output
```
1/15*(10*a+b*(3-2*p))*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(p+1)/a^2/f-1/5*cos(
f*x+e)^5*(a+b*sec(f*x+e)^2)^(p+1)/a/f-1/15*(15*a^2+b*(10*a+b*(3-2*p))*(1-2
*p))*cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x
+e)^2)^p/a^2/f/(((a+b*sec(f*x+e)^2)/a)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 3.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.42

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx$$
$$= \frac{2\cos(e+fx)\,(a+b\sec^2(e+fx))^p\sin^4(e+fx)\left(4(15a^2+10ab(1-2p)+b^2(3-8p+4p^2))\,\text{Hypergeom}\right.}{15a^2f\left(4\cos(2(e+fx))\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p-2^{-p}\left(3\right.\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

output
```
(2*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4*(4*(15*a^2 + 10*a*
b*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Hypergeometric2F1[-1/2, -p, 1/2, -((b
*Sec[e + f*x]^2)/a)] + (a + 2*b + a*Cos[2*(e + f*x)])*(-17*a - 6*b + 4*b*p
 + 3*a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(15*a^2*f*(4*C
os[2*(e + f*x)]*((a + b + b*Tan[e + f*x]^2)/a)^p - (3*(((a + 2*b + a*Cos[2
*(e + f*x)])*Sec[e + f*x]^2)/a)^p + 2^p*Cos[4*(e + f*x)]*((a + b + b*Tan[e
 + f*x]^2)/a)^p)/2^p))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4622, 365, 25, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(e+fx)^5\left(a+b\sec(e+fx)^2\right)^p dx$$
$$\Big\downarrow \text{4622}$$

$$\frac{\int \cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int -\cos^4(e+fx)(-5a\sec^2(e+fx)+10a+b(3-2p))\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{5a}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \cos^4(e+fx)(-5a\sec^2(e+fx)+10a+b(3-2p))\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{5a}}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{-\frac{-\frac{\left(15a^2+b(1-2p)(10a+b(3-2p))\right)\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{3a}-\frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{3a}}{5a}-\frac{\cos^5(e+fx)(a+b s}{5a}}{f}$$

$$\Big\downarrow \; 279$$

$$\frac{-\frac{-\frac{\left(15a^2+b(1-2p)(10a+b(3-2p))\right)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\int \cos^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p d\sec(e+fx)}{3a}-\frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+\right.}{3a}}{5a}}{f}$$

$$\Big\downarrow \; 278$$

$$\frac{-\frac{\frac{\left(15a^2+b(1-2p)(10a+b(3-2p))\right)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)}{3a}-\frac{(10a+b(3-2p))\cos^3}{}}{5a}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

output
```
(-1/5*(Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(1 + p))/a - (-1/3*((10*a + b
*(3 - 2*p))*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/a + ((15*a^2 +
b*(10*a + b*(3 - 2*p))*(1 - 2*p))*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p,
 1/2, -((b*Sec[e + f*x]^2)/a)]*(a + b*Sec[e + f*x]^2)^p)/(3*a*(1 + (b*Sec[
e + f*x]^2)/a)^p))/(5*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p} \sin \left(fx+e\right)^{5}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\sin^5(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p*
sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p\sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \int \sin\left(e+f\,x\right)^5\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^5(e+fx)\,dx=\int \left(\sec\left(fx+e\right)^2 b+a\right)^p \sin\left(fx+e\right)^5 dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sin(e + f*x)**5,x)
```

3.134 $\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = \frac{\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{1+p}}{3af}$$
$$-\frac{(3a+b-2bp)\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(\frac{a+b\text{se}}{}\right.}{3af}$$

output
```
1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(p+1)/a/f-1/3*(-2*b*p+3*a+b)*cos(f*x+e
)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^p/a/f/(
((a+b*sec(f*x+e)^2)/a)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.84 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.51

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx =$$
$$-\frac{\cos(e+fx)\,(a+b\sec^2(e+fx))^p\sin^2(e+fx)\left(-2(3a+b-2bp)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\right.\right.}{3af\left(-2\left(1+\frac{b\sec^2(e+fx)}{a}\right)^p+\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p+\cos(2(e+f\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

output
```
-1/3*(Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*(-2*(3*a + b -
2*b*p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)] + (a + 2*
b + a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(a*f*(-2*(1 + (
b*Sec[e + f*x]^2)/a)^p + ((a + b + b*Tan[e + f*x]^2)/a)^p + Cos[2*(e + f*x
)]*((a + b + b*Tan[e + f*x]^2)/a)^p))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4622, 25, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a-2bp+b)\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{3a}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{3a}}{f}$$

$$\Big\downarrow \text{279}$$

$$\frac{\frac{(3a-2bp+b)(a+b\sec^2(e+fx))^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\int\cos^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p d\sec(e+fx)}{3a}+\frac{\cos^3(e+fx)(a+b\sec^2(e+fx))^{p+1}}{3a}}{f}$$

$$\Big\downarrow \; 278$$

$$\frac{\frac{\cos^3(e+fx)(a+b\sec^2(e+fx))^{p+1}}{3a}-\frac{(3a-2bp+b)\cos(e+fx)(a+b\sec^2(e+fx))^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e}{a}\right.}{3a}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

output
```
((Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(3*a) - ((3*a + b - 2*b*p
)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(3*a*(1 + (b*Sec[e + f*x]^2)/a)^p))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p} \sin \left(fx+e\right)^{3}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx=\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}\sin \left(fx+e\right)^{3}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = \int \sin(e+f\,x)^3 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p dx$$

input
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = \int \left(\sec(fx+e)^2 b+a\right)^p \sin(fx+e)^3\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sin(e + f*x)**3,x)
```

3.135 $\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(\frac{a+b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
-cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^
2)^p/f/(((a+b*sec(f*x+e)^2)/a)^p)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*
(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4622, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx) \left(a+b\sec^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx) \left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \cos^2(e+fx) \left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(a+b\sec^2(e+fx)\right)^p \left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p} \int \cos^2(e+fx) \left(\frac{b\sec^2(e+fx)}{a}+1\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\cos(e+fx) \left(a+b\sec^2(e+fx)\right)^p \left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
```

output

```
-((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*
(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p} \sin \left(fx+e\right)dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

Fricas [F]

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx=\int (b\sec(fx+e)^2+a)^p \sin(fx+e)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx=\int (b\sec(fx+e)^2+a)^p \sin(fx+e)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Mupad [B] (verification not implemented)

Time = 14.84 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = \frac{\cos(e+fx)\left(a+\frac{b}{\cos(e+fx)^2}\right)^p {}_2F_1\left(\frac{1}{2}-p,-p;\ \frac{3}{2}-p;\ -\frac{a\cos(e+fx)^2}{b}\right)}{f\,(2p-1)\left(\frac{a\cos(e+fx)^2}{b}+1\right)^p}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p*hypergeom([1/2 - p, -p], 3/2 - p, -
(a*cos(e + f*x)^2)/b))/(f*(2*p - 1)*((a*cos(e + f*x)^2)/b + 1)^p)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = \int (\sec(fx+e)^2 b+a)^p \sin(fx+e)\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sin(e + f*x),x)
```

3.136 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{a+b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
-AppellF1(1/2,1,-p,3/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)*(a+b*sec
(f*x+e)^2)^p/f/(((a+b*sec(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1532 vs. 2(78) = 156.

Time = 15.45 (sec) , antiderivative size = 1532, normalized size of antiderivative = 19.64

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^p*Csc[e + f*x]*(Sec[e + f*x]^2)^p*(a + b*S
ec[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2
, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a
 + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/((a + b
+ b*Tan[e + f*x]^2)/(a + b))^p))/(2*f*(-(a*p*(a + 2*b + a*Cos[2*(e + f*x)]
)^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((2*AppellF1[-1/2 - p, -1/2
, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e
+ f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]
^2]) - (AppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))]*Tan[e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p)) + p*(a + 2*
b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((2*AppellF1[-1/
2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*
Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[C
sc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))]*Tan[e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p) +
 ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*((2*AppellF1[-1/2 -
p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[
e + f*x]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)
^p*Sqrt[Csc[e + f*x]^2]) + (4*(a + b)*p*AppellF1[-1/2 - p, -1/2, -p, 1/...
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4622, 25, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int -\frac{\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$-\frac{\left(a+b\sec^2(e+fx)\right)^p \left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p} \int \frac{\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p \left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[
e + f*x]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [F]

$$\int \csc \left(fx+e\right) \left(a+b\sec \left(fx+e\right)^{2}\right)^{p} dx$$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx=\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}\csc \left(fx+e\right) \, dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2 b+a\right)^p \csc\left(fx+e\right)dx$$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*csc(e + f*x),x)
```

3.137 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{a+b\sec^2(e+fx)}{a}\right)^{-p}}{3f}$$

output
```
1/3*AppellF1(3/2,2,-p,5/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)^3*(a+
b*sec(f*x+e)^2)^p/f/(((a+b*sec(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 266 vs. 2(82) = 164.

Time = 3.17 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.24

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2}-p,-\frac{1}{2},-p,\frac{3}{2}-p,-\cot^2(e+f\right.}{f(-1+2p)\left(-\frac{\left(2(a+b)p\operatorname{AppellF1}\left(\frac{3}{2}-p,-\frac{1}{2},1-p,\frac{5}{2}-p,-\cot^2(e+fx),-\frac{(a+b)\cot^2(e+fx)}{b}\right)+b\operatorname{AppellF1}\left(\frac{3}{2}-p,\frac{1}{2},-p,\frac{5}{2}-p,-\cot^2(e+fx),\right.\right.}{b(-3+2p)}\right.}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e +
f*x]^2)/b)]*Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(f*(-1 + 2*p)*(-(((2*
(a + b)*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2 - p, -Cot[e + f*x]^2, -(((a +
 b)*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2, -p, 5/2 - p, -Cot[e + f
*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)])*Cot[e + f*x]*Csc[e + f*x])/(b*(-3 +
 2*p))) + AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)
*Cot[e + f*x]^2)/b)]*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4622, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{4622}$$

$$\frac{\int \frac{\sec^2(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\int \frac{\sec^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{3f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e
+ f*x]^3*(a + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Maple [F]

$$\int \csc(fx+e)^3 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec(fx+e)^2+a\right)^p \csc(fx+e)^3 \, dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Giac [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\csc\left(fx+e\right)^3dx$$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*csc(e + f*x)**3,x)
```

3.138 $\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \frac{\operatorname{AppellF1}\left(\frac{5}{2},3,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e}{a+b}\right.}{5f}$$

output
```
1/5*AppellF1(5/2,3,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^
5*(a+b+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5878 vs. 2(90) = 180.

Time = 25.61 (sec) , antiderivative size = 5878, normalized size of antiderivative = 65.31

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4620, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 395$$

$$\frac{\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\tan^4(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 394$$

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\Big(\frac{5}{2},3,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}}{5f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
```

output
```
(AppellF1[5/2, 3, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)
/(a + b))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p} \sin \left(fx+e\right)^{4}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p dx$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \int (\sec(fx+e)^2 b+a)^p \sin(fx+e)^4\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sin(e + f*x)**4,x)
```

3.139 $\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\,(a+b+b\tan^2(e+fx))^p \left(\frac{a+b+b\tan^2(e}{a+b}\right.}{3f}$$

output
```
1/3*AppellF1(3/2,2,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^
3*(a+b+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3781 vs. 2(90) = 180.

Time = 19.79 (sec) , antiderivative size = 3781, normalized size of antiderivative = 42.01

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

output
```
(3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*(a +
 b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2
, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*A
ppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*App
ellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
- (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2])*Tan[e + f*x]^2)))/(f*(3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^
p*(Sec[e + f*x]^2)^(-1 + p)*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*Appell
F1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e
 + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Ta
n[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, ...
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \frac{\tan^2(e+fx)(b\tan^2(e+fx)+a+b)^p}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 395

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\tan^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 394

$$\frac{\tan^3(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

output
```
(AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)
/(a + b))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^{2}\right)^{p}\sin\left(fx+e\right)^{2}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\sin^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p\sin^2(e+fx)\,dx=\int \sin\left(e+f\,x\right)^2\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^p\sin^2(e+fx)\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\sin\left(fx+e\right)^2dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sin(e + f*x)**2,x)
```

3.140 $\int (a + b\sec^2(e + fx))^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int (a + b\sec^2(e + fx))^p \, dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 1, -p, \frac{3}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p \left(\frac{a+b+b\tan^2(e+}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2137 vs. 2(85) = 170.

Time = 14.23 (sec) , antiderivative size = 2137, normalized size of antiderivative = 25.14

$$\int (a + b\sec^2(e + fx))^p \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3
/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a +
b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(
a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)
/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*...
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{p}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a + b\sec^2(e + fx)\right)^p\, dx = \int \left(\sec\left(fx + e\right)^2 b + a\right)^p dx$$

input
```
int((a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p,x)
```

3.141 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e+fx)}{a+b}\right)}{f}$$

output
```
-cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f
*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
 b))]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4620, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4620}$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
 b))]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \csc (fx+e)^2 \left(a+b\sec (fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\csc\left(fx+e\right)^2\,dx$$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*csc(e + f*x)**2,x)
```

3.142 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=-\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f}-\frac{(3a+2b(1+p))\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+fx)\right)^p}{3(a+b)f}$$

output
```
-1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(p+1)/(a+b)/f-1/3*(3*a+2*b*(p+1))*c
ot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f*x
+e)^2)^p/(a+b)/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=-\frac{\cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left((3a+2b(1+p))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p\right.\right.}{3(a+b)f}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/3*(Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p*((3*a + 2*b*(1 + p))*Hypergeom
etric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))] + Cot[e + f*x]^2*(a
 + b + b*Tan[e + f*x]^2)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/((a + b)*f*(
1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4620, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a+2b(p+1))\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{3(a+b)}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{3(a+b)}}{f}$$

$$\Big\downarrow \text{279}$$

$$\frac{\frac{(3a+2b(p+1))\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p d\tan(e+fx)}{3(a+b)}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p}}{3(a+b)}}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\frac{(3a+2b(p+1))\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{3(a+b)}-\frac{\cot^3(e+fx)(a+b\tan}{3(a}}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(-1/3*(Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(a + b) - ((3*a
+ 2*b*(1 + p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e +
f*x]^2)/(a + b))]*(a + b + b*Tan[e + f*x]^2)^p)/(3*(a + b)*(1 + (b*Tan[e +
 f*x]^2)/(a + b))^p))/f
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \csc\left(fx+e\right)^{4} \left(a+b\sec\left(fx+e\right)^{2}\right)^{p} dx$$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{4}\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\csc\left(fx+e\right)^4dx$$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*csc(e + f*x)**4,x)
```

3.143 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f} - \frac{\left(15a^2+20ab(1+p)+4b^2(2+3p+p^2)\right)\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)(a}{15(a+b)^2f}$$

output
```
-1/15*(10*a+b*(7+2*p))*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(p+1)/(a+b)^2/f-1
/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(p+1)/(a+b)/f-1/15*(15*a^2+20*a*b*(p+
1)+4*b^2*(p^2+3*p+2))*cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^
2/(a+b))*(a+b+b*tan(f*x+e)^2)^p/(a+b)^2/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.77

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$

$$-\frac{\cot(e+fx)\left(3\cot^4(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-p,-\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)+10\cot^2(e+fx)\,\text{Hyper}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/15*(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -
p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] + 15*Hypergeometric2F1[-1/2, -p, 1
/2, -((b*Tan[e + f*x]^2)/(a + b))])*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*T
an[e + f*x]^2)/(a + b))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 365, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \ 4620$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 365$$

$$\frac{\frac{\int \cot^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p\left(5(a+b)\tan^2(e+fx)+10a+b(2p+7)\right)d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{5(a+b)}}{f}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{\frac{\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{3(a+b)}-\frac{(10a+b(2p+7))\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{3(a+b)}}{5(a+b)}-\frac{\cot^5(e+fx)(}{}}{f}$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\frac{\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p d\tan(e+fx)}{3(a+b)}-\frac{(10a+b(2p+7))\cot^3(e+fx)}{3(a+}}{5(a+b)}}{f}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\frac{-\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{3(a+b)}-\frac{(10a+b(2p+}{}}{5(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(-1/5*(Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(a + b) + (-1/3*
((10*a + b*(7 + 2*p))*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(
a + b) - ((15*a^2 + 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cot[e + f*x]*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b + b*
Tan[e + f*x]^2)^p)/(3*(a + b)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(5*(a +
 b)))/f
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \csc(fx+e)^6 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^6(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec(fx+e)^2+a\right)^p \csc(fx+e)^6 \, dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Giac [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6, x)
```

Reduce [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\csc\left(fx+e\right)^6dx$$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*csc(e + f*x)**6,x)
```

3.144 $\int (a-a\sec^2(c+dx))^4\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int (a-a\sec^2(c+dx))^4\,dx = a^4x-\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}$$

output
```
a^4*x-a^4*tan(d*x+c)/d+1/3*a^4*tan(d*x+c)^3/d-1/5*a^4*tan(d*x+c)^5/d+1/7*a
^4*tan(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int (a-a\sec^2(c+dx))^4\,dx = a^4\left(\frac{\arctan(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}\right)$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^4,x]
```

output
```
a^4*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[
c + d*x]^5/(5*d) + Tan[c + d*x]^7/(7*d))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sec^2(c+dx)\right)^4dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sec(c+dx)^2\right)^4dx$$

$$\Big\downarrow \text{4608}$$

$$a^4\int \tan^8(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^4\int \tan(c+dx)^8dx$$

$$\Big\downarrow \text{3954}$$

$$a^4\left(\frac{\tan^7(c+dx)}{7d}-\int \tan^6(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{\tan^7(c+dx)}{7d}-\int \tan(c+dx)^6dx\right)$$

$$\Big\downarrow \text{3954}$$

$$a^4\left(\int \tan^4(c+dx)dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\int \tan(c+dx)^4dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{ 3954}$$

$$a^4\left(-\int \tan^2(c+dx)dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$a^4\left(-\int \tan(c+dx)^2dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 3954}$$

$$a^4\left(\int 1dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}\right)$$

$$\Big\downarrow \text{ 24}$$

$$a^4\left(\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^4,x]
```

output
```
a^4*(x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*x]^5/(5*d) + Ta
n[c + d*x]^7/(7*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

method	result
risch	$a^4x - \frac{8ia^4(105\,e^{12i(dx+c)}+315\,e^{10i(dx+c)}+770\,e^{8i(dx+c)}+770\,e^{6i(dx+c)}+609\,e^{4i(dx+c)}+203\,e^{2i(dx+c)}+44)}{105d(e^{2i(dx+c)}+1)^7}$
derivativedivides	$\frac{a^4(dx+c)-4a^4\tan(dx+c)-6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-a^4}{d}$
default	$\frac{a^4(dx+c)-4a^4\tan(dx+c)-6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-a^4}{d}$
parts	$a^4x - \frac{a^4\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d} + \frac{4a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan}{d}$
parallelrisch	$\frac{a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}xd-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}xd+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}xd-\frac{44\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{3}-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}-a^4x+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{44a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{706a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}-\frac{3048a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{35d}+\frac{706a^4\tan\left(\right.}{15d}}{}$

input
```
int((a-sec(d*x+c)^2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x-8/105*I*a^4*(105*exp(12*I*(d*x+c))+315*exp(10*I*(d*x+c))+770*exp(8*I
*(d*x+c))+770*exp(6*I*(d*x+c))+609*exp(4*I*(d*x+c))+203*exp(2*I*(d*x+c))+4
4)/d/(exp(2*I*(d*x+c))+1)^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \left(a-a\sec^2(c+dx)\right)^4\,dx = \frac{105\,a^4dx\cos\left(dx+c\right)^7-\left(176\,a^4\cos\left(dx+c\right)^6-122\,a^4\cos\left(dx+c\right)^4+66\,a^4\cos\left(dx+c\right)^2-15\,a^4\right)\sin\left(d\right.}{105\,d\cos\left(dx+c\right)^7}$$

input
```
integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*(105*a^4*d*x*cos(d*x + c)^7 - (176*a^4*cos(d*x + c)^6 - 122*a^4*cos(
d*x + c)^4 + 66*a^4*cos(d*x + c)^2 - 15*a^4)*sin(d*x + c))/(d*cos(d*x + c)
^7)
```

Sympy [F]

$$\int \left(a-a\sec^2(c+dx)\right)^4\,dx = a^4\left(\int 1\,dx+\int \left(-4\sec^2\left(c+dx\right)\right)\,dx+\int 6\sec^4\left(c+dx\right)dx + \int \left(-4\sec^6\left(c+dx\right)\right)\,dx+\int \sec^8\left(c+dx\right)dx\right)$$

input
```
integrate((a-a*sec(d*x+c)**2)**4,x)
```

output
```
a**4*(Integral(1, x) + Integral(-4*sec(c + d*x)**2, x) + Integral(6*sec(c
+ d*x)**4, x) + Integral(-4*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**8
, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.74

$$\int \left(a-a\sec^2(c+dx)\right)^4\,dx$$
$$=a^4x+\frac{\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right)a^4}{35\,d}$$
$$-\frac{4\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)a^4}{15\,d}$$
$$+\frac{2\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^4}{d}-\frac{4\,a^4\tan\left(dx+c\right)}{d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
a^4*x + 1/35*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 3
5*tan(d*x + c))*a^4/d - 4/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*ta
n(d*x + c))*a^4/d + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - 4*a^4*tan(
d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \left(a-a\sec^2(c+dx)\right)^4\,dx$$
$$=\frac{15\,a^4\tan\left(dx+c\right)^7-21\,a^4\tan\left(dx+c\right)^5+35\,a^4\tan\left(dx+c\right)^3+105\,(dx+c)a^4-105\,a^4\tan\left(dx+c\right)}{105\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/105*(15*a^4*tan(d*x + c)^7 - 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)
^3 + 105*(d*x + c)*a^4 - 105*a^4*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \left(a - a\sec^2(c + dx)\right)^4 \, dx = \frac{\frac{a^4\tan(c+d\,x)^7}{7} - \frac{a^4\tan(c+d\,x)^5}{5} + \frac{a^4\tan(c+d\,x)^3}{3} - a^4\tan(c + d\,x) + d\,x\,a^4}{d}$$

input
```
int((a - a/cos(c + d*x)^2)^4,x)
```

output
```
((a^4*tan(c + d*x)^3)/3 - a^4*tan(c + d*x) - (a^4*tan(c + d*x)^5)/5 + (a^4
*tan(c + d*x)^7)/7 + a^4*d*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.04

$$\int \left(a - a\sec^2(c + dx)\right)^4 \, dx = \frac{a^4\left(105\cos\left(dx+c\right)\sin\left(dx+c\right)^6 dx - 315\cos\left(dx+c\right)\sin\left(dx+c\right)^4 dx + 315\cos\left(dx+c\right)\sin\left(dx+c\right)^2 d\right.}{105\cos\left(dx+c\right)d\left(\sin\left(dx+c\right)^6 - 3\,\mathrm{s}\right.}$$

input
```
int((a-a*sec(d*x+c)^2)^4,x)
```

output
```
(a**4*(105*cos(c + d*x)*sin(c + d*x)**6*d*x - 315*cos(c + d*x)*sin(c + d*x
)**4*d*x + 315*cos(c + d*x)*sin(c + d*x)**2*d*x - 105*cos(c + d*x)*d*x - 1
76*sin(c + d*x)**7 + 406*sin(c + d*x)**5 - 350*sin(c + d*x)**3 + 105*sin(c
 + d*x)))/(105*cos(c + d*x)*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin
(c + d*x)**2 - 1))
```

3.145 $\int (a-a\sec^2(c+dx))^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int (a-a\sec^2(c+dx))^3 \, dx = a^3x-\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan^5(c+dx)}{5d}$$

output
```
a^3*x-a^3*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)^3/d-1/5*a^3*tan(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int (a-a\sec^2(c+dx))^3 \, dx = -a^3\left(-\frac{\arctan(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}\right)$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^3,x]
```

output
```
-(a^3*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) +
 Tan[c + d*x]^5/(5*d)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sec^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sec(c+dx)^2\right)^3\,dx$$

$$\Big\downarrow \text{4608}$$

$$-a^3\int \tan^6(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3\int \tan(c+dx)^6dx$$

$$\Big\downarrow \text{3954}$$

$$-a^3\left(\frac{\tan^5(c+dx)}{5d}-\int \tan^4(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3\left(\frac{\tan^5(c+dx)}{5d}-\int \tan(c+dx)^4dx\right)$$

$$\Big\downarrow \text{3954}$$

$$-a^3\left(\int \tan^2(c+dx)dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3\left(\int \tan(c+dx)^2dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3954}$$

$$-a^3\left(-\int 1dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}\right)$$

$$\Big\downarrow \; 24$$

$$-a^3\left(\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^3,x]
```

output
```
-(a^3*(-x + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c + d*x]^5/(5*d)))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

method	result
risch	$a^{3}x-\frac{2ia^{3}\left(45\,e^{8i(dx+c)}+90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}+70\,e^{2i(dx+c)}+23\right)}{15d\left(e^{2i(dx+c)}+1\right)^{5}}$
derivativedivides	$\frac{a^{3}(dx+c)-3a^{3}\tan(dx+c)-3a^{3}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+a^{3}\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)}{d}$
default	$\frac{a^{3}(dx+c)-3a^{3}\tan(dx+c)-3a^{3}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+a^{3}\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)}{d}$
parts	$a^{3}x+\frac{a^{3}\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)}{d}-\frac{3a^{3}\tan(dx+c)}{d}-\frac{3a^{3}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)}{d}$
parallelrisch	$\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}xd-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}xd+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}xd-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{3}-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}xd+\cdots\right.}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cdots}$
norman	$\frac{a^{3}x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-a^{3}x+\frac{2a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{32a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3d}+\frac{356a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{15d}-\frac{32a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{3d}+\frac{2a^{3}\tan\left(\frac{dx}{2}+\cdots\right.}{d}\cdots}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{5}}$

input
```
int((a-sec(d*x+c)^2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x-2/15*I*a^3*(45*exp(8*I*(d*x+c))+90*exp(6*I*(d*x+c))+140*exp(4*I*(d*x
+c))+70*exp(2*I*(d*x+c))+23)/d/(exp(2*I*(d*x+c))+1)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \left(a-a\sec^{2}(c+dx)\right)^{3}\,dx =\frac{15\,a^{3}dx\cos\left(dx+c\right)^{5}-\left(23\,a^{3}\cos\left(dx+c\right)^{4}-11\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{5}}$$

input
```
integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(15*a^3*d*x*cos(d*x + c)^5 - (23*a^3*cos(d*x + c)^4 - 11*a^3*cos(d*x
+ c)^2 + 3*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \left(a - a\sec^2(c+dx)\right)^3\,dx = -a^3\left(\int (-1)\,dx + \int 3\sec^2(c+dx)\,dx + \int \left(-3\sec^4(c+dx)\right)\,dx + \int \sec^6(c+dx)\,dx\right)$$

input
```
integrate((a-a*sec(d*x+c)**2)**3,x)
```

output
```
-a**3*(Integral(-1, x) + Integral(3*sec(c + d*x)**2, x) + Integral(-3*sec(
c + d*x)**4, x) + Integral(sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \left(a - a\sec^2(c+dx)\right)^3\,dx = a^3x - \frac{\left(3\,\tan\left(dx+c\right)^5 + 10\,\tan\left(dx+c\right)^3 + 15\,\tan\left(dx+c\right)\right)a^3}{15\,d} + \frac{\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)a^3}{d} - \frac{3\,a^3\tan\left(dx+c\right)}{d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*x - 1/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3/
d + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3/d - 3*a^3*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \left(a - a\sec^2(c + dx)\right)^3 \, dx = -\frac{3\,a^3\tan\left(dx + c\right)^5 - 5\,a^3\tan\left(dx + c\right)^3 - 15\,(dx + c)a^3 + 15\,a^3\tan\left(dx + c\right)}{15\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/15*(3*a^3*tan(d*x + c)^5 - 5*a^3*tan(d*x + c)^3 - 15*(d*x + c)*a^3 + 15
*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 15.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \left(a - a\sec^2(c + dx)\right)^3 \, dx = -\frac{\frac{a^3\tan(c+d\,x)^5}{5} - \frac{a^3\tan(c+d\,x)^3}{3} + a^3\tan(c + d\,x) - d\,x\,a^3}{d}$$

input
```
int((a - a/cos(c + d*x)^2)^3,x)
```

output
```
-(a^3*tan(c + d*x) - (a^3*tan(c + d*x)^3)/3 + (a^3*tan(c + d*x)^5)/5 - a^3
*d*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.02

$$\int \left(a - a\sec^2(c + dx)\right)^3 \, dx = \frac{a^3\left(15\cos\left(dx + c\right)\sin\left(dx + c\right)^4 dx - 30\cos\left(dx + c\right)\sin\left(dx + c\right)^2 dx + 15\cos\left(dx + c\right)dx - 23\sin\left(dx + \right.\right.}{15\cos\left(dx + c\right)d\left(\sin\left(dx + c\right)^4 - 2\sin\left(dx + c\right)^2 + 1\right)}$$

input
```
int((a-a*sec(d*x+c)^2)^3,x)
```

output
```
(a**3*(15*cos(c + d*x)*sin(c + d*x)**4*d*x - 30*cos(c + d*x)*sin(c + d*x)*
*2*d*x + 15*cos(c + d*x)*d*x - 23*sin(c + d*x)**5 + 35*sin(c + d*x)**3 - 1
5*sin(c + d*x)))/(15*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 +
 1))
```

3.146 $\int (a-a\sec^2(c+dx))^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int (a-a\sec^2(c+dx))^2\,dx=a^2x-\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}$$

output
```
a^2*x-a^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int (a-a\sec^2(c+dx))^2\,dx=a^2\left(\frac{\arctan(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}\right)$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^2,x]
```

output
```
a^2*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sec^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sec(c+dx)^2\right)^2 dx$$

$$\Big\downarrow \text{4608}$$

$$a^2\int \tan^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \tan(c+dx)^4dx$$

$$\Big\downarrow \text{3954}$$

$$a^2\left(\frac{\tan^3(c+dx)}{3d}-\int \tan^2(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(\frac{\tan^3(c+dx)}{3d}-\int \tan(c+dx)^2dx\right)$$

$$\Big\downarrow \text{3954}$$

$$a^2\left(\int 1dx+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}\right)$$

$$\Big\downarrow \text{24}$$

$$a^2\left(\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^2,x]
```

output
```
a^2*(x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result
parts	$a^2x - \frac{a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d} - \frac{2a^2\tan(dx+c)}{d}$
derivativedivides	$\frac{a^2(dx+c)-2a^2\tan(dx+c)-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a^2(dx+c)-2a^2\tan(dx+c)-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$a^2x - \frac{4ia^2\left(3\,\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}+2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$
parallelrisch	$\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6xd-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4xd+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2xd-\frac{20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-dx+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$
norman	$\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-a^2x+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{20a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+3a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^2x\tan\left(\frac{dx}{2}+\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3}$

input
```
int((a-sec(d*x+c)^2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-a^2/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-2*a^2*tan(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \left(a-a\sec^2(c+dx)\right)^2\,dx=\frac{3\,a^2dx\cos\left(dx+c\right)^3-\left(4\,a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*d*x*cos(d*x + c)^3 - (4*a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))
/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \left(a-a\sec^2(c+dx)\right)^2\,dx = a^2\left(\int 1\,dx+\int \left(-2\sec^2{(c+dx)}\right)\,dx+\int \sec^4{(c+dx)}\,dx\right)$$

input
```
integrate((a-a*sec(d*x+c)**2)**2,x)
```

output
```
a**2*(Integral(1, x) + Integral(-2*sec(c + d*x)**2, x) + Integral(sec(c +
d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \left(a-a\sec^2(c+dx)\right)^2\,dx = a^2x+\frac{\left(\tan{(dx+c)}^3+3\,\tan{(dx+c)}\right)a^2}{3\,d}-\frac{2\,a^2\tan{(dx+c)}}{d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2/d - 2*a^2*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \left(a-a\sec^2(c+dx)\right)^2\,dx = \frac{a^2\tan{(dx+c)}^3+3\,(dx+c)a^2-3\,a^2\tan{(dx+c)}}{3\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(a^2*tan(d*x + c)^3 + 3*(d*x + c)*a^2 - 3*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \left(a - a\sec^2(c + dx)\right)^2 \, dx = a^2 x - \frac{a^2 \left(3\tan(c + d\,x) - \tan(c + d\,x)^3\right)}{3\,d}$$

input
```
int((a - a/cos(c + d*x)^2)^2,x)
```

output
```
a^2*x - (a^2*(3*tan(c + d*x) - tan(c + d*x)^3))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \left(a - a\sec^2(c + dx)\right)^2 \, dx = \frac{a^2\left(3\cos(dx + c)\sin(dx + c)^2 dx - 3\cos(dx + c)\,dx - 4\sin(dx + c)^3 + 3\sin(dx + c)\right)}{3\cos(dx + c)\,d\left(\sin(dx + c)^2 - 1\right)}$$

input
```
int((a-a*sec(d*x+c)^2)^2,x)
```

output
```
(a**2*(3*cos(c + d*x)*sin(c + d*x)**2*d*x - 3*cos(c + d*x)*d*x - 4*sin(c +
 d*x)**3 + 3*sin(c + d*x)))/(3*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.147 $\int (a - a\sec^2(c + dx))\, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int (a - a\sec^2(c + dx))\, dx = ax - \frac{a\tan(c + dx)}{d}$$

output

```
a*x-a*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int (a - a\sec^2(c + dx))\, dx = -a\left(-\frac{\arctan(\tan(c + dx))}{d} + \frac{\tan(c + dx)}{d}\right)$$

input

```
Integrate[a - a*Sec[c + d*x]^2,x]
```

output

```
-(a*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sec^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax-\frac{a\tan(c+dx)}{d}$$

input
```
Int[a - a*Sec[c + d*x]^2,x]
```

output
```
a*x - (a*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$ax-\frac{a\tan(dx+c)}{d}$	17
parts	$ax-\frac{a\tan(dx+c)}{d}$	17
derivativedivides	$\frac{(dx+c)a-a\tan(dx+c)}{d}$	22
risch	$ax-\frac{2ia}{d(e^{2i(dx+c)}+1)}$	25
parallelrisch	$-\frac{a\sin(dx+c)}{\cos(dx+c)d}+ax$	25
norman	$\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-ax+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}$	51

input
```
int(a-sec(d*x+c)^2*a,x,method=_RETURNVERBOSE)
```

output
```
a*x-a*tan(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \left(a-a\sec^2(c+dx)\right)\,dx=\frac{adx\cos(dx+c)-a\sin(dx+c)}{d\cos(dx+c)}$$

input
```
integrate(a-a*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
(a*d*x*cos(d*x + c) - a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a - a\sec^2(c + dx))\ dx = -a\left(\int (-1)\ dx + \int \sec^2{(c + dx)}\, dx\right)$$

input
```
integrate(a-a*sec(d*x+c)**2,x)
```

output
```
-a*(Integral(-1, x) + Integral(sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a - a\sec^2(c + dx))\ dx = ax - \frac{a\tan{(dx + c)}}{d}$$

input
```
integrate(a-a*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
a*x - a*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a - a\sec^2(c + dx))\ dx = ax - \frac{a\tan{(dx + c)}}{d}$$

input
```
integrate(a-a*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
a*x - a*tan(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 15.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a - a\sec^2(c + dx)) \, dx = a\,x - \frac{a\tan(c + d\,x)}{d}$$

input
```
int(a - a/cos(c + d*x)^2,x)
```

output
```
a*x - (a*tan(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int (a - a\sec^2(c + dx)) \, dx = \frac{a(\cos(dx + c)\,dx - \sin(dx + c))}{\cos(dx + c)\,d}$$

input
```
int(a-a*sec(d*x+c)^2,x)
```

output
```
(a*(cos(c + d*x)*d*x - sin(c + d*x)))/(cos(c + d*x)*d)
```

3.148 $\int \frac{1}{a-a\sec^2(c+dx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx=\frac{x}{a}+\frac{\cot(c+dx)}{ad}$$

output
```
x/a+cot(d*x+c)/a/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx=\frac{\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{ad}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-1),x]
```

output
```
(Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/(a*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4608, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sec^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-a\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{4608}$$

$$-\frac{\int \cot^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{-\int 1dx-\frac{\cot(c+dx)}{d}}{a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{-\frac{\cot(c+dx)}{d}-x}{a}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-1),x]
```

output
```
-((-x - Cot[c + d*x]/d)/a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{\arctan(\tan(dx+c))+\frac{1}{\tan(dx+c)}}{da}$	24
default	$\frac{\arctan(\tan(dx+c))+\frac{1}{\tan(dx+c)}}{da}$	24
risch	$\frac{x}{a}+\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}$	29
parallelrisch	$\frac{2dx-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}$	34
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{1}{2ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$	55

input
```
int(1/(a-sec(d*x+c)^2*a),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(arctan(tan(d*x+c))+1/tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{dx\sin(dx+c)+\cos(dx+c)}{ad\sin(dx+c)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="fricas")
```

output
```
(d*x*sin(d*x + c) + cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = -\frac{\int \frac{1}{\sec^2(c+dx)-1}\,dx}{a}$$

input
```
integrate(1/(a-a*sec(d*x+c)**2),x)
```

output
```
-Integral(1/(sec(c + d*x)**2 - 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{\frac{dx+c}{a}+\frac{1}{a\tan(dx+c)}}{d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="maxima")
```

output
```
((d*x + c)/a + 1/(a*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{\frac{2\,(dx+c)}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/a - tan(1/2*d*x + 1/2*c)/a + 1/(a*tan(1/2*d*x + 1/2*c)))/
d
```

Mupad [B] (verification not implemented)

Time = 15.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{x}{a} + \frac{\cot(c+d\,x)}{a\,d}$$

input
```
int(1/(a - a/cos(c + d*x)^2),x)
```

output
```
x/a + cot(c + d*x)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{\cos(dx+c) + \sin(dx+c)\,dx}{\sin(dx+c)\,ad}$$

input
```
int(1/(a-a*sec(d*x+c)^2),x)
```

output
```
(cos(c + d*x) + sin(c + d*x)*d*x)/(sin(c + d*x)*a*d)
```

3.149 $\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx=\frac{x}{a^2}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}$$

output
```
x/a^2+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3a^2d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-2),x]
```

output
```
-1/3*(Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(a
^2*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sec(c+dx)^2)^2}dx$$

$$\Big\downarrow \text{4608}$$

$$\frac{\int \cot^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-\int \cot^2(c+dx)dx-\frac{\cot^3(c+dx)}{3d}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{\cot^3(c+dx)}{3d}}{a^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int 1dx-\frac{\cot^3(c+dx)}{3d}+\frac{\cot(c+dx)}{d}}{a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{-\frac{\cot^3(c+dx)}{3d}+\frac{\cot(c+dx)}{d}+x}{a^2}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-2),x]
```

output
```
(x + Cot[c + d*x]/d - Cot[c + d*x]^3/(3*d))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{1}{3\tan(dx+c)^3}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^2}$	34
default	$\frac{-\frac{1}{3\tan(dx+c)^3}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^2}$	34
risch	$\frac{x}{a^2}+\frac{4i(3\,e^{4i(dx+c)}-3\,e^{2i(dx+c)}+2)}{3d\,a^2\left(e^{2i(dx+c)}-1\right)^3}$	53
parallelrisch	$\frac{-\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+24dx+15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^2}$	60
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}-\frac{1}{24ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{24ad}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$	98

input
```
int(1/(a-sec(d*x+c)^2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-1/3/tan(d*x+c)^3+1/tan(d*x+c)+arctan(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(35) = 70.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^2}\,dx = \frac{4\cos(dx+c)^3+3\left(dx\cos(dx+c)^2-dx\right)\sin(dx+c)-3\cos(dx+c)}{3\left(a^2d\cos(dx+c)^2-a^2d\right)\sin(dx+c)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(4*cos(d*x + c)^3 + 3*(d*x*cos(d*x + c)^2 - d*x)*sin(d*x + c) - 3*cos(
d*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{\int \frac{1}{\sec^4(c+dx)-2\sec^2(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**2,x)
```

output
```
Integral(1/(sec(c + d*x)**4 - 2*sec(c + d*x)**2 + 1), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{\frac{3(dx+c)}{a^2} + \frac{3\tan(dx+c)^2-1}{a^2\tan(dx+c)^3}}{3d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(3*(d*x + c)/a^2 + (3*tan(d*x + c)^2 - 1)/(a^2*tan(d*x + c)^3))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{\frac{24(dx+c)}{a^2} + \frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3} + \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{24d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(24*(d*x + c)/a^2 + (15*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/2*d*x
+ 1/2*c)^3) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

Mupad [B] (verification not implemented)

Time = 15.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{x}{a^2} + \frac{\tan(c+d\,x)^2-\frac{1}{3}}{a^2\,d\tan\left(c+d\,x\right)^3}$$

input
```
int(1/(a - a/cos(c + d*x)^2)^2,x)
```

output
```
x/a^2 + (tan(c + d*x)^2 - 1/3)/(a^2*d*tan(c + d*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{4\cos\left(dx+c\right)\sin\left(dx+c\right)^2-\cos\left(dx+c\right)+3\sin\left(dx+c\right)^3dx}{3\sin\left(dx+c\right)^3a^2d}$$

input
```
int(1/(a-a*sec(d*x+c)^2)^2,x)
```

output
```
(4*cos(c + d*x)*sin(c + d*x)**2 - cos(c + d*x) + 3*sin(c + d*x)**3*d*x)/(3
*sin(c + d*x)**3*a**2*d)
```

3.150 $\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx=\frac{x}{a^3}+\frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}$$

output
```
x/a^3+cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+1/5*cot(d*x+c)^5/a^3/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx=\frac{\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5a^3d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-3),x]
```

output
```
(Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*a^3*
d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sec(c+dx)^2)^3}dx$$

$$\Big\downarrow \text{4608}$$

$$-\frac{\int \cot^6(c+dx)dx}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^6dx}{a^3}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{-\int \cot^4(c+dx)dx-\frac{\cot^5(c+dx)}{5d}}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\int \tan\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{\cot^5(c+dx)}{5d}}{a^3}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\int \cot^2(c+dx)dx-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}}{a^3}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{-\int 1dx-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}-\frac{\cot(c+dx)}{d}}{a^3}$$

$$\Big\downarrow \text{24}$$

$$-\frac{-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}-\frac{\cot(c+dx)}{d}-x}{a^3}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-3),x]
```

output
```
-((-x - Cot[c + d*x]/d + Cot[c + d*x]^3/(3*d) - Cot[c + d*x]^5/(5*d))/a^3)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^3}$
default	$\frac{-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^3}$
risch	$\frac{x}{a^3}+\frac{2i\left(45\,\mathrm{e}^{8i(dx+c)}-90\,\mathrm{e}^{6i(dx+c)}+140\,\mathrm{e}^{4i(dx+c)}-70\,\mathrm{e}^{2i(dx+c)}+23\right)}{15d\,a^3\left(\mathrm{e}^{2i(dx+c)}-1\right)^5}$
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-35\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+480dx-330\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+330\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{480d\,a^3}$
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a}+\frac{1}{160ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{96ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16ad}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{96ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{160ad}}{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$

input
```
int(1/(a-sec(d*x+c)^2*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-1/3/tan(d*x+c)^3+1/5/tan(d*x+c)^5+1/tan(d*x+c)+arctan(tan(d*x+c)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(51) = 102$.

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.98

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx = \frac{23\cos(dx+c)^5-35\cos(dx+c)^3+15\left(dx\cos(dx+c)^4-2\,dx\cos(dx+c)^2+dx\right)\sin(dx+c)+15\,c}{15\left(a^3d\cos(dx+c)^4-2\,a^3d\cos(dx+c)^2+a^3d\right)\sin(dx+c)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(23*cos(d*x + c)^5 - 35*cos(d*x + c)^3 + 15*(d*x*cos(d*x + c)^4 - 2*d
*x*cos(d*x + c)^2 + d*x)*sin(d*x + c) + 15*cos(d*x + c))/((a^3*d*cos(d*x +
 c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a - a\sec^2(c + dx))^3}\,dx = -\frac{\int \frac{1}{\sec^6(c+dx)-3\sec^4(c+dx)+3\sec^2(c+dx)-1}\,dx}{a^3}$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**3,x)
```

output
```
-Integral(1/(sec(c + d*x)**6 - 3*sec(c + d*x)**4 + 3*sec(c + d*x)**2 - 1),
 x)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a - a\sec^2(c + dx))^3}\,dx = \frac{\frac{15\,(dx+c)}{a^3} + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{a^3\tan(dx+c)^5}}{15\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/15*(15*(d*x + c)/a^3 + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(a^3*t
an(d*x + c)^5))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(51) = 102.

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a - a\sec^2(c + dx))^3}\,dx = \frac{\frac{480\,(dx+c)}{a^3} + \frac{330\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 3}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 35\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 330\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{480\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/480*(480*(d*x + c)/a^3 + (330*tan(1/2*d*x + 1/2*c)^4 - 35*tan(1/2*d*x +
1/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) - (3*a^12*tan(1/2*d*x + 1/2*c)^
5 - 35*a^12*tan(1/2*d*x + 1/2*c)^3 + 330*a^12*tan(1/2*d*x + 1/2*c))/a^15)/
d
```

Mupad [B] (verification not implemented)

Time = 15.59 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx = \frac{x}{a^3} + \frac{\tan(c+d\,x)^4 - \frac{\tan(c+d\,x)^2}{3} + \frac{1}{5}}{a^3\,d\tan\left(c+d\,x\right)^5}$$

input
```
int(1/(a - a/cos(c + d*x)^2)^3,x)
```

output
```
x/a^3 + (tan(c + d*x)^4 - tan(c + d*x)^2/3 + 1/5)/(a^3*d*tan(c + d*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx = \frac{23\cos\left(dx+c\right)\sin\left(dx+c\right)^4 - 11\cos\left(dx+c\right)\sin\left(dx+c\right)^2 + 3\cos\left(dx+c\right) + 15\sin\left(dx+c\right)^5 dx}{15\sin\left(dx+c\right)^5 a^3 d}$$

input
```
int(1/(a-a*sec(d*x+c)^2)^3,x)
```

output
```
(23*cos(c + d*x)*sin(c + d*x)**4 - 11*cos(c + d*x)*sin(c + d*x)**2 + 3*cos
(c + d*x) + 15*sin(c + d*x)**5*d*x)/(15*sin(c + d*x)**5*a**3*d)
```

3.151 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx=\frac{x}{a^4}+\frac{\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^7(c+dx)}{7a^4d}$$

output
```
x/a^4+cot(d*x+c)/a^4/d-1/3*cot(d*x+c)^3/a^4/d+1/5*cot(d*x+c)^5/a^4/d-1/7*c
ot(d*x+c)^7/a^4/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx=-\frac{\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7a^4d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-4),x]
```

output
```
-1/7*(Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(a
^4*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sec(c+dx)^2)^4}dx$$

$$\Big\downarrow \text{4608}$$

$$\frac{\int \cot^8(c+dx)dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^8dx}{a^4}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-\int \cot^6(c+dx)dx-\frac{\cot^7(c+dx)}{7d}}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \tan\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{\cot^7(c+dx)}{7d}}{a^4}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int \cot^4(c+dx)dx-\frac{\cot^7(c+dx)}{7d}+\frac{\cot^5(c+dx)}{5d}}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{\cot^7(c+dx)}{7d}+\frac{\cot^5(c+dx)}{5d}}{a^4}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{-\int \cot^2(c+dx)dx-\frac{\cot^7(c+dx)}{7d}+\frac{\cot^5(c+dx)}{5d}-\frac{\cot^3(c+dx)}{3d}}{a^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{\cot^7(c+dx)}{7d}+\frac{\cot^5(c+dx)}{5d}-\frac{\cot^3(c+dx)}{3d}}{a^4}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\int 1dx-\frac{\cot^7(c+dx)}{7d}+\frac{\cot^5(c+dx)}{5d}-\frac{\cot^3(c+dx)}{3d}+\frac{\cot(c+dx)}{d}}{a^4}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{-\frac{\cot^7(c+dx)}{7d}+\frac{\cot^5(c+dx)}{5d}-\frac{\cot^3(c+dx)}{3d}+\frac{\cot(c+dx)}{d}+x}{a^4}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-4),x]
```

output
```
(x + Cot[c + d*x]/d - Cot[c + d*x]^3/(3*d) + Cot[c + d*x]^5/(5*d) - Cot[c
+ d*x]^7/(7*d))/a^4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{-\frac{1}{7\tan(dx+c)^7}-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^4}$
default	$\frac{-\frac{1}{7\tan(dx+c)^7}-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^4}$
risch	$\frac{x}{a^4}+\frac{8i\left(105\,\mathrm{e}^{12i(dx+c)}-315\,\mathrm{e}^{10i(dx+c)}+770\,\mathrm{e}^{8i(dx+c)}-770\,\mathrm{e}^{6i(dx+c)}+609\,\mathrm{e}^{4i(dx+c)}-203\,\mathrm{e}^{2i(dx+c)}+44\right)}{105d\,a^4\left(\mathrm{e}^{2i(dx+c)}-1\right)^7}$
parallelrisch	$\frac{-15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+189\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5-189\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-1295\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+1295\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{13440d\,a^4}$
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{a}-\frac{1}{896ad}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{640ad}-\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{384ad}+\frac{93\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{128ad}-\frac{93\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{128ad}+\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{384ad}-\frac{9\tan(}{}}{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}$

input
```
int(1/(a-sec(d*x+c)^2*a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(-1/7/tan(d*x+c)^7-1/3/tan(d*x+c)^3+1/5/tan(d*x+c)^5+1/tan(d*x+c)+
arctan(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(67) = 134.

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.01

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx$$
$$=\frac{176\,\cos\left(dx+c\right)^7-406\,\cos\left(dx+c\right)^5+350\,\cos\left(dx+c\right)^3+105\,\left(dx\cos\left(dx+c\right)^6-3\,dx\cos\left(dx+c\right)^4+\right.}{105\,\left(a^4d\cos\left(dx+c\right)^6-3\,a^4d\cos\left(dx+c\right)^4+3\,a^4d\cos\left(dx+c\right)^2-\right.}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*(176*cos(d*x + c)^7 - 406*cos(d*x + c)^5 + 350*cos(d*x + c)^3 + 105*
(d*x*cos(d*x + c)^6 - 3*d*x*cos(d*x + c)^4 + 3*d*x*cos(d*x + c)^2 - d*x)*s
in(d*x + c) - 105*cos(d*x + c))/((a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x +
 c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(a - a\sec^2(c + dx)\right)^4}\,dx = \frac{\int \frac{1}{\sec^8(c+dx)-4\sec^6(c+dx)+6\sec^4(c+dx)-4\sec^2(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**4,x)
```

output
```
Integral(1/(sec(c + d*x)**8 - 4*sec(c + d*x)**6 + 6*sec(c + d*x)**4 - 4*se
c(c + d*x)**2 + 1), x)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a - a\sec^2(c + dx)\right)^4}\,dx = \frac{\frac{105\,(dx+c)}{a^4} + \frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{a^4\tan(dx+c)^7}}{105\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
1/105*(105*(d*x + c)/a^4 + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*ta
n(d*x + c)^2 - 15)/(a^4*tan(d*x + c)^7))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(67) = 134$.

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.90

$$\int \frac{1}{(a - a\sec^2(c + dx))^4}\,dx$$
$$= \frac{\frac{13440\,(dx+c)}{a^4} + \frac{9765\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 1295\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 189\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 15}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7} + \frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 189\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{13440\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/13440*(13440*(d*x + c)/a^4 + (9765*tan(1/2*d*x + 1/2*c)^6 - 1295*tan(1/2
*d*x + 1/2*c)^4 + 189*tan(1/2*d*x + 1/2*c)^2 - 15)/(a^4*tan(1/2*d*x + 1/2*
c)^7) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*a^24*tan(1/2*d*x + 1/2*c)^5
+ 1295*a^24*tan(1/2*d*x + 1/2*c)^3 - 9765*a^24*tan(1/2*d*x + 1/2*c))/a^28)
/d
```

Mupad [B] (verification not implemented)

Time = 15.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a - a\sec^2(c + dx))^4}\,dx = \frac{x}{a^4} + \frac{\tan(c+d\,x)^6 - \frac{\tan(c+d\,x)^4}{3} + \frac{\tan(c+d\,x)^2}{5} - \frac{1}{7}}{a^4\,d\tan\left(c+d\,x\right)^7}$$

input
```
int(1/(a - a/cos(c + d*x)^2)^4,x)
```

output
```
x/a^4 + (tan(c + d*x)^2/5 - tan(c + d*x)^4/3 + tan(c + d*x)^6 - 1/7)/(a^4*
d*tan(c + d*x)^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx = \frac{176\cos(dx+c)\sin(dx+c)^6-122\cos(dx+c)\sin(dx+c)^4+66\cos(dx+c)\sin(dx+c)^2-15\cos(dx}{105\sin(dx+c)^7a^4d}$$

input
```
int(1/(a-a*sec(d*x+c)^2)^4,x)
```

output
```
(176*cos(c + d*x)*sin(c + d*x)**6 - 122*cos(c + d*x)*sin(c + d*x)**4 + 66*
cos(c + d*x)*sin(c + d*x)**2 - 15*cos(c + d*x) + 105*sin(c + d*x)**7*d*x)/
(105*sin(c + d*x)**7*a**4*d)
```

3.152 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(6a+5b)\text{arctanh}(\sin(e+fx))}{16f} + \frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f} + \frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b\sec^5(e+fx)\tan(e+fx)}{6f}$$

output
```
1/16*(6*a+5*b)*arctanh(sin(f*x+e))/f+1/16*(6*a+5*b)*sec(f*x+e)*tan(f*x+e)/
f+1/24*(6*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b*sec(f*x+e)^5*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{3a\text{arctanh}(\sin(e+fx))}{8f}+\frac{5b\text{arctanh}(\sin(e+fx))}{16f}+\frac{3a\sec(e+fx)\tan(e+fx)}{8f}+\frac{5b\sec(e+fx)\tan(e+fx)}{16f}+\frac{a\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{5b\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{b\sec^5(e+fx)\tan(e+fx)}{6f}$$

input
```
Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
(3*a*ArcTanh[Sin[e + f*x]])/(8*f) + (5*b*ArcTanh[Sin[e + f*x]])/(16*f) + (
3*a*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (5*b*Sec[e + f*x]*Tan[e + f*x])/(16
*f) + (a*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (5*b*Sec[e + f*x]^3*Tan[e +
f*x])/(24*f) + (b*Sec[e + f*x]^5*Tan[e + f*x])/(6*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 4534, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^5 \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \; 4534$$

$$\frac{1}{6}(6a+5b)\int \sec^5(e+fx)dx+\frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}(6a+5b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^5dx+\frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\int \sec^3(e+fx)dx+\frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right)+\frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\int \csc\left(e+fx+\frac{\pi}{2}\right)^3dx+\frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right)+\frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(e+fx)dx+\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right)+\frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right)+\frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(e+fx))}{2f}+\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right)+\frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

input
```
Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Sec[e + f*x]^5*Tan[e + f*x])/(6*f) + ((6*a + 5*b)*((Sec[e + f*x]^3*Tan[
e + f*x])/(4*f) + (3*(ArcTanh[Sin[e + f*x]]/(2*f) + (Sec[e + f*x]*Tan[e +
f*x])/(2*f)))/4))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+b\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(}{1}\right.\right.}{f}$
default	$\frac{a\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+b\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(}{1}\right.\right.}{f}$
parts	$\frac{a\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{b\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-5\,\mathrm{se}\right.\right.}{}$
parallelrisch	$\frac{-270\left(\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4fx+4e)}{5}+\cos(2fx+2e)+\frac{2}{3}\right)\left(a+\frac{5b}{6}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+270\left(\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4fx+4e)}{5}\right.}{48f(\cos(6fx+6e)+6\,\mathrm{co}}$
norman	$\frac{\frac{(2a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{(2a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{(10a+11b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{(10a+11b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{8f}-\frac{(42a-5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{24f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$
risch	$-\frac{i e^{i(fx+e)}\left(18a\,e^{10i(fx+e)}+15b\,e^{10i(fx+e)}+102a\,e^{8i(fx+e)}+85b\,e^{8i(fx+e)}+84a\,e^{6i(fx+e)}+198b\,e^{6i(fx+e)}-84a\,e^{4i(f}\right.}{24f\left(e^{2i(fx+e)}+1\right)^6}$

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+t
an(f*x+e)))+b*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(
f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{3\,(6\,a+5\,b)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-3\,(6\,a+5\,b)\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)+2}{96\,f\cos\left(fx+e\right)^6}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/96*(3*(6*a + 5*b)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*c
os(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(6*a + 5*b)*cos(f*x + e)^4 + 2
*(6*a + 5*b)*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\sec^5(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{3(6a+5b)\log(\sin(fx+e)+1) - 3(6a+5b)\log(\sin(fx+e)-1) - \frac{2\left(3(6a+5b)\sin(fx+e)^5 - 8(6a+5b)\sin(\right.}{\sin(fx+e)^6 - 3\sin(fx+e)^4}}{96f}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/96*(3*(6*a + 5*b)*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*log(sin(f*x + e)
 - 1) - 2*(3*(6*a + 5*b)*sin(f*x + e)^5 - 8*(6*a + 5*b)*sin(f*x + e)^3 + 3
*(10*a + 11*b)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*
x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{3\,(6\,a+5\,b)\log\left(|\sin\left(fx+e\right)+1|\right)-3\,(6\,a+5\,b)\log\left(|\sin\left(fx+e\right)-1|\right)-\frac{2\left(18\,a\sin(fx+e)^5+15\,b\sin(fx+e)^5-\right.}{}}{96\,f}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/96*(3*(6*a + 5*b)*log(abs(sin(f*x + e) + 1)) - 3*(6*a + 5*b)*log(abs(sin
(f*x + e) - 1)) - 2*(18*a*sin(f*x + e)^5 + 15*b*sin(f*x + e)^5 - 48*a*sin(
f*x + e)^3 - 40*b*sin(f*x + e)^3 + 30*a*sin(f*x + e) + 33*b*sin(f*x + e))/
(sin(f*x + e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 15.88 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\operatorname{atanh}(\sin\left(e+f\,x\right))\,\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)}{f}$$
$$-\frac{\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)\,\sin\left(e+f\,x\right)^5+\left(-a-\frac{5\,b}{6}\right)\,\sin\left(e+f\,x\right)^3+\left(\frac{5\,a}{8}+\frac{11\,b}{16}\right)\,\sin\left(e+f\,x\right)}{f\,\left(\sin\left(e+f\,x\right)^6-3\sin\left(e+f\,x\right)^4+3\sin\left(e+f\,x\right)^2-1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^5,x)
```

output
```
(atanh(sin(e + f*x))*((3*a)/8 + (5*b)/16))/f - (sin(e + f*x)^5*((3*a)/8 +
(5*b)/16) + sin(e + f*x)*((5*a)/8 + (11*b)/16) - sin(e + f*x)^3*(a + (5*b)
/6))/(f*(3*sin(e + f*x)^2 - 3*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 436, normalized size of antiderivative = 4.45

$$\int \sec^5(e+fx) \left(a+b\sec^2(e+fx)\right) dx$$
$$= \frac{-18\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^6 a-15\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^6 b+54\log\left(\tan\left(\frac{fx}{2}+\right.\right.}{}$$

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

output
```
( - 18*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6*a - 15*log(tan((e + f*x)/
2) - 1)*sin(e + f*x)**6*b + 54*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a
 + 45*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*b - 54*log(tan((e + f*x)/2
) - 1)*sin(e + f*x)**2*a - 45*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*b
+ 18*log(tan((e + f*x)/2) - 1)*a + 15*log(tan((e + f*x)/2) - 1)*b + 18*log
(tan((e + f*x)/2) + 1)*sin(e + f*x)**6*a + 15*log(tan((e + f*x)/2) + 1)*si
n(e + f*x)**6*b - 54*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*a - 45*log(
tan((e + f*x)/2) + 1)*sin(e + f*x)**4*b + 54*log(tan((e + f*x)/2) + 1)*sin
(e + f*x)**2*a + 45*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*b - 18*log(t
an((e + f*x)/2) + 1)*a - 15*log(tan((e + f*x)/2) + 1)*b - 18*sin(e + f*x)*
*5*a - 15*sin(e + f*x)**5*b + 48*sin(e + f*x)**3*a + 40*sin(e + f*x)**3*b
- 30*sin(e + f*x)*a - 33*sin(e + f*x)*b)/(48*f*(sin(e + f*x)**6 - 3*sin(e
+ f*x)**4 + 3*sin(e + f*x)**2 - 1))
```

3.153 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(4a+3b)\text{arctanh}(\sin(e+fx))}{8f}+\frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f}+\frac{b\sec^3(e+fx)\tan(e+fx)}{4f}$$

output
```
1/8*(4*a+3*b)*arctanh(sin(f*x+e))/f+1/8*(4*a+3*b)*sec(f*x+e)*tan(f*x+e)/f+
1/4*b*sec(f*x+e)^3*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\text{arctanh}(\sin(e+fx))}{2f} + \frac{3b\text{arctanh}(\sin(e+fx))}{8f} + \frac{a\sec(e+fx)\tan(e+fx)}{2f} + \frac{3b\sec(e+fx)\tan(e+fx)}{8f} + \frac{b\sec^3(e+fx)\tan(e+fx)}{4f}$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*ArcTanh[Sin[e + f*x]])/(2*f) + (3*b*ArcTanh[Sin[e + f*x]])/(8*f) + (a*S
ec[e + f*x]*Tan[e + f*x])/(2*f) + (3*b*Sec[e + f*x]*Tan[e + f*x])/(8*f) +
(b*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4534, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{4}(4a+3b)\int \sec^3(e+fx)dx+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}(4a+3b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^3dx+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{4}(4a+3b)\left(\frac{1}{2}\int \sec(e+fx)dx+\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}(4a+3b)\left(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{4}(4a+3b)\left(\frac{\text{arctanh}(\sin(e+fx))}{2f}+\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + ((4*a + 3*b)*(ArcTanh[Sin[e + f*x]
]/(2*f) + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{a\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(}{8}\right.}{f}$
default	$\frac{a\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(}{8}\right.}{f}$
parts	$\frac{a\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{b\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\mathrm{ta}}{8}\right.}{f}$
parallelrisch	$\frac{-8\left(a+\frac{3b}{4}\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+8\left(a+\frac{3b}{4}\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\ln\left(\mathrm{tan}\right.}{4f(\cos(4fx+4e)+4\cos(2fx+2e)+3)}$
norman	$\frac{-\frac{(4a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{4f}-\frac{(4a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{(4a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{(4a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{(4a+3b)\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{8f}$
risch	$-\frac{ie^{i(fx+e)}\left(4a\,e^{6i(fx+e)}+3b\,e^{6i(fx+e)}+4a\,e^{4i(fx+e)}+11b\,e^{4i(fx+e)}-4a\,e^{2i(fx+e)}-11b\,e^{2i(fx+e)}-4a-3b\right)}{4f\left(e^{2i(fx+e)}+1\right)^4}+\frac{\ln(e^{i(}}{}$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+b*(-(-1/4
*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(4a+3b)\cos(fx+e)^4\log(\sin(fx+e)+1)-(4a+3b)\cos(fx+e)^4\log(-\sin(fx+e)+1)+2\left((4\right.}{16f\cos(fx+e)^4}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/16*((4*a + 3*b)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (4*a + 3*b)*cos(f
*x + e)^4*log(-sin(f*x + e) + 1) + 2*((4*a + 3*b)*cos(f*x + e)^2 + 2*b)*si
n(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\sec^3(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(4a+3b)\log(\sin(fx+e)+1)-(4a+3b)\log(\sin(fx+e)-1)-\frac{2\left((4a+3b)\sin(fx+e)^3-(4a+5b)\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}}{16f}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/16*((4*a + 3*b)*log(sin(f*x + e) + 1) - (4*a + 3*b)*log(sin(f*x + e) - 1
) - 2*((4*a + 3*b)*sin(f*x + e)^3 - (4*a + 5*b)*sin(f*x + e))/(sin(f*x + e
)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{(4a+3b)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-(4a+3b)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\left(4a\sin(fx+e)^3+3b\sin(fx+e)^3-4a\sin(\right.}{\left(\sin(fx+e)^2-1\right)^2}}{16f}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/16*((4*a + 3*b)*log(abs(sin(f*x + e) + 1)) - (4*a + 3*b)*log(abs(sin(f*x
 + e) - 1)) - 2*(4*a*sin(f*x + e)^3 + 3*b*sin(f*x + e)^3 - 4*a*sin(f*x + e
) - 5*b*sin(f*x + e))/(sin(f*x + e)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\mathrm{atanh}(\sin\left(e+f\,x\right))\,\left(\frac{a}{2}+\frac{3\,b}{8}\right)}{f}-\frac{\sin\left(e+f\,x\right)^3\,\left(\frac{a}{2}+\frac{3\,b}{8}\right)-\sin\left(e+f\,x\right)\,\left(\frac{a}{2}+\frac{5\,b}{8}\right)}{f\,\left(\sin\left(e+f\,x\right)^4-2\sin\left(e+f\,x\right)^2+1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^3,x)
```

output
```
(atanh(sin(e + f*x))*(a/2 + (3*b)/8))/f - (sin(e + f*x)^3*(a/2 + (3*b)/8)
- sin(e + f*x)*(a/2 + (5*b)/8))/(f*(sin(e + f*x)^4 - 2*sin(e + f*x)^2 + 1)
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.46

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{-4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^4 a-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^4 b+8\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{}$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

output
```
( - 4*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a - 3*log(tan((e + f*x)/2)
 - 1)*sin(e + f*x)**4*b + 8*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a +
6*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*b - 4*log(tan((e + f*x)/2) - 1
)*a - 3*log(tan((e + f*x)/2) - 1)*b + 4*log(tan((e + f*x)/2) + 1)*sin(e +
f*x)**4*a + 3*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*b - 8*log(tan((e +
 f*x)/2) + 1)*sin(e + f*x)**2*a - 6*log(tan((e + f*x)/2) + 1)*sin(e + f*x)
**2*b + 4*log(tan((e + f*x)/2) + 1)*a + 3*log(tan((e + f*x)/2) + 1)*b - 4*
sin(e + f*x)**3*a - 3*sin(e + f*x)**3*b + 4*sin(e + f*x)*a + 5*sin(e + f*x
)*b)/(8*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.154 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(2a+b)\text{arctanh}(\sin(e+fx))}{2f}+\frac{b\sec(e+fx)\tan(e+fx)}{2f}$$

output
```
1/2*(2*a+b)*arctanh(sin(f*x+e))/f+1/2*b*sec(f*x+e)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{a\coth^{-1}(\sin(e+fx))}{f}+\frac{b\text{arctanh}(\sin(e+fx))}{2f}+\frac{b\sec(e+fx)\tan(e+fx)}{2f}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*ArcCoth[Sin[e + f*x]])/f + (b*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e +
 f*x]*Tan[e + f*x])/(2*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{ 4534}$$

$$\frac{1}{2}(2a+b)\int \sec(e+fx)dx+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}(2a+b)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{(2a+b)\text{arctanh}(\sin(e+fx))}{2f}+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
((2*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e + f*x]*Tan[e + f*x])/(2
*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result	siz
derivativedivides	$\frac{a\ln(\sec(fx+e)+\tan(fx+e))+b\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	55
default	$\frac{a\ln(\sec(fx+e)+\tan(fx+e))+b\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	55
parts	$\frac{a\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{b\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	57
parallelrisch	$\frac{-\left(a+\frac{b}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(a+\frac{b}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\sin(fx+e)b}{f(1+\cos(2fx+2e))}$	86
norman	$\frac{\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{(2a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{(2a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$	93
risch	$-\frac{ib\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}+\frac{\ln\left(e^{i(fx+e)}+i\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}+i\right)b}{2f}-\frac{\ln\left(e^{i(fx+e)}-i\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b}{2f}$	11

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*ln(sec(f*x+e)+tan(f*x+e))+b*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f
*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{(2\,a+b)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2\,a+b)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\,b\sin\left(\right.}{4\,f\cos\left(fx+e\right)^2}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*((2*a + b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a + b)*cos(f*x +
e)^2*log(-sin(f*x + e) + 1) + 2*b*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(2a+b)\log\left(\sin\left(fx+e\right)+1\right)-(2a+b)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2b\sin(fx+e)}{\sin(fx+e)^2-1}}{4f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*((2*a + b)*log(sin(f*x + e) + 1) - (2*a + b)*log(sin(f*x + e) - 1) - 2
*b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(2a+b)\log\left(|\sin\left(fx+e\right)+1|\right)-(2a+b)\log\left(|\sin\left(fx+e\right)-1|\right)-\frac{2b\sin(fx+e)}{\sin(fx+e)^2-1}}{4f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/4*((2*a + b)*log(abs(sin(f*x + e) + 1)) - (2*a + b)*log(abs(sin(f*x + e)
 - 1)) - 2*b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Mupad [B] (verification not implemented)

Time = 15.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\operatorname{atanh}(\sin\left(e+f\,x\right))\,\left(a+\frac{b}{2}\right)}{f}-\frac{b\,\sin\left(e+f\,x\right)}{2\,f\,\left({\sin\left(e+f\,x\right)}^2-1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x),x)
```

output
```
(atanh(sin(e + f*x))*(a + b/2))/f - (b*sin(e + f*x))/(2*f*(sin(e + f*x)^2
- 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.42

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^2 a-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^2 b+2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}{}$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

output
```
( - 2*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a - log(tan((e + f*x)/2) -
 1)*sin(e + f*x)**2*b + 2*log(tan((e + f*x)/2) - 1)*a + log(tan((e + f*x)/
2) - 1)*b + 2*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a + log(tan((e + f
*x)/2) + 1)*sin(e + f*x)**2*b - 2*log(tan((e + f*x)/2) + 1)*a - log(tan((e
 + f*x)/2) + 1)*b - sin(e + f*x)*b)/(2*f*(sin(e + f*x)**2 - 1))
```

3.155 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{a\sin(e+fx)}{f}$$

output
```
b*arctanh(sin(f*x+e))/f+a*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\coth^{-1}(\sin(e+fx))}{f}+\frac{a\cos(fx)\sin(e)}{f}+\frac{a\cos(e)\sin(fx)}{f}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*ArcCoth[Sin[e + f*x]])/f + (a*Cos[f*x]*Sin[e])/f + (a*Cos[e]*Sin[f*x])/
f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4533}$$

$$b\int \sec(e+fx)dx+\frac{a\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 3042}$$

$$b\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{a\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{a\sin(e+fx)}{f}+\frac{b\text{arctanh}(\sin(e+fx))}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*ArcTanh[Sin[e + f*x]])/f + (a*Sin[e + f*x])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\sin(fx+e)a+b\ln(\sec(fx+e)+\tan(fx+e))}{f}$	30
default	$\frac{\sin(fx+e)a+b\ln(\sec(fx+e)+\tan(fx+e))}{f}$	30
parallelrisch	$\frac{b\left(-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)+\sin(fx+e)a}{f}$	43
risch	$-\frac{ia\,e^{i(fx+e)}}{2f}+\frac{ia\,e^{-i(fx+e)}}{2f}+\frac{\ln(e^{i(fx+e)}+i)b}{f}-\frac{\ln(e^{i(fx+e)}-i)b}{f}$	71
norman	$\frac{-\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{f}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{f}$	101

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)*a+b*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\log\left(\sin\left(fx+e\right)+1\right)-b\log\left(-\sin\left(fx+e\right)+1\right)+2\,a\sin\left(fx+e\right)}{2\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*(b*log(sin(f*x + e) + 1) - b*log(-sin(f*x + e) + 1) + 2*a*sin(f*x + e)
)/f
```

Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cos\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b(\log\left(\sin\left(fx+e\right)+1\right)-\log\left(\sin\left(fx+e\right)-1\right))+2\,a\sin\left(fx+e\right)}{2\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)) + 2*a*sin(f*x + e))
/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\log\left(|\sin\left(fx+e\right)+1|\right)-b\log\left(|\sin\left(fx+e\right)-1|\right)+2\,a\sin\left(fx+e\right)}{2\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(b*log(abs(sin(f*x + e) + 1)) - b*log(abs(sin(f*x + e) - 1)) + 2*a*sin
(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\,\sin\left(e+f\,x\right)+b\,\mathrm{atanh}(\sin\left(e+f\,x\right))}{f}$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*sin(e + f*x) + b*atanh(sin(e + f*x)))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)b+\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)b+\sin\left(fx+e\right)a}{f}$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

output
```
( - log(tan((e + f*x)/2) - 1)*b + log(tan((e + f*x)/2) + 1)*b + sin(e + f*
x)*a)/f
```

3.156 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

output
```
(a+b)*sin(f*x+e)/f-1/3*a*sin(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\cos(fx)\sin(e)}{f}+\frac{b\cos(e)\sin(fx)}{f}+\frac{a\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Cos[f*x]*Sin[e])/f + (b*Cos[e]*Sin[f*x])/f + (a*Sin[e + f*x])/f - (a*Si
n[e + f*x]^3)/(3*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 4532}$$

$$\int \cos(e+fx)\left(a\cos^2(e+fx)+b\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(e+fx+\frac{\pi}{2}\right)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b\right)dx$$

$$\Big\downarrow \text{ 3492}$$

$$-\frac{\int \left(-a\sin^2(e+fx)+a+b\right)d(-\sin(e+fx))}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{1}{3}a\sin^3(e+fx)-(a+b)\sin(e+fx)}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((a + b)*Sin[e + f*x]) + (a*Sin[e + f*x]^3)/3)/f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{\sin(3fx+3e)a+9\left(a+\frac{4b}{3}\right)\sin(fx+e)}{12f}$	31
derivativedivides	$\frac{\frac{a\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}+\sin(fx+e)b}{f}$	33
default	$\frac{\frac{a\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}+\sin(fx+e)b}{f}$	33
risch	$\frac{3a\sin(fx+e)}{4f}+\frac{\sin(fx+e)b}{f}+\frac{a\sin(3fx+3e)}{12f}$	40
norman	$\frac{\frac{2(a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{2(a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	111

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(sin(3*f*x+3*e)*a+9*(a+4/3*b)*sin(f*x+e))/f
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\left(a\cos\left(fx+e\right)^2+2\,a+3\,b\right)\sin\left(fx+e\right)}{3\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(f*x + e)^2 + 2*a + 3*b)*sin(f*x + e)/f
```

Sympy [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\cos^3\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{a\sin\left(fx+e\right)^3-3\left(a+b\right)\sin\left(fx+e\right)}{3\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(a*sin(f*x + e)^3 - 3*(a + b)*sin(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=-\frac{a\sin\left(fx+e\right)^3-3\,a\sin\left(fx+e\right)-3\,b\sin\left(fx+e\right)}{3\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/3*(a*sin(f*x + e)^3 - 3*a*sin(f*x + e) - 3*b*sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{\frac{a\sin(e+f\,x)^3}{3}-\sin\left(e+f\,x\right)\,\left(a+b\right)}{f}$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

output
```
-((a*sin(e + f*x)^3)/3 - sin(e + f*x)*(a + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\sin\left(fx+e\right)\left(-\sin\left(fx+e\right)^2a+3a+3b\right)}{3f}$$

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

output
```
(sin(e + f*x)*( - sin(e + f*x)**2*a + 3*a + 3*b))/(3*f)
```

3.157 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(a+b)\sin(e+fx)}{f}-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{a\sin^5(e+fx)}{5f}$$

output
```
(a+b)*sin(f*x+e)/f-1/3*(2*a+b)*sin(f*x+e)^3/f+1/5*a*sin(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{a\sin(e+fx)}{f}+\frac{b\sin(e+fx)}{f}-\frac{2a\sin^3(e+fx)}{3f}-\frac{b\sin^3(e+fx)}{3f}+\frac{a\sin^5(e+fx)}{5f}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Sin[e + f*x])/f + (b*Sin[e + f*x])/f - (2*a*Sin[e + f*x]^3)/(3*f) - (b*
Sin[e + f*x]^3)/(3*f) + (a*Sin[e + f*x]^5)/(5*f)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4532, 3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4532}$$

$$\int \cos^3(e+fx)\left(a\cos^2(e+fx)+b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(e+fx+\frac{\pi}{2}\right)^3\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b\right)dx$$

$$\Big\downarrow \text{3492}$$

$$-\frac{\int \left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)d(-\sin(e+fx))}{f}$$

$$\Big\downarrow \text{290}$$

$$-\frac{\int \left(a\sin^4(e+fx)-(2a+b)\sin^2(e+fx)+a\left(\frac{b}{a}+1\right)\right)d(-\sin(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}(2a+b)\sin^3(e+fx)-(a+b)\sin(e+fx)-\frac{1}{5}a\sin^5(e+fx)}{f}$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((a + b)*Sin[e + f*x]) + ((2*a + b)*Sin[e + f*x]^3)/3 - (a*Sin[e + f*x
]^5)/5)/f)
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{(25a+20b)\sin(3fx+3e)+3\sin(5fx+5e)a+150\left(a+\frac{6b}{5}\right)\sin(fx+e)}{240f}$
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{b\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}}{f}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{b\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}}{f}$
risch	$\frac{5a\sin(fx+e)}{8f}+\frac{3\sin(fx+e)b}{4f}+\frac{a\sin(5fx+5e)}{80f}+\frac{5a\sin(3fx+3e)}{48f}+\frac{\sin(3fx+3e)b}{12f}$
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{f}-\frac{2(a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{2(a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3f}-\frac{4(19a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}+4}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/240*((25*a+20*b)*sin(3*f*x+3*e)+3*sin(5*f*x+5*e)*a+150*(a+6/5*b)*sin(f*x
+e))/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(3\,a\cos\left(fx+e\right)^4+\left(4\,a+5\,b\right)\cos\left(fx+e\right)^2+8\,a+10\,b\right)\sin\left(fx+e\right)}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/15*(3*a*cos(f*x + e)^4 + (4*a + 5*b)*cos(f*x + e)^2 + 8*a + 10*b)*sin(f*
x + e)/f
```

Sympy [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\cos^5{(e+fx)}\,dx$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{3\,a\sin{(fx+e)}^5-5\,(2\,a+b)\sin{(fx+e)}^3+15\,(a+b)\sin{(fx+e)}}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(3*a*sin(f*x + e)^5 - 5*(2*a + b)*sin(f*x + e)^3 + 15*(a + b)*sin(f*x
 + e))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{3\,a\sin{(fx+e)}^5-10\,a\sin{(fx+e)}^3-5\,b\sin{(fx+e)}^3+15\,a\sin{(fx+e)}+15\,b\sin{(fx+e)}}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(3*a*sin(f*x + e)^5 - 10*a*sin(f*x + e)^3 - 5*b*sin(f*x + e)^3 + 15*a
*sin(f*x + e) + 15*b*sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.43 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\frac{a\sin(e+f\,x)^5}{5}+\left(-\frac{2\,a}{3}-\frac{b}{3}\right)\,\sin\left(e+f\,x\right)^3+\left(a+b\right)\,\sin\left(e+f\,x\right)}{f}$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*sin(e + f*x)^5)/5 - sin(e + f*x)^3*((2*a)/3 + b/3) + sin(e + f*x)*(a +
 b))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\sin\left(fx+e\right)\left(3\sin\left(fx+e\right)^4a-10\sin\left(fx+e\right)^2a-5\sin\left(fx+e\right)^2b+15a+15b\right)}{15f}$$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

output
```
(sin(e + f*x)*(3*sin(e + f*x)**4*a - 10*sin(e + f*x)**2*a - 5*sin(e + f*x)
**2*b + 15*a + 15*b))/(15*f)
```

3.158 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\sec^6(e+fx)\tan(e+fx)}{7f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}+\frac{(7a+6b)\tan^5(e+fx)}{35f}$$

output
```
1/7*(7*a+6*b)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^6*tan(f*x+e)/f+2/21*(7*a+6*b)*
tan(f*x+e)^3/f+1/35*(7*a+6*b)*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{a\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f}$$
$$+\frac{b\left(\tan(e+fx)+\tan^3(e+fx)+\frac{3}{5}\tan^5(e+fx)+\frac{1}{7}\tan^7(e+fx)\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5))/f + (b*(Tan[e
 + f*x] + Tan[e + f*x]^3 + (3*Tan[e + f*x]^5)/5 + Tan[e + f*x]^7/7))/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4534, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^6\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{7}(7a+6b)\int \sec^6(e+fx)dx+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7a+6b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^6dx+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

$\downarrow$ 4254

$$\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}-\frac{(7a+6b)\int\left(\tan^4(e+fx)+2\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{7f}$$

$\downarrow$ 2009

$$\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}-\frac{(7a+6b)\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{7f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Sec[e + f*x]^6*Tan[e + f*x])/(7*f) - ((7*a + 6*b)*(-Tan[e + f*x] - (2*T
an[e + f*x]^3)/3 - Tan[e + f*x]^5/5))/(7*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}$
parts	$-\frac{a\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{b\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}$
risch	$\frac{16i\left(70a\,e^{8i(fx+e)}+175a\,e^{6i(fx+e)}+210b\,e^{6i(fx+e)}+147a\,e^{4i(fx+e)}+126b\,e^{4i(fx+e)}+49a\,e^{2i(fx+e)}+42b\,e^{2i(fx+e)}+7a\right.}{105f\left(e^{2i(fx+e)}+1\right)^7}$
parallelrisch	$\frac{(1176a+1008b)\sin(3fx+3e)+(392a+336b)\sin(5fx+5e)+(56a+48b)\sin(7fx+7e)+840\sin(fx+e)(a+2b)}{105f(\cos(7fx+7e)+7\cos(5fx+5e)+21\cos(3fx+3e)+35\cos(fx+e))}$
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{f}+\frac{4(5a+3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{4(5a+3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{3f}+\frac{8(91a+53b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{35f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^7}$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-b*(-16/35-1/
7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\left(8\,(7\,a+6\,b)\cos\left(fx+e\right)^6+4\,(7\,a+6\,b)\cos\left(fx+e\right)^4+3\,(7\,a+6\,b)\cos\left(fx+e\right)^2+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^7}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/105*(8*(7*a + 6*b)*cos(f*x + e)^6 + 4*(7*a + 6*b)*cos(f*x + e)^4 + 3*(7*
a + 6*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\sec^6\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{15\,b\tan\left(fx+e\right)^7+21\,(a+3\,b)\tan\left(fx+e\right)^5+35\,(2\,a+3\,b)\tan\left(fx+e\right)^3+105\,(a+b)\tan\left(fx+e\right)}{105\,f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/105*(15*b*tan(f*x + e)^7 + 21*(a + 3*b)*tan(f*x + e)^5 + 35*(2*a + 3*b)*
tan(f*x + e)^3 + 105*(a + b)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{15\,b\tan\left(fx+e\right)^7+21\,a\tan\left(fx+e\right)^5+63\,b\tan\left(fx+e\right)^5+70\,a\tan\left(fx+e\right)^3+105\,b\tan\left(fx+e\right)^3+}{105\,f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 + 63*b*tan(f*x + e)^5 + 7
0*a*tan(f*x + e)^3 + 105*b*tan(f*x + e)^3 + 105*a*tan(f*x + e) + 105*b*tan
(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\frac{b\tan(e+f\,x)^7}{7}+\left(\frac{a}{5}+\frac{3\,b}{5}\right)\,\tan(e+f\,x)^5+\left(\frac{2\,a}{3}+b\right)\,\tan(e+f\,x)^3+(a+b)\,\tan(e+f\,x)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^6,x)
```

output
```
(tan(e + f*x)^5*(a/5 + (3*b)/5) + (b*tan(e + f*x)^7)/7 + tan(e + f*x)^3*((
2*a)/3 + b) + tan(e + f*x)*(a + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\sin(fx+e)\left(56\sin(fx+e)^6 a+48\sin(fx+e)^6 b-196\sin(fx+e)^4 a-168\sin(fx+e)^4 b+245\sin\right.}{105\cos(fx+e) f\left(\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+\right.}$$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

output
```
(sin(e + f*x)*(56*sin(e + f*x)**6*a + 48*sin(e + f*x)**6*b - 196*sin(e + f
*x)**4*a - 168*sin(e + f*x)**4*b + 245*sin(e + f*x)**2*a + 210*sin(e + f*x
)**2*b - 105*a - 105*b))/(105*cos(e + f*x)*f*(sin(e + f*x)**6 - 3*sin(e +
f*x)**4 + 3*sin(e + f*x)**2 - 1))
```

3.159 $\int \sec^4(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \sec^4(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \frac{(5a+4b)\tan(e+fx)}{5f} + \frac{b\sec^4(e+fx)\tan(e+fx)}{5f} + \frac{(5a+4b)\tan^3(e+fx)}{15f}$$

output
```
1/5*(5*a+4*b)*tan(f*x+e)/f+1/5*b*sec(f*x+e)^4*tan(f*x+e)/f+1/15*(5*a+4*b)*
tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \sec^4(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \frac{a\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f} + \frac{b\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f + (b*(Tan[e + f*x] + (2*Tan[e + f*
x]^3)/3 + Tan[e + f*x]^5/5))/f
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4534, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{5}(5a+4b)\int \sec^4(e+fx)dx+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5a+4b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}-\frac{(5a+4b)\int\left(\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}-\frac{(5a+4b)\left(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{5f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Sec[e + f*x]^4*Tan[e + f*x])/(5*f) - ((5*a + 4*b)*(-Tan[e + f*x] - Tan[
e + f*x]^3/3))/(5*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{b\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$
parallelrisch	$\frac{(50a+40b)\sin(3fx+3e)+(10a+8b)\sin(5fx+5e)+40\sin(fx+e)(a+2b)}{15f(\cos(5fx+5e)+5\cos(3fx+3e)+10\cos(fx+e))}$
risch	$\frac{4i\left(15a\,\mathrm{e}^{6i(fx+e)}+35a\,\mathrm{e}^{4i(fx+e)}+40b\,\mathrm{e}^{4i(fx+e)}+25a\,\mathrm{e}^{2i(fx+e)}+20b\,\mathrm{e}^{2i(fx+e)}+5a+4b\right)}{15f\left(\mathrm{e}^{2i(fx+e)}+1\right)^5}$
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{f}+\frac{8(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{8(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3f}-\frac{4(25a+29b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b*(-8/15-1/5*sec(f*x+e)^4-4/15*
sec(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(2\,(5\,a+4\,b)\cos\left(fx+e\right)^4+(5\,a+4\,b)\cos\left(fx+e\right)^2+3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/15*(2*(5*a + 4*b)*cos(f*x + e)^4 + (5*a + 4*b)*cos(f*x + e)^2 + 3*b)*sin
(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\sec^4{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,b\tan\left(fx+e\right)^5+5\,(a+2\,b)\tan\left(fx+e\right)^3+15\,(a+b)\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(3*b*tan(f*x + e)^5 + 5*(a + 2*b)*tan(f*x + e)^3 + 15*(a + b)*tan(f*x
 + e))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,b\tan\left(fx+e\right)^5+5\,a\tan\left(fx+e\right)^3+10\,b\tan\left(fx+e\right)^3+15\,a\tan\left(fx+e\right)+15\,b\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 10*b*tan(f*x + e)^3 + 15*a
*tan(f*x + e) + 15*b*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.74 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\frac{b\tan(e+f\,x)^5}{5}+\left(\frac{a}{3}+\frac{2\,b}{3}\right)\,\tan(e+f\,x)^3+(a+b)\,\tan(e+f\,x)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^4,x)
```

output
```
(tan(e + f*x)^3*(a/3 + (2*b)/3) + (b*tan(e + f*x)^5)/5 + tan(e + f*x)*(a +
 b))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.42

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\sin(fx+e)\left(10\sin(fx+e)^4a+8\sin(fx+e)^4b-25\sin(fx+e)^2a-20\sin(fx+e)^2b+15a+15b\right)}{15\cos(fx+e)\,f\left(\sin(fx+e)^4-2\sin(fx+e)^2+1\right)}$$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

output
```
(sin(e + f*x)*(10*sin(e + f*x)**4*a + 8*sin(e + f*x)**4*b - 25*sin(e + f*x
)**2*a - 20*sin(e + f*x)**2*b + 15*a + 15*b))/(15*cos(e + f*x)*f*(sin(e +
f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.160 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\sec^2(e+fx)\tan(e+fx)}{3f}$$

output
```
1/3*(3*a+2*b)*tan(f*x+e)/f+1/3*b*sec(f*x+e)^2*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{a\tan(e+fx)}{f}+\frac{b\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Tan[e + f*x])/f + (b*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4534, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{3}(3a+2b)\int \sec^2(e+fx)dx+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(3a+2b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}-\frac{(3a+2b)\int 1d(-\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{24}$$

$$\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
((3*a + 2*b)*Tan[e + f*x])/(3*f) + (b*Sec[e + f*x]^2*Tan[e + f*x])/(3*f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{a\tan\left(fx+e\right)-b\left(-\frac{2}{3}-\frac{\sec\left(fx+e\right)^{2}}{3}\right)\tan\left(fx+e\right)}{f}$	35
default	$\frac{a\tan\left(fx+e\right)-b\left(-\frac{2}{3}-\frac{\sec\left(fx+e\right)^{2}}{3}\right)\tan\left(fx+e\right)}{f}$	35
parts	$\frac{a\tan\left(fx+e\right)}{f}-\frac{b\left(-\frac{2}{3}-\frac{\sec\left(fx+e\right)^{2}}{3}\right)\tan\left(fx+e\right)}{f}$	37
parallelrisch	$\frac{\left(3a+2b\right)\sin\left(3fx+3e\right)+3\sin\left(fx+e\right)\left(a+2b\right)}{3f\left(\cos\left(3fx+3e\right)+3\cos\left(fx+e\right)\right)}$	57
risch	$\frac{2i\left(3a\,{\mathrm e}^{4i\left(fx+e\right)}+6a\,{\mathrm e}^{2i\left(fx+e\right)}+6b\,{\mathrm e}^{2i\left(fx+e\right)}+3a+2b\right)}{3f\left({\mathrm e}^{2i\left(fx+e\right)}+1\right)^{3}}$	63
norman	$\frac{-\frac{2\left(a+b\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2\left(a+b\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{f}+\frac{4\left(3a+b\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$	75

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*tan(f*x+e)-b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\left((3\,a+2\,b)\cos\left(fx+e\right)^2+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*((3*a + 2*b)*cos(f*x + e)^2 + b)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\sec^2\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b+3\,a\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*b + 3*a*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan\left(fx+e\right)^3+3\,a\tan\left(fx+e\right)+3\,b\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/3*(b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 3*b*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)^3}{3\,f}+\frac{\tan(e+f\,x)\,\left(a+b\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x)^3)/(3*f) + (tan(e + f*x)*(a + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\sin\left(fx+e\right)\left(3\sin\left(fx+e\right)^2a+2\sin\left(fx+e\right)^2b-3a-3b\right)}{3\cos\left(fx+e\right)f\left(\sin\left(fx+e\right)^2-1\right)}$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
```

output
```
(sin(e + f*x)*(3*sin(e + f*x)**2*a + 2*sin(e + f*x)**2*b - 3*a - 3*b))/(3*
cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.161 $\int \left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan(e+fx)}{f}$$

output
```
a*x+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{b\tan(e+fx)}{f}$$

input
```
Int[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\tan(fx+e)}{f}$	16
parts	$ax+\frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
parallelrisch	$\frac{b\sin(fx+e)}{\cos(fx+e)f}+ax$	24
risch	$ax+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	25
norman	$\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-ax-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}$	51

input
```
int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*tan(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

output
```
(a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\,dx$$

input
```
integrate(a+b*sec(f*x+e)**2,x)
```

output
```
Integral(a + b*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan{(fx+e)}}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

output
```
a*x + b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan{(fx+e)}}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

output
```
a*x + b*tan(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 15.96 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\tan(e+f\,x)+a\,f\,x}{f}$$

input
```
int(a + b/cos(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x) + a*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\frac{\cos\left(fx+e\right)afx+\sin\left(fx+e\right)b}{\cos\left(fx+e\right)f}$$

input
```
int(a+b*sec(f*x+e)^2,x)
```

output
```
(cos(e + f*x)*a*f*x + sin(e + f*x)*b)/(cos(e + f*x)*f)
```

3.162 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{1}{2}(a+2b)x+\frac{a\cos(e+fx)\sin(e+fx)}{2f}$$

output
```
1/2*(a+2*b)*x+1/2*a*cos(f*x+e)*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=bx+\frac{a(e+fx)}{2f}+\frac{a\sin(2(e+fx))}{4f}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
b*x + (a*(e + f*x))/(2*f) + (a*Sin[2*(e + f*x)])/(4*f)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{2}(a+2b)\int 1dx+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}x(a+2b)+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b)*x)/2 + (a*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result
risch	$\frac{ax}{2}+xb+\frac{a\sin(2fx+2e)}{4f}$
parallelrisch	$\frac{a\sin(2fx+2e)+2(a+2b)xf}{4f}$
derivativedivides	$\frac{a\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(fx+e)}{f}$
default	$\frac{a\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(fx+e)}{f}$
norman	$\frac{(-\frac{a}{2}-b)x+(-\frac{a}{2}-b)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+(\frac{a}{2}+b)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(\frac{a}{2}+b)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x+x*b+1/4*a/f*sin(2*f*x+2*e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(a+2\,b)fx+a\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*((a + 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e))/f
```

Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = a\left(\begin{cases}\frac{x\sin^2(e+fx)}{2}+\frac{x\cos^2(e+fx)}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f} & \text{for } f\neq 0\\ x\cos^2(e) & \text{otherwise}\end{cases}\right)+bx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

output
```
a*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 + sin(e + f*x)*cos(
e + f*x)/(2*f), Ne(f, 0)), (x*cos(e)**2, True)) + b*x
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(a+2\,b)+\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(a+2\,b)+\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Mupad [B] (verification not implemented)

Time = 16.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\frac{a\,\sin(2\,e+2\,f\,x)}{4}+f\,x\,\left(\frac{a}{2}+b\right)}{f}$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*sin(2*e + 2*f*x))/4 + f*x*(a/2 + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\cos(fx+e)\sin(fx+e)\,a+afx+2bfx}{2f}$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
```

output
```
(cos(e + f*x)*sin(e + f*x)*a + a*f*x + 2*b*f*x)/(2*f)
```

3.163 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}$$

output
```
1/8*(3*a+4*b)*x+1/8*(3*a+4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*s
in(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{4(3a+4b)(e+fx)+8(a+b)\sin(2(e+fx))+a\sin(4(e+fx))}{32f}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(4*(3*a + 4*b)*(e + f*x) + 8*(a + b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)]
)/(32*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{4}(3a+4b)\int \cos^2(e+fx)dx+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(3a+4b)\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}(3a+4b)\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}(3a+4b)\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + ((3*a + 4*b)*(x/2 + (Cos[e + f*x]*
Sin[e + f*x])/(2*f)))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(8a+8b)\sin(2fx+2e)+a\sin(4fx+4e)+12x\left(a+\frac{4b}{3}\right)f}{32f}$
risch	$\frac{3ax}{8}+\frac{xb}{2}+\frac{a\sin(4fx+4e)}{32f}+\frac{a\sin(2fx+2e)}{4f}+\frac{\sin(2fx+2e)b}{4f}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
default	$\frac{a\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
norman	$\frac{\left(-\frac{3a}{8}-\frac{b}{2}\right)x+\left(-\frac{9a}{8}-\frac{3b}{2}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{3a}{4}-b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{3a}{4}+b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{3a}{8}+\frac{b}{2}\right)x\tan\left(\frac{fx}{2}+\frac{}{}\right.}{\left(1+\tan\left(\frac{fx}{2}\right.\right.}$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/32*((8*a+8*b)*sin(2*f*x+2*e)+a*sin(4*f*x+4*e)+12*x*(a+4/3*b)*f)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(3\,a+4\,b)fx+\left(2\,a\cos\left(fx+e\right)^3+(3\,a+4\,b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/8*((3*a + 4*b)*f*x + (2*a*cos(f*x + e)^3 + (3*a + 4*b)*cos(f*x + e))*sin
(f*x + e))/f
```

Sympy [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\cos^4\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(3a+4b)+\frac{(3a+4b)\tan(fx+e)^3+(5a+4b)\tan(fx+e)}{\tan(fx+e)^4+2\tan(fx+e)^2+1}}{8f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/8*((f*x + e)*(3*a + 4*b) + ((3*a + 4*b)*tan(f*x + e)^3 + (5*a + 4*b)*tan
(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(3a+4b)+\frac{3a\tan(fx+e)^3+4b\tan(fx+e)^3+5a\tan(fx+e)+4b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/8*((f*x + e)*(3*a + 4*b) + (3*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 5*
a*tan(f*x + e) + 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 15.97 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = x\left(\frac{3\,a}{8}+\frac{b}{2}\right)+\frac{\left(\frac{3\,a}{8}+\frac{b}{2}\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a}{8}+\frac{b}{2}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

output
```
x*((3*a)/8 + b/2) + (tan(e + f*x)^3*((3*a)/8 + b/2) + tan(e + f*x)*((5*a)/
8 + b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{-2\cos\left(fx+e\right)\sin\left(fx+e\right)^3a+5\cos\left(fx+e\right)\sin\left(fx+e\right)a+4\cos\left(fx+e\right)\sin\left(fx+e\right)b+3afx+4}{8f}$$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

output
```
( - 2*cos(e + f*x)*sin(e + f*x)**3*a + 5*cos(e + f*x)*sin(e + f*x)*a + 4*c
os(e + f*x)*sin(e + f*x)*b + 3*a*f*x + 4*b*f*x)/(8*f)
```

3.164 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{1}{16}(5a+6b)x + \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a\cos^5(e+fx)\sin(e+fx)}{6f}$$

output
```
1/16*(5*a+6*b)*x+1/16*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(5*a+6*b)*cos
(f*x+e)^3*sin(f*x+e)/f+1/6*a*cos(f*x+e)^5*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{60ae+72be+60afx+72bfx+(45a+48b)\sin(2(e+fx))+(9a+6b)\sin(4(e+fx))+a\sin(6(e+f}{192f}$$

input
```
Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(60*a*e + 72*b*e + 60*a*f*x + 72*b*f*x + (45*a + 48*b)*Sin[2*(e + f*x)] +
(9*a + 6*b)*Sin[4*(e + f*x)] + a*Sin[6*(e + f*x)])/(192*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4533, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{6}(5a+6b)\int \cos^4(e+fx)dx+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}(5a+6b)\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}(5a+6b)\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}(5a+6b)\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}(5a+6b)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}(5a+6b)\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

input
```
Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Cos[e + f*x]^5*Sin[e + f*x])/(6*f) + ((5*a + 6*b)*((Cos[e + f*x]^3*Sin[
e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4))/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{(45a+48b)\sin(2fx+2e)+(9a+6b)\sin(4fx+4e)+a\sin(6fx+6e)+60x\left(a+\frac{6b}{5}\right)f}{192f}$
risch	$\frac{5ax}{16}+\frac{3xb}{8}+\frac{a\sin(6fx+6e)}{192f}+\frac{3a\sin(4fx+4e)}{64f}+\frac{\sin(4fx+4e)b}{32f}+\frac{15a\sin(2fx+2e)}{64f}+\frac{\sin(2fx+2e)b}{4f}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3}{8}\right.}{f}$
default	$\frac{a\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3}{8}\right.}{f}$
norman	$\left(-\frac{5a}{16}-\frac{3b}{8}\right)x+\left(-\frac{45a}{16}-\frac{27b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{25a}{16}-\frac{15b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{25a}{16}-\frac{15b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{5a}{16}+\frac{3b}{8}\right.$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/192*((45*a+48*b)*sin(2*f*x+2*e)+(9*a+6*b)*sin(4*f*x+4*e)+a*sin(6*f*x+6*e
)+60*x*(a+6/5*b)*f)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{3\,(5\,a+6\,b)fx+\left(8\,a\cos\left(fx+e\right)^5+2\,(5\,a+6\,b)\cos\left(fx+e\right)^3+3\,(5\,a+6\,b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/48*(3*(5*a + 6*b)*f*x + (8*a*cos(f*x + e)^5 + 2*(5*a + 6*b)*cos(f*x + e)
^3 + 3*(5*a + 6*b)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,(fx+e)(5\,a+6\,b)+\frac{3\,(5\,a+6\,b)\tan(fx+e)^5+8\,(5\,a+6\,b)\tan(fx+e)^3+3\,(11\,a+10\,b)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/48*(3*(f*x + e)*(5*a + 6*b) + (3*(5*a + 6*b)*tan(f*x + e)^5 + 8*(5*a + 6
*b)*tan(f*x + e)^3 + 3*(11*a + 10*b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan
(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,(fx+e)(5\,a+6\,b)+\frac{15\,a\tan(fx+e)^5+18\,b\tan(fx+e)^5+40\,a\tan(fx+e)^3+48\,b\tan(fx+e)^3+33\,a\tan(fx+e)+30\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/48*(3*(f*x + e)*(5*a + 6*b) + (15*a*tan(f*x + e)^5 + 18*b*tan(f*x + e)^5
 + 40*a*tan(f*x + e)^3 + 48*b*tan(f*x + e)^3 + 33*a*tan(f*x + e) + 30*b*ta
n(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 16.78 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = x\left(\frac{5\,a}{16}+\frac{3\,b}{8}\right) + \frac{\left(\frac{5\,a}{16}+\frac{3\,b}{8}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a}{6}+b\right)\,\tan(e+f\,x)^3+\left(\frac{11\,a}{16}+\frac{5\,b}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

output
```
x*((5*a)/16 + (3*b)/8) + (tan(e + f*x)^5*((5*a)/16 + (3*b)/8) + tan(e + f*
x)*((11*a)/16 + (5*b)/8) + tan(e + f*x)^3*((5*a)/6 + b))/(f*(3*tan(e + f*x
)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{8\cos(fx+e)\sin(fx+e)^5 a - 26\cos(fx+e)\sin(fx+e)^3 a - 12\cos(fx+e)\sin(fx+e)^3 b + 33\cos(\ldots}{48f}$$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

output
```
(8*cos(e + f*x)*sin(e + f*x)**5*a - 26*cos(e + f*x)*sin(e + f*x)**3*a - 12
*cos(e + f*x)*sin(e + f*x)**3*b + 33*cos(e + f*x)*sin(e + f*x)*a + 30*cos(
e + f*x)*sin(e + f*x)*b + 15*a*f*x + 18*b*f*x)/(48*f)
```

3.165 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\begin{aligned}\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{(48a^2+80ab+35b^2)\operatorname{arctanh}(\sin(e+fx))}{128f} \\ &\quad+\frac{(48a^2+80ab+35b^2)\sec(e+fx)\tan(e+fx)}{128f} \\ &\quad+\frac{(48a^2+80ab+35b^2)\sec^3(e+fx)\tan(e+fx)}{192f} \\ &\quad+\frac{b(16a+7b)\sec^5(e+fx)\tan(e+fx)}{48f}+\frac{b^2\sec^7(e+fx)\tan(e+fx)}{8f}\end{aligned}$$

output
```
1/128*(48*a^2+80*a*b+35*b^2)*arctanh(sin(f*x+e))/f+1/128*(48*a^2+80*a*b+35
*b^2)*sec(f*x+e)*tan(f*x+e)/f+1/192*(48*a^2+80*a*b+35*b^2)*sec(f*x+e)^3*ta
n(f*x+e)/f+1/48*b*(16*a+7*b)*sec(f*x+e)^5*tan(f*x+e)/f+1/8*b^2*sec(f*x+e)^
7*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.69

$$\begin{aligned}&\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx\\&=\frac{3a^2\operatorname{arctanh}(\sin(e+fx))}{8f}+\frac{5ab\operatorname{arctanh}(\sin(e+fx))}{8f}\\&+\frac{35b^2\operatorname{arctanh}(\sin(e+fx))}{128f}+\frac{3a^2\sec(e+fx)\tan(e+fx)}{8f}\\&+\frac{5ab\sec(e+fx)\tan(e+fx)}{8f}+\frac{35b^2\sec(e+fx)\tan(e+fx)}{128f}\\&+\frac{a^2\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{5ab\sec^3(e+fx)\tan(e+fx)}{12f}\\&+\frac{35b^2\sec^3(e+fx)\tan(e+fx)}{192f}+\frac{ab\sec^5(e+fx)\tan(e+fx)}{3f}\\&+\frac{7b^2\sec^5(e+fx)\tan(e+fx)}{48f}+\frac{b^2\sec^7(e+fx)\tan(e+fx)}{8f}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[e + f*x]])/(8*f) + (5*a*b*ArcTanh[Sin[e + f*x]])/(8*f)
+ (35*b^2*ArcTanh[Sin[e + f*x]])/(128*f) + (3*a^2*Sec[e + f*x]*Tan[e + f*x
])/(8*f) + (5*a*b*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (35*b^2*Sec[e + f*x]*
Tan[e + f*x])/(128*f) + (a^2*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (5*a*b*S
ec[e + f*x]^3*Tan[e + f*x])/(12*f) + (35*b^2*Sec[e + f*x]^3*Tan[e + f*x])/
(192*f) + (a*b*Sec[e + f*x]^5*Tan[e + f*x])/(3*f) + (7*b^2*Sec[e + f*x]^5*
Tan[e + f*x])/(48*f) + (b^2*Sec[e + f*x]^7*Tan[e + f*x])/(8*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4635, 315, 25, 298, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(e+fx)^5\left(a+b\sec(e+fx)^2\right)^2\,dx$$

$$\Big\downarrow \; 4635$$

$$\frac{\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^2}{\left(1-\sin^2(e+fx)\right)^5}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 315$$

$$\frac{\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{8\left(1-\sin^2(e+fx)\right)^4}-\frac{1}{8}\int -\frac{(a+b)(8a+7b)-a(8a+5b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^4}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{8}\int \frac{(a+b)(8a+7b)-a(8a+5b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^4}d\sin(e+fx)+\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{8\left(1-\sin^2(e+fx)\right)^4}}{f}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+80ab+35b^2\right)\int \frac{1}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)+\frac{b(10a+7b)\sin(e+fx)}{6\left(1-\sin^2(e+fx)\right)^3}\right)+\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{8\left(1-\sin^2(e+fx)\right)^4}}{f}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+80ab+35b^2\right)\left(\frac{3}{4}\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)+\frac{\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right)+\frac{b(10a+7b)\sin(e+fx)}{6\left(1-\sin^2(e+fx)\right)^3}\right)+\frac{b\sin(e+f}{8(}}{f}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+80ab+35b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)+\frac{\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}\right)+\frac{\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right)+\frac{b(10a+7b)\sin}{6(1-\sin^2(e+}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+80ab+35b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(e+fx))+\frac{\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}\right)+\frac{\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right)+\frac{b(10a+7b)\sin(e+fx)}{6\left(1-\sin^2(e+fx)\right)^3}}{f}$$

input
```
Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(8*(1 - Sin[e + f*x]^2)^4) +
((b*(10*a + 7*b)*Sin[e + f*x])/(6*(1 - Sin[e + f*x]^2)^3) + ((48*a^2 + 80*
a*b + 35*b^2)*(Sin[e + f*x]/(4*(1 - Sin[e + f*x]^2)^2) + (3*(ArcTanh[Sin[e
 + f*x]]/2 + Sin[e + f*x]/(2*(1 - Sin[e + f*x]^2))))/4))/6)/8)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\,\mathrm{s}}{}\right.\right.}{}$
default	$\frac{a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\,\mathrm{s}}{}\right.\right.}{}$
parts	$\frac{a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{b^2\left(-\left(-\frac{\sec(fx+e)^7}{8}-\frac{7\sec(fx+e)^5}{48}-\right.\right.}{}$
parallelrisch	$\frac{-8064\left(a^2+\frac{5}{3}ab+\frac{35}{48}b^2\right)\left(\frac{5}{8}+\frac{\cos(8fx+8e)}{56}+\frac{\cos(6fx+6e)}{7}+\frac{\cos(4fx+4e)}{2}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+8064\left(a^2+\right.}{}$
norman	$\frac{\frac{\left(80a^2+176ab+93b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64f}+\frac{\left(80a^2+176ab+93b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{64f}-\frac{\left(432a^2+1360ab-1085b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{192f}-\frac{\left(432a^2+1\right.}{}}{}$
risch	$-\frac{i\,e^{i(fx+e)}\left(144a^2e^{14i(fx+e)}+240ab\,e^{14i(fx+e)}+105b^2e^{14i(fx+e)}+1104\,e^{12i(fx+e)}a^2+1840ab\,e^{12i(fx+e)}+805b^2e^{12i(}\right.}{}$

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)
+tan(f*x+e)))+2*a*b*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e)
)*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+b^2*(-(-1/8*sec(f*x+e)^7-7/48
*sec(f*x+e)^5-35/192*sec(f*x+e)^3-35/128*sec(f*x+e))*tan(f*x+e)+35/128*ln(
sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.10

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\cos\left(fx+e\right)^8\log\left(\sin\left(fx+e\right)+1\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\cos\left(fx+e\right)^8\log}{}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(sin(f*x + e) + 1) -
 3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(-sin(f*x + e) + 1) + 2*(3
*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^6 + 2*(48*a^2 + 80*a*b + 35*b^2)*
cos(f*x + e)^4 + 8*(16*a*b + 7*b^2)*cos(f*x + e)^2 + 48*b^2)*sin(f*x + e))
/(f*cos(f*x + e)^8)
```

Sympy [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^5\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.31

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\left(48\right.\right.}{}}{76}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(sin(f*x + e) + 1) - 3*(48*a^2 + 80
*a*b + 35*b^2)*log(sin(f*x + e) - 1) - 2*(3*(48*a^2 + 80*a*b + 35*b^2)*sin
(f*x + e)^7 - 11*(48*a^2 + 80*a*b + 35*b^2)*sin(f*x + e)^5 + (624*a^2 + 11
68*a*b + 511*b^2)*sin(f*x + e)^3 - 3*(80*a^2 + 176*a*b + 93*b^2)*sin(f*x +
 e))/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e
)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.44

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\left(14\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(abs(sin(f*x + e) + 1)) - 3*(48*a^2
 + 80*a*b + 35*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(144*a^2*sin(f*x + e)^7
 + 240*a*b*sin(f*x + e)^7 + 105*b^2*sin(f*x + e)^7 - 528*a^2*sin(f*x + e)^
5 - 880*a*b*sin(f*x + e)^5 - 385*b^2*sin(f*x + e)^5 + 624*a^2*sin(f*x + e)
^3 + 1168*a*b*sin(f*x + e)^3 + 511*b^2*sin(f*x + e)^3 - 240*a^2*sin(f*x +
e) - 528*a*b*sin(f*x + e) - 279*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^4)/
f
```

Mupad [B] (verification not implemented)

Time = 15.90 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.11

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{\left(-\frac{3a^2}{8}-\frac{5ab}{8}-\frac{35b^2}{128}\right)\sin(e+fx)^7+\left(\frac{11a^2}{8}+\frac{55ab}{24}+\frac{385b^2}{384}\right)\sin(e+fx)^5+\left(-\frac{13a^2}{8}-\frac{73ab}{24}-\frac{511b^2}{384}\right)\mathrm{s}}{f\left(\sin(e+fx)^8-4\sin(e+fx)^6+6\sin(e+fx)^4-4\sin(e+\right.}$$
$$+\frac{\operatorname{atanh}(\sin(e+fx))\left(\frac{3a^2}{8}+\frac{5ab}{8}+\frac{35b^2}{128}\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^5,x)
```

output
```
(sin(e + f*x)*((11*a*b)/8 + (5*a^2)/8 + (93*b^2)/128) - sin(e + f*x)^7*((5
*a*b)/8 + (3*a^2)/8 + (35*b^2)/128) + sin(e + f*x)^5*((55*a*b)/24 + (11*a^
2)/8 + (385*b^2)/384) - sin(e + f*x)^3*((73*a*b)/24 + (13*a^2)/8 + (511*b^
2)/384))/(f*(6*sin(e + f*x)^4 - 4*sin(e + f*x)^2 - 4*sin(e + f*x)^6 + sin(
e + f*x)^8 + 1)) + (atanh(sin(e + f*x))*((5*a*b)/8 + (3*a^2)/8 + (35*b^2)/
128))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 886, normalized size of antiderivative = 5.79

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 144*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**8*a**2 - 240*log(tan((e +
f*x)/2) - 1)*sin(e + f*x)**8*a*b - 105*log(tan((e + f*x)/2) - 1)*sin(e + f
*x)**8*b**2 + 576*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6*a**2 + 960*log
(tan((e + f*x)/2) - 1)*sin(e + f*x)**6*a*b + 420*log(tan((e + f*x)/2) - 1)
*sin(e + f*x)**6*b**2 - 864*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a**2
 - 1440*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a*b - 630*log(tan((e + f
*x)/2) - 1)*sin(e + f*x)**4*b**2 + 576*log(tan((e + f*x)/2) - 1)*sin(e + f
*x)**2*a**2 + 960*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*b + 420*log(
tan((e + f*x)/2) - 1)*sin(e + f*x)**2*b**2 - 144*log(tan((e + f*x)/2) - 1)
*a**2 - 240*log(tan((e + f*x)/2) - 1)*a*b - 105*log(tan((e + f*x)/2) - 1)*
b**2 + 144*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**8*a**2 + 240*log(tan((e
 + f*x)/2) + 1)*sin(e + f*x)**8*a*b + 105*log(tan((e + f*x)/2) + 1)*sin(e
+ f*x)**8*b**2 - 576*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**6*a**2 - 960*
log(tan((e + f*x)/2) + 1)*sin(e + f*x)**6*a*b - 420*log(tan((e + f*x)/2) +
 1)*sin(e + f*x)**6*b**2 + 864*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*a
**2 + 1440*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*a*b + 630*log(tan((e
+ f*x)/2) + 1)*sin(e + f*x)**4*b**2 - 576*log(tan((e + f*x)/2) + 1)*sin(e
+ f*x)**2*a**2 - 960*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a*b - 420*l
og(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*b**2 + 144*log(tan((e + f*x)/2) +
 1)*a**2 + 240*log(tan((e + f*x)/2) + 1)*a*b + 105*log(tan((e + f*x)/2)...
```

3.166 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(8a^2+12ab+5b^2)\operatorname{arctanh}(\sin(e+fx))}{16f} + \frac{(8a^2+12ab+5b^2)\sec(e+fx)\tan(e+fx)}{16f} + \frac{b(12a+5b)\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b^2\sec^5(e+fx)\tan(e+fx)}{6f}$$

output
```
1/16*(8*a^2+12*a*b+5*b^2)*arctanh(sin(f*x+e))/f+1/16*(8*a^2+12*a*b+5*b^2)*
sec(f*x+e)*tan(f*x+e)/f+1/24*b*(12*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b^
2*sec(f*x+e)^5*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.60

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{a^2\text{arctanh}(\sin(e+fx))}{2f} + \frac{3ab\text{arctanh}(\sin(e+fx))}{4f} + \frac{5b^2\text{arctanh}(\sin(e+fx))}{16f} + \frac{a^2\sec(e+fx)\tan(e+fx)}{2f} + \frac{3ab\sec(e+fx)\tan(e+fx)}{4f} + \frac{5b^2\sec(e+fx)\tan(e+fx)}{16f} + \frac{ab\sec^3(e+fx)\tan(e+fx)}{2f} + \frac{5b^2\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b^2\sec^5(e+fx)\tan(e+fx)}{6f}$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTanh[Sin[e + f*x]])/(2*f) + (3*a*b*ArcTanh[Sin[e + f*x]])/(4*f) +
(5*b^2*ArcTanh[Sin[e + f*x]])/(16*f) + (a^2*Sec[e + f*x]*Tan[e + f*x])/(2*
f) + (3*a*b*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (5*b^2*Sec[e + f*x]*Tan[e +
 f*x])/(16*f) + (a*b*Sec[e + f*x]^3*Tan[e + f*x])/(2*f) + (5*b^2*Sec[e + f
*x]^3*Tan[e + f*x])/(24*f) + (b^2*Sec[e + f*x]^5*Tan[e + f*x])/(6*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4635, 315, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^2}{\left(1-\sin^2(e+fx)\right)^4}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{6\left(1-\sin^2(e+fx)\right)^3}-\frac{1}{6}\int -\frac{(a+b)(6a+5b)-3a(2a+b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\int \frac{(a+b)(6a+5b)-3a(2a+b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)+\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{6\left(1-\sin^2(e+fx)\right)^3}}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+12ab+5b^2\right)\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)+\frac{b(8a+5b)\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right)+\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{6\left(1-\sin^2(e+fx)\right)^3}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+12ab+5b^2\right)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)+\frac{\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}\right)+\frac{b(8a+5b)\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right)+\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{6\left(1-\sin^2(e+fx)\right)^3}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+12ab+5b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(e+fx))+\frac{\sin(e+fx)}{2(1-\sin^2(e+fx))}\right)+\frac{b(8a+5b)\sin(e+fx)}{4(1-\sin^2(e+fx))^2}\right)+\frac{b\sin(e+fx)(-a\sin^2(e+fx)}{6(1-\sin^2(e+fx))^3}}{f}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(6*(1 - Sin[e + f*x]^2)^3) +
((b*(8*a + 5*b)*Sin[e + f*x])/(4*(1 - Sin[e + f*x]^2)^2) + (3*(8*a^2 + 12*
a*b + 5*b^2)*(ArcTanh[Sin[e + f*x]]/2 + Sin[e + f*x]/(2*(1 - Sin[e + f*x]^
2))))/4)/6)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\cdots}{8}\cdots}{f}$
default	$\frac{a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\cdots}{8}\cdots}{f}$
parts	$\frac{a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{b^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\cdots\right.}{f}$
parallelrisch	$\frac{-360\left(\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4fx+4e)}{5}+\cos(2fx+2e)+\frac{2}{3}\right)\left(a^2+\frac{3}{2}ab+\frac{5}{8}b^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+360\left(\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4\cdots}{\cdots}\right.}{48\cdots}$
norman	$\frac{\frac{\left(8a^2+4ab+15b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{\left(8a^2+4ab+15b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{\left(8a^2+20ab+11b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{\left(8a^2+20ab+11b^2\right)\tan\left(\cdots\right)}{8f}\cdots}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$
risch	$-\frac{i e^{i(fx+e)}\left(24\,e^{10i(fx+e)}a^2+36\,e^{10i(fx+e)}ab+15b^2e^{10i(fx+e)}+72a^2e^{8i(fx+e)}+204\,e^{8i(fx+e)}ab+85\,e^{8i(fx+e)}b^2+48a\cdots\right.}{\cdots}$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+2*a*b*(
-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e
)))+b^2*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)
+5/16*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\,(8\,a^2+12\,ab+5\,b^2)\cos{(fx+e)}^6\log{(\sin{(fx+e)}+1)}-3\,(8\,a^2+12\,ab+5\,b^2)\cos{(fx+e)}^6\log{(-\sin}}{96\,f\cos}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*
(8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(8*a
^2 + 12*a*b + 5*b^2)*cos(f*x + e)^4 + 2*(12*a*b + 5*b^2)*cos(f*x + e)^2 +
8*b^2)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int\left(a+b\sec^2{(e+fx)}\right)^2\sec^3{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.42

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$$
$$= \frac{3\,(8\,a^2+12\,ab+5\,b^2)\log\left(\sin\left(fx+e\right)+1\right)-3\,(8\,a^2+12\,ab+5\,b^2)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\,(8\,a^2+12\,a\cdots\right.}{\cdots}}{96\,f}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(sin(f*x + e) + 1) - 3*(8*a^2 + 12*a*b
 + 5*b^2)*log(sin(f*x + e) - 1) - 2*(3*(8*a^2 + 12*a*b + 5*b^2)*sin(f*x +
e)^5 - 8*(6*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)^3 + 3*(8*a^2 + 20*a*b + 11*
b^2)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 -
 1))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.56

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$$
$$= \frac{3\,(8\,a^2+12\,ab+5\,b^2)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3\,(8\,a^2+12\,ab+5\,b^2)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\left(24\,a^2\sin\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(abs(sin(f*x + e) + 1)) - 3*(8*a^2 + 1
2*a*b + 5*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(24*a^2*sin(f*x + e)^5 + 36*
a*b*sin(f*x + e)^5 + 15*b^2*sin(f*x + e)^5 - 48*a^2*sin(f*x + e)^3 - 96*a*
b*sin(f*x + e)^3 - 40*b^2*sin(f*x + e)^3 + 24*a^2*sin(f*x + e) + 60*a*b*si
n(f*x + e) + 33*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 16.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\operatorname{atanh}(\sin(e+f\,x))\,\left(\frac{a^2}{2}+\frac{3\,a\,b}{4}+\frac{5\,b^2}{16}\right)}{f}-\frac{\left(\frac{a^2}{2}+\frac{3\,a\,b}{4}+\frac{5\,b^2}{16}\right)\,\sin(e+f\,x)^5+\left(-a^2-2\,a\,b-\frac{5\,b^2}{6}\right)\,\sin(e+f\,x)^3+\left(\frac{a^2}{2}+\frac{5\,a\,b}{4}+\frac{11\,b^2}{16}\right)\,\sin(e+f}{f\,\left(\sin(e+f\,x)^6-3\sin(e+f\,x)^4+3\sin(e+f\,x)^2-1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^3,x)
```

output
```
(atanh(sin(e + f*x))*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/f - (sin(e + f*x)*(
(5*a*b)/4 + a^2/2 + (11*b^2)/16) - sin(e + f*x)^3*(2*a*b + a^2 + (5*b^2)/6
) + sin(e + f*x)^5*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/(f*(3*sin(e + f*x)^2
- 3*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 690, normalized size of antiderivative = 5.90

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\text{Too large to display}$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

output

```
( - 24*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6*a**2 - 36*log(tan((e + f*
x)/2) - 1)*sin(e + f*x)**6*a*b - 15*log(tan((e + f*x)/2) - 1)*sin(e + f*x)
**6*b**2 + 72*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a**2 + 108*log(tan
((e + f*x)/2) - 1)*sin(e + f*x)**4*a*b + 45*log(tan((e + f*x)/2) - 1)*sin(
e + f*x)**4*b**2 - 72*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a**2 - 108
*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*b - 45*log(tan((e + f*x)/2) -
 1)*sin(e + f*x)**2*b**2 + 24*log(tan((e + f*x)/2) - 1)*a**2 + 36*log(tan(
(e + f*x)/2) - 1)*a*b + 15*log(tan((e + f*x)/2) - 1)*b**2 + 24*log(tan((e
+ f*x)/2) + 1)*sin(e + f*x)**6*a**2 + 36*log(tan((e + f*x)/2) + 1)*sin(e +
 f*x)**6*a*b + 15*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**6*b**2 - 72*log(
tan((e + f*x)/2) + 1)*sin(e + f*x)**4*a**2 - 108*log(tan((e + f*x)/2) + 1)
*sin(e + f*x)**4*a*b - 45*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*b**2 +
 72*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a**2 + 108*log(tan((e + f*x)
/2) + 1)*sin(e + f*x)**2*a*b + 45*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**
2*b**2 - 24*log(tan((e + f*x)/2) + 1)*a**2 - 36*log(tan((e + f*x)/2) + 1)*
a*b - 15*log(tan((e + f*x)/2) + 1)*b**2 - 24*sin(e + f*x)**5*a**2 - 36*sin
(e + f*x)**5*a*b - 15*sin(e + f*x)**5*b**2 + 48*sin(e + f*x)**3*a**2 + 96*
sin(e + f*x)**3*a*b + 40*sin(e + f*x)**3*b**2 - 24*sin(e + f*x)*a**2 - 60*
sin(e + f*x)*a*b - 33*sin(e + f*x)*b**2)/(48*f*(sin(e + f*x)**6 - 3*sin(e
+ f*x)**4 + 3*sin(e + f*x)**2 - 1))
```

3.167 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(8a^2+8ab+3b^2)\,\text{arctanh}(\sin(e+fx))}{8f} + \frac{b(8a+3b)\sec(e+fx)\tan(e+fx)}{8f} + \frac{b^2\sec^3(e+fx)\tan(e+fx)}{4f}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctanh(sin(f*x+e))/f+1/8*b*(8*a+3*b)*sec(f*x+e)*t
an(f*x+e)/f+1/4*b^2*sec(f*x+e)^3*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx = \frac{a^2\coth^{-1}(\sin(e+fx))}{f} + \frac{ab\text{arctanh}(\sin(e+fx))}{f} + \frac{3b^2\text{arctanh}(\sin(e+fx))}{8f} + \frac{ab\sec(e+fx)\tan(e+fx)}{f} + \frac{3b^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{b^2\sec^3(e+fx)\tan(e+fx)}{4f}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcCoth[Sin[e + f*x]])/f + (a*b*ArcTanh[Sin[e + f*x]])/f + (3*b^2*Arc
Tanh[Sin[e + f*x]])/(8*f) + (a*b*Sec[e + f*x]*Tan[e + f*x])/f + (3*b^2*Sec
[e + f*x]*Tan[e + f*x])/(8*f) + (b^2*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4635, 315, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{(-a\sin^2(e+fx)+a+b)^2}{(1-\sin^2(e+fx))^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{b\sin(e+fx)(-a\sin^2(e+fx)+a+b)}{4(1-\sin^2(e+fx))^2}-\frac{1}{4}\int -\frac{(a+b)(4a+3b)-a(4a+b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\int \frac{(a+b)(4a+3b)-a(4a+b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^2}d\sin(e+fx)+\frac{b\sin(e+fx)(-a\sin^2(e+fx)+a+b)}{4(1-\sin^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(8a^2+8ab+3b^2)\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)+\frac{3b(2a+b)\sin(e+fx)}{2(1-\sin^2(e+fx))}\right)+\frac{b\sin(e+fx)(-a\sin^2(e+fx)+a+b)}{4(1-\sin^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(8a^2+8ab+3b^2)\text{arctanh}(\sin(e+fx))+\frac{3b(2a+b)\sin(e+fx)}{2(1-\sin^2(e+fx))}\right)+\frac{b\sin(e+fx)(-a\sin^2(e+fx)+a+b)}{4(1-\sin^2(e+fx))^2}}{f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(4*(1 - Sin[e + f*x]^2)^2) +
(((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]])/2 + (3*b*(2*a + b)*Sin[e
+ f*x])/(2*(1 - Sin[e + f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a^2 \ln(\sec(fx+e)+\tan(fx+e))+2ab\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e}{8}\right.\right.}{f}$
default	$\frac{a^2 \ln(\sec(fx+e)+\tan(fx+e))+2ab\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e}{8}\right.\right.}{f}$
parts	$\frac{a^2 \ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{b^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+ab$
parallelrisch	$\frac{-4\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(a^2+ab+\frac{3}{8}b^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+4\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(a^2+ab+\right.}{f(\cos(4fx+4e)+4\cos(2fx+2e)+3)}$
norman	$\frac{-\frac{b(8a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{4f}-\frac{b(8a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{b(8a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{b(8a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{(8a^2+8ab+3b^2)\ln}{8}$
risch	$-\frac{ib\,e^{i(fx+e)}\left(8a\,e^{6i(fx+e)}+3b\,e^{6i(fx+e)}+8a\,e^{4i(fx+e)}+11b\,e^{4i(fx+e)}-8a\,e^{2i(fx+e)}-11b\,e^{2i(fx+e)}-8a-3b\right)}{4f\left(e^{2i(fx+e)}+1\right)^4}-\ln(e$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*ln(sec(f*x+e)+tan(f*x+e))+2*a*b*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln
(sec(f*x+e)+tan(f*x+e)))+b^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+
e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{(8a^2+8ab+3b^2)\cos(fx+e)^4\log(\sin(fx+e)+1)-(8a^2+8ab+3b^2)\cos(fx+e)^4\log(-\sin(fx}{16f\cos(fx+e)^4}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/16*((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (8*a^
2 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*((8*a*b + 3*b
^2)*cos(f*x + e)^2 + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2 \, dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2 \sec\left(e+fx\right) dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2 \, dx$$
$$= \frac{(8\,a^2+8\,ab+3\,b^2)\log\left(\sin\left(fx+e\right)+1\right)-(8\,a^2+8\,ab+3\,b^2)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left((8\,ab+3\,b^2)\sin(fx+\right.}{\sin(fx+e)^4}}{16\,f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/16*((8*a^2 + 8*a*b + 3*b^2)*log(sin(f*x + e) + 1) - (8*a^2 + 8*a*b + 3*b
^2)*log(sin(f*x + e) - 1) - 2*((8*a*b + 3*b^2)*sin(f*x + e)^3 - (8*a*b + 5
*b^2)*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{(8\,a^2+8\,ab+3\,b^2)\log\left(|\sin\left(fx+e\right)+1|\right)-(8\,a^2+8\,ab+3\,b^2)\log\left(|\sin\left(fx+e\right)-1|\right)-\frac{2\left(8\,ab\sin(fx+e)^3\right.}{}}{16\,f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/16*((8*a^2 + 8*a*b + 3*b^2)*log(abs(sin(f*x + e) + 1)) - (8*a^2 + 8*a*b
+ 3*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(8*a*b*sin(f*x + e)^3 + 3*b^2*sin(
f*x + e)^3 - 8*a*b*sin(f*x + e) - 5*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)
^2)/f
```

Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\operatorname{atanh}(\sin\left(e+f\,x\right))\,\left(a^2+a\,b+\frac{3\,b^2}{8}\right)}{f}+\frac{\sin\left(e+f\,x\right)\,\left(\frac{5\,b^2}{8}+a\,b\right)-\sin\left(e+f\,x\right)^3\,\left(\frac{3\,b^2}{8}+a\,b\right)}{f\,\left(\sin\left(e+f\,x\right)^4-2\sin\left(e+f\,x\right)^2+1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x),x)
```

output
```
(atanh(sin(e + f*x))*(a*b + a^2 + (3*b^2)/8))/f + (sin(e + f*x)*(a*b + (5*
b^2)/8) - sin(e + f*x)^3*(a*b + (3*b^2)/8))/(f*(sin(e + f*x)^4 - 2*sin(e +
 f*x)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 470, normalized size of antiderivative = 5.80

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 8*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4*a**2 - 8*log(tan((e + f*x)
/2) - 1)*sin(e + f*x)**4*a*b - 3*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4
*b**2 + 16*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a**2 + 16*log(tan((e
+ f*x)/2) - 1)*sin(e + f*x)**2*a*b + 6*log(tan((e + f*x)/2) - 1)*sin(e + f
*x)**2*b**2 - 8*log(tan((e + f*x)/2) - 1)*a**2 - 8*log(tan((e + f*x)/2) -
1)*a*b - 3*log(tan((e + f*x)/2) - 1)*b**2 + 8*log(tan((e + f*x)/2) + 1)*si
n(e + f*x)**4*a**2 + 8*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*a*b + 3*l
og(tan((e + f*x)/2) + 1)*sin(e + f*x)**4*b**2 - 16*log(tan((e + f*x)/2) +
1)*sin(e + f*x)**2*a**2 - 16*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a*b
 - 6*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*b**2 + 8*log(tan((e + f*x)/
2) + 1)*a**2 + 8*log(tan((e + f*x)/2) + 1)*a*b + 3*log(tan((e + f*x)/2) +
1)*b**2 - 8*sin(e + f*x)**3*a*b - 3*sin(e + f*x)**3*b**2 + 8*sin(e + f*x)*
a*b + 5*sin(e + f*x)*b**2)/(8*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.168 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b(4a+b)\text{arctanh}(\sin(e+fx))}{2f}+\frac{a^2\sin(e+fx)}{f}+\frac{b^2\sec(e+fx)\tan(e+fx)}{2f}$$

output
```
1/2*b*(4*a+b)*arctanh(sin(f*x+e))/f+a^2*sin(f*x+e)/f+1/2*b^2*sec(f*x+e)*ta
n(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{2ab\coth^{-1}(\sin(e+fx))}{f}+\frac{b^2\text{arctanh}(\sin(e+fx))}{2f}+\frac{a^2\cos(fx)\sin(e)}{f}+\frac{a^2\cos(e)\sin(fx)}{f}+\frac{b^2\sec(e+fx)\tan(e+fx)}{2f}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(2*a*b*ArcCoth[Sin[e + f*x]])/f + (b^2*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2
*Cos[f*x]*Sin[e])/f + (a^2*Cos[e]*Sin[f*x])/f + (b^2*Sec[e + f*x]*Tan[e +
f*x])/(2*f)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^2}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(a^2+\frac{b(2a+b)-2ab\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^2}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\sin(e+fx)+\frac{1}{2}b(4a+b)\text{arctanh}(\sin(e+fx))+\frac{b^2\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b*(4*a + b)*ArcTanh[Sin[e + f*x]])/2 + a^2*Sin[e + f*x] + (b^2*Sin[e + f
*x])/(2*(1 - Sin[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\sin(fx+e)a^2+2ab\ln(\sec(fx+e)+\tan(fx+e))+b^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
default	$\frac{\sin(fx+e)a^2+2ab\ln(\sec(fx+e)+\tan(fx+e))+b^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parallelrisch	$\frac{-4\left(a+\frac{b}{4}\right)(1+\cos(2fx+2e))b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+4\left(a+\frac{b}{4}\right)(1+\cos(2fx+2e))b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\sin(3fx+3e)a^2}{2f(1+\cos(2fx+2e))}$
risch	$-\frac{ia^2e^{i(fx+e)}}{2f}+\frac{ia^2e^{-i(fx+e)}}{2f}-\frac{ib^2\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2\ln\left(e^{i(fx+e)}-i\right)ab}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b^2}{2f}+\frac{2\ln(e}{}$
norman	$\frac{\frac{(2a^2+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}+\frac{(6a^2-b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-\frac{(2a^2+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(6a^2-b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{b(4a+b)\ln\left(\tan\right.}{2f}$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)*a^2+2*a*b*ln(sec(f*x+e)+tan(f*x+e))+b^2*(1/2*sec(f*x+e)*ta
n(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.68

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{(4\,ab+b^2)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(4\,ab+b^2)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\,(2}{4\,f\cos\left(fx+e\right)^2}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/4*((4*a*b + b^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (4*a*b + b^2)*co
s(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(2*a^2*cos(f*x + e)^2 + b^2)*sin(f
*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cos\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{b^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-4\,ab(\log\left(\sin\left(fx+e\right)+1\right)-\log\left(s\right.}{4\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + l
og(sin(f*x + e) - 1)) - 4*a*b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) -
1)) - 4*a^2*sin(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{4\,a^2\sin\left(fx+e\right)+\left(4\,ab+b^2\right)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-\left(4\,ab+b^2\right)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\,b^2\sin(fx+e)}{\sin(fx+e)^2-1}}{4\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/4*(4*a^2*sin(f*x + e) + (4*a*b + b^2)*log(abs(sin(f*x + e) + 1)) - (4*a*
b + b^2)*log(abs(sin(f*x + e) - 1)) - 2*b^2*sin(f*x + e)/(sin(f*x + e)^2 -
 1))/f
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{a^2\sin\left(e+f\,x\right)+\frac{b\,\mathrm{atanh}(\sin(e+f\,x))\,(4\,a+b)}{2}-\frac{b^2\,\sin(e+f\,x)}{2\,\left(\sin(e+f\,x)^2-1\right)}}{f}$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(a^2*sin(e + f*x) + (b*atanh(sin(e + f*x))*(4*a + b))/2 - (b^2*sin(e + f*x
))/(2*(sin(e + f*x)^2 - 1)))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.84

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{-4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^2ab-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^2b^2+4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{}$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 4*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*b - log(tan((e + f*x)/2)
 - 1)*sin(e + f*x)**2*b**2 + 4*log(tan((e + f*x)/2) - 1)*a*b + log(tan((e
+ f*x)/2) - 1)*b**2 + 4*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a*b + lo
g(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*b**2 - 4*log(tan((e + f*x)/2) + 1)
*a*b - log(tan((e + f*x)/2) + 1)*b**2 + 2*sin(e + f*x)**3*a**2 - 2*sin(e +
 f*x)*a**2 - sin(e + f*x)*b**2)/(2*f*(sin(e + f*x)**2 - 1))
```

3.169 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\text{arctanh}(\sin(e+fx))}{f}+\frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}$$

output
```
b^2*arctanh(sin(f*x+e))/f+a*(a+2*b)*sin(f*x+e)/f-1/3*a^2*sin(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\coth^{-1}(\sin(e+fx))}{f}+\frac{2ab\cos(fx)\sin(e)}{f}+\frac{2ab\cos(e)\sin(fx)}{f}+\frac{a^2\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(b^2*ArcCoth[Sin[e + f*x]])/f + (2*a*b*Cos[f*x]*Sin[e])/f + (2*a*b*Cos[e]*
Sin[f*x])/f + (a^2*Sin[e + f*x])/f - (a^2*Sin[e + f*x]^3)/(3*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \ 4635$$

$$\frac{\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^2}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 300$$

$$\frac{\int \left(\frac{b^2}{1-\sin^2(e+fx)}-a^2\sin^2(e+fx)+a(a+2b)\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{3}a^2\sin^3(e+fx)+a(a+2b)\sin(e+fx)+b^2\text{arctanh}(\sin(e+fx))}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(b^2*ArcTanh[Sin[e + f*x]] + a*(a + 2*b)*Sin[e + f*x] - (a^2*Sin[e + f*x]^
3)/3)/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{a^2\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}+2\sin(fx+e)ab+b^2\ln(\sec(fx+e)+\tan(fx+e))}{f}$
default	$\frac{\frac{a^2\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}+2\sin(fx+e)ab+b^2\ln(\sec(fx+e)+\tan(fx+e))}{f}$
parallelrisch	$\frac{-12\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)b^2+12\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)b^2+\sin(3fx+3e)a^2+9\left(a+\frac{8b}{3}\right)\sin(fx+e)a}{12f}$
risch	$-\frac{3ia^2e^{i(fx+e)}}{8f}-\frac{ie^{i(fx+e)}ab}{f}+\frac{3ia^2e^{-i(fx+e)}}{8f}+\frac{ie^{-i(fx+e)}ab}{f}+\frac{\ln\left(e^{i(fx+e)}+i\right)b^2}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b^2}{f}+$
norman	$\frac{-\frac{4a(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\frac{4a(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}-\frac{2a(a+2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a(a+2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{f}+\frac{2a(7a+6b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{3f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*a^2*(2+cos(f*x+e)^2)*sin(f*x+e)+2*sin(f*x+e)*a*b+b^2*ln(sec(f*x+e
)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,b^2\log\left(\sin\left(fx+e\right)+1\right)-3\,b^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(a^2\cos\left(fx+e\right)^2+2\,a^2+6\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^2*log(sin(f*x + e) + 1) - 3*b^2*log(-sin(f*x + e) + 1) + 2*(a^2*c
os(f*x + e)^2 + 2*a^2 + 6*a*b)*sin(f*x + e))/f
```

Sympy [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cos^3\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{2a^2\sin(fx+e)^3-3b^2\log(\sin(fx+e)+1)+3b^2\log(\sin(fx+e)-1)-6(a^2+2ab)\sin(fx+e)}{6f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(sin(f*x + e) + 1) + 3*b^2*log(sin(f
*x + e) - 1) - 6*(a^2 + 2*a*b)*sin(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{2a^2\sin(fx+e)^3-3b^2\log(|\sin(fx+e)+1|)+3b^2\log(|\sin(fx+e)-1|)-6a^2\sin(fx+e)-12}{6f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(abs(sin(f*x + e) + 1)) + 3*b^2*log(
abs(sin(f*x + e) - 1)) - 6*a^2*sin(f*x + e) - 12*a*b*sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = -\frac{\sin\left(e+f\,x\right)\,\left(a^2-2\,a\,\left(a+b\right)\right)+\frac{a^2\,{\sin\left(e+f\,x\right)}^3}{3}-b^2\,\mathrm{atanh}\left(\sin\left(e+f\,x\right)\right)}{f}$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
-(sin(e + f*x)*(a^2 - 2*a*(a + b)) + (a^2*sin(e + f*x)^3)/3 - b^2*atanh(si
n(e + f*x)))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)b^2+3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)b^2-\sin\left(fx+e\right)^3a^2+3\sin\left(fx+e\right)a^2+6\sin\left(j\right.}{3f}$$

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*log(tan((e + f*x)/2) - 1)*b**2 + 3*log(tan((e + f*x)/2) + 1)*b**2 -
sin(e + f*x)**3*a**2 + 3*sin(e + f*x)*a**2 + 6*sin(e + f*x)*a*b)/(3*f)
```

3.170 $\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{(a+b)^2\sin(e+fx)}{f} - \frac{2a(a+b)\sin^3(e+fx)}{3f} + \frac{a^2\sin^5(e+fx)}{5f}$$

output
```
(a+b)^2*sin(f*x+e)/f-2/3*a*(a+b)*sin(f*x+e)^3/f+1/5*a^2*sin(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{b^2\cos(fx)\sin(e)}{f} + \frac{b^2\cos(e)\sin(fx)}{f} + \frac{a^2\sin(e+fx)}{f} + \frac{2ab\sin(e+fx)}{f} - \frac{2a^2\sin^3(e+fx)}{3f} - \frac{2ab\sin^3(e+fx)}{3f} + \frac{a^2\sin^5(e+fx)}{5f}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(b^2*Cos[f*x]*Sin[e])/f + (b^2*Cos[e]*Sin[f*x])/f + (a^2*Sin[e + f*x])/f +
 (2*a*b*Sin[e + f*x])/f - (2*a^2*Sin[e + f*x]^3)/(3*f) - (2*a*b*Sin[e + f*
x]^3)/(3*f) + (a^2*Sin[e + f*x]^5)/(5*f)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = $\{3042, 4635, 210, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^5}dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \left(-a\sin^2(e+fx)+a+b\right)^2d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\int \left(a^2\sin^4(e+fx)-2a^2\left(\frac{b}{a}+1\right)\sin^2(e+fx)+a^2\left(\frac{b(2a+b)}{a^2}+1\right)\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{5}a^2\sin^5(e+fx)-\frac{2}{3}a(a+b)\sin^3(e+fx)+(a+b)^2\sin(e+fx)}{f}$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + b)^2*Sin[e + f*x] - (2*a*(a + b)*Sin[e + f*x]^3)/3 + (a^2*Sin[e + f*
x]^5)/5)/f
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{2ab\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}+\sin(fx+e)b^2}{f}$
default	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{2ab\left(2+\cos(fx+e)^2\right)\sin(fx+e)}{3}+\sin(fx+e)b^2}{f}$
parallelrisch	$\frac{150\sin(fx+e)a^2+360\sin(fx+e)ab+240\sin(fx+e)b^2+3\sin(5fx+5e)a^2+25\sin(3fx+3e)a^2+40\sin(3fx+3e)ab}{240f}$
risch	$\frac{5a^2\sin(fx+e)}{8f}+\frac{3\sin(fx+e)ab}{2f}+\frac{\sin(fx+e)b^2}{f}+\frac{a^2\sin(5fx+5e)}{80f}+\frac{5a^2\sin(3fx+3e)}{48f}+\frac{\sin(3fx+3e)ab}{6f}$
norman	$\frac{-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{f}+\frac{2\left(5a^2+2ab-3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{2\left(5a^2+2ab-3b^2\right)\tan\left(\frac{fx}{2}+\right.}{3f}}{\left(1+\mathrm{ta}\right.}$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/5*a^2*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*x+e)+2/3*a*b*(2+cos
(f*x+e)^2)*sin(f*x+e)+sin(f*x+e)*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\left(3\,a^2\cos\left(fx+e\right)^4+2\left(2\,a^2+5\,ab\right)\cos\left(fx+e\right)^2+8\,a^2+20\,ab+15\,b^2\right)\sin\left(fx+e\right)}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/15*(3*a^2*cos(f*x + e)^4 + 2*(2*a^2 + 5*a*b)*cos(f*x + e)^2 + 8*a^2 + 20
*a*b + 15*b^2)*sin(f*x + e)/f
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^2\sin(fx+e)^5-10\,(a^2+ab)\sin(fx+e)^3+15\,(a^2+2\,ab+b^2)\sin(fx+e)}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(3*a^2*sin(f*x + e)^5 - 10*(a^2 + a*b)*sin(f*x + e)^3 + 15*(a^2 + 2*a
*b + b^2)*sin(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^2\sin(fx+e)^5-10\,a^2\sin(fx+e)^3-10\,ab\sin(fx+e)^3+15\,a^2\sin(fx+e)+30\,ab\sin(fx+e)+1}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/15*(3*a^2*sin(f*x + e)^5 - 10*a^2*sin(f*x + e)^3 - 10*a*b*sin(f*x + e)^3
 + 15*a^2*sin(f*x + e) + 30*a*b*sin(f*x + e) + 15*b^2*sin(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 14.99 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\sin\left(e+f\,x\right)\,\left(a+b\right)^2+\frac{a^2\,\sin(e+f\,x)^5}{5}-\frac{2\,a\,\sin(e+f\,x)^3\,(a+b)}{3}}{f}$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(sin(e + f*x)*(a + b)^2 + (a^2*sin(e + f*x)^5)/5 - (2*a*sin(e + f*x)^3*(a
+ b))/3)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\sin\left(fx+e\right)\left(3\sin\left(fx+e\right)^4a^2-10\sin\left(fx+e\right)^2a^2-10\sin\left(fx+e\right)^2ab+15a^2+30ab+15b^2\right)}{15f}$$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(sin(e + f*x)*(3*sin(e + f*x)**4*a**2 - 10*sin(e + f*x)**2*a**2 - 10*sin(e
 + f*x)**2*a*b + 15*a**2 + 30*a*b + 15*b**2))/(15*f)
```

3.171 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(a+b)^2\tan(e+fx)}{f} + \frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f} + \frac{(a^2+6ab+6b^2)\tan^5(e+fx)}{5f} + \frac{2b(a+2b)\tan^7(e+fx)}{7f} + \frac{b^2\tan^9(e+fx)}{9f}$$

output
```
(a+b)^2*tan(f*x+e)/f+2/3*(a+b)*(a+2*b)*tan(f*x+e)^3/f+1/5*(a^2+6*a*b+6*b^2
)*tan(f*x+e)^5/f+2/7*b*(a+2*b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{315(a+b)^2\tan(e+fx)+210(a^2+3ab+2b^2)\tan^3(e+fx)+63(a^2+6ab+6b^2)\tan^5(e+fx)+90b(a}{315f}$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(315*(a + b)^2*Tan[e + f*x] + 210*(a^2 + 3*a*b + 2*b^2)*Tan[e + f*x]^3 + 6
3*(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5 + 90*b*(a + 2*b)*Tan[e + f*x]^7 + 3
5*b^2*Tan[e + f*x]^9)/(315*f)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2dx$$
$$\Big\downarrow \text{4634}$$
$$\frac{\int \left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{290}$$
$$\frac{\int \left(b^2\tan^8(e+fx)+2b(a+2b)\tan^6(e+fx)+\left(a^2+6ba+6b^2\right)\tan^4(e+fx)+2(a+b)(a+2b)\tan^2(e+fx)-}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}(a^2+6ab+6b^2)\tan^5(e+fx)+\frac{2}{7}b(a+2b)\tan^7(e+fx)+\frac{2}{3}(a+b)(a+2b)\tan^3(e+fx)+(a+b)^2\tan(e+f}{f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + b)^2*Tan[e + f*x] + (2*(a + b)*(a + 2*b)*Tan[e + f*x]^3)/3 + ((a^2 +
 6*a*b + 6*b^2)*Tan[e + f*x]^5)/5 + (2*b*(a + 2*b)*Tan[e + f*x]^7)/7 + (b^
2*Tan[e + f*x]^9)/9)/f
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2ab\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)-}{f}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2ab\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)-}{f}$
parts	$-\frac{a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{b^2\left(-\frac{128}{315}-\frac{\sec(fx+e)^8}{9}-\frac{8\sec(fx+e)^6}{63}-\frac{16\sec(fx+e)^4}{105}-\frac{64\sec(f}{315}\right.}{f}$
parallelrisch	$\frac{(10752a^2+24192ab+10752b^2)\sin(3fx+3e)+(6048a^2+10368ab+4608b^2)\sin(5fx+5e)+(1512a^2+2592ab+1152b^2)\,\mathrm{si}}{315f(\cos(9fx+9e)+9\cos(7fx+7e)+36\cos(5fx+5e)+84\cos(}$
risch	$16i(210\,\mathrm{e}^{12i(fx+e)}a^2+945\,\mathrm{e}^{10i(fx+e)}a^2+1260\,\mathrm{e}^{10i(fx+e)}ab+1701a^2\mathrm{e}^{8i(fx+e)}+3276\,\mathrm{e}^{8i(fx+e)}ab+2016\,\mathrm{e}^{8i(fx+e)}b^2+$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-16
/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e)-b^2*(
-128/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4-64/315*sec
(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{(8\,(21\,a^2+36\,ab+16\,b^2)\cos(fx+e)^8+4\,(21\,a^2+36\,ab+16\,b^2)\cos(fx+e)^6+3\,(21\,a^2+36\,ab+16\,b}{315\,f\cos(fx+e)^9}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/315*(8*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^8 + 4*(21*a^2 + 36*a*b +
16*b^2)*cos(f*x + e)^6 + 3*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^4 + 10*
(9*a*b + 4*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*x + e)/(f*cos(f*x + e)^9)
```

Sympy [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int\left(a+b\sec^2(e+fx)\right)^2\sec^6(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{35\,b^2\tan(fx+e)^9+90\,(ab+2\,b^2)\tan(fx+e)^7+63\,(a^2+6\,ab+6\,b^2)\tan(fx+e)^5+210\,(a^2+3\,ab+\ldots}{315\,f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 + 90*(a*b + 2*b^2)*tan(f*x + e)^7 + 63*(a^2 +
 6*a*b + 6*b^2)*tan(f*x + e)^5 + 210*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^3
+ 315*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.43

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{35\,b^2\tan\left(fx+e\right)^9+90\,ab\tan\left(fx+e\right)^7+180\,b^2\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5+378\,ab\tan\left(fx+\right.}{}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 180*b^2*tan(f*x + e
)^7 + 63*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 378*b^2*tan(f*x + e
)^5 + 210*a^2*tan(f*x + e)^3 + 630*a*b*tan(f*x + e)^3 + 420*b^2*tan(f*x +
e)^3 + 315*a^2*tan(f*x + e) + 630*a*b*tan(f*x + e) + 315*b^2*tan(f*x + e))
/f
```

Mupad [B] (verification not implemented)

Time = 15.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{\tan(e+f\,x)\,\left(a+b\right)^2+\frac{b^2\tan(e+f\,x)^9}{9}+\tan(e+f\,x)^3\left(\frac{2\,a^2}{3}+2\,a\,b+\frac{4\,b^2}{3}\right)+\tan(e+f\,x)^5\left(\frac{a^2}{5}+\frac{6\,a\,b}{5}+\frac{6\,b}{5}\right.}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^6,x)
```

output
```
(tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^9)/9 + tan(e + f*x)^3*(2*a*b +
 (2*a^2)/3 + (4*b^2)/3) + tan(e + f*x)^5*((6*a*b)/5 + a^2/5 + (6*b^2)/5) +
 (2*b*tan(e + f*x)^7*(a + 2*b))/7)/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.15

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\sin(fx+e)\left(168\sin(fx+e)^8a^2+288\sin(fx+e)^8ab+128\sin(fx+e)^8b^2-756\sin(fx+e)^6a^2-1\right.}{}$$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(sin(e + f*x)*(168*sin(e + f*x)**8*a**2 + 288*sin(e + f*x)**8*a*b + 128*si
n(e + f*x)**8*b**2 - 756*sin(e + f*x)**6*a**2 - 1296*sin(e + f*x)**6*a*b -
 576*sin(e + f*x)**6*b**2 + 1323*sin(e + f*x)**4*a**2 + 2268*sin(e + f*x)*
*4*a*b + 1008*sin(e + f*x)**4*b**2 - 1050*sin(e + f*x)**2*a**2 - 1890*sin(
e + f*x)**2*a*b - 840*sin(e + f*x)**2*b**2 + 315*a**2 + 630*a*b + 315*b**2
))/(315*cos(e + f*x)*f*(sin(e + f*x)**8 - 4*sin(e + f*x)**6 + 6*sin(e + f*
x)**4 - 4*sin(e + f*x)**2 + 1))
```

3.172 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(a+b)^2\tan(e+fx)}{f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

output
```
(a+b)^2*tan(f*x+e)/f+1/3*(a+b)*(a+3*b)*tan(f*x+e)^3/f+1/5*b*(2*a+3*b)*tan(
f*x+e)^5/f+1/7*b^2*tan(f*x+e)^7/f
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{105(a+b)^2\tan(e+fx)+35(a^2+4ab+3b^2)\tan^3(e+fx)+21b(2a+3b)\tan^5(e+fx)+15b^2\tan^7(e+fx)}{105f}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(105*(a + b)^2*Tan[e + f*x] + 35*(a^2 + 4*a*b + 3*b^2)*Tan[e + f*x]^3 + 21
*b*(2*a + 3*b)*Tan[e + f*x]^5 + 15*b^2*Tan[e + f*x]^7)/(105*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(b^2\tan^6(e+fx)+b(2a+3b)\tan^4(e+fx)+(a+b)(a+3b)\tan^2(e+fx)+(a+b)^2\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}b(2a+3b)\tan^5(e+fx)+\frac{1}{3}(a+b)(a+3b)\tan^3(e+fx)+(a+b)^2\tan(e+fx)+\frac{1}{7}b^2\tan^7(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + b)^2*Tan[e + f*x] + ((a + b)*(a + 3*b)*Tan[e + f*x]^3)/3 + (b*(2*a +
 3*b)*Tan[e + f*x]^5)/5 + (b^2*Tan[e + f*x]^7)/7)/f
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2ab\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(\ldots}{\ldots}\right.}{f}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2ab\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(\ldots}{\ldots}\right.}{f}$
parts	$-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{b^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}-\frac{2ab\left(-\frac{8}{15}\ldots\right.}{\ldots}$
parallelrisch	$\frac{(1050a^2+2352ab+1008b^2)\sin(3fx+3e)+(490a^2+784ab+336b^2)\sin(5fx+5e)+(70a^2+112ab+48b^2)\sin(7fx+7e)+6\ldots}{105f(\cos(7fx+7e)+7\cos(5fx+5e)+21\cos(3fx+3e)+35\cos(fx+e))}$
risch	$\frac{4i(105\,e^{10i(fx+e)}a^2+455a^2e^{8i(fx+e)}+560\,e^{8i(fx+e)}ab+770a^2e^{6i(fx+e)}+1400ab\,e^{6i(fx+e)}+840b^2e^{6i(fx+e)}+630a^2e^{4\ldots}}{105f(e^{2i(f\ldots}}$
norman	$\frac{-\frac{2(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{f}+\frac{4(7a^2+10ab+3b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{4(7a^2+10ab+3b^2)\tan\left(\frac{fx}{2}\ldots\right.}{3f}}{\left(\tan\left(\frac{fx}{2}\ldots\right.\right.}$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-8/15-1/5*sec(f*x+e)^4
-4/15*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e
)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{(2\,(35\,a^2+56\,ab+24\,b^2)\cos{(fx+e)}^6+(35\,a^2+56\,ab+24\,b^2)\cos{(fx+e)}^4+6\,(7\,ab+3\,b^2)\cos{(fx+}}{105\,f\cos{(fx+e)}^7}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/105*(2*(35*a^2 + 56*a*b + 24*b^2)*cos(f*x + e)^6 + (35*a^2 + 56*a*b + 24
*b^2)*cos(f*x + e)^4 + 6*(7*a*b + 3*b^2)*cos(f*x + e)^2 + 15*b^2)*sin(f*x
+ e)/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2{(e+fx)}\right)^2\sec^4{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{15\,b^2\tan(fx+e)^7 + 21\,(2\,ab+3\,b^2)\tan(fx+e)^5 + 35\,(a^2+4\,ab+3\,b^2)\tan(fx+e)^3 + 105\,(a^2+2\,ab}{105\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + 3*b^2)*tan(f*x + e)^5 + 35*(a^2
 + 4*a*b + 3*b^2)*tan(f*x + e)^3 + 105*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{15\,b^2\tan(fx+e)^7 + 42\,ab\tan(fx+e)^5 + 63\,b^2\tan(fx+e)^5 + 35\,a^2\tan(fx+e)^3 + 140\,ab\tan(fx+e}{105\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)
^5 + 35*a^2*tan(f*x + e)^3 + 140*a*b*tan(f*x + e)^3 + 105*b^2*tan(f*x + e)
^3 + 105*a^2*tan(f*x + e) + 210*a*b*tan(f*x + e) + 105*b^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\tan(e+f\,x)\,\left(a+b\right)^2+\tan(e+f\,x)^3\,\left(\frac{a^2}{3}+\frac{4\,a\,b}{3}+b^2\right)+\frac{b^2\,\tan(e+f\,x)^7}{7}+\frac{b\,\tan(e+f\,x)^5\,(2\,a+3\,b)}{5}}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^4,x)
```

output
```
(tan(e + f*x)*(a + b)^2 + tan(e + f*x)^3*((4*a*b)/3 + a^2/3 + b^2) + (b^2*
tan(e + f*x)^7)/7 + (b*tan(e + f*x)^5*(2*a + 3*b))/5)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.25

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\sin(fx+e)\left(70\sin(fx+e)^6a^2+112\sin(fx+e)^6ab+48\sin(fx+e)^6b^2-245\sin(fx+e)^4a^2-392\cdots\right.}{105\cos(fx+e)\,f\,\left(\sin\cdots\right.}$$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(sin(e + f*x)*(70*sin(e + f*x)**6*a**2 + 112*sin(e + f*x)**6*a*b + 48*sin(
e + f*x)**6*b**2 - 245*sin(e + f*x)**4*a**2 - 392*sin(e + f*x)**4*a*b - 16
8*sin(e + f*x)**4*b**2 + 280*sin(e + f*x)**2*a**2 + 490*sin(e + f*x)**2*a*
b + 210*sin(e + f*x)**2*b**2 - 105*a**2 - 210*a*b - 105*b**2))/(105*cos(e
+ f*x)*f*(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e + f*x)**2 - 1))
```

3.173 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(a+b)^2\tan(e+fx)}{f}+\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

output
```
(a+b)^2*tan(f*x+e)/f+2/3*b*(a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{15(a+b)^2\tan(e+fx)+10b(a+b)\tan^3(e+fx)+3b^2\tan^5(e+fx)}{15f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(15*(a + b)^2*Tan[e + f*x] + 10*b*(a + b)*Tan[e + f*x]^3 + 3*b^2*Tan[e + f
*x]^5)/(15*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(e+fx)^2\left(a+b\sec(e+fx)^2\right)^2dx$$

$$\Big\downarrow \; 4634$$

$$\frac{\int \left(b\tan^2(e+fx)+a+b\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 210$$

$$\frac{\int \left(b^2\tan^4(e+fx)+2ab\left(\frac{b}{a}+1\right)\tan^2(e+fx)+a^2\left(\frac{b(2a+b)}{a^2}+1\right)\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2}{3}b(a+b)\tan^3(e+fx)+(a+b)^2\tan(e+fx)+\frac{1}{5}b^2\tan^5(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + b)^2*Tan[e + f*x] + (2*b*(a + b)*Tan[e + f*x]^3)/3 + (b^2*Tan[e + f*
x]^5)/5)/f
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{a^2\tan(fx+e)-2ab\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$
default	$\frac{a^2\tan(fx+e)-2ab\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$
parts	$\frac{a^2\tan(fx+e)}{f}-\frac{b^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{2ab\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
parallelrisch	$\frac{(45a^2+100ab+40b^2)\sin(3fx+3e)+(15a^2+20ab+8b^2)\sin(5fx+5e)+30\sin(fx+e)\left(a^2+\frac{8}{3}ab+\frac{8}{3}b^2\right)}{15f(\cos(5fx+5e)+5\cos(3fx+3e)+10\cos(fx+e))}$
risch	$\frac{2i\left(15a^2e^{8i(fx+e)}+60a^2e^{6i(fx+e)}+60ab\,e^{6i(fx+e)}+90a^2e^{4i(fx+e)}+140ab\,e^{4i(fx+e)}+80b^2e^{4i(fx+e)}+60a^2e^{2i(fx+e)}+\right.}{15f\left(e^{2i(fx+e)}+1\right)^5}$
norman	$\frac{-\frac{2(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a^2+2ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{f}+\frac{8(3a^2+4ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{8(3a^2+4ab+b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*tan(f*x+e)-2*a*b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-8/15-1/
5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \,dx = \frac{\left(\left(15\,a^2+20\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+2\left(5\,ab+2\,b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/15*((15*a^2 + 20*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(5*a*b + 2*b^2)*cos(f*x
 + e)^2 + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^2\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{10\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ab+\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)b^2+15}{15\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(10*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*b + (3*tan(f*x + e)^5 + 10*ta
n(f*x + e)^3 + 15*tan(f*x + e))*b^2 + 15*a^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,b^2\tan\left(fx+e\right)^5+10\,ab\tan\left(fx+e\right)^3+10\,b^2\tan\left(fx+e\right)^3+15\,a^2\tan\left(fx+e\right)+30\,ab\tan\left(fx+e\right)+}{15\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 10*b^2*tan(f*x + e)^3
 + 15*a^2*tan(f*x + e) + 30*a*b*tan(f*x + e) + 15*b^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{\tan(e+f\,x)\,\left(a+b\right)^2+\frac{b^2\,\tan(e+f\,x)^5}{5}+\frac{2\,b\,\tan(e+f\,x)^3\,\left(a+b\right)}{3}}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^2,x)
```

output
```
(tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^5)/5 + (2*b*tan(e + f*x)^3*(a
+ b))/3)/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.49

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{\sin(fx+e)\left(15\sin(fx+e)^4a^2+20\sin(fx+e)^4ab+8\sin(fx+e)^4b^2-30\sin(fx+e)^2a^2-50\sin(}{15\cos(fx+e)\,f\left(\sin(fx+e)^4-2\sin(fx+e)^2+\right.}$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(sin(e + f*x)*(15*sin(e + f*x)**4*a**2 + 20*sin(e + f*x)**4*a*b + 8*sin(e
+ f*x)**4*b**2 - 30*sin(e + f*x)**2*a**2 - 50*sin(e + f*x)**2*a*b - 20*sin
(e + f*x)**2*b**2 + 15*a**2 + 30*a*b + 15*b**2))/(15*cos(e + f*x)*f*(sin(e
 + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.174 $\int (a+b\sec^2(e+fx))^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int (a+b\sec^2(e+fx))^2 \, dx = a^2x + \frac{b(2a+b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

output
```
a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int (a+b\sec^2(e+fx))^2 \, dx = \frac{3a^2fx + 3b(2a+b)\tan(e+fx) + b^2\tan^3(e+fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(3*a^2*f*x + 3*b*(2*a + b)*Tan[e + f*x] + b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \; 4616$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 300$$

$$\frac{\int \left(\frac{a^2}{\tan^2(e+fx)+1}+b^2\tan^2(e+fx)+b(2a+b)\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2\arctan(\tan(e+fx))+b(2a+b)\tan(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] + b*(2*a + b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^3
)/3)/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result
parts	$a^2x-\frac{b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{2ab\tan(fx+e)}{f}$
derivativedivides	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
default	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
risch	$a^2x+\frac{4ib\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+3b\,e^{2i(fx+e)}+3a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
norman	$\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-a^2x+3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\frac{4b(}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
parallelrisch	$\frac{3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6a^2f+\left(-12ab-6b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-9x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4a^2f+\left(24ab+4b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+9x\tan\left(\frac{fx}{2}+\frac{e}{}\right.}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$

input
```
int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*tan(f*x+e)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^2fx\cos\left(fx+e\right)^3+\left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin
(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=a^2x+\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b^2}{3\,f}+\frac{2\,ab\tan\left(fx+e\right)}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan
(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\frac{b^2\tan(e+f\,x)^3}{3}-\tan(e+f\,x)\,\left(b^2-2\,b\,(a+b)\right)+a^2\,f\,x}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.65

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\cos(fx+e)\sin(fx+e)^2a^2fx-3\cos(fx+e)a^2fx+6\sin(fx+e)^3ab+2\sin(fx+e)^3b^2-6\sin(}{3\cos(fx+e)f\left(\sin(fx+e)^2-1\right)}$$

input
```
int((a+b*sec(f*x+e)^2)^2,x)
```

output
```
(3*cos(e + f*x)*sin(e + f*x)**2*a**2*f*x - 3*cos(e + f*x)*a**2*f*x + 6*sin
(e + f*x)**3*a*b + 2*sin(e + f*x)**3*b**2 - 6*sin(e + f*x)*a*b - 3*sin(e +
 f*x)*b**2)/(3*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.175 $\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{1}{2}a(a+4b)x + \frac{a^2\cos(e+fx)\sin(e+fx)}{2f} + \frac{b^2\tan(e+fx)}{f}$$

output
```
1/2*a*(a+4*b)*x+1/2*a^2*cos(f*x+e)*sin(f*x+e)/f+b^2*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = 2abx + \frac{a^2(e+fx)}{2f} + \frac{a^2\sin(2(e+fx))}{4f} + \frac{b^2\tan(e+fx)}{f}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
2*a*b*x + (a^2*(e + f*x))/(2*f) + (a^2*Sin[2*(e + f*x)])/(4*f) + (b^2*Tan[
e + f*x])/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^2+\frac{2ab\tan^2(e+fx)+a(a+2b)}{\left(\tan^2(e+fx)+1\right)^2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}+\frac{1}{2}a(a+4b)\arctan(\tan(e+fx))+b^2\tan(e+fx)}{f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a*(a + 4*b)*ArcTan[Tan[e + f*x]])/2 + b^2*Tan[e + f*x] + (a^2*Tan[e + f*
x])/(2*(1 + Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{a^2 \sin(2fx+2e)+4b^2 \tan(fx+e)+2fxa(a+4b)}{4f}$
derivativedivides	$\frac{a^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(fx+e)+b^2\tan(fx+e)}{f}$
default	$\frac{a^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(fx+e)+b^2\tan(fx+e)}{f}$
risch	$\frac{a^2x}{2}+2axb-\frac{ia^2 e^{2i(fx+e)}}{8f}+\frac{ia^2 e^{-2i(fx+e)}}{8f}+\frac{2ib^2}{f\left(e^{2i(fx+e)}+1\right)}$
norman	$\left(-\frac{1}{2}a^2-2ab\right)x+\left(-\frac{1}{2}a^2-2ab\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+\left(\frac{1}{2}a^2+2ab\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(\frac{1}{2}a^2+2ab\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-a^2-4ab$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a^2*sin(2*f*x+2*e)+4*b^2*tan(f*x+e)+2*f*x*a*(a+4*b))/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(a^2+4\,ab)fx\cos\left(fx+e\right)+\left(a^2\cos\left(fx+e\right)^2+2\,b^2\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 + 4*a*b)*f*x*cos(f*x + e) + (a^2*cos(f*x + e)^2 + 2*b^2)*sin(f*x
 + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cos^2\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2\,b^2\tan\left(fx+e\right)+(a^2+4\,ab)(fx+e)+\frac{a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(
f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2\,b^2\tan\left(fx+e\right)+(a^2+4\,ab)(fx+e)+\frac{a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(
f*x + e)^2 + 1))/f
```

Mupad [B] (verification not implemented)

Time = 16.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\tan(e+f\,x)}{f}+\frac{a^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{a\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)\,(a+4\,b)}{2\,\left(\frac{a^2}{2}+2\,b\,a\right)}\right)\,(a+4\,b)}{2\,f}$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2*tan(e + f*x))/f + (a^2*sin(2*e + 2*f*x))/(4*f) + (a*atan((a*tan(e + f
*x)*(a + 4*b))/(2*(2*a*b + a^2/2)))*(a + 4*b))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\cos\left(fx+e\right)a^2e+\cos\left(fx+e\right)a^2fx+4\cos\left(fx+e\right)abe+4\cos\left(fx+e\right)abfx-\sin\left(fx+e\right)^3a^2+\sin}{2\cos\left(fx+e\right)f}$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(cos(e + f*x)*a**2*e + cos(e + f*x)*a**2*f*x + 4*cos(e + f*x)*a*b*e + 4*co
s(e + f*x)*a*b*f*x - sin(e + f*x)**3*a**2 + sin(e + f*x)*a**2 + 2*sin(e +
f*x)*b**2)/(2*cos(e + f*x)*f)
```

3.176 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{1}{8}\left(3a^2+8ab+8b^2\right)x + \frac{a(3a+8b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2\cos^3(e+fx)\sin(e+fx)}{4f}$$

output
```
1/8*(3*a^2+8*a*b+8*b^2)*x+1/8*a*(3*a+8*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a^2*
cos(f*x+e)^3*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{4\left(3a^2+8ab+8b^2\right)(e+fx)+8a(a+2b)\sin(2(e+fx))+a^2\sin(4(e+fx))}{32f}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(4*(3*a^2 + 8*a*b + 8*b^2)*(e + f*x) + 8*a*(a + 2*b)*Sin[2*(e + f*x)] + a^
2*Sin[4*(e + f*x)])/(32*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 315, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^4}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\frac{1}{4}\int \frac{b(a+4b)\tan^2(e+fx)+(a+b)(3a+4b)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{a\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2+8ab+8b^2\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{3a(a+2b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{a\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2+8ab+8b^2\right)\arctan(\tan(e+fx))+\frac{3a(a+2b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{a\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(4*(1 + Tan[e + f*x]^2)^2) +
(((3*a^2 + 8*a*b + 8*b^2)*ArcTan[Tan[e + f*x]])/2 + (3*a*(a + 2*b)*Tan[e +
 f*x])/(2*(1 + Tan[e + f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{8a(a+2b)\sin(2fx+2e)+a^2\sin(4fx+4e)+12x\left(a^2+\frac{8}{3}ab+\frac{8}{3}b^2\right)f}{32f}$
risch	$\frac{3a^2x}{8}+axb+x\,b^2+\frac{a^2\sin(4fx+4e)}{32f}+\frac{\sin(2fx+2e)a^2}{4f}+\frac{\sin(2fx+2e)ab}{2f}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2(fx+e)}{f}$
default	$\frac{a^2\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2(fx+e)}{f}$
norman	$\left(-\frac{3}{8}a^2-ab-b^2\right)x+\left(-\frac{9}{8}a^2-3ab-3b^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+\left(-\frac{9}{8}a^2-3ab-3b^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+\left(-\frac{3}{8}a^2-ab-b^2\right)x\tan\left(\frac{f}{2}\right.$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(8*a*(a+2*b)*sin(2*f*x+2*e)+a^2*sin(4*f*x+4*e)+12*x*(a^2+8/3*a*b+8/3*
b^2)*f)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{(3\,a^2+8\,ab+8\,b^2)fx+\left(2\,a^2\cos\left(fx+e\right)^3+(3\,a^2+8\,ab)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/8*((3*a^2 + 8*a*b + 8*b^2)*f*x + (2*a^2*cos(f*x + e)^3 + (3*a^2 + 8*a*b)
*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2(e+fx)\right)^2\cos^4(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(3a^2+8ab+8b^2)(fx+e)+\frac{(3a^2+8ab)\tan(fx+e)^3+(5a^2+8ab)\tan(fx+e)}{\tan(fx+e)^4+2\tan(fx+e)^2+1}}{8f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + ((3*a^2 + 8*a*b)*tan(f*x + e)^3 +
 (5*a^2 + 8*a*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(3a^2+8ab+8b^2)(fx+e)+\frac{3a^2\tan(fx+e)^3+8ab\tan(fx+e)^3+5a^2\tan(fx+e)+8ab\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + (3*a^2*tan(f*x + e)^3 + 8*a*b*tan
(f*x + e)^3 + 5*a^2*tan(f*x + e) + 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1
)^2)/f
```

Mupad [B] (verification not implemented)

Time = 15.83 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=x\left(\frac{3\,a^2}{8}+a\,b+b^2\right)+\frac{\left(\frac{3\,a^2}{8}+b\,a\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a^2}{8}+b\,a\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
x*(a*b + (3*a^2)/8 + b^2) + (tan(e + f*x)*(a*b + (5*a^2)/8) + tan(e + f*x)
^3*(a*b + (3*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{-2\cos\left(fx+e\right)\sin\left(fx+e\right)^3a^2+5\cos\left(fx+e\right)\sin\left(fx+e\right)a^2+8\cos\left(fx+e\right)\sin\left(fx+e\right)ab+3a^2fx}{8f}$$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 2*cos(e + f*x)*sin(e + f*x)**3*a**2 + 5*cos(e + f*x)*sin(e + f*x)*a**2
 + 8*cos(e + f*x)*sin(e + f*x)*a*b + 3*a**2*f*x + 8*a*b*f*x + 8*b**2*f*x)/
(8*f)
```

3.177 $\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx = \frac{1}{16}\left(5a^2+12ab+8b^2\right)x + \frac{(5a^2+12ab+8b^2)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(5a+12b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^2\cos^5(e+fx)\sin(e+fx)}{6f}$$

output
```
1/16*(5*a^2+12*a*b+8*b^2)*x+1/16*(5*a^2+12*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e
)/f+1/24*a*(5*a+12*b)*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^2*cos(f*x+e)^5*sin(f
*x+e)/f
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{60a^2e+144abe+96b^2e+60a^2fx+144abfx+96b^2fx+(45a^2+96ab+48b^2)\sin(2(e+fx))+3a(3a}{192f}$$

input
```
Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(60*a^2*e + 144*a*b*e + 96*b^2*e + 60*a^2*f*x + 144*a*b*f*x + 96*b^2*f*x +
 (45*a^2 + 96*a*b + 48*b^2)*Sin[2*(e + f*x)] + 3*a*(3*a + 4*b)*Sin[4*(e +
f*x)] + a^2*Sin[6*(e + f*x)])/(192*f)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4634, 315, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^6}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{1}{6}\int \frac{3b(a+2b)\tan^2(e+fx)+(a+b)(5a+6b)}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)+\frac{a\tan(e+fx)(a+b\tan^2(e+fx)+b)}{6(\tan^2(e+fx)+1)^3}}{f}$$

$\Big\downarrow$ 298

$$\frac{\frac{1}{6}\left(\frac{3}{4}(5a^2+12ab+8b^2)\int \frac{1}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)+\frac{a(5a+8b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)+\frac{a\tan(e+fx)(a+b\tan^2(e+fx)+b)}{6(\tan^2(e+fx)+1)^3}}{f}$$

$\Big\downarrow$ 215

$$\frac{\frac{1}{6}\left(\frac{3}{4}(5a^2+12ab+8b^2)\left(\frac{1}{2}\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{a(5a+8b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)+\frac{a\tan(e+fx)(a+b}{6(\tan^2(e+}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{6}\left(\frac{3}{4}(5a^2+12ab+8b^2)\left(\frac{1}{2}\arctan(\tan(e+fx))+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{a(5a+8b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)+\frac{a\tan(e+fx)(a+b\tan^2(e+f}{6(\tan^2(e+fx)+1)^3}}{f}$$

input
```
Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(6*(1 + Tan[e + f*x]^2)^3) +
((a*(5*a + 8*b)*Tan[e + f*x])/(4*(1 + Tan[e + f*x]^2)^2) + (3*(5*a^2 + 12*
a*b + 8*b^2)*(ArcTan[Tan[e + f*x]]/2 + Tan[e + f*x]/(2*(1 + Tan[e + f*x]^2
))))/4)/6)/f
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{(45a^2+96ab+48b^2)\sin(2fx+2e)+(9a^2+12ab)\sin(4fx+4e)+a^2\sin(6fx+6e)+60xf\left(a^2+\frac{12}{5}ab+\frac{8}{5}b^2\right)}{192f}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}\right.}{f}$
default	$\frac{a^2\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}\right.}{f}$
risch	$\frac{5a^2x}{16}+\frac{3axb}{4}+\frac{xb^2}{2}+\frac{a^2\sin(6fx+6e)}{192f}+\frac{3a^2\sin(4fx+4e)}{64f}+\frac{\sin(4fx+4e)ab}{16f}+\frac{15\sin(2fx+2e)a^2}{64f}+\sin(2$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*((45*a^2+96*a*b+48*b^2)*sin(2*f*x+2*e)+(9*a^2+12*a*b)*sin(4*f*x+4*e)
+a^2*sin(6*f*x+6*e)+60*x*f*(a^2+12/5*a*b+8/5*b^2))/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,(5\,a^2+12\,ab+8\,b^2)fx+\left(8\,a^2\cos\left(fx+e\right)^5+2\,(5\,a^2+12\,ab)\cos\left(fx+e\right)^3+3\,(5\,a^2+12\,ab+8\,b^2)\,\mathrm{co}\right.}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*f*x + (8*a^2*cos(f*x + e)^5 + 2*(5*a^2 +
12*a*b)*cos(f*x + e)^3 + 3*(5*a^2 + 12*a*b + 8*b^2)*cos(f*x + e))*sin(f*x
+ e))/f
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.25

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\,(5\,a^2+12\,ab+8\,b^2)(fx+e)+\frac{3\,(5\,a^2+12\,ab+8\,b^2)\,\tan(fx+e)^5+8\,(5\,a^2+12\,ab+6\,b^2)\,\tan(fx+e)^3+3\,(11\,a^2+20\,ab+8\,b^2)\,\tan(fx}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (3*(5*a^2 + 12*a*b + 8*b^2)*t
an(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(11*a^2 + 20
*a*b + 8*b^2)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x
 + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.39

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\,(5\,a^2+12\,ab+8\,b^2)(fx+e)+\frac{15\,a^2\tan(fx+e)^5+36\,ab\tan(fx+e)^5+24\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3+96\,ab\tan(fx+e)^3+}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (15*a^2*tan(f*x + e)^5 + 36*a
*b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 96*a*b
*tan(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) + 60*a*b*tan
(f*x + e) + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = x\left(\frac{5\,a^2}{16}+\frac{3\,a\,b}{4}+\frac{b^2}{2}\right) + \frac{\left(\frac{5\,a^2}{16}+\frac{3\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a^2}{6}+2\,a\,b+b^2\right)\,\tan(e+f\,x)^3+\left(\frac{11\,a^2}{16}+\frac{5\,a\,b}{4}+\frac{b^2}{2}\right)\,\tan(e+f\,x}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
x*((3*a*b)/4 + (5*a^2)/16 + b^2/2) + (tan(e + f*x)*((5*a*b)/4 + (11*a^2)/1
6 + b^2/2) + tan(e + f*x)^3*(2*a*b + (5*a^2)/6 + b^2) + tan(e + f*x)^5*((3
*a*b)/4 + (5*a^2)/16 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + t
an(e + f*x)^6 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.22

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{8\cos\left(fx+e\right)\sin\left(fx+e\right)^5a^2-26\cos\left(fx+e\right)\sin\left(fx+e\right)^3a^2-24\cos\left(fx+e\right)\sin\left(fx+e\right)^3ab+33\,c}{}$$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(8*cos(e + f*x)*sin(e + f*x)**5*a**2 - 26*cos(e + f*x)*sin(e + f*x)**3*a**
2 - 24*cos(e + f*x)*sin(e + f*x)**3*a*b + 33*cos(e + f*x)*sin(e + f*x)*a**
2 + 60*cos(e + f*x)*sin(e + f*x)*a*b + 24*cos(e + f*x)*sin(e + f*x)*b**2 +
 15*a**2*f*x + 36*a*b*f*x + 24*b**2*f*x)/(48*f)
```

3.178 $\int (a+b\sec^2(c+dx))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int (a+b\sec^2(c+dx))^3\,dx = a^3x+\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

output
```
a^3*x+b*(3*a^2+3*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(3*a+2*b)*tan(d*x+c)^3/d+1/
5*b^3*tan(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (a+b\sec^2(c+dx))^3\,dx = \frac{15a^3dx+15b(3a^2+3ab+b^2)\tan(c+dx)+5b^2(3a+2b)\tan^3(c+dx)+3b^3\tan^5(c+dx)}{15d}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^3,x]
```

output
```
(15*a^3*d*x + 15*b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x] + 5*b^2*(3*a + 2*b)*
Tan[c + d*x]^3 + 3*b^3*Tan[c + d*x]^5)/(15*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(c+dx)^2\right)^3 dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a+b\right)^3}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^3\tan^4(c+dx)+b^2(3a+2b)\tan^2(c+dx)+b\left(3a^2+3ba+b^2\right)+\frac{a^3}{\tan^2(c+dx)+1}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\arctan(\tan(c+dx))+b\left(3a^2+3ab+b^2\right)\tan(c+dx)+\frac{1}{3}b^2(3a+2b)\tan^3(c+dx)+\frac{1}{5}b^3\tan^5(c+dx)}{d}$$

input
```
Int[(a + b*Sec[c + d*x]^2)^3,x]
```

output
```
(a^3*ArcTan[Tan[c + d*x]] + b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x] + (b^2*(3
*a + 2*b)*Tan[c + d*x]^3)/3 + (b^3*Tan[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^2b\tan(dx+c)-3ab^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
default	$\frac{a^3(dx+c)+3a^2b\tan(dx+c)-3ab^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
parts	$a^3x-\frac{b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{3a^2b\tan(dx+c)}{d}-\frac{3ab^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$a^3x+\frac{2ib\left(45a^2e^{8i(dx+c)}+180a^2e^{6i(dx+c)}+90ab\,e^{6i(dx+c)}+270a^2e^{4i(dx+c)}+210ab\,e^{4i(dx+c)}+80\,e^{4i(dx+c)}b^2+180a\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-a^3x+5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-10a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+10a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-\frac{2b(3a}{}}{}$
parallelrisch	$\frac{15a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}xd+(-90a^2b-90ab^2-30b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9-75a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8xd+360\left(a+\frac{b}{3}\right)^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+}{}$

input
```
int((a+sec(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)+3*a^2*b*tan(d*x+c)-3*a*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*
x+c)-b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx = \frac{15\,a^3dx\cos\left(dx+c\right)^5+\left(\left(45\,a^2b+30\,ab^2+8\,b^3\right)\cos\left(dx+c\right)^4+3\,b^3+\left(15\,ab^2+4\,b^3\right)\cos\left(dx+c\right)^2\right)\sin(}{15\,d\cos\left(dx+c\right)^5}$$

input
```
integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(15*a^3*d*x*cos(d*x + c)^5 + ((45*a^2*b + 30*a*b^2 + 8*b^3)*cos(d*x +
 c)^4 + 3*b^3 + (15*a*b^2 + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*
x + c)^5)
```

Sympy [F]

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx = \int \left(a+b\sec^2\left(c+dx\right)\right)^3\,dx$$

input
```
integrate((a+b*sec(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sec(c + d*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx = a^3x+\frac{\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)ab^2}{d}+\frac{\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)b^3}{15\,d}+\frac{3\,a^2b\tan\left(dx+c\right)}{d}$$

input
```
integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*x + (tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5
 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3/d + 3*a^2*b*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx = \frac{3\,b^3\tan\left(dx+c\right)^5+15\,ab^2\tan\left(dx+c\right)^3+10\,b^3\tan\left(dx+c\right)^3+15\left(dx+c\right)a^3+45\,a^2b\tan\left(dx+c\right)+45}{15\,d}$$

input
```
integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(3*b^3*tan(d*x + c)^5 + 15*a*b^2*tan(d*x + c)^3 + 10*b^3*tan(d*x + c)
^3 + 15*(d*x + c)*a^3 + 45*a^2*b*tan(d*x + c) + 45*a*b^2*tan(d*x + c) + 15
*b^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 16.77 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx$$
$$=\frac{\tan(c+d\,x)\,\left(3\,b\,(a+b)^2-3\,b^2\,(a+b)+b^3\right)+\frac{b^3\tan(c+d\,x)^5}{5}+\tan(c+d\,x)^3\,\left(b^2\,(a+b)-\frac{b^3}{3}\right)+a^3\,d\,x}{d}$$

input
```
int((a + b/cos(c + d*x)^2)^3,x)
```

output
```
(tan(c + d*x)*(3*b*(a + b)^2 - 3*b^2*(a + b) + b^3) + (b^3*tan(c + d*x)^5)
/5 + tan(c + d*x)^3*(b^2*(a + b) - b^3/3) + a^3*d*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.85

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx$$
$$=\frac{15\cos\left(dx+c\right)\sin\left(dx+c\right)^4a^3dx-30\cos\left(dx+c\right)\sin\left(dx+c\right)^2a^3dx+15\cos\left(dx+c\right)a^3dx+45\sin\left(dx\right.}{}$$

input
```
int((a+b*sec(d*x+c)^2)^3,x)
```

output
```
(15*cos(c + d*x)*sin(c + d*x)**4*a**3*d*x - 30*cos(c + d*x)*sin(c + d*x)**
2*a**3*d*x + 15*cos(c + d*x)*a**3*d*x + 45*sin(c + d*x)**5*a**2*b + 30*sin
(c + d*x)**5*a*b**2 + 8*sin(c + d*x)**5*b**3 - 90*sin(c + d*x)**3*a**2*b -
 75*sin(c + d*x)**3*a*b**2 - 20*sin(c + d*x)**3*b**3 + 45*sin(c + d*x)*a**
2*b + 45*sin(c + d*x)*a*b**2 + 15*sin(c + d*x)*b**3)/(15*cos(c + d*x)*d*(s
in(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.179 $\int \left(a+b\sec^2(c+dx)\right)^4 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int \left(a+b\sec^2(c+dx)\right)^4 \, dx = a^4x+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d} +\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d} +\frac{b^3(4a+3b)\tan^5(c+dx)}{5d}+\frac{b^4\tan^7(c+dx)}{7d}$$

output
```
a^4*x+b*(2*a+b)*(2*a^2+2*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(6*a^2+8*a*b+3*b^2)
*tan(d*x+c)^3/d+1/5*b^3*(4*a+3*b)*tan(d*x+c)^5/d+1/7*b^4*tan(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 3.64 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \left(a+b\sec^2(c+dx)\right)^4 \, dx = \frac{105a^4dx+105b\left(4a^3+6a^2b+4ab^2+b^3\right)\tan(c+dx)+35b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)+21b^3(4a+}{105d}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^4,x]
```

output
```
(105*a^4*d*x + 105*b*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*Tan[c + d*x] + 35*b
^2*(6*a^2 + 8*a*b + 3*b^2)*Tan[c + d*x]^3 + 21*b^3*(4*a + 3*b)*Tan[c + d*x
]^5 + 15*b^4*Tan[c + d*x]^7)/(105*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(c+dx)\right)^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(c+dx)^2\right)^4 dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a+b\right)^4}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^4\tan^6(c+dx)+b^3(4a+3b)\tan^4(c+dx)+b^2\left(6a^2+8ba+3b^2\right)\tan^2(c+dx)+b(2a+b)\left(2a^2+2ba+b^2\right)\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\arctan(\tan(c+dx))+\frac{1}{3}b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)+b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)+\frac{1}{5}b^3(4a}{d}$$

input
```
Int[(a + b*Sec[c + d*x]^2)^4,x]
```

output
```
(a^4*ArcTan[Tan[c + d*x]] + b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x]
 + (b^2*(6*a^2 + 8*a*b + 3*b^2)*Tan[c + d*x]^3)/3 + (b^3*(4*a + 3*b)*Tan[c
 + d*x]^5)/5 + (b^4*Tan[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-4a\,b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c}{d}$
default	$\frac{a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-4a\,b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c}{d}$
parts	$a^4x-\frac{b^4\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}+\frac{4b\,a^3\tan(dx+c)}{d}-\frac{6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c}{3}\right.}{d}$
risch	$a^4x+\frac{8ib(105a^3\mathrm{e}^{12i(dx+c)}+630a^3\mathrm{e}^{10i(dx+c)}+315a^2b\,\mathrm{e}^{10i(dx+c)}+1575a^3\mathrm{e}^{8i(dx+c)}+1365a^2b\,\mathrm{e}^{8i(dx+c)}+560a\,b^2\mathrm{e}^{8i(}}{}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}-a^4x+7a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-21a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+35a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-35a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+21a}{}$
parallelrisch	$\frac{105a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}xd+(-840b\,a^3-1260a^2b^2-840a\,b^3-210b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}-735a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}xd+(5040b\,a^3}{}$

input
```
int((a+sec(d*x+c)^2*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(d*x+c)+4*a^3*b*tan(d*x+c)-6*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(
d*x+c)-4*a*b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-b^4*(
-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \left(a+b\sec^2(c+dx)\right)^4\,dx$$
$$=\frac{105\,a^4dx\cos\left(dx+c\right)^7+\left(4\left(105\,a^3b+105\,a^2b^2+56\,ab^3+12\,b^4\right)\cos\left(dx+c\right)^6+2\left(105\,a^2b^2+56\,ab^3+1\right.\right.}{105\,d\cos\left(dx+c\right)^7}$$

input
```
integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*(105*a^4*d*x*cos(d*x + c)^7 + (4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3
 + 12*b^4)*cos(d*x + c)^6 + 2*(105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cos(d*x +
c)^4 + 15*b^4 + 6*(14*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^7)
```

Sympy [F]

$$\int \left(a+b\sec^2(c+dx)\right)^4\,dx=\int \left(a+b\sec^2\left(c+dx\right)\right)^4\,dx$$

input
```
integrate((a+b*sec(d*x+c)**2)**4,x)
```

output
```
Integral((a + b*sec(c + d*x)**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \left(a+b\sec^2(c+dx)\right)^4\,dx &= a^4x+\frac{2\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2b^2}{d}\\&\quad+\frac{4\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)ab^3}{15\,d}\\&\quad+\frac{\left(5\,\tan\left(dx+c\right)^7+21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right)b^4}{35\,d}\\&\quad+\frac{4\,a^3b\tan\left(dx+c\right)}{d}\end{aligned}$$

input
```
integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
a^4*x + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x +
c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3/d + 1/35*(5*tan(d*x + c)
^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*b^4/d + 4*a^
3*b*tan(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \left(a + b\sec^2(c + dx)\right)^4 \, dx$$
$$= \frac{15\, b^4 \tan\left(dx + c\right)^7 + 84\, ab^3 \tan\left(dx + c\right)^5 + 63\, b^4 \tan\left(dx + c\right)^5 + 210\, a^2 b^2 \tan\left(dx + c\right)^3 + 280\, ab^3 \tan\left(dx\right.}{}$$

input
```
integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/105*(15*b^4*tan(d*x + c)^7 + 84*a*b^3*tan(d*x + c)^5 + 63*b^4*tan(d*x +
c)^5 + 210*a^2*b^2*tan(d*x + c)^3 + 280*a*b^3*tan(d*x + c)^3 + 105*b^4*tan
(d*x + c)^3 + 105*(d*x + c)*a^4 + 420*a^3*b*tan(d*x + c) + 630*a^2*b^2*tan
(d*x + c) + 420*a*b^3*tan(d*x + c) + 105*b^4*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 15.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \left(a + b\sec^2(c + dx)\right)^4 \, dx$$
$$= \frac{\tan(c + d\,x)\,\left(4\,b\,(a+b)^3 + 4\,b^3\,(a+b) - 6\,b^2\,(a+b)^2 - b^4\right) + \tan(c + d\,x)^3\,\left(2\,b^2\,(a+b)^2 - \frac{4\,b^3\,(a+b)}{3} + \right.}{d}$$

input
```
int((a + b/cos(c + d*x)^2)^4,x)
```

output
```
(tan(c + d*x)*(4*b*(a + b)^3 + 4*b^3*(a + b) - 6*b^2*(a + b)^2 - b^4) + ta
n(c + d*x)^3*(2*b^2*(a + b)^2 - (4*b^3*(a + b))/3 + b^4/3) + (b^4*tan(c +
d*x)^7)/7 + tan(c + d*x)^5*((4*b^3*(a + b))/5 - b^4/5) + a^4*d*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.08

$$\int \left(a+b\sec^2(c+dx)\right)^4\,dx$$
$$= \frac{105\cos(dx+c)\sin(dx+c)^6 a^4dx - 315\cos(dx+c)\sin(dx+c)^4 a^4dx + 315\cos(dx+c)\sin(dx+c)^2 a}{}$$

input
```
int((a+b*sec(d*x+c)^2)^4,x)
```

output
```
(105*cos(c + d*x)*sin(c + d*x)**6*a**4*d*x - 315*cos(c + d*x)*sin(c + d*x)
**4*a**4*d*x + 315*cos(c + d*x)*sin(c + d*x)**2*a**4*d*x - 105*cos(c + d*x
)*a**4*d*x + 420*sin(c + d*x)**7*a**3*b + 420*sin(c + d*x)**7*a**2*b**2 +
224*sin(c + d*x)**7*a*b**3 + 48*sin(c + d*x)**7*b**4 - 1260*sin(c + d*x)**
5*a**3*b - 1470*sin(c + d*x)**5*a**2*b**2 - 784*sin(c + d*x)**5*a*b**3 - 1
68*sin(c + d*x)**5*b**4 + 1260*sin(c + d*x)**3*a**3*b + 1680*sin(c + d*x)*
*3*a**2*b**2 + 980*sin(c + d*x)**3*a*b**3 + 210*sin(c + d*x)**3*b**4 - 420
*sin(c + d*x)*a**3*b - 630*sin(c + d*x)*a**2*b**2 - 420*sin(c + d*x)*a*b**
3 - 105*sin(c + d*x)*b**4)/(105*cos(c + d*x)*d*(sin(c + d*x)**6 - 3*sin(c
+ d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.180 $\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(2a-b)\text{arctanh}(\sin(e+fx))}{2b^2f} + \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}f} + \frac{\sec(e+fx)\tan(e+fx)}{2bf}$$

output
```
-1/2*(2*a-b)*arctanh(sin(f*x+e))/b^2/f+a^(3/2)*arctanh(a^(1/2)*sin(f*x+e)/
(a+b)^(1/2))/b^2/(a+b)^(1/2)/f+1/2*sec(f*x+e)*tan(f*x+e)/b/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.69 (sec) , antiderivative size = 1195, normalized size of antiderivative = 13.90

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(4*a*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] - 2*b*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 4*a*Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*b*Log[Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2]] + (a^(3/2)*Cos[e]*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*
x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Sin[2*e + f*x]])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) - (a^(3/
2)*Cos[e]*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2
*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*
x]])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((2*I)*a^(3/2)*ArcTan[(2*
Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*Sqrt
[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e]
- I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos
[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[
e + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b
*Sin[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*Sqrt[(Cos[e] - I*Sin[e
])^2]*(Cos[e] + I*Sin[e]))/Sqrt[a + b] - (I*a^(3/2)*Log[a + 2*(a + b)*Cos[
2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqr...
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4635, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^5}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \textbf{316}$$

$$\frac{\frac{\int -\frac{a\sin^2(e+fx)+a-b}{\left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{2b}+\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}-\frac{\int \frac{a\sin^2(e+fx)+a-b}{\left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{2b}}{f}$$

$$\Big\downarrow \textbf{397}$$

$$\frac{\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}-\frac{\frac{(2a-b)\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)}{b}-\frac{2a^2\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{2b}}{f}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}-\frac{\frac{(2a-b)\text{arctanh}(\sin(e+fx))}{b}-\frac{2a^2\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{2b}}{f}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}-\frac{\frac{(2a-b)\text{arctanh}(\sin(e+fx))}{b}-\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{2b}}{f}$$

input
```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-1/2*(((2*a - b)*ArcTanh[Sin[e + f*x]])/b - (2*a^(3/2)*ArcTanh[(Sqrt[a]*S
in[e + f*x])/Sqrt[a + b]])/(b*Sqrt[a + b]))/b + Sin[e + f*x]/(2*b*(1 - Sin
[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{-\frac{1}{4b(\sin(fx+e)-1)}+\frac{(2a-b)\ln(\sin(fx+e)-1)}{4b^2}-\frac{1}{4b(\sin(fx+e)+1)}+\frac{(-2a+b)\ln(\sin(fx+e)+1)}{4b^2}+\frac{a^2\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{1}{4b(\sin(fx+e)-1)}+\frac{(2a-b)\ln(\sin(fx+e)-1)}{4b^2}-\frac{1}{4b(\sin(fx+e)+1)}+\frac{(-2a+b)\ln(\sin(fx+e)+1)}{4b^2}+\frac{a^2\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}}{f}$
risch	$-\frac{i\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{fb\left(e^{2i(fx+e)}+1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+i\right)a}{b^2f}+\frac{\ln\left(e^{i(fx+e)}+i\right)}{2bf}+\frac{\ln\left(e^{i(fx+e)}-i\right)a}{b^2f}-\frac{\ln\left(e^{i(fx+e)}-i\right)}{2bf}+\frac{\sqrt{a(a}}{}$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/4/b/(sin(f*x+e)-1)+1/4*(2*a-b)/b^2*ln(sin(f*x+e)-1)-1/4/b/(sin(f*x
+e)+1)+1/4/b^2*(-2*a+b)*ln(sin(f*x+e)+1)+a^2/b^2/(a*(a+b))^(1/2)*arctanh(a
*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.16

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[\frac{2\,a\sqrt{\frac{a}{a+b}}\cos(fx+e)^2\log\left(-\frac{a\cos(fx+e)^2-2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)-(2\,a-b)\cos(fx+e)^2\log(\sin(f}{4\,b^2f\cos(fx+e)^2}\right.$$
$$-\frac{4\,a\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{-\frac{a}{a+b}}\sin(fx+e)\right)\cos(fx+e)^2+(2\,a-b)\cos(fx+e)^2\log(\sin(fx+e)+1)}{4\,b^2f\cos(fx+e)^2}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*sqrt(a/(a + b))*cos(f*x + e)^2*log(-(a*cos(f*x + e)^2 - 2*(a + b
)*sqrt(a/(a + b))*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - (2*a -
 b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*a - b)*cos(f*x + e)^2*log(-s
in(f*x + e) + 1) + 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2), -1/4*(4*a*sqr
t(-a/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e))*cos(f*x + e)^2 + (2*a
- b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a - b)*cos(f*x + e)^2*log(-
sin(f*x + e) + 1) - 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{2a^2\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^2}+\frac{(2a-b)\log(\sin(fx+e)+1)}{b^2}-\frac{(2a-b)\log(\sin(fx+e)-1)}{b^2}+\frac{2\sin(fx+e)}{b\sin(fx+e)^2-b}}{4f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/4*(2*a^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt(
(a + b)*a)))/(sqrt((a + b)*a)*b^2) + (2*a - b)*log(sin(f*x + e) + 1)/b^2 -
 (2*a - b)*log(sin(f*x + e) - 1)/b^2 + 2*sin(f*x + e)/(b*sin(f*x + e)^2 -
b))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.31

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{\frac{4\,a^2\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b^2} + \frac{(2\,a-b)\log(|\sin(fx+e)+1|)}{b^2} - \frac{(2\,a-b)\log(|\sin(fx+e)-1|)}{b^2} + \frac{2\,\sin(fx+e)}{\left(\sin(fx+e)^2-1\right)b}}{4\,f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/4*(4*a^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b^2)
 + (2*a - b)*log(abs(sin(f*x + e) + 1))/b^2 - (2*a - b)*log(abs(sin(f*x +
e) - 1))/b^2 + 2*sin(f*x + e)/((sin(f*x + e)^2 - 1)*b))/f
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 591, normalized size of antiderivative = 6.87

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{b\left(a\,\sin\left(e+f\,x\right) - a\,\mathrm{atanh}(\sin\left(e+f\,x\right)) + a\sin\left(e+f\,x\right)^2\mathrm{atanh}(\sin\left(e+f\,x\right))\right) + b^2\left(\sin\left(e+f\,x\right) + a\right.}{}$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

output
```
(b*(a*sin(e + f*x) - a*atanh(sin(e + f*x)) + a*sin(e + f*x)^2*atanh(sin(e
+ f*x))) + atan((a^5*sin(e + f*x)*(a^3*b + a^4)^(1/2)*8i - b*sin(e + f*x)*
(a^3*b + a^4)^(3/2)*4i - a*sin(e + f*x)*(a^3*b + a^4)^(3/2)*8i - a^2*b^3*s
in(e + f*x)*(a^3*b + a^4)^(1/2)*2i + a^3*b^2*sin(e + f*x)*(a^3*b + a^4)^(1
/2)*1i + a*b^4*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a^4*b*sin(e + f*x)*(a
^3*b + a^4)^(1/2)*12i)/(a^3*b^4 - a^2*b^5 + 5*a^4*b^3 + 3*a^5*b^2))*(a^3*b
 + a^4)^(1/2)*2i + b^2*(sin(e + f*x) + atanh(sin(e + f*x)) - sin(e + f*x)^
2*atanh(sin(e + f*x))) - 2*a^2*atanh(sin(e + f*x)) - atan((a^5*sin(e + f*x
)*(a^3*b + a^4)^(1/2)*8i - b*sin(e + f*x)*(a^3*b + a^4)^(3/2)*4i - a*sin(e
 + f*x)*(a^3*b + a^4)^(3/2)*8i - a^2*b^3*sin(e + f*x)*(a^3*b + a^4)^(1/2)*
2i + a^3*b^2*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a*b^4*sin(e + f*x)*(a^3
*b + a^4)^(1/2)*1i + a^4*b*sin(e + f*x)*(a^3*b + a^4)^(1/2)*12i)/(a^3*b^4
- a^2*b^5 + 5*a^4*b^3 + 3*a^5*b^2))*sin(e + f*x)^2*(a^3*b + a^4)^(1/2)*2i
+ 2*a^2*sin(e + f*x)^2*atanh(sin(e + f*x)))/(f*(2*a*b^2 + 2*b^3 - 2*b^3*si
n(e + f*x)^2 - 2*a*b^2*sin(e + f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 493, normalized size of antiderivative = 5.73

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

output
```
( - sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
- 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a + sqrt(a)*sqrt(a + b)*log(
sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2)
)*a + sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b
) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a - sqrt(a)*sqrt(a + b)*lo
g(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/
2))*a + 2*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a**2 + log(tan((e + f*
x)/2) - 1)*sin(e + f*x)**2*a*b - log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2
*b**2 - 2*log(tan((e + f*x)/2) - 1)*a**2 - log(tan((e + f*x)/2) - 1)*a*b +
 log(tan((e + f*x)/2) - 1)*b**2 - 2*log(tan((e + f*x)/2) + 1)*sin(e + f*x)
**2*a**2 - log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a*b + log(tan((e + f*
x)/2) + 1)*sin(e + f*x)**2*b**2 + 2*log(tan((e + f*x)/2) + 1)*a**2 + log(t
an((e + f*x)/2) + 1)*a*b - log(tan((e + f*x)/2) + 1)*b**2 - sin(e + f*x)*a
*b - sin(e + f*x)*b**2)/(2*b**2*f*(sin(e + f*x)**2*a + sin(e + f*x)**2*b -
 a - b))
```

3.181 $\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\operatorname{arctanh}(\sin(e+fx))}{bf}-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}f}$$

output
```
arctanh(sin(f*x+e))/b/f-a^(1/2)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/b/
(a+b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\operatorname{arctanh}(\sin(e+fx))-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{bf}$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
(ArcTanh[Sin[e + f*x]] - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
b]])/Sqrt[a + b])/(b*f)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4635, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^3}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))(-a\sin^2(e+fx)+a+b)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 303}$$

$$\frac{\frac{\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)}{b}-\frac{a\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\text{arctanh}(\sin(e+fx))}{b}-\frac{a\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\text{arctanh}(\sin(e+fx))}{b}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{f}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
(ArcTanh[Sin[e + f*x]]/b - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a
+ b]])/(b*Sqrt[a + b]))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{\ln(\sin(fx+e)-1)}{2b}+\frac{\ln(\sin(fx+e)+1)}{2b}-\frac{a\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{\ln(\sin(fx+e)-1)}{2b}+\frac{\ln(\sin(fx+e)+1)}{2b}-\frac{a\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}}{f}$
risch	$\frac{\ln\left(e^{i(fx+e)}+i\right)}{bf}-\frac{\ln\left(e^{i(fx+e)}-i\right)}{bf}+\frac{\sqrt{a(a+b)}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{a(a+b)}\,e^{i(fx+e)}}{a}-1\right)}{2(a+b)fb}-\frac{\sqrt{a(a+b)}\,\ln\left(e^{2i(fx+e)}\right.}{2(a}$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/b*ln(sin(f*x+e)-1)+1/2/b*ln(sin(f*x+e)+1)-a/b/(a*(a+b))^(1/2)*ar
ctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.85

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos(fx+e)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+\log\left(\sin\left(fx+e\right)+1\right)-\log\left(-\sin\left(fx+e\right)+1\right)}{2\,bf},\right.$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*s
in(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + log(sin(f*x + e) + 1) - l
og(-sin(f*x + e) + 1))/(b*f), 1/2*(2*sqrt(-a/(a + b))*arctan(sqrt(-a/(a +
b))*sin(f*x + e)) + log(sin(f*x + e) + 1) - log(-sin(f*x + e) + 1))/(b*f)]
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{a\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b} + \frac{\log(\sin(fx+e)+1)}{b} - \frac{\log(\sin(fx+e)-1)}{b}}{2f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a +
b)*a)))/(sqrt((a + b)*a)*b) + log(sin(f*x + e) + 1)/b - log(sin(f*x + e) -
 1)/b)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\,a\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b} + \frac{\log(|\sin(fx+e)+1|)}{b} - \frac{\log(|\sin(fx+e)-1|)}{b}}{2f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(2*a*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b) + lo
g(abs(sin(f*x + e) + 1))/b - log(abs(sin(f*x + e) - 1))/b)/f
```

Mupad [B] (verification not implemented)

Time = 15.53 (sec) , antiderivative size = 456, normalized size of antiderivative = 8.29

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}(\sin(e+f\,x))}{b\,f} + \frac{\operatorname{atan}\left(\frac{\frac{\left(2\,a^3\,\sin(e+f\,x)+\frac{\left(2\,a^2\,b^2-\frac{\sin(e+f\,x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,(a+b)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,(a+b)}\,1\mathrm{i}}{b^2+a\,b}+\frac{\left(2\,a^3\,\sin(e+f\,x)-\frac{\left(2\,a^2\,b^2+\frac{\sin(e+f\,x)\,\left(16\,a\cdots\right.}{4\cdots}\right.}{2\cdots}\right.}{b\cdots}}{\frac{\left(2\,a^3\,\sin(e+f\,x)+\frac{\left(2\,a^2\,b^2-\frac{\sin(e+f\,x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{a\,(a+b)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,(a+b)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{a\,(a+b)}}{b^2+a\,b}-\frac{\left(2\,a^3\,\sin(e+f\,x)-\frac{\left(2\,a^2\,b^2+\frac{\sin(e+f\,x)\,\left(16\,a\cdots\right.}{4\cdots}\right.}{2\cdots}\right.}{b^2\cdots}}\right.\cdots}{f\,\left(b^2+a\,b\right)}$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

output
```
atanh(sin(e + f*x))/(b*f) + (atan((((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (s
in(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(
a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(a*b + b^2) + ((2
*a^3*sin(e + f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(
a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*
(a + b))^(1/2)*1i)/(a*b + b^2))/(((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (sin
(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*
(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2))/(a*b + b^2) - ((2*a^3*
sin(e + f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a
+ b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a +
b))^(1/2))/(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(f*(a*b + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.89

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\,\sqrt{a+b}\,\log\left(\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\sqrt{a+b}-2\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\sqrt{a}\,\sqrt{a+b}\,\log\left(\sqrt{a+b}\,\tan\left(\frac{fx}{2}\cdots\right.\right.}{\cdots}$$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

output
```
(sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2
*sqrt(a)*tan((e + f*x)/2)) - sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e +
f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2)) - 2*log(tan((e + f*
x)/2) - 1)*a - 2*log(tan((e + f*x)/2) - 1)*b + 2*log(tan((e + f*x)/2) + 1)
*a + 2*log(tan((e + f*x)/2) + 1)*b)/(2*b*f*(a + b))
```

3.182 $\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}f}$$

output
```
arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(1/2)/(a+b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}f}$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4635, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}f\sqrt{a+b}}$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{f\sqrt{a(a+b)}}$	28
default	$\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{f\sqrt{a(a+b)}}$	28
risch	$\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f}-\frac{\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f}$	102

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.25

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)}{2\sqrt{a^2+ab}f},\ -\frac{\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)}{(a^2+ab)f}\right]$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a
*cos(f*x + e)^2 + b))/(sqrt(a^2 + a*b)*f), -sqrt(-a^2 - a*b)*arctan(sqrt(-
a^2 - a*b)*sin(f*x + e)/(a + b))/((a^2 + a*b)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)
*a)))/(sqrt((a + b)*a)*f)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*f)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,f\,\sqrt{a+b}}$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)),x)
```

output
```
atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))/(a^(1/2)*f*(a + b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.56

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\,\sqrt{a+b}\left(-\log\left(\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\sqrt{a+b}-2\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\log\left(\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\cdots\right.\right.}{2af\,(a+b)}$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

output
```
(sqrt(a)*sqrt(a + b)*( - log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
 - 2*sqrt(a)*tan((e + f*x)/2)) + log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqr
t(a + b) + 2*sqrt(a)*tan((e + f*x)/2))))/(2*a*f*(a + b))
```

3.183 $\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)} dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx=-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}f}+\frac{\sin(e+fx)}{af}$$

output
```
-b*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(3/2)/(a+b)^(1/2)/f+sin(f*x+e
)/a/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+\sqrt{a}\sin(e+fx)}{a^{3/2}f}$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*
Sin[e + f*x])/(a^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(e+fx)\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\sin(e+fx)}{a}-\frac{b\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{a}}{f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\sin(e+fx)}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}}{f}$$

input
```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]))
+ Sin[e + f*x]/a)/f
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	si
derivativedivides	$\frac{\frac{\sin(fx+e)}{a}-\frac{b\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{f}$	45
default	$\frac{\frac{\sin(fx+e)}{a}-\frac{b\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{f}$	45
risch	$-\frac{i\mathrm{e}^{i(fx+e)}}{2af}+\frac{i\mathrm{e}^{-i(fx+e)}}{2af}+\frac{b\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,fa}-\frac{b\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,fa}$	14

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)/a-b/a/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.15

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{a^2+ab}\,b\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(a^2+ab\right)\sin\left(fx+e\right)}{2\left(a^3+a^2b\right)f}, \frac{\sqrt{-a^2-ab}\,b\arctan\left(\sqrt{-\right.}}{}\right.$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 + a*b)*b*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x
 + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b)*sin(f*x + e))/((a
^3 + a^2*b)*f), (sqrt(-a^2 - a*b)*b*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(
a + b)) + (a^2 + a*b)*sin(f*x + e))/((a^3 + a^2*b)*f)]
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cos\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a} + \frac{2\sin(fx+e)}{a}}{2f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(b*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a +
b)*a)))/(sqrt((a + b)*a)*a) + 2*sin(f*x + e)/a)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a} + \frac{\sin(fx+e)}{a}}{f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(b*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a) + sin(f*x
+ e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 16.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sin(e+fx)}{af} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}f\sqrt{a+b}}$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

output
```
sin(e + f*x)/(a*f) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(3
/2)*f*(a + b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.35

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\sqrt{a}\,\sqrt{a+b}\log\left(\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\sqrt{a+b}-2\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)b-\sqrt{a}\,\sqrt{a+b}\log\left(\sqrt{a+b}\,\tan\left(\frac{f}{}\right.\right.}{2a^2f\,(a+b)}$$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

output
```
(sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2
*sqrt(a)*tan((e + f*x)/2))*b - sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e
+ f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*b + 2*sin(e + f*x
)*a**2 + 2*sin(e + f*x)*a*b)/(2*a**2*f*(a + b))
```

3.184 $\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}f}+\frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}$$

output
```
b^2*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(5/2)/(a+b)^(1/2)/f+(a-b)*si
n(f*x+e)/a^2/f-1/3*sin(f*x+e)^3/a/f
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx =\frac{\frac{6b^2\left(-\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)+\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{\sqrt{a+b}}+3\sqrt{a}(3a-4b)\sin(e+fx)+a^{3/2}\sin(3(e+fx))}{12a^{5/2}f}$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
((6*b^2*(-Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] + Log[Sqrt[a + b] + Sqrt
[a]*Sin[e + f*x]]))/Sqrt[a + b] + 3*Sqrt[a]*(3*a - 4*b)*Sin[e + f*x] + a^(
3/2)*Sin[3*(e + f*x)])/(12*a^(5/2)*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(\frac{b^2}{a^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{\sin^2(e+fx)}{a}+\frac{a-b}{a^2}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b^2\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\sin(e+fx)}{a^2}-\frac{\sin^3(e+fx)}{3a}}{f}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
((b^2*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) +
 ((a - b)*Sin[e + f*x])/a^2 - Sin[e + f*x]^3/(3*a))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+\sin(fx+e)b}{a^2}+\frac{b^2\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+\sin(fx+e)b}{a^2}+\frac{b^2\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{f}$
risch	$-\frac{3i\mathrm{e}^{i(fx+e)}}{8af}+\frac{i\mathrm{e}^{i(fx+e)}b}{2a^2f}+\frac{3i\mathrm{e}^{-i(fx+e)}}{8af}-\frac{i\mathrm{e}^{-i(fx+e)}b}{2a^2f}+\frac{b^2\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f\,a^2}-\frac{b^2\ln\left(\mathrm{e}^{2}\right.}{}$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+sin(f*x+e)*b)+b^2/a^2/(a*(a+b
))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.03

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{3\sqrt{a^2+ab}\,b^2\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(2a^3-a^2b-3ab^2+(a^3+a^2b)\cos\left(fx+e\right)\cdots\right.}{6(a^4+a^3b)f}\right.$$
$$-\frac{3\sqrt{-a^2-ab}\,b^2\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(2a^3-a^2b-3ab^2+(a^3+a^2b)\cos\left(fx+e\right)^2\right)\sin\left(fx+\cdots\right.}{3(a^4+a^3b)f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a^2 + a*b)*b^2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin
(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3 - a^2*b - 3*a*b^2
+ (a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)*f), -1/3*(3*s
qrt(-a^2 - a*b)*b^2*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (2*a^3
 - a^2*b - 3*a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a
^3*b)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,b^2\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^2}+\frac{2\left(a\sin(fx+e)^3-3\,(a-b)\sin(fx+e)\right)}{a^2}}{6\,f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/6*(3*b^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt(
(a + b)*a)))/(sqrt((a + b)*a)*a^2) + 2*(a*sin(f*x + e)^3 - 3*(a - b)*sin(f
*x + e))/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,b^2\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a^2}+\frac{a^2\sin(fx+e)^3-3\,a^2\sin(fx+e)+3\,ab\sin(fx+e)}{a^3}}{3\,f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/3*(3*b^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^2)
 + (a^2*sin(f*x + e)^3 - 3*a^2*sin(f*x + e) + 3*a*b*sin(f*x + e))/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 16.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{a^{5/2}\,f\,\sqrt{a+b}}-\frac{\sin\left(e+f\,x\right)^3}{3\,a\,f} - \frac{\sin\left(e+f\,x\right)\,\left(\frac{a+b}{a^2}-\frac{2}{a}\right)}{f}$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

output
```
(b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(5/2)*f*(a + b)^(1/2)
) - sin(e + f*x)^3/(3*a*f) - (sin(e + f*x)*((a + b)/a^2 - 2/a))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.05

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-3\sqrt{a}\,\sqrt{a+b}\log\left(\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\sqrt{a+b}-2\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)b^2+3\sqrt{a}\,\sqrt{a+b}\log\left(\sqrt{a+b}\,\mathrm{t}\right.}{}$$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b
) - 2*sqrt(a)*tan((e + f*x)/2))*b**2 + 3*sqrt(a)*sqrt(a + b)*log(sqrt(a +
b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*b**2 -
2*sin(e + f*x)**3*a**3 - 2*sin(e + f*x)**3*a**2*b + 6*sin(e + f*x)*a**3 -
6*sin(e + f*x)*a*b**2)/(6*a**3*f*(a + b))
```

3.185 $\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}f}+\frac{(a^2-ab+b^2)\sin(e+fx)}{a^3f} - \frac{(2a-b)\sin^3(e+fx)}{3a^2f}+\frac{\sin^5(e+fx)}{5af}$$

output
```
-b^3*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(7/2)/(a+b)^(1/2)/f+(a^2-a*
b+b^2)*sin(f*x+e)/a^3/f-1/3*(2*a-b)*sin(f*x+e)^3/a^2/f+1/5*sin(f*x+e)^5/a/
f
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{120b^3\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{\sqrt{a+b}}+30\sqrt{a}(5a^2-6ab+8b^2)\sin(e+fx)+5a^{3/2}(5a-4b)\,\mathrm{s}}{240a^{7/2}f}$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((120*b^3*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a + b] + Sqr
t[a]*Sin[e + f*x]]))/Sqrt[a + b] + 30*Sqrt[a]*(5*a^2 - 6*a*b + 8*b^2)*Sin[
e + f*x] + 5*a^(3/2)*(5*a - 4*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f
*x)])/(240*a^(7/2)*f)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sec(e+fx)^5\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \; 4635$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^3}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 300$$

$$\frac{\int \left(\frac{\sin^4(e+fx)}{a}-\frac{(2a-b)\sin^2(e+fx)}{a^2}+\frac{a^2-ba+b^2}{a^3}-\frac{b^3}{a^3\left(-a\sin^2(e+fx)+a+b\right)}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}}-\frac{(2a-b)\sin^3(e+fx)}{3a^2}+\frac{\left(a^2-ab+b^2\right)\sin(e+fx)}{a^3}+\frac{\sin^5(e+fx)}{5a}}{f}$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((b^3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b])
) + ((a^2 - a*b + b^2)*Sin[e + f*x])/a^3 - ((2*a - b)*Sin[e + f*x]^3)/(3*a
^2) + Sin[e + f*x]^5/(5*a))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{ab\sin(fx+e)^3}{3}+\sin(fx+e)a^2-\sin(fx+e)ab+\sin(fx+e)b^2}{a^3}-\frac{b^3\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{f}$
default	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{ab\sin(fx+e)^3}{3}+\sin(fx+e)a^2-\sin(fx+e)ab+\sin(fx+e)b^2}{a^3}-\frac{b^3\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{f}$
risch	$-\frac{5i\mathrm{e}^{i(fx+e)}}{16af}+\frac{3i\mathrm{e}^{i(fx+e)}b}{8a^2f}-\frac{i\mathrm{e}^{i(fx+e)}b^2}{2f\,a^3}+\frac{5i\mathrm{e}^{-i(fx+e)}}{16af}-\frac{3i\mathrm{e}^{-i(fx+e)}b}{8a^2f}+\frac{i\mathrm{e}^{-i(fx+e)}b^2}{2f\,a^3}+\frac{b^3\ln\left(\mathrm{e}^{2i(fx+e)}\right.}{2\sqrt{}}$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*(1/5*a^2*sin(f*x+e)^5-2/3*a^2*sin(f*x+e)^3+1/3*a*b*sin(f*x+e)^3
+sin(f*x+e)*a^2-sin(f*x+e)*a*b+sin(f*x+e)*b^2)-b^3/a^3/(a*(a+b))^(1/2)*arc
tanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.82

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{15\sqrt{a^2+ab}b^3\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(3\left(a^4+a^3b\right)\cos\left(fx+e\right)^4+8a^4-2a^3b+\right.}{30\left(a^5+a^4b\right)f}\right.$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(a^2 + a*b)*b^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*s
in(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(3*(a^4 + a^3*b)*cos(f*
x + e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a^2
*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f), 1/15*(15*sqrt(-a^2
- a*b)*b^3*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (3*(a^4 + a^3*b
)*cos(f*x + e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b
 - 5*a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\,b^3\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^3}+\frac{2\left(3\,a^2\sin(fx+e)^5-5\,(2\,a^2-ab)\sin(fx+e)^3+15\,(a^2-ab+b^2)\sin(fx+e)\right)}{a^3}}{30\,f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/30*(15*b^3*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt
((a + b)*a)))/(sqrt((a + b)*a)*a^3) + 2*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2 -
 a*b)*sin(f*x + e)^3 + 15*(a^2 - a*b + b^2)*sin(f*x + e))/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\,b^3\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a^3}+\frac{3\,a^4\sin(fx+e)^5-10\,a^4\sin(fx+e)^3+5\,a^3b\sin(fx+e)^3+15\,a^4\sin(fx+e)-15\,a^3b\sin(fx+e)+15\,a^2b^2\sin(fx+e)}{a^5}}{15\,f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(15*b^3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^3
) + (3*a^4*sin(f*x + e)^5 - 10*a^4*sin(f*x + e)^3 + 5*a^3*b*sin(f*x + e)^3
 + 15*a^4*sin(f*x + e) - 15*a^3*b*sin(f*x + e) + 15*a^2*b^2*sin(f*x + e))/
a^5)/f
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sin(e+f\,x)\,\left(\frac{3}{a}+\frac{(a+b)\,\left(\frac{a+b}{a^2}-\frac{3}{a}\right)}{a}\right)}{f}+\frac{\sin(e+f\,x)^5}{5\,a\,f}+\frac{\sin(e+f\,x)^3\,\left(\frac{a+b}{3\,a^2}-\frac{1}{a}\right)}{f}-\frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{a^{7/2}\,f\,\sqrt{a+b}}$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

output
```
(sin(e + f*x)*(3/a + ((a + b)*((a + b)/a^2 - 3/a))/a))/f + sin(e + f*x)^5/
(5*a*f) + (sin(e + f*x)^3*((a + b)/(3*a^2) - 1/a))/f - (b^3*atanh((a^(1/2)
*sin(e + f*x))/(a + b)^(1/2)))/(a^(7/2)*f*(a + b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.84

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{15\sqrt{a}\,\sqrt{a+b}\log\left(\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\sqrt{a+b}-2\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)b^3-15\sqrt{a}\,\sqrt{a+b}\log\left(\sqrt{a+b}\right.}{}$$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

output
```
(15*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
- 2*sqrt(a)*tan((e + f*x)/2))*b**3 - 15*sqrt(a)*sqrt(a + b)*log(sqrt(a + b
)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*b**3 + 6
*sin(e + f*x)**5*a**4 + 6*sin(e + f*x)**5*a**3*b - 20*sin(e + f*x)**3*a**4
 - 10*sin(e + f*x)**3*a**3*b + 10*sin(e + f*x)**3*a**2*b**2 + 30*sin(e + f
*x)*a**4 + 30*sin(e + f*x)*a*b**3)/(30*a**4*f*(a + b))
```

3.186 $\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{a^2\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}f}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

output
```
a^2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+b)^(1/2)/f-(a-b)*tan
(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.91

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-3a^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)\right.}{6b^2\sqrt{a+b}f\,(}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(-3*a^2*ArcTan[(Sec[f*x]*(C
os[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(Sec[e]*(-3*a + 2*b + b*Sec[
e + f*x]^2)*Sin[f*x] + b*Sec[e + f*x]*Tan[e])))/(6*b^2*Sqrt[a + b]*f*(a +
b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^6}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\left(\tan^2(e+fx)+1\right)^2}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{a^2}{b^2\left(b\tan^2(e+fx)+a+b\right)}+\frac{\tan^2(e+fx)}{b}-\frac{a-b}{b^2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\tan(e+fx)}{b^2}+\frac{\tan^3(e+fx)}{3b}}{f}$$

input
```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a^2*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]) -
((a - b)*Tan[e + f*x])/b^2 + Tan[e + f*x]^3/(3*b))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)-b\tan(fx+e)}{b^2}+\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)-b\tan(fx+e)}{b^2}+\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2\sqrt{(a+b)b}}}{f}$
risch	$-\frac{2i\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-6b\,e^{2i(fx+e)}+3a-2b\right)}{3f\,b^2\left(e^{2i(fx+e)}+1\right)^3}-\frac{a^2\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f\,b^2}+$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)-b*tan(f*x+e))+a^2/b^2/((a+b)
*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(67) = 134$.

Time = 0.10 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.60

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[-\frac{3\sqrt{-ab-b^2}a^2\cos(fx+e)^3\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a+2b)\cos(fx+e)^3-b\cos(fx+\right.}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right.}{12(ab^3+b^4)f\cos(fx\right.$$
$$-\frac{3\sqrt{ab+b^2}a^2\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^3-2(ab^2+b^3-(3a^2b+ab^2-2b^3)\cos(}{6(ab^3+b^4)f\cos(fx+e)^3}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/12*(3*sqrt(-a*b - b^2)*a^2*cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*c
os(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x +
e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x +
 e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b^2 + b^3 - (3*a^2*b + a*b^2 -
 2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x + e)^3), -1
/6*(3*sqrt(a*b + b^2)*a^2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(
a*b + b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(a*b^2 + b^3 - (
3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*co
s(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{3\,a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}b^2} + \frac{b\tan(fx+e)^3-3\,(a-b)\tan(fx+e)}{b^2}}{3\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*a^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b^2) +
(b*tan(f*x + e)^3 - 3*(a - b)*tan(f*x + e))/b^2)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2}{\sqrt{ab+b^2}b^2} + \frac{b^2\tan(fx+e)^3-3\,ab\tan(fx+e)+3\,b^2\tan(fx+e)}{b^3}}{3\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*a^2/(sqrt(a*b + b^2)*b^2) + (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*
x + e) + 3*b^2*tan(f*x + e))/b^3)/f
```

Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\tan(e+f\,x)^3}{3\,b\,f}-\frac{\tan(e+f\,x)\,\left(\frac{a+b}{b^2}-\frac{2}{b}\right)}{f}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a+b}}\right)}{b^{5/2}\,f\,\sqrt{a+b}}$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

output
```
tan(e + f*x)^3/(3*b*f) - (tan(e + f*x)*((a + b)/b^2 - 2/b))/f + (a^2*atan(
(b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(5/2)*f*(a + b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.77

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{3\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)^2 a^2-3\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-}{\sqrt{b}}\right.}{}$$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

output
```
(3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*cos(e + f*x)*sin(e + f*x)**2*a**2 - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a
 + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*a**2 + 3*sqrt(b)*s
qrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e +
f*x)*sin(e + f*x)**2*a**2 - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
 + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*a**2 - 3*sin(e + f*x)**3*a**2*
b - sin(e + f*x)**3*a*b**2 + 2*sin(e + f*x)**3*b**3 + 3*sin(e + f*x)*a**2*
b - 3*sin(e + f*x)*b**3)/(3*cos(e + f*x)*b**3*f*(sin(e + f*x)**2*a + sin(e
 + f*x)**2*b - a - b))
```

3.187 $\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=-\frac{a\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}f}+\frac{\tan(e+fx)}{bf}$$

output
```
-a*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(1/2)/f+tan(f*x+e)
/b/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.69

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(a\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos\right.}{2b\sqrt{a+b}f\left(a+b\sec^2(e+fx)\right)\sqrt{b(\cos(e}$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(a*ArcTan[(Sec[f*x]*(Cos[2*
e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b
]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Se
c[e]*Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*Sin[f*x]))/(2*b*Sqrt[a + b
]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^4}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{a\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{b}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{a\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) +
 Tan[e + f*x]/b)/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}-\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b}-\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{f}$
risch	$\frac{2i}{fb\left(e^{2i(fx+e)}+1\right)}-\frac{a\ln\left(e^{2i(fx+e)}-\frac{2iba+2ib^2-a\sqrt{-ab-b^2}-2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,fb}+\frac{a\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}}{a\sqrt{-ab-b}}\right.}{2\sqrt{-ab-b^2}\,fb}$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/b-a/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(44) = 88$.

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 5.50

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[-\frac{\sqrt{-ab-b^2}a\cos(fx+e)\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{}}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right.}{4(ab^2+b^3)f\cos(fx+e)}\right.$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a*b - b^2)*a*cos(f*x + e)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
+ e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 -
b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 +
 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b + b^2)*sin(f*x + e))/((a*b^2 + b^3)
*f*cos(f*x + e)), 1/2*(sqrt(a*b + b^2)*a*arctan(1/2*((a + 2*b)*cos(f*x + e
)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*(a*
b + b^2)*sin(f*x + e))/((a*b^2 + b^3)*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}b} - \frac{\tan(fx+e)}{b}}{f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(a*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b) - tan(f*x +
 e)/b)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a}{\sqrt{ab+b^2}b} - \frac{\tan(fx+e)}{b}}{f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*a/(sqrt(a*b + b^2)*b) - tan(f*x + e)/b)/f
```

Mupad [B] (verification not implemented)

Time = 17.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\tan(e+f\,x)}{b\,f} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)}{b^{3/2}\,f\,\sqrt{a+b}}$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
```

output
```
tan(e + f*x)/(b*f) - (a*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(3/
2)*f*(a + b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.27

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\cos(fx+e)\,a - \sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\cos(fx+}{\cos(fx+e)\,b^2 f\,(a+b)}$$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*cos(e + f*x)*a - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/
2) + sqrt(a))/sqrt(b))*cos(e + f*x)*a + sin(e + f*x)*a*b + sin(e + f*x)*b*
*2)/(cos(e + f*x)*b**2*f*(a + b))
```

3.188 $\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f}$$

output
```
arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(1/2)/(a+b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f}$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4634, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(e+fx)^2}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \; 4634$$

$$\frac{\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}f\sqrt{a+b}}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	si
derivativedivides	$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{f\sqrt{(a+b)b}}$	28
default	$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{f\sqrt{(a+b)b}}$	28
risch	$-\frac{\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f}+\frac{\ln\left(e^{2i(fx+e)}+\frac{-2iba-2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f}$	17

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(28) = 56.

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 5.81

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[-\frac{\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a+2\,b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,(ab+b^2)\,f}\right.$$
$$\left.-\frac{\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\,\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)}{2\,\sqrt{ab+b^2}\,f}\right]$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*
b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*
sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2))/((a*b + b^2)*f), -1/2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))/(sqrt(a*b + b^2)*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*f)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))/(sqrt(a*b + b^2)*f)
```

Mupad [B] (verification not implemented)

Time = 18.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atan}\left(\frac{b\tan(e+f\,x)}{\sqrt{b^2+a\,b}}\right)}{f\,\sqrt{b^2+a\,b}}$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
```

output
```
atan((b*tan(e + f*x))/(a*b + b^2)^(1/2))/(f*(a*b + b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\,\sqrt{a+b}\,\left(atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)+atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\right)}{bf\,(a+b)}$$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

output
```
(sqrt(b)*sqrt(a + b)*(atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
)) + atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))))/(b*f*(a + b)
)
```

3.189 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{x}{a}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}$$

output
```
x/a+b^(1/2)*arctan((a+b)^(1/2)*cot(f*x+e)/b^(1/2))/a/(a+b)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx =\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(\ldots}{2\sqrt{a+\ldots}}\right.\right.}{2a\sqrt{a+b}f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e)\ldots}}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Co
s[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e
])^4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2
)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4615, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4615}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b}dx}{a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{b\int \frac{1}{(a+b)\cot^2(e+fx)+b}d\cot(e+fx)}{af}+\frac{x}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*
f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 4615
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Simp[b/a   Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f},
x] && NeQ[a + b, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
default	$\frac{-\frac{b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)fa}$	114

input
```
int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a*arctan(
tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,af}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a
 + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*
x + e)*sin(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \int \frac{1}{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(1/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)
/a)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 16.50 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{\operatorname{atan}\left(\frac{\frac{\left(2b^3\tan(e+fx) - \frac{\left(2a^2b^2 - \frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}\,1i}{a^2+ba} + \frac{\left(2b^3\tan(e+fx) + \frac{\left(2a^2b^2 + \frac{\tan(e+fx)\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\frac{\left(2b^3\tan(e+fx) - \frac{\left(2a^2b^2 - \frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{a^2+ba} - \frac{\left(2b^3\tan(e+fx) + \frac{\left(2a^2b^2 + \frac{\tan(e+fx)\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}}\right.}{f\left(a^2+ba\right)}$$

input
```
int(1/(a + b/cos(e + f*x)^2),x)
```

output
```
x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^
3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2
*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) +
 ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/
(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1
i)/(a*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^
2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2)
)/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x)
+ ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))
/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))
/(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.89

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right) - \sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right) + afx + bfx}{af\,(a+b)}$$

input
```
int(1/(a+b*sec(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b)) - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/s
qrt(b)) + a*f*x + b*f*x)/(a*f*(a + b))
```

3.190 $\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a-2b)x}{2a^2}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}f}+\frac{\cos(e+fx)\sin(e+fx)}{2af}$$

output
```
1/2*(a-2*b)*x/a^2+b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+b)
^(1/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{2(a-2b)(e+fx)+\frac{4b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+a\sin(2(e+fx))}{4a^2f}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
(2*(a - 2*b)*(e + f*x) + (4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b]])/Sqrt[a + b] + a*Sin[2*(e + f*x)])/(4*a^2*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4634, 316, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^2\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2b^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a-2b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{2a}+\frac{\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2b^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a-2b)\arctan(\tan(e+fx))}{a}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-2b)\arctan(\tan(e+fx))}{a}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
((((a - 2*b)*ArcTan[Tan[e + f*x]])/a + (2*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a) + Tan[e + f*x]/(2*a*(1 + Tan[e
+ f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b^{2} \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^{2}\sqrt{(a+b)b}}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^{2}}+\frac{(a-2b)\arctan(\tan(fx+e))}{2}}{a^{2}}}{f}$
default	$\frac{\frac{b^{2} \arctan\left(\frac{b \tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^{2}\sqrt{(a+b)b}}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^{2}}+\frac{(a-2b)\arctan(\tan(fx+e))}{2}}{a^{2}}}{f}$
risch	$\frac{x}{2a}-\frac{xb}{a^{2}}-\frac{i\mathrm{e}^{2i(fx+e)}}{8af}+\frac{i\mathrm{e}^{-2i(fx+e)}}{8af}+\frac{\sqrt{-(a+b)b}\, b\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)f\,a^{2}}-\frac{\sqrt{-(a+b)b}\, b\ln\left(\mathrm{e}^{2}\right.}{2}$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2/a^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^2*(1
/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a-2*b)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.63

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{2\,(a-2\,b)fx + 2\,a\cos\left(fx+e\right)\sin\left(fx+e\right) + b\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\,\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2-}{a^2\cos(f}\right.}{4\,a^2 f}\right.$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a - 2*b)*f*x + 2*a*cos(f*x + e)*sin(f*x + e) + b*sqrt(-b/(a + b))
*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e
)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*
sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)))/(a^2*f), 1/2*((a - 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e) -
 b*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b
))/(b*cos(f*x + e)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cos^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\,b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2} + \frac{(fx+e)(a-2\,b)}{a^2} + \frac{\tan(fx+e)}{a\tan(fx+e)^2+a}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(2*b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) +
(f*x + e)*(a - 2*b)/a^2 + tan(f*x + e)/(a*tan(f*x + e)^2 + a))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{\sqrt{ab+b^2}a^2} + \frac{(fx+e)(a-2\,b)}{a^2} + \frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^2/(sqrt(a*b + b^2)*a^2) + (f*x + e)*(a - 2*b)/a^2 + tan(f*x
+ e)/((tan(f*x + e)^2 + 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.97

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$
$$-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)-a\left(\frac{b\,\sin(2\,e+2\,f\,x)}{2}-b\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)\right)-a^2\left(\frac{\sin(2\,e+2\,f\,x)}{2}+\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)\right)+a}{}$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

output
```
-(atan((a*sin(e + f*x)*(- a*b^3 - b^4)^(3/2)*4i + b*sin(e + f*x)*(- a*b^3
- b^4)^(3/2)*8i + b^5*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*8i + a*b^4*sin(e
+ f*x)*(- a*b^3 - b^4)^(1/2)*12i + a^4*b*sin(e + f*x)*(- a*b^3 - b^4)^(1/2
)*1i + a^2*b^3*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*1i - a^3*b^2*sin(e + f*x
)*(- a*b^3 - b^4)^(1/2)*2i)/(3*a^2*b^5*cos(e + f*x) + 5*a^3*b^4*cos(e + f*
x) + a^4*b^3*cos(e + f*x) - a^5*b^2*cos(e + f*x)))*(- a*b^3 - b^4)^(1/2)*2
i + 2*b^2*atan(sin(e + f*x)/cos(e + f*x)) - a*((b*sin(2*e + 2*f*x))/2 - b*
atan(sin(e + f*x)/cos(e + f*x))) - a^2*(sin(2*e + 2*f*x)/2 + atan(sin(e +
f*x)/cos(e + f*x))))/(f*(2*a^2*b + 2*a^3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.95

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\frac{2\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)b+2\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)b+\cos\left(fx+e\right)\sin\left(f\right.}{2a^2f\left(a+b\right)}$$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

output
```
(2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*b + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a)
)/sqrt(b))*b + cos(e + f*x)*sin(e + f*x)*a**2 + cos(e + f*x)*sin(e + f*x)*
a*b + a**2*e + a**2*f*x - a*b*e - a*b*f*x - 2*b**2*e - 2*b**2*f*x)/(2*a**2
*f*(a + b))
```

3.191 $\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(3a^2-4ab+8b^2)\,x}{8a^3} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}f} + \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}$$

output
```
1/8*(3*a^2-4*a*b+8*b^2)*x/a^3-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2
))/a^3/(a+b)^(1/2)/f+1/8*(3*a-4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1/4*cos(f*x
+e)^3*sin(f*x+e)/a/f
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{4(3a^2-4ab+8b^2)(e+fx) - \frac{32b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 8a(a-b)\sin(2(e+fx)) + a^2\sin(4(e+fx))}{32a^3f}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(4*(3*a^2 - 4*a*b + 8*b^2)*(e + f*x) - (32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sin[2*(e + f*x)] + a^2*Sin[
4*(e + f*x)])/(32*a^3*f)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 4634, 316, 25, 402, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}-\frac{\int -\frac{3b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{3a^2-ba+4b^2+(3a-4b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\frac{\int \frac{3a^2-ba+4b^2+(3a-4b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}+\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \textbf{397}$$

$$\frac{\frac{\frac{\frac{\left(3a^2-4ab+8b^2\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{8b^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}+\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{\frac{\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{8b^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}+\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{\frac{\frac{\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a}+\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2) + ((((3*a^2 - 4*a*b + 8*b^2)*Ar
cTan[Tan[e + f*x]])/a - (8*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b]])/(a*Sqrt[a + b]))/(2*a) + ((3*a - 4*b)*Tan[e + f*x])/(2*a*(1 + Tan[e +
 f*x]^2)))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3+\left(-\frac{1}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3+\left(-\frac{1}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}}{f}$
risch	$\frac{3x}{8a}-\frac{xb}{2a^2}+\frac{x\,b^2}{a^3}-\frac{i\mathrm{e}^{2i(fx+e)}}{8af}+\frac{i\mathrm{e}^{2i(fx+e)}b}{8a^2f}+\frac{i\mathrm{e}^{-2i(fx+e)}}{8af}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{8a^2f}+\frac{\sqrt{-(a+b)b}\,b^2\ln\left(\mathrm{e}^{2i(fx+e)}-\right.}{2(a+b)f}$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*(((3/8*a^2-1/2*a*b)*tan(f*x+e)^3+(-1/2*a*b+5/8*a^2)*tan(f*x+e))
/(1+tan(f*x+e)^2)^2+1/8*(3*a^2-4*a*b+8*b^2)*arctan(tan(f*x+e)))-b^3/a^3/((
a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.93

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{2\,b^2\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+}}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{8\,a}\right.$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/8*(2*b^2*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(
f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (3*a^2 - 4*a*b + 8*b^2)*f*x +
(2*a^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f
), 1/8*(4*b^2*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (3*a^2 - 4*a*b + 8*b^2)*f*x
 + (2*a^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^
3*f)]
```

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=\int \frac{\cos^4{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{8\,b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3}-\frac{(3\,a-4\,b)\tan(fx+e)^3+(5\,a-4\,b)\tan(fx+e)}{a^2\tan(fx+e)^4+2\,a^2\tan(fx+e)^2+a^2}-\frac{(3\,a^2-4\,ab+8\,b^2)(fx+e)}{a^3}}{8\,f}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/8*(8*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3) -
 ((3*a - 4*b)*tan(f*x + e)^3 + (5*a - 4*b)*tan(f*x + e))/(a^2*tan(f*x + e)
^4 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)/a^3)/
f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{8\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^3}{\sqrt{ab+b^2}a^3}-\frac{(3a^2-4ab+8b^2)(fx+e)}{a^3}-\frac{3a\tan(fx+e)^3-4b\tan(fx+e)^3+5a\tan(fx+e)-4b\tan}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8f}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/8*(8*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^3/(sqrt(a*b + b^2)*a^3) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)
/a^3 - (3*a*tan(f*x + e)^3 - 4*b*tan(f*x + e)^3 + 5*a*tan(f*x + e) - 4*b*t
an(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 16.45 (sec) , antiderivative size = 1114, normalized size of antiderivative = 9.52

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

output
```
((tan(e + f*x)*(5*a - 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a - 4*b))/(8*a^2)
)/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((((-b^5*(a + b))^(1/
2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^
3))/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^
2)/2)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a + b))^(
1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^3*b + a^4) + ((-
b^5*(a + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^
3*b^4 + 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^
3)/2 + (3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*
(-b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^
3*b + a^4))/((((5*a*b^7)/4 - b^8 - (3*a^2*b^6)/4 + (9*a^3*b^5)/32)/a^6 + ((
-b^5*(a + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a
^3*b^4 + 9*a^4*b^3))/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b
^3)/2 + (3*a^8*b^2)/2)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)
*(-b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4))))/(a^3*
b + a^4) - ((-b^5*(a + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a
^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6
*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 +
 256*a^7*b^2)*(-b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b +
a^4))))/(a^3*b + a^4)))*(-b^5*(a + b))^(1/2)*1i)/(f*(a^3*b + a^4)) - (a...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.97

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{-8\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)b^2-8\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)b^2-2\cos\left(fx+e\right)}{}$$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*b**2 - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + s
qrt(a))/sqrt(b))*b**2 - 2*cos(e + f*x)*sin(e + f*x)**3*a**3 - 2*cos(e + f*
x)*sin(e + f*x)**3*a**2*b + 5*cos(e + f*x)*sin(e + f*x)*a**3 + cos(e + f*x
)*sin(e + f*x)*a**2*b - 4*cos(e + f*x)*sin(e + f*x)*a*b**2 + 3*a**3*e + 3*
a**3*f*x - a**2*b*e - a**2*b*f*x + 4*a*b**2*e + 4*a*b**2*f*x + 8*b**3*e +
8*b**3*f*x)/(8*a**3*f*(a + b))
```

3.192 $\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(5a^3-6a^2b+8ab^2-16b^3)\,x}{16a^4} + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4\sqrt{a+b}f} + \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af}$$

output
```
1/16*(5*a^3-6*a^2*b+8*a*b^2-16*b^3)*x/a^4+b^(7/2)*arctan(b^(1/2)*tan(f*x+e
)/(a+b)^(1/2))/a^4/(a+b)^(1/2)/f+1/16*(5*a^2-6*a*b+8*b^2)*cos(f*x+e)*sin(f
*x+e)/a^3/f+1/24*(5*a-6*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*cos(f*x+e)^5*
sin(f*x+e)/a/f
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{12(5a^3-6a^2b+8ab^2-16b^3)\,(e+fx)+\frac{192b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+3a(15a^2-16ab+16b^2)\sin(2(e+fx}{192a^4f}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(12*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*(e + f*x) + (192*b^(7/2)*ArcTan[(
Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/Sqrt[a + b] + 3*a*(15*a^2 - 16*a*b + 1
6*b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a - 2*b)*Sin[4*(e + f*x)] + a^3*Sin[6*(
e + f*x)])/(192*a^4*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4634, 316, 25, 402, 27, 402, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)}dx$$
$$\Big\downarrow \text{4634}$$
$$\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)^4\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\int -\frac{5b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{6a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{5b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}-\frac{\int -\frac{3\left(5a^2-ba+2b^2+(5a-6b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{5a^2-ba+2b^2+(5a-6b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{3\left(\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{5a^3-ba^2+2b^2a-8b^3+b\left(5a^2-6ba+8b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3\left(\frac{\int \frac{5a^3-ba^2+2b^2a-8b^3+b\left(5a^2-6ba+8b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}+\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{3\left(\frac{\frac{\left(5a^3-6a^2b+8ab^2-16b^3\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{2a}+\frac{16b^4\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{3\left(\frac{\frac{16b^4\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{\left(5a^3-6a^2b+8ab^2-16b^3\right)\arctan(\tan(e+fx))}{a}}{2a}+\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan}{6a(\tan^2(}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3\left(\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}+\frac{\frac{\left(5a^3-6a^2b+8ab^2-16b^3\right)\arctan(\tan(e+fx))}{a}+\frac{16b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx}{6a(\tan^2(e+fx)}}{f}$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3) + (((5*a - 6*b)*Tan[e + f*x])/(
4*a*(1 + Tan[e + f*x]^2)^2) + (3*((((5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*A
rcTan[Tan[e + f*x]])/a + (16*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b]])/(a*Sqrt[a + b]))/(2*a) + ((5*a^2 - 6*a*b + 8*b^2)*Tan[e + f*x])/(2*
a*(1 + Tan[e + f*x]^2))))/(4*a))/(6*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b^4 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^4\sqrt{(a+b)b}}+\frac{\frac{\left(\frac{5}{16}a^3-\frac{3}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^5+\left(ab^2+\frac{5}{6}a^3-a^2b\right)\tan(fx+e)^3+\left(-\frac{5}{8}a^2b+\frac{1}{2}ab^2+\frac{11}{16}a^3\right)\tan(fx+}{\left(1+\tan(fx+e)^2\right)^3}}{a^4}}{f}$
default	$\frac{\frac{b^4 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^4\sqrt{(a+b)b}}+\frac{\frac{\left(\frac{5}{16}a^3-\frac{3}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^5+\left(ab^2+\frac{5}{6}a^3-a^2b\right)\tan(fx+e)^3+\left(-\frac{5}{8}a^2b+\frac{1}{2}ab^2+\frac{11}{16}a^3\right)\tan(fx+}{\left(1+\tan(fx+e)^2\right)^3}}{a^4}}{f}$
risch	$\frac{5x}{16a}-\frac{3xb}{8a^2}+\frac{x\,b^2}{2a^3}-\frac{x\,b^3}{a^4}-\frac{15i\mathrm{e}^{2i(fx+e)}}{128af}+\frac{i\mathrm{e}^{2i(fx+e)}b}{8a^2f}-\frac{i\mathrm{e}^{2i(fx+e)}b^2}{8a^3f}+\frac{15i\mathrm{e}^{-2i(fx+e)}}{128af}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{8a^2f}$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^4/a^4/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^4*((
(5/16*a^3-3/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^5+(a*b^2+5/6*a^3-a^2*b)*tan(f*x+
e)^3+(-5/8*a^2*b+1/2*a*b^2+11/16*a^3)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*
(5*a^3-6*a^2*b+8*a*b^2-16*b^3)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.60

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{12\,b^3\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\,\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\ldots}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.\ldots}{\ldots}\right.$$

$$-\frac{24\,b^3\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)-3\,(5\,a^3-6\,a^2b+8\,ab^2-16\,b^3)fx-\left(8\,a^3\cos\left(fx+e\right)\ldots\right.}{48\,a^4f}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/48*(12*b^3*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2
 - 16*b^3)*f*x + (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^
3 + 3*(5*a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), -1/
48*(24*b^3*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(
b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) - 3*(5*a^3 - 6*a^2*b + 8*a*b^2 -
 16*b^3)*f*x - (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3
+ 3*(5*a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.16

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{48\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^4} + \frac{3\,(5\,a^2-6\,ab+8\,b^2)\tan(fx+e)^5+8\,(5\,a^2-6\,ab+6\,b^2)\tan(fx+e)^3+3\,(11\,a^2-10\,ab+8\,b^2)\tan(fx+e)}{a^3\tan(fx+e)^6+3\,a^3\tan(fx+e)^4+3\,a^3\tan(fx+e)^2+a^3} + \frac{3\,(5\,a^3-}{}}{48\,f}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/48*(48*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4)
+ (3*(5*a^2 - 6*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 - 6*a*b + 6*b^2)*ta
n(f*x + e)^3 + 3*(11*a^2 - 10*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f*x + e)
^6 + 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) + 3*(5*a^3 - 6*a^2
*b + 8*a*b^2 - 16*b^3)*(f*x + e)/a^4)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{48\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{\sqrt{ab+b^2}a^4} + \frac{3\,(5\,a^3-6\,a^2b+8\,ab^2-16\,b^3)(fx+e)}{a^4} + \frac{15\,a^2\tan(fx+e)^5-18\,ab\tan(fx+e)^5+24\,b^2\tan}{}}{48\,f}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/48*(48*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*b^4/(sqrt(a*b + b^2)*a^4) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 1
6*b^3)*(f*x + e)/a^4 + (15*a^2*tan(f*x + e)^5 - 18*a*b*tan(f*x + e)^5 + 24
*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 48*a*b*tan(f*x + e)^3 + 48*b
^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 30*a*b*tan(f*x + e) + 24*b^2*tan
(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 1979, normalized size of antiderivative = 12.14

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

output
```
((tan(e + f*x)*(11*a^2 - 10*a*b + 8*b^2))/(16*a^3) + (tan(e + f*x)^3*(5*a^
2 - 6*a*b + 6*b^2))/(6*a^3) + (tan(e + f*x)^5*(5*a^2 - 6*a*b + 8*b^2))/(16
*a^3))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (a
tan((((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 - (
tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i -
 b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4)
 - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^
4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i
 - b^3*16i)*1i)/(32*a^4) - (((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (
5*a^11*b^2)/4)/a^9 + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i
 - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5
i - b^3*16i))/(32*a^4) + (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7
- 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*
8i - a^2*b*6i + a^3*5i - b^3*16i)*1i)/(32*a^4))/((((((2*a^8*b^5 - (a^9*b^4
)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 - (tan(e + f*x)*(2048*a^8*b^3 + 1
024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2
*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - (tan(e + f*x)*(512*b^9 - 25
6*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^
3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - ((5*a*
b^10)/4 - b^11 - (11*a^2*b^9)/8 + (29*a^3*b^8)/32 - (15*a^4*b^7)/32 + (...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.04

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{48\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)b^3+48\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)b^3+8\cos\left(fx+e\right)}{}$$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

output
```
(48*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*b**3 + 48*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sq
rt(a))/sqrt(b))*b**3 + 8*cos(e + f*x)*sin(e + f*x)**5*a**4 + 8*cos(e + f*x
)*sin(e + f*x)**5*a**3*b - 26*cos(e + f*x)*sin(e + f*x)**3*a**4 - 14*cos(e
 + f*x)*sin(e + f*x)**3*a**3*b + 12*cos(e + f*x)*sin(e + f*x)**3*a**2*b**2
 + 33*cos(e + f*x)*sin(e + f*x)*a**4 + 3*cos(e + f*x)*sin(e + f*x)*a**3*b
- 6*cos(e + f*x)*sin(e + f*x)*a**2*b**2 + 24*cos(e + f*x)*sin(e + f*x)*a*b
**3 + 15*a**4*e + 15*a**4*f*x - 3*a**3*b*e - 3*a**3*b*f*x + 6*a**2*b**2*e
+ 6*a**2*b**2*f*x - 24*a*b**3*e - 24*a*b**3*f*x - 48*b**4*e - 48*b**4*f*x)
/(48*a**4*f*(a + b))
```

3.193 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\operatorname{arctanh}(\sin(e+fx))}{b^2f} - \frac{\sqrt{a}(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}f} - \frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
arctanh(sin(f*x+e))/b^2/f-1/2*a^(1/2)*(2*a+3*b)*arctanh(a^(1/2)*sin(f*x+e)
/(a+b)^(1/2))/b^2/(a+b)^(3/2)/f-1/2*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+
e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 980, normalized size of antiderivative = 9.61

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*I)*a*(2*a + 3*b)*ArcTa
n[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]
*Sqrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Co
s[e] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt
[(Cos[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a
*Cos[e + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e
] + b*Sin[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[
2*(e + f*x)])*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - a*(2*a + 3*b)*(a + 2*b +
a*Cos[2*(e + f*x)])*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I
)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin
[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + a*(2*a + 3*b)*(a + 2*b + a
*Cos[2*(e + f*x)])*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I
)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin
[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - 8*Sqrt[a]*(a + b)^(3/2)*(a
 + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sec[
e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] + 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b +
a*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sec[e + f*...
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4635, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^5}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(-a\sin^2(e+fx)+a+b\right)^2}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 316

$$\frac{-\frac{\int -\frac{a\sin^2(e+fx)+a+2b}{\left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{a\sin^2(e+fx)+a+2b}{\left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{2(a+b)\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)}{b}-\frac{a(2a+3b)\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{2(a+b)\text{arctanh}(\sin(e+fx))}{b}-\frac{a(2a+3b)\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{2(a+b)\text{arctanh}(\sin(e+fx))}{b}-\frac{\sqrt{a}(2a+3b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

input
```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((2*(a + b)*ArcTanh[Sin[e + f*x]])/b - (Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b]])/(b*Sqrt[a + b]))/(2*b*(a + b)) - (a*Sin[e
+ f*x])/(2*b*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{a\left(\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+e)^2\right)}-\frac{(2a+3b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}-\frac{\ln(\sin(fx+e)-1)}{2b^2}+\frac{\ln(\sin(fx+e)+1)}{2b^2}}{f}$
default	$\frac{\frac{a\left(\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+e)^2\right)}-\frac{(2a+3b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}-\frac{\ln(\sin(fx+e)-1)}{2b^2}+\frac{\ln(\sin(fx+e)+1)}{2b^2}}{f}$
risch	$\frac{ia\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{b(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{i(fx+e)}+i\right)}{b^2f}-\frac{\ln\left(e^{i(fx+e)}-i\right)}{b^2f}+\frac{\sqrt{a(a+b)}\,\ln\left(e^{2i(fx}\right.}{2}$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/b^2*a*(1/2/(a+b)*b*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(2*a+3*b)/(
a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))-1/2/b^2*ln(sin
(f*x+e)-1)+1/2/b^2*ln(sin(f*x+e)+1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.84

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{2\,ab\sin(fx+e) - \left(\left(2\,a^2+3\,ab\right)\cos(fx+e)^2 + 2\,ab + 3\,b^2\right)\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos(fx+e)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx}{a\cos(fx+e)^2+b}\right.}{4\left((a\right.}\right.$$

$$-\frac{ab\sin(fx+e) - \left(\left(2\,a^2+3\,ab\right)\cos(fx+e)^2 + 2\,ab + 3\,b^2\right)\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{-\frac{a}{a+b}}\sin(fx+e)\right) - (}{2\left(\left(a^2b^2+ab^3\right)\right.}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b + 3*b^
2)*sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*sin(
f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - 2*((a^2 + a*b)*cos(f*x + e)^
2 + a*b + b^2)*log(sin(f*x + e) + 1) + 2*((a^2 + a*b)*cos(f*x + e)^2 + a*b
 + b^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b
^3 + b^4)*f), -1/2*(a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2
*a*b + 3*b^2)*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e)) - ((a
^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*log(sin(f*x + e) + 1) + ((a^2 + a*b)
*cos(f*x + e)^2 + a*b + b^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*
cos(f*x + e)^2 + (a*b^3 + b^4)*f)]
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(2\,a+3\,b)a\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(ab^2+b^3)\sqrt{(a+b)a}} - \frac{2\,a\sin(fx+e)}{a^2b+2\,ab^2+b^3-(a^2b+ab^2)\sin(fx+e)^2} + \frac{2\,\log(\sin(fx+e)+1)}{b^2} - \frac{2\,\log(\sin(fx+e)-1)}{b^2}}{4\,f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*((2*a + 3*b)*a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e)
+ sqrt((a + b)*a)))/((a*b^2 + b^3)*sqrt((a + b)*a)) - 2*a*sin(f*x + e)/(a^
2*b + 2*a*b^2 + b^3 - (a^2*b + a*b^2)*sin(f*x + e)^2) + 2*log(sin(f*x + e)
 + 1)/b^2 - 2*log(sin(f*x + e) - 1)/b^2)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{(2\,a^2+3\,ab)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(ab^2+b^3)\sqrt{-a^2-ab}} + \frac{a\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(ab+b^2)} + \frac{\log(|\sin(fx+e)+1|)}{b^2} - \frac{\log(|\sin(fx+e)-1|)}{b^2}}{2\,f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((2*a^2 + 3*a*b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a*b^2 + b^3
)*sqrt(-a^2 - a*b)) + a*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a*b + b^
2)) + log(abs(sin(f*x + e) + 1))/b^2 - log(abs(sin(f*x + e) - 1))/b^2)/f
```

Mupad [B] (verification not implemented)

Time = 16.89 (sec) , antiderivative size = 2039, normalized size of antiderivative = 19.99

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(atan((((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)
) - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*
b^2*(2*a*b^3 + b^4 + a^2*b^2)))*1i)/(2*b^2) + (sin(e + f*x)*(20*a^4*b + 8*
a^5 + 13*a^3*b^2)*1i)/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((((4*a^2*b^6 +
 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)) + (sin(e + f*x)*(16*
a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a
^2*b^2)))*1i)/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2)*1i)/
(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2)/((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)
/(2*(2*a*b^4 + b^5 + a^2*b^3)) - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 +
80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a^2*b^2)))/(2*b^2) + (si
n(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/(4*(2*a*b^3 + b^4 + a^2*b^2)))
/b^2 + (((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3))
 + (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b
^2*(2*a*b^3 + b^4 + a^2*b^2)))/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 +
 13*a^3*b^2))/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((3*a^3*b)/2 + a^4)/(2*
a*b^4 + b^5 + a^2*b^3)))*1i)/(b^2*f) - (atan(((((sin(e + f*x)*(20*a^4*b +
8*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) + ((a*(a + b)^3)^(1/2)*
((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) - (sin(e +
f*x)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5
 + 32*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 879, normalized size of antiderivative = 8.62

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(2*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) -
 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2 + 3*sqrt(a)*sqrt(a + b)*
log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x
)/2))*sin(e + f*x)**2*a*b - 2*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e +
 f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**2 - 5*sqrt(a)*s
qrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*t
an((e + f*x)/2))*a*b - 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)
/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*b**2 - 2*sqrt(a)*sqrt(a
 + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e
 + f*x)/2))*sin(e + f*x)**2*a**2 - 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*t
an((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x
)**2*a*b + 2*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqr
t(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a**2 + 5*sqrt(a)*sqrt(a + b)*log(sq
rt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*
a*b + 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a +
 b) + 2*sqrt(a)*tan((e + f*x)/2))*b**2 - 4*log(tan((e + f*x)/2) - 1)*sin(e
 + f*x)**2*a**3 - 8*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a**2*b - 4*l
og(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*b**2 + 4*log(tan((e + f*x)/2) -
 1)*a**3 + 12*log(tan((e + f*x)/2) - 1)*a**2*b + 12*log(tan((e + f*x)/2) -
 1)*a*b**2 + 4*log(tan((e + f*x)/2) - 1)*b**3 + 4*log(tan((e + f*x)/2) ...
```

3.194 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}f}+\frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/2*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(1/2)/(a+b)^(3/2)/f+1/2*sin(
f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{3/2}}+\frac{2\sin(e+fx)}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2f}$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*(a + b)^(3/2)) + (2*
Sin[e + f*x])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*f)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4635, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^3}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{1}{(-a\sin^2(e+fx)+a+b)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{2(a+b)}+\frac{\sin(e+fx)}{2(a+b)(-a\sin^2(e+fx)+a+b)}}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}}+\frac{\sin(e+fx)}{2(a+b)(-a\sin^2(e+fx)+a+b)}}{f}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)) + S
in[e + f*x]/(2*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
 + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+e)^2\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+e)^2\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{f}$
risch	$-\frac{i\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{4\sqrt{a^2+ab}\,(a+b)f}-\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(\right.}{4\sqrt{a^2+a}}$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*sin(f*x+e)/(a+b)/(-a-b+a*sin(f*x+e)^2)+1/2/(a+b)/(a*(a+b))^(1/2)
*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.54

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \left[\frac{\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(a^2+ab\right)\sin\left(fx+e\right)}{4\left((a^4+2a^3b+a^2b^2)f\cos\left(fx+e\right)^2+(a^3b+2a^2b^2+ab^3)f\right)},\ -\frac{\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(a^2+ab\right)\sin\left(fx+e\right)}{2\left((a^4+2a^3b+a^2b^2)f\cos\left(fx+e\right)^2+(a^3b+2a^2b^2+ab^3)f\right)}\right]$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*((a*cos(f*x + e)^2 + b)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sq
rt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a
*b)*sin(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2
*a^2*b^2 + a*b^3)*f), -1/2*((a*cos(f*x + e)^2 + b)*sqrt(-a^2 - a*b)*arctan
(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (a^2 + a*b)*sin(f*x + e))/((a^4
+ 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f)]
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{2\sin(fx+e)}{(a^2+ab)\sin(fx+e)^2-a^2-2\,ab-b^2} + \frac{\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a+b)}}{4\,f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*sin(f*x + e)/((a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a*b - b^2) + lo
g((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(
sqrt((a + b)*a)*(a + b)))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}(a+b)} + \frac{\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(a+b)}}{2\,f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*(a + b)) +
 sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a + b)))/f
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\sin(e+fx)}{2f\,(a+b)\,\left(-a\sin(e+fx)^2+a+b\right)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,f\,(a+b)^{3/2}}$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
sin(e + f*x)/(2*f*(a + b)*(a + b - a*sin(e + f*x)^2)) + atanh((a^(1/2)*sin
(e + f*x))/(a + b)^(1/2))/(2*a^(1/2)*f*(a + b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 371, normalized size of antiderivative = 5.01

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{-\sqrt{a}\,\sqrt{a+b}\log\left(\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\sqrt{a+b}-2\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^2 a+\sqrt{a}\,\sqrt{a+b}\log}{}$$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
- 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a + sqrt(a)*sqrt(a + b)*log(
sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2)
)*a + sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b
) - 2*sqrt(a)*tan((e + f*x)/2))*b + sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*ta
n((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)
**2*a - sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a +
 b) + 2*sqrt(a)*tan((e + f*x)/2))*a - sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*
tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*b - 2*sin(
e + f*x)*a**2 - 2*sin(e + f*x)*a*b)/(4*a*f*(sin(e + f*x)**2*a**3 + 2*sin(e
 + f*x)**2*a**2*b + sin(e + f*x)**2*a*b**2 - a**3 - 3*a**2*b - 3*a*b**2 -
b**3))
```

3.195 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}f} - \frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/2*(2*a+b)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)/f-
1/2*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{2\sqrt{a}b\sin(e+fx)}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2a^{3/2}f}$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (
2*Sqrt[a]*b*Sin[e + f*x])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*a^(
3/2)*f)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \; 4635$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{(-a\sin^2(e+fx)+a+b)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{(2a+b)\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{2a(a+b)}-\frac{b\sin(e+fx)}{2a(a+b)(-a\sin^2(e+fx)+a+b)}}{f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sin(e+fx)}{2a(a+b)(-a\sin^2(e+fx)+a+b)}}{f}$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b
)^(3/2)) - (b*Sin[e + f*x])/(2*a*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{b\sin(fx+e)}{2a(a+b)\left(-a-b+a\sin(fx+e)^2\right)}+\frac{(2a+b)\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{f}$
default	$\frac{\frac{b\sin(fx+e)}{2a(a+b)\left(-a-b+a\sin(fx+e)^2\right)}+\frac{(2a+b)\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{f}$
risch	$\frac{ib\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{af(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,(a+b)f}+$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b/a/(a+b)*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)+1/2*(2*a+b)/a/(a+b)/(a
*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.63

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[\frac{\left((2\,a^2+ab)\cos\left(fx+e\right)^2+2\,ab+b^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\,\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)-2\,(a^2b+ab}{4\left((a^5+2\,a^4b+a^3b^2)f\cos\left(fx+e\right)^2+(a^4b+2\,a^3b^2+a^2b^3)f\right)} \right.$$
$$-\frac{\left((2\,a^2+ab)\cos\left(fx+e\right)^2+2\,ab+b^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)+(a^2b+ab^2)\sin\left(fx+e\right.}{2\left((a^5+2\,a^4b+a^3b^2)f\cos\left(fx+e\right)^2+(a^4b+2\,a^3b^2+a^2b^3)f\right)}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((2*a^2 + a*b)*cos(f*x + e)^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)*log(-(a
*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e
)^2 + b)) - 2*(a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*c
os(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/2*(((2*a^2 + a*b)*cos
(f*x + e)^2 + 2*a*b + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*
x + e)/(a + b)) + (a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)
*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{2\,b\sin(fx+e)}{a^3+2\,a^2b+ab^2-(a^3+a^2b)\sin(fx+e)^2} + \frac{(2\,a+b)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+ab)}}{4\,f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*sin(f*x + e)/(a^3 + 2*a^2*b + a*b^2 - (a^3 + a^2*b)*sin(f*x + e)
^2) + (2*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + s
qrt((a + b)*a)))/(sqrt((a + b)*a)*(a^2 + a*b)))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{(2\,a+b)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^2+ab)\sqrt{-a^2-ab}} - \frac{b\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(a^2+ab)}}{2\,f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((2*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + a*b)*sqrt(
-a^2 - a*b)) - b*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a^2 + a*b)))/f
```

Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(2\,a+b)}{2\,a^{3/2}\,f\,(a+b)^{3/2}} - \frac{b\,\sin\left(e+f\,x\right)}{2\,a\,f\,\left(a+b\right)\,\left(-a\sin\left(e+f\,x\right)^2+a+b\right)}$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(2*a + b))/(2*a^(3/2)*f*(a +
b)^(3/2)) - (b*sin(e + f*x))/(2*a*f*(a + b)*(a + b - a*sin(e + f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 583, normalized size of antiderivative = 7.02

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 2*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b
) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2 - sqrt(a)*sqrt(a + b)
*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*
x)/2))*sin(e + f*x)**2*a*b + 2*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e
+ f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**2 + 3*sqrt(a)*
sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*
tan((e + f*x)/2))*a*b + sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/
2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*b**2 + 2*sqrt(a)*sqrt(a
+ b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e
+ f*x)/2))*sin(e + f*x)**2*a**2 + sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan(
(e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**
2*a*b - 2*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a
 + b) + 2*sqrt(a)*tan((e + f*x)/2))*a**2 - 3*sqrt(a)*sqrt(a + b)*log(sqrt(
a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a*b
 - sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) +
 2*sqrt(a)*tan((e + f*x)/2))*b**2 + 2*sin(e + f*x)*a**2*b + 2*sin(e + f*x)
*a*b**2)/(4*a**2*f*(sin(e + f*x)**2*a**3 + 2*sin(e + f*x)**2*a**2*b + sin(
e + f*x)**2*a*b**2 - a**3 - 3*a**2*b - 3*a*b**2 - b**3))
```

3.196 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{b(4a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}f} + \frac{\sin(e+fx)}{a^2f} + \frac{b^2\sin(e+fx)}{2a^2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
-1/2*b*(4*a+3*b)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(5/2)/(a+b)^(3/
2)/f+sin(f*x+e)/a^2/f+1/2*b^2*sin(f*x+e)/a^2/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{-\frac{b(4a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \sqrt{a}\sin(e+fx)\left(2+\frac{b^2}{(a+b)\left(a+b-a\sin^2(e+fx)\right)}\right)}{2a^{5/2}f}$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-((b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a + b)^(3/
2)) + Sqrt[a]*Sin[e + f*x]*(2 + b^2/((a + b)*(a + b - a*Sin[e + f*x]^2))))
/(2*a^(5/2)*f)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(-a\sin^2(e+fx)+a+b\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{a^2}-\frac{b(2a+b)-2ab\sin^2(e+fx)}{a^2\left(-a\sin^2(e+fx)+a+b\right)^2}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b(4a+3b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}}+\frac{b^2\sin(e+fx)}{2a^2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{a^2}}{f}$$

input
```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/2*(b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)
*(a + b)^(3/2)) + Sin[e + f*x]/a^2 + (b^2*Sin[e + f*x])/(2*a^2*(a + b)*(a
+ b - a*Sin[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{a^2}+\frac{b\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+e)^2\right)}-\frac{(4a+3b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}}{f}$
default	$\frac{\frac{\sin(fx+e)}{a^2}+\frac{b\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+e)^2\right)}-\frac{(4a+3b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}}{f}$
risch	$-\frac{i e^{i(fx+e)}}{2a^2f}+\frac{i e^{-i(fx+e)}}{2a^2f}-\frac{i b^2\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(a+b)e^{i(f}}{\sqrt{a^2+ab}}\right.}{\sqrt{a^2+ab}\,(a+b)f}$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)/a^2+b/a^2*(-1/2/(a+b)*b*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1
/2*(4*a+3*b)/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.87

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \left[\frac{\left(4\,ab^2+3\,b^3+\left(4\,a^2b+3\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2+2\,\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(2\,a\cdots}{4\,\left(\left(a^6+2\,a^5b+a^4b^2\right)f\cos\left(fx+e\right)^2+\left(a^5b+2\cdots\right.\right.}\right.$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*((4*a*b^2 + 3*b^3 + (4*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt(a^2 + a*
b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*c
os(f*x + e)^2 + b)) + 2*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b
+ a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(
f*x + e)^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f), 1/2*((4*a*b^2 + 3*b^3 + (4*
a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*
sin(f*x + e)/(a + b)) + (2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b
+ a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(
f*x + e)^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f)]
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\cos\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{2\,b^2\sin(fx+e)}{a^4+2\,a^3b+a^2b^2-(a^4+a^3b)\sin(fx+e)^2}+\frac{(4\,ab+3\,b^2)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^3+a^2b)\sqrt{(a+b)a}}+\frac{4\sin(fx+e)}{a^2}}{4\,f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*b^2*sin(f*x + e)/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x +
 e)^2) + (4*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x
 + e) + sqrt((a + b)*a)))/((a^3 + a^2*b)*sqrt((a + b)*a)) + 4*sin(f*x + e)
/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{b^2\sin(fx+e)}{(a^3+a^2b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(4\,ab+3\,b^2)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^3+a^2b)\sqrt{-a^2-ab}}-\frac{2\sin(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*sin(f*x + e)/((a^3 + a^2*b)*(a*sin(f*x + e)^2 - a - b)) - (4*a*b
 + 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + a^2*b)*sqrt(-a^2
 - a*b)) - 2*sin(f*x + e)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sin(e+f\,x)}{a^2\,f} + \frac{b^2\,\sin(e+f\,x)}{2\,f\,(a+b)\,\left(-a^3\sin(e+f\,x)^2+a^3+b\,a^2\right)} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(4\,a+3\,b)}{2\,a^{5/2}\,f\,(a+b)^{3/2}}$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
sin(e + f*x)/(a^2*f) + (b^2*sin(e + f*x))/(2*f*(a + b)*(a^2*b + a^3 - a^3*
sin(e + f*x)^2)) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + 3
*b))/(2*a^(5/2)*f*(a + b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 665, normalized size of antiderivative = 6.58

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(4*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) -
 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b + 3*sqrt(a)*sqrt(a + b
)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f
*x)/2))*sin(e + f*x)**2*a*b**2 - 4*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan
((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**2*b - 7*sq
rt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sq
rt(a)*tan((e + f*x)/2))*a*b**2 - 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan
((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*b**3 - 4*sqrt
(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt
(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b - 3*sqrt(a)*sqrt(a + b)*log(s
qrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))
*sin(e + f*x)**2*a*b**2 + 4*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f
*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a**2*b + 7*sqrt(a)*s
qrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*t
an((e + f*x)/2))*a*b**2 + 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f
*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*b**3 + 4*sin(e + f*x
)**3*a**4 + 8*sin(e + f*x)**3*a**3*b + 4*sin(e + f*x)**3*a**2*b**2 - 4*sin
(e + f*x)*a**4 - 12*sin(e + f*x)*a**3*b - 14*sin(e + f*x)*a**2*b**2 - 6*si
n(e + f*x)*a*b**3)/(4*a**3*f*(sin(e + f*x)**2*a**3 + 2*sin(e + f*x)**2*a**
2*b + sin(e + f*x)**2*a*b**2 - a**3 - 3*a**2*b - 3*a*b**2 - b**3))
```

3.197 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{b^2(6a+5b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}f}+\frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}-\frac{b^3\sin(e+fx)}{2a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/2*b^2*(6*a+5*b)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(7/2)/(a+b)^(3
/2)/f+(a-2*b)*sin(f*x+e)/a^3/f-1/3*sin(f*x+e)^3/a^2/f-1/2*b^3*sin(f*x+e)/a
^3/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{-\frac{3b^2(6a+5b)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{3/2}}+3\sqrt{a}\left(3a-8b-\frac{4b^3}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)\sin(e+}{12a^{7/2}f}$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((-3*b^2*(6*a + 5*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
 + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*
b^3)/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))*Sin[e + f*x] + a^(3/2)*Sin[
3*(e + f*x)])/(12*a^(7/2)*f)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^3\,(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{(1-\sin^2(e+fx))^3}{(-a\sin^2(e+fx)+a+b)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-\frac{\sin^2(e+fx)}{a^2}+\frac{a-2b}{a^3}+\frac{b^2(3a+2b)-3ab^2\sin^2(e+fx)}{a^3(-a\sin^2(e+fx)+a+b)^2}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(6a+5b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}}-\frac{b^3\sin(e+fx)}{2a^3(a+b)(-a\sin^2(e+fx)+a+b)}+\frac{(a-2b)\sin(e+fx)}{a^3}-\frac{\sin^3(e+fx)}{3a^2}}{f}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b^2*(6*a + 5*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(7/2)*
(a + b)^(3/2)) + ((a - 2*b)*Sin[e + f*x])/a^3 - Sin[e + f*x]^3/(3*a^2) - (
b^3*Sin[e + f*x])/(2*a^3*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+2\sin(fx+e)b}{a^3}-\frac{b^2\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+e)^2\right)}-\frac{(6a+5b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
default	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+2\sin(fx+e)b}{a^3}-\frac{b^2\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+e)^2\right)}-\frac{(6a+5b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
risch	$-\frac{i\,e^{3i(fx+e)}}{24a^2f}-\frac{3i\,e^{i(fx+e)}}{8a^2f}+\frac{i\,e^{i(fx+e)}b}{a^3f}+\frac{3i\,e^{-i(fx+e)}}{8a^2f}-\frac{i\,e^{-i(fx+e)}b}{a^3f}+\frac{i\,e^{-3i(fx+e)}}{24a^2f}+\frac{ib^3\,(\mathrm{e}}{a^3(a+b)f\left(a\,\mathrm{e}^{4i(fx+}\right.}$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+2*sin(f*x+e)*b)-b^2/a^3*(-1/2
/(a+b)*b*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(6*a+5*b)/(a+b)/(a*(a+b))^(1
/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(115) = 230.

Time = 0.13 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.89

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[\frac{3\left(6\,ab^3+5\,b^4+(6\,a^2b^2+5\,ab^3)\cos(fx+e)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(}{12\left((a^7+2\,a^6b+a\right.}\right.$$
$$-\frac{3\left(6\,ab^3+5\,b^4+(6\,a^2b^2+5\,ab^3)\cos(fx+e)^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(4\,a^4b-4\,a^3b\right.}{6\left((a^7+2\,a^6b+a^5b^2)f\right.}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(3*(6*a*b^3 + 5*b^4 + (6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a^2
 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)
/(a*cos(f*x + e)^2 + b)) + 2*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4
+ 2*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^
2 - 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*
cos(f*x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f), -1/6*(3*(6*a*b^3 + 5*b^
4 + (6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^
2 - a*b)*sin(f*x + e)/(a + b)) - (4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*
b^4 + 2*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^
3*b^2 - 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2
)*f*cos(f*x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{6\,b^3\sin(fx+e)}{a^5+2\,a^4b+a^3b^2-(a^5+a^4b)\sin(fx+e)^2}+\frac{3\,(6\,ab^2+5\,b^3)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^4+a^3b)\sqrt{(a+b)a}}+\frac{4\left(a\sin(fx+e)^3-3\,(a-2\,b)\sin(fx+e)\right)}{a^3}}{12\,f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/12*(6*b^3*sin(f*x + e)/(a^5 + 2*a^4*b + a^3*b^2 - (a^5 + a^4*b)*sin(f*x
 + e)^2) + 3*(6*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*s
in(f*x + e) + sqrt((a + b)*a)))/((a^4 + a^3*b)*sqrt((a + b)*a)) + 4*(a*sin
(f*x + e)^3 - 3*(a - 2*b)*sin(f*x + e))/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{3\,b^3\sin(fx+e)}{(a^4+a^3b)\left(a\sin(fx+e)^2-a-b\right)} - \frac{3\,(6\,ab^2+5\,b^3)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^4+a^3b)\sqrt{-a^2-ab}} - \frac{2\left(a^4\sin(fx+e)^3-3\,a^4\sin(fx+e)+6\,a^3b\sin(fx+e)\right)}{a^6}}{6\,f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*b^3*sin(f*x + e)/((a^4 + a^3*b)*(a*sin(f*x + e)^2 - a - b)) - 3*(6*
a*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4 + a^3*b)*sqrt
(-a^2 - a*b)) - 2*(a^4*sin(f*x + e)^3 - 3*a^4*sin(f*x + e) + 6*a^3*b*sin(f
*x + e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 15.79 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(6\,a+5\,b)}{2\,a^{7/2}\,f\,(a+b)^{3/2}} - \frac{\sin\left(e+f\,x\right)^3}{3\,a^2\,f} - \frac{b^3\,\sin\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(-a^4\sin\left(e+f\,x\right)^2+a^4+b\,a^3\right)} - \frac{\sin\left(e+f\,x\right)\,\left(\frac{2\,(a+b)}{a^3}-\frac{3}{a^2}\right)}{f}$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(6*a + 5*b))/(2*a^(7/2)*f
*(a + b)^(3/2)) - sin(e + f*x)^3/(3*a^2*f) - (b^3*sin(e + f*x))/(2*f*(a +
b)*(a^3*b + a^4 - a^4*sin(e + f*x)^2)) - (sin(e + f*x)*((2*(a + b))/a^3 -
3/a^2))/f
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 746, normalized size of antiderivative = 5.92

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 18*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a +
b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b**2 - 15*sqrt(a)*sq
rt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*ta
n((e + f*x)/2))*sin(e + f*x)**2*a*b**3 + 18*sqrt(a)*sqrt(a + b)*log(sqrt(a
 + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**2
*b**2 + 33*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(
a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a*b**3 + 15*sqrt(a)*sqrt(a + b)*log(s
qrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))
*b**4 + 18*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(
a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b**2 + 15*sqrt(a
)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a
)*tan((e + f*x)/2))*sin(e + f*x)**2*a*b**3 - 18*sqrt(a)*sqrt(a + b)*log(sq
rt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*
a**2*b**2 - 33*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + s
qrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a*b**3 - 15*sqrt(a)*sqrt(a + b)*l
og(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)
/2))*b**4 - 4*sin(e + f*x)**5*a**5 - 8*sin(e + f*x)**5*a**4*b - 4*sin(e +
f*x)**5*a**3*b**2 + 16*sin(e + f*x)**3*a**5 + 12*sin(e + f*x)**3*a**4*b -
24*sin(e + f*x)**3*a**3*b**2 - 20*sin(e + f*x)**3*a**2*b**3 - 12*sin(e + f
*x)*a**5 - 12*sin(e + f*x)*a**4*b + 36*sin(e + f*x)*a**3*b**2 + 66*sin(...
```

3.198 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2} dx = -\frac{b^3(8a+7b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}f} + \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} + \frac{b^4\sin(e+fx)}{2a^4(a+b)f\,(a+b-a\sin^2(e+fx))}$$

output
```
-1/2*b^3*(8*a+7*b)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(9/2)/(a+b)^(
3/2)/f+(a^2-2*a*b+3*b^2)*sin(f*x+e)/a^4/f-2/3*(a-b)*sin(f*x+e)^3/a^3/f+1/5
*sin(f*x+e)^5/a^2/f+1/2*b^4*sin(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{60b^3(8a+7b)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{3/2}}+30\sqrt{a}\left(5a^2-12ab+8b^2\left(3+\frac{b^2}{(a+b)(a+2b+a\cos(2(e+f}\right.\right.}{240a^{9/2}f}$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((60*b^3*(8*a + 7*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
 + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 30*Sqrt[a]*(5*a^2 - 12*a*b
 + 8*b^2*(3 + b^2/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))*Sin[e + f*x]
+ 5*a^(3/2)*(5*a - 8*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)])/(24
0*a^(9/2)*f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sec(e+fx)^5\,(a+b\sec(e+fx)^2)^2}dx$$
$$\Big\downarrow \; 4635$$
$$\frac{\int \frac{(1-\sin^2(e+fx))^4}{(-a\sin^2(e+fx)+a+b)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 300$$

$$\frac{\int \left(\frac{\sin^4(e+fx)}{a^2}-\frac{2(a-b)\sin^2(e+fx)}{a^3}+\frac{a^2-2ba+3b^2}{a^4}-\frac{b^3(4a+3b)-4ab^3\sin^2(e+fx)}{a^4\left(-a\sin^2(e+fx)+a+b\right)^2}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b^3(8a+7b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}}+\frac{b^4\sin(e+fx)}{2a^4(a+b)\left(-a\sin^2(e+fx)+a+b\right)}-\frac{2(a-b)\sin^3(e+fx)}{3a^3}+\frac{\sin^5(e+fx)}{5a^2}+\frac{(a^2-2ab+3b^2)\sin(e+fx}{a^4}}{f}$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/2*(b^3*(8*a + 7*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(9/
2)*(a + b)^(3/2)) + ((a^2 - 2*a*b + 3*b^2)*Sin[e + f*x])/a^4 - (2*(a - b)*
Sin[e + f*x]^3)/(3*a^3) + Sin[e + f*x]^5/(5*a^2) + (b^4*Sin[e + f*x])/(2*a
^4*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{2ab\sin(fx+e)^3}{3}+\sin(fx+e)a^2-2\sin(fx+e)ab+3\sin(fx+e)b^2}{a^4}+\frac{b^3\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+\right.}\right.}{}}{f}$
default	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{2ab\sin(fx+e)^3}{3}+\sin(fx+e)a^2-2\sin(fx+e)ab+3\sin(fx+e)b^2}{a^4}+\frac{b^3\left(-\frac{b\sin(fx+e)}{2(a+b)\left(-a-b+a\sin(fx+\right.}\right.}{}}{f}$
risch	$-\frac{5ie^{3i(fx+e)}}{96a^2f}+\frac{5ie^{-i(fx+e)}}{16a^2f}+\frac{3ie^{i(fx+e)}b}{4a^3f}+\frac{5ie^{-3i(fx+e)}}{96a^2f}-\frac{3ie^{i(fx+e)}b^2}{2f\,a^4}-\frac{3ie^{-i(fx+e)}b}{4a^3f}-\frac{ie^{-3i(fx+e)}b}{12a^3f}$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(1/5*a^2*sin(f*x+e)^5-2/3*a^2*sin(f*x+e)^3+2/3*a*b*sin(f*x+e)^3
+sin(f*x+e)*a^2-2*sin(f*x+e)*a*b+3*sin(f*x+e)*b^2)+b^3/a^4*(-1/2/(a+b)*b*s
in(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(8*a+7*b)/(a+b)/(a*(a+b))^(1/2)*arctan
h(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.71

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \left[\frac{15\left(8\,ab^4+7\,b^5+\left(8\,a^2b^3+7\,ab^4\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\right.}{}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/60*(15*(8*a*b^4 + 7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(a^
2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b
)/(a*cos(f*x + e)^2 + b)) + 2*(6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^6
+ 16*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a^6 +
 a^5*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2 +
54*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b + a
^6*b^2)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f), 1/30*(15*(8*a
*b^4 + 7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arct
an(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*(a^6 + 2*a^5*b + a^4*b^2)*c
os(f*x + e)^6 + 16*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^
5 + 2*(4*a^6 + a^5*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 +
 11*a^4*b^2 + 54*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8
 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.17

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{30\,b^4\sin(fx+e)}{a^6+2\,a^5b+a^4b^2-(a^6+a^5b)\sin(fx+e)^2} + \frac{15\,(8\,ab^3+7\,b^4)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^5+a^4b)\sqrt{(a+b)a}} + \frac{4\left(3\,a^2\sin(fx+e)^5-10\,(a^2-ab)\sin(fx+e)^3+15\,(a}{a^4}}{60\,f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/60*(30*b^4*sin(f*x + e)/(a^6 + 2*a^5*b + a^4*b^2 - (a^6 + a^5*b)*sin(f*x
 + e)^2) + 15*(8*a*b^3 + 7*b^4)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*
sin(f*x + e) + sqrt((a + b)*a)))/((a^5 + a^4*b)*sqrt((a + b)*a)) + 4*(3*a^
2*sin(f*x + e)^5 - 10*(a^2 - a*b)*sin(f*x + e)^3 + 15*(a^2 - 2*a*b + 3*b^2
)*sin(f*x + e))/a^4)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{\frac{15\,b^4\sin(fx+e)}{(a^5+a^4b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{15\,(8\,ab^3+7\,b^4)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^5+a^4b)\sqrt{-a^2-ab}}-\frac{2\left(3\,a^8\sin(fx+e)^5-10\,a^8\sin(fx+e)^3+10\,a^7b\sin(fx+e)^3+15\right.}{a^{10}}}{30\,f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/30*(15*b^4*sin(f*x + e)/((a^5 + a^4*b)*(a*sin(f*x + e)^2 - a - b)) - 15
*(8*a*b^3 + 7*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^5 + a^4*b)*
sqrt(-a^2 - a*b)) - 2*(3*a^8*sin(f*x + e)^5 - 10*a^8*sin(f*x + e)^3 + 10*a
^7*b*sin(f*x + e)^3 + 15*a^8*sin(f*x + e) - 30*a^7*b*sin(f*x + e) + 45*a^6
*b^2*sin(f*x + e))/a^10)/f
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.10

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sin(e+f\,x)^5}{5\,a^2\,f} + \frac{\sin(e+f\,x)^3\left(\frac{2\,(a+b)}{3\,a^3}-\frac{4}{3\,a^2}\right)}{f} + \frac{\sin(e+f\,x)\,\left(\frac{6}{a^2}-\frac{(a+b)^2}{a^4}+\frac{2\,(a+b)\,\left(\frac{2\,(a+b)}{a^3}-\frac{4}{a^2}\right)}{a}\right)}{f} + \frac{b^4\,\sin(e+f\,x)}{2\,f\,\,(a+b)\,\,\left(-a^5\sin(e+f\,x)^2+a^5+b\,a^4\right)} - \frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(8\,a+7\,b)}{2\,a^{9/2}\,f\,(a+b)^{3/2}}$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
sin(e + f*x)^5/(5*a^2*f) + (sin(e + f*x)^3*((2*(a + b))/(3*a^3) - 4/(3*a^2
)))/f + (sin(e + f*x)*(6/a^2 - (a + b)^2/a^4 + (2*(a + b)*((2*(a + b))/a^3
 - 4/a^2))/a))/f + (b^4*sin(e + f*x))/(2*f*(a + b)*(a^4*b + a^5 - a^5*sin(
e + f*x)^2)) - (b^3*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(8*a + 7*b
))/(2*a^(9/2)*f*(a + b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 805, normalized size of antiderivative = 5.13

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(120*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
 - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b**3 + 105*sqrt(a)*sqr
t(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan
((e + f*x)/2))*sin(e + f*x)**2*a*b**4 - 120*sqrt(a)*sqrt(a + b)*log(sqrt(a
 + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**2
*b**3 - 225*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt
(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a*b**4 - 105*sqrt(a)*sqrt(a + b)*log
(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2
))*b**5 - 120*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sq
rt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b**3 - 105*sq
rt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sq
rt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a*b**4 + 120*sqrt(a)*sqrt(a + b)*l
og(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)
/2))*a**2*b**3 + 225*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)*
*2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a*b**4 + 105*sqrt(a)*sqrt(a
 + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e
 + f*x)/2))*b**5 + 12*sin(e + f*x)**7*a**6 + 24*sin(e + f*x)**7*a**5*b + 1
2*sin(e + f*x)**7*a**4*b**2 - 52*sin(e + f*x)**5*a**6 - 76*sin(e + f*x)**5
*a**5*b + 4*sin(e + f*x)**5*a**4*b**2 + 28*sin(e + f*x)**5*a**3*b**3 + 100
*sin(e + f*x)**3*a**6 + 80*sin(e + f*x)**3*a**5*b + 160*sin(e + f*x)**3...
```

3.199 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{a(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{b^2f} + \frac{a^2\tan(e+fx)}{2b^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-1/2*a*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/2
)/f+tan(f*x+e)/b^2/f+1/2*a^2*tan(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.02 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.48

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{a(3a+4b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+}{(a+b)^{3/2}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{8b^2f\,(a}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((a*(3*a + 4*b)*ArcTan[(Sec
[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x
)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
 + 2*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x] + (a*(-((
a + 2*b)*Sin[2*e]) + a*Sin[2*f*x]))/((a + b)*(Cos[e] - Sin[e])*(Cos[e] + S
in[e]))))/(8*b^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^6}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{(\tan^2(e+fx)+1)^2}{(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{b^2}-\frac{2ab\tan^2(e+fx)+a(a+2b)}{b^2(b\tan^2(e+fx)+a+b)^2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\tan(e+fx)}{2b^2(a+b)(a+b\tan^2(e+fx)+b)}-\frac{a(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}}+\frac{\tan(e+fx)}{b^2}}{f}$$

input
```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/2*(a*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*
(a + b)^(3/2)) + Tan[e + f*x]/b^2 + (a^2*Tan[e + f*x])/(2*b^2*(a + b)*(a +
 b + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{a\left(-\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^2}+\frac{\tan(fx+e)}{b^2}}{f}$
default	$\frac{-\frac{a\left(-\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^2}+\frac{\tan(fx+e)}{b^2}}{f}$
risch	$\frac{i\left(3a^2e^{4i(fx+e)}+4ab\,e^{4i(fx+e)}+6a^2e^{2i(fx+e)}+14ab\,e^{2i(fx+e)}+8b^2e^{2i(fx+e)}+3a^2+2ab\right)}{(a+b)b^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}+1\right)}-\frac{3a^2\ln\left(e^{2i(fx+e)}-\frac{2iba+}{}\right.}{4\sqrt{-ab-}}$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/b^2*a*(-1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+4*b)/
(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+tan(f*x+e)/b^2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(88) = 176.

Time = 0.12 (sec) , antiderivative size = 516, normalized size of antiderivative = 5.16

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[-\frac{\left((3a^3+4a^2b)\cos(fx+e)^3+(3a^2b+4ab^2)\cos(fx+e)\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3}{}\right.}{8\left((a^3b^3+2a^2b^4\right.}\right.$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e
))*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b +
 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqr
t(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2)) - 4*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b
^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x
 + e)^3 + (a^2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e)), 1/4*(((3*a^3 + 4*a^2*
b)*cos(f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e))*sqrt(a*b + b^2)*arct
an(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*
x + e))) + 2*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cos(f*x + e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x +
 e)^3 + (a^2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{a^2\tan(fx+e)}{a^2b^2+2\,ab^3+b^4+(ab^3+b^4)\tan(fx+e)^2} - \frac{(3\,a^2+4\,ab)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(ab^2+b^3)\sqrt{(a+b)b}} + \frac{2\,\tan(fx+e)}{b^2}}{2\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a^2*tan(f*x + e)/(a^2*b^2 + 2*a*b^3 + b^4 + (a*b^3 + b^4)*tan(f*x + e
)^2) - (3*a^2 + 4*a*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a*b^2 + b^
3)*sqrt((a + b)*b)) + 2*tan(f*x + e)/b^2)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{a^2\tan(fx+e)}{(ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)} - \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,a^2+4\,ab)}{(ab^2+b^3)\sqrt{ab+b^2}} + \frac{2\,\tan(fx+e)}{b^2}}{2\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2*tan(f*x + e)/((a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)) - (pi*flo
or((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3
*a^2 + 4*a*b)/((a*b^2 + b^3)*sqrt(a*b + b^2)) + 2*tan(f*x + e)/b^2)/f
```

Mupad [B] (verification not implemented)

Time = 15.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\tan(e+f\,x)}{b^2\,f} + \frac{a^2\tan(e+f\,x)}{2\,f\,(a+b)\,\left(b^3\tan\left(e+f\,x\right)^2+b^3+a\,b^2\right)} - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\tan(e+f\,x)\,(3\,a+4\,b)}{\sqrt{a+b}\,(3\,a^2+4\,b\,a)}\right)\,(3\,a+4\,b)}{2\,b^{5/2}\,f\,(a+b)^{3/2}}$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
tan(e + f*x)/(b^2*f) + (a^2*tan(e + f*x))/(2*f*(a + b)*(a*b^2 + b^3 + b^3*
tan(e + f*x)^2)) - (a*atan((a*b^(1/2)*tan(e + f*x)*(3*a + 4*b))/((a + b)^(
1/2)*(4*a*b + 3*a^2)))*(3*a + 4*b))/(2*b^(5/2)*f*(a + b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 622, normalized size of antiderivative = 6.22

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*cos(e + f*x)*sin(e + f*x)**2*a**3 - 4*sqrt(b)*sqrt(a + b)*atan((sqr
t(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*sin(e + f*x)**2
*a**2*b + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(
a))/sqrt(b))*cos(e + f*x)*a**3 + 7*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*t
an((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*a**2*b + 4*sqrt(b)*sqrt(a
 + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*
a*b**2 - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a
))/sqrt(b))*cos(e + f*x)*sin(e + f*x)**2*a**3 - 4*sqrt(b)*sqrt(a + b)*atan
((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*sin(e + f*
x)**2*a**2*b + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) +
sqrt(a))/sqrt(b))*cos(e + f*x)*a**3 + 7*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
 b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*a**2*b + 4*sqrt(b)*s
qrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e +
f*x)*a*b**2 + 3*sin(e + f*x)**3*a**3*b + 5*sin(e + f*x)**3*a**2*b**2 + 2*s
in(e + f*x)**3*a*b**3 - 3*sin(e + f*x)*a**3*b - 7*sin(e + f*x)*a**2*b**2 -
 6*sin(e + f*x)*a*b**3 - 2*sin(e + f*x)*b**4)/(2*cos(e + f*x)*b**3*f*(sin(
e + f*x)**2*a**3 + 2*sin(e + f*x)**2*a**2*b + sin(e + f*x)**2*a*b**2 - a**
3 - 3*a**2*b - 3*a*b**2 - b**3))
```

3.200 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx = \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}f} - \frac{a\tan(e+fx)}{2b(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/f-1
/2*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx = \frac{\frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2b^{3/2}f}$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a
*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*b^
(3/2)*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^4}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2b(a+b)}-\frac{a\tan(e+fx)}{2b(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}}-\frac{a\tan(e+fx)}{2b(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)
^(3/2)) - (a*Tan[e + f*x])/(2*b*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{a\tan(fx+e)}{2(a+b)b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{a\tan(fx+e)}{2(a+b)b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{f}$
risch	$-\frac{i\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{bf(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{a\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{4\sqrt{-ab-b^2}\,(a+b)fb}-$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*a/(a+b)/b*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(a+2*b)/(a+b)/b/((
a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 406, normalized size of antiderivative = 4.95

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[-\frac{4\,(a^2b+ab^2)\cos(fx+e)\sin(fx+e)+\left((a^2+2\,ab)\cos(fx+e)^2+ab+2\,b^2\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\cdots}{\cdots}\right.}{8\left((a^3b^2+2\,a^2b^3+ab^4)f\cos(fx+e)^2\cdots\right.}\right.$$
$$-\frac{2\,(a^2b+ab^2)\cos(fx+e)\sin(fx+e)+\left((a^2+2\,ab)\cos(fx+e)^2+ab+2\,b^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(\cdots}{2\sqrt{ab\cdots}}\right.}{4\left((a^3b^2+2\,a^2b^3+ab^4)f\cos(fx+e)^2+(a^2b^3+2\,ab^4+b^5)f\right)}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(a^2*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*
x + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*
x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3
- b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4
 + 2*a*b*cos(f*x + e)^2 + b^2)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x
+ e)^2 + (a^2*b^3 + 2*a*b^4 + b^5)*f), -1/4*(2*(a^2*b + a*b^2)*cos(f*x + e
)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b
^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e
)*sin(f*x + e))))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^2*b
^3 + 2*a*b^4 + b^5)*f)]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{a\tan(fx+e)}{a^2b+2\,ab^2+b^3+(ab^2+b^3)\tan(fx+e)^2} - \frac{(a+2\,b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(ab+b^2)}}{2\,f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(a*tan(f*x + e)/(a^2*b + 2*a*b^2 + b^3 + (a*b^2 + b^3)*tan(f*x + e)^2
) - (a + 2*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a*b
 + b^2)))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+2\,b)}{(ab+b^2)^{\frac{3}{2}}} - \frac{a\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(ab+b^2)}}{2\,f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(a + 2*b)/(a*b + b^2)^(3/2) - a*tan(f*x + e)/((b*tan(f*x + e)^2
+ a + b)*(a*b + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 15.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)\,(a+2\,b)}{2\,b^{3/2}\,f\,(a+b)^{3/2}}-\frac{a\tan(e+f\,x)}{2\,b\,f\,\left(a+b\right)\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 2*b))/(2*b^(3/2)*f*(a + b
)^(3/2)) - (a*tan(e + f*x))/(2*b*f*(a + b)*(a + b + b*tan(e + f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 483, normalized size of antiderivative = 5.89

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^2a^2+2\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx-\right.}{}$$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
)*sin(e + f*x)**2*a**2 + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b - sqrt(b)*sqrt(a + b)*atan
((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 - 3*sqrt(b)*sqrt(a
 + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b - 2*sqrt(
b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**2
 + sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(
b))*sin(e + f*x)**2*a**2 + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b - sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 - 3*sqrt(b)*sqrt
(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b - 2*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b*
*2 + cos(e + f*x)*sin(e + f*x)*a**2*b + cos(e + f*x)*sin(e + f*x)*a*b**2)/
(2*b**2*f*(sin(e + f*x)**2*a**3 + 2*sin(e + f*x)**2*a**2*b + sin(e + f*x)*
*2*a*b**2 - a**3 - 3*a**2*b - 3*a*b**2 - b**3))
```

3.201 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}f}+\frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(1/2)/(a+b)^(3/2)/f+1/2*tan(f
*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.89

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$=\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}}{8(a+b)f\left(a+b\sec^2(e+fx)\right)^2}$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-((ArcTan[(Sec[f*x]*(Cos[2
*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e
] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) + (-((a + 2*
b)*Sin[2*e]) + a*Sin[2*f*x])/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*
(a + b)*f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^2}{\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2(a+b)}+\frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}}+\frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{f}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)) + Ta
n[e + f*x]/(2*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}}{f}$
risch	$\frac{i\left(a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{2i(fx+e)}+a\right)}{af(a+b)\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}-\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{4\sqrt{-ab-b^2}\,(a+b)f}+\frac{\ln\Big(}{}$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*tan(f*x+e)/(a+b)/(a+b+b*tan(f*x+e)^2)+1/2/(a+b)/((a+b)*b)^(1/2)*a
rctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(61) = 122$.

Time = 0.10 (sec) , antiderivative size = 368, normalized size of antiderivative = 5.04

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \left[\frac{4\,(ab+b^2)\cos(fx+e)\sin(fx+e)-\left(a\cos(fx+e)^2+b\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,a}{}\right.}{8\left((a^3b+2\,a^2b^2+ab^3)f\cos(fx+e)^2+(a^2b^2+2\right.}\right.$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*sqr
t(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b
 - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 +
 b^4)*f), 1/4*(2*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2
 + b)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b
+ b^2)*cos(f*x + e)*sin(f*x + e))))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x
 + e)^2 + (a^2*b^2 + 2*a*b^3 + b^4)*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^2}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\tan(fx+e)}{(ab+b^2)\tan(fx+e)^2+a^2+2\,ab+b^2} + \frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(a+b)}}{2\,f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(tan(f*x + e)/((a*b + b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2) + arcta
n(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}(a+b)}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a+b)}}{2f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))/(sqrt(a*b + b^2)*(a + b)) + tan(f*x + e)/((b*tan(f*x + e)^2 + a
+ b)*(a + b)))/f
```

Mupad [B] (verification not implemented)

Time = 15.47 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\tan(e+f\,x)}{2\,f\,(a+b)\,\left(b\tan(e+f\,x)^2+a+b\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,(2\,a+2\,b)}{2\,(a+b)^{3/2}}\right)}{2\,\sqrt{b}\,f\,(a+b)^{3/2}}$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
tan(e + f*x)/(2*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + atan((b^(1/2)*tan(
e + f*x)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(2*b^(1/2)*f*(a + b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 4.36

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^2 a-\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)a-\sqrt{b}\,\sqrt{a}}{}$$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
)*sin(e + f*x)**2*a - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/
2) - sqrt(a))/sqrt(b))*a - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*b + sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a - sqrt(b)*sqrt(a + b)*a
tan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a - sqrt(b)*sqrt(a +
 b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b - cos(e + f*x
)*sin(e + f*x)*a*b - cos(e + f*x)*sin(e + f*x)*b**2)/(2*b*f*(sin(e + f*x)*
*2*a**3 + 2*sin(e + f*x)**2*a**2*b + sin(e + f*x)**2*a*b**2 - a**3 - 3*a**
2*b - 3*a*b**2 - b**3))
```

3.202 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{x}{a^2}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^2-1/2*b^(1/2)*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.61

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx)))+\frac{b(3a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(\cdots}{2\sqrt{a+\cdots}}\right.}{\cdots}\right.}{8a^2\left(a+b\sec^2(e+\cdots\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e
+ f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin
[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)
^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2
*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Se
c[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4616, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \ 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 316$$

$$\frac{\frac{\int \frac{-b\tan^2(e+fx)+2a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\frac{2(a+b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b(3a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{b(3a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
(((2*(a + b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*Tan[e +
f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{4(a+b)^2fa}+$

input
```
int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^2*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(
f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time = 0.11 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.73

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \left[\frac{8\,(a^2+ab)fx\cos{(fx+e)}^2 - 4\,ab\cos{(fx+e)}\sin{(fx+e)} + 8\,(ab+b^2)fx + \left((3\,a^2+2\,ab)\cos{(fx+\right.}{8\,\left((a^4+\right.}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
 8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*
b^2)*f), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*
x + e) + 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b
^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3
*b + a^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx=\int \frac{1}{\left(a+b\sec^2{(e+fx)}\right)^2}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{b\tan(fx+e)}{a^3+2\,a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2}+\frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)
^2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b
)*sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e
)/((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 17.68 (sec) , antiderivative size = 2056, normalized size of antiderivative = 22.35

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b
^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/
(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b
^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a
^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*
a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a
^2*b^2)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a
^3*b + a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/
(2*(2*a^4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 6
4*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) +
(tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*
b^2)))/a^2 + (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5
 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^
7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20
*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*
a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan((((tan(e + f*
x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(
a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*
b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5
*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.11

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*sin(e + f*x)**2*a**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 + 5*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b +
2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*b**2 - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(
a))/sqrt(b))*sin(e + f*x)**2*a**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 + 5*s
qrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*
a*b + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/
sqrt(b))*b**2 + cos(e + f*x)*sin(e + f*x)*a**2*b + cos(e + f*x)*sin(e + f*
x)*a*b**2 + 2*sin(e + f*x)**2*a**3*f*x + 4*sin(e + f*x)**2*a**2*b*f*x + 2*
sin(e + f*x)**2*a*b**2*f*x - 2*a**3*f*x - 6*a**2*b*f*x - 6*a*b**2*f*x - 2*
b**3*f*x)/(2*a**2*f*(sin(e + f*x)**2*a**3 + 2*sin(e + f*x)**2*a**2*b + sin
(e + f*x)**2*a*b**2 - a**3 - 3*a**2*b - 3*a*b**2 - b**3))
```

3.203 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\,(a+b+b\tan^2(e+fx))} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\,(a+b+b\tan^2(e+fx))}$$

output
```
1/2*(a-4*b)*x/a^3+1/2*b^(3/2)*(5*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(3/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+
1/2*b*(a+2*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{2(a-4b)(e+fx)+\frac{2b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\left(a+\frac{2ab^2}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)\sin(2(e+fx))}{4a^3f}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(2*(a - 4*b)*(e + f*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cos
[2*(e + f*x)])))*Sin[2*(e + f*x)])/(4*a^3*f)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4634, 316, 25, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^2\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}-\frac{\int -\frac{3b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{2\left(a^2-2ba-2b^2+b(a+2b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{a^2-2ba-2b^2+b(a+2b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a(a+b)}+\frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{b^2(5a+4b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a-4b)(a+b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{a(a+b)}+\frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\frac{b^2(5a+4b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a-4b)(a+b)\arctan(\tan(e+fx))}{a}}{a(a+b)}+\frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-4b)(a+b)\arctan(\tan(e+fx))}{a}}{a(a+b)}+\frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)) + ((((
a - 4*b)*(a + b)*ArcTan[Tan[e + f*x]])/a + (b^(3/2)*(5*a + 4*b)*ArcTan[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*(a + 2
*b)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(2*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-4b)\arctan(\tan(fx+e))}{2}}{a^3}+\frac{b^2\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(5a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
default	$\frac{\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-4b)\arctan(\tan(fx+e))}{2}}{a^3}+\frac{b^2\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(5a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
risch	$\frac{x}{2a^2}-\frac{2xb}{a^3}-\frac{ie^{2i(fx+e)}}{8a^2f}+\frac{ie^{-2i(fx+e)}}{8a^2f}+\frac{ib^2\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^3(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{5\sqrt{-(a+b}}{}$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*(1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a-4*b)*arctan(tan(f*x+e
)))+b^2/a^3*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(5*a+4*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.83

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \left[\frac{4\,(a^3-3\,a^2b-4\,ab^2)fx\cos\left(fx+e\right)^2+4\,(a^2b-3\,ab^2-4\,b^3)fx+\left(5\,ab^2+4\,b^3+(5\,a^2b+4\,ab^2)\cos\left(\right.\right.}{}\right.$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 4*(a^2*b - 3*a*b^2
- 4*b^3)*f*x + (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)) + 4*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 2*a
*b^2)*cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b
 + a^3*b^2)*f), 1/4*(2*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 2*(a
^2*b - 3*a*b^2 - 4*b^3)*f*x - (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f
*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(
b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*((a^3 + a^2*b)*cos(f*x + e)^
3 + (a^2*b + 2*a*b^2)*cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x
 + e)^2 + (a^4*b + a^3*b^2)*f)]
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(5ab^2+4b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}} + \frac{(ab+2b^2)\tan(fx+e)^3+(a^2+2ab+2b^2)\tan(fx+e)}{(a^3b+a^2b^2)\tan(fx+e)^4+a^4+2a^3b+a^2b^2+(a^4+3a^3b+2a^2b^2)\tan(fx+e)^2} + \frac{(fx+e)(a-4b)}{a^3}}{2f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((5*a*b^2 + 4*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + a^3*
b)*sqrt((a + b)*b)) + ((a*b + 2*b^2)*tan(f*x + e)^3 + (a^2 + 2*a*b + 2*b^2
)*tan(f*x + e))/((a^3*b + a^2*b^2)*tan(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^
2 + (a^4 + 3*a^3*b + 2*a^2*b^2)*tan(f*x + e)^2) + (f*x + e)*(a - 4*b)/a^3)
/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.35

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{(5\,ab^2+4\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+a^3b)\sqrt{ab+b^2}} + \frac{ab\tan(fx+e)^3+2\,b^2\tan(fx+e)^3+a^2\tan(fx+e)+2\,ab\tan(fx+e)+2\,b^2\tan(fx+}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2\,b\tan(fx+e)^2+a+b\right)(a^3+a^2b)}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((5*a*b^2 + 4*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan
(f*x + e)/sqrt(a*b + b^2)))/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (a*b*tan(f*x
 + e)^3 + 2*b^2*tan(f*x + e)^3 + a^2*tan(f*x + e) + 2*a*b*tan(f*x + e) + 2
*b^2*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e
)^2 + a + b)*(a^3 + a^2*b)) + (f*x + e)*(a - 4*b)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 18.40 (sec) , antiderivative size = 2401, normalized size of antiderivative = 16.91

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((tan(e + f*x)*(2*a*b + a^2 + 2*b^2))/(2*a^2*(a + b)) + (b*tan(e + f*x)^3*
(a + 2*b))/(2*a^2*(a + b)))/(f*(a + b + b*tan(e + f*x)^4 + tan(e + f*x)^2*
(a + 2*b))) - (atan(((((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a
^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^
4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*(a*1i
 - b*4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^
5*b + a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3) +
 (((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(
2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^
9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5
 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)
))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3))/((12*a*b^6 + 8*b^7 +
 (3*a^2*b^5)/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^6*b^2) - (((tan(e + f*x
)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^
6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b
 + a^8 + a^6*b^2) - (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5 + 80*a^7*b^4 +
 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a*1i - b*4i
))/(4*a^3))*(a*1i - b*4i))/(4*a^3) + (((tan(e + f*x)*(64*a*b^6 + 32*b^7 +
26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^
6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + ...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 792, normalized size of antiderivative = 5.58

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(5*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**2*a**2*b + 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((
e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**2 - 5*sqrt(b)*sqrt(a
+ b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*b - 9*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*
b**2 - 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))
/sqrt(b))*b**3 + 5*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2)
+ sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*b + 4*sqrt(b)*sqrt(a + b)*atan((s
qrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**2 - 5
*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b)
)*a**2*b - 9*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt
(a))/sqrt(b))*a*b**2 - 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*
x)/2) + sqrt(a))/sqrt(b))*b**3 + cos(e + f*x)*sin(e + f*x)**3*a**4 + 2*cos
(e + f*x)*sin(e + f*x)**3*a**3*b + cos(e + f*x)*sin(e + f*x)**3*a**2*b**2
- cos(e + f*x)*sin(e + f*x)*a**4 - 3*cos(e + f*x)*sin(e + f*x)*a**3*b - 4*
cos(e + f*x)*sin(e + f*x)*a**2*b**2 - 2*cos(e + f*x)*sin(e + f*x)*a*b**3 +
 sin(e + f*x)**2*a**4*e + sin(e + f*x)**2*a**4*f*x - 2*sin(e + f*x)**2*a**
3*b*e - 2*sin(e + f*x)**2*a**3*b*f*x - 7*sin(e + f*x)**2*a**2*b**2*e - 7*s
in(e + f*x)**2*a**2*b**2*f*x - 4*sin(e + f*x)**2*a*b**3*e - 4*sin(e + f*x)
**2*a*b**3*f*x - a**4*e - a**4*f*x + a**3*b*e + a**3*b*f*x + 9*a**2*b**...
```

3.204 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(3a^2-8ab+24b^2)\,x}{8a^4} - \frac{b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4(a+b)^{3/2}f} + \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-3b)b(3a+4b)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
1/8*(3*a^2-8*a*b+24*b^2)*x/a^4-1/2*b^(5/2)*(7*a+6*b)*arctan(b^(1/2)*tan(f*
x+e)/(a+b)^(1/2))/a^4/(a+b)^(3/2)/f+3/8*(a-2*b)*cos(f*x+e)*sin(f*x+e)/a^2/
f/(a+b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2
)+1/8*(a-3*b)*b*(3*a+4*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$=\frac{4(3a^2-8ab+24b^2)(e+fx)-\frac{16b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+8a(a-2b)\sin(2(e+fx))-\frac{16ab^3\sin(2(}{(a+b)(a+2b+a\,\mathrm{c}}}{32a^4f}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(4*(3*a^2 - 8*a*b + 24*b^2)*(e + f*x) - (16*b^(5/2)*(7*a + 6*b)*ArcTan[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) + 8*a*(a - 2*b)*Sin[2*(e +
 f*x)] - (16*a*b^3*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)
])) + a^2*Sin[4*(e + f*x)])/(32*a^4*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2}dx$$
$$\Big\downarrow \text{4634}$$
$$\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}-\frac{\int -\frac{5b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{5b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{\int -\frac{3a^2+ba+6b^2+9(a-2b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{\int \frac{3a^2+ba+6b^2+9(a-2b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\frac{\frac{\int \frac{2\left(3a^3-2ba^2+11b^2a+12b^3+(a-3b)b(3a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{\frac{\int \frac{3a^3-2ba^2+11b^2a+12b^3+(a-3b)b(3a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a(a+b)}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{\frac{(a+b)\left(3a^2-8ab+24b^2\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{4b^3(7a+6b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a(a+b)}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(}{2a\left(\tan^2(e+fx)+1\right)\left(a+\right.}}{4a}}{f}$$

$\big\downarrow$ 216

$$\frac{\frac{\frac{\frac{(a+b)\left(3a^2-8ab+24b^2\right)\arctan(\tan(e+fx))}{a}-\frac{4b^3(7a+6b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a(a+b)}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+\right.}}{4a}}{f}$$

$\big\downarrow$ 218

$$\frac{\frac{\frac{\frac{(a+b)\left(3a^2-8ab+24b^2\right)\arctan(\tan(e+fx))}{a}-\frac{4b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a(a+b)}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}}{4a}}{f}$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)) + ((
3*(a - 2*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x
]^2)) + ((((a + b)*(3*a^2 - 8*a*b + 24*b^2)*ArcTan[Tan[e + f*x]])/a - (4*b
^(5/2)*(7*a + 6*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a +
 b]))/(a*(a + b)) + ((a - 3*b)*b*(3*a + 4*b)*Tan[e + f*x])/(a*(a + b)*(a +
 b + b*Tan[e + f*x]^2)))/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-ab\right)\tan(fx+e)^3+\left(-ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-8ab+24b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{b^3\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(7a}{}\right.}{a^4}}{f}$
default	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-ab\right)\tan(fx+e)^3+\left(-ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-8ab+24b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{b^3\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(7a}{}\right.}{a^4}}{f}$
risch	$\frac{3x}{8a^2}-\frac{xb}{a^3}+\frac{3x\,b^2}{a^4}-\frac{i e^{4i(fx+e)}}{64a^2f}-\frac{i e^{2i(fx+e)}}{8a^2f}+\frac{i e^{2i(fx+e)}b}{4a^3f}+\frac{i e^{-2i(fx+e)}}{8a^2f}-\frac{i e^{-2i(fx+e)}b}{4a^3f}+\frac{i e^{-4i(fx+e)}}{64a^2f}$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(((3/8*a^2-a*b)*tan(f*x+e)^3+(-a*b+5/8*a^2)*tan(f*x+e))/(1+tan(
f*x+e)^2)^2+1/8*(3*a^2-8*a*b+24*b^2)*arctan(tan(f*x+e)))-b^3/a^4*(1/2*a/(a
+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(7*a+6*b)/(a+b)/((a+b)*b)^(1/2)*ar
ctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.23

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*cos(f*x + e)^2 + (3*a^
3*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + (7*a*b^3 + 6*b^4 + (7*a^2*b^2 +
 6*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(
f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*c
os(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) +
b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*(a^4 + a^3*b)
*cos(f*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b -
5*a^2*b^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x
 + e)^2 + (a^5*b + a^4*b^2)*f), 1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*
b^3)*f*x*cos(f*x + e)^2 + (3*a^3*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x +
2*(7*a*b^3 + 6*b^4 + (7*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))
*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) + (2*(a^4 + a^3*b)*cos(f*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2
*b^2)*cos(f*x + e)^3 + (3*a^3*b - 5*a^2*b^2 - 12*a*b^3)*cos(f*x + e))*sin(
f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.32

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx =$$

$$-\frac{\frac{4\,(7\,ab^3+6\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+a^4b)\sqrt{(a+b)b}} - \frac{(3\,a^2b-5\,ab^2-12\,b^3)\tan(fx+e)^5+(3\,a^3+3\,a^2b-16\,ab^2-24\,b^3)\tan(fx+e)^3+(5\,a^3+2\,a^2b-11\,ab^2-}{(a^4b+a^3b^2)\tan(fx+e)^6+a^5+2\,a^4b+a^3b^2+(a^5+4\,a^4b+3\,a^3b^2)\tan(fx+e)^4+(2\,a^5+5\,a^4b+3\,a^3b}}{8\,f}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*(4*(7*a*b^3 + 6*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + a
^4*b)*sqrt((a + b)*b)) - ((3*a^2*b - 5*a*b^2 - 12*b^3)*tan(f*x + e)^5 + (3
*a^3 + 3*a^2*b - 16*a*b^2 - 24*b^3)*tan(f*x + e)^3 + (5*a^3 + 2*a^2*b - 11
*a*b^2 - 12*b^3)*tan(f*x + e))/((a^4*b + a^3*b^2)*tan(f*x + e)^6 + a^5 + 2
*a^4*b + a^3*b^2 + (a^5 + 4*a^4*b + 3*a^3*b^2)*tan(f*x + e)^4 + (2*a^5 + 5
*a^4*b + 3*a^3*b^2)*tan(f*x + e)^2) - (3*a^2 - 8*a*b + 24*b^2)*(f*x + e)/a
^4)/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx =$$

$$-\frac{\frac{4\,b^3\tan(fx+e)}{(a^4+a^3b)\left(b\tan(fx+e)^2+a+b\right)} + \frac{4\,(7\,ab^3+6\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+a^4b)\sqrt{ab+b^2}} - \frac{(3\,a^2-8\,ab+24\,b^2)(fx+e)}{a^4} - \frac{3\,a\tan(}{}}{8\,f}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/8*(4*b^3*tan(f*x + e)/((a^4 + a^3*b)*(b*tan(f*x + e)^2 + a + b)) + 4*(7
*a*b^3 + 6*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x +
e)/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) - (3*a^2 - 8*a*b + 24
*b^2)*(f*x + e)/a^4 - (3*a*tan(f*x + e)^3 - 8*b*tan(f*x + e)^3 + 5*a*tan(f
*x + e) - 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 19.49 (sec) , antiderivative size = 2880, normalized size of antiderivative = 14.19

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(atan(((((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 +
 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (
((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2
)/2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a^2*3i - a*b*8i + b^2*24
i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*a^4*(
2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4))*(a^2*3i
- a*b*8i + b^2*24i)*1i)/(16*a^4) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9
+ 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2
*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2
+ (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*
x)*(a^2*3i - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3
 + 256*a^11*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i +
b^2*24i))/(16*a^4))*(a^2*3i - a*b*8i + b^2*24i)*1i)/(16*a^4))/(((135*a*b^9
)/4 + 27*b^10 - (9*a^2*b^8)/2 - (149*a^3*b^7)/32 + (219*a^4*b^6)/64 - (63*
a^5*b^5)/64)/(2*a^10*b + a^11 + a^9*b^2) - (((tan(e + f*x)*(2112*a*b^8 + 1
152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3)
)/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10
*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) - (ta
n(e + f*x)*(a^2*3i - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*
a^10*b^3 + 256*a^11*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i ...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 963, normalized size of antiderivative = 4.74

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

output

```
( - 28*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**2*a**2*b**2 - 24*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 + 28*sqrt(b
)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*
b**2 + 52*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*a*b**3 + 24*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)
/2) - sqrt(a))/sqrt(b))*b**4 - 28*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*b**2 - 24*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e +
 f*x)**2*a*b**3 + 28*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2
) + sqrt(a))/sqrt(b))*a**2*b**2 + 52*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b**3 + 24*sqrt(b)*sqrt(a + b)*atan
((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**4 - 2*cos(e + f*x)*s
in(e + f*x)**5*a**5 - 4*cos(e + f*x)*sin(e + f*x)**5*a**4*b - 2*cos(e + f*
x)*sin(e + f*x)**5*a**3*b**2 + 7*cos(e + f*x)*sin(e + f*x)**3*a**5 + 8*cos
(e + f*x)*sin(e + f*x)**3*a**4*b - 5*cos(e + f*x)*sin(e + f*x)**3*a**3*b**
2 - 6*cos(e + f*x)*sin(e + f*x)**3*a**2*b**3 - 5*cos(e + f*x)*sin(e + f*x)
*a**5 - 7*cos(e + f*x)*sin(e + f*x)*a**4*b + 9*cos(e + f*x)*sin(e + f*x)*a
**3*b**2 + 23*cos(e + f*x)*sin(e + f*x)*a**2*b**3 + 12*cos(e + f*x)*sin(e
+ f*x)*a*b**4 + 3*sin(e + f*x)**2*a**5*e + 3*sin(e + f*x)**2*a**5*f*x -...
```

3.205 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 278

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(5a^3-12a^2b+24ab^2-64b^3)\,x}{16a^5} + \frac{b^{7/2}(9a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5(a+b)^{3/2}f} + \frac{(15a^2-26ab+48b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\,(a+b+b\tan^2(e+fx))} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\,(a+b+b\tan^2(e+fx))} + \frac{b(5a^3-7a^2b+12ab^2+32b^3)\tan(e+fx)}{16a^4(a+b)f\,(a+b+b\tan^2(e+fx))}$$

output

```
1/16*(5*a^3-12*a^2*b+24*a*b^2-64*b^3)*x/a^5+1/2*b^(7/2)*(9*a+8*b)*arctan(b
^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(3/2)/f+1/48*(15*a^2-26*a*b+48*b^
2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(5*a-8*b)*cos(f*x
+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/
f/(a+b+b*tan(f*x+e)^2)+1/16*b*(5*a^3-7*a^2*b+12*a*b^2+32*b^3)*tan(f*x+e)/a
^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.79

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(12(5a^3-12a^2b+24ab^2-64b^3)\,x(a+2b+a\cos(2(e+fx)))\right.}{}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(12*(5*a^3 - 12*a^2*b + 24*
a*b^2 - 64*b^3)*x*(a + 2*b + a*Cos[2*(e + f*x)]) - (96*b^4*(9*a + 8*b)*Arc
Tan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e +
 f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*
(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*f*x]*(a + 2*b + a*Cos[2
*(e + f*x)])*Sin[2*e])/f + (3*a^2*(3*a - 4*b)*Cos[4*f*x]*(a + 2*b + a*Cos[
2*(e + f*x)])*Sin[4*e])/f + (a^3*Cos[6*f*x]*(a + 2*b + a*Cos[2*(e + f*x)])
*Sin[6*e])/f + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*e]*(a + 2*b + a*Cos[2
*(e + f*x)])*Sin[2*f*x])/f - (96*b^4*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/
((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) + (3*a^2*(3*a - 4*b)*Cos[4
*e]*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[4*f*x])/f + (a^3*Cos[6*e]*(a + 2*b
+ a*Cos[2*(e + f*x)])*Sin[6*f*x])/f))/(768*a^5*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \; 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\int -\frac{7b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{6a}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{7b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{(5a-8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{\int -\frac{15a^2-ba+8b^2+5(5a-8b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{15a^2-ba+8b^2+5(5a-8b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a}+\frac{(5a-8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\frac{\left(15a^2-26ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{\int -\frac{3\left(5a^3+3ba^2-2b^2a-16b^3+b\left(15a^2-26ba+48b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}}{4a}+\frac{(5a-8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+\cdots\right)}}{6a}\cdots}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{3\int \frac{5a^3+3ba^2-2b^2a-16b^3+b\left(15a^2-26ba+48b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}+\frac{\left(15a^2-26ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}}{4a}+\frac{(5a-8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)\cdots\right)}}{6a}\cdots}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\frac{3\left(\frac{\int \frac{2\left(5a^4-2ba^3+5b^2a^2-28b^3a-32b^4+b\left(5a^3-7ba^2+12b^2a+32b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{b\left(5a^3-7a^2b+12ab^2+32b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{2a}+\frac{\left(15a^2-26ab+\cdots\right.}{2a\left(\tan^2(e+fx)+1\right)\cdots}}{4a}\cdots}{6a}\cdots}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{3\left(\frac{\int \frac{5a^4-2ba^3+5b^2a^2-28b^3a-32b^4+b\left(5a^3-7ba^2+12b^2a+32b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a(a+b)}+\frac{b\left(5a^3-7a^2b+12ab^2+32b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{2a}+\frac{\left(15a^2-26ab+48b\cdots\right.}{2a\left(\tan^2(e+fx)+1\right)\left(a\cdots\right.}}{4a}\cdots}{6a}\cdots}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{3\left(\frac{\frac{(a+b)\left(5a^3-12a^2b+24ab^2-64b^3\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}+\frac{8b^4(9a+8b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a(a+b)}+\frac{b\left(5a^3-7a^2b+12ab^2+32b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{2a}\cdots}{4a}\cdots}{6a}\cdots}{f}$$

$\big\downarrow$ 216

$$\frac{\frac{\frac{3\left(\frac{\frac{8b^4(9a+8b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a+b)\left(5a^3-12a^2b+24ab^2-64b^3\right)\arctan(\tan(e+fx))}{a}}{a(a+b)}+\frac{b\left(5a^3-7a^2b+12ab^2+32b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{2a}+\frac{\left(15a\cdots\right.}{2a\left(\tan^2(\cdots\right.}}{4a}}{6a}}{f}$$

$\big\downarrow$ 218

$$\frac{\frac{\frac{\left(15a^2-26ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}+\frac{3\left(\frac{\frac{(a+b)\left(5a^3-12a^2b+24ab^2-64b^3\right)\arctan(\tan(e+fx))}{a}+\frac{8b^{7/2}(9a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a(a+b)}+\frac{b\left(5a^3-7a^2b+12\cdots\right.}{a(a+b)\left(a+\cdots\right.}\right.}{2a}}{4a}}{6a}}{f}$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)) + ((
(5*a - 8*b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f
*x]^2)) + (((15*a^2 - 26*a*b + 48*b^2)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x
]^2)*(a + b + b*Tan[e + f*x]^2)) + (3*((((a + b)*(5*a^3 - 12*a^2*b + 24*a*
b^2 - 64*b^3)*ArcTan[Tan[e + f*x]])/a + (8*b^(7/2)*(9*a + 8*b)*ArcTan[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*(5*a^3
- 7*a^2*b + 12*a*b^2 + 32*b^3)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e +
 f*x]^2))))/(2*a))/(4*a))/(6*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{b^4\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(9a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{5}{16}a^3-\frac{3}{4}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^5+\left(3ab^2+\frac{5}{6}a^3-2a^2b\right)\mathrm{ta}\cdots}{\left(1+\tan(fx+e)\cdots\right.}}{f}$
default	$\frac{\frac{b^4\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(9a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{5}{16}a^3-\frac{3}{4}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^5+\left(3ab^2+\frac{5}{6}a^3-2a^2b\right)\mathrm{ta}\cdots}{\left(1+\tan(fx+e)\cdots\right.}}{f}$
risch	$\frac{5x}{16a^2}-\frac{3xb}{4a^3}+\frac{3xb^2}{2a^4}-\frac{4xb^3}{a^5}+\frac{15ie^{-2i(fx+e)}}{128a^2f}-\frac{ie^{-4i(fx+e)}b}{32a^3f}-\frac{3ie^{4i(fx+e)}}{128a^2f}+\frac{3ie^{-4i(fx+e)}}{128a^2f}+\frac{ie^{4i(fx+e}\cdots}{32a^3f}$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^4/a^5*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(9*a+8*b)/(a
+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^5*(((5/16*a^
3-3/4*a^2*b+3/2*a*b^2)*tan(f*x+e)^5+(3*a*b^2+5/6*a^3-2*a^2*b)*tan(f*x+e)^3
+(-5/4*a^2*b+3/2*a*b^2+11/16*a^3)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*(5*a
^3-12*a^2*b+24*a*b^2-64*b^3)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.84

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/48*(3*(5*a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*
x + e)^2 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x +
6*(9*a*b^4 + 8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b)
)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2)) + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*
a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*co
s(f*x + e)^3 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e
))*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f), 1
/48*(3*(5*a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x
+ e)^2 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x - 12
*(9*a*b^4 + 8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*
arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*
sin(f*x + e))) + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*
a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*co
s(f*x + e)^3 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e
))*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.33

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{24\,(9\,ab^4+8\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+a^5b)\sqrt{(a+b)b}} + \frac{3\,(5\,a^3b-7\,a^2b^2+12\,ab^3+32\,b^4)\tan(fx+e)^7+(15\,a^4+34\,a^3b-41\,a^2b^2+156\,ab^3+288\,b^4)\tan(fx+e)^5+}{(a^5b+a^4b^2)\tan(fx+e)^8+(a^6+5\,a^5b+4\,a^4b^2)\tan(fx+e)^6+a^6+2\,a^5b+}}{}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/48*(24*(9*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 +
a^5*b)*sqrt((a + b)*b)) + (3*(5*a^3*b - 7*a^2*b^2 + 12*a*b^3 + 32*b^4)*tan
(f*x + e)^7 + (15*a^4 + 34*a^3*b - 41*a^2*b^2 + 156*a*b^3 + 288*b^4)*tan(f
*x + e)^5 + (40*a^4 + 17*a^3*b - 35*a^2*b^2 + 204*a*b^3 + 288*b^4)*tan(f*x
 + e)^3 + 3*(11*a^4 + 2*a^3*b - 5*a^2*b^2 + 28*a*b^3 + 32*b^4)*tan(f*x + e
))/((a^5*b + a^4*b^2)*tan(f*x + e)^8 + (a^6 + 5*a^5*b + 4*a^4*b^2)*tan(f*x
 + e)^6 + a^6 + 2*a^5*b + a^4*b^2 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f*x
+ e)^4 + (3*a^6 + 7*a^5*b + 4*a^4*b^2)*tan(f*x + e)^2) + 3*(5*a^3 - 12*a^2
*b + 24*a*b^2 - 64*b^3)*(f*x + e)/a^5)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.97

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{24\,b^4\tan(fx+e)}{(a^5+a^4b)\left(b\tan(fx+e)^2+a+b\right)} + \frac{24\,(9\,ab^4+8\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+a^5b)\sqrt{ab+b^2}} + \frac{3\,(5\,a^3-12\,a^2b+24\,ab^2-64\,b^3)(fx+e)}{a^5} -}{}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/48*(24*b^4*tan(f*x + e)/((a^5 + a^4*b)*(b*tan(f*x + e)^2 + a + b)) + 24*
(9*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x
+ e)/sqrt(a*b + b^2)))/((a^6 + a^5*b)*sqrt(a*b + b^2)) + 3*(5*a^3 - 12*a^2
*b + 24*a*b^2 - 64*b^3)*(f*x + e)/a^5 + (15*a^2*tan(f*x + e)^5 - 36*a*b*ta
n(f*x + e)^5 + 72*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 96*a*b*tan(
f*x + e)^3 + 144*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 60*a*b*tan(f*x
 + e) + 72*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

Mupad [B] (verification not implemented)

Time = 21.46 (sec) , antiderivative size = 3310, normalized size of antiderivative = 11.91

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((tan(e + f*x)*(28*a*b^3 + 2*a^3*b + 11*a^4 + 32*b^4 - 5*a^2*b^2))/(16*a^4
*(a + b)) + (tan(e + f*x)^5*(156*a*b^3 + 34*a^3*b + 15*a^4 + 288*b^4 - 41*
a^2*b^2))/(48*a^4*(a + b)) + (tan(e + f*x)^3*(204*a*b^3 + 17*a^3*b + 40*a^
4 + 288*b^4 - 35*a^2*b^2))/(48*a^4*(a + b)) + (b*tan(e + f*x)^7*(12*a*b^2
- 7*a^2*b + 5*a^3 + 32*b^3))/(16*a^4*(a + b)))/(f*(a + b + tan(e + f*x)^2*
(3*a + 4*b) + tan(e + f*x)^4*(3*a + 6*b) + b*tan(e + f*x)^8 + tan(e + f*x)
^6*(a + 4*b))) - (atan(-((((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 -
 (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12
*b^2) - (tan(e + f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^1
0*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b
 + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5)
- (tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64
*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a
^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*1i)/(3
2*a^5) - (((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4
- (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) + (tan(e +
 f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5120*a^1
1*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 + a^8*b^
2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5) + (tan(e + f*x)*
(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 56...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1142, normalized size of antiderivative = 4.11

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(216*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqr
t(b))*sin(e + f*x)**2*a**2*b**3 + 192*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**4 - 216*sqrt(b
)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*
b**3 - 408*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a
))/sqrt(b))*a*b**4 - 192*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*
x)/2) - sqrt(a))/sqrt(b))*b**5 + 216*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*b**3 + 192*sqrt
(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin
(e + f*x)**2*a*b**4 - 216*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f
*x)/2) + sqrt(a))/sqrt(b))*a**2*b**3 - 408*sqrt(b)*sqrt(a + b)*atan((sqrt(
a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b**4 - 192*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**5 + 8*cos(e +
 f*x)*sin(e + f*x)**7*a**6 + 16*cos(e + f*x)*sin(e + f*x)**7*a**5*b + 8*co
s(e + f*x)*sin(e + f*x)**7*a**4*b**2 - 34*cos(e + f*x)*sin(e + f*x)**5*a**
6 - 52*cos(e + f*x)*sin(e + f*x)**5*a**5*b - 2*cos(e + f*x)*sin(e + f*x)**
5*a**4*b**2 + 16*cos(e + f*x)*sin(e + f*x)**5*a**3*b**3 + 59*cos(e + f*x)*
sin(e + f*x)**3*a**6 + 60*cos(e + f*x)*sin(e + f*x)**3*a**5*b - 9*cos(e +
f*x)*sin(e + f*x)**3*a**4*b**2 + 38*cos(e + f*x)*sin(e + f*x)**3*a**3*b**3
 + 48*cos(e + f*x)*sin(e + f*x)**3*a**2*b**4 - 33*cos(e + f*x)*sin(e + ...
```

3.206 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\sin(e+fx)}{8(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
3/8*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(1/2)/(a+b)^(5/2)/f+1/4*sin(
f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+3/8*sin(f*x+e)/(a+b)^2/f/(a+b-a*sin(
f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{2(7a+10b+3a\cos(2(e+fx)))\sin(e+fx)}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8f}$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) +
 (2*(7*a + 10*b + 3*a*Cos[2*(e + f*x)])*Sin[e + f*x])/((a + b)^2*(a + 2*b
+ a*Cos[2*(e + f*x)])^2))/(8*f)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4635, 215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^5}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \frac{1}{(-a\sin^2(e+fx)+a+b)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{3\int \frac{1}{(-a\sin^2(e+fx)+a+b)^2}d\sin(e+fx)}{4(a+b)}+\frac{\sin(e+fx)}{4(a+b)(-a\sin^2(e+fx)+a+b)^2}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{2(a+b)}+\frac{\sin(e+fx)}{2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}\right)}{4(a+b)}+\frac{\sin(e+fx)}{4(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}}+\frac{\sin(e+fx)}{2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}\right)}{4(a+b)}+\frac{\sin(e+fx)}{4(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

input
```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Sin[e + f*x]/(4*(a + b)*(a + b - a*Sin[e + f*x]^2)^2) + (3*(ArcTanh[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)) + Sin[e + f*x]/(2
*(a + b)*(a + b - a*Sin[e + f*x]^2))))/(4*(a + b)))/f
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{4(a+b)\left(-a-b+a\sin(fx+e)^2\right)^2}+\frac{-\frac{3\sin(fx+e)}{8(a+b)\left(-a-b+a\sin(fx+e)^2\right)}+\frac{3\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a+b)\sqrt{a(a+b)}}}{a+b}}{f}$
default	$\frac{\frac{\sin(fx+e)}{4(a+b)\left(-a-b+a\sin(fx+e)^2\right)^2}+\frac{-\frac{3\sin(fx+e)}{8(a+b)\left(-a-b+a\sin(fx+e)^2\right)}+\frac{3\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a+b)\sqrt{a(a+b)}}}{a+b}}{f}$
risch	$-\frac{i\left(3a\,\mathrm{e}^{7i(fx+e)}+11a\,\mathrm{e}^{5i(fx+e)}+20b\,\mathrm{e}^{5i(fx+e)}-11a\,\mathrm{e}^{3i(fx+e)}-20b\,\mathrm{e}^{3i(fx+e)}-3a\,\mathrm{e}^{i(fx+e)}\right)}{4(a+b)^2f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)^2}+\frac{3\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a\cdots}{\cdots}\right.}{16\sqrt{a^2+ab}\,(\cdots}$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*sin(f*x+e)/(a+b)/(-a-b+a*sin(f*x+e)^2)^2+3/4/(a+b)*(-1/2*sin(f*x+
e)/(a+b)/(-a-b+a*sin(f*x+e)^2)+1/2/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x
+e)/(a*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(100) = 200$.

Time = 0.12 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.37

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(2\,\cdots}{16\,\left((a^6+3\,a^5b+3\,a^4b^2+a^3b^3)f\cos(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4)f\cos(\cdots}\right.$$
$$-\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(2\,a^3+7\,a^2b+5\cdots\right.}{8\left((a^6+3\,a^5b+3\,a^4b^2+a^3b^3)f\cos(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4)f\cos(fx+e\cdots\right.}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a^2 + a*b)
*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos
(f*x + e)^2 + b)) + 2*(2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x
 + e)^2)*sin(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x +
e)^4 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4
*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f), -1/8*(3*(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x
 + e)/(a + b)) - (2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x + e)
^2)*sin(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
+ 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2
+ 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f)]
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.66

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\left(3\,a\sin(fx+e)^3-5\,(a+b)\sin(fx+e)\right)}{(a^4+2\,a^3b+a^2b^2)\sin(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4-2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sin(fx+e)^2}+\frac{3\,\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+2\,ab+b^2)}}{16\,f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*(3*a*sin(f*x + e)^3 - 5*(a + b)*sin(f*x + e))/((a^4 + 2*a^3*b + a
^2*b^2)*sin(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sin(f*x + e)^2) + 3*log((a*sin(f*x + e) -
 sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(sqrt((a + b)*a)*(a^
2 + 2*a*b + b^2)))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\,\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{-a^2-ab}}+\frac{3\,a\sin(fx+e)^3-5\,a\sin(fx+e)-5\,b\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)^2(a^2+2\,ab+b^2)}}{8\,f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + 2*a*b + b^2)*sqrt(
-a^2 - a*b)) + (3*a*sin(f*x + e)^3 - 5*a*sin(f*x + e) - 5*b*sin(f*x + e))/
((a*sin(f*x + e)^2 - a - b)^2*(a^2 + 2*a*b + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$=\frac{\frac{5\sin(e+fx)}{8(a+b)}-\frac{3a\sin(e+fx)^3}{8(a+b)^2}}{f\left(2ab+a^2+b^2-\sin(e+fx)^2(2a^2+2ba)+a^2\sin(e+fx)^4\right)}+\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,f\,(a+b)^{5/2}}$$

input

```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

output

```
((5*sin(e + f*x))/(8*(a + b)) - (3*a*sin(e + f*x)^3)/(8*(a + b)^2))/(f*(2*
a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^2) + a^2*sin(e + f*x)^4)) +
(3*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(8*a^(1/2)*f*(a + b)^(5/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 834, normalized size of antiderivative = 7.72

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
( - 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b
) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**2 + 6*sqrt(a)*sqrt(a +
b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e +
f*x)/2))*sin(e + f*x)**2*a**2 + 6*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan(
(e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**
2*a*b - 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a
 + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**2 - 6*sqrt(a)*sqrt(a + b)*log(sqrt(
a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a*b
 - 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
 - 2*sqrt(a)*tan((e + f*x)/2))*b**2 + 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b
)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e +
f*x)**4*a**2 - 6*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 +
 sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2 - 6*sqrt(a
)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a
)*tan((e + f*x)/2))*sin(e + f*x)**2*a*b + 3*sqrt(a)*sqrt(a + b)*log(sqrt(a
 + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a**2
 + 6*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
 + 2*sqrt(a)*tan((e + f*x)/2))*a*b + 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)
*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*b**2 - 6*
sin(e + f*x)**3*a**3 - 6*sin(e + f*x)**3*a**2*b + 10*sin(e + f*x)*a**3 ...
```

3.207 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{(4a+b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}f} - \frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\sin(e+fx)}{8a(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/8*(4*a+b)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(3/2)/(a+b)^(5/2)/f-
1/4*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+1/8*(4*a+b)*sin(f*x+e)/a
/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.30

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(\frac{8\sin(e+fx)}{(a+b-a\sin^2(e+fx))^2}-(4a+b)\left(\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{4\sin(}{(a-}\right.\right.}{192af\,(a+b\sec^2(e+fx))^3}$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/192*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((8*Sin[e + f*x])/
(a + b - a*Sin[e + f*x]^2)^2 - (4*a + b)*((3*ArcTanh[(Sqrt[a]*Sin[e + f*x]
)/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (4*Sin[e + f*x]*(5*(a + b) - 3*a
*Sin[e + f*x]^2))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))))/(a*f*(a
+ b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4635, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(e+fx)^3}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \ 4635$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{(-a\sin^2(e+fx)+a+b)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(4a+b)\int \frac{1}{\left(-a\sin^2(e+fx)+a+b\right)^2}d\sin(e+fx)}{4a(a+b)}-\frac{b\sin(e+fx)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{(4a+b)\left(\frac{\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{2(a+b)}+\frac{\sin(e+fx)}{2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}\right)}{4a(a+b)}-\frac{b\sin(e+fx)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(4a+b)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}}+\frac{\sin(e+fx)}{2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}\right)}{4a(a+b)}-\frac{b\sin(e+fx)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Sin[e + f*x])/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^2) + ((4*a +
b)*(ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2))
+ Sin[e + f*x]/(2*(a + b)*(a + b - a*Sin[e + f*x]^2))))/(4*a*(a + b)))/f
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{-\frac{(4a+b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(4a-b)\sin(fx+e)}{8a(a+b)}}{\left(-a-b+a\sin(fx+e)^2\right)^2}+\frac{(4a+b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{a(a+b)}}}{f}$
default	$\frac{\frac{-\frac{(4a+b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(4a-b)\sin(fx+e)}{8a(a+b)}}{\left(-a-b+a\sin(fx+e)^2\right)^2}+\frac{(4a+b)\,\mathrm{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{a(a+b)}}}{f}$
risch	$-\frac{i\left(4a^2 e^{7i(fx+e)}+ab\,e^{7i(fx+e)}+4a^2 e^{5i(fx+e)}+9ab\,e^{5i(fx+e)}-4b^2 e^{5i(fx+e)}-4a^2 e^{3i(fx+e)}-9ab\,e^{3i(fx+e)}+4b^2 e^{3i(fx}\right.}{4a(a+b)^2 f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*((-1/8*(4*a+b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(4*a-b)/a/(a+b)*sin(f*
x+e))/(-a-b+a*sin(f*x+e)^2)^2+1/8*(4*a+b)/(a^2+2*a*b+b^2)/a/(a*(a+b))^(1/2
)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(117) = 234.

Time = 0.11 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.35

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{\left((4\,a^3+a^2b)\cos\left(fx+e\right)^4+4\,ab^2+b^3+2\,(4\,a^2b+ab^2)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\,\sqrt{a}}{a\cos(}\right.}{16\,\left((a^7+3\,a^6b+3\,a^5b^2+a^4b^3)f\cos\left(fx+e\right)^4+2\,(a^6b+3\,a^5b^2+3\,a^4\right.}\right.$$
$$-\frac{\left((4\,a^3+a^2b)\cos\left(fx+e\right)^4+4\,ab^2+b^3+2\,(4\,a^2b+ab^2)\cos\left(fx+e\right)^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin}{a+b}\right.}{8\,\left((a^7+3\,a^6b+3\,a^5b^2+a^4b^3)f\cos\left(fx+e\right)^4+2\,(a^6b+3\,a^5b^2+3\,a^4b^3+a^3\right.}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*b + a*b^
2)*cos(f*x + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a
*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3*b + a^2*b^2
 - a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^7
 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2
+ 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
 + a^2*b^5)*f), -1/8*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*
(4*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)
*sin(f*x + e)/(a + b)) - (2*a^3*b + a^2*b^2 - a*b^3 + (4*a^4 + 5*a^3*b + a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^
3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*
x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.70

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{(4a+b)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^3+2a^2b+ab^2)\sqrt{(a+b)a}}+\frac{2\left((4a^2+ab)\sin(fx+e)^3-(4a^2+3ab-b^2)\sin(fx+e)\right)}{a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4+(a^5+2a^4b+a^3b^2)\sin(fx+e)^4-2(a^5+3a^4b+3a^3b^2+a^2b^3)\sin(fx+}}{16f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*((4*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) +
sqrt((a + b)*a)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*a)) + 2*((4*a^2 +
a*b)*sin(f*x + e)^3 - (4*a^2 + 3*a*b - b^2)*sin(f*x + e))/(a^5 + 4*a^4*b +
 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 2*a^4*b + a^3*b^2)*sin(f*x + e)^4
- 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.24

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= -\frac{\frac{(4a+b)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^3+2a^2b+ab^2)\sqrt{-a^2-ab}}+\frac{4a^2\sin(fx+e)^3+ab\sin(fx+e)^3-4a^2\sin(fx+e)-3ab\sin(fx+e)+b^2\sin(fx+e)}{(a^3+2a^2b+ab^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{8f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((4*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + 2*a^2*b +
a*b^2)*sqrt(-a^2 - a*b)) + (4*a^2*sin(f*x + e)^3 + a*b*sin(f*x + e)^3 - 4*
a^2*sin(f*x + e) - 3*a*b*sin(f*x + e) + b^2*sin(f*x + e))/((a^3 + 2*a^2*b
+ a*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

Mupad [B] (verification not implemented)

Time = 16.75 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(4\,a+b)}{8\,a^{3/2}\,f\,(a+b)^{5/2}} - \frac{\frac{\sin(e+f\,x)^3\,(4\,a+b)}{8\,(a+b)^2} - \frac{\sin(e+f\,x)\,(4\,a-b)}{8\,a\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2-\sin\left(e+f\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)+a^2\sin\left(e+f\,x\right)^4\right)}$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
(atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + b))/(8*a^(3/2)*f*(a +
b)^(5/2)) - ((sin(e + f*x)^3*(4*a + b))/(8*(a + b)^2) - (sin(e + f*x)*(4*a
 - b))/(8*a*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^
2) + a^2*sin(e + f*x)^4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1184, normalized size of antiderivative = 9.47

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
( - 4*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b
) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**3 - sqrt(a)*sqrt(a + b)
*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*
x)/2))*sin(e + f*x)**4*a**2*b + 8*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan(
(e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**
2*a**3 + 10*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt
(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b + 2*sqrt(a)*s
qrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*t
an((e + f*x)/2))*sin(e + f*x)**2*a*b**2 - 4*sqrt(a)*sqrt(a + b)*log(sqrt(a
 + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**3
 - 9*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
 - 2*sqrt(a)*tan((e + f*x)/2))*a**2*b - 6*sqrt(a)*sqrt(a + b)*log(sqrt(a +
 b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a*b**2
 - sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) -
 2*sqrt(a)*tan((e + f*x)/2))*b**3 + 4*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*
tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*
x)**4*a**3 + sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqr
t(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**2*b - 8*sqrt(a)*
sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*
tan((e + f*x)/2))*sin(e + f*x)**2*a**3 - 10*sqrt(a)*sqrt(a + b)*log(sqr...
```

3.208 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{(8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}f} + \frac{b^2\sin(e+fx)}{4a^2(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{b(8a+5b)\sin(e+fx)}{8a^2(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(5/2)/(a
+b)^(5/2)/f+1/4*b^2*sin(f*x+e)/a^2/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2-1/8*b*(8
*a+5*b)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.41 (sec) , antiderivative size = 927, normalized size of antiderivative = 6.62

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(-2i(8a^2+8ab+3b^2)\arctan\left(\frac{(a+b)\sin(e}{(a+b)\cos(e)-\sqrt{a}\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e)}}\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((-2*I)*(8*a^2 + 8*a*b + 3*
b^2)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(a + 2*b + a*C
os[2*(e + f*x)])^2*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + (8*a^2 + 8*a*b + 3*b
^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*
(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] -
 I*Sin[e])^2]*Sin[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - (8*a^2 +
8*a*b + 3*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[-a - 2*(a + b)*Cos[2*e
] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sq
rt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e])
 + 2*(8*a^2 + 8*a*b + 3*b^2)*ArcTan[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2
*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*S
qrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin
[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqr
t[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)
*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e + 2*f*x] + I*a*Cos[3*e + 2*f*x] +
 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*Sin[3*e] + a*Sin[e + 2*f*x] - a*Si
n[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]*(I*Cos[e...
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4635, 315, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \text{ 4635}$$

$$\frac{\int \frac{(1-\sin^2(e+fx))^2}{(-a\sin^2(e+fx)+a+b)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{-\frac{\int -\frac{-((4a+3b)\sin^2(e+fx))+4a+b}{\left(-a\sin^2(e+fx)+a+b\right)^2}d\sin(e+fx)}{4a(a+b)}-\frac{b\sin(e+fx)(1-\sin^2(e+fx))}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{-((4a+3b)\sin^2(e+fx))+4a+b}{\left(-a\sin^2(e+fx)+a+b\right)^2}d\sin(e+fx)}{4a(a+b)}-\frac{b\sin(e+fx)(1-\sin^2(e+fx))}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{2a(a+b)}-\frac{3b(2a+b)\sin(e+fx)}{2a(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{4a(a+b)}-\frac{b\sin(e+fx)(1-\sin^2(e+fx))}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\frac{(8a^2+8ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{3b(2a+b)\sin(e+fx)}{2a(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{4a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Sin[e + f*x]*(1 - Sin[e + f*x]^2))/(a*(a + b)*(a + b - a*Sin[e +
f*x]^2)^2) + (((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt
[a + b]])/(2*a^(3/2)*(a + b)^(3/2)) - (3*b*(2*a + b)*Sin[e + f*x])/(2*a*(a
 + b)*(a + b - a*Sin[e + f*x]^2)))/(4*a*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{-\frac{b(8a+5b)\sin(fx+e)^3}{8a\left(a^2+2ab+b^2\right)}+\frac{(8a+3b)b\sin(fx+e)}{8a^2(a+b)}}{\left(-a-b+a\sin(fx+e)^2\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a^2\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{-\frac{b(8a+5b)\sin(fx+e)^3}{8a\left(a^2+2ab+b^2\right)}+\frac{(8a+3b)b\sin(fx+e)}{8a^2(a+b)}}{\left(-a-b+a\sin(fx+e)^2\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a^2\sqrt{a(a+b)}}}{f}$
risch	$\frac{ib\left(8a^2e^{7i(fx+e)}+5ab\,e^{7i(fx+e)}+8a^2e^{5i(fx+e)}+29ab\,e^{5i(fx+e)}+12b^2e^{5i(fx+e)}-8a^2e^{3i(fx+e)}-29ab\,e^{3i(fx+e)}-12b^2e^{3}\right.}{4a^2(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(8*a+3*b)/a^2*b
/(a+b)*sin(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2+1/8*(8*a^2+8*a*b+3*b^2)/(a^2+2*
a*b+b^2)/a^2/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(132) = 264$.

Time = 0.12 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.38

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{\left((8a^4+8a^3b+3a^2b^2)\cos(fx+e)^4+8a^2b^2+8ab^3+3b^4+2(8a^3b+8a^2b^2+3ab^3)\cos(fx+e)^2\right)\cdots}{16\left((a^8+3a^7b+3a^6b^2+a^5b^3)f\cos(fx+e)^4+2\cdots\right.}\right.$$
$$-\frac{\left((8a^4+8a^3b+3a^2b^2)\cos(fx+e)^4+8a^2b^2+8ab^3+3b^4+2(8a^3b+8a^2b^2+3ab^3)\cos(fx+e)^2\right)\cdots}{8\left((a^8+3a^7b+3a^6b^2+a^5b^3)f\cos(fx+e)^4+2(a^7b+\cdots\right.}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3
 + 3*b^4 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(a^2 + a*
b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*c
os(f*x + e)^2 + b)) - 2*(6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (8*a^4*b + 13*a
^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a^7*b + 3*a^6*
b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b
^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f), -1/
8*(((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3 + 3
*b^4 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*
arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*a^3*b^2 + 9*a^2*b^3 + 3
*a*b^4 + (8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/
((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6
*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^
4*b^4 + a^3*b^5)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^3}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.66

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=-\frac{\frac{\left(8\,a^{2}+8\,ab+3\,b^{2}\right)\log\left(\frac{a\sin\left(fx+e\right)-\sqrt{\left(a+b\right)a}}{a\sin\left(fx+e\right)+\sqrt{\left(a+b\right)a}}\right)}{\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)\sqrt{\left(a+b\right)a}}-\frac{2\left(\left(8\,a^{2}b+5\,ab^{2}\right)\sin\left(fx+e\right)^{3}-\left(8\,a^{2}b+11\,ab^{2}+3\,b^{3}\right)\sin\left(fx+e\right)\right)}{a^{6}+4\,a^{5}b+6\,a^{4}b^{2}+4\,a^{3}b^{3}+a^{2}b^{4}+\left(a^{6}+2\,a^{5}b+a^{4}b^{2}\right)\sin\left(fx+e\right)^{4}-2\left(a^{6}+3\,a^{5}b+3\,a^{4}b^{2}+a^{3}b\cdots\right.}}{16\,f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*((8*a^2 + 8*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*s
in(f*x + e) + sqrt((a + b)*a)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*a)
) - 2*((8*a^2*b + 5*a*b^2)*sin(f*x + e)^3 - (8*a^2*b + 11*a*b^2 + 3*b^3)*s
in(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^6 + 2*a
^5*b + a^4*b^2)*sin(f*x + e)^4 - 2*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*s
in(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.27

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(8\,a^2+8\,ab+3\,b^2)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^4+2\,a^3b+a^2b^2)\sqrt{-a^2-ab}}-\frac{8\,a^2b\sin(fx+e)^3+5\,ab^2\sin(fx+e)^3-8\,a^2b\sin(fx+e)-11\,ab^2\sin(fx+e)-3\,b^3\sin(fx+e)}{(a^4+2\,a^3b+a^2b^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{8\,f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((8*a^2 + 8*a*b + 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^
4 + 2*a^3*b + a^2*b^2)*sqrt(-a^2 - a*b)) - (8*a^2*b*sin(f*x + e)^3 + 5*a*b
^2*sin(f*x + e)^3 - 8*a^2*b*sin(f*x + e) - 11*a*b^2*sin(f*x + e) - 3*b^3*s
in(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{\sin(e+f\,x)^3\,(5\,b^2+8\,a\,b)}{8\,a\,(a+b)^2}-\frac{\sin(e+f\,x)\,(3\,b^2+8\,a\,b)}{8\,a^2\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2-\sin\left(e+f\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)+a^2\sin\left(e+f\,x\right)^4\right)} + \frac{\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(8\,a^2+8\,a\,b+3\,b^2)}{8\,a^{5/2}\,f\,(a+b)^{5/2}}$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
((sin(e + f*x)^3*(8*a*b + 5*b^2))/(8*a*(a + b)^2) - (sin(e + f*x)*(8*a*b +
 3*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b +
2*a^2) + a^2*sin(e + f*x)^4)) + (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2
))*(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*f*(a + b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1518, normalized size of antiderivative = 10.84

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 8*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b
) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**4 - 8*sqrt(a)*sqrt(a +
b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e +
f*x)/2))*sin(e + f*x)**4*a**3*b - 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*ta
n((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)
**4*a**2*b**2 + 16*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2
 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**4 + 32*sqr
t(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqr
t(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**3*b + 22*sqrt(a)*sqrt(a + b)*log
(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2
))*sin(e + f*x)**2*a**2*b**2 + 6*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((
e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2
*a*b**3 - 8*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt
(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**4 - 24*sqrt(a)*sqrt(a + b)*log(sq
rt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*
a**3*b - 27*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt
(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**2*b**2 - 14*sqrt(a)*sqrt(a + b)*l
og(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)
/2))*a*b**3 - 3*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 +
sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*b**4 + 8*sqrt(a)*sqrt(a + b)*...
```

3.209 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{3b(8a^2+12ab+5b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{a^3f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3b^2(4a+3b)\sin(e+fx)}{8a^3(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
-3/8*b*(8*a^2+12*a*b+5*b^2)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(7/2
)/(a+b)^(5/2)/f+sin(f*x+e)/a^3/f-1/4*b^3*sin(f*x+e)/a^3/(a+b)/f/(a+b-a*sin
(f*x+e)^2)^2+3/8*b^2*(4*a+3*b)*sin(f*x+e)/a^3/(a+b)^2/f/(a+b-a*sin(f*x+e)^
2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{-\frac{3b(8a^2+12ab+5b^2)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{4\sqrt{a}\sin(e+fx)(8a^4+32a^3b+60a^2b^2+51ab^3+15b^4-a(16a^3+48a^2b+60ab^2+25b^3)\sin^2(e+}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8a^{7/2}f}$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]
])/(a + b)^(5/2) + (4*Sqrt[a]*Sin[e + f*x]*(8*a^4 + 32*a^3*b + 60*a^2*b^2
+ 51*a*b^3 + 15*b^4 - a*(16*a^3 + 48*a^2*b + 60*a*b^2 + 25*b^3)*Sin[e + f*
x]^2 + 8*a^2*(a + b)^2*Sin[e + f*x]^4))/((a + b)^2*(a + 2*b + a*Cos[2*(e +
 f*x)])^2))/(8*a^(7/2)*f)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sec(e+fx)\left(a+b\sec(e+fx)^2\right)^3}dx$$
$$\Big\downarrow \; 4635$$
$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^3}{\left(-a\sin^2(e+fx)+a+b\right)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{a^3}-\frac{3a^2b\sin^4(e+fx)-3ab(2a+b)\sin^2(e+fx)+b(3a^2+3ba+b^2)}{a^3\left(-a\sin^2(e+fx)+a+b\right)^3}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{3b(4(a+b)^2+(2a+b)^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}}-\frac{b^3\sin(e+fx)}{4a^3(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sin(e+fx)}{8a^3(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{a^3}}{f}$$

input
```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
 b]])/(8*a^(7/2)*(a + b)^(5/2)) + Sin[e + f*x]/a^3 - (b^3*Sin[e + f*x])/(4
*a^3*(a + b)*(a + b - a*Sin[e + f*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin[e + f*
x])/(8*a^3*(a + b)^2*(a + b - a*Sin[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
 + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{a^3}+\frac{b\left(\frac{-\frac{3ab(4a+3b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{(12a+7b)b\sin(fx+e)}{8a+8b}}{(-a-b+a\sin(fx+e)^2)^2}-\frac{3(8a^2+12ab+5b^2)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a^2+2ab+b^2)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
default	$\frac{\frac{\sin(fx+e)}{a^3}+\frac{b\left(\frac{-\frac{3ab(4a+3b)\sin(fx+e)^3}{8(a^2+2ab+b^2)}+\frac{(12a+7b)b\sin(fx+e)}{8a+8b}}{(-a-b+a\sin(fx+e)^2)^2}-\frac{3(8a^2+12ab+5b^2)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a^2+2ab+b^2)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
risch	$-\frac{ie^{i(fx+e)}}{2a^3f}+\frac{ie^{-i(fx+e)}}{2a^3f}-\frac{ib^2(12a^2e^{7i(fx+e)}+9ab\,e^{7i(fx+e)}+12a^2e^{5i(fx+e)}+49ab\,e^{5i(fx+e)}+28b^2e^{5i(fx+e)}-12}{4a^3(a+b)^2f(a\,e^{4i(fx+e)}+2a\,e^{2i(f}}$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)/a^3+b/a^3*((-3/8*a*b*(4*a+3*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^
3+1/8*(12*a+7*b)*b/(a+b)*sin(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2-3/8*(8*a^2+12
*a*b+5*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))
^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(148) = 296.

Time = 0.13 (sec) , antiderivative size = 727, normalized size of antiderivative = 4.66

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3
)*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x + e)^2)*sq
rt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*
a - b)/(a*cos(f*x + e)^2 + b)) + 2*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 +
15*a*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(f*x + e)^4 + (16*a^
5*b + 60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/
((a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^4 + 2*(a^8*b + 3*a^7
*b^2 + 3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^
5*b^4 + a^4*b^5)*f), 1/8*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*
a^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)
*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a
+ b)) + (8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5 + 8*(a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b + 60*a^4*b^2 + 69*a^3*b
^3 + 25*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9 + 3*a^8*b + 3*a^7*b^2
 + a^6*b^3)*f*cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 + 3*a^6*b^3 + a^5*b^4)
*f*cos(f*x + e)^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\,(8\,a^2b+12\,ab^2+5\,b^3)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)a}} - \frac{2\left(3\,(4\,a^2b^2+3\,ab^3)\sin(fx+e)^3-(12\,a^2b^2+19\,ab^3+7\,b^4)\sin(fx+e)\right)}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(a^7+2\,a^6b+a^5b^2)\sin(fx+e)^4-2\,(a^7+3\,a^6b+3\,a^5b^2+a}}{16\,f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/16*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a)
)/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a +
 b)*a)) - 2*(3*(4*a^2*b^2 + 3*a*b^3)*sin(f*x + e)^3 - (12*a^2*b^2 + 19*a*b
^3 + 7*b^4)*sin(f*x + e))/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4
 + (a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2
 + a^4*b^3)*sin(f*x + e)^2) + 16*sin(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\,(8\,a^2b+12\,ab^2+5\,b^3)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{-a^2-ab}} - \frac{12\,a^2b^2\sin(fx+e)^3+9\,ab^3\sin(fx+e)^3-12\,a^2b^2\sin(fx+e)-19\,ab^3\sin(fx+e)-7\,b^4\sin(fx+e)}{(a^5+2\,a^4b+a^3b^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{8\,f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b)
)/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(-a^2 - a*b)) - (12*a^2*b^2*sin(f*x + e)^
3 + 9*a*b^3*sin(f*x + e)^3 - 12*a^2*b^2*sin(f*x + e) - 19*a*b^3*sin(f*x +
e) - 7*b^4*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*(a*sin(f*x + e)^2 - a
- b)^2) + 8*sin(f*x + e)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 17.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.12

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\sin(e+f\,x)}{a^3\,f} + \frac{\frac{\sin(e+f\,x)\,(7\,b^3+12\,a\,b^2)}{8\,(a+b)} - \frac{3\sin(e+f\,x)^3\,(4\,a^2\,b^2+3\,a\,b^3)}{8\,(a+b)^2}}{f\,\left(2\,a^4\,b-\sin(e+f\,x)^2\,(2\,a^5+2\,b\,a^4)+a^5+a^3\,b^2+a^5\sin(e+f\,x)^4\right)} - \frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(8\,a^2+12\,a\,b+5\,b^2)}{8\,a^{7/2}\,f\,(a+b)^{5/2}}$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
sin(e + f*x)/(a^3*f) + ((sin(e + f*x)*(12*a*b^2 + 7*b^3))/(8*(a + b)) - (3
*sin(e + f*x)^3*(3*a*b^3 + 4*a^2*b^2))/(8*(a + b)^2))/(f*(2*a^4*b - sin(e
+ f*x)^2*(2*a^4*b + 2*a^5) + a^5 + a^3*b^2 + a^5*sin(e + f*x)^4)) - (3*b*a
tanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(12*a*b + 8*a^2 + 5*b^2))/(8*a^
(7/2)*f*(a + b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1649, normalized size of antiderivative = 10.57

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
(24*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
- 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**4*b + 36*sqrt(a)*sqrt(a +
 b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e +
 f*x)/2))*sin(e + f*x)**4*a**3*b**2 + 15*sqrt(a)*sqrt(a + b)*log(sqrt(a +
b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e +
 f*x)**4*a**2*b**3 - 48*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/
2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**4*b -
 120*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
 - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**3*b**2 - 102*sqrt(a)*sqr
t(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan
((e + f*x)/2))*sin(e + f*x)**2*a**2*b**3 - 30*sqrt(a)*sqrt(a + b)*log(sqrt
(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*si
n(e + f*x)**2*a*b**4 + 24*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x
)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**4*b + 84*sqrt(a)*sq
rt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*ta
n((e + f*x)/2))*a**3*b**2 + 111*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e
 + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**2*b**3 + 66*s
qrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*s
qrt(a)*tan((e + f*x)/2))*a*b**4 + 15*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*t
an((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*b**5 - 2...
```

3.210 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\frac{b^2(48a^2+80ab+35b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}(a+b)^{5/2}f}+\frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}+\frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/8*b^2*(48*a^2+80*a*b+35*b^2)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a^(
9/2)/(a+b)^(5/2)/f+(a-3*b)*sin(f*x+e)/a^4/f-1/3*sin(f*x+e)^3/a^3/f+1/4*b^4
*sin(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2-1/8*b^3*(16*a+13*b)*sin(f*x
+e)/a^4/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{-\frac{3b^2(48a^2+80ab+35b^2)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{5/2}}+12\sqrt{a}\left(-12b-\frac{b^4(9a+22b+13a\cos(2(e+fx)))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}\right.}{48a^{9/2}f}$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((-3*b^2*(48*a^2 + 80*a*b + 35*b^2)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x
]] - Log[Sqrt[a + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(5/2) + 12*Sqrt[a]*
(-12*b - (b^4*(9*a + 22*b + 13*a*Cos[2*(e + f*x)]))/((a + b)^2*(a + 2*b +
a*Cos[2*(e + f*x)])^2) + a*(3 - (16*b^3)/((a + b)^2*(a + 2*b + a*Cos[2*(e
+ f*x)]))))*Sin[e + f*x] + 4*a^(3/2)*Sin[3*(e + f*x)])/(48*a^(9/2)*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^3\,(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \text{4635}$$

$$\frac{\int \frac{(1-\sin^2(e+fx))^4}{(-a\sin^2(e+fx)+a+b)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-\frac{\sin^2(e+fx)}{a^3}+\frac{a-3b}{a^4}+\frac{6a^2b^2\sin^4(e+fx)-4ab^2(3a+2b)\sin^2(e+fx)+b^2(6a^2+8ba+3b^2)}{a^4(-a\sin^2(e+fx)+a+b)^3}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^4\sin(e+fx)}{4a^4(a+b)(-a\sin^2(e+fx)+a+b)^2}-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4(a+b)^2(-a\sin^2(e+fx)+a+b)}+\frac{(a-3b)\sin(e+fx)}{a^4}-\frac{\sin^3(e+fx)}{3a^3}+\frac{b^2(48a^2+80ab+35b^2)\text{arctan}}{8a^{9/2}(a+b)^{5/}}}{f}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((b^2*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(9/2)*(a + b)^(5/2)) + ((a - 3*b)*Sin[e + f*x])/a^4 - Sin[e + f*x
]^3/(3*a^3) + (b^4*Sin[e + f*x])/(4*a^4*(a + b)*(a + b - a*Sin[e + f*x]^2)
^2) - (b^3*(16*a + 13*b)*Sin[e + f*x])/(8*a^4*(a + b)^2*(a + b - a*Sin[e +
 f*x]^2)))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+3\sin(fx+e)b}{a^4}-\frac{b^2\left(\frac{-\frac{ab(16a+13b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(16a+11b)b\sin(fx+e)}{8a+8b}}{\left(-a-b+a\sin(fx+e)^2\right)^2}-\frac{\left(48a^2+80ab+35b^2\right)\operatorname{arctan}\cdots}{8\left(a^2+2ab+b^2\right)\sqrt{\cdots}}\right.}{a^4}}{f}$
default	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+3\sin(fx+e)b}{a^4}-\frac{b^2\left(\frac{-\frac{ab(16a+13b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(16a+11b)b\sin(fx+e)}{8a+8b}}{\left(-a-b+a\sin(fx+e)^2\right)^2}-\frac{\left(48a^2+80ab+35b^2\right)\operatorname{arctan}\cdots}{8\left(a^2+2ab+b^2\right)\sqrt{\cdots}}\right.}{a^4}}{f}$
risch	$-\frac{i\mathrm{e}^{3i(fx+e)}}{24a^3f}-\frac{3i\mathrm{e}^{i(fx+e)}}{8a^3f}+\frac{3i\mathrm{e}^{i(fx+e)}b}{2a^4f}+\frac{3i\mathrm{e}^{-i(fx+e)}}{8a^3f}-\frac{3i\mathrm{e}^{-i(fx+e)}b}{2a^4f}+\frac{i\mathrm{e}^{-3i(fx+e)}}{24a^3f}+\frac{ib^3\left(16a^2\mathrm{e}^{7i(fx+e}\cdots\right.}{\cdots}$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^4*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+3*sin(f*x+e)*b)-b^2/a^4*((-1/
8*a*b*(16*a+13*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(16*a+11*b)*b/(a+b)*sin
(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2-1/8*(48*a^2+80*a*b+35*b^2)/(a^2+2*a*b+b^2
)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(171) = 342$.

Time = 0.15 (sec) , antiderivative size = 856, normalized size of antiderivative = 4.73

$$\int \frac{\cos^3(e + fx)}{(a + b\sec^2(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/48*(3*(48*a^2*b^4 + 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a
^2*b^4)*cos(f*x + e)^4 + 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x +
e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x +
 e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(16*a^5*b^2 - 24*a^4*b^3 - 210*
a^3*b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3
)*cos(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4)
*cos(f*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175
*a^2*b^5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7
*b^3)*f*cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos
(f*x + e)^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f), -1/24*(3*(48
*a^2*b^4 + 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^2*b^4)*cos(
f*x + e)^4 + 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(-
a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (16*a^5*b^2 - 2
4*a^4*b^3 - 210*a^3*b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a
^5*b^2 + a^4*b^3)*cos(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*
b^3 - 7*a^3*b^4)*cos(f*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 4
63*a^3*b^4 - 175*a^2*b^5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b +
 3*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3
+ a^6*b^4)*f*cos(f*x + e)^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*
f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.50

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{3\,(48\,a^2b^2+80\,ab^3+35\,b^4)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{(a+b)a}} - \frac{6\left((16\,a^2b^3+13\,ab^4)\sin(fx+e)^3-(16\,a^2b^3+27\,ab^4+11\,b^5)\sin(f}{a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4+(a^8+2\,a^7b+a^6b^2)\sin(fx+e)^4-2\,(a^8+3\,a^7b+3\,a}}{48\,f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/48*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*log((a*sin(f*x + e) - sqrt((a +
b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt
((a + b)*a)) - 6*((16*a^2*b^3 + 13*a*b^4)*sin(f*x + e)^3 - (16*a^2*b^3 + 2
7*a*b^4 + 11*b^5)*sin(f*x + e))/(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a
^4*b^4 + (a^8 + 2*a^7*b + a^6*b^2)*sin(f*x + e)^4 - 2*(a^8 + 3*a^7*b + 3*a
^6*b^2 + a^5*b^3)*sin(f*x + e)^2) + 16*(a*sin(f*x + e)^3 - 3*(a - 3*b)*sin
(f*x + e))/a^4)/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.27

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{3\left(48\,a^2b^2+80\,ab^3+35\,b^4\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\left(a^6+2\,a^5b+a^4b^2\right)\sqrt{-a^2-ab}}-\frac{3\left(16\,a^2b^3\sin(fx+e)^3+13\,ab^4\sin(fx+e)^3-16\,a^2b^3\sin(fx+e)-27\,ab^4\sin(fx+e)-11\,b^5\right.}{\left(a^6+2\,a^5b+a^4b^2\right)\left(a\sin(fx+e)^2-a-b\right)^2}}{24\,f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/24*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2
- a*b))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-a^2 - a*b)) - 3*(16*a^2*b^3*sin(f
*x + e)^3 + 13*a*b^4*sin(f*x + e)^3 - 16*a^2*b^3*sin(f*x + e) - 27*a*b^4*s
in(f*x + e) - 11*b^5*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(a*sin(f*x +
 e)^2 - a - b)^2) + 8*(a^6*sin(f*x + e)^3 - 3*a^6*sin(f*x + e) + 9*a^5*b*s
in(f*x + e))/a^9)/f
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx &= \frac{b^2\,\ln\left(\sqrt{a+b}+\sqrt{a}\,\sin\left(e+f\,x\right)\right)\,\left(3\,a^2+5\,a\,b+\frac{35\,b^2}{16}\right)}{a^{9/2}\,f\,{\left(a+b\right)}^{5/2}} \\ &\quad - \frac{\frac{\sin(e+f\,x)\,\left(11\,b^4+16\,a\,b^3\right)}{8\,(a+b)}-\frac{\sin(e+f\,x)^3\,\left(16\,a^2\,b^3+13\,a\,b^4\right)}{8\,{(a+b)}^2}}{f\,\left(2\,a^5\,b-{\sin\left(e+f\,x\right)}^2\,\left(2\,a^6+2\,b\,a^5\right)+a^6+a^4\,b^2+a^6\,{\sin\left(e+f\,x\right)}^4\right)} \\ &\quad - \frac{{\sin\left(e+f\,x\right)}^3}{3\,a^3\,f}-\frac{b^2\,\ln\left(\sqrt{a}\,\sin\left(e+f\,x\right)-\sqrt{a+b}\right)\,\left(48\,a^2+80\,a\,b+35\,b^2\right)}{16\,a^{9/2}\,f\,{\left(a+b\right)}^{5/2}} \\ &\quad - \frac{\sin\left(e+f\,x\right)\,\left(\frac{3\,(a+b)}{a^4}-\frac{4}{a^3}\right)}{f}\end{aligned}$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(b^2*log((a + b)^(1/2) + a^(1/2)*sin(e + f*x))*(5*a*b + 3*a^2 + (35*b^2)/1
6))/(a^(9/2)*f*(a + b)^(5/2)) - ((sin(e + f*x)*(16*a*b^3 + 11*b^4))/(8*(a
+ b)) - (sin(e + f*x)^3*(13*a*b^4 + 16*a^2*b^3))/(8*(a + b)^2))/(f*(2*a^5*
b - sin(e + f*x)^2*(2*a^5*b + 2*a^6) + a^6 + a^4*b^2 + a^6*sin(e + f*x)^4)
) - sin(e + f*x)^3/(3*a^3*f) - (b^2*log(a^(1/2)*sin(e + f*x) - (a + b)^(1/
2))*(80*a*b + 48*a^2 + 35*b^2))/(16*a^(9/2)*f*(a + b)^(5/2)) - (sin(e + f*
x)*((3*(a + b))/a^4 - 4/a^3))/f
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1766, normalized size of antiderivative = 9.76

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 144*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a +
 b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**4*b**2 - 240*sqrt(a)*
sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*
tan((e + f*x)/2))*sin(e + f*x)**4*a**3*b**3 - 105*sqrt(a)*sqrt(a + b)*log(
sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2)
)*sin(e + f*x)**4*a**2*b**4 + 288*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan(
(e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**
2*a**4*b**2 + 768*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2
+ sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**3*b**3 + 69
0*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) -
2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b**4 + 210*sqrt(a)*sqrt(a
 + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e
 + f*x)/2))*sin(e + f*x)**2*a*b**5 - 144*sqrt(a)*sqrt(a + b)*log(sqrt(a +
b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**4*b*
*2 - 528*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a
+ b) - 2*sqrt(a)*tan((e + f*x)/2))*a**3*b**3 - 729*sqrt(a)*sqrt(a + b)*log
(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2
))*a**2*b**4 - 450*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2
 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a*b**5 - 105*sqrt(a)*sqrt(a +
 b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((...
```

3.211 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b^3(80a^2+140ab+63b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}(a+b)^{5/2}f} + \frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{b^5\sin(e+fx)}{4a^5(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{b^4(20a+17b)\sin(e+fx)}{8a^5(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

output

```
-1/8*b^3*(80*a^2+140*a*b+63*b^2)*arctanh(a^(1/2)*sin(f*x+e)/(a+b)^(1/2))/a
^(11/2)/(a+b)^(5/2)/f+(a^2-3*a*b+6*b^2)*sin(f*x+e)/a^5/f-1/3*(2*a-3*b)*sin
(f*x+e)^3/a^4/f+1/5*sin(f*x+e)^5/a^3/f-1/4*b^5*sin(f*x+e)/a^5/(a+b)/f/(a+b
-a*sin(f*x+e)^2)^2+1/8*b^4*(20*a+17*b)*sin(f*x+e)/a^5/(a+b)^2/f/(a+b-a*sin
(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.26 (sec) , antiderivative size = 2670, normalized size of antiderivative = 12.48

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((5*a^2 - 18*a*b + 48*b^2)*Cos[f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
 + f*x]^6*Sin[e])/(64*a^5*f*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 14
0*a*b^4 - 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((I/128
)*ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin
[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*
e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*S
in[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*S
in[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e
+ f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])
/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*C
os[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e
] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e])/(a^(11/2)*Sqrt[a +
 b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) + (ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] +
 I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*C
os[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*
x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[
a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b
]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Co
s[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] +
 b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - ...
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sec(e+fx)^5\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \; 4635$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^5}{\left(-a\sin^2(e+fx)+a+b\right)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 300$$

$$\frac{\int \left(\frac{\sin^4(e+fx)}{a^3}-\frac{(2a-3b)\sin^2(e+fx)}{a^4}+\frac{a^2-3ba+6b^2}{a^5}-\frac{10a^2b^3\sin^4(e+fx)-5ab^3(4a+3b)\sin^2(e+fx)+b^3\left(10a^2+15ba+6b^2\right)}{a^5\left(-a\sin^2(e+fx)+a+b\right)^3}\right)d\sin(e+ \cdots}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{b^5\sin(e+fx)}{4a^5(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{b^4(20a+17b)\sin(e+fx)}{8a^5(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{(2a-3b)\sin^3(e+fx)}{3a^4}+\frac{\sin^5(e+fx)}{5a^3}-\frac{b^3\left(80a^2+140ab+63b^2\right)\mathrm{a}\cdots}{8a^{11/2}(a\cdots}}{f}$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/8*(b^3*(80*a^2 + 140*a*b + 63*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt
[a + b]])/(a^(11/2)*(a + b)^(5/2)) + ((a^2 - 3*a*b + 6*b^2)*Sin[e + f*x])/
a^5 - ((2*a - 3*b)*Sin[e + f*x]^3)/(3*a^4) + Sin[e + f*x]^5/(5*a^3) - (b^5
*Sin[e + f*x])/(4*a^5*(a + b)*(a + b - a*Sin[e + f*x]^2)^2) + (b^4*(20*a +
 17*b)*Sin[e + f*x])/(8*a^5*(a + b)^2*(a + b - a*Sin[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 6.95 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+ab\sin(fx+e)^3+\sin(fx+e)a^2-3\sin(fx+e)ab+6\sin(fx+e)b^2}{a^5}+\frac{b^3\left(\frac{-\frac{ab(20a+17b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+}{\left(-a-b+a\sin(f\right.}\right.}{}}{f}$
default	$\frac{\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+ab\sin(fx+e)^3+\sin(fx+e)a^2-3\sin(fx+e)ab+6\sin(fx+e)b^2}{a^5}+\frac{b^3\left(\frac{-\frac{ab(20a+17b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+}{\left(-a-b+a\sin(f\right.}\right.}{}}{f}$
risch	$\frac{5i\mathrm{e}^{-i(fx+e)}}{16a^3f}-\frac{i\mathrm{e}^{5i(fx+e)}}{160a^3f}+\frac{5i\mathrm{e}^{-3i(fx+e)}}{96a^3f}+\frac{3i\mathrm{e}^{-i(fx+e)}b^2}{f\,a^5}-\frac{5i\mathrm{e}^{3i(fx+e)}}{96a^3f}+\frac{9i\mathrm{e}^{i(fx+e)}b}{8a^4f}-\frac{ib^4\left(20a^2\mathrm{e}^{7i(fx+e}\right.}{}$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^5*(1/5*a^2*sin(f*x+e)^5-2/3*a^2*sin(f*x+e)^3+a*b*sin(f*x+e)^3+sin
(f*x+e)*a^2-3*sin(f*x+e)*a*b+6*sin(f*x+e)*b^2)+b^3/a^5*((-1/8*a*b*(20*a+17
*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+5/8*(4*a+3*b)*b/(a+b)*sin(f*x+e))/(-a-b+a
*sin(f*x+e)^2)^2-1/8*(80*a^2+140*a*b+63*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/
2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(202) = 404$.

Time = 0.18 (sec) , antiderivative size = 995, normalized size of antiderivative = 4.65

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/240*(15*(80*a^2*b^5 + 140*a*b^6 + 63*b^7 + (80*a^4*b^3 + 140*a^3*b^4 +
63*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f
*x + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(
f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(24*(a^8 + 3*a^7*b + 3*a^6
*b^2 + a^5*b^3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1
774*a^3*b^5 + 2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 -
 23*a^5*b^3 - 9*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2
+ 131*a^5*b^3 + 167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64
*a^6*b^2 + 360*a^5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f
*x + e)^2)*sin(f*x + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*
x + e)^4 + 2*(a^10*b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 +
 (a^9*b^2 + 3*a^8*b^3 + 3*a^7*b^4 + a^6*b^5)*f), 1/120*(15*(80*a^2*b^5 + 1
40*a*b^6 + 63*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 63*a^2*b^5)*cos(f*x + e)^4
 + 2*(80*a^3*b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x + e)^2)*sqrt(-a^2 - a*b
)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (24*(a^8 + 3*a^7*b + 3*a
^6*b^2 + a^5*b^3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 +
 1774*a^3*b^5 + 2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2
 - 23*a^5*b^3 - 9*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^
2 + 131*a^5*b^3 + 167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b -
64*a^6*b^2 + 360*a^5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.42

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{15\,(80\,a^2b^3+140\,ab^4+63\,b^5)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{(a+b)a}} - \frac{30\left((20\,a^2b^4+17\,ab^5)\sin(fx+e)^3-5\,(4\,a^2b^4+7\,ab^5+3\,b^6)\sin(fx\cdots\right.}{a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4+(a^9+2\,a^8b+a^7b^2)\sin(fx+e)^4-2\,(a^9+3\,a^8b+3\,a^7\cdots}}{240\,f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/240*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*log((a*sin(f*x + e) - sqrt((a
+ b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^7 + 2*a^6*b + a^5*b^2)*sq
rt((a + b)*a)) - 30*((20*a^2*b^4 + 17*a*b^5)*sin(f*x + e)^3 - 5*(4*a^2*b^4
 + 7*a*b^5 + 3*b^6)*sin(f*x + e))/(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 +
 a^5*b^4 + (a^9 + 2*a^8*b + a^7*b^2)*sin(f*x + e)^4 - 2*(a^9 + 3*a^8*b + 3
*a^7*b^2 + a^6*b^3)*sin(f*x + e)^2) + 16*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2
- 3*a*b)*sin(f*x + e)^3 + 15*(a^2 - 3*a*b + 6*b^2)*sin(f*x + e))/a^5)/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.27

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{15\,(80\,a^2b^3+140\,ab^4+63\,b^5)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{-a^2-ab}} - \frac{15\left(20\,a^2b^4\sin(fx+e)^3+17\,ab^5\sin(fx+e)^3-20\,a^2b^4\sin(fx+e)-35\,ab^5\sin(fx+e)-15\,b^6\cdots\right.}{(a^7+2\,a^6b+a^5b^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{120\cdots}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/120*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*arctan(a*sin(f*x + e)/sqrt(-a^
2 - a*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt(-a^2 - a*b)) - 15*(20*a^2*b^4*si
n(f*x + e)^3 + 17*a*b^5*sin(f*x + e)^3 - 20*a^2*b^4*sin(f*x + e) - 35*a*b^
5*sin(f*x + e) - 15*b^6*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*(a*sin(f*
x + e)^2 - a - b)^2) + 8*(3*a^12*sin(f*x + e)^5 - 10*a^12*sin(f*x + e)^3 +
 15*a^11*b*sin(f*x + e)^3 + 15*a^12*sin(f*x + e) - 45*a^11*b*sin(f*x + e)
+ 90*a^10*b^2*sin(f*x + e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{5\,\sin(e+f\,x)\,(3\,b^5+4\,a\,b^4)}{8\,(a+b)} - \frac{\sin(e+f\,x)^3\,(20\,a^2\,b^4+17\,a\,b^5)}{8\,(a+b)^2}}{f\,\left(2\,a^6\,b-\sin\left(e+f\,x\right)^2\,\left(2\,a^7+2\,b\,a^6\right)+a^7+a^5\,b^2+a^7\,\sin\left(e+f\,x\right)^4\right)}$$
$$+ \frac{\sin\left(e+f\,x\right)^5}{5\,a^3\,f} + \frac{\sin\left(e+f\,x\right)^3\,\left(\frac{a+b}{a^4}-\frac{5}{3\,a^3}\right)}{f}$$
$$+ \frac{\sin\left(e+f\,x\right)\,\left(\frac{10}{a^3}-\frac{3\,(a+b)^2}{a^5}+\frac{3\,(a+b)\,\left(\frac{3\,(a+b)}{a^4}-\frac{5}{a^3}\right)}{a}\right)}{f}$$
$$+ \frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sin(e+f\,x)\,1{}\mathrm{i}}{\sqrt{a+b}}\right)\,\left(80\,a^2+140\,a\,b+63\,b^2\right)\,1{}\mathrm{i}}{8\,a^{11/2}\,f\,(a+b)^{5/2}}$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((5*sin(e + f*x)*(4*a*b^4 + 3*b^5))/(8*(a + b)) - (sin(e + f*x)^3*(17*a*b^
5 + 20*a^2*b^4))/(8*(a + b)^2))/(f*(2*a^6*b - sin(e + f*x)^2*(2*a^6*b + 2*
a^7) + a^7 + a^5*b^2 + a^7*sin(e + f*x)^4)) + sin(e + f*x)^5/(5*a^3*f) + (
sin(e + f*x)^3*((a + b)/a^4 - 5/(3*a^3)))/f + (sin(e + f*x)*(10/a^3 - (3*(
a + b)^2)/a^5 + (3*(a + b)*((3*(a + b))/a^4 - 5/a^3))/a))/f + (b^3*atan((a
^(1/2)*sin(e + f*x)*1i)/(a + b)^(1/2))*(140*a*b + 80*a^2 + 63*b^2)*1i)/(8*
a^(11/2)*f*(a + b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 1887, normalized size of antiderivative = 8.82

$$\int \frac{\cos^5(e + fx)}{(a + b\sec^2(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
(1200*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b
) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**4*b**3 + 2100*sqrt(a)*s
qrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*t
an((e + f*x)/2))*sin(e + f*x)**4*a**3*b**4 + 945*sqrt(a)*sqrt(a + b)*log(s
qrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))
*sin(e + f*x)**4*a**2*b**5 - 2400*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan(
(e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**
2*a**4*b**3 - 6600*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2
 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**3*b**4 - 6
090*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
- 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b**5 - 1890*sqrt(a)*sqr
t(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan
((e + f*x)/2))*sin(e + f*x)**2*a*b**6 + 1200*sqrt(a)*sqrt(a + b)*log(sqrt(
a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**
4*b**3 + 4500*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sq
rt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a**3*b**4 + 6345*sqrt(a)*sqrt(a +
b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e +
f*x)/2))*a**2*b**5 + 3990*sqrt(a)*sqrt(a + b)*log(sqrt(a + b)*tan((e + f*x
)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a*b**6 + 945*sqrt(a)*s
qrt(a + b)*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a...
```

3.212 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}f} + \frac{a^2\tan(e+fx)}{4b^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{a(5a+8b)\tan(e+fx)}{8b^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/8*(3*a^2+8*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+
b)^(5/2)/f+1/4*a^2*tan(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*a*(5*
a+8*b)*tan(f*x+e)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+fx)))\sin(2(e+fx))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8b^{5/2}f}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a +
 b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))/(8
*b^(5/2)*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 315, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^6}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{(\tan^2(e+fx)+1)^2}{(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 315$$

$$\frac{\frac{\int \frac{(3a+4b)\tan^2(e+fx)+a+4b}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \ 298$$

$$\frac{\frac{\frac{\left(3a^2+8ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2b(a+b)}-\frac{3a(a+2b)\tan(e+fx)}{2b(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}}-\frac{3a(a+2b)\tan(e+fx)}{2b(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input

```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output

```
(-1/4*(a*Tan[e + f*x]*(1 + Tan[e + f*x]^2))/(b*(a + b)*(a + b + b*Tan[e +
f*x]^2)^2) + (((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a + b]])/(2*b^(3/2)*(a + b)^(3/2)) - (3*a*(a + 2*b)*Tan[e + f*x])/(2*b*(a
+ b)*(a + b + b*Tan[e + f*x]^2)))/(4*b*(a + b)))/f
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{-\frac{a(5a+8b)\tan(fx+e)^3}{8b\left(a^2+2ab+b^2\right)}-\frac{(3a+8b)a\tan(fx+e)}{8b^2(a+b)}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b^2\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{-\frac{a(5a+8b)\tan(fx+e)^3}{8b\left(a^2+2ab+b^2\right)}-\frac{(3a+8b)a\tan(fx+e)}{8b^2(a+b)}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b^2\sqrt{(a+b)b}}}{f}$
risch	$-\frac{i\left(3a^3e^{6i(fx+e)}+8a^2b\,e^{6i(fx+e)}+8a\,b^2e^{6i(fx+e)}+9a^3e^{4i(fx+e)}+42a^2b\,e^{4i(fx+e)}+72a\,b^2e^{4i(fx+e)}+48b^3e^{4i(fx+e)}+\right.}{4(a+b)^2b^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right.}$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*((-1/8*a*(5*a+8*b)/b/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/8*(3*a+8*b)*a/b^2/
(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2+8*a*b+8*b^2)/(a^2+2*a*
b+b^2)/b^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(124) = 248$.

Time = 0.12 (sec) , antiderivative size = 722, normalized size of antiderivative = 5.23

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^
3 + 8*b^4 + 2*(3*a^3*b + 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b -
b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
 + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*s
in(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*
(3*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^
3 + 8*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5
 + a^2*b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)
*f*cos(f*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f), -1/16*(((3*a
^4 + 8*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 2
*(3*a^3*b + 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/
2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e
))) + 2*(3*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 1
3*a^2*b^3 + 8*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3
*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6
+ a*b^7)*f*cos(f*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f)]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.52

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2b^2+2ab^3+b^4)\sqrt{(a+b)b}} - \frac{(5a^2b+8ab^2)\tan(fx+e)^3+(3a^3+11a^2b+8ab^2)\tan(fx+e)}{a^4b^2+4a^3b^3+6a^2b^4+4ab^5+b^6+(a^2b^4+2ab^5+b^6)\tan(fx+e)^4+2(a^3b^3+3a^2b^4+3ab^5+b^6)\tan(fx}}{8f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + 8*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*
b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)) - ((5*a^2*b + 8*a*b^2)*tan(f*x + e)^
3 + (3*a^3 + 11*a^2*b + 8*a*b^2)*tan(f*x + e))/(a^4*b^2 + 4*a^3*b^3 + 6*a^
2*b^4 + 4*a*b^5 + b^6 + (a^2*b^4 + 2*a*b^5 + b^6)*tan(f*x + e)^4 + 2*(a^3*
b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*tan(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.34

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3a^2+8ab+8b^2)}{(a^2b^2+2ab^3+b^4)\sqrt{ab+b^2}} - \frac{5a^2b\tan(fx+e)^3+8ab^2\tan(fx+e)^3+3a^3\tan(fx+e)+11a^2b\tan(fx+e)+}{(a^2b^2+2ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a^2 + 8*a*b + 8*b^2)/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt(a*b + b^
2)) - (5*a^2*b*tan(f*x + e)^3 + 8*a*b^2*tan(f*x + e)^3 + 3*a^3*tan(f*x + e
) + 11*a^2*b*tan(f*x + e) + 8*a*b^2*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b^
4)*(b*tan(f*x + e)^2 + a + b)^2))/f
```

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.08

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)(3a^2+8ab+8b^2)}{8b^{5/2}f(a+b)^{5/2}} - \frac{\frac{\tan(e+f\,x)^3(5a^2+8ba)}{8b(a+b)^2}+\frac{\tan(e+f\,x)(3a^2+8ba)}{8b^2(a+b)}}{f\left(2ab+a^2+b^2+\tan(e+f\,x)^2(2b^2+2ab)+b^2\tan(e+f\,x)^4\right)}$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
(atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(8*a*b + 3*a^2 + 8*b^2))/(8*b^
(5/2)*f*(a + b)^(5/2)) - ((tan(e + f*x)^3*(8*a*b + 5*a^2))/(8*b*(a + b)^2)
 + (tan(e + f*x)*(8*a*b + 3*a^2))/(8*b^2*(a + b)))/(f*(2*a*b + a^2 + b^2 +
 tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1296, normalized size of antiderivative = 9.39

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**4*a**4 + 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b + 8*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*
a**2*b**2 - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqr
t(a))/sqrt(b))*sin(e + f*x)**2*a**4 - 22*sqrt(b)*sqrt(a + b)*atan((sqrt(a
+ b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b - 32*sqrt
(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin
(e + f*x)**2*a**2*b**2 - 16*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
 f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 + 3*sqrt(b)*sqrt(a + b
)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 + 14*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3*b
 + 27*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*a**2*b**2 + 24*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/
2) - sqrt(a))/sqrt(b))*a*b**3 + 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4 + 3*sqrt(b)*sqrt(a + b)*atan((sqrt
(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4 + 8*sqrt
(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin
(e + f*x)**4*a**3*b + 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x
)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 - 6*sqrt(b)*sqrt(a +...
```

3.213 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{(a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}f} - \frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+4b)\tan(e+fx)}{8b(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/8*(a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(5/2)/f-1
/4*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(a+4*b)*tan(f*x+e)/b/
(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.17 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.30

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-\frac{(a+4b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+}{b\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{64(a+b)^2f\,(a+}$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-(((a + 4*b)*ArcTan[(Sec[f
*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2
*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)]
)^2*(Cos[2*e] - I*Sin[2*e]))/(b*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
) - (4*(a + b)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(
Cos[e] + Sin[e])) + ((a + 2*b + a*Cos[2*(e + f*x)])*((a + 4*b)*Sin[2*e] -
(a - 2*b)*Sin[2*f*x]))/(b*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(64*(a +
b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(e+fx)^4}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \; 4634$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{(a+4b)\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4b(a+b)}-\frac{a\tan(e+fx)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{(a+4b)\left(\frac{\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2(a+b)}+\frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4b(a+b)}-\frac{a\tan(e+fx)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{(a+4b)\left(\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}}+\frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4b(a+b)}-\frac{a\tan(e+fx)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(a*Tan[e + f*x])/(b*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + ((a + 4*
b)*(ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)) +
 Tan[e + f*x]/(2*(a + b)*(a + b + b*Tan[e + f*x]^2))))/(4*b*(a + b)))/f
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{\frac{(a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}-\frac{(a-4b)\tan(fx+e)}{8(a+b)b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\frac{(a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}-\frac{(a-4b)\tan(fx+e)}{8(a+b)b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b\sqrt{(a+b)b}}}{f}$
risch	$\frac{i\left(-a^3 e^{6i(fx+e)}-4a^2 b\, e^{6i(fx+e)}-3a^3 e^{4i(fx+e)}-2a^2 b\, e^{4i(fx+e)}+8a\, b^2 e^{4i(fx+e)}+16b^3 e^{4i(fx+e)}-3a^3 e^{2i(fx+e)}+4\, e^{2i(}\right.}{4a(a+b)^2 f\left(a\, e^{4i(fx+e)}+2a\, e^{2i(fx+e)}+4b\, e^{2i(fx+e)}+a\right)^2 b}$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*((1/8*(a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/8*(a-4*b)/(a+b)/b*tan(f*x
+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(a+4*b)/(a^2+2*a*b+b^2)/b/((a+b)*b)^(1/2)*
arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(109) = 218$.

Time = 0.13 (sec) , antiderivative size = 654, normalized size of antiderivative = 5.32

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b
^2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*
cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)) + 4*((a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^
3 - (a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^2 + 3*
a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3 + 3*a^3*b^4 +
 3*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^
7)*f), -1/16*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b +
 4*a*b^2)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x +
e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*((a^3*b - a^2*b
^2 - 2*a*b^3)*cos(f*x + e)^3 - (a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*s
in(f*x + e))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4
 + 2*(a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4
 + 3*a^2*b^5 + 3*a*b^6 + b^7)*f)]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.52

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2b+2ab^2+b^3)\sqrt{(a+b)b}} + \frac{(ab+4b^2)\tan(fx+e)^3-(a^2-3ab-4b^2)\tan(fx+e)}{a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5+(a^2b^3+2ab^4+b^5)\tan(fx+e)^4+2(a^3b^2+3a^2b^3+3ab^4+b^5)\tan(fx+e)^2}}{8f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((a + 4*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*b + 2*a*b^2 +
b^3)*sqrt((a + b)*b)) + ((a*b + 4*b^2)*tan(f*x + e)^3 - (a^2 - 3*a*b - 4*b
^2)*tan(f*x + e))/(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 + (a^2*b^
3 + 2*a*b^4 + b^5)*tan(f*x + e)^4 + 2*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
)*tan(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+4b)}{(a^2b+2ab^2+b^3)\sqrt{ab+b^2}}+\frac{ab\tan(fx+e)^3+4b^2\tan(fx+e)^3-a^2\tan(fx+e)+3ab\tan(fx+e)+4b^2\tan(fx+e)}{(a^2b+2ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(a + 4*b)/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b + b^2)) + (a*b*tan(f
*x + e)^3 + 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) + 3*a*b*tan(f*x + e) +
 4*b^2*tan(f*x + e))/((a^2*b + 2*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2
))/f
```

Mupad [B] (verification not implemented)

Time = 17.66 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{\tan(e+f\,x)^3\,(a+4\,b)}{8\,(a+b)^2}-\frac{\tan(e+f\,x)\,(a-4\,b)}{8\,b\,(a+b)}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)\,(a+4\,b)}{8\,b^{3/2}\,f\,(a+b)^{5/2}}$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
((tan(e + f*x)^3*(a + 4*b))/(8*(a + b)^2) - (tan(e + f*x)*(a - 4*b))/(8*b*
(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan
(e + f*x)^4)) + (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 4*b))/(8*
b^(3/2)*f*(a + b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1026, normalized size of antiderivative = 8.34

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
(sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
)*sin(e + f*x)**4*a**3 + 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b - 2*sqrt(b)*sqrt(a + b)
*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*
*3 - 10*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/
sqrt(b))*sin(e + f*x)**2*a**2*b - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*
tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**2 + sqrt(b)*sqrt
(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3 + 6*sq
rt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a
**2*b + 9*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*a*b**2 + 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/
2) - sqrt(a))/sqrt(b))*b**3 + sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
 + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3 + 4*sqrt(b)*sqrt(a + b
)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a
**2*b - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a)
)/sqrt(b))*sin(e + f*x)**2*a**3 - 10*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*b - 8*sqrt(b)*s
qrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e +
f*x)**2*a*b**2 + sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) +
sqrt(a))/sqrt(b))*a**3 + 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e...
```

3.214 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = \frac{3\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}(a+b)^{5/2}f} + \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\tan(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
3/8*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(1/2)/(a+b)^(5/2)/f+1/4*tan(f
*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+3/8*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f
*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.50

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-\frac{3\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{64(a+}$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((-3*ArcTan[(Sec[f*x]*(Cos[
2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*b*(a
+ b)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/a^2 + ((a + 2*b + a*Cos
[2*(e + f*x)])*Sec[2*e]*(-((5*a^2 + 16*a*b + 8*b^2)*Sin[2*e]) + a*(5*a + 2
*b)*Sin[2*f*x]))/a^2))/(64*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(e+fx)^2}{(a+b\sec(e+fx)^2)^3}dx$$

$\big\downarrow$ 4634

$$\frac{\int \frac{1}{(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$\big\downarrow$ 215

$$\frac{\frac{3\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4(a+b)}+\frac{\tan(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\big\downarrow$ 215

$$\frac{\frac{3\left(\frac{\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2(a+b)}+\frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)}+\frac{\tan(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\big\downarrow$ 218

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}}+\frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)}+\frac{\tan(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(4*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (3*(ArcTan[(Sqrt[
b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)) + Tan[e + f*x]/(2*
(a + b)*(a + b + b*Tan[e + f*x]^2))))/(4*(a + b)))/f
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{4(a+b)\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\frac{3\tan(fx+e)}{8(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a+b}}{f}$
default	$\frac{\frac{\tan(fx+e)}{4(a+b)\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\frac{3\tan(fx+e)}{8(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a+b}}{f}$
risch	$\frac{i(5a^3e^{6i(fx+e)}+16a^2b\,e^{6i(fx+e)}+8a\,b^2e^{6i(fx+e)}+15a^3e^{4i(fx+e)}+46a^2b\,e^{4i(fx+e)}+56a\,b^2e^{4i(fx+e)}+16b^3e^{4i(fx+e)}+}{4a^2(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*tan(f*x+e)/(a+b)/(a+b+b*tan(f*x+e)^2)^2+3/4/(a+b)*(1/2*tan(f*x+e)
/(a+b)/(a+b+b*tan(f*x+e)^2)+1/2/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/
((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(92) = 184$.

Time = 0.12 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.47

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[-\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\left(3\,ab+4\,b^2\right)\cos(fx+e}{a^2\cos(fx+}\right.}{32\left(\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^4+\right.}\right.$$
$$-\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2\,b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)-2\left(\left(5\,a^2b+\right.\right.}{16\left(\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^4+2\left(a^4b^2+3\,a^3b^3+3\,a^2b^4+ab^5\right)\right.}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((
5*a^2*b + 7*a*b^2 + 2*b^3)*cos(f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*
sin(f*x + e))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4
+ 2*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3
+ 3*a^2*b^4 + 3*a*b^5 + b^6)*f), -1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(
f*x + e)^2 + b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b
)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^2*b + 7*a*b^2 + 2
*b^3)*cos(f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f*x + e))/((a^5*b
 + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a^4*b^2 + 3*a^3*
b^3 + 3*a^2*b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5
 + b^6)*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.47

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{3\,b\tan(fx+e)^3+5\,(a+b)\tan(fx+e)}{(a^2b^2+2\,ab^3+b^4)\tan(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4+2\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\tan(fx+e)^2} + \frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}}{8\,f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b*tan(f*x + e)^3 + 5*(a + b)*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b
^4)*tan(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 2*(a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^2) + 3*arctan(b*tan(f*x + e)/sqr
t((a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^2+2\,ab+b^2)\sqrt{ab+b^2}}+\frac{3\,b\tan(fx+e)^3+5\,a\tan(fx+e)+5\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^2(a^2+2\,ab+b^2)}}{8\,f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^3 +
5*a*tan(f*x + e) + 5*b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*(a^2 +
2*a*b + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 15.73 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{5\tan(e+f\,x)}{8\,(a+b)}+\frac{3\,b\tan(e+f\,x)^3}{8\,(a+b)^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}
+ \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,f\,(a+b)^{5/2}}$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
((5*tan(e + f*x))/(8*(a + b)) + (3*b*tan(e + f*x)^3)/(8*(a + b)^2))/(f*(2*
a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) +
(3*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(8*b^(1/2)*f*(a + b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 752, normalized size of antiderivative = 7.09

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**4*a**2 - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2 - 6*sqrt(b)*sqrt(a + b)
*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*
b + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*a**2 + 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - s
qrt(a))/sqrt(b))*a*b + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*
x)/2) - sqrt(a))/sqrt(b))*b**2 + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*t
an((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2 - 6*sqrt(b)*sqrt(
a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)
**2*a**2 - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt
(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 + 6*sqrt(b)*sqrt(a + b)*atan((
sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b + 3*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**2 - 5*cos(e +
 f*x)*sin(e + f*x)**3*a**2*b - 7*cos(e + f*x)*sin(e + f*x)**3*a*b**2 - 2*c
os(e + f*x)*sin(e + f*x)**3*b**3 + 5*cos(e + f*x)*sin(e + f*x)*a**2*b + 10
*cos(e + f*x)*sin(e + f*x)*a*b**2 + 5*cos(e + f*x)*sin(e + f*x)*b**3)/(8*b
*f*(sin(e + f*x)**4*a**5 + 3*sin(e + f*x)**4*a**4*b + 3*sin(e + f*x)**4*a*
*3*b**2 + sin(e + f*x)**4*a**2*b**3 - 2*sin(e + f*x)**2*a**5 - 8*sin(e ...
```

3.215 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{x}{a^3}-\frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^3-1/8*b^(1/2)*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1
/8*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.53 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.31

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2+\frac{b(15a^2+20ab+8b^2)\arctan\left(\frac{\sec(fx)(\cdots}{\cdots}\right)\cdots}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e
+ f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*S
in[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*S
in[2*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2
*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e
])) + (b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e
] - 3*a*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] +
Sin[e]))))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4616, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^3}dx$$

$\big\downarrow$ 4616

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$\big\downarrow$ 316

$$\frac{\frac{\int \frac{-3b\tan^2(e+fx)+4a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{8a^2+9ba+4b^2-b(7a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\big\downarrow$ 397

$$\frac{\frac{\frac{\frac{8(a+b)^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\big\downarrow$ 216

$$\frac{\frac{\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\big\downarrow$ 218

$$\frac{\frac{\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
(-1/4*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (((8*(a
+ b)^2*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*
(7*a + 4*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*(
a + b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3 e^{6i(fx+e)}+28a^2 b\, e^{6i(fx+e)}+16a\, b^2 e^{6i(fx+e)}+27a^3 e^{4i(fx+e)}+90a^2 b\, e^{4i(fx+e)}+120a\, b^2 e^{4i(fx+e)}+48b^3 e\cdots\right.}{4a^3(a+b)^2 f\left(a\, e^{4i(fx+e)}+2a\, e^{2i(fx+e)}+4b\, e^{2i(f\cdots\right.}$

input
```
int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(9*a+4*b)
*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+20*a*b+8*b^2)/(a^2
+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*ar
ctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(130) = 260$.

Time = 0.14 (sec) , antiderivative size = 819, normalized size of antiderivative = 5.69

$$\int \frac{1}{(a + b\sec^2(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2
*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15
*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^
4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sq
rt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e
)^2 + b^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a
*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e
)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*
b^3 + a^3*b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 +
 32*(a^3*b + 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3
 + b^4)*f*x + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 20*a*b^3 + 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*
sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3
 + (7*a^2*b^2 + 4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5
*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2
+ (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]

$$\int \frac{1}{(a + b\sec^2(e + fx))^3}\,dx = \int \frac{1}{(a + b\sec^2{(e + fx)})^3}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}} + \frac{(7\,ab^2+4\,b^3)\tan(fx+e)^3+(9\,a^2b+13\,ab^2+4\,b^3)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^4b^2+2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+\cdots}}{8\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x
+ e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^
4 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x +
 e)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{ab+b^2}} + \frac{7\,ab^2\tan(fx+e)^3+4\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+13\,ab^2\tan(\cdots}{(a^4+2\,a^3b+a^2b^2)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a
*b + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(
f*x + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a
^2*b^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.25 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int \frac{1}{(a + b\sec^2(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
atan((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^1
0*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e
 + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a
^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 +
856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a
^6*b^2)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)
/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)
) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a
^5*b^3 + 6*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*
a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5
*b^3 + 6*a^6*b^2)))/a^3)/((((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*
b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b
^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*
a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*
b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^1
1*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(
2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 +
 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1496, normalized size of antiderivative = 10.39

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**4*a**4 - 20*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 8*sqrt(b)*sqrt
(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x
)**4*a**2*b**2 + 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2)
 - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4 + 70*sqrt(b)*sqrt(a + b)*atan((s
qrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b + 5
6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**2*a**2*b**2 + 16*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 - 15*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 - 50*
sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))
*a**3*b - 63*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt
(a))/sqrt(b))*a**2*b**2 - 36*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*a*b**3 - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a
+ b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4 - 15*sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4
- 20*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqr
t(b))*sin(e + f*x)**4*a**3*b - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan
((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 + 30*sqrt(b...
```

3.216 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 201

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{(a-6b)x}{2a^4}+\frac{b^{3/2}(35a^2+56ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/2*(a-6*b)*x/a^4+1/8*b^(3/2)*(35*a^2+56*a*b+24*b^2)*arctan(b^(1/2)*tan(f*
x+e)/(a+b)^(1/2))/a^4/(a+b)^(5/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*t
an(f*x+e)^2)^2+1/4*b*(2*a+3*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
^2+1/8*b*(4*a+3*b)*(a+4*b)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{4(a-6b)(e+fx)+\frac{b^{3/2}(35a^2+56ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+a\left(2+\frac{13ab^2}{(a+b)^2(a+2b+a\cos(2(e+fx)))}+\frac{2b^3(3a+8b+5a\,c}{(a+b)^2(a+2b+a\,c}\right.}{8a^4f}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(4*(a - 6*b)*(e + f*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[
b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^
2*(a + 2*b + a*Cos[2*(e + f*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cos[2*(e + f*x
)]))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))*Sin[2*(e + f*x)])/(8*a^
4*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 4634, 316, 25, 402, 27, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sec(e+fx)^2\,(a+b\sec(e+fx)^2)^3}dx$$
$$\Big\downarrow \ 4634$$
$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}-\frac{\int -\frac{5b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{2\left(2a^2-4ba-3b^2+3b(2a+3b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{2a^2-4ba-3b^2+3b(2a+3b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\frac{\int \frac{4a^3-12ba^2-25b^2a-12b^3+b(4a+3b)(a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\text{ta}\cdots}{2a(\tan^2(e+fx)+\cdots}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{\frac{b^2\left(35a^2+56ab+24b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{4(a-6b)(a+b)^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a(a+b)}+\frac{b(2a+3b)\tan(e\cdots}{2a(a+b)\left(a+b\tan^2(e\cdots\right.}}{2a}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\frac{b^2\left(35a^2+56ab+24b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{4(a-6b)(a+b)^2\arctan(\tan(e+fx))}{a}}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}}{f}$$

$\downarrow$ 218

$$\frac{\frac{\frac{\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{4(a-6b)(a+b)^2\arctan(\tan(e+fx))}{a}}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\ldots}{2\ldots}}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^2) + ((
b*(2*a + 3*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (
((4*(a - 6*b)*(a + b)^2*ArcTan[Tan[e + f*x]])/a + (b^(3/2)*(35*a^2 + 56*a*
b + 24*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(
2*a*(a + b)) + (b*(4*a + 3*b)*(a + 4*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b
+ b*Tan[e + f*x]^2)))/(2*a*(a + b)))/(2*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{b^2\left(\frac{\frac{ab(11a+8b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(13a+8b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+56ab+24b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^4}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-6b)\,a\cdots}{}}{a^4}}{f}$
default	$\frac{\frac{b^2\left(\frac{\frac{ab(11a+8b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(13a+8b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+56ab+24b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^4}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-6b)\,a\cdots}{}}{a^4}}{f}$
risch	$\frac{x}{2a^3}-\frac{3xb}{a^4}-\frac{i\,e^{2i(fx+e)}}{8a^3f}+\frac{i\,e^{-2i(fx+e)}}{8a^3f}+\frac{ib^2\left(13a^3e^{6i(fx+e)}+40a^2b\,e^{6i(fx+e)}+24a\,b^2e^{6i(fx+e)}+39a^3e^{4i(fx+e)}\cdots\right.}{4a^4\cdots}$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2/a^4*((1/8*a*b*(11*a+8*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(13*a+8
*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(35*a^2+56*a*b+24*b^2)/
(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^
4*(1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a-6*b)*arctan(tan(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(183) = 366$.

Time = 0.16 (sec) , antiderivative size = 970, normalized size of antiderivative = 4.83

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(16*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 32
*(a^4*b - 4*a^3*b^2 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^3*b
^2 - 4*a^2*b^3 - 11*a*b^4 - 6*b^5)*f*x + (35*a^2*b^3 + 56*a*b^4 + 24*b^5 +
 (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*
a^2*b^3 + 24*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8
*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b
+ 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(4*(
a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b
^3)*cos(f*x + e)^3 + (4*a^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin
(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*
b^2 + a^5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), 1/16
*(8*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 16*(a^4*
b - 4*a^3*b^2 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 8*(a^3*b^2 - 4*
a^2*b^3 - 11*a*b^4 - 6*b^5)*f*x - (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (35*a^
4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*b^3
 + 24*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x
 + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(4*(a^5 +
2*a^4*b + a^3*b^2)*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*co
s(f*x + e)^3 + (4*a^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{(35\,a^2b^2+56\,ab^3+24\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{(a+b)b}} + \frac{(4\,a^2b^2+19\,ab^3+12\,b^4)\tan(fx+e)^5+(8\,a^3b+37\,a^2b^2+56\,ab^3+24\,b^4)\tan(fx}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(a^5b^2+2\,a^4b^3+a^3b^4)\tan(fx+e)^6+(2\,a^6b+7\,a^5b^2+8\,a^4b^3+}}{8\,f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b
))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)) + ((4*a^2*b^2 + 19*a*b^3 +
12*b^4)*tan(f*x + e)^5 + (8*a^3*b + 37*a^2*b^2 + 56*a*b^3 + 24*b^4)*tan(f*
x + e)^3 + (4*a^4 + 16*a^3*b + 37*a^2*b^2 + 37*a*b^3 + 12*b^4)*tan(f*x + e
))/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (a^5*b^2 + 2*a^4*b^3
 + a^3*b^4)*tan(f*x + e)^6 + (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 3*a^3*b^4)
*tan(f*x + e)^4 + (a^7 + 6*a^6*b + 12*a^5*b^2 + 10*a^4*b^3 + 3*a^3*b^4)*ta
n(f*x + e)^2) + 4*(f*x + e)*(a - 6*b)/a^4)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{(35\,a^2b^2+56\,ab^3+24\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{ab+b^2}} + \frac{11\,ab^3\tan(fx+e)^3+8\,b^4\tan(fx+e)^3+13\,a^2b^2\tan(fx+e)+21\,ab^3\tan}{(a^5+2\,a^4b+a^3b^2)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
 + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt
(a*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 8*b^4*tan(f*x + e)^3 + 13*a^2*b^
2*tan(f*x + e) + 21*a*b^3*tan(f*x + e) + 8*b^4*tan(f*x + e))/((a^5 + 2*a^4
*b + a^3*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 4*(f*x + e)*(a - 6*b)/a^4 +
4*tan(f*x + e)/((tan(f*x + e)^2 + 1)*a^3))/f
```

Mupad [B] (verification not implemented)

Time = 19.59 (sec) , antiderivative size = 3708, normalized size of antiderivative = 18.45

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^5*(19*a*b^3 + 12*b^4 + 4*a^2*b^2))/(8*a^3*(a + b)^2) + (tan
(e + f*x)*(25*a*b^2 + 12*a^2*b + 4*a^3 + 12*b^3))/(8*a^3*(a + b)) + (b*tan
(e + f*x)^3*(56*a*b^2 + 37*a^2*b + 8*a^3 + 24*b^3))/(8*a^3*(a + b)^2))/(f*
(2*a*b + tan(e + f*x)^2*(4*a*b + a^2 + 3*b^2) + a^2 + b^2 + tan(e + f*x)^4
*(2*a*b + 3*b^2) + b^2*tan(e + f*x)^6)) + (atan((((((6*a^8*b^7 + (49*a^9*
b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*
b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(a*1i - b*6i
)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*
b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^
8*b^2)))*(a*1i - b*6i))/(4*a^4) - (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7
520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32
*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i)*1i)/(4
*a^4) - (((((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 +
2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*
b^2) + (tan(e + f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10
*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b +
a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) + (tan(e
+ f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5
 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 +
6*a^8*b^2)))*(a*1i - b*6i)*1i)/(4*a^4))/(((405*a*b^8)/4 + 27*b^9 + (261...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1949, normalized size of antiderivative = 9.70

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
(35*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*sin(e + f*x)**4*a**4*b + 56*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan
((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b**2 + 24*sqrt(b)*s
qrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e +
f*x)**4*a**2*b**3 - 70*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)
/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4*b - 182*sqrt(b)*sqrt(a + b)*a
tan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3
*b**2 - 160*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(
a))/sqrt(b))*sin(e + f*x)**2*a**2*b**3 - 48*sqrt(b)*sqrt(a + b)*atan((sqrt
(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**4 + 35*s
qrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*
a**4*b + 126*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt
(a))/sqrt(b))*a**3*b**2 + 171*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
 + f*x)/2) - sqrt(a))/sqrt(b))*a**2*b**3 + 104*sqrt(b)*sqrt(a + b)*atan((s
qrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b**4 + 24*sqrt(b)*sqrt(a
 + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**5 + 35*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*si
n(e + f*x)**4*a**4*b + 56*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f
*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b**2 + 24*sqrt(b)*sqrt(a +
 b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)...
```

3.217 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 269

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{3(a^2-4ab+16b^2)\,x}{8a^5} - \frac{3b^{5/2}(21a^2+36ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5(a+b)^{5/2}f} + \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(3a^2-7ab-12b^2)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{8a^4(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
3/8*(a^2-4*a*b+16*b^2)*x/a^5-3/8*b^(5/2)*(21*a^2+36*a*b+16*b^2)*arctan(b^(
1/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(5/2)/f+1/8*(3*a-8*b)*cos(f*x+e)*si
n(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b
+b*tan(f*x+e)^2)^2+1/8*b*(3*a^2-7*a*b-12*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+
b*tan(f*x+e)^2)^2+3/8*b*(a+2*b)*(a^2-4*a*b-4*b^2)*tan(f*x+e)/a^4/(a+b)^2/f
/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.91 (sec) , antiderivative size = 1134, normalized size of antiderivative = 4.22

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((96*b^3*(21*a^2 + 36*a*b +
 16*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) +
 a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*
b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Co
s[e] - I*Sin[e])^4]) + Sec[2*e]*(48*(a + b)^2*(3*a^4 - 4*a^3*b + 24*a^2*b^
2 + 96*a*b^3 + 128*b^4)*f*x*Cos[2*e] + 96*a*(a + b)^2*(a^3 - 2*a^2*b + 8*a
*b^2 + 32*b^3)*f*x*Cos[2*f*x] + 24*a^6*f*x*Cos[2*(e + 2*f*x)] - 48*a^5*b*f
*x*Cos[2*(e + 2*f*x)] + 216*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 672*a^3*b^3*f
*x*Cos[2*(e + 2*f*x)] + 384*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 96*a^6*f*x*Co
s[4*e + 2*f*x] + 480*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 4416*a^3*b^3*f*x*Cos[4
*e + 2*f*x] + 6912*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 3072*a*b^5*f*x*Cos[4*e +
 2*f*x] + 24*a^6*f*x*Cos[6*e + 4*f*x] - 48*a^5*b*f*x*Cos[6*e + 4*f*x] + 21
6*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 384*a^
2*b^4*f*x*Cos[6*e + 4*f*x] + 816*a^3*b^3*Sin[2*e] + 2848*a^2*b^4*Sin[2*e]
+ 3968*a*b^5*Sin[2*e] + 1792*b^6*Sin[2*e] + 44*a^6*Sin[2*f*x] + 104*a^5*b*
Sin[2*f*x] - 180*a^4*b^2*Sin[2*f*x] - 1696*a^3*b^3*Sin[2*f*x] - 3264*a^2*b
^4*Sin[2*f*x] - 1664*a*b^5*Sin[2*f*x] + 38*a^6*Sin[2*(e + 2*f*x)] + 60*a^5
*b*Sin[2*(e + 2*f*x)] - 170*a^4*b^2*Sin[2*(e + 2*f*x)] - 640*a^3*b^3*Sin[2
*(e + 2*f*x)] - 400*a^2*b^4*Sin[2*(e + 2*f*x)] + 44*a^6*Sin[4*e + 2*f*x] +
 104*a^5*b*Sin[4*e + 2*f*x] - 180*a^4*b^2*Sin[4*e + 2*f*x] - 608*a^3*b^...
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \ 4634$$

$$\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 316$$

$$\frac{\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\int -\frac{7b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{7b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{(3a-8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\int -\frac{3a^2+3ba+8b^2+5(3a-8b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}}{4a}+\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int \frac{3a^2+3ba+8b^2+5(3a-8b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}+\frac{(3a-8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\big\downarrow$ 402

$$\frac{\frac{\frac{\frac{\int \frac{12\left(a^3+5b^2a+4b^3+b\left(3a^2-7ba-12b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}+\frac{b\left(3a^2-7ab-12b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{(3a-8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}}{4a}+\frac{\cdots}{4a(\tan^2(\cdots}}{f}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{\frac{3\int \frac{a^3+5b^2a+4b^3+b\left(3a^2-7ba-12b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{a(a+b)}+\frac{b\left(3a^2-7ab-12b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{(3a-8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}}{4a}+\frac{\cdots}{4a(\tan^2(e+\cdots}}{f}$$

$\big\downarrow$ 402

$$\frac{\frac{\frac{\frac{3\left(\frac{\int \frac{2\left(a^4-ba^3+7b^2a^2+16b^3a+8b^4+b(a+2b)\left(a^2-4ba-4b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}+\frac{b\left(3a^2-7ab-12b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}\cdots}{4a}\cdots}{f}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{\frac{3\left(\frac{\int \frac{a^4-ba^3+7b^2a^2+16b^3a+8b^4+b(a+2b)\left(a^2-4ba-4b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a(a+b)}+\frac{b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}+\frac{b\left(3a^2-7ab-12b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\cdots}{2a\cdots}}{4a}\cdots}{f}$$

$\big\downarrow$ 397

$$\frac{\frac{\frac{3\left(\frac{\frac{(a+b)^2\left(a^2-4ab+16b^2\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^3\left(21a^2+36ab+16b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a(a+b)}+\frac{b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}+\frac{b(}{a(}}{2a}}{4a}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{3\left(\frac{\frac{(a+b)^2\left(a^2-4ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{b^3\left(21a^2+36ab+16b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a(a+b)}+\frac{b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}+\frac{b\left(3a^2-7ab-\right.}{a(a+b)\left(a+b\right.}}{2a}}{4a}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{3\left(\frac{\frac{(a+b)^2\left(a^2-4ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{b^{5/2}\left(21a^2+36ab+16b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a(a+b)}+\frac{b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}+\frac{b\left(3a^2-7ab-12b^2\right)t}{a(a+b)\left(a+b\tan^2(e-\right.}}{2a}}{4a}}{f}$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)^2) +
(((3*a - 8*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f
*x]^2)^2) + ((b*(3*a^2 - 7*a*b - 12*b^2)*Tan[e + f*x])/(a*(a + b)*(a + b +
 b*Tan[e + f*x]^2)^2) + (3*((((a + b)^2*(a^2 - 4*a*b + 16*b^2)*ArcTan[Tan[
e + f*x]])/a - (b^(5/2)*(21*a^2 + 36*a*b + 16*b^2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*(a + 2*b)*(a^2 - 4*
a*b - 4*b^2)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2))))/(a*(a
+ b)))/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)^3+\left(-\frac{3}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2-4ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b^3\left(\frac{\frac{3ab(5a+4b)\tan(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(17a \ldots}{}}{\left(a+b+b\tan(fx+\ldots\right.}\ldots\right.}{}}{f}$
default	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)^3+\left(-\frac{3}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2-4ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b^3\left(\frac{\frac{3ab(5a+4b)\tan(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(17a \ldots}{}}{\left(a+b+b\tan(fx+\ldots\right.}\ldots\right.}{}}{f}$
risch	$\frac{3x}{8a^3}-\frac{3xb}{2a^4}+\frac{6x\,b^2}{a^5}-\frac{ib^3\left(17a^3\mathrm{e}^{6i(fx+e)}+52a^2b\,\mathrm{e}^{6i(fx+e)}+32a\,b^2\mathrm{e}^{6i(fx+e)}+51a^3\mathrm{e}^{4i(fx+e)}+178a^2b\,\mathrm{e}^{4i(fx+e)}+2\ldots\right.}{4a^5(a+b)^2f\left(a\,\mathrm{e}^{4i(fx+e)}\ldots\right.}$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^5*(((3/8*a^2-3/2*a*b)*tan(f*x+e)^3+(-3/2*a*b+5/8*a^2)*tan(f*x+e))
/(1+tan(f*x+e)^2)^2+3/8*(a^2-4*a*b+16*b^2)*arctan(tan(f*x+e)))-b^3/a^5*((3
/8*a*b*(5*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(17*a+12*b)*a/(a+b)*tan(
f*x+e))/(a+b+b*tan(f*x+e)^2)^2+3/8*(21*a^2+36*a*b+16*b^2)/(a^2+2*a*b+b^2)/
((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(249) = 498$.

Time = 0.18 (sec) , antiderivative size = 1129, normalized size of antiderivative = 4.20

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(12*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*
x + e)^4 + 24*(a^5*b - 2*a^4*b^2 + 9*a^3*b^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*
cos(f*x + e)^2 + 12*(a^4*b^2 - 2*a^3*b^3 + 9*a^2*b^4 + 28*a*b^5 + 16*b^6)*
f*x + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21*a^4*b^2 + 36*a^3*b^3 + 16*a^
2*b^4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2*b^4 + 16*a*b^5)*cos(f*x + e
)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b
+ b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e
)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(2*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*
x + e)^7 + (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3*b^3)*cos(f*x + e)^5 + (6*
a^5*b - 10*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos(f*x + e)^3 + 3*(a^4*b^2
- 2*a^3*b^3 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*sin(f*x + e))/((a^9 + 2*
a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*cos(
f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), 1/16*(6*(a^6 - 2*a^5*b +
9*a^4*b^2 + 28*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x + e)^4 + 12*(a^5*b - 2*a^
4*b^2 + 9*a^3*b^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*cos(f*x + e)^2 + 6*(a^4*b^2
 - 2*a^3*b^3 + 9*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x + 3*(21*a^2*b^4 + 36*a*b
^5 + 16*b^6 + (21*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^4)*cos(f*x + e)^4 + 2*(2
1*a^3*b^3 + 36*a^2*b^4 + 16*a*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.72

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{3\,(21\,a^2b^3+36\,ab^4+16\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{(a+b)b}} - \frac{3\,(a^3b^2-2\,a^2b^3-12\,ab^4-8\,b^5)\tan(fx+e)^7+(6\,a^4b-a^3b^2-73\,a^2b^3-144\,ab^4-72\,b^5)\tan(}{(a^6b^2+2\,a^5b^3+a^4b^4)\tan(fx+e)^8+a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4+2\,(a^7b+4\,a^6b^2+5}}{}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)) - (3*(a^3*b^2 - 2*a^2*b^
3 - 12*a*b^4 - 8*b^5)*tan(f*x + e)^7 + (6*a^4*b - a^3*b^2 - 73*a^2*b^3 - 1
44*a*b^4 - 72*b^5)*tan(f*x + e)^5 + (3*a^5 + 10*a^4*b - 24*a^3*b^2 - 136*a
^2*b^3 - 180*a*b^4 - 72*b^5)*tan(f*x + e)^3 + (5*a^5 + 8*a^4*b - 18*a^3*b^
2 - 69*a^2*b^3 - 72*a*b^4 - 24*b^5)*tan(f*x + e))/((a^6*b^2 + 2*a^5*b^3 +
a^4*b^4)*tan(f*x + e)^8 + a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4
+ 2*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^6 + (a^8 + 8*
a^7*b + 19*a^6*b^2 + 18*a^5*b^3 + 6*a^4*b^4)*tan(f*x + e)^4 + 2*(a^8 + 5*a
^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^2) - 3*(a^2 - 4*a*b
 + 16*b^2)*(f*x + e)/a^5)/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.72

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{3\,(21\,a^2b^3+36\,ab^4+16\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{ab+b^2}}-\frac{3\,a^3b^2\tan(fx+e)^7-6\,a^2b^3\tan(fx+e)^7-36\,ab^4\tan(fx+e)^7-2\cdots}{\cdots}}{\cdots}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 2*a^6*b + a^5*b^2)*s
qrt(a*b + b^2)) - (3*a^3*b^2*tan(f*x + e)^7 - 6*a^2*b^3*tan(f*x + e)^7 - 3
6*a*b^4*tan(f*x + e)^7 - 24*b^5*tan(f*x + e)^7 + 6*a^4*b*tan(f*x + e)^5 -
a^3*b^2*tan(f*x + e)^5 - 73*a^2*b^3*tan(f*x + e)^5 - 144*a*b^4*tan(f*x + e
)^5 - 72*b^5*tan(f*x + e)^5 + 3*a^5*tan(f*x + e)^3 + 10*a^4*b*tan(f*x + e)
^3 - 24*a^3*b^2*tan(f*x + e)^3 - 136*a^2*b^3*tan(f*x + e)^3 - 180*a*b^4*ta
n(f*x + e)^3 - 72*b^5*tan(f*x + e)^3 + 5*a^5*tan(f*x + e) + 8*a^4*b*tan(f*
x + e) - 18*a^3*b^2*tan(f*x + e) - 69*a^2*b^3*tan(f*x + e) - 72*a*b^4*tan(
f*x + e) - 24*b^5*tan(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(b*tan(f*x + e)
^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2) - 3*(a^2 - 4*a*b +
16*b^2)*(f*x + e)/a^5)/f
```

Mupad [B] (verification not implemented)

Time = 20.13 (sec) , antiderivative size = 4158, normalized size of antiderivative = 15.46

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

output

```
(atan(((((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a
^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3
))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) - (3*((12*a^1
0*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3
*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 +
6*a^14*b^2) - (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16
*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536
*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6
*a^10*b^2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2
)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10
 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)
*(36*a*b + 21*a^2 + 16*b^2)*3i)/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4
+ 10*a^7*b^3 + 10*a^8*b^2)) + (((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 +
27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 -
 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a
^10*b^2)) + (3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b
^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a
^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2
)*(36*a*b + 21*a^2 + 16*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5
 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12...
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 2166, normalized size of antiderivative = 8.05

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
( - 63*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**4*a**4*b**2 - 108*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
 b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b**3 - 48*sq
rt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*s
in(e + f*x)**4*a**2*b**4 + 126*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((
e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4*b**2 + 342*sqrt(b)*sq
rt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f
*x)**2*a**3*b**3 + 312*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)
/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*b**4 + 96*sqrt(b)*sqrt(a + b)
*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*
b**5 - 63*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*a**4*b**2 - 234*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*a**3*b**3 - 327*sqrt(b)*sqrt(a + b)*atan((sqrt
(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*b**4 - 204*sqrt(b)*sqrt(
a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b**5 - 48*
sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))
*b**6 - 63*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a
))/sqrt(b))*sin(e + f*x)**4*a**4*b**2 - 108*sqrt(b)*sqrt(a + b)*atan((sqrt
(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b**3 - 4
8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqr...
```

3.218 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 352

$$\begin{aligned}\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx &= \frac{(5a^3-18a^2b+48ab^2-160b^3)\,x}{16a^6}\\
&\quad+\frac{b^{7/2}(99a^2+176ab+80b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6(a+b)^{5/2}f}\\
&\quad+\frac{(15a^2-34ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2}\\
&\quad+\frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2}\\
&\quad+\frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2}\\
&\quad+\frac{b(15a^3-29a^2b+64ab^2+120b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}\\
&\quad+\frac{b(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4)\tan(e+fx)}{16a^5(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output
```
1/16*(5*a^3-18*a^2*b+48*a*b^2-160*b^3)*x/a^6+1/8*b^(7/2)*(99*a^2+176*a*b+8
0*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^6/(a+b)^(5/2)/f+1/48*(15*a
^2-34*a*b+80*b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+5/24*
(a-2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e
)^5*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/48*b*(15*a^3-29*a^2*b+64*a*b^2
+120*b^3)*tan(f*x+e)/a^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/16*b*(5*a^4-8*a^
3*b+17*a^2*b^2+116*a*b^3+80*b^4)*tan(f*x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e
)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.48 (sec) , antiderivative size = 1770, normalized size of antiderivative = 5.03

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((99*a^2 + 176*a*b + 80*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]
^6*(-1/64*(b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] -
I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4
*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^6*S
qrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x
]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2
*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*S
in[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^6*Sqrt[a + b]*f*Sqrt[b*Cos[4*e]
- I*b*Sin[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Co
s[2*e + 2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(720*a^7*f*x*Cos[2*e] + 768*a^6*b*
f*x*Cos[2*e] + 1296*a^5*b^2*f*x*Cos[2*e] - 8352*a^4*b^3*f*x*Cos[2*e] - 641
28*a^3*b^4*f*x*Cos[2*e] - 158976*a^2*b^5*f*x*Cos[2*e] - 165888*a*b^6*f*x*C
os[2*e] - 61440*b^7*f*x*Cos[2*e] + 480*a^7*f*x*Cos[2*f*x] + 192*a^6*b*f*x*
Cos[2*f*x] + 96*a^5*b^2*f*x*Cos[2*f*x] - 4608*a^4*b^3*f*x*Cos[2*f*x] - 418
56*a^3*b^4*f*x*Cos[2*f*x] - 67584*a^2*b^5*f*x*Cos[2*f*x] - 30720*a*b^6*f*x
*Cos[2*f*x] + 480*a^7*f*x*Cos[4*e + 2*f*x] + 192*a^6*b*f*x*Cos[4*e + 2*f*x
] + 96*a^5*b^2*f*x*Cos[4*e + 2*f*x] - 4608*a^4*b^3*f*x*Cos[4*e + 2*f*x] -
41856*a^3*b^4*f*x*Cos[4*e + 2*f*x] - 67584*a^2*b^5*f*x*Cos[4*e + 2*f*x] -
30720*a*b^6*f*x*Cos[4*e + 2*f*x] + 120*a^7*f*x*Cos[2*e + 4*f*x] - 192*a^6*
b*f*x*Cos[2*e + 4*f*x] + 408*a^5*b^2*f*x*Cos[2*e + 4*f*x] - 1968*a^4*b^...
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 25, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}-\frac{\int -\frac{9b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{6a}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{9b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{5(a-2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\int -\frac{15a^2+ba+10b^2+35(a-2b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{15a^2+ba+10b^2+35(a-2b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a}+\frac{5(a-2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\frac{\left(15a^2-34ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\int -\frac{15a^3+21ba^2-26b^2a-80b^3+5b\left(15a^2-34ba+80b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}}{4a}+\frac{5(a-2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e\cdots\right.}}{6a}\cdots}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\frac{\int \frac{15a^3+21ba^2-26b^2a-80b^3+5b\left(15a^2-34ba+80b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}+\frac{\left(15a^2-34ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}}{4a}+\frac{5(a-2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+\cdots\right.}}{6a}\cdots}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\frac{\frac{\int \frac{12\left(5a^4+2ba^3+b^2a^2-48b^3a-40b^4+b\left(15a^3-29ba^2+64b^2a+120b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}+\frac{b\left(15a^3-29a^2b+64ab^2+120b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\left(15a^2-34a\cdots\right.}{2a\left(\tan^2(e+fx)+\cdots\right.}}{4a}\cdots}{6a}\cdots}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{\frac{3\int \frac{5a^4+2ba^3+b^2a^2-48b^3a-40b^4+b\left(15a^3-29ba^2+64b^2a+120b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{a(a+b)}+\frac{b\left(15a^3-29a^2b+64ab^2+120b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\left(15a^2-34ab+\cdots\right.}{2a\left(\tan^2(e+fx)+1\right)\left(\cdots\right.}}{4a}\cdots}{6a}\cdots}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\frac{3\left(\frac{\int \frac{2\left(5a^5-3ba^4+9b^2a^3-65b^3a^2-156b^4a-80b^5+b\left(5a^4-8ba^3+17b^2a^2+116b^3a+80b^4\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{b\left(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4\right)\tan(\cdots}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\cdots\right.}{a(a+b)}\cdots}{2a}\cdots}{4a}\cdots}{6a}\cdots$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\left(\frac{\int \frac{5a^5-3ba^4+9b^2a^3-65b^3a^2-156b^4a-80b^5+b\left(5a^4-8ba^3+17b^2a^2+116b^3a+80b^4\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a(a+b)}+\frac{b\left(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4\right)\tan(e+}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right.}{a(a+b)}}{2a}}{4a}}{6a}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{3\left(\frac{\frac{2b^4\left(99a^2+176ab+80b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a+b)^2\left(5a^3-18a^2b+48ab^2-160b^3\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{a(a+b)}+\frac{b\left(5a^4-8a^3b+17a^2b^2+116\right.}{a(a+b)\left(a+b\tan\right.}\right.}{a(a+b)}}{2a}}{4a}}{6}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{3\left(\frac{\frac{2b^4\left(99a^2+176ab+80b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a+b)^2\left(5a^3-18a^2b+48ab^2-160b^3\right)\arctan(\tan(e+fx))}{a}}{a(a+b)}+\frac{b\left(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4\right)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right.}\right.}{a(a+b)}}{2a}}{4a}}{6a}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\left(15a^2-34ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{\frac{b\left(15a^3-29a^2b+64ab^2+120b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{3\left(\frac{\frac{2b^{7/2}\left(99a^2+176ab+80b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a+b)^2\left(5a^3-\right.}{}}{a(a+b)}\right.}{}}{2a}}{4a}}{6a}$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)^2) +
((5*(a - 2*b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e +
 f*x]^2)^2) + (((15*a^2 - 34*a*b + 80*b^2)*Tan[e + f*x])/(2*a*(1 + Tan[e +
 f*x]^2)*(a + b + b*Tan[e + f*x]^2)^2) + ((b*(15*a^3 - 29*a^2*b + 64*a*b^2
 + 120*b^3)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (3*((
((a + b)^2*(5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*ArcTan[Tan[e + f*x]])/a
 + (2*b^(7/2)*(99*a^2 + 176*a*b + 80*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*(5*a^4 - 8*a^3*b + 17*a^2*b^
2 + 116*a*b^3 + 80*b^4)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2
))))/(a*(a + b)))/(2*a))/(4*a))/(6*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{\left(\frac{5}{16}a^3-\frac{9}{8}a^2b+3ab^2\right)\tan(fx+e)^5+\left(6ab^2+\frac{5}{6}a^3-3a^2b\right)\tan(fx+e)^3+\left(-\frac{15}{8}a^2b+3ab^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{\left(5a^3-18a^2b+48ab^2\cdots\right.}{}}{a^6}$
default	$\frac{\frac{\left(\frac{5}{16}a^3-\frac{9}{8}a^2b+3ab^2\right)\tan(fx+e)^5+\left(6ab^2+\frac{5}{6}a^3-3a^2b\right)\tan(fx+e)^3+\left(-\frac{15}{8}a^2b+3ab^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{\left(5a^3-18a^2b+48ab^2\cdots\right.}{}}{a^6}$
risch	$\frac{5x}{16a^3}-\frac{9xb}{8a^4}+\frac{3xb^2}{a^5}-\frac{10xb^3}{a^6}-\frac{3i e^{2i(fx+e)}b^2}{4a^5f}-\frac{3i e^{4i(fx+e)}}{128a^3f}+\frac{3i e^{-4i(fx+e)}}{128a^3f}+\frac{3i e^{4i(fx+e)}b}{64a^4f}-\frac{3i e^{-4i(fx}}{64a^4}$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^6*(((5/16*a^3-9/8*a^2*b+3*a*b^2)*tan(f*x+e)^5+(6*a*b^2+5/6*a^3-3*
a^2*b)*tan(f*x+e)^3+(-15/8*a^2*b+3*a*b^2+11/16*a^3)*tan(f*x+e))/(1+tan(f*x
+e)^2)^3+1/16*(5*a^3-18*a^2*b+48*a*b^2-160*b^3)*arctan(tan(f*x+e)))+b^4/a^
6*((1/8*a*b*(19*a+16*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(21*a+16*b)*a/(a+
b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(99*a^2+176*a*b+80*b^2)/(a^2+2*a
*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1296, normalized size of antiderivative = 3.68

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/96*(6*(5*a^7 - 8*a^6*b + 17*a^5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^
2*b^5)*f*x*cos(f*x + e)^4 + 12*(5*a^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*
b^4 - 272*a^2*b^5 - 160*a*b^6)*f*x*cos(f*x + e)^2 + 6*(5*a^5*b^2 - 8*a^4*b
^3 + 17*a^3*b^4 - 82*a^2*b^5 - 272*a*b^6 - 160*b^7)*f*x + 3*(99*a^2*b^5 +
176*a*b^6 + 80*b^7 + (99*a^4*b^3 + 176*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^
4 + 2*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*b^6)*cos(f*x + e)^2)*sqrt(-b/(a + b
))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e)
)*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
 + e)^2 + b^2)) + 2*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 10*(a^7
- 3*a^5*b^2 - 2*a^4*b^3)*cos(f*x + e)^7 + (15*a^7 - 4*a^6*b + 27*a^5*b^2 +
 126*a^4*b^3 + 80*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 19*a^5*b^2 + 43*
a^4*b^3 + 266*a^3*b^4 + 180*a^2*b^5)*cos(f*x + e)^3 + 3*(5*a^5*b^2 - 8*a^4
*b^3 + 17*a^3*b^4 + 116*a^2*b^5 + 80*a*b^6)*cos(f*x + e))*sin(f*x + e))/((
a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^
3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f), 1/48*(3*(5*a^7 -
 8*a^6*b + 17*a^5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2*b^5)*f*x*cos(f*
x + e)^4 + 6*(5*a^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^4 - 272*a^2*b^5
- 160*a*b^6)*f*x*cos(f*x + e)^2 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 -
82*a^2*b^5 - 272*a*b^6 - 160*b^7)*f*x - 3*(99*a^2*b^5 + 176*a*b^6 + 80*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.74

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{6\,(99\,a^2b^4+176\,ab^5+80\,b^6)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^8+2\,a^7b+a^6b^2)\sqrt{(a+b)b}} + \frac{3\,(5\,a^4b^2-8\,a^3b^3+17\,a^2b^4+116\,ab^5+80\,b^6)\tan(fx+e)^9+2\,(15\,a^5b+11\,a^4b^2-5\,a^3b^3+368\,\cdots}{(a^7b^2+2\,a^6b^3+a^5b^4)\tan(fx+e)^{10}+a^9+4\,a^8b+6\,a^{\cdots}}}{\cdots}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*arctan(b*tan(f*x + e)/sqrt((a +
b)*b))/((a^8 + 2*a^7*b + a^6*b^2)*sqrt((a + b)*b)) + (3*(5*a^4*b^2 - 8*a^3
*b^3 + 17*a^2*b^4 + 116*a*b^5 + 80*b^6)*tan(f*x + e)^9 + 2*(15*a^5*b + 11*
a^4*b^2 - 5*a^3*b^3 + 368*a^2*b^4 + 876*a*b^5 + 480*b^6)*tan(f*x + e)^7 +
(15*a^6 + 86*a^5*b + 3*a^4*b^2 + 240*a^3*b^3 + 1982*a^2*b^4 + 3168*a*b^5 +
 1440*b^6)*tan(f*x + e)^5 + 2*(20*a^6 + 41*a^5*b - 15*a^4*b^2 + 197*a^3*b^
3 + 980*a^2*b^4 + 1236*a*b^5 + 480*b^6)*tan(f*x + e)^3 + 3*(11*a^6 + 14*a^
5*b - 6*a^4*b^2 + 56*a^3*b^3 + 221*a^2*b^4 + 236*a*b^5 + 80*b^6)*tan(f*x +
 e))/((a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*tan(f*x + e)^10 + a^9 + 4*a^8*b + 6*
a^7*b^2 + 4*a^6*b^3 + a^5*b^4 + (2*a^8*b + 9*a^7*b^2 + 12*a^6*b^3 + 5*a^5*
b^4)*tan(f*x + e)^8 + (a^9 + 10*a^8*b + 27*a^7*b^2 + 28*a^6*b^3 + 10*a^5*b
^4)*tan(f*x + e)^6 + (3*a^9 + 18*a^8*b + 37*a^7*b^2 + 32*a^6*b^3 + 10*a^5*
b^4)*tan(f*x + e)^4 + (3*a^9 + 14*a^8*b + 24*a^7*b^2 + 18*a^6*b^3 + 5*a^5*
b^4)*tan(f*x + e)^2) + 3*(5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*(f*x + e)
/a^6)/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.99

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{6\,(99\,a^2b^4+176\,ab^5+80\,b^6)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^8+2\,a^7b+a^6b^2)\sqrt{ab+b^2}} + \frac{6\left(19\,ab^5\tan(fx+e)^3+16\,b^6\tan(fx+e)^3+21\,a^2b^4\tan(fx+e)+37\cdots\right.}{(a^7+2\,a^6b+a^5b^2)\left(b\tan(fx+e)^2+a\cdots\right.}}{\cdots}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sg
n(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 2*a^7*b + a^6*b^2)*
sqrt(a*b + b^2)) + 6*(19*a*b^5*tan(f*x + e)^3 + 16*b^6*tan(f*x + e)^3 + 21
*a^2*b^4*tan(f*x + e) + 37*a*b^5*tan(f*x + e) + 16*b^6*tan(f*x + e))/((a^7
 + 2*a^6*b + a^5*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 3*(5*a^3 - 18*a^2*b
+ 48*a*b^2 - 160*b^3)*(f*x + e)/a^6 + (15*a^2*tan(f*x + e)^5 - 54*a*b*tan(
f*x + e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 144*a*b*tan(
f*x + e)^3 + 288*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 90*a*b*tan(f*x
 + e) + 144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

Mupad [B] (verification not implemented)

Time = 20.38 (sec) , antiderivative size = 4594, normalized size of antiderivative = 13.05

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)*(156*a*b^4 + 3*a^4*b + 11*a^5 + 80*b^5 + 65*a^2*b^3 - 9*a^3
*b^2))/(16*a^5*(a + b)) + (tan(e + f*x)^7*(876*a*b^5 + 15*a^5*b + 480*b^6
+ 368*a^2*b^4 - 5*a^3*b^3 + 11*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x
)^3*(1236*a*b^5 + 41*a^5*b + 20*a^6 + 480*b^6 + 980*a^2*b^4 + 197*a^3*b^3
- 15*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x)^5*(3168*a*b^5 + 86*a^5*b
 + 15*a^6 + 1440*b^6 + 1982*a^2*b^4 + 240*a^3*b^3 + 3*a^4*b^2))/(48*a^5*(a
 + b)^2) + (b*tan(e + f*x)^9*(116*a*b^4 + 5*a^4*b + 80*b^5 + 17*a^2*b^3 -
8*a^3*b^2))/(16*a^5*(a + b)^2))/(f*(2*a*b + tan(e + f*x)^6*(8*a*b + a^2 +
10*b^2) + a^2 + b^2 + tan(e + f*x)^8*(2*a*b + 5*b^2) + b^2*tan(e + f*x)^10
 + tan(e + f*x)^2*(8*a*b + 3*a^2 + 5*b^2) + tan(e + f*x)^4*(12*a*b + 3*a^2
 + 10*b^2))) - (atan(-((((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 +
 (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^1
9*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e
+ f*x)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 + 9216*a
^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2)
)/(4096*a^6*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^
2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6) - (tan(e + f*x)*(199680*a
*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 3
6*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^
10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*...
```

Reduce [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 2455, normalized size of antiderivative = 6.97

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
(594*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqr
t(b))*sin(e + f*x)**4*a**4*b**3 + 1056*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b**4 + 480*sq
rt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*s
in(e + f*x)**4*a**2*b**5 - 1188*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4*b**3 - 3300*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e +
 f*x)**2*a**3*b**4 - 3072*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f
*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**2*b**5 - 960*sqrt(b)*sqrt(a
+ b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**
2*a*b**6 + 594*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sq
rt(a))/sqrt(b))*a**4*b**3 + 2244*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan
((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3*b**4 + 3186*sqrt(b)*sqrt(a + b)*ata
n((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2*b**5 + 2016*sqrt(
b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b*
*6 + 480*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))
/sqrt(b))*b**7 + 594*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2
) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4*b**3 + 1056*sqrt(b)*sqrt(a + b)
*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a*
*3*b**4 + 480*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + ...
```

3.219 $\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 204

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = \frac{x}{a^4} - \frac{\sqrt{b}(35a^3+70a^2b+56ab^2+16b^3)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} - \frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2} - \frac{b(19a^2+22ab+8b^2)\tan(c+dx)}{16a^3(a+b)^3d\left(a+b+b\tan^2(c+dx)\right)}$$

output
```
x/a^4-1/16*b^(1/2)*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)*arctan(b^(1/2)*tan(d*
x+c)/(a+b)^(1/2))/a^4/(a+b)^(7/2)/d-1/6*b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(
d*x+c)^2)^3-1/24*b*(11*a+6*b)*tan(d*x+c)/a^2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2
)^2-1/16*b*(19*a^2+22*a*b+8*b^2)*tan(d*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c
)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.65 (sec) , antiderivative size = 1411, normalized size of antiderivative = 6.92

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^(-4),x]
```

output
```
((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cos[2*c + 2*d*x])^4*
Sec[c + d*x]^8*((b*ArcTan[Sec[d*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c
] - I*b*Sin[4*c]]) - ((I/2)*Sin[2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*S
in[4*c]]))*(-(a*Sin[d*x]) - 2*b*Sin[d*x] + a*Sin[2*c + d*x])]*Cos[2*c])/(2
56*a^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/256)*b*ArcTan[
Sec[d*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/2
)*Sin[2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]))*(-(a*Sin[d*x])
- 2*b*Sin[d*x] + a*Sin[2*c + d*x])]*Sin[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Co
s[4*c] - I*b*Sin[4*c]])))/((a + b)^3*(a + b*Sec[c + d*x]^2)^4) + ((a + 2*b
 + a*Cos[2*c + 2*d*x])*Sec[2*c]*Sec[c + d*x]^8*(480*a^6*d*x*Cos[2*c] + 316
8*a^5*b*d*x*Cos[2*c] + 8928*a^4*b^2*d*x*Cos[2*c] + 14112*a^3*b^3*d*x*Cos[2
*c] + 13248*a^2*b^4*d*x*Cos[2*c] + 6912*a*b^5*d*x*Cos[2*c] + 1536*b^6*d*x*
Cos[2*c] + 360*a^6*d*x*Cos[2*d*x] + 2232*a^5*b*d*x*Cos[2*d*x] + 5688*a^4*b
^2*d*x*Cos[2*d*x] + 7272*a^3*b^3*d*x*Cos[2*d*x] + 4608*a^2*b^4*d*x*Cos[2*d
*x] + 1152*a*b^5*d*x*Cos[2*d*x] + 360*a^6*d*x*Cos[4*c + 2*d*x] + 2232*a^5*
b*d*x*Cos[4*c + 2*d*x] + 5688*a^4*b^2*d*x*Cos[4*c + 2*d*x] + 7272*a^3*b^3*
d*x*Cos[4*c + 2*d*x] + 4608*a^2*b^4*d*x*Cos[4*c + 2*d*x] + 1152*a*b^5*d*x*
Cos[4*c + 2*d*x] + 144*a^6*d*x*Cos[2*c + 4*d*x] + 720*a^5*b*d*x*Cos[2*c +
4*d*x] + 1296*a^4*b^2*d*x*Cos[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[2*c + 4*
d*x] + 288*a^2*b^4*d*x*Cos[2*c + 4*d*x] + 144*a^6*d*x*Cos[6*c + 4*d*x] ...
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4616, 316, 402, 27, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sec(c+dx)^2)^4}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)(b\tan^2(c+dx)+a+b)^4}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-5b\tan^2(c+dx)+6a+b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^3}d\tan(c+dx)}{6a(a+b)}-\frac{b\tan(c+dx)}{6a(a+b)(a+b\tan^2(c+dx)+b)^3}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3\left(8a^2+5ba+2b^2-b(11a+6b)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^2}d\tan(c+dx)}{4a(a+b)}-\frac{b(11a+6b)\tan(c+dx)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}}{6a(a+b)}-\frac{b\tan(c+dx)}{6a(a+b)(a+b\tan^2(c+dx)+b)^3}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{8a^2+5ba+2b^2-b(11a+6b)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^2}d\tan(c+dx)}{4a(a+b)}-\frac{b(11a+6b)\tan(c+dx)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}}{6a(a+b)}-\frac{b\tan(c+dx)}{6a(a+b)(a+b\tan^2(c+dx)+b)^3}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{3\left(\frac{\int \frac{16a^3+29ba^2+26b^2a+8b^3-b\left(19a^2+22ba+8b^2\right)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)}d\tan(c+dx)}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{2a(a+b)\left(a+b\tan^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)\tan(c+dx)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}}{6a(a+b)}-\frac{}{6a(a+b)}}{d}$$

$\downarrow$ 397

$$\frac{\frac{3\left(\frac{\frac{16(a+b)^3\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{a}-\frac{b\left(35a^3+70a^2b+56ab^2+16b^3\right)\int \frac{1}{b\tan^2(c+dx)+a+b}d\tan(c+dx)}{a}}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{2a(a+b)\left(a+b\tan^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6}{4a(a+b)\left(a+b\right.}}{6a(a+b)}}{d}$$

$\downarrow$ 216

$$\frac{\frac{3\left(\frac{\frac{16(a+b)^3\arctan(\tan(c+dx))}{a}-\frac{b\left(35a^3+70a^2b+56ab^2+16b^3\right)\int \frac{1}{b\tan^2(c+dx)+a+b}d\tan(c+dx)}{a}}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{2a(a+b)\left(a+b\tan^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)\tan(c+d}{4a(a+b)\left(a+b\tan^2(c+d\right.}}{6a(a+b)}}{d}$$

$\downarrow$ 218

$$\frac{\frac{3\left(\frac{\frac{16(a+b)^3\arctan(\tan(c+dx))}{a}-\frac{\sqrt{b}\left(35a^3+70a^2b+56ab^2+16b^3\right)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{2a(a+b)\left(a+b\tan^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)\tan(c+dx)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}}{6a(a+b)}}{d}$$

input
```
Int[(a + b*Sec[c + d*x]^2)^(-4),x]
```

output
```
(-1/6*(b*Tan[c + d*x])/(a*(a + b)*(a + b + b*Tan[c + d*x]^2)^3) + (-1/4*(b
*(11*a + 6*b)*Tan[c + d*x])/(a*(a + b)*(a + b + b*Tan[c + d*x]^2)^2) + (3*
(((16*(a + b)^3*ArcTan[Tan[c + d*x]])/a - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56
*a*b^2 + 16*b^3)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b
]))/(2*a*(a + b)) - (b*(19*a^2 + 22*a*b + 8*b^2)*Tan[c + d*x])/(2*a*(a + b
)*(a + b + b*Tan[c + d*x]^2))))/(4*a*(a + b)))/(6*a*(a + b)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{a^4}-\frac{b\left(\frac{\frac{b^2a\left(19a^2+22ab+8b^2\right)\tan(dx+c)^5}{16a^3+48a^2b+48a\,b^2+16b^3}+\frac{\left(17a^2+18ab+6b^2\right)ab\tan(dx+c)^3}{6a^2+12ab+6b^2}+\frac{\left(29a^2+26ab+8b^2\right)a\tan(dx+c)}{16a+16b}}{\left(a+b+b\tan(dx+c)^2\right)^3}+\right.}{a^4}}{d}$
default	$\frac{\frac{\arctan(\tan(dx+c))}{a^4}-\frac{b\left(\frac{\frac{b^2a\left(19a^2+22ab+8b^2\right)\tan(dx+c)^5}{16a^3+48a^2b+48a\,b^2+16b^3}+\frac{\left(17a^2+18ab+6b^2\right)ab\tan(dx+c)^3}{6a^2+12ab+6b^2}+\frac{\left(29a^2+26ab+8b^2\right)a\tan(dx+c)}{16a+16b}}{\left(a+b+b\tan(dx+c)^2\right)^3}+\right.}{a^4}}{d}$
risch	Expression too large to display

input
```
int(1/(a+sec(d*x+c)^2*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^4*arctan(tan(d*x+c))-b/a^4*((1/16*b^2*a*(19*a^2+22*a*b+8*b^2)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)^5+1/6*(17*a^2+18*a*b+6*b^2)*a*b/(a^2+2*a
*b+b^2)*tan(d*x+c)^3+1/16*(29*a^2+26*a*b+8*b^2)*a/(a+b)*tan(d*x+c))/(a+b+b
*tan(d*x+c)^2)^3+1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)/(a^3+3*a^2*b+3*a*b
^2+b^3)/((a+b)*b)^(1/2)*arctan(b*tan(d*x+c)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(188) = 376$.

Time = 0.18 (sec) , antiderivative size = 1323, normalized size of antiderivative = 6.49

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
[1/192*(192*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 576
*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*x*cos(d*x + c)^4 + 576*(a^4*b
^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*x*cos(d*x + c)^2 + 192*(a^3*b^3 + 3*
a^2*b^4 + 3*a*b^5 + b^6)*d*x + 3*((35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3
*b^3)*cos(d*x + c)^6 + 35*a^3*b^3 + 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35
*a^5*b + 70*a^4*b^2 + 56*a^3*b^3 + 16*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*
b^2 + 70*a^3*b^3 + 56*a^2*b^4 + 16*a*b^5)*cos(d*x + c)^2)*sqrt(-b/(a + b))
*log(((a^2 + 8*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a*b + 4*b^2)*cos(d*x + c
)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^3 - (a*b + b^2)*cos(d*x + c))*
sqrt(-b/(a + b))*sin(d*x + c) + b^2)/(a^2*cos(d*x + c)^4 + 2*a*b*cos(d*x +
 c)^2 + b^2)) - 4*((87*a^5*b + 116*a^4*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 +
2*(68*a^4*b^2 + 83*a^3*b^3 + 30*a^2*b^4)*cos(d*x + c)^3 + 3*(19*a^3*b^3 +
22*a^2*b^4 + 8*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8
*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*
b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*co
s(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d), 1/96*(96*(a
^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 288*(a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*x*cos(d*x + c)^4 + 288*(a^4*b^2 + 3*a^3*b^3
 + 3*a^2*b^4 + a*b^5)*d*x*cos(d*x + c)^2 + 96*(a^3*b^3 + 3*a^2*b^4 + 3*a*b
^5 + b^6)*d*x + 3*((35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3)*cos(d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c)**2)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(188) = 376$.

Time = 0.12 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.97

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = -\frac{\frac{3\,(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{(a+b)b}}\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{(a+b)b}} + \frac{3\,(19\,a^2b^3+22\,ab^4+8\,b^5)\tan(dx+c)^5+8\,(17\,a}{a^9+6\,a^8b+15\,a^7b^2+20\,a^6b^3+15\,a^5b^4+6\,a^4b^5+a^3b^6+(a^6b^3+3\,a^5b^4+3\,a^4b^5+a^3b}}{4}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
-1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*arctan(b*tan(d*x + c)
/sqrt((a + b)*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt((a + b)*b))
+ (3*(19*a^2*b^3 + 22*a*b^4 + 8*b^5)*tan(d*x + c)^5 + 8*(17*a^3*b^2 + 35*a
^2*b^3 + 24*a*b^4 + 6*b^5)*tan(d*x + c)^3 + 3*(29*a^4*b + 84*a^3*b^2 + 89*
a^2*b^3 + 42*a*b^4 + 8*b^5)*tan(d*x + c))/(a^9 + 6*a^8*b + 15*a^7*b^2 + 20
*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6 + (a^6*b^3 + 3*a^5*b^4 + 3*a^4
*b^5 + a^3*b^6)*tan(d*x + c)^6 + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^
4*b^5 + a^3*b^6)*tan(d*x + c)^4 + 3*(a^8*b + 5*a^7*b^2 + 10*a^6*b^3 + 10*a
^5*b^4 + 5*a^4*b^5 + a^3*b^6)*tan(d*x + c)^2) - 48*(d*x + c)/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = -\frac{\frac{3\,(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab+b^2}}\right)\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{ab+b^2}} + \frac{57\,a^2b^3\tan(dx+c)^5+66\,ab^4\tan(dx+c)^5+24\,b^5\tan(dx+}{}}{}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
-1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*(pi*floor((d*x + c)/p
i + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b + b^2)))/((a^7 + 3*a^6*b
+ 3*a^5*b^2 + a^4*b^3)*sqrt(a*b + b^2)) + (57*a^2*b^3*tan(d*x + c)^5 + 66*
a*b^4*tan(d*x + c)^5 + 24*b^5*tan(d*x + c)^5 + 136*a^3*b^2*tan(d*x + c)^3
+ 280*a^2*b^3*tan(d*x + c)^3 + 192*a*b^4*tan(d*x + c)^3 + 48*b^5*tan(d*x +
 c)^3 + 87*a^4*b*tan(d*x + c) + 252*a^3*b^2*tan(d*x + c) + 267*a^2*b^3*tan
(d*x + c) + 126*a*b^4*tan(d*x + c) + 24*b^5*tan(d*x + c))/((a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*(b*tan(d*x + c)^2 + a + b)^3) - 48*(d*x + c)/a^4)/d
```

Mupad [B] (verification not implemented)

Time = 19.89 (sec) , antiderivative size = 4506, normalized size of antiderivative = 22.09

$$\int \frac{1}{(a + b\sec^2(c + dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(c + d*x)^2)^4,x)
```

output
```
atan((((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4
 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^1
5 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) - (ta
n(c + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6
 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*
a^4*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*
a^10*b^2)))/(2*a^4) + (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 +
 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a^1
1*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2))
)/a^4 - ((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)
/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a
^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) + (
tan(c + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b
^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(51
2*a^4*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 1
5*a^10*b^2)))/(2*a^4) - (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7
 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a
^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2
)))/a^4)/((((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b
^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14...
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 2880, normalized size of antiderivative = 14.12

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sec(d*x+c)^2)^4,x)
```

output
```
( - 105*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((c + d*x)/2) - sqrt(a))/
sqrt(b))*sin(c + d*x)**6*a**6 - 210*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*
tan((c + d*x)/2) - sqrt(a))/sqrt(b))*sin(c + d*x)**6*a**5*b - 168*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((c + d*x)/2) - sqrt(a))/sqrt(b))*sin(c +
 d*x)**6*a**4*b**2 - 48*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((c + d*x
)/2) - sqrt(a))/sqrt(b))*sin(c + d*x)**6*a**3*b**3 + 315*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((c + d*x)/2) - sqrt(a))/sqrt(b))*sin(c + d*x)**4*
a**6 + 945*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((c + d*x)/2) - sqrt(a
))/sqrt(b))*sin(c + d*x)**4*a**5*b + 1134*sqrt(b)*sqrt(a + b)*atan((sqrt(a
 + b)*tan((c + d*x)/2) - sqrt(a))/sqrt(b))*sin(c + d*x)**4*a**4*b**2 + 648
*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((c + d*x)/2) - sqrt(a))/sqrt(b)
)*sin(c + d*x)**4*a**3*b**3 + 144*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((c + d*x)/2) - sqrt(a))/sqrt(b))*sin(c + d*x)**4*a**2*b**4 - 315*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((c + d*x)/2) - sqrt(a))/sqrt(b))*sin(c
+ d*x)**2*a**6 - 1260*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((c + d*x)/
2) - sqrt(a))/sqrt(b))*sin(c + d*x)**2*a**5*b - 2079*sqrt(b)*sqrt(a + b)*a
tan((sqrt(a + b)*tan((c + d*x)/2) - sqrt(a))/sqrt(b))*sin(c + d*x)**2*a**4
*b**2 - 1782*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((c + d*x)/2) - sqrt
(a))/sqrt(b))*sin(c + d*x)**2*a**3*b**3 - 792*sqrt(b)*sqrt(a + b)*atan((sq
rt(a + b)*tan((c + d*x)/2) - sqrt(a))/sqrt(b))*sin(c + d*x)**2*a**2*b**...
```

3.220 $\int (a-a\sec^2(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int (a-a\sec^2(c+dx))^{7/2}\,dx = -\frac{a^3\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}$$

output
```
-a^3*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^3*tan(d*x+c
)*(-a*tan(d*x+c)^2)^(1/2)/d+1/4*a^3*tan(d*x+c)^3*(-a*tan(d*x+c)^2)^(1/2)/d
-1/6*a^3*tan(d*x+c)^5*(-a*tan(d*x+c)^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int (a-a\sec^2(c+dx))^{7/2}\,dx = \frac{\cot^7(c+dx)\left(12\log(\cos(c+dx))+18\sec^2(c+dx)-9\sec^4(c+dx)+2\sec^6(c+dx)\right)(-a\,t}{12d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

output
```
(Cot[c + d*x]^7*(12*Log[Cos[c + d*x]] + 18*Sec[c + d*x]^2 - 9*Sec[c + d*x]
^4 + 2*Sec[c + d*x]^6)*(-(a*Tan[c + d*x]^2))^(7/2))/(12*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 4609, 3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sec(c+dx)^2\right)^{7/2}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \left(-a\tan^2(c+dx)\right)^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-a\tan(c+dx)^2\right)^{7/2}dx$$

$$\Big\downarrow \text{4141}$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan^7(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)^7dx$$

$$\Big\downarrow \text{3954}$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^6(c+dx)}{6d}-\int \tan^5(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^6(c+dx)}{6d}-\int \tan(c+dx)^5dx\right)$$

$$\Big\downarrow \text{3954}$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\int \tan^3(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\int \tan(c+dx)^3dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3954}$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(-\int \tan(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(-\int \tan(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3956}$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

output
```
-(a^3*Cot[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)]*(Log[Cos[c + d*x]]/d + Tan[c
+ d*x]^2/(2*d) - Tan[c + d*x]^4/(4*d) + Tan[c + d*x]^6/(6*d)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.07

method	result
default	$\frac{\left(-\cos(dx+c)^6\ln(-\cot(dx+c)+\csc(dx+c)+1)+\cos(dx+c)^6\ln\left(\frac{2}{\cos(dx+c)+1}\right)-\cos(dx+c)^6\ln(-\cot(dx+c)+\csc(dx+c)-1)+\frac{11\cos}{}\right.}{d}$
risch	$\frac{a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{2ia^3\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(9\,e^{10i(dx+c)}}{3(e^{2i(dx}}$

input
```
int((a-sec(d*x+c)^2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-cos(d*x+c)^6*ln(-cot(d*x+c)+csc(d*x+c)+1)+cos(d*x+c)^6*ln(2/(cos(d*x
+c)+1))-cos(d*x+c)^6*ln(-cot(d*x+c)+csc(d*x+c)-1)+11/12*cos(d*x+c)^6-3/2*c
os(d*x+c)^4+3/4*cos(d*x+c)^2-1/6)*a^3*(-a*tan(d*x+c)^2)^(1/2)*sec(d*x+c)^5
*csc(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx = -\frac{\left(12\,a^3\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)+18\,a^3\cos\left(dx+c\right)^4-9\,a^3\cos\left(dx+c\right)^2+2\,a^3\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{12\,d\cos\left(dx+c\right)^5\sin\left(dx+c\right)}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/12*(12*a^3*cos(d*x + c)^6*log(-cos(d*x + c)) + 18*a^3*cos(d*x + c)^4 -
9*a^3*cos(d*x + c)^2 + 2*a^3)*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/
(d*cos(d*x + c)^5*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a-a*sec(d*x+c)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx =$$

$$-\frac{2\sqrt{-a}a^3\tan(dx+c)^6-3\sqrt{-a}a^3\tan(dx+c)^4+6\sqrt{-a}a^3\tan(dx+c)^2-6\sqrt{-a}a^3\log\left(\tan(dx+c)^2\right.}{12\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/12*(2*sqrt(-a)*a^3*tan(d*x + c)^6 - 3*sqrt(-a)*a^3*tan(d*x + c)^4 + 6*s
qrt(-a)*a^3*tan(d*x + c)^2 - 6*sqrt(-a)*a^3*log(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.62

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx =$$

$$-\frac{6\sqrt{-a}a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right)-6\sqrt{-a}a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-\right.}{}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

output
```
-1/12*(6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^
2 + 2) - 6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c
)^2 - 2) + (11*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^3*sqrt(
-a)*a^3 - 90*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a
)*a^3 + 276*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a
^3 - 408*sqrt(-a)*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
- 2)^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx=\int \left(a-\frac{a}{\cos\left(c+d\,x\right)^2}\right)^{7/2}\,dx$$

input
```
int((a - a/cos(c + d*x)^2)^(7/2),x)
```

output
```
int((a - a/cos(c + d*x)^2)^(7/2), x)
```

Reduce [F]

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx=\sqrt{a}\,a^3\left(\int \sqrt{-\sec\left(dx+c\right)^2+1}dx-\left(\int \sqrt{-\sec\left(dx+c\right)^2+1}\,\sec\left(dx+c\right)^6dx\right)+3\left(\int \sqrt{-\sec\left(dx+c\right)^2+1}\,\sec\left(dx+c\right)^4dx\right)-3\left(\int \sqrt{-\sec\left(dx+c\right)^2+1}\,\sec\left(dx+c\right)^2dx\right)\right)$$

input
```
int((a-a*sec(d*x+c)^2)^(7/2),x)
```

output
```
sqrt(a)*a**3*(int(sqrt( - sec(c + d*x)**2 + 1),x) - int(sqrt( - sec(c + d*
x)**2 + 1)*sec(c + d*x)**6,x) + 3*int(sqrt( - sec(c + d*x)**2 + 1)*sec(c +
 d*x)**4,x) - 3*int(sqrt( - sec(c + d*x)**2 + 1)*sec(c + d*x)**2,x))
```

3.221 $\int (a - a\sec^2(c+dx))^{5/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int (a - a\sec^2(c+dx))^{5/2} \, dx = -\frac{a^2\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}$$

output
```
-a^2*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^2*tan(d*x+c
)*(-a*tan(d*x+c)^2)^(1/2)/d+1/4*a^2*tan(d*x+c)^3*(-a*tan(d*x+c)^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int (a - a\sec^2(c+dx))^{5/2} \, dx = \frac{a\csc^3(c+dx)(2+3\log(\cos(c+dx))+\cos(4(c+dx))\log(\cos(c+dx))+4\cos(2(c+dx))(}{8d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

output
```
(a*Csc[c + d*x]^3*(2 + 3*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c +
d*x]] + 4*Cos[2*(c + d*x)]*(1 + Log[Cos[c + d*x]]))*Sec[c + d*x]*(-(a*Tan[
c + d*x]^2))^(3/2))/(8*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 4609, 3042, 4141, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sec^2(c+dx)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sec(c+dx)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{4609}$$

$$\int \left(-a\tan^2(c+dx)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-a\tan(c+dx)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{4141}$$

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)^5dx$$

$$\Big\downarrow \text{3954}$$

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d}-\int \tan^3(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d}-\int\tan(c+dx)^3dx\right)$$

$\Big\downarrow$ 3954

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\int\tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}\right)$$

$\Big\downarrow$ 3042

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\int\tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}\right)$$

$\Big\downarrow$ 3956

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}-\frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

output
```
a^2*Cot[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)]*(-(Log[Cos[c + d*x]]/d) - Tan[c
 + d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

method	result
default	$\frac{\left(-\cos(dx+c)^4\ln(-\cot(dx+c)+\csc(dx+c)+1)+\cos(dx+c)^4\ln\left(\frac{2}{\cos(dx+c)+1}\right)-\cos(dx+c)^4\ln(-\cot(dx+c)+\csc(dx+c)-1)+\frac{3\cos\ldots}{}\right.}{d}$
risch	$\frac{a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{4ia^2\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{6i(dx+c)}+e\ldots\right.}{\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c}\ldots\right.}$

input
```
int((a-sec(d*x+c)^2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)+cos(d*x+c)^4*ln(2/(cos(d*x
+c)+1))-cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+3/4*cos(d*x+c)^4-cos(d*x
+c)^2+1/4)*a^2*(-a*tan(d*x+c)^2)^(1/2)*sec(d*x+c)^3*csc(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \left(a-a\sec^2(c+dx)\right)^{5/2}\,dx = -\frac{\left(4\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)+4\,a^2\cos\left(dx+c\right)^2-a^2\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{4\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/4*(4*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) + 4*a^2*cos(d*x + c)^2 - a^2
)*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x +
c))
```

Sympy [F]

$$\int \left(a-a\sec^2(c+dx)\right)^{5/2}\,dx = \int \left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a-a*sec(d*x+c)**2)**(5/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \left(a-a\sec^2(c+dx)\right)^{5/2}\,dx = \frac{\sqrt{-a}a^2\tan\left(dx+c\right)^4-2\sqrt{-a}a^2\tan\left(dx+c\right)^2+2\sqrt{-a}a^2\log\left(\tan\left(dx+c\right)^2+1\right)}{4\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(sqrt(-a)*a^2*tan(d*x + c)^4 - 2*sqrt(-a)*a^2*tan(d*x + c)^2 + 2*sqrt(
-a)*a^2*log(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.80

$$\int \left(a - a\sec^2(c + dx)\right)^{5/2} dx =$$

$$-\frac{2\sqrt{-a}a^2\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} + 2\right) - 2\sqrt{-a}a^2\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + \frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} - \right.}{4d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*(2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 + 2) - 2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)
^2 - 2) + (3*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a
)*a^2 - 20*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^
2 + 44*sqrt(-a)*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 -
2)^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(a - a\sec^2(c + dx)\right)^{5/2} dx = \int \left(a - \frac{a}{\cos\left(c + d\,x\right)^2}\right)^{5/2} dx$$

input
```
int((a - a/cos(c + d*x)^2)^(5/2),x)
```

output
```
int((a - a/cos(c + d*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(a - a\sec^2(c+dx)\right)^{5/2} dx = \sqrt{a}\, a^2 \left(\int \sqrt{-\sec(dx+c)^2+1}dx + \int \sqrt{-\sec(dx+c)^2+1}\,\sec(dx+c)^4\,dx - 2\left(\int \sqrt{-\sec(dx+c)^2+1}\,\sec(dx+c)^2\,dx\right)\right)$$

input
```
int((a-a*sec(d*x+c)^2)^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt( - sec(c + d*x)**2 + 1),x) + int(sqrt( - sec(c + d*
x)**2 + 1)*sec(c + d*x)**4,x) - 2*int(sqrt( - sec(c + d*x)**2 + 1)*sec(c +
 d*x)**2,x))
```

3.222 $\int (a - a\sec^2(c + dx))^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int (a - a\sec^2(c + dx))^{3/2} \, dx = -\frac{a\cot(c + dx)\log(\cos(c + dx))\sqrt{-a\tan^2(c + dx)}}{d} - \frac{a\tan(c + dx)\sqrt{-a\tan^2(c + dx)}}{2d}$$

output
```
-a*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a*tan(d*x+c)*(-
a*tan(d*x+c)^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int (a - a\sec^2(c + dx))^{3/2} \, dx = \frac{\cot^3(c + dx)\left(2\log(\cos(c + dx)) + \sec^2(c + dx)\right)\left(-a\tan^2(c + dx)\right)^{3/2}}{2d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

output
```
(Cot[c + d*x]^3*(2*Log[Cos[c + d*x]] + Sec[c + d*x]^2)*(-(a*Tan[c + d*x]^2
))^(3/2))/(2*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4609, 3042, 4141, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sec(c+dx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \left(-a\tan^2(c+dx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-a\tan(c+dx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4141}$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)^3dx$$

$$\Big\downarrow \text{3954}$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d}-\int\tan(c+dx)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

output
```
-(a*Cot[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)]*(Log[Cos[c + d*x]]/d + Tan[c +
d*x]^2/(2*d)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.56

method	result
default	$\frac{a\sqrt{-a\tan(dx+c)^2}\left(2\ln\left(\frac{2}{\cos(dx+c)+1}\right)\cot(dx+c)-2\ln(-\cot(dx+c)+\csc(dx+c)+1)\cot(dx+c)-2\ln(-\cot(dx+c)+\csc(dx+c)-1)\right.}{2d}$
risch	$-\frac{a\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(ie^{4i(dx+c)}\ln\left(e^{2i(dx+c)}+1\right)+e^{4i(dx+c)}dx+2\,e^{4i(dx+c)}c+2ie^{2i(dx+c)}\ln\left(e^{2i(dx+c)}+1\right)+2\,e^{2i(dx+c)}dx+2ie^{2}\right.}{\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)d}$

input
```
int((a-sec(d*x+c)^2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*a*(-a*tan(d*x+c)^2)^(1/2)*(2*ln(2/(cos(d*x+c)+1))*cot(d*x+c)-2*ln(-c
ot(d*x+c)+csc(d*x+c)+1)*cot(d*x+c)-2*ln(-cot(d*x+c)+csc(d*x+c)-1)*cot(d*x+
c)-tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=-\frac{\left(2\,a\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)+a\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*a*cos(d*x + c)^2*log(-cos(d*x + c)) + a)*sqrt((a*cos(d*x + c)^2 -
a)/cos(d*x + c)^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=\int \left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-a*sec(d*x+c)**2)**(3/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=-\frac{\sqrt{-a}a\tan\left(dx+c\right)^2-\sqrt{-a}a\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(-a)*a*tan(d*x + c)^2 - sqrt(-a)*a*log(tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.

Time = 0.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.14

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=$$

$$-\frac{\sqrt{-a}a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right)-\sqrt{-a}a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)}{2\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 + 2
) - sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)
+ ((tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a - 6*sqrt
(-a)*a)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=\int \left(a-\frac{a}{\cos\left(c+d\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int((a - a/cos(c + d*x)^2)^(3/2),x)
```

output
```
int((a - a/cos(c + d*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{-\sec\left(dx+c\right)^2+1}dx - \left(\int \sqrt{-\sec\left(dx+c\right)^2+1}\,\sec\left(dx+c\right)^2dx\right)\right)$$

input
```
int((a-a*sec(d*x+c)^2)^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt( - sec(c + d*x)**2 + 1),x) - int(sqrt( - sec(c + d*x)*
*2 + 1)*sec(c + d*x)**2,x))
```

3.223 $\int \sqrt{a-a\sec^2(c+dx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

output
```
-cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

input
```
Integrate[Sqrt[a - a*Sec[c + d*x]^2],x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4609, 3042, 4141, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \sqrt{-a\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-a\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4141}$$

$$\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

input
```
Int[Sqrt[a - a*Sec[c + d*x]^2],x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

method	result
default	$\frac{\sqrt{-a\tan(dx+c)^2}\,\ln\left(1+\tan(dx+c)^2\right)}{2d\tan(dx+c)}$
risch	$\frac{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)x}{e^{2i(dx+c)}-1}-\frac{2\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)(dx+c)}{(e^{2i(dx+c)}-1)d}-\frac{i\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)\ln(e^{2i(}}{(e^{2i(dx+c)}-1)d}$

input
```
int((a-sec(d*x+c)^2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(-a*tan(d*x+c)^2)^(1/2)/tan(d*x+c)*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \sqrt{a - a\sec^2(c + dx)}\, dx = -\frac{\sqrt{\frac{a\cos(dx+c)^2 - a}{\cos(dx+c)^2}}\cos(dx + c)\log(-\cos(dx + c))}{d\sin(dx + c)}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(-cos(d*x + c
))/(d*sin(d*x + c))
```

Sympy [F]

$$\int \sqrt{a - a\sec^2(c + dx)}\, dx = \int \sqrt{-a\sec^2(c + dx) + a}\, dx$$

input
```
integrate((a-a*sec(d*x+c)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*sec(c + d*x)**2 + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \sqrt{a - a\sec^2(c + dx)}\, dx = \frac{\sqrt{-a}\log\left(\tan(dx + c)^2 + 1\right)}{2\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a)*log(tan(d*x + c)^2 + 1)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(31) = 62$.

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 4.27

$$\int \sqrt{a - a\sec^2(c + dx)}\, dx =$$
$$-\frac{\left(\log\left(\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^2 + 1\right) \operatorname{sgn}\left(-\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^3 - \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)\right) - \log\left(\left|\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) + \right.\right.}{}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
-(log(tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(-tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*
x + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(-tan(1/2*d*x + 1/2*c)
^3 - tan(1/2*d*x + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(-tan(1
/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))*sqrt(-a)*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - a\sec^2(c + dx)}\, dx = \int \sqrt{a - \frac{a}{\cos\left(c + d\, x\right)^2}}\, dx$$

input
```
int((a - a/cos(c + d*x)^2)^(1/2),x)
```

output
```
int((a - a/cos(c + d*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a - a\sec^2(c + dx)}\, dx = \sqrt{a}\left(\int \sqrt{-\sec(dx + c)^2 + 1}dx\right)$$

input
```
int((a-a*sec(d*x+c)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt( - sec(c + d*x)**2 + 1),x)
```

3.224 $\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx=\frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}$$

output
```
ln(sin(d*x+c))*tan(d*x+c)/d/(-a*tan(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx=\frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

output
```
(Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 4609, 3042, 4141, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)^2}}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \frac{1}{\sqrt{-a\tan^2(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-a\tan(c+dx)^2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan(c+dx)\int \cot(c+dx)dx}{\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan(c+dx)\log(-\sin(c+dx))}{d\sqrt{-a\tan^2(c+dx)}}$$

input
```
Int[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

output
```
(Log[-Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\tan(dx+c)\left(2\ln(\tan(dx+c))-\ln\left(1+\tan(dx+c)^2\right)\right)}{2d\sqrt{-a\tan(dx+c)^2}}$	48
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}-\frac{i\left(e^{2i(dx+c)}-1\right)\ln\left(e^{2i(dx+c)}-1\right)}{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}$	194

input
```
int(1/(a-sec(d*x+c)^2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*tan(d*x+c)*(2*ln(tan(d*x+c))-ln(1+tan(d*x+c)^2))/(-a*tan(d*x+c)^2)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx=-\frac{\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}\cos(dx+c)\log\left(\frac{1}{2}\sin(dx+c)\right)}{ad\sin(dx+c)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(1/2*sin(d*x
+ c))/(a*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{a - a\sec^2(c + dx)}}\, dx = \int \frac{1}{\sqrt{-a\sec^2(c + dx) + a}}\, dx$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-a*sec(c + d*x)**2 + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a - a\sec^2(c + dx)}}\, dx = -\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}} - \frac{2\log(\tan(dx+c))}{\sqrt{-a}}}{2\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(log(tan(d*x + c)^2 + 1)/sqrt(-a) - 2*log(tan(d*x + c))/sqrt(-a))/d
```

Giac [F]

$$\int \frac{1}{\sqrt{a - a\sec^2(c + dx)}}\, dx = \int \frac{1}{\sqrt{-a\sec(dx + c)^2 + a}}\, dx$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-a*sec(d*x + c)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx=\int \frac{1}{\sqrt{a-\frac{a}{\cos(c+d\,x)^2}}}\,dx$$

input
```
int(1/(a - a/cos(c + d*x)^2)^(1/2),x)
```

output
```
int(1/(a - a/cos(c + d*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx=-\frac{\sqrt{a}\left(\int \frac{\sqrt{-\sec(dx+c)^2+1}}{\sec(dx+c)^2-1}dx\right)}{a}$$

input
```
int(1/(a-a*sec(d*x+c)^2)^(1/2),x)
```

output
```
( - sqrt(a)*int(sqrt( - sec(c + d*x)**2 + 1)/(sec(c + d*x)**2 - 1),x))/a
```

3.225 $\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx=\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{ad\sqrt{-a\tan^2(c+dx)}}$$

output
```
1/2*cot(d*x+c)/a/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a/d/(
-a*tan(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx=-\frac{(\csc^2(c+dx)+2\log(\sin(c+dx)))\tan^3(c+dx)}{2d\left(-a\tan^2(c+dx)\right)^{3/2}}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

output
```
-1/2*((Csc[c + d*x]^2 + 2*Log[Sin[c + d*x]])*Tan[c + d*x]^3)/(d*(-(a*Tan[c
 + d*x]^2))^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 4609, 3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sec(c+dx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \frac{1}{\left(-a\tan^2(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-a\tan(c+dx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$-\frac{\tan(c+dx)\int \cot^3(c+dx)dx}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3dx}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3dx}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}-\int -\cot(c+dx)dx\right)}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)\left(\int \cot(c+dx)dx+\frac{\cot^2(c+dx)}{2d}\right)}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^2(c+dx)}{2d}\right)}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}+\frac{\log(-\sin(c+dx))}{d}\right)}{a\sqrt{-a\tan^2(c+dx)}}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

output
```
((Cot[c + d*x]^2/(2*d) + Log[-Sin[c + d*x]]/d)*Tan[c + d*x])/(a*Sqrt[-(a*T
an[c + d*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

method	result
default	$\frac{4\ln(\csc(dx+c)-\cot(dx+c))\tan(dx+c)-4\ln\left(\frac{2}{\cos(dx+c)+1}\right)\tan(dx+c)+\cot(dx+c)+\sec(dx+c)\csc(dx+c)}{4d\sqrt{-a\tan(dx+c)^{2}}\,a}$
risch	$-\frac{i e^{4i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)+e^{4i(dx+c)}dx-2i e^{2i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)+2\,e^{4i(dx+c)}c-2\,e^{2i(dx+c)}dx-2i e^{2i(dx+c)}+i\ln(e^{2i(dx+c)}}{a\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^{2}}{\left(e^{2i(dx+c)}+1\right)^{2}}}\,d}$

input
```
int(1/(a-sec(d*x+c)^2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/(-a*tan(d*x+c)^2)^(1/2)/a*(4*ln(csc(d*x+c)-cot(d*x+c))*tan(d*x+c)-4*
ln(2/(cos(d*x+c)+1))*tan(d*x+c)+cot(d*x+c)+sec(d*x+c)*csc(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx = -\frac{\left(2\left(\cos\left(dx+c\right)^3-\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)\sin\left(dx+c\right)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) - cos(d*x +
c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^2*d*cos(d*x + c)^2 - a
^2*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**(3/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx = -\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a}-\frac{2\log(\tan(dx+c))}{\sqrt{-a}a}+\frac{\sqrt{-a}}{a^2\tan(dx+c)^2}}{2d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a) - 2*log(tan(d*x + c))/(sqrt(-a)
*a) + sqrt(-a)/(a^2*tan(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx = -\frac{\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{\sqrt{-a}a} - \frac{8\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{\sqrt{-a}a} + \frac{4\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)}{\sqrt{-a}a} - \frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}{\sqrt{-a}a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*(tan(1/2*d*x + 1/2*c)^2/(sqrt(-a)*a) - 8*log(tan(1/2*d*x + 1/2*c)^2 +
 1)/(sqrt(-a)*a) + 4*log(tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a) - (4*tan(1/2
*d*x + 1/2*c)^2 - 1)/(sqrt(-a)*a*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a-\frac{a}{\cos(c+d\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a - a/cos(c + d*x)^2)^(3/2),x)
```

output
```
int(1/(a - a/cos(c + d*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{-\sec(dx+c)^2+1}}{\sec(dx+c)^4-2\sec(dx+c)^2+1}dx\right)}{a^2}$$

input
```
int(1/(a-a*sec(d*x+c)^2)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt( - sec(c + d*x)**2 + 1)/(sec(c + d*x)**4 - 2*sec(c + d*x
)**2 + 1),x))/a**2
```

3.226 $\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

output
```
1/2*cot(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^2/d/(-a*ta
n(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)\left(-4\csc^2(c+dx)+\csc^4(c+dx)-4\log(\sin(c+dx))\right)\sqrt{-a\tan}}{4a^3d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

output
```
(Cot[c + d*x]*(-4*Csc[c + d*x]^2 + Csc[c + d*x]^4 - 4*Log[Sin[c + d*x]])*S
qrt[-(a*Tan[c + d*x]^2)])/(4*a^3*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {3042, 4609, 3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sec(c+dx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4609}$$

$$\int \frac{1}{(-a\tan^2(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-a\tan(c+dx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan(c+dx)\int \cot^5(c+dx)dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^5dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^5dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\int -\cot^3(c+dx)dx\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\left(\int \cot^3(c+dx)dx+\frac{\cot^4(c+dx)}{4d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\cot^4(c+dx)}{4d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3dx\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\tan(c+dx)\left(\int -\cot(c+dx)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\left(-\int \cot(c+dx)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tan(c+dx)\left(-\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\left(\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}-\frac{\log(-\sin(c+dx))}{d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

output
```
-(((-1/2*Cot[c + d*x]^2/d + Cot[c + d*x]^4/(4*d) - Log[-Sin[c + d*x]]/d)*T
an[c + d*x])/(a^2*Sqrt[-(a*Tan[c + d*x]^2)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

method	result
default	$\frac{\left(\ln(\csc(dx+c)-\cot(dx+c))\sin(dx+c)^4-\ln\left(\frac{2}{\cos(dx+c)+1}\right)\sin(dx+c)^4-\frac{13\cos(dx+c)^4}{32}-\frac{3\cos(dx+c)^2}{16}+\frac{11}{32}\right)\sec(dx+c)\csc(dx+c)^3}{d\sqrt{-a\tan(dx+c)^2}\,a^2}$
risch	$\frac{(\mathrm{e}^{2i(dx+c)}-1)x}{a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}}}-\frac{2(\mathrm{e}^{2i(dx+c)}-1)(dx+c)}{a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}}\,d}+\frac{4i\left(\mathrm{e}^{6i(dx+c)}-\mathrm{e}^{4i(dx+c)}+\mathrm{e}^{2i(dx+c)}\right)}{a^2\left(\mathrm{e}^{2i(dx+c)}-1\right)^3\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{2i(dx+c)}\right.}{\left(\mathrm{e}^{2i(dx+c)}\right.}}}$

input
```
int(1/(a-sec(d*x+c)^2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(ln(csc(d*x+c)-cot(d*x+c))*sin(d*x+c)^4-ln(2/(cos(d*x+c)+1))*sin(d*x+c
)^4-13/32*cos(d*x+c)^4-3/16*cos(d*x+c)^2+11/32)/(-a*tan(d*x+c)^2)^(1/2)/a^
2*sec(d*x+c)*csc(d*x+c)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx = \frac{\left(4\,\cos\left(dx+c\right)^3-4\left(\cos\left(dx+c\right)^5-2\,\cos\left(dx+c\right)^3+\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\right.\right.}{4\left(a^3d\cos\left(dx+c\right)^4-2\,a^3d\cos\left(dx+c\right)^2+\right.}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(4*cos(d*x + c)^3 - 4*(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c
))*log(1/2*sin(d*x + c)) - 3*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos
(d*x + c)^2)/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(
d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx=\int \frac{1}{\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**(5/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx=-\frac{\frac{2\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a^2}-\frac{4\log(\tan(dx+c))}{\sqrt{-a}a^2}+\frac{2\sqrt{-a}\tan(dx+c)^2-\sqrt{-a}}{a^3\tan(dx+c)^4}}{4\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(2*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^2) - 4*log(tan(d*x + c))/(sqrt
(-a)*a^2) + (2*sqrt(-a)*tan(d*x + c)^2 - sqrt(-a))/(a^3*tan(d*x + c)^4))/d
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx=\frac{\frac{64\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{\sqrt{-a}a^2}-\frac{32\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)}{\sqrt{-a}a^2}-\frac{\sqrt{-a}a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\sqrt{-a}a^2\tan}{a^5}}{64\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
1/64*(64*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2) - 32*log(tan(1/2*d
*x + 1/2*c)^2)/(sqrt(-a)*a^2) - (sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*
sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^2)/a^5 + (48*tan(1/2*d*x + 1/2*c)^4 - 12
*tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^4))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx=\int \frac{1}{\left(a-\frac{a}{\cos(c+d\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a - a/cos(c + d*x)^2)^(5/2),x)
```

output
```
int(1/(a - a/cos(c + d*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx=-\frac{\sqrt{a}\left(\int \frac{\sqrt{-\sec(dx+c)^2+1}}{\sec(dx+c)^6-3\sec(dx+c)^4+3\sec(dx+c)^2-1}dx\right)}{a^3}$$

input
```
int(1/(a-a*sec(d*x+c)^2)^(5/2),x)
```

output
```
( - sqrt(a)*int(sqrt( - sec(c + d*x)**2 + 1)/(sec(c + d*x)**6 - 3*sec(c +
d*x)**4 + 3*sec(c + d*x)**2 - 1),x))/a**3
```

3.227 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{7/2}}\,dx=\frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

output
```
1/2*cot(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^3/d/(-a*ta
n(d*x+c)^2)^(1/2)+1/6*cot(d*x+c)^5/a^3/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*
x+c))*tan(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{7/2}}\,dx=\frac{(18\csc^2(c+dx)-9\csc^4(c+dx)+2\csc^6(c+dx)+12\log(\sin(c+dx)))\,\mathrm{ta}}{12a^3d\sqrt{-a\tan^2(c+dx)}}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

output
```
((18*Csc[c + d*x]^2 - 9*Csc[c + d*x]^4 + 2*Csc[c + d*x]^6 + 12*Log[Sin[c +
d*x]])*Tan[c + d*x])/(12*a^3*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.118$, Rules used = {3042, 4609, 3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a-a\sec(c+dx)^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 4609}$$

$$\int \frac{1}{(-a\tan^2(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(-a\tan(c+dx)^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$-\frac{\tan(c+dx)\int \cot^7(c+dx)dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^7dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^7dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan(c+dx)\left(\frac{\cot^6(c+dx)}{6d}-\int -\cot^5(c+dx)dx\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)\left(\int \cot^5(c+dx)dx+\frac{\cot^6(c+dx)}{6d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{\cot^6(c+dx)}{6d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)\left(\frac{\cot^6(c+dx)}{6d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^5dx\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan(c+dx)\left(\int -\cot^3(c+dx)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)\left(-\int \cot^3(c+dx)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\left(-\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)\left(\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan(c+dx)\left(-\int -\cot(c+dx)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$\downarrow$ 25

$$\frac{\tan(c+dx)\left(\int \cot(c+dx)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$\downarrow$ 25

$$\frac{\tan(c+dx)\left(-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$\downarrow$ 3956

$$\frac{\tan(c+dx)\left(\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}+\frac{\log(-\sin(c+dx))}{d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

output
```
((Cot[c + d*x]^2/(2*d) - Cot[c + d*x]^4/(4*d) + Cot[c + d*x]^6/(6*d) + Log
[-Sin[c + d*x]]/d)*Tan[c + d*x])/(a^3*Sqrt[-(a*Tan[c + d*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

method	result
default	$\frac{\left(-\ln\left(\frac{2}{\cos(dx+c)+1}\right)\sin(dx+c)^6+\ln(\csc(dx+c)-\cot(dx+c))\sin(dx+c)^6+\frac{25\cos(dx+c)^6}{48}-\frac{\cos(dx+c)^4}{16}-\frac{11\cos(dx+c)^2}{16}+\frac{19}{48}\right)\sec(dx+}{d\sqrt{-a\tan(dx+c)^2}\,a^3}$
risch	$\frac{(e^{2i(dx+c)}-1)x}{a^3(e^{2i(dx+c)}+1)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}-\frac{2(e^{2i(dx+c)}-1)(dx+c)}{a^3(e^{2i(dx+c)}+1)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}+\frac{2i(9e^{10i(dx+c)}-18e^{8i(dx+c)}+34e^{6i(dx+c)}-18}{3a^3(e^{2i(dx+c)}-1)^5(e^{2i(dx+c)}+1)\sqrt{\frac{a(}{(}}}$

input
```
int(1/(a-sec(d*x+c)^2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-ln(2/(cos(d*x+c)+1))*sin(d*x+c)^6+ln(csc(d*x+c)-cot(d*x+c))*sin(d*x+
c)^6+25/48*cos(d*x+c)^6-1/16*cos(d*x+c)^4-11/16*cos(d*x+c)^2+19/48)/(-a*ta
n(d*x+c)^2)^(1/2)/a^3*sec(d*x+c)*csc(d*x+c)^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = \frac{\left(18\,\cos\left(dx+c\right)^5-27\,\cos\left(dx+c\right)^3-12\,\left(\cos\left(dx+c\right)^7-3\,\cos\left(dx+c\right)^5\right.\right.}{12\,\left(a^4d\cos\left(dx+c\right)^6-3\,a^4d\cos\left(dx\right.\right.}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/12*(18*cos(d*x + c)^5 - 27*cos(d*x + c)^3 - 12*(cos(d*x + c)^7 - 3*cos(d
*x + c)^5 + 3*cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) + 11*co
s(d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^4*d*cos(d*x +
c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x +
c))
```

Sympy [F]

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**(7/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(-7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{7/2}}\,dx = -\frac{\frac{6\,\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a^3}-\frac{12\,\log(\tan(dx+c))}{\sqrt{-a}a^3}+\frac{6\sqrt{-a}\tan(dx+c)^4-3\sqrt{-a}\tan(dx+c)^2+2\sqrt{-a}}{a^4\tan(dx+c)^6}}{12\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/12*(6*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^3) - 12*log(tan(d*x + c))/(sq
rt(-a)*a^3) + (6*sqrt(-a)*tan(d*x + c)^4 - 3*sqrt(-a)*tan(d*x + c)^2 + 2*s
qrt(-a))/(a^4*tan(d*x + c)^6))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{7/2}}\,dx = \frac{\frac{384\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{\sqrt{-a}a^3}-\frac{192\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)}{\sqrt{-a}a^3}+\frac{352\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-87\,\tan\left(\frac{1}{2}\,dx+\right.}{\sqrt{-a}a^3\tan\left(\frac{1}{2}\,d\right.}}{384}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

output
```
1/384*(384*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^3) - 192*log(tan(1/
2*d*x + 1/2*c)^2)/(sqrt(-a)*a^3) + (352*tan(1/2*d*x + 1/2*c)^6 - 87*tan(1/
2*d*x + 1/2*c)^4 + 12*tan(1/2*d*x + 1/2*c)^2 - 1)/(sqrt(-a)*a^3*tan(1/2*d*
x + 1/2*c)^6) - (a^7*tan(1/2*d*x + 1/2*c)^6 - 12*a^7*tan(1/2*d*x + 1/2*c)^
4 + 87*a^7*tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a^10))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - a\sec^2(c + dx))^{7/2}} \, dx = \int \frac{1}{\left(a - \frac{a}{\cos(c+d\,x)^2}\right)^{7/2}} \, dx$$

input
```
int(1/(a - a/cos(c + d*x)^2)^(7/2),x)
```

output
```
int(1/(a - a/cos(c + d*x)^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(a - a\sec^2(c + dx))^{7/2}} \, dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{-\sec(dx+c)^2+1}}{\sec(dx+c)^8-4\sec(dx+c)^6+6\sec(dx+c)^4-4\sec(dx+c)^2+1}dx\right)}{a^4}$$

input
```
int(1/(a-a*sec(d*x+c)^2)^(7/2),x)
```

output
```
(sqrt(a)*int(sqrt( - sec(c + d*x)**2 + 1)/(sec(c + d*x)**8 - 4*sec(c + d*x
)**6 + 6*sec(c + d*x)**4 - 4*sec(c + d*x)**2 + 1),x))/a**4
```

3.228 $\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 376

$$
\begin{aligned}
&\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= -\frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15b^2f} \\
&\quad+\frac{(2a^2-3ab-8b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15b^2f\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}} \\
&\quad-\frac{(a-8b)(a+b)\sqrt{\cos^2(e+fx)}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(a}}{15bf\left(a+b-a\sin^2(e+fx)\right)} \\
&\quad+\frac{(a+4b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15bf} \\
&\quad+\frac{\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f}
\end{aligned}
$$

output
```
-1/15*(2*a^2-3*a*b-8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)/b^2/f+1/15*(2*a^2-3*a*b-8*b^2)*(cos(f*x+e)^2)^(1/2)*EllipticE(sin(f*x
+e),(a/(a+b))^(1/2))*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b^2/f/((a+b
-a*sin(f*x+e)^2)/(a+b))^(1/2)-1/15*(a-8*b)*(a+b)*(cos(f*x+e)^2)^(1/2)*Elli
pticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)*(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(a+b-a*sin(f*x+e)^2)+1/15*(a+4*b)
*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/b/f+1/5*s
ec(f*x+e)^3*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

Mathematica [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3042, 4636, 2057, 2058, 314, 25, 402, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^5\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}{(1-\sin^2(e+fx))^3}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 2057

$$\frac{\int \frac{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}{(1-\sin^2(e+fx))^3}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 2058

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{(1-\sin^2(e+fx))^{7/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 314

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{5(1-\sin^2(e+fx))^{5/2}}-\frac{1}{5}\int -\frac{4(a+b)-3a\sin^2(e+fx)}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\int \frac{4(a+b)-3a\sin^2(e+fx)}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+}}{5(1-\sin^2(e+fx)}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{\int -\frac{a(a+4b)\sin^2(e+fx)+(a-8b)(a+b)}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}+\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(}}{3b(1-\sin^2(e+fx))}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\int \frac{a(a+4b)\sin^2(e+fx)+(a-8b)(a+b)}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d}{3b}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{a\left(2(a-2b)(a+b)-\left(2a^2-3ba-8b^2\right)\sin^2(e+\cdots}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b\cdots}}}{b}\cdots\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}\cdots}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a\cdots}}{b\sqrt{1-\sin^2(e+fx)}}\cdots}{\cdots}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}\cdots}$$

$\downarrow$ 27

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a\cdots}}{b\sqrt{1-\sin^2(e+fx)}}\cdots}{\cdots}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}\cdots}$$

$\downarrow$ 399

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a\cdots}}{b\sqrt{1-\sin^2(e+fx)}}\cdots}{\cdots}\right.\right.}{f\sqrt{-\cdots}}$$

$\downarrow$ 323

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a}}{b\sqrt{1-\sin^2(e+fx)}}}{}\right.\right.$$

$\Big\downarrow$ 321

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a}}{b\sqrt{1-\sin^2(e+fx)}}}{}\right.\right.$$

$f\sqrt{-}$

$\Big\downarrow$ 330

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a}}{b\sqrt{1-\sin^2(e+fx)}}}{}\right.\right.$$

$\Big\downarrow$ 327

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a}}{b\sqrt{1-\sin^2(e+fx)}}}{\ldots}\right.\right.$$

$$f$$

input
```
Int[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*(1 - Sin[e + f*x]^
2)^(5/2)) + (((a + 4*b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*b*
(1 - Sin[e + f*x]^2)^(3/2)) - (((2*a^2 - 3*a*b - 8*b^2)*Sin[e + f*x]*Sqrt[
a + b - a*Sin[e + f*x]^2])/(b*Sqrt[1 - Sin[e + f*x]^2]) - (a*(((2*a^2 - 3*
a*b - 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin
[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b)*b*(a +
 b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)
/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])))/b)/(3*b))/5))/(f*Sqrt[a +
b - a*Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 36.37 (sec) , antiderivative size = 3376, normalized size of antiderivative = 8.98

method	result	size
default	Expression too large to display	3376

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/15/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b
^2*(a+b*sec(f*x+e)^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)
*((-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b
)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*a^4*EllipticE(((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(-2*cos(f*x+e)^3-4*cos(f*
x+e)^2-2*cos(f*x+e))+(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(
1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*a^3*b*Elli
pticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(-co
s(f*x+e)^3-2*cos(f*x+e)^2-cos(f*x+e))+(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))
^(1/2)*a^2*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+
e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*(12*cos(f*x+e)^3+24*cos(f*x+e)^2+12*cos(f*x+e))+(-1/(a+b)*(
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+
e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 887, normalized size of antiderivative = 2.36

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/30*((2*(-2*I*a^3 + 3*I*a^2*b + 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*
cos(f*x + e)^4 - (-2*I*a^3 - I*a^2*b + 14*I*a*b^2 + 16*I*b^3)*sqrt(a)*cos(
f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsi
n(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x
+ e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2)
 + (2*(2*I*a^3 - 3*I*a^2*b - 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(
f*x + e)^4 - (2*I*a^3 + I*a^2*b - 14*I*a*b^2 - 16*I*b^3)*sqrt(a)*cos(f*x +
 e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqr
t((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))
), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*
((I*a^2*b + 4*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^4 + (I*a
^3 + I*a^2*b - 4*I*a*b^2 - 4*I*b^3)*sqrt(a)*cos(f*x + e)^4)*sqrt((2*a*sqrt
((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^
2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b
^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*((-I*a^2*b - 4*I*a*b^
2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^4 + (-I*a^3 - I*a^2*b + 4*I*
a*b^2 + 4*I*b^3)*sqrt(a)*cos(f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) -
 a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b
)*sqrt((a*b + b^2)/a^2))/a^2) - 2*((2*a^3 - 3*a^2*b - 8*a*b^2)*cos(f*x ...
```

Sympy [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\sec^5{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

Maxima [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Giac [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos(e+f\,x)^5}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5, x)
```

Reduce [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^5\,dx$$

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**5,x)
```

3.229 $\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 292

$$
\begin{aligned}
&\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3bf} \\
&\quad - \frac{(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3bf\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}} \\
&\quad + \frac{2(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f\left(a+b-a\sin^2(e+fx)\right)} \\
&\quad + \frac{\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f}
\end{aligned}
$$

output
```
1/3*(a+2*b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f-1/3*(
a+2*b)*(cos(f*x+e)^2)^(1/2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(sec(f*x
+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)+2
/3*(a+b)*(cos(f*x+e)^2)^(1/2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-
a*sin(f*x+e)^2)/(a+b))^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(
a+b-a*sin(f*x+e)^2)+1/3*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)*tan(f*x+e)/f
```

Mathematica [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4636, 2057, 2058, 314, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^3\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\left(1-\sin^2(e+fx)\right)^{5/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 314

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{1}{3}\int-\frac{2(a+b)-a\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\int\frac{2(a+b)-a\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+}}{3\left(1-\sin^2(e+fx)\right)}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 402

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{\int-\frac{a\left(-\left((a+2b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}+\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+}}{b\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\int\frac{a\left(-\left((a+2b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin}{b}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\int\frac{-\left((a+2b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin}{b}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 399

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx}{a}\right.}{}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 323

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx}{a}\right.}{}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a}}$$

$\downarrow$ 321

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx}{a}\right.}{}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 330

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2}{a}}}{\sqrt{1-\sin^2(}}}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right.}{}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+bE\left(\arcsin(\sin(\right.}}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right.}{}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*(1 - Sin[e + f*x]^
2)^(3/2)) + (((a + 2*b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(b*Sq
rt[1 - Sin[e + f*x]^2]) - (a*(((a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], a
/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(
a + b)]) - (2*b*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1
- (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])))/b)/3))
/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] && !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.59 (sec) , antiderivative size = 2475, normalized size of antiderivative = 8.48

method	result	size
default	Expression too large to display	2475

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b*
(a+b*sec(f*x+e)^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*((
-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*a^3*EllipticE(((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(cos(f*x+e)^3+2*cos(f*x+e)^2
+cos(f*x+e))+(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos
(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*a^2*b*EllipticE(((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(4*cos(f*x+e
)^3+8*cos(f*x+e)^2+4*cos(f*x+e))+(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2
)*a*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(
f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*(5*cos(f*x+e)^3+10*cos(f*x+e)^2+5*cos(f*x+e))+(-1/(a+b)*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2
)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.68

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*((2*(I*a^2 + 2*I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (
I*a^2 + 4*I*a*b + 4*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b
^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) -
 a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a
^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-I*a^2 - 2*I*a*b)*sqrt(a)*sq
rt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (-I*a^2 - 4*I*a*b - 4*I*b^2)*sqrt(a)*
cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(a
rcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(
f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/
a^2) - 2*(2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 + (-I*a^2 - 3
*I*a*b - 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2)
- a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)
/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*
b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*(-2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*c
os(f*x + e)^2 + (I*a^2 + 3*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((
2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((
a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a
*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*((a^2 + 2*a*b
)*cos(f*x + e)^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(a*b*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\sec^3{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\sec{(fx+e)}^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\sec{(fx+e)}^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{{\cos(e+f\,x)}^2}}}{{\cos\left(e+f\,x\right)}^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3, x)
```

Reduce [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^3\,dx$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**3,x)
```

3.230 $\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 222

$$\begin{aligned}\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f} \\ &\quad - \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}} \\ &\quad + \frac{(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\right.}}{f\left(a+b-a\sin^2(e+fx)\right)}\end{aligned}$$

output
```
sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-(cos(f*x+e)^2)^(1/2
)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/f/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)+(a+b)*(cos(f*x+e)^2)^(1/2)*El
lipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)*(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.522$, Rules used = {3042, 4636, 2057, 2058, 314, 25, 27, 389, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{(1-\sin^2(e+fx))^{3/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 314

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-\int-\frac{a\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\int\frac{a\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+}}{\sqrt{1-\sin^2(e+fx)}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\int\frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 389

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}-\frac{\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(}{a}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 323

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int\frac{\sqrt{-a\sin^2(e+}}{\sqrt{1-\sin^2(}}}{}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 321

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d}{a}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 330

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int}{a\sqrt{1-}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 327

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\sqrt{-a\sin^2(e+fx)+a+b}E}{a\sqrt{1-\frac{a}{}}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

input
```
Int[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/Sqrt[1 - Sin[e + f*x]
^2] + a*(-((EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[
e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])) + ((a + b)*Elliptic
F[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(
a*Sqrt[a + b - a*Sin[e + f*x]^2]))))/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.98 (sec) , antiderivative size = 1831, normalized size of antiderivative = 8.25

method	result	size
default	Expression too large to display	1831

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((1/
(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a^2*EllipticE(((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(-cos(f*x+e)^3-2*cos(f*x+e)^2-
cos(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f
*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a*b*EllipticE(((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(-2*cos(f*x+e)^
3-4*cos(f*x+e)^2-2*cos(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^
(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b
^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*
b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+
e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*(-cos(f*x+e)^3-2*cos(f*x+e)^2-cos(f*x+e))+I*a^(3/2)*b^(1/2)*(1/(a+b)*(
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.72

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{4i\,a^{\frac{3}{2}}\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}\sqrt{\frac{ab+b^2}{a^2}}F\left(\arcsin\left(\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))\right)\mid\frac{a^2+8\,ab+8\,b^2}{}\right.}{}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(4*I*a^(3/2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*sqrt((a*b
+ b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a
)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)
*sqrt((a*b + b^2)/a^2))/a^2) - 4*I*a^(3/2)*sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*sqrt((a*b + b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b
 + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b
+ 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt
((a*b + b^2)/a^2) + sqrt(a)*(I*a + 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b
)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a
*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqr
t(a)*(-I*a - 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*ellipti
c_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I
*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a
^2))/a^2) - 2*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/
(a*f)
```

Sympy [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\sec{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\sec{(fx+e)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec\left(fx+e\right)^2b+a}\,\sec\left(fx+e\right)dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x),x)
```

3.231 $\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

output
```
(cos(f*x+e)^2)^(1/2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)/f/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{2}\cos(e+fx)E\left(e+fx\left|\frac{a}{a+b}\right.\right)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

input
```
Integrate[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[2]*Cos[e + f*x]*EllipticE[e + f*x, a/(a + b)]*Sqrt[a + b*Sec[e + f*x
]^2])/(f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \sqrt{a+\frac{b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

input
```
Int[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[
(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f
*x]^2)/(a + b)])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 1770, normalized size of antiderivative = 21.59

method	result	size
default	Expression too large to display	1770

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)*cos(
f*x+e)*((cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))
^(1/2)*a^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-c
sc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))+(2*cos(f*x+e)^2+4*cos(f*x+e)+2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos
(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x
+e)))^(1/2)*a*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x
+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))+(cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+
b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(
f*x+e)))^(1/2)*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(
f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))+I*(-2*cos(f*x+e)^2-4*cos(f*x+e)-2)*a^(3/2)*b^(1/2)*(1/(
a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+co
s(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)
-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+...
```

Fricas [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Sympy [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2\left(e+fx\right)}\cos\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

Maxima [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Giac [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)dx$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x),x)
```

3.232 $\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f}$$
$$+ \frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3af\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$
$$- \frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(a+b-a}}{3af\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/3*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+1/
3*(2*a+b)*(cos(f*x+e)^2)^(1/2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2
)-1/3*b*(a+b)*(cos(f*x+e)^2)^(1/2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(
(a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)/a/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.13 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.61

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{6\sqrt{2}\sqrt{a+2b+a\cos(2(e+fx))}E\left(e+fx\middle|\frac{a}{a+b}\right)}{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}+\frac{\sqrt{-\frac{1}{b}}b\csc^2(2(e+fx))\sec(2(e+fx))\left(4i\sqrt{2}(a^2\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((6*Sqrt[2]*Sqrt[a + 2*b + a*Cos[
2*(e + f*x)]]*EllipticE[e + f*x, a/(a + b)])/Sqrt[(a + 2*b + a*Cos[2*(e +
f*x)])/(a + b)] + (Sqrt[-b^(-1)]*b*Csc[2*(e + f*x)]^2*Sec[2*(e + f*x)]*((4
*I)*Sqrt[2]*(a^2 + 3*a*b + 2*b^2)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticE[
I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(
a + b)]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)] + a*(a*Sqrt[-b^(-1)]*Sqrt[a + 2*b
 + a*Cos[2*(e + f*x)]]*(-1 + Cos[4*(e + f*x)]) - (4*I)*Sqrt[2]*(a + b)*Sqr
t[-((a*Cos[e + f*x]^2)/b)]*EllipticF[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b
 + a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(a + b)]*Sqrt[(a*Sin[e + f*x]^2)/(a +
b)]))*Sin[4*(e + f*x)])/(2*a^2)))/(12*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4636, 2057, 2058, 319, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\Big\downarrow \text{319}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{2}{3}\int \frac{-\left((2a+b)\sin^2(e+fx)\right)+2a+2b}{2\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{1}{3}\sqrt{1-\sin^2(e+fx)}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\int \frac{2(a+b)-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{1}{3}\sqrt{1-\sin^2(e+fx)}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+}}}{a}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{}{\sqrt{1-\sin^2(e+fx}}}{a\sqrt{-a\sin^2(e+fx}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+}}$$

$\downarrow$ 321

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\text{arcsin}\right.}{a\sqrt{-a\sin^2(e+fx)+a+}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 330

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+}{a+b}}}{a\sqrt{}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$\downarrow$ 327

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{a\sqrt{-a}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a}}$$

input
```
Int[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^
2])/3 + (((2*a + b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b
- a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)
*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/3))/(f*Sqrt[a + b - a*Sin[e +
f*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 319
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*(a + b*x^2)^p*((c + d*x^2)^q/(2*(p + q) + 1)), x] + Simp[2/(2*(p + q) +
 1)   Int[(a + b*x^2)^(p - 1)*(c + d*x^2)^(q - 1)*Simp[a*c*(p + q) + (q*(b*
c - a*d) + a*d*(p + q))*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 2, p, q,
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.78 (sec) , antiderivative size = 2386, normalized size of antiderivative = 9.54

method	result	size
default	Expression too large to display	2386

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)/a*
((2*cos(f*x+e)^2+4*cos(f*x+e)+2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)
*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2
)*a^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*
x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))+(5*cos(f*x+e)^2+10*cos(f*x+e)+5)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e))
)^(1/2)*a^2*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e
)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))+(4*cos(f*x+e)^2+8*cos(f*x+e)+4)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+
b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(
f*x+e)))^(1/2)*a*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(co
t(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))+(cos(f*x+e)^2+2*cos(f*x+e)+1)*(-1/(a+b)*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(
1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
+cos(f*x+e)))^(1/2)*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/...
```

Fricas [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Giac [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)^3 dx$$

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**3,x)
```

3.233 $\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 342

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{2(2a-b)\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af} + \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5af} + \frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15a^2f\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}} - \frac{2(2a-b)b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)}}{15a^2f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
2/15*(2*a-b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)/a/f+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)*(sec(f*x+e)^2*(a
+b-a*sin(f*x+e)^2))^(1/2)/a/f+1/15*(8*a^2+3*a*b-2*b^2)*(cos(f*x+e)^2)^(1/2
)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/a^2/f/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)-2/15*(2*a-b)*b*(a+b)*(cos
(f*x+e)^2)^(1/2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^
2)/(a+b))^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a^2/f/(a+b-a*sin
(f*x+e)^2)
```

Mathematica [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4636, 2057, 2058, 318, 25, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^5}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 318

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}(-a\sin^2(e+fx)+a+b)^{3/2}}{5a}-\frac{\int-\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-2(2a-b)\right.}{\sqrt{1-\sin^2(e+fx}}}{5a}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-2(2a-b)\sin^2(e+fx)+4a-b\right)}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{5a}+\frac{\sin(e+fx)\sqrt{1-\sin^2(e+f}}{}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 403

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{2}{3}(2a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}-\frac{1}{3}\int-\frac{(8a-b)(a+b)-\left(8a^2+3ba-2\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2}}}{5a}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{1}{3}\int\frac{(8a-b)(a+b)-\left(8a^2+3ba-2b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{2}{3}(2a-b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx}{5a}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 399

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(2a-b)(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin\ldots}}}{a}\ldots}{5a}\ldots\right.}{f\sqrt{-a\sin^2(\ldots}}$$

$\downarrow$ 323

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(2a-b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{\ldots}{\sqrt{1-\ldots}}}{a\sqrt{-a\sin^2(\ldots}}\ldots}{\ldots}\right.}{f\sqrt{-a\ldots}}$$

$\downarrow$ 321

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(2a-b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\ldots}{a\sqrt{-a\sin^2(e+fx)+\ldots}}\ldots}{5a}\ldots\right.}{f\sqrt{-a\sin^2(\ldots}}$$

$\downarrow$ 330

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(2a-b)(a+b)\sqrt{1-\frac{a\sin\ldots}{\ldots}}}{\ldots}\ldots}{\ldots}\right.}{f\sqrt{-\ldots}}$$

$$\big\downarrow 327$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(2a-b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{a\sqrt{-a}}\right.}{}\right.}{f\sqrt{-a}}$$

input
```
Int[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(
3/2))/(5*a) + ((2*(2*a - b)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a +
 b - a*Sin[e + f*x]^2])/3 + (((8*a^2 + 3*a*b - 2*b^2)*EllipticE[ArcSin[Sin
[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[
e + f*x]^2)/(a + b)]) - (2*(2*a - b)*b*(a + b)*EllipticF[ArcSin[Sin[e + f*
x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Si
n[e + f*x]^2]))/3)/(5*a)))/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.35 (sec) , antiderivative size = 3236, normalized size of antiderivative = 9.46

method	result	size
default	Expression too large to display	3236

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2/(2*I*a^(1/2)*b^(1/2)-a
+b)*((8*cos(f*x+e)^2+16*cos(f*x+e)+8)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))
^(1/2)*a^4*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-c
ot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))+(19*cos(f*x+e)^2+38*cos(f*x+e)+19)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+
b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(
f*x+e)))^(1/2)*a^3*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cs
c(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))+(12*cos(f*x+e)^2+24*cos(f*x+e)+12)*(1/(a+b)*(I*a^(1/2
)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/
2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-
b)/(1+cos(f*x+e)))^(1/2)*a^2*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+(-cos(f*x+e)^2-2*cos(f*x+e)-1)*(1/(a+b)*(
I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+
e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f
*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+...
```

Fricas [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Giac [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)^5 dx$$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**5,x)
```

3.234 $\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\left(a^2-2ab+5b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} + \frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(a-3b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{8b^2f} + \frac{\tan^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf}$$

output
```
1/16*(a+b)*(a^2-2*a*b+5*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^
2)^(1/2))/b^(5/2)/f+1/16*(a^2-2*a*b+5*b^2)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)
^(1/2)/b^2/f-1/8*(a-3*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b^2/f+1/6*t
an(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.23 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.43

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{ie^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\left(-3a^2\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-1/24*I)*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((
2*I)*(e + f*x))]*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e
+ f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*(-3*a^2*(1 + E^((2*I)*(e + f*x)))
^4 + 4*a*b*(1 + E^((2*I)*(e + f*x)))^2*(1 + 4*E^((2*I)*(e + f*x)) + E^((4*
I)*(e + f*x))) + b^2*(15 + 100*E^((2*I)*(e + f*x)) + 298*E^((4*I)*(e + f*x
)) + 100*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x)))) + 3*(a^3 - a^2*b +
 3*a*b^2 + 5*b^3)*(1 + E^((2*I)*(e + f*x)))^6*ArcTan[(Sqrt[b]*(-1 + E^((2*
I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2]])*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b^(5/2)*(1 + E^((2
*I)*(e + f*x)))^6*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4634, 318, 25, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^6\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{\int -\left(((3a-5b)\tan^2(e+fx)+a-5b)\sqrt{b\tan^2(e+fx)+a+b}\right)d\tan(e+fx)}{6b}+\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b}-\frac{\int ((3a-5b)\tan^2(e+fx)+a-5b)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{6b}}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b}-\frac{\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b}-\frac{3\left(a^2-2ab+5b^2\right)\int\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+f}{4b}}{6b}$$

$$f$$

$$\Big\downarrow \text{211}$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b}-\frac{\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b}-\frac{3\left(a^2-2ab+5b^2\right)\left(\frac{1}{2}(a+b)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}\right.}{4}}{6b}$$

$$f$$

$$\Big\downarrow \text{224}$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b}-\frac{\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b}-\frac{3\left(a^2-2ab+5b^2\right)\left(\frac{1}{2}(a+b)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+}}\right.}{}}{6b}$$

$$f$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b}-\frac{\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b}-\frac{3\left(a^2-2ab+5b^2\right)\left(\frac{(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(\dots}{\sqrt{a+b\tan^2\dots}}\right)}{2\sqrt{b}}\dots\right)}{4\dots}}{6b}}{f}$$

input
```
Int[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2))/(6*b
) - (((3*a - 5*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*b) - (
3*(a^2 - 2*a*b + 5*b^2)*(((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/2))/(4*b))/(6*b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. 2(158) = 316.

Time = 47.89 (sec) , antiderivative size = 1106, normalized size of antiderivative = 6.21

method	result	size
default	Expression too large to display	1106

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/96/f/b^(11/2)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(3*cos(f*x+e)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^3*b^3-3*cos(f*x+e)*ln(4*(
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))
*a^2*b^4+9*cos(f*x+e)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f
*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^5+15*cos(f*x+e)*ln(4*(b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^6+3*cos(f*x+e)*l
n(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+
e)-1))*a^3*b^3-3*cos(f*x+e)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^4+9*cos(f*x+e)*ln(-4*(b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^5+15*
cos(f*x+e)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a...
```

Fricas [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.63

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{3\left(a^3-a^2b+3\,ab^2+5\,b^3\right)\sqrt{b}\cos\left(fx+e\right)^5\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3\cdots\right.}{\cos(fx+e)^4}\right.}{\cdots}\right.$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2
 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b - 4*a*b^
2 - 15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)
^5), 1/96*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 -
 2*((3*a^2*b - 4*a*b^2 - 15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3
)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
)/(b^3*f*cos(f*x + e)^5)]
```

Sympy [F]

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sec^6(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(158) = 316$.

Time = 0.04 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.78

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{8\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}\tan(fx+e)^3}{b} + \frac{3\,(a+b)^2 a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \frac{3\,(a+b)^2\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} - \frac{12\,(a+b)a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}}{}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/48*(8*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)^3/b + 3*(a + b)^2*a*
arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)^2*arcsinh(b*ta
n(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 12*(a + b)*a*arcsinh(b*tan(f*x + e)/
sqrt((a + b)*b))/b^(3/2) - 12*(a + b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*
b))/sqrt(b) + 24*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 24*sq
rt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 24*sqrt(b*tan(f*x + e)^2 +
 a + b)*tan(f*x + e) - 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x
+ e)/b^2 + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)/b^2 + 2
4*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)/b - 12*sqrt(b*tan(f*x + e)
^2 + a + b)*(a + b)*tan(f*x + e)/b)/f
```

Giac [F]

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6, x)
```

Reduce [F]

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^6\,dx$$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**6,x)
```

3.235 $\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{(a-3b)(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf}$$

output
```
-1/8*(a-3*b)*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
b^(3/2)/f-1/8*(a-3*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f+1/4*tan(f*
x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.29 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.86

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{ie^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(-\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\left(a(1+\right.\right.}{4\sqrt{2}b^{3/2}\left(1+e^{2i(e+\right.}}$$

input
```
Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((I/4)*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)
*(e + f*x))]*(-(Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*(a*(1 + E^((2*I)*(e + f*x)))^2 + b*
(3 + 14*E^((2*I)*(e + f*x)) + 3*E^((4*I)*(e + f*x))))) + (a^2 - 2*a*b - 3*
b^2)*(1 + E^((2*I)*(e + f*x)))^4*ArcTan[(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))
))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])*Cos[e +
 f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b^(3/2)*(1 + E^((2*I)*(e + f*x)
))^4*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f*Sqrt[
a + 2*b + a*Cos[2*e + 2*f*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^4\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b}-\frac{(a-3b)\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{4b}}{f}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b}-\frac{(a-3b)\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)}{4b}}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b}-\frac{(a-3b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)}{4b}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b}-\frac{(a-3b)\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)}{4b}}{f}$$

input
```
Int[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*b) - ((a - 3*b)*(((a +
 b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqr
t[b]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2))/(4*b))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(106) = 212$.

Time = 31.99 (sec) , antiderivative size = 797, normalized size of antiderivative = 6.53

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(3b^{\frac{5}{2}}\cos(fx+e)\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)+2b^{\frac{3}{2}}\cos(fx\right.}{}$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/b^2*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(3*b^(5/2)*cos(f*x+e)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+2*b^(3/2)*cos(f*x+e)*ln(
4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+
1))*a-b^(1/2)*cos(f*x+e)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-si
n(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2+3*b^(5/2)*cos(f*x+e)*ln(-4*(b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))+2*b^(3/2)
*cos(f*x+e)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a
+b)/(sin(f*x+e)-1))*a-b^(1/2)*cos(f*x+e)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a*b*(2*sin(f*x+e)+2*tan(f*x+e))+(6*cos(f*x+e)^3+6*c
os(f*x+e)^2+4*cos(f*x+e)+4)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^
2*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.20

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{(a^2-2\,ab-3\,b^2)\sqrt{b}\cos\left(fx+e\right)^3\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(\right.}{\cos(fx+e)^4}\right.}{32\,b^2f\cos(}\right.$$

$$-\frac{(a^2-2\,ab-3\,b^2)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos\left(fx+e\right)^3-2\left((ab\right.}{16\,b^2f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b
^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((a*b + 3*b^2)*cos(f*x + e)^2
+ 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*
cos(f*x + e)^3), -1/16*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b
)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3
- 2*((a*b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2(e+fx)}\sec^4(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.42

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=$$
$$-\frac{\frac{(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{(a+b)\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{4\,a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-4\,\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)-4\,\sqrt{b}\cdots}{8\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*((a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + (a + b)*
arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 4*a*arcsinh(b*tan(f*x +
e)/sqrt((a + b)*b))/sqrt(b) - 4*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b
)*b)) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e) - 2*(b*tan(f*x + e)^
2 + a + b)^(3/2)*tan(f*x + e)/b + sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*t
an(f*x + e)/b)/f
```

Giac [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos(e+f\,x)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4, x)
```

Reduce [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec(fx+e)^2b+a}\,\sec(fx+e)^4\,dx$$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**4,x)
```

3.236 $\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f}+\frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
1/2*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(1/2)/f
+1/2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 210 vs. 2(76) = 152.

Time = 1.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.76

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$=\frac{\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\left(\sqrt{2}(a+b)\text{arctanh}\left(\frac{\sqrt{\frac{b\sin^2(e+fx)}{a+b}}}{\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}\right)\cos^2(e+fx)\sqrt{\frac{a+2b}{}}\right.}{\sqrt{2}f(a+2b+a\cos(2(e+fx)))^{3/2}\sqrt{\frac{b\sin}{}}}$$

input
```
Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*(Sqrt[2]*(a + b
)*ArcTanh[Sqrt[(b*Sin[e + f*x]^2)/(a + b)]/Sqrt[(a + b - a*Sin[e + f*x]^2)
/(a + b)]]*Cos[e + f*x]^2*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)] + (
a + 2*b + a*Cos[2*(e + f*x)])*Sqrt[(b*Sin[e + f*x]^2)/(a + b)])*Tan[e + f*
x])/(Sqrt[2]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sqrt[(b*Sin[e + f*x]^2
)/(a + b)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4634, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^2\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/
(2*Sqrt[b]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(64) = 128$.

Time = 23.77 (sec) , antiderivative size = 509, normalized size of antiderivative = 6.70

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)b^{\frac{3}{2}}\cos(fx+e)+\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+}{(1+}}}{}\right.\right.}{}$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/b*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a-a-b)/(sin(f*x+e)+1))*b^(3/2)*cos(f*x+e)+ln(4*(b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(1/2)*cos(f*x+e)*a+l
n(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+
e)-1))*b^(3/2)*cos(f*x+e)+ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(1/2)*cos(f*x+e)*a+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b*(2*sin(f*x+e)+2*tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(64) = 128$.

Time = 0.14 (sec) , antiderivative size = 320, normalized size of antiderivative = 4.21

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{(a+b)\sqrt{b}\cos(fx+e)\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(\cdots}{\cos(\cdots}}}{\cos(fx+e)^4}\right.}{8bf\cos(fx+e)}\right.$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8
*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*si
n(f*x + e))/(b*f*cos(f*x + e)), 1/4*((a + b)*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 2
*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x
+ e))]
```

Sympy [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sec^2(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\frac{a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} + \sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \sqrt{b\tan\left(fx+e\right)^2+a+b}\tan\left(fx+e\right)}{2\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + sqrt(b)*arcsinh(b
*tan(f*x + e)/sqrt((a + b)*b)) + sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x +
e))/f
```

Giac [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2, x)
```

Reduce [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\,dx$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2,x)
```

3.237 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

output
```
a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+b^(1/2)*ar
ctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.59

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{i\left(1+e^{2i(e+fx)}\right)\left(2\sqrt{b}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a}\text{arctanh}\left(\frac{a+2b+ae^{2i(e+fx)}}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\right.}{2\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\,f}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-1/2*I)*(1 + E^((2*I)*(e + f*x)))*(2*Sqrt[b]*ArcTan[(Sqrt[b]*(-1 + E^((2
*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + Sqrt[a]*ArcTanh[(a + 2*b + a*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])] - Sqrt[a]*ArcTanh
[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])])*Sqrt[a + b*Sec[e +
f*x]^2])/(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4616, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{a\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + S
qrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(67) = 134$.

Time = 4.98 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.47

method	result
default	$\frac{\left(\sqrt{b}\,\ln\left(\frac{-4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\sqrt{b}\,\ln\left(-\frac{4\left(-\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right.}{\ldots}\right.\right.}{\ldots}$

input
```
int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(b^(1/2)*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)+b^(1/2)*ln(-4*(-b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+2
*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a)*
(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time = 0.30 (sec) , antiderivative size = 1227, normalized size of antiderivative = 15.53

$$\int \sqrt{a + b\sec^2(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4))/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos
(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 -
256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3
*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
- (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a
^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*...
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 3227, normalized size of antiderivative = 40.85

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(a)*b^(3/2)*arctan2(a*sin(2*f*x + 2*e) + (a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^
2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*
arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), a*cos(2*f*x + 2*e) + (a^2*cos(4*
f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x
+ 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*
a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a
)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos
(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + a + 2*b) + a^(3/2)*sq
rt(b)*arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2
+ 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2
+ 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*...
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+\frac{b}{{\cos{(e+f\,x)}}^2}}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec{(fx+e)}^2 b+a}dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a),x)
```

3.238 $\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f+
1/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.66

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{2}\sqrt{a}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\sin(e+fx)\right)}{2\sqrt{2}\sqrt{a}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

input
```
Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[a + b]*ArcSin[(Sqrt[a]*Si
n[e + f*x])/Sqrt[a + b]] + Sqrt[2]*Sqrt[a]*Sqrt[(a + 2*b + a*Cos[2*(e + f*
x)])/(a + b)]*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[(a + 2*b + a*Cos[2*
(e + f*x)])/(a + b)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4634, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*Sqrt[a]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*(1 + Tan[e +
 f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(70) = 140.

Time = 5.56 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.39

method	result
default	$\frac{\left(\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a+\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\cdots\right.\right.}{2f\sqrt{-a}\,(1+\cos(fx+e))\cdots}$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*a+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*si
n(f*x+e)*a)*b+(1+cos(f*x+e))*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2))*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(70) = 140$.

Time = 0.23 (sec) , antiderivative size = 499, normalized size of antiderivative = 6.09

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \sqrt{-a}(a+b)\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\cos\right.\right.$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) - sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
 + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)))/(a*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e)*sin(f*x + e) - (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
 e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a
*f)]
```

Sympy [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cos^2(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**2, x)
```

Maxima [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\left(\frac{(a+b)\log\left(\left|-\sqrt{-a}\sin(fx+e)+\sqrt{-a\sin(fx+e)^2+a+b}\right|\right)}{\sqrt{-a}}-\sqrt{-a\sin(fx+e)^2+a+b}\sin(fx+e)\right)\mathrm{sgn}(\cos(fx+e))}{2f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*((a + b)*log(abs(-sqrt(-a)*sin(f*x + e) + sqrt(-a*sin(f*x + e)^2 + a
+ b)))/sqrt(-a) - sqrt(-a*sin(f*x + e)^2 + a + b)*sin(f*x + e))*sgn(cos(f*
x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)^2 dx$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**2,x)
```

3.239 $\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} + \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af}$$

output
```
1/8*(3*a-b)*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^
(3/2)/f+1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f+1
/4*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a/f
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}(3a-b)\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a}(4a+b+a\cos(2(e+fx\right.}{8a^{3/2}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

input
```
Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*(3*a - b)*Sqrt[a + b]*Ar
cSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]] + Sqrt[a]*(4*a + b + a*Cos[2*(e +
 f*x)])*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(8*a^(
3/2)*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4634, 296, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^4}dx$$
$$\Big\downarrow \text{4634}$$
$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \textcolor{blue}{296}$$

$$\frac{\frac{(3a-b)\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \textcolor{blue}{292}$$

$$\frac{\frac{(3a-b)\left(\frac{1}{2}(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \textcolor{blue}{291}$$

$$\frac{\frac{(3a-b)\left(\frac{1}{2}(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \textcolor{blue}{216}$$

$$\frac{\frac{(3a-b)\left(\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*a*(1 + Tan[e + f*x]^2)
^2) + ((3*a - b)*(((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(2*Sqrt[a]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(2*(1 + Tan[e + f*x]^2))))/(4*a))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(124) = 248.

Time = 9.62 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.15

method	result
default	$\frac{\left(3\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a^2+2\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e\right.\right.}{}$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/a/(-a)^(1/2)*(3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a^2+2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*a*b-ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*b^2+(2*cos(f*x+e)^3+2*cos(f*x+e)^2+3*cos(f*x+e)+3)*sin
(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+(1+cos(f*
x+e))*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b)
*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.05

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{(3a^2+2ab-b^2)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e)^6+32(5a^4-14a^3b+5\right.}{}\right.$$

$$-\frac{(3a^2+2ab-b^2)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8(a^2-ab)\cos(fx+e)^3+(a^2-6ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)-}{32a^2f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/64*((3*a^2 + 2*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)
^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2
*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
- 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + a*b
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/
(a^2*f), -1/32*((3*a^2 + 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2
*cos(f*x + e)^3 + (3*a^2 + a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e))/(a^2*f)]
```

Sympy [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cos^4(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)
```

Maxima [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\left(\sqrt{-a\sin(fx+e)^2+a+b}\left(2\sin(fx+e)^2-\frac{5a^2+ab}{a^2}\right)\sin(fx+e)+\frac{(3a^2+2ab-b^2)\log\left(\left|-\sqrt{-a}\sin(fx+e)+\right.\right.}{\sqrt{-a}a}\right.}{8f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(sqrt(-a*sin(f*x + e)^2 + a + b)*(2*sin(f*x + e)^2 - (5*a^2 + a*b)/a^
2)*sin(f*x + e) + (3*a^2 + 2*a*b - b^2)*log(abs(-sqrt(-a)*sin(f*x + e) + s
qrt(-a*sin(f*x + e)^2 + a + b)))/(sqrt(-a)*a))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^4\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)^4dx$$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**4,x)
```

3.240 $\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 196

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\left(5a^2-2ab+b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} + \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}$$

output
```
1/16*(a+b)*(5*a^2-2*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2
)^(1/2))/a^(5/2)/f+1/48*(3*a-b)*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan
(f*x+e)^2)^(1/2)/a^2/f+1/24*(5*a+b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x
+e)^2)^(1/2)/a/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 14.95 (sec) , antiderivative size = 1902, normalized size of antiderivative = 9.70

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)]*Cos[e + f*x]^10*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sqrt[a + b*Se
c[e + f*x]^2]*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1,
-1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((
3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^5*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/(3*(a + b)*App
ellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a
*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] +
 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2) - (12*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sqrt[a + 2*b + a*Co
s[2*(e + f*x)]]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1,
 -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2) +
 (3*(a + b)*Cos[e + f*x]^4*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sin[e + f*x]
*(-1/3*(a*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, -...
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4634, 314, 25, 402, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^6}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{(\tan^2(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{314}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6(\tan^2(e+fx)+1)^3}-\frac{1}{6}\int -\frac{4b\tan^2(e+fx)+5(a+b)}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\int \frac{4b\tan^2(e+fx)+5(a+b)}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{1}{6}\left(\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\int -\frac{2b(5a+b)\tan^2(e+fx)+(15a-b)(a+b)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}\right)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\left(\frac{\int \frac{2b(5a+b)\tan^2(e+fx)+(15a-b)(a+b)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{1}{6}\left(\frac{\frac{(3a-b)(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{3(a+b)\left(5a^2-2ba+b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{6}\left(\frac{\frac{3(a+b)\left(5a^2-2ab+b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{(3a-b)(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a}}{4a(\tan^2(e+}\right.}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{1}{6}\left(\frac{\frac{3(a+b)\left(5a^2-2ab+b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}+\frac{(3a-b)(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\,t}}{4a(\tan^2(e+fx)+}\right.}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{6}\left(\frac{\frac{3(a+b)\left(5a^2-2ab+b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}+\frac{(3a-b)(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right.}{f}$$

input
```
Int[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*(1 + Tan[e + f*x]^2)^3)
+ (((5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*a*(1 + Tan[e
 + f*x]^2)^2) + ((3*(a + b)*(5*a^2 - 2*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e +
f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*a^(3/2)) + ((3*a - b)*(5*a + 3*b
)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)))
/(4*a))/6)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(176) = 352$.

Time = 21.96 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.19

method	result
default	$\frac{\left(15\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a^3+9\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+\right.\right.}{}$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a^2/(-a)^(1/2)*(15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-4*sin(f*x+e)*a)*a^3+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)*a^2*b-3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3+(8*cos(f*x+e)^5+8*cos(f*x+e)^4+10*cos(
f*x+e)^3+10*cos(f*x+e)^2+15*cos(f*x+e)+15)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+(2*cos(f*x+e)^3+2*cos(f*x+e)^2+4*co
s(f*x+e)+4)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a*b+(-3*cos(f*x+e)-3)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b^2)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.27

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(3*(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2
)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 -
 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
+ e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5
+ 2*(5*a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos(f*x
+ e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), -
1/192*(3*(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))
*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^
4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*
a^3*cos(f*x + e)^5 + 2*(5*a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b
- 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(a^3*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

Giac [F]

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos(e+f\,x)^6\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^6\,dx$$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**6,x)
```

3.241 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 454

$$
\begin{aligned}
&\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \\
&-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{35b^2f} \\
&+\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+ \cdots\right.}}{35b^2f\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}} \\
&-\frac{(a+b)\left(a^2-16ab-16b^2\right)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e \cdots}}{35bf\left(a+b-a\sin^2(e+fx)\right)} \\
&+\frac{\left(a^2+11ab+8b^2\right)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35bf} \\
&+\frac{2(4a+3b)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35f} \\
&+\frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{7f}
\end{aligned}
$$

output
```
-2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)
^2))^(1/2)/b^2/f+2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*(cos(f*x+e)^2)^(1/2)*Ellip
ticE(sin(f*x+e),(a/(a+b))^(1/2))*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
/b^2/f/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)-1/35*(a+b)*(a^2-16*a*b-16*b^2)*(
cos(f*x+e)^2)^(1/2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+
e)^2)/(a+b))^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(a+b-a*si
n(f*x+e)^2)+1/35*(a^2+11*a*b+8*b^2)*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*
x+e)^2))^(1/2)*tan(f*x+e)/b/f+2/35*(4*a+3*b)*sec(f*x+e)^3*(sec(f*x+e)^2*(a
+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^5*(sec(f*x+e)^2*(a
+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

Mathematica [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

input
```
Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 4636, 2057, 2058, 315, 25, 402, 27, 402, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^5\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{9/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{7\left(1-\sin^2(e+fx)\right)^{7/2}}-\frac{1}{7}\int -\frac{(a+b)(7a+6b)-a(7a+5b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{7/2}\sqrt{-a\sin^2(e+fx)+a+b}}d s$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\int \frac{(a+b)(7a+6b)-a(7a+5b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{7/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{b\sin(e+fx)\sqrt{-a\sin^2(e}{7\left(1-\sin^2(e+fx)\right.}$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{\int \frac{3b\left((a+b)(9a+8b)-2a(4a+3b)\sin^2(e+fx)\right)}{\left(1-\sin^2(e+fx)\right)^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{5b}+\frac{2(4a+3b)\sin(e+fx)\sqrt{-a\sin^2}{5\left(1-\sin^2(e+fx)\right)^{5}}\right.\right.}{}$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\int \frac{(a+b)(9a+8b)-2a(4a+3b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{2(4a+3b)\sin(e+fx}{5(1-\sin}\right.\right.}{}$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\int -\frac{a\left(a^2+11ba+8b^2\right)\sin^2(e+fx)+(a+b)\left(a^2-16ba-16b^2\right)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}+\frac{(a^2+11ab+8b}{}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\int \frac{a\left(a^2+11ba+8b^2\right)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-}}}{}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)}}$$

$\downarrow$ 402

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\int -\frac{a\left((a+b)\left(2a^2-5ba-8b^2\right)-2\right.}{\sqrt{1-\sin^2(e+fx}}}{}\right.\right.\right.}{}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+f}{b\sqrt{1-\sin^2(e}}}{}\right.\right.\right.}{}$$

$\downarrow$ 27

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+f}{b\sqrt{1-\sin^2(e}}}{}\right.\right.\right.}{}$$

$\downarrow$ 399

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\frac{2(a+2b)(a^2-4ab-4b^2)\sin(e+f}{b\sqrt{1-\sin^2(e}}}{}\right.\right.\right.$$

$\downarrow$ 323

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\frac{2(a+2b)(a^2-4ab-4b^2)\sin(e+f}{b\sqrt{1-\sin^2(e}}}{}\right.\right.\right.$$

$\downarrow$ 321

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\frac{2(a+2b)(a^2-4ab-4b^2)\sin(e+f}{b\sqrt{1-\sin^2(e}}}{}\right.\right.\right.$$

$\downarrow$ 330

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+f}{b\sqrt{1-\sin^2(e}}\right.\right.\right.$$

$\downarrow$ 327

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+f}{b\sqrt{1-\sin^2(e}}\right.\right.\right.$$

input
```
Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((b*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(7*(1 - Sin[e + f*x
]^2)^(7/2)) + ((2*(4*a + 3*b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])
/(5*(1 - Sin[e + f*x]^2)^(5/2)) + (3*(((a^2 + 11*a*b + 8*b^2)*Sin[e + f*x]
*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*b*(1 - Sin[e + f*x]^2)^(3/2)) - ((2*(a
 + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])
/(b*Sqrt[1 - Sin[e + f*x]^2]) - (a*((2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Ell
ipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a
*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(a^2 - 16*a*b - 16*b^2
)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(
a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])))/b)/(3*b)))/5)/7))/(f*Sqrt[a
+ b - a*Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 45.56 (sec) , antiderivative size = 3995, normalized size of antiderivative = 8.80

method	result	size
default	Expression too large to display	3995

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b
^2*(a+b*sec(f*x+e)^2)^(3/2)/(1+cos(f*x+e))/(cos(f*x+e)^4*a^2+2*cos(f*x+e)^
2*a*b+b^2)*((16*cos(f*x+e)^8+8*cos(f*x+e)^7+16*cos(f*x+e)^6+9*cos(f*x+e)^5
+9*cos(f*x+e)^4+7*cos(f*x+e)^3+7*cos(f*x+e)^2-5*cos(f*x+e)-5)*((I*b^(1/2)+
a^(1/2))^2/(a+b))^(1/2)*a*b^4*tan(f*x+e)*sec(f*x+e)^3+(8*cos(f*x+e)^6+3*co
s(f*x+e)^5-17*cos(f*x+e)^4-8*cos(f*x+e)^3-8*cos(f*x+e)^2-13*cos(f*x+e)-13)
*((I*b^(1/2)+a^(1/2))^2/(a+b))^(1/2)*a^2*b^3*tan(f*x+e)*sec(f*x+e)+2*I*sin
(f*x+e)*(9+24*cos(f*x+e)^4+11*cos(f*x+e)^3+15*cos(f*x+e)^2+9*cos(f*x+e))*a
^(5/2)*b^(5/2)*((I*b^(1/2)+a^(1/2))^2/(a+b))^(1/2)-4*I*a^(9/2)*b^(1/2)*((I
*b^(1/2)+a^(1/2))^2/(a+b))^(1/2)*cos(f*x+e)^4*sin(f*x+e)+sin(f*x+e)*cos(f*
x+e)^2*(-6*cos(f*x+e)^2-cos(f*x+e)+1)*((I*b^(1/2)+a^(1/2))^2/(a+b))^(1/2)*
a^4*b+30*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+
e)*a+b)/(1+cos(f*x+e)))^(1/2)*((-I*a^(1/2)*b^(1/2)*cos(f*x+e)+I*a^(1/2)*b^
(1/2)+cos(f*x+e)*a+b)/(a+b)/(1+cos(f*x+e)))^(1/2)*a^3*b^2*EllipticE(((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(1/(a+b)^2*(-4*I
*a^(3/2)*b^(1/2)+4*I*a^(1/2)*b^(3/2)+a^2-6*a*b+b^2))^(1/2))*(cos(f*x+e)^5+
2*cos(f*x+e)^4+cos(f*x+e)^3)+68*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*((-I*a^(1/2)*b^(1/2)*
cos(f*x+e)+I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(a+b)/(1+cos(f*x+e)))^(1/2)*a
^2*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-co...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.16

$$\int \sec^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/35*((2*(-I*a^4 + 2*I*a^3*b + 12*I*a^2*b^2 + 8*I*a*b^3)*sqrt(a)*sqrt((a*b
 + b^2)/a^2)*cos(f*x + e)^6 - (-I*a^4 + 16*I*a^2*b^2 + 32*I*a*b^3 + 16*I*b
^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*
elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x +
 e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b +
 b^2)/a^2))/a^2) + (2*(I*a^4 - 2*I*a^3*b - 12*I*a^2*b^2 - 8*I*a*b^3)*sqrt(
a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^6 - (I*a^4 - 16*I*a^2*b^2 - 32*I*a*b
^3 - 16*I*b^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a
 - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)
*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*
sqrt((a*b + b^2)/a^2))/a^2) + (2*(-I*a^3*b - 11*I*a^2*b^2 - 8*I*a*b^3)*sqr
t(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^6 - (2*I*a^4 + I*a^3*b - 19*I*a^2*
b^2 - 34*I*a*b^3 - 16*I*b^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b +
 b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*
(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(I*a^3*b + 11*I*a^2*b^2 + 8
*I*a*b^3)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^6 - (-2*I*a^4 - I*a^3
*b + 19*I*a^2*b^2 + 34*I*a*b^3 + 16*I*b^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2
*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a
*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8...
```

Sympy [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^5\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**5, x)
```

Maxima [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Giac [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5, x)
```

Reduce [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^7 dx\right)b+\left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^5 dx\right)a$$

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**7,x)*b + int(sqrt(sec(e + f*
x)**2*b + a)*sec(e + f*x)**5,x)*a
```

3.242 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 375

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf}$$
$$-\frac{(3a^2+13ab+8b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{(a+b)(9a+8b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(a+}}{15f\left(a+b-a\sin^2(e+fx)\right)}$$
$$+\frac{2(3a+2b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15f}$$
$$+\frac{b\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f}$$

output
```
1/15*(3*a^2+13*a*b+8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)/b/f-1/15*(3*a^2+13*a*b+8*b^2)*(cos(f*x+e)^2)^(1/2)*EllipticE(sin(f*x+
e),(a/(a+b))^(1/2))*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/((a+b-a*
sin(f*x+e)^2)/(a+b))^(1/2)+1/15*(a+b)*(9*a+8*b)*(cos(f*x+e)^2)^(1/2)*Ellip
ticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)*(sec(f
*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(a+b-a*sin(f*x+e)^2)+2/15*(3*a+2*b)*
sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f+1/5*b*se
c(f*x+e)^3*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

Mathematica [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3042, 4636, 2057, 2058, 315, 25, 402, 27, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}{(1-\sin^2(e+fx))^2}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 2057

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}{(1-\sin^2(e+fx))^2}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 2058

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{(-a\sin^2(e+fx)+a+b)^{3/2}}{(1-\sin^2(e+fx))^{7/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 315

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{5(1-\sin^2(e+fx))^{5/2}}-\frac{1}{5}\int -\frac{(a+b)(5a+4b)-a(5a+3b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\int \frac{(a+b)(5a+4b)-a(5a+3b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{b\sin(e+fx)\sqrt{-a\sin^2(e}}{5(1-\sin^2(e+fx)}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{\int \frac{b\left((a+b)(9a+8b)-2a(3a+2b)\sin^2(e+fx)\right)}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}+\frac{2(3a+2b)\sin(e+fx)\sqrt{-a\sin^2}}{3(1-\sin^2(e+fx))^{3}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{(a+b)(9a+8b)-2a(3a+2b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{2(3a+2b)\sin(e+fx}{3(1-\sin}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int -\frac{a\left((a+b)(3a+4b)-\left(3a^2+13ba+8b^2\right)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}+\frac{(3a^2+13ab+8b^2)\,\mathrm{s}}{b\sqrt{}}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\int \frac{a\left((a+b)(3a+4b)-\left(3a^2+13b\right.\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\,\mathrm{si}}}}{}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)}\ -}$$

$\downarrow$ 27

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\int \frac{(a+b)(3a+4b)-\left(3a^2+13ba\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\,\mathrm{si}}}}{}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$\downarrow$ 399

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{\left(3a^2+13ab+8b^2\right)\int \frac{\sqrt{-a\,\mathrm{s}}}{\sqrt{1}}}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 323

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{\left(3a^2+13ab+8b^2\right)\int \frac{\sqrt{-a\,\mathrm{s}}}{\sqrt{1}}}{}\right.}{}\right.\right.\right.$$

$\Big\downarrow$ 321

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(3a^2+13ab+8b^2)\int\frac{\sqrt{-a\sin\ldots}}{\sqrt{1\ldots}}}{\ldots}\right.}{\ldots}\right.\right.\right.}{\ldots}$$

$\Big\downarrow$ 330

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(3a^2+13ab+8b^2)\sqrt{-a\sin^2\ldots}}{\ldots}\right.}{\ldots}\right.\right.\right.}{\ldots}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(3a^2+13ab+8b^2)\sqrt{-a\sin^2\ldots}}{a\sqrt{\ldots}}\right.}{\ldots}\right.\right.\right.}{\ldots}$$

input

```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((b*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*(1 - Sin[e + f*x
]^2)^(5/2)) + ((2*(3*a + 2*b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])
/(3*(1 - Sin[e + f*x]^2)^(3/2)) + (((3*a^2 + 13*a*b + 8*b^2)*Sin[e + f*x]*
Sqrt[a + b - a*Sin[e + f*x]^2])/(b*Sqrt[1 - Sin[e + f*x]^2]) - (a*(((3*a^2
 + 13*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b -
 a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*
(9*a + 8*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e +
 f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])))/b)/3)/5))/(f*Sqrt[
a + b - a*Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 33.15 (sec) , antiderivative size = 3442, normalized size of antiderivative = 9.18

method	result	size
default	Expression too large to display	3442

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b
*(a+b*sec(f*x+e)^2)^(3/2)/(cos(f*x+e)^4*(1+cos(f*x+e))*a^2+cos(f*x+e)^2*(2
*cos(f*x+e)+2)*a*b+(1+cos(f*x+e))*b^2)*((1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)
))^(1/2)*a^4*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)
-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*(3*cos(f*x+e)^5+6*cos(f*x+e)^4+3*cos(f*x+e)^3)+(1/(a+b)*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))
^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e
)*a-b)/(1+cos(f*x+e)))^(1/2)*a^3*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(19*cos(f*x+e)^5+38*cos(f*x+e)^4+19*cos
(f*x+e)^3)+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*
x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*
a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a^2*b^2*EllipticE(((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(37*cos(f*x+
e)^5+74*cos(f*x+e)^4+37*cos(f*x+e)^3)+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.37

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/30*((2*(3*I*a^3 + 13*I*a^2*b + 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*
cos(f*x + e)^4 - (3*I*a^3 + 19*I*a^2*b + 34*I*a*b^2 + 16*I*b^3)*sqrt(a)*co
s(f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arc
sin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*
x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^
2) + (2*(-3*I*a^3 - 13*I*a^2*b - 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*
cos(f*x + e)^4 - (-3*I*a^3 - 19*I*a^2*b - 34*I*a*b^2 - 16*I*b^3)*sqrt(a)*c
os(f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(ar
csin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f
*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a
^2) - 2*(4*(3*I*a^2*b + 2*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x +
 e)^4 + (-3*I*a^3 - 13*I*a^2*b - 18*I*a*b^2 - 8*I*b^3)*sqrt(a)*cos(f*x + e
)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt(
(2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))),
 (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*(4
*(-3*I*a^2*b - 2*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^4 + (
3*I*a^3 + 13*I*a^2*b + 18*I*a*b^2 + 8*I*b^3)*sqrt(a)*cos(f*x + e)^4)*sqrt(
(2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt(
(a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*
a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*((3*a^3 +...
```

Sympy [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^3\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^3} \,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3, x)
```

Reduce [F]

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^5 dx\right) b + \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^3 dx\right) a$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**5,x)*b + int(sqrt(sec(e + f*
x)**2*b + a)*sec(e + f*x)**3,x)*a
```

3.243 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 294

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f} - \frac{2(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}} + \frac{(a+b)(3a+2b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+\ldots\right.}}{3f\left(a+b-a\sin^2(e+fx)\right)} + \frac{b\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f}$$

output

```
2/3*(2*a+b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-2/3*(2*
a+b)*(cos(f*x+e)^2)^(1/2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(sec(f*x+e
)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)+1/3*(
a+b)*(3*a+2*b)*(cos(f*x+e)^2)^(1/2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*
((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)/f/(a+b-a*sin(f*x+e)^2)+1/3*b*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)
^2))^(1/2)*tan(f*x+e)/f
```

Mathematica [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx=\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int\frac{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{5/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 315

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{1}{3}\int-\frac{(a+b)(3a+2b)-a(3a+b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\,s\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\int\frac{(a+b)(3a+2b)-a(3a+b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{b\sin(e+fx)\sqrt{-a\sin^2(e}{3\left(1-\sin^2(e+fx\right)}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 402

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{\int-\frac{ab\left(-2(2a+b)\sin^2(e+fx)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}+\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+}{\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-\frac{\int\frac{ab\left(-2(2a+b)\sin^2(e+fx)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin}{b}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\int\frac{-2(2a+b)\sin^2(e+fx)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 399

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(}{a}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$\downarrow$ 323

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(}{a}\right.\right.\right.}{f\sqrt{-a\sin^2(e+}}$$

$\downarrow$ 321

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(}{a}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)-}}$$

$\downarrow$ 330

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1}}{\sqrt{}}}{a\sqrt{1-\frac{a\sin^2(e+}{a+b}}}\right.\right.\right.}{f\sqrt{-a\sin^2(e}}$$

$\downarrow$ 327

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\text{ar}\right.}{a\sqrt{1-\frac{a\sin^2(e+f}{a+b}}}\right.\right.\right.}{f\sqrt{-a\sin^2(e+}}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((b*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*(1 - Sin[e + f*x
]^2)^(3/2)) + ((2*(2*a + b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/S
qrt[1 - Sin[e + f*x]^2] - a*((2*(2*a + b)*EllipticE[ArcSin[Sin[e + f*x]],
a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/
(a + b)]) - ((a + b)*(3*a + 2*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])
))/3))/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 22.91 (sec) , antiderivative size = 2743, normalized size of antiderivative = 9.33

method	result	size
default	Expression too large to display	2743

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((
1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1
+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1
/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a^3*EllipticE(((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(4*cos(f*x+e)^5+8*cos(f*x+e)
^4+4*cos(f*x+e)^3)+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a^2*b*Ellipt
icE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(10*co
s(f*x+e)^5+20*cos(f*x+e)^4+10*cos(f*x+e)^3)+(1/(a+b)*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e)))^(1/2)*a*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(
f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*(8*cos(f*x+e)^5+16*cos(f*x+e)^4+8*cos(f*x+e)^3)+(1/(a+b
)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.71

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((2*(2*I*a^2 + I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (
2*I*a^2 + 5*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b +
 b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*
(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-2*I*a^2 - I*a*b)*sqrt(a)*
sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (-2*I*a^2 - 5*I*a*b - 2*I*b^2)*sqrt
(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic
_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*
sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^
2))/a^2) + (2*(-3*I*a^2 - I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e
)^2 - (-I*a^2 - 3*I*a*b - 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt(
(a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2
)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^
2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(3*I*a^2 + I*a*b)*sqr
t(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (I*a^2 + 3*I*a*b + 2*I*b^2)*sq
rt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*ellipt
ic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) -
I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/
a^2))/a^2) + (2*(2*a^2 + a*b)*cos(f*x + e)^2 + a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)} dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\, \sec\left(fx+e\right)^3 dx\right) b + \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\, \sec\left(fx+e\right) dx\right) a$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**3,x)*b + int(sqrt(sec(e + f*
x)**2*b + a)*sec(e + f*x),x)*a
```

3.244 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 228

$$\begin{aligned}\int \cos(e+fx)\,(a&+b\sec^2(e+fx))^{3/2}\,dx=\frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f}\\&+\frac{(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}\\&+\frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\mathrm{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f\left(a+b-a\sin^2(e+fx)\right)}\end{aligned}$$

output
```
b*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+(a-b)*(cos(f*x+e)
^2)^(1/2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(sec(f*x+e)^2*(a+b-a*sin(f
*x+e)^2))^(1/2)/f/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)+b*(a+b)*(cos(f*x+e)^2
)^(1/2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))
^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int\frac{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 315

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-\int-\frac{a(-((a-b)\sin^2(e+fx))+a+b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 25

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\int\frac{a(-((a-b)\sin^2(e+fx))+a+b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\int\frac{-((a-b)\sin^2(e+fx))+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+f}}{\sqrt{1-\sin^2(e+fx)}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 399

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{b(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}+\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}}{a}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 323

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{}{\sqrt{1-\sin^2(e+fx)}}}{a\sqrt{-a\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 321

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 330

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+f}{a+b}}}{a\sqrt{-}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\downarrow$ 327

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}+\frac{(a-b)\sqrt{-a\sin^2(e+fx)}}{a\sqrt{1}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((b*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/Sqrt[1 - Sin[e + f*
x]^2] + a*(((a - b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b
- a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (b*(a + b)
*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))))/(f*Sqrt[a + b - a*Sin[e + f*
x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.90 (sec) , antiderivative size = 1987, normalized size of antiderivative = 8.71

method	result	size
default	Expression too large to display	1987

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(
f*x+e)^2*((1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a
^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a^3*EllipticE(((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(cos(f*x+e)^3+2*c
os(f*x+e)^2+cos(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a^2*b*E
llipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(
cos(f*x+e)^3+2*cos(f*x+e)^2+cos(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)
))^(1/2)*a*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+
e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*(-cos(f*x+e)^3-2*cos(f*x+e)^2-cos(f*x+e))+(1/(a+b)*(I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1
/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e)))^(1/2)*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b...
```

Fricas [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)*sec(f*x + e)^2 + a*cos(f*x + e))*sqrt(b*sec(f*x +
 e)^2 + a), x)
```

Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^{\frac{3}{2}}\cos{(e+fx)}\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*cos(e + f*x), x)
```

Maxima [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec{(fx+e)}^2+a\right)^{\frac{3}{2}}\cos{(fx+e)}\;dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

Giac [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec{(fx+e)}^2+a\right)^{\frac{3}{2}}\cos{(fx+e)}\;dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)\sec\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)dx\right)a$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)*sec(e + f*x)**2,x)*b + int(sq
rt(sec(e + f*x)**2*b + a)*cos(e + f*x),x)*a
```

3.245 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 245

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f}+\frac{2(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/3*a*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+
2/3*(a+2*b)*(cos(f*x+e)^2)^(1/2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2
)-1/3*b*(a+b)*(cos(f*x+e)^2)^(1/2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(
(a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(4\sqrt{2}(a^2+3ab+2b^2)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}E\left(e+fx\middle|\frac{a}{a+b}\right) \right.}{6f(a+}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(4*Sqrt[2]*(a^2 + 3*a*b + 2*b^2)*
Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)]
 - 2*Sqrt[2]*b*(a + b)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Ellipt
icF[e + f*x, a/(a + b)] + a*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)
]))/(6*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4636, 2057, 2058, 318, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \; 4636$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}-\frac{1}{3}\int \frac{2a(a+2}{\sqrt{1-si}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(2(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-b(a+b)\int \frac{}{\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(2(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int -}{\sqrt{-}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(2(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{Elli}}{\sqrt{-a\sin^2}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e-}{a+b}}}{\sqrt{}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)-}}$$

$\downarrow$ 327

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx}{a+b}}}{\sqrt{-a}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((a*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x
]^2])/3 + ((2*(a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a
+ b - a*Sin[e + f*x]^2])/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)] - (b*(a + b)
*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/Sqrt[a + b - a*Sin[e + f*x]^2])/3))/(f*Sqrt[a + b - a*Sin[e + f*x]
^2])
```

Defintions of rubi rules used

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 2625, normalized size of antiderivative = 10.71

method	result	size
default	Expression too large to display	2625

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)*co
s(f*x+e)^3*((2*cos(f*x+e)^2+4*cos(f*x+e)+2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*
x+e)))^(1/2)*a^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*
x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))+(8*cos(f*x+e)^2+16*cos(f*x+e)+8)*(1/(a+b)*(I*a^(1/2)*b^(1
/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1
/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+
cos(f*x+e)))^(1/2)*a^2*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))+(10*cos(f*x+e)^2+20*cos(f*x+e)+10)*(1/(a+b)*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))
^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e
)*a-b)/(1+cos(f*x+e)))^(1/2)*a*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+(4*cos(f*x+e)^2+8*cos(f*x+e)+4)*(1/(a+b
)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)...
```

Fricas [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^3*sec(f*x + e)^2 + a*cos(f*x + e)^3)*sqrt(b*sec(f
*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

Giac [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)^3\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}\,dx=\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)^3\sec\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)^3dx\right)a$$

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**3*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**3,x)*a
```

3.246 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 323

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{2(a-3(a+b))\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f} + \frac{a\cos^4(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5f} + \frac{(8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}} - \frac{b(a+b)(4a+3b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)(a}}{15af\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
-2/15*(-2*a-3*b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)/f+1/5*a*cos(f*x+e)^4*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)/f+1/15*(8*a^2+13*a*b+3*b^2)*(cos(f*x+e)^2)^(1/2)*EllipticE(sin(f*
x+e),(a/(a+b))^(1/2))*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/((a+b-
a*sin(f*x+e)^2)/(a+b))^(1/2)-1/15*b*(a+b)*(4*a+3*b)*(cos(f*x+e)^2)^(1/2)*E
llipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)*(s
ec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/(a+b-a*sin(f*x+e)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.28 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.10

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\cos^3(e+fx)\csc(2(e+fx))\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-8i\sqrt{2}(8a^3+21a^2b+16ab^2+3b^3)\sqrt{}\right.}{}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]^3*Csc[2*(e + f*x)]*(a + b*Sec[e + f*x]^2)^(3/2)*((-8*I)*Sqrt
[2]*(8*a^3 + 21*a^2*b + 16*a*b^2 + 3*b^3)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*El
lipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[
2]], b/(a + b)]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)] + a*((8*I)*Sqrt[2]*(8*a^2
 + 17*a*b + 9*b^2)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticF[I*ArcSinh[(Sqrt
[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(a + b)]*Sqrt[(a
*Sin[e + f*x]^2)/(a + b)] + a*Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*
x)]]*(11*a + 12*b + 3*a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]^2)))/(30*a^2*Sq
rt[-b^(-1)]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4636, 2057, 2058, 318, 403, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^5}dx$$

$\Big\downarrow$ 4636

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 2057

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 2058

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 318

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}a\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}-\frac{1}{5}\int \frac{\sqrt{}}{}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$\Big\downarrow$ 403

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{\int \frac{a\left((a+b)(8a+9b)-\left(8a^2+13ba+3b^2\right)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}-\frac{2}{3}(a-3(a+b))\sin(\right.\right.}{f\sqrt{-a\,\mathrm{s}}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{(a+b)(8a+9b)-\left(8a^2+13ba+3b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)-\frac{2}{3}(a-3(a+b)\right.\right.}{f\sqrt{-a}}$$

$\Big\downarrow$ 399

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(8a^2+13ab+3b^2)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(4a+3b)\int\frac{}{\sqrt{1-\sin^2(}}\right.\right.\right.$$

$\downarrow$ 323

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(8a^2+13ab+3b^2)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(4a+3b)\sqrt{1-\frac{a\sin^2}{a}}}{}\right.\right.\right.$$

$\downarrow$ 321

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(8a^2+13ab+3b^2)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(4a+3b)\sqrt{1-\frac{a\sin^2(}{a}}}{a\sqrt{-}}\right.\right.\right.$$

$\downarrow$ 330

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(8a^2+13ab+3b^2)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)}{}\right.\right.\right.$$

$\downarrow$ 327

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(8a^2+13ab+3b^2)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)(4a}{}\right.\right.\right.$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((a*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(3/2)*Sqrt[a + b - a*Sin[e + f
*x]^2])/5 + ((-2*(a - 3*(a + b))*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqr
t[a + b - a*Sin[e + f*x]^2])/3 + (((8*a^2 + 13*a*b + 3*b^2)*EllipticE[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (
a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(4*a + 3*b)*EllipticF[ArcSin[Sin[
e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b
- a*Sin[e + f*x]^2]))/3)/5))/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 3284, normalized size of antiderivative = 10.17

method	result	size
default	Expression too large to display	3284

input

```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/15/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a/(2*I*a^(1/2)*b^(1/2)-a+b)
*((8*cos(f*x+e)^2+16*cos(f*x+e)+8)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)
-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/
2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1
/2)*a^4*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(
f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))+(29*cos(f*x+e)^2+58*cos(f*x+e)+29)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e)))^(1/2)*a^3*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f
*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))+(37*cos(f*x+e)^2+74*cos(f*x+e)+37)*(-1/(a+b)*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)
*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/
(1+cos(f*x+e)))^(1/2)*a^2*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))+(19*cos(f*x+e)^2+38*cos(f*x+e)+19)*(-1/(a+b)
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(
f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*a*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)...
```

Fricas [F]

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^5 dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^5*sec(f*x + e)^2 + a*cos(f*x + e)^5)*sqrt(b*sec(f
*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^5 dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

Giac [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)^5\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)^5\sec\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)^5dx\right)a$$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**5*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**5,x)*a
```

3.247 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{(a+b)^2\left(3a^2-10ab+35b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} + \frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2f} - \frac{(3a-13b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f} + \frac{\tan^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf}$$

output
```
1/128*(a+b)^2*(3*a^2-10*a*b+35*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(
f*x+e)^2)^(1/2))/b^(5/2)/f+1/128*(a+b)*(3*a^2-10*a*b+35*b^2)*tan(f*x+e)*(a
+b+b*tan(f*x+e)^2)^(1/2)/b^2/f+1/192*(3*a^2-10*a*b+35*b^2)*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^(3/2)/b^2/f-1/48*(3*a-13*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^
2)^(5/2)/b^2/f+1/8*tan(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(5/2)/b/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.12 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.16

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-9a^3\left(1+e^{2i(e+fx)}\right)^6+\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(-9*a^3*(1
 + E^((2*I)*(e + f*x)))^6 + 3*a^2*b*(1 + E^((2*I)*(e + f*x)))^4*(5 + 18*E^
((2*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x))) + a*b^2*(1 + E^((2*I)*(e + f*x)
))^2*(145 + 948*E^((2*I)*(e + f*x)) + 2758*E^((4*I)*(e + f*x)) + 948*E^((6
*I)*(e + f*x)) + 145*E^((8*I)*(e + f*x))) + b^3*(105 + 910*E^((2*I)*(e + f
*x)) + 3591*E^((4*I)*(e + f*x)) + 8644*E^((6*I)*(e + f*x)) + 3591*E^((8*I)
*(e + f*x)) + 910*E^((10*I)*(e + f*x)) + 105*E^((12*I)*(e + f*x)))))/(1 +
E^((2*I)*(e + f*x)))^8 - (3*(a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*Log[(-4*Sq
rt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) +
a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^(
(2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^
(3/2))/(96*Sqrt[2]*b^(5/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4634, 318, 25, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{\int -\left(((3a-7b)\tan^2(e+fx)+a-7b)\left(b\tan^2(e+fx)+a+b\right)^{3/2}\right)d\tan(e+fx)}{8b}+\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b}-\frac{\int((3a-7b)\tan^2(e+fx)+a-7b)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{8b}}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b}-\frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{\left(3a^2-10ab+35b^2\right)\int\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan}{6b}}{8b}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b}-\frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{\left(3a^2-10ab+35b^2\right)\left(\frac{3}{4}(a+b)\int\sqrt{b\tan^2(e+fx)+a+}\right.}{}}{8b}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b}-\frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{\left(3a^2-10ab+35b^2\right)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int\frac{}{\sqrt{b\tan^2(}}\right.\right.}{}}{}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b}-\frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{\left(3a^2-10ab+35b^2\right)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int\frac{b\tan}{1-\frac{b\tan^2}{}}\right.\right.}{}}{}}{f}$$

$\downarrow$ 219

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b}-\frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{\left(3a^2-10ab+35b^2\right)\left(\frac{3}{4}(a+b)\left(\frac{(a+b)\text{arctanh}\left(\right.}{}\right.\right.}{}}{}}{f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(5/2))/(8*b
) - (((3*a - 7*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*b) - (
(3*a^2 - 10*a*b + 35*b^2)*((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))
/4 + (3*(a + b)*(((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(2*Sqrt[b]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/2))/4))/(6*b))/(8*b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(213) = 426$.

Time = 63.99 (sec) , antiderivative size = 1474, normalized size of antiderivative = 6.22

method	result	size
default	Expression too large to display	1474

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/768/f/b^(13/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(9*cos(f*x+e)^3
*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x
+e)+1))*a^4*b^4-12*cos(f*x+e)^3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^3*b^5+54*cos(f*x+e)^3*ln(4*(b^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*
b^6+180*cos(f*x+e)^3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*
x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^7+105*cos(f*x+e)^3*ln(4*(b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^8+9*cos(f*x+e)
^3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(
f*x+e)-1))*a^4*b^4-12*cos(f*x+e)^3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^3*b^5+54*cos(f*x+e)^3*ln(-4*(
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*...
```

Fricas [A] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.39

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(3a^4-4a^3b+18a^2b^2+60ab^3+35b^4\right)\sqrt{b}\cos(fx+e)^7\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/1536*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*sqrt(b)*cos(
f*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x
 + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) -
4*((9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*cos(f*x + e)^6 - 2*(3*a^2*
b^2 + 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 + 7*b^4)*cos
(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^
3*f*cos(f*x + e)^7), 1/768*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 3
5*b^4)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
b^2)*sin(f*x + e)))*cos(f*x + e)^7 - 2*((9*a^3*b - 15*a^2*b^2 - 145*a*b^3
- 105*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 + 46*a*b^3 + 35*b^4)*cos(f*x + e)
^4 - 48*b^4 - 8*(9*a*b^3 + 7*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7)]
```

Sympy [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^6\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.75

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\frac{48\left(b\tan(fx+e)^2+a+b\right)^{\frac{5}{2}}\tan(fx+e)^3}{b}+\frac{9\,(a+b)^3 a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{9\,(a+b)^3\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{48\,(a+b)^2 \cdots}{\cdots}}{\cdots}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/384*(48*(b*tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)^3/b + 9*(a + b)^3*
a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 9*(a + b)^3*arcsinh(b*
tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*a*arcsinh(b*tan(f*x +
 e)/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a
 + b)*b))/sqrt(b) + 144*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/
sqrt(b) + 144*(a + b)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 96
*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e) + 144*sqrt(b*tan(f*x + e)^2
 + a + b)*(a + b)*tan(f*x + e) - 24*(b*tan(f*x + e)^2 + a + b)^(5/2)*(a +
b)*tan(f*x + e)/b^2 + 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x
 + e)/b^2 + 9*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3*tan(f*x + e)/b^2 +
128*(b*tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b - 32*(b*tan(f*x + e)^2
 + a + b)^(3/2)*(a + b)*tan(f*x + e)/b - 48*sqrt(b*tan(f*x + e)^2 + a + b)
*(a + b)^2*tan(f*x + e)/b)/f
```

Giac [F]

$$\int \sec^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b \sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \sec\left(fx+e\right)^6 \, dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^6} \, dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6, x)
```

Reduce [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^8\,dx\right)b+\left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^6\,dx\right)a$$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**8,x)*b + int(sqrt(sec(e + f*
x)**2*b + a)*sec(e + f*x)**6,x)*a
```

3.248 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{(a-5b)(a+b)^2\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} - \frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf}$$

output
```
-1/16*(a-5*b)*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/b^(3/2)/f-1/16*(a-5*b)*(a+b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f-
1/24*(a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f+1/6*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^(5/2)/b/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.84 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.42

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(3a^2\left(1+e^{2i(e+fx)}\right)^4+2a\right.}{}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(3*a^2*(1
+ E^((2*I)*(e + f*x)))^4 + 2*a*b*(1 + E^((2*I)*(e + f*x)))^2*(11 + 50*E^((
2*I)*(e + f*x)) + 11*E^((4*I)*(e + f*x))) + b^2*(15 + 100*E^((2*I)*(e + f*
x)) + 298*E^((4*I)*(e + f*x)) + 100*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e +
 f*x)))))/(1 + E^((2*I)*(e + f*x)))^6 + (3*(a - 5*b)*(a + b)^2*Log[(-4*Sqr
t[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a
*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((
2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(
3/2))/(12*Sqrt[2]*b^(3/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4634, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$\downarrow$ 3042

$$\int \sec(e+fx)^4 \left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$\downarrow$ 4634

$$\frac{\int \left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$\downarrow$ 299

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\int \left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{6b}}{f}$$

$\downarrow$ 211

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\left(\frac{3}{4}(a+b)\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)+\frac{1}{4}\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{6b}}{f}$$

$\downarrow$ 211

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\text{ta}}{6b}}{f}$$

$\downarrow$ 224

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right.\right.}{6b}}{f}$$

$\downarrow$ 219

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\left(\frac{3}{4}(a+b)\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\tan(\right.}{6b}}{f}$$

input

```
Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*b) - ((a - 5*b)*((Tan[
e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/4 + (3*(a + b)*(((a + b)*ArcTan
h[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (T
an[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2))/4))/(6*b))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1150 vs. $2(145) = 290$.

Time = 48.30 (sec) , antiderivative size = 1151, normalized size of antiderivative = 6.98

method	result	size
default	Expression too large to display	1151

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/f/b^(9/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(3*cos(f*x+e)^3*
ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(
f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x
+e)-1))*a^3*b^3-9*cos(f*x+e)^3*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a^2*b^4-27*cos(f*x+e)^3*ln(4*(-b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a*b
^5-15*cos(f*x+e)^3*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-a-b)/(sin(f*x+e)-1))*b^6+3*cos(f*x+e)^3*ln(-4*(-b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*a^3*b^3-9*cos(f*x+e
)^3*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(si
n(f*x+e)+1))*a^2*b^4-27*cos(f*x+e)^3*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*a*b^5-15*cos(f*x+e)^3*ln(-4*
(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*...
```

Fricas [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.85

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{b}\cos(fx+e)^5\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2-}{}\right.}{}\right.$$

$$-\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)^5 -}{96\,b^2 f\cos}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((
a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b + 22*
a*b^2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b^3)*cos(f*x + e)^
2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*
x + e)^5), -1/96*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2
*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x
 + e)^5 - 2*((3*a^2*b + 22*a*b^2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a
*b^2 + 5*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))/(b^2*f*cos(f*x + e)^5)]
```

Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^4\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.47

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{\frac{3\,(a+b)^2a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{3\,(a+b)^2\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{18\,(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-18\,(a+b)\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(}{\sqrt{(a}}\right.$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(a + b)^2*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + 3*(
a + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*a*ar
csinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*sqrt(b)*arcsinh
(b*tan(f*x + e)/sqrt((a + b)*b)) - 12*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan
(f*x + e) - 18*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e) - 8*(b*
tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b + 2*(b*tan(f*x + e)^2 + a + b
)^(3/2)*(a + b)*tan(f*x + e)/b + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^
2*tan(f*x + e)/b)/f
```

Giac [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4, x)
```

Reduce [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\sec\left(fx+e\right)^6dx\right)b+\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\sec\left(fx+e\right)^4dx\right)a$$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**6,x)*b + int(sqrt(sec(e + f*
x)**2*b + a)*sec(e + f*x)**4,x)*a
```

3.249 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{3(a+b)^2\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

output

```
3/8*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(1/2)
/f+3/8*(a+b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f+1/4*tan(f*x+e)*(a+b+b
*tan(f*x+e)^2)^(3/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{(a+b)^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}\sin(2(e+fx))}{f(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + b)^2*Hypergeometric2F1[1/2, 3, 3/2, (b*Sin[e + f*x]^2)/(a + b - a*Si
n[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(f*(a + 2*b +
a*Cos[2*(e + f*x)]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{3}{4}(a+b)\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)+\frac{1}{4}\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{f}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\tan(e+}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\tan(e+fx)\left(a+b\tan\right.}{f}$$

input

```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output

```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/4 + (3*(a + b)*(((a + b)*
ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]
) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2))/4)/f
```

Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(95) = 190$.

Time = 40.88 (sec) , antiderivative size = 829, normalized size of antiderivative = 7.47

method	result	size
default	Expression too large to display	829

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/b^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(3*cos(f*x+e)^3*l
n(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e
)+1))*a^2*b^2+6*cos(f*x+e)^3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^3+3*cos(f*x+e)^3*ln(4*(b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^4+3*cos(
f*x+e)^3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)
/(sin(f*x+e)-1))*a^2*b^2+6*cos(f*x+e)^3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3+3*cos(f*x+e)^3*ln(-4
*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1
))*b^4+(6*cos(f*x+e)^3+6*cos(f*x+e)^2+4*cos(f*x+e)+4)*b^(7/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)+10*sin(f*x+e)*cos(f*x+e)*(1+cos
(f*x+e))*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.51

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{b}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx\right.}{\cos(fx+e)}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)
^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b + 3*b^2)*cos(f*x + e)^2
 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*c
os(f*x + e)^3), 1/16*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(-1/2*((a - b)*
cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 +
2*((5*a*b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^{\frac{3}{2}}\sec^2{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\frac{3\,(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+3\,(a+b)\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+2\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}\tan}{8\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(3*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 3*(a +
b)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 2*(b*tan(f*x + e)^2 +
 a + b)^(3/2)*tan(f*x + e) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(
f*x + e))/f
```

Giac [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2, x)
```

Reduce [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^4\,dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\,dx\right)a$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**4,x)*b + int(sqrt(sec(e + f*
x)**2*b + a)*sec(e + f*x)**2,x)*a
```

3.250 $\int (a+b\sec^2(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*b^(1/2
)*(3*a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*b*t
an(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.47

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib(-1+e^{2i(e+fx)})}{(1+e^{2i(e+fx)})^2}+\frac{2a^{3/2}fx-ia^{3/2}\mathrm{lo}\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^
((2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)
*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f
*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2]] - 3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqr
t[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)
*(1 + E^((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*f + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]*f)/(b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(
a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$\Big\downarrow$ 3042

$$\int \left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$\Big\downarrow$ 4616

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 318

$$\frac{\frac{1}{2}\int \frac{b(3a+b)\tan^2(e+fx)+(a+b)(2a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\Big\downarrow$ 398

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}b\tan(e-}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}b\,\mathrm{ta}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+f}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{2}\left(2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
+ Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(100) = 200$.

Time = 10.14 (sec) , antiderivative size = 677, normalized size of antiderivative = 5.74

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(\cos(fx+e)^3 b^{\frac{5}{2}}\ln\left(\frac{-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+3\,\mathrm{co}}{}$

input
```
int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(cos(f*x+e)^3
*b^(5/2)*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)
/(sin(f*x+e)-1))*(-a)^(1/2)+3*cos(f*x+e)^3*b^(3/2)*ln(4*(-b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)*a+co
s(f*x+e)^3*b^(5/2)*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+3*cos(f*x+e)^3*b^(3/2)*ln(-4*(-b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a
)^(1/2)*a+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*
x+e)*a)*a^2*b*cos(f*x+e)^3+(2*cos(f*x+e)+2)*sin(f*x+e)*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(100) = 200$.

Time = 0.43 (sec) , antiderivative size = 1457, normalized size of antiderivative = 12.35

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*
b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4
 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*
b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqr
t(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^
8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e
)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)...
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2{\left(e+fx\right)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2} dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}dx\right)a+\left(\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\sec\left(fx+e\right)^2dx\right)b$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a),x)*a + int(sqrt(sec(e + f*x)**2*b + a)*sec
(e + f*x)**2,x)*b
```

3.251 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output

```
1/2*a^(1/2)*(a+3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
f+b^(3/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*a*c
os(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.41 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.76

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-ia\left(-1+e^{2i(e+fx)}\right)+\frac{2e^{2i(e+fx)}\left(2\right.}{}\right.}{}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f
*x]^3*((-I)*a*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a^(3/
2)*f*x + 6*Sqrt[a]*b*f*x - I*Sqrt[a]*(a + 3*b)*Log[(a + 2*b + a*E^((2*I)*(
e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2])/E^((2*I)*e)] + I*Sqrt[a]*(a + 3*b)*Log[(a + a*E^((2*I)*(e + f*x))
 + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
 E^((2*I)*(e + f*x)))^2])/E^((2*I)*e)] - 4*b^(3/2)*Log[-1/2*(E^((3*I)*e)*(
Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
 + E^((2*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b
*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]
^2)^(3/2))/(2*Sqrt[2]*E^(I*(e + f*x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/
2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4634, 315, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\frac{1}{2}\int \frac{2b^2\tan^2(e+fx)+(a+b)(a+2b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{a\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{1}{2}\left(2b^2\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a(a+3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{a\tan(e+fx)\ldots}{2(\mathrm{ta}\ldots}}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{2}\left(2b^2\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+a(a+3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{a\tan(\ldots}{\ldots}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}\left(a(a+3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{a\tan(e+fx)\sqrt{a\ldots}}{2(\tan^2(e\ldots}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{1}{2}\left(a(a+3b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{a\tan(e+fx)\sqrt{a+b\tan^2(e\ldots}}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{1}{2}\left(\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{a\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]] + 2*b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 + (a*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*(1 + Tan[
e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(106) = 212$.

Time = 21.64 (sec) , antiderivative size = 507, normalized size of antiderivative = 4.09

method	result
default	$\left(\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)\sqrt{-a}\,b^{\frac{3}{2}}+\ln\left(-\frac{4\left(\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+\right.}{}\right.\right.$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e
)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*b^(3/2)+ln(-4*(b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*b^(3/2)+ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+3*l
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b+(
1+cos(f*x+e))*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^
(1/2)*a)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(106) = 212$.

Time = 0.45 (sec) , antiderivative size = 1403, normalized size of antiderivative = 11.31

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)
*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 -
a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*
x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)) + 4*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^
4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
 + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4))/f, 1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e)*sin(f*x + e) + 8*sqrt(-b)*b*arctan(-1/2*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*(a + 3*b)*l
og(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a
*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16
*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f...
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^2 dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^2 dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \cos\left(e+f\,x\right)^2 \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2} dx$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\cos(fx+e)^2\sec(fx+e)^2 dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\cos(fx+e)^2 dx\right)a$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**2*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**2,x)*a
```

3.252 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 125

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f}+\frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}+\frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

output
```
3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/
f+3/8*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f+1/4*cos(f*x
+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.53

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\cos(e+fx)\left(b+a\cos^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\left(3(a+b)^{3/2}\right.}{2\sqrt{a}f(a+2b+a\cos(2(e+fx}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*(b + a*Cos[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b
 - a*Sin[e + f*x]^2]*(3*(a + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a
 + b]] + Sqrt[a]*(4*a + 5*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]*Sqrt[(a + b
 - a*Sin[e + f*x]^2)/(a + b)]))/(2*Sqrt[a]*f*(a + 2*b + a*Cos[2*(e + f*x)]
)^(3/2)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4634, 292, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^4}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{\frac{3}{4}(a+b)\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)(a+b\,\mathrm{ta}}{4(\tan^2(e-}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)(a+b\tan^2(e+fx)}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*(1 + Tan[e + f*x]^2)^2
) + (3*(a + b)*(((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/(2*Sqrt[a]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/
(2*(1 + Tan[e + f*x]^2))))/4)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(109) = 218$.

Time = 5.83 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.71

method	result
default	$\frac{\left(3\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a^2+6\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e\right.}{}$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(-a)^(1/2)*(3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
4*sin(f*x+e)*a)*a^2+6*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*b^2+(2*cos(f*x+e)^3+2*cos(f*x+e)^2+3*cos(f*x+e)+3)*sin
(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+(5*cos(f*
x+e)+5)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*
b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(109) = 218$.

Time = 0.35 (sec) , antiderivative size = 563, normalized size of antiderivative = 4.50

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{-a}\log\Big(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a\right.\cdots}{\cdots}\right.$$

$$-\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4\,(\cdots}{32\,af}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7
*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + 5*
a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a*f), -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
 e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^
2*cos(f*x + e)^3 + (3*a^2 + 5*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^4(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^4 \, dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)^4\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)}^{3/2}\,dx$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)^4\sec\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)^4dx\right)a$$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**4*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**4,x)*a
```

3.253 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{(5a-b)(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af}$$

output
```
1/16*(5*a-b)*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(3/2)/f+1/16*(5*a-b)*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(
1/2)/a/f+1/24*(5*a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a
/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/a/f
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{3\sqrt{2}\sqrt{a+b}(5a^2+4ab-b^2)\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}+\sqrt{a}(23a^2+29ab+\right.}{48a^{3/2}f}$$

input
```
Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((3*Sqrt[2]*Sqrt[a + b]*(5*a^2 +
4*a*b - b^2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[(a + 2*b + a
*Cos[2*(e + f*x)])/(a + b)] + Sqrt[a]*(23*a^2 + 29*a*b + 3*b^2 + a*(9*a +
7*b)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Sin[e + f*x]))/(48*a^(3/2)*f
)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4634, 296, 292, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^6}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 296$$

$$\frac{\frac{(5a-b)\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{6a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \ 292$$

$$\frac{\frac{(5a-b)\left(\frac{3}{4}(a+b)\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}\right)}{6a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \ 292$$

$$\frac{\frac{(5a-b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}\right)}{6a}}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{(5a-b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}\right)}{6a}+}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{(5a-b)\left(\frac{3}{4}(a+b)\left(\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}\right)}{6a}+\frac{\tan(e+fx)(a+b}{6a(\tan^2(}}{f}$$

input

```
Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*a*(1 + Tan[e + f*x]^2)
^3) + ((5*a - b)*((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*(1 +
Tan[e + f*x]^2)^2) + (3*(a + b)*(((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sq
rt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[a]) + (Tan[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(2*(1 + Tan[e + f*x]^2))))/4))/(6*a))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(173) = 346$.

Time = 10.57 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.36

method	result
default	$\frac{\left(15\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a^3+27\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+\right.\right.}{}$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a/(-a)^(1/2)*(15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*a^3+27*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*sin(f*x+e)*a)*a^2*b+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2-3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3+(8*cos(f*x+e)^5+8*cos(f*x+e)^4+10*cos(f
*x+e)^3+10*cos(f*x+e)^2+15*cos(f*x+e)+15)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+2*(7*cos(f*x+e)^3+7*cos(f*x+e)^2+11*
cos(f*x+e)+11)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a*b+3*(1+cos(f*x+e))*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^2)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)
*b+b)
```

Fricas [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.35

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
 e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
- 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5
 + 2*(5*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a*b^2)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f
), -1/192*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
 + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) -
 4*(8*a^3*cos(f*x + e)^5 + 2*(5*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 +
22*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

Giac [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)^6\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^6 \sec(fx+e)^2 dx\right) b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^6 dx\right) a$$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**6*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**6,x)*a
```

3.254 $\int (a+b\sec^2(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 166

$$\int (a+b\sec^2(c+dx))^{5/2}\,dx = \frac{a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{d} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{8d} + \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d}$$

output
```
a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/d+1/8*b^(1/2
)*(15*a^2+10*a*b+3*b^2)*arctanh(b^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1
/2))/d+1/8*b*(7*a+3*b)*tan(d*x+c)*(a+b+b*tan(d*x+c)^2)^(1/2)/d+1/4*b*tan(d
*x+c)*(a+b+b*tan(d*x+c)^2)^(3/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.78 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.25

$$\int (a+b\sec^2(c+dx))^{5/2}\,dx = \frac{e^{i(c+dx)}\sqrt{4b+ae^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2}\cos^5(c+dx)\left(-\frac{ib\left(-1+e^{2i(c+dx)}\right)\left(9a\left(1+e^{2i(c+dx)}\right)^2+b(3+14\right.}{\left(1+e^{2i(c+dx)}\right)^4}\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

output
```
(E^(I*(c + d*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(c + d*x)))^2)/E^((2*I)*(c +
d*x))]*Cos[c + d*x]^5*(((-I)*b*(-1 + E^((2*I)*(c + d*x)))*(9*a*(1 + E^((2*
I)*(c + d*x)))^2 + b*(3 + 14*E^((2*I)*(c + d*x)) + 3*E^((4*I)*(c + d*x))))
)/(1 + E^((2*I)*(c + d*x)))^4 + (8*a^(5/2)*d*x - (4*I)*a^(5/2)*Log[a + 2*b
 + a*E^((2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E
^((2*I)*(c + d*x)))^2]] + (4*I)*a^(5/2)*Log[a + a*E^((2*I)*(c + d*x)) + 2*
b*E^((2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((
2*I)*(c + d*x)))^2]] - 15*a^2*Sqrt[b]*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c
+ d*x))) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x
)))^2])/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))] - 10*a*b^
(3/2)*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x))) + (4*I)*d*Sqrt[4*b*E^((
2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(b*(15*a^2 + 10*a*b + 3*
b^2)*(1 + E^((2*I)*(c + d*x))))] - 3*b^(5/2)*Log[(-4*Sqrt[b]*d*(-1 + E^((2
*I)*(c + d*x))) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(
c + d*x)))^2])/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))])/S
qrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])*(a + b*Sec[c
 + d*x]^2)^(5/2))/(Sqrt[2]*d*(a + 2*b + a*Cos[2*c + 2*d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 4616, 318, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(c+dx)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a+b\right)^{5/2}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{1}{4}\int \frac{\sqrt{b\tan^2(c+dx)+a+b}\,(b(7a+3b)\tan^2(c+dx)+(a+b)(4a+3b))}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{1}{4}b\tan(c+dx)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}{d}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{b(15a^2+10ba+3b^2)\tan^2(c+dx)+(a+b)(8a^2+7ba+3b^2)}{(\tan^2(c+dx)+1)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)+\frac{1}{2}b(7a+3b)\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}\right.}{d}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{(\tan^2(c+dx)+1)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)+b\left(15a^2+10ab+3b^2\right)\int \frac{1}{\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)\right.\right.}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{(\tan^2(c+dx)+1)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)+b\left(15a^2+10ab+3b^2\right)\int \frac{1}{1-\frac{b\tan^2(c+dx)}{b\tan^2(c+dx)+a+b}}d\frac{\tan(c+dx)}{\sqrt{b\tan^2(c+dx)}}\right.\right.}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{(\tan^2(c+dx)+1)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)+\sqrt{b}(15a^2+10ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)\right) -}{d}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{\frac{a\tan^2(c+dx)}{b\tan^2(c+dx)+a+b}+1}d\frac{\tan(c+dx)}{\sqrt{b\tan^2(c+dx)+a+b}}+\sqrt{b}(15a^2+10ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)\right)+\frac{1}{2}b(7a}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)+\sqrt{b}(15a^2+10ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)\right)+\frac{1}{2}b(7a+3b)\tan}{d}$$

input
```
Int[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

output
```
((b*Tan[c + d*x]*(a + b + b*Tan[c + d*x]^2)^(3/2))/4 + ((8*a^(5/2)*ArcTan[
(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]] + Sqrt[b]*(15*a^2 +
 10*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x
]^2]])/2 + (b*(7*a + 3*b)*Tan[c + d*x]*Sqrt[a + b + b*Tan[c + d*x]^2])/2)/
4)/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(144) = 288$.

Time = 33.53 (sec) , antiderivative size = 988, normalized size of antiderivative = 5.95

method	result	size
default	Expression too large to display	988

input
```
int((a+sec(d*x+c)^2*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/d/(-a)^(1/2)/b^2*(a+sec(d*x+c)^2*b)^(5/2)/((cos(d*x+c)^2*a+b)/(cos(d*
x+c)+1)^2)^(1/2)/(cos(d*x+c)+1)/(cos(d*x+c)^4*a^2+2*cos(d*x+c)^2*a*b+b^2)*
(3*b^(9/2)*ln(4*(b^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d
*x+c)+b^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)-a*sin(d*x+c)-a-b
)/(1+sin(d*x+c)))*cos(d*x+c)^5*(-a)^(1/2)+10*b^(7/2)*ln(4*(b^(1/2)*((cos(d
*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+b^(1/2)*((cos(d*x+c)^2*a+b
)/(cos(d*x+c)+1)^2)^(1/2)-a*sin(d*x+c)-a-b)/(1+sin(d*x+c)))*cos(d*x+c)^5*(
-a)^(1/2)*a+15*b^(5/2)*ln(4*(b^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)
^(1/2)*cos(d*x+c)+b^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)-a*si
n(d*x+c)-a-b)/(1+sin(d*x+c)))*cos(d*x+c)^5*(-a)^(1/2)*a^2+3*b^(9/2)*ln(-4*
(b^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+b^(1/2)*((
cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)-a*sin(d*x+c)+a+b)/(-1+sin(d*x+c)
))*cos(d*x+c)^5*(-a)^(1/2)+10*b^(7/2)*ln(-4*(b^(1/2)*((cos(d*x+c)^2*a+b)/(
cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+b^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+
1)^2)^(1/2)-a*sin(d*x+c)+a+b)/(-1+sin(d*x+c)))*cos(d*x+c)^5*(-a)^(1/2)*a+1
5*b^(5/2)*ln(-4*(b^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d
*x+c)+b^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)-a*sin(d*x+c)+a+b
)/(-1+sin(d*x+c)))*cos(d*x+c)^5*(-a)^(1/2)*a^2+16*ln(4*(-a)^(1/2)*((cos(d*
x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+4*(-a)^(1/2)*((cos(d*x+c)^2
*a+b)/(cos(d*x+c)+1)^2)^(1/2)-4*a*sin(d*x+c))*cos(d*x+c)^5*a^3*b^2+18*s...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(144) = 288$.

Time = 1.05 (sec) , antiderivative size = 1611, normalized size of antiderivative = 9.70

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/32*(4*sqrt(-a)*a^2*cos(d*x + c)^3*log(128*a^4*cos(d*x + c)^8 - 256*(a^4
 - a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
^2*b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*
b)*cos(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 -
 7*a^2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 +
b)/cos(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10*a*b + 3*b^2)*sqrt(b)*cos(d
*x + c)^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 8*(a*b - b^2)*cos(d*x
+ c)^2 + 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(b)*sqrt((a*cos
(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b^2)/cos(d*x + c)^4) + 4
*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(
d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3), 1/16*(2*sqrt(-a)*a^2*cos(d*x
 + c)^3*log(128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 + 32
*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*b
^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^
2 - 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3
- 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x +
c)) + (15*a^2 + 10*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(d*x + c)
^3 + 2*b*cos(d*x + c))*sqrt(-b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c...
```

Sympy [F]

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(a+b\sec^{2}\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c)**2)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(b\sec\left(dx+c\right)^{2}+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(b\sec\left(dx+c\right)^{2}+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)^2}\right)^{5/2}\,dx$$

input
```
int((a + b/cos(c + d*x)^2)^(5/2),x)
```

output
```
int((a + b/cos(c + d*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\left(\int \sqrt{\sec\left(dx+c\right)^2 b+a}dx\right)a^2 +\left(\int \sqrt{\sec\left(dx+c\right)^2 b+a}\,\sec\left(dx+c\right)^4 dx\right)b^2 +2\left(\int \sqrt{\sec\left(dx+c\right)^2 b+a}\,\sec\left(dx+c\right)^2 dx\right)ab$$

input
```
int((a+b*sec(d*x+c)^2)^(5/2),x)
```

output
```
int(sqrt(sec(c + d*x)**2*b + a),x)*a**2 + int(sqrt(sec(c + d*x)**2*b + a)*
sec(c + d*x)**4,x)*b**2 + 2*int(sqrt(sec(c + d*x)**2*b + a)*sec(c + d*x)**
2,x)*a*b
```

3.255 $\int (1+\sec^2(x))^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int (1+\sec^2(x))^{3/2}\,dx = 2\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right) + \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}$$

output
```
2*arcsinh(1/2*tan(x)*2^(1/2))+arctan(tan(x)/(2+tan(x)^2)^(1/2))+1/2*tan(x)
*(2+tan(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.60

$$\int (1+\sec^2(x))^{3/2}\,dx = \frac{(1+\cos^2(x))\sec(x)\sqrt{1+\sec^2(x)}\left(4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{3+\cos(2x)}}\right)\cos^2(x) - 2i\sqrt{2}\cos^2(x)\log\right.}{(3+\cos(2x))^{3/2}}$$

input
```
Integrate[(1 + Sec[x]^2)^(3/2),x]
```

output
```
((1 + Cos[x]^2)*Sec[x]*Sqrt[1 + Sec[x]^2]*(4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sin[
x])/Sqrt[3 + Cos[2*x]]]*Cos[x]^2 - (2*I)*Sqrt[2]*Cos[x]^2*Log[Sqrt[3 + Cos
[2*x]] + I*Sqrt[2]*Sin[x]] + Sqrt[3 + Cos[2*x]]*Sin[x]))/(3 + Cos[2*x])^(3
/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4616, 318, 27, 398, 222, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sec^2(x)+1\right)^{3/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sec(x)^2+1\right)^{3/2}dx$$

$$\Big\downarrow \text{4616}$$

$$\int \frac{\left(\tan^2(x)+2\right)^{3/2}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{318}$$

$$\frac{1}{2}\int \frac{2\left(2\tan^2(x)+3\right)}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+2}\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{2\tan^2(x)+3}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+2}\tan(x)$$

$$\Big\downarrow \text{398}$$

$$2\int \frac{1}{\sqrt{\tan^2(x)+2}}d\tan(x)+\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+2}\tan(x)$$

$$\Big\downarrow \text{222}$$

$$\int \frac{1}{(\tan^2(x)+1)\sqrt{\tan^2(x)+2}}d\tan(x) + 2\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{\frac{\tan^2(x)}{\tan^2(x)+2}+1}d\frac{\tan(x)}{\sqrt{\tan^2(x)+2}} + 2\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}$$

$$\Big\downarrow \text{216}$$

$$2\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right) + \frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}$$

input
```
Int[(1 + Sec[x]^2)^(3/2),x]
```

output
```
2*ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]] + (Tan[x]*Sq
rt[2 + Tan[x]^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(35) = 70$.

Time = 9.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.31

method	result
default	$-\frac{\sqrt{2}\,\sqrt{2+2\sec(x)^2}\left(2\cos(x)\arctan\left(\frac{2\cot(x)-2\csc(x)}{\sqrt{2\cot(x)^4-8\cot(x)^3\csc(x)+12\cot(x)^2\csc(x)^2-8\cot(x)\csc(x)^3+2\csc(x)^4+2}}\right)+2\cos(x)\operatorname{arctanh}\right.}{4(\cos(x)+1)\sqrt{\frac{\cos(x)}{(\cos(x)}}}$

input
```
int((1+sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)*(2+2*sec(x)^2)^(1/2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(
1/2)*(2*cos(x)*arctan(2*(cot(x)-csc(x))/(2*cot(x)^4-8*cot(x)^3*csc(x)+12*c
ot(x)^2*csc(x)^2-8*cot(x)*csc(x)^3+2*csc(x)^4+2)^(1/2))+2*cos(x)*arctanh((
sin(x)-2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))+2*cos(x)*arctanh((
sin(x)+2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))+((cos(x)^2+1)/(cos
(x)+1)^2)^(1/2)*(-sin(x)-tan(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(35) = 70$.

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int (1+\sec^2(x))^{3/2} \, dx = \frac{\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)\cos(x) - \arctan\left(\frac{\sin(x)}{\cos(x)}\right)\cos(x) + 2\cos(x)\,\mathrm{lo}}{}$$

input
```
integrate((1+sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x)
)/(cos(x)^4 + cos(x)^2 - 1))*cos(x) - arctan(sin(x)/cos(x))*cos(x) + 2*cos
(x)*log(cos(x)^2 + cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)
^2 + 1)/cos(x)^2) + 1) - 2*cos(x)*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2
 - cos(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) + 1) + sqrt((cos(x)^2 + 1)
/cos(x)^2)*sin(x))/cos(x)
```

Sympy [F]

$$\int (1+\sec^2(x))^{3/2} \, dx = \int \left(\sec^2{(x)} + 1\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((1+sec(x)**2)**(3/2),x)
```

output
```
Integral((sec(x)**2 + 1)**(3/2), x)
```

Maxima [F]

$$\int \left(1+\sec^2(x)\right)^{3/2} dx = \int \left(\sec(x)^2+1\right)^{\frac{3}{2}} dx$$

input
```
integrate((1+sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((sec(x)^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \left(1+\sec^2(x)\right)^{3/2} dx = \int \left(\sec(x)^2+1\right)^{\frac{3}{2}} dx$$

input
```
integrate((1+sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((sec(x)^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+\sec^2(x)\right)^{3/2} dx = \int \left(\frac{1}{\cos(x)^2}+1\right)^{3/2} dx$$

input
```
int((1/cos(x)^2 + 1)^(3/2),x)
```

output
```
int((1/cos(x)^2 + 1)^(3/2), x)
```

Reduce [F]

$$\int \left(1+\sec^2(x)\right)^{3/2} \, dx = \int \sqrt{\sec(x)^2+1}dx + \int \sqrt{\sec(x)^2+1}\,\sec(x)^2\,dx$$

input
```
int((1+sec(x)^2)^(3/2),x)
```

output
```
int(sqrt(sec(x)**2 + 1),x) + int(sqrt(sec(x)**2 + 1)*sec(x)**2,x)
```

3.256 $\int \sqrt{1+\sec^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \sqrt{1+\sec^2(x)}\,dx = \operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right)+\arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$

output
```
arcsinh(1/2*tan(x)*2^(1/2))+arctan(tan(x)/(2+tan(x)^2)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(24) = 48.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \sqrt{1+\sec^2(x)}\,dx = \frac{\sqrt{2}\left(\arcsin\left(\frac{\sin(x)}{\sqrt{2}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{3+\cos(2x)}}\right)\right)\cos(x)\sqrt{1+\sec^2(x)}}{\sqrt{3+\cos(2x)}}$$

input
```
Integrate[Sqrt[1 + Sec[x]^2],x]
```

output
```
(Sqrt[2]*(ArcSin[Sin[x]/Sqrt[2]] + ArcTanh[(Sqrt[2]*Sin[x])/Sqrt[3 + Cos[2
*x]]])*Cos[x]*Sqrt[1 + Sec[x]^2])/Sqrt[3 + Cos[2*x]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4616, 301, 222, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec^2(x)+1} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\sec(x)^2+1}dx$$

$$\Big\downarrow \; 4616$$

$$\int \frac{\sqrt{\tan^2(x)+2}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \; 301$$

$$\int \frac{1}{\sqrt{\tan^2(x)+2}}d\tan(x)+\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)$$

$$\Big\downarrow \; 222$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)+\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

$$\Big\downarrow \; 291$$

$$\int \frac{1}{\frac{\tan^2(x)}{\tan^2(x)+2}+1}d\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}+\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

$$\Big\downarrow \; 216$$

$$\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right)+\arctan\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

input
```
Int[Sqrt[1 + Sec[x]^2],x]
```

output
```
ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(21) = 42.

Time = 7.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 5.62

method	result
default	$-\frac{\sqrt{2}\,\sqrt{2+2\sec(x)^2}\,\left((1-\cos(x))^2\csc(x)^2-1\right)\left(2\arctan\left(\frac{2\csc(x)-2\cot(x)}{\sqrt{2(1-\cos(x))^4\csc(x)^4+2}}\right)-\operatorname{arctanh}\left(\frac{\sin(x)-2}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)-\operatorname{arctan}\right.}{8\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}$

input
```
int((1+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*2^(1/2)*(2+2*sec(x)^2)^(1/2)*((1-cos(x))^2*csc(x)^2-1)*(2*arctan(2/(2
*(1-cos(x))^4*csc(x)^4+2)^(1/2)*(csc(x)-cot(x)))-arctanh((sin(x)-2)/(cos(x
)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))-arctanh((sin(x)+2)/(cos(x)+1)/((co
s(x)^2+1)/(cos(x)+1)^2)^(1/2)))/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.46

$$\int \sqrt{1+\sec^2(x)}\,dx = \frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right) - \frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right) + \frac{1}{2}\log\left(\cos(x)^2+\cos(x)\sin(x) + \left(\cos(x)^2+\cos(x)\sin(x)\right)\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right) - \frac{1}{2}\log\left(\cos(x)^2-\cos(x)\sin(x) + \left(\cos(x)^2-\cos(x)\sin(x)\right)\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right)$$

input
```
integrate((1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))
/(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x)) + 1/2*log(cos(x)^2
 + cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2
) + 1) - 1/2*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - cos(x)*sin(x))*sqr
t((cos(x)^2 + 1)/cos(x)^2) + 1)
```

Sympy [F]

$$\int \sqrt{1+\sec^2(x)}\, dx = \int \sqrt{\sec^2(x)+1}\, dx$$

input
```
integrate((1+sec(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(sec(x)**2 + 1), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 1391, normalized size of antiderivative = 57.96

$$\int \sqrt{1+\sec^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate((1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arctan2(2*(2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 +
 sin(4*x)^2 + 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4
)*sin(1/2*arctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)), 2*(2
*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*
sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2
(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 8) + 3/2*arctan2(2*(
2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12
*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan
2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*sin(2*x), 2*(2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin
(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(si
n(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*cos(2*x) + 6) - arcta
n2((2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2
+ 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*ar
ctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + sin(2*x), (2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin
(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(si
n(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + cos(2*x) + 3) - 1/2*log
(-(2*sqrt(2)*(abs(2*e^(2*I*x) + 2)^4 + 4*cos(2*x)^4 + 4*sin(2*x)^4 - 4*(co
s(2*x)^2 - sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^2 - 16*cos...
```

Giac [F]

$$\int \sqrt{1+\sec^2(x)}\,dx=\int \sqrt{\sec(x)^2+1}\,dx$$

input
```
integrate((1+sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(x)^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sec^2(x)}\,dx = \int \sqrt{\frac{1}{\cos(x)^2}+1}\,dx$$

input
```
int((1/cos(x)^2 + 1)^(1/2),x)
```

output
```
int((1/cos(x)^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1+\sec^2(x)}\,dx = \int \sqrt{\sec(x)^2+1}dx$$

input
```
int((1+sec(x)^2)^(1/2),x)
```

output
```
int(sqrt(sec(x)**2 + 1),x)
```

3.257 $\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 334

$$\begin{aligned}\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{2(a-b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\
&\quad-\frac{(a-2b)\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\
&\quad-\frac{2(a-b)\sec(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3b^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\
&\quad+\frac{\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{3bf\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\end{aligned}$$

output
```
2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/f
/(cos(f*x+e)^2)^(1/2)/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+
b-a*sin(f*x+e)^2))^(1/2)-1/3*(a-2*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))
*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2
*(a+b-a*sin(f*x+e)^2))^(1/2)-2/3*(a-b)*sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan
(f*x+e)/b^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*sec(f*x+e)^3*(
a+b-a*sin(f*x+e)^2)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)
```

Mathematica [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3042, 4636, 2057, 2058, 316, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(e+fx)^5}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \ 4636$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int -\frac{a\sin^2(e+fx)+a-2b}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\int \frac{a\sin^2(e+fx)+a-2b}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\frac{\int -\frac{a\left(-2(a-b)\sin^2(e+fx)+2a-b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}+\frac{2(a-b)\sin(e+fx)\sqrt{\ldots}}{b\sqrt{1-\sin\ldots}}}{3b}\ldots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\int\frac{a\left(-2(a-b)\sin^2(e+fx)+2a-\cdots\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx\cdots}}}{b}}{3b}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\int\frac{-2(a-b)\sin^2(e+fx)+2a\cdots}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+\cdots}}}{b}}{3b}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+\cdots}}{\sqrt{1-\sin^2(e+fx)}}}{a}\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow 323$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+\cdots}}{\sqrt{1-\sin^2(e+fx)}}}{a}\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+\cdots}}{\sqrt{1-\sin^2(e+fx)}}}{a}\cdots\right.}{\cdots}}{\cdots}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\int\cdots}{a\sqrt{1-\frac{a\,\mathrm{s}\cdots}{\cdots}}}\cdots\right.}{\cdots}}{\cdots}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 327

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}E\cdots}{a\sqrt{1-\frac{a\sin^2\cdots}{a\cdots}}}\cdots\right.}{\cdots}}{\cdots}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]
^2])/(3*b*(1 - Sin[e + f*x]^2)^(3/2)) - ((2*(a - b)*Sin[e + f*x]*Sqrt[a +
b - a*Sin[e + f*x]^2])/(b*Sqrt[1 - Sin[e + f*x]^2]) - (a*((2*(a - b)*Ellip
ticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*S
qrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 2*b)*b*EllipticF[ArcSin[Sin[e
 + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b -
 a*Sin[e + f*x]^2])))/b)/(3*b)))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b -
 a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] && !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.51 (sec) , antiderivative size = 2359, normalized size of antiderivative = 7.06

method	result	size
default	Expression too large to display	2359

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2/(2*I*a^(1/2)*b^(1/2)-a+
b)/(1+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(1/2)*((1/(a+b)*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e)))^(1/2)*a^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*
x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*(2*cos(f*x+e)+4+2*sec(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a
+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos
(f*x+e)))^(1/2)*a^2*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(c
sc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*(2*cos(f*x+e)+4+2*sec(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^
(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(
-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(
1+cos(f*x+e)))^(1/2)*a*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*(-2*cos(f*x+e)-4-2*sec(f*x+e))+(1/(a+b)*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(
1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e)))^(1/2)*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.35

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output

```
1/3*((2*(-I*a^2 + I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (-
I*a^2 - I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2
)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a
 - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2
 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(I*a^2 - I*a*b)*sqrt(a)*sqrt((a
*b + b^2)/a^2)*cos(f*x + e)^2 - (I*a^2 + I*a*b - 2*I*b^2)*sqrt(a)*cos(f*x
+ e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sq
rt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e)
)), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (
2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 + (2*I*a^2 + 3*I*a*b -
2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b
)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(
f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((
a*b + b^2)/a^2))/a^2) - (-2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)
^2 + (-2*I*a^2 - 3*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt
((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^
2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b
^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (2*(a^2 - a*b)*cos(f*x
+ e)^2 - a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a
*b^2*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec^5{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)^5\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^5}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**5)/(sec(e + f*x)**2*b + a),
x)
```

3.258 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a}\sqrt{a+b}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\sec(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output
```
-a^(1/2)*(a+b)^(1/2)*EllipticE(a^(1/2)*sin(f*x+e)/(a+b)^(1/2),((a+b)/a)^(1
/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan(f*x+e
)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4636, 2057, 2058, 316, 25, 27, 331, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^3}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int -\frac{a\sqrt{1-\sin^2(e+fx)}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\int\frac{a\sqrt{1-\sin^2(e+fx)}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\int\frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \ 331$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{b\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\sqrt{a}\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{b\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]
^2])/(b*Sqrt[1 - Sin[e + f*x]^2]) - (Sqrt[a]*Sqrt[a + b]*EllipticE[ArcSin[
(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]], (a + b)/a]*Sqrt[1 - (a*Sin[e + f*x]^2
)/(a + b)])/(b*Sqrt[a + b - a*Sin[e + f*x]^2])))/(f*Sqrt[1 - Sin[e + f*x]^
2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.36 (sec) , antiderivative size = 1518, normalized size of antiderivative = 8.83

method	result	size
default	Expression too large to display	1518

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b/(2*I*a^(1/2)*b^(1/2)-a+b)/(1
+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(1/2)*((1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x
+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
)^(1/2)*a^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-
cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*(cos(f*x+e)+2+sec(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+
e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))
^(1/2)*a*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-c
ot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*(2*cos(f*x+e)+4+2*sec(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
)))^(1/2)*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e
)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*(cos(f*x+e)+2+sec(f*x+e))+I*a^(3/2)*b^(1/2)*(1/(a+b)*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(
1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e)))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.45

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$

$$-\frac{2\sqrt{a}(-i\,a-2i\,b)\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}\,F\left(\arcsin\left(\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}(\cos(fx+e)+i\,\sin(fx+e))\right) \mid \frac{a^2+8\,a\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(a)*(-I*a - 2*I*b)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*
x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*
b + b^2)/a^2))/a^2) + 2*sqrt(a)*(I*a + 2*I*b)*sqrt((2*a*sqrt((a*b + b^2)/a
^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -
2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 +
2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) +
 sqrt(a)*(I*a + 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elli
ptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e)
+ I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2
)/a^2))/a^2) + (2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqrt(a)*(-I*a - 2*I*b)
)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*
a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a
^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*a*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b*f)
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^3}{\sec(fx+e)^2 b+a}dx$$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**3)/(sec(e + f*x)**2*b + a),
x)
```

3.259 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output
```
EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/f
/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\operatorname{EllipticF}\left(e+fx,\frac{a}{a+b}\right)\sec(e+fx)}{\sqrt{2}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)
]*Sec[e + f*x])/(Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4636, 2057, 2058, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - S
in[e + f*x]^2)])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.92 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.80

method	result
default	$\frac{2\sqrt{\frac{i\sqrt{a}\,\sqrt{b}\,\cos(fx+e)-i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(a+b)(1+\cos(fx+e))}}\,\sqrt{\frac{-i\sqrt{a}\,\sqrt{b}\,\cos(fx+e)+i\sqrt{a}\,\sqrt{b}+\cos(fx+e)a+b}{(a+b)(1+\cos(fx+e))}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}\,(\csc(fx+e)-\mathrm{co}\right.}{f\sqrt{\frac{2i\sqrt{a}\,\sqrt{b}+a-b}{a+b}}\,\sqrt{a+b\sec(fx+e)^2}}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*((-I*a^(1
/2)*b^(1/2)*cos(f*x+e)+I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(a+b)/(1+cos(f*x+
e)))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-c
ot(f*x+e)),(1/(a+b)^2*(-4*I*a^(3/2)*b^(1/2)+4*I*a^(1/2)*b^(3/2)+a^2-6*a*b+
b^2))^(1/2))/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.70

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$

$$-\frac{\left(2i\,a^{\frac{3}{2}}\sqrt{\frac{ab+b^2}{a^2}}+\sqrt{a}(i\,a+2i\,b)\right)\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}F\left(\arcsin\left(\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}(\cos(fx+e)+i\,\sin(fx\right.\right.}{}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-((2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqrt(a)*(I*a + 2*I*b))*sqrt((2*a*sq
rt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b +
b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8
*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt((a
*b + b^2)/a^2) + sqrt(a)*(-I*a - 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) -
 a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b
)*sqrt((a*b + b^2)/a^2))/a^2))/(a^2*f)
```

Sympy [F]

$$\int \frac{\sec(e + fx)}{\sqrt{a + b\sec^2(e + fx)}}\,dx = \int \frac{\sec(e + fx)}{\sqrt{a + b\sec^2(e + fx)}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sec(e + fx)}{\sqrt{a + b\sec^2(e + fx)}}\,dx = \int \frac{\sec(fx + e)}{\sqrt{b\sec(fx + e)^2 + a}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+fx)\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)}{\sec(fx+e)^2 b+a}dx$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x))/(sec(e + f*x)**2*b + a),x)
```

3.260 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+b}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{\sqrt{a}f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

output
```
(a+b)^(1/2)*EllipticE(a^(1/2)*sin(f*x+e)/(a+b)^(1/2),((a+b)/a)^(1/2))*((a+
b-a*sin(f*x+e)^2)/(a+b))^(1/2)/a^(1/2)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^
2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.27 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.56

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{\csc(2(e+fx))\sin(e+fx)\left(a^2\sqrt{-\frac{1}{b}}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\operatorname{EllipticF}\left(e+fx,\frac{a}{a+b}\right)-2i\sqrt{-\frac{a\cos^2(e+fx)}{b}}\sqrt{a+\right.}{}$$

input
```
Integrate[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Csc[2*(e + f*x)]*Sin[e + f*x]*(a^2*Sqrt[-b^(-1)]*Sqrt[(a + 2*b + a*Cos[2*
(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)] - (2*I)*Sqrt[-((a*Cos[e
 + f*x]^2)/b)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Csc[2*(e + f*x)]*(2*(a +
 b)*EllipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
/Sqrt[2]], b/(a + b)] - a*EllipticF[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b
+ a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(a + b)])*Sqrt[(a*Sin[e + f*x]^2)/(a +
b)]))/(Sqrt[2]*a^2*Sqrt[-b^(-1)]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 331, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{1}{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1}{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{\sqrt{a}f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]], (a + b)
/a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(Sqrt[a]*f*Sqrt[1 - Sin[e + f*x]
^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.54 (sec) , antiderivative size = 1526, normalized size of antiderivative = 14.26

method	result	size
default	Expression too large to display	1526

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a/(2*I*a^(1/2)*b^(1/2)-a+b)/(
1+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(1/2)*((-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
))^(1/2)*a^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)
-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*(-cos(f*x+e)-2-sec(f*x+e))+(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f
*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*
a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)
))^(1/2)*a*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)
-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*(-2*cos(f*x+e)-4-2*sec(f*x+e))+(-1/(a+b)*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*
x+e)))^(1/2)*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*
x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*(-cos(f*x+e)-2-sec(f*x+e))+I*a^(3/2)*b^(1/2)*(-1/(a+b)*(I
*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e
)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x
+e)*a+b)/(1+cos(f*x+e)))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+...
```

Fricas [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)}{\sec(fx+e)^2 b+a}dx$$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x))/(sec(e + f*x)**2*b + a),x)
```

3.261 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 259

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{2(a-b)E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\left(a+b-a\sin^2(e+fx)\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{(a-2b)b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

output
```
1/3*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)+2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^
2)/a^2/f/(cos(f*x+e)^2)^(1/2)/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-1/3*(a-2*b)*b*EllipticF(sin(f*x+e),(a/(a+
b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(
sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4636, 2057, 2058, 318, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^3\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\left(1-\sin^2(e+fx)\right)^{3/2}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 318

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int-\frac{-2(a-b)\sin^2(e+fx)+2a-b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int\frac{-2(a-b)\sin^2(e+fx)+2a-b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3a}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-2b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}}{3a}+\frac{\sqrt{1-\ldots}}{\ldots}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+\ldots}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\ldots\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}+\frac{\sqrt{1-}}{}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 327

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+}\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sq
rt[a + b - a*Sin[e + f*x]^2])/(3*a) + ((2*(a - b)*EllipticE[ArcSin[Sin[e +
 f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e +
f*x]^2)/(a + b)]) - ((a - 2*b)*b*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])
)/(3*a)))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 -
 Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 2331, normalized size of antiderivative = 9.00

method	result	size
default	Expression too large to display	2331

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+
b)/(1+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(1/2)*((1/(a+b)*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e)))^(1/2)*a^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*
x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*(-2*cos(f*x+e)-4-2*sec(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(
a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e)))^(1/2)*a^2*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(
cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*(-2*cos(f*x+e)-4-2*sec(f*x+e))+(1/(a+b)*(I*a^(1/2)*
b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)
*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)
/(1+cos(f*x+e)))^(1/2)*a*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*(2*cos(f*x+e)+4+2*sec(f*x+e))+(1/(a+b)*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^
(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)
*a-b)/(1+cos(f*x+e)))^(1/2)*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a...
```

Fricas [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)^3}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**3)/(sec(e + f*x)**2*b + a),
x)
```

3.262 $\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 349

$$\begin{aligned}
&\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx \\
&= \frac{4(a-b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad + \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5af\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad + \frac{(8a^2-7ab+8b^2)\,E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad - \frac{b(4a^2-3ab+8b^2)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}
\end{aligned}$$

output
```
4/15*(a-b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/f/(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec
(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/15*(8*a^2-7*a*b+8*b^2)*EllipticE(s
in(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/f/(cos(f*x+e)^2)^(1/2)
/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1
/2)-1/15*b*(4*a^2-3*a*b+8*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b
-a*sin(f*x+e)^2)/(a+b))^(1/2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+
b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4636, 2057, 2058, 318, 25, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(e+fx)^5\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 4636}$$

$$\frac{\int \frac{(1-\sin^2(e+fx))^2}{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{(1-\sin^2(e+fx))^2}{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{(1-\sin^2(e+fx))^{5/2}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}{5a}-\frac{\int -\frac{\sqrt{1-\sin^2(e+fx)}\left(-4(a-b)\sin^2(e+fx)+4a-b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin}{5a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\sqrt{1-\sin^2(e+fx)}\left(-4(a-b)\sin^2(e+fx)+4a-b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{5a}+\frac{\sin(e+fx)(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx}}{5a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{4(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int -\frac{8a^2-3ba+4b^2-\left(8a^2-7ba+8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}}{5a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int \frac{8a^2-3ba+4b^2-\left(8a^2-7ba+8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{4(a-b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3a}}{5a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(8a^2-7ab+8b^2\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b\left(4a^2-3ab+8b^2\right)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin}{a}}{3a}}{5a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2}{1-\sin}}}$$

$\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(8a^2-7ab+8b^2\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin}{}}}}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}}{5a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{}}$$

$\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(8a^2-7ab+8b^2\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}}{5a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2}{1-\sin}}}$$

$\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(8a^2-7ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{Elliptic}}{a\sqrt{-a\sin^2(e+fx)+}}}{3a}}{5a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{}}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(8a^2-7ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\text{arc}\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}}{5a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{}}$$

input
```
Int[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(3/2)*
Sqrt[a + b - a*Sin[e + f*x]^2])/(5*a) + ((4*(a - b)*Sin[e + f*x]*Sqrt[1 -
Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a) + (((8*a^2 - 7*a*b +
 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e +
f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a^2 - 3*a*b + 8*
b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2
)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a))/(5*a)))/(f*Sqrt[1 -
 Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.44 (sec) , antiderivative size = 3198, normalized size of antiderivative = 9.16

method	result	size
default	Expression too large to display	3198

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/a^3/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a
+b)/(1+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(1/2)*((-1/(a+b)*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b
)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e)))^(1/2)*a^4*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f
*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*(-8*cos(f*x+e)-16-8*sec(f*x+e))+(-1/(a+b)*(I*a^(1/2)*b^(
1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1
/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+
cos(f*x+e)))^(1/2)*a^3*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*(-9*cos(f*x+e)-18-9*sec(f*x+e))+(-1/(a+b)*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(
1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e)))^(1/2)*a^2*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(-2*cos(f*x+e)-4-2*sec(f*x+e))+(-1/(a+b)
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(
f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*a*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)...
```

Fricas [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(e+f\,x\right)^5}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)^5}{\sec\left(fx+e\right)^2b+a}\,dx$$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**5)/(sec(e + f*x)**2*b + a),
x)
```

3.263 $\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(3a^2-2ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{(3a-5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}$$

output

```
1/8*(3*a^2-2*a*b+3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/b^(5/2)/f-1/8*(3*a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b^2/f+1
/4*tan(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.45

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\sqrt{a+2b+a\cos(2e+2fx)}\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-3a\left(1+e^{2i(e+fx)}\right)^2+\right.}{\left(1+e^{2i(e+fx)}\right)}\right.}{8\sqrt{2}b^{5/2}f\sqrt{a+}}$$

input
```
Integrate[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e
 + f*x)))*(-3*a*(1 + E^((2*I)*(e + f*x)))^2 + b*(3 + 14*E^((2*I)*(e + f*x)
) + 3*E^((4*I)*(e + f*x)))))/(1 + E^((2*I)*(e + f*x)))^4 - ((3*a^2 - 2*a*b
 + 3*b^2)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^
((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f
*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sec
[e + f*x])/(8*Sqrt[2]*b^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4634, 318, 25, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(e+fx)^6}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$\big\downarrow$ 4634

$$\frac{\int \frac{\left(\tan^2(e+fx)+1\right)^2}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$\big\downarrow$ 318

$$\frac{\frac{\int -\frac{3(a-b)\tan^2(e+fx)+a-3b}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}+\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b}}{f}$$

$\big\downarrow$ 25

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\int \frac{3(a-b)\tan^2(e+fx)+a-3b}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}}{f}$$

$\big\downarrow$ 299

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^2-2ab+3b^2\right)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{4b}}{f}$$

$\big\downarrow$ 224

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^2-2ab+3b^2\right)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{4b}}{f}$$

$\big\downarrow$ 219

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^2-2ab+3b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}}}{4b}}{f}$$

input

```
Int[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*b)
- (-1/2*((3*a^2 - 2*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/b^(3/2) + (3*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Ta
n[e + f*x]^2])/(2*b))/(4*b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(117) = 234$.

Time = 29.72 (sec) , antiderivative size = 879, normalized size of antiderivative = 6.61

method	result	size
default	Expression too large to display	879

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/b^(9/2)/(a+b*sec(f*x+e)^2)^(1/2)*(3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a^2*b^2*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*
x+e)*a-a-b)/(sin(f*x+e)+1))*(sec(f*x+e)+1)+2*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a*b^3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*
x+e)*a-a-b)/(sin(f*x+e)+1))*(-1-sec(f*x+e))+3*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^4*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x
+e)*a-a-b)/(sin(f*x+e)+1))*(sec(f*x+e)+1)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a^2*b^2*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(
f*x+e)*a+a+b)/(sin(f*x+e)-1))*(sec(f*x+e)+1)+2*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a*b^3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin
(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-1-sec(f*x+e))+3*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*b^4*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin
(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(sec(f*x+e)+1)+2*(2+3*cos(f*x+e)^2)*b^(9/2)
*tan(f*x+e)*sec(f*x+e)^4+2*b^(7/2)*a*tan(f*x+e)*(3-sec(f*x+e)^2)-6*b^(5...
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.98

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{(3a^2-2ab+3b^2)\sqrt{b}\cos(fx+e)^3\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(f\right.}{\cos(fx+e)^4}\right.}{32b^3f\cos(}\right.$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b +
b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x +
e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(3*(a*b - b^2)*cos(f*x + e)^2
 - 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f
*cos(f*x + e)^3), 1/16*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^
3 - 2*(3*(a*b - b^2)*cos(f*x + e)^2 - 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.20

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$=\frac{\frac{2\sqrt{b\tan(fx+e)^2+a+b}\tan(fx+e)^3}{b}+\frac{3\,(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{3\,(a+b)\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{8\,a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-3\sqrt{b}}{8\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)^3/b + 3*(a + b)*a*arcsi
nh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)*arcsinh(b*tan(f*x +
 e)/sqrt((a + b)*b))/b^(3/2) - 8*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))
/b^(3/2) - 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)/b^2 + 8*s
qrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)/b)/f
```

Giac [F]

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)^6\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^6}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**6)/(sec(e + f*x)**2*b + a),
x)
```

3.264 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}$$

output
```
-1/2*(a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/
f+1/2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.19 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.02

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{\sqrt{a+2b+a\cos(2e+2fx)}\sec^4(e+fx)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\tan(e+fx)\left(\frac{16b^2(b+a\cos^2(e+fx))\,\text{Hypergeometric2F}}{}\right.}{30\sqrt{2}f\sqrt{a+b\sec^2(e-}}$$

input
```
Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]^4*(1 - (a*Sin[e + f*x]^2)
/(a + b))*Tan[e + f*x]*((16*b^2*(b + a*Cos[e + f*x]^2)*Hypergeometric2F1[2
, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4*Sqrt[-((b*Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])/(a + b)^3 +
 (15*(3*b + a*(3 - 2*Sin[e + f*x]^2))*(ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a
 + b))]] - Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x
]^2)/(a + b)^2)]))/(a + b)))/(30*Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt
[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Tan[
e + f*x]^2)/(a + b)))^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4634, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^4}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}}}{f}$$

input
```
Int[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/2*((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^
2]])/b^(3/2) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(69) = 138$.

Time = 23.80 (sec) , antiderivative size = 572, normalized size of antiderivative = 7.06

method	result
default	$-\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, ab\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)(\sec(fx+e)+1)}{4}-\frac{\sqrt{\frac{b+a\cos(fx}{(1+\cos(fx+}}}{}$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*ln(4*(b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(sec(f*x+
e)+1)-1/4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*ln(4*(b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-1-sec(f
*x+e))-1/4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*ln(-4*(b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(sec(f*
x+e)+1)-1/4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*ln(-4*(b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-1-se
c(f*x+e))+1/2*b^(3/2)*a*tan(f*x+e)+1/2*b^(5/2)*tan(f*x+e)*sec(f*x+e)^2)/b^
(5/2)/(a+b*sec(f*x+e)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(69) = 138$.

Time = 0.16 (sec) , antiderivative size = 324, normalized size of antiderivative = 4.00

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{(a-b)\sqrt{b}\cos(fx+e)\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos\ldots}{\ldots}}\ldots}{\cos(fx+e)^4}\ldots\right.}{8\,b^2f\cos(fx+e)}\ldots\right.$$

$$-\frac{(a-b)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)-2\,b\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\ldots}{4\,b^2f\cos(fx+e)}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^
4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
 + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(b^2*f*cos(f*x + e)), -1/4*((a - b)*sqrt(-b)*arctan(-1/2*((a
- b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)
 - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*co
s(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sec^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{\sqrt{b\tan(fx+e)^2+a+b}\tan(fx+e)}{b}}{2\,f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - arcsinh(b*tan(f*
x + e)/sqrt((a + b)*b))/sqrt(b) - sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x +
 e)/b)/f
```

Giac [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^4\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2\,b+a}\,\sec(fx+e)^4}{\sec(fx+e)^2\,b+a}dx$$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**4)/(sec(e + f*x)**2*b + a),
x)
```

3.265 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 39

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

output
```
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(1/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{b}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2
*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[b]*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4634, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^2}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

input
```
Int[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]*f)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(33) = 66$.

Time = 9.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.18

method	result
default	$\frac{\left(\ln\left(\frac{-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)+\ln\left(-\frac{4\left(-\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+\sin\right.}{\sin(fx-}\right.}{2f\sqrt{b}\sqrt{a+b\sec(fx+e)^2}}$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/b^(1/2)*(ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-a-b)/(sin(f*x+e)-1))+ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)+a+b)/(sin(f*x+e)+1)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(33) = 66$.

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 5.51

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \left[\frac{\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\,\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8\,b^2}{\cos(fx+e)^4}\right)}{4\,\sqrt{b}f},\right.$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^
2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)/(sqrt(b)
*f), 1/2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
+ b^2)*sin(f*x + e)))/(b*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt(b)*f)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^2}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2)/(sec(e + f*x)**2*b + a),
x)
```

3.266 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4616, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a+b\sec(e+fx)^2}}dx$$
$$\Big\downarrow \text{4616}$$
$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{291}$$
$$\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

input
```
Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 3.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

method	result	size
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)}{f\sqrt{-a}\,\sqrt{a+b\sec(fx+e)^2}}$	138

input
```
int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)/(a+b*sec(f*x+e)^2)^
(1/2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(33) = 66$.

Time = 0.19 (sec) , antiderivative size = 408, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos\left(fx+\right.\right.}{}\right.$$
$$\left.-\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\,\sqrt{a}f}\right]$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(33) = 66$.

Time = 0.26 (sec) , antiderivative size = 992, normalized size of antiderivative = 25.44

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*
f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b
)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x +
2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
 + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4
*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*
b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*co
s(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(
4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt
(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*...
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}}{\sec(fx+e)^2 b+a}dx$$

input
```
int(1/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)/(sec(e + f*x)**2*b + a),x)
```

3.267 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f}+\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

output
```
1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+
1/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$=\frac{\sqrt{a+2b+a\cos(2(e+fx))}\left((a-b)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sec(e+fx)+\sqrt{a}\sqrt{a+b-a\sin^2(e+f}\right.}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*((a - b)*ArcTan[(Sqrt[a]*Sin[e + f*x])
/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sec[e + f*x] + Sqrt[a]*Sqrt[a + b - a*Sin
[e + f*x]^2]*Tan[e + f*x]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2
])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4634, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(e+fx)^2\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 296}$$

$$\frac{\frac{(a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{(a-b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(3/2)) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e
 + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(75) = 150.

Time = 6.21 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.39

method	result
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a \ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)+\sqrt{\frac{b+a\cos(f}{(1+\cos(f}}}{}$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)/a/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*s
in(f*x+e)*a)*(sec(f*x+e)+1)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b*
ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-1-
sec(f*x+e))+sin(f*x+e)*cos(f*x+e)*(-a)^(1/2)*a+(-a)^(1/2)*b*tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(75) = 150.

Time = 0.23 (sec) , antiderivative size = 502, normalized size of antiderivative = 5.77

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+\sqrt{-a}(a-b)\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\,\mathrm{co}\right.}{}\right.$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) + sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e)))/(a^2*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)*sin(f*x + e) - (a - b)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
 + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/
(a^2*f)]
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos\left(e+f\,x\right)^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)^2}{\sec\left(fx+e\right)^2b+a}dx$$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**2)/(sec(e + f*x)**2*b + a),
x)
```

3.268 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} + \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}$$

output
```
1/8*(3*a^2-2*a*b+3*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/
2))/a^(5/2)/f+3/8*(a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a
^2/f+1/4*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 14.51 (sec) , antiderivative size = 1840, normalized size of antiderivative = 12.87

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]
]*Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2
, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a
 + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)]*Cos[e + f*x]^5)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF
1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*Appe
llF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a
 + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Sin[e + f*x]^2)) - (12*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a +
 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a
 + b)*Cos[e + f*x]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b
)) - (4*f*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2...
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4634, 316, 25, 402, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^4\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\int -\frac{2b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{3a^2-2ba+3b^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{3a^2-2ba+3b^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\left(3a^2-2ab+3b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{\left(3a^2-2ab+3b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}+\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\left(3a^2-2ab+3b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}+\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*a*(1 + Tan[e + f*x]^2)^2
) + (((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/(2*a^(3/2)) + (3*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Ta
n[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(127) = 254$.

Time = 9.98 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.24

method	result
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a^2 \ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(3+3\sec(fx+e))+\sqrt{\frac{b+a\cos}{(1+\cos}}}{}$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(-a)^(1/2)/a^2/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)*(3+3*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)
*a)*(-2-2*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*ln(4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(3+3*sec
(f*x+e))+sin(f*x+e)*cos(f*x+e)*(2*cos(f*x+e)^2+3)*a^2*(-a)^(1/2)+(-cos(f*x
+e)^2+3)*(-a)^(1/2)*a*b*tan(f*x+e)-3*(-a)^(1/2)*b^2*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.97

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{(3a^2-2ab+3b^2)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256(a^4-a^3b)\cos(fx+e)^6+32(5a^4-14a^3b-\cdots\right.}{\cdots}\right.$$

$$-\frac{(3a^2-2ab+3b^2)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8(a^2-ab)\cos(fx+e)^3+(a^2-6ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)\cdots}{32a^3f}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/64*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*
(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a
^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + 3*(a^2 -
a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^3*f), -1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f
*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(
2*a^2*cos(f*x + e)^3 + 3*(a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cos(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2\,b+a}\,\cos(fx+e)^4}{\sec(fx+e)^2\,b+a}dx$$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**4)/(sec(e + f*x)**2*b + a),
x)
```

3.269 $\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(a-b)\left(5a^2+2ab+5b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} + \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}$$

output
```
1/16*(a-b)*(5*a^2+2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)
^2)^(1/2))/a^(7/2)/f+1/48*(15*a^2-14*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+
b+b*tan(f*x+e)^2)^(1/2)/a^3/f+5/24*(a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*ta
n(f*x+e)^2)^(1/2)/a^2/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(
1/2)/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 14.13 (sec) , antiderivative size = 1739, normalized size of antiderivative = 8.52

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/
2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(
a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Cos[e + f*x]^7)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*Appell
F1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*App
ellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(
a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)])*Sin[e + f*x]^2)) - (18*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a
+ 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(
a + b)*Cos[e + f*x]^6*Sin[e + f*x]*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a +
b)) - 2*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^6\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\int -\frac{4b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{6a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{4b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{6a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}-\frac{\int -\frac{15a^2-4ba+5b^2+10(a-b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}}{6a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{15a^2-4ba+5b^2+10(a-b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\frac{\left(15a^2-14ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{3(a-b)\left(5a^2+2ba+5b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\tan\cdots}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{3(a-b)\left(5a^2+2ab+5b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{\left(15a^2-14ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\,\mathrm{t}\cdots}}{4a\left(\tan^2(e+fx)+\cdots\right.}}{6a}\cdots}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{\frac{3(a-b)\left(5a^2+2ab+5b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}+\frac{\left(15a^2-14ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(\cdots}}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}\cdots}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{\left(15a^2-14ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}+\frac{3(a-b)\left(5a^2+2ab+5b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\mathrm{ta}\cdots}{f}$$

input

```
Int[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*(1 + Tan[e + f*x]^2)^3
) + ((5*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*a*(1 + Tan
[e + f*x]^2)^2) + ((3*(a - b)*(5*a^2 + 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[
e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*a^(3/2)) + ((15*a^2 - 14*a*b
 + 15*b^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e +
f*x]^2)))/(4*a))/(6*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(184) = 368.

Time = 22.08 (sec) , antiderivative size = 636, normalized size of antiderivative = 3.12

method	result
default	$\frac{15\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^3\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)+9\sqrt{\frac{b+a}{(1+c}}}{}$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/(-a)^(1/2)/a^3/(a+b*sec(f*x+e)^2)^(1/2)*(15*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-4*sin(f*x+e)*a)*(sec(f*x+e)+1)+9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a^2*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)*(-1-sec(f*x+e))+9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*
b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*
(sec(f*x+e)+1)+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*ln(4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-1-sec(f*x+
e))+sin(f*x+e)*cos(f*x+e)*(8*cos(f*x+e)^4+10*cos(f*x+e)^2+15)*a^3*(-a)^(1/
2)+(-2*cos(f*x+e)^4-4*cos(f*x+e)^2+15)*(-a)^(1/2)*a^2*b*tan(f*x+e)+(5*cos(
f*x+e)^2-14)*(-a)^(1/2)*a*b^2*tan(f*x+e)+15*(-a)^(1/2)*b^3*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 643, normalized size of antiderivative = 3.15

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(-a)*log(128*a^4*cos(f*x
 + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*
b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a
^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)
^5 + 10*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a*b^2)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f
), -1/192*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(a)*arctan(1/4*(8*a^2
*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f
*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f
*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
 - 4*(8*a^3*cos(f*x + e)^5 + 10*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 1
4*a^2*b + 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^4*f)]
```

Sympy [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(e+f\,x\right)^6}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}}\,dx$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)^6}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**6)/(sec(e + f*x)**2*b + a),
x)
```

3.270 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 293

$$\begin{aligned}
\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx &= \frac{a(2a+b)\sin(e+fx)}{b^2(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&- \frac{(2a+b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{b^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&+ \frac{\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&+ \frac{\sec(e+fx)\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}$$

output
```
a*(2*a+b)*sin(f*x+e)/b^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
-(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/(a
+b)/f/(cos(f*x+e)^2)^(1/2)/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^
2*(a+b-a*sin(f*x+e)^2))^(1/2)+EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-
a*sin(f*x+e)^2)/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)+sec(f*x+e)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f
*x+e)^2))^(1/2)
```

Mathematica [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4636, 2057, 2058, 316, 25, 27, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^5}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 4636}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2057}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{\left(1-\sin^2(e+fx)\right)^{3/2}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 316

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int -\frac{a\left(\sin^2(e+fx)+1\right)}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{b}+\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int \frac{a\left(\sin^2(e+fx)+1\right)}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\int \frac{\sin^2(e+fx)+1}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(-\frac{\int -\frac{2(a+b)-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{(2a+ \cdots}{b(a \cdots}\right.}{b}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{\int\frac{2(a+b)-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{(2a+b)\sin}{b(a+b)}\right.}{b}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{\frac{(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\int\frac{}{\sqrt{1-\sin^2}}}{}}{b(a+b)}\right.}{}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{\frac{(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2}{a}}}{}}{b(a+b)}\right.}{}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{\frac{(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2}{a}}}{}}{b(a+b)}\right)}{}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-}{b(a+}\right)}{}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 327

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+}{}}{b(a+b)}\right)}{}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(Sin[e + f*x]/(b*Sqrt[1 - Sin[e + f*x]^2]*
Sqrt[a + b - a*Sin[e + f*x]^2]) - (a*(-(((2*a + b)*Sin[e + f*x]*Sqrt[1 - S
in[e + f*x]^2])/(b*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((2*a + b)*
EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])
/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*EllipticF[ArcSin[Si
n[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a +
b - a*Sin[e + f*x]^2]))/(b*(a + b))))/b))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt
[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.24 (sec) , antiderivative size = 3242, normalized size of antiderivative = 11.06

method	result	size
default	Expression too large to display	3242

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/b^(5/2)/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/(a+b)/(1+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(3/2)*(b^(9/2)*(-1/(a+b)*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
)^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e)))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*(-sec(f*x+e)-2*sec(f*x+e)^2-sec(f*x+e)^3)+b^(
7/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2
)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*a*EllipticE(((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(-cos(f*x+e)-2-5*sec(f*x
+e)-8*sec(f*x+e)^2-4*sec(f*x+e)^3)+b^(5/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e)))^(1/2)*a^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x
+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*(-4*cos(f*x+e)-8-9*sec(f*x+e)-10*sec(f*x+e)^2-5*sec(f*x+e)
^3)+b^(3/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(
f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 960, normalized size of antiderivative = 3.28

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((2*(2*I*a^2*b + I*a*b^2 + (2*I*a^3 + I*a^2*b)*cos(f*x + e)^2)*sqrt(a)
*sqrt((a*b + b^2)/a^2) - (2*I*a^2*b + 5*I*a*b^2 + 2*I*b^3 + (2*I*a^3 + 5*I
*a^2*b + 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^
2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2
*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2
*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-2*I*a^2*b - I*a*b^2 + (-2*I*a^3 -
 I*a^2*b)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (-2*I*a^2*b - 5*
I*a*b^2 - 2*I*b^3 + (-2*I*a^3 - 5*I*a^2*b - 2*I*a*b^2)*cos(f*x + e)^2)*sqr
t(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt
((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e)))
, (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*(
(-I*a^2*b*cos(f*x + e)^2 - I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (-I*a^
2*b - 3*I*a*b^2 - 2*I*b^3 + (-I*a^3 - 3*I*a^2*b - 2*I*a*b^2)*cos(f*x + e)^
2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsi
n(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x
+ e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2)
 - 4*((I*a^2*b*cos(f*x + e)^2 + I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (
I*a^2*b + 3*I*a*b^2 + 2*I*b^3 + (I*a^3 + 3*I*a^2*b + 2*I*a*b^2)*cos(f*x +
e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(ar
csin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*si...
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^5}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^5}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^5\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^5}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**5)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.271 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 152

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{a\sin(e+fx)}{b(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output
```
-a*sin(f*x+e)/b/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+Elliptic
E(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b/(a+b)/f/(cos(f*x+e)^2
)^(1/2)/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)
^2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{2}(a+b)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}E(\right.}{4b(a+b)f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[2]*(a + b)*Sqrt[(a +
2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] - a*Sin[2
*(e + f*x)]))/(4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4636, 2057, 2058, 316, 25, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^3}{\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b(a+b)}-\frac{a\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b(a+b)}-\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \; 330$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-((a*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2
])/(b*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (EllipticE[ArcSin[Sin[e +
 f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(b*(a + b)*Sqrt[1 - (a*
Sin[e + f*x]^2)/(a + b)])))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Si
n[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.40 (sec) , antiderivative size = 1783, normalized size of antiderivative = 11.73

method	result	size
default	Expression too large to display	1783

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+
b)/b*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*
(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^3/(a+b*sec(f*x+e)^2)^(3/2)*(I*(2*cot(f*x+e)-2
*csc(f*x+e)+2*(1-cos(f*x+e))^3*csc(f*x+e)^3)*((I*b^(1/2)+a^(1/2))^2/(a+b))
^(1/2)*b^(1/2)*a^(3/2)+I*(-2*cot(f*x+e)+2*csc(f*x+e)+2*(1-cos(f*x+e))^3*cs
c(f*x+e)^3)*((I*b^(1/2)+a^(1/2))^2/(a+b))^(1/2)*b^(3/2)*a^(1/2)+(2*cot(f*x
+e)-2*csc(f*x+e))*b*a*((I*b^(1/2)+a^(1/2))^2/(a+b))^(1/2)+(-(1-cos(f*x+e))
^3*csc(f*x+e)^3-cot(f*x+e)+csc(f*x+e))*a^2*((I*b^(1/2)+a^(1/2))^2/(a+b))^(
1/2)+((1-cos(f*x+e))^3*csc(f*x+e)^3-cot(f*x+e)+csc(f*x+e))*b^2*((I*b^(1/2)
+a^(1/2))^2/(a+b))^(1/2)+4*I*a^(1/2)*b^(3/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*((-I*a^(
1/2)*b^(1/2)*cos(f*x+e)+I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(a+b)/(1+cos(f*x
+e)))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-
cot(f*x+e)),(1/(a+b)^2*(-4*I*a^(3/2)*b^(1/2)+4*I*a^(1/2)*b^(3/2)+a^2-6*a*b
+b^2))^(1/2))+4*I*a^(3/2)*b^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I
*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*((-I*a^(1/2)*b^(1/2
)*cos(f*x+e)+I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(a+b)/(1+cos(f*x+e)))^(1/2)
*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))
,(1/(a+b)^2*(-4*I*a^(3/2)*b^(1/2)+4*I*a^(1/2)*b^(3/2)+a^2-6*a*b+b^2))^(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 894, normalized size of antiderivative = 5.88

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*a^3*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2*sin
(f*x + e) - (2*(-I*a^3*cos(f*x + e)^2 - I*a^2*b)*sqrt(a)*sqrt((a*b + b^2)/
a^2) - (-I*a^2*b - 2*I*a*b^2 + (-I*a^3 - 2*I*a^2*b)*cos(f*x + e)^2)*sqrt(a
))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2
*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (
a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (2*(I*
a^3*cos(f*x + e)^2 + I*a^2*b)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (I*a^2*b + 2
*I*a*b^2 + (I*a^3 + 2*I*a^2*b)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*
b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a
^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 +
 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*(2*(I*a^2*b*cos(f*x + e)^
2 + I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (I*a^2*b + 3*I*a*b^2 + 2*I*b^
3 + (I*a^3 + 3*I*a^2*b + 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqr
t((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b
^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*
b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*(2*(-I*a^2*b*cos(f*x
 + e)^2 - I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (-I*a^2*b - 3*I*a*b^2 -
 2*I*b^3 + (-I*a^3 - 3*I*a^2*b - 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqrt(
(2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt(
(a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 +...
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^3{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec{(fx+e)}^3}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec{(fx+e)}^3}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{1}{\cos(e+f\,x)^3\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{\sec(fx+e)^2\,b+a}\,\sec(fx+e)^3}{\sec(fx+e)^4\,b^2+2\sec(fx+e)^2\,ab+a^2}dx$$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**3)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.272 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 233

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\sin(e+fx)}{(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
sin(f*x+e)/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-EllipticE(sin
(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a/(a+b)/f/(cos(f*x+e)^2)^(1/
2)/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)+EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(
1/2)/a/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.80 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.88

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec(e+fx)\left(a^2\sqrt{-\frac{1}{b}}+2i\sqrt{2}(a+b)\sqrt{-\frac{a\cos^2(}{b}}\right.}{}$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]*(a^2*Sqrt[-b^(-1)] + (2*I)*Sq
rt[2]*(a + b)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*Sqrt[a + 2*b + a*Cos[2*(e + f*
x)]]*Csc[2*(e + f*x)]^2*EllipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b +
a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(a + b)]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)]
)*Tan[e + f*x])/(2*a^2*Sqrt[-b^(-1)]*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2
))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4636, 2057, 2058, 314, 25, 389, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \; 4636$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2057}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\sin^2(e+fx)}}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{ 314}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int -\frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a+b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a+b}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{ 389}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}-\frac{\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}}{a+b}+\frac{\sqrt{1-\sin^2(e+}}{(a+b)\sqrt{-a\sin}}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}}{a+b}+\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}}{a+b}+\frac{\sqrt{1-\sin^2(e+}}{(a+b)\sqrt{-a\sin}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}}{a+b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}}{a+b}+\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(
(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2]) + (-((EllipticE[ArcSin[Sin[e + f*x
]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]
^2)/(a + b)])) + ((a + b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[
1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(a +
b)))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[
e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.64 (sec) , antiderivative size = 1783, normalized size of antiderivative = 7.65

method	result	size
default	Expression too large to display	1783

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b
)/a/(-(1-cos(f*x+e))^2*csc(f*x+e)^2+1)^3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+
b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-c
os(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b*sec(f*x+e)^2)^(3/2)*(I*(2*cot(f*x+e)-2
*csc(f*x+e)-2*(1-cos(f*x+e))^3*csc(f*x+e)^3)*((I*b^(1/2)+a^(1/2))^2/(a+b))
^(1/2)*b^(3/2)*a^(1/2)+I*(-2*cot(f*x+e)+2*csc(f*x+e)-2*(1-cos(f*x+e))^3*cs
c(f*x+e)^3)*((I*b^(1/2)+a^(1/2))^2/(a+b))^(1/2)*b^(1/2)*a^(3/2)+(cot(f*x+e
)-csc(f*x+e)+(1-cos(f*x+e))^3*csc(f*x+e)^3)*a^2*((I*b^(1/2)+a^(1/2))^2/(a+
b))^(1/2)+(-(1-cos(f*x+e))^3*csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))*b^2*((I*b
^(1/2)+a^(1/2))^2/(a+b))^(1/2)+(-2*cot(f*x+e)+2*csc(f*x+e))*b*a*((I*b^(1/2
)+a^(1/2))^2/(a+b))^(1/2)+4*I*a^(3/2)*b^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*
cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*((-I*a^
(1/2)*b^(1/2)*cos(f*x+e)+I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(a+b)/(1+cos(f*
x+e)))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)
-cot(f*x+e)),(1/(a+b)^2*(-4*I*a^(3/2)*b^(1/2)+4*I*a^(1/2)*b^(3/2)+a^2-6*a*
b+b^2))^(1/2))+4*I*a^(1/2)*b^(3/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-
I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*((-I*a^(1/2)*b^(1/
2)*cos(f*x+e)+I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(a+b)/(1+cos(f*x+e)))^(1/2
)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)
),(1/(a+b)^2*(-4*I*a^(3/2)*b^(1/2)+4*I*a^(1/2)*b^(3/2)+a^2-6*a*b+b^2))^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 762, normalized size of antiderivative = 3.27

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*a^2*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2*sin(
f*x + e) - 4*(I*a^2*cos(f*x + e)^2 + I*a*b)*sqrt(a)*sqrt((2*a*sqrt((a*b +
b^2)/a^2) - a - 2*b)/a)*sqrt((a*b + b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*
sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2
 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*(-I*a^2
*cos(f*x + e)^2 - I*a*b)*sqrt(a)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b
)/a)*sqrt((a*b + b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^
2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 +
4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(I*a^2*cos(f*x + e)^2 + I
*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((I*a^2 + 2*I*a*b)*cos(f*x + e)^2 +
I*a*b + 2*I*b^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*el
liptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e
) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b
^2)/a^2))/a^2) + (2*(-I*a^2*cos(f*x + e)^2 - I*a*b)*sqrt(a)*sqrt((a*b + b^
2)/a^2) - ((-I*a^2 - 2*I*a*b)*cos(f*x + e)^2 - I*a*b - 2*I*b^2)*sqrt(a))*s
qrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*s
qrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2
+ 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2))/((a^4 + a^3
*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x))/(sec(e + f*x)**4*b**2 + 2*s
ec(e + f*x)**2*a*b + a**2),x)
```

3.273 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 244

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{b\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(a+2b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{2b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output
```
-b*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+(a+2*b)*
EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(co
s(f*x+e)^2)^(1/2)/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*
sin(f*x+e)^2))^(1/2)-2*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin
(f*x+e)^2)/(a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*si
n(f*x+e)^2))^(1/2)
```

Mathematica [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sec(e+fx)\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \ 4636$$

$$\frac{\int \frac{1}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 2057$$

$$\frac{\int \frac{1}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 315

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{-\left((a+2b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{-\left((a+2b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}}{a(a+b)}-\frac{b\sin \cdots}{a(a+\cdots}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+\cdots}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a(a+b)}-\frac{b\sin}{a(a+}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+}\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input

```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output

```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2
])/(a*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((a + 2*b)*EllipticE[Arc
Sin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)]) - (2*b*(a + b)*EllipticF[ArcSin[Sin[e + f*x]]
, a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e
 + f*x]^2]))/(a*(a + b))))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin
[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.56 (sec) , antiderivative size = 3227, normalized size of antiderivative = 13.23

method	result	size
default	Expression too large to display	3227

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/(1+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(3/2)*((1/(a+b)*(I*a^(1/2)*b^(1/2
)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(
a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+co
s(f*x+e)))^(1/2)*a^4*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cs
c(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*(cos(f*x+e)+2+sec(f*x+e))+(1/(a+b)*(I*a^(1/2)*b^(1/2)
*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a
+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos
(f*x+e)))^(1/2)*a^3*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(c
sc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*(4*cos(f*x+e)+8+5*sec(f*x+e)+2*sec(f*x+e)^2+sec(f*x+
e)^3)+(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/
2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a^2*b^2*EllipticE(((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(5*cos(f*x+e)+10+
9*sec(f*x+e)+8*sec(f*x+e)^2+4*sec(f*x+e)^3)+(1/(a+b)*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos...
```

Fricas [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^2*sec(f*x + e)^4 + 2*a
*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\cos\left(e+f\,x\right)}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}}\,dx$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^{2}b+a}\,\cos\left(fx+e\right)}{\sec\left(fx+e\right)^{4}b^{2}+2\sec\left(fx+e\right)^{2}ab+a^{2}}dx$$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x))/(sec(e + f*x)**4*b**2 + 2*s
ec(e + f*x)**2*a*b + a**2),x)
```

3.274 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 339

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cos^2(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(a+4b)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{3a^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(2a^2-3ab-8b^2)\,E\left(\arcsin(\sin(e+fx))\left|\frac{a}{a+b}\right.\right)\left(a+b-a\sin^2(e+fx)\right)}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{(a-8b)b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

output

```
-b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)+1/3*(a+4*b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2
*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(2*a^2-3*a*b-8*b^2)*EllipticE(sin(f*x+e),
(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/((a
+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-
1/3*(a-8*b)*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/
(a+b))^(1/2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)
```

Mathematica [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.520$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 4636$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 2057$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{5/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\big\downarrow$ 315

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((a+4b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\big\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((a+4b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\big\downarrow$ 403

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+4b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int -\frac{2(a-2b)(a+b)-\left(2a^2-3ba-8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}}{a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\big\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int \frac{2(a-2b)(a+b)-\left(2a^2-3ba-8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{(a+4b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3a}}{a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\big\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(2a^2-3ab-8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-8b)(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e\ldots}{a}}{3a}}{a(a+b)}\ldots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+\ldots}{1-\sin^2(e+fx)}}\ldots}$$

$\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(2a^2-3ab-8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-8b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2\ldots}{a\ldots}}}\ldots}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}}{a(a+b)}\ldots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+f\ldots}{1-\sin^2(e\ldots}}\ldots}$$

$\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(2a^2-3ab-8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-8b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{\ldots}{a\ldots}\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}}{a(a+b)}\ldots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+\ldots}{1-\sin^2(e+fx)}}\ldots}$$

$\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(2a^2-3ab-8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a-8b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(a\ldots\right.}{a\sqrt{-a\sin^2(e+fx)+a+\ldots}}}{3a}}{a(a+b)}\ldots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+\ldots}{1-\sin^2(\ldots}}\ldots}$$

$\downarrow$ 327

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(2a^2-3ab-8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a-8b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}}{a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx}{1-\sin^2(e+}}}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(3
/2))/(a*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((a + 4*b)*Sin[e + f*x
]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a) + (((2*a^
2 - 3*a*b - 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b -
 a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b)*
b*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f
*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a))/(a*(a + b))))/
(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f
*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.40 (sec) , antiderivative size = 4192, normalized size of antiderivative = 12.37

method	result	size
default	Expression too large to display	4192

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/a^3/(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*((-cos(f*x+e)^4-2*cos(f*x+e)^3-3*cos(f*x+e)^2-4*cos(f*x+e)-2)*(1/
(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+c
os(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a^4*b*EllipticE(((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+I*sin(f*x+e)*cos(f*x+e)^4*(-
2*cos(f*x+e)^2-2*cos(f*x+e)-4)*a^(9/2)*b^(1/2)*((I*b^(1/2)+a^(1/2))^2/(a+b
))^(1/2)+I*sin(f*x+e)*cos(f*x+e)^2*(-2*cos(f*x+e)^4-2*cos(f*x+e)^3+2*cos(f
*x+e)^2-4*cos(f*x+e)-8)*a^(7/2)*b^(3/2)*((I*b^(1/2)+a^(1/2))^2/(a+b))^(1/2
)+(12*cos(f*x+e)^4+24*cos(f*x+e)^3+11*cos(f*x+e)^2-2*cos(f*x+e)-1)*(1/(a+b
)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f
*x+e)))^(1/2)*(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-co
s(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*a^3*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+(19*cos(f*x+e)^4+38*cos(f*x+e)
^3+31*cos(f*x+e)^2+24*cos(f*x+e)+12)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e
)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(-1/(a+b)*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^
(1/2)*a^2*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*...
```

Fricas [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^2*sec(f*x + e)^4 + 2
*a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^3}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**3)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.275 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 440

$$\begin{aligned}\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= -\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&+ \frac{(4a^2-5ab-24b^2)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&+ \frac{(a+6b)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{5a^2(a+b)f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&+ \frac{(8a^3-9a^2b+16ab^2+48b^3)\,E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{15a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&- \frac{4b(a^2-2ab+12b^2)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\end{aligned}$$

output
```
-b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)+1/15*(4*a^2-5*a*b-24*b^2)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f
/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/5*(a+6*b)*cos(f*x+e)^2*sin(f*
x+e)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)+1/15*(8*a^3-9*a^2*b+16*a*b^2+48*b^3)*EllipticE(sin(f*x+e),(a/(a+b))^
(1/2))*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/((a+b-a*sin(f
*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-4/15*b*(a^
2-2*a*b+12*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2
)/(a+b))^(1/2)/a^4/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)
```

Mathematica [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.600$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 403, 25, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sec(e+fx)^5\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{7/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left(-\left((a+6b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{5/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left(-\left((a+6b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{5/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+6b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}{5a}-\frac{\int -\frac{\sqrt{1-\sin^2(e+fx)}\left(2(2a-3b)(a+b)-\left(4a^2-5ba-24b^2\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}}{5a}}{a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int\frac{\sqrt{1-\sin^2(e+fx)}\left(2(2a-3b)(a+b)-\left(4a^2-5ba-24b^2\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{5a}+\frac{(a+6b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\ldots}{5a}}{a(a+b)}\ldots\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 403

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(4a^2-5ab-24b^2\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int-\frac{(a+b)\left(8a^2-13ba+24b^2\right)-\left(8a^3-9ba^2+16b^2a\ldots\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx\ldots}}}{3a}}{5a}}{a(a+b)}\ldots\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin\ldots}{1-\ldots}}}$$

$\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\int\frac{(a+b)\left(8a^2-13ba+24b^2\right)-\left(8a^3-9ba^2+16b^2a+48b^3\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{\left(4a^2-5ab-24b^2\right)\sqrt{1-\sin^2(e+fx)}\sin\ldots}{3a}}{5a}}{a(a+b)}\ldots\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin\ldots}{1-\ldots}}}$$

$\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{4b(a+b)\left(a^2-2ab+12b^2\right)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2\ldots}}}{a}}{3a}}{5a}\ldots\right)}{f\sqrt{1\ldots}}$$

$\downarrow$ 323

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{4b(a+b)\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{\ \ }{\sqrt{1-\sin\ldots}}}{a\sqrt{-a\sin^2(e+fx)\ldots}}}{3a}}{5a}\right.$$

$$\Big\downarrow \text{ 321}$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{4b(a+b)\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\ldots}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}}{5a}\right.$$

$$f\sqrt{1\ldots}$$

$$\Big\downarrow \text{ 330}$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{4b(a+b)\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin\ldots}{\ldots}}}{a\sqrt{-a\ldots}}}{3a}}{5a}\right.$$

$$\Big\downarrow \text{ 327}$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\left(4a^2-5ab-24b^2\right)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3a}+\frac{\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\sqrt{-a\sin^2(e+fx)+a+b}\ldots}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\ldots}{\ldots}}{5a}\right.$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(5
/2))/(a*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((a + 6*b)*Sin[e + f*x
]*(1 - Sin[e + f*x]^2)^(3/2)*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*a) + (((4*
a^2 - 5*a*b - 24*b^2)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a
*Sin[e + f*x]^2])/(3*a) + (((8*a^3 - 9*a^2*b + 16*a*b^2 + 48*b^3)*Elliptic
E[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt
[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (4*b*(a + b)*(a^2 - 2*a*b + 12*b^2)*El
lipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a))/(5*a))/(a*(a + b))))/(f*Sq
rt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2
)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.53 (sec) , antiderivative size = 5206, normalized size of antiderivative = 11.83

method	result	size
default	Expression too large to display	5206

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^2*sec(f*x + e)^4 + 2
*a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos(e+f\,x)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^5}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**5)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.276 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} + \frac{a^2\tan(e+fx)}{b^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2f}$$

output

```
-1/2*(3*a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2
)/f+a^2*tan(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*tan(f*x+e)*(
a+b+b*tan(f*x+e)^2)^(1/2)/b^2/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.05 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.14

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^8(e+fx)\left(-7(a+b)\cos^2(e+fx)\left(15b^2+10ab(2+\cos(2(e+fx)))+a^2\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/210*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^8*(-7*(a + b)*Cos[e +
f*x]^2*(15*b^2 + 10*a*b*(2 + Cos[2*(e + f*x)]) + a^2*(8 + 6*Cos[2*(e + f*x
)] + Cos[4*(e + f*x)]))*Hypergeometric2F1[1, 2, 7/2, -((b*Tan[e + f*x]^2)/
(a + b))] + 4*b*(a + 2*b + a*Cos[2*(e + f*x)])^2*HypergeometricPFQ[{2, 2,
3}, {1, 9/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2 + 16*b*Hyperge
ometric2F1[2, 3, 9/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(4*(a
+ b)^2 - 7*a*(a + b)*Sin[e + f*x]^2 + 3*a^2*Sin[e + f*x]^4))*Tan[e + f*x])
/((a + b)^4*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 315, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^6}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{(\tan^2(e+fx)+1)^2}{(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\frac{\int \frac{(3a+b)\tan^2(e+fx)+a+b}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{b(a+b)}-\frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(3a-b)(a+b)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{b(a+b)}-\frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(3a-b)(a+b)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{b(a+b)}-\frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(3a-b)(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}}}{b(a+b)}-\frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-((a*Tan[e + f*x]*(1 + Tan[e + f*x]^2))/(b*(a + b)*Sqrt[a + b + b*Tan[e +
 f*x]^2])) + (-1/2*((3*a - b)*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a + b + b*Tan[e + f*x]^2]])/b^(3/2) + ((3*a + b)*Tan[e + f*x]*Sqrt[a + b +
 b*Tan[e + f*x]^2])/(2*b))/(b*(a + b)))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. $2(107) = 214$.

Time = 25.35 (sec) , antiderivative size = 1285, normalized size of antiderivative = 10.62

method	result	size
default	Expression too large to display	1285

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(a+b)/b^(7/2)/(a+b*sec(f*x+e)^2)^(3/2)*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a^3*b*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(3+3*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^2*b^2*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(2+2*sec(f*x+e)+3*sec(f*x+e)^2+3*sec(f*x
+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*ln(4*(-b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-1-sec(f
*x+e)+2*sec(f*x+e)^2+2*sec(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^4*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-
b)/(sin(f*x+e)-1))*(-sec(f*x+e)^2-sec(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^3*b*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(3+3*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*a^2*b^2*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(2+2*sec(f*x+e)+3*sec(f*x+e)^2+3*s...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(107) = 214$.

Time = 0.28 (sec) , antiderivative size = 524, normalized size of antiderivative = 4.33

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{\left((3\,a^3+2\,a^2b-ab^2)\cos(fx+e)^3+(3\,a^2b+2\,ab^2-b^3)\cos(fx+e)\right)\sqrt{\cdots}}{\cdots}\right.$$

$$-\frac{\left((3\,a^3+2\,a^2b-ab^2)\cos(fx+e)^3+(3\,a^2b+2\,ab^2-b^3)\cos(fx+e)\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cdots\right.}{2\left(ab\cos(f\cdots\right.}\right.}{4\left((a^2b^3+ab^4)f\cos(fx+e)^3+\cdots\right.}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(((3*a^3 + 2*a^2*b - a*b^2)*cos(f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^
3)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b
- b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(
f*x + e)^4) - 4*(a*b^2 + b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(
f*x + e)^3 + (a*b^4 + b^5)*f*cos(f*x + e)), -1/4*(((3*a^3 + 2*a^2*b - a*b^
2)*cos(f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e))*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) -
 2*(a*b^2 + b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x + e)^3 +
(a*b^4 + b^5)*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.33

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\frac{\tan(fx+e)^3}{\sqrt{b\tan(fx+e)^2+a+b}\,b} - \frac{3\,(a+b)\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \frac{4\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{2\tan(}{\sqrt{b\tan(fx+e}}}{2\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(tan(f*x + e)^3/(sqrt(b*tan(f*x + e)^2 + a + b)*b) - 3*(a + b)*arcsinh
(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 4*arcsinh(b*tan(f*x + e)/sqrt((
a + b)*b))/b^(3/2) + 2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b
)) + 3*(a + b)*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b^2) - 4*tan(f
*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b))/f
```

Giac [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\sec(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{1}{\cos(e+f\,x)^6\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^6}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**6)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.277 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-a*tan(f*x
+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.36 (sec) , antiderivative size = 405, normalized size of antiderivative = 5.26

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{(a+2b+a\cos(2e+2fx))^{3/2}\sec^4(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}\tan(e+fx)\left(15\arcsin\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\mathrm{se}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/15*((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^4*Sqrt[1 - (2*a*S
in[e + f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*(15*ArcSin[Sqrt[-((b*Tan[e + f*x]
^2)/(a + b))]]*Sec[e + f*x]^2*(3*b^2 + a*b*(6 - 5*Sin[e + f*x]^2) + a^2*(3
 - 5*Sin[e + f*x]^2 + 2*Sin[e + f*x]^4)) + 15*(a + b)*(-3*b + a*(-3 + 2*Si
n[e + f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e +
 f*x]^2)/(a + b)^2)] + 4*b*(a + b)*Hypergeometric2F1[2, 2, 7/2, -((b*Tan[e
 + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(-((b*Sec[e + f*x]^2*(a + b - a*Sin[e
+ f*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(3/2)))/((a + b)^2*(2*a + 2*b)*f*(a
+ b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[2*a +
2*b - 2*a*Sin[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*(-((b*Tan[e
 + f*x]^2)/(a + b)))^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4634, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^4}{\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{b}-\frac{a\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{b}-\frac{a\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}}-\frac{a\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/b^(3/2) -
(a*Tan[e + f*x])/(b*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(69) = 138$.

Time = 9.94 (sec) , antiderivative size = 890, normalized size of antiderivative = 11.56

method	result	size
default	Expression too large to display	890

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b*ln(4*(b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(sec(f*x
+e)+1)+1/2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2*ln(4*(b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(1+sec
(f*x+e)+sec(f*x+e)^2+sec(f*x+e)^3)+1/2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a
-b)/(sin(f*x+e)+1))*(sec(f*x+e)^2+sec(f*x+e)^3)+1/2*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a^2*b*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(sec(f*x+e)+1)+1/2*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a*b^2*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(1+sec(f*x+e)+sec(f*x+e)^2+sec(f*x+
e)^3)+1/2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*ln(-4*(b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(sec(f*x
+e)^2+sec(f*x+e)^3)-b^(3/2)*a^2*tan(f*x+e)-b^(5/2)*a*tan(f*x+e)*sec(f*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(69) = 138$.

Time = 0.15 (sec) , antiderivative size = 410, normalized size of antiderivative = 5.32

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{4\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \left((a^2+ab)\cos(fx+e)^2\right.}{4}\right.$$

$$-\frac{2\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \left((a^2+ab)\cos(fx+e)^2+ab+b^2\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)c\right.}{}\right.}{2\left((a^2b^2+ab^3)f\cos(fx+e)^2+(ab^3+b^4)f\right)}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(
f*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(b)*log(((a^2 - 6*
a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/((a^2*b^2 + a*b^3)*f*cos(f
*x + e)^2 + (a*b^3 + b^4)*f), -1/2*(2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
b^2)*sin(f*x + e))))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 + b^4)*f
)]
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)} - \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}b}}{f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
(arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + tan(f*x + e)/(sqrt(b*ta
n(f*x + e)^2 + a + b)*(a + b)) - tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a +
 b)*b))/f
```

Giac [F]

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sec(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^4\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2\,b+a}\,\sec(fx+e)^4}{\sec(fx+e)^4\,b^2+2\sec(fx+e)^2\,ab+a^2}dx$$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**4)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.278 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\tan(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*(a + b)*f*
(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4634, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^2}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Tan[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{a\tan(fx+e)+b\tan(fx+e)\sec(fx+e)^{2}}{f(a+b)\left(a+b\sec(fx+e)^{2}\right)^{\frac{3}{2}}}$	49

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(a+b)/(a+b*sec(f*x+e)^2)^(3/2)*(a*tan(f*x+e)+b*tan(f*x+e)*sec(f*x+e)^2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.03

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\frac{\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)}{(a^2+ab)f\cos(fx+e)^2+(ab+b^2)f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e)/((a^
2 + a*b)*f*cos(f*x + e)^2 + (a*b + b^2)*f)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(30) = 60.

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.22

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{2\,a^2b^2\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}{(a^3b^2+a^2b^3)\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\right.}}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
2*a^2*b^2*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)/((a^3*b^2 + a^2*b^3)*sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*f)
```

Mupad [B] (verification not implemented)

Time = 17.73 (sec) , antiderivative size = 199, normalized size of antiderivative = 6.22

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{\frac{a+2\,b+a\,\cos(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}\,(5\,a\,\sin(2\,e+2\,f\,x)+4\,a\,\sin(}{f\,(a+b)\,(24\,a\,b+10\,a^2+16\,b^2+15\,a^2\,\cos(2\,e+2\,f\,x)+6\,a^2\,\cos(4\,e+4}$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
(((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2)*(5*a*sin(2*
e + 2*f*x) + 4*a*sin(4*e + 4*f*x) + a*sin(6*e + 6*f*x) + 8*b*sin(2*e + 2*f
*x) + 4*b*sin(4*e + 4*f*x)))/(f*(a + b)*(24*a*b + 10*a^2 + 16*b^2 + 15*a^2
*cos(2*e + 2*f*x) + 6*a^2*cos(4*e + 4*f*x) + a^2*cos(6*e + 6*f*x) + 16*b^2
*cos(2*e + 2*f*x) + 32*a*b*cos(2*e + 2*f*x) + 8*a*b*cos(4*e + 4*f*x)))
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2\,b+a}\,\sec(fx+e)^2}{\sec(fx+e)^4\,b^2+2\sec(fx+e)^2\,ab+a^2}dx$$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.279 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+
e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. $2(77) = 154$.

Time = 1.00 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+\right.}{4a^{3/2}(a+b)f\left(a+b\sec^2(e+f\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt
[a]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/
2)*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/
(a + b)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4616, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/a^(3/2) - (
b*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(69) = 138$.

Time = 4.49 (sec) , antiderivative size = 450, normalized size of antiderivative = 5.84

method	result
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a^2 \ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, \cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)+\sqrt{\frac{b+a\cos(\ldots}{(1+\cos(\ldots}}}{}$

input
```
int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(a+b)/a/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(3/2)*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-4*sin(f*x+e)*a)*(sec(f*x+e)+1)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)
*a)*(1+sec(f*x+e)+sec(f*x+e)^2+sec(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4
*sin(f*x+e)*a)*(sec(f*x+e)^2+sec(f*x+e)^3)-b*a*(-a)^(1/2)*tan(f*x+e)-(-a)^
(1/2)*b^2*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(69) = 138$.

Time = 0.26 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.81

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f
*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e
)^2 + a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^
2 + (a^3*b + a^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. 2(69) = 138.

Time = 0.37 (sec) , antiderivative size = 2055, normalized size of antiderivative = 26.69

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2
*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2
*e) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2
*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2
*(a*b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) +
2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x +
 2*e) + a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b +
4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 +
 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))^(1/4)*(((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(
2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 +
(a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*si
n(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a
^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a
^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*...
```

Giac [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2b+a}}{\sec\left(fx+e\right)^4b^2+2\sec\left(fx+e\right)^2ab+a^2}dx$$

input
```
int(1/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)/(sec(e + f*x)**4*b**2 + 2*sec(e + f*x)**2*
a*b + a**2),x)
```

3.280 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}} dx = \frac{(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
1/2*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/
f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*b*(a+3*b)*t
an(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 14.09 (sec) , antiderivative size = 2059, normalized size of antiderivative = 15.72

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}} dx = \text{Result too large to show}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^6*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x
)]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*App
ellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*
a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
- 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqr
t[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*Ap
pellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3
*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5)/(2*Sqrt[a + 2*b +
a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2
, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/
2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*A
ppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Si
n[e + f*x]^2)) - (6*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b ...
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4634, 316, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^2\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$\Big\downarrow$ 4634

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 316

$$\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int -\frac{2b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{2b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{(a-3b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b(a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{(a-3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b(a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{(a-3b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b(a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{b(a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]/(2*a*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2]) +
(((a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/a^(3/2) + (b*(a + 3*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*
x]^2]))/(2*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(115) = 230$.

Time = 7.39 (sec) , antiderivative size = 666, normalized size of antiderivative = 5.08

method	result
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a^3 \ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)+\sqrt{\frac{b+a\cos(\ldots}{(1+\cos(\ldots}}}{}$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(a+b)/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(3/2)*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*sin(f*x+e)*a)*(sec(f*x+e)+1)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a^2*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)*(-2-2*sec(f*x+e)+sec(f*x+e)^2+sec(f*x+e)^3)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)*(-3-3*sec(f*x+e)-2*sec(f*x+e)^2-2*sec(f*x+e)^3)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*ln(4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-3*sec(f*x+e)^2-3*sec(f*x+e)^
3)+(-a)^(1/2)*a^3*cos(f*x+e)*sin(f*x+e)+(2+cos(f*x+e)^2)*(-a)^(1/2)*a^2*b*
tan(f*x+e)+(-a)^(1/2)*a*b^2*(4*tan(f*x+e)+tan(f*x+e)*sec(f*x+e)^2)+3*(-a)^
(1/2)*b^3*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(115) = 230$.

Time = 0.45 (sec) , antiderivative size = 699, normalized size of antiderivative = 5.34

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^2
)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 +
 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*
cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e)) + 8*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*
f*cos(f*x + e)^2 + (a^4*b + a^3*b^2)*f), -1/8*((a^2*b - 2*a*b^2 - 3*b^3 +
(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*
x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e)
)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*((
a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e
)^2 + (a^4*b + a^3*b^2)*f)]
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos\left(e+f\,x\right)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2b+a}\,\cos\left(fx+e\right)^2}{\sec\left(fx+e\right)^4b^2+2\sec\left(fx+e\right)^2ab+a^2}dx$$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**2)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.281 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 194

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{3(a^2-2ab+5b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} + \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)\tan(e+fx)}{8a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
3/8*(a^2-2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2)
)/a^(7/2)/f+1/8*(3*a-5*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)
^(1/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/8*(a-3
*b)*b*(3*a+5*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 14.33 (sec) , antiderivative size = 2046, normalized size of antiderivative = 10.55

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)]*Cos[e + f*x]^10*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*((a + b)*Appell
F1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*App
ellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(
a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)])*Sin[e + f*x]^2)*((a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2
*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*((a + b)*AppellF1[1/
2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1
[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b
)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])
*Sin[e + f*x]^2)) + ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7)/(2*Sqrt[a + 2*b + a*Cos[2*(e +
f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2,
3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) -
(3*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*...
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int -\frac{4b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{4b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int -\frac{3a^2+5b^2+2(3a-5b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{3a^2+5b^2+2(3a-5b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}+\frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\frac{\int \frac{3(a+b)\left(a^2-2ba+5b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b(a-3b)(3a+5b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{3\left(a^2-2ab+5b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b(a-3b)(3a+5b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\downarrow$ 291

$$\frac{\frac{\frac{\frac{3\left(a^2-2ab+5b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b(a-3b)(3a+5b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\downarrow$ 216

$$\frac{\frac{\frac{\frac{3\left(a^2-2ab+5b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{b(a-3b)(3a+5b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input

```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*Sqrt[a + b + b*Tan[e + f*x]^2])
+ (((3*a - 5*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan
[e + f*x]^2]) + ((3*(a^2 - 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sq
rt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) + ((a - 3*b)*b*(3*a + 5*b)*Tan[e +
f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(174) = 348$.

Time = 11.56 (sec) , antiderivative size = 858, normalized size of antiderivative = 4.42

method	result	size
default	Expression too large to display	858

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(a+b)/a^3/(-a)^(1/2)*(cos(f*x+e)^2*(3*cos(f*x+e)+3)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)+(3*cos(f*x+e)+3)*sin(f*x+e)^2*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)+(9*cos(f*x+e)^3+9*cos(f*x+e)^2-3*cos(f*x+e)-
3)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*ln(4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+(15*cos(f*x+e)^3+15*cos
(f*x+e)^2+9*cos(f*x+e)+9)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^
3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+(1
5*cos(f*x+e)+15)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^4*ln(4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+sin(f*x+e)*co
s(f*x+e)^4*(2*cos(f*x+e)^2+3)*(-a)^(1/2)*a^4+sin(f*x+e)*cos(f*x+e)^2*(2*co
s(f*x+e)^4+6)*(-a)^(1/2)*a^3*b+(-3*cos(f*x+e)^4-6*cos(f*x+e)^2+3)*sin(f*x+
e)*(-a)^(1/2)*a^2*b^2+(-20*cos(f*x+e)^2-4)*sin(f*x+e)*(-a)^(1/2)*a*b^3-15*
(-a)^(1/2)*b^4*sin(f*x+e))/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 811, normalized size of antiderivative = 4.18

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^3*b - a^2*b^2 + 3*a*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 +
5*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4
 - 2*a^3*b - 5*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((
a^6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f), -1/32*(3*(a^3*b - a^
2*b^2 + 3*a*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5*a*b^3)*cos(f*x + e)
^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^
3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))) - 4*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4
 - 2*a^3*b - 5*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((
a^6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^4}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**4)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.282 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 271

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(5a^3-9a^2b+15ab^2-35b^3\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} + \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b\left(15a^3-17a^2b+25ab^2+105b^3\right)\tan(e+fx)}{48a^4(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
1/16*(5*a^3-9*a^2*b+15*a*b^2-35*b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(
f*x+e)^2)^(1/2))/a^(9/2)/f+1/48*(15*a^2-22*a*b+35*b^2)*cos(f*x+e)*sin(f*x+
e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/24*(5*a-7*b)*cos(f*x+e)^3*sin(f*x+e)
/a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a+b+b*t
an(f*x+e)^2)^(1/2)+1/48*b*(15*a^3-17*a^2*b+25*a*b^2+105*b^3)*tan(f*x+e)/a^
4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.03 (sec) , antiderivative size = 2068, normalized size of antiderivative = 7.63

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^14*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*
x)]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Ap
pellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3
*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(Sq
rt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*A
ppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (
3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9)/(2*Sqrt[a + 2*b +
 a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -
4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3
/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*
AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*S
in[e + f*x]^2)) - (12*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*...
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int -\frac{6b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{6a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{6b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{(5a-7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int -\frac{15a^2-2ba+7b^2+4(5a-7b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{4a}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{15a^2-2ba+7b^2+4(5a-7b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{4a}+\frac{(5a-7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int -\frac{15a^3+3ba^2+b^2a-35b^3+2b\left(15a^2-22ba+35b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}}{4a}+\frac{(5a-7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx\cdots}}}{6a}\cdots}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\frac{\int \frac{15a^3+3ba^2+b^2a-35b^3+2b\left(15a^2-22ba+35b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}+\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a}+\frac{(5a-7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)\cdots}}}{6a}\cdots}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\frac{\frac{\int \frac{3(a+b)\left(5a^3-9ba^2+15b^2a-35b^3\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b\left(15a^3-17a^2b+25ab^2+105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a}+\frac{(5a\cdots}{4a\left(\tan^2(e+fx)\cdots\right.}}{6a}\cdots}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{\frac{3\left(5a^3-9a^2b+15ab^2-35b^3\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b\left(15a^3-17a^2b+25ab^2+105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\left(15a^2-22ab+35b^2\right)\tan(e\cdots}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2\cdots}}}{4a}\cdots}{6a}\cdots}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{\frac{\frac{3\left(5a^3-9a^2b+15ab^2-35b^3\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b\left(15a^3-17a^2b+25ab^2+105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+f}}}{4a}}{6a}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\frac{b\left(15a^3-17a^2b+25ab^2+105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{3\left(5a^3-9a^2b+15ab^2-35b^3\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}}{2a}}{4a}+\frac{}{4a\left(\tan^2\right.}}{6a}}{f}$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3*Sqrt[a + b + b*Tan[e + f*x]^2])
+ (((5*a - 7*b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*Sqrt[a + b + b*T
an[e + f*x]^2]) + (((15*a^2 - 22*a*b + 35*b^2)*Tan[e + f*x])/(2*a*(1 + Tan
[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((3*(5*a^3 - 9*a^2*b + 15*a
*b^2 - 35*b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2
]])/a^(3/2) + (b*(15*a^3 - 17*a^2*b + 25*a*b^2 + 105*b^3)*Tan[e + f*x])/(a
*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/(2*a))/(4*a))/(6*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(247) = 494$.

Time = 24.63 (sec) , antiderivative size = 1099, normalized size of antiderivative = 4.06

method	result	size
default	Expression too large to display	1099

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a^4/(-a)^(1/2)/(a+b)*(15*cos(f*x+e)^2*(1+cos(f*x+e))*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)+3*(5-4*cos(f*x+e)^3-4*cos(f*x+e)^2+5*cos(f*x
+e))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b*ln(4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+6*(3*cos(f*x+e)^3+3*cos
(f*x+e)^2-2*cos(f*x+e)-2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*
b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+
6*(3-10*cos(f*x+e)^3-10*cos(f*x+e)^2+3*cos(f*x+e))*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^2*b^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-4*sin(f*x+e)*a)+15*(-7*cos(f*x+e)^3-7*cos(f*x+e)^2-4*cos(f*x+e)-4
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^4*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+105*(-1-cos(f*x+e))*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+sin(f*x+e)*cos(f*x+e)^4*(8*cos(f*...
```

Fricas [A] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 941, normalized size of antiderivative = 3.47

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 + (5*a^5 -
4*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(
128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14
*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^
3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^
3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*s
qrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*(
a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b - 7*a^3*b^2)*cos(f*x + e)
^5 + (15*a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*cos(f*x + e)^3 + (15*a^4
*b - 17*a^3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 +
(a^6*b + a^5*b^2)*f), -1/192*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^
4 - 35*b^5 + (5*a^5 - 4*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x
 + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2
*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2
*(5*a^5 - 2*a^4*b - 7*a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 7*a^4*b + 13*a^3
*b^2 + 35*a^2*b^3)*cos(f*x + e)^3 + (15*a^4*b - 17*a^3*b^2 + 25*a^2*b^3...
```

Sympy [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cos(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cos\left(e+f\,x\right)^6}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)^6}{\sec\left(fx+e\right)^4 b^2+2\sec\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**6)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.283 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 325

$$\begin{aligned}\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx =& -\frac{2a(a+2b)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\ &-\frac{a\sin(e+fx)}{3b(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\ &+\frac{2(a+2b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3b^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\\ &-\frac{\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{3b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}\end{aligned}$$

output
```
-2/3*a*(a+2*b)*sin(f*x+e)/b^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)-1/3*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b
-a*sin(f*x+e)^2))^(1/2)+2/3*(a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*
(a+b-a*sin(f*x+e)^2)/b^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/((a+b-a*sin(f*x+e)
^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-1/3*EllipticF(s
in(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/b/(a+b)/f/(c
os(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [A] (verified)

Time = 3.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.51

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\sqrt{2}(a+b)^2\left(\frac{a+2b+a\cos(2(e+fx))}{a+b}\right)^{3}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(Sqrt[2]*(a + b)^2*((a + 2*
b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(2*(a + 2*b)*EllipticE[e + f*x, a/(
a + b)] - b*EllipticF[e + f*x, a/(a + b)]) - 2*a*(a^2 + 5*a*b + 5*b^2 + a*
(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(24*b^2*(a + b)^2*f*(a + b*
Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4636, 2057, 2058, 316, 25, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^5}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 4636}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2057}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^3\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 2058

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{5/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 316

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{a\sin^2(e+fx)+a+3b}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3b(a+b)}-\frac{a\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3b(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{a\sin^2(e+fx)+a+3b}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3b(a+b)}-\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{-\frac{\int -\frac{(a+b)(2a+3b)-2a(a+2b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{2a(a+2b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3b(a+b)}-\frac{a\sin(e+fx)\sqrt{\cdots}}{3b(a+b)\left(-a\sin^2\cdots\right.}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int \frac{(a+b)(2a+3b)-2a(a+2b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{2a(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3b(a+b)}-\frac{a\sin(e+fx)\sqrt{1-\cdots}}{3b(a+b)\left(-a\sin^2(e+\cdots\right.}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-b(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{2a(a+2b)\cdots}{b(a+b\cdots}}{3b(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\cdots}$$

$\Big\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+\cdots}{\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}\cdots}{3b(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\cdots}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}-\frac{2a(a+2b\cdots}{b(a+\cdots}}{3b(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\cdots}$$

$\Big\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)\cdots\right.}{\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}\cdots}{3b(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\cdots}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+}\right.}{\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}}{3b(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input

```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output

```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(a*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x
]^2])/(b*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((-2*a*(a + 2*b)*Sin[
e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(b*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^
2]) + ((2*(a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b
- a*Sin[e + f*x]^2])/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)] - (b*(a + b)*Ell
ipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b
)])/Sqrt[a + b - a*Sin[e + f*x]^2])/(b*(a + b)))/(3*b*(a + b))))/(f*Sqrt[1
 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 316

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.74 (sec) , antiderivative size = 4473, normalized size of antiderivative = 13.76

method	result	size
default	Expression too large to display	4473

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a^2+2*a*b+b^2)/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/b^2/(1+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(5/2)*((-3*cos(f*x+e)
^4+9*cos(f*x+e)^3+4*cos(f*x+e)^2-2*cos(f*x+e)+2)*((I*b^(1/2)+a^(1/2))^2/(a
+b))^(1/2)*a^2*b^3*tan(f*x+e)*sec(f*x+e)^4+(-1/(a+b)*(I*a^(1/2)*b^(1/2)*co
s(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*
(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x
+e)))^(1/2)*b^5*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x
+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*(-6*sec(f*x+e)^3-12*sec(f*x+e)^4-6*sec(f*x+e)^5)+(-1/(a+b)
*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(
f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*b^5*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(4*sec(f*x+e)^3+8*sec(f*x+e)^4+4*sec
(f*x+e)^5)+4*I*a^(9/2)*b^(1/2)*((I*b^(1/2)+a^(1/2))^2/(a+b))^(1/2)*sin(f*x
+e)+(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a
-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)
*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*a^5*EllipticE(((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(2*cos(f*x+e)+4+2*se...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 1374, normalized size of antiderivative = 4.23

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(-I*a^3*b^2 - 2*I*a^2*b^3 + (-I*a^5 - 2*I*a^4*b)*cos(f*x + e)^4 -
2*(I*a^4*b + 2*I*a^3*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) -
(-I*a^3*b^2 - 4*I*a^2*b^3 - 4*I*a*b^4 + (-I*a^5 - 4*I*a^4*b - 4*I*a^3*b^2)
*cos(f*x + e)^4 + 2*(-I*a^4*b - 4*I*a^3*b^2 - 4*I*a^2*b^3)*cos(f*x + e)^2)
*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(
sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x +
e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) +
 (2*(I*a^3*b^2 + 2*I*a^2*b^3 + (I*a^5 + 2*I*a^4*b)*cos(f*x + e)^4 - 2*(-I*
a^4*b - 2*I*a^3*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (I*a^
3*b^2 + 4*I*a^2*b^3 + 4*I*a*b^4 + (I*a^5 + 4*I*a^4*b + 4*I*a^3*b^2)*cos(f*
x + e)^4 + 2*(I*a^4*b + 4*I*a^3*b^2 + 4*I*a^2*b^3)*cos(f*x + e)^2)*sqrt(a)
)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*
a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a
^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-I*
a^2*b^3 - 3*I*a*b^4 + (-I*a^4*b - 3*I*a^3*b^2)*cos(f*x + e)^4 - 2*(I*a^3*b
^2 + 3*I*a^2*b^3)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (2*I*a^3
*b^2 + 9*I*a^2*b^3 + 13*I*a*b^4 + 6*I*b^5 + (2*I*a^5 + 9*I*a^4*b + 13*I*a^
3*b^2 + 6*I*a^2*b^3)*cos(f*x + e)^4 + 2*(2*I*a^4*b + 9*I*a^3*b^2 + 13*I*a^
2*b^3 + 6*I*a*b^4)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2
) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -...
```

Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^5\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^5}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a b^2+3\sec(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**5)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.284 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 323

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{(a-b)\sin(e+fx)}{3b(a+b)^2 f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\sin(e+fx)}{3(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(a-b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3ab(a+b)^2 f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

output
```
-1/3*(a-b)*sin(f*x+e)/b/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)+1/3*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x
+e)^2))^(1/2)+1/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f
*x+e)^2)/a/b/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/((a+b-a*sin(f*x+e)^2)/(a+b))^(
1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*EllipticF(sin(f*x+e),(a
/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/a/(a+b)/f/(cos(f*x+e)^2)
^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.45 (sec) , antiderivative size = 1204, normalized size of antiderivative = 3.73

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-1/24*((-2*Sqrt[-b^(
-1)]*(-a - a*Cos[2*e + 2*f*x])*(2*a^2*(a + 3*b + a*Cos[2*e + 2*f*x]) + b*(
2*b^2 + 3*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - 2*(a + 2*b + a*Cos[2*e + 2*f*
x])^2) + a*(4*b^2 + 5*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b + a*Cos[
2*e + 2*f*x])^2)) + (2*I)*(a^2 + 3*a*b + 2*b^2)*Sqrt[(a - a*Cos[2*e + 2*f*
x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2*(a + 2*b + a
*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a
*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)] - I*(2*a^2 + 5*a*b + 3*b^2)*(a +
2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[(4*a + 4*b - 2*(a + 2*b + a*Cos[2*e +
 2*f*x]))/(a + b)]*Sqrt[2 - (a + 2*b + a*Cos[2*e + 2*f*x])/b]*EllipticF[I*
ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a
+ b)])*Sin[2*e + 2*f*x])/(a*Sqrt[-b^(-1)]*b^2*(a + b)^2*f*Sqrt[((a - a*Cos
[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*(a + 2*b + a*Cos[2*e + 2*f*x
])^(3/2)*Sqrt[1 - Cos[2*e + 2*f*x]^2]) + (Cos[2*(e + f*x)]*(-2*Sqrt[-b^(-1
)]*(-a - a*Cos[2*e + 2*f*x])*(4*b^4 - b^2*(a + 2*b + a*Cos[2*e + 2*f*x])^2
 + 2*a^3*(a + 3*b + a*Cos[2*e + 2*f*x]) + a*b*(10*b^2 + b*(a + 2*b + a*Cos
[2*e + 2*f*x]) - (a + 2*b + a*Cos[2*e + 2*f*x])^2) + a^2*(8*b^2 + 3*b*(a +
 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b + a*Cos[2*e + 2*f*x])^2)) + (2*I)*(a
^3 + 2*a^2*b + 2*a*b^2 + b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[(4
*a + 4*b - 2*(a + 2*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a + 2*b...
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4636, 2057, 2058, 314, 25, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^3}{\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\sin^2(e+fx)}}{\left(-a\sin^2(e+fx)+a+b\right)^{5/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}-\frac{\int -\frac{2-\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{2-\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3(a+b)}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\big\downarrow$ 402

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{-\frac{\int -\frac{-\left((a-b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{(a-b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3(a+b)}+\frac{\sqrt{1-\sin^2(e+fx)}}{3(a+b)\left(-a\sin^2(e+\right.}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\big\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int \frac{-\left((a-b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3(a+b)}+\frac{\sqrt{1-\sin^2(e+fx)}\sin}{3(a+b)\left(-a\sin^2(e+fx)\right.}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\big\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{b(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}+\frac{(a-b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}}{b(a+b)}-\frac{(a-b)\sin(}{b(a+b)\sqrt{}}}{3(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\big\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+f x)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}}{3(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}-\frac{(a-b)\sin(}{b(a+b)\sqrt{}}}{3(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}}{3(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 327

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}+\frac{(a-b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}}{b(a+b)}}{3(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(
3*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (-(((a - b)*Sin[e + f*x]*Sqr
t[1 - Sin[e + f*x]^2])/(b*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((a
- b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x
]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (b*(a + b)*EllipticF[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt
[a + b - a*Sin[e + f*x]^2]))/(b*(a + b)))/(3*(a + b))))/(f*Sqrt[1 - Sin[e
+ f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.99 (sec) , antiderivative size = 3241, normalized size of antiderivative = 10.03

method	result	size
default	Expression too large to display	3241

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/b/a/(a^2+2*a*b+b^2)/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/(1+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(5/2)*((-1/(a+b)*(I*a
^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e))
)^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e
)*a+b)/(1+cos(f*x+e)))^(1/2)*a^5*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(cos(f*x+e)+2*sec(f*x+e))+(-1/(a+b)*(I*a^
(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))
^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)
*a+b)/(1+cos(f*x+e)))^(1/2)*a^4*b*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(2+cos(f*x+e)+3*sec(f*x+e)+4*sec(f*x+e)^
2+2*sec(f*x+e)^3)-(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2
)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*
x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+2
-sec(f*x+e)-4*sec(f*x+e)^2-3*sec(f*x+e)^3-2*sec(f*x+e)^4-sec(f*x+e)^5)*a^3
*b^2*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x
+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))-(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*
a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 1244, normalized size of antiderivative = 3.85

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
1/6*((2*((-I*a^4 + I*a^3*b)*cos(f*x + e)^4 - I*a^2*b^2 + I*a*b^3 - 2*(I*a^
3*b - I*a^2*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-I*a^4
- I*a^3*b + 2*I*a^2*b^2)*cos(f*x + e)^4 - I*a^2*b^2 - I*a*b^3 + 2*I*b^4 +
2*(-I*a^3*b - I*a^2*b^2 + 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sq
rt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b +
b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8
*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*((I*a^4 - I*a^3*b)
*cos(f*x + e)^4 + I*a^2*b^2 - I*a*b^3 - 2*(-I*a^3*b + I*a^2*b^2)*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((I*a^4 + I*a^3*b - 2*I*a^2*b^2)*co
s(f*x + e)^4 + I*a^2*b^2 + I*a*b^3 - 2*I*b^4 + 2*(I*a^3*b + I*a^2*b^2 - 2*
I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*
b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos
(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt(
(a*b + b^2)/a^2))/a^2) - 2*(4*(I*a^3*b*cos(f*x + e)^4 + 2*I*a^2*b^2*cos(f*
x + e)^2 + I*a*b^3)*sqrt(a)*sqrt((a*b + b^2)/a^2) + ((I*a^4 + 3*I*a^3*b +
2*I*a^2*b^2)*cos(f*x + e)^4 + I*a^2*b^2 + 3*I*a*b^3 + 2*I*b^4 + 2*(I*a^3*b
 + 3*I*a^2*b^2 + 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b +
 b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*
(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*(4*(-I*a^3*b*cos(f*x + e)...
```

Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^3\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^3}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**3)/(sec(e + f*x)**6*b**3 + 
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.285 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 331

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{2(2a+b)\sin(e+fx)}{3a(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{b\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{2(2a+b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^2(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(3a+2b)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{3a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output
```
2/3*(2*a+b)*sin(f*x+e)/a/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)-1/3*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*si
n(f*x+e)^2))^(1/2)-2/3*(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-
a*sin(f*x+e)^2)/a^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/((a+b-a*sin(f*x+e)^2)/(
a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(3*a+2*b)*Ellipt
icF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/a^2/(a+
b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 402, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\left(1-\sin^2(e+fx)\right)^{3/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{5/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 315

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int-\frac{-\left((3a+2b)\sin^2(e+fx)\right)+3a+b}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int\frac{-\left((3a+2b)\sin^2(e+fx)\right)+3a+b}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int\frac{b\left(-2(2a+b)\sin^2(e+fx)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}}{3a(a+b)}-\frac{b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int\frac{-2(2a+b)\sin^2(e+fx)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a+b}}{3a(a+b)}-\frac{b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\Big\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{(a+b)(3a+2b)\int\frac{}{\sqrt{1-\sin^2(e+fx)}\sqrt{\cdots}}}{\cdots}}{a+b}}{3a(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\cdots}$$

$\Big\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{(a+b)(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\cdots}{a\sqrt{-\cdots}}}{a+b}}{3a(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\cdots}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{(a+b)(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\mathrm{F}\cdots}{a\sqrt{-a\sin^2(\cdots}}}{a+b}}{3a(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\cdots}$$

$\Big\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{(a+b)(3a+2b)\sqrt{\cdots}}{\cdots}}{a+b}}{3a(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\cdots}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{(a+b)(3a+2b)\sqrt{1-\frac{a}{}}}{}}{a+b}}{3a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(b*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x
]^2])/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((2*(2*a + b)*Sin[e +
 f*x]*Sqrt[1 - Sin[e + f*x]^2])/((a + b)*Sqrt[a + b - a*Sin[e + f*x]^2]) -
 ((2*(2*a + b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*S
in[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a + b)*(3*a +
 2*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^
2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(a + b))/(3*a*(a + b))))/
(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f
*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.36 (sec) , antiderivative size = 4473, normalized size of antiderivative = 13.51

method	result	size
default	Expression too large to display	4473

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^2+2*a*b+b^2)/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/a^2/(1+cos(f*x+e))/(a+b*sec(f*x+e)^2)^(5/2)*(-4*I*a^(1/2)*b^
(9/2)*((I*b^(1/2)+a^(1/2))^2/(a+b))^(1/2)*tan(f*x+e)*sec(f*x+e)^4-2*((I*b^
(1/2)+a^(1/2))^2/(a+b))^(1/2)*b^5*tan(f*x+e)*sec(f*x+e)^4+(-1/(a+b)*(I*a^(
1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^
(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*
a+b)/(1+cos(f*x+e)))^(1/2)*a^2*b^3*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(-2*cos(f*x+e)-4-18*sec(f*x+e)-32*sec(f
*x+e)^2-26*sec(f*x+e)^3-20*sec(f*x+e)^4-10*sec(f*x+e)^5)+(-1/(a+b)*(I*a^(1
/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(
1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a
+b)/(1+cos(f*x+e)))^(1/2)*a^4*b*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*(12*cos(f*x+e)+24+24*sec(f*x+e)+24*sec(f*x
+e)^2+12*sec(f*x+e)^3)+(-1/(a+b)*(I*a^(1/2)*b^(1/2)*cos(f*x+e)-I*a^(1/2)*b
^(1/2)-cos(f*x+e)*a-b)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(I*a^(1/2)*b^(1/2)*c
os(f*x+e)-I*a^(1/2)*b^(1/2)+cos(f*x+e)*a+b)/(1+cos(f*x+e)))^(1/2)*a^4*b*El
lipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+e)-csc(f*x+e)),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.91

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*((2*I*a^4 + I*a^3*b)*cos(f*x + e)^4 + 2*I*a^2*b^2 + I*a*b^3 - 2*(-
2*I*a^3*b - I*a^2*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((2
*I*a^4 + 5*I*a^3*b + 2*I*a^2*b^2)*cos(f*x + e)^4 + 2*I*a^2*b^2 + 5*I*a*b^3
 + 2*I*b^4 + 2*(2*I*a^3*b + 5*I*a^2*b^2 + 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(
a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((
2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))),
(a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*((
-2*I*a^4 - I*a^3*b)*cos(f*x + e)^4 - 2*I*a^2*b^2 - I*a*b^3 - 2*(2*I*a^3*b
+ I*a^2*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-2*I*a^4 -
5*I*a^3*b - 2*I*a^2*b^2)*cos(f*x + e)^4 - 2*I*a^2*b^2 - 5*I*a*b^3 - 2*I*b^
4 + 2*(-2*I*a^3*b - 5*I*a^2*b^2 - 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt
((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt
((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8
*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*((-3*I*a^4
 - I*a^3*b)*cos(f*x + e)^4 - 3*I*a^2*b^2 - I*a*b^3 - 2*(3*I*a^3*b + I*a^2*
b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-I*a^4 - 3*I*a^3*b
- 2*I*a^2*b^2)*cos(f*x + e)^4 - I*a^2*b^2 - 3*I*a*b^3 - 2*I*b^4 + 2*(-I*a^
3*b - 3*I*a^2*b^2 - 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*
b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a
^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^...
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^{2}b+a}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)^{6}b^{3}+3\sec\left(fx+e\right)^{4}a\,b^{2}+3\sec\left(fx+e\right)^{2}a^{2}b+a^{3}}dx$$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x))/(sec(e + f*x)**6*b**3 + 3*s
ec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.286 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 353

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{2b(3a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b\cos^2(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} + \frac{(3a^2+13ab+8b^2)\,E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{3a^3(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} - \frac{b(9a+8b)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

output
```
-2/3*b*(3*a+2*b)*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)-1/3*b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(3*a^2+13*a*b+8*b^2)*EllipticE(
sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(cos(f*x+e)
^2)^(1/2)/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+
e)^2))^(1/2)-1/3*b*(9*a+8*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a
*sin(f*x+e)^2)/(a+b))^(1/2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2
*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.522$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 401, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{1}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\left(1-\sin^2(e+fx)\right)^{5/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{5/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 315

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int-\frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((3a+4b)\sin^2(e+fx)\right)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 25

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int\frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((3a+4b)\sin^2(e+fx)\right)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 401

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int\frac{(a+b)(3a+4b)-\left(3a^2+13ba+8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{2b(3a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3a(a+b)}-\frac{b\sin(e+fx)}{3a(a+b)(-a\mathrm{s}}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 399

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(9a+8b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(}{a}}{a(a+b)}}{3a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin\cdots}{\cdots}}}\cdots}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a(a+b)}\cdots}{3a(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+\cdots}{1-\sin^2(e+fx)}}\cdots}$$

$\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx))\cdots\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a(a+b)}\cdots}{3a(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\cdots}$$

$\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(3a^2+13ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\cdots}{a\sqrt{-a\sin^2(e+fx)+a\cdots}}}{a(a+b)}\cdots}{3a(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+\cdots}{1-\sin^2(e+fx)}}\cdots}$$

$\downarrow$ 327

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{\left(3a^2+13ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\,E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\text{arcsi}\cdots\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a(a+b)}\cdots}{3a(a+b)}\cdots\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+\cdots}{1-\sin^2(e+fx)}}\cdots}$$

input
```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)
^(3/2))/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((-2*b*(3*a + 2*b)*
Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(a*(a + b)*Sqrt[a + b - a*Sin[e + f
*x]^2]) + (((3*a^2 + 13*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a
+ b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)]) - (b*(a + b)*(9*a + 8*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*S
qrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(
a*(a + b)))/(3*a*(a + b))))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Si
n[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.84 (sec) , antiderivative size = 5176, normalized size of antiderivative = 14.66

method	result	size
default	Expression too large to display	5176

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^3*sec(f*x + e)^6 + 3*a
*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\cos\left(e+f\,x\right)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\cos\left(fx+e\right)}{\sec\left(fx+e\right)^6 b^3+3\sec\left(fx+e\right)^4 a\,b^2+3\sec\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x))/(sec(e + f*x)**6*b**3 + 3*s
ec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.287 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 445

$$
\begin{aligned}
\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =& -\frac{2b(4a+3b)\cos^2(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
& - \frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
& + \frac{(a^2+11ab+8b^2)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3a^3(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
& + \frac{2(a+2b)\,(a^2-4ab-4b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^4(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
& - \frac{b(a^2-16ab-16b^2)\,\mathrm{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{3a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}
$$

output
```
-2/3*b*(4*a+3*b)*cos(f*x+e)^2*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-
a*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(
f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(a^2+11*a*b+8*b^2)
*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*
x+e)^2))^(1/2)+2/3*(a+2*b)*(a^2-4*a*b-4*b^2)*EllipticE(sin(f*x+e),(a/(a+b)
)^(1/2))*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/((a+b-a*s
in(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-1/3*b*
(a^2-16*a*b-16*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*((a+b-a*sin(f*x+
e)^2)/(a+b))^(1/2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*s
in(f*x+e)^2))^(1/2)
```

Mathematica [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4636, 2057, 2058, 315, 25, 401, 27, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{7/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{5/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left(-3(a+2b)\sin^2(e+fx)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{5/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left(-3(a+2b)\sin^2(e+fx)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{5/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{401}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int \frac{3\sqrt{1-\sin^2(e+fx)}\left((a+b)(a+2b)-\left(a^2+11ba+8b^2\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{2b(4a+3b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{3\int\frac{\sqrt{1-\sin^2(e+fx)}\left((a+b)(a+2b)-\left(a^2+11ba+8b^2\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{2b(4a+3b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \textbf{403}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{3\left(\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int-\frac{(a+b)\left(2a^2-5ba-8b^2\right)-2(a+2b)\left(a^2-4ba-4\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)}}}{3a}\right.}{a(a+b)}}{3a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx}{1-\sin^2(e+}}}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{3\left(\frac{\int\frac{(a+b)\left(2a^2-5ba-8b^2\right)-2(a+2b)\left(a^2-4ba-4b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{\left(a^2+11ab+8b^2\right)\sqrt{1-\sin^2(e+fx)}\sin(e+}{3a}\right.}{a(a+b)}}{3a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)}{1-\sin^2(e+}}}$$

$$\Big\downarrow \textbf{399}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(a^2-16ab-16b^2\right)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2 \cdots}}}{a}}{3a} \cdots\right.}{a(a+b)} \cdots\right.}{f\sqrt{1-\sin \cdots}}$$

$\downarrow$ 323

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{\cdots}{\sqrt{1-\sin \cdots}}}{a\sqrt{-a\sin^2(e+fx) \cdots}}}{3a} \cdots\right.}{a(a+b)} \cdots\right.}{f\sqrt{1 \cdots}}$$

$\downarrow$ 321

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}}{a(a+b)}\right)}{f\sqrt{1-\sin}}$$

$\downarrow$ 330

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin}{}}}{a\sqrt{-a}}}{3a}}{a(a+}\right)}{f\sqrt{}}$$

$\downarrow$ 327

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(e+}{a+b}}}{a\sqrt{-a\sin^2(}}}{3a}\right.}{a(a+b)}\right.}{f\sqrt{1}}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)
^(5/2))/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((-2*b*(4*a + 3*b)*
Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(3/2))/(a*(a + b)*Sqrt[a + b - a*Sin[e +
 f*x]^2]) + (3*(((a^2 + 11*a*b + 8*b^2)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]
^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a) + ((2*(a + 2*b)*(a^2 - 4*a*b - 4
*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*
x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(a^2 - 16*a*b
 - 16*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e +
f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a)))/(a*(a + b)))
/(3*a*(a + b))))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^
2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.99 (sec) , antiderivative size = 6007, normalized size of antiderivative = 13.50

method	result	size
default	Expression too large to display	6007

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^3*sec(f*x + e)^6 + 3
*a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^3}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**3)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.288 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 563

$$\begin{aligned}
\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx &= -\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad - \frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad + \frac{2(2a^3-3a^2b-42ab^2-32b^3)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^4(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad + \frac{(3a^2+61ab+48b^2)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad + \frac{(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{15a^5(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}} \\
&\quad - \frac{b(4a^3-9a^2b+120ab^2+128b^3)\,\mathrm{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}{15a^5(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}
\end{aligned}$$

output
```
-2/3*b*(5*a+4*b)*cos(f*x+e)^4*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-
a*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^6*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(
f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+2/15*(2*a^3-3*a^2*b-42
*a*b^2-32*b^3)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(sec(f*x+e)^2
*(a+b-a*sin(f*x+e)^2))^(1/2)+1/15*(3*a^2+61*a*b+48*b^2)*cos(f*x+e)^2*sin(f
*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)+1/15*(8*a^4-11*a^3*b+27*a^2*b^2+184*a*b^3+128*b^4)*EllipticE(sin(
f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^5/(a+b)^2/f/(cos(f*x+e)^2)^
(1/2)/((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)-1/15*b*(4*a^3-9*a^2*b+120*a*b^2+128*b^3)*EllipticF(sin(f*x+e),(a/
(a+b))^(1/2))*((a+b-a*sin(f*x+e)^2)/(a+b))^(1/2)/a^5/(a+b)/f/(cos(f*x+e)^2
)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

Mathematica [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 531, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.640$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 401, 403, 27, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^5 \left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{9/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{5/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\left(1-\sin^2(e+fx)\right)^{5/2}\left(-\left((3a+8b)\sin^2(e+fx)\right)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{7/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^{5/2}\left(-\left((3a+8b)\sin^2(e+fx)\right)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{7/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\Big\downarrow \text{401}$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int\frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left((a+b)(3a+8b)-\left(3a^2+61ba+48b^2\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{2b(5a+4b)\sin(e+fx)\left(1-\sin^2(e+\right.}{a(a+b)\sqrt{-a\sin^2(e+fx)+a-}}}{3a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 403

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\left(3a^2+61ab+48b^2\right)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}{5a}-\frac{\int-\frac{3\sqrt{1-\sin^2(e+fx)}\left((a+b)\left(4a^2-7ba-16b\right.\right.}{\sqrt{-a\sin^2}}}{}}{a(a+b)}}{3a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{-}}$$

$\downarrow$ 27

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\frac{3\int\frac{\sqrt{1-\sin^2(e+fx)}\left((a+b)\left(4a^2-7ba-16b^2\right)-2\left(2a^3-3ba^2-42b^2a-32b^3\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{5a}+\frac{\left(3a^2+61ab+48b\right.}{}}{a(a+b)}}{3a(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{-}}$$

$\downarrow$ 403

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{3\left(\frac{2\left(2a^3-3a^2b-42ab^2-32b^3\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int-\frac{(a+b)\left(8a^3-15ba^2+36b^2a+64b^3\right.}{\sqrt{1-\sin^2}}}{}\right.}{5a}}{}\right.$$

$\downarrow$ 25

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{3\left(\frac{\int \frac{(a+b)\left(8a^3-15ba^2+36b^2a+64b^3\right)-\left(8a^4-11ba^3+27b^2a^2+184b^3a+128b^4\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{2\left(2a^3-3a^2b\cdots\right.}{\cdots}\right)}{5a}}{\cdots}\right.$$

$$\Big\downarrow \text{399}$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{3\left(\frac{\left(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(4a^3-9a^2b+120ab^2+128b^3\right)\int\cdots}{3a}\cdots\right)}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{323}$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{3\left(\frac{\frac{\left(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(4a^3-9a^2b+120ab^2+128b^3\right)\sqrt{\cdots}}{\cdots}}{3a}\cdots\right)}{\cdots}}{\cdots}\right.$$

$\downarrow$ 321

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\left(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(4a^3-9a^2b+120ab^2+128b^3\right)\sqrt{}}{a\sqrt{-}}\right)}{3a}\right.$$

$\downarrow$ 330

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\left(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\left(4a^3-9a^2b\right.}{}\right)}{3a}\right.$$

$\downarrow$ 327

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\left(3a^2+61ab+48b^2\right)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}{5a}+\frac{3\left(\frac{2\left(2a^3-3a^2b-42ab^2-32b^3\right)\sqrt{1-\sin^2(e+\ldots}}{3a}\right.}{\ldots}}{\ldots}\right.$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)
^(7/2))/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((-2*b*(5*a + 4*b)*
Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(5/2))/(a*(a + b)*Sqrt[a + b - a*Sin[e +
 f*x]^2]) + (((3*a^2 + 61*a*b + 48*b^2)*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^
(3/2)*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*a) + (3*((2*(2*a^3 - 3*a^2*b - 42
*a*b^2 - 32*b^3)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[
e + f*x]^2])/(3*a) + (((8*a^4 - 11*a^3*b + 27*a^2*b^2 + 184*a*b^3 + 128*b^
4)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^
2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(4*a^3 - 9*a^2*b
 + 120*a*b^2 + 128*b^3)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1
- (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a)))
/(5*a))/(a*(a + b)))/(3*a*(a + b))))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a +
 b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.63 (sec) , antiderivative size = 7179, normalized size of antiderivative = 12.75

method	result	size
default	Expression too large to display	7179

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^3*sec(f*x + e)^6 + 3
*a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos\left(e+f\,x\right)^5}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^5}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2 b+a^3}dx$$

input

```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output

```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**5)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.289 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} + \frac{a^2\tan(e+fx)}{3b^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2a(2a+3b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+1/3*a^2*t
an(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-2/3*a*(2*a+3*b)*tan(f*x+e
)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.89 (sec) , antiderivative size = 592, normalized size of antiderivative = 4.66

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2e+2fx))^{5/2}\sec^6(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}\tan(e+fx)\Big(}{}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^6*Sqrt[1 - (2*a*Sin[e +
 f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*((-24*b*Cos[e + f*x]^2*HypergeometricPF
Q[{2, 2, 2}, {1, 9/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(-((b
*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(5/
2))/(a + b) - (Sec[e + f*x]^6*(24*b^3*Hypergeometric2F1[2, 2, 9/2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6*(4*b^2 + a*b*(8 - 7*Sin[e + f*x]^2)
 + a^2*(4 - 7*Sin[e + f*x]^2 + 3*Sin[e + f*x]^4))*Sqrt[-((b*Sec[e + f*x]^2
*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)] + 35*(a + b)*Cos[e
 + f*x]^2*(15*b^2 + 10*a*b*(3 - 2*Sin[e + f*x]^2) + a^2*(15 - 20*Sin[e + f
*x]^2 + 8*Sin[e + f*x]^4))*(-3*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]
*(a + b - a*Sin[e + f*x]^2)^2 - (a + b)*Cos[e + f*x]^2*(-3*a*Cos[e + f*x]^
2 - b*(3 + Sin[e + f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)*Tan[e + f*x]^2)/(a + b)^2)])))/(a + b)^5))/(315*(2*a + 2*b)^2*f*(a +
 b*Sec[e + f*x]^2)^(5/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[2*a + 2
*b - 2*a*Sin[e + f*x]^2]*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2)*(-((b*Tan[
e + f*x]^2)/(a + b)))^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 315, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^6}{\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{(\tan^2(e+fx)+1)^2}{(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 315$$

$$\frac{\frac{\int \frac{3(a+b)\tan^2(e+fx)+a+3b}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3b(a+b)}-\frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{3b(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \ 298$$

$$\frac{\frac{\frac{3(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{b}-\frac{a(3a+5b)\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3b(a+b)}-\frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{3b(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{\frac{3(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{b}-\frac{a(3a+5b)\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3b(a+b)}-\frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{3b(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{3(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}}-\frac{a(3a+5b)\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3b(a+b)}-\frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{3b(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(a*Tan[e + f*x]*(1 + Tan[e + f*x]^2))/(b*(a + b)*(a + b + b*Tan[e +
f*x]^2)^(3/2)) + ((3*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/b^(3/2) - (a*(3*a + 5*b)*Tan[e + f*x])/(b*(a + b)*Sqrt[
a + b + b*Tan[e + f*x]^2]))/(3*b*(a + b)))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(113) = 226$.

Time = 21.56 (sec) , antiderivative size = 1637, normalized size of antiderivative = 12.89

method	result	size
default	Expression too large to display	1637

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f/(a+b)^2/b^(7/2)*(cos(f*x+e)^4*(3*cos(f*x+e)+3)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a^4*b*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+cos(f*x+e)^2*(6*cos(f*x+e)^3+6*cos(f*x
+e)^2+6*cos(f*x+e)+6)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*
ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+
e)+1))+(3*cos(f*x+e)^5+3*cos(f*x+e)^4+12*cos(f*x+e)^3+12*cos(f*x+e)^2+3*co
s(f*x+e)+3)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^3*ln(4*(b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+(6*
cos(f*x+e)^3+6*cos(f*x+e)^2+6*cos(f*x+e)+6)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a*b^4*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x
+e)*a-a-b)/(sin(f*x+e)+1))+(3*cos(f*x+e)+3)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^5*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e
)*a-a-b)/(sin(f*x+e)+1))+cos(f*x+e)^4*(3*cos(f*x+e)+3)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^4*b*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(113) = 226$.

Time = 0.32 (sec) , antiderivative size = 688, normalized size of antiderivative = 5.42

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(b)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f*c
os(f*x + e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^2*b^
5 + 2*a*b^6 + b^7)*f), 1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))) - 2*((3*a^3*b + 5*a^2*b^2)*cos(f*x + e)^3 + 2*(2*a^2*b^2 + 3*a*
b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^3*b^4 + 2*a^2*
b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^2*b^5 + 2*a*b^6 + b^7)*f)]
```

Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(113) = 226.

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.17

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(\frac{3\tan(fx+e)^2}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}+\frac{2a}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b^2}+\frac{2}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}\right)\tan(fx+e)-\frac{3\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}-\frac{}{\sqrt{}}}{}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*((3*tan(f*x + e)^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) + 2*a/((b*tan
(f*x + e)^2 + a + b)^(3/2)*b^2) + 2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b))*
tan(f*x + e) - 3*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) - 2*tan(f
*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - tan(f*x + e)/((b*tan(
f*x + e)^2 + a + b)^(3/2)*(a + b)) + 3*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2
 + a + b)*b^2) - 2*a*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*
b^2) + 2*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) - 4*tan(f*x + e
)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*b))/f
```

Giac [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^6\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2\,b+a}\,\sec(fx+e)^6}{\sec(fx+e)^6\,b^3+3\sec(fx+e)^4\,a\,b^2+3\sec(fx+e)^2\,a^2b+a^3}dx$$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**6)/(sec(e + f*x)**6*b**3 + 
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.290 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{a\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a+3b)\tan(e+fx)}{3b(a+b)^2 f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-1/3*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/3*(a+3*b)*tan(f*x
+e)/b/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 3.47 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))(2a+3b+a\cos(2(e+fx)))\sec^4(e+fx)\tan(e}{6(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*(2*a + 3*b + a*Cos[2*(e + f*x)])*Sec[e + f
*x]^4*Tan[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4634, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(e+fx)^4}{(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\Big\downarrow \text{ 4634}$$
$$\frac{\int \frac{\tan^2(e+fx)+1}{(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{ 292}$$
$$\frac{\frac{2\int \frac{1}{(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{3(a+b)}+\frac{\tan(e+fx)(\tan^2(e+fx)+1)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$
$$\Big\downarrow \text{ 208}$$
$$\frac{\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(\tan^2(e+fx)+1)\tan(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2))/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)
^(3/2)) + (2*Tan[e + f*x])/(3*(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\left(b+a\cos\left(fx+e\right)^{2}\right)\left(2a\cos\left(fx+e\right)^{2}+a+3b\right)\tan\left(fx+e\right)\sec\left(fx+e\right)^{4}}{3f\left(a^{2}+2ab+b^{2}\right)\left(a+b\sec\left(fx+e\right)^{2}\right)^{\frac{5}{2}}}$	75

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^2+2*a*b+b^2)*(b+a*cos(f*x+e)^2)*(2*a*cos(f*x+e)^2+a+3*b)/(a+b*sec
(f*x+e)^2)^(5/2)*tan(f*x+e)*sec(f*x+e)^4
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\left(2\,a\cos\left(fx+e\right)^3+\left(a+3\,b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\mathrm{si}}{3\left(\left(a^4+2\,a^3b+a^2b^2\right)f\cos\left(fx+e\right)^4+2\left(a^3b+2\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)\right.}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*a*cos(f*x + e)^3 + (a + 3*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 +
 b^4)*f)
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.41

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\frac{2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)}-\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}+\frac{}{\sqrt{b\,t}}}{3\,f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
 e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)) - tan(f*x + e)/((b*tan(f*x
+ e)^2 + a + b)^(3/2)*b) + tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a
 + b)*b))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(75) = 150$.

Time = 0.75 (sec) , antiderivative size = 336, normalized size of antiderivative = 4.05

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{2\left(\left(\frac{3\,(a^6b^4\mathrm{sgn}(\cos(fx+e))+2\,a^5b^5\mathrm{sgn}(\cos(fx+e))+a^4b^6\mathrm{sgn}(\cos(fx+e)))\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^7b^4+3\,a^6b^5+3\,a^5b^6+a^4b^7} - \frac{2\,(\cdots}{\cdots}\right.\right.}{3\left(a\tan\left(\frac{1}{2}\,fx+\cdots\right.\right.}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*((3*(a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(cos(f*x + e)) + a^4*b^6
*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^
6 + a^4*b^7) - 2*(a^6*b^4*sgn(cos(f*x + e)) - 2*a^5*b^5*sgn(cos(f*x + e))
- 3*a^4*b^6*sgn(cos(f*x + e)))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7)
)*tan(1/2*f*x + 1/2*e)^2 + 3*(a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(co
s(f*x + e)) + a^4*b^6*sgn(cos(f*x + e)))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6
+ a^4*b^7))*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b)^(3/2)*f)
```

Mupad [B] (verification not implemented)

Time = 26.84 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{2\left(\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-1\right)\sqrt{a+\frac{b}{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)^2}}\,\left(a\,1\mathrm{i}+a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,4\mathrm{i}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}+b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,6\mathrm{i}\right)}{3\,f\,(a+b)^2\,\left(a+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)^2}$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
-(2*(exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2)^2)^(1/2)*(a*1i + a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*
1i + b*exp(e*2i + f*x*2i)*6i))/(3*f*(a + b)^2*(a + 2*a*exp(e*2i + f*x*2i)
+ a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)
```

Reduce [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^4}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**4)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.291 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\frac{\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
1/3*tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)+2/3*tan(f*x+e)/(a+b)^2/f
/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 5.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\frac{(a+2b+a\cos(2(e+fx)))(3a+5b+(3a+b)\cos(2(e+fx)))\sec^4(e+f}{12(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*(3*a + 5*b + (3*a + b)*Cos[2*(e + f*x)])*S
ec[e + f*x]^4*Tan[e + f*x])/(12*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4634, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^2}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)}+\frac{\tan(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*
x])/(3*(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 5.87 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\left(3a\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)^{2}b+2b\right)\left(b+a\cos\left(fx+e\right)^{2}\right)\tan\left(fx+e\right)\sec\left(fx+e\right)^{4}}{3f\left(a^{2}+2ab+b^{2}\right)\left(a+b\sec\left(fx+e\right)^{2}\right)^{\frac{5}{2}}}$	84

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^2+2*a*b+b^2)*(3*a*cos(f*x+e)^2+cos(f*x+e)^2*b+2*b)*(b+a*cos(f*x+e
)^2)/(a+b*sec(f*x+e)^2)^(5/2)*tan(f*x+e)*sec(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(63) = 126$.

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\left((3\,a+b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\mathrm{si}}{3\,\left(\left(a^4+2\,a^3b+a^2b^2\right)f\cos\left(fx+e\right)^4+2\left(a^3b+2\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((3*a + b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 +
 b^4)*f)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\frac{2\,\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)}}{3\,f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
 e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(63) = 126$.

Time = 0.72 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.75

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{2\left(\left(\frac{3\,(a^6b^4\text{sgn}(\cos(fx+e))+2\,a^5b^5\text{sgn}(\cos(fx+e))+a^4b^6\text{sgn}(\cos(fx+e)))\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^7b^4+3\,a^6b^5+3\,a^5b^6+a^4b^7} - \frac{2\,(3}{}\right.\right.}{3\left(a\tan\left(\frac{1}{2}\,fx+\right.\right.}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*((3*(a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(cos(f*x + e)) + a^4*b^6
*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^
6 + a^4*b^7) - 2*(3*a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(cos(f*x + e)
) - a^4*b^6*sgn(cos(f*x + e)))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7)
)*tan(1/2*f*x + 1/2*e)^2 + 3*(a^6*b^4*sgn(cos(f*x + e)) + 2*a^5*b^5*sgn(co
s(f*x + e)) + a^4*b^6*sgn(cos(f*x + e)))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6
+ a^4*b^7))*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b)^(3/2)*f)
```

Mupad [B] (verification not implemented)

Time = 26.72 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.42

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-1\right)\sqrt{a+\frac{b}{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)^2}}\,\left(a\,3\mathrm{i}+b\,1\mathrm{i}+a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,6\mathrm{i}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,3\mathrm{i}+b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,10\mathrm{i}\right.}{3\,f\,(a+b)^2\,\left(a+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)^2}$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
-((exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2)^2)^(1/2)*(a*3i + b*1i + a*exp(e*2i + f*x*2i)*6i + a*exp(e*4i + f*x
*4i)*3i + b*exp(e*2i + f*x*2i)*10i + b*exp(e*4i + f*x*4i)*1i))/(3*f*(a + b
)^2*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*
x*2i))^2)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.292 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*tan(
f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*b*(5*a+3*b)*tan(f*x+e)/a^2
/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 6.35 (sec) , antiderivative size = 1927, normalized size of antiderivative = 15.42

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^
(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7
/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[
3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*
x]^2)*((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^5)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*Appe
llF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a
*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt
[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, ...
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4616, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 4616}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-2b\tan^2(e+fx)+3a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{3(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{3(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output
```
(-1/3*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*
(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(
3/2) - (b*(5*a + 3*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]
^2]))/(3*a*(a + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(111) = 222$.

Time = 6.88 (sec) , antiderivative size = 861, normalized size of antiderivative = 6.89

method	result	size
default	Expression too large to display	861

input
```
int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*ln(4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-3-3*sec(f*x+e))-1/
3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-6-6*sec(f*x+e)-6*sec(f*x
+e)^2-6*sec(f*x+e)^3)-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*
b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*
(-3-3*sec(f*x+e)-12*sec(f*x+e)^2-12*sec(f*x+e)^3-3*sec(f*x+e)^4-3*sec(f*x+
e)^5)-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-6*sec(f*x+e)^2-6
*sec(f*x+e)^3-6*sec(f*x+e)^4-6*sec(f*x+e)^5)-1/3*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*b^4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-4*sin(f*x+e)*a)*(-3*sec(f*x+e)^4-3*sec(f*x+e)^5)-2*(-a)^(1/2)*a^3*b*tan
(f*x+e)-1/3*(-a)^(1/2)*a^2*b^2*(4*tan(f*x+e)+11*tan(f*x+e)*sec(f*x+e)^2)-1
/3*(7*cos(f*x+e)^2+5)*(-a)^(1/2)*a*b^3*tan(f*x+e)*sec(f*x+e)^4-(-a)^(1/2)*
b^4*tan(f*x+e)*sec(f*x+e)^4)/(a+b)^2/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(111) = 222$.

Time = 0.53 (sec) , antiderivative size = 881, normalized size of antiderivative = 7.05

$$\int \frac{1}{(a + b\sec^2(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 +
b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b +
 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2
)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a
^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(
f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*
x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a
^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x
+ e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 +
 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^...
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^{2}b+a}}{\sec\left(fx+e\right)^{6}b^{3}+3\sec\left(fx+e\right)^{4}a\,b^{2}+3\sec\left(fx+e\right)^{2}a^{2}b+a^{3}}dx$$

input
```
int(1/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)/(sec(e + f*x)**6*b**3 + 3*sec(e + f*x)**4*
a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.293 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a-5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a^2+22ab+15b^2)\tan(e+fx)}{6a^3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
1/2*(a-5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/
f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*b*(3*a+5*b)
*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*b*(3*a^2+22*a*b+15*
b^2)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 15.07 (sec) , antiderivative size = 1775, normalized size of antiderivative = 9.49

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^
(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7
/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[
3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*
x]^2)*((15*a*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^7)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*Appe
llF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a
*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)])*Sin[e + f*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(2*Sq
rt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4634, 316, 25, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\int -\frac{4b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{4b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3a^2-6ba-5b^2+2b(3a+5b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3(a-5b)(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{3(a-5b)(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{\frac{3(a-5b)(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{3(a-5b)(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2))
 + ((b*(3*a + 5*b)*Tan[e + f*x])/(3*a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3
/2)) + ((3*(a - 5*b)*(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*
Tan[e + f*x]^2]])/a^(3/2) + (b*(3*a^2 + 22*a*b + 15*b^2)*Tan[e + f*x])/(a*
(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/(3*a*(a + b)))/(2*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1106 vs. $2(167) = 334$.

Time = 9.10 (sec) , antiderivative size = 1107, normalized size of antiderivative = 5.92

method	result	size
default	Expression too large to display	1107

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/6*cos(f*x+e)^4*(3*cos(f*x+e)+3)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)+1/6*cos(f*x+e)^2*(-9*cos(f*x+e)^3-9*cos(f*x+e)^2+6*cos(f*x+e)+6)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b*ln(4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+1/6*(-27*cos(f*x+e)^5-27*cos
(f*x+e)^4-18*cos(f*x+e)^3-18*cos(f*x+e)^2+3*cos(f*x+e)+3)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+1/6*(-15*cos(f*x+e)^5-15*cos(f*x+e)^4-54*c
os(f*x+e)^3-54*cos(f*x+e)^2-9*cos(f*x+e)-9)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a^2*b^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)+1/6*(-30*cos(f*x+e)^3-30*cos(f*x+e)^2-27*cos(f*x+e)-27)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^4*ln(4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+1/6*(-15*cos(f*x+e)-15)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(167) = 334$.

Time = 1.48 (sec) , antiderivative size = 1023, normalized size of antiderivative = 5.47

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(a^3*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b
^2 - 5*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^
4)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)
*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 -
a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*
x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)) + 8*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5 +
2*(3*a^4*b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^2
*b^3 + 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*
a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f),
-1/24*(3*(a^3*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b
^2 - 5*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^
4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(3*(a^5 + 2*a^4*b + a^3*b
^2)*cos(f*x + e)^5 + 2*(3*a^4*b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)...
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos^2{(e+fx)}}{\left(a+b\sec^2{(e+fx)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos{(fx+e)}^2}{\left(b\sec{(fx+e)}^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos{(fx+e)}^2}{\left(b\sec{(fx+e)}^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{{\cos\left(e+f\,x\right)}^2}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}} \,d x$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^{2} b+a}\,\cos\left(fx+e\right)^{2}}{\sec\left(fx+e\right)^{6} b^{3}+3\sec\left(fx+e\right)^{4} a\,b^{2}+3\sec\left(fx+e\right)^{2} a^{2}b+a^{3}}dx$$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**2)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.294 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 261

$$\begin{aligned}\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx &= \frac{(3a^2-10ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} \\ &+ \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\,(a+b+b\tan^2(e+fx))^{3/2}} \\ &+ \frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} \\ &+ \frac{b(9a^3-15a^2b-145ab^2-105b^3)\tan(e+fx)}{24a^4(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}\end{aligned}$$

output

```
1/8*(3*a^2-10*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(
1/2))/a^(9/2)/f+1/8*(3*a-7*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e
)^2)^(3/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/24
*b*(9*a^2-18*a*b-35*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
+1/24*b*(9*a^3-15*a^2*b-145*a*b^2-105*b^3)*tan(f*x+e)/a^4/(a+b)^2/f/(a+b+b
*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.13 (sec) , antiderivative size = 1777, normalized size of antiderivative = 6.81

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)
^(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2,
3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4,
7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1
[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2)*((15*a*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*S
in[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b
)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
Cos[e + f*x]^9)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*App
ellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*
a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
- 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2)) - (3*Sqrt[2]*(a + b)*AppellF1[1/2, -4, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]
^2)/((a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2,...
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\int -\frac{6b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{6b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{(3a-7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int -\frac{3a^2+2ba+7b^2+4(3a-7b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{3a^2+2ba+7b^2+4(3a-7b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}+\frac{(3a-7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}\cdots}{f}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\frac{\int \frac{9a^3-3ba^2+39b^2a+35b^3+2b\left(9a^2-18ba-35b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}+\frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{(3a-7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{4a}\cdots}{f}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\frac{\frac{\int \frac{3(a+b)^2\left(3a^2-10ba+35b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b\left(9a^3-15a^2b-145ab^2-105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{(3a-7b)\cdots}{2a\left(\tan^2(e+fx)+1\right)\left(a\cdots\right.}}{4a}\cdots}{f}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{\frac{3(a+b)\left(3a^2-10ab+35b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b\left(9a^3-15a^2b-145ab^2-105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\cdots}{2a}\cdots}{4a}\cdots}{f}$$

$\downarrow$ 291

$$\frac{\frac{\frac{\frac{\frac{3(a+b)\left(3a^2-10ab+35b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b\left(9a^3-15a^2b-145ab^2-105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\cdots}{4a}\cdots}{f}$$

$\downarrow$ 216

$$\frac{\frac{\frac{\frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{\frac{3(a+b)\left(3a^2-10ab+35b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{b\left(9a^3-15a^2b-145ab^2-105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}}{2a}+\frac{(3a- \cdots}{2a\left(\tan^2(e+fx)+ \cdots\right.}}{4a}}{f}$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)^(3/2)
) + (((3*a - 7*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e
 + f*x]^2)^(3/2)) + ((b*(9*a^2 - 18*a*b - 35*b^2)*Tan[e + f*x])/(3*a*(a +
b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*(a + b)*(3*a^2 - 10*a*b + 35*b^
2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2)
+ (b*(9*a^3 - 15*a^2*b - 145*a*b^2 - 105*b^3)*Tan[e + f*x])/(a*(a + b)*Sqr
t[a + b + b*Tan[e + f*x]^2]))/(3*a*(a + b)))/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. 2(237) = 474.

Time = 13.80 (sec) , antiderivative size = 1342, normalized size of antiderivative = 5.14

method	result	size
default	Expression too large to display	1342

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/f/a^4/(-a)^(1/2)/(a+b)^2*(cos(f*x+e)^4*(9*cos(f*x+e)+9)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^6*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)-4*sin(f*x+e)*a)+cos(f*x+e)^2*(-12*cos(f*x+e)^3-12*cos(f*x+e
)^2+18*cos(f*x+e)+18)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^5*b*ln
(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+(54*co
s(f*x+e)^5+54*cos(f*x+e)^4-24*cos(f*x+e)^3-24*cos(f*x+e)^2+9*cos(f*x+e)+9)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b^2*ln(4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+(180*cos(f*x+e)^5+180*cos
(f*x+e)^4+108*cos(f*x+e)^3+108*cos(f*x+e)^2-12*cos(f*x+e)-12)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+(105*cos(f*x+e)^5+105*cos(f*x+e)^4+360
*cos(f*x+e)^3+360*cos(f*x+e)^2+54*cos(f*x+e)+54)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a^2*b^4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*sin(f*x+e)*a)+(210*cos(f*x+e)^3+210*cos(f*x+e)^2+180*cos(f*x+e)+1
80)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^5*ln(4*(-a)^(1/2)*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(237) = 474$.

Time = 4.80 (sec) , antiderivative size = 1187, normalized size of antiderivative = 4.55

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(3*a^4*b^2 - 4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^
6 - 4*a^5*b + 18*a^4*b^2 + 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*
a^5*b - 4*a^4*b^2 + 18*a^3*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sq
rt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*
(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^
2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(
f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
)) - 8*(6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11
*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^
3 - 70*a^2*b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 1
05*a*b^5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/((a^9 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2
 + a^6*b^3)*f*cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), -1/96*(
3*(3*a^4*b^2 - 4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6 - 4*a^5
*b + 18*a^4*b^2 + 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^5*b - 4
*a^4*b^2 + 18*a^3*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(a)*arc
tan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*
b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)...
```

Sympy [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cos\left(e+f\,x\right)^4}{\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^4}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**4)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.295 $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 332

$$\begin{aligned}\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx &= \frac{5(a-3b)\,(a^2+7b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f}\\ &+\frac{(5a^2-10ab+21b^2)\cos(e+fx)\sin(e+fx)}{16a^3f\,(a+b+b\tan^2(e+fx))^{3/2}}\\ &+\frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\,(a+b+b\tan^2(e+fx))^{3/2}}+\frac{\cos^5(e+fx)\sin(e+fx)}{6af\,(a+b+b\tan^2(e+fx))^{3/2}}\\ &+\frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}}\\ &+\frac{b(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4)\tan(e+fx)}{48a^5(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}\end{aligned}$$

output
```
5/16*(a-3*b)*(a^2+7*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/a^(11/2)/f+1/16*(5*a^2-10*a*b+21*b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+
b+b*tan(f*x+e)^2)^(3/2)+1/24*(5*a-9*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+
b*tan(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)
^(3/2)+1/48*b*(15*a^3-25*a^2*b+49*a*b^2+105*b^3)*tan(f*x+e)/a^4/(a+b)/f/(a
+b+b*tan(f*x+e)^2)^(3/2)+1/48*b*(15*a^4-20*a^3*b+38*a^2*b^2+420*a*b^3+315*
b^4)*tan(f*x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 18.26 (sec) , antiderivative size = 1776, normalized size of antiderivative = 5.35

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^16*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)
^(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2,
3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5,
7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1
[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2)*((15*a*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^11*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*
Sin[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a +
b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*Cos[e + f*x]^11)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*A
ppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5
*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)])*Sin[e + f*x]^2)) - (15*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(
2*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5,...
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4634, 316, 25, 402, 27, 402, 25, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\int -\frac{8b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{6a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{8b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{(5a-9b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int -\frac{3\left(5a^2+3b^2+2(5a-9b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{5a^2+3b^2+2(5a-9b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a}+\frac{(5a-9b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\frac{3\left(\frac{\left(5a^2-10ab+21b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int -\frac{5a^3+5ba^2-5b^2a-21b^3+4b\left(5a^2-10ba+21b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}\right)}{4a}+\frac{(5a-9b)\tan(e+f}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan\right.}}{6a}}{f}$$

$\downarrow$ 25

$$\frac{\frac{\frac{3\left(\frac{\int \frac{5a^3+5ba^2-5b^2a-21b^3+4b\left(5a^2-10ba+21b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}+\frac{\left(5a^2-10ab+21b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{4a}+\frac{(5a-9b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(\right.}}{6a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{3\left(\frac{\frac{\int \frac{15a^4+10b^2a^2-112b^3a-105b^4+2b\left(15a^3-25ba^2+49b^2a+105b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}+\frac{b\left(15a^3-25a^2b+49ab^2+105b^3\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\left(5a^2-10ab\right.}{2a\left(\tan^2(e+fx)+1\right.}\right.}{4a}}{6a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\frac{3\left(\frac{\frac{\frac{\int \frac{15(a-3b)(a+b)^2\left(a^2+7b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b\left(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(15a^3-25a^2b+49ab^2+105b^3\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\cdots\right.}{4a}\cdots}{6a}\cdots}{f}$$

$\downarrow$ 27

$$\frac{\frac{3\left(\frac{\frac{\frac{15(a-3b)(a+b)\left(a^2+7b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b\left(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(15a^3-25a^2b+49a\cdots\right.}{3a(a+b)\left(a+b\tan\cdots\right.}}{2a}\cdots\right.}{4a}\cdots}{6a}\cdots$$

$\downarrow$ 291

$$\frac{\frac{3\left(\frac{\frac{\frac{15(a-3b)(a+b)\left(a^2+7b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b\left(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(15a^3-25a^2b+49ab^2+1\cdots\right.}{3a(a+b)\left(a+b\tan^2(e\cdots\right.}}{2a}\cdots\right.}{4a}\cdots}{6a}\cdots$$

$\downarrow$ 216

$$\frac{\frac{\frac{3\left(\frac{(5a^2-10ab+21b^2)\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}+\frac{\frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}+\frac{\frac{15(a-3b)(a+b)(a^2+7b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{b(15\cdots}{\cdots}}{3a(a+b)}}{2a}\right.}{4a}}{6a}}{f}$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)^(3/2)
) + (((5*a - 9*b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan
[e + f*x]^2)^(3/2)) + (3*(((5*a^2 - 10*a*b + 21*b^2)*Tan[e + f*x])/(2*a*(1
 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((b*(15*a^3 - 25*a^
2*b + 49*a*b^2 + 105*b^3)*Tan[e + f*x])/(3*a*(a + b)*(a + b + b*Tan[e + f*
x]^2)^(3/2)) + ((15*(a - 3*b)*(a + b)*(a^2 + 7*b^2)*ArcTan[(Sqrt[a]*Tan[e
+ f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) + (b*(15*a^4 - 20*a^3*b +
 38*a^2*b^2 + 420*a*b^3 + 315*b^4)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b
*Tan[e + f*x]^2]))/(3*a*(a + b)))/(2*a)))/(4*a))/(6*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1601 vs. $2(304) = 608$.

Time = 27.99 (sec) , antiderivative size = 1602, normalized size of antiderivative = 4.83

method	result	size
default	Expression too large to display	1602

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a^5/(-a)^(1/2)/(a+b)^2*(15*cos(f*x+e)^4*(1+cos(f*x+e))*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^7*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+15*cos(f*x+e)^2*(-cos(f*x+e)^3-cos(f*x+e)^
2+2*cos(f*x+e)+2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^6*b*ln(4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+15*(1+2*co
s(f*x+e)^5+2*cos(f*x+e)^4-2*cos(f*x+e)^3-2*cos(f*x+e)^2+cos(f*x+e))*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^5*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+15*(-10*cos(f*x+e)^5-10*cos(f*x+
e)^4+4*cos(f*x+e)^3+4*cos(f*x+e)^2-cos(f*x+e)-1)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a^4*b^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*sin(f*x+e)*a)+15*(-35*cos(f*x+e)^5-35*cos(f*x+e)^4-20*cos(f*x+e)^
3-20*cos(f*x+e)^2+2*cos(f*x+e)+2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a^3*b^4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*
x+e)*a)+15*(-21*cos(f*x+e)^5-21*cos(f*x+e)^4-70*cos(f*x+e)^3-70*cos(f*x+e)
^2-10*cos(f*x+e)-10)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^...
```

Fricas [A] (verification not implemented)

Time = 14.67 (sec) , antiderivative size = 1337, normalized size of antiderivative = 4.03

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b^5 - 35*a*b^6 - 21*b^7
 + (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4 - 21*a^2*b^5)*cos(f*
x + e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4 - 35*a^2*b^5 - 21*a
*b^6)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3
*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^
4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos
(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) + 8*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9
 + 2*(5*a^7 + a^6*b - 13*a^5*b^2 - 9*a^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 +
6*a^5*b^2 + 32*a^4*b^3 + 21*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 15*a^5
*b^2 + 31*a^4*b^3 + 287*a^3*b^4 + 210*a^2*b^5)*cos(f*x + e)^3 + (15*a^5*b^
2 - 20*a^4*b^3 + 38*a^3*b^4 + 420*a^2*b^5 + 315*a*b^6)*cos(f*x + e))*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 2*a^9*b + a^
8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2
 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f), -1/192*(15*(a^5*b^2 - a^4*b^3 + 2*a
^3*b^4 - 10*a^2*b^5 - 35*a*b^6 - 21*b^7 + (a^7 - a^6*b + 2*a^5*b^2 - 10*a^
4*b^3 - 35*a^3*b^4 - 21*a^2*b^5)*cos(f*x + e)^4 + 2*(a^6*b - a^5*b^2 + 2*a
^4*b^3 - 10*a^3*b^4 - 35*a^2*b^5 - 21*a*b^6)*cos(f*x + e)^2)*sqrt(a)*ar...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cos(fx+e)^6}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cos(e + f*x)**6)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.296 $\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 179

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} - \frac{b(33a^2+40ab+15b^2)\tan(c+dx)}{15a^3(a+b)^3d\sqrt{a+b+b\tan^2(c+dx)}}$$

output
```
arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/a^(7/2)/d-1/5*b*tan(
d*x+c)/a/(a+b)/d/(a+b+b*tan(d*x+c)^2)^(5/2)-1/15*b*(9*a+5*b)*tan(d*x+c)/a^
2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2)^(3/2)-1/15*b*(33*a^2+40*a*b+15*b^2)*tan(d
*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 15.04 (sec) , antiderivative size = 1777, normalized size of antiderivative = 9.93

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/
(a + b)]*Cos[c + d*x]^6*Sin[c + d*x])/(8*Sqrt[2]*d*(a + b*Sec[c + d*x]^2)^
(7/2)*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3
/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9
/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[
3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*
x]^2)*((21*a*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
+ d*x]^2)/(a + b)]*Cos[c + d*x]^7*Sin[c + d*x]^2)/(8*Sqrt[2]*(a + b - a*Si
n[c + d*x]^2)^(9/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2,
 (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*
x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2,
Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)
*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*C
os[c + d*x]^7)/(8*Sqrt[2]*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*Appe
llF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a
*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] -
 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/
(a + b)])*Sin[c + d*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c
+ d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^5*Sin[c + d*x]^2)/(4*Sq
rt[2]*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2...
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4616, 316, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a+b\sec(c+dx)^2\right)^{7/2}}dx$$

$$\Big\downarrow \; 4616$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)(b\tan^2(c+dx)+a+b)^{7/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\int \frac{-4b\tan^2(c+dx)+5a+b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^{5/2}}d\tan(c+dx)}{5a(a+b)}-\frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b)^{5/2}}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{15a^2+12ba+5b^2-2b(9a+5b)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^{3/2}}d\tan(c+dx)}{3a(a+b)}-\frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b)^{5/2}}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\frac{\int \frac{15(a+b)^3}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)}{a(a+b)}-\frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{a(a+b)\sqrt{a+b\tan^2(c+dx)+b}}}{3a(a+b)}-\frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{15(a+b)^2\int\frac{1}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)}{a}-\frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{a(a+b)\sqrt{a+b\tan^2(c+dx)+b}}}{3a(a+b)}-\frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b)^{5/2}}}{d}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{\frac{15(a+b)^2\int\frac{1}{\frac{a\tan^2(c+dx)}{b\tan^2(c+dx)+a+b}+1}d\frac{\tan(c+dx)}{\sqrt{b\tan^2(c+dx)+a+b}}}{a}-\frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{a(a+b)\sqrt{a+b\tan^2(c+dx)+b}}}{3a(a+b)}-\frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b)^{5/2}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{\frac{15(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{a^{3/2}}-\frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{a(a+b)\sqrt{a+b\tan^2(c+dx)+b}}}{3a(a+b)}-\frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b)^{5/2}}}{d}$$

input
```
Int[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

output
```
(-1/5*(b*Tan[c + d*x])/(a*(a + b)*(a + b + b*Tan[c + d*x]^2)^(5/2)) + (-1/
3*(b*(9*a + 5*b)*Tan[c + d*x])/(a*(a + b)*(a + b + b*Tan[c + d*x]^2)^(3/2)
) + ((15*(a + b)^2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*
x]^2]])/a^(3/2) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tan[c + d*x])/(a*(a + b)*S
qrt[a + b + b*Tan[c + d*x]^2]))/(3*a*(a + b)))/(5*a*(a + b)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1291 vs. 2(161) = 322.

Time = 11.17 (sec) , antiderivative size = 1292, normalized size of antiderivative = 7.22

method	result	size
default	Expression too large to display	1292

input
```
int(1/(a+sec(d*x+c)^2*b)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d/(a+b)^3/a^3/(-a)^(1/2)*(cos(d*x+c)^6*(-15*cos(d*x+c)-15)*((cos(d*x
+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*a^6*ln(4*(-a)^(1/2)*((cos(d*x+c)^2*a+b)
/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+4*(-a)^(1/2)*((cos(d*x+c)^2*a+b)/(cos(
d*x+c)+1)^2)^(1/2)-4*a*sin(d*x+c))+cos(d*x+c)^4*(-45*cos(d*x+c)^3-45*cos(d
*x+c)^2-45*cos(d*x+c)-45)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*a^5*
b*ln(4*(-a)^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+4
*(-a)^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)-4*a*sin(d*x+c))+co
s(d*x+c)^2*(-45*cos(d*x+c)^5-45*cos(d*x+c)^4-135*cos(d*x+c)^3-135*cos(d*x+
c)^2-45*cos(d*x+c)-45)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*a^4*b^2
*ln(4*(-a)^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+4*
(-a)^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)-4*a*sin(d*x+c))+(-1
5*cos(d*x+c)^7-15*cos(d*x+c)^6-135*cos(d*x+c)^5-135*cos(d*x+c)^4-135*cos(d
*x+c)^3-135*cos(d*x+c)^2-15*cos(d*x+c)-15)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)
+1)^2)^(1/2)*a^3*b^3*ln(4*(-a)^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)
^(1/2)*cos(d*x+c)+4*(-a)^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)
-4*a*sin(d*x+c))+(-45*cos(d*x+c)^5-45*cos(d*x+c)^4-135*cos(d*x+c)^3-135*co
s(d*x+c)^2-45*cos(d*x+c)-45)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*a
^2*b^4*ln(4*(-a)^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x
+c)+4*(-a)^(1/2)*((cos(d*x+c)^2*a+b)/(cos(d*x+c)+1)^2)^(1/2)-4*a*sin(d*x+c
))+(-45*cos(d*x+c)^3-45*cos(d*x+c)^2-45*cos(d*x+c)-45)*((cos(d*x+c)^2*a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(161) = 322$.

Time = 1.86 (sec) , antiderivative size = 1241, normalized size of antiderivative = 6.93

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

output
```
[-1/120*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 + a^3*b^
3 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4
)*cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*x + c
)^2)*sqrt(-a)*log(128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x
 + c)^2 + 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^
3)*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(
d*x + c)) + 8*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*cos(d*x + c)^5 + (75*a
^4*b^2 + 94*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (33*a^3*b^3 + 40*a^2*b^
4 + 15*a*b^5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*si
n(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(
a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3
*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 +
3*a^5*b^5 + a^4*b^6)*d), -1/60*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*
cos(d*x + c)^6 + a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^
2 + 3*a^3*b^3 + a^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b
^4 + a*b^5)*cos(d*x + c)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(
a^2 - a*b)*cos(d*x + c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*s...
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\sec^2{(c+dx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c)**2)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x)**2)**(-7/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}^2+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c)^2 + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x)^2)^(7/2),x)
```

output
```
int(1/(a + b/cos(c + d*x)^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)^2 b+a}}{\sec(dx+c)^8 b^4+4\sec(dx+c)^6 a\,b^3+6\sec(dx+c)^4 a^2b^2+4\sec(dx+$$

input
```
int(1/(a+b*sec(d*x+c)^2)^(7/2),x)
```

output
```
int(sqrt(sec(c + d*x)**2*b + a)/(sec(c + d*x)**8*b**4 + 4*sec(c + d*x)**6*
a*b**3 + 6*sec(c + d*x)**4*a**2*b**2 + 4*sec(c + d*x)**2*a**3*b + a**4),x)
```

3.297 $\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$

output
```
arctan(tan(x)/(2+tan(x)^2)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(14) = 28.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \frac{\arcsin\left(\frac{\sin(x)}{\sqrt{2}}\right)\sqrt{3+\cos(2x)}\sec(x)}{\sqrt{2}\sqrt{1+\sec^2(x)}}$$

input
```
Integrate[1/Sqrt[1 + Sec[x]^2],x]
```

output
```
(ArcSin[Sin[x]/Sqrt[2]]*Sqrt[3 + Cos[2*x]]*Sec[x])/(Sqrt[2]*Sqrt[1 + Sec[x
]^2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4616, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec^2(x)+1}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sec(x)^2+1}}dx$$

$$\Big\downarrow \text{ 4616}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)$$

$$\Big\downarrow \text{ 291}$$

$$\int \frac{1}{\frac{\tan^2(x)}{\tan^2(x)+2}+1}d\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}$$

$$\Big\downarrow \text{ 216}$$

$$\arctan\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

input
```
Int[1/Sqrt[1 + Sec[x]^2],x]
```

output
```
ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(12) = 24$.

Time = 1.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 5.43

method	result	size
default	$-\frac{2\sqrt{2}\,\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}\,\arctan\left(\frac{2\csc(x)-2\cot(x)}{\sqrt{2(1-\cos(x))^4\csc(x)^4+2}}\right)}{\sqrt{2+2\sec(x)^2}\,\left((1-\cos(x))^2\csc(x)^2-1\right)}$	76

input
```
int(1/(1+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*2^(1/2)/(2+2*sec(x)^2)^(1/2)/((1-cos(x))^2*csc(x)^2-1)*((cos(x)^2+1)/(c
os(x)+1)^2)^(1/2)*arctan(2/(2*(1-cos(x))^4*csc(x)^4+2)^(1/2)*(csc(x)-cot(x
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right) - \frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

input
```
integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))
/(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \int \frac{1}{\sqrt{\sec^2(x)+1}}\,dx$$

input
```
integrate(1/(1+sec(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(sec(x)**2 + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 388, normalized size of antiderivative = 27.71

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arctan2(2*(2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 +
 sin(4*x)^2 + 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4
)*sin(1/2*arctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)), 2*(2
*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*
sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2
(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 8) + 1/2*arctan2(2*(
2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12
*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan
2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*sin(2*x), 2*(2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin
(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(si
n(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*cos(2*x) + 6)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx=\int \frac{1}{\sqrt{\sec\left(x\right)^2+1}}\,dx$$

input
```
integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sec(x)^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(x)^2}+1}}\,dx$$

input
```
int(1/(1/cos(x)^2 + 1)^(1/2),x)
```

output
```
int(1/(1/cos(x)^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \int \frac{\sqrt{\sec(x)^2+1}}{\sec(x)^2+1}dx$$

input
```
int(1/(1+sec(x)^2)^(1/2),x)
```

output
```
int(sqrt(sec(x)**2 + 1)/(sec(x)**2 + 1),x)
```

3.298 $\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{m}{2},\frac{1}{2},-p,\frac{2+m}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\cot(e+fx)(d\sec(e+fx))^m\left(a+b\sec^2(e+fx)\right)^p}{fm}$$

output
```
AppellF1(1/2*m,1/2,-p,1+1/2*m,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*cot(f*x+e)*(
d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p*(-tan(f*x+e)^2)^(1/2)/f/m/(((a+b*sec(
f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2195 vs. 2(112) = 224.

Time = 16.95 (sec) , antiderivative size = 2195, normalized size of antiderivative = 19.60

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(d*Sec[e + f*x])^m*(Sec[
e + f*x]^2)^(-1 + m/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a
 + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((
3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(m/2 + p
))/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 -
 m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[
e + f*x]^2)^(-1 + m/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*Appel
lF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + ...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \; 3042$$

$$\int (d\sec(e+fx))^m \left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \; 4638$$

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4638
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*sec[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sec[e + f*x])^m*(a +
 b*(c*Sec[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [F]

$$\int (d\sec (fx+e))^{m} \left(a+b\sec (fx+e)^{2}\right)^{p} dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec (fx+e)^{2}+a\right)^{p} (d\sec (fx+e))^{m} \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*sec(e + f*x)**2)**p, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec(fx+e)^2+a\right)^p (d\sec(fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec(fx+e)^2+a\right)^p (d\sec(fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$
$$= \int \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p \left(\frac{d}{\cos(e+f\,x)}\right)^m \, dx$$

input
```
int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$
$$= d^m \left(\int \sec(fx+e)^m \left(\sec(fx+e)^2 b+a\right)^p dx\right)$$

input
```
int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
d**m*int(sec(e + f*x)**m*(sec(e + f*x)**2*b + a)**p,x)
```

3.299 $\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},2+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\frac{a+b-a\sin^2(e+fx)}{a+b}\right)^{-p}(\sec^2(}{f}$$

output
```
AppellF1(1/2,2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^
p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/(((a+b-a*sin(f*x+e)^2
)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1392 vs. 2(105) = 210.

Time = 15.47 (sec) , antiderivative size = 1392, normalized size of antiderivative = 13.26

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))]*Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*(3*(a + b)*Appell
F1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2))/(f*(AppellF1[1/2, -1/2, -p, 3/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*(3*(a + b)
*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (2*a*p*AppellF1[1/2, -1/
2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[2*(e + f*x
)]*Tan[e + f*x]*(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2,
-p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2))
/(a + 2*b + a*Cos[2*(e + f*x)]) + 2*(1/2 + p)*AppellF1[1/2, -1/2, -p, 3/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2*(3*(a + b)
*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[...
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 2057

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$\Big\downarrow$ 2058

$$\frac{\left(1-\sin^2(e+fx)\right)^p\left(-a\sin^2(e+fx)+a+b\right)^{-p}\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\int\left(1-\sin^2(e+fx)\right)^{-p-2}\left(-a\sin^2(e+fx\right.}{f}$$

$\Big\downarrow$ 334

$$\frac{\left(1-\sin^2(e+fx)\right)^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\int\left(1-\sin^2(e+fx)\right)^{-p-2}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(}{f}$$

$\Big\downarrow$ 333

$$\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},p+2,-p,\frac{3}{2},\sin^2(e+fx\right.}{f}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin[
e + f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sec(fx+e)^3 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)^3 \, dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3, x)
```

Reduce [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\sec\left(fx+e\right)^3dx$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sec(e + f*x)**3,x)
```

3.300 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{1}{2},1+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\frac{a+b-a\sin^2(e+fx)}{a+b}\right)^{-p}(\sec^2(}{f}$$

output
```
AppellF1(1/2,p+1,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^
p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/(((a+b-a*sin(f*x+e)^2
)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1394 vs. 2(105) = 210.

Time = 15.16 (sec) , antiderivative size = 1394, normalized size of antiderivative = 13.28

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*(3*(a + b)*AppellF1[
1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*
p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))])*Tan[e + f*x]^2))/(f*(AppellF1[1/2, 1/2, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*(3*(a + b)*Appel
lF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan
[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (2*a*p*AppellF1[1/2, 1/2, -p, 3/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[2*(e + f*x)]*Tan[e
+ f*x]*(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2))/(a + 2*b +
 a*Cos[2*(e + f*x)]) + 2*(-1/2 + p)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2*(3*(a + b)*AppellF1[1
/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p
*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(...
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(e+fx)\left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{ 4636}$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2057}$$

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{(1-\sin^2(e+fx))^p\left(-a\sin^2(e+fx)+a+b\right)^{-p}\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\int (1-\sin^2(e+fx))^{-p-1}\left(-a\sin^2(e+fx\right.}{f}$$

$$\Big\downarrow \text{ 334}$$

$$\frac{(1-\sin^2(e+fx))^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\int (1-\sin^2(e+fx))^{-p-1}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(}{f}$$

$$\Big\downarrow \text{ 333}$$

$$\frac{\sin(e+fx)\,(1-\sin^2(e+fx))^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},p+1,-p,\frac{3}{2},\sin^2(e+fx\right.}{f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin[
e + f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sec(fx+e)\left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(a+b\sec^2(e+fx)\right)^p\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\sec\left(fx+e\right)dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sec(e + f*x),x)
```

3.301 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\frac{a+b-a\sin^2(e+fx)}{a+b}\right)^{-p}(\sec^2(e+}{f}$$

output
```
AppellF1(1/2,p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p*
sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/(((a+b-a*sin(f*x+e)^2)/
(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1983 vs. $2(103) = 206$.

Time = 15.16 (sec) , antiderivative size = 1983, normalized size of antiderivative = 19.25

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)
*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 3/2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF
1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] +
3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])
^p*(Sec[e + f*x]^2)^(-1/2 + p))/(-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 3/2,
 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a + b)*A
ppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
)*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 +
p)*(Sec[e + f*x]^2)^(-3/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*
AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-3/2 + p)*Ap
pellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)...
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{(1-\sin^2(e+fx))^p\,(-a\sin^2(e+fx)+a+b)^{-p}\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \int (1-\sin^2(e+fx))^{-p}\,(-a\sin^2(e+fx)\ -}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{(1-\sin^2(e+fx))^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\int (1-\sin^2(e+fx))^{-p}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(e\ -}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sin(e+fx)\,(1-\sin^2(e+fx))^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\,\text{si}}{}\right.}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Sin
[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin[e +
f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cos(fx+e)\left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Giac [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\cos\left(fx+e\right)dx$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*cos(e + f*x),x)
```

3.302 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},-1+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\frac{a+b-a\sin^2(e+fx)}{a+b}\right)^{-p}(\sec}{f}$$

output
```
AppellF1(1/2,-1+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)
^p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/(((a+b-a*sin(f*x+e)^
2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1987 vs. 2(105) = 210.

Time = 15.44 (sec) , antiderivative size = 1987, normalized size of antiderivative = 18.92

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)
*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 5/2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF
1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] +
5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])
^p*(Sec[e + f*x]^2)^(-3/2 + p))/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2,
 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*A
ppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
)*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 +
p)*(Sec[e + f*x]^2)^(-5/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*
AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-5/2 + p)*Ap
pellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)...
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 2057$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 2058$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p\left(-a\sin^2(e+fx)+a+b\right)^{-p}\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\int\left(1-\sin^2(e+fx)\right)^{1-p}\left(-a\sin^2(e+fx)\right.}{f}$$

$$\Big\downarrow \; 334$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\int\left(1-\sin^2(e+fx)\right)^{1-p}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(e}{f}$$

$$\Big\downarrow \; 333$$

$$\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},p-1,-p,\frac{3}{2},\sin^2(e+fx\right.}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin
[e + f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cos (fx+e)^3 \left(a+b\sec (fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \cos\left(fx+e\right)^3 \, dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \cos\left(fx+e\right)^3 \, dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Giac [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^3\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\cos\left(fx+e\right)^3dx$$

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*cos(e + f*x)**3,x)
```

3.303 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},-2+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\frac{a+b-a\sin^2(e+fx)}{a+b}\right)^{-p}(\sec}{f}$$

output
```
AppellF1(1/2,-2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)
^p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/(((a+b-a*sin(f*x+e)^
2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1997 vs. 2(105) = 210.

Time = 15.52 (sec) , antiderivative size = 1997, normalized size of antiderivative = 19.02

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Cos[e + f*x]^4*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x
]^2)^(-7/2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*Appe
llF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] +
(-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 7/2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Co
s[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p))/(-3*(a + b)*AppellF1[1/2, 7
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*App
ellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e
+ f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-7/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x
])/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b
)*(-7/2 + p)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + ...
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx) \left(a+b\sec^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)^5}dx$$

$$\Big\downarrow \text{4636}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2 \left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2057}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2 \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2058}$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p \left(-a\sin^2(e+fx)+a+b\right)^{-p} \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \int \left(1-\sin^2(e+fx)\right)^{2-p} \left(-a\sin^2(e+fx)\right.}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p} \int \left(1-\sin^2(e+fx)\right)^{2-p} \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(e}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)^p \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p} \text{AppellF1}\left(\frac{1}{2},p-2,-p,\frac{3}{2},\sin^2(e+fx\right.}{f}$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin
[e + f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cos(fx+e)^5 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cos^5(e + fx) \left(a + b \sec^2(e + fx)\right)^p \, dx = \int \left(b \sec\left(fx + e\right)^2 + a\right)^p \cos\left(fx + e\right)^5 \, dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e + fx) \left(a + b \sec^2(e + fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^5(e + fx) \left(a + b \sec^2(e + fx)\right)^p \, dx = \int \left(b \sec\left(fx + e\right)^2 + a\right)^p \cos\left(fx + e\right)^5 \, dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Giac [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^5\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\cos\left(fx+e\right)^5dx$$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*cos(e + f*x)**5,x)
```

3.304 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{(3a-b(7+4p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\tan^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{(3a^2-4ab(1+p)+4b^2(2+3p+p^2))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b\right.}{b^2f(3+2p)(5+2p)}$$

output
```
-(3*a-b*(7+4*p))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(p+1)/b^2/f/(3+2*p)/(5+2*
p)+tan(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(p+1)/b/f/(5+2*p)+(3*a^2-4*a*b*(p+1)+
4*b^2*(p^2+3*p+2))*hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*
x+e)*(a+b+b*tan(f*x+e)^2)^p/b^2/f/(3+2*p)/(5+2*p)/(((a+b+b*tan(f*x+e)^2)/(
a+b))^p)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.70

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{(a+b\sec^2(e+fx))^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left(15\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\right.}{}$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*(15*Hypergeometric2F1[1/2, -p, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b))] + 10*Hypergeometric2F1[3/2, -p, 5/2, -((b*
Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/2, -p, 7/
2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4))/(15*f*(1 + (b*Tan[e + f
*x]^2)/(a + b))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4634, 318, 25, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4634}$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^p\,d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{\int -\left(b\tan^2(e+fx)+a+b\right)^p\left((3a-2b(p+2))\tan^2(e+fx)+a-2b(p+2)\right)d\tan(e+fx)}{b(2p+5)}+\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)}}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)}-\frac{\int \left(b\tan^2(e+fx)+a+b\right)^p\left((3a-2b(p+2))\tan^2(e+fx)+a-2b(p+2)\right)d\tan(e+fx)}{b(2p+5)}}{f}$$

$\Big\downarrow$ 299

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)}-\frac{\frac{(3a-2b(p+2))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\int \left(b\tan\right.}{b(2p+3)}}{b(2p+5)}}{f}$$

$\Big\downarrow$ 238

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)}-\frac{\frac{(3a-2b(p+2))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\left(a+b\tan\right.}{}}{b(2p+5)}}{f}$$

$\Big\downarrow$ 237

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)}-\frac{\frac{(3a-2b(p+2))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\tan(e+}{}}{b(2p}}{f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b
*(5 + 2*p)) - (((3*a - 2*b*(2 + p))*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2
)^(1 + p))/(b*(3 + 2*p)) - ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2)
)*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f
*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p))/(b*(5 + 2*p)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \sec(fx+e)^6 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Giac [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6, x)
```

Reduce [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\sec\left(fx+e\right)^6dx$$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sec(e + f*x)**6,x)
```

3.305 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)}$$
$$-\frac{(a-2b(1+p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a}{}\right.}{bf(3+2p)}$$

output

```
tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(p+1)/b/f/(3+2*p)-(a-2*b*(p+1))*hypergeom(
[1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^p/b
/f/(3+2*p)/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\left(a+b\sec^2(e+fx)\right)^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left((-a+2b(1+p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\right.\right.}{bf(3+2p)}$$

input

```
Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*((-a + 2*b*(1 + p))*Hypergeometric2
F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b + b*Tan[e + f*x]^
2)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x
]^2)/(a + b))^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^p\,d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{(a-2b(p+1))\int\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{b(2p+3)}}{f}$$

$$\Big\downarrow \text{ 238}$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{(a-2b(p+1))\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p d\tan(e+fx)}{b(2p+3)}}{f}$$

$$\Big\downarrow \text{ 237}$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{(a-2b(p+1))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{b(2p+3)}}{f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b*(3 + 2*p)) - ((a - 2
*b*(1 + p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b*(3 + 2*p)*(1 + (b*Tan[e + f
*x]^2)/(a + b))^p))/f
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \sec(fx+e)^4 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)^4 dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4, x)
```

Reduce [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\sec\left(fx+e\right)^4dx$$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sec(e + f*x)**4,x)
```

3.306 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

output
```
hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x
+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b*Sec
[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4634, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx) \left(a+b\sec^2(e+fx)\right)^p dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(e+fx)^2 \left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \; 4634$$

$$\frac{\int \left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 238$$

$$\frac{\left(a+b\tan^2(e+fx)+b\right)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \int \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 237$$

$$\frac{\tan(e+fx) \left(a+b\tan^2(e+fx)+b\right)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*
x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \sec(fx+e)^2 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^p\sec^2\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x)**2, x)
```

Maxima [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2, x)
```

Reduce [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*sec(e + f*x)**2,x)
```

3.307 $\int (a+b\sec^2(e+fx))^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int (a+b\sec^2(e+fx))^p \, dx = \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e+}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2137 vs. $2(85) = 170$.

Time = 6.18 (sec) , antiderivative size = 2137, normalized size of antiderivative = 25.14

$$\int (a+b\sec^2(e+fx))^p \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3
/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a +
b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(
a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)
/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*...
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{p}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^pdx$$

input
```
int((a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p,x)
```

3.308 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e+}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,2,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1343 vs. 2(85) = 170.

Time = 14.16 (sec) , antiderivative size = 1343, normalized size of antiderivative = 15.80

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(3*(a + b)*AppellF1[1/
2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Ap
pellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2))/(f*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*(3*(a + b)*AppellF1[1/2,
 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Appe
llF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] -
 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))])*Tan[e + f*x]^2) - (2*a*p*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[2*(e + f*x)]*Tan[e + f*x]*(3*(a +
 b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2))/(a + 2*b + a*Cos[2*(e + f*x)
]) + 2*(-2 + p)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*Tan[e + f*x]^2*(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*Appel...
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\sec(e+fx)^2)^p}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^p}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \cos \left(fx+e\right)^{2} \left(a+b\sec \left(fx+e\right)^{2}\right)^{p} dx$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p} \cos \left(fx+e\right)^{2} \, dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^2\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\cos\left(fx+e\right)^2dx$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*cos(e + f*x)**2,x)
```

3.309 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$=\frac{\operatorname{AppellF1}\left(\frac{1}{2},3,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e+}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,3,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1342 vs. 2(85) = 170.

Time = 14.90 (sec) , antiderivative size = 1342, normalized size of antiderivative = 15.79

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(3*(a + b)*AppellF1[
1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*
AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))])*Tan[e + f*x]^2))/(f*(AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*(3*(a + b)*AppellF1[1/
2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Ap
pellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2) - (2*a*p*AppellF1[1/2, 3, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[2*(e + f*x)]*Tan[e + f*x]*(3*(a
 + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2))/(a + 2*b + a*Cos[2*(e + f*
x)]) + 2*(-3 + p)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))]*Tan[e + f*x]^2*(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 -
p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*App...
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)^4}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^p}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},3,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \cos(fx+e)^4 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^4\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\cos\left(fx+e\right)^4dx$$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*cos(e + f*x)**4,x)
```

3.310 $\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},4,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p \left(\frac{a+b+b\tan^2(e+}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,4,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1914 vs. 2(85) = 170.

Time = 15.20 (sec) , antiderivative size = 1914, normalized size of antiderivative = 22.52

$$\int \cos^6(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*Cos[e + f*x]^5*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2
)^(-4 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1
/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*A
ppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^
p*(Sec[e + f*x]^2)^(-3 + p))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3
/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f
*x]^2)^(-4 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 4,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF
1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*
(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-4 + p)*AppellF1[1/2, 4, -p, 3/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + ...
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)^6}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^p}{(\tan^2(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 334

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{(\tan^2(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 333

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},4,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

Maple [F]

$$\int \cos(fx+e)^6 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Giac [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^6\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\cos\left(fx+e\right)^6dx$$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*cos(e + f*x)**6,x)
```

3.311 $\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx=-\frac{a\log(\cos(e+fx))}{f}-\frac{(2a-b)\sec^2(e+fx)}{2f}+\frac{(a-2b)\sec^4(e+fx)}{4f}+\frac{b\sec^6(e+fx)}{6f}$$

output
```
-a*ln(cos(f*x+e))/f-1/2*(2*a-b)*sec(f*x+e)^2/f+1/4*(a-2*b)*sec(f*x+e)^4/f+
1/6*b*sec(f*x+e)^6/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx=-\frac{a\log(\cos(e+fx))}{f}-\frac{a\sec^2(e+fx)}{f}+\frac{a\sec^4(e+fx)}{4f}+\frac{b\tan^6(e+fx)}{6f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

output
```
-((a*Log[Cos[e + f*x]])/f) - (a*Sec[e + f*x]^2)/f + (a*Sec[e + f*x]^4)/(4*
f) + (b*Tan[e + f*x]^6)/(6*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4626, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)\sec^7(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)\sec^4(e+fx)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{85}$$

$$-\frac{\int \left(b\sec^4(e+fx)+(a-2b)\sec^3(e+fx)+(b-2a)\sec^2(e+fx)+a\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{2}(a-2b)\sec^2(e+fx)+(2a-b)\sec(e+fx)+a\log\left(\cos^2(e+fx)\right)-\frac{1}{3}b\sec^3(e+fx)}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

output
```
-1/2*(a*Log[Cos[e + f*x]^2] + (2*a - b)*Sec[e + f*x] - ((a - 2*b)*Sec[e +
f*x]^2)/2 - (b*Sec[e + f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

method	result
parts	$\frac{a\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\tan(fx+e)^6}{6f}$
derivativedivides	$\frac{\frac{\sec(fx+e)^6 b}{6}+\frac{\sec(fx+e)^4 a}{4}-\frac{b\sec(fx+e)^4}{2}-a\sec(fx+e)^2+\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{\sec(fx+e)^6 b}{6}+\frac{\sec(fx+e)^4 a}{4}-\frac{b\sec(fx+e)^4}{2}-a\sec(fx+e)^2+\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$
risch	$iax+\frac{2iae}{f}+\frac{-4a\,e^{10i(fx+e)}+2b\,e^{10i(fx+e)}-12a\,e^{8i(fx+e)}-16a\,e^{6i(fx+e)}+\frac{20b\,e^{6i(fx+e)}}{3}-12a\,e^{4i(fx+e)}-4a\,e^{2i(}}{f\left(e^{2i(fx+e)}+1\right)^6}$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
a/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f*x+e)^2))+1/6*b/f*tan
(f*x+e)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx = -\frac{12\,a\cos\left(fx+e\right)^6\log\left(-\cos\left(fx+e\right)\right)+6\left(2\,a-b\right)\cos\left(fx+e\right)^4-3\left(a-2\,b\right)\cos\left(fx+e\right)^2-2\,b}{12\,f\cos\left(fx+e\right)^6}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/12*(12*a*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(2*a - b)*cos(f*x + e)^4
 - 3*(a - 2*b)*cos(f*x + e)^2 - 2*b)/(f*cos(f*x + e)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(56) = 112$.

Time = 0.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int (a + b\sec^2(e + fx))\tan^5(e + fx)\,dx$$
$$= \begin{cases} \frac{a\log(\tan^2(e+fx)+1)}{2f} + \frac{a\tan^4(e+fx)}{4f} - \frac{a\tan^2(e+fx)}{2f} + \frac{b\tan^4(e+fx)\sec^2(e+fx)}{6f} - \frac{b\tan^2(e+fx)\sec^2(e+fx)}{6f} + \frac{b\sec^2(e+fx)}{6f} \\ x(a + b\sec^2(e))\tan^5(e) \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**5,x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*
tan(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**2/(6*f) - b*tan(e
+ f*x)**2*sec(e + f*x)**2/(6*f) + b*sec(e + f*x)**2/(6*f), Ne(f, 0)), (x*(
a + b*sec(e)**2)*tan(e)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int (a + b\sec^2(e + fx))\tan^5(e + fx)\,dx$$
$$= -\frac{6\,a\log\left(\sin(fx+e)^2 - 1\right) - \frac{6\,(2\,a-b)\sin(fx+e)^4 - 3\,(7\,a-2\,b)\sin(fx+e)^2 + 9\,a - 2\,b}{\sin(fx+e)^6 - 3\,\sin(fx+e)^4 + 3\,\sin(fx+e)^2 - 1}}{12\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/12*(6*a*log(sin(f*x + e)^2 - 1) - (6*(2*a - b)*sin(f*x + e)^4 - 3*(7*a
- 2*b)*sin(f*x + e)^2 + 9*a - 2*b)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*
sin(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx = -\frac{a\log\left(|\cos\left(fx+e\right)|\right)}{f}-\frac{6\left(2a-b\right)\cos\left(fx+e\right)^4-3\left(a-2b\right)\cos\left(fx+e\right)^2-2b}{12f\cos\left(fx+e\right)^6}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
-a*log(abs(cos(f*x + e)))/f - 1/12*(6*(2*a - b)*cos(f*x + e)^4 - 3*(a - 2*
b)*cos(f*x + e)^2 - 2*b)/(f*cos(f*x + e)^6)
```

Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx = \frac{\frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2}-\frac{a\tan(e+f\,x)^2}{2}+\frac{a\tan(e+f\,x)^4}{4}+\frac{b\tan(e+f\,x)^6}{6}}{f}$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*log(tan(e + f*x)^2 + 1))/2 - (a*tan(e + f*x)^2)/2 + (a*tan(e + f*x)^4)
/4 + (b*tan(e + f*x)^6)/6)/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \left(a + b\sec^2(e + fx)\right)\tan^5(e + fx)\,dx$$
$$= \frac{6\log\left(\tan\left(fx+e\right)^2+1\right)a+2\sec\left(fx+e\right)^2\tan\left(fx+e\right)^4b-2\sec\left(fx+e\right)^2\tan\left(fx+e\right)^2b+2\sec\left(fx\right.}{12f}$$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x)
```

output
```
(6*log(tan(e + f*x)**2 + 1)*a + 2*sec(e + f*x)**2*tan(e + f*x)**4*b - 2*se
c(e + f*x)**2*tan(e + f*x)**2*b + 2*sec(e + f*x)**2*b + 3*tan(e + f*x)**4*
a - 6*tan(e + f*x)**2*a)/(12*f)
```

3.312 $\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx = \frac{a\log(\cos(e+fx))}{f} + \frac{(a-b)\sec^2(e+fx)}{2f} + \frac{b\sec^4(e+fx)}{4f}$$

output
```
a*ln(cos(f*x+e))/f+1/2*(a-b)*sec(f*x+e)^2/f+1/4*b*sec(f*x+e)^4/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx = \frac{a(2\log(\cos(e+fx))+\sec^2(e+fx))}{2f} + \frac{b\tan^4(e+fx)}{4f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
```

output
```
(a*(2*Log[Cos[e + f*x]] + Sec[e + f*x]^2))/(2*f) + (b*Tan[e + f*x]^4)/(4*f
)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)\sec^5(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)\sec^3(e+fx)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{85}$$

$$-\frac{\int \left(b\sec^3(e+fx)+(a-b)\sec^2(e+fx)-a\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-(a-b)\sec(e+fx)-a\log\left(\cos^2(e+fx)\right)-\frac{1}{2}b\sec^2(e+fx)}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
```

output
```
-1/2*(-(a*Log[Cos[e + f*x]^2]) - (a - b)*Sec[e + f*x] - (b*Sec[e + f*x]^2)
/2)/f
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

method	result	s
parts	$\frac{a\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\tan(fx+e)^4}{4f}$	4
derivativedivides	$\frac{\frac{b\sec(fx+e)^4}{4}+\frac{a\sec(fx+e)^2}{2}-\frac{b\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$	4
default	$\frac{\frac{b\sec(fx+e)^4}{4}+\frac{a\sec(fx+e)^2}{2}-\frac{b\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$	4
risch	$-iax-\frac{2iae}{f}-\frac{2\left(-a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}-a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^4}+\frac{a\ln\left(e^{2i(fx+e)}+1\right)}{f}$	1

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
a/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+1/4*b/f*tan(f*x+e)^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx = \frac{4\,a\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)+2\left(a-b\right)\cos\left(fx+e\right)^2+b}{4\,f\cos\left(fx+e\right)^4}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/4*(4*a*cos(f*x + e)^4*log(-cos(f*x + e)) + 2*(a - b)*cos(f*x + e)^2 + b)
/(f*cos(f*x + e)^4)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx$$
$$=\begin{cases} -\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{4f}-\frac{b\sec^2\left(e+fx\right)}{4f} & \text{for } f\neq 0 \\ x(a+b\sec^2\left(e\right))\tan^3\left(e\right) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**3,x)
```

output
```
Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b
*tan(e + f*x)**2*sec(e + f*x)**2/(4*f) - b*sec(e + f*x)**2/(4*f), Ne(f, 0)
), (x*(a + b*sec(e)**2)*tan(e)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx$$
$$=\frac{2\,a\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{2\,(a-b)\sin(fx+e)^2-2\,a+b}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}}{4\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/4*(2*a*log(sin(f*x + e)^2 - 1) - (2*(a - b)*sin(f*x + e)^2 - 2*a + b)/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx = \frac{a\log(|\cos(fx+e)|)}{f} + \frac{2(a-b)\cos(fx+e)^2+b}{4f\cos(fx+e)^4}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
a*log(abs(cos(f*x + e)))/f + 1/4*(2*(a - b)*cos(f*x + e)^2 + b)/(f*cos(f*x
 + e)^4)
```

Mupad [B] (verification not implemented)

Time = 16.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx = \frac{a\tan(e+fx)^2}{2f} - \frac{a\ln\left(\tan(e+fx)^2+1\right)}{2f} + \frac{b\tan(e+fx)^4}{4f}$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*tan(e + f*x)^2)/(2*f) - (a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e +
f*x)^4)/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx = \frac{-2\log\left(\tan(fx+e)^2+1\right)a+\sec(fx+e)^2\tan(fx+e)^2b-\sec(fx+e)^2b+2\tan(fx+e)^2a}{4f}$$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x)
```

output
```
( - 2*log(tan(e + f*x)**2 + 1)*a + sec(e + f*x)**2*tan(e + f*x)**2*b - sec
(e + f*x)**2*b + 2*tan(e + f*x)**2*a)/(4*f)
```

3.313 $\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx = -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f}$$

output
```
-a*ln(cos(f*x+e))/f+1/2*b*sec(f*x+e)^2/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx = -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

output
```
-((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4626, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)\sec^3(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(b\sec^3(e+fx)+a\sec(e+fx)\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\log(\cos(e+fx))-\frac{1}{2}b\sec^2(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

output
```
-((a*Log[Cos[e + f*x]] - (b*Sec[e + f*x]^2)/2)/f)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$	26
default	$\frac{\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$	26
parts	$\frac{a\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b\sec(fx+e)^2}{2f}$	33
risch	$iax+\frac{2iae}{f}+\frac{2b\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{a\ln\left(e^{2i(fx+e)}+1\right)}{f}$	61

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b*sec(f*x+e)^2+a*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx=-\frac{2\,a\cos\left(fx+e\right)^2\log\left(-\cos\left(fx+e\right)\right)-b}{2\,f\cos\left(fx+e\right)^2}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/2*(2*a*cos(f*x + e)^2*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx=\begin{cases}\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b\sec^2\left(e+fx\right)}{2f} & \text{for}\ f\neq 0\\ x(a+b\sec^2\left(e\right))\tan\left(e\right) & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e),x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**2/(2*f), Ne(
f, 0)), (x*(a + b*sec(e)**2)*tan(e), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx=-\frac{a\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{b}{\sin(fx+e)^2-1}}{2\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/2*(a*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx=-\frac{a\log\left(\left|\cos\left(fx+e\right)\right|\right)}{f}+\frac{b}{2\,f\cos\left(fx+e\right)^2}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="giac")
```

output
```
-a*log(abs(cos(f*x + e)))/f + 1/2*b/(f*cos(f*x + e)^2)
```

Mupad [B] (verification not implemented)

Time = 16.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx=\frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}+\frac{b\tan(e+f\,x)^2}{2\,f}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e + f*x)^2)/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx=\frac{\log\left(\tan\left(fx+e\right)^2+1\right)a+\sec\left(fx+e\right)^2b}{2f}$$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e),x)
```

output
```
(log(tan(e + f*x)**2 + 1)*a + sec(e + f*x)**2*b)/(2*f)
```

3.314 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{b\log(\cos(e+fx))}{f}+\frac{(a+b)\log(\sin(e+fx))}{f}$$

output
```
-b*ln(cos(f*x+e))/f+(a+b)*ln(sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{b\log(\cos(e+fx))}{f}+\frac{a\log(\sin(e+fx))}{f}+\frac{b\log(\sin(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((b*Log[Cos[e + f*x]])/f) + (a*Log[Sin[e + f*x]])/f + (b*Log[Sin[e + f*x]
])/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)\sec(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)\sec(e+fx)}{1-\cos^2(e+fx)}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{86}$$

$$-\frac{\int \left(\frac{-a-b}{\cos^2(e+fx)-1}+b\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\log\left(\cos^2(e+fx)\right)-(a+b)\log\left(1-\cos^2(e+fx)\right)}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(b*Log[Cos[e + f*x]^2] - (a + b)*Log[1 - Cos[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{a \ln(\sin(fx+e))+b \ln(\tan(fx+e))}{f}$	24
default	$\frac{a \ln(\sin(fx+e))+b \ln(\tan(fx+e))}{f}$	24
risch	$-iax-\frac{2iae}{f}+\frac{\ln(\mathrm{e}^{2i(fx+e)}-1)a}{f}+\frac{\ln(\mathrm{e}^{2i(fx+e)}-1)b}{f}-\frac{b\ln(\mathrm{e}^{2i(fx+e)}+1)}{f}$	67

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*ln(sin(f*x+e))+b*ln(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \cot(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = -\frac{b \log \left(\cos \left(fx+e\right)^{2}\right) - (a+b) \log \left(-\frac{1}{4} \, \cos \left(fx+e\right)^{2} + \frac{1}{4}\right)}{2 \, f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*log(cos(f*x + e)^2) - (a + b)*log(-1/4*cos(f*x + e)^2 + 1/4))/f
```

Sympy [F]

$$\int \cot(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = \int \left(a+b\sec^{2}\left(e+fx\right)\right) \cot \left(e+fx\right)\, dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{b\log\left(\sin\left(fx+e\right)^2-1\right)-(a+b)\log\left(\sin\left(fx+e\right)^2\right)}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(b*log(sin(f*x + e)^2 - 1) - (a + b)*log(sin(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(a+b)\log\left(\left|\cos\left(fx+e\right)^2-1\right|\right)}{2\,f} - \frac{b\log\left(\left|\cos\left(fx+e\right)\right|\right)}{f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(a + b)*log(abs(cos(f*x + e)^2 - 1))/f - b*log(abs(cos(f*x + e)))/f
```

Mupad [B] (verification not implemented)

Time = 15.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,(a+b)}{f} - \frac{a\,\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,f}$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

output
```
(log(tan(e + f*x))*(a + b))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.71

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)b-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)b+\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a}{f}$$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

output
```
( - log(tan((e + f*x)/2)**2 + 1)*a - log(tan((e + f*x)/2) - 1)*b - log(tan
((e + f*x)/2) + 1)*b + log(tan((e + f*x)/2))*a + log(tan((e + f*x)/2))*b)/
f
```

3.315 $\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = -\frac{(a+b)\csc^2(e+fx)}{2f} - \frac{a\log(\sin(e+fx))}{f}$$

output
```
-1/2*(a+b)*csc(f*x+e)^2/f-a*ln(sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = -\frac{a\csc^2(e+fx)}{2f} - \frac{b\csc^2(e+fx)}{2f} - \frac{a\log(\sin(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(a*Csc[e + f*x]^2)/f - (b*Csc[e + f*x]^2)/(2*f) - (a*Log[Sin[e + f*x]
])/f
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos(e+fx)\left(a\cos^2(e+fx)+b\right)}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{353}$$

$$-\frac{\int \frac{a\cos^2(e+fx)+b}{\left(1-\cos^2(e+fx)\right)^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(\frac{a}{\cos^2(e+fx)-1}+\frac{a+b}{\left(\cos^2(e+fx)-1\right)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a+b}{1-\cos^2(e+fx)}+a\log\left(1-\cos^2(e+fx)\right)}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*((a + b)/(1 - Cos[e + f*x]^2) + a*Log[1 - Cos[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{b}{2\sin(fx+e)^2}}{f}$	39
default	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{b}{2\sin(fx+e)^2}}{f}$	39
risch	$iax+\frac{2iae}{f}+\frac{2(a+b)\mathrm{e}^{2i(fx+e)}}{f(\mathrm{e}^{2i(fx+e)}-1)^2}-\frac{\ln(\mathrm{e}^{2i(fx+e)}-1)a}{f}$	63

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))-1/2*b/sin(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2\left(a\cos\left(fx+e\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(fx+e\right)\right)-a-b}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*(a*cos(f*x + e)^2 - a)*log(1/2*sin(f*x + e)) - a - b)/(f*cos(f*x +
 e)^2 - f)
```

Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cot^3\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\log\left(\sin\left(fx+e\right)^2\right)+\frac{a+b}{\sin(fx+e)^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(a*log(sin(f*x + e)^2) + (a + b)/sin(f*x + e)^2)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\log\left(|\sin\left(fx+e\right)|\right)}{f}-\frac{a+b}{2\,f\sin\left(fx+e\right)^2}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-a*log(abs(sin(f*x + e)))/f - 1/2*(a + b)/(f*sin(f*x + e)^2)
```

Mupad [B] (verification not implemented)

Time = 15.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}-\frac{a\,\ln\left(\tan(e+f\,x)\right)}{f} - \frac{\cot(e+f\,x)^2\,\left(\frac{a}{2}+\frac{b}{2}\right)}{f}$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*log(tan(e + f*x)^2 + 1))/(2*f) - (a*log(tan(e + f*x)))/f - (cot(e + f*x
)^2*(a/2 + b/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.69

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2a-4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(fx+e\right)^2a+\sin\left(fx+e\right)^2a+\sin\left(f\right.}{4\sin\left(fx+e\right)^2f}$$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

output
```
(4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a - 4*log(tan((e + f*x)/2)
)*sin(e + f*x)**2*a + sin(e + f*x)**2*a + sin(e + f*x)**2*b - 2*a - 2*b)/(
4*sin(e + f*x)**2*f)
```

3.316 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(2a+b)\csc^2(e+fx)}{2f}-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{a\log(\sin(e+fx))}{f}$$

output
```
1/2*(2*a+b)*csc(f*x+e)^2/f-1/4*(a+b)*csc(f*x+e)^4/f+a*ln(sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{b\cot^4(e+fx)}{4f}+\frac{a\csc^2(e+fx)}{f}-\frac{a\csc^4(e+fx)}{4f}+\frac{a\log(\sin(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/4*(b*Cot[e + f*x]^4)/f + (a*Csc[e + f*x]^2)/f - (a*Csc[e + f*x]^4)/(4*f
) + (a*Log[Sin[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \; 4626$$

$$-\frac{\int \frac{\cos^3(e+fx)\left(a\cos^2(e+fx)+b\right)}{\left(1-\cos^2(e+fx)\right)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 354$$

$$-\frac{\int \frac{\cos^2(e+fx)\left(a\cos^2(e+fx)+b\right)}{\left(1-\cos^2(e+fx)\right)^3}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \; 86$$

$$-\frac{\int \left(-\frac{a}{\cos^2(e+fx)-1}+\frac{-2a-b}{\left(\cos^2(e+fx)-1\right)^2}+\frac{-a-b}{\left(\cos^2(e+fx)-1\right)^3}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{a+b}{2\left(1-\cos^2(e+fx)\right)^2}-\frac{2a+b}{1-\cos^2(e+fx)}-a\log\left(1-\cos^2(e+fx)\right)}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*((a + b)/(2*(1 - Cos[e + f*x]^2)^2) - (2*a + b)/(1 - Cos[e + f*x]^2)
- a*Log[1 - Cos[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result	s
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{b\cos(fx+e)^4}{4\sin(fx+e)^4}}{f}$	5
default	$\frac{a\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{b\cos(fx+e)^4}{4\sin(fx+e)^4}}{f}$	5
risch	$-iax-\frac{2iae}{f}-\frac{2(2a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)})}{f(e^{2i(fx+e)}-1)^4}+\frac{\ln(e^{2i(fx+e)}-1)a}{f}$	1

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/4*cot(f*x+e)^4+1/2*cot(f*x+e)^2+ln(sin(f*x+e)))-1/4*b/sin(f*x+e
)^4*cos(f*x+e)^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2\,(2\,a+b)\cos\left(fx+e\right)^2-4\left(a\cos\left(fx+e\right)^4-2\,a\cos\left(fx+e\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)-3\,a-b}{4\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/4*(2*(2*a + b)*cos(f*x + e)^2 - 4*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^
2 + a)*log(1/2*sin(f*x + e)) - 3*a - b)/(f*cos(f*x + e)^4 - 2*f*cos(f*x +
e)^2 + f)
```

Sympy [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\cot^5{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{2\,a\log\left(\sin\left(fx+e\right)^2\right)+\frac{2\,(2\,a+b)\sin(fx+e)^2-a-b}{\sin(fx+e)^4}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*(2*a*log(sin(f*x + e)^2) + (2*(2*a + b)*sin(f*x + e)^2 - a - b)/sin(f*
x + e)^4)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{a\log\left(\left|\cos\left(fx+e\right)^2-1\right|\right)}{2\,f}-\frac{2\,(2\,a+b)\cos\left(fx+e\right)^2-3\,a-b}{4\,\left(\cos\left(fx+e\right)^2-1\right)^2f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*a*log(abs(cos(f*x + e)^2 - 1))/f - 1/4*(2*(2*a + b)*cos(f*x + e)^2 - 3
*a - b)/((cos(f*x + e)^2 - 1)^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.64 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{a\ln\left(\tan(e+f\,x)\right)}{f}-\frac{a\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}-\frac{-\frac{a\tan(e+f\,x)^2}{2}+\frac{a}{4}+\frac{b}{4}}{f\tan\left(e+f\,x\right)^4}$$

input
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*log(tan(e + f*x)))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f) - (a/4 + b/4 -
 (a*tan(e + f*x)^2)/2)/(f*tan(e + f*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{-32\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^4a+32\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(fx+e\right)^4a-13\sin\left(fx+e\right)^4a-}{32\sin\left(fx+e\right)^4f}$$

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

output
```
( - 32*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a + 32*log(tan((e + f*
x)/2))*sin(e + f*x)**4*a - 13*sin(e + f*x)**4*a - 5*sin(e + f*x)**4*b + 32
*sin(e + f*x)**2*a + 16*sin(e + f*x)**2*b - 8*a - 8*b)/(32*sin(e + f*x)**4
*f)
```

3.317 $\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx=-ax+\frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}$$

output
```
-a*x+a*tan(f*x+e)/f-1/3*a*tan(f*x+e)^3/f+1/5*a*tan(f*x+e)^5/f+1/7*b*tan(f*
x+e)^7/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx=-\frac{a\arctan(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

output
```
-((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3
*f) + (a*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4629, 2075, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^6\left(a+b\sec(e+fx)^2\right)dx$$

$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\tan^6(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 363$$

$$\frac{a\int \frac{\tan^6(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{1}{7}b\tan^7(e+fx)}{f}$$

$$\Big\downarrow \; 254$$

$$\frac{a\int \left(\tan^4(e+fx)-\tan^2(e+fx)-\frac{1}{\tan^2(e+fx)+1}+1\right)d\tan(e+fx)+\frac{1}{7}b\tan^7(e+fx)}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\left(-\arctan(\tan(e+fx))+\frac{1}{5}\tan^5(e+fx)-\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)+\frac{1}{7}b\tan^7(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

output
```
((b*Tan[e + f*x]^7)/7 + a*(-ArcTan[Tan[e + f*x]] + Tan[e + f*x] - Tan[e +
f*x]^3/3 + Tan[e + f*x]^5/5))/f
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result
parts	$\frac{a\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}+\frac{b\tan(fx+e)^7}{7f}$
derivativedivides	$\frac{a\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{b\sin(fx+e)^7}{7\cos(fx+e)^7}}{f}$
default	$\frac{a\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{b\sin(fx+e)^7}{7\cos(fx+e)^7}}{f}$
risch	$-ax-\frac{2i(-315a\,e^{12i(fx+e)}+105b\,e^{12i(fx+e)}-1260a\,e^{10i(fx+e)}-2555a\,e^{8i(fx+e)}+525b\,e^{8i(fx+e)}-3080a\,e^{6i(fx+e}}{105f\left(e^{2i(fx+e)}+1\right)^7}$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
a/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-arctan(tan(f*x+e)))+1/7*
b*tan(f*x+e)^7/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.39

$$\int\left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx=$$
$$-\frac{105\,afx\cos\left(fx+e\right)^7-\left((161\,a-15\,b)\cos\left(fx+e\right)^6-(77\,a-45\,b)\cos\left(fx+e\right)^4+3\,(7\,a-15\,b)\cos\right.}{105\,f\cos\left(fx+e\right)^7}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="fricas")
```

output
```
-1/105*(105*a*f*x*cos(f*x + e)^7 - ((161*a - 15*b)*cos(f*x + e)^6 - (77*a
- 45*b)*cos(f*x + e)^4 + 3*(7*a - 15*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e
))/(f*cos(f*x + e)^7)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx = a\left(\begin{cases} -x+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f} & \text{for } f\neq 0\\ x\tan^6(e) & \text{otherwise}\end{cases}\right) + b\left(\begin{cases} x\tan^6(e)\sec^2(e) & \text{for } f=0\\ \frac{\tan^7(e+fx)}{7f} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**6,x)
```

output
```
a*Piecewise((-x + tan(e + f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e +
f*x)/f, Ne(f, 0)), (x*tan(e)**6, True)) + b*Piecewise((x*tan(e)**6*sec(e)*
*2, Eq(f, 0)), (tan(e + f*x)**7/(7*f), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx = \frac{15\,b\tan(fx+e)^7+21\,a\tan(fx+e)^5-35\,a\tan(fx+e)^3-105\,(fx+e)a+105\,a\tan(fx+e)}{105\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="maxima")
```

output
```
1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 1
05*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx = -\frac{(fx+e)a}{f} + \frac{15\,bf^6\tan\left(fx+e\right)^7+21\,af^6\tan\left(fx+e\right)^5-35\,af^6\tan\left(fx+e\right)^3+105\,af^6\tan\left(fx+e\right)}{105\,f^7}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="giac")
```

output
```
-(f*x + e)*a/f + 1/105*(15*b*f^6*tan(f*x + e)^7 + 21*a*f^6*tan(f*x + e)^5
- 35*a*f^6*tan(f*x + e)^3 + 105*a*f^6*tan(f*x + e))/f^7
```

Mupad [B] (verification not implemented)

Time = 15.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx = \frac{\frac{b\tan(e+f\,x)^7}{7}+\frac{a\tan(e+f\,x)^5}{5}-\frac{a\tan(e+f\,x)^3}{3}+a\tan(e+f\,x)-a\,f\,x}{f}$$

input
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*tan(e + f*x) - (a*tan(e + f*x)^3)/3 + (a*tan(e + f*x)^5)/5 + (b*tan(e +
 f*x)^7)/7 - a*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 393, normalized size of antiderivative = 6.14

$$\int (a + b\sec^2(e + fx)) \tan^6(e + fx) \, dx$$
$$= \frac{21\cos(fx+e)\sin(fx+e)^6\tan(fx+e)^5 a - 35\cos(fx+e)\sin(fx+e)^6\tan(fx+e)^3 a + 105\cos(fx$$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x)
```

output
```
(21*cos(e + f*x)*sin(e + f*x)**6*tan(e + f*x)**5*a - 35*cos(e + f*x)*sin(e
 + f*x)**6*tan(e + f*x)**3*a + 105*cos(e + f*x)*sin(e + f*x)**6*tan(e + f*
x)*a - 105*cos(e + f*x)*sin(e + f*x)**6*a*f*x - 63*cos(e + f*x)*sin(e + f*
x)**4*tan(e + f*x)**5*a + 105*cos(e + f*x)*sin(e + f*x)**4*tan(e + f*x)**3
*a - 315*cos(e + f*x)*sin(e + f*x)**4*tan(e + f*x)*a + 315*cos(e + f*x)*si
n(e + f*x)**4*a*f*x + 63*cos(e + f*x)*sin(e + f*x)**2*tan(e + f*x)**5*a -
105*cos(e + f*x)*sin(e + f*x)**2*tan(e + f*x)**3*a + 315*cos(e + f*x)*sin(
e + f*x)**2*tan(e + f*x)*a - 315*cos(e + f*x)*sin(e + f*x)**2*a*f*x - 21*c
os(e + f*x)*tan(e + f*x)**5*a + 35*cos(e + f*x)*tan(e + f*x)**3*a - 105*co
s(e + f*x)*tan(e + f*x)*a + 105*cos(e + f*x)*a*f*x - 15*sin(e + f*x)**7*b)
/(105*cos(e + f*x)*f*(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e + f*x)
**2 - 1))
```

3.318 $\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = ax-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

output
```
a*x-a*tan(f*x+e)/f+1/3*a*tan(f*x+e)^3/f+1/5*b*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = \frac{a\arctan(\tan(e+fx))}{f}-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

output
```
(a*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f)
 + (b*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4629, 2075, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)dx$$

$$\Big\downarrow \text{ 4629}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 2075}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 363}$$

$$\frac{a\int \frac{\tan^4(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{1}{5}b\tan^5(e+fx)}{f}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{a\int \left(\tan^2(e+fx)+\frac{1}{\tan^2(e+fx)+1}-1\right)d\tan(e+fx)+\frac{1}{5}b\tan^5(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a\left(\arctan(\tan(e+fx))+\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)+\frac{1}{5}b\tan^5(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

output
```
((b*Tan[e + f*x]^5)/5 + a*(ArcTan[Tan[e + f*x]] - Tan[e + f*x] + Tan[e + f
*x]^3/3))/f
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

method	result
parts	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{b\tan(fx+e)^5}{5f}$
derivativedivides	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{b\sin(fx+e)^5}{5\cos(fx+e)^5}}{f}$
default	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{b\sin(fx+e)^5}{5\cos(fx+e)^5}}{f}$
risch	$ax+\frac{2i\left(-30a\,\mathrm{e}^{8i(fx+e)}+15b\,\mathrm{e}^{8i(fx+e)}-90a\,\mathrm{e}^{6i(fx+e)}-110a\,\mathrm{e}^{4i(fx+e)}+30b\,\mathrm{e}^{4i(fx+e)}-70a\,\mathrm{e}^{2i(fx+e)}-20a+3b\right)}{15f\left(\mathrm{e}^{2i(fx+e)}+1\right)^5}$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
a/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+1/5*b*tan(f*x+e)^5/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx$$
$$=\frac{15\,afx\cos\left(fx+e\right)^5-\left(\left(20\,a-3\,b\right)\cos\left(fx+e\right)^4-\left(5\,a-6\,b\right)\cos\left(fx+e\right)^2-3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/15*(15*a*f*x*cos(f*x + e)^5 - ((20*a - 3*b)*cos(f*x + e)^4 - (5*a - 6*b)
*cos(f*x + e)^2 - 3*b)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx = a\left(\begin{cases} x+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f} & \text{for } f\neq 0 \\ x\tan^4(e) & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} x\tan^4(e)\sec^2(e) & \text{for } f=0 \\ \frac{\tan^5(e+fx)}{5f} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**4,x)
```

output
```
a*Piecewise((x + tan(e + f*x)**3/(3*f) - tan(e + f*x)/f, Ne(f, 0)), (x*tan
(e)**4, True)) + b*Piecewise((x*tan(e)**4*sec(e)**2, Eq(f, 0)), (tan(e + f
*x)**5/(5*f), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx = \frac{3\,b\tan(fx+e)^5+5\,a\tan(fx+e)^3+15\,(fx+e)a-15\,a\tan(fx+e)}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(
f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx$$
$$=\frac{(fx+e)a}{f}+\frac{3\,bf^4\tan{(fx+e)}^5+5\,af^4\tan{(fx+e)}^3-15\,af^4\tan{(fx+e)}}{15\,f^5}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
(f*x + e)*a/f + 1/15*(3*b*f^4*tan(f*x + e)^5 + 5*a*f^4*tan(f*x + e)^3 - 15
*a*f^4*tan(f*x + e))/f^5
```

Mupad [B] (verification not implemented)

Time = 15.57 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx=\frac{\frac{b\tan(e+f\,x)^5}{5}+\frac{a\tan(e+f\,x)^3}{3}-a\tan(e+f\,x)+a\,f\,x}{f}$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*tan(e + f*x)^3)/3 - a*tan(e + f*x) + (b*tan(e + f*x)^5)/5 + a*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 4.67

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx$$
$$=\frac{5\cos{(fx+e)}\sin{(fx+e)}^4\tan{(fx+e)}^3a-15\cos{(fx+e)}\sin{(fx+e)}^4\tan{(fx+e)}a+15\cos{(fx+}}{}$$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x)
```

output
```
(5*cos(e + f*x)*sin(e + f*x)**4*tan(e + f*x)**3*a - 15*cos(e + f*x)*sin(e
+ f*x)**4*tan(e + f*x)*a + 15*cos(e + f*x)*sin(e + f*x)**4*a*f*x - 10*cos(
e + f*x)*sin(e + f*x)**2*tan(e + f*x)**3*a + 30*cos(e + f*x)*sin(e + f*x)*
*2*tan(e + f*x)*a - 30*cos(e + f*x)*sin(e + f*x)**2*a*f*x + 5*cos(e + f*x)
*tan(e + f*x)**3*a - 15*cos(e + f*x)*tan(e + f*x)*a + 15*cos(e + f*x)*a*f*
x + 3*sin(e + f*x)**5*b)/(15*cos(e + f*x)*f*(sin(e + f*x)**4 - 2*sin(e + f
*x)**2 + 1))
```

3.319 $\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx=-ax+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

output
```
-a*x+a*tan(f*x+e)/f+1/3*b*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int (a+b\sec^2(e+fx))\tan^2(e+fx)\,dx=-\frac{a\arctan(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

output
```
-((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3
*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4629, 2075, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{363}$$

$$\frac{a\int \frac{\tan^2(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{1}{3}b\tan^3(e+fx)}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{a\left(\tan(e+fx)-\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)\right)+\frac{1}{3}b\tan^3(e+fx)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{a(\tan(e+fx)-\arctan(\tan(e+fx)))+\frac{1}{3}b\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

output
```
((b*Tan[e + f*x]^3)/3 + a*(-ArcTan[Tan[e + f*x]] + Tan[e + f*x]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
parts	$\frac{a(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{b\tan(fx+e)^3}{3f}$	37
derivativedivides	$\frac{a(\tan(fx+e)-fx-e)+\frac{b\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$	41
default	$\frac{a(\tan(fx+e)-fx-e)+\frac{b\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$	41
risch	$-ax-\frac{2i\left(-3a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}-3a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	66

input

```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output

```
a/f*(tan(f*x+e)-arctan(tan(f*x+e)))+1/3*b*tan(f*x+e)^3/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx$$
$$=-\frac{3\,afx\cos\left(fx+e\right)^3-\left((3\,a-b)\cos\left(fx+e\right)^2+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input

```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output

```
-1/3*(3*a*f*x*cos(f*x + e)^3 - ((3*a - b)*cos(f*x + e)^2 + b)*sin(f*x + e)
)/(f*cos(f*x + e)^3)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = a\left(\begin{cases} -x+\frac{\tan(e+fx)}{f} & \text{for } f\neq 0 \\ x\tan^2(e) & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} x\tan^2(e)\sec^2(e) & \text{for } f=0 \\ \frac{\tan^3(e+fx)}{3f} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**2,x)
```

output
```
a*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True)) + b*Piec
ewise((x*tan(e)**2*sec(e)**2, Eq(f, 0)), (tan(e + f*x)**3/(3*f), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = \frac{b\tan(fx+e)^3 - 3(fx+e)a + 3a\tan(fx+e)}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = -\frac{(fx+e)a}{f} + \frac{bf^2\tan(fx+e)^3 + 3af^2\tan(fx+e)}{3f^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
-(f*x + e)*a/f + 1/3*(b*f^2*tan(f*x + e)^3 + 3*a*f^2*tan(f*x + e))/f^3
```

Mupad [B] (verification not implemented)

Time = 15.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx=\frac{\frac{b\tan(e+f\,x)^3}{3}+a\tan(e+f\,x)-a\,f\,x}{f}$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*tan(e + f*x) + (b*tan(e + f*x)^3)/3 - a*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.28

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx$$
$$=\frac{3\cos(fx+e)\sin(fx+e)^2\tan(fx+e)\,a-3\cos(fx+e)\sin(fx+e)^2\,afx-3\cos(fx+e)\tan(fx+\ldots}{3\cos(fx+e)\,f\left(\sin(fx+e)^2-1\right)}$$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x)
```

output
```
(3*cos(e + f*x)*sin(e + f*x)**2*tan(e + f*x)*a - 3*cos(e + f*x)*sin(e + f*
x)**2*a*f*x - 3*cos(e + f*x)*tan(e + f*x)*a + 3*cos(e + f*x)*a*f*x - sin(e
 + f*x)**3*b)/(3*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.320 $\int \left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan(e+fx)}{f}$$

output
```
a*x+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$ax+\frac{b\tan(e+fx)}{f}$$

input
```
Int[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\tan(fx+e)}{f}$	16
parts	$ax+\frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
parallelrisch	$\frac{b\sin(fx+e)}{\cos(fx+e)f}+ax$	24
risch	$ax+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	25
norman	$\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-ax-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}$	51

input
```
int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*tan(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

output
```
(a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)\,dx$$

input
```
integrate(a+b*sec(f*x+e)**2,x)
```

output
```
Integral(a + b*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan{(fx+e)}}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

output
```
a*x + b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=ax+\frac{b\tan{(fx+e)}}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

output
```
a*x + b*tan(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 15.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\frac{b\tan(e+f\,x)+a\,f\,x}{f}$$

input
```
int(a + b/cos(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x) + a*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(a+b\sec^2(e+fx)\right)\,dx=\frac{\cos\left(fx+e\right)afx+\sin\left(fx+e\right)b}{\cos\left(fx+e\right)f}$$

input
```
int(a+b*sec(f*x+e)^2,x)
```

output
```
(cos(e + f*x)*a*f*x + sin(e + f*x)*b)/(cos(e + f*x)*f)
```

3.321 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right) dx = -ax-\frac{(a+b)\cot(e+fx)}{f}$$

output
```
-a*x-(a+b)*cot(f*x+e)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right) dx = -\frac{b\cot(e+fx)}{f}-\frac{a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((b*Cot[e + f*x])/f) - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -T
an[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4629, 2075, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{-a\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-((a+b)\cot(e+fx))}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{-a\arctan(\tan(e+fx))-(a+b)\cot(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-(a*ArcTan[Tan[e + f*x]]) - (a + b)*Cot[e + f*x])/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

method	result	size
derivativedivides	$\frac{a(-\cot(fx+e)-fx-e)-b\cot(fx+e)}{f}$	33
default	$\frac{a(-\cot(fx+e)-fx-e)-b\cot(fx+e)}{f}$	33
risch	$-ax-\frac{2ia}{f\left(\mathrm{e}^{2i(fx+e)}-1\right)}-\frac{2ib}{f\left(\mathrm{e}^{2i(fx+e)}-1\right)}$	46

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-cot(f*x+e)-f*x-e)-b*cot(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{afx\sin(fx+e)+(a+b)\cos(fx+e)}{f\sin(fx+e)}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-(a*f*x*sin(f*x + e) + (a + b)*cos(f*x + e))/(f*sin(f*x + e))
```

Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2(e+fx)\right)\cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{(fx+e)a+\frac{a+b}{\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-((f*x + e)*a + (a + b)/tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(19) = 38.

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=-\frac{2\,(fx+e)a-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\frac{a+b}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(2*(f*x + e)*a - a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e) + (a
 + b)/tan(1/2*f*x + 1/2*e))/f
```

Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-a\,x-\frac{\cot(e+f\,x)\,(a+b)}{f}$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

output
```
- a*x - (cot(e + f*x)*(a + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{-\cos\left(fx+e\right)a-\cos\left(fx+e\right)b-\sin\left(fx+e\right)afx}{\sin\left(fx+e\right)f}$$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
```

output
```
( - (cos(e + f*x)*a + cos(e + f*x)*b + sin(e + f*x)*a*f*x))/(sin(e + f*x)*
f)
```

3.322 $\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = ax + \frac{a\cot(e+fx)}{f} - \frac{(a+b)\cot^3(e+fx)}{3f}$$

output
```
a*x+a*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = -\frac{b\cot^3(e+fx)}{3f} - \frac{a\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3f}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/3*(b*Cot[e + f*x]^3)/f - (a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -
1/2, -Tan[e + f*x]^2])/(3*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4629, 2075, 359, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{-a\int \frac{\cot^2(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{1}{3}(a+b)\cot^3(e+fx)}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{-a\left(-\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-\cot(e+fx)\right)-\frac{1}{3}(a+b)\cot^3(e+fx)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{-a(-\arctan(\tan(e+fx))-\cot(e+fx))-\frac{1}{3}(a+b)\cot^3(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-(a*(-ArcTan[Tan[e + f*x]] - Cot[e + f*x])) - ((a + b)*Cot[e + f*x]^3)/3)
/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)-\frac{b\cos(fx+e)^3}{3\sin(fx+e)^3}}{f}$	48
default	$\frac{a\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)-\frac{b\cos(fx+e)^3}{3\sin(fx+e)^3}}{f}$	48
risch	$ax+\frac{2i\left(6a\,\mathrm{e}^{4i(fx+e)}+3b\,\mathrm{e}^{4i(fx+e)}-6a\,\mathrm{e}^{2i(fx+e)}+4a+b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}-1\right)^3}$	65

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-1/3*b/sin(f*x+e)^3*cos(f*x+e)^
3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(31) = 62.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.30

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{(4\,a+b)\cos\left(fx+e\right)^3-3\,a\cos\left(fx+e\right)+3\left(afx\cos\left(fx+e\right)^2-afx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*((4*a + b)*cos(f*x + e)^3 - 3*a*cos(f*x + e) + 3*(a*f*x*cos(f*x + e)^2
 - a*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)\cot^4\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{3\,(fx+e)a+\frac{3\,a\tan(fx+e)^2-a-b}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)*a + (3*a*tan(f*x + e)^2 - a - b)/tan(f*x + e)^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+24\,(fx+e)a-15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+}{24\,f}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/24*(a*tan(1/2*f*x + 1/2*e)^3 + b*tan(1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a
 - 15*a*tan(1/2*f*x + 1/2*e) - 3*b*tan(1/2*f*x + 1/2*e) + (15*a*tan(1/2*f*
x + 1/2*e)^2 + 3*b*tan(1/2*f*x + 1/2*e)^2 - a - b)/tan(1/2*f*x + 1/2*e)^3)
/f
```

Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = a\,x-\frac{-a\tan(e+f\,x)^2+\frac{a}{3}+\frac{b}{3}}{f\tan\left(e+f\,x\right)^3}$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

output
```
a*x - (a/3 + b/3 - a*tan(e + f*x)^2)/(f*tan(e + f*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{4\cos\left(fx+e\right)\sin\left(fx+e\right)^2a+\cos\left(fx+e\right)\sin\left(fx+e\right)^2b-\cos\left(fx+e\right)a-\cos\left(fx+e\right)b+3\sin\left(fx\right.}{3\sin\left(fx+e\right)^3f}$$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

output
```
(4*cos(e + f*x)*sin(e + f*x)**2*a + cos(e + f*x)*sin(e + f*x)**2*b - cos(e
 + f*x)*a - cos(e + f*x)*b + 3*sin(e + f*x)**3*a*f*x)/(3*sin(e + f*x)**3*f
)
```

3.323 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-ax-\frac{a\cot(e+fx)}{f}+\frac{a\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}$$

output
```
-a*x-a*cot(f*x+e)/f+1/3*a*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=-\frac{b\cot^5(e+fx)}{5f}-\frac{a\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5f}$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/5*(b*Cot[e + f*x]^5)/f - (a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -
3/2, -Tan[e + f*x]^2])/(5*f)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4629, 2075, 359, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{-a\int \frac{\cot^4(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{1}{5}(a+b)\cot^5(e+fx)}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{-a\left(-\int \frac{\cot^2(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{1}{3}\cot^3(e+fx)\right)-\frac{1}{5}(a+b)\cot^5(e+fx)}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{-a\left(\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{1}{3}\cot^3(e+fx)+\cot(e+fx)\right)-\frac{1}{5}(a+b)\cot^5(e+fx)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{-a\left(\arctan(\tan(e+fx))-\frac{1}{3}\cot^3(e+fx)+\cot(e+fx)\right)-\frac{1}{5}(a+b)\cot^5(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-1/5*((a + b)*Cot[e + f*x]^5) - a*(ArcTan[Tan[e + f*x]] + Cot[e + f*x] -
Cot[e + f*x]^3/3))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{b\cos(fx+e)^5}{5\sin(fx+e)^5}}{f}$
default	$\frac{a\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{b\cos(fx+e)^5}{5\sin(fx+e)^5}}{f}$
risch	$-ax-\frac{2i\left(45a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-90a\,e^{6i(fx+e)}+140a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-70a\,e^{2i(fx+e)}+23a+3b\right)}{15f\left(e^{2i(fx+e)}-1\right)^5}$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-1/5*b/sin(f*x
+e)^5*cos(f*x+e)^5)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(47) = 94$.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx =$$
$$-\frac{(23\,a+3\,b)\cos(fx+e)^5-35\,a\cos(fx+e)^3+15\,a\cos(fx+e)+15\left(afx\cos(fx+e)^4-2\,afx\cos\right.}{15\left(f\cos(fx+e)^4-2\,f\cos(fx+e)^2+f\right)\sin(fx+e)}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/15*((23*a + 3*b)*cos(f*x + e)^5 - 35*a*cos(f*x + e)^3 + 15*a*cos(f*x +
e) + 15*(a*f*x*cos(f*x + e)^4 - 2*a*f*x*cos(f*x + e)^2 + a*f*x)*sin(f*x +
e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f*x + e))
```

Sympy [F]

$$\int \cot^6(e + fx) \left(a + b\sec^2(e + fx)\right) \, dx = \int \left(a + b\sec^2{(e + fx)}\right) \cot^6{(e + fx)} \, dx$$

input
```
integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right) \, dx = -\frac{15\,(fx+e)a + \frac{15\,a\tan(fx+e)^4 - 5\,a\tan(fx+e)^2 + 3\,a + 3\,b}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*(f*x + e)*a + (15*a*tan(f*x + e)^4 - 5*a*tan(f*x + e)^2 + 3*a +
3*b)/tan(f*x + e)^5)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(47) = 94$.

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.33

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-480\,(fx}{}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/480*(3*a*tan(1/2*f*x + 1/2*e)^5 + 3*b*tan(1/2*f*x + 1/2*e)^5 - 35*a*tan(
1/2*f*x + 1/2*e)^3 - 15*b*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a + 330*a
*tan(1/2*f*x + 1/2*e) + 30*b*tan(1/2*f*x + 1/2*e) - (330*a*tan(1/2*f*x + 1
/2*e)^4 + 30*b*tan(1/2*f*x + 1/2*e)^4 - 35*a*tan(1/2*f*x + 1/2*e)^2 - 15*b
*tan(1/2*f*x + 1/2*e)^2 + 3*a + 3*b)/tan(1/2*f*x + 1/2*e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 15.96 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -a\,x-\frac{a\tan(e+f\,x)^4-\frac{a\tan(e+f\,x)^2}{3}+\frac{a}{5}+\frac{b}{5}}{f\tan\left(e+f\,x\right)^5}$$

input
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

output
```
- a*x - (a/5 + b/5 - (a*tan(e + f*x)^2)/3 + a*tan(e + f*x)^4)/(f*tan(e + f
*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.22

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{-23\cos(fx+e)\sin(fx+e)^4 a-3\cos(fx+e)\sin(fx+e)^4 b+11\cos(fx+e)\sin(fx+e)^2 a+6\cos}{15\sin(fx+e)^5 f}$$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

output
```
( - 23*cos(e + f*x)*sin(e + f*x)**4*a - 3*cos(e + f*x)*sin(e + f*x)**4*b +
 11*cos(e + f*x)*sin(e + f*x)**2*a + 6*cos(e + f*x)*sin(e + f*x)**2*b - 3*
cos(e + f*x)*a - 3*cos(e + f*x)*b - 15*sin(e + f*x)**5*a*f*x)/(15*sin(e +
f*x)**5*f)
```

3.324 $\int (a+b\sec^2(e+fx))^2\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\sec^2(e+fx))^2\tan^5(e+fx)\,dx = -\frac{a^2\log(\cos(e+fx))}{f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{(a^2-4ab+b^2)\sec^4(e+fx)}{4f}+\frac{(a-b)b\sec^6(e+fx)}{3f}+\frac{b^2\sec^8(e+fx)}{8f}$$

output
```
-a^2*ln(cos(f*x+e))/f-a*(a-b)*sec(f*x+e)^2/f+1/4*(a^2-4*a*b+b^2)*sec(f*x+e
)^4/f+1/3*(a-b)*b*sec(f*x+e)^6/f+1/8*b^2*sec(f*x+e)^8/f
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.26

$$\int (a+b\sec^2(e+fx))^2\tan^5(e+fx)\,dx = -\frac{\cos^4(e+fx)\,(a+b\sec^2(e+fx))^2\,(24a^2\log(\cos(e+fx))+24a(a-b)\sec^2(e+fx)-6(a^2-4ab+b}{6f(a+2b+a\cos(2e+2fx))^2}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

output
```
-1/6*(Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(24*a^2*Log[Cos[e + f*x]] +
24*a*(a - b)*Sec[e + f*x]^2 - 6*(a^2 - 4*a*b + b^2)*Sec[e + f*x]^4 - 8*(a
- b)*b*Sec[e + f*x]^6 - 3*b^2*Sec[e + f*x]^8))/(f*(a + 2*b + a*Cos[2*e + 2
*f*x])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2\sec^9(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2\sec^5(e+fx)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(b^2\sec^5(e+fx)+2(a-b)b\sec^4(e+fx)+\left(a^2-4ba+b^2\right)\sec^3(e+fx)-2a(a-b)\sec^2(e+fx)+a^2\sec(e}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{2}\left(a^2-4ab+b^2\right)\sec^2(e+fx)+a^2\log\left(\cos^2(e+fx)\right)-\frac{2}{3}b(a-b)\sec^3(e+fx)+2a(a-b)\sec(e+fx)-\frac{1}{4}b}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

output
```
-1/2*(a^2*Log[Cos[e + f*x]^2] + 2*a*(a - b)*Sec[e + f*x] - ((a^2 - 4*a*b +
 b^2)*Sec[e + f*x]^2)/2 - (2*(a - b)*b*Sec[e + f*x]^3)/3 - (b^2*Sec[e + f*
x]^4)/4)/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

method	result
parts	$\frac{a^{2}\left(\frac{\tan(fx+e)^{4}}{4}-\frac{\tan(fx+e)^{2}}{2}+\frac{\ln\left(1+\tan(fx+e)^{2}\right)}{2}\right)}{f}+\frac{b^{2}\left(\frac{\tan(fx+e)^{8}}{8}+\frac{\tan(fx+e)^{6}}{6}\right)}{f}+\frac{ab\tan(fx+e)^{6}}{3f}$
derivativedivides	$\frac{\frac{\sec(fx+e)^{8}b^{2}}{8}+\frac{\sec(fx+e)^{6}ab}{3}-\frac{b^{2}\sec(fx+e)^{6}}{3}+\frac{\sec(fx+e)^{4}a^{2}}{4}-ab\sec(fx+e)^{4}+\frac{\sec(fx+e)^{4}b^{2}}{4}-a^{2}\sec(fx+e)^{2}+\sec(fx+e)}{f}$
default	$\frac{\frac{\sec(fx+e)^{8}b^{2}}{8}+\frac{\sec(fx+e)^{6}ab}{3}-\frac{b^{2}\sec(fx+e)^{6}}{3}+\frac{\sec(fx+e)^{4}a^{2}}{4}-ab\sec(fx+e)^{4}+\frac{\sec(fx+e)^{4}b^{2}}{4}-a^{2}\sec(fx+e)^{2}+\sec(fx+e)}{f}$
risch	$ia^{2}x+\frac{2ia^{2}e}{f}+\frac{-4a^{2}e^{14i(fx+e)}+4ab\,e^{14i(fx+e)}-20\,e^{12i(fx+e)}a^{2}+8ab\,e^{12i(fx+e)}+4b^{2}e^{12i(fx+e)}-44\,e^{10i(fx+e)}a}{}$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f*x+e)^2))+b^2/f*(1/
8*tan(f*x+e)^8+1/6*tan(f*x+e)^6)+1/3*a*b/f*tan(f*x+e)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx = -\frac{24\,a^{2}\cos\left(fx+e\right)^{8}\log\left(-\cos\left(fx+e\right)\right)+24\left(a^{2}-ab\right)\cos\left(fx+e\right)^{6}-6\left(a^{2}-4\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}}{24\,f\cos\left(fx+e\right)^{8}}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/24*(24*a^2*cos(f*x + e)^8*log(-cos(f*x + e)) + 24*(a^2 - a*b)*cos(f*x +
 e)^6 - 6*(a^2 - 4*a*b + b^2)*cos(f*x + e)^4 - 8*(a*b - b^2)*cos(f*x + e)^
2 - 3*b^2)/(f*cos(f*x + e)^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(85) = 170$.

Time = 1.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.90

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx$$
$$= \begin{cases} \frac{a^2\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{a^2\tan^4(e+fx)}{4f}-\frac{a^2\tan^2(e+fx)}{2f}+\frac{ab\tan^4(e+fx)\sec^2(e+fx)}{3f}-\frac{ab\tan^2(e+fx)\sec^2(e+fx)}{3f}+\frac{ab\sec^2}{3} \\ x\left(a+b\sec^2(e)\right)^2\tan^5(e) \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**5,x)
```

output
```
Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f
) - a**2*tan(e + f*x)**2/(2*f) + a*b*tan(e + f*x)**4*sec(e + f*x)**2/(3*f)
 - a*b*tan(e + f*x)**2*sec(e + f*x)**2/(3*f) + a*b*sec(e + f*x)**2/(3*f) +
 b**2*tan(e + f*x)**4*sec(e + f*x)**4/(8*f) - b**2*tan(e + f*x)**2*sec(e +
 f*x)**4/(12*f) + b**2*sec(e + f*x)**4/(24*f), Ne(f, 0)), (x*(a + b*sec(e)
**2)**2*tan(e)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx =$$
$$-\frac{12\,a^2\log\left(\sin(fx+e)^2-1\right)-\frac{24\,(a^2-ab)\sin(fx+e)^6-6\,(11\,a^2-8\,ab-b^2)\sin(fx+e)^4+4\,(15\,a^2-8\,ab-b^2)\sin(fx+e)^2-18\,a^2+}{\sin(fx+e)^8-4\,\sin(fx+e)^6+6\,\sin(fx+e)^4-4\,\sin(fx+e)^2+1}}{24\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/24*(12*a^2*log(sin(f*x + e)^2 - 1) - (24*(a^2 - a*b)*sin(f*x + e)^6 - 6
*(11*a^2 - 8*a*b - b^2)*sin(f*x + e)^4 + 4*(15*a^2 - 8*a*b - b^2)*sin(f*x
+ e)^2 - 18*a^2 + 8*a*b + b^2)/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(
f*x + e)^4 - 4*sin(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx=-\frac{a^2\log\left(|\cos\left(fx+e\right)|\right)}{f}$$
$$-\frac{24\left(a^2-ab\right)\cos\left(fx+e\right)^6-6\left(a^2-4ab+b^2\right)\cos\left(fx+e\right)^4-8\left(ab-b^2\right)\cos\left(fx+e\right)^2-3b^2}{24f\cos\left(fx+e\right)^8}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
-a^2*log(abs(cos(f*x + e)))/f - 1/24*(24*(a^2 - a*b)*cos(f*x + e)^6 - 6*(a
^2 - 4*a*b + b^2)*cos(f*x + e)^4 - 8*(a*b - b^2)*cos(f*x + e)^2 - 3*b^2)/(
f*cos(f*x + e)^8)
```

Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.24

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx=\frac{\tan(e+f\,x)^4\left(\frac{(a+b)^2}{4}+\frac{b^2}{4}-\frac{b\,(a+b)}{2}\right)}{f}$$
$$-\frac{\tan(e+f\,x)^2\left(\frac{(a+b)^2}{2}+\frac{b^2}{2}-b\,(a+b)\right)}{f}$$
$$-\frac{\tan(e+f\,x)^6\left(\frac{b^2}{6}-\frac{b\,(a+b)}{3}\right)}{f}$$
$$+\frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}$$
$$+\frac{b^2\tan(e+f\,x)^8}{8\,f}$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)^4*((a + b)^2/4 + b^2/4 - (b*(a + b))/2))/f - (tan(e + f*x)^2
*((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^6*(b^2/6 - (b*(a + b
))/3))/f + (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e + f*x)^8)/(8*f
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.54

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx$$
$$= \frac{12\log\left(\tan\left(fx+e\right)^2+1\right)a^2+3\sec\left(fx+e\right)^4\tan\left(fx+e\right)^4b^2-2\sec\left(fx+e\right)^4\tan\left(fx+e\right)^2b^2+\sec\left(f\right.}{}$$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x)
```

output
```
(12*log(tan(e + f*x)**2 + 1)*a**2 + 3*sec(e + f*x)**4*tan(e + f*x)**4*b**2
 - 2*sec(e + f*x)**4*tan(e + f*x)**2*b**2 + sec(e + f*x)**4*b**2 + 8*sec(e
 + f*x)**2*tan(e + f*x)**4*a*b - 8*sec(e + f*x)**2*tan(e + f*x)**2*a*b + 8
*sec(e + f*x)**2*a*b + 6*tan(e + f*x)**4*a**2 - 12*tan(e + f*x)**2*a**2)/(
24*f)
```

3.325 $\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx=\frac{a^2\log(\cos(e+fx))}{f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{(2a-b)b\sec^4(e+fx)}{4f}+\frac{b^2\sec^6(e+fx)}{6f}$$

output
```
a^2*ln(cos(f*x+e))/f+1/2*a*(a-2*b)*sec(f*x+e)^2/f+1/4*(2*a-b)*b*sec(f*x+e)
^4/f+1/6*b^2*sec(f*x+e)^6/f
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(12a^2\log(\cos(e+fx))+6a(a-2b)\sec^2(e+fx)+3(2a-b)b\sec^4(e\right.}{3f(a+2b+a\cos(2e+2fx))^2}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

output
```
(Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a^2*Log[Cos[e + f*x]] + 6*a*(
a - 2*b)*Sec[e + f*x]^2 + 3*(2*a - b)*b*Sec[e + f*x]^4 + 2*b^2*Sec[e + f*x
]^6))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{ 4626}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2\sec^7(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 85}$$

$$-\frac{\int \left(b^2\sec^4(e+fx)+(2a-b)b\sec^3(e+fx)+a(a-2b)\sec^2(e+fx)-a^2\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-a^2\log\left(\cos^2(e+fx)\right)-\frac{1}{2}b(2a-b)\sec^2(e+fx)-a(a-2b)\sec(e+fx)-\frac{1}{3}b^2\sec^3(e+fx)}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

output
```
-1/2*(-(a^2*Log[Cos[e + f*x]^2]) - a*(a - 2*b)*Sec[e + f*x] - ((2*a - b)*b
*Sec[e + f*x]^2)/2 - (b^2*Sec[e + f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

method	result
parts	$\frac{a^2\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b^2\left(\frac{\sec(fx+e)^6}{6}-\frac{\sec(fx+e)^4}{4}\right)}{f}+\frac{ab\tan(fx+e)^4}{2f}$
derivativedivides	$\frac{\frac{b^2\sec(fx+e)^6}{6}+\frac{ab\sec(fx+e)^4}{2}-\frac{\sec(fx+e)^4b^2}{4}+\frac{a^2\sec(fx+e)^2}{2}-\sec(fx+e)^2ab-a^2\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{b^2\sec(fx+e)^6}{6}+\frac{ab\sec(fx+e)^4}{2}-\frac{\sec(fx+e)^4b^2}{4}+\frac{a^2\sec(fx+e)^2}{2}-\sec(fx+e)^2ab-a^2\ln(\sec(fx+e))}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}-\frac{2\left(-3\,e^{10i(fx+e)}a^2+6\,e^{10i(fx+e)}ab-12a^2e^{8i(fx+e)}+12\,e^{8i(fx+e)}ab+6\,e^{8i(fx+e)}b^2-18a^2e^{6i(fx}\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+b^2/f*(1/6*sec(f*x+e)^6-1/
4*sec(f*x+e)^4)+1/2*a*b/f*tan(f*x+e)^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\frac{12\,a^2\cos\left(fx+e\right)^6\log\left(-\cos\left(fx+e\right)\right)+6\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4+3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+2\,b^2}{12\,f\cos\left(fx+e\right)^6}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/12*(12*a^2*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(a^2 - 2*a*b)*cos(f*x +
 e)^4 + 3*(2*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)/(f*cos(f*x + e)^6)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.66

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\begin{cases} -\frac{a^2\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{a^2\tan^2(e+fx)}{2f}+\frac{ab\tan^2(e+fx)\sec^2(e+fx)}{2f}-\frac{ab\sec^2(e+fx)}{2f}+\frac{b^2\tan^2(e+fx)\sec^4(e+fx)}{6f}-b^2\sec \\ x\left(a+b\sec^2(e)\right)^2\tan^3(e) \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**3,x)
```

output
```
Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*
f) + a*b*tan(e + f*x)**2*sec(e + f*x)**2/(2*f) - a*b*sec(e + f*x)**2/(2*f)
 + b**2*tan(e + f*x)**2*sec(e + f*x)**4/(6*f) - b**2*sec(e + f*x)**4/(12*f
), Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\frac{6a^2\log\left(\sin(fx+e)^2-1\right)-\frac{6(a^2-2ab)\sin(fx+e)^4-3(4a^2-6ab-b^2)\sin(fx+e)^2+6a^2-6ab-b^2}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}}{12f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/12*(6*a^2*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 2*a*b)*sin(f*x + e)^4 - 3*
(4*a^2 - 6*a*b - b^2)*sin(f*x + e)^2 + 6*a^2 - 6*a*b - b^2)/(sin(f*x + e)^
6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx = \frac{a^2\log\left(\left|\cos\left(fx+e\right)\right|\right)}{f} + \frac{6\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4+3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+2\,b^2}{12\,f\cos\left(fx+e\right)^6}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
a^2*log(abs(cos(f*x + e)))/f + 1/12*(6*(a^2 - 2*a*b)*cos(f*x + e)^4 + 3*(2
*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)/(f*cos(f*x + e)^6)
```

Mupad [B] (verification not implemented)

Time = 15.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx = \frac{\tan(e+f\,x)^2\left(\frac{(a+b)^2}{2}+\frac{b^2}{2}-b\,(a+b)\right)}{f} - \frac{\tan(e+f\,x)^4\left(\frac{b^2}{4}-\frac{b\,(a+b)}{2}\right)}{f} - \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} + \frac{b^2\tan(e+f\,x)^6}{6\,f}$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)^2*((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^4*(b^
2/4 - (b*(a + b))/2))/f - (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e
 + f*x)^6)/(6*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\frac{-6\log\left(\tan(fx+e)^2+1\right)a^2+2\sec(fx+e)^4\tan(fx+e)^2b^2-\sec(fx+e)^4b^2+6\sec(fx+e)^2\tan(}{12f}$$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x)
```

output
```
( - 6*log(tan(e + f*x)**2 + 1)*a**2 + 2*sec(e + f*x)**4*tan(e + f*x)**2*b*
*2 - sec(e + f*x)**4*b**2 + 6*sec(e + f*x)**2*tan(e + f*x)**2*a*b - 6*sec(
e + f*x)**2*a*b + 6*tan(e + f*x)**2*a**2)/(12*f)
```

3.326 $\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx = -\frac{a^2\log(\cos(e+fx))}{f} + \frac{ab\sec^2(e+fx)}{f} + \frac{b^2\sec^4(e+fx)}{4f}$$

output
```
-a^2*ln(cos(f*x+e))/f+a*b*sec(f*x+e)^2/f+1/4*b^2*sec(f*x+e)^4/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx = -\frac{\cos^4(e+fx)\,(a+b\sec^2(e+fx))^2\,(4a^2\log(\cos(e+fx))-4ab\sec^2(e+fx)-b^2\sec^4(e+fx))}{f(a+2b+a\cos(2e+2fx))^2}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

output
```
-((Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(4*a^2*Log[Cos[e + f*x]] - 4*a*
b*Sec[e + f*x]^2 - b^2*Sec[e + f*x]^4))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^
2))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)^2\sec^5(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)^2\sec^3(e+fx)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(b^2\sec^3(e+fx)+2ab\sec^2(e+fx)+a^2\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\log\left(\cos^2(e+fx)\right)-2ab\sec(e+fx)-\frac{1}{2}b^2\sec^2(e+fx)}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

output
```
-1/2*(a^2*Log[Cos[e + f*x]^2] - 2*a*b*Sec[e + f*x] - (b^2*Sec[e + f*x]^2)/
2)/f
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 1.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\sec(fx+e)^4 b^2}{4}+\sec(fx+e)^2 ab+a^2\ln(\sec(fx+e))}{f}$	41
default	$\frac{\frac{\sec(fx+e)^4 b^2}{4}+\sec(fx+e)^2 ab+a^2\ln(\sec(fx+e))}{f}$	41
parts	$\frac{a^2\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b^2\sec(fx+e)^4}{4f}+\frac{ab\sec(fx+e)^2}{f}$	51
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{4b\left(a\,\mathrm{e}^{6i(fx+e)}+2a\,\mathrm{e}^{4i(fx+e)}+b\,\mathrm{e}^{4i(fx+e)}+a\,\mathrm{e}^{2i(fx+e)}\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^4}-\frac{a^2\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{f}$	104

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*sec(f*x+e)^4*b^2+sec(f*x+e)^2*a*b+a^2*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan(e+fx)\,dx = -\frac{4\,a^2\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)-4\,ab\cos\left(fx+e\right)^2-b^2}{4\,f\cos\left(fx+e\right)^4}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/4*(4*a^2*cos(f*x + e)^4*log(-cos(f*x + e)) - 4*a*b*cos(f*x + e)^2 - b^2
)/(f*cos(f*x + e)^4)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan(e+fx)\,dx = \begin{cases} \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{ab\sec^2\left(e+fx\right)}{f}+\frac{b^2\sec^4\left(e+fx\right)}{4f} & \text{for } f\neq 0 \\ x\left(a+b\sec^2\left(e\right)\right)^2\tan\left(e\right) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e),x)
```

output
```
Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a*b*sec(e + f*x)**2/f + b
**2*sec(e + f*x)**4/(4*f), Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan(e+fx)\,dx = -\frac{2\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{4\,ab\sin\left(fx+e\right)^2-4\,ab-b^2}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2+1}}{4\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/4*(2*a^2*log(sin(f*x + e)^2 - 1) + (4*a*b*sin(f*x + e)^2 - 4*a*b - b^2)
/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx = -\frac{a^2\log(|\cos(fx+e)|)}{f} + \frac{4\,ab\cos(fx+e)^2+b^2}{4\,f\cos(fx+e)^4}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="giac")
```

output
```
-a^2*log(abs(cos(f*x + e)))/f + 1/4*(4*a*b*cos(f*x + e)^2 + b^2)/(f*cos(f*
x + e)^4)
```

Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx = \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} - \frac{\tan(e+f\,x)^2\,\left(\frac{b^2}{2}-b\,(a+b)\right)}{f} + \frac{b^2\tan(e+f\,x)^4}{4\,f}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (tan(e + f*x)^2*(b^2/2 - b*(a + b)))
/f + (b^2*tan(e + f*x)^4)/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int (a+b\sec^2(e+fx))^2\tan(e+fx)\,dx = \frac{2\log\left(\tan(fx+e)^2+1\right)a^2+\sec(fx+e)^4b^2+4\sec(fx+e)^2ab}{4f}$$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x)
```

output
```
(2*log(tan(e + f*x)**2 + 1)*a**2 + sec(e + f*x)**4*b**2 + 4*sec(e + f*x)**
2*a*b)/(4*f)
```

3.327 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{b(2a+b)\log(\cos(e+fx))}{f} + \frac{(a+b)^2\log(\sin(e+fx))}{f} + \frac{b^2\sec^2(e+fx)}{2f}$$

output
```
-b*(2*a+b)*ln(cos(f*x+e))/f+(a+b)^2*ln(sin(f*x+e))/f+1/2*b^2*sec(f*x+e)^2/
f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{2(b^2+2\cos^2(e+fx)\left(-b(2a+b)\log(\cos(e+fx))+(a+b)^2\log(\sin(e+fx))\right))\left(a\cos(e+fx)+b\sec(e\right.}{f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(2*(b^2 + 2*Cos[e + f*x]^2*(-(b*(2*a + b)*Log[Cos[e + f*x]]) + (a + b)^2*L
og[Sin[e + f*x]]))*(a*Cos[e + f*x] + b*Sec[e + f*x])^2)/(f*(a + 2*b + a*Co
s[2*(e + f*x)])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec^3(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(-\frac{(a+b)^2}{\cos^2(e+fx)-1}+b^2\sec^2(e+fx)+b(2a+b)\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b(2a+b)\log\left(\cos^2(e+fx)\right)-(a+b)^2\log\left(1-\cos^2(e+fx)\right)+b^2(-\sec(e+fx))}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(b*(2*a + b)*Log[Cos[e + f*x]^2] - (a + b)^2*Log[1 - Cos[e + f*x]^2]
- b^2*Sec[e + f*x])/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^{2} \ln(\sin(fx+e))+2ab\ln(\tan(fx+e))+b^{2}\left(\frac{1}{2\cos(fx+e)^{2}}+\ln(\tan(fx+e))\right)}{f}$
default	$\frac{a^{2} \ln(\sin(fx+e))+2ab\ln(\tan(fx+e))+b^{2}\left(\frac{1}{2\cos(fx+e)^{2}}+\ln(\tan(fx+e))\right)}{f}$
risch	$-ia^{2}x-\frac{2ia^{2}e}{f}+\frac{2b^{2}e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^{2}}+\frac{\ln\left(e^{2i(fx+e)}-1\right)a^{2}}{f}+\frac{2\ln\left(e^{2i(fx+e)}-1\right)ab}{f}+\frac{\ln\left(e^{2i(fx+e)}-1\right)b^{2}}{f}-$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*ln(sin(f*x+e))+2*a*b*ln(tan(f*x+e))+b^2*(1/2/cos(f*x+e)^2+ln(tan(
f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(2\,ab+b^2)\cos\left(fx+e\right)^2\log\left(\cos\left(fx+e\right)^2\right)-(a^2+2\,ab+b^2)\cos\left(fx+e\right)^2\log\left(-\frac{1}{4}\,\cos\left(fx+e\right)^2+\frac{1}{4}\right.}{2\,f\cos\left(fx+e\right)^2}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((2*a*b + b^2)*cos(f*x + e)^2*log(cos(f*x + e)^2) - (a^2 + 2*a*b + b^
2)*cos(f*x + e)^2*log(-1/4*cos(f*x + e)^2 + 1/4) - b^2)/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int\left(a+b\sec^2(e+fx)\right)^2\cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx= -\frac{(2\,ab+b^2)\log\left(\sin(fx+e)^2-1\right)-(a^2+2\,ab+b^2)\log\left(\sin(fx+e)^2\right)+\frac{b^2}{\sin(fx+e)^2-1}}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (a^2 + 2*a*b + b^2)*log(sin(
f*x + e)^2) + b^2/(sin(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{(a^2+2\,ab+b^2)\log\left(\left|\cos(fx+e)^2-1\right|\right)}{2\,f}-\frac{(2\,ab+b^2)\log\left(\left|\cos(fx+e)\right|\right)}{f}+\frac{b^2}{2\,f\cos(fx+e)^2}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2 + 2*a*b + b^2)*log(abs(cos(f*x + e)^2 - 1))/f - (2*a*b + b^2)*log
(abs(cos(f*x + e)))/f + 1/2*b^2/(f*cos(f*x + e)^2)
```

Mupad [B] (verification not implemented)

Time = 15.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+2\,a\,b+b^2\right)}{f} - \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} + \frac{b^2\tan(e+f\,x)^2}{2\,f}$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(log(tan(e + f*x))*(2*a*b + a^2 + b^2))/f - (a^2*log(tan(e + f*x)^2 + 1))/
(2*f) + (b^2*tan(e + f*x)^2)/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 353, normalized size of antiderivative = 6.66

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2a^2+2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a^2-4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin}{}$$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**2 + 2*log(tan((e + f
*x)/2)**2 + 1)*a**2 - 4*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*b - 2*
log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*b**2 + 4*log(tan((e + f*x)/2) -
1)*a*b + 2*log(tan((e + f*x)/2) - 1)*b**2 - 4*log(tan((e + f*x)/2) + 1)*si
n(e + f*x)**2*a*b - 2*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*b**2 + 4*l
og(tan((e + f*x)/2) + 1)*a*b + 2*log(tan((e + f*x)/2) + 1)*b**2 + 2*log(ta
n((e + f*x)/2))*sin(e + f*x)**2*a**2 + 4*log(tan((e + f*x)/2))*sin(e + f*x
)**2*a*b + 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*b**2 - 2*log(tan((e + f
*x)/2))*a**2 - 4*log(tan((e + f*x)/2))*a*b - 2*log(tan((e + f*x)/2))*b**2
- sin(e + f*x)**2*b**2)/(2*f*(sin(e + f*x)**2 - 1))
```

3.328 $\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = -\frac{(a+b)^2\csc^2(e+fx)}{2f} - \frac{b^2\log(\cos(e+fx))}{f} - \frac{(a^2-b^2)\log(\sin(e+fx))}{f}$$

output
```
-1/2*(a+b)^2*csc(f*x+e)^2/f-b^2*ln(cos(f*x+e))/f-(a^2-b^2)*ln(sin(f*x+e))/
f
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = -\frac{2(b+a\cos^2(e+fx))^2\left((a+b)^2\csc^2(e+fx)+2b^2\log(\cos(e+fx))+2(a^2-b^2)\log(\sin(e+fx))\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-2*(b + a*Cos[e + f*x]^2)^2*((a + b)^2*Csc[e + f*x]^2 + 2*b^2*Log[Cos[e +
 f*x]] + 2*(a^2 - b^2)*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)
])^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec(e+fx)}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec(e+fx)}{\left(1-\cos^2(e+fx)\right)^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\sec(e+fx)b^2+\frac{a^2-b^2}{\cos^2(e+fx)-1}+\frac{(a+b)^2}{\left(\cos^2(e+fx)-1\right)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a^2-b^2)\log\left(1-\cos^2(e+fx)\right)+\frac{(a+b)^2}{1-\cos^2(e+fx)}+b^2\log\left(\cos^2(e+fx)\right)}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*((a + b)^2/(1 - Cos[e + f*x]^2) + b^2*Log[Cos[e + f*x]^2] + (a^2 - b^
2)*Log[1 - Cos[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{ab}{\sin(fx+e)^2}+b^2\left(-\frac{1}{2\sin(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
default	$\frac{a^2\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{ab}{\sin(fx+e)^2}+b^2\left(-\frac{1}{2\sin(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{2(a^2+2ab+b^2)\,\mathrm{e}^{2i(fx+e)}}{f\left(\mathrm{e}^{2i(fx+e)}-1\right)^2}-\frac{b^2\ln(\mathrm{e}^{2i(fx+e)}+1)}{f}-\frac{\ln(\mathrm{e}^{2i(fx+e)}-1)a^2}{f}+\frac{\ln(\mathrm{e}^{2i(fx+e)}-1)b^2}{f}$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))-a*b/sin(f*x+e)^2+b^2*(-1/2/sin
(f*x+e)^2+ln(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{a^2+2\,ab+b^2-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(\cos\left(fx+e\right)^2\right)-\left(\left(a^2-b^2\right)\cos\left(fx+e\right)^2-a^2+b^2\right)\log\left(-\ldots\right)}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a^2 + 2*a*b + b^2 - (b^2*cos(f*x + e)^2 - b^2)*log(cos(f*x + e)^2) -
((a^2 - b^2)*cos(f*x + e)^2 - a^2 + b^2)*log(-1/4*cos(f*x + e)^2 + 1/4))/(
f*cos(f*x + e)^2 - f)
```

Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cot^3\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{b^2\log\left(\sin\left(fx+e\right)^2-1\right)+\left(a^2-b^2\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{a^2+2\,ab+b^2}{\sin(fx+e)^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - b^2)*log(sin(f*x + e)^2) + (a^2
 + 2*a*b + b^2)/sin(f*x + e)^2)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{b^2\log\left(\left|\cos\left(fx+e\right)\right|\right)}{f}-\frac{\left(a^2-b^2\right)\log\left(\left|\cos\left(fx+e\right)^2-1\right|\right)}{2\,f}+\frac{a^2+2\,ab+b^2}{2\left(\cos\left(fx+e\right)^2-1\right)f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-b^2*log(abs(cos(f*x + e)))/f - 1/2*(a^2 - b^2)*log(abs(cos(f*x + e)^2 - 1
))/f + 1/2*(a^2 + 2*a*b + b^2)/((cos(f*x + e)^2 - 1)*f)
```

Mupad [B] (verification not implemented)

Time = 15.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{a^2\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} - \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2-b^2\right)}{f} - \frac{\cot(e+f\,x)^2\,\left(\frac{a^2}{2}+a\,b+\frac{b^2}{2}\right)}{f}$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (log(tan(e + f*x))*(a^2 - b^2))/f -
(cot(e + f*x)^2*(a*b + a^2/2 + b^2/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.28

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2a^2-4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^2b^2-4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{}$$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**2 - 4*log(tan((e + f*x)
/2) - 1)*sin(e + f*x)**2*b**2 - 4*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**
2*b**2 - 4*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**2 + 4*log(tan((e + f*x
)/2))*sin(e + f*x)**2*b**2 + sin(e + f*x)**2*a**2 + 2*sin(e + f*x)**2*a*b
+ sin(e + f*x)**2*b**2 - 2*a**2 - 4*a*b - 2*b**2)/(4*sin(e + f*x)**2*f)
```

3.329 $\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{a(a+b)\csc^2(e+fx)}{f} - \frac{(a+b)^2\csc^4(e+fx)}{4f} + \frac{a^2\log(\sin(e+fx))}{f}$$

output
```
a*(a+b)*csc(f*x+e)^2/f-1/4*(a+b)^2*csc(f*x+e)^4/f+a^2*ln(sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = -\frac{\left(b+a\cos^2(e+fx)\right)^2 \left(-4a(a+b)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)-4a^2\log(\sin(e+fx))\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-(((b + a*Cos[e + f*x]^2)^2*(-4*a*(a + b)*Csc[e + f*x]^2 + (a + b)^2*Csc[e
 + f*x]^4 - 4*a^2*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos(e+fx)\left(a\cos^2(e+fx)+b\right)^2}{\left(1-\cos^2(e+fx)\right)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{353}$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2}{\left(1-\cos^2(e+fx)\right)^3}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(-\frac{a^2}{\cos^2(e+fx)-1}-\frac{2(a+b)a}{\left(\cos^2(e+fx)-1\right)^2}-\frac{(a+b)^2}{\left(\cos^2(e+fx)-1\right)^3}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(-\log\left(1-\cos^2(e+fx)\right)\right)-\frac{2a(a+b)}{1-\cos^2(e+fx)}+\frac{(a+b)^2}{2\left(1-\cos^2(e+fx)\right)^2}}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*((a + b)^2/(2*(1 - Cos[e + f*x]^2)^2) - (2*a*(a + b))/(1 - Cos[e + f*
x]^2) - a^2*Log[1 - Cos[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{ab\cos(fx+e)^4}{2\sin(fx+e)^4}-\frac{b^2}{4\sin(fx+e)^4}}{f}$
default	$\frac{a^2\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{ab\cos(fx+e)^4}{2\sin(fx+e)^4}-\frac{b^2}{4\sin(fx+e)^4}}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}-\frac{4(a^2e^{6i(fx+e)}+ab\,e^{6i(fx+e)}-a^2e^{4i(fx+e)}+b^2e^{4i(fx+e)}+a^2e^{2i(fx+e)}+ab\,e^{2i(fx+e)})}{f(e^{2i(fx+e)}-1)^4}+\frac{\ln(e^{2i}$

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/4*cot(f*x+e)^4+1/2*cot(f*x+e)^2+ln(sin(f*x+e)))-1/2*a*b/sin(f
*x+e)^4*cos(f*x+e)^4-1/4*b^2/sin(f*x+e)^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{4\left(a^2+ab\right)\cos\left(fx+e\right)^2-3\,a^2-2\,ab+b^2-4\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\log\left(\frac{1}{2}\sin\right.}{4\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - 3*a^2 - 2*a*b + b^2 - 4*(a^2*cos(f*x
+ e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*log(1/2*sin(f*x + e)))/(f*cos(f*x + e
)^4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^5\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2\,a^2\log\left(\sin\left(fx+e\right)^2\right)+\frac{4\,(a^2+ab)\sin(fx+e)^2-a^2-2\,ab-b^2}{\sin(fx+e)^4}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*a^2*log(sin(f*x + e)^2) + (4*(a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a
*b - b^2)/sin(f*x + e)^4)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{a^2\log\left(\left|\cos\left(fx+e\right)^2-1\right|\right)}{2\,f}-\frac{4\,(a^2+ab)\cos\left(fx+e\right)^2-3\,a^2-2\,ab+b^2}{4\,\left(\cos\left(fx+e\right)^2-1\right)^2f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*a^2*log(abs(cos(f*x + e)^2 - 1))/f - 1/4*(4*(a^2 + a*b)*cos(f*x + e)^2
 - 3*a^2 - 2*a*b + b^2)/((cos(f*x + e)^2 - 1)^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{a^2\,\ln\left(\tan(e+f\,x)\right)}{f} - \frac{\frac{a\,b}{2}+\frac{a^2}{4}+\frac{b^2}{4}-\tan(e+f\,x)^2\left(\frac{a^2}{2}-\frac{b^2}{2}\right)}{f\tan\left(e+f\,x\right)^4} - \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}$$

input
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(a^2*log(tan(e + f*x)))/f - ((a*b)/2 + a^2/4 + b^2/4 - tan(e + f*x)^2*(a^2
/2 - b^2/2))/(f*tan(e + f*x)^4) - (a^2*log(tan(e + f*x)^2 + 1))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.76

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{-32\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^4a^2+32\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(fx+e\right)^4a^2-13\sin\left(fx+e\right)^4a}{32\sin\left(f\right.}$$

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 32*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**2 + 32*log(tan((e +
 f*x)/2))*sin(e + f*x)**4*a**2 - 13*sin(e + f*x)**4*a**2 - 10*sin(e + f*x)
**4*a*b + 3*sin(e + f*x)**4*b**2 + 32*sin(e + f*x)**2*a**2 + 32*sin(e + f*
x)**2*a*b - 8*a**2 - 16*a*b - 8*b**2)/(32*sin(e + f*x)**4*f)
```

3.330 $\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx=-a^2x+\frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

output
```
-a^2*x+a^2*tan(f*x+e)/f-1/3*a^2*tan(f*x+e)^3/f+1/5*a^2*tan(f*x+e)^5/f+1/7*
b*(2*a+b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 275 vs. 2(95) = 190.

Time = 2.31 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.89

$$\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx = -\frac{4(b+a\cos^2(e+fx))^2\sec^9(e+fx)\left(315a^2fx\cos^9(e+fx)-35b^2\sec(e)\sin(fx)-5(18a-19b)b\cos^2\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

output
```
(-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^9*(315*a^2*f*x*Cos[e + f*x]^9 -
35*b^2*Sec[e]*Sin[f*x] - 5*(18*a - 19*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x]
- 3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^4*Sec[e]*Sin[f*x] + (231*a^2 -
 270*a*b + 5*b^2)*Cos[e + f*x]^6*Sec[e]*Sin[f*x] - (483*a^2 - 90*a*b - 10*
b^2)*Cos[e + f*x]^8*Sec[e]*Sin[f*x] - 35*b^2*Cos[e + f*x]*Tan[e] - 5*(18*a
 - 19*b)*b*Cos[e + f*x]^3*Tan[e] - 3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*
x]^5*Tan[e] + (231*a^2 - 270*a*b + 5*b^2)*Cos[e + f*x]^7*Tan[e]))/(315*f*(
a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2dx$$

$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\tan^6(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 364$$

$$\frac{\int \left(b^2 \tan^8(e+fx)+b(2a+b)\tan^6(e+fx)+a^2\tan^4(e+fx)-a^2\tan^2(e+fx)+a^2-\frac{a^2}{\tan^2(e+fx)+1}\right)d\tan(e+}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^2\arctan(\tan(e+fx))+\frac{1}{5}a^2\tan^5(e+fx)-\frac{1}{3}a^2\tan^3(e+fx)+a^2\tan(e+fx)+\frac{1}{7}b(2a+b)\tan^7(e+fx)+}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

output
```
(-(a^2*ArcTan[Tan[e + f*x]]) + a^2*Tan[e + f*x] - (a^2*Tan[e + f*x]^3)/3 +
 (a^2*Tan[e + f*x]^5)/5 + (b*(2*a + b)*Tan[e + f*x]^7)/7 + (b^2*Tan[e + f*
x]^9)/9)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 7.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result
parts	$\frac{a^{2}\left(\frac{\tan(fx+e)^{5}}{5}-\frac{\tan(fx+e)^{3}}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}+\frac{b^{2}\left(\frac{\tan(fx+e)^{9}}{9}+\frac{\tan(fx+e)^{7}}{7}\right)}{f}+\frac{2ab\tan(fx+e}{7f}$
derivativedivides	$\frac{a^{2}\left(\frac{\tan(fx+e)^{5}}{5}-\frac{\tan(fx+e)^{3}}{3}+\tan(fx+e)-fx-e\right)+\frac{2ab\sin(fx+e)^{7}}{7\cos(fx+e)^{7}}+b^{2}\left(\frac{\sin(fx+e)^{7}}{9\cos(fx+e)^{9}}+\frac{2\sin(fx+e)^{7}}{63\cos(fx+e)^{7}}\right)}{f}$
default	$\frac{a^{2}\left(\frac{\tan(fx+e)^{5}}{5}-\frac{\tan(fx+e)^{3}}{3}+\tan(fx+e)-fx-e\right)+\frac{2ab\sin(fx+e)^{7}}{7\cos(fx+e)^{7}}+b^{2}\left(\frac{\sin(fx+e)^{7}}{9\cos(fx+e)^{9}}+\frac{2\sin(fx+e)^{7}}{63\cos(fx+e)^{7}}\right)}{f}$
risch	$-a^{2}x-\frac{2i\left(10b^{2}-483a^{2}+90ab-28350\,\mathrm{e}^{10i(fx+e)}a^{2}+3150b^{2}\mathrm{e}^{10i(fx+e)}-32508a^{2}\mathrm{e}^{8i(fx+e)}-1890\,\mathrm{e}^{8i(fx+e)}b^{2}+189\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-arctan(tan(f*x+e)))+b^
2/f*(1/9*tan(f*x+e)^9+1/7*tan(f*x+e)^7)+2/7*a*b/f*tan(f*x+e)^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.44

$$\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx =$$
$$-\frac{315\,a^{2}fx\cos(fx+e)^{9}-\left((483\,a^{2}-90\,ab-10\,b^{2})\cos(fx+e)^{8}-(231\,a^{2}-270\,ab+5\,b^{2})\cos(fx+e\right.}{315\,f\cos(f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="fricas")
```

output
```
-1/315*(315*a^2*f*x*cos(f*x + e)^9 - ((483*a^2 - 90*a*b - 10*b^2)*cos(f*x
+ e)^8 - (231*a^2 - 270*a*b + 5*b^2)*cos(f*x + e)^6 + 3*(21*a^2 - 90*a*b +
 25*b^2)*cos(f*x + e)^4 + 5*(18*a*b - 19*b^2)*cos(f*x + e)^2 + 35*b^2)*sin
(f*x + e))/(f*cos(f*x + e)^9)
```

Sympy [F]

$$\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx=\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**6,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx = \frac{35\,b^2\tan(fx+e)^9+45\,(2\,ab+b^2)\tan(fx+e)^7+63\,a^2\tan(fx+e)^5-105\,a^2\tan(fx+e)^3-315\,(fx}{315\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="maxima")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b + b^2)*tan(f*x + e)^7 + 63*a^2*ta
n(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 315*a^2*tan(f*
x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx = -\frac{(fx+e)a^2}{f} + \frac{35\,b^2f^8\tan\left(fx+e\right)^9+90\,abf^8\tan\left(fx+e\right)^7+45\,b^2f^8\tan\left(fx+e\right)^7+63\,a^2f^8\tan\left(fx+e\right)^5-105\,a^2}{315\,f^9}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="giac")
```

output
```
-(f*x + e)*a^2/f + 1/315*(35*b^2*f^8*tan(f*x + e)^9 + 90*a*b*f^8*tan(f*x +
 e)^7 + 45*b^2*f^8*tan(f*x + e)^7 + 63*a^2*f^8*tan(f*x + e)^5 - 105*a^2*f^
8*tan(f*x + e)^3 + 315*a^2*f^8*tan(f*x + e))/f^9
```

Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx = \frac{\tan(e+f\,x)\,\left(\left(a+b\right)^2+b^2-2\,b\,\left(a+b\right)\right)-\tan(e+f\,x)^3\left(\frac{(a+b)^2}{3}+\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)+\tan(e+f\,x)^5\left(\frac{(a+b)}{5}\right.}{f}$$

input
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*((a + b)^2/
3 + b^2/3 - (2*b*(a + b))/3) + tan(e + f*x)^5*((a + b)^2/5 + b^2/5 - (2*b*
(a + b))/5) - tan(e + f*x)^7*(b^2/7 - (2*b*(a + b))/7) + (b^2*tan(e + f*x)
^9)/9 - a^2*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 574, normalized size of antiderivative = 6.04

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx = \frac{63\cos(fx+e)\sin(fx+e)^8\tan(fx+e)^5a^2-105\cos(fx+e)\sin(fx+e)^8\tan(fx+e)^3a^2+315\cos}{}$$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x)
```

output
```
(63*cos(e + f*x)*sin(e + f*x)**8*tan(e + f*x)**5*a**2 - 105*cos(e + f*x)*s
in(e + f*x)**8*tan(e + f*x)**3*a**2 + 315*cos(e + f*x)*sin(e + f*x)**8*tan
(e + f*x)*a**2 - 315*cos(e + f*x)*sin(e + f*x)**8*a**2*f*x - 252*cos(e + f
*x)*sin(e + f*x)**6*tan(e + f*x)**5*a**2 + 420*cos(e + f*x)*sin(e + f*x)**
6*tan(e + f*x)**3*a**2 - 1260*cos(e + f*x)*sin(e + f*x)**6*tan(e + f*x)*a*
*2 + 1260*cos(e + f*x)*sin(e + f*x)**6*a**2*f*x + 378*cos(e + f*x)*sin(e +
 f*x)**4*tan(e + f*x)**5*a**2 - 630*cos(e + f*x)*sin(e + f*x)**4*tan(e + f
*x)**3*a**2 + 1890*cos(e + f*x)*sin(e + f*x)**4*tan(e + f*x)*a**2 - 1890*c
os(e + f*x)*sin(e + f*x)**4*a**2*f*x - 252*cos(e + f*x)*sin(e + f*x)**2*ta
n(e + f*x)**5*a**2 + 420*cos(e + f*x)*sin(e + f*x)**2*tan(e + f*x)**3*a**2
 - 1260*cos(e + f*x)*sin(e + f*x)**2*tan(e + f*x)*a**2 + 1260*cos(e + f*x)
*sin(e + f*x)**2*a**2*f*x + 63*cos(e + f*x)*tan(e + f*x)**5*a**2 - 105*cos
(e + f*x)*tan(e + f*x)**3*a**2 + 315*cos(e + f*x)*tan(e + f*x)*a**2 - 315*
cos(e + f*x)*a**2*f*x - 90*sin(e + f*x)**9*a*b - 10*sin(e + f*x)**9*b**2 +
 90*sin(e + f*x)**7*a*b + 45*sin(e + f*x)**7*b**2)/(315*cos(e + f*x)*f*(si
n(e + f*x)**8 - 4*sin(e + f*x)**6 + 6*sin(e + f*x)**4 - 4*sin(e + f*x)**2
+ 1))
```

3.331 $\int (a+b\sec^2(e+fx))^2\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int (a+b\sec^2(e+fx))^2\tan^4(e+fx)\,dx = a^2x-\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

output
```
a^2*x-a^2*tan(f*x+e)/f+1/3*a^2*tan(f*x+e)^3/f+1/5*b*(2*a+b)*tan(f*x+e)^5/f
+1/7*b^2*tan(f*x+e)^7/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 395 vs. 2(77) = 154.

Time = 1.47 (sec) , antiderivative size = 395, normalized size of antiderivative = 5.13

$$\int (a+b\sec^2(e+fx))^2\tan^4(e+fx)\,dx = \frac{\sec(e)\sec^7(e+fx)\left(3675a^2fx\cos(fx)+3675a^2fx\cos(2e+fx)+2205a^2fx\cos(2e+3fx)+2205a^2fx\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

output
```
(Sec[e]*Sec[e + f*x]^7*(3675*a^2*f*x*Cos[f*x] + 3675*a^2*f*x*Cos[2*e + f*x
] + 2205*a^2*f*x*Cos[2*e + 3*f*x] + 2205*a^2*f*x*Cos[4*e + 3*f*x] + 735*a^
2*f*x*Cos[4*e + 5*f*x] + 735*a^2*f*x*Cos[6*e + 5*f*x] + 105*a^2*f*x*Cos[6*
e + 7*f*x] + 105*a^2*f*x*Cos[8*e + 7*f*x] - 5320*a^2*Sin[f*x] + 1680*a*b*S
in[f*x] + 840*b^2*Sin[f*x] + 4480*a^2*Sin[2*e + f*x] - 1260*a*b*Sin[2*e +
f*x] + 420*b^2*Sin[2*e + f*x] - 3780*a^2*Sin[2*e + 3*f*x] + 924*a*b*Sin[2*
e + 3*f*x] - 168*b^2*Sin[2*e + 3*f*x] + 2100*a^2*Sin[4*e + 3*f*x] - 840*a*
b*Sin[4*e + 3*f*x] - 420*b^2*Sin[4*e + 3*f*x] - 1540*a^2*Sin[4*e + 5*f*x]
+ 168*a*b*Sin[4*e + 5*f*x] + 84*b^2*Sin[4*e + 5*f*x] + 420*a^2*Sin[6*e + 5
*f*x] - 420*a*b*Sin[6*e + 5*f*x] - 280*a^2*Sin[6*e + 7*f*x] + 84*a*b*Sin[6
*e + 7*f*x] + 12*b^2*Sin[6*e + 7*f*x]))/(13440*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4 \left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(b^2 \tan^6(e+fx) + b(2a+b)\tan^4(e+fx) + a^2\tan^2(e+fx) - a^2 + \frac{a^2}{\tan^2(e+fx)+1}\right) d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\arctan(\tan(e+fx)) + \frac{1}{3}a^2\tan^3(e+fx) - a^2\tan(e+fx) + \frac{1}{5}b(2a+b)\tan^5(e+fx) + \frac{1}{7}b^2\tan^7(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] - a^2*Tan[e + f*x] + (a^2*Tan[e + f*x]^3)/3 + (b
*(2*a + b)*Tan[e + f*x]^5)/5 + (b^2*Tan[e + f*x]^7)/7)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 4.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

method	result
parts	$\frac{a^{2}\left(\frac{\tan(fx+e)^{3}}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{b^{2}\left(\frac{\tan(fx+e)^{7}}{7}+\frac{\tan(fx+e)^{5}}{5}\right)}{f}+\frac{2ab\tan(fx+e)^{5}}{5f}$
derivativedivides	$\frac{a^{2}\left(\frac{\tan(fx+e)^{3}}{3}-\tan(fx+e)+fx+e\right)+\frac{2ab\sin(fx+e)^{5}}{5\cos(fx+e)^{5}}+b^{2}\left(\frac{\sin(fx+e)^{5}}{7\cos(fx+e)^{7}}+\frac{2\sin(fx+e)^{5}}{35\cos(fx+e)^{5}}\right)}{f}$
default	$\frac{a^{2}\left(\frac{\tan(fx+e)^{3}}{3}-\tan(fx+e)+fx+e\right)+\frac{2ab\sin(fx+e)^{5}}{5\cos(fx+e)^{5}}+b^{2}\left(\frac{\sin(fx+e)^{5}}{7\cos(fx+e)^{7}}+\frac{2\sin(fx+e)^{5}}{35\cos(fx+e)^{5}}\right)}{f}$
risch	$a^{2}x+\frac{4i\left(-105\,\mathrm{e}^{12i(fx+e)}a^{2}+105ab\,\mathrm{e}^{12i(fx+e)}-525\,\mathrm{e}^{10i(fx+e)}a^{2}+210\,\mathrm{e}^{10i(fx+e)}ab+105b^{2}\mathrm{e}^{10i(fx+e)}-1120a^{2}\mathrm{e}^{8i(\ldots}$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+b^2/f*(1/7*tan(f*x+
e)^7+1/5*tan(f*x+e)^5)+2/5*a*b/f*tan(f*x+e)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx = \frac{105\,a^{2}fx\cos\left(fx+e\right)^{7}-\left(2\left(70\,a^{2}-21\,ab-3\,b^{2}\right)\cos\left(fx+e\right)^{6}-\left(35\,a^{2}-84\,ab+3\,b^{2}\right)\cos\left(fx+e\right)^{4}-\ldots}{105\,f\cos\left(fx+e\right)^{7}}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/105*(105*a^2*f*x*cos(f*x + e)^7 - (2*(70*a^2 - 21*a*b - 3*b^2)*cos(f*x +
 e)^6 - (35*a^2 - 84*a*b + 3*b^2)*cos(f*x + e)^4 - 6*(7*a*b - 4*b^2)*cos(f
*x + e)^2 - 15*b^2)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\tan^4\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**4,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx$$
$$=\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab+b^2\right)\tan\left(fx+e\right)^5+35\,a^2\tan\left(fx+e\right)^3+105\left(fx+e\right)a^2-105\,a^2\tan\left(f\right.}{105\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + b^2)*tan(f*x + e)^5 + 35*a^2*ta
n(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx=\frac{(fx+e)a^2}{f}+\frac{15\,b^2f^6\tan\left(fx+e\right)^7+42\,abf^6\tan\left(fx+e\right)^5+21\,b^2f^6\tan\left(fx+e\right)^5+35\,a^2f^6\tan\left(fx+e\right)^3-105\,a^2}{105\,f^7}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
(f*x + e)*a^2/f + 1/105*(15*b^2*f^6*tan(f*x + e)^7 + 42*a*b*f^6*tan(f*x +
e)^5 + 21*b^2*f^6*tan(f*x + e)^5 + 35*a^2*f^6*tan(f*x + e)^3 - 105*a^2*f^6
*tan(f*x + e))/f^7
```

Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx=\frac{\tan(e+f\,x)^3\left(\frac{(a+b)^2}{3}+\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)-\tan(e+f\,x)\,\left((a+b)^2+b^2-2\,b\,(a+b)\right)-\tan(e+f\,x)^5\left(\frac{b^2}{5}-\right.}{f}$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)^3*((a + b)^2/3 + b^2/3 - (2*b*(a + b))/3) - tan(e + f*x)*((a
 + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^5*(b^2/5 - (2*b*(a + b))/5) +
(b^2*tan(e + f*x)^7)/7 + a^2*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.73

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx$$
$$= \frac{35\cos(fx+e)\sin(fx+e)^6\tan(fx+e)^3a^2-105\cos(fx+e)\sin(fx+e)^6\tan(fx+e)a^2+105\cos(}{}$$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x)
```

output
```
(35*cos(e + f*x)*sin(e + f*x)**6*tan(e + f*x)**3*a**2 - 105*cos(e + f*x)*s
in(e + f*x)**6*tan(e + f*x)*a**2 + 105*cos(e + f*x)*sin(e + f*x)**6*a**2*f
*x - 105*cos(e + f*x)*sin(e + f*x)**4*tan(e + f*x)**3*a**2 + 315*cos(e + f
*x)*sin(e + f*x)**4*tan(e + f*x)*a**2 - 315*cos(e + f*x)*sin(e + f*x)**4*a
**2*f*x + 105*cos(e + f*x)*sin(e + f*x)**2*tan(e + f*x)**3*a**2 - 315*cos(
e + f*x)*sin(e + f*x)**2*tan(e + f*x)*a**2 + 315*cos(e + f*x)*sin(e + f*x)
**2*a**2*f*x - 35*cos(e + f*x)*tan(e + f*x)**3*a**2 + 105*cos(e + f*x)*tan
(e + f*x)*a**2 - 105*cos(e + f*x)*a**2*f*x + 42*sin(e + f*x)**7*a*b + 6*si
n(e + f*x)**7*b**2 - 42*sin(e + f*x)**5*a*b - 21*sin(e + f*x)**5*b**2)/(10
5*cos(e + f*x)*f*(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e + f*x)**2
- 1))
```

3.332 $\int (a+b\sec^2(e+fx))^2\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int (a+b\sec^2(e+fx))^2\tan^2(e+fx)\,dx = -a^2x+\frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

output
```
-a^2*x+a^2*tan(f*x+e)/f+1/3*b*(2*a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/
f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(59) = 118.

Time = 1.10 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.76

$$\int (a+b\sec^2(e+fx))^2\tan^2(e+fx)\,dx = -\frac{\sec(e)\sec^5(e+fx)\left(150a^2fx\cos(fx)+150a^2fx\cos(2e+fx)+75a^2fx\cos(2e+3fx)+75a^2fx\cos(\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

output
```
-1/480*(Sec[e]*Sec[e + f*x]^5*(150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e
+ f*x] + 75*a^2*f*x*Cos[2*e + 3*f*x] + 75*a^2*f*x*Cos[4*e + 3*f*x] + 15*a^
2*f*x*Cos[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*Sin[f*x] +
80*a*b*Sin[f*x] - 20*b^2*Sin[f*x] + 120*a^2*Sin[2*e + f*x] - 120*a*b*Sin[2
*e + f*x] - 60*b^2*Sin[2*e + f*x] - 120*a^2*Sin[2*e + 3*f*x] + 40*a*b*Sin[
2*e + 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] + 30*a^2*Sin[4*e + 3*f*x] - 60*a*b*
Sin[4*e + 3*f*x] - 30*a^2*Sin[4*e + 5*f*x] + 20*a*b*Sin[4*e + 5*f*x] + 4*b
^2*Sin[4*e + 5*f*x]))/f
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(b^2\tan^4(e+fx)+b(2a+b)\tan^2(e+fx)+a^2-\frac{a^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^2\arctan(\tan(e+fx))+a^2\tan(e+fx)+\frac{1}{3}b(2a+b)\tan^3(e+fx)+\frac{1}{5}b^2\tan^5(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

output
```
(-(a^2*ArcTan[Tan[e + f*x]]) + a^2*Tan[e + f*x] + (b*(2*a + b)*Tan[e + f*x
]^3)/3 + (b^2*Tan[e + f*x]^5)/5)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

method	result
parts	$\frac{a^2(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{b^2\left(\frac{\tan(fx+e)^5}{5}+\frac{\tan(fx+e)^3}{3}\right)}{f}+\frac{2ab\tan(fx+e)^3}{3f}$
derivativedivides	$\frac{a^2(\tan(fx+e)-fx-e)+\frac{2ab\sin(fx+e)^3}{3\cos(fx+e)^3}+b^2\left(\frac{\sin(fx+e)^3}{5\cos(fx+e)^5}+\frac{2\sin(fx+e)^3}{15\cos(fx+e)^3}\right)}{f}$
default	$\frac{a^2(\tan(fx+e)-fx-e)+\frac{2ab\sin(fx+e)^3}{3\cos(fx+e)^3}+b^2\left(\frac{\sin(fx+e)^3}{5\cos(fx+e)^5}+\frac{2\sin(fx+e)^3}{15\cos(fx+e)^3}\right)}{f}$
risch	$-a^2x-\frac{2i(-15a^2e^{8i(fx+e)}+30\,e^{8i(fx+e)}ab-60a^2e^{6i(fx+e)}+60ab\,e^{6i(fx+e)}+30b^2e^{6i(fx+e)}-90a^2e^{4i(fx+e)}+40ab}{15f(e^{2i(fx+e)}+}$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(tan(f*x+e)-arctan(tan(f*x+e)))+b^2/f*(1/5*tan(f*x+e)^5+1/3*tan(f*x+
e)^3)+2/3*a*b/f*tan(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = -\frac{15\,a^2fx\cos\left(fx+e\right)^5-\left(\left(15\,a^2-10\,ab-2\,b^2\right)\cos\left(fx+e\right)^4+\left(10\,ab-b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(\right.}{15\,f\cos\left(fx+e\right)^5}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/15*(15*a^2*f*x*cos(f*x + e)^5 - ((15*a^2 - 10*a*b - 2*b^2)*cos(f*x + e)
^4 + (10*a*b - b^2)*cos(f*x + e)^2 + 3*b^2)*sin(f*x + e))/(f*cos(f*x + e)^
5)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx=\int \left(a+b\sec^2{(e+fx)}\right)^2\tan^2{(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = \frac{3\,b^2\tan\left(fx+e\right)^5+5\,\left(2\,ab+b^2\right)\tan\left(fx+e\right)^3-15\,\left(fx+e\right)a^2+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b + b^2)*tan(f*x + e)^3 - 15*(f*x + e)
*a^2 + 15*a^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = -\frac{\left(fx+e\right)a^2}{f} + \frac{3\,b^2f^4\tan\left(fx+e\right)^5+10\,abf^4\tan\left(fx+e\right)^3+5\,b^2f^4\tan\left(fx+e\right)^3+15\,a^2f^4\tan\left(fx+e\right)}{15\,f^5}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
-(f*x + e)*a^2/f + 1/15*(3*b^2*f^4*tan(f*x + e)^5 + 10*a*b*f^4*tan(f*x + e
)^3 + 5*b^2*f^4*tan(f*x + e)^3 + 15*a^2*f^4*tan(f*x + e))/f^5
```

Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = \frac{\tan(e+f\,x)\,\left((a+b)^2+b^2-2\,b\,(a+b)\right)-\tan(e+f\,x)^3\,\left(\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)+\frac{b^2\,\tan(e+f\,x)^5}{5}-a^2\,f\,x}{f}$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*(b^2/3 - (2
*b*(a + b))/3) + (b^2*tan(e + f*x)^5)/5 - a^2*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.53

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx = \frac{15\cos(fx+e)\sin(fx+e)^4\tan(fx+e)\,a^2-15\cos(fx+e)\sin(fx+e)^4\,a^2fx-30\cos(fx+e)\sin(f}{}$$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x)
```

output
```
(15*cos(e + f*x)*sin(e + f*x)**4*tan(e + f*x)*a**2 - 15*cos(e + f*x)*sin(e
 + f*x)**4*a**2*f*x - 30*cos(e + f*x)*sin(e + f*x)**2*tan(e + f*x)*a**2 +
30*cos(e + f*x)*sin(e + f*x)**2*a**2*f*x + 15*cos(e + f*x)*tan(e + f*x)*a*
*2 - 15*cos(e + f*x)*a**2*f*x - 10*sin(e + f*x)**5*a*b - 2*sin(e + f*x)**5
*b**2 + 10*sin(e + f*x)**3*a*b + 5*sin(e + f*x)**3*b**2)/(15*cos(e + f*x)*
f*(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.333 $\int (a+b\sec^2(e+fx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int (a+b\sec^2(e+fx))^2\,dx = a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int (a+b\sec^2(e+fx))^2\,dx = \frac{3a^2fx+3b(2a+b)\tan(e+fx)+b^2\tan^3(e+fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(3*a^2*f*x + 3*b*(2*a + b)*Tan[e + f*x] + b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{a^2}{\tan^2(e+fx)+1}+b^2\tan^2(e+fx)+b(2a+b)\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\arctan(\tan(e+fx))+b(2a+b)\tan(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] + b*(2*a + b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^3
)/3)/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result
parts	$a^{2} x -\frac{b^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x +e \right)^{2}}{3}\right) \tan \left(f x +e \right)}{f}+\frac{2 a b \tan \left(f x +e \right)}{f}$
derivativedivides	$\frac{a^{2} \left(f x +e \right)+2 a b \tan \left(f x +e \right)-b^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x +e \right)^{2}}{3}\right) \tan \left(f x +e \right)}{f}$
default	$\frac{a^{2} \left(f x +e \right)+2 a b \tan \left(f x +e \right)-b^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x +e \right)^{2}}{3}\right) \tan \left(f x +e \right)}{f}$
risch	$a^{2} x +\frac{4 i b \left(3 a \,{\mathrm e}^{4 i \left(f x +e \right)}+6 a \,{\mathrm e}^{2 i \left(f x +e \right)}+3 b \,{\mathrm e}^{2 i \left(f x +e \right)}+3 a +b \right)}{3 f \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)^{3}}$
norman	$\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-a^{2} x +3 a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-3 a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-\frac{2 b \left(2 a +b \right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{f}-\frac{2 b \left(2 a +b \right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{f}+\frac{4 b (}{}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$
parallelrisch	$\frac{3 x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6} a^{2} f +\left(-12 a b -6 b^{2}\right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-9 x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4} a^{2} f +\left(24 a b +4 b^{2}\right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+9 x \tan \left(\frac{f x}{2}+\frac{e}{2}\right.}{3 f \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{3} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}$

input
```
int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*tan(f*x+e)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^2fx\cos\left(fx+e\right)^3+\left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin
(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int (a+b\sec^2(e+fx))^2 \, dx = a^2x + \frac{(\tan(fx+e)^3 + 3\tan(fx+e))b^2}{3f} + \frac{2ab\tan(fx+e)}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int (a+b\sec^2(e+fx))^2 \, dx = \frac{b^2\tan(fx+e)^3 + 3(fx+e)a^2 + 6ab\tan(fx+e) + 3b^2\tan(fx+e)}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan
(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int (a+b\sec^2(e+fx))^2 \, dx = \frac{\frac{b^2\tan(e+fx)^3}{3} - \tan(e+fx)\,(b^2 - 2b(a+b)) + a^2fx}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.65

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\cos(fx+e)\sin(fx+e)^2a^2fx-3\cos(fx+e)\,a^2fx+6\sin(fx+e)^3ab+2\sin(fx+e)^3b^2-6\sin(}{3\cos(fx+e)\,f\left(\sin(fx+e)^2-1\right)}$$

input
```
int((a+b*sec(f*x+e)^2)^2,x)
```

output
```
(3*cos(e + f*x)*sin(e + f*x)**2*a**2*f*x - 3*cos(e + f*x)*a**2*f*x + 6*sin
(e + f*x)**3*a*b + 2*sin(e + f*x)**3*b**2 - 6*sin(e + f*x)*a*b - 3*sin(e +
 f*x)*b**2)/(3*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.334 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-a^2x-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

output
```
-a^2*x-(a+b)^2*cot(f*x+e)/f+b^2*tan(f*x+e)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

Time = 2.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.28

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{4(b+a\cos^2(e+fx))^2\sec(e+fx)\left(a^2fx\cos(e+fx)-((a+b)^2\cot(e+fx)\csc(e)+b^2\sec(e))\sin(f}{f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]*(a^2*f*x*Cos[e + f*x] - ((a + b)
^2*Cot[e + f*x]*Csc[e] + b^2*Sec[e])*Sin[f*x]))/(f*(a + 2*b + a*Cos[2*(e +
 f*x)])^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(-\frac{a^2}{\tan^2(e+fx)+1}+b^2+(a+b)^2\cot^2(e+fx)\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(-\arctan(\tan(e+fx)))-(a+b)^2\cot(e+fx)+b^2\tan(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-(a^2*ArcTan[Tan[e + f*x]]) - (a + b)^2*Cot[e + f*x] + b^2*Tan[e + f*x])/
f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

method	result	size
derivativedivides	$\frac{a^2(-\cot(fx+e)-fx-e)-2ab\cot(fx+e)+b^2\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	66
default	$\frac{a^2(-\cot(fx+e)-fx-e)-2ab\cot(fx+e)+b^2\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	66
risch	$-a^2x-\frac{2i\left(a^2 e^{2i(fx+e)}+2ab\,e^{2i(fx+e)}+a^2+2ab+2b^2\right)}{f\left(e^{2i(fx+e)}-1\right)\left(e^{2i(fx+e)}+1\right)}$	79

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-cot(f*x+e)-f*x-e)-2*a*b*cot(f*x+e)+b^2*(1/sin(f*x+e)/cos(f*x+e)
-2*cot(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{a^2fx\cos(fx+e)\sin(fx+e)+(a^2+2\,ab+2\,b^2)\cos(fx+e)^2-b^2}{f\cos(fx+e)\sin(fx+e)}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-(a^2*f*x*cos(f*x + e)*sin(f*x + e) + (a^2 + 2*a*b + 2*b^2)*cos(f*x + e)^2
 - b^2)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^2\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(fx+e)a^2-b^2\tan\left(fx+e\right)+\frac{a^2+2\,ab+b^2}{\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=-\frac{(fx+e)a^2-b^2\tan\left(fx+e\right)+\frac{a^2+2\,ab+b^2}{\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{b^2\tan(e+f\,x)}{f}-a^2\,x-\frac{a^2+2\,a\,b+b^2}{f\tan\left(e+f\,x\right)}$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2*tan(e + f*x))/f - a^2*x - (2*a*b + a^2 + b^2)/(f*tan(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.53

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{-\cos\left(fx+e\right)\sin\left(fx+e\right)a^2fx+\sin\left(fx+e\right)^2a^2+2\sin\left(fx+e\right)^2ab+2\sin\left(fx+e\right)^2b^2-a^2-2ab-}{\cos\left(fx+e\right)\sin\left(fx+e\right)f}$$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - cos(e + f*x)*sin(e + f*x)*a**2*f*x + sin(e + f*x)**2*a**2 + 2*sin(e +
f*x)**2*a*b + 2*sin(e + f*x)**2*b**2 - a**2 - 2*a*b - b**2)/(cos(e + f*x)*
sin(e + f*x)*f)
```

3.335 $\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 45

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = a^2x+\frac{(a^2-b^2)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f}$$

output
```
a^2*x+(a^2-b^2)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(45) = 90.

Time = 0.98 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.56

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{\csc(e)\csc^3(e+fx)\left(9a^2fx\cos(fx)-9a^2fx\cos(2e+fx)-3a^2fx\cos(2e+3fx)+3a^2fx\cos(4e+3fx\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Csc[e]*Csc[e + f*x]^3*(9*a^2*f*x*Cos[f*x] - 9*a^2*f*x*Cos[2*e + f*x] - 3*
a^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*x] +
12*b^2*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 12*a*b*Sin[2*e + f*x] + 8*a^2*Si
n[2*e + 3*f*x] + 4*a*b*Sin[2*e + 3*f*x] - 4*b^2*Sin[2*e + 3*f*x]))/(24*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left((a+b)^2\cot^4(e+fx)+\left(b^2-a^2\right)\cot^2(e+fx)+\frac{a^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\arctan(\tan(e+fx))+\left(a^2-b^2\right)\cot(e+fx)-\frac{1}{3}(a+b)^2\cot^3(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] + (a^2 - b^2)*Cot[e + f*x] - ((a + b)^2*Cot[e +
f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

method	result	size
derivativedivides	$\frac{a^{2}\left(-\frac{\cot(fx+e)^{3}}{3}+\cot(fx+e)+fx+e\right)-\frac{2ab\cos(fx+e)^{3}}{3\sin(fx+e)^{3}}+b^{2}\left(-\frac{2}{3}-\frac{\csc(fx+e)^{2}}{3}\right)\cot(fx+e)}{f}$	73
default	$\frac{a^{2}\left(-\frac{\cot(fx+e)^{3}}{3}+\cot(fx+e)+fx+e\right)-\frac{2ab\cos(fx+e)^{3}}{3\sin(fx+e)^{3}}+b^{2}\left(-\frac{2}{3}-\frac{\csc(fx+e)^{2}}{3}\right)\cot(fx+e)}{f}$	73
risch	$a^{2}x+\frac{4i\left(3a^{2}e^{4i(fx+e)}+3ab\,e^{4i(fx+e)}-3a^{2}e^{2i(fx+e)}+3b^{2}e^{2i(fx+e)}+2a^{2}+ab-b^{2}\right)}{3f\left(e^{2i(fx+e)}-1\right)^{3}}$	95

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-2/3*a*b/sin(f*x+e)^3*cos(f*x
+e)^3+b^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(43) = 86.

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.18

$$\int \cot^{4}(e+fx)\left(a+b\sec^{2}(e+fx)\right)^{2}\,dx$$
$$=\frac{2\left(2a^{2}+ab-b^{2}\right)\cos\left(fx+e\right)^{3}-3\left(a^{2}-b^{2}\right)\cos\left(fx+e\right)+3\left(a^{2}fx\cos\left(fx+e\right)^{2}-a^{2}fx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^{2}-f\right)\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(2*(2*a^2 + a*b - b^2)*cos(f*x + e)^3 - 3*(a^2 - b^2)*cos(f*x + e) + 3
*(a^2*f*x*cos(f*x + e)^2 - a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*
sin(f*x + e))
```

Sympy [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2(e+fx)\right)^2\cot^4(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{3\,(fx+e)a^2+\frac{3\,(a^2-b^2)\tan(fx+e)^2-a^2-2\,ab-b^2}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)*a^2 + (3*(a^2 - b^2)*tan(f*x + e)^2 - a^2 - 2*a*b - b^2)/
tan(f*x + e)^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(43) = 86.

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.91

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+24\,(fx+e)a^2-15\,a^2\tan\left(\frac{1}{2}\,fx+\right.}{}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 + 2*a*b*tan(1/2*f*x + 1/2*e)^3 + b^2*tan(
1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a^2 - 15*a^2*tan(1/2*f*x + 1/2*e) - 6*a*
b*tan(1/2*f*x + 1/2*e) + 9*b^2*tan(1/2*f*x + 1/2*e) + (15*a^2*tan(1/2*f*x
+ 1/2*e)^2 + 6*a*b*tan(1/2*f*x + 1/2*e)^2 - 9*b^2*tan(1/2*f*x + 1/2*e)^2 -
 a^2 - 2*a*b - b^2)/tan(1/2*f*x + 1/2*e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = a^2\,x-\frac{\frac{2\,a\,b}{3}-\tan(e+f\,x)^2\,\left(a^2-b^2\right)+\frac{a^2}{3}+\frac{b^2}{3}}{f\,\tan\left(e+f\,x\right)^3}$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
a^2*x - ((2*a*b)/3 - tan(e + f*x)^2*(a^2 - b^2) + a^2/3 + b^2/3)/(f*tan(e
+ f*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.60

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{4\cos\left(fx+e\right)\sin\left(fx+e\right)^2a^2+2\cos\left(fx+e\right)\sin\left(fx+e\right)^2ab-2\cos\left(fx+e\right)\sin\left(fx+e\right)^2b^2-\cos\left(f\right.}{3\sin\left(fx+e\right)^3f}$$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(4*cos(e + f*x)*sin(e + f*x)**2*a**2 + 2*cos(e + f*x)*sin(e + f*x)**2*a*b
- 2*cos(e + f*x)*sin(e + f*x)**2*b**2 - cos(e + f*x)*a**2 - 2*cos(e + f*x)
*a*b - cos(e + f*x)*b**2 + 3*sin(e + f*x)**3*a**2*f*x)/(3*sin(e + f*x)**3*
f)
```

3.336 $\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = -a^2x-\frac{a^2\cot(e+fx)}{f}+\frac{(a^2-b^2)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

output
```
-a^2*x-a^2*cot(f*x+e)/f+1/3*(a^2-b^2)*cot(f*x+e)^3/f-1/5*(a+b)^2*cot(f*x+e
)^5/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 256 vs. 2(65) = 130.

Time = 2.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.94

$$\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^2 \, dx = \frac{\csc(e)\csc^5(e+fx)\left(-150a^2fx\cos(fx)+150a^2fx\cos(2e+fx)+75a^2fx\cos(2e+3fx)-75a^2fx\cos(4\right.}{}$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Csc[e]*Csc[e + f*x]^5*(-150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e + f*x]
 + 75*a^2*f*x*Cos[2*e + 3*f*x] - 75*a^2*f*x*Cos[4*e + 3*f*x] - 15*a^2*f*x*
Cos[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] + 280*a^2*Sin[f*x] + 120*a*
b*Sin[f*x] + 20*b^2*Sin[f*x] + 180*a^2*Sin[2*e + f*x] - 60*b^2*Sin[2*e + f
*x] - 140*a^2*Sin[2*e + 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] - 90*a^2*Sin[4*e
+ 3*f*x] - 60*a*b*Sin[4*e + 3*f*x] + 46*a^2*Sin[4*e + 5*f*x] + 12*a*b*Sin[
4*e + 5*f*x] - 4*b^2*Sin[4*e + 5*f*x]))/(480*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left((a+b)^2\cot^6(e+fx)+\left(b^2-a^2\right)\cot^4(e+fx)+a^2\cot^2(e+fx)-\frac{a^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^2 \arctan(\tan(e+fx)) + \frac{1}{3}(a^2-b^2)\cot^3(e+fx) - a^2\cot(e+fx) - \frac{1}{5}(a+b)^2\cot^5(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-(a^2*ArcTan[Tan[e + f*x]]) - a^2*Cot[e + f*x] + ((a^2 - b^2)*Cot[e + f*x
]^3)/3 - ((a + b)^2*Cot[e + f*x]^5)/5)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{2ab\cos(fx+e)^5}{5\sin(fx+e)^5}+b^2\left(-\frac{\cos(fx+e)^3}{5\sin(fx+e)^5}-\frac{2\cos(fx+e)^3}{15\sin(fx+e)^3}\right)}{f}$
default	$\frac{a^2\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{2ab\cos(fx+e)^5}{5\sin(fx+e)^5}+b^2\left(-\frac{\cos(fx+e)^3}{5\sin(fx+e)^5}-\frac{2\cos(fx+e)^3}{15\sin(fx+e)^3}\right)}{f}$
risch	$-a^2x-\frac{2i\left(45a^2e^{8i(fx+e)}+30\,e^{8i(fx+e)}ab-90a^2e^{6i(fx+e)}+30b^2e^{6i(fx+e)}+140a^2e^{4i(fx+e)}+60ab\,e^{4i(fx+e)}+10b^2e\right.}{15f\left(e^{2i(fx+e)}-1\right)^5}$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-2/5*a*b/sin
(f*x+e)^5*cos(f*x+e)^5+b^2*(-1/5/sin(f*x+e)^5*cos(f*x+e)^3-2/15*cos(f*x+e)
^3/sin(f*x+e)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(61) = 122$.

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.09

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(23\,a^2+6\,ab-2\,b^2)\cos\left(fx+e\right)^5-5\left(7\,a^2-b^2\right)\cos\left(fx+e\right)^3+15\,a^2\cos\left(fx+e\right)+15\left(a^2fx\cos\left(fx\right.\right.}{15\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)\sin\left(fx+\right.}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*((23*a^2 + 6*a*b - 2*b^2)*cos(f*x + e)^5 - 5*(7*a^2 - b^2)*cos(f*x +
 e)^3 + 15*a^2*cos(f*x + e) + 15*(a^2*f*x*cos(f*x + e)^4 - 2*a^2*f*x*cos(f
*x + e)^2 + a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2
 + f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{15\,(fx+e)a^2+\frac{15\,a^2\tan(fx+e)^4-5\,(a^2-b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/15*(15*(f*x + e)*a^2 + (15*a^2*tan(f*x + e)^4 - 5*(a^2 - b^2)*tan(f*x +
 e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(61) = 122.

Time = 0.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.20

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-30\,ab}{}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/480*(3*a^2*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*tan(1/2*f*x + 1/2*e)^5 + 3*b^2
*tan(1/2*f*x + 1/2*e)^5 - 35*a^2*tan(1/2*f*x + 1/2*e)^3 - 30*a*b*tan(1/2*f
*x + 1/2*e)^3 + 5*b^2*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a^2 + 330*a^2
*tan(1/2*f*x + 1/2*e) + 60*a*b*tan(1/2*f*x + 1/2*e) - 30*b^2*tan(1/2*f*x +
 1/2*e) - (330*a^2*tan(1/2*f*x + 1/2*e)^4 + 60*a*b*tan(1/2*f*x + 1/2*e)^4
- 30*b^2*tan(1/2*f*x + 1/2*e)^4 - 35*a^2*tan(1/2*f*x + 1/2*e)^2 - 30*a*b*t
an(1/2*f*x + 1/2*e)^2 + 5*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*a^2 + 6*a*b + 3*b
^2)/tan(1/2*f*x + 1/2*e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= -a^2\,x - \frac{\frac{2\,a\,b}{5}+\frac{a^2}{5}+\frac{b^2}{5}-\tan(e+f\,x)^2\,\left(\frac{a^2}{3}-\frac{b^2}{3}\right)+a^2\tan(e+f\,x)^4}{f\tan\left(e+f\,x\right)^5}$$

input
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
- a^2*x - ((2*a*b)/5 + a^2/5 + b^2/5 - tan(e + f*x)^2*(a^2/3 - b^2/3) + a^
2*tan(e + f*x)^4)/(f*tan(e + f*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.65

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{-23\cos\left(fx+e\right)\sin\left(fx+e\right)^4a^2-6\cos\left(fx+e\right)\sin\left(fx+e\right)^4ab+2\cos\left(fx+e\right)\sin\left(fx+e\right)^4b^2+11\,c}{}$$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 23*cos(e + f*x)*sin(e + f*x)**4*a**2 - 6*cos(e + f*x)*sin(e + f*x)**4*
a*b + 2*cos(e + f*x)*sin(e + f*x)**4*b**2 + 11*cos(e + f*x)*sin(e + f*x)**
2*a**2 + 12*cos(e + f*x)*sin(e + f*x)**2*a*b + cos(e + f*x)*sin(e + f*x)**
2*b**2 - 3*cos(e + f*x)*a**2 - 6*cos(e + f*x)*a*b - 3*cos(e + f*x)*b**2 -
15*sin(e + f*x)**5*a**2*f*x)/(15*sin(e + f*x)**5*f)
```

3.337 $\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b)\log(\cos(e+fx))}{b^2f} - \frac{(a+b)^2\log\left(b+a\cos^2(e+fx)\right)}{2ab^2f} + \frac{\sec^2(e+fx)}{2bf}$$

output
```
(a+2*b)*ln(cos(f*x+e))/b^2/f-1/2*(a+b)^2*ln(b+a*cos(f*x+e)^2)/a/b^2/f+1/2*
sec(f*x+e)^2/b/f
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(2a(a+2b)\log(\cos(e+fx))-(a+b)^2\log\left(a+b-a\sin^2(e+\right.\right.}{4ab^2f\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(2*a*(a + 2*b)*Log[Cos[e +
f*x]] - (a + b)^2*Log[a + b - a*Sin[e + f*x]^2] + a*b*Sec[e + f*x]^2))/(4*
a*b^2*f*(a + b*Sec[e + f*x]^2))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(e+fx)^5}{a+b\sec(e+fx)^2}dx$$
$$\Big\downarrow \; 4626$$
$$-\frac{\int \frac{(1-\cos^2(e+fx))^2\sec^3(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{f}$$
$$\Big\downarrow \; 354$$
$$-\frac{\int \frac{(1-\cos^2(e+fx))^2\sec^2(e+fx)}{a\cos^2(e+fx)+b}d\cos^2(e+fx)}{2f}$$
$$\Big\downarrow \; 99$$
$$-\frac{\int \left(\frac{(a+b)^2}{b^2(a\cos^2(e+fx)+b)}+\frac{\sec^2(e+fx)}{b}+\frac{(-a-2b)\sec(e+fx)}{b^2}\right)d\cos^2(e+fx)}{2f}$$
$$\Big\downarrow \; 2009$$
$$-\frac{\frac{(a+b)^2\log(a\cos^2(e+fx)+b)}{ab^2}-\frac{(a+2b)\log(\cos^2(e+fx))}{b^2}-\frac{\sec(e+fx)}{b}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(-(((a + 2*b)*Log[Cos[e + f*x]^2])/b^2) + ((a + b)^2*Log[b + a*Cos[e
+ f*x]^2])/(a*b^2) - Sec[e + f*x]/b)/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{\left(a^{2}+2ab+b^{2}\right)\ln\left(b+a\cos\left(fx+e\right)^{2}\right)}{2b^{2}a}+\frac{\left(a+2b\right)\ln\left(\cos\left(fx+e\right)\right)}{b^{2}}+\frac{1}{2b\cos\left(fx+e\right)^{2}}}{f}$
default	$\frac{-\frac{\left(a^{2}+2ab+b^{2}\right)\ln\left(b+a\cos\left(fx+e\right)^{2}\right)}{2b^{2}a}+\frac{\left(a+2b\right)\ln\left(\cos\left(fx+e\right)\right)}{b^{2}}+\frac{1}{2b\cos\left(fx+e\right)^{2}}}{f}$
risch	$\frac{ix}{a}+\frac{2ie}{af}+\frac{2\,\mathrm{e}^{2i(fx+e)}}{fb\left(\mathrm{e}^{2i(fx+e)}+1\right)^{2}}+\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)a}{b^{2}f}+\frac{2\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{bf}-\frac{a\ln\left(\mathrm{e}^{4i(fx+e)}+\frac{2(a+2b)\mathrm{e}^{2i(fx+e)}}{a}\right.}{2b^{2}f}$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*(a^2+2*a*b+b^2)/b^2/a*ln(b+a*cos(f*x+e)^2)+(a+2*b)/b^2*ln(cos(f*
x+e))+1/2/b/cos(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a^2+2\,ab+b^2)\cos\left(fx+e\right)^2\log\left(a\cos\left(fx+e\right)^2+b\right)-2\,(a^2+2\,ab)\cos\left(fx+e\right)^2\log\left(-\cos\left(fx+e\right.\right.}{2\,ab^2f\cos\left(fx+e\right)^2}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2*log(a*cos(f*x + e)^2 + b) - 2*(a^
2 + 2*a*b)*cos(f*x + e)^2*log(-cos(f*x + e)) - a*b)/(a*b^2*f*cos(f*x + e)^
2)
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx=\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a+2b)\log\left(\sin(fx+e)^2-1\right)}{b^2}-\frac{1}{b\sin(fx+e)^2-b}-\frac{(a^2+2ab+b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{ab^2}}{2f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*((a + 2*b)*log(sin(f*x + e)^2 - 1)/b^2 - 1/(b*sin(f*x + e)^2 - b) - (a
^2 + 2*a*b + b^2)*log(a*sin(f*x + e)^2 - a - b)/(a*b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b)\log\left(|\cos(fx+e)|\right)}{b^2f} - \frac{(a^2+2ab+b^2)\log\left(\left|a\cos(fx+e)^2+b\right|\right)}{2ab^2f} + \frac{1}{2bf\cos(fx+e)^2}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(a + 2*b)*log(abs(cos(f*x + e)))/(b^2*f) - 1/2*(a^2 + 2*a*b + b^2)*log(abs
(a*cos(f*x + e)^2 + b))/(a*b^2*f) + 1/2/(b*f*cos(f*x + e)^2)
```

Mupad [B] (verification not implemented)

Time = 16.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{b\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f} + \frac{\tan(e+f\,x)^2}{2\,b\,f} - \frac{a\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b^2\,f}$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*a*f) - log(a + b + b*tan(e + f*x)^2)/(b*f) - lo
g(a + b + b*tan(e + f*x)^2)/(2*a*f) + tan(e + f*x)^2/(2*b*f) - (a*log(a +
b + b*tan(e + f*x)^2))/(2*b^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 766, normalized size of antiderivative = 11.10

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

output

```
(2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*b**2 - 2*log(tan((e + f*x)
/2)**2 + 1)*b**2 + 2*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a**2 + 4*lo
g(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*a*b - 2*log(tan((e + f*x)/2) - 1)*
a**2 - 4*log(tan((e + f*x)/2) - 1)*a*b + 2*log(tan((e + f*x)/2) + 1)*sin(e
 + f*x)**2*a**2 + 4*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a*b - 2*log(
tan((e + f*x)/2) + 1)*a**2 - 4*log(tan((e + f*x)/2) + 1)*a*b - log(sqrt(a
+ b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e
 + f*x)**2*a**2 - 2*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*
sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a*b - log(sqrt(a + b)*tan((e + f
*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*b**2
 + log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e +
f*x)/2))*a**2 + 2*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sq
rt(a)*tan((e + f*x)/2))*a*b + log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a
 + b) - 2*sqrt(a)*tan((e + f*x)/2))*b**2 - log(sqrt(a + b)*tan((e + f*x)/2
)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2 - 2*
log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x
)/2))*sin(e + f*x)**2*a*b - log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a +
 b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*b**2 + log(sqrt(a + b)*t
an((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a**2 + 2*lo
g(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*...
```

3.338 $\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 45

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log(\cos(e+fx))}{bf}+\frac{(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}$$

output
```
-ln(cos(f*x+e))/b/f+1/2*(a+b)*ln(b+a*cos(f*x+e)^2)/a/b/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-2a\log(\cos(e+fx))+(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-2*a*Log[Cos[e + f*x]] + (a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{(1-\cos^2(e+fx))\sec(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{(1-\cos^2(e+fx))\sec(e+fx)}{a\cos^2(e+fx)+b}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{86}$$

$$-\frac{\int \left(\frac{-a-b}{b(a\cos^2(e+fx)+b)}+\frac{\sec(e+fx)}{b}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\log(\cos^2(e+fx))}{b}-\frac{(a+b)\log(a\cos^2(e+fx)+b)}{ab}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(Log[Cos[e + f*x]^2]/b - ((a + b)*Log[b + a*Cos[e + f*x]^2])/(a*b))/f
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{\ln(\cos(fx+e))}{b}+\frac{(a+b)\ln\left(b+a\cos(fx+e)^2\right)}{2ba}}{f}$
default	$\frac{-\frac{\ln(\cos(fx+e))}{b}+\frac{(a+b)\ln\left(b+a\cos(fx+e)^2\right)}{2ba}}{f}$
risch	$-\frac{ix}{a}-\frac{2ie}{af}-\frac{\ln\left(e^{2i(fx+e)}+1\right)}{bf}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2bf}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af}$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/b*ln(cos(f*x+e))+1/2*(a+b)/b/a*ln(b+a*cos(f*x+e)^2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\,a\log\left(-\cos\left(fx+e\right)\right)}{2\,abf}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*((a + b)*log(a*cos(f*x + e)^2 + b) - 2*a*log(-cos(f*x + e)))/(a*b*f)
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a+b)\log\left(a\sin(fx+e)^2-a-b\right)}{ab} - \frac{\log\left(\sin(fx+e)^2-1\right)}{b}}{2\,f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*((a + b)*log(a*sin(f*x + e)^2 - a - b)/(a*b) - log(sin(f*x + e)^2 - 1)
/b)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\log\left(\left|a\cos(fx+e)^2+b\right|\right)}{2\,abf} - \frac{\log\left(\left|\cos(fx+e)\right|\right)}{bf}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(a + b)*log(abs(a*cos(f*x + e)^2 + b))/(a*b*f) - log(abs(cos(f*x + e))
)/(b*f)
```

Mupad [B] (verification not implemented)

Time = 16.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f} + \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b\,f} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f}$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

output
```
log(a + b + b*tan(e + f*x)^2)/(2*a*f) + log(a + b + b*tan(e + f*x)^2)/(2*b
*f) - log(tan(e + f*x)^2 + 1)/(2*a*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.60

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)b-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)a-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)a+\log\left(\sqrt{a+b}\tan\right.}{}$$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 2*log(tan((e + f*x)/2)**2 + 1)*b - 2*log(tan((e + f*x)/2) - 1)*a - 2*l
og(tan((e + f*x)/2) + 1)*a + log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a
+ b) - 2*sqrt(a)*tan((e + f*x)/2))*a + log(sqrt(a + b)*tan((e + f*x)/2)**2
 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*b + log(sqrt(a + b)*tan((e +
f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a + log(sqrt(a + b)
*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*b)/(2*a*b
*f)
```

3.339 $\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(b+a\cos^2(e+fx)\right)}{2af}$$

output
```
-1/2*ln(b+a*cos(f*x+e)^2)/a/f
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log(a+2b+a\cos(2(e+fx)))}{2af}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*Log[a + 2*b + a*Cos[2*(e + f*x)]]/(a*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4626, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4626}$$

$$-\frac{\int \frac{\cos(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 240}$$

$$-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2af}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*Log[b + a*Cos[e + f*x]^2]/(a*f)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{\frac{\ln(\sec(fx+e))}{a}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2a}}{f}$	35
default	$\frac{\frac{\ln(\sec(fx+e))}{a}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2a}}{f}$	35
risch	$\frac{ix}{a}+\frac{2ie}{af}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af}$	58

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a*ln(sec(f*x+e))-1/2/a*ln(a+b*sec(f*x+e)^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx=-\frac{\log\left(a\cos\left(fx+e\right)^2+b\right)}{2\,af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*log(a*cos(f*x + e)^2 + b)/(a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(19) = 38$.

Time = 5.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.09

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = \begin{cases} \frac{\tilde{\infty}x\tan(e)}{\sec^2(e)} & \text{for } a=0\wedge b=0\wedge f=0 \\ -\frac{1}{2bf\sec^2(e+fx)} & \text{for } a=0 \\ \frac{\log(\tan^2(e+fx)+1)}{2af} & \text{for } b=0 \\ \frac{x\tan(e)}{a+b\sec^2(e)} & \text{for } f=0 \\ -\frac{\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{2af} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{2af} + \frac{\log(\tan^2(e+fx)+1)}{2af} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x*tan(e)/sec(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(2
*b*f*sec(e + f*x)**2), Eq(a, 0)), (log(tan(e + f*x)**2 + 1)/(2*a*f), Eq(b,
 0)), (x*tan(e)/(a + b*sec(e)**2), Eq(f, 0)), (-log(-sqrt(-a/b) + sec(e +
f*x))/(2*a*f) - log(sqrt(-a/b) + sec(e + f*x))/(2*a*f) + log(tan(e + f*x)*
*2 + 1)/(2*a*f), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(a\sin(fx+e)^2-a-b\right)}{2\,af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*log(a*sin(f*x + e)^2 - a - b)/(a*f)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\,af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*log(abs(a*cos(f*x + e)^2 + b))/(a*f)
```

Mupad [B] (verification not implemented)

Time = 16.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.74

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}\left(\frac{a}{2\left(\frac{3\,a}{2}+2\,b+\frac{a\,\cos(2\,e+2\,f\,x)}{2}\right)}-\frac{a\,\cos(2\,e+2\,f\,x)}{2\left(\frac{3\,a}{2}+2\,b+\frac{a\,\cos(2\,e+2\,f\,x)}{2}\right)}\right)}{a\,f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

output
```
atanh(a/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)) - (a*cos(2*e + 2*f*x)
)/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\log\left(\tan\left(fx+e\right)^2+1\right)-\log\left(\sec\left(fx+e\right)^2b+a\right)}{2af}$$

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

output
```
(log(tan(e + f*x)**2 + 1) - log(sec(e + f*x)**2*b + a))/(2*a*f)
```

3.340 $\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{b\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)f}+\frac{\log(\sin(e+fx))}{(a+b)f}$$

output
```
1/2*b*ln(b+a*cos(f*x+e)^2)/a/(a+b)/f+ln(sin(f*x+e))/(a+b)/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{2a\log(\sin(e+fx))+b\log\left(a+b-a\sin^2(e+fx)\right)}{2a^2f+2abf}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
(2*a*Log[Sin[e + f*x]] + b*Log[a + b - a*Sin[e + f*x]^2])/(2*a^2*f + 2*a*b
*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos^3(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos^2(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{86}$$

$$-\frac{\int \left(\frac{1}{(-a-b)(\cos^2(e+fx)-1)}-\frac{b}{(a+b)(a\cos^2(e+fx)+b)}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\log(1-\cos^2(e+fx))}{a+b}-\frac{b\log(a\cos^2(e+fx)+b)}{a(a+b)}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(-(Log[1 - Cos[e + f*x]^2]/(a + b)) - (b*Log[b + a*Cos[e + f*x]^2])/(
a*(a + b)))/f
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result	s
derivativedivides	$\frac{\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\frac{b\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)a}+\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$	6
default	$\frac{\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\frac{b\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)a}+\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$	6
risch	$\frac{ix}{a}-\frac{2ix}{a+b}-\frac{2ie}{f(a+b)}-\frac{2ibx}{a(a+b)}-\frac{2ibe}{af(a+b)}+\frac{\ln(\mathrm{e}^{2i(fx+e)}-1)}{f(a+b)}+\frac{b\ln\left(\mathrm{e}^{4i(fx+e)}+\frac{2(a+2b)\mathrm{e}^{2i(fx+e)}}{a}+1\right)}{2af(a+b)}$	1

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(2*a+2*b)*ln(-1+cos(f*x+e))+1/2/(a+b)*b/a*ln(b+a*cos(f*x+e)^2)+1/(2
*a+2*b)*ln(1+cos(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\log\left(a\cos\left(fx+e\right)^2+b\right)+2\,a\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{2\,(a^2+ab)f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*(b*log(a*cos(f*x + e)^2 + b) + 2*a*log(1/2*sin(f*x + e)))/((a^2 + a*b)
*f)
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\log\left(a\sin(fx+e)^2-a-b\right)}{a^2+ab}+\frac{\log\left(\sin(fx+e)^2\right)}{a+b}}{2f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(b*log(a*sin(f*x + e)^2 - a - b)/(a^2 + a*b) + log(sin(f*x + e)^2)/(a
+ b))/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\left(a^2f+abf\right)}+\frac{\log\left(\left|\cos\left(fx+e\right)^2-1\right|\right)}{2\left(af+bf\right)}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*b*log(abs(a*cos(f*x + e)^2 + b))/(a^2*f + a*b*f) + 1/2*log(abs(cos(f*x
 + e)^2 - 1))/(a*f + b*f)
```

Mupad [B] (verification not implemented)

Time = 16.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)}{f\,\left(a+b\right)}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f}+\frac{b\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,f\,\left(a^2+b\,a\right)}$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

output
```
log(tan(e + f*x))/(f*(a + b)) - log(tan(e + f*x)^2 + 1)/(2*a*f) + (b*log(a
 + b + b*tan(e + f*x)^2))/(2*f*(a*b + a^2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.93

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)b+\log\left(\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\sqrt{a+b}-2\sqrt{\ }\right.}{2af\,(}$$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 2*log(tan((e + f*x)/2)**2 + 1)*a - 2*log(tan((e + f*x)/2)**2 + 1)*b +
log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x
)/2))*b + log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*ta
n((e + f*x)/2))*b + 2*log(tan((e + f*x)/2))*a)/(2*a*f*(a + b))
```

3.341 $\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\csc^2(e+fx)}{2(a+b)f} - \frac{b^2\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^2f} - \frac{(a+2b)\log(\sin(e+fx))}{(a+b)^2f}$$

output
```
-1/2*csc(f*x+e)^2/(a+b)/f-1/2*b^2*ln(b+a*cos(f*x+e)^2)/a/(a+b)^2/f-(a+2*b)
*ln(sin(f*x+e))/(a+b)^2/f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(a(a+b)\csc^2(e+fx)+2a(a+2b)\log(\sin(e+fx))+b^2\log\left(a+b-a\,\mathrm{si}\right.\right.}{4a(a+b)^2f\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a*(a + b)*Csc[e + f*x]^2 + 2*a*(a +
2*b)*Log[Sin[e + f*x]] + b^2*Log[a + b - a*Sin[e + f*x]^2])*Sec[e + f*x]^2
)/(a*(a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos^5(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos^4(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{b^2}{(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{a+2b}{(a+b)^2\left(\cos^2(e+fx)-1\right)}+\frac{1}{(a+b)\left(\cos^2(e+fx)-1\right)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^2\log\left(a\cos^2(e+fx)+b\right)}{a(a+b)^2}+\frac{1}{(a+b)\left(1-\cos^2(e+fx)\right)}+\frac{(a+2b)\log\left(1-\cos^2(e+fx)\right)}{(a+b)^2}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(1/((a + b)*(1 - Cos[e + f*x]^2)) + ((a + 2*b)*Log[1 - Cos[e + f*x]^2
])/(a + b)^2 + (b^2*Log[b + a*Cos[e + f*x]^2])/(a*(a + b)^2))/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-a-2b)\ln(-1+\cos(fx+e))}{2(a+b)^2}-\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a-2b)\ln(1+\cos(fx+e))}{2(a+b)^2}-\frac{b^2\ln\left(b+a\cos(fx\right.}{2(a+b)^2a}}{f}$
default	$\frac{\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-a-2b)\ln(-1+\cos(fx+e))}{2(a+b)^2}-\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a-2b)\ln(1+\cos(fx+e))}{2(a+b)^2}-\frac{b^2\ln\left(b+a\cos(fx\right.}{2(a+b)^2a}}{f}$
risch	$-\frac{ix}{a}+\frac{2iax}{a^2+2ab+b^2}+\frac{2iae}{f(a^2+2ab+b^2)}+\frac{4ibx}{a^2+2ab+b^2}+\frac{4ibe}{f(a^2+2ab+b^2)}+\frac{2ib^2x}{a(a^2+2ab+b^2)}+\frac{2ib^2e}{af(a^2+2ab+b^2)}$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(4*a+4*b)/(-1+cos(f*x+e))+1/2*(-a-2*b)/(a+b)^2*ln(-1+cos(f*x+e))-1/
(4*a+4*b)/(1+cos(f*x+e))+1/2*(-a-2*b)/(a+b)^2*ln(1+cos(f*x+e))-1/2*b^2/(a+
b)^2/a*ln(b+a*cos(f*x+e)^2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{a^2+ab-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\left(\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2-a^2-2\,ab\right)\log}{2\left(\left(a^3+2\,a^2b+ab^2\right)f\cos\left(fx+e\right)^2-\left(a^3+2\,a^2b+ab^2\right)f\right)}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*(a^2 + a*b - (b^2*cos(f*x + e)^2 - b^2)*log(a*cos(f*x + e)^2 + b) - 2*
((a^2 + 2*a*b)*cos(f*x + e)^2 - a^2 - 2*a*b)*log(1/2*sin(f*x + e)))/((a^3
+ 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b^2\log\left(a\sin(fx+e)^2-a-b\right)}{a^3+2a^2b+ab^2} + \frac{(a+2b)\log\left(\sin(fx+e)^2\right)}{a^2+2ab+b^2} + \frac{1}{(a+b)\sin(fx+e)^2}}{2f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(b^2*log(a*sin(f*x + e)^2 - a - b)/(a^3 + 2*a^2*b + a*b^2) + (a + 2*b
)*log(sin(f*x + e)^2)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\begin{aligned}\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = &-\frac{b^2\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\left(a^3f+2a^2bf+ab^2f\right)} \\ &-\frac{(a+2b)\log\left(\left|-\cos\left(fx+e\right)^2+1\right|\right)}{2\left(a^2f+2abf+b^2f\right)} \\ &+\frac{1}{2\left(\cos\left(fx+e\right)^2-1\right)(a+b)f}\end{aligned}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*b^2*log(abs(a*cos(f*x + e)^2 + b))/(a^3*f + 2*a^2*b*f + a*b^2*f) - 1/
2*(a + 2*b)*log(abs(-cos(f*x + e)^2 + 1))/(a^2*f + 2*a*b*f + b^2*f) + 1/2/
((cos(f*x + e)^2 - 1)*(a + b)*f)
```

Mupad [B] (verification not implemented)

Time = 16.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f} - \frac{\cot(e+f\,x)^2}{2\,f\,(a+b)} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a+2\,b)}{f\,(a^2+2\,a\,b+b^2)} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f\,(a+b)^2}$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*a*f) - cot(e + f*x)^2/(2*f*(a + b)) - (log(tan(
e + f*x))*(a + 2*b))/(f*(2*a*b + a^2 + b^2)) - (b^2*log(a + b + b*tan(e +
f*x)^2))/(2*a*f*(a + b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.82

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin(fx+e)^2a^2+8\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin(fx+e)^2ab+4\log\left(\tan\left(\frac{fx}{2}+\right.\right.}{}$$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

output

```
(4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**2 + 8*log(tan((e + f*x)
/2)**2 + 1)*sin(e + f*x)**2*a*b + 4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f
*x)**2*b**2 - 2*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt
(a)*tan((e + f*x)/2))*sin(e + f*x)**2*b**2 - 2*log(sqrt(a + b)*tan((e + f*
x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*b**2
- 4*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**2 - 8*log(tan((e + f*x)/2))*s
in(e + f*x)**2*a*b + sin(e + f*x)**2*a**2 + sin(e + f*x)**2*a*b - 2*a**2 -
 2*a*b)/(4*sin(e + f*x)**2*a*f*(a**2 + 2*a*b + b**2))
```

3.342 $\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(2a+3b)\csc^2(e+fx)}{2(a+b)^2f} - \frac{\csc^4(e+fx)}{4(a+b)f} + \frac{b^3\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^3f} + \frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{(a+b)^3f}$$

output
```
1/2*(2*a+3*b)*csc(f*x+e)^2/(a+b)^2/f-1/4*csc(f*x+e)^4/(a+b)/f+1/2*b^3*ln(b
+a*cos(f*x+e)^2)/a/(a+b)^3/f+(a^2+3*a*b+3*b^2)*ln(sin(f*x+e))/(a+b)^3/f
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.28

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2e+2fx))\left(\frac{2(2a+3b)\csc^2(e+fx)}{(a+b)^2} - \frac{\csc^4(e+fx)}{a+b} + \frac{4(a^2+3ab+3b^2)\log(\sin(e+fx))}{(a+b)^3} + \frac{2b^3\log(a+b-a\sin^2(e+ }{a(a+b)^3}\right.}{8f\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*e + 2*f*x])*((2*(2*a + 3*b)*Csc[e + f*x]^2)/(a + b)^2
- Csc[e + f*x]^4/(a + b) + (4*(a^2 + 3*a*b + 3*b^2)*Log[Sin[e + f*x]])/(a
+ b)^3 + (2*b^3*Log[a + b - a*Sin[e + f*x]^2])/(a*(a + b)^3))*Sec[e + f*x]
^2)/(8*f*(a + b*Sec[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos^7(e+fx)}{\left(1-\cos^2(e+fx)\right)^3\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos^6(e+fx)}{\left(1-\cos^2(e+fx)\right)^3\left(a\cos^2(e+fx)+b\right)}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(-\frac{b^3}{(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{-a^2-3ba-3b^2}{(a+b)^3\left(\cos^2(e+fx)-1\right)}+\frac{-2a-3b}{(a+b)^2\left(\cos^2(e+fx)-1\right)^2}-\frac{1}{(a+b)\left(\cos^2(e+fx)-1\right)^3}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{(a^2+3ab+3b^2)\log(1-\cos^2(e+fx))}{(a+b)^3}-\frac{b^3\log(a\cos^2(e+fx)+b)}{a(a+b)^3}-\frac{2a+3b}{(a+b)^2(1-\cos^2(e+fx))}+\frac{1}{2(a+b)(1-\cos^2(e+fx))^2}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(1/(2*(a + b)*(1 - Cos[e + f*x]^2)^2) - (2*a + 3*b)/((a + b)^2*(1 - C
os[e + f*x]^2)) - ((a^2 + 3*a*b + 3*b^2)*Log[1 - Cos[e + f*x]^2])/(a + b)^
3 - (b^3*Log[b + a*Cos[e + f*x]^2])/(a*(a + b)^3))/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{-\frac{1}{2(8a+8b)(1+\cos(fx+e))^2}-\frac{-7a-11b}{16(a+b)^2(1+\cos(fx+e))}+\frac{(a^2+3ab+3b^2)\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{b^3\ln(b+a\cos(fx+e)^2)}{2(a+b)^3a}-\frac{}{2(8a+8b)(-}}{f}$
default	$\frac{-\frac{1}{2(8a+8b)(1+\cos(fx+e))^2}-\frac{-7a-11b}{16(a+b)^2(1+\cos(fx+e))}+\frac{(a^2+3ab+3b^2)\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{b^3\ln(b+a\cos(fx+e)^2)}{2(a+b)^3a}-\frac{}{2(8a+8b)(-}}{f}$
risch	$\frac{ix}{a}-\frac{2ia^2x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2ia^2e}{f(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6iabx}{a^3+3a^2b+3a\,b^2+b^3}-\frac{6iabe}{f(a^3+3a^2b+3a\,b^2+b^3)}-\frac{}{a^3+3a}$

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/(8*a+8*b)/(1+cos(f*x+e))^2-1/16*(-7*a-11*b)/(a+b)^2/(1+cos(f*x+e
))+1/2*(a^2+3*a*b+3*b^2)/(a+b)^3*ln(1+cos(f*x+e))+1/2*b^3/(a+b)^3/a*ln(b+a
*cos(f*x+e)^2)-1/2/(8*a+8*b)/(-1+cos(f*x+e))^2-1/16*(7*a+11*b)/(a+b)^2/(-1
+cos(f*x+e))+1/2*(a^2+3*a*b+3*b^2)/(a+b)^3*ln(-1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(102) = 204.

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.45

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\frac{3\,a^3+8\,a^2b+5\,ab^2-2\,(2\,a^3+5\,a^2b+3\,ab^2)\cos{(fx+e)}^2+2\,\big(b^3\cos{(fx+e)}^4-2\,b^3\cos{(fx+e)}^2+b^3}{4\,\big((a^4+3\,a^3b+3\,a^2b^2+ab^3)f\cos{(fx+}}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*(3*a^3 + 8*a^2*b + 5*a*b^2 - 2*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e
)^2 + 2*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*log(a*cos(f*x +
e)^2 + b) + 4*((a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^4 + a^3 + 3*a^2*b +
3*a*b^2 - 2*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)
))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*
b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*f)
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.34

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2b^3\log\left(a\sin(fx+e)^2-a-b\right)}{a^4+3a^3b+3a^2b^2+ab^3} + \frac{2(a^2+3ab+3b^2)\log\left(\sin(fx+e)^2\right)}{a^3+3a^2b+3ab^2+b^3} + \frac{2(2a+3b)\sin(fx+e)^2-a-b}{(a^2+2ab+b^2)\sin(fx+e)^4}}{4f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*(2*b^3*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3) + 2*(a^2 + 3*a*b + 3*b^2)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2
+ b^3) + (2*(2*a + 3*b)*sin(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*sin(f
*x + e)^4))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.50

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b^3\log\left(\left|a\sin\left(fx+e\right)^2-a-b\right|\right)}{2\left(a^4f+3\,a^3bf+3\,a^2b^2f+ab^3f\right)} + \frac{\left(a^2+3\,ab+3\,b^2\right)\log\left(\left|\sin\left(fx+e\right)\right|\right)}{a^3f+3\,a^2bf+3\,ab^2f+b^3f} + \frac{2\left(2\,a^2+5\,ab+3\,b^2\right)\sin\left(fx+e\right)^2-a^2-2\,ab-b^2}{4\left(a+b\right)^3f\sin\left(fx+e\right)^4}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*b^3*log(abs(a*sin(f*x + e)^2 - a - b))/(a^4*f + 3*a^3*b*f + 3*a^2*b^2*
f + a*b^3*f) + (a^2 + 3*a*b + 3*b^2)*log(abs(sin(f*x + e)))/(a^3*f + 3*a^2
*b*f + 3*a*b^2*f + b^3*f) + 1/4*(2*(2*a^2 + 5*a*b + 3*b^2)*sin(f*x + e)^2
- a^2 - 2*a*b - b^2)/((a + b)^3*f*sin(f*x + e)^4)
```

Mupad [B] (verification not implemented)

Time = 15.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.48

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+3\,a\,b+3\,b^2\right)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)} - \frac{\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,a\,f} - \frac{\ln\left(b\,{\tan(e+f\,x)}^2+a+b\right)\,\left(\frac{b}{2\,{(a+b)}^2}+\frac{1}{2\,(a+b)}+\frac{b^2}{2\,{(a+b)}^3}-\frac{1}{2\,a}\right)}{f} - \frac{{\cot(e+f\,x)}^4\,\left(\frac{1}{4\,(a+b)}-\frac{{\tan(e+f\,x)}^2\,(a+2\,b)}{2\,{(a+b)}^2}\right)}{f}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

output
```
(log(tan(e + f*x))*(3*a*b + a^2 + 3*b^2))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^
3)) - log(tan(e + f*x)^2 + 1)/(2*a*f) - (log(a + b + b*tan(e + f*x)^2)*(b/
(2*(a + b)^2) + 1/(2*(a + b)) + b^2/(2*(a + b)^3) - 1/(2*a)))/f - (cot(e +
 f*x)^4*(1/(4*(a + b)) - (tan(e + f*x)^2*(a + 2*b))/(2*(a + b)^2)))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.83

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{-32\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin(fx+e)^4 a^3 - 96\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin(fx+e)^4 a^2 b - 96\log\left(\tan\right.}{}$$

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 32*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**3 - 96*log(tan((e +
 f*x)/2)**2 + 1)*sin(e + f*x)**4*a**2*b - 96*log(tan((e + f*x)/2)**2 + 1)*
sin(e + f*x)**4*a*b**2 - 32*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*b
**3 + 16*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan
((e + f*x)/2))*sin(e + f*x)**4*b**3 + 16*log(sqrt(a + b)*tan((e + f*x)/2)*
*2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*b**3 + 32*l
og(tan((e + f*x)/2))*sin(e + f*x)**4*a**3 + 96*log(tan((e + f*x)/2))*sin(e
 + f*x)**4*a**2*b + 96*log(tan((e + f*x)/2))*sin(e + f*x)**4*a*b**2 - 13*s
in(e + f*x)**4*a**3 - 34*sin(e + f*x)**4*a**2*b - 21*sin(e + f*x)**4*a*b**
2 + 32*sin(e + f*x)**2*a**3 + 80*sin(e + f*x)**2*a**2*b + 48*sin(e + f*x)*
*2*a*b**2 - 8*a**3 - 16*a**2*b - 8*a*b**2)/(32*sin(e + f*x)**4*a*f*(a**3 +
 3*a**2*b + 3*a*b**2 + b**3))
```

3.343 $\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a}+\frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f} - \frac{(a+2b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

output

```
-x/a+(a+b)^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(5/2)/f-(a+2*b
)*tan(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.76

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-\frac{3x}{a}-\frac{3(a+b)^{5/2}\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)}{ab^2f\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{6\left(a+b\sec^2(e+fx)\right.}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-3*x)/a - (3*(a + b)^(5/2
)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[
2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*
Sin[2*e]))/(a*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - ((3*a + 7*b)*Sec[e]*S
ec[e + f*x]*Sin[f*x])/(b^2*f) + (Sec[e]*Sec[e + f*x]^3*Sin[f*x])/(b*f) + (
Sec[e + f*x]^2*Tan[e])/(b*f)))/(6*(a + b*Sec[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4629, 2075, 381, 27, 444, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^6}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{381}$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\int \frac{3\tan^2(e+fx)\left((a+2b)\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3b}}{f}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\int \frac{\tan^2(e+fx)\left((a+2b)\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{b}}{f}$$

$$\Big\downarrow \textbf{444}$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+2b)\tan(e+fx)}{b}-\frac{\int \frac{\left(a^2+3ba+3b^2\right)\tan^2(e+fx)+(a+b)(a+2b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{b}}{b}}{f}$$

$$\Big\downarrow \textbf{397}$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+2b)\tan(e+fx)}{b}-\frac{\frac{(a+b)^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-\frac{b^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{b}}{b}}{f}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+2b)\tan(e+fx)}{b}-\frac{\frac{(a+b)^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-\frac{b^2\arctan(\tan(e+fx))}{a}}{b}}{b}}{f}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+2b)\tan(e+fx)}{b}-\frac{\frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}-\frac{b^2\arctan(\tan(e+fx))}{a}}{b}}{b}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]^3/(3*b) - (-((-((b^2*ArcTan[Tan[e + f*x]])/a) + ((a + b)^(5/
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/b) + ((a + 2*b
)*Tan[e + f*x])/b)/b)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)+2b\tan(fx+e)}{b^2}+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
default	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)+2b\tan(fx+e)}{b^2}+\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
risch	$-\frac{x}{a}-\frac{2i\left(3a\,e^{4i(fx+e)}+9b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+12b\,e^{2i(fx+e)}+3a+7b\right)}{3f\,b^2\left(e^{2i(fx+e)}+1\right)^3}-\frac{\sqrt{-(a+b)b}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a\cdots}{\cdots}\right.}{2b^3f}$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)+2*b*tan(f*x+e))+1/b^2*(a^3+3
*a^2*b+3*a*b^2+b^3)/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
-1/a*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(73) = 146$.

Time = 0.12 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.49

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[-\frac{12\,b^2fx\cos\left(fx+e\right)^3 - 3\left(a^2+2\,ab+b^2\right)\sqrt{-\frac{a+b}{b}}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4 - 2\left(3\,ab+4\,b^2\right)\cos}{a}\right.}{12\,ab^2}\right.$$
$$-\frac{6\,b^2fx\cos\left(fx+e\right)^3 + 3\left(a^2+2\,ab+b^2\right)\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)\cos\left(fx+e\right)^3 + 2\left(\left(\right.\right.}{6\,ab^2f\cos\left(fx+e\right)^3}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/12*(12*b^2*f*x*cos(f*x + e)^3 - 3*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/b)
*cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b
^2)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*s
qrt(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
e)^2 + b^2)) + 4*((3*a^2 + 7*a*b)*cos(f*x + e)^2 - a*b)*sin(f*x + e))/(a*b
^2*f*cos(f*x + e)^3), -1/6*(6*b^2*f*x*cos(f*x + e)^3 + 3*(a^2 + 2*a*b + b^
2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/
b)/((a + b)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((3*a^2 + 7*a*b
)*cos(f*x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{\frac{3(fx+e)}{a} - \frac{b\tan(fx+e)^3 - 3(a+2b)\tan(fx+e)}{b^2} - \frac{3(a^3+3a^2b+3ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}ab^2}}{3f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*(f*x + e)/a - (b*tan(f*x + e)^3 - 3*(a + 2*b)*tan(f*x + e))/b^2 -
3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(
sqrt((a + b)*b)*a*b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{fx+e}{af} + \frac{(a^3+3a^2b+3ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}ab^2f}$$
$$+ \frac{b^2f^2\tan(fx+e)^3 - 3abf^2\tan(fx+e) - 6b^2f^2\tan(fx+e)}{3b^3f^3}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-(f*x + e)/(a*f) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(f*x + e)/s
qrt(a*b + b^2))/(sqrt(a*b + b^2)*a*b^2*f) + 1/3*(b^2*f^2*tan(f*x + e)^3 -
3*a*b*f^2*tan(f*x + e) - 6*b^2*f^2*tan(f*x + e))/(b^3*f^3)
```

Mupad [B] (verification not implemented)

Time = 15.34 (sec) , antiderivative size = 1109, normalized size of antiderivative = 13.36

$$\int \frac{\tan^6(e + fx)}{a + b\sec^2(e + fx)}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

output
```
tan(e + f*x)^3/(3*b*f) - atan((40*a^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*
b^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a^3*tan(e + f*x))/(30
*a*b^2 + 40*a^2*b + 30*a^3 + 10*b^3 + (12*a^4)/b + (2*a^5)/b^2) + (12*a^4*
tan(e + f*x))/(30*a*b^3 + 30*a^3*b + 12*a^4 + 10*b^4 + 40*a^2*b^2 + (2*a^5
)/b) + (2*a^5*tan(e + f*x))/(30*a*b^4 + 12*a^4*b + 2*a^5 + 10*b^5 + 40*a^2
*b^3 + 30*a^3*b^2) + (10*b^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*b^2 + (30
*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a*b*tan(e + f*x))/(30*a*b + 40
*a^2 + 10*b^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3))/(a*f) - (tan(e +
 f*x)*(a + 2*b))/(b^2*f) - (atan((((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)
*(6*a*b^5 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))
/b^3 + ((-b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 +
 (tan(e + f*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(
2*a*b^5))*1i)/(2*a*b^5) + ((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^
5 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 - (
(-b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e
+ f*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5)
)*1i)/(2*a*b^5))/((2*(12*a*b^4 + 6*a^4*b + a^5 + 3*b^5 + 19*a^2*b^3 + 15*a
^3*b^2))/b^3 - ((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b
 + a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 + ((-b^5*(a +
b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 654, normalized size of antiderivative = 7.88

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

output
```
(3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*cos(e + f*x)*sin(e + f*x)**2*a**2 + 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a
 + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*sin(e + f*x)**2*a*
b + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*cos(e + f*x)*sin(e + f*x)**2*b**2 - 3*sqrt(b)*sqrt(a + b)*atan((sqr
t(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*a**2 - 6*sqrt(b
)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e
 + f*x)*a*b - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - s
qrt(a))/sqrt(b))*cos(e + f*x)*b**2 + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*sin(e + f*x)**2*a**2
+ 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt
(b))*cos(e + f*x)*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a
 + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*sin(e + f*x)**2*b*
*2 - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/s
qrt(b))*cos(e + f*x)*a**2 - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
 + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*a*b - 3*sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*b**2 - 3
*cos(e + f*x)*sin(e + f*x)**2*b**3*f*x + 3*cos(e + f*x)*b**3*f*x - 3*sin(e
 + f*x)**3*a**2*b - 7*sin(e + f*x)**3*a*b**2 + 3*sin(e + f*x)*a**2*b + 6*s
in(e + f*x)*a*b**2)/(3*cos(e + f*x)*a*b**3*f*(sin(e + f*x)**2 - 1))
```

3.344 $\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{x}{a}-\frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f}+\frac{\tan(e+fx)}{bf}$$

output
```
x/a-(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(3/2)/f+tan(f*x
+e)/b/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.49

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left((a+b)^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx)}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.\right.}{2ab\sqrt{a+b}f\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((a + b)^2*ArcTan[(Sec[f*x]
*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sq
rt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a
 + b]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(b*f*x + a*Sec[e]*Sec[e + f*x]*Sin[f*x
])))/(2*a*b*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4629, 2075, 381, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{381}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{\int \frac{(a+2b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{b}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{\frac{(a+b)^2\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-\frac{b\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{b}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{\frac{(a+b)^2\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-\frac{b\arctan(\tan(e+fx))}{a}}{b}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{\frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}-\frac{b\arctan(\tan(e+fx))}{a}}{b}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((-((b*ArcTan[Tan[e + f*x]])/a) + ((a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/b) + Tan[e + f*x]/b)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(-a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{ab\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(-a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{ab\sqrt{(a+b)b}}}{f}$
risch	$\frac{x}{a}+\frac{2i}{fb\left(e^{2i(fx+e)}+1\right)}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2b^2f}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)}}{a}\right.}{2bfa}$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/b+1/a*arctan(tan(f*x+e))+(-a^2-2*a*b-b^2)/a/b/((a+b)*b)^(1
/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 5.03

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{4\,bfx\cos(fx+e)+(a+b)\sqrt{-\frac{a+b}{b}}\cos(fx+e)\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((ab+2\right.}{a^2\cos(fx+e)^4+2\,ab\cos}\right.}{4\,abf\cos(fx+e)} \right.$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*b*f*x*cos(f*x + e) + (a + b)*sqrt(-(a + b)/b)*cos(f*x + e)*log(((a
^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*
((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin(f*x
 + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*a*sin(
f*x + e))/(a*b*f*cos(f*x + e)), 1/2*(2*b*f*x*cos(f*x + e) + (a + b)*sqrt((
a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a +
b)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*a*sin(f*x + e))/(a*b*f*cos
(f*x + e))]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{fx+e}{a}+\frac{\tan(fx+e)}{b}-\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}ab}}{f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
((f*x + e)/a + tan(f*x + e)/b - (a^2 + 2*a*b + b^2)*arctan(b*tan(f*x + e)/
sqrt((a + b)*b))/(sqrt((a + b)*b)*a*b))/f
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{fx+e}{af}+\frac{\tan(fx+e)}{bf}-\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}abf}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(f*x + e)/(a*f) + tan(f*x + e)/(b*f) - (a^2 + 2*a*b + b^2)*arctan(b*tan(f*
x + e)/sqrt(a*b + b^2))/(sqrt(a*b + b^2)*a*b*f)
```

Mupad [B] (verification not implemented)

Time = 15.14 (sec) , antiderivative size = 410, normalized size of antiderivative = 6.95

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atan}\left(\frac{8a^2\tan(e+fx)}{12ab+8a^2+6b^2+\frac{2a^3}{b}}+\frac{2a^3\tan(e+fx)}{2a^3+8a^2b+12ab^2+6b^3}+\frac{6b^2\tan(e+fx)}{12ab+8a^2+6b^2+\frac{2a^3}{b}}+\frac{12ab\tan(e+fx)}{12ab+8a^2+6b^2+\frac{2a^3}{b}}\right)}{af} + \frac{\tan(e+fx)}{bf} + \frac{\operatorname{atanh}\left(\frac{6\tan(e+fx)\sqrt{-a^3b^3-3a^2b^4-3ab^5-b^6}}{18ab^2+20a^2b+10a^3+6b^3+\frac{2a^4}{b}}+\frac{6a\tan(e+fx)\sqrt{-a^3b^3-3a^2b^4-3ab^5-b^6}}{2a^4+10a^3b+20a^2b^2+18ab^3+6b^4}+\frac{2a^2\tan(e+fx)\sqrt{-a^3b^3-3a^2b^4-\ldots}}{2a^4b+10a^3b^2+20a^2b^3+18ab\ldots}\right.}{ab^3f}$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

output
```
atan((8*a^2*tan(e + f*x))/(12*a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (2*a^3*ta
n(e + f*x))/(12*a*b^2 + 8*a^2*b + 2*a^3 + 6*b^3) + (6*b^2*tan(e + f*x))/(1
2*a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (12*a*b*tan(e + f*x))/(12*a*b + 8*a^2
 + 6*b^2 + (2*a^3)/b))/(a*f) + tan(e + f*x)/(b*f) + (atanh((6*tan(e + f*x)
*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^2 + 20*a^2*b + 10*
a^3 + 6*b^3 + (2*a^4)/b) + (6*a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4
- a^3*b^3)^(1/2))/(18*a*b^3 + 10*a^3*b + 2*a^4 + 6*b^4 + 20*a^2*b^2) + (2*
a^2*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^4
+ 2*a^4*b + 6*b^5 + 20*a^2*b^3 + 10*a^3*b^2))*(-b^3*(a + b)^3)^(1/2))/(a*b
^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.32

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-\sqrt{b}\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\cos(fx+e)\,a-\sqrt{b}\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\cos(fx+\ldots}{}$$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*cos(e + f*x)*a - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/
2) - sqrt(a))/sqrt(b))*cos(e + f*x)*b - sqrt(b)*sqrt(a + b)*atan((sqrt(a +
 b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*a - sqrt(b)*sqrt(a +
 b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*b
+ cos(e + f*x)*b**2*f*x + sin(e + f*x)*a*b)/(cos(e + f*x)*a*b**2*f)
```

3.345 $\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a}+\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}f}$$

output
```
-x/a+(a+b)^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.00

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+(a+b)\arctan\left(\frac{\sec(fx)(\cos\cdots}{\cdots}\right.\right.}{2a\sqrt{a+b}f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\,\mathrm{s}\cdots}}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[
b*(Cos[e] - I*Sin[e])^4] + (a + b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e]
)*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e])))/(a*Sqrt[a + b]*f*(a + b*Sec[e
 + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4629, 2075, 383, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{383}$$

$$\frac{\frac{(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-\frac{\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-\frac{\arctan(\tan(e+fx))}{a}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}-\frac{\arctan(\tan(e+fx))}{a}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-(ArcTan[Tan[e + f*x]]/a) + (Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(a*Sqrt[b]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$	48
default	$\frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$	48
risch	$-\frac{x}{a}-\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2bfa}+\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2bfa}$	111

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((a+b)/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1/a*arct
an(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 4.91

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,fx-\sqrt{-\frac{a+b}{b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\left((ab+2\,b^2)\cos(fx+e)^3-b^2\cos(fx+e)\right)\sqrt{-\frac{a+b}{b}}\,\mathrm{si}\cdots}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,af}\right.$$

$$\left. -\frac{2\,fx+\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)}{2\,af}\right]$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*(4*f*x - sqrt(-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
- 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2
*cos(f*x + e))*sqrt(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 +
2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), -1/2*(2*f*x + sqrt((a + b)/b)*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*s
in(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^2{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
((a + b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x
 + e)/a)/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}af} - \frac{fx+e}{af}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(a + b)*arctan(b*tan(f*x + e)/sqrt(a*b + b^2))/(sqrt(a*b + b^2)*a*f) - (f*
x + e)/(a*f)
```

Mupad [B] (verification not implemented)

Time = 15.64 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\mathrm{atan}\left(\frac{2\,a\,b^2\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2} + \frac{2\,a^2\,b\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2}\right)}{a\,f} - \frac{\mathrm{atanh}\left(\frac{2\,a\,b^2\tan(e+f\,x)\sqrt{-b^2-a\,b}}{2\,a^2\,b^2+2\,a\,b^3}\right)\sqrt{-b\,(a+b)}}{a\,b\,f}$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

output
```
- atan((2*a*b^2*tan(e + f*x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(e + f*x))
/(2*a*b^2 + 2*a^2*b))/(a*f) - (atanh((2*a*b^2*tan(e + f*x)*(- a*b - b^2)^(
1/2))/(2*a*b^3 + 2*a^2*b^2))*(-b*(a + b))^(1/2))/(a*b*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)+\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)-bfx}{abf}$$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

output
```
(sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
) + sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt
(b)) - b*f*x)/(a*b*f)
```

3.346 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx=\frac{x}{a}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}$$

output
```
x/a+b^(1/2)*arctan((a+b)^(1/2)*cot(f*x+e)/b^(1/2))/a/(a+b)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(}{2\sqrt{a+}}\right.\right.}{2a\sqrt{a+b}f\,(a+b\sec^2(e+fx))\,\sqrt{b(\cos(e)-i\sin(e)}}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Co
s[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e
])^4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2
)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4615, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4615}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b}dx}{a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{b\int \frac{1}{(a+b)\cot^2(e+fx)+b}d\cot(e+fx)}{af}+\frac{x}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*
f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 4615
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Simp[b/a   Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f},
x] && NeQ[a + b, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$	46
default	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$	46
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)fa}$	114

input
```
int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a*arctan(tan(f*x+e))-b/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\,\left(3\,ab+4\,b^2\right)\cos(fx+e)^2+4\,\left(\left(a^2+3\,ab+2\,b^2\right)\cos(fx+e)^3-\left(ab+b^2\right)\cos(fx+e)\right)\sqrt{\cdots}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,af}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a
 + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*
x + e)*sin(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \int \frac{1}{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(1/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)
/a)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 16.38 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{\operatorname{atan}\left(\frac{\frac{\left(2b^3\tan(e+fx) - \frac{\left(2a^2b^2 - \frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}\,1\mathrm{i}}{a^2+ba} + \frac{\left(2b^3\tan(e+fx) + \frac{\left(2a^2b^2 + \frac{\tan(e+fx)}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\frac{\left(2b^3\tan(e+fx) - \frac{\left(2a^2b^2 - \frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{a^2+ba} - \frac{\left(2b^3\tan(e+fx) + \frac{\left(2a^2b^2 + \frac{\tan(e+fx)}{\cdots}\right.}{\cdots}\right.}{\cdots}}\right.}{f\left(a^2+ba\right)}$$

input
```
int(1/(a + b/cos(e + f*x)^2),x)
```

output
```
x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^
3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2
*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) +
 ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/
(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1
i)/(a*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^
2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2)
)/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x)
+ ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))
/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))
/(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.89

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right) - \sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right) + afx + bfx}{af\,(a+b)}$$

input
```
int(1/(a+b*sec(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b)) - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/s
qrt(b)) + a*f*x + b*f*x)/(a*f*(a + b))
```

3.347 $\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}f}-\frac{\cot(e+fx)}{(a+b)f}$$

output
```
-x/a+b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(3/2)/f-cot(f*
x+e)/(a+b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.29

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(b^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cdots}{2a(a+b)^{3/2}f\left(a+b\sec^2(e+f\cdots\right.}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b^2*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqr
t[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a
+ b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*((a + b)*f*x - a*Csc[e]*Csc[e + f*x]*Si
n[f*x])))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin
[e])^4])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4629, 2075, 382, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int -\frac{b\tan^2(e+fx)+a+2b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{a+b}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{b\tan^2(e+fx)+a+2b}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{a+b}}{f}$$

$\Big\downarrow$ 397

$$\frac{-\frac{\frac{(a+b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\cot(e+fx)}{a+b}}{f}$$

$\Big\downarrow$ 216

$$\frac{-\frac{\frac{(a+b)\arctan(\tan(e+fx))}{a}-\frac{b^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\cot(e+fx)}{a+b}}{f}$$

$\Big\downarrow$ 218

$$\frac{-\frac{\frac{(a+b)\arctan(\tan(e+fx))}{a}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{\cot(e+fx)}{a+b}}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((((a + b)*ArcTan[Tan[e + f*x]])/a - (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - Cot[e + f*x]/(a + b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)\tan(fx+e)}+\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
default	$\frac{-\frac{1}{(a+b)\tan(fx+e)}+\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
risch	$-\frac{x}{a}-\frac{2i}{f(a+b)\left(e^{2i(fx+e)}-1\right)}-\frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)^2fa}+\frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2i(fx+e)}-\frac{2}{}\right.}{2(a+b)^2fa}$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a+b)/tan(f*x+e)+1/(a+b)*b^2/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)
/((a+b)*b)^(1/2))-1/a*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(54) = 108.

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 5.00

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,(a+b)fx\sin(fx+e)-b\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\left((a^2+3\,ab+2\,b^2)\cos(f\right.}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2}\right.}{4\,(a^2+ab)f\sin(fx+e)}\right.$$

$$\left.-\frac{2\,(a+b)fx\sin(fx+e)+b\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)+2\,a\cos(fx+e)}{2\,(a^2+ab)f\sin(fx+e)}\right]$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*(4*(a + b)*f*x*sin(f*x + e) - b*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*
b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*
x + e) + 4*a*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), -1/2*(2*(a + b)*f
*x*sin(f*x + e) + b*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*a*cos(
f*x + e))/((a^2 + a*b)*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx=\int \frac{\cot^2{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx=\frac{\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+ab)\sqrt{(a+b)b}}-\frac{fx+e}{a}-\frac{1}{(a+b)\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
(b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + a*b)*sqrt((a + b)*b))
- (f*x + e)/a - 1/((a + b)*tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{(a^2+ab)\sqrt{ab+b^2}}-\frac{fx+e}{a}-\frac{1}{(a+b)\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b^2/((a^2 + a*b)*sqrt(a*b + b^2)) - (f*x + e)/a - 1/((a + b)*tan(f*x
 + e)))/f
```

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 637, normalized size of antiderivative = 10.27

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{a\,b^2+2\,a^2\,b+a^3+a^3\tan(e+f\,x)\,\operatorname{atan}(\tan(e+f\,x))+b^3\tan(e+f\,x)\,\operatorname{atan}(\tan(e+f\,x))+3\,a\,b^2\,\mathrm{t}}{}$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

output
```
-(a*b^2 + 2*a^2*b + a^3 - atan((a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^
4 - a^3*b^3)^(3/2)*1i + b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*
b^3)^(3/2)*2i + b^7*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(
1/2)*2i + a*b^6*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)
*10i + a^6*b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*1i
 + a^2*b^5*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*21i
+ a^3*b^4*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*24i +
 a^4*b^3*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*16i +
a^5*b^2*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*6i)/(3*
a*b^9 + 18*a^2*b^8 + 46*a^3*b^7 + 65*a^4*b^6 + 55*a^5*b^5 + 28*a^6*b^4 + 8
*a^7*b^3 + a^8*b^2))*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^
(1/2)*1i + a^3*tan(e + f*x)*atan(tan(e + f*x)) + b^3*tan(e + f*x)*atan(tan
(e + f*x)) + 3*a*b^2*tan(e + f*x)*atan(tan(e + f*x)) + 3*a^2*b*tan(e + f*x
)*atan(tan(e + f*x)))/(a^4*f*tan(e + f*x) + a*b^3*f*tan(e + f*x) + 3*a^3*b
*f*tan(e + f*x) + 3*a^2*b^2*f*tan(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.65

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin(fx+e)\,b+\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\sin(fx+e)}{\sin(fx+e)\,af\,(a}$$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

output
```
(sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
)*sin(e + f*x)*b + sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2)
+ sqrt(a))/sqrt(b))*sin(e + f*x)*b - cos(e + f*x)*a**2 - cos(e + f*x)*a*b
- sin(e + f*x)*a**2*f*x - 2*sin(e + f*x)*a*b*f*x - sin(e + f*x)*b**2*f*x)/
(sin(e + f*x)*a*f*(a**2 + 2*a*b + b**2))
```

3.348 $\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a}-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}f} + \frac{(a+2b)\cot(e+fx)}{(a+b)^2f}-\frac{\cot^3(e+fx)}{3(a+b)f}$$

output
```
x/a-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(5/2)/f+(a+2*b)
*cot(f*x+e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.53

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3b^3\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(c\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*b^3*ArcTan[(Sec[f*x]*(Co
s[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a +
b]*Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(9*(a + b)^2*f*x*Co
s[f*x] - 9*(a + b)^2*f*x*Cos[2*e + f*x] - 3*a^2*f*x*Cos[2*e + 3*f*x] - 6*a
*b*f*x*Cos[2*e + 3*f*x] - 3*b^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e +
 3*f*x] + 6*a*b*f*x*Cos[4*e + 3*f*x] + 3*b^2*f*x*Cos[4*e + 3*f*x] - 12*a^2
*Sin[f*x] - 24*a*b*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 18*a*b*Sin[2*e + f*x
] + 8*a^2*Sin[2*e + 3*f*x] + 14*a*b*Sin[2*e + 3*f*x]))/8))/(6*a*(a + b)^(5
/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4629, 2075, 382, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int -\frac{3\cot^2(e+fx)\left(b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3(a+b)}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{\int \frac{a^2+3ba+3b^2+b(a+2b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{(a+2b)\cot(e+fx)}{a+b}}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{-\frac{-\frac{\frac{(a+b)^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{(a+2b)\cot(e+fx)}{a+b}}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{-\frac{-\frac{\frac{(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{(a+2b)\cot(e+fx)}{a+b}}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{-\frac{\frac{(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{(a+2b)\cot(e+fx)}{a+b}}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/(a + b) - (-((((a + b)^2*ArcTan[Tan[e + f*x]])/a - (b
^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b
)) - ((a + 2*b)*Cot[e + f*x])/(a + b))/(a + b))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2a\sqrt{(a+b)b}}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{-a-2b}{(a+b)^2\tan(fx+e)}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2a\sqrt{(a+b)b}}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{-a-2b}{(a+b)^2\tan(fx+e)}}{f}$
risch	$\frac{x}{a}+\frac{2i\left(6a\,e^{4i(fx+e)}+9b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}-12b\,e^{2i(fx+e)}+4a+7b\right)}{3f(a+b)^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-(a+b)b}\,b^2\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b}}{a}\right.}{2(a+b)^3fa}$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a*arctan(tan(f*x+e))-1/(a+b)^2*b^3/a/((a+b)*b)^(1/2)*arctan(b*tan(f
*x+e)/((a+b)*b)^(1/2))-1/3/(a+b)/tan(f*x+e)^3-1/(a+b)^2*(-a-2*b)/tan(f*x+e
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(76) = 152.

Time = 0.12 (sec) , antiderivative size = 533, normalized size of antiderivative = 6.20

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[\frac{4\,(4\,a^2+7\,ab)\cos{(fx+e)}^3+3\,(b^2\cos{(fx+e)}^2-b^2)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos}{}\right.}{}\right.$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/12*(4*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)^2 - b^2)*sqr
t(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*c
os(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(a^2 + 2*a*b)*cos(f*x + e) +
12*((a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f*x)*sin(
f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*
b^2)*f)*sin(f*x + e)), 1/6*(2*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(
f*x + e)^2 - b^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b
)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a^2 + 2
*a*b)*cos(f*x + e) + 6*((a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*
a*b + b^2)*f*x)*sin(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 -
 (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx=\int \frac{\cot^4{(e+fx)}}{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{(a+b)b}} - \frac{3\,(fx+e)}{a} - \frac{3\,(a+2\,b)\tan(fx+e)^2-a-b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 2*a^2*b + a*b^2
)*sqrt((a + b)*b)) - 3*(f*x + e)/a - (3*(a + 2*b)*tan(f*x + e)^2 - a - b)/
((a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^3}{(a^3+2\,a^2b+ab^2)\sqrt{ab+b^2}} - \frac{3\,(fx+e)}{a} - \frac{3\,a\tan(fx+e)^2+6\,b\tan(fx+e)^2-a-b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^3/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b + b^2)) - 3*(f*x + e)/a
 - (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*
tan(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 19.75 (sec) , antiderivative size = 2644, normalized size of antiderivative = 30.74

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

output
```
atan((10*b^12*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*
b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4
+ 22*a^9*b^3 + 2*a^10*b^2) + (80*a*b^11*tan(e + f*x))/(80*a*b^11 + 10*b^12
 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 +
330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (290*a^2*b^10*tan(e
 + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 +
 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^
10*b^2) + (630*a^3*b^9*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 +
 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110
*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (912*a^4*b^8*tan(e + f*x))/(80*a*b^1
1 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660
*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (922*a^5
*b^7*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912
*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*
b^3 + 2*a^10*b^2) + (660*a^6*b^6*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*
a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7
*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (330*a^7*b^5*tan(e + f*x))
/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5
*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2)
+ (110*a^8*b^4*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.26

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{-3\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin\left(fx+e\right)^3 b^2-3\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\sin\left(f\right.}{}$$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*sin(e + f*x)**3*b**2 - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**3*b**2 + 4*cos(e + f*x)*sin
(e + f*x)**2*a**3 + 11*cos(e + f*x)*sin(e + f*x)**2*a**2*b + 7*cos(e + f*x
)*sin(e + f*x)**2*a*b**2 - cos(e + f*x)*a**3 - 2*cos(e + f*x)*a**2*b - cos
(e + f*x)*a*b**2 + 3*sin(e + f*x)**3*a**3*f*x + 9*sin(e + f*x)**3*a**2*b*f
*x + 9*sin(e + f*x)**3*a*b**2*f*x + 3*sin(e + f*x)**3*b**3*f*x)/(3*sin(e +
 f*x)**3*a*f*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

3.349 $\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a}+\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{7/2}f} - \frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f}$$

output
```
-x/a+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(7/2)/f-(a^2+3
*a*b+3*b^2)*cot(f*x+e)/(a+b)^3/f+1/3*(a+2*b)*cot(f*x+e)^3/(a+b)^2/f-1/5*co
t(f*x+e)^5/(a+b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.57 (sec) , antiderivative size = 671, normalized size of antiderivative = 5.59

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-\frac{480b^4\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-480*b^4*ArcTan[(Sec[f*x]
*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sq
rt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a
 + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + Csc[e]*Csc[e + f*x]^5*(-150*(a + b)
^3*f*x*Cos[f*x] + 150*(a + b)^3*f*x*Cos[2*e + f*x] + 75*a^3*f*x*Cos[2*e +
3*f*x] + 225*a^2*b*f*x*Cos[2*e + 3*f*x] + 225*a*b^2*f*x*Cos[2*e + 3*f*x] +
 75*b^3*f*x*Cos[2*e + 3*f*x] - 75*a^3*f*x*Cos[4*e + 3*f*x] - 225*a^2*b*f*x
*Cos[4*e + 3*f*x] - 225*a*b^2*f*x*Cos[4*e + 3*f*x] - 75*b^3*f*x*Cos[4*e +
3*f*x] - 15*a^3*f*x*Cos[4*e + 5*f*x] - 45*a^2*b*f*x*Cos[4*e + 5*f*x] - 45*
a*b^2*f*x*Cos[4*e + 5*f*x] - 15*b^3*f*x*Cos[4*e + 5*f*x] + 15*a^3*f*x*Cos[
6*e + 5*f*x] + 45*a^2*b*f*x*Cos[6*e + 5*f*x] + 45*a*b^2*f*x*Cos[6*e + 5*f*
x] + 15*b^3*f*x*Cos[6*e + 5*f*x] + 280*a^3*Sin[f*x] + 780*a^2*b*Sin[f*x] +
 680*a*b^2*Sin[f*x] + 180*a^3*Sin[2*e + f*x] + 540*a^2*b*Sin[2*e + f*x] +
480*a*b^2*Sin[2*e + f*x] - 140*a^3*Sin[2*e + 3*f*x] - 420*a^2*b*Sin[2*e +
3*f*x] - 400*a*b^2*Sin[2*e + 3*f*x] - 90*a^3*Sin[4*e + 3*f*x] - 240*a^2*b*
Sin[4*e + 3*f*x] - 180*a*b^2*Sin[4*e + 3*f*x] + 46*a^3*Sin[4*e + 5*f*x] +
132*a^2*b*Sin[4*e + 5*f*x] + 116*a*b^2*Sin[4*e + 5*f*x])))/(960*a*(a + b)^
3*f*(a + b*Sec[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4629, 2075, 382, 27, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\left(a+b\sec(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int -\frac{5\cot^4(e+fx)\left(b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\cot^4(e+fx)\left(b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(a^2+3ba+3b^2+b(a+2b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3(a+b)}-\frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(a^2+3ba+3b^2+b(a+2b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$\Big\downarrow$ 445

$$-\frac{-\frac{-\frac{-\frac{\int \frac{b\left(a^2+3ba+3b^2\right)\tan^2(e+fx)+(a+2b)\left(a^2+2ba+2b^2\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(a^2+3ab+3b^2\right)\cot(e+fx)}{a+b}}{a+b}-\frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$\Big\downarrow$ 397

$$-\frac{-\frac{-\frac{-\frac{\frac{(a+b)^3\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^4\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(a^2+3ab+3b^2\right)\cot(e+fx)}{a+b}}{a+b}-\frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$\Big\downarrow$ 216

$$-\frac{-\frac{-\frac{-\frac{\frac{(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^4\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(a^2+3ab+3b^2\right)\cot(e+fx)}{a+b}}{a+b}-\frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$\Big\downarrow$ 218

$$-\frac{-\frac{-\frac{-\frac{\left(a^2+3ab+3b^2\right)\cot(e+fx)}{a+b}-\frac{\frac{(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}}{a+b}-\frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

input

```
Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output

```
(-1/5*Cot[e + f*x]^5/(a + b) - (-1/3*((a + 2*b)*Cot[e + f*x]^3)/(a + b) -
(-((((a + b)^3*ArcTan[Tan[e + f*x]])/a - (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((a^2 + 3*a*b + 3*b^2)*Cot
[e + f*x])/(a + b))/(a + b))/(a + b))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{-a-2b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(fx+e)}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
default	$\frac{-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{-a-2b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(fx+e)}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
risch	$-\frac{x}{a}-\frac{2i(45a^2e^{8i(fx+e)}+120\,e^{8i(fx+e)}ab+90\,e^{8i(fx+e)}b^2-90a^2e^{6i(fx+e)}-270ab\,e^{6i(fx+e)}-240b^2e^{6i(fx+e)}+140a^2}{15f(a+b)}$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/5/(a+b)/tan(f*x+e)^5-1/3*(-a-2*b)/(a+b)^2/tan(f*x+e)^3-(a^2+3*a*b+
3*b^2)/(a+b)^3/tan(f*x+e)+1/(a+b)^3*b^4/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x
+e)/((a+b)*b)^(1/2))-1/a*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(108) = 216$.

Time = 0.13 (sec) , antiderivative size = 833, normalized size of antiderivative = 6.94

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/60*(4*(23*a^3 + 66*a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 20*(7*a^3 + 21*a
^2*b + 20*a*b^2)*cos(f*x + e)^3 - 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x +
 e)^2 + b^3)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos
(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(a^3 + 3*a^2*
b + 3*a*b^2)*cos(f*x + e) + 60*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*
x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x
+ e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), -1/30*(2*(2
3*a^3 + 66*a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 10*(7*a^3 + 21*a^2*b + 20*a*
b^2)*cos(f*x + e)^3 + 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))
/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^3 + 3*a^2*b + 3*a*b^2
)*cos(f*x + e) + 30*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^4 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*co
s(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.34

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{(a+b)b}} - \frac{15\,(fx+e)}{a} - \frac{15\,(a^2+3\,ab+3\,b^2)\tan(fx+e)^4-5\,(a^2+3\,ab+2\,b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(15*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*sqrt((a + b)*b)) - 15*(f*x + e)/a - (15*(a^2 + 3*a*b + 3*b^
2)*tan(f*x + e)^4 - 5*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b
 + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.77

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab+b^2}} - \frac{15\,(fx+e)}{a} - \frac{15\,a^2\tan(fx+e)^4+45\,ab\tan(fx+e)^4+45\,b^2\tan(fx+e)^4-5\,a^2\tan(fx+e}{(a^3+3\,a^2b+3\,ab^2+b^3)\,\mathrm{ta}}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*b^4/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b + b^2)) -
15*(f*x + e)/a - (15*a^2*tan(f*x + e)^4 + 45*a*b*tan(f*x + e)^4 + 45*b^2*t
an(f*x + e)^4 - 5*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 - 10*b^2*tan(
f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f
*x + e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 20.83 (sec) , antiderivative size = 4324, normalized size of antiderivative = 36.03

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

output
```
(atan(((((tan(e + f*x)*(48*a*b^17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*b^15
+ 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12888*
a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6 +
210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 - ((-b^7*(a + b)^7)^(1/2)*(8*a
^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 14
168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^
11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32
*a^16*b^3 + 2*a^17*b^2 - (tan(e + f*x)*(-b^7*(a + b)^7)^(1/2)*(16*a^2*b^18
 + 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25480*a^6*b^14 + 58968*a
^7*b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*a^10*b^10 + 131560*a^
11*b^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*b^6 + 5320*a^15*b^5
+ 1080*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a + b)^7)))/(2*a*(a +
b)^7))*(-b^7*(a + b)^7)^(1/2)*1i)/(a*(a + b)^7) + (((tan(e + f*x)*(48*a*b^
17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*b^15 + 2982*a^4*b^14 + 6258*a^5*b^13
 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12888*a^8*b^10 + 10012*a^9*b^9 + 6006
*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6 + 210*a^13*b^5 + 30*a^14*b^4 + 2*
a^15*b^3))/2 + ((-b^7*(a + b)^7)^(1/2)*(8*a^2*b^17 + 108*a^3*b^16 + 680*a^
4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 14168*a^7*b^12 + 21296*a^8*b^11 +
 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^11*b^8 + 8712*a^12*b^7 + 3638*a
^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32*a^16*b^3 + 2*a^17*b^2 + (ta...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.52

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{15\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin(fx+e)^5 b^3+15\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}}{\sqrt{b}}\right)\sin(f}{}$$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

output
```
(15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*sin(e + f*x)**5*b**3 + 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((
e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**5*b**3 - 23*cos(e + f*x)*sin
(e + f*x)**4*a**4 - 89*cos(e + f*x)*sin(e + f*x)**4*a**3*b - 124*cos(e + f
*x)*sin(e + f*x)**4*a**2*b**2 - 58*cos(e + f*x)*sin(e + f*x)**4*a*b**3 + 1
1*cos(e + f*x)*sin(e + f*x)**2*a**4 + 38*cos(e + f*x)*sin(e + f*x)**2*a**3
*b + 43*cos(e + f*x)*sin(e + f*x)**2*a**2*b**2 + 16*cos(e + f*x)*sin(e + f
*x)**2*a*b**3 - 3*cos(e + f*x)*a**4 - 9*cos(e + f*x)*a**3*b - 9*cos(e + f*
x)*a**2*b**2 - 3*cos(e + f*x)*a*b**3 - 15*sin(e + f*x)**5*a**4*f*x - 60*si
n(e + f*x)**5*a**3*b*f*x - 90*sin(e + f*x)**5*a**2*b**2*f*x - 60*sin(e + f
*x)**5*a*b**3*f*x - 15*sin(e + f*x)**5*b**4*f*x)/(15*sin(e + f*x)**5*a*f*(
a**4 + 4*a**3*b + 6*a**2*b**2 + 4*a*b**3 + b**4))
```

3.350 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{(a+b)^2}{2a^2bf\left(b+a\cos^2(e+fx)\right)} - \frac{\log(\cos(e+fx))}{b^2f} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)}{2f}$$

output
```
-1/2*(a+b)^2/a^2/b/f/(b+a*cos(f*x+e)^2)-ln(cos(f*x+e))/b^2/f-1/2*(1/a^2-1/
b^2)*ln(b+a*cos(f*x+e)^2)/f
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.42

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{(a+2b+a\cos(2e+2fx))^2\left(\frac{(a+b)^2}{a^2b(b+a\cos^2(e+fx))}+\frac{2\log(\cos(e+fx))}{b^2}+\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)\right)\text{se}}{8f\left(a+b\sec^2(e+fx)\right)^2}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/8*((a + 2*b + a*Cos[2*e + 2*f*x])^2*((a + b)^2/(a^2*b*(b + a*Cos[e + f*
x]^2)) + (2*Log[Cos[e + f*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cos[e + f
*x]^2])*Sec[e + f*x]^4)/(f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{(1-\cos^2(e+fx))^2\sec(e+fx)}{(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{(1-\cos^2(e+fx))^2\sec(e+fx)}{(a\cos^2(e+fx)+b)^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(-\frac{(a+b)^2}{ab(a\cos^2(e+fx)+b)^2}+\frac{\sec(e+fx)}{b^2}+\frac{b^2-a^2}{ab^2(a\cos^2(e+fx)+b)}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)+\frac{(a+b)^2}{a^2b(a\cos^2(e+fx)+b)}+\frac{\log(\cos^2(e+fx))}{b^2}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*((a + b)^2/(a^2*b*(b + a*Cos[e + f*x]^2)) + Log[Cos[e + f*x]^2]/b^2 +
 (a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{(a+b)\left(-\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(a-b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2b^2}-\frac{\ln(\cos(fx+e))}{b^2}}{f}$
default	$\frac{\frac{(a+b)\left(-\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(a-b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2b^2}-\frac{\ln(\cos(fx+e))}{b^2}}{f}$
risch	$\frac{ix}{a^2}+\frac{2ie}{a^2f}-\frac{2(a^2+2ab+b^2)\mathrm{e}^{2i(fx+e)}}{a^2bf(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a)}-\frac{\ln(\mathrm{e}^{2i(fx+e)}+1)}{b^2f}+\frac{\ln\left(\mathrm{e}^{4i(fx+e)}+\frac{2(a+2b)\mathrm{e}^{2i(\ldots}}{a}\right.}{2b^2f}$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/b^2*(a+b)*(-(a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(a-b)/a^2*ln(b+a*cos(f
*x+e)^2))-1/b^2*ln(cos(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{a^2b+2\,ab^2+b^3-\left(a^2b-b^3+(a^3-ab^2)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+2\left(a^3\cos\left(fx+e\right)^2\right.}{2\left(a^3b^2f\cos\left(fx+e\right)^2+a^2b^3f\right)}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*b + 2*a*b^2 + b^3 - (a^2*b - b^3 + (a^3 - a*b^2)*cos(f*x + e)^2)
*log(a*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(-cos(f*x +
 e)))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{a^2+2ab+b^2}{a^3b\sin(fx+e)^2-a^3b-a^2b^2} - \frac{\log\left(\sin(fx+e)^2-1\right)}{b^2} + \frac{(a^2-b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{a^2b^2}}{2f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((a^2 + 2*a*b + b^2)/(a^3*b*sin(f*x + e)^2 - a^3*b - a^2*b^2) - log(si
n(f*x + e)^2 - 1)/b^2 + (a^2 - b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^2*b^2
))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\log(|\cos(fx+e)|)}{b^2f} + \frac{(a^2-b^2)\log\left(\left|a\cos(fx+e)^2+b\right|\right)}{2a^2b^2f} - \frac{a^2b+2ab^2+b^3}{2\left(a\cos(fx+e)^2+b\right)a^2b^2f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-log(abs(cos(f*x + e)))/(b^2*f) + 1/2*(a^2 - b^2)*log(abs(a*cos(f*x + e)^2
 + b))/(a^2*b^2*f) - 1/2*(a^2*b + 2*a*b^2 + b^3)/((a*cos(f*x + e)^2 + b)*a
^2*b^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.21

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b^2\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a^2\,f} + \frac{a^2}{2\,f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)} + \frac{b^2}{2\,f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)} + \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} + \frac{a\,b}{f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)}$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
log(a + b + b*tan(e + f*x)^2)/(2*b^2*f) - log(a + b + b*tan(e + f*x)^2)/(2
*a^2*f) + a^2/(2*f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + b^2/(2*f*(a
*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + log(tan(e + f*x)^2 + 1)/(2*a^2*f
) + (a*b)/(f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 756, normalized size of antiderivative = 9.82

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

output

```
(2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a*b**2 - 2*log(tan((e + f*
x)/2)**2 + 1)*a*b**2 - 2*log(tan((e + f*x)/2)**2 + 1)*b**3 - 2*log(tan((e
+ f*x)/2) - 1)*sin(e + f*x)**2*a**3 + 2*log(tan((e + f*x)/2) - 1)*a**3 + 2
*log(tan((e + f*x)/2) - 1)*a**2*b - 2*log(tan((e + f*x)/2) + 1)*sin(e + f*
x)**2*a**3 + 2*log(tan((e + f*x)/2) + 1)*a**3 + 2*log(tan((e + f*x)/2) + 1
)*a**2*b + log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*t
an((e + f*x)/2))*sin(e + f*x)**2*a**3 - log(sqrt(a + b)*tan((e + f*x)/2)**
2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a*b**2 - log
(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2
))*a**3 - log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*ta
n((e + f*x)/2))*a**2*b + log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
 - 2*sqrt(a)*tan((e + f*x)/2))*a*b**2 + log(sqrt(a + b)*tan((e + f*x)/2)**
2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*b**3 + log(sqrt(a + b)*tan((
e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2
*a**3 - log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan(
(e + f*x)/2))*sin(e + f*x)**2*a*b**2 - log(sqrt(a + b)*tan((e + f*x)/2)**2
 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a**3 - log(sqrt(a + b)*tan((e
 + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a**2*b + log(sqr
t(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a
*b**2 + log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*t...
```

3.351 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{a+b}{2a^2f\,(b+a\cos^2(e+fx))} + \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

output
```
1/2*(a+b)/a^2/f/(b+a*cos(f*x+e)^2)+1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.59

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{2(a+b)+(a+2b)\log(a+2b+a\cos(2(e+fx)))+a\cos(2(e+fx))\log(a+2b+a\cos(2(e+fx)))}{2a^2f(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*
x)]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(2*a^2*f*(a + 2*b + a*Cos[2*(e + f*
x)]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos(e+fx)(1-\cos^2(e+fx))}{(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{353}$$

$$-\frac{\int \frac{1-\cos^2(e+fx)}{(a\cos^2(e+fx)+b)^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(\frac{a+b}{a(a\cos^2(e+fx)+b)^2}-\frac{1}{a(a\cos^2(e+fx)+b)}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{a+b}{a^2(a\cos^2(e+fx)+b)}-\frac{\log(a\cos^2(e+fx)+b)}{a^2}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(-((a + b)/(a^2*(b + a*Cos[e + f*x]^2))) - Log[b + a*Cos[e + f*x]^2]/
a^2)/f
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^2}-\frac{-a-b}{2a^2\left(b+a\cos(fx+e)^2\right)}}{f}$	50
default	$\frac{\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^2}-\frac{-a-b}{2a^2\left(b+a\cos(fx+e)^2\right)}}{f}$	50
risch	$-\frac{ix}{a^2}-\frac{2ie}{a^2f}+\frac{2(a+b)\,\mathrm{e}^{2i(fx+e)}}{a^2f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}+\frac{\ln\left(\mathrm{e}^{4i(fx+e)}+\frac{2(a+2b)\mathrm{e}^{2i(fx+e)}}{a}+1\right)}{2a^2f}$	117

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/a^2*ln(b+a*cos(f*x+e)^2)-1/2*(-a-b)/a^2/(b+a*cos(f*x+e)^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+a+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + a + b)/(a^3*f*cos(
f*x + e)^2 + a^2*b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{a+b}{a^3\sin(fx+e)^2-a^3-a^2b} - \frac{\log\left(a\sin(fx+e)^2-a-b\right)}{a^2}}{2f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((a + b)/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 -
a - b)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\,a^2f} + \frac{a+b}{2\left(a\cos\left(fx+e\right)^2+b\right)a^2f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*log(abs(a*cos(f*x + e)^2 + b))/(a^2*f) + 1/2*(a + b)/((a*cos(f*x + e)^
2 + b)*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{4\,b^2\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan(e+f\,x)^2+\frac{8\,b^3\tan(e+f\,x)^2}{a}}\right)}{a^2\,f} - \frac{a+b}{2\,a\,b\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
- atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 +
 (8*b^3*tan(e + f*x)^2)/a))/(a^2*f) - (a + b)/(2*a*b*f*(a + b + b*tan(e +
f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.10

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{-\log\left(\tan\left(fx+e\right)^2+1\right)\sec\left(fx+e\right)^2b-\log\left(\tan\left(fx+e\right)^2+1\right)a+\log\left(\sec\left(fx+e\right)^2b+a\right)\sec\left(fx+\right.}{2a^2f\left(\sec\left(fx+e\right)^2b+a\right)}$$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - log(tan(e + f*x)**2 + 1)*sec(e + f*x)**2*b - log(tan(e + f*x)**2 + 1)*
a + log(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*b + log(sec(e + f*x)**2*b +
 a)*a + tan(e + f*x)**2*a)/(2*a**2*f*(sec(e + f*x)**2*b + a))
```

3.352 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{b}{2a^2f(b+a\cos^2(e+fx))} - \frac{\log(b+a\cos^2(e+fx))}{2a^2f}$$

output
```
-1/2*b/a^2/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.61

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{2b+(a+2b)\log(a+2b+a\cos(2(e+fx)))+a\cos(2(e+fx))\log(a+2b+a\cos(2(e+fx)))}{2a^2f(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x
)]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(a^2*f*(a + 2*b + a*Cos[2*(e + f*x)]
))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos^3(e+fx)}{(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int \frac{\cos^2(e+fx)}{(a\cos^2(e+fx)+b)^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(\frac{1}{a(a\cos^2(e+fx)+b)}-\frac{b}{a(a\cos^2(e+fx)+b)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b}{a^2(a\cos^2(e+fx)+b)}+\frac{\log(a\cos^2(e+fx)+b)}{a^2}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(b/(a^2*(b + a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/a^2)/f
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\ln(\sec(fx+e))}{f\,a^2}+\frac{1}{2fa\left(a+b\sec(fx+e)^2\right)}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2f\,a^2}$	59
default	$\frac{\ln(\sec(fx+e))}{f\,a^2}+\frac{1}{2fa\left(a+b\sec(fx+e)^2\right)}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2f\,a^2}$	59
risch	$\frac{ix}{a^2}+\frac{2ie}{a^2f}-\frac{2b\,e^{2i(fx+e)}}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2a^2f}$	115

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f/a^2*ln(sec(f*x+e))+1/2/f/a/(a+b*sec(f*x+e)^2)-1/2/f/a^2*ln(a+b*sec(f*x
+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + b)/(a^3*f*cos(f*x
 + e)^2 + a^2*b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{b}{a^3\sin(fx+e)^2-a^3-a^2b}-\frac{\log\left(a\sin(fx+e)^2-a-b\right)}{a^2}}{2f}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(b/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a - b)/
a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\,a^2f}-\frac{b}{2\left(a\cos\left(fx+e\right)^2+b\right)a^2f}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*log(abs(a*cos(f*x + e)^2 + b))/(a^2*f) - 1/2*b/((a*cos(f*x + e)^2 + b
)*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\mathrm{atanh}\left(\frac{4\,b^2\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan(e+f\,x)^2+\frac{8\,b^3\tan(e+f\,x)^2}{a}}\right)}{a^2\,f}+\frac{1}{2\,a\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (
8*b^3*tan(e + f*x)^2)/a))/(a^2*f) + 1/(2*a*f*(a + b + b*tan(e + f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.20

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\log\left(\tan\left(fx+e\right)^2+1\right)\sec\left(fx+e\right)^2 b+\log\left(\tan\left(fx+e\right)^2+1\right)a-\log\left(\sec\left(fx+e\right)^2 b+a\right)\sec\left(fx+e\right.}{2a^2 f\left(\sec\left(fx+e\right)^2 b+a\right)}$$

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(log(tan(e + f*x)**2 + 1)*sec(e + f*x)**2*b + log(tan(e + f*x)**2 + 1)*a -
 log(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*b - log(sec(e + f*x)**2*b + a)
*a - sec(e + f*x)**2*b)/(2*a**2*f*(sec(e + f*x)**2*b + a))
```

3.353 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{b^2}{2a^2(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{b(2a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^2f} + \frac{\log(\sin(e+fx))}{(a+b)^2f}$$

output
```
1/2*b^2/a^2/(a+b)/f/(b+a*cos(f*x+e)^2)+1/2*b*(2*a+b)*ln(b+a*cos(f*x+e)^2)/
a^2/(a+b)^2/f+ln(sin(f*x+e))/(a+b)^2/f
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(2\log(\sin(e+fx)) + \frac{b(2a+b)\log\left(a+b-a\sin^2(e+fx)\right)}{a^2} + \frac{b^2(a+b)}{a^2\left(a+b-a\sin^2(e\right.}\right.}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^2}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(2*Log[Sin[e + f*x]] + (b
*(2*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (b^2*(a + b))/(a^2*(a + b
- a*Sin[e + f*x]^2))))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos^5(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{b^2}{a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{(2a+b)b}{a(a+b)^2(a\cos^2(e+fx)+b)}-\frac{1}{(a+b)^2(\cos^2(e+fx)-1)}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b^2}{a^2(a+b)(a\cos^2(e+fx)+b)}-\frac{b(2a+b)\log(a\cos^2(e+fx)+b)}{a^2(a+b)^2}-\frac{\log(1-\cos^2(e+fx))}{(a+b)^2}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(-(b^2/(a^2*(a + b)*(b + a*Cos[e + f*x]^2))) - Log[1 - Cos[e + f*x]^2
]/(a + b)^2 - (b*(2*a + b)*Log[b + a*Cos[e + f*x]^2])/(a^2*(a + b)^2))/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(2a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^2}}{f}$
default	$\frac{\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(2a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^2}}{f}$
risch	$\frac{ix}{a^2}-\frac{2ix}{a^2+2ab+b^2}-\frac{2ie}{f(a^2+2ab+b^2)}-\frac{4ibx}{a(a^2+2ab+b^2)}-\frac{4ibe}{af(a^2+2ab+b^2)}-\frac{2ib^2x}{a^2(a^2+2ab+b^2)}-\frac{2ib^2e}{a^2f(a^2+2ab}$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/(a+b)^2*ln(1+cos(f*x+e))+1/2/(a+b)^2*ln(-1+cos(f*x+e))+1/2*b/(a+b
)^2*((a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(2*a+b)/a^2*ln(b+a*cos(f*x+e)^2)))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$=\frac{ab^2+b^3+\left(2\,ab^2+b^3+\left(2\,a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+2\left(a^3\cos\left(fx+e\right)^2+a^2b\right.}{2\left(\left(a^5+2\,a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^4b+2\,a^3b^2+a^2b^3\right)f\right)}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a*b^2 + b^3 + (2*a*b^2 + b^3 + (2*a^2*b + a*b^2)*cos(f*x + e)^2)*log(
a*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(1/2*sin(f*x + e
)))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2
*b^3)*f)
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.41

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{b^2}{a^4+2a^3b+a^2b^2-(a^4+a^3b)\sin(fx+e)^2} + \frac{(2ab+b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{a^4+2a^3b+a^2b^2} + \frac{\log\left(\sin(fx+e)^2\right)}{a^2+2ab+b^2}}{2f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x + e)^2) + (2*a*b
 + b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 2*a^3*b + a^2*b^2) + log(sin(
f*x + e)^2)/(a^2 + 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.42

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(2ab+b^2)\log\left(\left|a\cos(fx+e)^2+b\right|\right)}{2(a^4f+2a^3bf+a^2b^2f)} + \frac{\log\left(\left|-\cos(fx+e)^2+1\right|\right)}{2(a^2f+2abf+b^2f)} + \frac{ab^2+b^3}{2\left(a\cos(fx+e)^2+b\right)(a+b)^2a^2f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(2*a*b + b^2)*log(abs(a*cos(f*x + e)^2 + b))/(a^4*f + 2*a^3*b*f + a^2*
b^2*f) + 1/2*log(abs(-cos(f*x + e)^2 + 1))/(a^2*f + 2*a*b*f + b^2*f) + 1/2
*(a*b^2 + b^3)/((a*cos(f*x + e)^2 + b)*(a + b)^2*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\ln(\tan(e+f\,x))}{f\,(a^2+2\,a\,b+b^2)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} - \frac{b}{2\,a\,f\,(a+b)\,\left(b\tan(e+f\,x)^2+a+b\right)} + \frac{b\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(2\,a+b)}{2\,a^2\,f\,(a+b)^2}$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
log(tan(e + f*x))/(f*(2*a*b + a^2 + b^2)) - log(tan(e + f*x)^2 + 1)/(2*a^2
*f) - b/(2*a*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + (b*log(a + b + b*tan(
e + f*x)^2)*(2*a + b))/(2*a^2*f*(a + b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 740, normalized size of antiderivative = 8.92

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**3 - 4*log(tan((e + f
*x)/2)**2 + 1)*sin(e + f*x)**2*a**2*b - 2*log(tan((e + f*x)/2)**2 + 1)*sin
(e + f*x)**2*a*b**2 + 2*log(tan((e + f*x)/2)**2 + 1)*a**3 + 6*log(tan((e +
 f*x)/2)**2 + 1)*a**2*b + 6*log(tan((e + f*x)/2)**2 + 1)*a*b**2 + 2*log(ta
n((e + f*x)/2)**2 + 1)*b**3 + 2*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt
(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b + log(sqrt(a
+ b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e
 + f*x)**2*a*b**2 - 2*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) -
2*sqrt(a)*tan((e + f*x)/2))*a**2*b - 3*log(sqrt(a + b)*tan((e + f*x)/2)**2
 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*a*b**2 - log(sqrt(a + b)*tan(
(e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*b**3 + 2*log(s
qrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))
*sin(e + f*x)**2*a**2*b + log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b
) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a*b**2 - 2*log(sqrt(a + b)
*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*a**2*b -
3*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f
*x)/2))*a*b**2 - log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqr
t(a)*tan((e + f*x)/2))*b**3 + 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**3
 - 2*log(tan((e + f*x)/2))*a**3 - 2*log(tan((e + f*x)/2))*a**2*b - sin(e +
 f*x)**2*a*b**2)/(2*a**2*f*(sin(e + f*x)**2*a**3 + 2*sin(e + f*x)**2*a*...
```

3.354 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{b^3}{2a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\csc^2(e+fx)}{2(a+b)^2f} - \frac{b^2(3a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^3f} - \frac{(a+3b)\log(\sin(e+fx))}{(a+b)^3f}$$

output
```
-1/2*b^3/a^2/(a+b)^2/f/(b+a*cos(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^2/f-1/2*b
^2*(3*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^3/f-(a+3*b)*ln(sin(f*x+e))/(a+b)
^3/f
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))^2\left((a+b)\csc^2(e+fx)+2(a+3b)\log(\sin(e+fx))+\frac{b^2\left(\frac{2b(a+b)}{a+2b+a\cos(2(e+fx))}+\right.}{}\right.}{8(a+b)^3f\left(a+b\sec^2(e+fx)\right)^2}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/8*((a + 2*b + a*Cos[2*(e + f*x)])^2*((a + b)*Csc[e + f*x]^2 + 2*(a + 3*
b)*Log[Sin[e + f*x]] + (b^2*((2*b*(a + b))/(a + 2*b + a*Cos[2*(e + f*x)])
+ (3*a + b)*Log[a + b - a*Sin[e + f*x]^2]))/a^2)*Sec[e + f*x]^4)/((a + b)^
3*f*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^3\,(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{ 4626}$$

$$-\frac{\int \frac{\cos^7(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 99}$$

$$-\frac{\int \left(-\frac{b^3}{a(a+b)^2(a\cos^2(e+fx)+b)^2}+\frac{(3a+b)b^2}{a(a+b)^3(a\cos^2(e+fx)+b)}+\frac{a+3b}{(a+b)^3(\cos^2(e+fx)-1)}+\frac{1}{(a+b)^2(\cos^2(e+fx)-1)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{b^3}{a^2(a+b)^2(a\cos^2(e+fx)+b)}+\frac{b^2(3a+b)\log(a\cos^2(e+fx)+b)}{a^2(a+b)^3}+\frac{1}{(a+b)^2(1-\cos^2(e+fx))}+\frac{(a+3b)\log(1-\cos^2(e+fx))}{(a+b)^3}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(1/((a + b)^2*(1 - Cos[e + f*x]^2)) + b^3/(a^2*(a + b)^2*(b + a*Cos[e
 + f*x]^2)) + ((a + 3*b)*Log[1 - Cos[e + f*x]^2])/(a + b)^3 + (b^2*(3*a +
b)*Log[b + a*Cos[e + f*x]^2])/(a^2*(a + b)^3))/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{-\frac{b^2\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^3}-\frac{1}{4(a+b)^2(1+\cos(fx+e))}+\frac{(-a-3b)\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{1}{4(a+b)^2(-1+\cdots}}{f}$
default	$\frac{-\frac{b^2\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^3}-\frac{1}{4(a+b)^2(1+\cos(fx+e))}+\frac{(-a-3b)\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{1}{4(a+b)^2(-1+\cdots}}{f}$
risch	$-\frac{ix}{a^2}+\frac{2iax}{a^3+3a^2b+3a\,b^2+b^3}+\frac{2iae}{f(a^3+3a^2b+3a\,b^2+b^3)}+\frac{6ibx}{a^3+3a^2b+3a\,b^2+b^3}+\frac{6ibe}{f(a^3+3a^2b+3a\,b^2+b^3)}+\frac{\cdots}{a(a\cdots}$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*b^2/(a+b)^3*((a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(3*a+b)/a^2*ln(b+a*c
os(f*x+e)^2))-1/4/(a+b)^2/(1+cos(f*x+e))+1/2*(-a-3*b)/(a+b)^3*ln(1+cos(f*x
+e))+1/4/(a+b)^2/(-1+cos(f*x+e))+1/2*(-a-3*b)/(a+b)^3*ln(-1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(105) = 210$.

Time = 0.34 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.81

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{a^3b+a^2b^2+ab^3+b^4+(a^4+a^3b-ab^3-b^4)\cos(fx+e)^2-((3\,a^2b^2+ab^3)\cos(fx+e)^4-3\,ab^3-b^4\cdots}{2\,((a^6+3\,a^5b+3\,a^4b^2+a^3b^3)f\cdots}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a^3*b + a^2*b^2 + a*b^3 + b^4 + (a^4 + a^3*b - a*b^3 - b^4)*cos(f*x +
 e)^2 - ((3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - 3*a*b^3 - b^4 - (3*a^2*b^2 -
 2*a*b^3 - b^4)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 2*((a^4 + 3*a^
3*b)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - (a^4 + 2*a^3*b - 3*a^2*b^2)*cos(
f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*
f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2
- (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\int \frac{\cot^3{(e+fx)}}{(a+b\sec^2{(e+fx)})^2}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.73

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{(3\,ab^2+b^3)\log\left(a\sin(fx+e)^2-a-b\right)}{a^5+3\,a^4b+3\,a^3b^2+a^2b^3}+\frac{(a+3\,b)\log\left(\sin(fx+e)^2\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{a^3+a^2b-(a^3-b^3)\sin(fx+e)^2}{(a^5+2\,a^4b+a^3b^2)\sin(fx+e)^4-(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sin(fx+e)^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^5 + 3*a^4*b + 3*a^3
*b^2 + a^2*b^3) + (a + 3*b)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3) - (a^3 + a^2*b - (a^3 - b^3)*sin(f*x + e)^2)/((a^5 + 2*a^4*b + a^3*b
^2)*sin(f*x + e)^4 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2)
)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.56

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{(3\,ab^2+b^3)\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\left(a^5f+3\,a^4bf+3\,a^3b^2f+a^2b^3f\right)} - \frac{(a+3\,b)\log\left(\left|-\cos\left(fx+e\right)^2+1\right|\right)}{2\left(a^3f+3\,a^2bf+3\,ab^2f+b^3f\right)} + \frac{a^2b+b^3+(a^3-b^3)\cos\left(fx+e\right)^2}{2\left(a\cos\left(fx+e\right)^2+b\right)\left(\cos\left(fx+e\right)^2-1\right)(a+b)^2a^2f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a*b^2 + b^3)*log(abs(a*cos(f*x + e)^2 + b))/(a^5*f + 3*a^4*b*f + 3
*a^3*b^2*f + a^2*b^3*f) - 1/2*(a + 3*b)*log(abs(-cos(f*x + e)^2 + 1))/(a^3
*f + 3*a^2*b*f + 3*a*b^2*f + b^3*f) + 1/2*(a^2*b + b^3 + (a^3 - b^3)*cos(f
*x + e)^2)/((a*cos(f*x + e)^2 + b)*(cos(f*x + e)^2 - 1)*(a + b)^2*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} - \frac{\frac{1}{2\,(a+b)}+\frac{\tan(e+f\,x)^2\,(a\,b-b^2)}{2\,a\,(a+b)^2}}{f\,\left(b\tan\left(e+f\,x\right)^4+(a+b)\,\tan\left(e+f\,x\right)^2\right)} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a+3\,b)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(3\,a+b)}{2\,a^2\,f\,(a+b)^3}$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*a^2*f) - (1/(2*(a + b)) + (tan(e + f*x)^2*(a*b
- b^2))/(2*a*(a + b)^2))/(f*(tan(e + f*x)^2*(a + b) + b*tan(e + f*x)^4)) -
 (log(tan(e + f*x))*(a + 3*b))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (b^2*
log(a + b + b*tan(e + f*x)^2)*(3*a + b))/(2*a^2*f*(a + b)^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1057, normalized size of antiderivative = 9.52

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**4 + 6*log(tan((e + f*x)
/2)**2 + 1)*sin(e + f*x)**4*a**3*b + 6*log(tan((e + f*x)/2)**2 + 1)*sin(e
+ f*x)**4*a**2*b**2 + 2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a*b**
3 - 2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**4 - 8*log(tan((e + f
*x)/2)**2 + 1)*sin(e + f*x)**2*a**3*b - 12*log(tan((e + f*x)/2)**2 + 1)*si
n(e + f*x)**2*a**2*b**2 - 8*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a
*b**3 - 2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*b**4 - 3*log(sqrt(a
 + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(
e + f*x)**4*a**2*b**2 - log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
- 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a*b**3 + 3*log(sqrt(a + b)*t
an((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x
)**2*a**2*b**2 + 4*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*s
qrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a*b**3 + log(sqrt(a + b)*tan((e +
 f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*b*
*4 - 3*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((
e + f*x)/2))*sin(e + f*x)**4*a**2*b**2 - log(sqrt(a + b)*tan((e + f*x)/2)*
*2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a*b**3 + 3*
log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x
)/2))*sin(e + f*x)**2*a**2*b**2 + 4*log(sqrt(a + b)*tan((e + f*x)/2)**2 +
sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a*b**3 + log(...
```

3.355 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{b^4}{2a^2(a+b)^3f\left(b+a\cos^2(e+fx)\right)}+\frac{(a+2b)\csc^2(e+fx)}{(a+b)^3f} - \frac{\csc^4(e+fx)}{4(a+b)^2f}+\frac{b^3(4a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^4f} + \frac{(a^2+4ab+6b^2)\log(\sin(e+fx))}{(a+b)^4f}$$

output
```
1/2*b^4/a^2/(a+b)^3/f/(b+a*cos(f*x+e)^2)+(a+2*b)*csc(f*x+e)^2/(a+b)^3/f-1/
4*csc(f*x+e)^4/(a+b)^2/f+1/2*b^3*(4*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^4/
f+(a^2+4*a*b+6*b^2)*ln(sin(f*x+e))/(a+b)^4/f
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(4(a+b)(a+2b)\csc^2(e+fx)-(a+b)^2\csc^4(e+fx)+4(a^2\right.}{16(a+b)^4f\left(a+b\sec^2(e+\right.}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(4*(a + b)*(a + 2*b)*Csc[
e + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 4*a*b + 6*b^2)*Log[Sin[e
+ f*x]] + (2*b^3*(4*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (2*b^4*(a
+ b))/(a^2*(a + b - a*Sin[e + f*x]^2))))/(16*(a + b)^4*f*(a + b*Sec[e + f*
x]^2)^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos^9(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos^8(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{b^4}{a(a+b)^3(a\cos^2(e+fx)+b)^2}-\frac{(4a+b)b^3}{a(a+b)^4(a\cos^2(e+fx)+b)}+\frac{-a^2-4ba-6b^2}{(a+b)^4(\cos^2(e+fx)-1)}-\frac{2(a+2b)}{(a+b)^3(\cos^2(e+fx)-1)^2}-\frac{1}{(a+b)^2(\cos^2(e+f}\right.}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b^4}{a^2(a+b)^3(a\cos^2(e+fx)+b)}-\frac{b^3(4a+b)\log(a\cos^2(e+fx)+b)}{a^2(a+b)^4}-\frac{(a^2+4ab+6b^2)\log(1-\cos^2(e+fx))}{(a+b)^4}-\frac{2(a+2b)}{(a+b)^3(1-\cos^2(e+fx))}+\frac{}{2(a-}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(1/(2*(a + b)^2*(1 - Cos[e + f*x]^2)^2) - (2*(a + 2*b))/((a + b)^3*(1
 - Cos[e + f*x]^2)) - b^4/(a^2*(a + b)^3*(b + a*Cos[e + f*x]^2)) - ((a^2 +
 4*a*b + 6*b^2)*Log[1 - Cos[e + f*x]^2])/(a + b)^4 - (b^3*(4*a + b)*Log[b
+ a*Cos[e + f*x]^2])/(a^2*(a + b)^4))/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 7.77 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{-\frac{1}{16(a+b)^2(-1+\cos(fx+e))^2}-\frac{7a+15b}{16(a+b)^3(-1+\cos(fx+e))}+\frac{(a^2+4ab+6b^2)\ln(-1+\cos(fx+e))}{2(a+b)^4}+\frac{b^3\left(\frac{(a+b)b}{a^2(b+a\cos(fx+e)^2)}+\frac{(4a}{}\right)}{2(a+b)}}{f}$
default	$\frac{-\frac{1}{16(a+b)^2(-1+\cos(fx+e))^2}-\frac{7a+15b}{16(a+b)^3(-1+\cos(fx+e))}+\frac{(a^2+4ab+6b^2)\ln(-1+\cos(fx+e))}{2(a+b)^4}+\frac{b^3\left(\frac{(a+b)b}{a^2(b+a\cos(fx+e)^2)}+\frac{(4a}{}\right)}{2(a+b)}}{f}$
risch	$-\frac{12ib^2e}{f(a^4+4b\,a^3+6a^2b^2+4a\,b^3+b^4)}-\frac{2ib^4x}{a^2(a^4+4b\,a^3+6a^2b^2+4a\,b^3+b^4)}-\frac{8ib^3x}{a(a^4+4b\,a^3+6a^2b^2+4a\,b^3+b^4)}-\frac{}{a^2f(a^4}$

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/16/(a+b)^2/(-1+cos(f*x+e))^2-1/16*(7*a+15*b)/(a+b)^3/(-1+cos(f*x+e
))+1/2*(a^2+4*a*b+6*b^2)/(a+b)^4*ln(-1+cos(f*x+e))+1/2*b^3/(a+b)^4*((a+b)*
b/a^2/(b+a*cos(f*x+e)^2)+(4*a+b)/a^2*ln(b+a*cos(f*x+e)^2))-1/16/(a+b)^2/(1
+cos(f*x+e))^2-1/16*(-7*a-15*b)/(a+b)^3/(1+cos(f*x+e))+1/2*(a^2+4*a*b+6*b^
2)/(a+b)^4*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(134) = 268.

Time = 0.63 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.98

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3\,a^4b+10\,a^3b^2+7\,a^2b^3+2\,ab^4+2\,b^5-2\,(2\,a^5+6\,a^4b+4\,a^3b^2-ab^4-b^5)\cos{(fx+e)}^4+(3\,a^5+6\,a^4b}{}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/4*(3*a^4*b + 10*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 2*b^5 - 2*(2*a^5 + 6*a^4
*b + 4*a^3*b^2 - a*b^4 - b^5)*cos(f*x + e)^4 + (3*a^5 + 6*a^4*b - 5*a^3*b^
2 - 8*a^2*b^3 - 4*a*b^4 - 4*b^5)*cos(f*x + e)^2 + 2*((4*a^2*b^3 + a*b^4)*c
os(f*x + e)^6 + 4*a*b^4 + b^5 - (8*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4
 + (4*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b)
 + 4*((a^5 + 4*a^4*b + 6*a^3*b^2)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a
^2*b^3 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 - 6*a^2*b^3)*cos(f*x + e)^4 + (a^5 +
 2*a^4*b - 2*a^3*b^2 - 12*a^2*b^3)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/
((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a
^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)
^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos
(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(134) = 268$.

Time = 0.04 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.99

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{2\,(4\,ab^3+b^4)\log\left(a\sin(fx+e)^2-a-b\right)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4} + \frac{2\,(a^2+4\,ab+6\,b^2)\log\left(\sin(fx+e)^2\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{2\,(2\,a^4+4\,a^3b-b^4)\sin(fx+e)^4+a^4+2\,a^3b+a^2b^2-(5\,a^4+13\,\cdots}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sin(fx+e)^6-(a^6+4\,a^5b+6\,a^4b^2+4\,\cdots}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*(4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 4*a^5*b + 6*a^
4*b^2 + 4*a^3*b^3 + a^2*b^4) + 2*(a^2 + 4*a*b + 6*b^2)*log(sin(f*x + e)^2)
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*(2*a^4 + 4*a^3*b - b^4)*
sin(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - (5*a^4 + 13*a^3*b + 8*a^2*b^2)*
sin(f*x + e)^2)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sin(f*x + e)^6 - (a
^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sin(f*x + e)^4))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(134) = 268$.

Time = 0.18 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.20

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(4\,ab^3+b^4)\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\left(a^6f+4\,a^5bf+6\,a^4b^2f+4\,a^3b^3f+a^2b^4f\right)} + \frac{(a^2+4\,ab+6\,b^2)\log\left(\left|-\cos\left(fx+e\right)^2+1\right|\right)}{2\left(a^4f+4\,a^3bf+6\,a^2b^2f+4\,ab^3f+b^4f\right)} - \frac{\frac{2\,(2\,a^5+6\,a^4b+4\,a^3b^2-ab^4-b^5)\cos(fx+e)^4}{a} - \frac{(3\,a^5+6\,a^4b-5\,a^3b^2-8\,a^2b^3-4\,ab^4-4\,b^5)\cos(fx+e)^2}{a} - \frac{3\,a^4b+10\,a^3b^2+7\,a^2b^3+2\,ab^4+2\,\cdots}{a}}{4\left(a\cos\left(fx+e\right)^2+b\right)\left(\cos\left(fx+e\right)^2-1\right)^2(a+b)^4af}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(4*a*b^3 + b^4)*log(abs(a*cos(f*x + e)^2 + b))/(a^6*f + 4*a^5*b*f + 6*
a^4*b^2*f + 4*a^3*b^3*f + a^2*b^4*f) + 1/2*(a^2 + 4*a*b + 6*b^2)*log(abs(-
cos(f*x + e)^2 + 1))/(a^4*f + 4*a^3*b*f + 6*a^2*b^2*f + 4*a*b^3*f + b^4*f)
 - 1/4*(2*(2*a^5 + 6*a^4*b + 4*a^3*b^2 - a*b^4 - b^5)*cos(f*x + e)^4/a - (
3*a^5 + 6*a^4*b - 5*a^3*b^2 - 8*a^2*b^3 - 4*a*b^4 - 4*b^5)*cos(f*x + e)^2/
a - (3*a^4*b + 10*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 2*b^5)/a)/((a*cos(f*x +
e)^2 + b)*(cos(f*x + e)^2 - 1)^2*(a + b)^4*a*f)
```

Mupad [B] (verification not implemented)

Time = 16.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.47

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\tan(e+f\,x)^2\,(2\,a+5\,b)}{4\,(a+b)^2}-\frac{1}{4\,(a+b)}+\frac{\tan(e+f\,x)^4\,\left(a^2\,b+3\,a\,b^2-b^3\right)}{2\,a\,(a+b)^3}}{f\,\left(b\tan\left(e+f\,x\right)^6+\left(a+b\right)\,\tan\left(e+f\,x\right)^4\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} + \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+4\,a\,b+6\,b^2\right)}{f\,\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)} + \frac{b^3\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,\left(4\,a+b\right)}{2\,a^2\,f\,(a+b)^4}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((tan(e + f*x)^2*(2*a + 5*b))/(4*(a + b)^2) - 1/(4*(a + b)) + (tan(e + f*x
)^4*(3*a*b^2 + a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^4*(a + b) +
 b*tan(e + f*x)^6)) - log(tan(e + f*x)^2 + 1)/(2*a^2*f) + (log(tan(e + f*x
))*(4*a*b + a^2 + 6*b^2))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))
+ (b^3*log(a + b + b*tan(e + f*x)^2)*(4*a + b))/(2*a^2*f*(a + b)^4)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 1312, normalized size of antiderivative = 9.37

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 64*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a**5 - 256*log(tan((e
+ f*x)/2)**2 + 1)*sin(e + f*x)**6*a**4*b - 384*log(tan((e + f*x)/2)**2 + 1
)*sin(e + f*x)**6*a**3*b**2 - 256*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x
)**6*a**2*b**3 - 64*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a*b**4 +
64*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**5 + 320*log(tan((e + f*
x)/2)**2 + 1)*sin(e + f*x)**4*a**4*b + 640*log(tan((e + f*x)/2)**2 + 1)*si
n(e + f*x)**4*a**3*b**2 + 640*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4
*a**2*b**3 + 320*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a*b**4 + 64*
log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*b**5 + 128*log(sqrt(a + b)*ta
n((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)
**6*a**2*b**3 + 32*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*s
qrt(a)*tan((e + f*x)/2))*sin(e + f*x)**6*a*b**4 - 128*log(sqrt(a + b)*tan(
(e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**
4*a**2*b**3 - 160*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sq
rt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a*b**4 - 32*log(sqrt(a + b)*tan((e
 + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*
b**5 + 128*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*t
an((e + f*x)/2))*sin(e + f*x)**6*a**2*b**3 + 32*log(sqrt(a + b)*tan((e + f
*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**6*a*b*
*4 - 128*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) + 2*sqrt(a)*...
```

3.356 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=-\frac{x}{a^2}-\frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f}+\frac{(3a+b)\tan(e+fx)}{2ab^2f}-\frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
-x/a^2-1/2*(3*a-2*b)*(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^
2/b^(5/2)/f+1/2*(3*a+b)*tan(f*x+e)/a/b^2/f-1/2*(a+b)*tan(f*x+e)^3/a/b/f/(a
+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.07 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.40

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{2x(a+2b+a\cos(2(e+fx)))}{a^2}+\frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))}{2\sqrt{a+b}\sqrt{}}\right.}{}\right.}{}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(
e + f*x)]))/a^2 + ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sec[f*x]*(Cos[2*e] -
I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqr
t[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*
Sin[2*e]))/(a^2*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos
[2*(e + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x])/(b^2*f) - ((a + b)^2*((a + 2*
b)*Sin[2*e] - a*Sin[2*f*x]))/(a^2*b^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]
))))/(8*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4629, 2075, 372, 444, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^6}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 2075}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\frac{\int \frac{\tan^2(e+fx)\left((3a+b)\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2ab}-\frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{\frac{\frac{(3a+b)\tan(e+fx)}{b}-\frac{\int \frac{\left(3a^2+4ba-b^2\right)\tan^2(e+fx)+(a+b)(3a+b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{b}}{2ab}-\frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{(3a+b)\tan(e+fx)}{b}-\frac{\frac{2b^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}+\frac{(3a-2b)(a+b)^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{b}}{2ab}-\frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\frac{(3a+b)\tan(e+fx)}{b}-\frac{\frac{(3a-2b)(a+b)^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{2b^2\arctan(\tan(e+fx))}{a}}{b}}{2ab}-\frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{(3a+b)\tan(e+fx)}{b}-\frac{\frac{2b^2\arctan(\tan(e+fx))}{a}+\frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}}{b}}{2ab}-\frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((-(((2*b^2*ArcTan[Tan[e + f*x]])/a + ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/b) + ((3*a + b)*Tan[e + f*
x])/b)/(2*a*b) - ((a + b)*Tan[e + f*x]^3)/(2*a*b*(a + b + b*Tan[e + f*x]^2
)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{\frac{\left(-\frac{1}{2}a^3-a^2b-\frac{1}{2}a\,b^2\right)\tan(fx+e)}{a+b+b\tan(fx+e)^2}+\frac{\left(3a^3+4a^2b-a\,b^2-2b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2b^2}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{\frac{\left(-\frac{1}{2}a^3-a^2b-\frac{1}{2}a\,b^2\right)\tan(fx+e)}{a+b+b\tan(fx+e)^2}+\frac{\left(3a^3+4a^2b-a\,b^2-2b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2b^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(3a^3\mathrm{e}^{4i(fx+e)}+4a^2b\,\mathrm{e}^{4i(fx+e)}+5a\,b^2\mathrm{e}^{4i(fx+e)}+2b^3\mathrm{e}^{4i(fx+e)}+6a^3\mathrm{e}^{2i(fx+e)}+14\,\mathrm{e}^{2i(fx+e)}a^2b+6\,\mathrm{e}^{2i(fx+e)}\right.}{a^2b^2f\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)}$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/b^2-1/a^2*arctan(tan(f*x+e))-1/a^2/b^2*((-1/2*a^3-a^2*b-1/
2*a*b^2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a^3+4*a^2*b-a*b^2-2*b^3)/(
(a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(105) = 210$.

Time = 0.12 (sec) , antiderivative size = 514, normalized size of antiderivative = 4.32

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[-\frac{8\,ab^2fx\cos{(fx+e)}^3 + 8\,b^3fx\cos{(fx+e)} + \left((3\,a^3 + a^2b - 2\,ab^2)\cos{(fx+e)}^3 + (3\,a^2b + ab^2 - 2\,b\right.}{} \right.$$

$$-\frac{4\,ab^2fx\cos{(fx+e)}^3 + 4\,b^3fx\cos{(fx+e)} - \left((3\,a^3 + a^2b - 2\,ab^2)\cos{(fx+e)}^3 + (3\,a^2b + ab^2 - 2\,b^3\right.}{4\,(a^3b^2f\cos{(f}}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(8*a*b^2*f*x*cos(f*x + e)^3 + 8*b^3*f*x*cos(f*x + e) + ((3*a^3 + a^2
*b - 2*a*b^2)*cos(f*x + e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqr
t(-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqr
t(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2)) - 4*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*
x + e))/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e)), -1/4*(4*a*b^2
*f*x*cos(f*x + e)^3 + 4*b^3*f*x*cos(f*x + e) - ((3*a^3 + a^2*b - 2*a*b^2)*
cos(f*x + e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a + b)/b)*a
rctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x
+ e)*sin(f*x + e))) - 2*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^
2)*sin(f*x + e))/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(a^2+2ab+b^2)\tan(fx+e)}{ab^3\tan(fx+e)^2+a^2b^2+ab^3} - \frac{2(fx+e)}{a^2} + \frac{2\tan(fx+e)}{b^2} - \frac{(3a^3+4a^2b-ab^2-2b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2b^2}}{2f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((a^2 + 2*a*b + b^2)*tan(f*x + e)/(a*b^3*tan(f*x + e)^2 + a^2*b^2 + a*
b^3) - 2*(f*x + e)/a^2 + 2*tan(f*x + e)/b^2 - (3*a^3 + 4*a^2*b - a*b^2 - 2
*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2*b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{fx+e}{a^2f} + \frac{\tan(fx+e)}{b^2f} - \frac{(3a^3+4a^2b-ab^2-2b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{2\sqrt{ab+b^2}\,a^2b^2f} + \frac{a^2\tan(fx+e)+2ab\tan(fx+e)+b^2\tan(fx+e)}{2\left(b\tan(fx+e)^2+a+b\right)ab^2f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-(f*x + e)/(a^2*f) + tan(f*x + e)/(b^2*f) - 1/2*(3*a^3 + 4*a^2*b - a*b^2 -
 2*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b + b^2))/(sqrt(a*b + b^2)*a^2*b^2*f)
 + 1/2*(a^2*tan(f*x + e) + 2*a*b*tan(f*x + e) + b^2*tan(f*x + e))/((b*tan(
f*x + e)^2 + a + b)*a*b^2*f)
```

Mupad [B] (verification not implemented)

Time = 16.08 (sec) , antiderivative size = 765, normalized size of antiderivative = 6.43

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
tan(e + f*x)/(b^2*f) - atan((5*tan(e + f*x))/((12*a)/b - (10*b)/a - (15*b^
2)/(2*a^2) + (9*a^2)/(2*b^2) + 5) - (10*tan(e + f*x))/((5*a)/b - (15*b)/(2
*a) + (12*a^2)/b^2 + (9*a^3)/(2*b^3) - 10) + (12*a*tan(e + f*x))/(12*a + 5
*b - (10*b^2)/a + (9*a^2)/(2*b) - (15*b^3)/(2*a^2)) - (15*b*tan(e + f*x))/
(2*((5*a^2)/b - (15*b)/2 - 10*a + (12*a^3)/b^2 + (9*a^4)/(2*b^3))) + (9*a^
2*tan(e + f*x))/(2*(12*a*b + (9*a^2)/2 + 5*b^2 - (10*b^3)/a - (15*b^4)/(2*
a^2))))/(a^2*f) + (tan(e + f*x)*(2*a*b + a^2 + b^2))/(2*a*f*(a*b^2 + b^3 +
 b^3*tan(e + f*x)^2)) - (atan((tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 -
 a^3*b^5)^(1/2)*15i)/(2*((25*a*b^3)/4 - (49*a^3*b)/2 + 9*a^4 + (15*b^4)/2
- (85*a^2*b^2)/4 + (81*a^5)/(4*b) + (27*a^6)/(4*b^2))) - (tan(e + f*x)*(-
3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*35i)/(4*(9*a^3*b - (85*a*b^3)/4
 + (81*a^4)/4 + (25*b^4)/4 - (49*a^2*b^2)/2 + (15*b^5)/(2*a) + (27*a^5)/(4
*b))) - (tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*45i)/(
4*((81*a^3*b)/4 - (49*a*b^3)/2 + (27*a^4)/4 - (85*b^4)/4 + 9*a^2*b^2 + (25
*b^5)/(4*a) + (15*b^6)/(2*a^2))) + (a^2*tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*
a^2*b^6 - a^3*b^5)^(1/2)*27i)/(4*(9*a^2*b^4 - (85*b^6)/4 - (49*a*b^5)/2 +
(81*a^3*b^3)/4 + (27*a^4*b^2)/4 + (25*b^7)/(4*a) + (15*b^8)/(2*a^2))) + (a
*tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*27i)/(4*((27*a
^4*b)/4 - (49*a*b^4)/2 - (85*b^5)/4 + 9*a^2*b^3 + (81*a^3*b^2)/4 + (25*b^6
)/(4*a) + (15*b^7)/(2*a^2))))*(-b^5*(a + b)^3)^(1/2)*(3*a - 2*b)*1i)/(2...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 799, normalized size of antiderivative = 6.71

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*cos(e + f*x)*sin(e + f*x)**2*a**3 - sqrt(b)*sqrt(a + b)*atan((sqrt(
a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*sin(e + f*x)**2*a
**2*b + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*cos(e + f*x)*sin(e + f*x)**2*a*b**2 + 3*sqrt(b)*sqrt(a + b)*ata
n((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*a**3 + 4*
sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))
*cos(e + f*x)*a**2*b - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)
/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*a*b**2 - 2*sqrt(b)*sqrt(a + b)*atan((
sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*cos(e + f*x)*b**3 - 3*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*co
s(e + f*x)*sin(e + f*x)**2*a**3 - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*sin(e + f*x)**2*a**2*b + 2
*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b)
)*cos(e + f*x)*sin(e + f*x)**2*a*b**2 + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a
 + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*a**3 + 4*sqrt(b)*s
qrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e +
f*x)*a**2*b - sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqr
t(a))/sqrt(b))*cos(e + f*x)*a*b**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*cos(e + f*x)*b**3 - 2*cos(e + f...
```

3.357 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx=\frac{x}{a^2}+\frac{(a-2b)\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f}-\frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^2+1/2*(a-2*b)*(a+b)^(1/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/b
^(3/2)/f-1/2*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.77

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$=\frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx)))+\frac{(-a^2+ab+2b^2)\arctan\left(\frac{\sec(fx)(\cos(2e}{\cdots}\right)}{\cdots}\right.}{8a^2\left(a+b\sec^2(e\cdots\right.}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e
+ f*x)]) + ((-a^2 + a*b + 2*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*
(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
- I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(
b*Sqrt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + ((a + b)*((a + 2*b)*Sin[2
*e] - a*Sin[2*f*x]))/(b*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a
 + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4629, 2075, 372, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{(a+b\sec(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{(a-b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2ab}-\frac{(a+b)\tan(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{2b\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}+\frac{(a-2b)(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2ab}-\frac{(a+b)\tan(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{(a-2b)(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{2b\arctan(\tan(e+fx))}{a}}{2ab}-\frac{(a+b)\tan(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{2b\arctan(\tan(e+fx))}{a}+\frac{(a-2b)\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}}{2ab}-\frac{(a+b)\tan(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((2*b*ArcTan[Tan[e + f*x]])/a + ((a - 2*b)*Sqrt[a + b]*ArcTan[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/(2*a*b) - ((a + b)*Tan[e + f*x])/(2
*a*b*(a + b + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}+\frac{(a+b)\left(-\frac{a\tan(fx+e)}{2b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2b\sqrt{(a+b)b}}\right)}{a^2}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}+\frac{(a+b)\left(-\frac{a\tan(fx+e)}{2b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2b\sqrt{(a+b)b}}\right)}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{i\left(a^2 e^{2i(fx+e)}+3ab\,e^{2i(fx+e)}+2b^2 e^{2i(fx+e)}+a^2+ab\right)}{a^2bf\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{4b^2fa}-$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^2*arctan(tan(f*x+e))+(a+b)/a^2*(-1/2*a/b*tan(f*x+e)/(a+b+b*tan(f*
x+e)^2)+1/2*(a-2*b)/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(78) = 156.

Time = 0.11 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.37

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$=\left[\frac{8\,abfx\cos(fx+e)^2+8\,b^2fx-4\,(a^2+ab)\cos(fx+e)\sin(fx+e)-\left((a^2-2\,ab)\cos(fx+e)^2+ab\right.}{8\left(a^3bf\cos\right.}\right.$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*a*b*f*x*cos(f*x + e)^2 + 8*b^2*f*x - 4*(a^2 + a*b)*cos(f*x + e)*si
n(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 + a*b - 2*b^2)*sqrt(-(a + b)/b)
*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e
)^2 + 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)
*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(
a^3*b*f*cos(f*x + e)^2 + a^2*b^2*f), 1/4*(4*a*b*f*x*cos(f*x + e)^2 + 4*b^2
*f*x - 2*(a^2 + a*b)*cos(f*x + e)*sin(f*x + e) - ((a^2 - 2*a*b)*cos(f*x +
e)^2 + a*b - 2*b^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))))/(a^3*b*f*cos(f*x
 + e)^2 + a^2*b^2*f)]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\int \frac{\tan^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^2}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{(a+b)\tan(fx+e)}{ab^2\tan(fx+e)^2+a^2b+ab^2}-\frac{2(fx+e)}{a^2}-\frac{(a^2-ab-2b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2b}}{2f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((a + b)*tan(f*x + e)/(a*b^2*tan(f*x + e)^2 + a^2*b + a*b^2) - 2*(f*x
 + e)/a^2 - (a^2 - a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sq
rt((a + b)*b)*a^2*b))/f
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{fx+e}{a^2f} + \frac{(a^2-ab-2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{2\,\sqrt{ab+b^2}a^2bf} - \frac{a\tan\left(fx+e\right)+b\tan\left(fx+e\right)}{2\left(b\tan\left(fx+e\right)^2+a+b\right)abf}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
(f*x + e)/(a^2*f) + 1/2*(a^2 - a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2))/(sqrt(a*b + b^2)*a^2*b*f) - 1/2*(a*tan(f*x + e) + b*tan(f*x + e))
/((b*tan(f*x + e)^2 + a + b)*a*b*f)
```

Mupad [B] (verification not implemented)

Time = 15.60 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.17

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)}{\frac{3\,b}{2\,a}-\frac{a}{2\,b}+1}-\frac{\tan(e+f\,x)}{2\left(\frac{b}{a}+\frac{3\,b^2}{2\,a^2}-\frac{1}{2}\right)}+\frac{3\,b\tan(e+f\,x)}{2\left(a+\frac{3\,b}{2}-\frac{a^2}{2\,b}\right)}\right)}{a^2\,f} - \frac{\tan(e+f\,x)\,\left(a+b\right)}{2\,a\,b\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)} - \frac{\mathrm{atanh}\left(\frac{3\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{2\left(\frac{a\,b}{4}-a^2+\frac{3\,b^2}{2}+\frac{a^3}{4\,b}\right)}-\frac{5\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{4\left(\frac{a^2}{4}-a\,b+\frac{b^2}{4}+\frac{3\,b^3}{2\,a}\right)}+\frac{\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{4\left(\frac{a\,b}{4}-b^2+\frac{b^3}{4\,a}+\frac{3\,b^4}{2\,a^2}\right)}\right)\,\sqrt{-b^3\,\left(a+b\right)}\,(a-2\,b)}{2\,a^2\,b^3\,f}$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atan(tan(e + f*x)/((3*b)/(2*a) - a/(2*b) + 1) - tan(e + f*x)/(2*(b/a + (3*
b^2)/(2*a^2) - 1/2)) + (3*b*tan(e + f*x))/(2*(a + (3*b)/2 - a^2/(2*b))))/(
a^2*f) - (tan(e + f*x)*(a + b))/(2*a*b*f*(a + b + b*tan(e + f*x)^2)) - (at
anh((3*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(2*((a*b)/4 - a^2 + (3*b^2)/2 +
 a^3/(4*b))) - (5*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*(a^2/4 - a*b + b^
2/4 + (3*b^3)/(2*a))) + (tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*((a*b)/4 -
 b^2 + b^3/(4*a) + (3*b^4)/(2*a^2))))*(-b^3*(a + b))^(1/2)*(a - 2*b))/(2*a
^2*b^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 470, normalized size of antiderivative = 5.22

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin(fx+e)^2\,a^2 - 2\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin(fx}{}$$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
)*sin(e + f*x)**2*a**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b - sqrt(b)*sqrt(a + b)*atan
((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 + sqrt(b)*sqrt(a +
 b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b + 2*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**2 +
 sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b)
)*sin(e + f*x)**2*a**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b - sqrt(b)*sqrt(a + b)*atan
((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 + sqrt(b)*sqrt(a +
 b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b + 2*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b**2 +
 cos(e + f*x)*sin(e + f*x)*a**2*b + cos(e + f*x)*sin(e + f*x)*a*b**2 + 2*s
in(e + f*x)**2*a*b**2*f*x - 2*a*b**2*f*x - 2*b**3*f*x)/(2*a**2*b**2*f*(sin
(e + f*x)**2*a - a - b))
```

3.358 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{x}{a^2}+\frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}f}+\frac{\tan(e+fx)}{2af\,(a+b+b\tan^2(e+fx))}$$

output

```
-x/a^2+1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/b^(1/2)/(a+b
)^(1/2)/f+1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.38 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.07

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(-\frac{16x+\frac{\left(-a^3+6a^2b+24ab^2+16b^3\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))-}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}}{}\right.}{64\,(a+}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(-((16*x + ((-a^3 + 6*a^2
*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e
])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*
(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e
])))/a^2) + (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b
)^(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*
x)])))/(b^(3/2)*f)))/(64*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4629, 2075, 373, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\tan(e+fx)}{2a(a+b\tan^2(e+fx)+b)}-\frac{\int \frac{1-\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\tan(e+fx)}{2a(a+b\tan^2(e+fx)+b)}-\frac{\frac{2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{(a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\tan(e+fx)}{2a(a+b\tan^2(e+fx)+b)}-\frac{\frac{2\arctan(\tan(e+fx))}{a}-\frac{(a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\tan(e+fx)}{2a(a+b\tan^2(e+fx)+b)}-\frac{\frac{2\arctan(\tan(e+fx))}{a}-\frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\sqrt{a+b}}}{2a}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/2*((2*ArcTan[Tan[e + f*x]])/a - ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(a*Sqrt[b]*Sqrt[a + b]))/a + Tan[e + f*x]/(2*a*(a + b + b
*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{-2iba-2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{4\sqrt{-ab-b^2}\,fa}$ -

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2*arctan(tan(f*x+e))+1/a^2*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2
)+1/2*(a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(73) = 146$.

Time = 0.11 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.39

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \left[-\frac{8\,(a^2b+ab^2)fx\cos{(fx+e)}^2+8\,(ab^2+b^3)fx-4\,(a^2b+ab^2)\cos{(fx+e)}\sin{(fx+e)}+\left((a^2+2\,ab\right.}{8\left((a^4b\right.}\right.$$

$$-\frac{4\,(a^2b+ab^2)fx\cos{(fx+e)}^2+4\,(ab^2+b^3)fx-2\,(a^2b+ab^2)\cos{(fx+e)}\sin{(fx+e)}+\left((a^2+2\,ab\right.}{4\left((a^4b+a^3b^2)f\cos{(fx+e)}^2+(a^3b^2+\right.}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(8*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 8*(a*b^2 + b^3)*f*x - 4*(a^2
*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*
b + 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*
(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x +
 e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(
f*x + e)^2 + b^2)))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^3*b^2 + a^2*b
^3)*f), -1/4*(4*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b^2 + b^3)*f*x -
 2*(a^2*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)
^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b
)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))))/((a^4*b + a^3*b^2)*f*cos(f
*x + e)^2 + (a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\frac{\frac{\tan(fx+e)}{ab\tan(fx+e)^2+a^2+ab}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2}-\frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(tan(f*x + e)/(a*b*tan(f*x + e)^2 + a^2 + a*b) + (a + 2*b)*arctan(b*ta
n(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx=\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{2\sqrt{ab+b^2}a^2f}-\frac{fx+e}{a^2f}+\frac{\tan(fx+e)}{2\left(b\tan(fx+e)^2+a+b\right)af}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a + 2*b)*arctan(b*tan(f*x + e)/sqrt(a*b + b^2))/(sqrt(a*b + b^2)*a^2*
f) - (f*x + e)/(a^2*f) + 1/2*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)*a*f)
```

Mupad [B] (verification not implemented)

Time = 15.57 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.36

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
tan(e + f*x)/(2*a*f*(a + b + b*tan(e + f*x)^2)) - x/a^2 - (atan(((((tan(e
+ f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*
a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^
2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a
 + b))^(1/2)*1i)/(4*(a^3*b + a^2*b^2)) + (((tan(e + f*x)*(4*a*b^2 + a^2*b
+ 8*b^3))/(2*a^2) + ((2*a*b^2 + (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a
 + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a +
b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2)*1i)/(4*(a^3
*b + a^2*b^2)))/(((a*b)/2 + b^2)/a^3 - (((tan(e + f*x)*(4*a*b^2 + a^2*b +
8*b^3))/(2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a +
 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b)
)^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b +
a^2*b^2)) + (((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) + ((2*a*b^2
 + (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(
8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^
2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2))))*(a + 2*b)*(-b*(
a + b))^(1/2)*1i)/(2*f*(a^3*b + a^2*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 519, normalized size of antiderivative = 6.11

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin(fx+e)^2 a^2+2\sqrt{b}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\sqrt{a}}{\sqrt{b}}\right)\sin(fx-}{}$$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
)*sin(e + f*x)**2*a**2 + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b - sqrt(b)*sqrt(a + b)*atan
((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 - 3*sqrt(b)*sqrt(a
 + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b - 2*sqrt(
b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**2
 + sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(
b))*sin(e + f*x)**2*a**2 + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b - sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 - 3*sqrt(b)*sqrt
(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a*b - 2*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*b*
*2 - cos(e + f*x)*sin(e + f*x)*a**2*b - cos(e + f*x)*sin(e + f*x)*a*b**2 -
 2*sin(e + f*x)**2*a**2*b*f*x - 2*sin(e + f*x)**2*a*b**2*f*x + 2*a**2*b*f*
x + 4*a*b**2*f*x + 2*b**3*f*x)/(2*a**2*b*f*(sin(e + f*x)**2*a**2 + sin(e +
 f*x)**2*a*b - a**2 - 2*a*b - b**2))
```

3.359 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{x}{a^2}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^2-1/2*b^(1/2)*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.61

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx)))+\frac{b(3a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(}{2\sqrt{a+}}\right.}{}\right.}{8a^2\left(a+b\sec^2(e+\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e
+ f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin
[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)
^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2
*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Se
c[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4616, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+b\sec(e+fx)^2)^2}dx$$
$$\Big\downarrow \text{4616}$$
$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{316}$$
$$\frac{\frac{\int \frac{-b\tan^2(e+fx)+2a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$
$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2(a+b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b(3a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{b(3a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
(((2*(a + b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*Tan[e +
f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{4(a+b)^2fa}+$

input
```
int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^2*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(
f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time = 0.12 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.73

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2-4\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+8\left(ab+b^2\right)fx+\left(\left(3\,a^2+2\,ab\right)\cos\left(fx+\right.\right.}{8\left(\left(a^4+\right.\right.}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
 8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*
b^2)*f), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*
x + e) + 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b
^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3
*b + a^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{b\tan(fx+e)}{a^3+2\,a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2} + \frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)
^2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b
)*sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}} + \frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e
)/((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 2056, normalized size of antiderivative = 22.35

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b
^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/
(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b
^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a
^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*
a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a
^2*b^2)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a
^3*b + a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/
(2*(2*a^4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 6
4*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) +
(tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*
b^2)))/a^2 + (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5
 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^
7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20
*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*
a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan((((tan(e + f*
x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(
a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*
b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5
*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.11

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*sin(e + f*x)**2*a**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**2 + 5*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a*b +
2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*b**2 - 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(
a))/sqrt(b))*sin(e + f*x)**2*a**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b + 3*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*a**2 + 5*s
qrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*
a*b + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/
sqrt(b))*b**2 + cos(e + f*x)*sin(e + f*x)*a**2*b + cos(e + f*x)*sin(e + f*
x)*a*b**2 + 2*sin(e + f*x)**2*a**3*f*x + 4*sin(e + f*x)**2*a**2*b*f*x + 2*
sin(e + f*x)**2*a*b**2*f*x - 2*a**3*f*x - 6*a**2*b*f*x - 6*a*b**2*f*x - 2*
b**3*f*x)/(2*a**2*f*(sin(e + f*x)**2*a**3 + 2*sin(e + f*x)**2*a**2*b + sin
(e + f*x)**2*a*b**2 - a**3 - 3*a**2*b - 3*a*b**2 - b**3))
```

3.360 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{x}{a^2}+\frac{b^{3/2}(5a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}f} - \frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-x/a^2+1/2*b^(3/2)*(5*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a
+b)^(5/2)/f-1/2*(2*a-b)*cot(f*x+e)/a/(a+b)^2/f-1/2*b*cot(f*x+e)/a/(a+b)/f/
(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.29 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.38

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{2x(a+2b+a\cos(2(e+fx)))}{a^2}-\frac{b^2(5a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+}{2\sqrt{a+b}\sqrt{b(\cos(e)}}\right.}{a^2(a+}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(
e + f*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e]
)*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))
/(a^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos
[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x])/((a + b)^2*f) + (b^2*(-((a +
2*b)*Sin[2*e]) + a*Sin[2*f*x]))/(a^2*(a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e]
 + Sin[e]))))/(8*(a + b*Sec[e + f*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4629, 2075, 374, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^2}dx$$

↓ 4629

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}d\tan(e+fx)}{f}$$

↓ 2075

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

↓ 374

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left(-3b\tan^2(e+fx)+2a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

↓ 445

$$\frac{\frac{-\frac{\int \frac{2a^2+6ba+b^2+(2a-b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{(2a-b)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

↓ 397

$$\frac{\frac{-\frac{\frac{2(a+b)^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^2(5a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{(2a-b)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

↓ 216

$$\frac{\frac{-\frac{\frac{2(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b^2(5a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{(2a-b)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

↓ 218

$$\frac{\frac{-\frac{\frac{2(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b^{3/2}(5a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{(2a-b)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((-(((2*(a + b)^2*ArcTan[Tan[e + f*x]])/a - (b^(3/2)*(5*a + 2*b)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((2*a - b)*
Cot[e + f*x])/(a + b))/(2*a*(a + b)) - (b*Cot[e + f*x])/(2*a*(a + b)*(a +
b + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{b^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(5a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^2}-\frac{1}{(a+b)^2\tan(fx+e)}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{b^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(5a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^2}-\frac{1}{(a+b)^2\tan(fx+e)}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(-2a^3e^{4i(fx+e)}+a\,b^2e^{4i(fx+e)}+2b^3e^{4i(fx+e)}-4a^3e^{2i(fx+e)}-8\,e^{2i(fx+e)}a^2b-2b^3e^{2i(fx+e)}-2a^3-a\,b^2\right)}{a^2(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}-1\right)}-$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2*arctan(tan(f*x+e))+b^2/a^2/(a+b)^2*(1/2*a*tan(f*x+e)/(a+b+b*ta
n(f*x+e)^2)+1/2*(5*a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1
/2)))-1/(a+b)^2/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(107) = 214$.

Time = 0.13 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.99

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(2*a^3 + a*b^2)*cos(f*x + e)^3 - (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*
a*b^2)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(
f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2
)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 4*(2*a
^2*b - a*b^2)*cos(f*x + e) + 8*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2
 + (a^2*b + 2*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*
f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), -1/4*(2
*(2*a^3 + a*b^2)*cos(f*x + e)^3 + (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a*b^2)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*s
qrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(2*a^2*b -
a*b^2)*cos(f*x + e) + 4*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^2
*b + 2*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f
*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(5ab^2+2b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+2a^3b+a^2b^2)\sqrt{(a+b)b}} - \frac{(2ab-b^2)\tan(fx+e)^2+2a^2+2ab}{(a^3b+2a^2b^2+ab^3)\tan(fx+e)^3+(a^4+3a^3b+3a^2b^2+ab^3)\tan(fx+e)} - \frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((5*a*b^2 + 2*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 2*a^
3*b + a^2*b^2)*sqrt((a + b)*b)) - ((2*a*b - b^2)*tan(f*x + e)^2 + 2*a^2 +
2*a*b)/((a^3*b + 2*a^2*b^2 + a*b^3)*tan(f*x + e)^3 + (a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*tan(f*x + e)) - 2*(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(5\,ab^2+2\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+2\,a^3b+a^2b^2)\sqrt{ab+b^2}} - \frac{2\,ab\tan(fx+e)^2-b^2\tan(fx+e)^2+2\,a^2+2\,ab}{\left(b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)\right)(a^3+2\,a^2b+ab^2)} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((5*a*b^2 + 2*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan
(f*x + e)/sqrt(a*b + b^2)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b + b^2)) -
(2*a*b*tan(f*x + e)^2 - b^2*tan(f*x + e)^2 + 2*a^2 + 2*a*b)/((b*tan(f*x +
e)^3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^3 + 2*a^2*b + a*b^2)) - 2*(f*x
+ e)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 19.78 (sec) , antiderivative size = 3146, normalized size of antiderivative = 26.00

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(atan((((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 1616
0*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^
6 + 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) - ((-b^3*(a + b)^5)^(1/2)*
(5*a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 +
29568*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a
^14*b^4 + 1408*a^15*b^3 + 128*a^16*b^2 - (tan(e + f*x)*(-b^3*(a + b)^5)^(1
/2)*(5*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^1
0*b^10 + 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14
*b^6 + 53760*a^15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4
*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*
a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a +
b)^5)^(1/2)*(5*a + 2*b)*1i)/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a
^4*b^3 + 10*a^5*b^2)) + ((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 616
0*a^5*b^11 + 16160*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^
7 + 10400*a^10*b^6 + 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) + ((-b^3*
(a + b)^5)^(1/2)*(5*a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 +
 15360*a^9*b^9 + 29568*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*
a^13*b^5 + 6720*a^14*b^4 + 1408*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(-
b^3*(a + b)^5)^(1/2)*(5*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9
*b^11 + 72960*a^10*b^10 + 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 784, normalized size of antiderivative = 6.48

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

output
```
(5*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**3*a**2*b + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((
e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*a*b**2 - 5*sqrt(b)*sqrt(a
+ b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*a
**2*b - 7*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*sin(e + f*x)*a*b**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*t
an((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*b**3 + 5*sqrt(b)*sqrt(a +
 b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**3
*a**2*b + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(
a))/sqrt(b))*sin(e + f*x)**3*a*b**2 - 5*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
 b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)*a**2*b - 7*sqrt(b)*s
qrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e +
f*x)*a*b**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + s
qrt(a))/sqrt(b))*sin(e + f*x)*b**3 - 2*cos(e + f*x)*sin(e + f*x)**2*a**4 -
 2*cos(e + f*x)*sin(e + f*x)**2*a**3*b - cos(e + f*x)*sin(e + f*x)**2*a**2
*b**2 - cos(e + f*x)*sin(e + f*x)**2*a*b**3 + 2*cos(e + f*x)*a**4 + 4*cos(
e + f*x)*a**3*b + 2*cos(e + f*x)*a**2*b**2 - 2*sin(e + f*x)**3*a**4*f*x -
6*sin(e + f*x)**3*a**3*b*f*x - 6*sin(e + f*x)**3*a**2*b**2*f*x - 2*sin(e +
 f*x)**3*a*b**3*f*x + 2*sin(e + f*x)*a**4*f*x + 8*sin(e + f*x)*a**3*b*f*x
+ 12*sin(e + f*x)*a**2*b**2*f*x + 8*sin(e + f*x)*a*b**3*f*x + 2*sin(e +...
```

3.361 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}f} + \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^2-1/2*b^(5/2)*(7*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(7/2)/f+1/2*(2*a^2+6*a*b-b^2)*cot(f*x+e)/a/(a+b)^3/f-1/6*(2*a-3*b)*cot(
f*x+e)^3/a/(a+b)^2/f-1/2*b*cot(f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.44 (sec) , antiderivative size = 1588, normalized size of antiderivative = 9.92

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((48*b^3*(7*a + 2*b)*ArcTan
[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*
x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e
+ f*x)])*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4]) + Csc[e]*Csc[e + f*x]^3*Sec[2*e]*(-6*(a + b)^3*(a + 6*b)*f*x*Cos[f*x]
+ 3*(a - 4*b)*(a + b)^3*f*x*Cos[3*f*x] + 6*a^4*f*x*Cos[2*e - f*x] + 54*a^3
*b*f*x*Cos[2*e - f*x] + 126*a^2*b^2*f*x*Cos[2*e - f*x] + 114*a*b^3*f*x*Cos
[2*e - f*x] + 36*b^4*f*x*Cos[2*e - f*x] + 6*a^4*f*x*Cos[2*e + f*x] + 54*a^
3*b*f*x*Cos[2*e + f*x] + 126*a^2*b^2*f*x*Cos[2*e + f*x] + 114*a*b^3*f*x*Co
s[2*e + f*x] + 36*b^4*f*x*Cos[2*e + f*x] - 6*a^4*f*x*Cos[4*e + f*x] - 54*a
^3*b*f*x*Cos[4*e + f*x] - 126*a^2*b^2*f*x*Cos[4*e + f*x] - 114*a*b^3*f*x*C
os[4*e + f*x] - 36*b^4*f*x*Cos[4*e + f*x] - 3*a^4*f*x*Cos[2*e + 3*f*x] + 3
*a^3*b*f*x*Cos[2*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[2*e + 3*f*x] + 33*a*b^3*f
*x*Cos[2*e + 3*f*x] + 12*b^4*f*x*Cos[2*e + 3*f*x] + 3*a^4*f*x*Cos[4*e + 3*
f*x] - 3*a^3*b*f*x*Cos[4*e + 3*f*x] - 27*a^2*b^2*f*x*Cos[4*e + 3*f*x] - 33
*a*b^3*f*x*Cos[4*e + 3*f*x] - 12*b^4*f*x*Cos[4*e + 3*f*x] - 3*a^4*f*x*Cos[
6*e + 3*f*x] + 3*a^3*b*f*x*Cos[6*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[6*e + 3*f
*x] + 33*a*b^3*f*x*Cos[6*e + 3*f*x] + 12*b^4*f*x*Cos[6*e + 3*f*x] - 3*a^4*
f*x*Cos[2*e + 5*f*x] - 9*a^3*b*f*x*Cos[2*e + 5*f*x] - 9*a^2*b^2*f*x*Cos[2*
e + 5*f*x] - 3*a*b^3*f*x*Cos[2*e + 5*f*x] + 3*a^4*f*x*Cos[4*e + 5*f*x] ...
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4629, 2075, 374, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-5b\tan^2(e+fx)+2a-3b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(2a^2+6ba-b^2+(2a-3b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3(a+b)}-\frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)}-\frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2a^2+6ba-b^2+(2a-3b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)}-\frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$\Big\downarrow$ 445

$$\frac{-\frac{-\frac{\int \frac{2a^3+8ba^2+12b^2a+b^3+b\left(2a^2+6ba-b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{a+b}}{a+b}-\frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)}-\frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$\Big\downarrow$ 397

$$\frac{-\frac{-\frac{\frac{2(a+b)^3\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^3(7a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{a+b}}{a+b}-\frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)}-\frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$\Big\downarrow$ 216

$$\frac{-\frac{-\frac{\frac{2(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^3(7a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{a+b}}{a+b}-\frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)}-\frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$\Big\downarrow$ 218

$$\frac{-\frac{-\frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{a+b}-\frac{\frac{2(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^{5/2}(7a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}}{a+b}-\frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)}-\frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input

```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output

```
((-1/3*((2*a - 3*b)*Cot[e + f*x]^3)/(a + b) - (-(((2*(a + b)^3*ArcTan[Tan[
e + f*x]])/a - (b^(5/2)*(7*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b]])/(a*Sqrt[a + b]))/(a + b)) - ((2*a^2 + 6*a*b - b^2)*Cot[e + f*x])/(a
+ b))/(a + b))/(2*a*(a + b)) - (b*Cot[e + f*x]^3)/(2*a*(a + b)*(a + b + b*
Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 5.81 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{b^{3}\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^{2}}+\frac{(7a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^{2}(a+b)^{3}}+\frac{\arctan(\tan(fx+e))}{a^{2}}-\frac{1}{3(a+b)^{2}\tan(fx+e)^{3}}-\frac{-a-3b}{(a+b)^{3}\tan(fx+e)}}{f}$
default	$\frac{-\frac{b^{3}\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^{2}}+\frac{(7a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^{2}(a+b)^{3}}+\frac{\arctan(\tan(fx+e))}{a^{2}}-\frac{1}{3(a+b)^{2}\tan(fx+e)^{3}}-\frac{-a-3b}{(a+b)^{3}\tan(fx+e)}}{f}$
risch	$\frac{x}{a^{2}}-\frac{i\left(-12a^{4}e^{8i(fx+e)}-24a^{3}b\,e^{8i(fx+e)}+3a\,b^{3}e^{8i(fx+e)}+6b^{4}e^{8i(fx+e)}-12a^{4}e^{6i(fx+e)}-60a^{3}b\,e^{6i(fx+e)}-96a^{2}b^{2}e^{6}\right.}{}$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b^3/a^2/(a+b)^3*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(7*a+2*b)
/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(f*
x+e))-1/3/(a+b)^2/tan(f*x+e)^3-(-a-3*b)/(a+b)^3/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(144) = 288$.

Time = 0.14 (sec) , antiderivative size = 979, normalized size of antiderivative = 6.12

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/24*(4*(8*a^4 + 20*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 8*(3*a^4 + 5*a^3*b
- 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x
+ e)^4 - 7*a*b^3 - 2*b^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*s
qrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b
^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)
*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 +
2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(2*a^3*b + 6*a^2*b^2 - a*b^
3)*cos(f*x + e) + 24*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)
^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b
^2 + 3*a*b^3 + b^4)*f*x)*sin(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x +
e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e)), 1/12*(2
*(8*a^4 + 20*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 4*(3*a^4 + 5*a^3*b - 10*a^2
*b^2 + 3*a*b^3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 -
 7*a*b^3 - 2*b^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt(b/(a
 + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*
x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(f
*x + e) + 12*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^
4 + 2*a^3*b - 2*a*b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4)*f*x)*sin(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*...
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.47

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3(7ab^3+2b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+3a^4b+3a^3b^2+a^2b^3)\sqrt{(a+b)b}} - \frac{3(2a^2b+6ab^2-b^3)\tan(fx+e)^4-2a^3-4a^2b-2ab^2+2(3a^3+11a^2b+8ab^2)\tan(fx+e)^2}{(a^4b+3a^3b^2+3a^2b^3+ab^4)\tan(fx+e)^5+(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)\tan(fx+e)^3} - \frac{6(fx+e)}{a^2}}{6f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(7*a*b^3 + 2*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*b)) - (3*(2*a^2*b + 6*a*b^2 - b
^3)*tan(f*x + e)^4 - 2*a^3 - 4*a^2*b - 2*a*b^2 + 2*(3*a^3 + 11*a^2*b + 8*a
*b^2)*tan(f*x + e)^2)/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(f*x + e
)^5 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*tan(f*x + e)^3) - 6*
(f*x + e)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.32

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx =$$

$$-\frac{\frac{3\,b^3\tan(fx+e)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\left(b\tan(fx+e)^2+a+b\right)}+\frac{3\,(7\,ab^3+2\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\sqrt{ab+b^2}}-\frac{6\,(fx+e)}{a^2}-\frac{2\,(3\,a\tan}{(a^3+}}{6\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*b^3*tan(f*x + e)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(b*tan(f*x +
 e)^2 + a + b)) + 3*(7*a*b^3 + 2*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
 + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*sqrt(a*b + b^2)) - 6*(f*x + e)/a^2 - 2*(3*a*tan(f*x + e)^2 + 9*b*t
an(f*x + e)^2 - a - b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 21.96 (sec) , antiderivative size = 4987, normalized size of antiderivative = 31.17

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((tan(e + f*x)^2*(3*a + 8*b))/(3*(a + b)^2) - 1/(3*(a + b)) + (tan(e + f*x
)^4*(6*a*b^2 + 2*a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b)
 + b*tan(e + f*x)^5)) + atan((560*a^3*b^16*tan(e + f*x))/(560*a^3*b^16 + 7
280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440
*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a
^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3
 + 64*a^17*b^2) + (7280*a^4*b^15*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^
15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11
+ 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 +
116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17
*b^2) + (42560*a^5*b^14*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 4256
0*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*
a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^
13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (
149184*a^6*b^13*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^
14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10
 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 +
 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (351904*a
^7*b^12*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149
184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 69...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 993, normalized size of antiderivative = 6.21

$$\int \frac{\cot^4(e + fx)}{(a + b\sec^2(e + fx))^2} \, dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

output

```
( - 21*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**5*a**2*b**2 - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a*b**3 + 21*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e
+ f*x)**3*a**2*b**2 + 27*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*
x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*a*b**3 + 6*sqrt(b)*sqrt(a + b)*a
tan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*b**4
 - 21*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sq
rt(b))*sin(e + f*x)**5*a**2*b**2 - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**5*a*b**3 + 21*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e +
 f*x)**3*a**2*b**2 + 27*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x
)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**3*a*b**3 + 6*sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**3*b**4
+ 8*cos(e + f*x)*sin(e + f*x)**4*a**5 + 28*cos(e + f*x)*sin(e + f*x)**4*a*
*4*b + 20*cos(e + f*x)*sin(e + f*x)**4*a**3*b**2 + 3*cos(e + f*x)*sin(e +
f*x)**4*a**2*b**3 + 3*cos(e + f*x)*sin(e + f*x)**4*a*b**4 - 10*cos(e + f*x
)*sin(e + f*x)**2*a**5 - 40*cos(e + f*x)*sin(e + f*x)**2*a**4*b - 50*cos(e
 + f*x)*sin(e + f*x)**2*a**3*b**2 - 20*cos(e + f*x)*sin(e + f*x)**2*a**2*b
**3 + 2*cos(e + f*x)*a**5 + 6*cos(e + f*x)*a**4*b + 6*cos(e + f*x)*a**3...
```

3.362 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{x}{a^2} + \frac{b^{7/2}(9a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{9/2}f} - \frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2a(a+b)^4f} + \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f\,(a+b+b\tan^2(e+fx))}$$

output
```
-x/a^2+1/2*b^(7/2)*(9*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a
+b)^(9/2)/f-1/2*(2*a^3+8*a^2*b+12*a*b^2-b^3)*cot(f*x+e)/a/(a+b)^4/f+1/6*(2
*a^2+6*a*b-3*b^2)*cot(f*x+e)^3/a/(a+b)^3/f-1/10*(2*a-5*b)*cot(f*x+e)^5/a/(
a+b)^2/f-1/2*b*cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.14 (sec) , antiderivative size = 3028, normalized size of antiderivative = 14.63

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((9*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-1/8*(b^4*Ar
cTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) -
((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f
*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^2*Sqrt[a + b]*f*Sqrt
[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/8)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqr
t[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*
Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*
e + f*x])]*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))
/((a + b)^4*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Cs
c[e]*Csc[e + f*x]^5*Sec[2*e]*Sec[e + f*x]^4*(75*a^5*f*x*Cos[f*x] + 900*a^4
*b*f*x*Cos[f*x] + 2850*a^3*b^2*f*x*Cos[f*x] + 3900*a^2*b^3*f*x*Cos[f*x] +
2475*a*b^4*f*x*Cos[f*x] + 600*b^5*f*x*Cos[f*x] - 15*a^5*f*x*Cos[3*f*x] + 2
40*a^4*b*f*x*Cos[3*f*x] + 1110*a^3*b^2*f*x*Cos[3*f*x] + 1740*a^2*b^3*f*x*C
os[3*f*x] + 1185*a*b^4*f*x*Cos[3*f*x] + 300*b^5*f*x*Cos[3*f*x] - 75*a^5*f*
x*Cos[2*e - f*x] - 900*a^4*b*f*x*Cos[2*e - f*x] - 2850*a^3*b^2*f*x*Cos[2*e
 - f*x] - 3900*a^2*b^3*f*x*Cos[2*e - f*x] - 2475*a*b^4*f*x*Cos[2*e - f*x]
- 600*b^5*f*x*Cos[2*e - f*x] - 75*a^5*f*x*Cos[2*e + f*x] - 900*a^4*b*f*x*C
os[2*e + f*x] - 2850*a^3*b^2*f*x*Cos[2*e + f*x] - 3900*a^2*b^3*f*x*Cos[2*e
 + f*x] - 2475*a*b^4*f*x*Cos[2*e + f*x] - 600*b^5*f*x*Cos[2*e + f*x] + 75*
a^5*f*x*Cos[4*e + f*x] + 900*a^4*b*f*x*Cos[4*e + f*x] + 2850*a^3*b^2*f*...
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4629, 2075, 374, 445, 27, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(-7b\tan^2(e+fx)+2a-5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b\cot^5(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int \frac{5\cot^4(e+fx)\left(2a^2+6ba-3b^2+(2a-5b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{5(a+b)}-\frac{(2a-5b)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)}-\frac{b\cot^5(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{\int \frac{\cot^4(e+fx)\left(2a^2+6ba-3b^2+(2a-5b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{(2a-5b)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)}-\frac{b\cot^5(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(2a^3+8ba^2+12b^2a-b^3+b\left(2a^2+6ba-3b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3(a+b)}-\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{3(a+b)}}{a+b}-\frac{(2a-5b)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)}-\frac{b\,\mathrm{co}}{2a(a+b)(a+}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2a^3+8ba^2+12b^2a-b^3+b\left(2a^2+6ba-3b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{3(a+b)}}{a+b}-\frac{(2a-5b)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)}-\frac{b\cot}{2a(a+b)(a+b}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{-\frac{-\frac{\int \frac{2a^4+10ba^3+20b^2a^2+20b^3a+b^4+b\left(2a^3+8ba^2+12b^2a-b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{a+b}}{a+b}-\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx}{3(a+b)}}{a+b}}{2a(a+b)}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{-\frac{-\frac{-\frac{-\frac{2(a+b)^4\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^4(9a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{a+b}}{a+b}-\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx}{3(a+b)}}{a+b}}{2a(a+b)}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{-\frac{-\frac{-\frac{\frac{2(a+b)^4\arctan(\tan(e+fx))}{a}-\frac{b^4(9a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{a+b}}{a+b}-\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{3(a+b)}-\frac{(2a-5b}{}}{a+b}}{2a(a+b)}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{-\frac{-\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{3(a+b)}-\frac{-\frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{a+b}-\frac{\frac{2(a+b)^4\arctan(\tan(e+fx))}{a}-\frac{b^{7/2}(9a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}}{a+b}}{a+b}-\frac{(2a-5b)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((-1/5*((2*a - 5*b)*Cot[e + f*x]^5)/(a + b) - (-1/3*((2*a^2 + 6*a*b - 3*b^
2)*Cot[e + f*x]^3)/(a + b) - (-(((2*(a + b)^4*ArcTan[Tan[e + f*x]])/a - (b
^(7/2)*(9*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a +
 b]))/(a + b)) - ((2*a^3 + 8*a^2*b + 12*a*b^2 - b^3)*Cot[e + f*x])/(a + b)
)/(a + b))/(a + b))/(2*a*(a + b)) - (b*Cot[e + f*x]^5)/(2*a*(a + b)*(a + b
 + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397

```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 10.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\frac{b^4\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(9a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4a^2}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{-a-3b}{3(a+b)^3\tan(fx+e)^3}-\frac{a^2+4ab+6b^2}{(a+b)^4\tan(fx+e)}}{f}$
default	$\frac{\frac{b^4\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(9a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4a^2}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{-a-3b}{3(a+b)^3\tan(fx+e)^3}-\frac{a^2+4ab+6b^2}{(a+b)^4\tan(fx+e)}}{f}$
risch	$-\frac{x}{a^2}-\frac{i(46a^5+15a\,b^4+172b\,a^4+216a^3b^2+90a^5\,\mathrm{e}^{12i(fx+e)}+150b^5\,\mathrm{e}^{10i(fx+e)}+300b^5\,\mathrm{e}^{6i(fx+e)}+240a^5\,\mathrm{e}^{6i(fx+e)}-30}{}$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^4/(a+b)^4/a^2*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(9*a+2*b)/
((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/5/(a+b)^2/tan(f*x+
e)^5-1/3*(-a-3*b)/(a+b)^3/tan(f*x+e)^3-(a^2+4*a*b+6*b^2)/(a+b)^4/tan(f*x+e
)-1/a^2*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(189) = 378.

Time = 0.17 (sec) , antiderivative size = 1505, normalized size of antiderivative = 7.27

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/120*(4*(46*a^5 + 172*a^4*b + 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 -
4*(70*a^5 + 234*a^4*b + 218*a^3*b^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)
^5 + 20*(6*a^5 + 10*a^4*b - 20*a^3*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e
)^3 - 15*((9*a^2*b^3 + 2*a*b^4)*cos(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2
*b^3 - 5*a*b^4 - 2*b^5)*cos(f*x + e)^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*co
s(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(2*a^4*b + 8
*a^3*b^2 + 12*a^2*b^3 - a*b^4)*cos(f*x + e) + 120*((a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2
 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*x*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*
b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*x*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2
 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^7 + 4*a^6*b + 6*a^5*
b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2
 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*
a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4
*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e)), -1/60*(2*(46
*a^5 + 172*a^4*b + 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 2*(70*a^5 + 23
4*a^4*b + 218*a^3*b^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + 10*(6*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.54

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{15\,(9\,ab^4+2\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\sqrt{(a+b)b}} - \frac{15\,(2\,a^3b+8\,a^2b^2+12\,ab^3-b^4)\tan(fx+e)^6+10\,(3\,a^4+14\,a^3b+26\,a^2b^2+15\,ab^3)\tan(fx+e)^4+6\,a}{(a^5b+4\,a^4b^2+6\,a^3b^3+4\,a^2b^4+ab^5)\tan(fx+e)^7+(a^6+5\,a^5b+10}}{30\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/30*(15*(9*a*b^4 + 2*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 +
4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt((a + b)*b)) - (15*(2*a^3*b
 + 8*a^2*b^2 + 12*a*b^3 - b^4)*tan(f*x + e)^6 + 10*(3*a^4 + 14*a^3*b + 26*
a^2*b^2 + 15*a*b^3)*tan(f*x + e)^4 + 6*a^4 + 18*a^3*b + 18*a^2*b^2 + 6*a*b
^3 - 2*(5*a^4 + 22*a^3*b + 29*a^2*b^2 + 12*a*b^3)*tan(f*x + e)^2)/((a^5*b
+ 4*a^4*b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*tan(f*x + e)^7 + (a^6 + 5*a^5
*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*tan(f*x + e)^5) - 30*(f*
x + e)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.44

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{15\,b^4\tan(fx+e)}{(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\left(b\tan(fx+e)^2+a+b\right)} + \frac{15\,(9\,ab^4+2\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\sqrt{ab+b^2}} - \frac{30\,(fx+e)}{a^2} - \frac{2\,(}{}}{30\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/30*(15*b^4*tan(f*x + e)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)
*(b*tan(f*x + e)^2 + a + b)) + 15*(9*a*b^4 + 2*b^5)*(pi*floor((f*x + e)/pi
 + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 4*a^5*b +
 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b + b^2)) - 30*(f*x + e)/a^2 - 2*
(15*a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 - 5
*a^2*tan(f*x + e)^2 - 20*a*b*tan(f*x + e)^2 - 15*b^2*tan(f*x + e)^2 + 3*a^
2 + 6*a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x +
e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 21.51 (sec) , antiderivative size = 6017, normalized size of antiderivative = 29.07

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atan(((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 +
997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b
^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874
112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8
+ 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 +
2688*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^2
2 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*
b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 1182
43840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^
19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 644
9664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632
*a^27*b^3 + 256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) + tan(e + f*x)*(128*a^3
*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19
 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*
b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154
592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 3
10080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23
*b^3))/(2*a^2) - (((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*
a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 1780
9920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^1...
```

Reduce [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 1216, normalized size of antiderivative = 5.87

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

output

```
(135*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqr
t(b))*sin(e + f*x)**7*a**2*b**3 + 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**7*a*b**4 - 135*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e
+ f*x)**5*a**2*b**3 - 165*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f
*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a*b**4 - 30*sqrt(b)*sqrt(a + b)
*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*b*
*5 + 135*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))
/sqrt(b))*sin(e + f*x)**7*a**2*b**3 + 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a
+ b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**7*a*b**4 - 135*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*si
n(e + f*x)**5*a**2*b**3 - 165*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
 + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**5*a*b**4 - 30*sqrt(b)*sqrt(a
+ b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**
5*b**5 - 46*cos(e + f*x)*sin(e + f*x)**6*a**6 - 218*cos(e + f*x)*sin(e + f
*x)**6*a**5*b - 388*cos(e + f*x)*sin(e + f*x)**6*a**4*b**2 - 216*cos(e + f
*x)*sin(e + f*x)**6*a**3*b**3 - 15*cos(e + f*x)*sin(e + f*x)**6*a**2*b**4
- 15*cos(e + f*x)*sin(e + f*x)**6*a*b**5 + 68*cos(e + f*x)*sin(e + f*x)**4
*a**6 + 350*cos(e + f*x)*sin(e + f*x)**4*a**5*b + 712*cos(e + f*x)*sin(e +
 f*x)**4*a**4*b**2 + 646*cos(e + f*x)*sin(e + f*x)**4*a**3*b**3 + 216*c...
```

3.363 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+b)^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{a+b}{a^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

output
```
1/4*(a+b)^2/a^3/f/(b+a*cos(f*x+e)^2)^2-(a+b)/a^3/f/(b+a*cos(f*x+e)^2)-1/2*
ln(b+a*cos(f*x+e)^2)/a^3/f
```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.74

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{2(a^2+4ab+3b^2)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(\ldots}{2a^3f(a+2b+a\cos(2(e-\ldots}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x
)]] + a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2
*(e + f*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^
3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos(e+fx)\left(1-\cos^2(e+fx)\right)^2}{\left(a\cos^2(e+fx)+b\right)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{353}$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2}{\left(a\cos^2(e+fx)+b\right)^3}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(\frac{(a+b)^2}{a^2\left(a\cos^2(e+fx)+b\right)^3}-\frac{2(a+b)}{a^2\left(a\cos^2(e+fx)+b\right)^2}+\frac{1}{a^2\left(a\cos^2(e+fx)+b\right)}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{(a+b)^2}{2a^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{2(a+b)}{a^3\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{a^3}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(-1/2*(a + b)^2/(a^3*(b + a*Cos[e + f*x]^2)^2) + (2*(a + b))/(a^3*(b
+ a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/a^3)/f
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 11.79 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{\ln\left(b+a\cos(fx+e)^{2}\right)}{2a^{3}}+\frac{a^{2}+2ab+b^{2}}{4a^{3}\left(b+a\cos(fx+e)^{2}\right)^{2}}+\frac{-2a-2b}{2a^{3}\left(b+a\cos(fx+e)^{2}\right)}}{f}$
default	$\frac{-\frac{\ln\left(b+a\cos(fx+e)^{2}\right)}{2a^{3}}+\frac{a^{2}+2ab+b^{2}}{4a^{3}\left(b+a\cos(fx+e)^{2}\right)^{2}}+\frac{-2a-2b}{2a^{3}\left(b+a\cos(fx+e)^{2}\right)}}{f}$
risch	$\frac{ix}{a^{3}}+\frac{2ie}{a^{3}f}-\frac{4\left(a^{2}e^{6i(fx+e)}+ab\,e^{6i(fx+e)}+a^{2}e^{4i(fx+e)}+4ab\,e^{4i(fx+e)}+3b^{2}e^{4i(fx+e)}+a^{2}e^{2i(fx+e)}+ab\,e^{2i(fx+e)}\right)}{a^{3}f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^{2}}$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/a^3*ln(b+a*cos(f*x+e)^2)+1/4*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)
^2)^2+1/2*(-2*a-2*b)/a^3/(b+a*cos(f*x+e)^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{4\left(a^{2}+ab\right)\cos\left(fx+e\right)^{2}-a^{2}+2\,ab+3\,b^{2}+2\left(a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\log\left(a\cos\left(\right.\right.}{4\left(a^{5}f\cos\left(fx+e\right)^{4}+2\,a^{4}bf\cos\left(fx+e\right)^{2}+a^{3}b^{2}f\right)}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
-1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - a^2 + 2*a*b + 3*b^2 + 2*(a^2*cos(f*x
+ e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos
(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{4\,(a^2+ab)\sin(fx+e)^2-3\,a^2-6\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2} - \frac{2\,\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((4*(a^2 + a*b)*sin(f*x + e)^2 - 3*a^2 - 6*a*b - 3*b^2)/(a^5*sin(f*x +
 e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(
a*sin(f*x + e)^2 - a - b)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\,a^3f} - \frac{4\,(a+b)\cos\left(fx+e\right)^2-\frac{a^2-2\,ab-3\,b^2}{a}}{4\,\left(a\cos\left(fx+e\right)^2+b\right)^2a^2f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/2*log(abs(a*cos(f*x + e)^2 + b))/(a^3*f) - 1/4*(4*(a + b)*cos(f*x + e)^
2 - (a^2 - 2*a*b - 3*b^2)/a)/((a*cos(f*x + e)^2 + b)^2*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 16.50 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.13

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\operatorname{atanh}\left(\frac{4\,b^2\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan(e+f\,x)^2+\frac{8\,b^3\tan(e+f\,x)^2}{a}}\right)}{a^3\,f} + \frac{\frac{-a^3+3\,a\,b^2+2\,b^3}{4\,a^2\,b^2}-\frac{\tan(e+f\,x)^2\,(a^2-b^2)}{2\,a^2\,b}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)}$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (
8*b^3*tan(e + f*x)^2)/a))/(a^3*f) + ((3*a*b^2 - a^3 + 2*b^3)/(4*a^2*b^2) -
 (tan(e + f*x)^2*(a^2 - b^2))/(2*a^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f
*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.77

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{2\log\left(\tan\left(fx+e\right)^2+1\right)\sec\left(fx+e\right)^4b^2+4\log\left(\tan\left(fx+e\right)^2+1\right)\sec\left(fx+e\right)^2ab+2\log\left(\tan\left(fx+e\right.\right.}{}$$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
(2*log(tan(e + f*x)**2 + 1)*sec(e + f*x)**4*b**2 + 4*log(tan(e + f*x)**2 +
 1)*sec(e + f*x)**2*a*b + 2*log(tan(e + f*x)**2 + 1)*a**2 - 2*log(sec(e +
f*x)**2*b + a)*sec(e + f*x)**4*b**2 - 4*log(sec(e + f*x)**2*b + a)*sec(e +
 f*x)**2*a*b - 2*log(sec(e + f*x)**2*b + a)*a**2 - 2*sec(e + f*x)**2*tan(e
 + f*x)**2*a*b + tan(e + f*x)**4*a**2 - 2*tan(e + f*x)**2*a**2)/(4*a**3*f*
(sec(e + f*x)**4*b**2 + 2*sec(e + f*x)**2*a*b + a**2))
```

3.364 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b(a+b)}{4a^3f\,(b+a\cos^2(e+fx))^2} + \frac{a+2b}{2a^3f\,(b+a\cos^2(e+fx))} + \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

output
```
-1/4*b*(a+b)/a^3/f/(b+a*cos(f*x+e)^2)^2+1/2*(a+2*b)/a^3/f/(b+a*cos(f*x+e)^
2)+1/2*ln(b+a*cos(f*x+e)^2)/a^3/f
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{2(a^2+3ab+3b^2)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e}{2a^3f(a+2b+a\cos(2(e+fx))}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] +
 a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*(a + 2*b)*
Cos[2*(e + f*x)]*(1 + Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(2*a^3*f*(a + 2*
b + a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^3}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \; 4626$$

$$-\frac{\int \frac{\cos^3(e+fx)(1-\cos^2(e+fx))}{(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 354$$

$$-\frac{\int \frac{\cos^2(e+fx)(1-\cos^2(e+fx))}{(a\cos^2(e+fx)+b)^3}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \; 86$$

$$-\frac{\int \left(-\frac{b(a+b)}{a^2(a\cos^2(e+fx)+b)^3}-\frac{1}{a^2(a\cos^2(e+fx)+b)}+\frac{a+2b}{a^2(a\cos^2(e+fx)+b)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{b(a+b)}{2a^3(a\cos^2(e+fx)+b)^2}-\frac{a+2b}{a^3(a\cos^2(e+fx)+b)}-\frac{\log(a\cos^2(e+fx)+b)}{a^3}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*((b*(a + b))/(2*a^3*(b + a*Cos[e + f*x]^2)^2) - (a + 2*b)/(a^3*(b + a
*Cos[e + f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/a^3)/f
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 6.82 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{(a+b)b}{4a^3\left(b+a\cos(fx+e)^2\right)^2}-\frac{-a-2b}{2a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^3}}{f}$
default	$\frac{-\frac{(a+b)b}{4a^3\left(b+a\cos(fx+e)^2\right)^2}-\frac{-a-2b}{2a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^3}}{f}$
risch	$-\frac{ix}{a^3}-\frac{2ie}{a^3f}+\frac{2a^2e^{6i(fx+e)}+4ab\,e^{6i(fx+e)}+4a^2e^{4i(fx+e)}+12ab\,e^{4i(fx+e)}+12b^2e^{4i(fx+e)}+2a^2e^{2i(fx+e)}+4ab\,e^{2i(}}{a^3f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/4*(a+b)*b/a^3/(b+a*cos(f*x+e)^2)^2-1/2*(-a-2*b)/a^3/(b+a*cos(f*x+e
)^2)+1/2/a^3*ln(b+a*cos(f*x+e)^2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{2\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2+ab+3\,b^2+2\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\log\left(a\cos\left(fx+e\right)\right.}{4\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 3*b^2 + 2*(a^2*cos(f*x + e)^4
+ 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x +
e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\,(a^2+2\,ab)\sin(fx+e)^2-2\,a^2-5\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2}-\frac{2\,\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((2*(a^2 + 2*a*b)*sin(f*x + e)^2 - 2*a^2 - 5*a*b - 3*b^2)/(a^5*sin(f*
x + e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*l
og(a*sin(f*x + e)^2 - a - b)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\,a^3f}+\frac{2\,(a+2\,b)\cos\left(fx+e\right)^2+\frac{ab+3\,b^2}{a}}{4\,\left(a\cos\left(fx+e\right)^2+b\right)^2a^2f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/2*log(abs(a*cos(f*x + e)^2 + b))/(a^3*f) + 1/4*(2*(a + 2*b)*cos(f*x + e)
^2 + (a*b + 3*b^2)/a)/((a*cos(f*x + e)^2 + b)^2*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.89

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{a^2+3\,a\,b+2\,b^2}{4\,a^2\,b}+\frac{b\,\tan(e+f\,x)^2}{2\,a^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\,\tan\left(e+f\,x\right)^4\right)} - \frac{\mathrm{atanh}\left(\frac{4\,b^2\,\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\,\tan(e+f\,x)^2+\frac{8\,b^3\,\tan(e+f\,x)^2}{a}}\right)}{a^3\,f}$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
- ((3*a*b + a^2 + 2*b^2)/(4*a^2*b) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b
 + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) - ata
nh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b
^3*tan(e + f*x)^2)/a))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.79

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{-2\log\left(\tan\left(fx+e\right)^2+1\right)\sec\left(fx+e\right)^4b^2-4\log\left(\tan\left(fx+e\right)^2+1\right)\sec\left(fx+e\right)^2ab-2\log\left(\tan\left(fx+\right.\right.}{}$$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
( - 2*log(tan(e + f*x)**2 + 1)*sec(e + f*x)**4*b**2 - 4*log(tan(e + f*x)**
2 + 1)*sec(e + f*x)**2*a*b - 2*log(tan(e + f*x)**2 + 1)*a**2 + 2*log(sec(e
 + f*x)**2*b + a)*sec(e + f*x)**4*b**2 + 4*log(sec(e + f*x)**2*b + a)*sec(
e + f*x)**2*a*b + 2*log(sec(e + f*x)**2*b + a)*a**2 + sec(e + f*x)**4*b**2
 + sec(e + f*x)**2*tan(e + f*x)**2*a*b + sec(e + f*x)**2*a*b + 2*tan(e + f
*x)**2*a**2)/(4*a**3*f*(sec(e + f*x)**4*b**2 + 2*sec(e + f*x)**2*a*b + a**
2))
```

3.365 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{b^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b}{a^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

output
```
1/4*b^2/a^3/f/(b+a*cos(f*x+e)^2)^2-b/a^3/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*c
os(f*x+e)^2)/a^3/f
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.74

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{2b(2a+3b)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e+}{2a^3f(a+2b+a\cos(2(e+fx))}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a^
2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(e + f*
x)]*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^3*f*(a + 2*b +
 a*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos^5(e+fx)}{(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(a\cos^2(e+fx)+b)^3}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(\frac{b^2}{a^2(a\cos^2(e+fx)+b)^3}-\frac{2b}{a^2(a\cos^2(e+fx)+b)^2}+\frac{1}{a^2(a\cos^2(e+fx)+b)}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b^2}{2a^3(a\cos^2(e+fx)+b)^2}+\frac{2b}{a^3(a\cos^2(e+fx)+b)}+\frac{\log(a\cos^2(e+fx)+b)}{a^3}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(-1/2*b^2/(a^3*(b + a*Cos[e + f*x]^2)^2) + (2*b)/(a^3*(b + a*Cos[e +
f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/a^3)/f
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 6.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{1}{4fa\left(a+b\sec(fx+e)^2\right)^2}+\frac{1}{2f\,a^2\left(a+b\sec(fx+e)^2\right)}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2f\,a^3}+\frac{\ln(\sec(fx+e))}{f\,a^3}$
default	$\frac{1}{4fa\left(a+b\sec(fx+e)^2\right)^2}+\frac{1}{2f\,a^2\left(a+b\sec(fx+e)^2\right)}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2f\,a^3}+\frac{\ln(\sec(fx+e))}{f\,a^3}$
risch	$\frac{ix}{a^3}+\frac{2ie}{a^3f}-\frac{4b\left(a\,e^{6i(fx+e)}+2a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}+a\,e^{2i(fx+e)}\right)}{a^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2f}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2a^3f}$

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/f/a/(a+b*sec(f*x+e)^2)^2+1/2/f/a^2/(a+b*sec(f*x+e)^2)-1/2/f/a^3*ln(a+b
*sec(f*x+e)^2)+1/f/a^3*ln(sec(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{4\,ab\cos{(fx+e)}^2+3\,b^2+2\,\left(a^2\cos{(fx+e)}^4+2\,ab\cos{(fx+e)}^2+b^2\right)\log\left(a\cos{(fx+e)}^2+b\right)}{4\,\left(a^5f\cos{(fx+e)}^4+2\,a^4bf\cos{(fx+e)}^2+a^3b^2f\right)}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
-1/4*(4*a*b*cos(f*x + e)^2 + 3*b^2 + 2*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
 + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*
f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\frac{\frac{4\,ab\sin(fx+e)^2-4\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2}-\frac{2\,\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((4*a*b*sin(f*x + e)^2 - 4*a*b - 3*b^2)/(a^5*sin(f*x + e)^4 + a^5 + 2*
a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*sin(f*x + e)^2
 - a - b)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=-\frac{\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\,a^3f}-\frac{4\,b\cos\left(fx+e\right)^2+\frac{3\,b^2}{a}}{4\,\left(a\cos\left(fx+e\right)^2+b\right)^2a^2f}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/2*log(abs(a*cos(f*x + e)^2 + b))/(a^3*f) - 1/4*(4*b*cos(f*x + e)^2 + 3*
b^2/a)/((a*cos(f*x + e)^2 + b)^2*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 17.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.92

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\,a+2\,b}{4\,a^2}+\frac{b\tan(e+f\,x)^2}{2\,a^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)} + \frac{\mathrm{atanh}\left(\frac{4\,b^2\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan(e+f\,x)^2+\frac{8\,b^3\tan(e+f\,x)^2}{a}}\right)}{a^3\,f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((3*a + 2*b)/(4*a^2) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b + a^2 + b^2 +
 tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + atanh((4*b^2*tan(
e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b^3*tan(e + f*x
)^2)/a))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.65

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{2\log\left(\tan\left(fx+e\right)^2+1\right)\sec\left(fx+e\right)^4b^2+4\log\left(\tan\left(fx+e\right)^2+1\right)\sec\left(fx+e\right)^2ab+2\log\left(\tan\left(fx+e\right.\right.}{}$$

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
(2*log(tan(e + f*x)**2 + 1)*sec(e + f*x)**4*b**2 + 4*log(tan(e + f*x)**2 +
 1)*sec(e + f*x)**2*a*b + 2*log(tan(e + f*x)**2 + 1)*a**2 - 2*log(sec(e +
f*x)**2*b + a)*sec(e + f*x)**4*b**2 - 4*log(sec(e + f*x)**2*b + a)*sec(e +
 f*x)**2*a*b - 2*log(sec(e + f*x)**2*b + a)*a**2 - 3*sec(e + f*x)**4*b**2
- 4*sec(e + f*x)**2*a*b)/(4*a**3*f*(sec(e + f*x)**4*b**2 + 2*sec(e + f*x)*
*2*a*b + a**2))
```

3.366 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b^3}{4a^3(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2f\left(b+a\cos^2(e+fx)\right)} + \frac{b(3a^2+3ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^3f} + \frac{\log(\sin(e+fx))}{(a+b)^3f}$$

output
```
-1/4*b^3/a^3/(a+b)/f/(b+a*cos(f*x+e)^2)^2+1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/f/
(b+a*cos(f*x+e)^2)+1/2*b*(3*a^2+3*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^
3/f+ln(sin(f*x+e))/(a+b)^3/f
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(4\log(\sin(e+fx))+\frac{2b(3a^2+3ab+b^2)\log(a+b-a\sin^2(e+fx))}{a^3}-\frac{}{a^3(a+b}\right.}{32(a+b)^3f\,(a+b\sec^2(e+fx))^3}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(4*Log[Sin[e + f*x]] + (2
*b*(3*a^2 + 3*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3 - (b^3*(a + b)
^2)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^3*
(a + b - a*Sin[e + f*x]^2))))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\tan(e+fx)\,(a+b\sec(e+fx)^2)^3}dx$$
$$\Big\downarrow \; 4626$$
$$-\frac{\int \frac{\cos^7(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$
$$\Big\downarrow \; 354$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^3}d\cos^2(e+fx)}{2f}$$

$\Big\downarrow$ 99

$$-\frac{\int \left(-\frac{b^3}{a^2(a+b)(a\cos^2(e+fx)+b)^3}+\frac{(3a+2b)b^2}{a^2(a+b)^2(a\cos^2(e+fx)+b)^2}-\frac{(3a^2+3ba+b^2)b}{a^2(a+b)^3(a\cos^2(e+fx)+b)}-\frac{1}{(a+b)^3(\cos^2(e+fx)-1)}\right)d\cos^2(e+fx)}{2f}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{b^3}{2a^3(a+b)(a\cos^2(e+fx)+b)^2}-\frac{b^2(3a+2b)}{a^3(a+b)^2(a\cos^2(e+fx)+b)}-\frac{b(3a^2+3ab+b^2)\log(a\cos^2(e+fx)+b)}{a^3(a+b)^3}-\frac{\log(1-\cos^2(e+fx))}{(a+b)^3}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(b^3/(2*a^3*(a + b)*(b + a*Cos[e + f*x]^2)^2) - (b^2*(3*a + 2*b))/(a^
3*(a + b)^2*(b + a*Cos[e + f*x]^2)) - Log[1 - Cos[e + f*x]^2]/(a + b)^3 -
(b*(3*a^2 + 3*a*b + b^2)*Log[b + a*Cos[e + f*x]^2])/(a^3*(a + b)^3))/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 9.88 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}+\frac{b\left(\frac{b(3a^2+5ab+2b^2)}{a^3(b+a\cos(fx+e)^2)}+\frac{(3a^2+3ab+b^2)\ln(b+a\cos(fx+e)^2)}{a^3}-\frac{b^2(a^2+2ab+b^2)}{2a^3(b+a\cos(fx+e)^2)^2}\right)}{2(a+b)^3}+\frac{\ln(1+\cos(fx+e}{2(a+b)^3}}{f}$
default	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}+\frac{b\left(\frac{b(3a^2+5ab+2b^2)}{a^3(b+a\cos(fx+e)^2)}+\frac{(3a^2+3ab+b^2)\ln(b+a\cos(fx+e)^2)}{a^3}-\frac{b^2(a^2+2ab+b^2)}{2a^3(b+a\cos(fx+e)^2)^2}\right)}{2(a+b)^3}+\frac{\ln(1+\cos(fx+e}{2(a+b)^3}}{f}$
risch	$\frac{ix}{a^3}-\frac{2ix}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2ie}{f(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6ibx}{a(a^3+3a^2b+3a\,b^2+b^3)}-\frac{6ibe}{af(a^3+3a^2b+3a\,b^2+b^3)}-$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/(a+b)^3*ln(-1+cos(f*x+e))+1/2*b/(a+b)^3*(1/a^3*b*(3*a^2+5*a*b+2*b
^2)/(b+a*cos(f*x+e)^2)+(3*a^2+3*a*b+b^2)/a^3*ln(b+a*cos(f*x+e)^2)-1/2*b^2*
(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2)+1/2/(a+b)^3*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(124) = 248$.

Time = 0.41 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.36

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{5\,a^2b^3+8\,ab^4+3\,b^5+2\,(3\,a^3b^2+5\,a^2b^3+2\,ab^4)\cos\left(fx+e\right)^2+2\left(3\,a^2b^3+3\,ab^4+b^5+(3\,a^4b+3\,a^3b^2\right.}{4\left((a^8+3\,a^7b+3\,a^6b^2+a^5b^3)f\cos\left(f\right.\right.}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(5*a^2*b^3 + 8*a*b^4 + 3*b^5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*cos
(f*x + e)^2 + 2*(3*a^2*b^3 + 3*a*b^4 + b^5 + (3*a^4*b + 3*a^3*b^2 + a^2*b^
3)*cos(f*x + e)^4 + 2*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*log(
a*cos(f*x + e)^2 + b) + 4*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a
^3*b^2)*log(1/2*sin(f*x + e)))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*co
s(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^
2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.87

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{2\,(3\,a^2b+3\,ab^2+b^3)\log\left(a\sin(fx+e)^2-a-b\right)}{a^6+3\,a^5b+3\,a^4b^2+a^3b^3}+\frac{6\,a^2b^2+9\,ab^3+3\,b^4-2\,(3\,a^2b^2+2\,ab^3)\sin(fx+e)^2}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4+(a^7+2\,a^6b+a^5b^2)\sin(fx+e)^4-2\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\,\mathrm{s}}}{4\,f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3) + (6*a^2*b^2 + 9*a*b^3 + 3*b^4 - 2*(3*a^2*b^2 +
 2*a*b^3)*sin(f*x + e)^2)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4
 + (a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2
 + a^4*b^3)*sin(f*x + e)^2) + 2*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.46

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(3\,a^2b+3\,ab^2+b^3)\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\,(a^6f+3\,a^5bf+3\,a^4b^2f+a^3b^3f)} + \frac{\log\left(\left|-\cos\left(fx+e\right)^2+1\right|\right)}{2\,(a^3f+3\,a^2bf+3\,ab^2f+b^3f)} + \frac{2\,(3\,a^2b^2+5\,ab^3+2\,b^4)\cos\left(fx+e\right)^2+\frac{5\,a^2b^3+8\,ab^4+3\,b^5}{a}}{4\left(a\cos\left(fx+e\right)^2+b\right)^2(a+b)^3a^2f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(abs(a*cos(f*x + e)^2 + b))/(a^6*f + 3*a^
5*b*f + 3*a^4*b^2*f + a^3*b^3*f) + 1/2*log(abs(-cos(f*x + e)^2 + 1))/(a^3*
f + 3*a^2*b*f + 3*a*b^2*f + b^3*f) + 1/4*(2*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*
cos(f*x + e)^2 + (5*a^2*b^3 + 8*a*b^4 + 3*b^5)/a)/((a*cos(f*x + e)^2 + b)^
2*(a + b)^3*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 18.95 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.46

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\ln(\tan(e+f\,x))}{f\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)} - \frac{\frac{2\,b^2+5\,a\,b}{4\,a^2\,(a+b)}+\frac{b\tan(e+f\,x)^2\,(b^2+2\,a\,b)}{2\,a^2\,(a+b)^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f} + \frac{b\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(3\,a^2+3\,a\,b+b^2)}{2\,a^3\,f\,(a+b)^3}$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
log(tan(e + f*x))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - ((5*a*b + 2*b^2)/(
4*a^2*(a + b)) + (b*tan(e + f*x)^2*(2*a*b + b^2))/(2*a^2*(a + b)^2))/(f*(2
*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) -
 log(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b*log(a + b + b*tan(e + f*x)^2)*(3*a
*b + 3*a^2 + b^2))/(2*a^3*f*(a + b)^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1871, normalized size of antiderivative = 14.39

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
( - 4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**5 - 12*log(tan((e +
f*x)/2)**2 + 1)*sin(e + f*x)**4*a**4*b - 12*log(tan((e + f*x)/2)**2 + 1)*s
in(e + f*x)**4*a**3*b**2 - 4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*
a**2*b**3 + 8*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**5 + 32*log(t
an((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**4*b + 48*log(tan((e + f*x)/2)**
2 + 1)*sin(e + f*x)**2*a**3*b**2 + 32*log(tan((e + f*x)/2)**2 + 1)*sin(e +
 f*x)**2*a**2*b**3 + 8*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a*b**4
 - 4*log(tan((e + f*x)/2)**2 + 1)*a**5 - 20*log(tan((e + f*x)/2)**2 + 1)*a
**4*b - 40*log(tan((e + f*x)/2)**2 + 1)*a**3*b**2 - 40*log(tan((e + f*x)/2
)**2 + 1)*a**2*b**3 - 20*log(tan((e + f*x)/2)**2 + 1)*a*b**4 - 4*log(tan((
e + f*x)/2)**2 + 1)*b**5 + 6*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a
+ b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**4*b + 6*log(sqrt(a +
 b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e
+ f*x)**4*a**3*b**2 + 2*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
- 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**2*b**3 - 12*log(sqrt(a +
b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e +
 f*x)**2*a**4*b - 24*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2
*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**2*a**3*b**2 - 16*log(sqrt(a + b)*
tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*
x)**2*a**2*b**3 - 4*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) -...
```

3.367 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{b^4}{4a^3(a+b)^2f\,(b+a\cos^2(e+fx))^2} - \frac{b^3(2a+b)}{a^3(a+b)^3f\,(b+a\cos^2(e+fx))} - \frac{\csc^2(e+fx)}{2(a+b)^3f} - \frac{b^2(6a^2+4ab+b^2)\log(b+a\cos^2(e+fx))}{2a^3(a+b)^4f} - \frac{(a+4b)\log(\sin(e+fx))}{(a+b)^4f}$$

output
```
1/4*b^4/a^3/(a+b)^2/f/(b+a*cos(f*x+e)^2)^2-b^3*(2*a+b)/a^3/(a+b)^3/f/(b+a*
cos(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^3/f-1/2*b^2*(6*a^2+4*a*b+b^2)*ln(b+a*
cos(f*x+e)^2)/a^3/(a+b)^4/f-(a+4*b)*ln(sin(f*x+e))/(a+b)^4/f
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(2(a+b)\csc^2(e+fx)+4(a+4b)\log(\sin(e+fx))+\frac{2b^2(6a\cdots}{\cdots}\right.}{32(a+b)^4f\,(a+b\sec^2(e+fx))\cdots}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/32*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*Csc[e +
f*x]^2 + 4*(a + 4*b)*Log[Sin[e + f*x]] + (2*b^2*(6*a^2 + 4*a*b + b^2)*Log[
a + b - a*Sin[e + f*x]^2])/a^3 - (b^4*(a + b)^2)/(a^3*(a + b - a*Sin[e + f
*x]^2)^2) + (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/(
(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\tan(e+fx)^3\,(a+b\sec(e+fx)^2)^3}dx$$
$$\Big\downarrow \ 4626$$
$$-\frac{\int \frac{\cos^9(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos^8(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^3}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{b^4}{a^2(a+b)^2(a\cos^2(e+fx)+b)^3}-\frac{2(2a+b)b^3}{a^2(a+b)^3(a\cos^2(e+fx)+b)^2}+\frac{(6a^2+4ba+b^2)b^2}{a^2(a+b)^4(a\cos^2(e+fx)+b)}+\frac{a+4b}{(a+b)^4(\cos^2(e+fx)-1)}+\frac{}{(a+b)^3(\cos}\right.}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b^4}{2a^3(a+b)^2(a\cos^2(e+fx)+b)^2}+\frac{2b^3(2a+b)}{a^3(a+b)^3(a\cos^2(e+fx)+b)}+\frac{b^2(6a^2+4ab+b^2)\log(a\cos^2(e+fx)+b)}{a^3(a+b)^4}+\frac{1}{(a+b)^3(1-\cos^2(e+fx))}+\frac{(a}{}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(1/((a + b)^3*(1 - Cos[e + f*x]^2)) - b^4/(2*a^3*(a + b)^2*(b + a*Cos
[e + f*x]^2)^2) + (2*b^3*(2*a + b))/(a^3*(a + b)^3*(b + a*Cos[e + f*x]^2))
 + ((a + 4*b)*Log[1 - Cos[e + f*x]^2])/(a + b)^4 + (b^2*(6*a^2 + 4*a*b + b
^2)*Log[b + a*Cos[e + f*x]^2])/(a^3*(a + b)^4))/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 17.36 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^3(-1+\cos(fx+e))}+\frac{(-a-4b)\ln(-1+\cos(fx+e))}{2(a+b)^4}-\frac{b^2\left(-\frac{b^2(a^2+2ab+b^2)}{2a^3(b+a\cos(fx+e)^2)^2}+\frac{(6a^2+4ab+b^2)\ln(b+a\cos(fx+e)^2)}{a^3}+\right.}{2(a+b)^4}}{f}$
default	$\frac{\frac{1}{4(a+b)^3(-1+\cos(fx+e))}+\frac{(-a-4b)\ln(-1+\cos(fx+e))}{2(a+b)^4}-\frac{b^2\left(-\frac{b^2(a^2+2ab+b^2)}{2a^3(b+a\cos(fx+e)^2)^2}+\frac{(6a^2+4ab+b^2)\ln(b+a\cos(fx+e)^2)}{a^3}+\right.}{2(a+b)^4}}{f}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4/(a+b)^3/(-1+cos(f*x+e))+1/2*(-a-4*b)/(a+b)^4*ln(-1+cos(f*x+e))-1/
2*b^2/(a+b)^4*(-1/2*b^2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2+(6*a^2+4*
a*b+b^2)/a^3*ln(b+a*cos(f*x+e)^2)+2/a^3*b*(2*a^2+3*a*b+b^2)/(b+a*cos(f*x+e
)^2))-1/4/(a+b)^3/(1+cos(f*x+e))+1/2*(-a-4*b)/(a+b)^4*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(148) = 296.

Time = 0.76 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.79

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{2\,a^4b^2 + 2\,a^3b^3 + 7\,a^2b^4 + 10\,ab^5 + 3\,b^6 + 2\,(a^6 + a^5b - 4\,a^3b^3 - 6\,a^2b^4 - 2\,ab^5)\cos(fx+e)^4 + (4\,a^5b + 4}{}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*a^4*b^2 + 2*a^3*b^3 + 7*a^2*b^4 + 10*a*b^5 + 3*b^6 + 2*(a^6 + a^5*b
 - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 + (4*a^5*b + 4*a^4*b^2
+ 8*a^3*b^3 + 5*a^2*b^4 - 6*a*b^5 - 3*b^6)*cos(f*x + e)^2 - 2*((6*a^4*b^2
+ 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (6*a^4
*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (12*a^3*b^3 + 2*a
^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 4*((a^
6 + 4*a^5*b)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - (a^6 + 2*a^5*b - 8*a^4
*b^2)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 - 4*a^3*b^3)*cos(f*x + e)^2)*l
og(1/2*sin(f*x + e)))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f
*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a
^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 -
2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 +
 4*a^4*b^5 + a^3*b^6)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(148) = 296$.

Time = 0.04 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.23

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$
$$-\frac{\frac{2\,(6\,a^2b^2+4\,ab^3+b^4)\log\left(a\sin(fx+e)^2-a-b\right)}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4}+\frac{2\,(a+4\,b)\log\left(\sin(fx+e)^2\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\,a^5+4\,a^4b+2\,a^3b^2+2\,(a^5-4\,a^2b^3-}{(a^8+3\,a^7b+3\,a^6b^2+a^5b^3)\sin(fx+e)^6-2\,(a^8+4\,a^7b+6\,a}}{4\,f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^7 + 4
*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 2*(a + 4*b)*log(sin(f*x + e)^2
)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*a^5 + 4*a^4*b + 2*a^3*b
^2 + 2*(a^5 - 4*a^2*b^3 - 2*a*b^4)*sin(f*x + e)^4 - (4*a^5 + 4*a^4*b - 8*a
^2*b^3 - 11*a*b^4 - 3*b^5)*sin(f*x + e)^2)/((a^8 + 3*a^7*b + 3*a^6*b^2 + a
^5*b^3)*sin(f*x + e)^6 - 2*(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^
4)*sin(f*x + e)^4 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 +
 a^3*b^5)*sin(f*x + e)^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(148) = 296$.

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.98

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{(6\,a^2b^2+4\,ab^3+b^4)\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\,(a^7f+4\,a^6bf+6\,a^5b^2f+4\,a^4b^3f+a^3b^4f)}$$
$$-\frac{(a+4\,b)\log\left(\left|-\cos\left(fx+e\right)^2+1\right|\right)}{2\,(a^4f+4\,a^3bf+6\,a^2b^2f+4\,ab^3f+b^4f)}$$
$$+\frac{2\,(a^5+a^4b-4\,a^2b^3-6\,ab^4-2\,b^5)\cos\left(fx+e\right)^4+\frac{(4\,a^5b+4\,a^4b^2+8\,a^3b^3+5\,a^2b^4-6\,ab^5-3\,b^6)\cos(fx+e)^2}{a}+\frac{2\,a^4b^2+2}{}}{4\,\left(a\cos\left(fx+e\right)^2+b\right)^2\left(\cos\left(fx+e\right)^2-1\right)(a+b)^4a^2f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(abs(a*cos(f*x + e)^2 + b))/(a^7*f + 4
*a^6*b*f + 6*a^5*b^2*f + 4*a^4*b^3*f + a^3*b^4*f) - 1/2*(a + 4*b)*log(abs(
-cos(f*x + e)^2 + 1))/(a^4*f + 4*a^3*b*f + 6*a^2*b^2*f + 4*a*b^3*f + b^4*f
) + 1/4*(2*(a^5 + a^4*b - 4*a^2*b^3 - 6*a*b^4 - 2*b^5)*cos(f*x + e)^4 + (4
*a^5*b + 4*a^4*b^2 + 8*a^3*b^3 + 5*a^2*b^4 - 6*a*b^5 - 3*b^6)*cos(f*x + e)
^2/a + (2*a^4*b^2 + 2*a^3*b^3 + 7*a^2*b^4 + 10*a*b^5 + 3*b^6)/a)/((a*cos(f
*x + e)^2 + b)^2*(cos(f*x + e)^2 - 1)*(a + b)^4*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 19.69 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.77

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{\tan(e+f\,x)^2\,\left(-4\,a^2\,b+7\,a\,b^2+2\,b^3\right)}{4\,a^2\,\left(a^2+2\,a\,b+b^2\right)}-\frac{1}{2\,(a+b)}+\frac{\tan(e+f\,x)^4\,\left(-a^2\,b^2+3\,a\,b^3+b^4\right)}{2\,a^2\,(a+b)\,\left(a^2+2\,a\,b+b^2\right)}}{f\,\left(\tan\left(e+f\,x\right)^2\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^4\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^6\right)} + \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a+4\,b)}{f\,\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,\left(6\,a^2+4\,a\,b+b^2\right)}{2\,a^3\,f\,(a+b)^4}$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^2*(7*a*b^2 - 4*a^2*b + 2*b^3))/(4*a^2*(2*a*b + a^2 + b^2))
- 1/(2*(a + b)) + (tan(e + f*x)^4*(3*a*b^3 + b^4 - a^2*b^2))/(2*a^2*(a + b
)*(2*a*b + a^2 + b^2)))/(f*(tan(e + f*x)^2*(2*a*b + a^2 + b^2) + tan(e + f
*x)^4*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^6)) + log(tan(e + f*x)^2 + 1)/(2*
a^3*f) - (log(tan(e + f*x))*(a + 4*b))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 +
 6*a^2*b^2)) - (b^2*log(a + b + b*tan(e + f*x)^2)*(4*a*b + 6*a^2 + b^2))/(
2*a^3*f*(a + b)^4)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 2392, normalized size of antiderivative = 15.53

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
(4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a**6 + 16*log(tan((e + f*x
)/2)**2 + 1)*sin(e + f*x)**6*a**5*b + 24*log(tan((e + f*x)/2)**2 + 1)*sin(
e + f*x)**6*a**4*b**2 + 16*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a*
*3*b**3 + 4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a**2*b**4 - 8*log
(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**6 - 40*log(tan((e + f*x)/2)**
2 + 1)*sin(e + f*x)**4*a**5*b - 80*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*
x)**4*a**4*b**2 - 80*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**3*b**
3 - 40*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**2*b**4 - 8*log(tan(
(e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a*b**5 + 4*log(tan((e + f*x)/2)**2 +
1)*sin(e + f*x)**2*a**6 + 24*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*
a**5*b + 60*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**4*b**2 + 80*lo
g(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**3*b**3 + 60*log(tan((e + f*x
)/2)**2 + 1)*sin(e + f*x)**2*a**2*b**4 + 24*log(tan((e + f*x)/2)**2 + 1)*s
in(e + f*x)**2*a*b**5 + 4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*b**
6 - 12*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((
e + f*x)/2))*sin(e + f*x)**6*a**4*b**2 - 8*log(sqrt(a + b)*tan((e + f*x)/2
)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**6*a**3*b**3
 - 2*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e
+ f*x)/2))*sin(e + f*x)**6*a**2*b**4 + 24*log(sqrt(a + b)*tan((e + f*x)/2)
**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**4*a**4*b*...
```

3.368 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b^5}{4a^3(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^4(5a+2b)}{2a^3(a+b)^4f\left(b+a\cos^2(e+fx)\right)} + \frac{(2a+5b)\csc^2(e+fx)}{2(a+b)^4f} - \frac{\csc^4(e+fx)}{4(a+b)^3f} + \frac{b^3(10a^2+5ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^5f} + \frac{(a^2+5ab+10b^2)\log(\sin(e+fx))}{(a+b)^5f}$$

output
```
-1/4*b^5/a^3/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+1/2*b^4*(5*a+2*b)/a^3/(a+b)^4/
f/(b+a*cos(f*x+e)^2)+1/2*(2*a+5*b)*csc(f*x+e)^2/(a+b)^4/f-1/4*csc(f*x+e)^4
/(a+b)^3/f+1/2*b^3*(10*a^2+5*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^5/f+(
a^2+5*a*b+10*b^2)*ln(sin(f*x+e))/(a+b)^5/f
```

Mathematica [A] (verified)

Time = 4.90 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.08

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(2(a+b)(2a+5b)\csc^2(e+fx)-(a+b)^2\csc^4(e+fx)+4(a\right.}{32(a+b)^5 f\,(}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*(2*a + 5*b)*Cs
c[e + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 5*a*b + 10*b^2)*Log[Sin
[e + f*x]] + (2*b^3*(10*a^2 + 5*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/
a^3 - (b^5*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^4*(a + b)*
(5*a + 2*b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/(32*(a + b)^5*f*(a + b*Sec
[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\tan(e+fx)^5\,(a+b\sec(e+fx)^2)^3}dx$$
$$\Big\downarrow \ 4626$$
$$-\frac{\int \frac{\cos^{11}(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos^{10}(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^3}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(-\frac{b^5}{a^2(a+b)^3(a\cos^2(e+fx)+b)^3}+\frac{(5a+2b)b^4}{a^2(a+b)^4(a\cos^2(e+fx)+b)^2}-\frac{(10a^2+5ba+b^2)b^3}{a^2(a+b)^5(a\cos^2(e+fx)+b)}+\frac{-a^2-5ba-10b^2}{(a+b)^5(\cos^2(e+fx)-1)}+\frac{}{(a+b)^4(c}\right.}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^5}{2a^3(a+b)^3(a\cos^2(e+fx)+b)^2}-\frac{b^4(5a+2b)}{a^3(a+b)^4(a\cos^2(e+fx)+b)}-\frac{(a^2+5ab+10b^2)\log(1-\cos^2(e+fx))}{(a+b)^5}-\frac{b^3(10a^2+5ab+b^2)\log(a\cos^2(e+f}{a^3(a+b)^5}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(1/(2*(a + b)^3*(1 - Cos[e + f*x]^2)^2) - (2*a + 5*b)/((a + b)^4*(1 -
 Cos[e + f*x]^2)) + b^5/(2*a^3*(a + b)^3*(b + a*Cos[e + f*x]^2)^2) - (b^4*
(5*a + 2*b))/(a^3*(a + b)^4*(b + a*Cos[e + f*x]^2)) - ((a^2 + 5*a*b + 10*b
^2)*Log[1 - Cos[e + f*x]^2])/(a + b)^5 - (b^3*(10*a^2 + 5*a*b + b^2)*Log[b
 + a*Cos[e + f*x]^2])/(a^3*(a + b)^5))/f
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 29.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{7a+19b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{(a^2+5ab+10b^2)\ln(-1+\cos(fx+e))}{2(a+b)^5}+\frac{b^3\left(\frac{(10a^2+5ab+b^2)\ln(b+a\cos}{a^3}\right.}{}}{}$
default	$\frac{-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{7a+19b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{(a^2+5ab+10b^2)\ln(-1+\cos(fx+e))}{2(a+b)^5}+\frac{b^3\left(\frac{(10a^2+5ab+b^2)\ln(b+a\cos}{a^3}\right.}{}}{}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/16/(a+b)^3/(-1+cos(f*x+e))^2-1/16*(7*a+19*b)/(a+b)^4/(-1+cos(f*x+e
))+1/2*(a^2+5*a*b+10*b^2)/(a+b)^5*ln(-1+cos(f*x+e))+1/2*b^3/(a+b)^5*((10*a
^2+5*a*b+b^2)/a^3*ln(b+a*cos(f*x+e)^2)+1/a^3*b*(5*a^2+7*a*b+2*b^2)/(b+a*co
s(f*x+e)^2)-1/2*b^2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2)-1/16/(a+b)^3
/(1+cos(f*x+e))^2-1/16*(-7*a-19*b)/(a+b)^4/(1+cos(f*x+e))+1/2*(a^2+5*a*b+1
0*b^2)/(a+b)^5*ln(1+cos(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(182) = 364$.

Time = 1.42 (sec) , antiderivative size = 859, normalized size of antiderivative = 4.47

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(3*a^5*b^2 + 12*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 + 12*a*b^6 + 3*b^7 - 2
*(2*a^7 + 7*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 7*a^2*b^5 - 2*a*b^6)*cos(f*x +
 e)^6 + (3*a^7 + 4*a^6*b - 19*a^5*b^2 - 20*a^4*b^3 - 20*a^3*b^4 - 19*a^2*b
^5 + 4*a*b^6 + 3*b^7)*cos(f*x + e)^4 + 2*(3*a^6*b + 10*a^5*b^2 + 2*a^4*b^3
 - 2*a^2*b^5 - 10*a*b^6 - 3*b^7)*cos(f*x + e)^2 + 2*((10*a^4*b^3 + 5*a^3*b
^4 + a^2*b^5)*cos(f*x + e)^8 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(10*a^4*b^3
- 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (10*a^4*b^3 - 35*a^3*b^4
 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(10*a^3*b^4 - 5*a^2*b^5 - 4
*a*b^6 - b^7)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*((a^7 + 5*a^6*
b + 10*a^5*b^2)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 - 2*(a^7
 + 4*a^6*b + 5*a^5*b^2 - 10*a^4*b^3)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5
*b^2 - 35*a^4*b^3 + 10*a^3*b^4)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*
a^4*b^3 - 10*a^3*b^4)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^10 + 5*a^
9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*
(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x +
 e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 +
a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5
*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 +
10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(182) = 364.

Time = 0.04 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.36

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{2\,(10\,a^2b^3+5\,ab^4+b^5)\log\left(a\sin(fx+e)^2-a-b\right)}{a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5}+\frac{2\,(a^2+5\,ab+10\,b^2)\log\left(\sin(fx+e)^2\right)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5}+\frac{2\,(2\,a^6+5\,a^5b-5\,a^2b^4-2\,ab^5)\sin(fx+e)^6-a^6-3}{(a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4)\sin(fx+e)^8-}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a*sin(f*x + e)^2 - a - b)/(a^8 + 5
*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 2*(a^2 + 5*a*b +
 10*b^2)*log(sin(f*x + e)^2)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5) + (2*(2*a^6 + 5*a^5*b - 5*a^2*b^4 - 2*a*b^5)*sin(f*x + e)^6 -
 a^6 - 3*a^5*b - 3*a^4*b^2 - a^3*b^3 - (9*a^6 + 29*a^5*b + 20*a^4*b^2 - 10
*a^2*b^4 - 13*a*b^5 - 3*b^6)*sin(f*x + e)^4 + 2*(3*a^6 + 11*a^5*b + 13*a^4
*b^2 + 5*a^3*b^3)*sin(f*x + e)^2)/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3
+ a^5*b^4)*sin(f*x + e)^8 - 2*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5
*a^5*b^4 + a^4*b^5)*sin(f*x + e)^6 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*
b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*sin(f*x + e)^4))/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.87

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(10\,a^2b^3+5\,ab^4+b^5)\log\left(\left|a\cos\left(fx+e\right)^2+b\right|\right)}{2\,(a^8f+5\,a^7bf+10\,a^6b^2f+10\,a^5b^3f+5\,a^4b^4f+a^3b^5f)}$$
$$+\frac{(a^2+5\,ab+10\,b^2)\log\left(\left|\cos\left(fx+e\right)^2-1\right|\right)}{2\,(a^5f+5\,a^4bf+10\,a^3b^2f+10\,a^2b^3f+5\,ab^4f+b^5f)}$$
$$-\frac{2\,(2\,a^6+5\,a^5b-5\,a^2b^4-2\,ab^5)\cos\left(fx+e\right)^6-3\,a^4b^2-9\,a^3b^3-9\,ab^5-3\,b^6-(3\,a^6+a^5b-20\,a^4b^2-}{4\left(a\cos\left(fx+e\right)^2+b\right)^2(\cos}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/2*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(abs(a*cos(f*x + e)^2 + b))/(a^8*f + 5
*a^7*b*f + 10*a^6*b^2*f + 10*a^5*b^3*f + 5*a^4*b^4*f + a^3*b^5*f) + 1/2*(a
^2 + 5*a*b + 10*b^2)*log(abs(cos(f*x + e)^2 - 1))/(a^5*f + 5*a^4*b*f + 10*
a^3*b^2*f + 10*a^2*b^3*f + 5*a*b^4*f + b^5*f) - 1/4*(2*(2*a^6 + 5*a^5*b -
5*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^6 - 3*a^4*b^2 - 9*a^3*b^3 - 9*a*b^5 - 3*
b^6 - (3*a^6 + a^5*b - 20*a^4*b^2 - 20*a^2*b^4 + a*b^5 + 3*b^6)*cos(f*x +
e)^4 - 2*(3*a^5*b + 7*a^4*b^2 - 5*a^3*b^3 + 5*a^2*b^4 - 7*a*b^5 - 3*b^6)*c
os(f*x + e)^2)/((a*cos(f*x + e)^2 + b)^2*(cos(f*x + e)^2 - 1)^2*(a + b)^4*
a^3*f)
```

Mupad [B] (verification not implemented)

Time = 19.04 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.70

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\ln(\tan(e+fx))\,(a^2+5ab+10b^2)}{f\,(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5)} - \frac{\frac{1}{4(a+b)} - \frac{\tan(e+fx)^2(a+3b)}{2(a+b)^2} + \frac{\tan(e+fx)^4(-4a^3b-15a^2b^2+9ab^3+2b^4)}{4a^2(a+b)(a^2+2ab+b^2)} + \frac{\tan(e+fx)^6(-a^3b^2-4a^2b^3+4ab^4+b^5)}{2a^2(a+b)^2(a^2+2ab+b^2)}}{f\left(\tan(e+fx)^4(a^2+2ab+b^2)+\tan(e+fx)^6(2b^2+2ab)+b^2\tan(e+fx)^8\right)} - \frac{\ln\left(\tan(e+fx)^2+1\right)}{2a^3f} + \frac{b^3\ln\left(b\tan(e+fx)^2+a+b\right)(10a^2+5ab+b^2)}{2a^3f(a+b)^5}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(log(tan(e + f*x))*(5*a*b + a^2 + 10*b^2))/(f*(5*a*b^4 + 5*a^4*b + a^5 + b
^5 + 10*a^2*b^3 + 10*a^3*b^2)) - (1/(4*(a + b)) - (tan(e + f*x)^2*(a + 3*b
))/(2*(a + b)^2) + (tan(e + f*x)^4*(9*a*b^3 - 4*a^3*b + 2*b^4 - 15*a^2*b^2
))/(4*a^2*(a + b)*(2*a*b + a^2 + b^2)) + (tan(e + f*x)^6*(4*a*b^4 + b^5 -
4*a^2*b^3 - a^3*b^2))/(2*a^2*(a + b)^2*(2*a*b + a^2 + b^2)))/(f*(tan(e + f
*x)^4*(2*a*b + a^2 + b^2) + tan(e + f*x)^6*(2*a*b + 2*b^2) + b^2*tan(e + f
*x)^8)) - log(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b^3*log(a + b + b*tan(e + f
*x)^2)*(5*a*b + 10*a^2 + b^2))/(2*a^3*f*(a + b)^5)
```

Reduce [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 2744, normalized size of antiderivative = 14.29

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
( - 16*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**8*a**7 - 80*log(tan((e +
 f*x)/2)**2 + 1)*sin(e + f*x)**8*a**6*b - 160*log(tan((e + f*x)/2)**2 + 1)
*sin(e + f*x)**8*a**5*b**2 - 160*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)
**8*a**4*b**3 - 80*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**8*a**3*b**4
- 16*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**8*a**2*b**5 + 32*log(tan((
e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a**7 + 192*log(tan((e + f*x)/2)**2 + 1
)*sin(e + f*x)**6*a**6*b + 480*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**
6*a**5*b**2 + 640*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a**4*b**3 +
 480*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a**3*b**4 + 192*log(tan(
(e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a**2*b**5 + 32*log(tan((e + f*x)/2)**
2 + 1)*sin(e + f*x)**6*a*b**6 - 16*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*
x)**4*a**7 - 112*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**6*b - 336
*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**5*b**2 - 560*log(tan((e +
 f*x)/2)**2 + 1)*sin(e + f*x)**4*a**4*b**3 - 560*log(tan((e + f*x)/2)**2 +
 1)*sin(e + f*x)**4*a**3*b**4 - 336*log(tan((e + f*x)/2)**2 + 1)*sin(e + f
*x)**4*a**2*b**5 - 112*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a*b**6
 - 16*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*b**7 + 80*log(sqrt(a +
b)*tan((e + f*x)/2)**2 + sqrt(a + b) - 2*sqrt(a)*tan((e + f*x)/2))*sin(e +
 f*x)**8*a**4*b**3 + 40*log(sqrt(a + b)*tan((e + f*x)/2)**2 + sqrt(a + b)
- 2*sqrt(a)*tan((e + f*x)/2))*sin(e + f*x)**8*a**3*b**4 + 8*log(sqrt(a ...
```

3.369 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3}+\frac{\sqrt{a+b}(3a^2-4ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-x/a^3+1/8*(a+b)^(1/2)*(3*a^2-4*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)
^(1/2))/a^3/b^(5/2)/f-1/4*(a+b)*tan(f*x+e)^3/a/b/f/(a+b+b*tan(f*x+e)^2)^2-
1/8*(3*a-4*b)*(a+b)*tan(f*x+e)/a^2/b^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.12 (sec) , antiderivative size = 523, normalized size of antiderivative = 3.56

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(\frac{2(3a^3-a^2b+4ab^2+8b^3)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(\cdots}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\cdots\right.}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^{\cdots}}}\cdots}{\cdots}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/128*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((2*(3*a^3 - a^2*b +
 4*a*b^2 + 8*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Si
n[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*S
qrt[b*(Cos[e] - I*Sin[e])^4]) + Sec[2*e]*(8*b^2*(3*a^2 + 8*a*b + 8*b^2)*f*
x*Cos[2*e] + 16*a*b^2*(a + 2*b)*f*x*Cos[2*f*x] + 4*a^2*b^2*f*x*Cos[2*(e +
2*f*x)] + 16*a^2*b^2*f*x*Cos[4*e + 2*f*x] + 32*a*b^3*f*x*Cos[4*e + 2*f*x]
+ 4*a^2*b^2*f*x*Cos[6*e + 4*f*x] - 9*a^4*Sin[2*e] - 15*a^3*b*Sin[2*e] + 18
*a^2*b^2*Sin[2*e] + 72*a*b^3*Sin[2*e] + 48*b^4*Sin[2*e] + 9*a^4*Sin[2*f*x]
 + 13*a^3*b*Sin[2*f*x] - 28*a^2*b^2*Sin[2*f*x] - 32*a*b^3*Sin[2*f*x] + 3*a
^4*Sin[2*(e + 2*f*x)] - 3*a^3*b*Sin[2*(e + 2*f*x)] - 6*a^2*b^2*Sin[2*(e +
2*f*x)] - 3*a^4*Sin[4*e + 2*f*x] + a^3*b*Sin[4*e + 2*f*x] + 20*a^2*b^2*Sin
[4*e + 2*f*x] + 16*a*b^3*Sin[4*e + 2*f*x])))/(a^3*b^2*f*(a + b*Sec[e + f*x
]^2)^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4629, 2075, 372, 440, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\tan(e+fx)^6}{(a+b\sec(e+fx)^2)^3}dx$$

$\Big\downarrow$ 4629

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^3}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 2075

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 372

$$\frac{\frac{\int \frac{\tan^2(e+fx)\left((3a-b)\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4ab}-\frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\Big\downarrow$ 440

$$\frac{\frac{-\frac{\int -\frac{\left(3a^2-ba+4b^2\right)\tan^2(e+fx)+(3a-4b)(a+b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2ab}-\frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{\left(3a^2-ba+4b^2\right)\tan^2(e+fx)+(3a-4b)(a+b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2ab}-\frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{(a+b)\left(3a^2-4ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-\frac{8b^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{2ab}-\frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{(a+b)\left(3a^2-4ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-\frac{8b^2\arctan(\tan(e+fx))}{a}}{2ab}-\frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}-\frac{8b^2\arctan(\tan(e+fx))}{a}}{2ab}-\frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*((a + b)*Tan[e + f*x]^3)/(a*b*(a + b + b*Tan[e + f*x]^2)^2) + (((-8*
b^2*ArcTan[Tan[e + f*x]])/a + (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTan[
(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/(2*a*b) - ((3*a - 4*b)*(
a + b)*Tan[e + f*x])/(2*a*b*(a + b + b*Tan[e + f*x]^2)))/(4*a*b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 15.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{(a+b)\left(\frac{-\frac{a(5a-4b)\tan(fx+e)^3}{8b}-\frac{a\left(3a^2-ab-4b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b^2\sqrt{(a+b)b}}\right)}{a^3}-\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{\frac{(a+b)\left(\frac{-\frac{a(5a-4b)\tan(fx+e)^3}{8b}-\frac{a\left(3a^2-ab-4b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b^2\sqrt{(a+b)b}}\right)}{a^3}-\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(-3a^4 e^{6i(fx+e)}+a^3 b\, e^{6i(fx+e)}+20a^2b^2e^{6i(fx+e)}+16a\,b^3e^{6i(fx+e)}-9a^4e^{4i(fx+e)}-15a^3b\,e^{4i(fx+e)}+18a^2b\right.}{4a^3fb^2\left(a\,e^{4i(\right.}$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*((a+b)/a^3*((-1/8*a*(5*a-4*b)/b*tan(f*x+e)^3-1/8*a*(3*a^2-a*b-4*b^2)/b
^2*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2-4*a*b+8*b^2)/b^2/((a+b)*b
)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/a^3*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(133) = 266.

Time = 0.13 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.52

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(32*a^2*b^2*f*x*cos(f*x + e)^4 + 64*a*b^3*f*x*cos(f*x + e)^2 + 32*b
^4*f*x - ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 - 4*a*b
^3 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-(a +
b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*
x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a +
b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2
)) + 4*(3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 -
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3
*f*cos(f*x + e)^2 + a^3*b^4*f), -1/16*(16*a^2*b^2*f*x*cos(f*x + e)^4 + 32*
a*b^3*f*x*cos(f*x + e)^2 + 16*b^4*f*x + ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos
(f*x + e)^4 + 3*a^2*b^2 - 4*a*b^3 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b
^3)*cos(f*x + e)^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(a^4 - a^3
*b - 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3)*cos(f*x + e
))*sin(f*x + e))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 +
a^3*b^4*f)]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.31

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(5a^2b+ab^2-4b^3)\tan(fx+e)^3+(3a^3+2a^2b-5ab^2-4b^3)\tan(fx+e)}{a^2b^4\tan(fx+e)^4+a^4b^2+2a^3b^3+a^2b^4+2(a^3b^3+a^2b^4)\tan(fx+e)^2}+\frac{8(fx+e)}{a^3}-\frac{(3a^3-a^2b+4ab^2+8b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3b^2}}{8f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(((5*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e)^3 + (3*a^3 + 2*a^2*b - 5*a*b
^2 - 4*b^3)*tan(f*x + e))/(a^2*b^4*tan(f*x + e)^4 + a^4*b^2 + 2*a^3*b^3 +
a^2*b^4 + 2*(a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) + 8*(f*x + e)/a^3 - (3*a^3
 - a^2*b + 4*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((
a + b)*b)*a^3*b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.28

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{fx+e}{a^3f} + \frac{(3\,a^3-a^2b+4\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{8\,\sqrt{ab+b^2}a^3b^2f} - \frac{5\,a^2b\tan\left(fx+e\right)^3+ab^2\tan\left(fx+e\right)^3-4\,b^3\tan\left(fx+e\right)^3+3\,a^3\tan\left(fx+e\right)+2\,a^2b\tan\left(fx+e\right)-}{8\,\left(b\tan\left(fx+e\right)^2+a+b\right)^2a^2b^2f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-(f*x + e)/(a^3*f) + 1/8*(3*a^3 - a^2*b + 4*a*b^2 + 8*b^3)*arctan(b*tan(f*
x + e)/sqrt(a*b + b^2))/(sqrt(a*b + b^2)*a^3*b^2*f) - 1/8*(5*a^2*b*tan(f*x
 + e)^3 + a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e)
 + 2*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e) - 4*b^3*tan(f*x + e))/((b*t
an(f*x + e)^2 + a + b)^2*a^2*b^2*f)
```

Mupad [B] (verification not implemented)

Time = 15.69 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.18

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
- atan((25*tan(e + f*x))/(32*((5*b)/(4*a) - (3*a)/(16*b) + (9*a^2)/(32*b^2
) + 25/32)) - (3*tan(e + f*x))/(16*((9*a)/(32*b) + (25*b)/(32*a) + (5*b^2)
/(4*a^2) - 3/16)) + (9*tan(e + f*x))/(32*((25*b^2)/(32*a^2) - (3*b)/(16*a)
 + (5*b^3)/(4*a^3) + 9/32)) + (5*tan(e + f*x))/(4*((25*a)/(32*b) - (3*a^2)
/(16*b^2) + (9*a^3)/(32*b^3) + 5/4)))/(a^3*f) - ((tan(e + f*x)^3*(a*b + 5*
a^2 - 4*b^2))/(8*a^2*b) - (tan(e + f*x)*(a + b)*(a*b - 3*a^2 + 4*b^2))/(8*
a^2*b^2))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan
(e + f*x)^4)) - (atanh((27*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(256*((27*a
*b^2)/256 - (27*b^3)/128 + (171*b^4)/(256*a) - (7*b^5)/(64*a^2) + (5*b^6)/
(32*a^3) + (5*b^7)/(4*a^4))) - (81*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(25
6*((27*a^2*b)/256 - (27*a*b^2)/128 + (171*b^3)/256 - (7*b^4)/(64*a) + (5*b
^5)/(32*a^2) + (5*b^6)/(4*a^3))) - (35*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))
/(32*((171*a^2*b)/256 - (7*a*b^2)/64 - (27*a^3)/128 + (5*b^3)/32 + (5*b^4)
/(4*a) + (27*a^4)/(256*b))) + (5*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(4*((
5*a*b^2)/32 - (7*a^2*b)/64 + (171*a^3)/256 + (5*b^3)/4 - (27*a^4)/(128*b)
+ (27*a^5)/(256*b^2))) + (63*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(64*((171
*a*b^2)/256 - (27*a^2*b)/128 + (27*a^3)/256 - (7*b^3)/64 + (5*b^4)/(32*a)
+ (5*b^5)/(4*a^2))))*(-b^5*(a + b))^(1/2)*(3*a^2 - 4*a*b + 8*b^2))/(8*a^3*
b^5*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1259, normalized size of antiderivative = 8.56

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**4*a**4 - 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b + 8*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*
a**2*b**2 - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqr
t(a))/sqrt(b))*sin(e + f*x)**2*a**4 + 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
 b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b - 8*sqrt(b
)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e
 + f*x)**2*a**2*b**2 - 16*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f
*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 + 3*sqrt(b)*sqrt(a + b)*
atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 + 2*sqrt(b)*sq
rt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3*b +
3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*a**2*b**2 + 12*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) -
 sqrt(a))/sqrt(b))*a*b**3 + 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
 + f*x)/2) - sqrt(a))/sqrt(b))*b**4 + 3*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
 b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4 - 4*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e +
 f*x)**4*a**3*b + 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2)
 + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 - 6*sqrt(b)*sqrt(a + b)*...
```

3.370 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3} \, dx = \frac{x}{a^3} + \frac{(a^2-4ab-8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}f} - \frac{(a+b)\tan(e+fx)}{4abf\,(a+b+b\tan^2(e+fx))^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\,(a+b+b\tan^2(e+fx))}$$

output
```
x/a^3+1/8*(a^2-4*a*b-8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3/b^(
3/2)/(a+b)^(1/2)/f-1/4*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(
a-4*b)*tan(f*x+e)/a^2/b/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.95 (sec) , antiderivative size = 1333, normalized size of antiderivative = 9.73

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((6*a*(a + 2*b)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (4*(3*a^2 + 8*a*b + 8*b
^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (4*a*Sqrt[
b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) - (2*Sqrt[b]*(3*a^3 + 1
4*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e + f*x)])*
Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (Sqrt[b]*
((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*Ar
cTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e
+ f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[
2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]*(256*b^2*(a
 + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b)^2*(a + 2*
b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3*b^3*f*x*C
os[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*b^2*f*x*C
os[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4*f*x*Cos
[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*Cos[6*e
+ 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*e + 4*
f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 2880*a^
3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^6*Si
n[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*x...
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4629, 2075, 372, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{(a-3b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4ab}-\frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{(a+b)\left((a-4b)\tan^2(e+fx)+a+4b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{(a-4b)\tan^2(e+fx)+a+4b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}+\frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{\left(a^2-4ab-8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{8b\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{2a}+\frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{\left(a^2-4ab-8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{8b\arctan(\tan(e+fx))}{a}}{2a}+\frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\sqrt{a+b}}+\frac{8b\arctan(\tan(e+fx))}{a}}{2a}+\frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*((a + b)*Tan[e + f*x])/(a*b*(a + b + b*Tan[e + f*x]^2)^2) + (((8*b*A
rcTan[Tan[e + f*x]])/a + ((a^2 - 4*a*b - 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b]])/(a*Sqrt[b]*Sqrt[a + b]))/(2*a) + ((a - 4*b)*Tan[e + f*x]
)/(2*a*(a + b + b*Tan[e + f*x]^2)))/(4*a*b))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 9.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\left(\frac{1}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3-\frac{a\left(a^2+5ab+4b^2\right)\tan(fx+e)}{8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b\sqrt{(a+b)b}}}{a^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\left(\frac{1}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3-\frac{a\left(a^2+5ab+4b^2\right)\tan(fx+e)}{8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b\sqrt{(a+b)b}}}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{i\left(a^3e^{6i(fx+e)}+12a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+3a^3e^{4i(fx+e)}+26a^2b\,e^{4i(fx+e)}+56a\,b^2e^{4i(fx+e)}+48b^3e^{4i(f}\right.}{4a^3f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right.}$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*arctan(tan(f*x+e))+1/a^3*(((1/8*a^2-1/2*a*b)*tan(f*x+e)^3-1/8*a
*(a^2+5*a*b+4*b^2)/b*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(a^2-4*a*b-8*b
^2)/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(123) = 246$.

Time = 0.13 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.57

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^2*b^3 + a*b^4)*f*
x*cos(f*x + e)^2 + 32*(a*b^4 + b^5)*f*x + ((a^4 - 4*a^3*b - 8*a^2*b^2)*cos
(f*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8*a*b^3)*
cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(
f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b
*cos(f*x + e)^2 + b^2)) - 4*((a^4*b + 7*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^
3 - (a^3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^2
+ a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^
4*b^4 + a^3*b^5)*f), 1/16*(16*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 32*
(a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 16*(a*b^4 + b^5)*f*x - ((a^4 - 4*a^
3*b - 8*a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4
*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*
cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((a^4
*b + 7*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^3*b^2 - 3*a^2*b^3 - 4*a*b^
4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(
a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(ab-4\,b^2)\tan(fx+e)^3-(a^2+5\,ab+4\,b^2)\tan(fx+e)}{a^2b^3\tan(fx+e)^4+a^4b+2\,a^3b^2+a^2b^3+2\,(a^3b^2+a^2b^3)\tan(fx+e)^2}+\frac{8\,(fx+e)}{a^3}+\frac{(a^2-4\,ab-8\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3b}}{8\,f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(((a*b - 4*b^2)*tan(f*x + e)^3 - (a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(
a^2*b^3*tan(f*x + e)^4 + a^4*b + 2*a^3*b^2 + a^2*b^3 + 2*(a^3*b^2 + a^2*b^
3)*tan(f*x + e)^2) + 8*(f*x + e)/a^3 + (a^2 - 4*a*b - 8*b^2)*arctan(b*tan(
f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3*b))/f
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{fx+e}{a^3f} + \frac{(a^2-4\,ab-8\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{8\sqrt{ab+b^2}a^3bf} + \frac{ab\tan(fx+e)^3-4\,b^2\tan(fx+e)^3-a^2\tan(fx+e)-5\,ab\tan(fx+e)-4\,b^2\tan(fx+e)}{8\left(b\tan(fx+e)^2+a+b\right)^2a^2bf}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
(f*x + e)/(a^3*f) + 1/8*(a^2 - 4*a*b - 8*b^2)*arctan(b*tan(f*x + e)/sqrt(a
*b + b^2))/(sqrt(a*b + b^2)*a^3*b*f) + 1/8*(a*b*tan(f*x + e)^3 - 4*b^2*tan
(f*x + e)^3 - a^2*tan(f*x + e) - 5*a*b*tan(f*x + e) - 4*b^2*tan(f*x + e))/
((b*tan(f*x + e)^2 + a + b)^2*a^2*b*f)
```

Mupad [B] (verification not implemented)

Time = 15.69 (sec) , antiderivative size = 1117, normalized size of antiderivative = 8.15

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^3*(a - 4*b))/(8*a^2) - (tan(e + f*x)*(a + b)*(a + 4*b))/(8*
a^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e
 + f*x)^4)) - atan(tan(e + f*x)/(32*(b/(4*a) - 1/32)) + tan(e + f*x)/(4*(a
/(32*b) - 1/4)))/(a^3*f) + (atan(-(((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (
a^7*b^2)/2)/(a^6*b) - (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a +
 b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b
- a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) - (tan(e + f*x)*(64*a*b^3 - 8*a^3
*b + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2
)*1i)/(16*(a^3*b^4 + a^4*b^3)) - ((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^
7*b^2)/2)/(a^6*b) + (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b
))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b -
a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) + (tan(e + f*x)*(64*a*b^3 - 8*a^3*b
 + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2)*
1i)/(16*(a^3*b^4 + a^4*b^3)))/(((a*b^2)/4 - (a^2*b)/4 + a^3/32 + b^3)/(a^6
*b) + ((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) - (tan(e
+ f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b
^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4
+ a^4*b^3)) - (tan(e + f*x)*(64*a*b^3 - 8*a^3*b + a^4 + 128*b^4))/(32*a^4*
b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) +
 ((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) + (tan(e + ...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1337, normalized size of antiderivative = 9.76

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
(sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b)
)*sin(e + f*x)**4*a**4 - 4*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 8*sqrt(b)*sqrt(a + b)
*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a*
*2*b**2 - 2*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(
a))/sqrt(b))*sin(e + f*x)**2*a**4 + 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b
)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b + 24*sqrt(b)
*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e
+ f*x)**2*a**2*b**2 + 16*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*
x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 + sqrt(b)*sqrt(a + b)*ata
n((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 - 2*sqrt(b)*sqrt(
a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3*b - 15*
sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))
*a**2*b**2 - 20*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - s
qrt(a))/sqrt(b))*a*b**3 - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
 f*x)/2) - sqrt(a))/sqrt(b))*b**4 + sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*
tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4 - 4*sqrt(b)*sqrt
(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x
)**4*a**3*b - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + s
qrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 - 2*sqrt(b)*sqrt(a + b)*atan...
```

3.371 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3}+\frac{(3a^2+12ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-x/a^3+1/8*(3*a^2+12*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3
/b^(1/2)/(a+b)^(3/2)/f+1/4*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a+
4*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.91 (sec) , antiderivative size = 1334, normalized size of antiderivative = 9.67

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((-6*a*(a + 2*b)*ArcTan[(
Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (4*(3*a^2 + 8*a*b + 8*
b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (4*a*Sqrt
[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e +
f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (2*Sqrt[b]*(3*a^3 +
14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e + f*x)])
*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) - (Sqrt[b]
*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*A
rcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e
 + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin
[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]*(256*b^2*(
a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b)^2*(a + 2
*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3*b^3*f*x*
Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*b^2*f*x*
Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4*f*x*Co
s[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*Cos[6*e
 + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*e + 4
*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 2880*a
^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^6*S
in[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4629, 2075, 373, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^2}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \text{ 4629}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 2075}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 373}$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\int \frac{1-3\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a}}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\int \frac{-\left((3a+4b)\tan^2(e+fx)\right)+5a+4b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{(3a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a}}{f}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\frac{8(a+b)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{(3a^2+12ab+8b^2)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{(3a+4b)\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{4a}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\frac{8(a+b)\arctan(\tan(e+fx))}{a}-\frac{(3a^2+12ab+8b^2)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{(3a+4b)\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{4a}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\frac{8(a+b)\arctan(\tan(e+fx))}{a}-\frac{(3a^2+12ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\sqrt{a+b}}}{2a(a+b)}-\frac{(3a+4b)\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{4a}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(4*a*(a + b + b*Tan[e + f*x]^2)^2) - (((8*(a + b)*ArcTan[Tan
[e + f*x]])/a - ((3*a^2 + 12*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(a*Sqrt[b]*Sqrt[a + b]))/(2*a*(a + b)) - ((3*a + 4*b)*Tan[e +
f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 6.91 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\frac{ab(3a+4b)\tan(fx+e)^3}{8a+8b}+\frac{(5a+4b)a\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{\frac{ab(3a+4b)\tan(fx+e)^3}{8a+8b}+\frac{(5a+4b)a\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(5a^3e^{6i(fx+e)}+20a^2b\,e^{6i(fx+e)}+16a\,b^2e^{6i(fx+e)}+15a^3e^{4i(fx+e)}+58a^2b\,e^{4i(fx+e)}+88a\,b^2e^{4i(fx+e)}+48b^3e\right.}{4a^3(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(\right.}}$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*arctan(tan(f*x+e))+1/a^3*((1/8*a*b*(3*a+4*b)/(a+b)*tan(f*x+e)^
3+1/8*(5*a+4*b)*a*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2+12*a*b+8*b
^2)/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(124) = 248.

Time = 0.14 (sec) , antiderivative size = 860, normalized size of antiderivative = 6.23

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(32*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^3*b^2
+ 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 32*(a^2*b^3 + 2*a*b^4 + b^5)*f*x
 + ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 12*a*b^3 +
 8*b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((
5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3
+ 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^7*b + 2*a^6*b^2 + a^5*b^3)*f*co
s(f*x + e)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^5*b
^3 + 2*a^4*b^4 + a^3*b^5)*f), -1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*
cos(f*x + e)^4 + 32*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 16*
(a^2*b^3 + 2*a*b^4 + b^5)*f*x + ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x +
e)^4 + 3*a^2*b^2 + 12*a*b^3 + 8*b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*c
os(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(
sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^4*b + 11*a^3*b^2 + 6
*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^4)*cos(f*x + e))
*sin(f*x + e))/((a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b^
2 + 2*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5
)*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\tan^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^3}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.38

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(3\,a^2+12\,ab+8\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}} + \frac{(3\,ab+4\,b^2)\tan(fx+e)^3+(5\,a^2+9\,ab+4\,b^2)\tan(fx+e)}{a^5+3\,a^4b+3\,a^3b^2+a^2b^3+(a^3b^2+a^2b^3)\tan(fx+e)^4+2\,(a^4b+2\,a^3b^2+a^2b^3)\tan(fx+e)^2} - \frac{8\,(fx+\cdots}{a^3}}{8\,f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + 12*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
 + a^3*b)*sqrt((a + b)*b)) + ((3*a*b + 4*b^2)*tan(f*x + e)^3 + (5*a^2 + 9*
a*b + 4*b^2)*tan(f*x + e))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^3*b^2
 + a^2*b^3)*tan(f*x + e)^4 + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(f*x + e)^
2) - 8*(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(3\,a^2+12\,ab+8\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{8\,(a^4f+a^3bf)\sqrt{ab+b^2}} + \frac{3\,ab\tan{(fx+e)}^3+4\,b^2\tan{(fx+e)}^3+5\,a^2\tan{(fx+e)}+9\,ab\tan{(fx+e)}+4\,b^2\tan{(fx+e)}}{8\,(a^3f+a^2bf)\left(b\tan{(fx+e)}^2+a+b\right)^2} - \frac{fx+e}{a^3f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*a^2 + 12*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b + b^2))/((a^4*
f + a^3*b*f)*sqrt(a*b + b^2)) + 1/8*(3*a*b*tan(f*x + e)^3 + 4*b^2*tan(f*x
+ e)^3 + 5*a^2*tan(f*x + e) + 9*a*b*tan(f*x + e) + 4*b^2*tan(f*x + e))/((a
^3*f + a^2*b*f)*(b*tan(f*x + e)^2 + a + b)^2) - (f*x + e)/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 18.13 (sec) , antiderivative size = 2405, normalized size of antiderivative = 17.43

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)*(5*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a*b + 4*b^2))/(8*
a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2
*tan(e + f*x)^4)) - atan((((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1
i)/(2*(2*a^7*b + a^8 + a^6*b^2)) - (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b
^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))/(2*
a^3) + (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3
*b^2))/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3 - ((((2*a^6*b^4 + (9*a^7*b^3)/2
 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2)) + (tan(e + f*x)*(512*a
^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a
^6 + a^4*b^2)))/(2*a^3) - (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 2
56*a^2*b^3 + 72*a^3*b^2))/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3)/(((9*a*b^3)
/4 + (9*a^3*b)/32 + b^4 + (3*a^2*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) + (((((
2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2
)) - (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^
2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))*1i)/(2*a^3) + (tan(e + f*x)*(320*
a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2)*1i)/(64*(2*a^5*b + a
^6 + a^4*b^2)))/a^3 + (((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/
(2*(2*a^7*b + a^8 + a^6*b^2)) + (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4
+ 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))*1i)/(2
*a^3) - (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1422, normalized size of antiderivative = 10.30

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
(3*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(
b))*sin(e + f*x)**4*a**4 + 12*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
 + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b + 8*sqrt(b)*sqrt(a +
 b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4
*a**2*b**2 - 6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sq
rt(a))/sqrt(b))*sin(e + f*x)**2*a**4 - 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a
 + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b - 40*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*si
n(e + f*x)**2*a**2*b**2 - 16*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 + 3*sqrt(b)*sqrt(a +
b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 + 18*sqrt(b
)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**3*
b + 35*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*a**2*b**2 + 28*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)
/2) - sqrt(a))/sqrt(b))*a*b**3 + 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*t
an((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4 + 3*sqrt(b)*sqrt(a + b)*atan((sqr
t(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4 + 12*sq
rt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*s
in(e + f*x)**4*a**3*b + 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f
*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 - 6*sqrt(b)*sqrt(a...
```

3.372 $\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\,(a+b+b\tan^2(e+fx))}$$

output
```
x/a^3-1/8*b^(1/2)*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1
/8*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.07 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.31

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2+\frac{b(15a^2+20ab+8b^2)\arctan\left(\frac{\sec(fx)(c}{}\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e
+ f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*S
in[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*S
in[2*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2
*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e
])) + (b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e
] - 3*a*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] +
Sin[e]))))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^3}dx$$

$$\Big\downarrow \ 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 316$$

$$\frac{\frac{\int \frac{-3b\tan^2(e+fx)+4a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{\int \frac{8a^2+9ba+4b^2-b(7a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx}}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
(-1/4*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (((8*(a
+ b)^2*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*
(7*a + 4*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*(
a + b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3 e^{6i(fx+e)}+28a^2 b\, e^{6i(fx+e)}+16a\, b^2 e^{6i(fx+e)}+27a^3 e^{4i(fx+e)}+90a^2 b\, e^{4i(fx+e)}+120a\, b^2 e^{4i(fx+e)}+48b^3 e \cdots\right.}{4a^3(a+b)^2 f\left(a\, e^{4i(fx+e)}+2a\, e^{2i(fx+e)}+4b\, e^{2i(f \cdots\right.}$

input
```
int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(9*a+4*b)
*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+20*a*b+8*b^2)/(a^2
+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*ar
ctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(130) = 260$.

Time = 0.14 (sec) , antiderivative size = 819, normalized size of antiderivative = 5.69

$$\int \frac{1}{(a + b\sec^2(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2
*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15
*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^
4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sq
rt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e
)^2 + b^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a
*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e
)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*
b^3 + a^3*b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 +
 32*(a^3*b + 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3
 + b^4)*f*x + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 20*a*b^3 + 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*
sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3
 + (7*a^2*b^2 + 4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5
*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2
+ (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]

$$\int \frac{1}{(a + b\sec^2(e + fx))^3}\,dx = \int \frac{1}{(a + b\sec^2{(e + fx)})^3}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}}+\frac{(7\,ab^2+4\,b^3)\tan(fx+e)^3+(9\,a^2b+13\,ab^2+4\,b^3)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^4b^2+2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3 \ldots}}{8\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x
+ e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^
4 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x +
 e)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan(fx+e)^3+4\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+13\,ab^2\tan(\ldots}{(a^4+2\,a^3b+a^2b^2)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a
*b + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(
f*x + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a
^2*b^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.72 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
atan(((((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^1
0*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e
 + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a
^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 +
856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a
^6*b^2)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)
/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)
) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a
^5*b^3 + 6*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*
a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5
*b^3 + 6*a^6*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*
b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b
^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*
a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*
b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^1
1*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(
2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 +
 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1496, normalized size of antiderivative = 10.39

$$\int \frac{1}{(a + b\sec^2(e + fx))^3}\, dx = \text{Too large to display}$$

input
```
int(1/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)**4*a**4 - 20*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**3*b - 8*sqrt(b)*sqrt
(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x
)**4*a**2*b**2 + 30*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2)
 - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**4 + 70*sqrt(b)*sqrt(a + b)*atan((s
qrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a**3*b + 5
6*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b
))*sin(e + f*x)**2*a**2*b**2 + 16*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*ta
n((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**2*a*b**3 - 15*sqrt(b)*sqr
t(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*a**4 - 50*
sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))
*a**3*b - 63*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt
(a))/sqrt(b))*a**2*b**2 - 36*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e
+ f*x)/2) - sqrt(a))/sqrt(b))*a*b**3 - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a
+ b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*b**4 - 15*sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**4
- 20*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqr
t(b))*sin(e + f*x)**4*a**3*b - 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan
((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 + 30*sqrt(b...
```

3.373 $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3} + \frac{b^{3/2}(35a^2+28ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}f} - \frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2(a+b)^3f} - \frac{b\cot(e+fx)}{4a(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2f\,(a+b+b\tan^2(e+fx))}$$

output
```
-x/a^3+1/8*b^(3/2)*(35*a^2+28*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(
1/2))/a^3/(a+b)^(7/2)/f-1/8*(8*a^2-11*a*b-4*b^2)*cot(f*x+e)/a^2/(a+b)^3/f-
1/4*b*cot(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(9*a+4*b)*cot(f*x+
e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.68 (sec) , antiderivative size = 2089, normalized size of antiderivative = 11.54

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*(-1/64*(b^2*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*
b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e
]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^3*Sqr
t[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^2*ArcTan[Sec[f*x]*
(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e
])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin
[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] -
I*b*Sin[4*e]])))/((a + b)^3*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[
2*e + 2*f*x])*Csc[e]*Csc[e + f*x]*Sec[2*e]*Sec[e + f*x]^6*(8*a^5*f*x*Cos[f
*x] + 56*a^4*b*f*x*Cos[f*x] + 184*a^3*b^2*f*x*Cos[f*x] + 296*a^2*b^3*f*x*C
os[f*x] + 224*a*b^4*f*x*Cos[f*x] + 64*b^5*f*x*Cos[f*x] - 12*a^5*f*x*Cos[3*
f*x] - 68*a^4*b*f*x*Cos[3*f*x] - 132*a^3*b^2*f*x*Cos[3*f*x] - 108*a^2*b^3*
f*x*Cos[3*f*x] - 32*a*b^4*f*x*Cos[3*f*x] - 8*a^5*f*x*Cos[2*e - f*x] - 56*a
^4*b*f*x*Cos[2*e - f*x] - 184*a^3*b^2*f*x*Cos[2*e - f*x] - 296*a^2*b^3*f*x
*Cos[2*e - f*x] - 224*a*b^4*f*x*Cos[2*e - f*x] - 64*b^5*f*x*Cos[2*e - f*x]
 - 8*a^5*f*x*Cos[2*e + f*x] - 56*a^4*b*f*x*Cos[2*e + f*x] - 184*a^3*b^2*f*
x*Cos[2*e + f*x] - 296*a^2*b^3*f*x*Cos[2*e + f*x] - 224*a*b^4*f*x*Cos[2*e
+ f*x] - 64*b^5*f*x*Cos[2*e + f*x] + 8*a^5*f*x*Cos[4*e + f*x] + 56*a^4*b*f
*x*Cos[4*e + f*x] + 184*a^3*b^2*f*x*Cos[4*e + f*x] + 296*a^2*b^3*f*x*Co...
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 4629, 2075, 374, 441, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 374$$

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left(-5b\tan^2(e+fx)+4a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}-\frac{b\cot(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 441$$

$$\frac{\frac{\frac{\int \frac{\cot^2(e+fx)\left(8a^2-11ba-4b^2-3b(9a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\cot(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{-\frac{\frac{\int \frac{8a^3+32ba^2+13b^2a+4b^3+b\left(8a^2-11ba-4b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\cot(e}{4a(a+b)(a+b\tan}}{f}$$

$\downarrow$ 397

$$\frac{\frac{-\frac{\frac{8(a+b)^3\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^2\left(35a^2+28ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+\right.}}{4a(a+b)}}{f}$$

$\downarrow$ 216

$$\frac{\frac{-\frac{\frac{8(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^2\left(35a^2+28ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{}{4a(}}{f}$$

$\downarrow$ 218

$$\frac{\frac{-\frac{\frac{8(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{}{4a(a+b)(}}{f}$$

input

```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output

```
(-1/4*(b*Cot[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + ((-(((8*
(a + b)^3*ArcTan[Tan[e + f*x]])/a - (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*Arc
Tan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((8*a
^2 - 11*a*b - 4*b^2)*Cot[e + f*x])/(a + b))/(2*a*(a + b)) - (b*(9*a + 4*b)
*Cot[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*(a + b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 13.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{1}{2}a\,b^2\right)\tan(fx+e)^3+\frac{a\left(13a^2+17ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+28ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{a^3(a+b)^3}-\frac{\arctan(\tan(fx}{a^3}}{f}$
default	$\frac{\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{1}{2}a\,b^2\right)\tan(fx+e)^3+\frac{a\left(13a^2+17ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+28ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{a^3(a+b)^3}-\frac{\arctan(\tan(fx}{a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(-8a^5e^{8i(fx+e)}+13a^3b^2e^{8i(fx+e)}+36a^2b^3e^{8i(fx+e)}+16a\,b^4e^{8i(fx+e)}-32a^5e^{6i(fx+e)}-64a^4b\,e^{6i(fx+e)}+2\right.}{}$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2/a^3/(a+b)^3*(((11/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^3+1/8*a*(13*a^2+1
7*a*b+4*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(35*a^2+28*a*b+8*b^2)/
((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/a^3*arctan(tan(f*x
+e))-1/(a+b)^3/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(165) = 330$.

Time = 0.16 (sec) , antiderivative size = 1060, normalized size of antiderivative = 5.86

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(4*(8*a^5 + 13*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 4*(16*a^4*b -
13*a^3*b^2 + 5*a^2*b^3 + 4*a*b^4)*cos(f*x + e)^3 - (35*a^2*b^3 + 28*a*b^4
+ 8*b^5 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b
^2 + 28*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*
a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 +
 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b)
)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*s
in(f*x + e) + 4*(8*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 32*((a^5
 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^
2 + 3*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4
 + b^5)*f*x)*sin(f*x + e))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f
*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 +
 (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), -1/16*(2*(8
*a^5 + 13*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 2*(16*a^4*b - 13*a^3*b^2 +
 5*a^2*b^3 + 4*a*b^4)*cos(f*x + e)^3 + (35*a^2*b^3 + 28*a*b^4 + 8*b^5 + (3
5*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 28*a^2*
b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f
*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e)
 + 2*(8*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 16*((a^5 + 3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.72

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(35\,a^2b^2+28\,ab^3+8\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{(a+b)b}} - \frac{(8\,a^2b^2-11\,ab^3-4\,b^4)\tan(fx+e)^4+8\,a^4+16\,a^3b+8\,a^2b^2+(16\,a^3b+3}{(a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5)\tan(fx+e)^5+2\,(a^6b+4\,a^5b^2+6\,a^4b^3+4\,a^3b^4+a^2b^5)\tan(fx+e)^3}}{8\,f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b)
)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)) - ((8*a^2*b^2 -
11*a*b^3 - 4*b^4)*tan(f*x + e)^4 + 8*a^4 + 16*a^3*b + 8*a^2*b^2 + (16*a^3*
b + 3*a^2*b^2 - 17*a*b^3 - 4*b^4)*tan(f*x + e)^2)/((a^5*b^2 + 3*a^4*b^3 +
3*a^3*b^4 + a^2*b^5)*tan(f*x + e)^5 + 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4
*a^3*b^4 + a^2*b^5)*tan(f*x + e)^3 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*
b^3 + 5*a^3*b^4 + a^2*b^5)*tan(f*x + e)) - 8*(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.36

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(35\,a^2b^2+28\,ab^3+8\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{ab+b^2}} + \frac{11\,ab^3\tan(fx+e)^3+4\,b^4\tan(fx+e)^3+13\,a^2b^2\tan(fx+e)+17\,ab^3\tan}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\left(b\tan(fx+e)^2+a+b\right)}}{8\,f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
+ arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^
3*b^3)*sqrt(a*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 4*b^4*tan(f*x + e)^3
+ 13*a^2*b^2*tan(f*x + e) + 17*a*b^3*tan(f*x + e) + 4*b^4*tan(f*x + e))/((
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*
x + e)/a^3 - 8/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

Mupad [B] (verification not implemented)

Time = 22.08 (sec) , antiderivative size = 4890, normalized size of antiderivative = 27.02

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^4*(11*a*b^3 + 4*b^4 - 8*a^2*b^2))/(8*a^2*(a + b)^3) - 1/(a
+ b) + (tan(e + f*x)^2*(13*a*b^2 - 16*a^2*b + 4*b^3))/(8*a^2*(a + b)^2))/(
f*(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b + a^2 + b^2) + b^2
*tan(e + f*x)^5)) - atan((286720*a^6*b^15*tan(e + f*x))/(286720*a^6*b^15 +
 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10
*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 22
1663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b
^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (3619840*a^7*b^14*tan(e + f*x))/
(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^1
2 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 33434
4192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^
5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (21052416*a^8*
b^13*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13
 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 3693465
60*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6
 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^
2) + (74346496*a^9*b^12*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14
+ 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*
a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 +
 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1851, normalized size of antiderivative = 10.23

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
(35*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*sin(e + f*x)**5*a**4*b + 28*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan
((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a**3*b**2 + 8*sqrt(b)*sq
rt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f
*x)**5*a**2*b**3 - 70*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/
2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*a**4*b - 126*sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*a**3*
b**2 - 72*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*sin(e + f*x)**3*a**2*b**3 - 16*sqrt(b)*sqrt(a + b)*atan((sqrt(a
 + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*a*b**4 + 35*sqr
t(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*si
n(e + f*x)*a**4*b + 98*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)
/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*a**3*b**2 + 99*sqrt(b)*sqrt(a + b)*at
an((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*a**2*b**
3 + 44*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/s
qrt(b))*sin(e + f*x)*a*b**4 + 8*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)*b**5 + 35*sqrt(b)*sqrt(a + b
)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**5*a
**4*b + 28*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a
))/sqrt(b))*sin(e + f*x)**5*a**3*b**2 + 8*sqrt(b)*sqrt(a + b)*atan((sqr...
```

3.374 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 230

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3}-\frac{b^{5/2}(63a^2+36ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}f} + \frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{b\cot^3(e+fx)}{4a(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\,(a+b+b\tan^2(e+fx))}$$

output

```
x/a^3-1/8*b^(5/2)*(63*a^2+36*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(9/2)/f+1/8*(8*a^3+32*a^2*b-15*a*b^2-4*b^3)*cot(f*x+e)/a^2/
(a+b)^4/f-1/24*(8*a^2-39*a*b-12*b^2)*cot(f*x+e)^3/a^2/(a+b)^3/f-1/4*b*cot(
f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+4*b)*cot(f*x+e)^3/a^
2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.37 (sec) , antiderivative size = 3340, normalized size of antiderivative = 14.52

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*((b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[
4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(
-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*Sqrt[a
 + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*b^3*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/
(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*
x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b
*Sin[4*e]])))/((a + b)^4*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e
 + 2*f*x])*Csc[e]*Csc[e + f*x]^3*Sec[2*e]*Sec[e + f*x]^6*(-36*a^6*f*x*Cos[
f*x] - 336*a^5*b*f*x*Cos[f*x] - 1560*a^4*b^2*f*x*Cos[f*x] - 3600*a^3*b^3*f
*x*Cos[f*x] - 4260*a^2*b^4*f*x*Cos[f*x] - 2496*a*b^5*f*x*Cos[f*x] - 576*b^
6*f*x*Cos[f*x] + 36*a^6*f*x*Cos[3*f*x] + 240*a^5*b*f*x*Cos[3*f*x] + 408*a^
4*b^2*f*x*Cos[3*f*x] - 48*a^3*b^3*f*x*Cos[3*f*x] - 732*a^2*b^4*f*x*Cos[3*f
*x] - 672*a*b^5*f*x*Cos[3*f*x] - 192*b^6*f*x*Cos[3*f*x] + 36*a^6*f*x*Cos[2
*e - f*x] + 336*a^5*b*f*x*Cos[2*e - f*x] + 1560*a^4*b^2*f*x*Cos[2*e - f*x]
 + 3600*a^3*b^3*f*x*Cos[2*e - f*x] + 4260*a^2*b^4*f*x*Cos[2*e - f*x] + 249
6*a*b^5*f*x*Cos[2*e - f*x] + 576*b^6*f*x*Cos[2*e - f*x] + 36*a^6*f*x*Cos[2
*e + f*x] + 336*a^5*b*f*x*Cos[2*e + f*x] + 1560*a^4*b^2*f*x*Cos[2*e + f*x]
 + 3600*a^3*b^3*f*x*Cos[2*e + f*x] + 4260*a^2*b^4*f*x*Cos[2*e + f*x] + ...
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4629, 2075, 374, 441, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-7b\tan^2(e+fx)+4a-3b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}-\frac{b\cot^3(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{441}$$

$$\frac{\frac{\frac{\int \frac{\cot^4(e+fx)\left(8a^2-39ba-12b^2-5b(11a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(11a+4b)\cot^3(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\cot^3(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\frac{\int \frac{3\cot^2(e+fx)\left(8a^3+32ba^2-15b^2a-4b^3+b\left(8a^2-39ba-12b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3(a+b)}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)}-\frac{b(11a+4b)\cot^3(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{\frac{\int \frac{\cot^2(e+fx)\left(8a^3+32ba^2-15b^2a-4b^3+b\left(8a^2-39ba-12b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)}-\frac{b(11a+4b)\cot^3(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-}{f}$$

$\Big\downarrow$ 445

$$\frac{\frac{-\frac{\frac{-\frac{\int \frac{8a^4+40ba^3+80b^2a^2+17b^3a+4b^4+b\left(8a^3+32ba^2-15b^2a-4b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{a+b}}{a+b}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(}{3(a+b)}}{2a(a+b)}}{4a(a+b)}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{-\frac{\frac{-\frac{\frac{8(a+b)^4\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^3\left(63a^2+36ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{a+b}}{a+b}-\frac{\left(8a^2-39ab-12b^2\right.}{3(a+}}{2a(a+b)}}{4a(a+b)}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{-\frac{\frac{-\frac{\frac{8(a+b)^4\arctan(\tan(e+fx))}{a}-\frac{b^3\left(63a^2+36ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{a+b}}{a+b}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+}{3(a+b)}}{2a(a+b)}}{4a(a+b)}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{-\frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{3(a+b)}-\frac{-\frac{\frac{8(a+b)^4\arctan(\tan(e+fx))}{a}-\frac{b^{5/2}(63a^2+36ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}}{a+b}-\frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{a+b}}{2a(a+b)}-\cdots}{4a(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Cot[e + f*x]^3)/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + ((-1/3
*((8*a^2 - 39*a*b - 12*b^2)*Cot[e + f*x]^3)/(a + b) - (-(((8*(a + b)^4*Arc
Tan[Tan[e + f*x]])/a - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((8*a^3 + 32*a^2*b
 - 15*a*b^2 - 4*b^3)*Cot[e + f*x])/(a + b))/(a + b))/(2*a*(a + b)) - (b*(1
1*a + 4*b)*Cot[e + f*x]^3)/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*
(a + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 22.67 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{-a-4b}{(a+b)^4\tan(fx+e)}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\frac{a\left(17a^2+21ab+4b^2\right)\tan(fx+\dots}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}\dots\right)}{a^3(a+b)^4}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{-a-4b}{(a+b)^4\tan(fx+e)}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\frac{a\left(17a^2+21ab+4b^2\right)\tan(fx+\dots}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}\dots\right)}{a^3(a+b)^4}}{f}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*arctan(tan(f*x+e))-1/3/(a+b)^3/tan(f*x+e)^3-(-a-4*b)/(a+b)^4/ta
n(f*x+e)-b^3/a^3/(a+b)^4*(((15/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^3+1/8*a*(17*a
^2+21*a*b+4*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(63*a^2+36*a*b+8*b
^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(212) = 424$.

Time = 0.19 (sec) , antiderivative size = 1649, normalized size of antiderivative = 7.17

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/96*(4*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 4
*(24*a^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*co
s(f*x + e)^5 - 4*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a
*b^5)*cos(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x +
e)^6 - 63*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b
^4 - 16*a*b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^
6)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e
)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x
+ e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a
^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(8*a^4*
b^2 + 32*a^3*b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 96*((a^6 + 4*a^5*b
 + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b -
2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b
 + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 -
 (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a
^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 +
2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4
- (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*co
s(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*
sin(f*x + e)), 1/48*(2*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.78

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{3\,(63\,a^2b^3+36\,ab^4+8\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\sqrt{(a+b)b}} - \frac{3\,(8\,a^3b^2+32\,a^2b^3-15\,ab^4-4\,b^5)\tan(fx+e)^6-8\,a^5-24\,a^4b-24\,a^3b^2-8\,a^2b^3+(48\,a^4b+2}{(a^6b^2+4\,a^5b^3+6\,a^4b^4+4\,a^3b^5+a^2b^6)\tan(fx+e)^7+2\,(a^7b+5\,a^6b^2+10\,a^5b^3+10\,a^4b^4+5}}{24\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt((a + b)*b))
- (3*(8*a^3*b^2 + 32*a^2*b^3 - 15*a*b^4 - 4*b^5)*tan(f*x + e)^6 - 8*a^5 -
24*a^4*b - 24*a^3*b^2 - 8*a^2*b^3 + (48*a^4*b + 232*a^3*b^2 + 133*a^2*b^3
- 63*a*b^4 - 12*b^5)*tan(f*x + e)^4 + 8*(3*a^5 + 16*a^4*b + 23*a^3*b^2 + 1
0*a^2*b^3)*tan(f*x + e)^2)/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 +
 a^2*b^6)*tan(f*x + e)^7 + 2*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4
+ 5*a^3*b^5 + a^2*b^6)*tan(f*x + e)^5 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a
^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*tan(f*x + e)^3) - 24*(f*x + e)/
a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.31

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{3\,(63\,a^2b^3+36\,ab^4+8\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\sqrt{ab+b^2}} + \frac{3\left(15\,ab^4\tan(fx+e)^3+4\,b^5\tan(fx+e)^3+17\,a^2b^3\tan(fx+e)+21\right.}{(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\left(b\tan(fx\right.}}{24\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 4*a^6*b + 6*a^5*b^2
+ 4*a^4*b^3 + a^3*b^4)*sqrt(a*b + b^2)) + 3*(15*a*b^4*tan(f*x + e)^3 + 4*b
^5*tan(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) + 21*a*b^4*tan(f*x + e) + 4*b^
5*tan(f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*(b*tan(
f*x + e)^2 + a + b)^2) - 24*(f*x + e)/a^3 - 8*(3*a*tan(f*x + e)^2 + 12*b*t
an(f*x + e)^2 - a - b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*
x + e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 22.91 (sec) , antiderivative size = 7057, normalized size of antiderivative = 30.68

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
atan((860160*a^6*b^20*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 49658112
00*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*
a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^1
7*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 8
7162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2)
+ (14515200*a^7*b^19*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 496581120
0*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a
^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17
*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87
162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) +
 (115347456*a^8*b^18*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 496581120
0*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a
^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17
*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87
162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) +
 (570587136*a^9*b^17*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 496581...
```

Reduce [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 2154, normalized size of antiderivative = 9.37

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

output
```
( - 189*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/
sqrt(b))*sin(e + f*x)**7*a**4*b**2 - 108*sqrt(b)*sqrt(a + b)*atan((sqrt(a
+ b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**3*b**3 - 24*s
qrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*
sin(e + f*x)**7*a**2*b**4 + 378*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a**4*b**2 + 594*sqrt(b)*s
qrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e +
f*x)**5*a**3*b**3 + 264*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x
)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a**2*b**4 + 48*sqrt(b)*sqrt(a + b
)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a
*b**5 - 189*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(
a))/sqrt(b))*sin(e + f*x)**3*a**4*b**2 - 486*sqrt(b)*sqrt(a + b)*atan((sqr
t(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*a**3*b**3 -
429*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt
(b))*sin(e + f*x)**3*a**2*b**4 - 156*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)
*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**3*a*b**5 - 24*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e +
 f*x)**3*b**6 - 189*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2)
 + sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**4*b**2 - 108*sqrt(b)*sqrt(a + b)*a
tan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**7*a...
```

3.375 $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 285

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3} + \frac{b^{7/2}(99a^2+44ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{11/2}f} - \frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2(a+b)^4f} - \frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\,(a+b+b\tan^2(e+fx))^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\,(a+b+b\tan^2(e+fx))}$$

output

```
-x/a^3+1/8*b^(7/2)*(99*a^2+44*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(
1/2))/a^3/(a+b)^(11/2)/f-1/8*(8*a^4+40*a^3*b+80*a^2*b^2-19*a*b^3-4*b^4)*co
t(f*x+e)/a^2/(a+b)^5/f+1/24*(8*a^3+32*a^2*b-51*a*b^2-12*b^3)*cot(f*x+e)^3/
a^2/(a+b)^4/f-1/40*(8*a^2-75*a*b-20*b^2)*cot(f*x+e)^5/a^2/(a+b)^3/f-1/4*b*
cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(13*a+4*b)*cot(f*x+e)^
5/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.91 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.92

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-\frac{120x(a+2b+a\cos(2(e+fx)))^2}{a^3}+\frac{8(11a+26b)(a+2b+a\cos(2(e+fx)))^2\cot(e)\operatorname{cs}}{(a+b)^4f}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((-120*x*(a + 2*b + a*Cos[2
*(e + f*x)])^2)/a^3 + (8*(11*a + 26*b)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Co
t[e]*Csc[e + f*x]^2)/((a + b)^4*f) - (24*(a + 2*b + a*Cos[2*(e + f*x)])^2*
Cot[e]*Csc[e + f*x]^4)/((a + b)^3*f) - (15*b^4*(99*a^2 + 44*a*b + 8*b^2)*A
rcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e
 + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[
2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(a^3*(a + b)^(11/2)*f*Sqrt[b*(Cos
[e] - I*Sin[e])^4]) + (8*(23*a^2 + 106*a*b + 173*b^2)*(a + 2*b + a*Cos[2*(
e + f*x)])^2*Csc[e]*Csc[e + f*x]*Sin[f*x])/((a + b)^5*f) - (8*(11*a + 26*b
)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^3*Sin[f*x])/((a + b
)^4*f) + (24*(a + 2*b + a*Cos[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^5*Sin[f*
x])/((a + b)^3*f) + (60*b^5*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/
(a^3*(a + b)^4*f) - (15*b^4*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[2*e]*((21*a
^2 + 52*a*b + 16*b^2)*Sin[2*e] - 3*a*(7*a + 2*b)*Sin[2*f*x]))/(a^3*(a + b)
^5*f)))/(960*(a + b*Sec[e + f*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4629, 2075, 374, 441, 445, 27, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^6(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^6(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(-9b\tan^2(e+fx)+4a-5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}-\frac{b\cot^5(e+fx)}{4a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{f}$$

$$\Big\downarrow \text{441}$$

$$\frac{\frac{\frac{\int \frac{\cot^6(e+fx)\left(8a^2-75ba-20b^2-7b(13a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(13a+4b)\cot^5(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\cot^5(e+fx)}{4a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{\frac{-\frac{\int \frac{5\cot^4(e+fx)\left(8a^3+32ba^2-51b^2a-12b^3+b\left(8a^2-75ba-20b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{5(a+b)}-\frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)}-\frac{b(13a+4b)\cot^5(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\cdots}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{-\frac{\int \frac{\cot^4(e+fx)\left(8a^3+32ba^2-51b^2a-12b^3+b\left(8a^2-75ba-20b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)}-\frac{b(13a+4b)\cot^5(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\cdots}{f}$$

$$\Big\downarrow \ 445$$

$$\frac{\frac{\frac{-\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(8a^4+40ba^3+80b^2a^2-19b^3a-4b^4+b\left(8a^3+32ba^2-51b^2a-12b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3(a+b)}-\frac{\left(8a^3+32a^2b-51ab^2-12b^3\right)\cot^3(e+fx)}{3(a+b)}-\frac{\left(8a^2\cdots\right.}{\cdots}}{a+b}\cdots}{2a(a+b)}\cdots}{4a(a+b)}\cdots}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(8a^4+40ba^3+80b^2a^2-19b^3a-4b^4+b\left(8a^3+32ba^2-51b^2a-12b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(8a^3+32a^2b-51ab^2-12b^3\right)\cot^3(e+fx)}{3(a+b)}-\frac{\left(8a^2-\cdots\right.}{\cdots}}{a+b}\cdots}{2a(a+b)}\cdots}{4a(a+b)}\cdots}{f}$$

$$\Big\downarrow \ 445$$

$$\frac{\frac{\frac{-\frac{-\frac{-\frac{\int \frac{8a^5+48ba^4+120b^2a^3+160b^3a^2+21b^4a+4b^5+b\left(8a^4+40ba^3+80b^2a^2-19b^3a-4b^4\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4\right)\cot(e+\cdots}{a+b}\cdots}{a+b}\cdots}{a+b}\cdots}{2a(a+b)}\cdots}{4a(a+b)}\cdots}{f}$$

$$\Big\downarrow \ 397$$

$$-\frac{-\frac{-\frac{-\frac{\frac{8(a+b)^5\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^4(99a^2+44ab+8b^2)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}}{a+b}-\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{a+b}}{a+b}-\frac{(8a^3\cdots}{}}{2a(a+b)}}{4a(a+b)}}{f}$$

$\downarrow$ 216

$$-\frac{-\frac{-\frac{-\frac{\frac{8(a+b)^5\arctan(\tan(e+fx))}{a}-\frac{b^4(99a^2+44ab+8b^2)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}}{a+b}-\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{a+b}}{a+b}-\frac{(8a^3+32a^2b-51\cdots}{}}{2a(a+b)}}{4a(a+b)}}{f}$$

$\downarrow$ 218

$$-\frac{-\frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{5(a+b)}-\frac{-\frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{3(a+b)}-\frac{-\frac{\frac{8(a+b)^5\arctan(\tan(e+fx))}{a}-\frac{b^{7/2}(99a^2+44ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}\cdots}{a+b}}{a+b}}{2a(a+b)}}{4a(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Cot[e + f*x]^5)/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + ((-1/5
*((8*a^2 - 75*a*b - 20*b^2)*Cot[e + f*x]^5)/(a + b) - (-1/3*((8*a^3 + 32*a
^2*b - 51*a*b^2 - 12*b^3)*Cot[e + f*x]^3)/(a + b) - (-(((8*(a + b)^5*ArcTa
n[Tan[e + f*x]])/a - (b^(7/2)*(99*a^2 + 44*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((8*a^4 + 40*a^3*b +
 80*a^2*b^2 - 19*a*b^3 - 4*b^4)*Cot[e + f*x])/(a + b))/(a + b))/(a + b))/(
2*a*(a + b)) - (b*(13*a + 4*b)*Cot[e + f*x]^5)/(2*a*(a + b)*(a + b + b*Tan
[e + f*x]^2)))/(4*a*(a + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [A] (verified)

Time = 45.07 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{5(a+b)^3\tan(fx+e)^5}-\frac{-a-4b}{3(a+b)^4\tan(fx+e)^3}-\frac{a^2+5ab+10b^2}{(a+b)^5\tan(fx+e)}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\cdots}{(a+b+b\tan(fx\cdots}\right.}{\cdots}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{5(a+b)^3\tan(fx+e)^5}-\frac{-a-4b}{3(a+b)^4\tan(fx+e)^3}-\frac{a^2+5ab+10b^2}{(a+b)^5\tan(fx+e)}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\cdots}{(a+b+b\tan(fx\cdots}\right.}{\cdots}}{f}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*arctan(tan(f*x+e))-1/5/(a+b)^3/tan(f*x+e)^5-1/3*(-a-4*b)/(a+b)
^4/tan(f*x+e)^3-(a^2+5*a*b+10*b^2)/(a+b)^5/tan(f*x+e)+b^4/a^3/(a+b)^5*(((1
9/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^3+1/8*a*(21*a^2+25*a*b+4*b^2)*tan(f*x+e))/
(a+b+b*tan(f*x+e)^2)^2+1/8*(99*a^2+44*a*b+8*b^2)/((a+b)*b)^(1/2)*arctan(b*
tan(f*x+e)/((a+b)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(265) = 530$.

Time = 0.23 (sec) , antiderivative size = 2229, normalized size of antiderivative = 7.82

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/480*(4*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)
*cos(f*x + e)^9 - 4*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 9
45*a^3*b^4 - 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 4*(120*a^7 + 40*a^6*b
 - 1416*a^5*b^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*c
os(f*x + e)^5 + 20*(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 1
53*a^2*b^5 + 36*a*b^6)*cos(f*x + e)^3 - 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a
^2*b^5)*cos(f*x + e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 5
5*a^3*b^4 - 36*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b
^4 - 69*a^2*b^5 + 12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^
2*b^5 - 36*a*b^6 - 8*b^7)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a
*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 +
3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))
*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*si
n(f*x + e) + 60*(8*a^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^
6)*cos(f*x + e) + 480*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^
4 + a^2*b^5)*f*x*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4
 - 4*a^2*b^5 - a*b^6)*f*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a
^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6
*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2
 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.82

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{15\,(99\,a^2b^4+44\,ab^5+8\,b^6)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5)\sqrt{(a+b)b}} - \frac{15\,(8\,a^4b^2+40\,a^3b^3+80\,a^2b^4-19\,ab^5-4\,b^6)\tan(fx+e)^8+5\,(48\,a^5b+280\,a^4b^2+680\,a^3b^3}{(a^7b^2+5\,a^6b^3+10\,a^5b^4+10\,a^4b^5+5\,a^3b^6+a^2}}{}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*arctan(b*tan(f*x + e)/sqrt((a +
b)*b))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sq
rt((a + b)*b)) - (15*(8*a^4*b^2 + 40*a^3*b^3 + 80*a^2*b^4 - 19*a*b^5 - 4*b
^6)*tan(f*x + e)^8 + 5*(48*a^5*b + 280*a^4*b^2 + 680*a^3*b^3 + 385*a^2*b^4
 - 75*a*b^5 - 12*b^6)*tan(f*x + e)^6 + 24*a^6 + 96*a^5*b + 144*a^4*b^2 + 9
6*a^3*b^3 + 24*a^2*b^4 + 8*(15*a^6 + 95*a^5*b + 258*a^4*b^2 + 291*a^3*b^3
+ 113*a^2*b^4)*tan(f*x + e)^4 - 8*(5*a^6 + 29*a^5*b + 57*a^4*b^2 + 47*a^3*
b^3 + 14*a^2*b^4)*tan(f*x + e)^2)/((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*
a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*tan(f*x + e)^9 + 2*(a^8*b + 6*a^7*b^2 + 15*
a^6*b^3 + 20*a^5*b^4 + 15*a^4*b^5 + 6*a^3*b^6 + a^2*b^7)*tan(f*x + e)^7 +
(a^9 + 7*a^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3
*b^6 + a^2*b^7)*tan(f*x + e)^5) - 120*(f*x + e)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.36

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{15\,(99\,a^2b^4+44\,ab^5+8\,b^6)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5)\sqrt{ab+b^2}} + \frac{15\left(19\,ab^5\tan(fx+e)^3+4\,b^6\tan(fx+e)^3+21\,a^2b^4\tan(fx+e)+25\,a}{(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5)\left(b\tan(\right.}}{}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sg
n(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 5*a^7*b + 10*a^6*b^
2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sqrt(a*b + b^2)) + 15*(19*a*b^5*tan(
f*x + e)^3 + 4*b^6*tan(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) + 25*a*b^5*tan
(f*x + e) + 4*b^6*tan(f*x + e))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3
+ 5*a^3*b^4 + a^2*b^5)*(b*tan(f*x + e)^2 + a + b)^2) - 120*(f*x + e)/a^3 -
 8*(15*a^2*tan(f*x + e)^4 + 75*a*b*tan(f*x + e)^4 + 150*b^2*tan(f*x + e)^4
 - 5*a^2*tan(f*x + e)^2 - 25*a*b*tan(f*x + e)^2 - 20*b^2*tan(f*x + e)^2 +
3*a^2 + 6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5)*tan(f*x + e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 23.61 (sec) , antiderivative size = 7460, normalized size of antiderivative = 26.18

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
atan(((((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478
080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*
a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*
a^19*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280
*a^22*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 6215385743
36*a^25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 842304716
80*a^28*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31
*b^6 + 340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*
a^35*b^2 + (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 16384000
0*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^
17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a
^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 31938471526
40*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188
774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 110
2610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74
281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 76021760
0*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(
2*a^3))*1i)/(2*a^3) + tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 4
0960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*
a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 13447268454...
```

Reduce [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 2455, normalized size of antiderivative = 8.61

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

output

```
(1485*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sq
rt(b))*sin(e + f*x)**9*a**4*b**3 + 660*sqrt(b)*sqrt(a + b)*atan((sqrt(a +
b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**9*a**3*b**4 + 120*sq
rt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*s
in(e + f*x)**9*a**2*b**5 - 2970*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan(
(e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**4*b**3 - 4290*sqrt(b)*
sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e +
 f*x)**7*a**3*b**4 - 1560*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f
*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**7*a**2*b**5 - 240*sqrt(b)*sqrt(a
+ b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**
7*a*b**6 + 1485*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - s
qrt(a))/sqrt(b))*sin(e + f*x)**5*a**4*b**3 + 3630*sqrt(b)*sqrt(a + b)*atan
((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a**3*b*
*4 + 2925*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a)
)/sqrt(b))*sin(e + f*x)**5*a**2*b**5 + 900*sqrt(b)*sqrt(a + b)*atan((sqrt(
a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*sin(e + f*x)**5*a*b**6 + 120*s
qrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) - sqrt(a))/sqrt(b))*
sin(e + f*x)**5*b**7 + 1485*sqrt(b)*sqrt(a + b)*atan((sqrt(a + b)*tan((e +
 f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f*x)**9*a**4*b**3 + 660*sqrt(b)*sqrt(
a + b)*atan((sqrt(a + b)*tan((e + f*x)/2) + sqrt(a))/sqrt(b))*sin(e + f...
```

3.376 $\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}$$

output
```
-a^(1/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+(a+b*sec(f*x+e)^2)^(1
/2)/f-1/3*(a+2*b)*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f+1/5*(a+b*sec(f*x+e)^2)^(5
/2)/b^2/f
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.25

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$
$$=\frac{15b^2(a+b\sec^2(e+fx))-5a(a+b\sec^2(e+fx))^2-10b(a+b\sec^2(e+fx))^2+3(a+b\sec^2(e+fx))^3}{15b^2f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```

output
```
(15*b^2*(a + b*Sec[e + f*x]^2) - 5*a*(a + b*Sec[e + f*x]^2)^2 - 10*b*(a +
b*Sec[e + f*x]^2)^2 + 3*(a + b*Sec[e + f*x]^2)^3 - 15*a*b^2*ArcTanh[Sqrt[1
 + (b*Sec[e + f*x]^2)/a]]*Sqrt[1 + (b*Sec[e + f*x]^2)/a])/(15*b^2*f*Sqrt[a
 + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)}{2f}$$

$\Big\downarrow$ 99

$$\frac{\int \left(\frac{(b\sec^2(e+fx)+a)^{3/2}}{b}+\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}+\frac{(-a-2b)\sqrt{b\sec^2(e+fx)+a}}{b}\right)d\sec^2(e+fx)}{2f}$$

$\Big\downarrow$ 2009

$$\frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)+\frac{2(a+b\sec^2(e+fx))^{5/2}}{5b^2}-\frac{2(a+2b)(a+b\sec^2(e+fx))^{3/2}}{3b^2}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sec
[e + f*x]^2] - (2*(a + 2*b)*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b^2) + (2*(a
+ b*Sec[e + f*x]^2)^(5/2))/(5*b^2))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(95) = 190$.

Time = 28.32 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.90

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)^2}\left(15\sqrt{a}\,\ln\left(4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\cos(fx+e)a\right)b^2\cos(fx+e)+(2\cos(\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/b^2*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(15*a^(1/2)*ln(4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)+4*cos(f*x+e)*a)*b^2*cos(f*x+e)+(2*cos(f*x+e)+2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
a*b*(10*cos(f*x+e)+10-sec(f*x+e)-sec(f*x+e)^2)+b^2*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(-15*cos(f*x+e)-15+10*sec(f*x+e)+10*sec(f*x+e)^2-3*se
c(f*x+e)^3-3*sec(f*x+e)^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(95) = 190$.

Time = 1.68 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.11

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

$$= \left[\frac{15\sqrt{a}b^2\cos(fx+e)^4\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab\right.}{}\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
[1/120*(15*sqrt(a)*b^2*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3
*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 +
 b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f
*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)) - 8*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*
cos(f*x + e)^2 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*
f*cos(f*x + e)^4), 1/60*(15*sqrt(-a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x
+ e)^4 - 4*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(
f*x + e)^2 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*co
s(f*x + e)^4)]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**5, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. 2(95) = 190.

Time = 0.63 (sec) , antiderivative size = 1280, normalized size of antiderivative = 11.53

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
2/15*(15*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2
*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
 1/2*e)^2 + a + b))^9*a - 165*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*a + 20*(sqrt(a
+ b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b))^7*(27*a^2 - 5*a*b - 16*b^2) - 20*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2
*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(33*a^2 - 83*a*b
+ 32*b^2)*sqrt(a + b) - 2*(15*a^3 + 1230*a^2*b - 625*a*b^2 - 416*b^3)*(sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))^5 + 10*(81*a^3 + 90*a^2*b - 391*a*b^2 + 256*b^3)*(sqrt(a + b)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))
^4*sqrt(a + b) - 20*(33*a^4 - 45*a^3*b - 157*a^2*b^2 + 161*a*b^3 - 16*b...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx=\int \tan(e+f\,x)^5\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$
$$= \frac{3\sqrt{\sec\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^4-4\sqrt{\sec\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^2+8\sqrt{\sec\left(fx+e\right)^2 b+a}-}{}$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x)
```

output
```
(3*sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**4 - 4*sqrt(sec(e + f*x)**2*b
+ a)*tan(e + f*x)**2 + 8*sqrt(sec(e + f*x)**2*b + a) - 3*int((sqrt(sec(e +
 f*x)**2*b + a)*sec(e + f*x)**2*tan(e + f*x)**5)/(sec(e + f*x)**2*b + a),x
)*b*f - 4*int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**3)/(sec(e + f*x)*
*2*b + a),x)*a*f + 8*int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x))/(sec(e
 + f*x)**2*b + a),x)*a*f)/(12*f)
```

3.377 $\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}$$

output
```
a^(1/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-(a+b*sec(f*x+e)^2)^(1/
2)/f+1/3*(a+b*sec(f*x+e)^2)^(3/2)/b/f
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\frac{3\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a+b\sec^2(e+fx)}\left(a-3b+b\sec^2(e+fx)\right)}{3bf}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

output
```
(3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sec[
e + f*x]^2]*(a - 3*b + b*Sec[e + f*x]^2))/(3*b*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4627, 25, 354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\cos(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)-\frac{2\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b}}{2f}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)-\frac{2\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{\frac{2a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}-\frac{2\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)-\frac{2\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

output
```
-1/2*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a +
b*Sec[e + f*x]^2] - (2*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(68) = 136$.

Time = 10.57 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.92

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(3\sqrt{a}\,\ln\left(4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\cos(fx+e)a\right)b\cos(fx+e)+(1+\cos(fx+\cdots\right.}{3fb(1+\cos(fx+e))\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}}$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/f/b*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(3*a^(1/2)*ln(4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)+4*cos(f*x+e)*a)*b*cos(f*x+e)+(1+cos(f*x+e))*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a+b*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-3*cos(f
*x+e)-3+sec(f*x+e)+sec(f*x+e)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(68) = 136$.

Time = 0.41 (sec) , antiderivative size = 386, normalized size of antiderivative = 4.82

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$$

$$= \left[\frac{3\sqrt{a}b\cos(fx+e)^2\log\left(128a^4\cos(fx+e)^8+256a^3b\cos(fx+e)^6+160a^2b^2\cos(fx+e)^4+32ab^3\right.}{}\right.$$

$$-\frac{3\sqrt{-a}b\arctan\left(\frac{\left(8a^2\cos(fx+e)^4+8ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4+3a^2b\cos(fx+e)^2+ab^2\right)}\right)\cos(fx+e)^2-4\left((a-3b)\cos(fx\right.}{12bf\cos(fx+e)^2}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(a)*b*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*c
os(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
 e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) + 8*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2))/(b*f*cos(f*x + e)^2), -1/12*(3*sqrt(-a)*b*arctan(1/4*(8*a^2*c
os(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*
b^2))*cos(f*x + e)^2 - 4*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\tan^3{(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\tan{(fx+e)}^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(68) = 136$.

Time = 0.45 (sec) , antiderivative size = 800, normalized size of antiderivative = 10.00

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
-2/3*(3*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*
(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^5*a - 3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(3*a - 4*b)*sqrt(a + b) + 2*(sq
rt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^
2 + a + b))^3*(3*a^2 - 9*a*b + 8*b^2) + 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(a^2 + a*b - 4*
b^2)*sqrt(a + b) - 3*(3*a^3 - 2*a^2*b - 5*a*b^2 + 16*b^3)*(sqrt(a + b)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) +
 (3*a^3 - 6*a^2*b + 19*a*b^2 - 20*b^3)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx=\int \tan(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$$
$$=\frac{\sqrt{\sec\left(fx+e\right)^2b+a}\,\tan\left(fx+e\right)^2-2\sqrt{\sec\left(fx+e\right)^2b+a}-\left(\int\frac{\sqrt{\sec(fx+e)^2b+a}\,\sec(fx+e)^2\tan(fx+e)^3}{\sec(fx+e)^2b+a}dx\right)}{2f}$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x)
```

output
```
(sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**2 - 2*sqrt(sec(e + f*x)**2*b +
a) - int((sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*tan(e + f*x)**3)/(se
c(e + f*x)**2*b + a),x)*b*f - 2*int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f
*x))/(sec(e + f*x)**2*b + a),x)*a*f)/(2*f)
```

3.378 $\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx=-\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}$$

output
```
-a^(1/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+(a+b*sec(f*x+e)^2)^(1
/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 119 vs. 2(54) = 108.

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.20

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx=\frac{\left(-2\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)\cos(e+fx)+\sqrt{2}\sqrt{b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{b}}\right)\sqrt{a+b\sec^2(e+fx)}}{\sqrt{2}\sqrt{b}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{b}}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

output
```
((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]]*Cos[e + f*x] + Sqrt[2
]*Sqrt[b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])*Sqrt[a + b*Sec[e + f*x]^
2])/(Sqrt[2]*Sqrt[b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4627, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \; 4627$$

$$\frac{\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 243$$

$$\frac{\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow 221$$

$$\frac{2\sqrt{a+b\sec^2(e+fx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sec
[e + f*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\sqrt{a+b\sec(fx+e)^2}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f}$	58
default	$\frac{\sqrt{a+b\sec(fx+e)^2}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f}$	58

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*((a+b*sec(f*x+e)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)
^(1/2))/sec(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(46) = 92.

Time = 0.18 (sec) , antiderivative size = 312, normalized size of antiderivative = 5.78

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2+\right.}{}\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*
a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos
(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^
2)) + 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx = \int \sqrt{a+b\sec^2\left(e+fx\right)}\tan\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x), x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(46) = 92$.

Time = 0.30 (sec) , antiderivative size = 377, normalized size of antiderivative = 6.98

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$$
$$= \frac{2\left(\frac{a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{\ldots}{\left(\sqrt{a+b}\ldots\right.}\right.}{\ldots}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
2*(a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + 2*((sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))*b + sqrt(a + b)*b)/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
+ b) + a - 3*b))*sgn(cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 16.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx=\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{f}-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{f}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
(a + b/cos(e + f*x)^2)^(1/2)/f - (a^(1/2)*atanh((a + b/cos(e + f*x)^2)^(1/
2)/a^(1/2)))/f
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx =\frac{\sqrt{\sec\left(fx+e\right)^{2}b+a}+\left(\int \frac{\sqrt{\sec(fx+e)^{2}b+a}\,\tan(fx+e)}{\sec(fx+e)^{2}b+a}dx\right)af}{f}$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x)
```

output
```
(sqrt(sec(e + f*x)**2*b + a) + int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*
x))/(sec(e + f*x)**2*b + a),x)*a*f)/f
```

3.379 $\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

output
```
a^(1/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-(a+b)^(1/2)*arctanh((a
+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

input
```
Integrate[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (Sqrt[a + b]*Arc
Tanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4627, 25, 354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)}dx$$

$$\Big\downarrow \; 4627$$

$$\frac{\int -\frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 354$$

$$-\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 94$$

$$-\frac{(a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{2f}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

input
```
Int[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(58) = 116$.

Time = 1.50 (sec) , antiderivative size = 540, normalized size of antiderivative = 7.71

method	result
default	$\frac{\left(2\sqrt{a}\,\ln\left(4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\cos(fx+e)a\right)\sqrt{a+b}+\ln\left(\frac{2\sqrt{a+b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+}{\sqrt{a-}}\right.\right.}{}$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(a+b)^(1/2)*(2*a^(1/2)*ln(4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)+4*cos(f*x+e)*a)*(a+b)^(1/2)+ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a+b*ln(2/(a+b)^(1/
2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+
b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+co
s(f*x+e)))-ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x
+e)*a+b)/(-1+cos(f*x+e)))*a-ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b)*(a+b*sec(f*x+e)^2)^(1/2)*cos
(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(58) = 116$.

Time = 0.25 (sec) , antiderivative size = 963, normalized size of antiderivative = 13.76

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output

```
[1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*
a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(a +
b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x
 + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x
 + e)^2 + 1)))/f, 1/8*(4*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2
 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b
)*cos(f*x + e)^2 + a*b + b^2)) + sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*
a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*co
s(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)))/f, -1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*
cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - sqrt(a + b)*lo
g(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)
^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)))/f, -1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co...
```

Sympy [F]

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot(e+fx)\,dx$$

input

```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output

```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 3317, normalized size of antiderivative = 47.39

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(a^(3/2)*log(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 +
4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 +
2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a
*b)*cos(2*f*x + 2*e))*a*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*s
in(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2
 + 4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b
 + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^
2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x
+ 2*e))*a*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e
), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 16*(a^2*cos
(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f
*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a
+ b)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x +
2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 16*a^2 + 3
2*a*b + 16*b^2) - 2*sqrt(a + b)*a*log(4*(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(58) = 116$.

Time = 0.52 (sec) , antiderivative size = 402, normalized size of antiderivative = 5.74

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\left(\frac{4\,a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{a}\right.}{}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(4*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + sq
rt(a + b)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))) - sqrt(a + b)*log(abs(-sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b) - sqrt(a + b))) + sqrt(a + b)*log(abs((sqrt(a + b)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a
*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - s
qrt(a + b)*(a - b))))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)\,\sqrt{a+\frac{b}{{\cos\left(e+f\,x\right)}^2}}\,dx$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\cot\left(fx+e\right)dx$$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x),x)
```

3.380 $\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a+b)\,\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}f} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

output

```
-a^(1/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/2*(2*a+b)*arctanh((
a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)/f-1/2*cot(f*x+e)^2*(a+b*s
ec(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.82 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.83

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(\frac{1+e^{2i(e+fx)}}{\left(-1+e^{2i(e+fx)}\right)^2}-\frac{-2i\sqrt{a}\sqrt{a+b}fx+(2a+b)\log\left(1-e^{2i(e+fx)}\right)+}{}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]*((1 + E^((2*I)*(e + f*x)))/(-1 + E^((2*I)*(e + f*x)))^
2 - ((-2*I)*Sqrt[a]*Sqrt[a + b]*f*x + (2*a + b)*Log[1 - E^((2*I)*(e + f*x)
)] + Sqrt[a]*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqr
t[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[a]*Sqrt
[a + b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*
Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*a*Log[a
 + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - b*Log[a + b + a*
E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((
2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]
^2])/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4627, 354, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \; 4627$$

$$\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 354$$

$$\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^2}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 110$$

$$\frac{\frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}-\int -\frac{\cos(e+fx)(b\sec^2(e+fx)+2a)}{2(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\int \frac{\cos(e+fx)(b\sec^2(e+fx)+2a)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{1}{2}\left((2a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)\right)+\frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{2}\left(\frac{2(2a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{4a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}\right)+\frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{2}\left(\frac{2(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

input
```
Int[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + (2*(2*a + b)*Ar
cTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/2 + Sqrt[a + b
*Sec[e + f*x]^2]/(1 - Sec[e + f*x]^2))/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(91) = 182$.

Time = 1.59 (sec) , antiderivative size = 1200, normalized size of antiderivative = 11.01

method	result	size
default	Expression too large to display	1200

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/(a+b)^(5/2)*((4*cos(f*x+e)-4)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^2+(-4*cos
(f*x+e)+4)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x
+e)*a+b)/(-1+cos(f*x+e)))*a*b^2+(5*cos(f*x+e)-5)*ln(2/(a+b)^(1/2)*((a+b)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*
a^2*b+(-5*cos(f*x+e)+5)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b+(4*cos(f*x+e)-4)*a^(1/2)*(a+b
)^(3/2)*ln(4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*b+
(-2+2*cos(f*x+e))*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3+(-1+cos(f*x+e))*ln(2/(a+b)^
(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+
(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1
+cos(f*x+e)))*b^3+(-2*cos(f*x+e)+2)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(91) = 182$.

Time = 0.37 (sec) , antiderivative size = 1342, normalized size of antiderivative = 12.31

$$\int \cot^3(e + fx)\sqrt{a + b\sec^2(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
 + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 2
56*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x +
e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2
*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)) + ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a + b)*log(2*
((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 +
 b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 +
 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a + b)*cos(f*x + e)^
2 - 2*a - b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x +
e)^2 + a*b + b^2)) + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*
cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 3
2*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x
 + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^
2 + 2*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f...
```

Sympy [F]

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\cot^3{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

Maxima [F]

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}^2+a}\cot{(fx+e)}^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(91) = 182$.

Time = 0.64 (sec) , antiderivative size = 574, normalized size of antiderivative = 5.27

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(16*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 4*
(2*a + b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) + 2*(2*a + b)*
log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f
*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/sqrt(a + b) + sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*((sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b)
- (a + b)^(3/2))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)^3\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\cot\left(fx+e\right)^3 dx$$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**3,x)
```

3.381 $\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(8a^2+12ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f} + \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

output

```
a^(1/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/8*(8*a^2+12*a*b+3*b^
2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f+1/8*(4*a+3*
b)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f-1/4*cot(f*x+e)^4*(a+b*sec
(f*x+e)^2)^(1/2)/f
```

Mathematica [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.440$, Rules used = {3042, 4627, 25, 354, 110, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}d\sec^2(e+fx)}{2f}$$

$\Big\downarrow$ 110

$$-\frac{\frac{\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}-\frac{1}{2}\int -\frac{\cos(e+fx)(3b\sec^2(e+fx)+4a)}{2(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$\Big\downarrow$ 27

$$-\frac{\frac{1}{4}\int \frac{\cos(e+fx)(3b\sec^2(e+fx)+4a)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}}{2f}$$

$\Big\downarrow$ 168

$$-\frac{\frac{1}{4}\left(\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}-\frac{\int -\frac{\cos(e+fx)\left(b(4a+3b)\sec^2(e+fx)+8a(a+b)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a+b}\right)+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}}{2f}$$

$\Big\downarrow$ 27

$$-\frac{\frac{1}{4}\left(\frac{\int \frac{\cos(e+fx)\left(b(4a+3b)\sec^2(e+fx)+8a(a+b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}\right)+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}}{2f}$$

$\Big\downarrow$ 174

$$-\frac{\frac{1}{4}\left(\frac{\left(8a^2+12ab+3b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+8a(a+b)\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}\right.}{2f}$$

$\Big\downarrow$ 73

$$-\frac{\frac{1}{4}\left(\frac{\frac{2\left(8a^2+12ab+3b^2\right)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{16a(a+b)\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{2(a+b)}+\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}\right)+}{2f}$$

$\Big\downarrow$ 221

$$-\frac{\frac{1}{4}\left(\frac{\frac{2(8a^2+12ab+3b^2)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-16\sqrt{a}(a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{2(a+b)}+\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}\right)+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}}{2f}$$

input
```
Int[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(Sqrt[a + b*Sec[e + f*x]^2]/(2*(1 - Sec[e + f*x]^2)^2) + ((-16*Sqrt[a
]*(a + b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + (2*(8*a^2 + 12*a*b
 + 3*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(2
*(a + b)) + ((4*a + 3*b)*Sqrt[a + b*Sec[e + f*x]^2])/((a + b)*(1 - Sec[e +
 f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1943 vs. $2(139) = 278$.

Time = 1.62 (sec) , antiderivative size = 1944, normalized size of antiderivative = 12.07

method	result	size
default	Expression too large to display	1944

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/(a+b)^(9/2)*((-8*cos(f*x+e)+8)*sin(f*x+e)^2*ln(2/(a+b)^(1/2)*((a+b)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e))
)*a^5+(-36*cos(f*x+e)+36)*sin(f*x+e)^2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b+(-63
*cos(f*x+e)+63)*sin(f*x+e)^2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b^2+(-53*cos(f*x
+e)+53)*sin(f*x+e)^2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^3+(-21*cos(f*x+e)+21)*
sin(f*x+e)^2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^4+(-3*cos(f*x+e)+3)*sin(f*x+e)^2
*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*
x+e)*a+b)/(1+cos(f*x+e)))*b^5+(8*cos(f*x+e)-8)*sin(f*x+e)^2*ln(-4*((a+b)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(139) = 278$.

Time = 1.16 (sec) , antiderivative size = 1953, normalized size of antiderivative = 12.13

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f
*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^
3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2
+ b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(
f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b
 + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(a + b)*log(2*((8*a
^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2
- 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
- 4*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos
(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b +
b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*
a*b + b^2)*f), 1/16*(((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 +
 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-a - b)*arc
tan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2
+ 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2
 + 2*a*b + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e
)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(1...
```

Sympy [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\cot^5{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)
```

Maxima [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\cot{(fx+e)}^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(139) = 278$.

Time = 1.17 (sec) , antiderivative size = 866, normalized size of antiderivative = 5.38

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*(tan(1/2*f*x + 1/2
*e)^2 - (11*a + 9*b)/(a + b)) - 128*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2
*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a +
 b))/sqrt(-a))/sqrt(-a) + 8*(8*a^2 + 12*a*b + 3*b^2)*arctan(-(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)/sqrt(-a - b))/((a + b)*sqrt(-a - b)) - 4*(8*a^2 + 12*a*b + 3*b^2)*log(ab
s((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/
2*e)^2 + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a + b)^(3/2) + 4*(2*(sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))^3*(3*a^2 - 2*b^2) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(7*a^2 + 10*a*b + 3*b^2)*s
qrt(a + b) - 2*(2*a^3 + 2*a^2*b - 3*a*b^2 - 3*b^3)*(sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + 5*(...
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cot(e+f\,x)^5\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^5\,dx$$

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**5,x)
```

3.382 $\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 219

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+5a^2b+15ab^2-5b^3)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} - \frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}$$

output
```
-a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/16*(a^3
+5*a^2*b+15*a*b^2-5*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(
1/2))/b^(5/2)/f-1/16*(a-b)*(a+5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b
^2/f+1/24*(a-5*b)*tan(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f+1/6*tan(f*x+
e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = -\frac{\left(16\sqrt{a}b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(a^3+5a^2b+15ab^2-5b^3)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos(e+fx)\sqrt{a+}}{8\sqrt{2}b^2f\sqrt{a+2b+a\cos(2e+2fx)}} - \frac{(9a^2+34ab-59b^2+4(3a^2+12ab-7b^2)\cos(2(e+fx))+(3a^2+14ab-33b^2)\cos(4(e+fx)))\sec^4(}{384b^2f}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

output
```
-1/8*((16*Sqrt[a]*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e +
 f*x]^2]] - ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Sin[e + f
*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec
[e + f*x]^2])/(Sqrt[2]*b^2*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) - ((9*a^2
 + 34*a*b - 59*b^2 + 4*(3*a^2 + 12*a*b - 7*b^2)*Cos[2*(e + f*x)] + (3*a^2
+ 14*a*b - 33*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x
]^2]*Tan[e + f*x])/(384*b^2*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 4629, 2075, 380, 444, 27, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^6\sqrt{a+b\sec(e+fx)^2}dx$$

$\downarrow$ 4629

$$\frac{\int \frac{\tan^6(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\downarrow$ 2075

$$\frac{\int \frac{\tan^6(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\downarrow$ 380

$$\frac{\frac{1}{6}\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{6}\int \frac{\tan^4(e+fx)(5(a+b)-(a-5b)\tan^2(e+fx))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$\downarrow$ 444

$$\frac{\frac{1}{6}\left(\frac{\int -\frac{3\tan^2(e+fx)\left((a-b)(a+5b)\tan^2(e+fx)+(a-5b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}+\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}\right)+\frac{1}{6}\tan^5(e+fx)\sqrt{}}{f}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\int \frac{\tan^2(e+fx)\left((a-b)(a+5b)\tan^2(e+fx)+(a-5b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}\right)+\frac{1}{6}\tan^5(e+fx)\sqrt{}}{f}$$

$\downarrow$ 444

$$\frac{\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int \frac{\left(a^3+5ba^2+15b^2a-5b^3\right)\tan^2(e+fx)+(a+5b)\left(a^2-\right.}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}}{2b}\right.}{4b}\right.}{f}$$

$\downarrow$ 398

$$\frac{\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(a^3+5a^2b+15ab^2-5b^3\right)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan}{}\right.}{4b}\right.}{f}$$

$\downarrow$ 224

$$\frac{\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(a^3+5a^2b+15ab^2-5b^3\right)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{}{\sqrt{}}}{}\right.}{4b}\right.}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{\left(a^3+5a^2b+15ab^2-5b^3\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+f}{\sqrt{a+b\tan^2(e+}}\right.}{\sqrt{b}}}{}\right.}{4b}\right.}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{\left(a^3+5a^2b+15ab^2-5b^3\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+f}{\sqrt{a+b\tan^2(e+}}\right.}{\sqrt{b}}}{}\right.}{4b}\right.}{f}$$

$\downarrow$ 216

$$\frac{\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{\left(a^3+5a^2b+15ab^2-5b^3\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+f}{\sqrt{a+b\tan^2(e+}}\right.}{\sqrt{b}}}{2b}\right.}{4b}\right.}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

output
```
((Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/6 + (((a - 5*b)*Tan[e + f
*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*b) - (3*(-1/2*(-16*Sqrt[a]*b^2*Ar
cTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((a^3 + 5*a^
2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/Sqrt[b])/b + ((a - b)*(a + 5*b)*Tan[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(2*b)))/(4*b))/6)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1256 vs. $2(193) = 386$.

Time = 49.50 (sec) , antiderivative size = 1257, normalized size of antiderivative = 5.74

method	result	size
default	Expression too large to display	1257

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/96/f/(-a)^(1/2)/b^(11/2)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-3*cos(f*x+e)*(-a)^(1/2)*ln(4*(b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^3
*b^3-15*cos(f*x+e)*(-a)^(1/2)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^4-45*cos(f*x+e)*(-a)^(1/2)*ln(4
*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1
))*a*b^5+15*cos(f*x+e)*(-a)^(1/2)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^6-3*cos(f*x+e)*(-a)^(1/2)*ln(-4
*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1
))*a^3*b^3-15*cos(f*x+e)*(-a)^(1/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^4-45*cos(f*x+e)*(-a)^(1/
2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(
f*x+e)-1))*a*b^5+15*cos(f*x+e)*(-a)^(1/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+...
```

Fricas [A] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 1775, normalized size of antiderivative = 8.11

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

output
```
[1/192*(24*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7
*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*
sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4) - 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a
*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*b^3*cos(f*x + e)^
5*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*
(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) +
3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(...
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\tan^6{(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**6, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx=\int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx=\int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx=\int \tan(e+f\,x)^6\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \int \sqrt{\sec\left(fx+e\right)^{2}b+a}\,\tan\left(fx+e\right)^{6}dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**6,x)
```

3.383 $\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^2+6ab-3b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

output
```
a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/8*(a^2+6
*a*b-3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)
/f+1/8*(a-3*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f+1/4*tan(f*x+e)^3*
(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.26

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx$$
$$= \frac{\left(8\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(a^2+6ab-3b^2)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos(e+fx)\sqrt{a+b\sec^2(e+}{4\sqrt{2}bf\sqrt{a+2b+a\cos(2e+2fx)}}$$
$$+\frac{(a-b+(a-5b)\cos(2(e+fx)))\sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\tan(e+fx)}{16bf}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

output
```
((8*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
] - ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*S
in[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(4*Sqrt
[2]*b*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) + ((a - b + (a - 5*b)*Cos[2*(e
 + f*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(16*b*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4629, 2075, 380, 444, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^4(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 2075

$$\frac{\int \frac{\tan^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 380

$$\frac{\frac{1}{4}\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{4}\int \frac{\tan^2(e+fx)(3(a+b)-(a-3b)\tan^2(e+fx))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 444

$$\frac{\frac{1}{4}\left(\frac{\int -\frac{\left(a^2+6ba-3b^2\right)\tan^2(e+fx)+(a-3b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}+\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}\right)+\frac{1}{4}\tan^3(e+fx)\sqrt{a+b\tan}}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int \frac{\left(a^2+6ba-3b^2\right)\tan^2(e+fx)+(a-3b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}\right)+\frac{1}{4}\tan^3(e+fx)\sqrt{a+b\tan^2(}}{f}$$

$\Big\downarrow$ 398

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(a^2+6ab-3b^2\right)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-8ab\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\,ta}{2b}\right.}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(a^2+6ab-3b^2\right)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-8ab\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fa}}}{2b}\right.}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{(a^2+6ab-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8ab\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan}{2b}\right.}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{(a^2+6ab-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8ab\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+}}}{2b}\right.}{f}$$

$\downarrow$ 216

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{(a^2+6ab-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8\sqrt{a}b\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b}\right)+\frac{1}{4}\mathrm{t}}{f}$$

input

```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

output

```
((Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/4 + (-1/2*(-8*Sqrt[a]*b*A
rcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((a^2 + 6*a
*b - 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/Sqrt[b])/b + ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
b))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(143) = 286$.

Time = 42.96 (sec) , antiderivative size = 933, normalized size of antiderivative = 5.65

method	result	size
default	Expression too large to display	933

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/f/(-a)^(1/2)/b^2*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(3*cos(f*x+e)*b^(5/2)*ln(4*(b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)-6*
cos(f*x+e)*b^(3/2)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*a-cos(f*x+e)*b^(1/2)*ln(4*(b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2
)*a^2+3*cos(f*x+e)*b^(5/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)-6*cos(f*x+e)*b^(3/2)*ln(-4*(
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))
*(-a)^(1/2)*a-cos(f*x+e)*b^(1/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*a^2+16*cos(f*x+e)*ln(4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+2*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b*(sin(f*x+e)+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(143) = 286$.

Time = 1.05 (sec) , antiderivative size = 1621, normalized size of antiderivative = 9.82

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
[1/32*(4*sqrt(-a)*b^2*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b - 3*b^2)*sqrt(b)*cos(f*x +
 e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)
^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((a
*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*b^2*cos(f*x +
 e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(
5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
- 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f
*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
) - (a^2 + 6*a*b - 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(...
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\tan^4{(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^4\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**4,x)
```

3.384 $\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
-a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(a-b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(1/2)/f+1/2*tan(
f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.46

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(-\frac{i\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{-2\sqrt{a}\sqrt{b}fx+i\sqrt{a}\sqrt{b}\log\left(a+2b+ae^{2i(e+f}\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e +
f*x)))^2 + (-2*Sqrt[a]*Sqrt[b]*f*x + I*Sqrt[a]*Sqrt[b]*Log[a + 2*b + a*E^(
(2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*
(e + f*x)))^2]] - I*Sqrt[a]*Sqrt[b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^
((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]] - a*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 +
E^((2*I)*(e + f*x))))] + b*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*
Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)
*(1 + E^((2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*f*Sqrt[a
 + 2*b + a*Cos[2*e + 2*f*x]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4629, 2075, 380, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^2(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{380}$$

$$\frac{\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{2}\int \frac{-((a-b)\tan^2(e+fx))+a+b}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-2a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}\tan(e+fx}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-2a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}\tan(e}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-2a\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}\tan(e+fx)\sqrt{a+b}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{1}{2}\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-2a\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{2}\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

output
```
((-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
 + ((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/Sqrt[b])/2 + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(100) = 200$.

Time = 24.11 (sec) , antiderivative size = 636, normalized size of antiderivative = 5.39

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(-\cos(fx+e)\sqrt{-a}\,\ln\left(\frac{-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)b^{\frac{3}{2}}+\cos\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(-cos(f*x+e)*(-a)^(1/2)*ln(4*(-b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*b^(3/2)+cos(f
*x+e)*(-a)^(1/2)*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-a-b)/(sin(f*x+e)-1))*b^(1/2)*a-cos(f*x+e)*(-a)^(1/2)*ln(-4*(-b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*b^(3/2)+
cos(f*x+e)*(-a)^(1/2)*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)+a+b)/(sin(f*x+e)+1))*b^(1/2)*a-4*cos(f*x+e)*ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b+(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b*(2*sin(f*x+e)+2*tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(100) = 200$.

Time = 0.43 (sec) , antiderivative size = 1471, normalized size of antiderivative = 12.47

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*
b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4
 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*
b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x
 + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*(a - b)*sqrt(
-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x
 + e)))*cos(f*x + e) + sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8
- 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos
(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a
^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(
a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^
3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2...
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\tan^2{(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^2\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**2,x)
```

3.385 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

output
```
a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+b^(1/2)*ar
ctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.59

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{i\left(1+e^{2i(e+fx)}\right)\left(2\sqrt{b}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a}\operatorname{arctanh}\left(\frac{a+2b+ae^{2i(e+fx)}}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\right.}{2\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\,f}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-1/2*I)*(1 + E^((2*I)*(e + f*x)))*(2*Sqrt[b]*ArcTan[(Sqrt[b]*(-1 + E^((2
*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + Sqrt[a]*ArcTanh[(a + 2*b + a*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])] - Sqrt[a]*ArcTanh
[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])])*Sqrt[a + b*Sec[e +
f*x]^2])/(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4616, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{a\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + S
qrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(67) = 134$.

Time = 5.03 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.47

method	result
default	$\left(\sqrt{b}\,\ln\left(\frac{-4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\sqrt{b}\,\ln\left(-\frac{4\left(-\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right.}{}\right.\right.$

input
```
int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(b^(1/2)*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)+b^(1/2)*ln(-4*(-b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+2
*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a)*
(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time = 0.30 (sec) , antiderivative size = 1227, normalized size of antiderivative = 15.53

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4))/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos
(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 -
256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3
*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
- (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a
^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*...
```

Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+b\sec^2{(e+fx)}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 3227, normalized size of antiderivative = 40.85

$$\int \sqrt{a + b\sec^2(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(a)*b^(3/2)*arctan2(a*sin(2*f*x + 2*e) + (a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^
2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*
arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), a*cos(2*f*x + 2*e) + (a^2*cos(4*
f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x
+ 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*
a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a
)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos
(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + a + 2*b) + a^(3/2)*sq
rt(b)*arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2
+ 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2
+ 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*...
```

Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)^2+a}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec\left(fx+e\right)^2b+a}dx$$

input
```
int((a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a),x)
```

3.386 $\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

output
```
-a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.88

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\Bigg(\sqrt{a+b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}+\sqrt{2}\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\sin(e+fx}{\sqrt{a+b}\,f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

input
```
Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[a + b]*Sqrt[(a + 2*b + a*
Cos[2*(e + f*x)])/(a + b)] + Sqrt[2]*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])
/Sqrt[a + b]]*Sin[e + f*x]))/(Sqrt[a + b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f
*x)])/(a + b)]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 377, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^2(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{377}$$

$$\frac{\int -\frac{a}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)+b}\right)-\int \frac{a}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)+b}\right)-a\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)+b}\right)-a\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{-\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
- Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(61) = 122$.

Time = 4.00 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.83

method	result
default	$-\frac{\left(\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a\sin(fx+e)+\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,(1+\cos\right.}{f\sqrt{-a}\,(1+\cos(fx+e))\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}}$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(-a)^(1/2)*(ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*s
in(f*x+e)*a)*a*sin(f*x+e)+(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(1+cos(f*x+e)))*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(61) = 122$.

Time = 0.24 (sec) , antiderivative size = 499, normalized size of antiderivative = 7.23

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\cos(fx+e)^6 + 32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos(fx+e)\right.}{}\right.$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))*sin(f*x + e) - 8*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e))/(f*sin(f*x + e)), 1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^
5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^
2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e)
- 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x +
 e))]
```

Sympy [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cot(e+f\,x)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\cot\left(fx+e\right)^2 dx$$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**2,x)
```

3.387 $\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}$$

output
```
a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/3*(3*a+2
*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.54

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{\csc^3(e+fx)(a+b-a\sin^2(e+fx))^{3/2}}{a+b} - 3\csc(e+fx)\sqrt{a+b-a\sin^2(e+\cdots}\right.}{3f\sqrt{a+2b+a\cos(2e+2fx)}}$$

input
```
Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/3*(Sqrt[2]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((Csc[e + f*x]^3*(a
+ b - a*Sin[e + f*x]^2)^(3/2))/(a + b) - 3*Csc[e + f*x]*Sqrt[a + b - a*Sin
[e + f*x]^2]*(1 + (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[
e + f*x])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))))/(f*Sqrt[a
+ 2*b + a*Cos[2*e + 2*f*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 377, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{377}$$

$$\frac{\frac{1}{3}\int \frac{\cot^2(e+fx)(-2b\tan^2(e+fx)+b-3(a+b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{1}{3}\left(\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-\frac{\int -\frac{3a(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(}}{f}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{1}{3}\left(3a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+}}{f}$$

$$\Big\downarrow \textbf{291}$$

$$\frac{\frac{1}{3}\left(3a\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e}}{f}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{1}{3}\left(3\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{f}$$

input
```
Int[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/3*(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2]) + (3*Sqrt[a]*ArcTan[
(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((3*a + 2*b)*Cot[
e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/3)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(100) = 200$.

Time = 5.08 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.15

method	result
default	$-\frac{\left((3\cos(fx+e)-3)\sin(fx+e)a^2\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)+(3\cos(fx+\right.}{}$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a+b)/(-a)^(1/2)*((3*cos(f*x+e)-3)*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+(3*cos(f*x+e)-3)*sin(
f*x+e)*a*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*b+(4*cos(f*x+e)^2-3)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a+(3*cos(f*x+e)^2-2)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cot(f*x+e)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(100) = 200$.

Time = 0.57 (sec) , antiderivative size = 629, normalized size of antiderivative = 5.52

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*log(128*a^4*cos(f*x + e
)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
 e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((4*a + 3*
b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), -1
/12*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x
+ e) - 4*((4*a + 3*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f
)*sin(f*x + e))]
```

Sympy [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2{(e+fx)}}\cot^4{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

Maxima [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}^2+a}\cot{(fx+e)}^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \cot(e+f\,x)^4\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec(fx+e)^2b+a}\,\cot(fx+e)^4\,dx$$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**4,x)
```

3.388 $\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}$$

output
```
-a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/15*(15*
a^2+25*a*b+8*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/15*(-4
*b-5*a)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/5*cot(f*x+e)^5*(
a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$
$$-\frac{\cot(e+fx)\left(23a^2+40ab+15b^2-(11a^2+21ab+10b^2)\csc^2(e+fx)+3(a+b)^2\csc^4(e+fx)\right)\sqrt{a+}}{15(a+b)^2f}$$

input
```
Integrate[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*
x]^2]]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*
e + 2*f*x]])) - (Cot[e + f*x]*(23*a^2 + 40*a*b + 15*b^2 - (11*a^2 + 21*a*b
 + 10*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sqrt[a + b*Sec[e +
 f*x]^2])/(15*(a + b)^2*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4629, 2075, 377, 445, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^6(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^6(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{377}$$

$$\frac{\frac{1}{5}\int \frac{\cot^4(e+fx)(-4b\tan^2(e+fx)+b-5(a+b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{5}\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{1}{5}\left(-\frac{\int -\frac{\cot^2(e+fx)\left(15a^2+25ba+8b^2-2b(b-5(a+b))\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}\right)-\frac{1}{5}\cot^5(e}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{5}\left(\frac{\int \frac{\cot^2(e+fx)\left(15a^2+25ba+8b^2-2b(b-5(a+b))\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}\right)-\frac{1}{5}\cot^5(e+f}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{1}{5}\left(\frac{-\frac{\int \frac{15a(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e}}{3(a+b)}\right.}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\left(\frac{-15a(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{}}{3(a+b)}\right.}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{5}\left(\frac{-15a(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}\right) \over f$$

$\downarrow$ 216

$$\frac{1}{5}\left(\frac{-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-15\sqrt{a}(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}\right) \over f$$

input
```
Int[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2]) + (-1/3*((b - 5*(a +
 b))*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) + (-15*Sqrt[a]
*(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] - (
(15*a^2 + 25*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a
+ b))/(3*(a + b)))/5)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(149) = 298.

Time = 6.35 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.28

method	result
default	$-\frac{\left(\sin(fx+e)^3(-15\cos(fx+e)+15)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a^3+\sin(fx\right.}{}$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a+b)^2/(-a)^(1/2)*(sin(f*x+e)^3*(-15*cos(f*x+e)+15)*ln(4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3+sin(f*x+e)^3*
(-30*cos(f*x+e)+30)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
4*sin(f*x+e)*a)*a^2*b+sin(f*x+e)^3*(-15*cos(f*x+e)+15)*ln(4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+(23*cos(f*x+e)^4-
35*cos(f*x+e)^2+15)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^2+(40*cos(f*x+e)^4-59*cos(f*x+e)^2+25)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a*b+(15*cos(f*x+e)^4-20*cos(f*x+e)^2+8)*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)*(a+b*sec(f*x+e)^2)^(1/2)/
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)*csc(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(149) = 298.

Time = 2.05 (sec) , antiderivative size = 849, normalized size of antiderivative = 5.08

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/120*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos
(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b
+ 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
 a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (
a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8*((23*a^2 + 40*a*b +
 15*b^2)*cos(f*x + e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*
a^2 + 25*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*co
s(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/60*(15*((a^2 + 2*a*
b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a
*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^
2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^2 + 40*a*b + 1
5*b^2)*cos(f*x + e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^
2 + 25*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*c...
```

Sympy [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot^6(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

Maxima [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Giac [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)^6\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx=\int \sqrt{\sec\left(fx+e\right)^2b+a}\,\cot\left(fx+e\right)^6dx$$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**6,x)
```

3.389 $\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx=-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{(a+b\sec^2(e+fx))^{3/2}}{3f}-\frac{(a+2b)\,(a+b\sec^2(e+fx))^{5/2}}{5b^2f}+\frac{(a+b\sec^2(e+fx))^{7/2}}{7b^2f}$$

output
```
-a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+a*(a+b*sec(f*x+e)^2)^
(1/2)/f+1/3*(a+b*sec(f*x+e)^2)^(3/2)/f-1/5*(a+2*b)*(a+b*sec(f*x+e)^2)^(5/2
)/b^2/f+1/7*(a+b*sec(f*x+e)^2)^(7/2)/b^2/f
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx=\frac{\sqrt{a+b\sec^2(e+fx)}\Big(35b^2(a+b\sec^2(e+fx))-42b(a+b\sec^2(e+fx))^2+15(a+b\sec^2(e+ \ldots}{105b^2f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

output
```
(Sqrt[a + b*Sec[e + f*x]^2]*(35*b^2*(a + b*Sec[e + f*x]^2) - 42*b*(a + b*S
ec[e + f*x]^2)^2 + 15*(a + b*Sec[e + f*x]^2)^3 - (105*a*b^2*ArcTanh[Sqrt[(
a + b*Sec[e + f*x]^2)/a]])/Sqrt[(a + b*Sec[e + f*x]^2)/a] + 21*a*(5*b^2 -
(a + b*Sec[e + f*x]^2)^2)))/(105*b^2*f)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5 \left(a+b\sec(e+fx)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2 \left(b\sec^2(e+fx)+a\right)^{3/2} d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2 \left(b\sec^2(e+fx)+a\right)^{3/2} d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\left(b\sec^2(e+fx)+a\right)^{5/2}}{b}+\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}+\frac{(-a-2b)\left(b\sec^2(e+fx)+a\right)^{3/2}}{b}\right) d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)+\frac{2(a+b\sec^2(e+fx))^{7/2}}{7b^2}-\frac{2(a+2b)(a+b\sec^2(e+fx))^{5/2}}{5b^2}+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}+2a}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

output
```
(-2*a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*a*Sqrt[a + b*S
ec[e + f*x]^2] + (2*(a + b*Sec[e + f*x]^2)^(3/2))/3 - (2*(a + 2*b)*(a + b*
Sec[e + f*x]^2)^(5/2))/(5*b^2) + (2*(a + b*Sec[e + f*x]^2)^(7/2))/(7*b^2))
/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(115) = 230$.

Time = 36.38 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.24

method	result
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(105a^{\frac{3}{2}}\ln\left(4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\cos(fx+e)a\right)b^2\cos(fx+e)^3+6\sqrt{}\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/105/f/b^2*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(105*a^(3/2)*ln(4*a
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*a^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*b^2*cos(f*x+e)^3
+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*(cos(f*x+e)^3+cos(f*x+e
)^2)+3*(14*cos(f*x+e)^3+14*cos(f*x+e)^2-cos(f*x+e)-1)*a^2*b*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*a*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(21-35*cos(f*x+e)^3-35*cos(f*x+e)^2+21*cos(f*x+e)-6*sec(f*x+e)-6*s
ec(f*x+e)^2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*(-35*cos(f*x+
e)-35+42*sec(f*x+e)+42*sec(f*x+e)^2-15*sec(f*x+e)^3-15*sec(f*x+e)^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(115) = 230$.

Time = 6.88 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.90

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \left[\frac{105\,a^{\frac{3}{2}}b^2\cos(fx+e)^6\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)\right.}{}\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
[1/840*(105*a^(3/2)*b^2*cos(f*x + e)^6*log(128*a^4*cos(f*x + e)^8 + 256*a^
3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2
+ b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(
f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) - 8*(2*(3*a^3 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b
 - 84*a*b^2 + 35*b^3)*cos(f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*
x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)
^6), 1/420*(105*sqrt(-a)*a*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^6 - 4
*(2*(3*a^3 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84*a*b^2 + 3
5*b^3)*cos(f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)^2)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6)]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\tan^5(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**5, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2026 vs. 2(115) = 230.

Time = 2.12 (sec) , antiderivative size = 2026, normalized size of antiderivative = 15.01

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
2/105*(105*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)
^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(
f*x + e))/sqrt(-a) - 2*(105*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^13*a^2*sgn(cos(f*x + e)) - 1575
*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*t
an(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))^12*sqrt(a + b)*a^2*sgn(cos(f*x + e)) + 70*(129*a^3 + 21*a^
2*b - 96*a*b^2 - 32*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^11*sgn(cos(f*x + e)) - 70*(387*a^3
 - 417*a^2*b - 96*a*b^2 + 128*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^10*sqrt(a + b)*sgn(cos(f
*x + e)) + 7*(6525*a^4 - 11790*a^3*b - 2235*a^2*b^2 + 9088*a*b^3 - 1344*b^
4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^9*sgn(cos(f*x + e)) - 7*(5355*a^4 - 28290*a^3*b + 28995*
a^2*b^2 - 3008*a*b^3 - 896*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s...
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\int \tan(e+f\,x)^5\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}dx$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \frac{60\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\tan(fx+e)^4 b-48\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2 \tan}{}$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x)
```

output
```
(60*sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*tan(e + f*x)**4*b - 48*sqr
t(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*tan(e + f*x)**2*b + 32*sqrt(sec(e
 + f*x)**2*b + a)*sec(e + f*x)**2*b + 105*sqrt(sec(e + f*x)**2*b + a)*tan(
e + f*x)**4*a - 156*sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**2*a + 344*sq
rt(sec(e + f*x)**2*b + a)*a - 45*int((sqrt(sec(e + f*x)**2*b + a)*sec(e +
f*x)**2*tan(e + f*x)**5)/(sec(e + f*x)**2*b + a),x)*a*b*f - 108*int((sqrt(
sec(e + f*x)**2*b + a)*tan(e + f*x)**3)/(sec(e + f*x)**2*b + a),x)*a**2*f
+ 312*int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x))/(sec(e + f*x)**2*b +
a),x)*a**2*f)/(420*f)
```

3.390 $\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+b\sec^2(e+fx))^{3/2}}{3f} + \frac{(a+b\sec^2(e+fx))^{5/2}}{5bf}$$

output
```
a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-a*(a+b*sec(f*x+e)^2)^(
1/2)/f-1/3*(a+b*sec(f*x+e)^2)^(3/2)/f+1/5*(a+b*sec(f*x+e)^2)^(5/2)/b/f
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{15a^{3/2}b\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right) - 15ab\sqrt{a+b\sec^2(e+fx)} - 5b(a+b\sec^2(e+fx))^{3/2} + 3(a}{15bf}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

output
```
(15*a^(3/2)*b*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] - 15*a*b*Sqrt[a
+ b*Sec[e + f*x]^2] - 5*b*(a + b*Sec[e + f*x]^2)^(3/2) + 3*(a + b*Sec[e +
f*x]^2)^(5/2))/(15*b*f)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4627, 25, 354, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec^2(e+fx)-\frac{2\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b}}{2f}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)-\frac{2(a+b\sec^2(e+fx))^{5/2}}{5b}+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\left(a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2\sqrt{a+b\sec^2(e+fx)}\right)-\frac{2(a+b\sec^2(e+fx))^{5/2}}{5b}+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(\frac{2a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+2\sqrt{a+b\sec^2(e+fx)}\right)-\frac{2(a+b\sec^2(e+fx))^{5/2}}{5b}+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a\left(2\sqrt{a+b\sec^2(e+fx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right)-\frac{2(a+b\sec^2(e+fx))^{5/2}}{5b}+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

output
```
-1/2*((2*(a + b*Sec[e + f*x]^2)^(3/2))/3 - (2*(a + b*Sec[e + f*x]^2)^(5/2)
)/(5*b) + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sq
rt[a + b*Sec[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(88) = 176$.

Time = 32.08 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.24

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(15a^{\frac{3}{2}}\ln\left(4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\cos(fx+e)a\right)b\cos(fx+e)^3+\sqrt{\frac{b+a\,\mathrm{c}}{(1+\mathrm{co}}}\right.}{15fb}$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15/f/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(15*a^(3/2)*ln(4*a^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*b*cos(f*x+e)^3+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*(3*cos(f*x+e)^3+3*cos(f*x+e)^2)+
(-20*cos(f*x+e)^3-20*cos(f*x+e)^2+6*cos(f*x+e)+6)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*a*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*(-
5*cos(f*x+e)-5+3*sec(f*x+e)+3*sec(f*x+e)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(88) = 176$.

Time = 1.70 (sec) , antiderivative size = 443, normalized size of antiderivative = 4.26

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \left[\frac{15\,a^{\frac{3}{2}}b\cos\left(fx+e\right)^4\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4\right.}{}\right.$$

$$-\frac{15\sqrt{-a}ab\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right)\cos\left(fx+e\right)^4-4\left(\left(3\,a^2-20\,ab\right)\cos\right.}{60\,bf\cos\left(fx+e\right)^4}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
[1/120*(15*a^(3/2)*b*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b
*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
 + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)) + 8*((3*a^2 - 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x +
e)^2 + 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x +
e)^4), -1/60*(15*sqrt(-a)*a*b*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos
(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 - 4*
((3*a^2 - 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2 + 3*b^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4)]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^3\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1431 vs. $2(88) = 176$.

Time = 1.37 (sec) , antiderivative size = 1431, normalized size of antiderivative = 13.76

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
-2/15*(15*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f
*x + e))/sqrt(-a) - 2*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*a^2*sgn(cos(f*x + e)) - 15*(sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))^8*(7*a^2 - 8*a*b - 4*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + 20*(1
5*a^3 - 21*a^2*b - 12*a*b^2 + 8*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*sgn(cos(f*x + e)) -
20*(21*a^3 - 63*a^2*b + 60*a*b^2 - 4*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*sqrt(a + b)*sgn
(cos(f*x + e)) + 2*(105*a^4 - 630*a^3*b + 1065*a^2*b^2 + 360*a*b^3 - 16*b^
4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^5*sgn(cos(f*x + e)) + 10*(21*a^4 + 42*a^3*b - 303*a^2*b^
2 + 336*a*b^3 + 4*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan...
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\int \tan(e+f\,x)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}dx$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \frac{6\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\tan(fx+e)^2 b-4\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2 b+1}{}$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x)
```

output
```
(6*sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*tan(e + f*x)**2*b - 4*sqrt(
sec(e + f*x)**2*b + a)*sec(e + f*x)**2*b + 15*sqrt(sec(e + f*x)**2*b + a)*
tan(e + f*x)**2*a - 34*sqrt(sec(e + f*x)**2*b + a)*a - 9*int((sqrt(sec(e +
 f*x)**2*b + a)*sec(e + f*x)**2*tan(e + f*x)**3)/(sec(e + f*x)**2*b + a),x
)*a*b*f - 30*int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x))/(sec(e + f*x)*
*2*b + a),x)*a**2*f)/(30*f)
```

3.391 $\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx=-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{(a+b\sec^2(e+fx))^{3/2}}{3f}$$

output
```
-a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+a*(a+b*sec(f*x+e)^2)^
(1/2)/f+1/3*(a+b*sec(f*x+e)^2)^(3/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx=\frac{2b\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{a\cos^2(e+fx)}{b}\right)(a+b\sec^2(e+fx))^{3/2}}{3f\sqrt{1+\frac{a\cos^2(e+fx)}{b}}(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

output
```
(2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cos[e + f*x]^2)/b)]*(a + b*S
ec[e + f*x]^2)^(3/2))/(3*f*Sqrt[1 + (a*Cos[e + f*x]^2)/b]*(a + 2*b + a*Cos
[2*(e + f*x)]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4627, 243, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2\sqrt{a+b\sec^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{a\left(\frac{2a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+2\sqrt{a+b\sec^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{a\left(2\sqrt{a+b\sec^2(e+fx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

output
```
((2*(a + b*Sec[e + f*x]^2)^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec
[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sec[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\left(a+b\sec\left(fx+e\right)^{2}\right)^{\frac{3}{2}}}{3f}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec\left(fx+e\right)^{2}}}{\sec\left(fx+e\right)}\right)}{f}+\frac{a\sqrt{a+b\sec\left(fx+e\right)^{2}}}{f}$	81
default	$\frac{\left(a+b\sec\left(fx+e\right)^{2}\right)^{\frac{3}{2}}}{3f}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec\left(fx+e\right)^{2}}}{\sec\left(fx+e\right)}\right)}{f}+\frac{a\sqrt{a+b\sec\left(fx+e\right)^{2}}}{f}$	81

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a+b*sec(f*x+e)^2)^(3/2)/f-1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+
e)^2)^(1/2))/sec(f*x+e))+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(66) = 132.

Time = 0.41 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.78

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \left[\frac{3\,a^{\frac{3}{2}}\cos\left(fx+e\right)^{2}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\cos\left(fx+e\right)^{4}+\right.}{}\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
[1/24*(3*a^(3/2)*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos
(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 -
 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e
)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) + 8*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/(f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f
*x + e)^2 + 4*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \int \left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(66) = 132$.

Time = 0.84 (sec) , antiderivative size = 867, normalized size of antiderivative = 11.12

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
2/3*(3*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x
+ e))/sqrt(-a) + 2*(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*(2*a*b + b^2)*sgn(cos(f*x + e)) - 3
*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*t
an(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))^4*(6*a*b - b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + 2*(6*a^2*b
 - 15*a*b^2 - b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b
*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) + 6*(2*a^2*b + 7*a*b
^2 + b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*
f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 3*(6*a^3*b - a^
2*b^2 - 28*a*b^3 - 5*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) + (6*a^3*b - 21
*a^2*b^2 + 28*a*b^3 + 7*b^4)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x ...
```

Mupad [B] (verification not implemented)

Time = 18.61 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx=\frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{3\,f}-\frac{a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{f}+\frac{a\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{f}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
(a + b/cos(e + f*x)^2)^(3/2)/(3*f) - (a^(3/2)*atanh((a + b/cos(e + f*x)^2)
^(1/2)/a^(1/2)))/f + (a*(a + b/cos(e + f*x)^2)^(1/2))/f
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx=\frac{\sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2 b+4\sqrt{\sec(fx+e)^2 b+a}\,a+3\left(\int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)}{\sec(fx+e)^2 b+a}dx\right.}{3f}$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x)
```

output
```
(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*b + 4*sqrt(sec(e + f*x)**2*b
+ a)*a + 3*int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x))/(sec(e + f*x)**2
*b + a),x)*a**2*f)/(3*f)
```

3.392 $\int \cot(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cot(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f} + \frac{b\sqrt{a+b\sec^2(e+fx)}}{f}$$

output
```
a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-(a+b)^(3/2)*arctanh((a
+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f+b*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.47 (sec) , antiderivative size = 506, normalized size of antiderivative = 5.56

$$\int \cot(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{2b}{1+e^{2i(e+fx)}}+\frac{-2ia^{3/2}fx+2(a+b)^{3/2}\log\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*((2*b)/(1 + E^((2*I)*(e + f*x))) + ((-2*I)*a
^(3/2)*f*x + 2*(a + b)^(3/2)*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Log[a
+ 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2]] + a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b
*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2]] - 2*a*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) +
b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] - 2*b*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x
)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*
(1 + E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e
+ 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4627, 25, 354, 95, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 354

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{2f}$$

$\Big\downarrow$ 95

$$-\frac{-\int -\frac{\cos(e+fx)\left(a^2+b(2a+b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)-2b\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{\cos(e+fx)\left(a^2+b(2a+b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)-2b\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$\Big\downarrow$ 174

$$-\frac{a^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+(a+b)^2\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)-2b\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$\Big\downarrow$ 73

$$-\frac{\frac{2a^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2(a+b)^2\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}-2b\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$\Big\downarrow$ 221

$$-\frac{-2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)+2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)-2b\sqrt{a+b\sec^2(e+fx)}}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(-2*a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*(a + b)^(
3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Sec[
e + f*x]^2])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1300 vs. $2(77) = 154$.

Time = 6.80 (sec) , antiderivative size = 1301, normalized size of antiderivative = 14.30

method	result	size
default	Expression too large to display	1301

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(a+b)^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/(1+cos(f*x+e))/(b+a*cos(f*x+e)^
2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(2*a^(3/2)*(a+b)^(5/2)*ln(4
*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*a^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*cos(f*x+e)^3+l
n(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+
e)*a+b)/(1+cos(f*x+e)))*a^4*cos(f*x+e)^3+4*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b*
cos(f*x+e)^3+6*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^2*cos(f*x+e)^3+4*ln(2/(a+b)^
(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+
(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1
+cos(f*x+e)))*a*b^3*cos(f*x+e)^3+ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b^4*cos(f*x+e)^3
-ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(
-1+cos(f*x+e)))*a^4*cos(f*x+e)^3-4*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(77) = 154$.

Time = 0.40 (sec) , antiderivative size = 1075, normalized size of antiderivative = 11.81

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*
a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*(a + b)^(
3/2)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f
*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a +
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f
*x + e)^2 + 1)) + 8*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/8*
(4*(a + b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a
- b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)
^2 + a*b + b^2)) + a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x
+ e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(1
6*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 +
 b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
+ 8*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, -1/4*(sqrt(-a)*a*arc
tan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos
(f*x + e)^2 + a*b^2)) - (a + b)^(3/2)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x
 + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*...
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

Maxima [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(77) = 154$.

Time = 4.01 (sec) , antiderivative size = 718, normalized size of antiderivative = 7.89

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(4*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x
 + e))/sqrt(-a) - (a + b)^(3/2)*log(abs(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))*sgn(c
os(f*x + e)) + (a + b)^(3/2)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 +
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))*sgn(cos
(f*x + e)) + (a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a
+ b)*(a - b)))*sgn(cos(f*x + e))/sqrt(a + b) - 8*((sqrt(a + b)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2
*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*b^2*sgn(c
os(f*x + e)) + sqrt(a + b)*b^2*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2...
```

Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \int \cot(e+f\,x)\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2} \,dx$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot(e+fx) \left(a + b\sec^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sec(fx+e)^2 b + a}\, \cot(fx+e) \sec(fx+e)^2 dx\right) b + \left(\int \sqrt{\sec(fx+e)^2 b + a}\, \cot(fx+e)\, dx\right) a$$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)*sec(e + f*x)**2,x)*b + int(sq
rt(sec(e + f*x)**2*b + a)*cot(e + f*x),x)*a
```

3.393 $\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = -\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a-b)\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f} - \frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

output
```
-a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/2*(2*a-b)*(a+b)^(1/
2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f-1/2*(a+b)*cot(f*x+e)^2*
(a+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 622, normalized size of antiderivative = 5.46

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{(a+b)\left(1+e^{2i(e+fx)}\right)}{\left(-1+e^{2i(e+fx)}\right)^2}-\frac{-2ia^{3/2}\sqrt{a+b}fx+}{}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((a + b)*(1 + E^((2*I)*(e + f*x))))/(-1 + E
^((2*I)*(e + f*x)))^2 - ((-2*I)*a^(3/2)*Sqrt[a + b]*f*x + (2*a^2 + a*b - b
^2)*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Sqrt[a + b]*Log[a + 2*b + a*E^(
(2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*
(e + f*x)))^2]] + a^(3/2)*Sqrt[a + b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*
E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2]] - 2*a^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*
(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
 + f*x)))^2]] - a*b*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*
x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + b^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + S
qrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])
/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2
]))*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4627, 354, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^2}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{109}$$

$$\frac{\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}-\int -\frac{\cos(e+fx)\left(2a^2+(a-b)b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{\cos(e+fx)\left(2a^2+(a-b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{1}{2}\left(2a^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+(2a-b)(a+b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)\right)+\frac{(a+b)\sqrt{a}}{1-\text{se}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{2}\left(\frac{4a^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2(2a-b)(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}\right)+\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{2}\left(2(2a-b)\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)-4a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((-4*a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*(2*a - b)*Sqr
t[a + b]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/2 + ((a + b)*Sqr
t[a + b*Sec[e + f*x]^2])/(1 - Sec[e + f*x]^2))/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(96) = 192$.

Time = 1.58 (sec) , antiderivative size = 901, normalized size of antiderivative = 7.90

method	result	size
default	Expression too large to display	901

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/(a+b)^(3/2)*((3*cos(f*x+e)-3)*a^2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b+(-3*cos
(f*x+e)+3)*a^2*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos
(f*x+e)*a+b)/(-1+cos(f*x+e)))*b+(-2+2*cos(f*x+e))*ln(2/(a+b)^(1/2)*((a+b)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))
*a^3+(1-cos(f*x+e))*b^3*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))+(-2*cos(f*x+e)+2)*ln(-4*(
(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f
*x+e)))*a^3+(-1+cos(f*x+e))*b^3*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))+(4*cos(f*x+e)-4)*a^(3/2)*(a
+b)^(3/2)*ln(4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)-
2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(3/2)*a-2*c
os(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(3/2)*b)*(a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(96) = 192$.

Time = 0.42 (sec) , antiderivative size = 1300, normalized size of antiderivative = 11.40

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
 + (a*cos(f*x + e)^2 - a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*c
os(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
 e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 +
8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*(
(2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*c
os(f*x + e)^2 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)^2 - 2*((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a - b)*
arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a*co
s(f*x + e)^2 - a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x +
 e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16
*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 +
b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/
(f*cos(f*x + e)^2 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e)^2 + 2*(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*
a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^3 \sec(fx+e)^2 dx\right) b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^3 dx\right) a$$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**3*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**3,x)*a
```

3.394 $\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(8a^2+4ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}f} + \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

output
```
a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/8*(8*a^2+4*a*b-b^2)*
arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)/f+1/8*(4*a-b)*co
t(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/f-1/4*(a+b)*cot(f*x+e)^4*(a+b*sec(f*x+
e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.00 (sec) , antiderivative size = 684, normalized size of antiderivative = 4.30

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{\left(1+e^{2i(e+fx)}\right)\left(b\left(1+6e^{2i(e+fx)}+e^{4i(e+fx)}\right)+\cdots\right.}{\left(-1+e^{2i(e+fx)}\right)\cdots}\right.}{\cdots}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]^3*(-(((1 + E^((2*I)*(e + f*x)))*(b*(1 + 6*E^((2*I)*(e
+ f*x)) + E^((4*I)*(e + f*x))) + a*(6 - 4*E^((2*I)*(e + f*x)) + 6*E^((4*I)
*(e + f*x)))))/(-1 + E^((2*I)*(e + f*x)))^4) + ((-8*I)*a^(3/2)*Sqrt[a + b]
*f*x + (8*a^2 + 4*a*b - b^2)*Log[1 - E^((2*I)*(e + f*x))] + 4*a^(3/2)*Sqrt
[a + b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e
 + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + 4*a^(3/2)*Sqrt[a + b]*Log[a +
 a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 8*a^2*Log[a + b + a*E^((
2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e
 + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*a*b*Log[a + b + a*E^((2*I)*
(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*
x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + b^2*Log[a + b + a*E^((2*I)*(e + f*
x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a
*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x))
+ a*(1 + E^((2*I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2
]*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4627, 25, 354, 109, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{109}$$

$$-\frac{\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)^2}-\frac{1}{2}\int -\frac{\cos(e+fx)\left(4a^2+(3a-b)b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{4}\int \frac{\cos(e+fx)\left(4a^2+(3a-b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)^2}}{2f}$$

$$\Big\downarrow \text{168}$$

$$-\frac{\frac{1}{4}\left(\frac{(4a-b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}-\frac{\int -\frac{(a+b)\cos(e+fx)\left(8a^2+(4a-b)b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a+b}\right)+\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}}{2f}$$

$\Big\downarrow$ 27

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{\cos(e+fx)(8a^2+(4a-b)b\sec^2(e+fx))}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\frac{(4a-b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}\right)+\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}}{2f}$$

$\Big\downarrow$ 174

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left((8a^2+4ab-b^2)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+8a^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)\right)+\frac{(4}{}\right.}{2f}$$

$\Big\downarrow$ 73

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{2(8a^2+4ab-b^2)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{16a^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}\right)+\frac{(4a-b)\sqrt{a+b\sec^2(e+f}}{1-\sec^2(e+fx)}\right.}{2f}$$

$\Big\downarrow$ 221

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{2(8a^2+4ab-b^2)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-16a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{(4a-b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}\right)+\frac{(}{}}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(((a + b)*Sqrt[a + b*Sec[e + f*x]^2])/(2*(1 - Sec[e + f*x]^2)^2) + ((
-16*a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + (2*(8*a^2 + 4*a*
b - b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/2 +
 ((4*a - b)*Sqrt[a + b*Sec[e + f*x]^2])/(1 - Sec[e + f*x]^2))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1450 vs. $2(137) = 274$.

Time = 1.61 (sec) , antiderivative size = 1451, normalized size of antiderivative = 9.13

method	result	size
default	Expression too large to display	1451

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/(a+b)^(5/2)*((-16*cos(f*x+e)+16)*sin(f*x+e)^2*(a+b)^(5/2)*a^(3/2)*l
n(4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*a^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)+cos(f*x+e)*
(-12*cos(f*x+e)^2+8)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(5/
2)*a+cos(f*x+e)*(-2*cos(f*x+e)^2-2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(5/2)*b+(8*cos(f*x+e)-8)*sin(f*x+e)^2*ln(-4*((a+b)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^4+(20*c
os(f*x+e)-20)*sin(f*x+e)^2*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3*b+(15*cos(f*x+e)-15)*sin(f*x
+e)^2*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a
+b)/(-1+cos(f*x+e)))*a^2*b^2+(-2+2*cos(f*x+e))*sin(f*x+e)^2*ln(-4*((a+b)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))
*a*b^3+(1-cos(f*x+e))*sin(f*x+e)^2*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^4+(-8*cos(f*x+e)+8)*si
n(f*x+e)^2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(137) = 274$.

Time = 1.38 (sec) , antiderivative size = 1801, normalized size of antiderivative = 11.33

$$\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(4*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2
+ a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*
x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
 + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - ((8*a^2 +
4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a
^2 + 4*a*b - b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4
+ 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b
*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^2 + 7*a*b + b^2)*cos(f*
x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2
 + (a + b)*f), 1/16*(((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*
a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*(
(2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + a*b)*co
s(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*log(128*a
^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4
+ 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(
f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^5(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \mathrm{cot}{\left(e+f\,x\right)}^5\,{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{3/2}\,dx$$

input
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec(fx+e)^2b+a}\,\cot(fx+e)^5\sec(fx+e)^2\,dx\right)b+\left(\int \sqrt{\sec(fx+e)^2b+a}\,\cot(fx+e)^5\,dx\right)a$$

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**5*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**5,x)*a
```

3.395 $\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 290

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} - \frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}$$

output

```
-a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/128*(3*
a^4+20*a^3*b+90*a^2*b^2-60*a*b^3-5*b^4)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*
tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/128*(3*a^3+17*a^2*b-55*a*b^2-5*b^3)*tan(f
*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b^2/f+1/192*(3*a^2-50*a*b-5*b^2)*tan(f*x+
e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f+1/48*(9*a+b)*tan(f*x+e)^5*(a+b+b*tan(f
*x+e)^2)^(1/2)/f+1/8*b*tan(f*x+e)^7*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 4.37 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx =$$

$$-\frac{\left(128a^{3/2}b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\,\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos^3(e+f}{32\sqrt{2}b^2f(a+2b+a\cos(2e+2fx))^{3/2}}$$

$$-\frac{(90a^3+498a^2b-1594ab^2-626b^3+(135a^3+759a^2b-2303ab^2+513b^3)\cos(2(e+fx))+2(27a^3+15}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

output
```
-1/32*((128*a^(3/2)*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
 + f*x]^2]] - ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[
(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f
*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/(Sqrt[2]*b^2*f*(a + 2*b + a*Cos[2*e +
2*f*x])^(3/2)) - ((90*a^3 + 498*a^2*b - 1594*a*b^2 - 626*b^3 + (135*a^3 +
759*a^2*b - 2303*a*b^2 + 513*b^3)*Cos[2*(e + f*x)] + 2*(27*a^3 + 159*a^2*b
 - 523*a*b^2 - 191*b^3)*Cos[4*(e + f*x)] + 9*a^3*Cos[6*(e + f*x)] + 57*a^2
*b*Cos[6*(e + f*x)] - 337*a*b^2*Cos[6*(e + f*x)] + 15*b^3*Cos[6*(e + f*x)]
)*Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(12288*b^2*f)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4629, 2075, 379, 444, 27, 444, 27, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^6 \left(a+b\sec(e+fx)^2\right)^{3/2} dx$$

$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\tan^6(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 379$$

$$\frac{\frac{1}{8}\int \frac{\tan^6(e+fx)\left(b(9a+b)\tan^2(e+fx)+(a+b)(8a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{8}b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \; 444$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}(9a+b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{\int \frac{b\tan^4(e+fx)\left(5(a+b)(9a+b)-\left(3a^2-50ba-5b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{6b}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}(9a+b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{6}\int \frac{\tan^4(e+fx)\left(5(a+b)(9a+b)-\left(3a^2-50ba-5b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e\right.}{f}$$

$$\Big\downarrow \; 444$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{\int -\frac{3\tan^2(e+fx)\left(\left(3a^3+17ba^2-55b^2a-5b^3\right)\tan^2(e+fx)+(a+b)\left(3a^2-50ba-5b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}+\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(\right.}{4b}\right.\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\int \frac{\tan^2(e+fx)\left(\left(3a^3+17ba^2-55b^2a-5b^3\right)\tan^2(e+fx)+(a+b)\left(3a^2-50ba-5b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan\cdots}{4b}\cdots\right)\cdots\right)}{f}$$

$\downarrow$ 444

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int\left(3a^4+20ba^3+90b^2a^2\cdots\right.}{\cdots}\cdots\right)}{4b}\cdots\right)\cdots\right)$$

$\downarrow$ 398

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^4+20a^3b+90a^2b^2-\cdots\right.}{\cdots}\cdots\right)}{\cdots}\cdots\right)\cdots\right)$$

$\downarrow$ 224

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^4+20a^3b+90a^2b^2-\cdots\right.}{\cdots}\cdots\right)}{\cdots}\cdots\right)\cdots\right)$$

$\downarrow$ 219

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^4+20a^3b+90a^2b^2-\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.\right.$$

$\downarrow$ 291

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^4+20a^3b+90a^2b^2-\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.\right.$$

$\downarrow$ 216

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^4+20a^3b+90a^2b^2-\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.\right.$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

output
```
((b*Tan[e + f*x]^7*Sqrt[a + b + b*Tan[e + f*x]^2])/8 + (((9*a + b)*Tan[e +
 f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/6 + (((3*a^2 - 50*a*b - 5*b^2)*Tan
[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*b) - (3*(-1/2*(-128*a^(3/2)
*b^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((3*a
^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[b])/b + ((3*a^3 + 17*a^2*b - 55*
a*b^2 - 5*b^3)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b)))/(4*b))
/6)/8)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1643 vs. $2(260) = 520$.

Time = 42.32 (sec) , antiderivative size = 1644, normalized size of antiderivative = 5.67

method	result	size
default	Expression too large to display	1644

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/768/f/(-a)^(1/2)/b^(13/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(-9
*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x
+e)+1))*cos(f*x+e)^3*(-a)^(1/2)*a^4*b^4-60*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)^3*(-a)^(1/2)*
a^3*b^5-270*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-
b)/(sin(f*x+e)+1))*cos(f*x+e)^3*(-a)^(1/2)*a^2*b^6+180*ln(4*(b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)^3
*(-a)^(1/2)*a*b^7+15*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*
x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)^3*(-a)^(1/2)*b^8-9*ln(-4*(b^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+
e)^3*(-a)^(1/2)*a^4*b^4-60*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)^3*(-a)^(1/2)*a^3*b^5-270*...
```

Fricas [A] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 1973, normalized size of antiderivative = 6.80

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

output
```
[1/1536*(192*sqrt(-a)*a*b^3*cos(f*x + e)^7*log(128*a^4*cos(f*x + e)^8 - 25
6*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
 + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b
 + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3
- a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 -
(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2
- 60*a*b^3 - 5*b^4)*sqrt(b)*cos(f*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*
x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*
cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e) + 8*b^2)/cos(f*x + e)^4) - 4*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*
b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 -
48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7), 1/768*(96*sqrt(-a)*a
*b^3*cos(f*x + e)^7*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x
 + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*
b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*c
os(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e...
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^6\left(e+fx\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**6,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**6, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx=\int \tan(e+f\,x)^6\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int (a + b\sec^2(e + fx))^{3/2} \tan^6(e + fx)\, dx = \left(\int \sqrt{\sec (fx + e)^2 b + a}\, \sec (fx + e)^2 \tan (fx + e)^6\, dx\right) b + \left(\int \sqrt{\sec (fx + e)^2 b + a}\, \tan (fx + e)^6\, dx\right) a$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*tan(e + f*x)**6,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**6,x)*a
```

3.396 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$
$$-\frac{(a-b)\left(a^2+10ab+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}$$
$$+\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf}$$
$$+\frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f}$$
$$+\frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}$$

output
```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/16*(a-b)
*(a^2+10*a*b+b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b
^(3/2)/f+1/16*(a^2-8*a*b-b^2)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f+1/
24*(7*a+b)*tan(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f+1/6*b*tan(f*x+e)^5*(a
+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \frac{\left(16a^{3/2}b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(a-b)(a^2+10ab+b^2)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos^3(e+fx)\,(}{4\sqrt{2}bf(a+2b+a\cos(2e+2fx))^{3/2}} + \frac{(9a^2-58ab+17b^2+4(3a^2-24ab-11b^2)\cos(2(e+fx))+(3a^2-38ab+3b^2)\cos(4(e+fx)))\sec^4(e+}{384bf}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

output
```
((16*a^(3/2)*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2
]] - ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a +
 b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3
/2))/(4*Sqrt[2]*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)) + ((9*a^2 - 58*a
*b + 17*b^2 + 4*(3*a^2 - 24*a*b - 11*b^2)*Cos[2*(e + f*x)] + (3*a^2 - 38*a
*b + 3*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*Ta
n[e + f*x])/(384*b*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4629, 2075, 379, 444, 27, 444, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^4 \left(a+b\sec(e+fx)^2\right)^{3/2} dx$$

$\big\downarrow$ 4629

$$\frac{\int \frac{\tan^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 2075

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 379

$$\frac{\frac{1}{6}\int \frac{\tan^4(e+fx)\left(b(7a+b)\tan^2(e+fx)+(a+b)(6a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{6}b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\big\downarrow$ 444

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{\int \frac{3b\tan^2(e+fx)\left((a+b)(7a+b)-\left(a^2-8ba-b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}\right)+$$

$\big\downarrow$ 27

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\int \frac{\tan^2(e+fx)\left((a+b)(7a+b)-\left(a^2-8ba-b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+f\right.}{f}$$

$\big\downarrow$ 444

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(-\frac{\int -\frac{(a-b)\left(a^2+10ba+b^2\right)\tan^2(e+fx)+(a+b)\left(a^2-8ba-b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}\right.\right.}{f}$$

$\big\downarrow$ 25

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{\int\frac{(a-b)\left(a^2+10ba+b^2\right)\tan^2(e+fx)+(a+b)\left(a^2-8ba-b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}-\right.\right.}{f}$$

$\downarrow$ 398

$$\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{(a-b)(a^2+10ab+b^2)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-16a^2b\int\frac{}{\left(\tan\right.}}{2b}\right.\right.$$

$\downarrow$ 224

$$\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{(a-b)(a^2+10ab+b^2)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-16a}{2b}\right.\right.$$

$\downarrow$ 219

$$\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{\frac{(a-b)\left(a^2+10ab+b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-16a^2b\int\frac{}{\left(\tan^2(e\right.}}{2b}\right.\right.$$

$\downarrow$ 291

$$\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{\frac{(a-b)\left(a^2+10ab+b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-16a^2b\int\frac{\frac{a\tan}{b\tan^2(}}{}}{2b}\right.\right.$$

$\downarrow$ 216

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{\frac{(a-b)\left(a^2+10ab+b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-16a^{3/2}b\arctan}{2b}\right.\right.}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

output
```
((b*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/6 + (((7*a + b)*Tan[e +
 f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/4 - (3*((-16*a^(3/2)*b*ArcTan[(Sqr
t[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((a - b)*(a^2 + 10*a*
b + b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/S
qrt[b])/(2*b) - ((a^2 - 8*a*b - b^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f
*x]^2])/(2*b)))/4)/6)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(188) = 376$.

Time = 16.06 (sec) , antiderivative size = 1306, normalized size of antiderivative = 6.10

method	result	size
default	Expression too large to display	1306

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/96/f/b^(9/2)/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(-3*ln
(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*
x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e
)-1))*cos(f*x+e)^3*(-a)^(1/2)*a^3*b^3-27*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*cos(f*x+e)^3*(-a)^(1/2)*a
^2*b^4+27*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b
)/(sin(f*x+e)-1))*cos(f*x+e)^3*(-a)^(1/2)*a*b^5+3*ln(4*(-b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*cos(f*x+e)^3*(-a
)^(1/2)*b^6-3*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)+a+b)/(sin(f*x+e)+1))*cos(f*x+e)^3*(-a)^(1/2)*a^3*b^3-27*ln(-4*(-b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*cos(f*x
+e)^3*(-a)^(1/2)*a^2*b^4+27*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*cos(f*x+e)^3*(-a)^(1/2)*a*b^5+3*ln...
```

Fricas [A] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 1777, normalized size of antiderivative = 8.30

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
[1/192*(24*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*
(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a
^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*s
qrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b
^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) + 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*
b^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/(b^2*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*a*b^2*cos(f*x + e)^5*
log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4
- 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*
a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e
))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*
(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x +...
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^4\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\tan(fx+e)^4\,dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^4\,dx\right)a$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*tan(e + f*x)**4,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**4,x)*a
```

3.397 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=-\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a^2-6ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}+\frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

output
```
-a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/8*(3*a^
2-6*a*b-b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(1/2
)/f+1/8*(5*a+b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f+1/4*b*tan(f*x+e)^3
*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.23

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(-1+e^{2i(e+fx)}\right)\left(5a\left(1+e^{2i(e+fx)}\right)^2-b\left(1-6e^{2i(e+\cdots\right.\right.}{\left(1+e^{2i(e+fx)}\right)^4}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]^3*(((-I)*(-1 + E^((2*I)*(e + f*x)))*(5*a*(1 + E^((2*I)
*(e + f*x)))^2 - b*(1 - 6*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(1
+ E^((2*I)*(e + f*x)))^4 + (-8*a^(3/2)*Sqrt[b]*f*x + (4*I)*a^(3/2)*Sqrt[b]
*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)
) + a*(1 + E^((2*I)*(e + f*x)))^2]] - (4*I)*a^(3/2)*Sqrt[b]*Log[a + a*E^((
2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e +
 f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 3*a^2*Log[(4*(Sqrt[b]*(-1 + E^(
(2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x))))] + 6*a*b*L
og[(4*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x))
 + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)
*(e + f*x))))] + b^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*
b - b^2)*(1 + E^((2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)
) + a*(1 + E^((2*I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt
[2]*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4629, 2075, 379, 444, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{379}$$

$$\frac{\frac{1}{4}\int \frac{\tan^2(e+fx)\left(b(5a+b)\tan^2(e+fx)+(a+b)(4a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{4}b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{\int \frac{b\left((a+b)(5a+b)-\left(3a^2-6ba-b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}\right)+\frac{1}{4}b\tan^3(e}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{2}\int \frac{(a+b)(5a+b)-\left(3a^2-6ba-b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{4}b\text{ ta}}{f}$$

$\Big\downarrow$ 398

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2-6ab-b^2\right)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-8a^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\right.}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2-6ab-b^2\right)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-8a^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+f\right.\right.}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{(3a^2-6ab-b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8a^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}(5a+b)\right.}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{(3a^2-6ab-b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8a^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}(5a+b)\tan(e+\right.}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{(3a^2-6ab-b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}(5a+b)\tan(e+fx)\sqrt{a}\right.}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

output
```
((b*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/4 + ((-8*a^(3/2)*ArcTan
[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((3*a^2 - 6*a*b
- b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqr
t[b])/2 + ((5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/4)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(144) = 288$.

Time = 11.24 (sec) , antiderivative size = 971, normalized size of antiderivative = 5.85

method	result	size
default	Expression too large to display	971

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16/f/b^(5/2)/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(-3*(
-a)^(1/2)*cos(f*x+e)^3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(
f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2+6*(-a)^(1/2)*cos(f*x+e)^3*ln(4*(b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b
^3+(-a)^(1/2)*cos(f*x+e)^3*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^4-3*(-a)^(1/2)*cos(f*x+e)^3*ln(-4*(b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^
2*b^2+6*(-a)^(1/2)*cos(f*x+e)^3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3+(-a)^(1/2)*cos(f*x+e)^3*ln(-
4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-
1))*b^4+16*b^(5/2)*cos(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+(2*cos(f*x+e)^3+2*cos(f*x+e)^2-4*cos(f*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(144) = 288$.

Time = 1.06 (sec) , antiderivative size = 1627, normalized size of antiderivative = 9.80

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
[1/32*(4*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)) - (3*a^2 - 6*a*b - b^2)*sqrt(b)*cos(f*x +
 e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)
^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5
*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*a*b*cos(f*x + e
)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*
a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 +
8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14
*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x
 + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))
+ (3*a^2 - 6*a*b - b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b
*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a...
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^2\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\sec(fx+e)^2\tan(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^2\,dx\right)a$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*sec(e + f*x)**2*tan(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**2,x)*a
```

3.398 $\int (a+b\sec^2(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output

```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*b^(1/2
)*(3*a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*b*t
an(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.47

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\,\mathrm{lo}}{}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^
((2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)
*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f
*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2]] - 3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqr
t[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)
*(1 + E^((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*f + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]*f)/(b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(
a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$\big\downarrow$ 3042

$$\int \left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$\big\downarrow$ 4616

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 318

$$\frac{\frac{1}{2}\int \frac{b(3a+b)\tan^2(e+fx)+(a+b)(2a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\big\downarrow$ 398

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}b\tan(e-}{f}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}b\,\mathrm{ta}}{f}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+f}{f}$$

$\big\downarrow$ 291

$$\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+}}{f}$$

$\big\downarrow$ 216

$$\frac{\frac{1}{2}\left(2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
+ Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(100) = 200$.

Time = 6.28 (sec) , antiderivative size = 673, normalized size of antiderivative = 5.70

method	result
default	$\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(b^{\frac{5}{2}}\sqrt{-a}\,\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)\cos(fx+e)^3+3b^{\frac{3}{2}}\right.$

input
```
int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(a*cos(f*x+e)^3+a*cos(f*x+e)^2+cos(f*x+e)*b+b)*(b^(5/2)*(-a)
^(1/2)*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(s
in(f*x+e)+1))*cos(f*x+e)^3+3*b^(3/2)*(-a)^(1/2)*ln(4*(b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)^3*a+b^(5
/2)*(-a)^(1/2)*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*
a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)^3+3*b^(3/2)*(-a)^(1/2)*ln(-4*(b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e
)^3*a+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)
*a)*cos(f*x+e)^3*a^2*b+(2*cos(f*x+e)+2)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(100) = 200$.

Time = 0.43 (sec) , antiderivative size = 1457, normalized size of antiderivative = 12.35

$$\int \left(a + b\sec^2(e + fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*
b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4
 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*
b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqr
t(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^
8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e
)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)...
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} \, dx=\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}dx\right)a + \left(\int \sqrt{\sec\left(fx+e\right)^2 b+a}\,\sec\left(fx+e\right)^2 dx\right)b$$

input
```
int((a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a),x)*a + int(sqrt(sec(e + f*x)**2*b + a)*sec
(e + f*x)**2,x)*b
```

3.399 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

output

```
-a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+b^(3/2)*a
rctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-(a+b)*cot(f*x+e)*(
a+b+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.69

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{2i(a+b)}{-1+e^{2i(e+fx)}}+\frac{ia^{3/2}\log\left(a+2b+ae^{2i}\right.}{}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-2*I)*(a + b))/(-1 + E^((2*I)*(e + f*x)))
 + (I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2
*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - I*a^(3/2)*Log[a + a*E^(
(2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e
+ f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*(a^(3/2)*f*x + b^(3/2)*Log[(
(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x
]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 376, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$\big\downarrow$ 3042

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^2}dx$$

$\big\downarrow$ 4629

$$\frac{\int \frac{\cot^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 2075

$$\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 376

$$\frac{\int -\frac{a^2-b^2-b^2\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\big\downarrow$ 25

$$\frac{-\int \frac{a^2-b^2-b^2\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\left((a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)}{f}$$

$\big\downarrow$ 398

$$\frac{a^2\left(-\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+b^2\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-(a+b)\cot(e+fx}{f}$$

$\big\downarrow$ 224

$$\frac{a^2\left(-\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+b^2\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-(a+b)\cot(e}{f}$$

$\big\downarrow$ 219

$$\frac{a^2\left(-\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a+b)\cot(e+fx)\sqrt{}}{f}$$

$\Big\downarrow$ 291

$$\frac{a^2\left(-\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\Big\downarrow$ 216

$$\frac{a^{3/2}\left(-\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
+ b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] -
 (a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(97) = 194$.

Time = 3.14 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.30

method	result
default	$\left(\ln\left(-\frac{4\left(\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-\sin(fx+e)a+a+b\right)}{\sin(fx+e)-1}\right)b^{\frac{3}{2}}\sqrt{-a}\,\sin(fx+e)+\ln\left(\frac{4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}}{\cdots}\right.\right.$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a+a+b)/(sin(f*x+e)-1))*b^(3/2)*(-a)^(1/2)*sin(f*x+e)+ln(4*(b^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(3/2)*(-a
)^(1/2)*sin(f*x+e)-2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)*a^2*sin(f*x+e)+(-2*cos(f*x+e)-2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+(-2*cos(f*x+e)-2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e
)^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(1+cos(f*x+e))*cos(f*x+e)
^2*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(97) = 194$.

Time = 0.46 (sec) , antiderivative size = 1446, normalized size of antiderivative = 13.03

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b +
 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(
f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -
 b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))*sin(f*x + e) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f
*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
+ 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e)), 1/8*(4*sqrt(-b)*b*arct
an(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*
sin(f*x + e) + sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 +...
```

Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot^2\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^2 dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^2 dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \int \cot(e+f\,x)^2 \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2} dx$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^2\sec(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^2\,dx\right)a$$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**2*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**2,x)*a
```

3.400 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}-\frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}$$

output

```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/3*(3*a-b
)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*(a+b)*cot(f*x+e)^3*(a+b+b*ta
n(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{2(a+b)\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{a\sin^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f(a+2b+a\cos(2(e+fx)))\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-2*(a + b)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (a*Sin[e +
f*x]^2)/(a + b)]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*(e
+ f*x)])*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 376, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^4(e+fx)(b\tan^2(e+fx)+a+b)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{376}$$

$$\frac{\frac{1}{3}\int -\frac{\cot^2(e+fx)((2a-b)b\tan^2(e+fx)+(3a-b)(a+b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{3}(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{1}{3}\int \frac{\cot^2(e+fx)((2a-b)b\tan^2(e+fx)+(3a-b)(a+b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{3}(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{1}{3}\left(\frac{\int \frac{3a^2(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}+(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{1}{3}(a+b)\cot^3(e+}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\left(3a^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{1}{3}(a+b}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{3}\left(3a^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{1}{3}(a+b)\cot^3(e}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{3}\left(3a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{1}{3}(a+b)\cot^3(e+fx)\sqrt{}}{f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/3*((a + b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2]) + (3*a^(3/2)
*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + (3*a - b)
*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/3)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(98) = 196$.

Time = 3.66 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.23

method	result
default	$-\frac{\Big((3\cos(fx+e)-3)\sin(fx+e)a^2\ln\Big(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\Big)+\Big(4\cos(fx+}{3f\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\Big(b+a\cos(fx-}$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(-a)^(1/2)*((3*cos(f*x+e)-3)*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+(4*cos(f*x+e)^2-3)*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(-a)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)*cot(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(98) = 196$.

Time = 0.65 (sec) , antiderivative size = 597, normalized size of antiderivative = 5.33

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a*cos(f*x + e)^2 - a)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*
(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a
^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*(4*a*cos(f*x + e)^3
- (3*a - b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f
*cos(f*x + e)^2 - f)*sin(f*x + e)), -1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(a
)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e
)^2)*sin(f*x + e)))*sin(f*x + e) - 4*(4*a*cos(f*x + e)^3 - (3*a - b)*cos(f
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^2 -
 f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)^4\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^4\sec(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^4\,dx\right)a$$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**4*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**4,x)*a
```

3.401 $\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \cot^6(e+fx) \left(a+b\sec^2(e+fx)\right)^{3/2} dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} + \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f} - \frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}$$

output
```
-a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/15*(15*
a^2+10*a*b-2*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f+1/15*(5*a-
b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/5*(a+b)*cot(f*x+e)^5*(a+b+b
*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.97 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.84

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{2\cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-\frac{3}{4}(a+2b+a\cos(2(e+fx)))^2\csc^2(e+fx)+\frac{5(a+\cdots}{\cdots}\right.}{15(a+b)f(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(2*Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-3*(a + 2*b + a*Cos[2*(e
+ f*x)])^2*Csc[e + f*x]^2)/4 + (5*(a + b)^2*Hypergeometric2F1[-3/2, -3/2,
-1/2, (a*Sin[e + f*x]^2)/(a + b)])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)
]))/(15*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4629, 2075, 376, 25, 445, 27, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{ 4629}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 2075

$$\frac{\int \frac{\cot^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 376

$$\frac{\frac{1}{5}\int -\frac{\cot^4(e+fx)\left((4a-b)b\tan^2(e+fx)+(5a-b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{5}(a+b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\big\downarrow$ 25

$$\frac{-\frac{1}{5}\int \frac{\cot^4(e+fx)\left((4a-b)b\tan^2(e+fx)+(5a-b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{5}(a+b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$\big\downarrow$ 445

$$\frac{\frac{1}{5}\left(\frac{\int \frac{(a+b)\cot^2(e+fx)\left(15a^2+10ba-2b^2+2(5a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}+\frac{1}{3}(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-}{f}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\cot^2(e+fx)\left(15a^2+10ba-2b^2+2(5a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{3}(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+}\right.}{f}$$

$\big\downarrow$ 445

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-\frac{\int \frac{15a^2(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)+\frac{1}{3}(5a-b)\cot^3(e\right.}{f}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-15a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)+\frac{1}{3}(5a}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-15a^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)+\frac{1}{3}(5a-b)\cot^3}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-15a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)+\frac{1}{3}(5a-b)\cot^3(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/5*((a + b)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2]) + (((5*a - b
)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/3 + (-15*a^(3/2)*ArcTan[(
Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] - ((15*a^2 + 10*a*b
- 2*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/3)/5)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(147) = 294$.

Time = 4.70 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.73

method	result
default	$-\frac{\left(\sin(fx+e)^3(-15\cos(fx+e)+15)a^3 \ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)+\sin(fx\right.}{}$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a+b)/(-a)^(1/2)*(sin(f*x+e)^3*(-15*cos(f*x+e)+15)*a^3*ln(4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)+sin(f*x+e)^3*(-
15*cos(f*x+e)+15)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*a^2*b+(23*cos(f*x+e)^4-35*cos(f*x+e)^2+15)*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+(20*cos(f*x+e)^4-24*cos(f*x+e)^2
+10)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+(5*cos(f*x
+e)^2-2)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)*(a+b*
sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(b+a*cos(f
*x+e)^2)*cot(f*x+e)^3*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(147) = 294$.

Time = 2.66 (sec) , antiderivative size = 767, normalized size of antiderivative = 4.65

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/120*(15*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^
2 + a*b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
 e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b
+ 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos
(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
- b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))*sin(f*x + e) - 8*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2
 + 24*a*b - 5*b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4
- 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f)*sin(f*x + e)), 1/60*(15*((a^2 +
a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*ar
ctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a
*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)
*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a
^2 + 24*a*b - 5*b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^
4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Giac [F]

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)^6\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sec(fx+e)^2b+a}\,\cot(fx+e)^6\sec(fx+e)^2\,dx\right)b+\left(\int \sqrt{\sec(fx+e)^2b+a}\,\cot(fx+e)^6\,dx\right)a$$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**6*sec(e + f*x)**2,x)*b + int
(sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**6,x)*a
```

3.402 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3b^2f}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f-(a+2*b)*(a+b*sec(f*x+
e)^2)^(1/2)/b^2/f+1/3*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{-2a(a+3b)-b(a+6b)\sec^2(e+fx)+b^2\sec^4(e+fx)-3b^2\operatorname{arctanh}\left(\sqrt{1+\frac{b\sec^2(e+fx)}{a}}\right)\sqrt{1+\frac{b\sec^2(e+}{a}}}{3b^2f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-2*a*(a + 3*b) - b*(a + 6*b)*Sec[e + f*x]^2 + b^2*Sec[e + f*x]^4 - 3*b^2*
ArcTanh[Sqrt[1 + (b*Sec[e + f*x]^2)/a]]*Sqrt[1 + (b*Sec[e + f*x]^2)/a])/(3
*b^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{-a-2b}{b\sqrt{b\sec^2(e+fx)+a}}+\frac{\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\right)d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2}-\frac{2(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2}}{2f}$$

input
```
Int[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] - (2*(a + 2*b)*S
qrt[a + b*Sec[e + f*x]^2])/b^2 + (2*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b^2))
/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(77) = 154$.

Time = 11.88 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.17

method	result
default	$\frac{\left(-\frac{\sec(fx+e)^2}{3}-2\right)b\,a^{\frac{3}{2}}+\left(\frac{\sec(fx+e)^4}{3}-2\sec(fx+e)^2\right)b^2\sqrt{a}-\frac{2a^{\frac{5}{2}}}{3}-\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{a}\,\cos(fx+e)+4\sqrt{a}\,\sqrt{b}\right.}{3}}{f\,b^2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((-1/3*sec(f*x+e)^2-2)*b*a^(3/2)+(1/3*sec(f*x+e)^4-2*sec(f*x+e)^2)*b^2
*a^(1/2)-2/3*a^(5/2)-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*b^2*(3+3*sec(f*
x+e)))/b^2/a^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(77) = 154$.

Time = 0.47 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.61

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{3\sqrt{a}b^2\cos(fx+e)^2\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab^3\right.}{}\right.$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(a)*b^2*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b
*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
 + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)) - 8*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*b^2*arct
an(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(
f*x + e)^2 + a*b^2))*cos(f*x + e)^2 - 4*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 -
a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2)
]
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan^5{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan{(fx+e)}^5}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(77) = 154$.

Time = 0.69 (sec) , antiderivative size = 765, normalized size of antiderivative = 8.60

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b
*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*(3*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*ta
n(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*
e)^2 + a + b))^5 - 21*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*sqrt(a + b) + 2*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(3
*a - 17*b) + 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b))^2*(5*a + 9*b)*sqrt(a + b) - 3*(sqrt(a + b)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))
*(3*a^2 - 18*a*b - 37*b^2) - (9*a^2 + 10*a*b - 47*b^2)*sqrt(a + b))/((sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2
*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
+ a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^5}{\sec(fx+e)^2 b+a}dx$$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**5)/(sec(e + f*x)**2*b + a),
x)
```

3.403 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{bf}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f+(a+b*sec(f*x+e)^2)^(1/
2)/b/f
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{bf}$$

input
```
Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) + Sqrt[a + b*Sec[e
 + f*x]^2]/(b*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4627, 25, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^3}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 4627}$$

$$\frac{\int -\frac{\cos(e+fx)(1-\sec^2(e+fx))}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 90}$$

$$-\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)-\frac{2\sqrt{a+b\sec^2(e+fx)}}{b}}{2f}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}-\frac{2\sqrt{a+b\sec^2(e+fx)}}{b}}{2f}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{a+b\sec^2(e+fx)}}{b}}{2f}$$

input
```
Int[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] - (2*Sqrt[a
 + b*Sec[e + f*x]^2])/b)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(48) = 96$.

Time = 4.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.75

method	result
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{a}\,\cos(fx+e)+4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\cos(fx+e)a\right)b(\sec(fx+e)+1)+a^{\frac{3}{2}}+\sqrt{a}\,b\sec(f}{fb\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/b/a^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x
+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*
b*(sec(f*x+e)+1)+a^(3/2)+a^(1/2)*b*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(48) = 96$.

Time = 0.19 (sec) , antiderivative size = 328, normalized size of antiderivative = 5.86

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{\sqrt{a}b\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2+\right.}{}\right.$$

$$\left. -\frac{\sqrt{-a}b\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right)-4\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\,abf}\right]$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 16
0*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f
*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*
x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*a*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f), -1/4*(sqrt(-a)*b*arctan(1
/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x
+ e)^2 + a*b^2)) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f
)]
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^3\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(48) = 96$.

Time = 0.50 (sec) , antiderivative size = 373, normalized size of antiderivative = 6.66

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{2\left(\frac{\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\cdots}{\left(\sqrt{a\cdots}\right.}\cdots\right)}{\cdots}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*(arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*ta
n(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*(sqrt(
a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
 a + b) + sqrt(a + b))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) +
a - 3*b))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^3}{\sec(fx+e)^2 b+a}dx$$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**3)/(sec(e + f*x)**2*b + a),
x)
```

3.404 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Integrate[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4627, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{bf}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Int[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^{2}}}{\sec(fx+e)}\right)}{f\sqrt{a}}$	42
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^{2}}}{\sec(fx+e)}\right)}{f\sqrt{a}}$	42

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(27) = 54$.

Time = 0.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 7.91

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2+b^4-\right.}{8}\right.$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*c
os(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 +
 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(sqrt(a)*f), 1/4*sqrt(
-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^
2*b*cos(f*x + e)^2 + a*b^2))/(a*f)]
```

Sympy [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(27) = 54.

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}f\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(
1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*f*sgn(cos(f
*x + e)))
```

Mupad [B] (verification not implemented)

Time = 16.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
-atanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(1/2)*f)
```

Reduce [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x))/(sec(e + f*x)**2*b + a),x)
```

3.405 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}f}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f-arctanh((a+b*sec(f*x+e
)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.20

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\left(-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\left(1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a+b}\left(\operatorname{arctanh}\left(\frac{\ldots}{\sqrt{a}\sqrt{4b\ldots}}\right.\right.\right.}{2\sqrt{a}\sqrt{a+b}\left(1+e^{2i(e+fx)}\right)f\sqrt{a+b\sec^2(e\ldots}}$$

input
```
Integrate[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*(-2*Sqrt[a]
*ArcTanh[(Sqrt[a + b]*(1 + E^((2*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*
x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[a + b]*(ArcTanh[(a + 2*b + a*
E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2])] + ArcTanh[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e
 + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x
)))^2])])))/(2*Sqrt[a]*Sqrt[a + b]*(1 + E^((2*I)*(e + f*x)))*f*Sqrt[a + b*
Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4627, 25, 354, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(e+fx)\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \; 4627$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 354$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 97$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{2f}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2f}$$

input
```
Int[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*ArcTan
h[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.90

method	result
default	$-\frac{\left(\ln\left(-\frac{4\left(\sqrt{a+b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{a+b}+\cos(fx+e)a+b\right)}{\cos(fx+e)-1}\right)\sqrt{a}-\ln\left(\frac{2\sqrt{a+b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+\cdots}{\sqrt{a+\cdots}}\right.\right.}{\cdots}$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(a+b)^(1/2)/a^(1/2)*(ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+cos(f*x+e)*a+b)/(cos(f*x+e)-1))*a^(1/2)-ln(2/(a+b)^(1/2)*((a+b)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))
*a^(1/2)-2*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*
x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)
*(a+b)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x+e)^
2)^(1/2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(58) = 116$.

Time = 0.24 (sec) , antiderivative size = 1015, normalized size of antiderivative = 14.50

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((a + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^
6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3
*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*
cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*s
qrt(a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^
2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*
sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 -
 2*cos(f*x + e)^2 + 1)))/((a^2 + a*b)*f), 1/8*(4*a*sqrt(-a - b)*arctan(1/2
*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a + b)*sqrt(a)
*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f
*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*
a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^2 + a*b)*f), -1/4*(sq
rt(-a)*(a + b)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b
^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x +
e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - sqrt(a + b)*a*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2
*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a...
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input

```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output

```
integrate(cot(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(58) = 116.

Time = 0.66 (sec) , antiderivative size = 403, normalized size of antiderivative = 5.76

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{4\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\log\left(\left|-\sqrt{\cdots}\right|\right)}{\cdots}}{\cdots}$$

input

```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output

```
-1/2*(4*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + log(
abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
1/2*e)^2 + a + b) + sqrt(a + b)))/sqrt(a + b) - log(abs(-sqrt(a + b)*tan(1
/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)
^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sq
rt(a + b)))/sqrt(a + b) + log(abs((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - sqrt(a + b)*(a
- b)))/sqrt(a + b))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2\,b+a}\,\cot(fx+e)}{\sec(fx+e)^2\,b+a}dx$$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x))/(sec(e + f*x)**2*b + a),x)
```

3.406 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f+1/2*(2*a+3*b)*arctanh
((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-1/2*cot(f*x+e)^2*(a+b
*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4627, 354, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{114}$$

$$\frac{\frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}-\frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+2a+2b\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a+b}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+2(a+b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{(2a+3b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2(a+b)\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{2(2a+3b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{4(a+b)\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{2(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{2(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{4(a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

input

```
Int[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output

```
(((-4*(a + b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*(2
*a + 3*b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(2
*(a + b)) + Sqrt[a + b*Sec[e + f*x]^2]/((a + b)*(1 - Sec[e + f*x]^2)))/(2*
f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1047 vs. $2(98) = 196$.

Time = 0.43 (sec) , antiderivative size = 1048, normalized size of antiderivative = 9.03

method	result	size
default	Expression too large to display	1048

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(a+b)^(5/2)/a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+
b*sec(f*x+e)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)/(1-cos(f*x+e))^2*(
2*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/
(cos(f*x+e)-1))*a^(5/2)*(1-cos(f*x+e))^2-2*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^(5/2
)*(1-cos(f*x+e))^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(1-cos(f*x+
e))^2*(a+b)^(3/2)*a^(1/2)-4*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)+4*cos(f*x+e)*a)*(1-cos(f*x+e))^2*(a+b)^(3/2)*a-4*ln(4*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*(1-cos(f*x+e))^2*(a+b)^(3/2)*b+5*
ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(c
os(f*x+e)-1))*(1-cos(f*x+e))^2*a^(3/2)*b-5*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*(1-cos
(f*x+e))^2*a^(3/2)*b+3*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(98) = 196$.

Time = 0.42 (sec) , antiderivative size = 1550, normalized size of antiderivative = 13.36

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)^2 + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*lo
g(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x
+ e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2
*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^2 + 3*a*b)*cos(f*x +
e)^2 - 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e
)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4
 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3 + 2*a^2*b + a*b^2)*f*c
os(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a^2 + 3*a*b)*cos(f
*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)
^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a
*b)*cos(f*x + e)^2 + a*b + b^2)) + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a
^2 - 2*a*b - b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x +
 e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16
*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 +
b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/
((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f),...
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^3{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot{(fx+e)}^3}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(98) = 196$.

Time = 0.71 (sec) , antiderivative size = 595, normalized size of antiderivative = 5.13

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(16*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*
b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 4*(2
*a + 3*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) + 2*
(2*a + 3*b)*log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2
*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/(a + b
)^(3/2) + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)/(a + b) - 2*((
sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e
)^2 + a + b))*(a - b) - (a + b)^(3/2))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b)*(a + b)
))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^3}{\sec(fx+e)^2 b+a}dx$$

input

```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output

```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**3)/(sec(e + f*x)**2*b + a),
x)
```

3.407 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{(8a^2+20ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f} + \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f-1/8*(8*a^2+20*a*b+15*b
^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f+1/8*(4*a+7
*b)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2/f-1/4*cot(f*x+e)^4*(a+b*
sec(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4627, 25, 354, 114, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^5\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 4627}$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}-\frac{\int -\frac{\cos(e+fx)\left(3b\sec^2(e+fx)+4(a+b)\right)}{2\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(3b\sec^2(e+fx)+4(a+b)\right)}{\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}}{2f}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{\frac{\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}-\frac{\int -\frac{\cos(e+fx)\left(8(a+b)^2+b(4a+7b)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a+b}}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}}{2f}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^2+b(4a+7b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}}{2f}$$

$$\Big\downarrow \text{ 174}$$

$$-\frac{\frac{\frac{\left(8a^2+20ab+15b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+8(a+b)^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}}{2f}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{\frac{\frac{2\left(8a^2+20ab+15b^2\right)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{16(a+b)^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{2(a+b)}+\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}}{2f}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{\frac{\frac{2(8a^2+20ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{16(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2(a+b)}+\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}}{2f}$$

input
```
Int[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(Sqrt[a + b*Sec[e + f*x]^2]/(2*(a + b)*(1 - Sec[e + f*x]^2)^2) + (((-
16*(a + b)^2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*(8*
a^2 + 20*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sq
rt[a + b])/(2*(a + b)) + ((4*a + 7*b)*Sqrt[a + b*Sec[e + f*x]^2])/((a + b)
*(1 - Sec[e + f*x]^2)))/(4*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1977 vs. $2(144) = 288$.

Time = 0.49 (sec) , antiderivative size = 1978, normalized size of antiderivative = 11.92

method	result	size
default	Expression too large to display	1978

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f/(a+b)^(9/2)/a^(1/2)*((8*cos(f*x+e)+8)*sin(f*x+e)^4*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(9/2)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(cos(f*x+e)-1))+(-8*cos(f*x+e)-8)*sin(f
*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(9/2)*ln(2/(a+b)^(1/
2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+co
s(f*x+e)))+(36*cos(f*x+e)+36)*sin(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a^(7/2)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+cos(f*x+e)*a+b)/(cos(f*x+e)-1))*b+(-36*cos(f*x+e)-36)*sin(f*x+e)^4*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(7/2)*ln(2/(a+b)^(1/2)*((a+b)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b
+(63*cos(f*x+e)+63)*sin(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a^(5/2)*ln(-4*((a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x
+e)*a+b)/(cos(f*x+e)-1))*b^2+(-63*cos(f*x+e)-63)*sin(f*x+e)^4*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(5/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(144) = 288$.

Time = 1.15 (sec) , antiderivative size = 2257, normalized size of antiderivative = 13.60

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b +
 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)
*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f
*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*
a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((8*a^3 + 20*a^2*b + 15*
a*b^2)*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b
+ 15*a*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x
+ e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*(3*(2*a^3 + 5*a^2*b
 + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 +
 a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x
 + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), 1/16*(((8*a^3 + 20*a^2*b
 + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*
a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f
*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + ...
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(144) = 288$.

Time = 1.05 (sec) , antiderivative size = 924, normalized size of antiderivative = 5.57

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*((a + b)*tan(1/2*f
*x + 1/2*e)^2/(a^2 + 2*a*b + b^2) - (11*a + 17*b)/(a^2 + 2*a*b + b^2)) - 1
28*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + 8*(8*a^2
+ 20*a*b + 15*b^2)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)
^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a^2 + 2*a*b + b^
2)*sqrt(-a - b)) - 4*(8*a^2 + 20*a*b + 15*b^2)*log(abs((sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a +
 b) - sqrt(a + b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 4*(2*(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2
*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
+ a + b))^3*(3*a^2 + 2*a*b - 4*b^2) - 7*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2
*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(a^2 + 2*a*b + b^
2)*sqrt(a + b) - 2*(2*a^3 + 4*a^2*b - 3*a*b^2 - 5*b^3)*(sqrt(a + b)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^5}{\sec(fx+e)^2 b+a}dx$$

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**5)/(sec(e + f*x)**2*b + a),
x)
```

3.408 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a^2+10ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f+1/8*(3*a^
2+10*a*b+15*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^
(5/2)/f-1/8*(3*a+7*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b^2/f+1/4*tan(
f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f
```

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.33

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$

$$-\frac{\left(\frac{8b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}-\frac{(3a^2+10ab+15b^2)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec}{8\sqrt{2}b^2f\sqrt{a+b\sec^2(e+fx)}}$$

$$-\frac{(a+2b+a\cos(2(e+fx)))(3a+5b+3(a+3b)\cos(2(e+fx)))\sec^4(e+fx)\tan(e+fx)}{32b^2f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/8*(((8*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
])/Sqrt[a] - ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqr
t[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*
Sec[e + f*x])/(Sqrt[2]*b^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - ((a + 2*b + a*C
os[2*(e + f*x)])*(3*a + 5*b + 3*(a + 3*b)*Cos[2*(e + f*x)])*Sec[e + f*x]^4
*Tan[e + f*x])/(32*b^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 381, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^6}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$\Big\downarrow$ 4629

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 2075

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 381

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\int \frac{\tan^2(e+fx)\left((3a+7b)\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}}{f}$$

$\Big\downarrow$ 444

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int \frac{\left(3a^2+10ba+15b^2\right)\tan^2(e+fx)+(a+b)(3a+7b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{4b}}{f}$$

$\Big\downarrow$ 398

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-8b^2\int \frac{}{\left(\tan^2(e+fx\right.}}{2b}}{4b}}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-8b^2\int}{2b}}{4b}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{\left(3a^2+10ab+15b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8b^2\int \frac{}{\left(\tan^2(e+f\right.}}{2b}}{4b}}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{\left(3a^2+10ab+15b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8b^2\int\frac{\frac{a\tan^2(e\cdots}{b\tan^2(e+\cdots}}{\cdots}}{2b}}{4b}}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{\left(3a^2+10ab+15b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-\frac{8b^2\arctan\left(\frac{\cdots}{\sqrt{a\cdots}}\right)}{\cdots}}{2b}}{4b}}{f}$$

input
```
Int[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*b) - (-1/2*((-8*b^2*Ar
cTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a] + ((3
*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/Sqrt[b])/b + ((3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(2*b))/(4*b))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(151) = 302$.

Time = 13.00 (sec) , antiderivative size = 1054, normalized size of antiderivative = 6.09

method	result	size
default	Expression too large to display	1054

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f/b^(9/2)/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(3*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a^2*b^2*ln(4*(b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-1-sec(f*x+e))+10*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^3*ln(4*(b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-1-sec(f*
x+e))+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^4*ln(4*(
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))
*(-1-sec(f*x+e))+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*
a^2*b^2*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/
(sin(f*x+e)-1))*(-1-sec(f*x+e))+10*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(-a)^(1/2)*a*b^3*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(
f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-1-sec(f*x+e))+15*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^4*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-1-sec(f*x+e))+16*b^(9/2)*((...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(151) = 302$.

Time = 1.20 (sec) , antiderivative size = 1673, normalized size of antiderivative = 9.67

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(4*sqrt(-a)*b^3*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)
^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2
*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
- 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*
cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4
) - 4*(2*a*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), -1/16*(2*sq
rt(-a)*b^3*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2
*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +...
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^6{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan{(fx+e)}^6}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan{(fx+e)}^6}{\sqrt{b\sec{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan(e+f\,x)^6}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^6}{\sec(fx+e)^2 b+a}dx$$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**6)/(sec(e + f*x)**2*b + a),
x)
```

3.409 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f-1/2*(a+3*b
)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f+1/2*tan
(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/b/f
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.63

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$=\frac{\left(\frac{2b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}-\frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{2\sqrt{2}bf\sqrt{a+b\sec^2(e+fx)}}$$
$$+\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{4bf\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((2*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt
[a] - ((a + 3*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x
]^2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(2*Sqrt[2
]*b*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e
+ f*x]^2*Tan[e + f*x])/(4*b*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4629, 2075, 381, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(e+fx)^4}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{381}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int \frac{(a+3b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(a+3b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-2b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(a+3b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-2b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-2b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-2b\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-\frac{2b\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{2b}}{f}$$

input
```
Int[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/2*((-2*b*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/Sqrt[a] + ((a + 3*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/Sqrt[b])/b + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2
*b))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(102) = 204$.

Time = 4.36 (sec) , antiderivative size = 732, normalized size of antiderivative = 6.10

method	result
default	$\frac{\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,ab\ln\left(\frac{4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)(-1-\sec(fx+e))}{4}+$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*ln(4*(
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))
*(-1-sec(f*x+e))+1/4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^2*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(si
n(f*x+e)+1))*(-3-3*sec(f*x+e))+1/4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a*b*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*
x+e)*a+a+b)/(sin(f*x+e)-1))*(-1-sec(f*x+e))+1/4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-3-3*sec(f*x+e))+1/4*b^(5/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(4+4*sec(f*x+e))+1/2*b^(3/2)*(-a)^(1/
2)*a*tan(f*x+e)+1/2*(-a)^(1/2)*b^(5/2)*tan(f*x+e)*sec(f*x+e)^2)/(-a)^(1/2)
/b^(5/2)/(a+b*sec(f*x+e)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(102) = 204$.

Time = 0.49 (sec) , antiderivative size = 1507, normalized size of antiderivative = 12.56

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a)*b^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b
)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*
(sqrt(-a)*b^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
 + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) + 2*(a^2 + 3*a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos
(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*...
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan^4{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^4}{\sec(fx+e)^2 b+a}dx$$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**4)/(sec(e + f*x)**2*b + a),
x)
```

3.410 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f+arctanh(b
^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(1/2)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.70

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \frac{i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\left(-2\sqrt{a}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{b}\,\text{arctanh}\left(\frac{a+2b+\cdots}{\sqrt{a}\sqrt{4be^{2i(e+fx)}\cdots}}\right.\right.}{2\sqrt{a}\sqrt{b}\left(1+e^{2i(e+fx)}\right)f\sqrt{a+b\sec^2(e+f\cdots}}$$

input
```
Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((I/2)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*(-2*S
qrt[a]*ArcTan[(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[b]*ArcTanh[(a + 2*b + a*E^(
(2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*
(e + f*x)))^2])] - Sqrt[b]*ArcTanh[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*
I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
+ f*x)))^2])]))/(Sqrt[a]*Sqrt[b]*(1 + E^((2*I)*(e + f*x)))*f*Sqrt[a + b*Se
c[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 385, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{385}$$

$$\frac{\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \textbf{291}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{f}$$

input

```
Int[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output

```
(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/Sqrt[a])
+ ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/Sqrt[b])/
f
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(68) = 136$.

Time = 2.19 (sec) , antiderivative size = 342, normalized size of antiderivative = 4.28

method	result
default	$\left(\ln\left(\frac{4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)\sqrt{-a}+\ln\left(-\frac{4\left(\sqrt{b}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+\cdots\right.}{\sin(\cdots}\right)\right)$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)/b^(1/2)*(ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-s
in(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)+ln(-4*(b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)-2*ln(4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^(1/2))*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+
e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(68) = 136$.

Time = 0.33 (sec) , antiderivative size = 1259, normalized size of antiderivative = 15.74

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
 e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b
+ 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos
(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
- b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e)) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a
*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4))/(a*b*f), 1/8*(4*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x +
e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*b*log(128*a^4*co
s(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5
*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*
cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*b*f), 1/4*(sqr
t(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co...
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^2{(e+fx)}}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\tan(e+f\,x)^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2\,b+a}\,\tan(fx+e)^2}{\sec(fx+e)^2\,b+a}dx$$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**2)/(sec(e + f*x)**2*b + a),
x)
```

3.411 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4616, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

input
```
Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 1.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

method	result	size
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)}{f\sqrt{-a}\,\sqrt{a+b\sec(fx+e)^2}}$	138

input
```
int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)/(a+b*sec(f*x+e)^2)^
(1/2)*(sec(f*x+e)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(33) = 66$.

Time = 0.20 (sec) , antiderivative size = 408, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{a + b\sec^2(e + fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8 - 256\,(a^4 - a^3b)\cos\left(fx+e\right)^6 + 32\,(5\,a^4 - 14\,a^3b + 5\,a^2b^2)\cos\left(fx+\right.\right.}{}\right.$$

$$\left. -\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5 - 8\,(a^2-ab)\cos(fx+e)^3 + (a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4 - a^2b + ab^2 - (a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\sqrt{a}f}\right]$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec^2{(e+fx)}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(33) = 66.

Time = 0.26 (sec) , antiderivative size = 992, normalized size of antiderivative = 25.44

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*
f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b
)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x +
2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
 + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4
*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*
b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*co
s(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(
4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt
(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*...
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}}{\sec\left(fx+e\right)^2 b+a}dx$$

input
```
int(1/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)/(sec(e + f*x)**2*b + a),x)
```

3.412 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left((a+b)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)+\sqrt{a}\csc(e+fx)\sqrt{a+b}\right.}{\sqrt{2}\sqrt{a}(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] + Sqrt[a]*Csc[e + f*x]*Sqr
t[a + b - a*Sin[e + f*x]^2]))/(Sqrt[2]*Sqrt[a]*(a + b)*f*Sqrt[a + b*Sec[e
+ f*x]^2]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 382, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int -\frac{a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{-\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

input
```
Int[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/Sqrt[a])
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(66) = 132$.

Time = 2.02 (sec) , antiderivative size = 298, normalized size of antiderivative = 4.03

method	result
default	$\frac{-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a(\sec(fx+e)+1)-\sqrt{\frac{b+a\cos}{(1+\cos(}}}{f(}$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*(sec(f*x+e)+1)-((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)*b*(sec(f*x+e)+1)-a*(-a)^(1/2)*cot(f*x+e)-b*(-
a)^(1/2)*sec(f*x+e)*csc(f*x+e))/(a+b)/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(66) = 132$.

Time = 0.24 (sec) , antiderivative size = 525, normalized size of antiderivative = 7.09

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}(a+b)\log\left(128\,a^4\cos\left(fx+e\right)^8 - 256\,(a^4-a^3b)\cos\left(fx+e\right)^6 + 32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos\right.}{}\right.$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^
3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*
a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), 1/4*((a + b)*sqrt(a)*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6
*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e)))*sin(f*x + e) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2\,b+a}\,\cot(fx+e)^2}{\sec(fx+e)^2\,b+a}dx$$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**2)/(sec(e + f*x)**2*b + a),
x)
```

3.413 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f+1/3*(3*a+5
*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e)^3*(a+b+
b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.41

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}} - \frac{(a+2b+a\cos(2(e+fx)))(-a-2b+(2a+3b)\cos(2(e+fx)))\csc^3(e+fx)\sec(e+fx)}{6(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2]) - ((a + 2*b + a*Cos[2*(e + f*x)])*(-a - 2*b + (2*a + 3*b)*Cos[2
*(e + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 382, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$\big\downarrow$ 2075

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$\big\downarrow$ 382

$$\frac{\frac{\int -\frac{\cot^2(e+fx)\left(2b\tan^2(e+fx)+3a+5b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$\big\downarrow$ 25

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2b\tan^2(e+fx)+3a+5b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$\big\downarrow$ 445

$$\frac{-\frac{-\frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$\big\downarrow$ 27

$$\frac{-\frac{-3(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$\big\downarrow$ 291

$$\frac{-\frac{-3(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$\big\downarrow$ 216

$$\frac{-\frac{-\frac{3(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}-\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/3*(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - ((-3*(a +
b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a]
- ((3*a + 5*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/(3*(a
 + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(105) = 210$.

Time = 5.05 (sec) , antiderivative size = 505, normalized size of antiderivative = 4.24

method	result
default	$-\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a^2(-3-3\sec(fx+e))+\sqrt{\cdots}}{}$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a+b)^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a^2*(-3-3*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+
e)*a)*a*b*(-6-6*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^2*(-
3-3*sec(f*x+e))+(-a)^(1/2)*a^2*(4*cot(f*x+e)^3-3*cot(f*x+e)*csc(f*x+e)^2)+
(6*cos(f*x+e)^4-cos(f*x+e)^2-3)*(-a)^(1/2)*a*b*sec(f*x+e)*csc(f*x+e)^3+(6*
cos(f*x+e)^2-5)*(-a)^(1/2)*b^2*sec(f*x+e)*csc(f*x+e)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(105) = 210$.

Time = 0.48 (sec) , antiderivative size = 723, normalized size of antiderivative = 6.08

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a
)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*
(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*si
n(f*x + e) - 8*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x
 + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b
^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e)), -1/12*(3*
((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*arctan(1/
4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e)))*sin(f*x + e) - 4*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a
*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*
a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e))
]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx=\int \frac{\cot^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^4}{\sec(fx+e)^2 b+a}dx$$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**4)/(sec(e + f*x)**2*b + a),
x)
```

3.414 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\begin{aligned}\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} \\ &\quad -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} \\ &\quad +\frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} \\ &\quad -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f}\end{aligned}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(1/2)/f-1/15*(15*
a^2+40*a*b+33*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f+1/15*(5
*a+9*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot(f*x+e)^5
*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\csc(e+fx)\left(23a^2+60ab+45b^2-(11a^2+26ab+15b^2)\csc^2(e+fx)+3\right.}{30(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a +
2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[
e + f*x]^2])) - ((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*(23*a^2 + 60*
a*b + 45*b^2 - (11*a^2 + 26*a*b + 15*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc
[e + f*x]^4)*Sec[e + f*x])/(30*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 382, 25, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\tan(e+fx)^6\sqrt{a+b\sec(e+fx)^2}}\,dx$$
$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 382$$

$$\frac{\frac{\int -\frac{\cot^4(e+fx)\left(4b\tan^2(e+fx)+5a+9b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{\cot^4(e+fx)\left(4b\tan^2(e+fx)+5a+9b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(15a^2+40ba+33b^2+2b(5a+9b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{5(a+b)}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{15(a+b)^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{-15(a+b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+}}{3(a+b)}}{5(a+b)}}{f}$$

$$\Big\downarrow \; 291$$

$$-\frac{-\frac{-15(a+b)^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{3(a+b)}}{5(a+b)}}{f}$$

$\downarrow$ 216

$$-\frac{-\frac{-\frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-\frac{15(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{3(a+b)}-\frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)}-\frac{\cot^5(e+}{}}{f}$$

input
```
Int[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - (-1/3*((5*
a + 9*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - ((-15*(a
 + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqr
t[a] - ((15*a^2 + 40*a*b + 33*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(a + b))/(3*(a + b)))/(5*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(154) = 308$.

Time = 7.65 (sec) , antiderivative size = 741, normalized size of antiderivative = 4.31

method	result
default	$-\frac{\left(\sin(fx+e)^5(15\cos(fx+e)+15)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ \ln\left(4\sqrt{-a}\ \sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ \cos(fx+e)+4\sqrt{-a}\ \sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+\right.\right.}{}$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a+b)^3/(-a)^(1/2)*(sin(f*x+e)^5*(15*cos(f*x+e)+15)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)*a^3+sin(f*x+e)^5*(45*cos(f*x+e)+45)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b+sin(f*x+e)^5*(45*cos(f*x+e)+45)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+sin(f*x+e)^5*(15*cos(f*x+e)+15)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3+cos(f*x+e)^2*(23*cos(f*x+e)^4-35*c
os(f*x+e)^2+15)*(-a)^(1/2)*a^3+(60*cos(f*x+e)^6-71*cos(f*x+e)^4+5*cos(f*x+
e)^2+15)*(-a)^(1/2)*a^2*b+(45*cos(f*x+e)^6-15*cos(f*x+e)^4-61*cos(f*x+e)^2
+40)*(-a)^(1/2)*a*b^2+(45*cos(f*x+e)^4-75*cos(f*x+e)^2+33)*(-a)^(1/2)*b^3)
/(1+cos(f*x+e))^2/(a+b*sec(f*x+e)^2)^(1/2)/(cos(f*x+e)^2-2*cos(f*x+e)+1)*s
ec(f*x+e)*csc(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(154) = 308$.

Time = 1.58 (sec) , antiderivative size = 987, normalized size of antiderivative = 5.74

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/120*(15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*
b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt
(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
- 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 -
 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 1
4*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*
x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))
*sin(f*x + e) + 8*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a^3
 + 94*a^2*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)
), 1/60*(15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*
b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt
(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2
 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
 e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos
(f*x + e)^5 - (35*a^3 + 94*a^2*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 +...
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^6}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^6}{\sec(fx+e)^2 b+a}dx$$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**6)/(sec(e + f*x)**2*b + a),
x)
```

3.415 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+(a+b)^2/a/b^2/f/(a+b*
sec(f*x+e)^2)^(1/2)+(a+b*sec(f*x+e)^2)^(1/2)/b^2/f
```

Mathematica [A] (verified)

Time = 3.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.38

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(a+2b+a\cos(2(e+fx)))\left(2a^2+4ab+b^2+(2a^2+2ab+b^2)\cos(2(e+fx))\right)\sec^4(e+fx)}{4ab^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ((a + 2*b + a
*Cos[2*(e + f*x)])*(2*a^2 + 4*a*b + b^2 + (2*a^2 + 2*a*b + b^2)*Cos[2*(e +
 f*x)])*Sec[e + f*x]^4)/(4*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))^2}{(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))^2}{(b\sec^2(e+fx)+a)^{3/2}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{98}$$

$$\frac{\int \left(-\frac{(a+b)^2}{ab(b\sec^2(e+fx)+a)^{3/2}}+\frac{\cos(e+fx)}{a\sqrt{b\sec^2(e+fx)+a}}+\frac{1}{b\sqrt{b\sec^2(e+fx)+a}}\right)d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{2(a+b)^2}{ab^2\sqrt{a+b\sec^2(e+fx)}}+\frac{2\sqrt{a+b\sec^2(e+fx)}}{b^2}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) + (2*(a + b)^2)/
(a*b^2*Sqrt[a + b*Sec[e + f*x]^2]) + (2*Sqrt[a + b*Sec[e + f*x]^2])/b^2)/(
2*f)
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(78) = 156.

Time = 7.12 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.83

method	result
default	$\frac{2a^{\frac{9}{2}}+a^{\frac{7}{2}}b\left(2+3\sec(fx+e)^{2}\right)+a^{\frac{5}{2}}b^{2}\left(1+2\sec(fx+e)^{2}+\sec(fx+e)^{4}\right)+a^{\frac{3}{2}}b^{3}\sec(fx+e)^{2}+\sqrt{\frac{b+a\cos(fx+e)^{2}}{(1+\cos(fx+e))^{2}}}\ \ln\left(4\sqrt{\frac{b+a\cos(fx+e)^{2}}{(1+\cos(fx+e))^{2}}}\right.}{}$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/b^2/a^(5/2)/(a+b*sec(f*x+e)^2)^(3/2)*(2*a^(9/2)+a^(7/2)*b*(2+3*sec(f*x
+e)^2)+a^(5/2)*b^2*(1+2*sec(f*x+e)^2+sec(f*x+e)^4)+a^(3/2)*b^3*sec(f*x+e)^
2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^2*b^2*(-1-sec(f*x+e))+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)+4*cos(f*x+e)*a)*a*b^3*(-sec(f*x+e)^2-sec(f*x+e)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(78) = 156.

Time = 0.63 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.20

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[\frac{\left(ab^2\cos\left(fx+e\right)^2+b^3\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+\right.\right.}{}\right.$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 25
6*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e
)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*
cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)) + 8*(a^2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*
b^3*f), 1/4*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f
*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)
) + 4*(a^2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)]
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^5{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan{(fx+e)}^5}{(b\sec{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(78) = 156$.

Time = 0.99 (sec) , antiderivative size = 560, normalized size of antiderivative = 6.36

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-(((a^4*b*sgn(cos(f*x + e)) + 2*a^3*b^2*sgn(cos(f*x + e)) + a^2*b^3*sgn(co
s(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^3*b^3) - (a^4*b*sgn(cos(f*x + e)) +
 2*a^3*b^2*sgn(cos(f*x + e)) + a^2*b^3*sgn(cos(f*x + e)))/(a^3*b^3))/sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*arctan(-1/2*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
 + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(f*x + e))) - 4*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
 + sqrt(a + b))/(((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a - 3*
b)*b*sgn(cos(f*x + e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^5}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**5)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.416 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-(a+b)/a/b/f/(a+b*sec(f
*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\frac{\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{a\sqrt{a+b\sec^2(e+fx)}}}{bf}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((b*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2/Sqrt[a]])/a^(3/2) - (a + b)/(a*Sqrt
[a + b*Sec[e + f*x]^2]))/(b*f)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4627, 25, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$-\frac{\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a}+\frac{2(a+b)}{ab\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{ab}+\frac{2(a+b)}{ab\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{2(a+b)}{ab\sqrt{a+b\sec^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) + (2*(a + b
))/(a*b*Sqrt[a + b*Sec[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(55) = 110$.

Time = 0.44 (sec) , antiderivative size = 874, normalized size of antiderivative = 13.87

method	result	size
default	Expression too large to display	874

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a^(3/2)/((-a*b)^(1/2)-a)/((-a*b)^(1/2)+a)/b/(cos(f*x+e)^2*a^2+(cos(f*
x+e)^2+1)*a*b+b^2)/(a+b*sec(f*x+e)^2)^(3/2)*(ln(4*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^5*b*(cos(f*x+e)^2+cos(f*x+e))+ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)+4*cos(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4
*b^2*(2*cos(f*x+e)^2+2*cos(f*x+e)+2+2*sec(f*x+e))+ln(4*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a^3*b^3*(cos(f*x+e)^2+cos(f*x+e)+4+4*sec(f*x+e)+sec(f*x+e)^2+sec
(f*x+e)^3)+ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*
x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^4*(2+2*sec(f*x+e)+2*sec
(f*x+e)^2+2*sec(f*x+e)^3)+ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+
4*cos(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^5*(sec(f*x
+e)^2+sec(f*x+e)^3)-a^(13/2)*cos(f*x+e)^2+(-3*cos(f*x+e)^2-2)*a^(11/2)*b+a
^(9/2)*b^2*(-3*cos(f*x+e)^2-6-sec(f*x+e)^2)+a^(7/2)*b^3*(-cos(f*x+e)^2-6-3
*sec(f*x+e)^2)+a^(5/2)*b^4*(-2-3*sec(f*x+e)^2)-a^(3/2)*b^5*sec(f*x+e)^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(55) = 110$.

Time = 0.21 (sec) , antiderivative size = 417, normalized size of antiderivative = 6.62

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{8\,(a^2+ab)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)^2-\left(ab\cos(fx+e)^2+b^2\right)\sqrt{a}\log\cdots}{\cdots}\right.$$

$$-\frac{4\,(a^2+ab)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)^2+\left(ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)\cdots\right.}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2\cdots\right.}\right.}{4\left(a^3bf\cos(fx+e)^2+a^2b^2f\right)}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e)^2 - (a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 25
6*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e
)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*
cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*f), -1/4*(4*(a^2 + a*b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*b*cos(f*x
 + e)^2 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
 e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)))/(a^3*b*f*cos(f*x + e)^2
+ a^2*b^2*f)]
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^3{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2532 vs. $2(55) = 110$.

Time = 0.40 (sec) , antiderivative size = 2532, normalized size of antiderivative = 40.19

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 4*a*b*cos(
1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(1/2*arctan2(a*sin(4*f*x +
 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a))^2 - 4*(a^2 + a*b)*sin(2*f*x + 2*e)*sin(1/2*arctan2(a*s
in(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) - 4*(a^2 + 2*a*b + (a^2 + a*b)*cos(2*f*x + 2
*e))*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*
cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^
2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4
*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e)
)*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((b*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + b*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*co
s(2*f*x + 2*e) + a))^2)*log(4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2
*e)^2 + 4*a^2 + 16*a*b + 16*b^2 + 8*(a^2 + 2*a*b)*cos(2*f*x + 2*e) + 8*(a^
2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(55) = 110$.

Time = 0.84 (sec) , antiderivative size = 251, normalized size of antiderivative = 3.98

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\frac{\frac{\left(a^3\mathrm{sgn}(\cos(fx+e))+a^2b\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^3b}-\frac{a^3\mathrm{sgn}(\cos(fx+e))+a^2b\mathrm{sgn}(\cos(fx+e))}{a^3b}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}} - \frac{2\,\mathrm{arcta}}{}}{}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
(((a^3*sgn(cos(f*x + e)) + a^2*b*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2
/(a^3*b) - (a^3*sgn(cos(f*x + e)) + a^2*b*sgn(cos(f*x + e)))/(a^3*b))/sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*arctan(-1/2*(sqrt(a + b)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(f*x + e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^3}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**3)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.417 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sec(f*x+e)
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 6.70

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{3/2}\sec^2(e+fx)\left(-\frac{2}{b\sqrt{a+2b+a\cos(2(e+fx))}} + \frac{\sqrt{2}e^{i(\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sec[e + f*x]^2*(-2/(b*Sqrt[a + 2*b +
 a*Cos[2*(e + f*x)]]) + (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*
I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*((Sqrt[a]*(a + 4*b)*(1 + E^((2*I)*(
e + f*x))))/(b*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2))
+ ((4*I)*f*x - 2*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^
((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*Log[a + a*E^((2*I)
*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
+ E^((2*I)*(e + f*x)))^2])*Sec[e + f*x])/a^(3/2)))/(16*f*(a + b*Sec[e + f*
x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4627, 243, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \frac{\cos(e+fx)}{(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{\cos(e+fx)}{(b\sec^2(e+fx)+a)^{3/2}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\Big\downarrow$ 73

$$\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\Big\downarrow$ 221

$$\frac{\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a +
b*Sec[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\frac{1}{a\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{a^{\frac{3}{2}}}}{f}$	62
default	$\frac{\frac{1}{a\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{a^{\frac{3}{2}}}}{f}$	62

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a/(a+b*sec(f*x+e)^2)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x
+e)^2)^(1/2))/sec(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(49) = 98$.

Time = 0.19 (sec) , antiderivative size = 392, normalized size of antiderivative = 6.88

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)^2 + \left(a\cos\left(fx+e\right)^2+b\right)\sqrt{a}\log\left(128\,a^4\cos\right.}{}\right.$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*
cos(f*x + e)^2 + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x
 + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(
16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
+ b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
)/(a^3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*cos(f*x + e)^2 + (a*cos(f*x + e)^2 + b)*sqrt(-a)*arctan(1/4
*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x +
e)^2 + a*b^2)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [A] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{2af\sqrt{a+b\sec^2(e+fx)}} + \frac{b\operatorname{atan}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{-a}}\right)}{2af\sqrt{-a}}\right)}{b} & \text{for } b \neq 0 \\ \frac{\log\left(\sec^2(e+fx)\right)}{2a^{\frac{3}{2}}f} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Piecewise((2*(b/(2*a*f*sqrt(a + b*sec(e + f*x)**2)) + b*atan(sqrt(a + b*se
c(e + f*x)**2)/sqrt(-a))/(2*a*f*sqrt(-a)))/b, Ne(b, 0)), (log(sec(e + f*x)
**2)/(2*a**(3/2)*f), True))
```

Maxima [F]

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(49) = 98$.

Time = 0.65 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.79

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{\frac{\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}-\frac{1}{a\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}}{\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}}-\frac{2\,\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\right.}}{}\right.}{}}{f}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-((tan(1/2*f*x + 1/2*e)^2/(a*sgn(cos(f*x + e))) - 1/(a*sgn(cos(f*x + e))))
/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*arctan(-1/2*(sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(f*x + e))))/f
```

Mupad [B] (verification not implemented)

Time = 17.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{1}{a\,f\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{a^{3/2}\,f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
1/(a*f*(a + b/cos(e + f*x)^2)^(1/2)) - atanh((a + b/cos(e + f*x)^2)^(1/2)/
a^(1/2))/(a^(3/2)*f)
```

Reduce [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x))/(sec(e + f*x)**4*b**2 + 2*s
ec(e + f*x)**2*a*b + a**2),x)
```

3.418 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-arctanh((a+b*sec(f*x+e
)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 4627, 25, 354, 96, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{96}$$

$$-\frac{\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{\cos(e+fx)\left(-b\sec^2(e+fx)+a+b\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}}{2f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(-b\sec^2(e+fx)+a+b\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{174}$$

$$-\frac{\frac{a\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+(a+b)\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{\frac{2a\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2(a+b)\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{2a\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

input

```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output

```
-1/2*(((-2*(a + b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] +
(2*a*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a +
 b)) + (2*b)/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2948 vs. $2(86) = 172$.

Time = 0.50 (sec) , antiderivative size = 2949, normalized size of antiderivative = 29.49

method	result	size
default	Expression too large to display	2949

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/a^(3/2)/((-a*b)^(1/2)-a)/((-a*b)^(1/2)+a)/(a+b)^(5/2)/(cos(f*x+e)^2*
a^2+(cos(f*x+e)^2+1)*a*b+b^2)/(a+b*sec(f*x+e)^2)^(3/2)*(2*cos(f*x+e)^2*(a+
b)^(3/2)*a^(11/2)*b+(4*cos(f*x+e)^2+4)*(a+b)^(3/2)*a^(9/2)*b^2+(a+b)^(3/2)
*a^(7/2)*b^3*(2*cos(f*x+e)^2+8+2*sec(f*x+e)^2)+(a+b)^(3/2)*a^(5/2)*b^4*(4+
4*sec(f*x+e)^2)+2*(a+b)^(3/2)*a^(3/2)*b^5*sec(f*x+e)^2+(a+b)^(3/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)+4*cos(f*x+e)*a)*a^6*(-2*cos(f*x+e)^2-2*cos(f*x+e))+(a+b)^(3/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^5*b*(-6*cos(f*x+e)^2-6*cos(f*x+e)-4-4*
sec(f*x+e))+(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^4*b^2*(-6*cos(
f*x+e)^2-6*cos(f*x+e)-12-12*sec(f*x+e)-2*sec(f*x+e)^2-2*sec(f*x+e)^3)+(a+b
)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^3*b^3*(-2*cos(f*x+e)^2-2*cos(f
*x+e)-12-12*sec(f*x+e)-6*sec(f*x+e)^2-6*sec(f*x+e)^3)+(a+b)^(3/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(86) = 172$.

Time = 0.43 (sec) , antiderivative size = 1569, normalized size of antiderivative = 15.69

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)^2 - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^
2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2
*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(a^3*cos(f*x
 + e)^2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 +
 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*
cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(
cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(
f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/8*(8*(a^2*b + a*b^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 4*(a^3*cos(f*x +
 e)^2 + a^2*b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt
(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
+ e)^2 + a*b + b^2)) - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*co
s(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^
6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3
*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*
cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^
5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)...
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x))/(sec(e + f*x)**4*b**2 + 2*s
ec(e + f*x)**2*a*b + a**2),x)
```

3.419 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(2a+5b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/2*(2*a+5*b)*arctanh
((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/2*(a-2*b)*b/a/(a+b)
^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2)^
(1/2)
```

Mathematica [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4627, 354, 114, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^3\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 4627}$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{3/2}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{\cos(e+fx)\left(3b\sec^2(e+fx)+2a+2b\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a+b}}{2f}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\cos(e+fx)\left(3b\sec^2(e+fx)+2(a+b)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\downarrow$ 169

$$\frac{\frac{\frac{2\int \frac{\cos(e+fx)\left(2(a+b)^2+(a-2b)b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(2(a+b)^2+(a-2b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\downarrow$ 174

$$\frac{\frac{\frac{a(2a+5b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2(a+b)^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\downarrow$ 73

$$\frac{\frac{\frac{\frac{4(a+b)^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2a(2a+5b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\downarrow$ 221

$$\frac{\frac{\frac{\frac{2a(2a+5b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{4(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(1/((a + b)*(1 - Sec[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]) + (((-4*(a +
b)^2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a*(2*a + 5*
b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b
)) - (2*(a - 2*b)*b)/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(2*(a + b)))/
(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3945 vs. $2(131) = 262$.

Time = 1.13 (sec) , antiderivative size = 3946, normalized size of antiderivative = 25.79

method	result	size
default	Expression too large to display	3946

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/a^(3/2)/(a+b)^(11/2)/((-a*b)^(1/2)+a)/((-a*b)^(1/2)-a)/(b+a*cos(f*x+
e)^2)/(a+b*sec(f*x+e)^2)^(3/2)*(-4*(a+b)^(5/2)*a^(3/2)*b^6*sec(f*x+e)^2+2*
(a+b)^(5/2)*a^(15/2)*cos(f*x+e)^2*cot(f*x+e)^2+a^(19/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*((a+b)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*(2*cos(f*x+e)^2+2*cos(f
*x+e))+a^(19/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*((a+b)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(cos(f*x+e)-1))*
(-2*cos(f*x+e)^2-2*cos(f*x+e))+(8*cos(f*x+e)^6+6*cos(f*x+e)^4+4*cos(f*x+e)
^2+2)*(a+b)^(5/2)*a^(9/2)*b^3*sec(f*x+e)^2*csc(f*x+e)^2+(a+b)^(5/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)+4*cos(f*x+e)*a)*a^4*b^3*(16*cos(f*x+e)^2+16*cos(f*x+e)+48+48*s
ec(f*x+e)+16*sec(f*x+e)^2+16*sec(f*x+e)^3)+(a+b)^(5/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4
*cos(f*x+e)*a)*a^3*b^4*(4*cos(f*x+e)^2+4*cos(f*x+e)+32+32*sec(f*x+e)+24*se
c(f*x+e)^2+24*sec(f*x+e)^3)+(a+b)^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(131) = 262$.

Time = 1.34 (sec) , antiderivative size = 2347, normalized size of antiderivative = 15.34

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*
b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt
(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*co
s(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 +
24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^4 + 5*a^3*b)*co
s(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x
 + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*
b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
+ e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*c
os(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*co
s(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a
^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), -1/8*(2*((2*a^4 + 5*a^3*b)*cos
(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x
+ e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2
+ a*b + b^2)) - ((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*
b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x...
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1062 vs. $2(131) = 262$.

Time = 1.40 (sec) , antiderivative size = 1062, normalized size of antiderivative = 6.94

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*((((a^7*b*sgn(cos(f*x + e)) + 4*a^6*b^2*sgn(cos(f*x + e)) + 6*a^5*b^3*
sgn(cos(f*x + e)) + 4*a^4*b^4*sgn(cos(f*x + e)) + a^3*b^5*sgn(cos(f*x + e)
))*tan(1/2*f*x + 1/2*e)^2/(a^8*b + 5*a^7*b^2 + 10*a^6*b^3 + 10*a^5*b^4 + 5
*a^4*b^5 + a^3*b^6) - 2*(a^7*b*sgn(cos(f*x + e)) + 2*a^6*b^2*sgn(cos(f*x +
 e)) + 4*a^5*b^3*sgn(cos(f*x + e)) + 10*a^4*b^4*sgn(cos(f*x + e)) + 11*a^3
*b^5*sgn(cos(f*x + e)) + 4*a^2*b^6*sgn(cos(f*x + e)))/(a^8*b + 5*a^7*b^2 +
 10*a^6*b^3 + 10*a^5*b^4 + 5*a^4*b^5 + a^3*b^6))*tan(1/2*f*x + 1/2*e)^2 +
(a^7*b*sgn(cos(f*x + e)) + 4*a^6*b^2*sgn(cos(f*x + e)) + 14*a^5*b^3*sgn(co
s(f*x + e)) + 28*a^4*b^4*sgn(cos(f*x + e)) + 25*a^3*b^5*sgn(cos(f*x + e))
+ 8*a^2*b^6*sgn(cos(f*x + e)))/(a^8*b + 5*a^7*b^2 + 10*a^6*b^3 + 10*a^5*b^
4 + 5*a^4*b^5 + a^3*b^6))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
) - 4*(2*a + 5*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/((a^2 + 2*a*b + b^2
)*sqrt(-a - b)*sgn(cos(f*x + e))) + 2*(2*a + 5*b)*log(abs(-(sqrt(a + b)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*
sqrt(a + b) + a - b))/((a^2 + 2*a*b + b^2)*sqrt(a + b)*sgn(cos(f*x + e)))
+ 16*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cot(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^3}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**3)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.420 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 213

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{(8a^2+28ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} + \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/8*(8*a^2+28*a*b+35*b
^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f+1/8*b*(4*a
^2+11*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)+1/8*(4*a+9*b)*cot(f*
x+e)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b*se
c(f*x+e)^2)^(1/2)
```

Mathematica [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.520$, Rules used = {3042, 4627, 25, 354, 114, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec^2(e+fx)}{2f}$$

$\Big\downarrow$ 114

$$-\frac{\frac{1}{2(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{\cos(e+fx)\left(5b\sec^2(e+fx)+4(a+b)\right)}{2\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{2(a+b)}}{2f}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(5b\sec^2(e+fx)+4(a+b)\right)}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\Big\downarrow$ 168

$$-\frac{\frac{\frac{4a+9b}{(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{\cos(e+fx)\left(8(a+b)^2+3b(4a+9b)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a+b}}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^2+3b(4a+9b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{2(a+b)}+\frac{4a+9b}{(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$\Big\downarrow$ 169

$$-\frac{\frac{\frac{\frac{2\int \frac{\cos(e+fx)\left(8(a+b)^3+b\left(4a^2+11ba-8b^2\right)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^2+11ab-8b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{4a+9b}{(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}}{4(a+b)}+\frac{1}{2(a+b)(1\ldots}}{2f}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^3+b\left(4a^2+11ba-8b^2\right)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^2+11ab-8b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{4a+9b}{(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}}{4(a+b)}+\frac{1}{2(a+b)(1-\cdots}}{2f}$$

$\Big\downarrow$ 174

$$-\frac{\frac{\frac{\frac{a\left(8a^2+28ab+35b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+8(a+b)^3\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^2+11ab-8b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{\cdots}{(a+b)\left(1-\sec\cdots\right.}}{4(a+b)}}{2f}$$

$\Big\downarrow$ 73

$$-\frac{\frac{\frac{\frac{\frac{2a\left(8a^2+28ab+35b^2\right)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{16(a+b)^3\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}-\frac{2b\left(4a^2+11ab-8b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{\cdots}{(a+b)\left(1-\sec^2\cdots\right.}}{4(a+b)}}{2f}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\frac{\frac{\frac{2a\left(8a^2+28ab+35b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{16(a+b)^3\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}-\frac{2b\left(4a^2+11ab-8b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{4a+9b}{(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{\cdots}}}{4(a+b)}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(1/(2*(a + b)*(1 - Sec[e + f*x]^2)^2*Sqrt[a + b*Sec[e + f*x]^2]) + ((
4*a + 9*b)/((a + b)*(1 - Sec[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]) + (((
-16*(a + b)^3*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a*
(8*a^2 + 28*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])
/Sqrt[a + b])/(a*(a + b)) - (2*b*(4*a^2 + 11*a*b - 8*b^2))/(a*(a + b)*Sqrt
[a + b*Sec[e + f*x]^2]))/(2*(a + b)))/(4*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4961 vs. $2(187) = 374$.

Time = 1.21 (sec) , antiderivative size = 4962, normalized size of antiderivative = 23.30

method	result	size
default	Expression too large to display	4962

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f/(a+b)^(15/2)/a^(3/2)/((-a*b)^(1/2)+a)/((-a*b)^(1/2)-a)/(b+a*cos(f*
x+e)^2)/(a+b*sec(f*x+e)^2)^(3/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(7/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*c
os(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+
e)*a)*a^6*b^2*(160*cos(f*x+e)^2+160*cos(f*x+e)+160+160*sec(f*x+e)+16*sec(f
*x+e)^2+16*sec(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(7/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)
+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^5
*b^3*(160*cos(f*x+e)^2+160*cos(f*x+e)+320+320*sec(f*x+e)+80*sec(f*x+e)^2+8
0*sec(f*x+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*ln
(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^4*b^4*(80*
cos(f*x+e)^2+80*cos(f*x+e)+320+320*sec(f*x+e)+160*sec(f*x+e)^2+160*sec(f*x
+e)^3)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*ln(4*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^3*b^5*(16*cos(f*x+e
)^2+16*cos(f*x+e)+160+160*sec(f*x+e)+160*sec(f*x+e)^2+160*sec(f*x+e)^3)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*ln(4*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^2*b^6*(32+32*sec(f*x+e)+8...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 829 vs. $2(187) = 374$.

Time = 4.40 (sec) , antiderivative size = 3501, normalized size of antiderivative = 16.44

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1771 vs. $2(187) = 374$.

Time = 2.18 (sec) , antiderivative size = 1771, normalized size of antiderivative = 8.31

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/64*(((((a^10*b + 7*a^9*b^2 + 21*a^8*b^3 + 35*a^7*b^4 + 35*a^6*b^5 + 21*a
^5*b^6 + 7*a^4*b^7 + a^3*b^8)*tan(1/2*f*x + 1/2*e)^2/(a^11*b*sgn(cos(f*x +
 e)) + 8*a^10*b^2*sgn(cos(f*x + e)) + 28*a^9*b^3*sgn(cos(f*x + e)) + 56*a^
8*b^4*sgn(cos(f*x + e)) + 70*a^7*b^5*sgn(cos(f*x + e)) + 56*a^6*b^6*sgn(co
s(f*x + e)) + 28*a^5*b^7*sgn(cos(f*x + e)) + 8*a^4*b^8*sgn(cos(f*x + e)) +
 a^3*b^9*sgn(cos(f*x + e))) - (13*a^10*b + 101*a^9*b^2 + 333*a^8*b^3 + 605
*a^7*b^4 + 655*a^6*b^5 + 423*a^5*b^6 + 151*a^4*b^7 + 23*a^3*b^8)/(a^11*b*s
gn(cos(f*x + e)) + 8*a^10*b^2*sgn(cos(f*x + e)) + 28*a^9*b^3*sgn(cos(f*x +
 e)) + 56*a^8*b^4*sgn(cos(f*x + e)) + 70*a^7*b^5*sgn(cos(f*x + e)) + 56*a^
6*b^6*sgn(cos(f*x + e)) + 28*a^5*b^7*sgn(cos(f*x + e)) + 8*a^4*b^8*sgn(cos
(f*x + e)) + a^3*b^9*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 + (23*a^10
*b + 145*a^9*b^2 + 331*a^8*b^3 + 349*a^7*b^4 + 245*a^6*b^5 + 323*a^5*b^6 +
 425*a^4*b^7 + 271*a^3*b^8 + 64*a^2*b^9)/(a^11*b*sgn(cos(f*x + e)) + 8*a^1
0*b^2*sgn(cos(f*x + e)) + 28*a^9*b^3*sgn(cos(f*x + e)) + 56*a^8*b^4*sgn(co
s(f*x + e)) + 70*a^7*b^5*sgn(cos(f*x + e)) + 56*a^6*b^6*sgn(cos(f*x + e))
+ 28*a^5*b^7*sgn(cos(f*x + e)) + 8*a^4*b^8*sgn(cos(f*x + e)) + a^3*b^9*sgn
(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 - (11*a^10*b + 91*a^9*b^2 + 315*a^
8*b^3 + 659*a^7*b^4 + 985*a^6*b^5 + 1081*a^5*b^6 + 801*a^4*b^7 + 345*a^3*b
^8 + 64*a^2*b^9)/(a^11*b*sgn(cos(f*x + e)) + 8*a^10*b^2*sgn(cos(f*x + e))
+ 28*a^9*b^3*sgn(cos(f*x + e)) + 56*a^8*b^4*sgn(cos(f*x + e)) + 70*a^7*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^5}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**5)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.421 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{(3a+5b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-1/2*(3*a+
5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-(a+b
)*tan(f*x+e)^3/a/b/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*(3*a+2*b)*tan(f*x+e)*(
a+b+b*tan(f*x+e)^2)^(1/2)/a/b^2/f
```

Mathematica [A] (verified)

Time = 6.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(\frac{2b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}+\frac{a(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+f}{4\sqrt{2}ab^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

$$+\frac{(a+2b+a\cos(2(e+fx)))\,(3a^2+6ab+2b^2+(3a^2+4ab+2b^2)\cos(2(e+fx)))\sec^4(e+fx)\tan(e+f}{8ab^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*(((2*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
])/Sqrt[a] + (a*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*
Sin[e + f*x]^2]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*
x]^3)/(Sqrt[2]*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2
*(e + f*x)])*(3*a^2 + 6*a*b + 2*b^2 + (3*a^2 + 4*a*b + 2*b^2)*Cos[2*(e + f
*x)])*Sec[e + f*x]^4*Tan[e + f*x])/(8*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 372, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^6}{\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\int \frac{\tan^2(e+fx)\left((3a+2b)\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{(a+b)\tan^3(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \; 444$$

$$\frac{\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int \frac{a(3a+5b)\tan^2(e+fx)+(a+b)(3a+2b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{ab}-\frac{(a+b)\tan^3(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{2b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a(3a+5b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{ab}-\frac{(a+b}{ab\sqrt{a+}}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{2b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a(3a+5b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{ab}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{2b^2\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{a(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{2b}}{ab}-\frac{(a+}{ab\sqrt{a}}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{2b^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{a(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{2b}}{ab}-\frac{(a+b)\,\text{ta}}{ab\sqrt{a+b\,\text{ta}}}}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{2b^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}+\frac{a(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{2b}}{ab}-\frac{(a+b)\tan^3(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(((a + b)*Tan[e + f*x]^3)/(a*b*Sqrt[a + b + b*Tan[e + f*x]^2])) + (-1/2*
((2*b^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqr
t[a] + (a*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
+ f*x]^2]])/Sqrt[b])/b + ((3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
f*x]^2])/(2*b))/(a*b))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1295 vs. $2(152) = 304$.

Time = 13.58 (sec) , antiderivative size = 1296, normalized size of antiderivative = 7.53

method	result	size
default	Expression too large to display	1296

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((cos(f*x+e)^4+2*cos(f*x+e)^2+1/2)*tan(f*x+e)*sec(f*x+e)^4*a*(-a)^(1/2
)*b^(7/2)+(2*sec(f*x+e)^2*tan(f*x+e)+2*tan(f*x+e))*a^2*(-a)^(1/2)*b^(5/2)+
3/2*b^(3/2)*(-a)^(1/2)*a^3*tan(f*x+e)+b^(9/2)*(-a)^(1/2)*tan(f*x+e)*sec(f*
x+e)^2-1/4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a^3*b*ln
(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)
+1))*(3+3*sec(f*x+e))-1/4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)
^(1/2)*a^2*b^2*ln(4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a
-a-b)/(sin(f*x+e)+1))*(5+5*sec(f*x+e)+3*sec(f*x+e)^2+3*sec(f*x+e)^3)-1/4*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^3*ln(4*(b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(5*sec(
f*x+e)^2+5*sec(f*x+e)^3)-1/4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
-a)^(1/2)*a^3*b*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)
*a+a+b)/(sin(f*x+e)-1))*(3+3*sec(f*x+e))-1/4*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(-a)^(1/2)*a^2*b^2*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(5+5*sec(f*x+e)+3*sec(f*x+e)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(152) = 304$.

Time = 1.49 (sec) , antiderivative size = 1895, normalized size of antiderivative = 11.02

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32
*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*co
s(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - ((3*a^4 + 5*a^3*b)
*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*log(((a^2 -
6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(a^2*b^2 + (3*a^3*b +
 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)),
 -1/8*(2*((3*a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x
 + e))*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
b^2)*sin(f*x + e))) + (a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*l
og(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a
*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*...
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^6{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^6}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**6)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.422 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-(a+b)*tan(f*x+e)/a/
b/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 3.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.73

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\left(\frac{b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e}{2\sqrt{2}abf\,(a+b\sec^2(e+fx))^{3/2}}$$
$$-\frac{(a+b)(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2abf\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(((b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[a
] + (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqr
t[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a*b*
f*(a + b*Sec[e + f*x]^2)^(3/2)) - ((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[e + f*x]^2*Tan[e + f*x])/(2*a*b*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4629, 2075, 372, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(e+fx)^4}{(a+b\sec(e+fx)^2)^{3/2}}dx$$
$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{a\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{a\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{a\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{b\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{b\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(((b*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a
] + (a*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqr
t[b])/(a*b) - ((a + b)*Tan[e + f*x])/(a*b*Sqrt[a + b + b*Tan[e + f*x]^2]))
/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(102) = 204$.

Time = 6.14 (sec) , antiderivative size = 921, normalized size of antiderivative = 7.94

method	result	size
default	Expression too large to display	921

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((-sec(f*x+e)^2*tan(f*x+e)-tan(f*x+e))*b^(5/2)*(-a)^(1/2)*a-tan(f*x+e)
*a^2*b^(3/2)*(-a)^(1/2)-b^(7/2)*(-a)^(1/2)*tan(f*x+e)*sec(f*x+e)^2+1/2*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a^2*b*ln(4*(b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(sec(f*x+
e)+1)+1/2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^2*ln(
4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+
1))*(sec(f*x+e)^2+sec(f*x+e)^3)+1/2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(-a)^(1/2)*a^2*b*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin
(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(sec(f*x+e)+1)+1/2*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^2*ln(-4*(b^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(sec(f*x+e)^2+sec(f*x+e)^3)+1
/2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)*ln(4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(2*sec(f*x+e)^2+2*sec(f
*x+e)^3)+1/2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)*a*ln(4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(102) = 204$.

Time = 0.57 (sec) , antiderivative size = 1655, normalized size of antiderivative = 14.27

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 2*(a^3*cos(f*x
+ e)^2 + a^2*b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f
*x + e)^4))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f), -1/8*(8*(a^2*b + a*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) -
4*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + (a*b^2*cos(f*x + e)^2 + b^3)
*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 +
32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2
*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5...
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^4}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**4)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.423 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}+\frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+tan(f*x+e
)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(71) = 142.

Time = 1.69 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.38

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx)))-\sqrt{2}\sqrt{a}\sqrt{a}\right.}{4a^{3/2}\sqrt{a+b}f\,(a+b\sec^2(e+fx))^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(ArcSin[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a]*Sqr
t[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(a^(3
/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x
]^2)/(a + b)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4629, 2075, 373, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(e+fx)^2}{(a+b\sec(e+fx)^2)^{3/2}}dx$$
$$\Big\downarrow \text{4629}$$
$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{2075}$$
$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{373}$$
$$\frac{\frac{\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}}{f}$$

input

```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output

```
(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/a^(3/2))
+ Tan[e + f*x]/(a*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(63) = 126$.

Time = 2.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.97

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\sqrt{\frac{b+a\cos(fx+}{(1+\cos(fx+}}}{fa\sqrt{-a}\,(}$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(cos(f*x+e)*ln(4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)-(-a)^(1/2)*sin(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*
a))/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(63) = 126$.

Time = 0.26 (sec) , antiderivative size = 548, normalized size of antiderivative = 7.72

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \left(a\cos(fx+e)^2+b\right)\sqrt{-a}\,\mathrm{l}\right.$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) - (a*cos(f*x + e)^2 + b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^
3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), 1/4
*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e
) + (a*cos(f*x + e)^2 + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a
^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*
b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^3*f*cos(f*x + e)^
2 + a^2*b*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2005 vs. $2(63) = 126$.

Time = 0.35 (sec) , antiderivative size = 2005, normalized size of antiderivative = 28.24

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*a*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e)
, a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 2*a*cos(1/2*
arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e) + 2*(a*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 - a*cos(2*f*x + 2*e))*sin(1/2*ar
ctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e
) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin
(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*
a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x
 + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4
*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((cos(1/2*arctan2(a*sin(4*f*
x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*
cos(2*f*x + 2*e) + a))^2 + sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b
)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)
)^2)*arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*
f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b
)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x +
2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
 + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*si...
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^2}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**2)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.424 $\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+
e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.

Time = 0.94 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+\cdots\right.}{4a^{3/2}(a+b)f\left(a+b\sec^2(e+f\cdots\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt
[a]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/
2)*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/
(a + b)])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4616, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/a^(3/2) - (
b*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(69) = 138$.

Time = 3.42 (sec) , antiderivative size = 465, normalized size of antiderivative = 6.04

method	result
default	$-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(-1-\sec(fx+e))-\sqrt{\frac{b+a}{(1+}}$

input
```
int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-1-sec(f*x+e))-((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-1-sec(f*x+e)-sec(f*x+e)^2-sec(f*x+e)
^3)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(-sec(f*x+e)^2-sec(f*x+e)^
3)-(-a)^(1/2)*a*b*tan(f*x+e)-b^2*(-a)^(1/2)*tan(f*x+e)*sec(f*x+e)^2)/(a+b)
/a/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(69) = 138$.

Time = 0.26 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.81

$$\int \frac{1}{(a + b\sec^2(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f
*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e
)^2 + a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^
2 + (a^3*b + a^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a + b\sec^2(e + fx))^{3/2}}\,dx = \int \frac{1}{(a + b\sec^2(e + fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. $2(69) = 138$.

Time = 0.37 (sec) , antiderivative size = 2055, normalized size of antiderivative = 26.69

$$\int \frac{1}{(a + b\sec^2(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2
*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2
*e) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2
*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2
*(a*b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) +
2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x +
 2*e) + a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b +
4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 +
 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))^(1/4)*(((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(
2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 +
(a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*si
n(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a
^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a
^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*...
```

Giac [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2b+a}}{\sec\left(fx+e\right)^4b^2+2\sec\left(fx+e\right)^2ab+a^2}dx$$

input
```
int(1/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)/(sec(e + f*x)**4*b**2 + 2*sec(e + f*x)**2*
a*b + a**2),x)
```

3.425 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x
+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-(a-b)*cot(f*x+e)*(a+b+b*tan(f*x+e
)^2)^(1/2)/a/(a+b)^2/f
```

Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.53

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}a^{3/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(a+2b+a\cos(2(e+fx)))\left(a^2+2ab-b^2+(a^2+b^2)\cos(2(e+fx))\right)\csc(e+fx)\sec^3(e+fx)}{4a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2
*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*S
ec[e + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(a^2 + 2*a*b - b^2
 + (a^2 + b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x]^3)/(4*a*(a + b)
^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 374, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)\left(a+b(\tan^2(e+fx)+1)\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left(-2b\tan^2(e+fx)+a-b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \ 445$$

$$\frac{\frac{-\frac{\int \frac{(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{-(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{-\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input

```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output

```
(-((b*Cot[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) + (-(((a +
 b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a]
) - ((a - b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/(a*(a +
 b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(109) = 218$.

Time = 5.64 (sec) , antiderivative size = 661, normalized size of antiderivative = 5.55

method	result
default	$-\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a^3(\sec(fx+e)+1)+\sqrt{\frac{b+a\,\mathrm{c}}{(1+\mathrm{c}}}}{}$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a+b)^2/a/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(3/2)*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a^3*(sec(f*x+e)+1)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*
a)*a^2*b*(sec(f*x+e)^3+sec(f*x+e)^2+2*sec(f*x+e)+2)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a*b^2*(1+sec(f*x+e)+2*sec(f*x+e)^2+2*sec(f*x+e)^3)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3*(sec(f*x+e)^2+sec(f*x+e)^3)+(-a)^(1
/2)*a^3*cot(f*x+e)+2*a^2*b*(-a)^(1/2)*sec(f*x+e)*csc(f*x+e)+(cos(f*x+e)^4-
cos(f*x+e)^2+1)*(-a)^(1/2)*a*b^2*sec(f*x+e)^3*csc(f*x+e)-(-a)^(1/2)*b^3*ta
n(f*x+e)*sec(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(109) = 218.

Time = 0.56 (sec) , antiderivative size = 741, normalized size of antiderivative = 6.23

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*((a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sq
rt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*
(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^
2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(
f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))*sin(f*x + e) + 8*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*cos(f*
x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b + a^
3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)),
1/4*((a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt
(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2
 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
 e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^
2*b - a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((
(a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)
*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^2{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^2}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**2)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.426 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^3f}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^2f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+
e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(3*a-b)*(a+3*b)*cot(f*x+e)*(
a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/3*(a-3*b)*cot(f*x+e)^3*(a+b+b*tan(
f*x+e)^2)^(1/2)/a/(a+b)^2/f
```

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}a^{3/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(a+2b+a\cos(2e+2fx))^2\sec^3(e+fx)\left(\frac{(4a+9b)\csc(e+fx)}{12(a+b)^3f}-\frac{\csc^3(e+fx)}{12(a+b)^2f}-\frac{b^3\sin(e+fx)}{2a(a+b)^3f(a+2b+a\cos(2e+2fx))}\right)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b +
a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a^(3/2)*f*(a + b*Sec[
e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^3*(((4
*a + 9*b)*Csc[e + f*x])/(12*(a + b)^3*f) - Csc[e + f*x]^3/(12*(a + b)^2*f)
 - (b^3*Sin[e + f*x])/(2*a*(a + b)^3*f*(a + 2*b + a*Cos[2*e + 2*f*x]))))/(
a + b*Sec[e + f*x]^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 374, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}d\tan(e+fx)}{f}$$

$\downarrow$ 2075

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$\downarrow$ 374

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-4b\tan^2(e+fx)+a-3b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b\cot^3(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\downarrow$ 445

$$\frac{\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2(a-3b)b\tan^2(e+fx)+(3a-b)(a+3b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)}-\frac{b\cot^3(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$\downarrow$ 445

$$\frac{\frac{-\frac{-\frac{\int \frac{3(a+b)^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)}-\frac{\cdots}{a(a+b\cdots}}{f}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{-3(a+b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)}-\cdots}{f}$$

$\downarrow$ 291

$$\frac{\frac{-\frac{-3(a+b)^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)}-\frac{\cdots}{a(a\cdots}}{f}$$

$\downarrow$ 216

$$\frac{-\frac{-\frac{-\frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}-\frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)}-\frac{b\cot^3(e+}{a(a+b)\sqrt{a+b\tan}}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-((b*Cot[e + f*x]^3)/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) + (-1/3*
((a - 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - ((-3*(
a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sq
rt[a] - ((3*a - b)*(a + 3*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/
(a + b))/(3*(a + b)))/(a*(a + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(158) = 316$.

Time = 10.02 (sec) , antiderivative size = 918, normalized size of antiderivative = 5.28

method	result	size
default	Expression too large to display	918

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((2*sec(f*x+e)*csc(f*x+e)^3-3*cot(f*x+e)^3)*b*a^3*(-a)^(1/2)+(-6*cos(f
*x+e)^4+4*cos(f*x+e)^2+1)*sec(f*x+e)^3*csc(f*x+e)^3*b^2*a^2*(-a)^(1/2)+(-c
os(f*x+e)^6+2*cos(f*x+e)^4-4*cos(f*x+e)^2+8/3)*sec(f*x+e)^3*csc(f*x+e)^3*b
^3*a*(-a)^(1/2)+(-4/3*cot(f*x+e)^3+cot(f*x+e)*csc(f*x+e)^2)*a^4*(-a)^(1/2)
-(-a)^(1/2)*b^4*tan(f*x+e)*sec(f*x+e)^2-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)*a^4*(-3-3*sec(f*x+e))-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a
)*a^3*b*(-9-9*sec(f*x+e)-3*sec(f*x+e)^2-3*sec(f*x+e)^3)-1/3*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2*(-9-9*sec(f*x+e)-9*sec(f*x+e)^2-9*sec(f*
x+e)^3)-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3*(-3-3*sec(f*x+e)
-9*sec(f*x+e)^2-9*sec(f*x+e)^3)-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(158) = 316$.

Time = 1.81 (sec) , antiderivative size = 1061, normalized size of antiderivative = 6.10

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*
a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*
sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 3
2*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*
b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)
^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*co
s(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))*sin(f*x + e) - 8*((4*a^4 + 9*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - (3*a^4
 + 4*a^3*b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (3*a^3*b + 8*a^2*b^2 -
3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^6
 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^
3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b
^4)*f)*sin(f*x + e)), -1/12*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*
x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 -
b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a
*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 -
(a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*a^4 +
9*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - (3*a^4 + 4*a^3*b - 9*a^2*b^2 + 6*a*...
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^4}{\sec(fx+e)^4 b^2+2\sec(fx+e)^2 ab+a^2}dx$$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**4)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.427 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 241

$$\begin{aligned}\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} \\ &\quad - \frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} \\ &\quad - \frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^4f} \\ &\quad + \frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^3f} \\ &\quad - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f}\end{aligned}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x
+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^3+55*a^2*b+73*a*b^2-
15*b^3)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^4/f+1/15*(5*a^2+14*a
*b-15*b^2)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/5*(a-5*b)
*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^2/f
```

Mathematica [A] (verified)

Time = 4.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.98

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}a^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(a+2b+a\cos(2(e+fx)))^2\left(\frac{30b^4}{a(a+2b+a\cos(2(e+fx)))}-(23a^2+80ab+90b^2)\csc^2(e+fx)+(a+b)(11a+2\right.}{60(a+b)^4f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2
*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*S
ec[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^2*((30*b^4)/(a*(a
+ 2*b + a*Cos[2*(e + f*x)])) - (23*a^2 + 80*a*b + 90*b^2)*Csc[e + f*x]^2 +
 (a + b)*(11*a + 20*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e
+ f*x]^2*Tan[e + f*x])/(60*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 374, 445, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(-6b\tan^2(e+fx)+a-5b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b\cot^5(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int \frac{\cot^4(e+fx)\left(5a^2+14ba-15b^2+4(a-5b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)}-\frac{(a-5b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{a(a+b)}-\frac{b\cot^5(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(15a^3+55ba^2+73b^2a-15b^3+2b\left(5a^2+14ba-15b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)}-\frac{(a-5b)\,\mathrm{c}}{}}{a(a+b)}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{-\frac{-\frac{\int \frac{15(a+b)^4}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a-}}{3(a+b)}}{5(a+b)}}{a(a+b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{-\frac{-15(a+b)^3\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e\cdots}{3(a\cdots}}{5(a+b)}}{a(a+b)}}{f}$$

$\big\downarrow$ 291

$$-\frac{-\frac{-\frac{-15(a+b)^3\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\cdots}{3(a+b)}}{5(a+b)}}{a(a+b)}}{f}$$

$\big\downarrow$ 216

$$-\frac{-\frac{-\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}-\frac{-\frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-\frac{15(a+b)^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}\cdots}\right.}{\sqrt{a}}}{3(a+b)}}{5(a+b)}}{a(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-((b*Cot[e + f*x]^5)/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) + (-1/5*
((a - 5*b)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - (-1/3*
((5*a^2 + 14*a*b - 15*b^2)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/
(a + b) - ((-15*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/Sqrt[a] - ((15*a^3 + 55*a^2*b + 73*a*b^2 - 15*b^3)*Cot[e +
f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/(3*(a + b)))/(5*(a + b)))/(a
*(a + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(221) = 442$.

Time = 15.24 (sec) , antiderivative size = 1161, normalized size of antiderivative = 4.82

method	result	size
default	Expression too large to display	1161

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a+b)^4/a/(-a)^(1/2)*(sin(f*x+e)^5*cos(f*x+e)^2*(15*cos(f*x+e)+15)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^5+sin(f*x+e)^5*(60*cos(f*x+e)^3
+60*cos(f*x+e)^2+15*cos(f*x+e)+15)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)
*a^4*b+sin(f*x+e)^5*(90*cos(f*x+e)^3+90*cos(f*x+e)^2+60*cos(f*x+e)+60)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b^2+sin(f*x+e)^5*(60*cos(f*x+e)^3
+60*cos(f*x+e)^2+90*cos(f*x+e)+90)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)
*a^2*b^3+sin(f*x+e)^5*(15*cos(f*x+e)^3+15*cos(f*x+e)^2+60*cos(f*x+e)+60)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^4+sin(f*x+e)^5*(15*cos(f*x+e)+1
5)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(221) = 442$.

Time = 6.57 (sec) , antiderivative size = 1517, normalized size of antiderivative = 6.29

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/120*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^
6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a
^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^
3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*
cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4
- 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*
x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2
)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8
*((23*a^5 + 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - (35*a^5 + 1
06*a^4*b + 80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + (15*a^5 +
20*a^4*b - 56*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^3 + (15*a^4*b
 + 55*a^3*b^2 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4
)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4
- a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a
^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*
a^3*b^4 + a^2*b^5)*f)*sin(f*x + e)), 1/60*(15*((a^5 + 4*a^4*b + 6*a^3*b^2
+ 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4...
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\cot\left(fx+e\right)^6}{\sec\left(fx+e\right)^4 b^2+2\sec\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**6)/(sec(e + f*x)**4*b**2 +
2*sec(e + f*x)**2*a*b + a**2),x)
```

3.428 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(a+b)^2}{3ab^2f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3*(a+b)^2/a/b^2/f/(
a+b*sec(f*x+e)^2)^(3/2)+(1/a^2-1/b^2)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.93

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{4(a+}{3f\,(a+b\sec^2(e+fx))^{5/2}\left(8(a+b)\operatorname{AppellF1}\left(3,\frac{1}{2},\frac{5}{2},4,\sin^2(e+fx),\frac{a\sin^2}{a}\right.\right.}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(4*(a + b)*AppellF1[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Tan[e + f*x]^6)/(3*f*(a + b*Sec[e + f*x]^2)^(5/2)*(8*(a + b)*AppellF
1[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*Appel
lF1[4, 1/2, 7/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*
AppellF1[4, 3/2, 5/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[
e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^5}{\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4627$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 354$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 98$$

$$\frac{\int \left(-\frac{(a+b)^2}{ab\left(b\sec^2(e+fx)+a\right)^{5/2}}+\frac{\cos(e+fx)}{a^2\sqrt{b\sec^2(e+fx)+a}}+\frac{a^2-b^2}{a^2b\left(b\sec^2(e+fx)+a\right)^{3/2}}\right)d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)}{\sqrt{a+b\sec^2(e+fx)}}+\frac{2(a+b)^2}{3ab^2(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(5/2) + (2*(a + b)^2)/
(3*a*b^2*(a + b*Sec[e + f*x]^2)^(3/2)) + (2*(a^(-2) - b^(-2)))/Sqrt[a + b*
Sec[e + f*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(85) = 170$.

Time = 5.46 (sec) , antiderivative size = 1570, normalized size of antiderivative = 16.19

method	result	size
default	Expression too large to display	1570

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/a^(5/2)/((-a*b)^(1/2)-a)^2/((-a*b)^(1/2)+a)^2/b^2/(cos(f*x+e)^2*a^3
+(2*cos(f*x+e)^2+1)*a^2*b+(cos(f*x+e)^2+2)*b^2*a+b^3)/(a+b*sec(f*x+e)^2)^(
5/2)*(3*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e
)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^9*b^2*(cos(f*x+e)^2+cos(f*x+e)
)+3*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*
a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^8*b^3*(3+4*cos(f*x+e)^2+4*cos(f*x+
e)+3*sec(f*x+e))+9*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2
)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f
*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^7*b^4*(4+2*cos(f*x+
e)^2+2*cos(f*x+e)+4*sec(f*x+e)+sec(f*x+e)^2+sec(f*x+e)^3)+3*ln(4*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*a^6*b^5*(18+4*cos(f*x+e)^2+4*cos(f*x+e)+18*sec(f*x+e)+
12*sec(f*x+e)^2+12*sec(f*x+e)^3+sec(f*x+e)^4+sec(f*x+e)^5)+3*ln(4*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^5*b^6*(4*sec(f*x+e)^5+4*sec(f*x+e)^4+18*sec(f*x+e)^
3+18*sec(f*x+e)^2+cos(f*x+e)^2+12*sec(f*x+e)+cos(f*x+e)+12)+9*ln(4*((b+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(85) = 170$.

Time = 0.79 (sec) , antiderivative size = 564, normalized size of antiderivative = 5.81

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(a^2b^2\cos\left(fx+e\right)^4+2\,ab^3\cos\left(fx+e\right)^2+b^4\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+\right.\right.}{}\right.$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b^4)*sqrt(a)*l
og(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x
 + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^
2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(2*(a^4 - a^3*b - 2*a^2*
b^2)*cos(f*x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x
 + e)^2 + a^3*b^4*f), 1/12*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x +
e)^2 + b^4)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)
^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(
f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 4*(2*(a^4 - a^3*b - 2*a^2*
b^2)*cos(f*x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x
 + e)^2 + a^3*b^4*f)]
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(85) = 170.

Time = 1.56 (sec) , antiderivative size = 466, normalized size of antiderivative = 4.80

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*(((((2*a^11*sgn(cos(f*x + e)) + a^10*b*sgn(cos(f*x + e)) - 4*a^9*b^2*s
gn(cos(f*x + e)) - 3*a^8*b^3*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^
10*b^2) - 3*(2*a^11*sgn(cos(f*x + e)) - 3*a^10*b*sgn(cos(f*x + e)) - 4*a^9
*b^2*sgn(cos(f*x + e)) + a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))*tan(1/2*f*
x + 1/2*e)^2 + 3*(2*a^11*sgn(cos(f*x + e)) - 3*a^10*b*sgn(cos(f*x + e)) -
4*a^9*b^2*sgn(cos(f*x + e)) + a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))*tan(1
/2*f*x + 1/2*e)^2 - (2*a^11*sgn(cos(f*x + e)) + a^10*b*sgn(cos(f*x + e)) -
 4*a^9*b^2*sgn(cos(f*x + e)) - 3*a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))/(a
*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)^(3/2) + 6*arctan(-1/2*(sqrt(a
+ b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(f*x + e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^5}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^5}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**5)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.429 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-1/3*(a+b)/a/b/f/(a+b*s
ec(f*x+e)^2)^(3/2)-1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.79 (sec) , antiderivative size = 522, normalized size of antiderivative = 5.87

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(-\frac{4(a+3b+a\cos(2(e+fx)))}{b^2(a+2b+a\cos(2(e+fx)))^{3/2}}+\frac{2(\cdots}{b^2\cdots}\right.}{\cdots}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*((-4*(a + 3*b + a*Cos
[2*(e + f*x)]))/(b^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) + (2*(a + b + (
a - 2*b)*Cos[2*(e + f*x)]))/(b^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (
Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e +
 f*x)) + a^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x
)))^2 - 6*a^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x -
 12*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x))
+ 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(
e + f*x)))^2])*Sec[e + f*x])/a^(5/2)))/(192*f*(a + b*Sec[e + f*x]^2)^(5/2)
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4627, 25, 354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 354

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{5/2}}d\sec^2(e+fx)}{2f}$$

$\Big\downarrow$ 87

$$-\frac{\frac{\int \frac{\cos(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a}+\frac{2(a+b)}{3ab(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 61

$$-\frac{\frac{\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{a}+\frac{2(a+b)}{3ab(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 73

$$-\frac{\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{a}+\frac{2(a+b)}{3ab(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2(a+b)}{3ab(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/2*((2*(a + b))/(3*a*b*(a + b*Sec[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt
[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sec[e + f*x]^2]
))/a)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(77) = 154$.

Time = 1.41 (sec) , antiderivative size = 1541, normalized size of antiderivative = 17.31

method	result	size
default	Expression too large to display	1541

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/a^(5/2)/((-a*b)^(1/2)-a)^2/((-a*b)^(1/2)+a)^2/b/(cos(f*x+e)^2*a^3+(2
*cos(f*x+e)^2+1)*a^2*b+(cos(f*x+e)^2+2)*b^2*a+b^3)/(a+b*sec(f*x+e)^2)^(5/2
)*(3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^9*b*(cos(f*x+e)^2+cos(f*x+e))+3*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^8*b^2*(3+4*cos(f*x+e)^2+4*cos(f*x+e)+3*
sec(f*x+e))+9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)*a)*a^7*b^3*(4+2*cos(f*x+e)^2+
2*cos(f*x+e)+4*sec(f*x+e)+sec(f*x+e)^2+sec(f*x+e)^3)+3*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4
*cos(f*x+e)*a)*a^6*b^4*(18+4*cos(f*x+e)^2+4*cos(f*x+e)+18*sec(f*x+e)+12*se
c(f*x+e)^2+12*sec(f*x+e)^3+sec(f*x+e)^4+sec(f*x+e)^5)+3*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^(1/2)*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+
4*cos(f*x+e)*a)*a^5*b^5*(4*sec(f*x+e)^5+4*sec(f*x+e)^4+18*sec(f*x+e)^3+18*
sec(f*x+e)^2+cos(f*x+e)^2+12*sec(f*x+e)+cos(f*x+e)+12)+9*((b+a*cos(f*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(77) = 154.

Time = 0.65 (sec) , antiderivative size = 522, normalized size of antiderivative = 5.87

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(a^2b\cos(fx+e)^4+2\,ab^2\cos(fx+e)^2+b^3\right)\sqrt{a}\log\left(128\,a^4\cos(fx+e\right.}{}\right.$$

$$-\frac{3\left(a^2b\cos(fx+e)^4+2\,ab^2\cos(fx+e)^2+b^3\right)\sqrt{-a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e}{\cos(fx+e}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right.}{12\left(a^5bf\cos(fx+e)^4+2\,a^4b^2f\cos(fx+\right.}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log
(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x +
 e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*
b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(3*a*b^2*cos(f*x + e)^2 +
(a^3 + 4*a^2*b)*cos(f*x + e)^4)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
))/(a^5*b*f*cos(f*x + e)^4 + 2*a^4*b^2*f*cos(f*x + e)^2 + a^3*b^3*f), -1/1
2*(3*(a^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan
(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*
x + e)^2 + a*b^2)) + 4*(3*a*b^2*cos(f*x + e)^2 + (a^3 + 4*a^2*b)*cos(f*x +
 e)^4)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*b*f*cos(f*x + e)^
4 + 2*a^4*b^2*f*cos(f*x + e)^2 + a^3*b^3*f)]
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(77) = 154.

Time = 1.30 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.70

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\left(\left(\frac{\left(a^{10}b\operatorname{sgn}(\cos(fx+e))+4\,a^9b^2\operatorname{sgn}(\cos(fx+e))+3\,a^8b^3\operatorname{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^{10}b^2} - \frac{3\left(a^{10}b\operatorname{sgn}(\cos(fx+\right.}{}\right.\right.}{}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*(((((a^10*b*sgn(cos(f*x + e)) + 4*a^9*b^2*sgn(cos(f*x + e)) + 3*a^8*b^
3*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2/(a^10*b^2) - 3*(a^10*b*sgn(cos
(f*x + e)) + 4*a^9*b^2*sgn(cos(f*x + e)) - a^8*b^3*sgn(cos(f*x + e)))/(a^1
0*b^2))*tan(1/2*f*x + 1/2*e)^2 + 3*(a^10*b*sgn(cos(f*x + e)) + 4*a^9*b^2*s
gn(cos(f*x + e)) - a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))*tan(1/2*f*x + 1/
2*e)^2 - (a^10*b*sgn(cos(f*x + e)) + 4*a^9*b^2*sgn(cos(f*x + e)) + 3*a^8*b
^3*sgn(cos(f*x + e)))/(a^10*b^2))/(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b)^(3/2) - 6*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-
a)*a^2*sgn(cos(f*x + e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^3}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2 b+a^3}\,dx$$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**3)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.430 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sec(f*x+
e)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.54 (sec) , antiderivative size = 521, normalized size of antiderivative = 6.28

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(-\frac{4(a+3b+a\cos(2(e+fx)))}{b^2(a+2b+a\cos(2(e+fx)))^{3/2}} + \frac{6(\cdots}{b^2\cdots}\right.}{\cdots}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*((-4*(a + 3*b + a*Cos
[2*(e + f*x)]))/(b^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) + (6*(a + b + (
a - 2*b)*Cos[2*(e + f*x)]))/(b^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) + (
Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e +
 f*x)) + a^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x
)))^2 - 6*a^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x -
 12*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x))
+ 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(
e + f*x)))^2])*Sec[e + f*x])/a^(5/2)))/(192*f*(a + b*Sec[e + f*x]^2)^(5/2)
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4627, 243, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \frac{\cos(e+fx)}{(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{\cos(e+fx)}{(b\sec^2(e+fx)+a)^{5/2}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 61$$

$$\frac{\frac{\int \frac{\cos(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a}+\frac{2}{3a(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 61$$

$$\frac{\frac{\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{a}+\frac{2}{3a(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{a}+\frac{2}{3a(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(2/(3*a*(a + b*Sec[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Sec[e + f*
x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sec[e + f*x]^2]))/a)/(2*f)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{1}{3af\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{a^2f\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f\,a^{\frac{5}{2}}}$	86
default	$\frac{1}{3af\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{a^2f\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f\,a^{\frac{5}{2}}}$	86

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a/f/(a+b*sec(f*x+e)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/f/a^(5
/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(71) = 142$.

Time = 0.53 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.95

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{a}\log\left(128\,a^4\cos(fx+e)^8\right.}{}\right.$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128
*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^
4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*co
s(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*(4*a^2*cos(f*x + e)^4 + 3*a*b*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*f*cos(f*
x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), 1/12*(3*(a^2*cos(f*x + e
)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^
4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(4
*a^2*cos(f*x + e)^4 + 3*a*b*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*
f)]
```

Sympy [A] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{6af\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}+\frac{b}{2a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{-a}}\right)}{2a^2f\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left(\sec^2(e+fx)\right)}{2a^{\frac{5}{2}}f} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Piecewise((2*(b/(6*a*f*(a + b*sec(e + f*x)**2)**(3/2)) + b/(2*a**2*f*sqrt(
a + b*sec(e + f*x)**2)) + b*atan(sqrt(a + b*sec(e + f*x)**2)/sqrt(-a))/(2*
a**2*f*sqrt(-a)))/b, Ne(b, 0)), (log(sec(e + f*x)**2)/(2*a**(5/2)*f), True
))
```

Maxima [F]

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tan\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(71) = 142$.

Time = 1.07 (sec) , antiderivative size = 370, normalized size of antiderivative = 4.46

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=$$

$$-\frac{\left(\left(\left(\frac{\left(4\,a^9b^2\mathrm{sgn}(\cos(fx+e))+3\,a^8b^3\mathrm{sgn}(\cos(fx+e))\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^{10}b^2}-\frac{3\left(4\,a^9b^2\mathrm{sgn}(\cos(fx+e))-a^8b^3\mathrm{sgn}(\cos(fx+e))\right)}{a^{10}b^2}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\frac{3\left(4\,a^9b^2\mathrm{sgn}(\mathrm{co}\right.}{}}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\right.}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(((((4*a^9*b^2*sgn(cos(f*x + e)) + 3*a^8*b^3*sgn(cos(f*x + e)))*tan(1
/2*f*x + 1/2*e)^2/(a^10*b^2) - 3*(4*a^9*b^2*sgn(cos(f*x + e)) - a^8*b^3*sg
n(cos(f*x + e)))/(a^10*b^2))*tan(1/2*f*x + 1/2*e)^2 + 3*(4*a^9*b^2*sgn(cos
(f*x + e)) - a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))*tan(1/2*f*x + 1/2*e)^2
 - (4*a^9*b^2*sgn(cos(f*x + e)) + 3*a^8*b^3*sgn(cos(f*x + e)))/(a^10*b^2))
/(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)^(3/2) - 6*arctan(-1/2*(sqrt
(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2
*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
+ a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(f*x + e))))/f
```

Mupad [B] (verification not implemented)

Time = 19.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\frac{a+\frac{b}{\cos(e+f\,x)^2}}{a^2}+\frac{1}{3\,a}}{f\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}} - \frac{\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{a^{5/2}\,f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
((a + b/cos(e + f*x)^2)/a^2 + 1/(3*a))/(f*(a + b/cos(e + f*x)^2)^(3/2)) -
atanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(5/2)*f)
```

Reduce [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)}{\sec\left(fx+e\right)^6 b^3+3\sec\left(fx+e\right)^4 a\,b^2+3\sec\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x))/(sec(e + f*x)**6*b**3 + 3*s
ec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.431 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-arctanh((a+b*sec(f*x+e
)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/3*b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(
3/2)-b*(2*a+b)/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4627, 25, 354, 96, 25, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(e+fx)\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 4627$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 354$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 96$$

$$-\frac{-\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}-\frac{\int -\frac{\cos(e+fx)\left(-b\sec^2(e+fx)+a+b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a(a+b)}}{2f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(-b\sec^2(e+fx)+a+b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{2\int \frac{\cos(e+fx)\left((a+b)^2-b(2a+b)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\int \frac{\cos(e+fx)\left((a+b)^2-b(2a+b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 174$$

$$-\frac{\frac{\frac{a^2\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+(a+b)^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx)}}{2f}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{\frac{\frac{2a^2\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2(a+b)^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^3}}{2f}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{\frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/2*((2*b)/(3*a*(a + b)*(a + b*Sec[e + f*x]^2)^(3/2)) + (((-2*(a + b)^2*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a^2*ArcTanh[Sqrt[
a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b)) + (2*b*(2*a +
 b))/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(a*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 9175 vs. 2(119) = 238.

Time = 10.53 (sec) , antiderivative size = 9176, normalized size of antiderivative = 66.98

method	result	size
default	Expression too large to display	9176

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(119) = 238.

Time = 1.25 (sec) , antiderivative size = 2279, normalized size of antiderivative = 16.64

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos
(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*
cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*c
os(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 6*(a
^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((
8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1
)) - 8*((7*a^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 +
 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b
+ 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3
 + 3*a^4*b^4 + a^3*b^5)*f), 1/24*(12*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x
 + e)^2 + a^3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*
sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(
f*x + e)^2 + a*b + b^2)) + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*
a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 2...
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cot\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x))/(sec(e + f*x)**6*b**3 + 3*s
ec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.432 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(2a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} - \frac{(3a-2b)b}{6a(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{b(a^2-6ab-2b^2)}{2a^2(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(2*a+7*b)*arctanh
((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f-1/6*(3*a-2*b)*b/a/(a+
b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2
)^(3/2)-1/2*b*(a^2-6*a*b-2*b^2)/a^2/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4627, 354, 114, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{5/2}}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{114}$$

$$\frac{\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}-\frac{\int -\frac{\cos(e+fx)\left(5b\sec^2(e+fx)+2a+2b\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{a+b}}{2f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\cos(e+fx)\left(5b\sec^2(e+fx)+2(a+b)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 169

$$\frac{\frac{\frac{2\int \frac{3\cos(e+fx)\left(2(a+b)^2+(3a-2b)b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{3a(a+b)}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(2(a+b)^2+(3a-2b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 169

$$\frac{\frac{\frac{\frac{2\int \frac{\cos(e+fx)\left(2(a+b)^3+b\left(a^2-6ba-2b^2\right)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec}}{2f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(2(a+b)^3+b\left(a^2-6ba-2b^2\right)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec}}{2f}$$

$\Big\downarrow$ 174

$$\frac{\frac{\frac{\frac{a^2(2a+7b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2(a+b)^3\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{2f}$$

$\downarrow$ 73

$$\frac{\frac{\frac{\frac{\frac{2a^2(2a+7b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{4(a+b)^3\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+}{2f}$$

$\downarrow$ 221

$$\frac{\frac{\frac{\frac{\frac{2a^2(2a+7b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{4(a+b)^3\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{}{(a+b)(1-}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(1/((a + b)*(1 - Sec[e + f*x]^2)*(a + b*Sec[e + f*x]^2)^(3/2)) + ((2*b*(2/
a - 5/(a + b)))/(3*(a + b*Sec[e + f*x]^2)^(3/2)) + (((-4*(a + b)^3*ArcTanh
[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a^2*(2*a + 7*b)*ArcTanh
[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b)) - (2*b*
(a^2 - 6*a*b - 2*b^2))/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(a*(a + b))
)/(2*(a + b)))/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 12013 vs. $2(174) = 348$.

Time $= 8.63$ (sec) , antiderivative size $= 12014$, normalized size of antiderivative $= 60.07$

method	result	size
default	Expression too large to display	12014

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(174) = 348$.

Time = 4.48 (sec) , antiderivative size = 3507, normalized size of antiderivative = 17.54

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2160 vs. $2(174) = 348$.

Time = 2.14 (sec) , antiderivative size = 2160, normalized size of antiderivative = 10.80

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/24*(((((3*(a^20*b^2*sgn(cos(f*x + e)) + 10*a^19*b^3*sgn(cos(f*x + e)) +
45*a^18*b^4*sgn(cos(f*x + e)) + 120*a^17*b^5*sgn(cos(f*x + e)) + 210*a^16*
b^6*sgn(cos(f*x + e)) + 252*a^15*b^7*sgn(cos(f*x + e)) + 210*a^14*b^8*sgn(
cos(f*x + e)) + 120*a^13*b^9*sgn(cos(f*x + e)) + 45*a^12*b^10*sgn(cos(f*x
+ e)) + 10*a^11*b^11*sgn(cos(f*x + e)) + a^10*b^12*sgn(cos(f*x + e)))*tan(
1/2*f*x + 1/2*e)^2/(a^21*b^2 + 11*a^20*b^3 + 55*a^19*b^4 + 165*a^18*b^5 +
330*a^17*b^6 + 462*a^16*b^7 + 462*a^15*b^8 + 330*a^14*b^9 + 165*a^13*b^10
+ 55*a^12*b^11 + 11*a^11*b^12 + a^10*b^13) - 4*(3*a^20*b^2*sgn(cos(f*x + e
)) + 24*a^19*b^3*sgn(cos(f*x + e)) + 101*a^18*b^4*sgn(cos(f*x + e)) + 330*
a^17*b^5*sgn(cos(f*x + e)) + 900*a^16*b^6*sgn(cos(f*x + e)) + 1896*a^15*b^
7*sgn(cos(f*x + e)) + 2898*a^14*b^8*sgn(cos(f*x + e)) + 3132*a^13*b^9*sgn(
cos(f*x + e)) + 2355*a^12*b^10*sgn(cos(f*x + e)) + 1200*a^11*b^11*sgn(cos(
f*x + e)) + 393*a^10*b^12*sgn(cos(f*x + e)) + 74*a^9*b^13*sgn(cos(f*x + e)
) + 6*a^8*b^14*sgn(cos(f*x + e)))/(a^21*b^2 + 11*a^20*b^3 + 55*a^19*b^4 +
165*a^18*b^5 + 330*a^17*b^6 + 462*a^16*b^7 + 462*a^15*b^8 + 330*a^14*b^9 +
 165*a^13*b^10 + 55*a^12*b^11 + 11*a^11*b^12 + a^10*b^13))*tan(1/2*f*x + 1
/2*e)^2 + 6*(3*a^20*b^2*sgn(cos(f*x + e)) + 22*a^19*b^3*sgn(cos(f*x + e))
+ 111*a^18*b^4*sgn(cos(f*x + e)) + 460*a^17*b^5*sgn(cos(f*x + e)) + 1330*a
^16*b^6*sgn(cos(f*x + e)) + 2516*a^15*b^7*sgn(cos(f*x + e)) + 3094*a^14*b^
8*sgn(cos(f*x + e)) + 2432*a^13*b^9*sgn(cos(f*x + e)) + 1135*a^12*b^10*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{{\mathrm{cot}\left(e+f\,x\right)}^3}{{\left(a+\frac{b}{{\cos\left(e+f\,x\right)}^2}\right)}^{5/2}} \,dx$$

input

```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output

```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{\sec \left(f x +e \right)^{2} b +a}\, \cot \left(f x +e \right)^{3}}{\sec \left(f x +e \right)^{6} b^{3}+3 \sec \left(f x +e \right)^{4} a \,b^{2}+3 \sec \left(f x +e \right)^{2} a^{2} b +a^{3}}d x$$

input

```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output

```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**3)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.433 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 268

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{(8a^2+36ab+63b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} + \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2(a+b)^4f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-1/8*(8*a^2+36*a*b+63*b
^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(9/2)/f+1/24*b*(12
*a^2+39*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)+1/8*(4*a+11*b)*cot
(f*x+e)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b
*sec(f*x+e)^2)^(3/2)+1/8*b*(4*a^3+15*a^2*b-32*a*b^2-8*b^3)/a^2/(a+b)^4/f/(
a+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4627, 25, 354, 114, 27, 168, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}d\sec^2(e+fx)}{2f}$$

$\downarrow$ 114

$$-\frac{\frac{1}{2(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}-\frac{\int -\frac{\cos(e+fx)\left(7b\sec^2(e+fx)+4(a+b)\right)}{2\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{2(a+b)}}{2f}$$

$\downarrow$ 27

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(7b\sec^2(e+fx)+4(a+b)\right)}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$\downarrow$ 168

$$-\frac{\frac{\frac{4a+11b}{(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{\cos(e+fx)\left(8(a+b)^2+5b(4a+11b)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{a+b}}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e-}}{2f}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^2+5b(4a+11b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{2(a+b)}+\frac{4a+11b}{(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+}}{2f}$$

$\downarrow$ 169

$$-\frac{\frac{\frac{\frac{2\int \frac{3\cos(e+fx)\left(8(a+b)^3+b\left(12a^2+39ba-8b^2\right)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{3a(a+b)}-\frac{2b\left(12a^2+39ab-8b^2\right)}{3a(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{4a+11b}{(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{4(a+b)}+}{2f}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^3+b\left(12a^2+39ba-8b^2\right)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(12a^2+39ab-8b^2\right)}{3a(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{4a+11b}{(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{4(a+b)}+\frac{\cdots}{2\cdots}}{2f}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{\frac{\frac{\frac{2\int \frac{\cos(e+fx)\left(8(a+b)^4+b\left(4a^3+15ba^2-32b^2a-8b^3\right)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}-\frac{2b\left(12a^2+39ab-8b^2\right)}{3a(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{\cdots}{(a+b)\left(1-\cdots\right.}}{4(a+b)}\cdots}{2f}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^4+b\left(4a^3+15ba^2-32b^2a-8b^3\right)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}-\frac{2b\left(12a^2+39ab-8b^2\right)}{3a(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{\cdots}{(a+b)\left(1-\mathrm{se}\cdots\right.}}{4(a+b)}\cdots}{2f}$$

$$\Big\downarrow \; 174$$

$$-\frac{\frac{\frac{\frac{\frac{a^2\left(8a^2+36ab+63b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+8(a+b)^4\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}-\frac{2b\cdots}{3a(a+\cdots}}{2(a+b)}\cdots}{4(a+b)}\cdots}{2f}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{\frac{\frac{\frac{\frac{2a^2\left(8a^2+36ab+63b^2\right)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{16(a+b)^4\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}-\frac{2b\left(1\cdots\right.}{3a(a+b)\cdots}}{2(a+b)}\cdots}{4(a+b)}\cdots}{2f}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{\frac{\frac{\frac{2a^2\left(8a^2+36ab+63b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{16(a+b)^4\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}-\frac{2b\left(12a^2+39ab-\cdots\right.}{3a(a+b)\left(a+b\sec^2(e\cdots\right.}}{2(a+b)}\cdots}{4(a+b)}\cdots}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/2*(1/(2*(a + b)*(1 - Sec[e + f*x]^2)^2*(a + b*Sec[e + f*x]^2)^(3/2)) +
((4*a + 11*b)/((a + b)*(1 - Sec[e + f*x]^2)*(a + b*Sec[e + f*x]^2)^(3/2))
+ ((-2*b*(12*a^2 + 39*a*b - 8*b^2))/(3*a*(a + b)*(a + b*Sec[e + f*x]^2)^(3
/2)) + (((-16*(a + b)^4*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[
a] + (2*a^2*(8*a^2 + 36*a*b + 63*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/S
qrt[a + b]])/Sqrt[a + b])/(a*(a + b)) - (2*b*(4*a^3 + 15*a^2*b - 32*a*b^2
- 8*b^3))/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(a*(a + b)))/(2*(a + b))
)/(4*(a + b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 15442 vs. $2(238) = 476$.

Time = 8.18 (sec) , antiderivative size = 15443, normalized size of antiderivative = 57.62

method	result	size
default	Expression too large to display	15443

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. 2(238) = 476.

Time = 17.47 (sec) , antiderivative size = 4751, normalized size of antiderivative = 17.73

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3397 vs. $2(238) = 476$.

Time = 2.89 (sec) , antiderivative size = 3397, normalized size of antiderivative = 12.68

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/192*(((((3*((a^25*b^2 + 15*a^24*b^3 + 105*a^23*b^4 + 455*a^22*b^5 + 1365
*a^21*b^6 + 3003*a^20*b^7 + 5005*a^19*b^8 + 6435*a^18*b^9 + 6435*a^17*b^10
 + 5005*a^16*b^11 + 3003*a^15*b^12 + 1365*a^14*b^13 + 455*a^13*b^14 + 105*
a^12*b^15 + 15*a^11*b^16 + a^10*b^17)*tan(1/2*f*x + 1/2*e)^2/(a^26*b^2*sgn
(cos(f*x + e)) + 16*a^25*b^3*sgn(cos(f*x + e)) + 120*a^24*b^4*sgn(cos(f*x
+ e)) + 560*a^23*b^5*sgn(cos(f*x + e)) + 1820*a^22*b^6*sgn(cos(f*x + e)) +
 4368*a^21*b^7*sgn(cos(f*x + e)) + 8008*a^20*b^8*sgn(cos(f*x + e)) + 11440
*a^19*b^9*sgn(cos(f*x + e)) + 12870*a^18*b^10*sgn(cos(f*x + e)) + 11440*a^
17*b^11*sgn(cos(f*x + e)) + 8008*a^16*b^12*sgn(cos(f*x + e)) + 4368*a^15*b
^13*sgn(cos(f*x + e)) + 1820*a^14*b^14*sgn(cos(f*x + e)) + 560*a^13*b^15*s
gn(cos(f*x + e)) + 120*a^12*b^16*sgn(cos(f*x + e)) + 16*a^11*b^17*sgn(cos(
f*x + e)) + a^10*b^18*sgn(cos(f*x + e))) - (15*a^25*b^2 + 239*a^24*b^3 + 1
771*a^23*b^4 + 8099*a^22*b^5 + 25571*a^21*b^6 + 59059*a^20*b^7 + 103103*a^
19*b^8 + 138567*a^18*b^9 + 144573*a^17*b^10 + 117117*a^16*b^11 + 73073*a^1
5*b^12 + 34489*a^14*b^13 + 11921*a^13*b^14 + 2849*a^12*b^15 + 421*a^11*b^1
6 + 29*a^10*b^17)/(a^26*b^2*sgn(cos(f*x + e)) + 16*a^25*b^3*sgn(cos(f*x +
e)) + 120*a^24*b^4*sgn(cos(f*x + e)) + 560*a^23*b^5*sgn(cos(f*x + e)) + 18
20*a^22*b^6*sgn(cos(f*x + e)) + 4368*a^21*b^7*sgn(cos(f*x + e)) + 8008*a^2
0*b^8*sgn(cos(f*x + e)) + 11440*a^19*b^9*sgn(cos(f*x + e)) + 12870*a^18*b^
10*sgn(cos(f*x + e)) + 11440*a^17*b^11*sgn(cos(f*x + e)) + 8008*a^16*b^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\text{Hanged}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^5}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**5)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.434 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}} dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{3abf\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tan(e+fx)}{f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+arctanh(b
^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*(a+b)*tan(f*x+
e)^3/a/b/f/(a+b+b*tan(f*x+e)^2)^(3/2)+(1/a^2-1/b^2)*tan(f*x+e)/f/(a+b+b*ta
n(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 316 vs. $2(157) = 314$.

Time = 9.48 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.01

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\left(\frac{b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} - \frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^2b^2f\,(a+b\sec^2(e+fx))^{5/2}} + \frac{(a+2b+a\cos(2e+2fx))^3\sec^5(e+fx)\left(\frac{-a^2\sin(e+fx)-2ab\sin(e+fx)-b^2\sin(e+fx)}{6a^2bf(a+2b+a\cos(2e+2fx))^2} + \frac{-3a^2\sin(e+fx)+ab\sin(e+fx)+4b^2\sin(e+fx)}{12a^2b^2f(a+2b+a\cos(2e+2fx))}\right)}{(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/4*(((b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])
/Sqrt[a] - (a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]
^2]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[
2]*a^2*b^2*f*(a + b*Sec[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x
])^3*Sec[e + f*x]^5*((-(a^2*Sin[e + f*x]) - 2*a*b*Sin[e + f*x] - b^2*Sin[e
 + f*x])/(6*a^2*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + (-3*a^2*Sin[e + f*
x] + a*b*Sin[e + f*x] + 4*b^2*Sin[e + f*x])/(12*a^2*b^2*f*(a + 2*b + a*Cos
[2*e + 2*f*x]))))/(a + b*Sec[e + f*x]^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4629, 2075, 372, 27, 440, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^6}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{3\tan^2(e+fx)\left(a\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\tan^2(e+fx)\left(a\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{440}$$

$$\frac{\frac{-\frac{\int -\frac{\tan^2(e+fx)a^2+a^2-b^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{\tan^2(e+fx)a^2+a^2-b^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{\frac{a^2\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{a^2\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-b^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-\frac{b^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input

```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*((a + b)*Tan[e + f*x]^3)/(a*b*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((
-((b^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt
[a]) + (a^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/Sqrt[b])/(a*b) - ((a^2 - b^2)*Tan[e + f*x])/(a*b*Sqrt[a + b + b*Tan[e +
f*x]^2]))/(a*b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1422 vs. $2(139) = 278$.

Time = 5.41 (sec) , antiderivative size = 1423, normalized size of antiderivative = 9.06

method	result	size
default	Expression too large to display	1423

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f/b^(7/2)/(-a)^(1/2)/a^2*(cos(f*x+e)^4*(3*cos(f*x+e)+3)*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b*ln(4*(-b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))+cos(f*x+e)^2*(6*cos
(f*x+e)+6)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*
ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(
f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x
+e)-1))+(3*cos(f*x+e)+3)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*a^2*b^3*ln(4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-a-b)/(sin(f*x+e)-1))+cos(f*x+e)^4*(3*cos(f*x+e)+3)*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))+cos(f*x+e)^2*(6*cos(f*x+e)
+6)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*ln(-4*(
-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+sin(f*x+e)
*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1)
)+(3*cos(f*x+e)+3)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
a^2*b^3*ln(-4*(-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*
x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(139) = 278$.

Time = 1.86 (sec) , antiderivative size = 2035, normalized size of antiderivative = 12.96

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)
*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4
 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^
2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 6
*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2
 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*((3*a^4*b - a^3*b^
2 - 4*a^2*b^3)*cos(f*x + e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*c
os(f*x + e)^4 + 2*a^4*b^4*f*cos(f*x + e)^2 + a^3*b^5*f), 1/24*(12*(a^5*cos
(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(a^2*b^3*
cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*log(128*a^4*cos(f*
x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2
*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 3...
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^6{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan{(fx+e)}^6}{\left(b\sec{(fx+e)}^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan{(fx+e)}^6}{\left(b\sec{(fx+e)}^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\tan(fx+e)^6}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**6)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.435 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}-\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*(a+b)*
tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/3*(a-3*b)*tan(f*x+e)/a^2/b/f
/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. $2(120) = 240$.

Time = 4.30 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.41

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*((Sqrt[2]*Csc[e + f*x
]*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*S
in[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Si
n[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]
^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b -
 a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x]
)/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^
3))/(a + b - a*Sin[e + f*x]^2)^(3/2) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])
*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (12*(b +
 (3*a + 2*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2
*(e + f*x)])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 372, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^4}{\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\int \frac{(a-2b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3ab}-\frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{3b(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{(a-3b)\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{3ab}-\frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{(a-3b)\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{3ab}-\frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{3b\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{(a-3b)\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{3ab}-\frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{3b\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{(a-3b)\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{3ab}-\frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*((a + b)*Tan[e + f*x])/(a*b*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*
b*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) +
 ((a - 3*b)*Tan[e + f*x])/(a*Sqrt[a + b + b*Tan[e + f*x]^2]))/(3*a*b))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(106) = 212$.

Time = 2.84 (sec) , antiderivative size = 489, normalized size of antiderivative = 4.08

method	result
default	$-\frac{\left(\cos(fx+e)^4(-3\cos(fx+e)-3)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a^2 \ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(f\right.\right.}{}$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/a^2/(-a)^(1/2)*(cos(f*x+e)^4*(-3*cos(f*x+e)-3)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-4*sin(f*x+e)*a)+cos(f*x+e)^2*(-6*cos(f*x+e)-6)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-4*sin(f*x+e)*a)+(-3*cos(f*x+e)-3)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*s
in(f*x+e)*a)+sin(f*x+e)*cos(f*x+e)^2*(4*cos(f*x+e)^2-1)*a^2*(-a)^(1/2)+(7*
cos(f*x+e)^2-1)*sin(f*x+e)*(-a)^(1/2)*a*b+3*sin(f*x+e)*(-a)^(1/2)*b^2)/(a+
b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(106) = 212$.

Time = 0.78 (sec) , antiderivative size = 661, normalized size of antiderivative = 5.51

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*log(1
28*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*
a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3
 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3
*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(4*a^
2*cos(f*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x +
 e)^2 + a^3*b^2*f), -1/12*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)
^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*
cos(f*x + e)^2)*sin(f*x + e))) + 4*(4*a^2*cos(f*x + e)^3 - (a^2 - 3*a*b)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^
5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^4}{\sec\left(fx+e\right)^6 b^3+3\sec\left(fx+e\right)^4 a\,b^2+3\sec\left(fx+e\right)^2 a^2 b+a^3}dx$$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output

```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**4)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.436 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}} dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*tan(f
*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/3*(2*a+3*b)*tan(f*x+e)/a^2/(a+b)/f/
(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 410 vs. 2(119) = 238.

Time = 3.16 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.45

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(-\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*(-((Sqrt[2]*Csc[e + f
*x]*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])
*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*
Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*
x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b
 - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*
x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/
a^3))/(a + b - a*Sin[e + f*x]^2)^(3/2)) + (8*(2*a + 3*b + a*Cos[2*(e + f*x
)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (4*(b
 + (3*a + 2*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos
[2*(e + f*x)])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 373, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\int \frac{1-2\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{\int \frac{3(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{(2a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{(2a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{3\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{(2a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{(2a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(3*a*(a + b + b*Tan[e + f*x]^2)^(3/2)) - ((3*ArcTan[(Sqrt[a]
*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) - ((2*a + 3*b)*Tan
[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/(3*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(105) = 210$.

Time = 3.88 (sec) , antiderivative size = 692, normalized size of antiderivative = 5.82

method	result
default	$-\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, a^3 \ln\left(4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\, \cos(fx+e)+4\sqrt{-a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(3+3\sec(fx+e))}{3} - \sqrt{\frac{b+a\cos(fx+e)}{(1+\cos(fx+e))}}$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*ln(4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(3+3*sec(f*x+e))-1/3
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b*ln(4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(3+3*sec(f*x+e)+6*sec(f*x+e
)^2+6*sec(f*x+e)^3)-1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2*
ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(6*s
ec(f*x+e)^2+6*sec(f*x+e)^3+3*sec(f*x+e)^4+3*sec(f*x+e)^5)-1/3*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(3*sec(f*x+e)^4+3*sec(f*x+e)^5)+(-a)^(1/2)
*a^3*tan(f*x+e)-1/3*(-a)^(1/2)*a^2*b*(-4*tan(f*x+e)-5*sec(f*x+e)^2*tan(f*x
+e))-1/3*(-7*cos(f*x+e)^2-2)*(-a)^(1/2)*a*b^2*tan(f*x+e)*sec(f*x+e)^4+(-a)
^(1/2)*b^3*tan(f*x+e)*sec(f*x+e)^4)/(a+b)/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2
)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(105) = 210$.

Time = 0.65 (sec) , antiderivative size = 773, normalized size of antiderivative = 6.50

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(a^2*b + a*b^2)*
cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28
*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b
^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
 e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e)) - 8*((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (2*a^2*b + 3*
a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/((a^6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^5*b + a^4*b^2)*f*cos(f*x + e)^
2 + (a^4*b^2 + a^3*b^3)*f), 1/12*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2
+ b^3 + 2*(a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*
x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e)
)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*((
3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f
*x + e)^4 + 2*(a^5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b^2 + a^3*b^3)*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^2}{\sec\left(fx+e\right)^6 b^3+3\sec\left(fx+e\right)^4 a\,b^2+3\sec\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*tan(e + f*x)**2)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.437 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}} dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*tan(
f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*b*(5*a+3*b)*tan(f*x+e)/a^2
/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 6.29 (sec) , antiderivative size = 1927, normalized size of antiderivative = 15.42

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}} dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^
(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7
/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[
3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*
x]^2)*((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^5)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*Appe
llF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a
*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt
[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, ...
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4616, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-2b\tan^2(e+fx)+3a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{3(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{3(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output
```
(-1/3*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*
(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(
3/2) - (b*(5*a + 3*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]
^2]))/(3*a*(a + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(111) = 222$.

Time = 6.07 (sec) , antiderivative size = 861, normalized size of antiderivative = 6.89

method	result	size
default	Expression too large to display	861

input
```
int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*ln(4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(3+3*sec(f*x+e))+1/3*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*ln(4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(6+6*sec(f*x+e)+6*sec(f*x+e)
^2+6*sec(f*x+e)^3)+1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2
*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(3+
3*sec(f*x+e)+12*sec(f*x+e)^2+12*sec(f*x+e)^3+3*sec(f*x+e)^4+3*sec(f*x+e)^5
)+1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*(6*sec(f*x+e)^2+6*sec(
f*x+e)^3+6*sec(f*x+e)^4+6*sec(f*x+e)^5)+1/3*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*(3*sec(f*x+e)^4+3*sec(f*x+e)^5)-2*(-a)^(1/2)*a^3*b*tan(f*x+e
)+1/3*(-a)^(1/2)*a^2*b^2*(-4*tan(f*x+e)-11*sec(f*x+e)^2*tan(f*x+e))+1/3*(-
7*cos(f*x+e)^2-5)*(-a)^(1/2)*a*b^3*tan(f*x+e)*sec(f*x+e)^4-(-a)^(1/2)*b^4*
tan(f*x+e)*sec(f*x+e)^4)/(a+b)^2/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(5/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(111) = 222$.

Time = 0.55 (sec) , antiderivative size = 881, normalized size of antiderivative = 7.05

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 +
b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b +
 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2
)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a
^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(
f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*
x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a
^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x
+ e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 +
 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^...
```

Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}}{\sec\left(fx+e\right)^6 b^3+3\sec\left(fx+e\right)^4 a\,b^2+3\sec\left(fx+e\right)^2 a^2 b+a^3}dx$$

input
```
int(1/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int(sqrt(sec(e + f*x)**2*b + a)/(sec(e + f*x)**6*b**3 + 3*sec(e + f*x)**4*
a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.438 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*cot
(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*b*(7*a+3*b)*cot(f*x+e)/a^
2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a-3*b)*(3*a+b)*cot(f*x+e)*(a+b
+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f
```

Mathematica [A] (verified)

Time = 4.20 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.42

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx =$$
$$-\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\left(3(3a^4+8a^3b+5a^2b^2-12ab^3-4b^4)+4(3a^4+6a^3b+8ab^3+3b^4)\cos(2(e\right.}{48a^2(a+b)^3f\left(a+b\sec^2(e+fx)\right)^5}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2
*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*S
ec[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(3*a^4 + 8*a^3*
b + 5*a^2*b^2 - 12*a*b^3 - 4*b^4) + 4*(3*a^4 + 6*a^3*b + 8*a*b^3 + 3*b^4)*
Cos[2*(e + f*x)] + a*(3*a^3 + 9*a*b^2 + 4*b^3)*Cos[4*(e + f*x)])*Csc[e + f
*x]*Sec[e + f*x]^5)/(48*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 374, 441, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$
$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$\downarrow$ 2075

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$\downarrow$ 374

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left(-4b\tan^2(e+fx)+3a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}-\frac{b\cot(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\downarrow$ 441

$$\frac{\frac{\frac{\int \frac{\cot^2(e+fx)\left((a-3b)(3a+b)-2b(7a+3b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cot(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$\downarrow$ 445

$$\frac{\frac{\frac{-\frac{\int \frac{3(a+b)^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cot(e+}{3a(a+b)(a+b\tan^2}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\frac{-3(a+b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b}{3a(a+b)(a-}}{f}$$

$\downarrow$ 291

$$\frac{\frac{\frac{-3(a+b)^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cot(}{3a(a+b)(a+b\tan}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{-\frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cot(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Cot[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (-((
b*(7*a + 3*b)*Cot[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) +
((-3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2
]])/Sqrt[a] - ((a - 3*b)*(3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(a + b))/(a*(a + b)))/(3*a*(a + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(156) = 312$.

Time = 9.80 (sec) , antiderivative size = 1109, normalized size of antiderivative = 6.37

method	result	size
default	Expression too large to display	1109

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a+b)^3/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(5/2)*(ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a^5*(3+3*sec(f*x+e))+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^4*b*(9+9*sec(f*x+e)+6*sec(f*x+e)^2+6*sec(f*x+e)^3)+ln(4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*(9+9*sec(f*x+e)+18*sec(f*x+e)^2+18*sec
(f*x+e)^3+3*sec(f*x+e)^4+3*sec(f*x+e)^5)+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a^2*b^3*(3+3*sec(f*x+e)+18*sec(f*x+e)^2+18*sec(f*x+e)^3+9*sec(f*x+e
)^4+9*sec(f*x+e)^5)+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^4*(6*sec(f
*x+e)^2+6*sec(f*x+e)^3+9*sec(f*x+e)^4+9*sec(f*x+e)^5)+ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(156) = 312$.

Time = 1.84 (sec) , antiderivative size = 1097, normalized size of antiderivative = 6.30

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^
2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*co
s(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e
)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a
*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e))*sin(f*x + e) + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f
*x + e)^5 + (6*a^4*b - 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (
3*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4
 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2
 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), 1/12*(3*(a^3*b^2 + 3
*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(f*x +
 e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(a)*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6
*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e)))*sin(f*x + e) - 4*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos...
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cot(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)^2 b+a}\,\cot\left(fx+e\right)^2}{\sec\left(fx+e\right)^6 b^3+3\sec\left(fx+e\right)^4 a\,b^2+3\sec\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**2)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.439 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 236

$$\begin{aligned}\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx &= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} \\ &- \frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} \\ &+ \frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^4f} \\ &- \frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f}\end{aligned}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*cot(
f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-b*(3*a+b)*cot(f*x+e)^3/a^2/(
a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(a-b)*(3*a^2+14*a*b+3*b^2)*cot(f*x
+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^4/f-1/3*(a^2-10*a*b-3*b^2)*cot(f*
x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f
```

Mathematica [A] (verified)

Time = 6.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.99

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f\,(a+b\sec^2(e+fx))^{5/2}} - \frac{(a+2b+a\cos(2(e+fx)))^3\sec^5(e+fx)\left(-4(a+3b)\csc(e+fx)+(a+b)\csc^3(e+fx)+\frac{4b^3(6a^2+13ab+\cdots}{a\cdots}\right.}{24(a+b)^4f\,(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b +
a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(4*Sqrt[2]*a^(5/2)*f*(a + b*Sec[
e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^5*(-4*
(a + 3*b)*Csc[e + f*x] + (a + b)*Csc[e + f*x]^3 + (4*b^3*(6*a^2 + 13*a*b +
 3*b^2 + 2*a*(3*a + b)*Cos[2*(e + f*x)])*Sin[e + f*x])/(a^2*(a + 2*b + a*C
os[2*(e + f*x)])^2)))/(24*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4629, 2075, 374, 27, 441, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(e+fx)^4\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 374$$

$$\frac{\frac{\int \frac{3\cot^4(e+fx)\left(-2b\tan^2(e+fx)+a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}-\frac{b\cot^3(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-2b\tan^2(e+fx)+a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{a(a+b)}-\frac{b\cot^3(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 441$$

$$\frac{\frac{\frac{\int \frac{\cot^4(e+fx)\left(a^2-10ba-3b^2-4b(3a+b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b(3a+b)\cot^3(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{a(a+b)}-\frac{b\cot^3(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2b\left(a^2-10ba-3b^2\right)\tan^2(e+fx)+(a-b)\left(3a^2+14ba+3b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)}-\frac{b(3a+b)\cot^3(\cdots}{a(a+b)\sqrt{a+b\tan^2\cdots}}}{a(a+b)}\cdots}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{\frac{-\frac{-\frac{\int \frac{3(a+b)^4}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e\cdots}}{3(a+b)}}{a(a+b)}\cdots}{a(a+b)}\cdots}{f}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{-\frac{-3(a+b)^3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b}}{3(a+b)}}{a(a+b)}}{a(a+b)}}{f}$$

$\Big\downarrow$ 291

$$-\frac{\frac{-\frac{-3(a+b)^3\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2}}{3(a+b)}}{a(a+b)}}{a(a+b)}}{f}$$

$\Big\downarrow$ 216

$$-\frac{\frac{-\frac{-\frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-\frac{3(a+b)^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{3(a+b)}-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)}-\frac{}{a(}}{a(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Cot[e + f*x]^3)/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (-
((b*(3*a + b)*Cot[e + f*x]^3)/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))
+ (-1/3*((a^2 - 10*a*b - 3*b^2)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]
^2])/(a + b) - ((-3*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/Sqrt[a] - ((a - b)*(3*a^2 + 14*a*b + 3*b^2)*Cot[e + f*x
]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/(3*(a + b)))/(a*(a + b)))/(a*(a
 + b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. $2(216) = 432$.

Time = 13.35 (sec) , antiderivative size = 1397, normalized size of antiderivative = 5.92

method	result	size
default	Expression too large to display	1397

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a+b)^4/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(5/2)*(((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-4*sin(f*x+e)*a)*a^6*(-3-3*sec(f*x+e))+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(
f*x+e)*a)*a^5*b*(-12-12*sec(f*x+e)-6*sec(f*x+e)^2-6*sec(f*x+e)^3)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*b^2*(-18-18*sec(f*x+e)-24*sec(f*x+e)^2
-24*sec(f*x+e)^3-3*sec(f*x+e)^4-3*sec(f*x+e)^5)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*a^3*b^3*(-12-12*sec(f*x+e)-36*sec(f*x+e)^2-36*sec(f*x+e)^3-1
2*sec(f*x+e)^4-12*sec(f*x+e)^5)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^
2*b^4*(-3-3*sec(f*x+e)-24*sec(f*x+e)^2-24*sec(f*x+e)^3-18*sec(f*x+e)^4-18*
sec(f*x+e)^5)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(216) = 432$.

Time = 6.71 (sec) , antiderivative size = 1579, normalized size of antiderivative = 6.69

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^
6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a
^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^
4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*lo
g(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*
b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*
a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b
 + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*
x + e) - 8*(4*(a^6 + 3*a^5*b + 3*a^3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^
6 + a^5*b - 8*a^4*b^2 + 8*a^3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a
^5*b + 3*a^4*b^2 - 4*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*
b^2 + 11*a^3*b^3 - 11*a^2*b^4 - 3*a*b^5)*cos(f*x + e))*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b
^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4
- 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b
^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*
b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e)), -1/12*(3*((a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6...
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx=\int \frac{\cot^4{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\cot\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx=\text{Hanged}$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^4}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**4)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.440 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 315

$$\begin{aligned}\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} \\ &- \frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} \\ &- \frac{(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^5f} \\ &+ \frac{(5a^3+19a^2b-65ab^2-15b^3)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^4f} \\ &- \frac{(a^2-20ab-5b^2)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a^2(a+b)^3f}\end{aligned}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*cot
(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*b*(11*a+3*b)*cot(f*x+e)
^5/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^4+70*a^3*b+128*a^2*
b^2-70*a*b^3-15*b^4)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^5/f+1
/15*(5*a^3+19*a^2*b-65*a*b^2-15*b^3)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/
2)/a^2/(a+b)^4/f-1/5*(a^2-20*a*b-5*b^2)*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^
(1/2)/a^2/(a+b)^3/f
```

Mathematica [A] (verified)

Time = 9.59 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.86

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f\,(a+b\sec^2(e+fx))^{5/2}}$$

$$+\frac{(a+2b+a\cos(2(e+fx)))^3\left(-\frac{20b^5(a+b)}{a^2(a+2b+a\cos(2(e+fx)))^2}+\frac{10b^4(15a+4b)}{a^2(a+2b+a\cos(2(e+fx)))}-(23a^2+100ab+150b^2)\,\text{cs}\right.}{120(a+b)^5f\,(a+b\,\text{se}}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2
*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*S
ec[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^3*((-20*b^5*(a + b
))/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (10*b^4*(15*a + 4*b))/(a^2*(a
+ 2*b + a*Cos[2*(e + f*x)])) - (23*a^2 + 100*a*b + 150*b^2)*Csc[e + f*x]^2
 + (a + b)*(11*a + 25*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[
e + f*x]^4*Tan[e + f*x])/(120*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 4629, 2075, 374, 441, 27, 445, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 4629$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2075$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 374$$

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(-8b\tan^2(e+fx)+3a-5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}-\frac{b\cot^5(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 441$$

$$\frac{\frac{\frac{\int \frac{3\cot^6(e+fx)\left(a^2-20ba-5b^2-2b(11a+3b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b(11a+3b)\cot^5(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cot^5(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\int \frac{\cot^6(e+fx)\left(a^2-20ba-5b^2-2b(11a+3b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}-\frac{b(11a+3b)\cot^5(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cot^5(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{3\left(-\frac{\int \frac{\cot^4(e+fx)\left(5a^3+19ba^2-65b^2a-15b^3+4b\left(a^2-20ba-5b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)}-\frac{\left(a^2-20ab-5b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}\right)}{a(a+b)}-\frac{b(11a+3}{a(a+b)\sqrt{a-}}}{3a(a+b)}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{\frac{3\left(-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(15a^4+70ba^3+128b^2a^2-70b^3a-15b^4+2b\left(5a^3+19ba^2-65b^2a-15b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{\left(5a^3+19a^2b-65ab^2-15b^3\right)\cot^3(e+fx)\sqrt{\cdots}}{3(a+b)}}{5(a+b)}\cdots\right)}{a(a+b)}\cdots}{3a(a+b)}\cdots}{f}$$

$$\Big\downarrow \ 445$$

$$\frac{\frac{3\left(-\frac{-\frac{-\frac{\int \frac{15(a+b)^5}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\left(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(5a^3+19a^2b-65ab^2-\cdots\right.}{\cdots}}{5(a+b)}\cdots\right)}{a(a+b)}\cdots}{3a(a+b)}\cdots$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\left(-\frac{-\frac{-15(a+b)^4\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\left(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(5a^3+19a^2b\cdots\right.}{\cdots}}{5(a+b)}\cdots\right)}{a(a+b)}\cdots}{3a(a+b)}\cdots$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{3\left(-\frac{-\frac{-15(a+b)^4\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{\left(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(5a^3+19a^2b-65a\cdots\right.}{\cdots}}{5(a+b)}\cdots\right)}{a(a+b)}\cdots}{3a(a+b)}\cdots$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{3\left(-\frac{\left(a^2-20ab-5b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}-\frac{-\frac{\left(5a^3+19a^2b-65ab^2-15b^3\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}-\frac{-\frac{\left(15a^4+70a^3b+128a^2b^2-70ab^3-15\right.}{a-}}{}}{5(a+b)}\right)}{a(a+b)}}{3a(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Cot[e + f*x]^5)/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (-
((b*(11*a + 3*b)*Cot[e + f*x]^5)/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]
)) + (3*(-1/5*((a^2 - 20*a*b - 5*b^2)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(a + b) - (-1/3*((5*a^3 + 19*a^2*b - 65*a*b^2 - 15*b^3)*Cot[e +
 f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - ((-15*(a + b)^4*ArcTan[(
Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a] - ((15*a^4
+ 70*a^3*b + 128*a^2*b^2 - 70*a*b^3 - 15*b^4)*Cot[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(a + b))/(3*(a + b)))/(5*(a + b))))/(a*(a + b)))/(3*a*(a
+ b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(289) = 578$.

Time = 17.25 (sec) , antiderivative size = 1698, normalized size of antiderivative = 5.39

method	result	size
default	Expression too large to display	1698

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/15*sin(f*x+e)^5*cos(f*x+e)^4*(15*cos(f*x+e)+15)*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a^7-1/15*sin(f*x+e)^5*cos(f*x+e)^2*(75*cos(f*x+e)^3+
75*cos(f*x+e)^2+30*cos(f*x+e)+30)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
a^6*b-1/15*sin(f*x+e)^5*(150*cos(f*x+e)^5+150*cos(f*x+e)^4+150*cos(f*x+e)^
3+150*cos(f*x+e)^2+15*cos(f*x+e)+15)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a^5*b^2-1/15*sin(f*x+e)^5*(150*cos(f*x+e)^5+150*cos(f*x+e)^4+300*cos(f*
x+e)^3+300*cos(f*x+e)^2+75*cos(f*x+e)+75)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a^4*b^3-1/15*sin(f*x+e)^5*(75*cos(f*x+e)^5+75*cos(f*x+e)^4+300*cos
(f*x+e)^3+300*cos(f*x+e)^2+150*cos(f*x+e)+150)*ln(4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a^3*b^4-1/15*sin(f*x+e)^5*(15*cos(f*x+e)^5+15*cos(f*x+e)^4...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(289) = 578$.

Time = 22.98 (sec) , antiderivative size = 2059, normalized size of antiderivative = 6.54

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/120*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^
5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^
6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*co
s(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*
b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a
^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^
8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e
)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^7 + 1
00*a^6*b + 150*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*cos(f*x + e)^9 - (35*a^7
 + 118*a^6*b + 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b^4 - 10*a^2*b^5 - 15*a*
b^6)*cos(f*x + e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^4*b^3 + 125*a^3*b^4 -
50*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 105*a^5*b^2 + 92*a^4*b
^3 - 350*a^3*b^4 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e)^3 + (15*a^5*b^2 +
70*a^4*b^3 + 128*a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7
*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^...
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^6{(e+fx)}}{(a+b\sec^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot{(fx+e)}^6}{(b\sec{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)^2 b+a}\,\cot(fx+e)^6}{\sec(fx+e)^6 b^3+3\sec(fx+e)^4 a\,b^2+3\sec(fx+e)^2 a^2b+a^3}dx$$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)**2*b + a)*cot(e + f*x)**6)/(sec(e + f*x)**6*b**3 +
3*sec(e + f*x)**4*a*b**2 + 3*sec(e + f*x)**2*a**2*b + a**3),x)
```

3.441 $\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m \, dx$

Optimal result

Integrand size = 25, antiderivative size = 107

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m \, dx = \frac{\text{AppellF1}\left(\frac{1+m}{2},1,-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)(d\tan(e+fx))^{1+m}(a+b+b\tan^2(e+fx))^p}{df(1+m)}$$

output
```
AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*(d*
tan(f*x+e))^(1+m)*(a+b+b*tan(f*x+e)^2)^p/d/f/(1+m)/(((a+b+b*tan(f*x+e)^2)/
(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(107) = 214.

Time = 2.97 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.42

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m \, dx = \frac{\text{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)\cos(e+fx)(a\cdots}{f(1+m)\left(\text{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)+\frac{2\left(bp\,\text{AppellF1}\left(\frac{3+m}{2},1-p,1,\frac{5+m}{2},-\frac{b\tan^2(\cdots}{a+\cdots}\right.\cdots}{\cdots}\right.\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

output
```
(AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*Tan[e +
 f*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2] + (2*(b*p*AppellF1[(3 + m)/2, 1 - p, 1,
(5 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[(3 + m)/2, -p, 2, (5 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2])*Tan[e + f*x]^2)/((a + b)*(3 + m)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(e+fx))^m \left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{(d\tan(e+fx))^m\left(a+b\left(\tan^2(e+fx)+1\right)\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{(d\tan(e+fx))^m\left(b\tan^2(e+fx)+a+b\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{(d\tan(e+fx))^m\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{m+1}{2},1,-p,\frac{m+3}{2},-\tan^2(e+fx),-\right.}{df(m+1)}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

output
```
(AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(d*Tan[e + f*x])^(1 + m)*(a + b + b*Tan[e + f*x]^2)^p)/(d*f*(
1 + m)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [F]

$$\int (a+b\sec (fx+e)^2)^p (d\tan (fx+e))^m dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

Fricas [F]

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m dx = \int (b\sec (fx+e)^2+a)^p (d\tan (fx+e))^m dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p (d\tan\left(fx+e\right))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p (d\tan\left(fx+e\right))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx = \int (d\tan(e+f\,x))^m \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m \, dx$$
$$= d^m \left(\int \tan(fx+e)^m \left(\sec(fx+e)^2 b+a\right)^p dx \right)$$

input
```
int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

output
```
d**m*int(tan(e + f*x)**m*(sec(e + f*x)**2*b + a)**p,x)
```

3.442 $\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\begin{aligned}\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx &= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)} \\ &\quad - \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)} \\ &\quad + \frac{\left(a+b\sec^2(e+fx)\right)^{2+p}}{2b^2f(2+p)}\end{aligned}$$

output
```
-1/2*(a+2*b)*(a+b*sec(f*x+e)^2)^(p+1)/b^2/f/(p+1)-1/2*hypergeom([1, p+1],[
2+p],(a+b*sec(f*x+e)^2)/a)*(a+b*sec(f*x+e)^2)^(p+1)/a/f/(p+1)+1/2*(a+b*sec
(f*x+e)^2)^(2+p)/b^2/f/(2+p)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx = -\frac{(a+b\sec^2(e+fx))^{1+p}\left(b^2(2+p)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)+a(a+2b(2\right.}{2ab^2f(1+p)(2+p)}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

output
```
-1/2*((a + b*Sec[e + f*x]^2)^(1 + p)*(b^2*(2 + p)*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a] + a*(a + 2*b*(2 + p) - b*(1 + p)*Sec[
e + f*x]^2)))/(a*b^2*f*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^p d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p+\frac{(-a-2b)\left(b\sec^2(e+fx)+a\right)^p}{b}+\frac{\left(b\sec^2(e+fx)+a\right)^{p+1}}{b}\right)d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{p+1}}{b^2(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+2}}{b^2(p+2)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{a(p+1)}}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

output
```
(-(((a + 2*b)*(a + b*Sec[e + f*x]^2)^(1 + p))/(b^2*(1 + p))) - (Hypergeome
tric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^
(1 + p))/(a*(1 + p)) + (a + b*Sec[e + f*x]^2)^(2 + p)/(b^2*(2 + p)))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^{2}\right)^{p}\tan\left(fx+e\right)^{5}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\tan^5(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p\tan^5(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx=\int (b\sec(fx+e)^2+a)^p\tan(fx+e)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx=\int (b\sec(fx+e)^2+a)^p\tan(fx+e)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p\tan^5(e+fx)\,dx=\int \tan(e+f\,x)^5\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx = \text{Too large to display}$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

output
```
((sec(e + f*x)**2*b + a)**p*tan(e + f*x)**4*p**2 + (sec(e + f*x)**2*b + a)
**p*tan(e + f*x)**4*p - 2*(sec(e + f*x)**2*b + a)**p*tan(e + f*x)**2*p + 2
*(sec(e + f*x)**2*b + a)**p + 2*int(((sec(e + f*x)**2*b + a)**p*tan(e + f*
x)**5)/(sec(e + f*x)**2*b*p**2 + 3*sec(e + f*x)**2*b*p + 2*sec(e + f*x)**2
*b + a*p**2 + 3*a*p + 2*a),x)*a*f*p**5 + 8*int(((sec(e + f*x)**2*b + a)**p
*tan(e + f*x)**5)/(sec(e + f*x)**2*b*p**2 + 3*sec(e + f*x)**2*b*p + 2*sec(
e + f*x)**2*b + a*p**2 + 3*a*p + 2*a),x)*a*f*p**4 + 10*int(((sec(e + f*x)*
*2*b + a)**p*tan(e + f*x)**5)/(sec(e + f*x)**2*b*p**2 + 3*sec(e + f*x)**2*
b*p + 2*sec(e + f*x)**2*b + a*p**2 + 3*a*p + 2*a),x)*a*f*p**3 + 4*int(((se
c(e + f*x)**2*b + a)**p*tan(e + f*x)**5)/(sec(e + f*x)**2*b*p**2 + 3*sec(e
 + f*x)**2*b*p + 2*sec(e + f*x)**2*b + a*p**2 + 3*a*p + 2*a),x)*a*f*p**2 -
 4*int(((sec(e + f*x)**2*b + a)**p*tan(e + f*x)**3)/(sec(e + f*x)**2*b*p**
2 + 3*sec(e + f*x)**2*b*p + 2*sec(e + f*x)**2*b + a*p**2 + 3*a*p + 2*a),x)
*a*f*p**4 - 12*int(((sec(e + f*x)**2*b + a)**p*tan(e + f*x)**3)/(sec(e + f
*x)**2*b*p**2 + 3*sec(e + f*x)**2*b*p + 2*sec(e + f*x)**2*b + a*p**2 + 3*a
*p + 2*a),x)*a*f*p**3 - 8*int(((sec(e + f*x)**2*b + a)**p*tan(e + f*x)**3)
/(sec(e + f*x)**2*b*p**2 + 3*sec(e + f*x)**2*b*p + 2*sec(e + f*x)**2*b + a
*p**2 + 3*a*p + 2*a),x)*a*f*p**2 + 4*int(((sec(e + f*x)**2*b + a)**p*tan(e
 + f*x))/(sec(e + f*x)**2*b*p**2 + 3*sec(e + f*x)**2*b*p + 2*sec(e + f*x)*
*2*b + a*p**2 + 3*a*p + 2*a),x)*a*f*p**3 + 12*int(((sec(e + f*x)**2*b +...
```

3.443 $\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \frac{(a+b\sec^2(e+fx))^{1+p}}{2bf(1+p)} + \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

output
```
1/2*(a+b*sec(f*x+e)^2)^(p+1)/b/f/(p+1)+1/2*hypergeom([1, p+1],[2+p],(a+b*s
ec(f*x+e)^2)/a)*(a+b*sec(f*x+e)^2)^(p+1)/a/f/(p+1)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \frac{\left(a+b\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)\right)(a+b\sec^2(e+fx))^{1+p}}{2abf(1+p)}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

output
```
((a + b*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a])*(a +
 b*Sec[e + f*x]^2)^(1 + p))/(2*a*b*f*(1 + p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4627, 25, 354, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3 \left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec^2(e+fx)-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{b(p+1)}}{2f}$$

$$\Big\downarrow \text{75}$$

$$-\frac{-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{a(p+1)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{b(p+1)}}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

output
```
-1/2*(-((a + b*Sec[e + f*x]^2)^(1 + p)/(b*(1 + p))) - (Hypergeometric2F1[1
, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/
(a*(1 + p)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p} \tan \left(fx+e\right)^{3}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\tan^3(e+fx)\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\tan^3(e+fx)\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^p\tan^3\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p\tan^3(e+fx)\,dx=\int (b\sec(fx+e)^2+a)^p\tan(fx+e)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p\tan^3(e+fx)\,dx=\int (b\sec(fx+e)^2+a)^p\tan(fx+e)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p\tan^3(e+fx)\,dx=\int \tan(e+f\,x)^3\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\tan^3(e+fx)\,dx$$
$$= \frac{\left(\sec(fx+e)^2b+a\right)^p\tan(fx+e)^2p-\left(\sec(fx+e)^2b+a\right)^p+2\left(\int\frac{\left(\sec(fx+e)^2b+a\right)^p\tan(fx+e)^3}{\sec(fx+e)^2bp+\sec(fx+e)^2b+ap+a}dx\right)af}{}$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

output
```
((sec(e + f*x)**2*b + a)**p*tan(e + f*x)**2*p - (sec(e + f*x)**2*b + a)**p
 + 2*int(((sec(e + f*x)**2*b + a)**p*tan(e + f*x)**3)/(sec(e + f*x)**2*b*p
 + sec(e + f*x)**2*b + a*p + a),x)*a*f*p**3 + 2*int(((sec(e + f*x)**2*b +
a)**p*tan(e + f*x)**3)/(sec(e + f*x)**2*b*p + sec(e + f*x)**2*b + a*p + a)
,x)*a*f*p**2 - 2*int(((sec(e + f*x)**2*b + a)**p*tan(e + f*x))/(sec(e + f*
x)**2*b*p + sec(e + f*x)**2*b + a*p + a),x)*a*f*p**2 - 2*int(((sec(e + f*x
)**2*b + a)**p*tan(e + f*x))/(sec(e + f*x)**2*b*p + sec(e + f*x)**2*b + a*
p + a),x)*a*f*p)/(2*f*p*(p + 1))
```

3.444 $\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

output
```
-1/2*hypergeom([1, p+1],[2+p],(a+b*sec(f*x+e)^2)/a)*(a+b*sec(f*x+e)^2)^(p+
1)/a/f/(p+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
```

output
```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*
Sec[e + f*x]^2)^(1 + p))/(a*f*(1 + p))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4627, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \; 4627$$

$$\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 243$$

$$\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 75$$

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
```

output
```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*
Sec[e + f*x]^2)^(1 + p))/(a*f*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p}\tan \left(fx+e\right)dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

Fricas [F]

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Sympy [F]

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^p \tan\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x), x)
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Giac [F]

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx=\int (b\sec(fx+e)^2+a)^p\tan(fx+e)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx=\int \tan(e+f\,x)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx$$
$$=\frac{(\sec(fx+e)^2b+a)^p+2\left(\int\frac{(\sec(fx+e)^2b+a)^p\tan(fx+e)}{\sec(fx+e)^2b+a}dx\right)afp}{2fp}$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

output
```
((sec(e + f*x)**2*b + a)**p + 2*int(((sec(e + f*x)**2*b + a)**p*tan(e + f*
x))/(sec(e + f*x)**2*b + a),x)*a*f*p)/(2*f*p)
```

3.445 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)} - \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f(1+p)}$$

output
```
1/2*hypergeom([1, p+1],[2+p],(a+b*sec(f*x+e)^2)/a)*(a+b*sec(f*x+e)^2)^(p+1
)/a/f/(p+1)-1/2*hypergeom([1, p+1],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*se
c(f*x+e)^2)^(p+1)/(a+b)/f/(p+1)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b+b\tan^2(e+fx)}{a}\right)-a\,\text{Hypergeo}\right.}{4a(a+b)f(1+p)}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Hypergeometric2F1[1, 1 + p, 2 + p
, (a + b + b*Tan[e + f*x]^2)/a] - a*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
 (b*Tan[e + f*x]^2)/(a + b)])*Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(4*
a*(a + b)*f*(1 + p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4627, 25, 354, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{97}$$

$$-\frac{\int \frac{(b\sec^2(e+fx)+a)^p}{1-\sec^2(e+fx)}d\sec^2(e+fx)+\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \text{75}$$

$$-\frac{\int \frac{(b\sec^2(e+fx)+a)^p}{1-\sec^2(e+fx)}d\sec^2(e+fx)-\frac{(a+b\sec^2(e+fx))^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{a(p+1)}}{2f}$$

$$\Big\downarrow \text{78}$$

$$-\frac{\frac{(a+b\sec^2(e+fx))^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)+a}{a+b}\right)}{(p+1)(a+b)}-\frac{(a+b\sec^2(e+fx))^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+}{a}\right.}{a(p+1)}}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/2*((Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*
(a + b*Sec[e + f*x]^2)^(1 + p))/((a + b)*(1 + p)) - (Hypergeometric2F1[1,
1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(a
*(1 + p)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [F]

$$\int \cot(fx+e)\left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^p\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*cot(e + f*x), x)
```

Maxima [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,2,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%%{2,[0,0,0,2]%%%} Error: Bad Argument
Value
```

Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cot(e+f\,x)\,\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(\sec\left(fx+e\right)^2 b+a\right)^p \cot\left(fx+e\right)dx$$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*cot(e + f*x),x)
```

3.446 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 158

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=-\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f}-\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}+\frac{(a+b-bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)^2f(1+p)}$$

output

```
-1/2*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(p+1)/(a+b)/f-1/2*hypergeom([1, p+1],
[2+p],(a+b*sec(f*x+e)^2)/a)*(a+b*sec(f*x+e)^2)^(p+1)/a/f/(p+1)+1/2*(-b*p+a
+b)*hypergeom([1, p+1],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*sec(f*x+e)^2)^
(p+1)/(a+b)^2/f/(p+1)
```

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.88

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$
$$-\frac{(b+(a+b)\cot^2(e+fx))\left(a(a+b)(1+p)\cot^2(e+fx)+(a+b)^2\,\text{Hypergeometric2F1}\left(1,1+p,2+\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/2*((b + (a + b)*Cot[e + f*x]^2)*(a*(a + b)*(1 + p)*Cot[e + f*x]^2 + (a
+ b)^2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b + b*Tan[e + f*x]^2)/a] -
a*(a + b - b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/
(a + b)])*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2)/(a*(a + b)^2*f*(1 + p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4627, 354, 114, 25, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{\left(1-\sec^2(e+fx)\right)^2}d\sec^2(e+fx)}{2f}$$

$$\Big\downarrow \; 114$$

$$\frac{\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{(a+b)\left(1-\sec^2(e+fx)\right)}-\frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p\left(-bp\sec^2(e+fx)+a+b\right)}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{a+b}}{2f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p\left(-bp\sec^2(e+fx)+a+b\right)}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{a+b}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{2f}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{(a-bp+b)\int \frac{\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec^2(e+fx)+(a+b)\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec^2(e+fx)}{a+b}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{2f}$$

$$\Big\downarrow \; 75$$

$$\frac{\frac{(a-bp+b)\int \frac{\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec^2(e+fx)-\frac{(a+b)\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{a(p+1)}}{a+b}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{2f}$$

$$\Big\downarrow \; 78$$

$$\frac{\frac{\frac{(a-bp+b)\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)+a}{a+b}\right)}{(p+1)(a+b)}-\frac{(a+b)\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}\right.}{a(p+1)}}{a+b}}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + b*Sec[e + f*x]^2)^(1 + p)/((a + b)*(1 - Sec[e + f*x]^2)) + (((a + b
- b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*
(a + b*Sec[e + f*x]^2)^(1 + p))/((a + b)*(1 + p)) - ((a + b)*Hypergeometri
c2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1
+ p))/(a*(1 + p)))/(a + b))/(2*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [F]

$$\int \cot (fx+e)^3 \left(a+b\sec (fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec (fx+e)^2+a\right)^p \cot (fx+e)^3 \, dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^3(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,2,0]%%%}+%%%{2,[0,1,0,0]%%%} / %%%{2,[0,0,0,2]%%%} Error: Bad Argument
Value
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\cot\left(fx+e\right)^3dx$$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*cot(e + f*x)**3,x)
```

3.447 $\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx = \frac{\text{AppellF1}\left(\frac{5}{2},1,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e}{a+b}\right.}{5f}$$

output
```
1/5*AppellF1(5/2,1,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^
5*(a+b+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2777 vs. 2(90) = 180.

Time = 16.13 (sec) , antiderivative size = 2777, normalized size of antiderivative = 30.86

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2
)^p*Tan[e + f*x]^5*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF
1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a
 + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e +
f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(
a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f*(((a +
2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1 + p)*((9*(a + b)*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f
*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^
2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeomet
ric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/
2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e +
 f*x]^2)/(a + b))^p))/3 - (2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(
Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*Tan[e + f*x]*((9*(a + b)*AppellF1[1/2,
-p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x...
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\tan^4(e+fx)(a+b(\tan^2(e+fx)+1))^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^4(e+fx)(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\tan^4(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$\frac{\tan^5(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{2},1,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

output
```
(AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)
/(a + b))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [F]

$$\int \left(a+b\sec \left(fx+e\right)^{2}\right)^{p} \tan \left(fx+e\right)^{4}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

Fricas [F]

$$\int (a + b\sec^2(e + fx))^p \tan^4(e + fx)\,dx = \int (b\sec(fx + e)^2 + a)^p \tan(fx + e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\sec^2(e + fx))^p \tan^4(e + fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + b\sec^2(e + fx))^p \tan^4(e + fx)\,dx = \int (b\sec(fx + e)^2 + a)^p \tan(fx + e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0]%%%}+%%%{-1,[0,1,0,0]%%%} / %%%{1,[0,0,0,1]%%%}
 Error: B
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx = \int \left(\sec(fx+e)^2 b+a\right)^p \tan(fx+e)^4\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*tan(e + f*x)**4,x)
```

3.448 $\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \frac{\text{AppellF1}\left(\frac{3}{2},1,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e+fx)}{a+b}\right)^{-p}}{3f}$$

output
```
1/3*AppellF1(3/2,1,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^
3*(a+b+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2465 vs. 2(90) = 180.

Time = 15.15 (sec) , antiderivative size = 2465, normalized size of antiderivative = 27.39

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2
)^p*Tan[e + f*x]^3*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(
a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p,
 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(
3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a +
b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*((a + 2*b + a*Cos[2*
(e + f*x)])^p*(Sec[e + f*x]^2)^(1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -(
(b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a +
b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^
2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 2*
a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[2*(e +
f*x)]*Tan[e + f*x]*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(
a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p,
 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(
3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan...
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{ 4629}$$

$$\frac{\int \frac{\tan^2(e+fx)(a+b(\tan^2(e+fx)+1))^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\tan^2(e+fx)(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\tan^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$\frac{\tan^3(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},1,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

output
```
(AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)
/(a + b))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^{2}\right)^{p}\tan\left(fx+e\right)^{2}dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
```

Fricas [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^p \tan^2\left(e+fx\right)dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \left(\sec(fx+e)^2 b+a\right)^p \tan(fx+e)^2\,dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*tan(e + f*x)**2,x)
```

3.449 $\int (a+b\sec^2(e+fx))^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int (a+b\sec^2(e+fx))^p \, dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e+}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1512 vs. 2(85) = 170.

Time = 5.85 (sec) , antiderivative size = 1512, normalized size of antiderivative = 17.79

$$\int (a+b\sec^2(e+fx))^p \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(3*(a + b)*AppellF1[1/
2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*Ap
pellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
 - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2])*Tan[e + f*x]^2))/(f*(AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(3*(a + b)*AppellF1[1/2, -
p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*Appell
F1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (
a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2])*Tan[e + f*x]^2) - AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*
AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])
*Tan[e + f*x]^2) + 2*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
- p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])...
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sec(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4616}$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 334

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\big\downarrow$ 333

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

Maple [F]

$$\int \left(a + b\sec\left(fx + e\right)^{2}\right)^{p} dx$$

input
```
int((a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \left(a + b\sec^2(e + fx)\right)^{p} \, dx = \int \left(b\sec\left(fx + e\right)^{2} + a\right)^{p} dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F]

$$\int \left(a + b\sec^2(e + fx)\right)^{p} \, dx = \int \left(a + b\sec^{2}\left(e + fx\right)\right)^{p} \, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p, x)
```

Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p dx$$

input
```
int((a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p,x)
```

3.450 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$
$$-\frac{\operatorname{AppellF1}\left(-\frac{1}{2},1,-p,\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

output
```
-AppellF1(-1/2,1,-p,1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+e)*(a
+b+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2469 vs. 2(86) = 172.

Time = 15.55 (sec) , antiderivative size = 2469, normalized size of antiderivative = 28.71

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*(a + b
*Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2
)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*AppellF1[1/2,
 -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^
2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Ta
n[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*(2*p*(a + 2*b +
a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/
2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3
*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p
, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
) - (a + 2*b + a*Cos[2*(e + f*x)])^p*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*(-(
Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Ta
n[e + f*x]^2)/(a + b))^p) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(...
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sec(e+fx)^2)^p}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^2(e+fx)(a+b(\tan^2(e+fx)+1))^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^2(e+fx)(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},1,-p,\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+f}{a+b}\right.}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))]*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/
(a + b))^p))
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [F]

$$\int \cot(fx+e)^2 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\int \left(\sec\left(fx+e\right)^2b+a\right)^p\cot\left(fx+e\right)^2dx$$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((sec(e + f*x)**2*b + a)**p*cot(e + f*x)**2,x)
```

3.451 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\operatorname{AppellF1}\left(-\frac{3}{2},1,-p,-\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(\frac{a+b+\cdots}{\cdots}\right)}{3f}$$

output
```
-1/3*AppellF1(-3/2,1,-p,-1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+
e)^3*(a+b+b*tan(f*x+e)^2)^p/f/(((a+b+b*tan(f*x+e)^2)/(a+b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3033 vs. 2(90) = 180.

Time = 16.48 (sec) , antiderivative size = 3033, normalized size of antiderivative = 33.70

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^7*(Sec[e + f*x]^2)^p*(a + b
*Sec[e + f*x]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] +
 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2,
 -((b*Tan[e + f*x]^2)/(a + b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*
Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))
^p))/(3*f*((2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^2*(Sec[e + f
*x]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/
2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*Ap
pellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
 - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2])*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[
e + f*x]^2)/(a + b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*
x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 - (
a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^2*Csc[e + f*x]^2*(Sec[e + f*x
]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + ...
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4629}$$

$$\frac{\int \frac{\cot^4(e+fx)(a+b(\tan^2(e+fx)+1))^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{\cot^4(e+fx)(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\cot^4(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\cot^3(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(-\frac{3}{2},1,-p,-\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e}{a+}\right.}{3f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/3*(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))]*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*
x]^2)/(a + b))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Maple [F]

$$\int \cot (fx+e)^4 \left(a+b\sec (fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \cot\left(fx+e\right)^4 \, dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^4(e+fx) \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \cot\left(fx+e\right)^4 \, dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cot(e+f\,x)^4\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p\,dx$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cot\left(fx+e\right)^4\left(\sec\left(fx+e\right)^2b+a\right)^p\,dx$$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

3.452 $\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx=-\frac{a\log(\cos(e+fx))}{f}-\frac{a\sec^2(e+fx)}{f}+\frac{b\sec^3(e+fx)}{3f}+\frac{a\sec^4(e+fx)}{4f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^7(e+fx)}{7f}$$

output
```
-a*ln(cos(f*x+e))/f-a*sec(f*x+e)^2/f+1/3*b*sec(f*x+e)^3/f+1/4*a*sec(f*x+e)
^4/f-2/5*b*sec(f*x+e)^5/f+1/7*b*sec(f*x+e)^7/f
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx=-\frac{a\log(\cos(e+fx))}{f}-\frac{a\sec^2(e+fx)}{f}+\frac{b\sec^3(e+fx)}{3f}+\frac{a\sec^4(e+fx)}{4f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^7(e+fx)}{7f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

output
```
-((a*Log[Cos[e + f*x]])/f) - (a*Sec[e + f*x]^2)/f + (b*Sec[e + f*x]^3)/(3*
f) + (a*Sec[e + f*x]^4)/(4*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*
x]^7)/(7*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4626, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^5\left(a+b\sec(e+fx)^3\right)dx$$

$$\Big\downarrow \; 4626$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^3(e+fx)+b\right)\sec^8(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 2333$$

$$-\frac{\int \left(b\sec^8(e+fx)-2b\sec^6(e+fx)+a\sec^5(e+fx)+b\sec^4(e+fx)-2a\sec^3(e+fx)+a\sec(e+fx)\right)d\cos}{f}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{1}{4}a\sec^4(e+fx)+a\sec^2(e+fx)+a\log(\cos(e+fx))-\frac{1}{7}b\sec^7(e+fx)+\frac{2}{5}b\sec^5(e+fx)-\frac{1}{3}b\sec^3(e+fx}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

output
```
-((a*Log[Cos[e + f*x]] + a*Sec[e + f*x]^2 - (b*Sec[e + f*x]^3)/3 - (a*Sec[
e + f*x]^4)/4 + (2*b*Sec[e + f*x]^5)/5 - (b*Sec[e + f*x]^7)/7)/f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{b\sec(fx+e)^7}{7}-\frac{2b\sec(fx+e)^5}{5}+\frac{a\sec(fx+e)^4}{4}+\frac{b\sec(fx+e)^3}{3}-a\sec(fx+e)^2+a\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{b\sec(fx+e)^7}{7}-\frac{2b\sec(fx+e)^5}{5}+\frac{a\sec(fx+e)^4}{4}+\frac{b\sec(fx+e)^3}{3}-a\sec(fx+e)^2+a\ln(\sec(fx+e))}{f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\left(\frac{\sec(fx+e)^7}{7}-\frac{2\sec(fx+e)^5}{5}+\frac{\sec(fx+e)^3}{3}\right)}{f}$
risch	$iax+\frac{2iae}{f}-\frac{4\left(105a\,e^{12i(fx+e)}-70b\,e^{11i(fx+e)}+420a\,e^{10i(fx+e)}+56b\,e^{9i(fx+e)}+735a\,e^{8i(fx+e)}-228b\,e^{7i(fx+e)}\right.}{105f\left(e^{2i(fx+e)}+1\right)}$

input
```
int((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/7*b*sec(f*x+e)^7-2/5*b*sec(f*x+e)^5+1/4*a*sec(f*x+e)^4+1/3*b*sec(f*
x+e)^3-a*sec(f*x+e)^2+a*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx = -\frac{420\,a\cos\left(fx+e\right)^7\log\left(-\cos\left(fx+e\right)\right)+420\,a\cos\left(fx+e\right)^5-140\,b\cos\left(fx+e\right)^4-105\,a\cos\left(fx+e\right.}{420\,f\cos\left(fx+e\right)^7}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/420*(420*a*cos(f*x + e)^7*log(-cos(f*x + e)) + 420*a*cos(f*x + e)^5 - 1
40*b*cos(f*x + e)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/
(f*cos(f*x + e)^7)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.29

$$\int \left(a+b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx = \begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^4\left(e+fx\right)}{4f}-\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^4\left(e+fx\right)\sec^3\left(e+fx\right)}{7f}-\frac{4b\tan^2\left(e+fx\right)\sec^3\left(e+fx\right)}{35f}+\frac{8b\sec^3\left(e+f\right.}{105f} \\ x\left(a+b\sec^3\left(e\right)\right)\tan^5\left(e\right) \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**5,x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*
tan(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**3/(7*f) - 4*b*tan(
e + f*x)**2*sec(e + f*x)**3/(35*f) + 8*b*sec(e + f*x)**3/(105*f), Ne(f, 0)
), (x*(a + b*sec(e)**3)*tan(e)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int (a + b\sec^3(e + fx))\tan^5(e + fx)\,dx = -\frac{420\,a\log\left(\cos\left(fx + e\right)\right) + \frac{420\,a\cos(fx+e)^5 - 140\,b\cos(fx+e)^4 - 105\,a\cos(fx+e)^3 + 168\,b\cos(fx+e)^2 - 60\,b}{\cos(fx+e)^7}}{420\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/420*(420*a*log(cos(f*x + e)) + (420*a*cos(f*x + e)^5 - 140*b*cos(f*x +
e)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/cos(f*x + e)^7)
/f
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int (a + b\sec^3(e + fx))\tan^5(e + fx)\,dx = -\frac{420\,a\log\left(|\cos\left(fx + e\right)|\right) + \frac{420\,a\cos(fx+e)^5 - 140\,b\cos(fx+e)^4 - 105\,a\cos(fx+e)^3 + 168\,b\cos(fx+e)^2 - 60\,b}{\cos(fx+e)^7}}{420\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
-1/420*(420*a*log(abs(cos(f*x + e))) + (420*a*cos(f*x + e)^5 - 140*b*cos(f
*x + e)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/cos(f*x +
e)^7)/f
```

Mupad [B] (verification not implemented)

Time = 18.51 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.47

$$\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}{f}$$
$$-\frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-14\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+\left(32\,a+\frac{32\,b}{3}\right)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+\left(\frac{16\,b}{3}-32\,a\right)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\tan\left(\frac{e}{2}\right.\right.}$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^3),x)
```

output
```
(2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((16*b)/105 - tan(e/2 + (f*x)/2)^2*(
2*a + (16*b)/15) + tan(e/2 + (f*x)/2)^4*(14*a + (16*b)/5) - tan(e/2 + (f*x
)/2)^6*(32*a - (16*b)/3) + tan(e/2 + (f*x)/2)^8*(32*a + (32*b)/3) - 14*a*t
an(e/2 + (f*x)/2)^10 + 2*a*tan(e/2 + (f*x)/2)^12)/(f*(7*tan(e/2 + (f*x)/2)
^2 - 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x
)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f
*x)/2)^14 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int (a+b\sec^3(e+fx))\tan^5(e+fx)\,dx$$
$$=\frac{210\log\left(\tan(fx+e)^2+1\right)a+60\sec(fx+e)^3\tan(fx+e)^4b-48\sec(fx+e)^3\tan(fx+e)^2b+32\,\mathrm{se}}{420f}$$

input
```
int((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x)
```

output
```
(210*log(tan(e + f*x)**2 + 1)*a + 60*sec(e + f*x)**3*tan(e + f*x)**4*b - 4
8*sec(e + f*x)**3*tan(e + f*x)**2*b + 32*sec(e + f*x)**3*b + 105*tan(e + f
*x)**4*a - 210*tan(e + f*x)**2*a)/(420*f)
```

3.453 $\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx=\frac{a\log(\cos(e+fx))}{f}+\frac{a\sec^2(e+fx)}{2f}-\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+fx)}{5f}$$

output
```
a*ln(cos(f*x+e))/f+1/2*a*sec(f*x+e)^2/f-1/3*b*sec(f*x+e)^3/f+1/5*b*sec(f*x
+e)^5/f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx=-\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+fx)}{5f}+\frac{a(2\log(\cos(e+fx))+\sec^2(e+fx))}{2f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

output
```
-1/3*(b*Sec[e + f*x]^3)/f + (b*Sec[e + f*x]^5)/(5*f) + (a*(2*Log[Cos[e + f
*x]] + Sec[e + f*x]^2))/(2*f)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4626, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^3\left(a+b\sec(e+fx)^3\right)dx$$

$$\Big\downarrow \text{ 4626}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^3(e+fx)+b\right)\sec^6(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 2333}$$

$$-\frac{\int \left(b\sec^6(e+fx)-b\sec^4(e+fx)+a\sec^3(e+fx)-a\sec(e+fx)\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{1}{2}a\sec^2(e+fx)-a\log(\cos(e+fx))-\frac{1}{5}b\sec^5(e+fx)+\frac{1}{3}b\sec^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

output
```
-((-(a*Log[Cos[e + f*x]]) - (a*Sec[e + f*x]^2)/2 + (b*Sec[e + f*x]^3)/3 -
(b*Sec[e + f*x]^5)/5)/f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{b\sec(fx+e)^5}{5}-\frac{b\sec(fx+e)^3}{3}+\frac{a\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{b\sec(fx+e)^5}{5}-\frac{b\sec(fx+e)^3}{3}+\frac{a\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\left(\frac{\sec(fx+e)^5}{5}-\frac{\sec(fx+e)^3}{3}\right)}{f}$
risch	$-iax-\frac{2iae}{f}+\frac{2a\,e^{8i(fx+e)}-\frac{8b\,e^{7i(fx+e)}}{3}+6a\,e^{6i(fx+e)}+\frac{16b\,e^{5i(fx+e)}}{15}+6\,e^{4i(fx+e)}a-\frac{8b\,e^{3i(fx+e)}}{3}+2\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^5}$

input
```
int((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/5*b*sec(f*x+e)^5-1/3*b*sec(f*x+e)^3+1/2*a*sec(f*x+e)^2-a*ln(sec(f*x
+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \left(a+b\sec^3(e+fx)\right)\tan^3(e+fx)\,dx = \frac{30\,a\cos\left(fx+e\right)^5\log\left(-\cos\left(fx+e\right)\right)+15\,a\cos\left(fx+e\right)^3-10\,b\cos\left(fx+e\right)^2+6\,b}{30\,f\cos\left(fx+e\right)^5}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/30*(30*a*cos(f*x + e)^5*log(-cos(f*x + e)) + 15*a*cos(f*x + e)^3 - 10*b*
cos(f*x + e)^2 + 6*b)/(f*cos(f*x + e)^5)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \left(a+b\sec^3(e+fx)\right)\tan^3(e+fx)\,dx = \begin{cases} -\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^2\left(e+fx\right)\sec^3\left(e+fx\right)}{5f}-\frac{2b\sec^3\left(e+fx\right)}{15f} & \text{for } f\neq 0 \\ x(a+b\sec^3\left(e\right))\tan^3\left(e\right) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**3,x)
```

output
```
Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b
*tan(e + f*x)**2*sec(e + f*x)**3/(5*f) - 2*b*sec(e + f*x)**3/(15*f), Ne(f,
 0)), (x*(a + b*sec(e)**3)*tan(e)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx = \frac{30\,a\log\left(\cos\left(fx+e\right)\right)+\frac{15\,a\cos\left(fx+e\right)^3-10\,b\cos\left(fx+e\right)^2+6\,b}{\cos\left(fx+e\right)^5}}{30\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/30*(30*a*log(cos(f*x + e)) + (15*a*cos(f*x + e)^3 - 10*b*cos(f*x + e)^2
+ 6*b)/cos(f*x + e)^5)/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx = \frac{30\,a\log\left(|\cos\left(fx+e\right)|\right)+\frac{15\,a\cos\left(fx+e\right)^3-10\,b\cos\left(fx+e\right)^2+6\,b}{\cos\left(fx+e\right)^5}}{30\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
1/30*(30*a*log(abs(cos(f*x + e))) + (15*a*cos(f*x + e)^3 - 10*b*cos(f*x +
e)^2 + 6*b)/cos(f*x + e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 18.45 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.74

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx$$
$$= \frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+(-6\,a-4\,b)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\left(6\,a-\frac{4\,b}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\left(-2\,a-\frac{4\,b}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$
$$-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}{f}$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

output
```
((4*b)/15 - tan(e/2 + (f*x)/2)^2*(2*a + (4*b)/3) - tan(e/2 + (f*x)/2)^6*(6
*a + 4*b) + tan(e/2 + (f*x)/2)^4*(6*a - (4*b)/3) + 2*a*tan(e/2 + (f*x)/2)^
8)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*
x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1)) - (2*a*atan
h(tan(e/2 + (f*x)/2)^2))/f
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx$$
$$= \frac{-15\log\left(\tan(fx+e)^2+1\right)a+6\sec(fx+e)^3\tan(fx+e)^2b-4\sec(fx+e)^3b+15\tan(fx+e)^2a}{30f}$$

input
```
int((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x)
```

output
```
( - 15*log(tan(e + f*x)**2 + 1)*a + 6*sec(e + f*x)**3*tan(e + f*x)**2*b -
4*sec(e + f*x)**3*b + 15*tan(e + f*x)**2*a)/(30*f)
```

3.454 $\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx=-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}$$

output
```
-a*ln(cos(f*x+e))/f+1/3*b*sec(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx=-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
```

output
```
-((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4626, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^3\right)dx$$

$$\Big\downarrow \ 4626$$

$$-\frac{\int \left(a\cos^3(e+fx)+b\right)\sec^4(e+fx)d\cos(e+fx)}{f}$$

$$\Big\downarrow \ 802$$

$$-\frac{\int \left(b\sec^4(e+fx)+a\sec(e+fx)\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a\log(\cos(e+fx))-\frac{1}{3}b\sec^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
```

output
```
-((a*Log[Cos[e + f*x]] - (b*Sec[e + f*x]^3)/3)/f)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{b\sec(fx+e)^3}{3}+a\ln(\sec(fx+e))}{f}$	26
default	$\frac{\frac{b\sec(fx+e)^3}{3}+a\ln(\sec(fx+e))}{f}$	26
parts	$\frac{a\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b\sec(fx+e)^3}{3f}$	33
risch	$iax+\frac{2iae}{f}+\frac{8b\,\mathrm{e}^{3i(fx+e)}}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}-\frac{a\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{f}$	61

input
```
int((a+b*sec(f*x+e)^3)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*b*sec(f*x+e)^3+a*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx = -\frac{3\,a\cos\left(fx+e\right)^3\log\left(-\cos\left(fx+e\right)\right)-b}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/3*(3*a*cos(f*x + e)^3*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^3)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx = \begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b\sec^3\left(e+fx\right)}{3f} & \text{for } f\neq 0 \\ x(a+b\sec^3\left(e\right))\tan\left(e\right) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**3)*tan(f*x+e),x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**3/(3*f), Ne(
f, 0)), (x*(a + b*sec(e)**3)*tan(e), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx = -\frac{a\log\left(\cos\left(fx+e\right)^3\right)-\frac{b}{\cos(fx+e)^3}}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/3*(a*log(cos(f*x + e)^3) - b/cos(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx = -\frac{3\,a\log\left(\left|\cos\left(fx+e\right)\right|\right)-\frac{a\cos(fx+e)^3+b}{\cos(fx+e)^3}}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="giac")
```

output
```
-1/3*(3*a*log(abs(cos(f*x + e))) - (a*cos(f*x + e)^3 + b)/cos(f*x + e)^3)/
f
```

Mupad [B] (verification not implemented)

Time = 15.98 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int (a+b\sec^3(e+fx))\tan(e+fx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}{f} - \frac{2\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{2\,b}{3}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

output
```
(2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((2*b)/3 + 2*b*tan(e/2 + (f*x)/2)^4)
/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^
6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx=\frac{3\log\left(\tan\left(fx+e\right)^2+1\right)a+2\sec\left(fx+e\right)^3b}{6f}$$

input
```
int((a+b*sec(f*x+e)^3)*tan(f*x+e),x)
```

output
```
(3*log(tan(e + f*x)**2 + 1)*a + 2*sec(e + f*x)**3*b)/(6*f)
```

3.455 $\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=\frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(1+\cos(e+fx))}{2f}+\frac{b\sec(e+fx)}{f}$$

output
```
1/2*(a+b)*ln(1-cos(f*x+e))/f+1/2*(a-b)*ln(1+cos(f*x+e))/f+b*sec(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{a\log(\sin(e+fx))}{f}+\frac{b\sec(e+fx)}{f}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

output
```
-((b*Log[Cos[(e + f*x)/2]])/f) + (b*Log[Sin[(e + f*x)/2]])/f + (a*Log[Sin[
e + f*x]])/f + (b*Sec[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4626, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(e+fx)^3}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{(a\cos^3(e+fx)+b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2333}$$

$$-\frac{\int \left(b\sec^2(e+fx)+\frac{-a-b}{2(\cos(e+fx)-1)}+\frac{b-a}{2(\cos(e+fx)+1)}\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{2}(a+b)\log(1-\cos(e+fx))-\frac{1}{2}(a-b)\log(\cos(e+fx)+1)-b\sec(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

output
```
-((-1/2*((a + b)*Log[1 - Cos[e + f*x]]) - ((a - b)*Log[1 + Cos[e + f*x]])/
2 - b*Sec[e + f*x])/f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a\ln(\sin(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$
default	$\frac{a\ln(\sin(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$
risch	$-iax-\frac{2iae}{f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)b}{f}+\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}\right.}{f}$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*ln(sin(f*x+e))+b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{(a-b)\cos(fx+e)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+(a+b)\cos(fx+e)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+2\,b}{2\,f\cos(fx+e)}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
1/2*((a - b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) + (a + b)*cos(f*x +
e)*log(-1/2*cos(f*x + e) + 1/2) + 2*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \int \left(a+b\sec^3(e+fx)\right)\cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**3),x)
```

output
```
Integral((a + b*sec(e + f*x)**3)*cot(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{(a-b)\log(\cos(fx+e)+1)+(a+b)\log(\cos(fx+e)-1)+\frac{2\,b}{\cos(fx+e)}}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
1/2*((a - b)*log(cos(f*x + e) + 1) + (a + b)*log(cos(f*x + e) - 1) + 2*b/c
os(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{(a-b)\log\left(\left|\cos\left(fx+e\right)+1\right|\right)}{2\,f} + \frac{(a+b)\log\left(\left|\cos\left(fx+e\right)-1\right|\right)}{2\,f} + \frac{b}{f\cos\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
1/2*(a - b)*log(abs(cos(f*x + e) + 1))/f + 1/2*(a + b)*log(abs(cos(f*x + e
) - 1))/f + b/(f*cos(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f} - \frac{a\,\ln\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}{f} - \frac{2\,b}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)} + \frac{b\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

output
```
(a*log(tan(e/2 + (f*x)/2)))/f - (a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (2*b
)/(f*(tan(e/2 + (f*x)/2)^2 - 1)) + (b*log(tan(e/2 + (f*x)/2)))/f
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$
$$=\frac{-\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)a+\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}\right.\right.}{\cos\left(fx+e\right)f}$$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^3),x)
```

output
```
( - cos(e + f*x)*log(tan((e + f*x)/2)**2 + 1)*a + cos(e + f*x)*log(tan((e
+ f*x)/2))*a + cos(e + f*x)*log(tan((e + f*x)/2))*b - cos(e + f*x)*b + b)/
(cos(e + f*x)*f)
```

3.456 $\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=-\frac{b\text{arctanh}(\cos(e+fx))}{2f}-\frac{b\cot(e+fx)\csc(e+fx)}{2f}-\frac{a\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}$$

output
```
-1/2*b*arctanh(cos(f*x+e))/f-1/2*b*cot(f*x+e)*csc(f*x+e)/f-1/2*a*csc(f*x+e
)^2/f-a*ln(sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = -\frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{a\csc^2(e+fx)}{2f}-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}-\frac{a\log(\sin(e+fx))}{f}+\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

output
```
-1/8*(b*Csc[(e + f*x)/2]^2)/f - (a*Csc[e + f*x]^2)/(2*f) - (b*Log[Cos[(e +
 f*x)/2]])/(2*f) + (b*Log[Sin[(e + f*x)/2]])/(2*f) - (a*Log[Sin[e + f*x]])
/f + (b*Sec[(e + f*x)/2]^2)/(8*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4626, 2345, 25, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(e+fx)^3}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{a\cos^3(e+fx)+b}{(1-\cos^2(e+fx))^2}d\cos(e+fx)}{f}$$

$\Big\downarrow$ 2345

$$-\frac{\frac{a+b\cos(e+fx)}{2(1-\cos^2(e+fx))}-\frac{1}{2}\int -\frac{b-2a\cos(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$\Big\downarrow$ 25

$$-\frac{\frac{1}{2}\int \frac{b-2a\cos(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{a+b\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

$\Big\downarrow$ 452

$$-\frac{\frac{1}{2}\left(b\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-2a\int \frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)+\frac{a+b\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

$\Big\downarrow$ 219

$$-\frac{\frac{1}{2}\left(b\text{arctanh}(\cos(e+fx))-2a\int \frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)+\frac{a+b\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

$\Big\downarrow$ 240

$$-\frac{\frac{1}{2}\left(a\log\left(1-\cos^2(e+fx)\right)+b\text{arctanh}(\cos(e+fx))\right)+\frac{a+b\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

output
```
-(((a + b*Cos[e + f*x])/(2*(1 - Cos[e + f*x]^2)) + (b*ArcTanh[Cos[e + f*x]
] + a*Log[1 - Cos[e + f*x]^2])/2)/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.80 (sec), antiderivative size = 63, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
default	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
risch	$iax+\frac{2iae}{f}+\frac{b\,e^{3i(fx+e)}+2\,e^{2i(fx+e)}a+b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)b}{2f}-\frac{\ln\left(e^{i(fx+e)}+1\right)}{f}$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))+b*(-1/2*csc(f*x+e)*cot(f*x+e)+1/
2*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec), antiderivative size = 99, normalized size of antiderivative = 1.52

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$
$$=\frac{2\,b\cos\left(fx+e\right)-\left((2\,a+b)\cos\left(fx+e\right)^2-2\,a-b\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-\left((2\,a-b)\cos\left(fx+e\right)^2\right.}{4\left(f\cos\left(fx+e\right)^2-f\right)}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
1/4*(2*b*cos(f*x + e) - ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*log(1/2*cos(f
*x + e) + 1/2) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*log(-1/2*cos(f*x + e
) + 1/2) + 2*a)/(f*cos(f*x + e)^2 - f)
```

Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=\int \left(a+b\sec^3\left(e+fx\right)\right)\cot^3\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**3),x)
```

output
```
Integral((a + b*sec(e + f*x)**3)*cot(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$
$$=-\frac{(2\,a+b)\log\left(\cos\left(fx+e\right)+1\right)+(2\,a-b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,(b\cos(fx+e)+a)}{\cos(fx+e)^2-1}}{4\,f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
-1/4*((2*a + b)*log(cos(f*x + e) + 1) + (2*a - b)*log(cos(f*x + e) - 1) -
2*(b*cos(f*x + e) + a)/(cos(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx=-\frac{(2\,a+b)\log\left(|\cos\left(fx+e\right)+1|\right)}{4\,f}-\frac{(2\,a-b)\log\left(|\cos\left(fx+e\right)-1|\right)}{4\,f}+\frac{b\cos\left(fx+e\right)+a}{2\,f(\cos\left(fx+e\right)+1)(\cos\left(fx+e\right)-1)}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
-1/4*(2*a + b)*log(abs(cos(f*x + e) + 1))/f - 1/4*(2*a - b)*log(abs(cos(f*
x + e) - 1))/f + 1/2*(b*cos(f*x + e) + a)/(f*(cos(f*x + e) + 1)*(cos(f*x +
 e) - 1))
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{a\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a}{8}-\frac{b}{8}\right)}{f} - \frac{\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a}{8}+\frac{b}{8}\right)}{f} - \frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(a-\frac{b}{2}\right)}{f}$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

output
```
(a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (tan(e/2 + (f*x)/2)^2*(a/8 - b/8))/f
 - (cot(e/2 + (f*x)/2)^2*(a/8 + b/8))/f - (log(tan(e/2 + (f*x)/2))*(a - b/
2))/f
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.58

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{-2\cos(fx+e)\,b+4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin(fx+e)^2 a-4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^2 a+2\mathrm{lo}}{4\sin(fx+e)^2 f}$$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x)
```

output
```
( - 2*cos(e + f*x)*b + 4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a -
4*log(tan((e + f*x)/2))*sin(e + f*x)**2*a + 2*log(tan((e + f*x)/2))*sin(e
+ f*x)**2*b + sin(e + f*x)**2*a - 2*a)/(4*sin(e + f*x)**2*f)
```

3.457 $\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\begin{aligned}\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx &= -\frac{\left(a^{2/3}+2b^{2/3}\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}f}\\ &\quad-\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{4/3}f}\\ &\quad+\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{4/3}f}\\ &\quad-\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}+\frac{\sec(e+fx)}{bf}\end{aligned}$$

output
```
-1/3*(a^(2/3)+2*b^(2/3))*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))*3^(1/2)
/b^(1/3))*3^(1/2)/a^(1/3)/b^(4/3)/f-1/3*(a^(2/3)-2*b^(2/3))*ln(b^(1/3)+a^(
1/3)*cos(f*x+e))/a^(1/3)/b^(4/3)/f+1/6*(a^(2/3)-2*b^(2/3))*ln(b^(2/3)-a^(1
/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/b^(4/3)/f-1/3*ln(b+a*
cos(f*x+e)^3)/a/f+sec(f*x+e)/b/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.15

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$= \frac{3b\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right) - \text{RootSum}\left[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\&, \frac{-4a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+\right.\right.}{}\right.}{}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

output
```
(3*b*Log[Sec[(e + f*x)/2]^2] - RootSum[-8*a + 12*a*#1 - 6*a*#1^2 + a*#1^3
- b*#1^3 & , (-4*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 4*a*b*Log[1 - #1 +
 Tan[(e + f*x)/2]^2] + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 - 8*a*b*L
og[1 - #1 + Tan[(e + f*x)/2]^2]*#1 + a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*
#1^2 - b^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 -
 b*#1^2) & ] + 3*a*Sec[e + f*x])/(3*a*b*f)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4626, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\tan(e+fx)^5}{a+b\sec(e+fx)^3}dx$$
$$\Big\downarrow \text{ 4626}$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2\sec^2(e+fx)}{a\cos^3(e+fx)+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2373}$$

$$-\frac{\int \left(\frac{\sec^2(e+fx)}{b}+\frac{b\cos^2(e+fx)-a\cos(e+fx)-2b}{b(a\cos^3(e+fx)+b)}\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\left(a^{2/3}+2b^{2/3}\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}-\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}b^{4/3}}+\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{\right.}{3\sqrt[3]{a}}}{f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

output
```
-((((a^(2/3) + 2*b^(2/3))*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[
3]*b^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) + ((a^(2/3) - 2*b^(2/3))*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*b^(4/3)) - ((a^(2/3) - 2*b^(2/3))*L
og[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2])/(6*a^
(1/3)*b^(4/3)) + Log[b + a*Cos[e + f*x]^3]/(3*a) - Sec[e + f*x]/b)/f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.06

method	result
risch	$\frac{ix}{a}+\frac{2ie}{af}+\frac{2\,e^{i(fx+e)}}{fb\left(e^{2i(fx+e)}+1\right)}-i\left(\sum_{\_R=\mathrm{RootOf}\left(27a^3b^4f^3\_Z^3+27ia^2b^4f^2\_Z^2+(-18a^3b^2f-9a\,b^4f)\_Z-ia^4+2\right.}\right.$
derivativedivides	$\frac{1}{b\cos(fx+e)}+\frac{2b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right.}{}$
default	$\frac{1}{b\cos(fx+e)}+\frac{2b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right.}{}$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
I*x/a+2*I/a/f*e+2*exp(I*(f*x+e))/f/b/(exp(2*I*(f*x+e))+1)-I*sum(_R*ln(exp(
2*I*(f*x+e))+(-18/(a^3+8*a*b^2)*a^2*b^3*f^2*_R^2-36*I/(a^3+8*a*b^2)*b^3*f*
a*_R+8/(a^3+8*a*b^2)*a^2*b+10/(a^3+8*a*b^2)*b^3)*exp(I*(f*x+e))+1),_R=Root
Of(27*a^3*b^4*f^3*_Z^3+27*I*a^2*b^4*f^2*_Z^2+(-18*a^3*b^2*f-9*a*b^4*f)*_Z-
I*a^4+2*I*a^2*b^2-I*b^4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 4427, normalized size of antiderivative = 20.21

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**3),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{\frac{2\sqrt{3}\left(2\,ab\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{b}{a}\right)+3\,a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+2\,b^2\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\,\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{3\left(2\,b\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)-a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)\right)}{ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{18\,f}$$

input

```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output

```
1/18*(2*sqrt(3)*(2*a*b*(3*(b/a)^(1/3) - b/a) + 3*a^2*(b/a)^(2/3) + 2*b^2)*
arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b^2) -
3*(2*b*((b/a)^(2/3) + 1) - a*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)
*cos(f*x + e) + (b/a)^(2/3))/(a*b*(b/a)^(2/3)) - 6*(b*((b/a)^(2/3) - 2) +
a*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a*b*(b/a)^(2/3)) + 18/(b*c
os(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx &= -\frac{\log\left(\left|a\cos\left(fx+e\right)^3+b\right|\right)}{3\,af} \\ &+ \frac{\sqrt{3}\left(2\left(-a^2b\right)^{\frac{1}{3}}b-\left(-a^2b\right)^{\frac{2}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2\,\cos(fx+e)\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab^2f} \\ &+ \frac{\left(2\left(-a^2b\right)^{\frac{1}{3}}b+\left(-a^2b\right)^{\frac{2}{3}}\right)\log\left(\cos\left(fx+e\right)^2+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\cos\left(fx+e\right)+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,ab^2f} \\ &+ \frac{1}{bf\cos\left(fx+e\right)}-\frac{\left(a^2bf\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2\,ab^2f\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\cos\left(fx+e\right)\right|\right)}{3\,ab^3f^2}\end{aligned}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
-1/3*log(abs(a*cos(f*x + e)^3 + b))/(a*f) + 1/3*sqrt(3)*(2*(-a^2*b)^(1/3)*
b - (-a^2*b)^(2/3))*arctan(1/3*sqrt(3)*((-b/a)^(1/3) + 2*cos(f*x + e))/(-b
/a)^(1/3))/(a*b^2*f) + 1/6*(2*(-a^2*b)^(1/3)*b + (-a^2*b)^(2/3))*log(cos(f
*x + e)^2 + (-b/a)^(1/3)*cos(f*x + e) + (-b/a)^(2/3))/(a*b^2*f) + 1/(b*f*c
os(f*x + e)) - 1/3*(a^2*b*f*(-b/a)^(1/3) + 2*a*b^2*f)*(-b/a)^(1/3)*log(abs
(-(-b/a)^(1/3) + cos(f*x + e)))/(a*b^3*f^2)
```

Mupad [B] (verification not implemented)

Time = 17.54 (sec) , antiderivative size = 7402, normalized size of antiderivative = 33.80

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^3),x)
```

output
```
symsum(log(-(262144*(148*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*b^17 - 1920*a*b^15 - 156*b^1
6*cos(e + f*x) + 300*b^16 + 16*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*
b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*b^18 + 5232*a^2*b^14
 - 7872*a^3*b^13 + 7080*a^4*b^12 - 3840*a^5*b^11 + 1200*a^6*b^10 - 192*a^7
*b^9 + 12*a^8*b^8 - 5916*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^2*b^15 + 4820*root(27*a^3*
b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^
4, z, k)*a^3*b^14 + 5933*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^4*b^13 - 12882*root(27*a^3
*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a
^4, z, k)*a^5*b^12 + 8891*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
 + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^6*b^11 - 2872*root(27*a^3
*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a
^4, z, k)*a^7*b^10 + 447*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^8*b^9 - 26*root(27*a^3*b^4
*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4,
z, k)*a^9*b^8 + root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*
b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^10*b^7 + 1396*root(27*a^3*b^4*z^3 +
 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, ...
```

Reduce [F]

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{too large to display}$$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x)
```

output
```
( - 36*cos(e + f*x)**2*a**3*b - 72*cos(e + f*x)**2*a**2*b**2 - 30*cos(e +
f*x)*int(sin(e + f*x)**5/(cos(e + f*x)*sin(e + f*x)**4*a**3 + 4*cos(e + f*
x)*sin(e + f*x)**4*a**2*b + 4*cos(e + f*x)*sin(e + f*x)**4*a*b**2 - 2*cos(
e + f*x)*sin(e + f*x)**2*a**3 - 8*cos(e + f*x)*sin(e + f*x)**2*a**2*b - 8*
cos(e + f*x)*sin(e + f*x)**2*a*b**2 + cos(e + f*x)*a**3 + 4*cos(e + f*x)*a
**2*b + 4*cos(e + f*x)*a*b**2 - sin(e + f*x)**2*a**2*b - 4*sin(e + f*x)**2
*a*b**2 - 4*sin(e + f*x)**2*b**3 + a**2*b + 4*a*b**2 + 4*b**3),x)*a**6*b*f
 - 111*cos(e + f*x)*int(sin(e + f*x)**5/(cos(e + f*x)*sin(e + f*x)**4*a**3
 + 4*cos(e + f*x)*sin(e + f*x)**4*a**2*b + 4*cos(e + f*x)*sin(e + f*x)**4*
a*b**2 - 2*cos(e + f*x)*sin(e + f*x)**2*a**3 - 8*cos(e + f*x)*sin(e + f*x)
**2*a**2*b - 8*cos(e + f*x)*sin(e + f*x)**2*a*b**2 + cos(e + f*x)*a**3 + 4
*cos(e + f*x)*a**2*b + 4*cos(e + f*x)*a*b**2 - sin(e + f*x)**2*a**2*b - 4*
sin(e + f*x)**2*a*b**2 - 4*sin(e + f*x)**2*b**3 + a**2*b + 4*a*b**2 + 4*b*
*3),x)*a**5*b**2*f - 48*cos(e + f*x)*int(sin(e + f*x)**5/(cos(e + f*x)*sin
(e + f*x)**4*a**3 + 4*cos(e + f*x)*sin(e + f*x)**4*a**2*b + 4*cos(e + f*x)
*sin(e + f*x)**4*a*b**2 - 2*cos(e + f*x)*sin(e + f*x)**2*a**3 - 8*cos(e +
f*x)*sin(e + f*x)**2*a**2*b - 8*cos(e + f*x)*sin(e + f*x)**2*a*b**2 + cos(
e + f*x)*a**3 + 4*cos(e + f*x)*a**2*b + 4*cos(e + f*x)*a*b**2 - sin(e + f*
x)**2*a**2*b - 4*sin(e + f*x)**2*a*b**2 - 4*sin(e + f*x)**2*b**3 + a**2*b
+ 4*a*b**2 + 4*b**3),x)*a**4*b**3*f + 192*cos(e + f*x)*int(sin(e + f*x)...
```

3.458 $\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}f} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

output
```
1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))*3^(1/2)/b^(1/3))*3^(1/2)/a^(
1/3)/b^(2/3)/f-1/3*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/b^(2/3)/f+1/6*ln
(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/b^(2/3)/
f+1/3*ln(b+a*cos(f*x+e)^3)/a/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.46

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$= \frac{-3\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right)+\text{RootSum}\Big[-a-b+3a\#1-3b\#1-3a\#1^2-3b\#1^2+a\#1^3-b\#1^3\&,\frac{-a}{\ }\ldots}{}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

output
```
(-3*Log[Sec[(e + f*x)/2]^2] + RootSum[-a - b + 3*a*#1 - 3*b*#1 - 3*a*#1^2
- 3*b*#1^2 + a*#1^3 - b*#1^3 & , (-(a*Log[-#1 + Tan[(e + f*x)/2]^2]) - b*L
og[-#1 + Tan[(e + f*x)/2]^2] - 4*a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1 - 2*b*
Log[-#1 + Tan[(e + f*x)/2]^2]*#1 + a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1^2 -
b*Log[-#1 + Tan[(e + f*x)/2]^2]*#1^2)/(a - b - 2*a*#1 - 2*b*#1 + a*#1^2 -
b*#1^2) & ])/(3*a*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4626, 2410, 750, 16, 792, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\tan(e+fx)^3}{a+b\sec(e+fx)^3}dx$$
$$\Big\downarrow \text{ 4626}$$

$$-\frac{\int \frac{1-\cos^2(e+fx)}{a\cos^3(e+fx)+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 2410}$$

$$-\frac{\int \frac{1}{a\cos^3(e+fx)+b}d\cos(e+fx)-\int \frac{\cos^2(e+fx)}{a\cos^3(e+fx)+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 750}$$

$$-\frac{\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}}d\cos(e+fx)}{3b^{2/3}}-\int \frac{\cos^2(e+fx)}{a\cos^3(e+fx)+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{3b^{2/3}}-\int \frac{\cos^2(e+fx)}{a\cos^3(e+fx)+b}d\cos(e+fx)+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}}{f}$$

$$\Big\downarrow \text{ 792}$$

$$-\frac{\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3a}}{f}$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)\right)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+f\right.}{3\sqrt[3]{a}b^{2/}}}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)+\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)\right)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx\right.}{3\sqrt[3]{a}b^{2/3}}}{f}$$

$$\downarrow 27$$

$$-\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+\right.}{3\sqrt[3]{a}b}}{f}$$

$$\downarrow 1082$$

$$-\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{}\right.}{3\sqrt[3]{a}b^{2/3}}}{f}$$

$$\downarrow 217$$

$$-\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(a\cos^3(e+\right.}{3a}}{f}$$

$$\downarrow 1103$$

$$-\frac{\frac{-\frac{\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3a}}{f}$$

input

```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

output

```
-((Log[b^(1/3) + a^(1/3)*Cos[e + f*x]]/(3*a^(1/3)*b^(2/3)) + (-((Sqrt[3]*A
rcTan[(1 - (2*a^(1/3)*Cos[e + f*x])/b^(1/3))/Sqrt[3]])/a^(1/3)) - Log[b^(2
/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2]/(2*a^(1/3)))/
(3*b^(2/3)) - Log[b + a*Cos[e + f*x]^3]/(3*a))/f)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, 
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.70 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{ix}{a}-\frac{2ie}{af}+i\left(\sum_{\_R=\text{RootOf}\left(27b^2a^3f^3\_Z^3+27ib^2a^2f^2\_Z^2-9\_Zab^2f+ia^2-ib^2\right)}\_R\ln\left(e^{2i(fx+e)}+(-\right.\right.$
derivativedivides	$\frac{-\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(b+a\cos(fx\right.}{3a}}{f}$
default	$\frac{-\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(b+a\cos(fx\right.}{3a}}{f}$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
-I*x/a-2*I/a/f*e+I*sum(_R*ln(exp(2*I*(f*x+e))+(-6*I*b*f*_R+2*b/a)*exp(I*(f
*x+e))+1),_R=RootOf(27*b^2*a^3*f^3*_Z^3+27*I*b^2*a^2*f^2*_Z^2-9*_Z*a*b^2*f
+I*a^2-I*b^2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 1052, normalized size of antiderivative = 6.34

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^
2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f*log(-1/2*(3*(I*sqrt(3) + 1)*(
-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3)
 - 2/(a*f))*a*b*f - a*cos(f*x + e) - b) - ((3*(I*sqrt(3) + 1)*(-1/54/(a^3*
f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))
*a*f + 3*sqrt(1/3)*a*f*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(
a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 +
4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2
)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)) + 6)*log(1/2*(3*(I*s
qrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^
2*f^3))^(1/3) - 2/(a*f))*a*b*f + 3/2*sqrt(1/3)*a*b*f*sqrt(-((3*(I*sqrt(3)
+ 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))
^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54
/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(
a^2*f^2)) - 2*a*cos(f*x + e) + b) - ((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) +
 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f -
 3*sqrt(1/3)*a*f*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*
f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(
I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3
*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)) + 6)*log(-1/2*(3*(I*sqr...
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx=\int \frac{\tan^3{(e+fx)}}{a+b\sec^3{(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{2b}{a}\right)+2b\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{3\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-}{18f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
-1/18*(2*sqrt(3)*(a*(3*(b/a)^(1/3) - 2*b/a) + 2*b)*arctan(-1/3*sqrt(3)*((b
/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b) - 3*(2*(b/a)^(2/3) + 1)*log
(cos(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/a)^(2/3))/(a*(b/a)^(2/3))
- 6*((b/a)^(2/3) - 1)*log((b/a)^(1/3) + cos(f*x + e))/(a*(b/a)^(2/3)))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx &= \frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\cos(fx+e)\right|\right)}{3\,bf}+\frac{\log\left(\left|a\cos(fx+e)^3+b\right|\right)}{3\,af} \\ &\quad -\frac{\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2\cos(fx+e)\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,abf} \\ &\quad -\frac{(-a^2b)^{\frac{1}{3}}\log\left(\cos(fx+e)^2+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,abf}\end{aligned}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
1/3*(-b/a)^(1/3)*log(abs(-(-b/a)^(1/3) + cos(f*x + e)))/(b*f) + 1/3*log(ab
s(a*cos(f*x + e)^3 + b))/(a*f) - 1/3*sqrt(3)*(-a^2*b)^(1/3)*arctan(1/3*sqr
t(3)*((-b/a)^(1/3) + 2*cos(f*x + e))/(-b/a)^(1/3))/(a*b*f) - 1/6*(-a^2*b)^
(1/3)*log(cos(f*x + e)^2 + (-b/a)^(1/3)*cos(f*x + e) + (-b/a)^(2/3))/(a*b*
f)
```

Mupad [B] (verification not implemented)

Time = 18.91 (sec) , antiderivative size = 1620, normalized size of antiderivative = 9.76

$$\int \frac{\tan^3(e + fx)}{a + b\sec^3(e + fx)}\, dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

output
```
symsum(log(262144*(a - b)^2*(8*a - 8*b + 4*root(27*a^3*b^2*z^3 - 27*a^2*b^
2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^2 + 4*root(27*a^3*b^2*z^3 - 27*a^2*
b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^2 - 3*root(27*a^3*b^2*z^3 - 27*a^
2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3 - 24*root(27*a^3*b^2*z^3 -
27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^2 - 36*root(27*a^3*b^2
*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^3*b + 28*root(27*
a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b + 36*root(
27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^2*b^2)*
(16*a^2*tan(e/2 + (f*x)/2)^2 + 32*b^2*tan(e/2 + (f*x)/2)^2 - 4*root(27*a^3
*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^3 - 4*root(27*a
^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^3 - 8*a^2 + 8
*b^2 + 3*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z,
k)^2*a^4 - 3*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2,
 z, k)^2*a^4*tan(e/2 + (f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^
2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^3 + 3*root(27*a^3*b^2*z^3 - 27*a^2*
b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3*b + 36*root(27*a^3*b^2*z^3 -
27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^4*b - 48*a*b*tan(e/2 + (
f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^
2, z, k)^2*a^2*b^2 - 36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z +
 a^2 - b^2, z, k)^3*a^2*b^3 + 14*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 +...
```

Reduce [F]

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan\left(fx+e\right)^3}{\sec\left(fx+e\right)^3 b+a}dx$$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x)
```

output
```
int(tan(e + f*x)**3/(sec(e + f*x)**3*b + a),x)
```

3.459 $\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

output
```
-1/3*ln(b+a*cos(f*x+e)^3)/a/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

output
```
-1/3*Log[b + a*Cos[e + f*x]^3]/(a*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4626, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\tan(e+fx)}{a+b\sec(e+fx)^3}dx$$
$$\Big\downarrow \text{ 4626}$$
$$-\frac{\int \frac{\cos^2(e+fx)}{a\cos^3(e+fx)+b}d\cos(e+fx)}{f}$$
$$\Big\downarrow \text{ 792}$$
$$-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

output
```
-1/3*Log[b + a*Cos[e + f*x]^3]/(a*f)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(a+b\sec\left(fx+e\right)^{3}\right)}{3a}+\frac{\ln\left(\sec\left(fx+e\right)\right)}{a}}{f}$	35
default	$\frac{-\frac{\ln\left(a+b\sec\left(fx+e\right)^{3}\right)}{3a}+\frac{\ln\left(\sec\left(fx+e\right)\right)}{a}}{f}$	35
risch	$\frac{ix}{a}+\frac{2ie}{af}-\frac{\ln\left({\mathrm e}^{6i\left(fx+e\right)}+3\,{\mathrm e}^{4i\left(fx+e\right)}+\frac{8b\,{\mathrm e}^{3i\left(fx+e\right)}}{a}+3\,{\mathrm e}^{2i\left(fx+e\right)}+1\right)}{3af}$	76

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3/a*ln(a+b*sec(f*x+e)^3)+1/a*ln(sec(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx=-\frac{\log\left(a\cos\left(fx+e\right)^{3}+b\right)}{3\,af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
-1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(19) = 38$.

Time = 17.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.04

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = \begin{cases} \frac{\tilde{\infty}x\tan{(e)}}{\sec^3{(e)}} & \text{for } a=0 \wedge b= \\ -\frac{1}{3bf\sec^3{(e+fx)}} & \text{for } a=0 \\ \frac{\log{(\tan^2{(e+fx)}+1)}}{2af} & \text{for } b=0 \\ \frac{x\tan{(e)}}{a+b\sec^3{(e)}} & \text{for } f=0 \\ -\frac{\log{\left(-\sqrt[3]{-\frac{a}{b}}+\sec{(e+fx)}\right)}}{3af}+\frac{\log{(\tan^2{(e+fx)}+1)}}{2af}-\frac{\log{\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\sec{(e+fx)}+4\sec^2{(e+fx)}\right)}}{3af} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

output
```
Piecewise((zoo*x*tan(e)/sec(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(3
*b*f*sec(e + f*x)**3), Eq(a, 0)), (log(tan(e + f*x)**2 + 1)/(2*a*f), Eq(b,
 0)), (x*tan(e)/(a + b*sec(e)**3), Eq(f, 0)), (-log(-(-a/b)**(1/3) + sec(e
 + f*x))/(3*a*f) + log(tan(e + f*x)**2 + 1)/(2*a*f) - log(4*(-a/b)**(2/3)
+ 4*(-a/b)**(1/3)*sec(e + f*x) + 4*sec(e + f*x)**2)/(3*a*f), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(a\cos\left(fx+e\right)^3+b\right)}{3\,af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
-1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(\left|a\cos\left(fx+e\right)^3+b\right|\right)}{3\,af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
-1/3*log(abs(a*cos(f*x + e)^3 + b))/(a*f)
```

Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.96

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$= \frac{3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)-\ln\left(a+b-3\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+3\,b\tan}{3\,a\,f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^3),x)
```

output
```
(3*log(tan(e/2 + (f*x)/2)^2 + 1) - log(a + b - 3*a*tan(e/2 + (f*x)/2)^2 +
3*a*tan(e/2 + (f*x)/2)^4 - a*tan(e/2 + (f*x)/2)^6 + 3*b*tan(e/2 + (f*x)/2)
^2 + 3*b*tan(e/2 + (f*x)/2)^4 + b*tan(e/2 + (f*x)/2)^6))/(3*a*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.13

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$= \frac{3\log\left(\tan\left(fx+e\right)^2+1\right)-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}\sec\left(fx+e\right)+b^{\frac{2}{3}}\sec\left(fx+e\right)^2\right)-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\sec\left(fx+\right.\right.}{6af}$$

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^3),x)
```

output
```
(3*log(tan(e + f*x)**2 + 1) - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*sec(e + f
*x) + b**(2/3)*sec(e + f*x)**2) - 2*log(a**(1/3) + b**(1/3)*sec(e + f*x)))
/(6*a*f)
```

3.460 $\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 295

$$\begin{aligned}
\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx &= -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)f}+\frac{\log(1-\cos(e+fx))}{2(a+b)f} \\
&\quad+\frac{\log(1+\cos(e+fx))}{2(a-b)f}-\frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)f} \\
&\quad+\frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)f} \\
&\quad-\frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)f}
\end{aligned}$$

output
```
-1/3*b^(2/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))*3^(1/2)/b^(1/3))*3^
(1/2)/a^(1/3)/(a^(4/3)+a^(2/3)*b^(2/3)+b^(4/3))/f+1/2*ln(1-cos(f*x+e))/(a+
b)/f+1/2*ln(1+cos(f*x+e))/(a-b)/f-1/3*(a^(2/3)+b^(2/3))*b^(2/3)*ln(b^(1/3)
+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2-b^2)/f+1/6*(a^(2/3)+b^(2/3))*b^(2/3)*ln(
b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/(a^2-b^2)
/f-1/3*b^2*ln(b+a*cos(f*x+e)^3)/a/(a^2-b^2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.73

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(a-b)f} + \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(a+b)f} + \frac{b\Big(3b\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right) + (-a+b)\text{RootSum}\Big[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\&, \frac{-4a\log\big(1-}{}}{3\left(a^3-ab^2\right)f}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

output
```
Log[Cos[(e + f*x)/2]]/((a - b)*f) + Log[Sin[(e + f*x)/2]]/((a + b)*f) + (b
*(3*b*Log[Sec[(e + f*x)/2]^2] + (-a + b)*RootSum[-8*a + 12*a*#1 - 6*a*#1^2
 + a*#1^3 - b*#1^3 & , (-4*a*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 2*a*Log[1
- #1 + Tan[(e + f*x)/2]^2]*#1 + b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(
4*a - 4*a*#1 + a*#1^2 - b*#1^2) & ]))/(3*(a^3 - a*b^2)*f)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4626, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\sec(e+fx)^3\right)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))(a\cos^3(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{\int \left(-\frac{b(b\cos^2(e+fx)-a\cos(e+fx)+b)}{(b^2-a^2)(a\cos^3(e+fx)+b)}-\frac{1}{2(a+b)(\cos(e+fx)-1)}-\frac{1}{2(a-b)(\cos(e+fx)+1)}\right)d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}(a^{2/3}b^{2/3}+a^{4/3}+b^{4/3})}+\frac{b^2\log(a\cos^3(e+fx)+b)}{3a(a^2-b^2)}-\frac{b^{2/3}(a^{2/3}+b^{2/3})\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}(a^2-b^2)}+}{f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

output
```
-(((b^(2/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))])/
(Sqrt[3]*a^(1/3)*(a^(4/3) + a^(2/3)*b^(2/3) + b^(4/3))) - Log[1 - Cos[e +
f*x]]/(2*(a + b)) - Log[1 + Cos[e + f*x]]/(2*(a - b)) + ((a^(2/3) + b^(2/3
))*b^(2/3)*Log[b^(1/3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)) -
((a^(2/3) + b^(2/3))*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] +
a^(2/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)) + (b^2*Log[b + a*Cos[e +
f*x]^3])/(3*a*(a^2 - b^2)))/f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.10 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.98

method	result
risch	$\frac{ix}{a}-\frac{ix}{a+b}-\frac{ie}{f(a+b)}-\frac{ix}{a-b}-\frac{ie}{f(a-b)}-\frac{2ia^2b^2f^3x}{-a^5f^3+a^3b^2f^3}-\frac{2ia^2b^2f^2e}{-a^5f^3+a^3b^2f^3}+\frac{\ln\left(e^{i(fx+e)}-1\right)}{f(a+b)}+\frac{\ln\left(e^{i(f\ldots}\right.}{f(a\ldots}$
derivativedivides	$\frac{\frac{\ln(\cos(fx+e)-1)}{2a+2b}+\frac{\ln(1+\cos(fx+e))}{2a-2b}+\frac{-b\left(\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\ldots}{\ldots}\right.\right.}{\ldots}}{\ldots}$
default	$\frac{\frac{\ln(\cos(fx+e)-1)}{2a+2b}+\frac{\ln(1+\cos(fx+e))}{2a-2b}+\frac{-b\left(\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\ldots}{\ldots}\right.\right.}{\ldots}}{\ldots}$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
I*x/a-I/(a+b)*x-I/f/(a+b)*e-I/(a-b)*x-I/f/(a-b)*e-2*I*a^2*b^2*f^3/(-a^5*f^
3+a^3*b^2*f^3)*x-2*I*a^2*b^2*f^2/(-a^5*f^3+a^3*b^2*f^3)*e+1/f/(a+b)*ln(exp
(I*(f*x+e))-1)+1/f/(a-b)*ln(exp(I*(f*x+e))+1)+I*sum(_R*ln(exp(2*I*(f*x+e))
+((-18*a^3/b*f^2+18*a*b*f^2)*_R^2+18*I*b*f*_R-4*b/a)*exp(I*(f*x+e))+1),_R=
RootOf((27*a^5*f^3-27*a^3*b^2*f^3)*_Z^3-27*I*b^2*a^2*f^2*_Z^2+9*_Z*a*b^2*f
+I*b^2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 6482, normalized size of antiderivative = 21.97

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.04

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\frac{2\sqrt{3}\left(ab^2\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{2b}{a}\right)-3a^2b\left(\frac{b}{a}\right)^{\frac{2}{3}}-2b^3\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}-a^2b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{3\left(b^2\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)-ab\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos(fx+e)^2\right.}{a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}-ab^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{1}$$

input

```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output

```
-1/18*(2*sqrt(3)*(a*b^2*(3*(b/a)^(1/3) + 2*b/a) - 3*a^2*b*(b/a)^(2/3) - 2*
b^3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^4
*(b/a)^(2/3) - a^2*b^2*(b/a)^(2/3))*(b/a)^(1/3)) + 3*(b^2*(2*(b/a)^(2/3) -
 1) - a*b*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/
a)^(2/3))/(a^3*(b/a)^(2/3) - a*b^2*(b/a)^(2/3)) + 6*(b^2*((b/a)^(2/3) + 1)
 + a*b*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^3*(b/a)^(2/3) - a*b
^2*(b/a)^(2/3)) - 9*log(cos(f*x + e) + 1)/(a - b) - 9*log(cos(f*x + e) - 1
)/(a + b))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.12

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{b^2\log\left(\left|a\cos\left(fx+e\right)^3+b\right|\right)}{3\left(a^3f-ab^2f\right)} - \frac{\left(a^4bf\left(-\frac{b}{a}\right)^{\frac{1}{3}} - a^2b^3f\left(-\frac{b}{a}\right)^{\frac{1}{3}} - a^3b^2f + ab^4f\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\cos\left(fx+e\right)\right|\right)}{3\left(a^5bf^2-2\,a^3b^3f^2+ab^5f^2\right)} - \frac{\left(\left(-a^2b\right)^{\frac{1}{3}}b + \left(-a^2b\right)^{\frac{2}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2\cos(fx+e)\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(\sqrt{3}a^3-\sqrt{3}ab^2\right)f} - \frac{\left(\left(-a^2b\right)^{\frac{1}{3}}b - \left(-a^2b\right)^{\frac{2}{3}}\right)\log\left(\cos\left(fx+e\right)^2 + \left(-\frac{b}{a}\right)^{\frac{1}{3}}\cos\left(fx+e\right) + \left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(a^3-ab^2\right)f} + \frac{\log\left(\left|\cos\left(fx+e\right)+1\right|\right)}{2\left(af-bf\right)} + \frac{\log\left(\left|\cos\left(fx+e\right)-1\right|\right)}{2\left(af+bf\right)}$$

input

```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output

```
-1/3*b^2*log(abs(a*cos(f*x + e)^3 + b))/(a^3*f - a*b^2*f) - 1/3*(a^4*b*f*(
-b/a)^(1/3) - a^2*b^3*f*(-b/a)^(1/3) - a^3*b^2*f + a*b^4*f)*(-b/a)^(1/3)*l
og(abs(-(-b/a)^(1/3) + cos(f*x + e)))/(a^5*b*f^2 - 2*a^3*b^3*f^2 + a*b^5*f
^2) - ((-a^2*b)^(1/3)*b + (-a^2*b)^(2/3))*arctan(1/3*sqrt(3)*((-b/a)^(1/3)
 + 2*cos(f*x + e))/(-b/a)^(1/3))/((sqrt(3)*a^3 - sqrt(3)*a*b^2)*f) - 1/6*(
(-a^2*b)^(1/3)*b - (-a^2*b)^(2/3))*log(cos(f*x + e)^2 + (-b/a)^(1/3)*cos(f
*x + e) + (-b/a)^(2/3))/((a^3 - a*b^2)*f) + 1/2*log(abs(cos(f*x + e) + 1))
/(a*f - b*f) + 1/2*log(abs(cos(f*x + e) - 1))/(a*f + b*f)
```

Mupad [B] (verification not implemented)

Time = 18.41 (sec) , antiderivative size = 11182, normalized size of antiderivative = 37.91

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^3),x)
```

output
```
log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))/(f*(a + b)) - log(1/cos(e/2 + (
f*x)/2)^2)/(f*(a + b)) + (a*symsum(log((262144*(832*root(27*a^3*b^2*z^3 -
27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*b^7 - 22*a*b^5 - 840*
b^6*cos(e + f*x) + 440*b^6 - 264*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2
*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8 + 16*root(27*a^3*b^2*z^3 - 27*a^5*
z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*b^9 + 1823*root(27*a^3*b^2
*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^2*b^5 - 21*r
oot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*
a^3*b^4 - 8864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2
*z - b^2, z, k)^2*a*b^7 + 3092*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a*b^8 - 192*root(27*a^3*b^2*z^3 - 27*a^5
*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a*b^9 + 88*root(27*a^3*b^
2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8*cos(e +
 f*x) - a^2*b^4*cos(e + f*x) + 65221*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27
*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^2*b^6 - 32708*root(27*a^3*b^2*z^
3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^3*b^5 + 2859*
root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)
^2*a^4*b^4 - 9*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2
*z - b^2, z, k)^2*a^5*b^3 + 26274*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^
2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^2*b^7 - 212230*root(27*a^3*b^2*z...
```

Reduce [F]

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{too large to display}$$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^3),x)
```

output
```
( - 4*int(tan((e + f*x)/2)/(tan((e + f*x)/2)**6*a**3 + 3*tan((e + f*x)/2)*
*6*a**2*b - 9*tan((e + f*x)/2)**6*a*b**2 + 5*tan((e + f*x)/2)**6*b**3 - 3*
tan((e + f*x)/2)**4*a**3 - 15*tan((e + f*x)/2)**4*a**2*b + 3*tan((e + f*x)
/2)**4*a*b**2 + 15*tan((e + f*x)/2)**4*b**3 + 3*tan((e + f*x)/2)**2*a**3 +
 9*tan((e + f*x)/2)**2*a**2*b - 27*tan((e + f*x)/2)**2*a*b**2 + 15*tan((e
+ f*x)/2)**2*b**3 - a**3 - 5*a**2*b + a*b**2 + 5*b**3),x)*a**4*b*f + 84*in
t(tan((e + f*x)/2)/(tan((e + f*x)/2)**6*a**3 + 3*tan((e + f*x)/2)**6*a**2*
b - 9*tan((e + f*x)/2)**6*a*b**2 + 5*tan((e + f*x)/2)**6*b**3 - 3*tan((e +
 f*x)/2)**4*a**3 - 15*tan((e + f*x)/2)**4*a**2*b + 3*tan((e + f*x)/2)**4*a
*b**2 + 15*tan((e + f*x)/2)**4*b**3 + 3*tan((e + f*x)/2)**2*a**3 + 9*tan((
e + f*x)/2)**2*a**2*b - 27*tan((e + f*x)/2)**2*a*b**2 + 15*tan((e + f*x)/2
)**2*b**3 - a**3 - 5*a**2*b + a*b**2 + 5*b**3),x)*a**2*b**3*f - 80*int(tan
((e + f*x)/2)/(tan((e + f*x)/2)**6*a**3 + 3*tan((e + f*x)/2)**6*a**2*b - 9
*tan((e + f*x)/2)**6*a*b**2 + 5*tan((e + f*x)/2)**6*b**3 - 3*tan((e + f*x)
/2)**4*a**3 - 15*tan((e + f*x)/2)**4*a**2*b + 3*tan((e + f*x)/2)**4*a*b**2
 + 15*tan((e + f*x)/2)**4*b**3 + 3*tan((e + f*x)/2)**2*a**3 + 9*tan((e + f
*x)/2)**2*a**2*b - 27*tan((e + f*x)/2)**2*a*b**2 + 15*tan((e + f*x)/2)**2*
b**3 - a**3 - 5*a**2*b + a*b**2 + 5*b**3),x)*a*b**4*f + 4*int(1/(tan((e +
f*x)/2)**7*a**3 + 3*tan((e + f*x)/2)**7*a**2*b - 9*tan((e + f*x)/2)**7*a*b
**2 + 5*tan((e + f*x)/2)**7*b**3 - 3*tan((e + f*x)/2)**5*a**3 - 15*tan(...
```

3.461 $\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 393

$$\begin{aligned}\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx &= \frac{b^{4/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 f} \\ &\quad-\frac{1}{4(a+b)f(1-\cos(e+fx))}-\frac{1}{4(a-b)f(1+\cos(e+fx))} \\ &\quad-\frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2 f}-\frac{(2a-5b)\log(1+\cos(e+fx))}{4(a-b)^2 f} \\ &\quad-\frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)^2 f} \\ &\quad+\frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)^2 f} \\ &\quad-\frac{b^2\left(2a^2+b^2\right)\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)^2 f}\end{aligned}$$

output
```
1/3*b^(4/3)*(a^2-3*a^(2/3)*b^(4/3)+2*b^2)*arctan(1/3*(b^(1/3)-2*a^(1/3)*co
s(f*x+e))*3^(1/2)/b^(1/3))*3^(1/2)/a^(1/3)/(a^2-b^2)^2/f-1/4/(a+b)/f/(1-co
s(f*x+e))-1/4/(a-b)/f/(1+cos(f*x+e))-1/4*(2*a+5*b)*ln(1-cos(f*x+e))/(a+b)^
2/f-1/4*(2*a-5*b)*ln(1+cos(f*x+e))/(a-b)^2/f-1/3*b^(4/3)*(a^2+3*a^(2/3)*b^
(4/3)+2*b^2)*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2-b^2)^2/f+1/6*b^(4
/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^
(2/3)*cos(f*x+e)^2)/a^(1/3)/(a^2-b^2)^2/f-1/3*b^2*(2*a^2+b^2)*ln(b+a*cos(f
*x+e)^3)/a/(a^2-b^2)^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.65 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.85

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$= \frac{-\frac{3\csc^2\left(\frac{1}{2}(e+fx)\right)}{a+b} + \frac{12(-2a+5b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(a-b)^2} - \frac{12(2a+5b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(a+b)^2} + \frac{8b^2\left(3(2a^2+b^2)\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right)+(-a+b\right.}{}}{}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

output
```
((-3*Csc[(e + f*x)/2]^2)/(a + b) + (12*(-2*a + 5*b)*Log[Cos[(e + f*x)/2]])
/(a - b)^2 - (12*(2*a + 5*b)*Log[Sin[(e + f*x)/2]])/(a + b)^2 + (8*b^2*(3*
(2*a^2 + b^2)*Log[Sec[(e + f*x)/2]^2] + (-a + b)*RootSum[-8*a + 12*a*#1 -
6*a*#1^2 + a*#1^3 - b*#1^3 & , (8*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 4
*a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 6*a^2*Log[1 - #1 + Tan[(e + f*x)/2
]^2]*#1 + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2 + b^2*Log[1 - #1 + T
an[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^2) & ]))/(a*(a^2 -
b^2)^2) - (3*Sec[(e + f*x)/2]^2)/(a - b))/(24*f)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4626, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\,(a+b\sec(e+fx)^3)}dx$$

$$\Big\downarrow \text{4626}$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^3(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{\int \left(\frac{(a^2-3b\cos(e+fx)a+2b^2+(2a^2+b^2)\cos^2(e+fx))b^2}{(a^2-b^2)^2(a\cos^3(e+fx)+b)}+\frac{2a+5b}{4(a+b)^2(\cos(e+fx)-1)}+\frac{2a-5b}{4(a-b)^2(\cos(e+fx)+1)}+\frac{1}{4(a+b)(\cos(e+fx)-1)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^2(2a^2+b^2)\log(a\cos^3(e+fx)+b)}{3a(a^2-b^2)^2}-\frac{b^{4/3}(-3a^{2/3}b^{4/3}+a^2+2b^2)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}(a^2-b^2)^2}-\frac{b^{4/3}(3a^{2/3}b^{4/3}+a^2+2b^2)\log\left(a^{2/3}\cos\right.}{6\sqrt[3]{a}(a}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

output
```
-((-((b^(4/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(b^(1/3) - 2*a^(1/3
)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2)) + 1/(
4*(a + b)*(1 - Cos[e + f*x])) + 1/(4*(a - b)*(1 + Cos[e + f*x])) + ((2*a +
 5*b)*Log[1 - Cos[e + f*x]])/(4*(a + b)^2) + ((2*a - 5*b)*Log[1 + Cos[e +
f*x]])/(4*(a - b)^2) + (b^(4/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(1
/3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)^2) - (b^(4/3)*(a^2 + 3
*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(
2/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2) + (b^2*(2*a^2 + b^2)*Log[b
 + a*Cos[e + f*x]^3])/(3*a*(a^2 - b^2)^2))/f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 5.51 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{1}{(4a-4b)(1+\cos(fx+e))}+\frac{(-2a+5b)\ln(1+\cos(fx+e))}{4(a-b)^2}+\frac{1}{(4a+4b)(\cos(fx+e)-1)}+\frac{(-2a-5b)\ln(\cos(fx+e)-1)}{4(a+b)^2}-\frac{\left((a^2+2b^2)\left(\ln \ldots\right.\right.}{}$
default	$-\frac{1}{(4a-4b)(1+\cos(fx+e))}+\frac{(-2a+5b)\ln(1+\cos(fx+e))}{4(a-b)^2}+\frac{1}{(4a+4b)(\cos(fx+e)-1)}+\frac{(-2a-5b)\ln(\cos(fx+e)-1)}{4(a+b)^2}-\frac{\left((a^2+2b^2)\left(\ln \ldots\right.\right.}{}$
risch	$-\frac{5ibx}{2(a^2-2ab+b^2)}-i\left(\sum_{\_R=\text{RootOf}\left((27a^7f^3-54a^5b^2f^3+27a^3b^4f^3)\_Z^3+(54ia^4b^2f^2+27ia^2b^4f^2)\_Z^2-9\_Zab^4f \ldots\right.}\right.$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(4*a-4*b)/(1+cos(f*x+e))+1/4/(a-b)^2*(-2*a+5*b)*ln(1+cos(f*x+e))+1
/(4*a+4*b)/(cos(f*x+e)-1)+1/4/(a+b)^2*(-2*a-5*b)*ln(cos(f*x+e)-1)-((a^2+2*
b^2)*(1/3/a/(b/a)^(2/3)*ln(cos(f*x+e)+(b/a)^(1/3))-1/6/a/(b/a)^(2/3)*ln(co
s(f*x+e)^2-(b/a)^(1/3)*cos(f*x+e)+(b/a)^(2/3))+1/3/a/(b/a)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*cos(f*x+e)-1)))-3*a*b*(-1/3/a/(b/a)^(1/3)
*ln(cos(f*x+e)+(b/a)^(1/3))+1/6/a/(b/a)^(1/3)*ln(cos(f*x+e)^2-(b/a)^(1/3)*
cos(f*x+e)+(b/a)^(2/3))+1/3*3^(1/2)/a/(b/a)^(1/3)*arctan(1/3*3^(1/2)*(2/(b
/a)^(1/3)*cos(f*x+e)-1)))+1/3*(2*a^2+b^2)/a*ln(b+a*cos(f*x+e)^3))*b^2/(a-b
)^2/(a+b)^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 10746, normalized size of antiderivative = 27.34

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.24

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
1/36*(4*sqrt(3)*(a^2*b^3*(9*(b/a)^(2/3) + 4) - a^3*b^2*(3*(b/a)^(1/3) + 4*
b/a) - 2*a*b^4*(3*(b/a)^(1/3) + b/a) + 2*b^5)*arctan(-1/3*sqrt(3)*((b/a)^(
1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^6*(b/a)^(2/3) - 2*a^4*b^2*(b/a)^(2
/3) + a^2*b^4*(b/a)^(2/3))*(b/a)^(1/3)) - 6*(a^2*b^2*(4*(b/a)^(2/3) - 1) +
 2*b^4*((b/a)^(2/3) - 1) - 3*a*b^3*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)
^(1/3)*cos(f*x + e) + (b/a)^(2/3))/(a^5*(b/a)^(2/3) - 2*a^3*b^2*(b/a)^(2/3
) + a*b^4*(b/a)^(2/3)) - 12*(a^2*b^2*(2*(b/a)^(2/3) + 1) + b^4*((b/a)^(2/3
) + 2) + 3*a*b^3*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^5*(b/a)^(
2/3) - 2*a^3*b^2*(b/a)^(2/3) + a*b^4*(b/a)^(2/3)) - 9*(2*a - 5*b)*log(cos(
f*x + e) + 1)/(a^2 - 2*a*b + b^2) - 9*(2*a + 5*b)*log(cos(f*x + e) - 1)/(a
^2 + 2*a*b + b^2) - 18*(b*cos(f*x + e) - a)/((a^2 - b^2)*cos(f*x + e)^2 -
a^2 + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.39

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\left(3\,a^6b^3f\left(-\frac{b}{a}\right)^{\frac{1}{3}}-6\,a^4b^5f\left(-\frac{b}{a}\right)^{\frac{1}{3}}+3\,a^2b^7f\left(-\frac{b}{a}\right)^{\frac{1}{3}}-a^7b^2f+3\,a^3b^6f-2\,ab^8f\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}-\right.\right.}{3\left(a^9bf^2-4\,a^7b^3f^2+6\,a^5b^5f^2-4\,a^3b^7f^2+ab^9f^2\right)}$$
$$-\frac{(2\,a^2b^2+b^4)\log\left(\left|a\cos\left(fx+e\right)^3+b\right|\right)}{3\left(a^5f-2\,a^3b^2f+ab^4f\right)}-\frac{(2\,a+5\,b)\log\left(\left|-\cos\left(fx+e\right)+1\right|\right)}{4\left(a^2f+2\,abf+b^2f\right)}$$
$$-\frac{(2\,a-5\,b)\log\left(\left|-\cos\left(fx+e\right)-1\right|\right)}{4\left(a^2f-2\,abf+b^2f\right)}$$
$$-\frac{\left(3\left(-a^2b\right)^{\frac{2}{3}}b^2+(a^2b+2\,b^3)\left(-a^2b\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2\cos(fx+e)\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(\sqrt{3}a^5-2\,\sqrt{3}a^3b^2+\sqrt{3}ab^4\right)f}$$
$$+\frac{\left(3\left(-a^2b\right)^{\frac{2}{3}}b^2-(a^2b+2\,b^3)\left(-a^2b\right)^{\frac{1}{3}}\right)\log\left(\cos\left(fx+e\right)^2+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\cos\left(fx+e\right)+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(a^5-2\,a^3b^2+ab^4\right)f}$$
$$+\frac{a^3-ab^2-(a^2b-b^3)\cos\left(fx+e\right)}{2\left(a+b\right)^2\left(a-b\right)^2f\left(\cos\left(fx+e\right)+1\right)\left(\cos\left(fx+e\right)-1\right)}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
-1/3*(3*a^6*b^3*f*(-b/a)^(1/3) - 6*a^4*b^5*f*(-b/a)^(1/3) + 3*a^2*b^7*f*(-
b/a)^(1/3) - a^7*b^2*f + 3*a^3*b^6*f - 2*a*b^8*f)*(-b/a)^(1/3)*log(abs(-(-
b/a)^(1/3) + cos(f*x + e)))/(a^9*b*f^2 - 4*a^7*b^3*f^2 + 6*a^5*b^5*f^2 - 4
*a^3*b^7*f^2 + a*b^9*f^2) - 1/3*(2*a^2*b^2 + b^4)*log(abs(a*cos(f*x + e)^3
 + b))/(a^5*f - 2*a^3*b^2*f + a*b^4*f) - 1/4*(2*a + 5*b)*log(abs(-cos(f*x
+ e) + 1))/(a^2*f + 2*a*b*f + b^2*f) - 1/4*(2*a - 5*b)*log(abs(-cos(f*x +
e) - 1))/(a^2*f - 2*a*b*f + b^2*f) - (3*(-a^2*b)^(2/3)*b^2 + (a^2*b + 2*b^
3)*(-a^2*b)^(1/3))*arctan(1/3*sqrt(3)*((-b/a)^(1/3) + 2*cos(f*x + e))/(-b/
a)^(1/3))/((sqrt(3)*a^5 - 2*sqrt(3)*a^3*b^2 + sqrt(3)*a*b^4)*f) + 1/6*(3*(
-a^2*b)^(2/3)*b^2 - (a^2*b + 2*b^3)*(-a^2*b)^(1/3))*log(cos(f*x + e)^2 + (
-b/a)^(1/3)*cos(f*x + e) + (-b/a)^(2/3))/((a^5 - 2*a^3*b^2 + a*b^4)*f) + 1
/2*(a^3 - a*b^2 - (a^2*b - b^3)*cos(f*x + e))/((a + b)^2*(a - b)^2*f*(cos(
f*x + e) + 1)*(cos(f*x + e) - 1))
```

Mupad [B] (verification not implemented)

Time = 30.03 (sec) , antiderivative size = 58699, normalized size of antiderivative = 149.36

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

output
```
-(a^3*cos(e/2 + (f*x)/2)^4 + a^3*sin(e/2 + (f*x)/2)^4 - a*b^2*cos(e/2 + (f
*x)/2)^4 + a*b^2*sin(e/2 + (f*x)/2)^4 + 2*a^2*b*sin(e/2 + (f*x)/2)^4 - 8*a
^3*log((cos(e/2 + (f*x)/2)^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)
*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2 + 8*b^3*log((cos(e/2 + (f*x)/2)
^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*cos(e/2 + (f*x)/2)^2*sin(
e/2 + (f*x)/2)^2 + 8*a^3*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/cos(e
/2 + (f*x)/2))*sin(e/2 + (f*x)/2)^2 - 8*a^4*cos(e/2 + (f*x)/2)^2*sin(e/2 +
 (f*x)/2)^2*symsum(log((131072*(980*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*s
in(e/2 + (f*x)/2)^2 + 1764*a^2*b^9*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(
e/2 + (f*x)/2)^2 + 640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 +
 (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^
4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*sin(e/2 + (f*x)
/2)^2 - 1176*a^2*b^9*sin(e/2 + (f*x)/2)^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)
^2 + 952*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*cos(e/2 + (f*x)/2)^2 + 23
52*a*b^10*cos(e/2 + (f*x)/2)^2 + 1944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b
^12*sin(e/2 + (f*x)/2)^2 - 56*a*b^10*sin(e/2 + (f*x)/2)^2 + 304*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z...
```

Reduce [F]

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{too large to display}$$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x)
```

output
```
(128*int(1/(tan((e + f*x)/2)**9*a**5 + 19*tan((e + f*x)/2)**9*a**4*b - 50*
tan((e + f*x)/2)**9*a**3*b**2 + 26*tan((e + f*x)/2)**9*a**2*b**3 + 17*tan(
(e + f*x)/2)**9*a*b**4 - 13*tan((e + f*x)/2)**9*b**5 - 3*tan((e + f*x)/2)*
*7*a**5 - 63*tan((e + f*x)/2)**7*a**4*b + 30*tan((e + f*x)/2)**7*a**3*b**2
 + 102*tan((e + f*x)/2)**7*a**2*b**3 - 27*tan((e + f*x)/2)**7*a*b**4 - 39*
tan((e + f*x)/2)**7*b**5 + 3*tan((e + f*x)/2)**5*a**5 + 57*tan((e + f*x)/2
)**5*a**4*b - 150*tan((e + f*x)/2)**5*a**3*b**2 + 78*tan((e + f*x)/2)**5*a
**2*b**3 + 51*tan((e + f*x)/2)**5*a*b**4 - 39*tan((e + f*x)/2)**5*b**5 - t
an((e + f*x)/2)**3*a**5 - 21*tan((e + f*x)/2)**3*a**4*b + 10*tan((e + f*x)
/2)**3*a**3*b**2 + 34*tan((e + f*x)/2)**3*a**2*b**3 - 9*tan((e + f*x)/2)**
3*a*b**4 - 13*tan((e + f*x)/2)**3*b**5),x)*tan((e + f*x)/2)**2*a**7*b**2*f
 + 2400*int(1/(tan((e + f*x)/2)**9*a**5 + 19*tan((e + f*x)/2)**9*a**4*b -
50*tan((e + f*x)/2)**9*a**3*b**2 + 26*tan((e + f*x)/2)**9*a**2*b**3 + 17*t
an((e + f*x)/2)**9*a*b**4 - 13*tan((e + f*x)/2)**9*b**5 - 3*tan((e + f*x)/
2)**7*a**5 - 63*tan((e + f*x)/2)**7*a**4*b + 30*tan((e + f*x)/2)**7*a**3*b
**2 + 102*tan((e + f*x)/2)**7*a**2*b**3 - 27*tan((e + f*x)/2)**7*a*b**4 -
39*tan((e + f*x)/2)**7*b**5 + 3*tan((e + f*x)/2)**5*a**5 + 57*tan((e + f*x
)/2)**5*a**4*b - 150*tan((e + f*x)/2)**5*a**3*b**2 + 78*tan((e + f*x)/2)**
5*a**2*b**3 + 51*tan((e + f*x)/2)**5*a*b**4 - 39*tan((e + f*x)/2)**5*b**5
- tan((e + f*x)/2)**3*a**5 - 21*tan((e + f*x)/2)**3*a**4*b + 10*tan((e ...
```

3.462 $\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx = \text{Int}((a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m, x)$$

output
```
Defer(Int)((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

Mathematica [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx = \int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
```

output
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m (a+b(c\sec(e+fx))^n)^p \, dx$$

$$\Big\downarrow \; 3042$$

$$\int (d\tan(e+fx))^m (a+b(c\sec(e+fx))^n)^p dx$$

$$\Big\downarrow \; 4632$$

$$\int (d\tan(e+fx))^m (a+b(c\sec(e+fx))^n)^p dx$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec(fx+e))^n)^p (d\tan(fx+e))^m dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx = \int ((c\sec(fx+e))^n b+a)^p (d\tan(fx+e))^m dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [N/A]

Not integrable

Time = 98.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m dx = \int (d\tan(e+fx))^m (a+b(c\sec(e+fx))^n)^p dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*(d*tan(f*x+e))**m,x)
```

output
```
Integral((d*tan(e + f*x))**m*(a + b*(c*sec(e + f*x))**n)**p, x)
```

Maxima [N/A]

Not integrable

Time = 5.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx = \int ((c\sec(fx+e))^n b+a)^p (d\tan(fx+e))^m \, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Giac [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx = \int ((c\sec(fx+e))^n b+a)^p (d\tan(fx+e))^m \, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [N/A]

Not integrable

Time = 14.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx = \int (d\tan(e+f\,x))^m \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx = d^m \left(\int \tan(fx+e)^m (c^n \sec(fx+e)^n b+a)^p \, dx\right)$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

output
```
d**m*int(tan(e + f*x)**m*(c**n*sec(e + f*x)**n*b + a)**p,x)
```

3.463 $\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\, dx$

Optimal result

Integrand size = 25, antiderivative size = 229

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx$$

$$= -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$

$$-\frac{\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(\frac{a+b(c\sec(e+fx))}{a}\right.}{f}$$

$$+\frac{\text{Hypergeometric2F1}\left(\frac{4}{n},-p,\frac{4+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^4(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(\frac{a+b(c\sec(e+fx))}{a}\right.}{4f}$$

output
```
-hypergeom([1, p+1],[2+p],(a+b*(c*sec(f*x+e))^n)/a)*(a+b*(c*sec(f*x+e))^n)
^(p+1)/a/f/n/(p+1)-hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)*se
c(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/(((a+b*(c*sec(f*x+e))^n)/a)^p)+1/4*h
ypergeom([-p, 4/n],[(4+n)/n],-b*(c*sec(f*x+e))^n/a)*sec(f*x+e)^4*(a+b*(c*s
ec(f*x+e))^n)^p/f/(((a+b*(c*sec(f*x+e))^n)/a)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 8.59 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx$$

$$= \frac{(a+b(c\sec(e+fx))^n)^p \left(1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)^{-p} \left(-4an(1+p)\,\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b\left(c\right.}{}\right.\right.}{}$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

output
```
((a + b*(c*Sec[e + f*x])^n)^p*(-4*a*n*(1 + p)*Hypergeometric2F1[2/n, -p, (
2 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)]*Sec[e + f*x]^2 + a*n*(1 + p
)*Hypergeometric2F1[4/n, -p, (4 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a
)]*Sec[e + f*x]^4 - 4*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sqrt[Se
c[e + f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n)*(1 + (b*(c*Sqrt[Se
c[e + f*x]^2])^n)/a)^p))/(4*a*f*n*(1 + p)*(1 + (b*(c*Sqrt[Sec[e + f*x]^2])
^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4627, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^5\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{ 4627}$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b(c\sec(e+fx))^n+a\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(\sec^3(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p+\cos(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p-2\sec(e+fx)\left(b(c\sec(e+fx)\right)\right.}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\sec^4(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{n},-p,\frac{n+4}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)}{}$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

output
```
(-((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a +
b*(c*Sec[e + f*x])^n)^(1 + p))/(a*n*(1 + p))) - (Hypergeometric2F1[2/n, -p
, (2 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e +
 f*x])^n)^p)/(1 + (b*(c*Sec[e + f*x])^n)/a)^p + (Hypergeometric2F1[4/n, -p
, (4 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^4*(a + b*(c*Sec[e +
 f*x])^n)^p)/(4*(1 + (b*(c*Sec[e + f*x])^n)/a)^p))/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int (a+b(c\sec (fx+e))^n)^p \tan (fx+e)^5 \, dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

Fricas [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx) \, dx = \int ((c\sec (fx+e))^n b + a)^p \tan (fx+e)^5 \, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)^5\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Giac [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)^5\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int \tan(e+f\,x)^5 \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \text{Too large to display}$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

output
```
((c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x)**4*n**2*p**2 + 2*(c**n*sec(e
 + f*x)**n*b + a)**p*tan(e + f*x)**4*n*p - 4*(c**n*sec(e + f*x)**n*b + a)*
*p*tan(e + f*x)**2*n*p + 8*(c**n*sec(e + f*x)**n*b + a)**p + int(((c**n*se
c(e + f*x)**n*b + a)**p*tan(e + f*x)**5)/(c**n*sec(e + f*x)**n*b*n**2*p**2
 + 6*c**n*sec(e + f*x)**n*b*n*p + 8*c**n*sec(e + f*x)**n*b + a*n**2*p**2 +
 6*a*n*p + 8*a),x)*a*f*n**5*p**5 + 8*int(((c**n*sec(e + f*x)**n*b + a)**p*
tan(e + f*x)**5)/(c**n*sec(e + f*x)**n*b*n**2*p**2 + 6*c**n*sec(e + f*x)**
n*b*n*p + 8*c**n*sec(e + f*x)**n*b + a*n**2*p**2 + 6*a*n*p + 8*a),x)*a*f*n
**4*p**4 + 20*int(((c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x)**5)/(c**n*
sec(e + f*x)**n*b*n**2*p**2 + 6*c**n*sec(e + f*x)**n*b*n*p + 8*c**n*sec(e
+ f*x)**n*b + a*n**2*p**2 + 6*a*n*p + 8*a),x)*a*f*n**3*p**3 + 16*int(((c**
n*sec(e + f*x)**n*b + a)**p*tan(e + f*x)**5)/(c**n*sec(e + f*x)**n*b*n**2*
p**2 + 6*c**n*sec(e + f*x)**n*b*n*p + 8*c**n*sec(e + f*x)**n*b + a*n**2*p*
*2 + 6*a*n*p + 8*a),x)*a*f*n**2*p**2 - 4*int(((c**n*sec(e + f*x)**n*b + a)
**p*tan(e + f*x)**3)/(c**n*sec(e + f*x)**n*b*n**2*p**2 + 6*c**n*sec(e + f*
x)**n*b*n*p + 8*c**n*sec(e + f*x)**n*b + a*n**2*p**2 + 6*a*n*p + 8*a),x)*a
*f*n**4*p**4 - 24*int(((c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x)**3)/(c
**n*sec(e + f*x)**n*b*n**2*p**2 + 6*c**n*sec(e + f*x)**n*b*n*p + 8*c**n*se
c(e + f*x)**n*b + a*n**2*p**2 + 6*a*n*p + 8*a),x)*a*f*n**3*p**3 - 32*int((
(c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x)**3)/(c**n*sec(e + f*x)**n*...
```

3.464 $\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)} + \frac{\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(\frac{a+b(c\sec(e+fx))}{a}\right.}{2f}$$

output
```
hypergeom([1, p+1],[2+p],(a+b*(c*sec(f*x+e))^n)/a)*(a+b*(c*sec(f*x+e))^n)^
(p+1)/a/f/n/(p+1)+1/2*hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)
*sec(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/(((a+b*(c*sec(f*x+e))^n)/a)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 3.97 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx$$

$$= \frac{(a+b(c\sec(e+fx))^n)^p \left(\frac{2\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)\left(a+b\left(c\sqrt{\sec^2(e+fx)}\right)^n\right)}{an(1+p)} + \text{Hypergeom}\right.}{2f}$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

output
```
((a + b*(c*Sec[e + f*x])^n)^p*((2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
b*(c*Sqrt[Sec[e + f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n))/(a*n*
(1 + p)) + (Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*(c*Sqrt[Sec[e + f*x
]^2])^n)/a)]*Sec[e + f*x]^2)/(1 + (b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)^p))/(2
*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int -\cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b(c\sec(e+fx))^n+a\right)^pd\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b(c\sec(e+fx))^n+a\right)^pd\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 7293}$$

$$-\frac{\int \left(\cos(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p-\sec(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}\sec^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{n+2}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)}{f}$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

output
```
((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*n*(1 + p)) + (Hypergeometric2F1[2/n, -p, (
2 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f*
x])^n)^p)/(2*(1 + (b*(c*Sec[e + f*x])^n)/a)^p))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int (a+b(c\sec(fx+e))^n)^p \tan(fx+e)^3\,dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

Fricas [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)^3\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Sympy [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx=\int (a+b(c\sec{(e+fx)})^n)^p\tan^3{(e+fx)}\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**3,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx=\int ((c\sec{(fx+e)})^n b+a)^p\tan{(fx+e)}^3\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Giac [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx=\int ((c\sec{(fx+e)})^n b+a)^p\tan{(fx+e)}^3\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3\left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p\,dx$$

input
```
int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (a+b(c\sec(e+fx))^n)^p\tan^3(e+fx)\,dx = \frac{(c^n\sec(fx+e)^n b+a)^p\tan(fx+e)^2 np-2(c^n\sec(fx+e)^n b+a)^p+\left(\int \frac{(c^n\sec(fx+e)^n b+a)^p\tan(fx+e}{c^n\sec(fx+e)^n bnp+2c^n\sec(fx+e)^n b+}\right.}{}$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

output
```
((c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x)**2*n*p - 2*(c**n*sec(e + f*x
)**n*b + a)**p + int(((c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x)**3)/(c*
*n*sec(e + f*x)**n*b*n*p + 2*c**n*sec(e + f*x)**n*b + a*n*p + 2*a),x)*a*f*
n**3*p**3 + 2*int(((c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x)**3)/(c**n*
sec(e + f*x)**n*b*n*p + 2*c**n*sec(e + f*x)**n*b + a*n*p + 2*a),x)*a*f*n**
2*p**2 - 2*int(((c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x))/(c**n*sec(e
+ f*x)**n*b*n*p + 2*c**n*sec(e + f*x)**n*b + a*n*p + 2*a),x)*a*f*n**2*p**2
 - 4*int(((c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x))/(c**n*sec(e + f*x)
**n*b*n*p + 2*c**n*sec(e + f*x)**n*b + a*n*p + 2*a),x)*a*f*n*p)/(f*n*p*(n*
p + 2))
```

3.465 $\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 60

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$

output
```
-hypergeom([1, p+1],[2+p],(a+b*(c*sec(f*x+e))^n)/a)*(a+b*(c*sec(f*x+e))^n)
^(p+1)/a/f/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

output
```
-((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b
*(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4627, 891, 27, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{4627}$$

$$\frac{\int \cos(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p\,d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{891}$$

$$\frac{\int c\cos(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p\,d(c\sec(e+fx))}{cf}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cos(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p\,d(c\sec(e+fx))}{f}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \cos(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p\,d(c\sec(e+fx))^n}{fn}$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(a+b(c\sec(e+fx))^n\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

output
```
-((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b
*(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

Maple [F]

$$\int (a+b(c\sec(fx+e))^{n})^{p}\tan(fx+e)\,dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

Fricas [F]

$$\int (a+b(c\sec(e+fx))^{n})^{p}\tan(e+fx)\,dx = \int ((c\sec(fx+e))^{n}b+a)^{p}\tan(fx+e)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Sympy [F]

$$\int (a+b(c\sec(e+fx))^{n})^{p}\tan(e+fx)\,dx = \int (a+b(c\sec(e+fx))^{n})^{p}\tan(e+fx)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e),x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x), x)
```

Maxima [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Giac [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int \tan(e+f\,x)\,\left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \frac{(c^n\sec(fx+e)^n b+a)^p + \left(\int \frac{(c^n\sec(fx+e)^n b+a)^p\tan(fx+e)}{c^n\sec(fx+e)^n b+a}dx\right) afnp}{fnp}$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

output
```
((c**n*sec(e + f*x)**n*b + a)**p + int(((c**n*sec(e + f*x)**n*b + a)**p*ta
n(e + f*x))/(c**n*sec(e + f*x)**n*b + a),x)*a*f*n*p)/(f*n*p)
```

3.466 $\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\operatorname{Int}\left(\cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

output
```
Defer(Int)(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

input
```
Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b(c\sec(e+fx))^n)^p}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4632}$$

$$\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Int[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cot\left(fx+e\right)\left(a+b(c\sec\left(fx+e\right))^{n}\right)^{p} dx$$

input
```
int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^{n}\right)^{p}\,dx=\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 9.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^{n}\right)^{p}\,dx=\int \left(a+b(c\sec\left(e+fx\right))^{n}\right)^{p}\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))**n)**p,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 3.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left((c\sec\left(fx+e\right))^n b+a\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left((c\sec\left(fx+e\right))^n b+a\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 15.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot(e+f\,x)\,\left(a+b\,{\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^n\right)^p\,dx$$

input
```
int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \left(c^n\sec\left(fx+e\right)^n b+a\right)^p\cot\left(fx+e\right)dx$$

input
```
int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int((c**n*sec(e + f*x)**n*b + a)**p*cot(e + f*x),x)
```

3.467 $\int \cot^3(e+fx) \left(a+b(c\sec(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \cot^3(e+fx) \left(a+b(c\sec(e+fx))^n\right)^p \, dx = \operatorname{Int}\left(\cot^3(e+fx) \left(a+b(c\sec(e+fx))^n\right)^p, x\right)$$

output
```
Defer(Int)(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 19.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx) \left(a+b(c\sec(e+fx))^n\right)^p \, dx = \int \cot^3(e+fx) \left(a+b(c\sec(e+fx))^n\right)^p \, dx$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b(c\sec(e+fx))^n\right)^p}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4632}$$

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p dx$$

input
```
Int[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cot \left(fx+e\right)^{3} \left(a+b \left(c \sec \left(fx+e\right)\right)^{n}\right)^{p}dx$$

input
```
int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx) \left(a+b(c\sec(e+fx))^n\right)^p \, dx=\int { \left(\left(c \sec \left(fx+e\right)\right)^{n} b+a\right)^{p} \cot \left(fx+e\right)^{3} \,d x}$$

input
```
integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^3(e+fx) \left(a+b(c\sec(e+fx))^n\right)^p \, dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**3*(a+b*(c*sec(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 9.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left((c\sec\left(fx+e\right))^n b+a\right)^p\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left((c\sec\left(fx+e\right))^n b+a\right)^p\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Mupad [N/A]

Not integrable

Time = 15.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$
$$=\int \cot(e+f\,x)^3\left(a+b\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^n\right)^p\,dx$$

input
```
int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \cot\left(fx+e\right)^3\left(a+b(\sec\left(fx+e\right)c)^n\right)^p dx$$

input
```
int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

3.468 $\int (a + b(c\sec(e + fx))^n)^p \tan^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \text{Int}\left((a+b(c\sec(e+fx))^n)^p \tan^2(e+fx),x\right)$$

output
```
Defer(Int)((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
```

output
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{4632}$$

$$\int \tan^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec (fx+e))^{n})^{p} \tan (fx+e)^{2} dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\, dx = \int ((c\sec (fx+e))^{n} b + a)^{p} \tan (fx+e)^{2} \, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Sympy [N/A]

Not integrable

Time = 18.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\, dx = \int (a+b(c\sec (e+fx))^{n})^{p} \tan^{2} (e+fx)\, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**2,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 6.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^2\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^2\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 14.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2 \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int (c^n \sec(fx+e)^n b+a)^p \tan(fx+e)^2\,dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

output
```
int((c**n*sec(e + f*x)**n*b + a)**p*tan(e + f*x)**2,x)
```

3.469 $\int (a+b(c\sec(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \text{Int}((a+b(c\sec(e+fx))^n)^p, x)$$

output
```
Defer(Int)((a+b*(c*sec(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int (a+b(c\sec(e+fx))^n)^p \, dx$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b(c\sec(e+fx))^n)^p\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+b(c\sec(e+fx))^n)^p\,dx$$
$$\Big\downarrow \text{4619}$$
$$\int (a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4619
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Unintegrable[(a + b*(c*Sec[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, e, f, n, p
}, x]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec (fx+e))^n)^p \, dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int ((c\sec (fx+e))^n b + a)^p \, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (a+b(c\sec(e+fx))^n)^p \, dx = \int (a+b(c\sec (e+fx))^n)^p \, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p, x)
```

Maxima [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p\,dx=\int ((c\sec(fx+e))^n b+a)^p\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p\,dx=\int ((c\sec(fx+e))^n b+a)^p\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 14.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (a+b(c\sec(e+fx))^n)^p\,dx=\int \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p\,dx$$

input
```
int((a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int((a + b*(c/cos(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int (a + b(c\sec(e + fx))^n)^p \, dx = \int (c^n \sec(fx + e)^n b + a)^p \, dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int((c**n*sec(e + f*x)**n*b + a)**p,x)
```

3.470 $\int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx=\text{Int}\big(\cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,,x\big)$$

output
```
Defer(Int)(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx=\int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b(c\sec(e+fx))^n\right)^p}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4632}$$

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p dx$$

input
```
Int[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cot (fx+e)^2 (a+b(c\sec (fx+e))^n)^p \, dx$$

input
```
int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx) (a+b(c\sec(e+fx))^n)^p \, dx = \int ((c\sec (fx+e))^n b+a)^p \cot (fx+e)^2 \, dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Sympy [N/A]

Not integrable

Time = 48.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \cot^2(e+fx) (a+b(c\sec(e+fx))^n)^p \, dx = \int (a+b(c\sec (e+fx))^n)^p \cot^2 (e+fx) \, dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*(c*sec(f*x+e))**n)**p,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 6.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx) (a+b(c\sec(e+fx))^n)^p \, dx = \int ((c\sec(fx+e))^n b+a)^p \cot(fx+e)^2 \, dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx) (a+b(c\sec(e+fx))^n)^p \, dx = \int ((c\sec(fx+e))^n b+a)^p \cot(fx+e)^2 \, dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 15.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \cot^2(e+fx) (a+b(c\sec(e+fx))^n)^p \, dx = \int \cot(e+f\,x)^2 \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p \, dx$$

input
```
int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left(c^n\sec\left(fx+e\right)^n b+a\right)^p\cot\left(fx+e\right)^2dx$$

input
```
int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int((c**n*sec(e + f*x)**n*b + a)**p*cot(e + f*x)**2,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file