

Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.5-Secant/242-4.5.4.11

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [83]. This is test number [242].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (83)	0.00 (0)
Mathematica	95.18 (79)	4.82 (4)
Fricas	75.90 (63)	24.10 (20)
Maple	61.45 (51)	38.55 (32)
Maxima	57.83 (48)	42.17 (35)
Mupad	56.63 (47)	43.37 (36)
Reduce	55.42 (46)	44.58 (37)
Giac	51.81 (43)	48.19 (40)
Sympy	44.58 (37)	55.42 (46)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

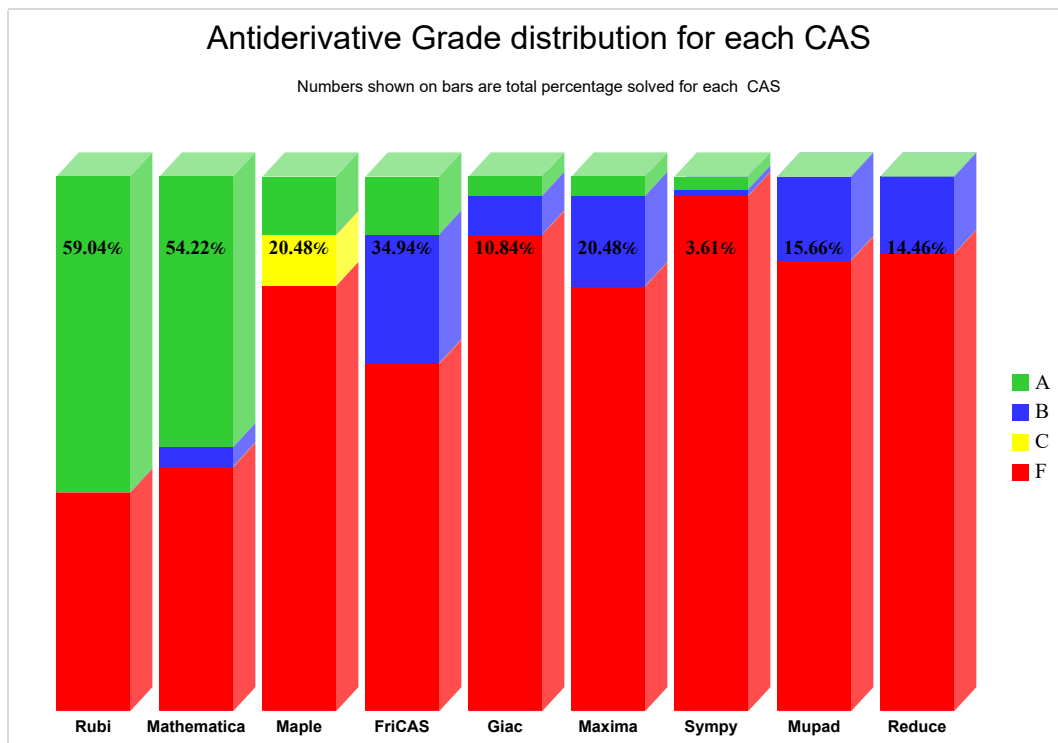
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

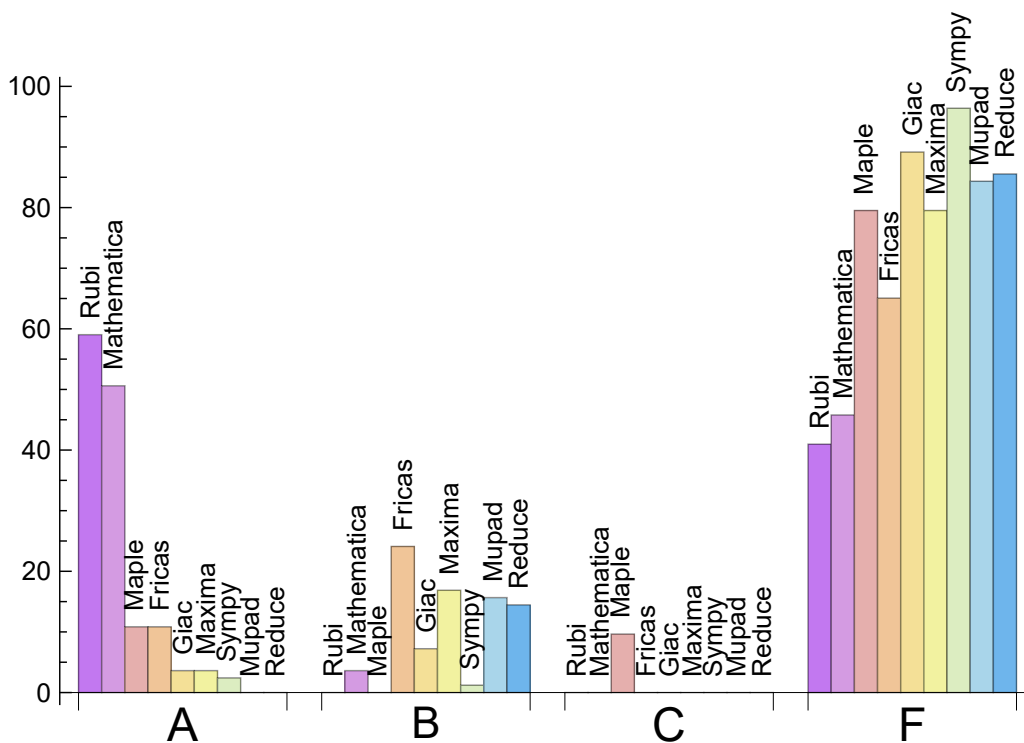
System	% A grade	% B grade	% C grade	% F grade
Rubi	59.036	0.000	0.000	40.964
Mathematica	50.602	3.614	0.000	45.783
Maple	10.843	0.000	9.639	79.518
Fricas	10.843	24.096	0.000	65.060
Giac	3.614	7.229	0.000	89.157
Maxima	3.614	16.867	0.000	79.518
Sympy	2.410	1.205	0.000	96.386
Mupad	0.000	15.663	0.000	84.337
Reduce	0.000	14.458	0.000	85.542

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	20	100.00	0.00	0.00
Maple	32	100.00	0.00	0.00
Maxima	35	54.29	8.57	37.14
Mupad	36	0.00	100.00	0.00
Reduce	37	100.00	0.00	0.00
Giac	40	100.00	0.00	0.00
Sympy	46	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.11
Reduce	0.23
Maple	0.34
Giac	0.48
Rubi	0.68
Maxima	2.25
Sympy	5.08
Mathematica	7.74
Mupad	15.53

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	21.19	1.01	17.00	0.94
Giac	45.81	1.29	20.00	1.11
Mupad	74.91	1.58	22.00	1.22
Maple	164.00	1.44	18.00	1.00
Rubi	301.94	0.99	69.00	1.00
Mathematica	346.82	1.13	54.00	1.10
Fricas	411.59	2.29	44.00	2.11
Maxima	1447.38	38.13	287.50	7.92
Reduce	5577.48	264.20	102.00	3.80

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

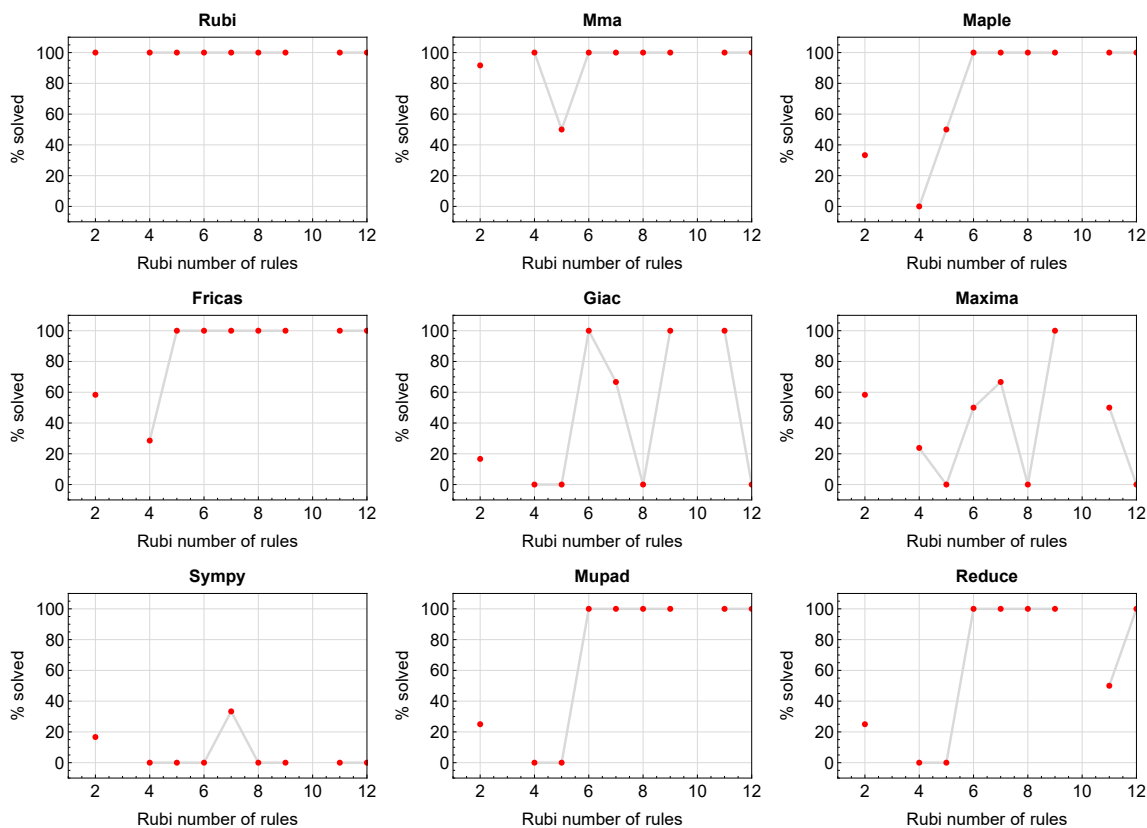


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

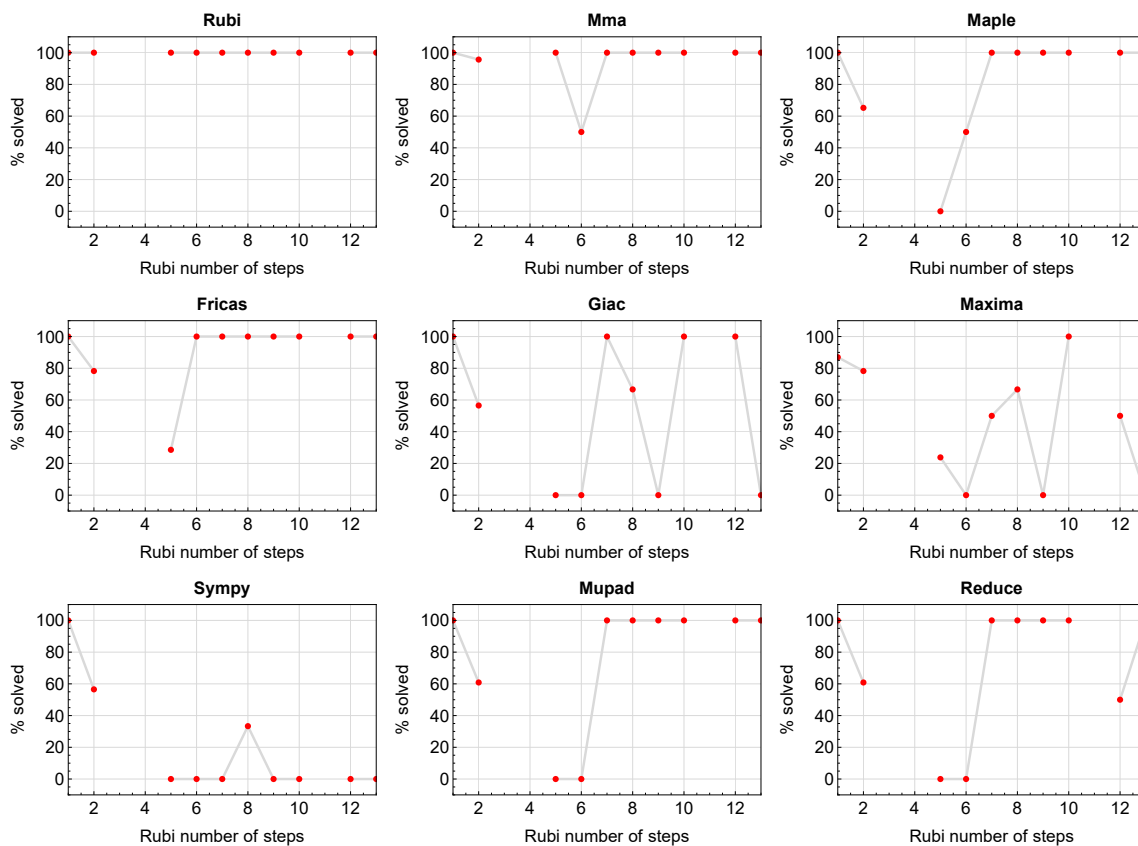


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the percentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

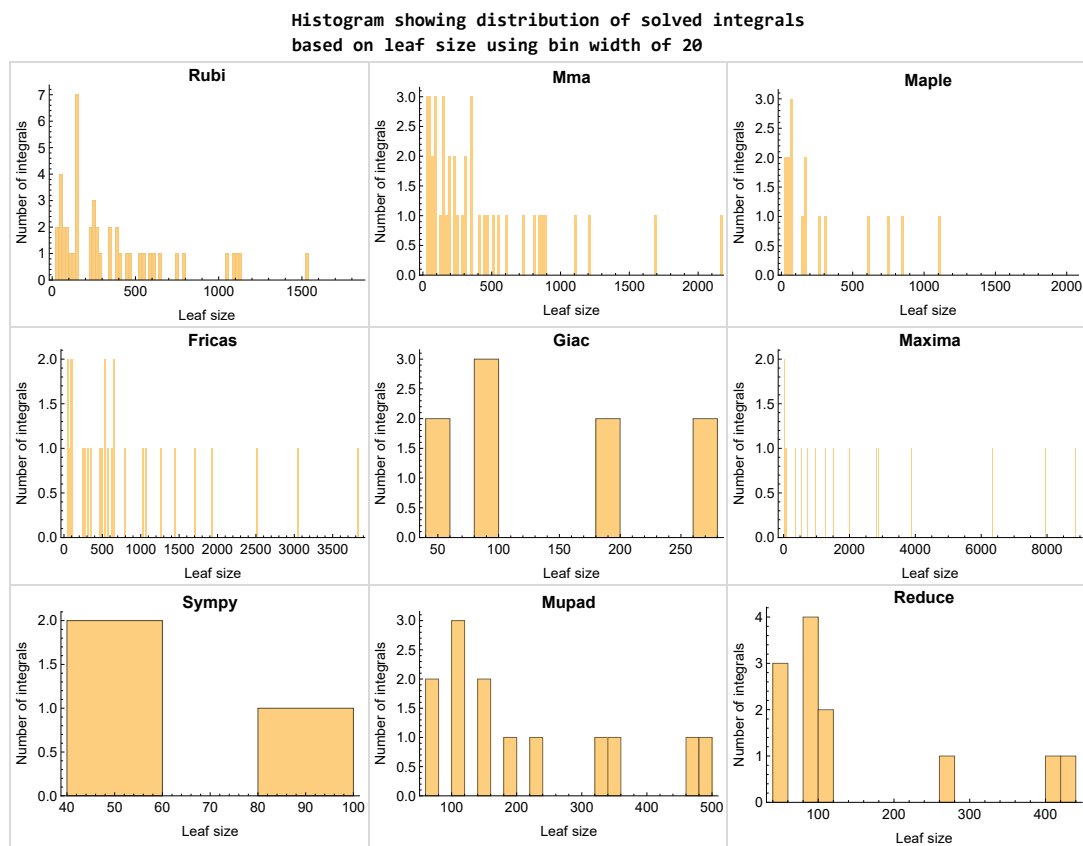


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

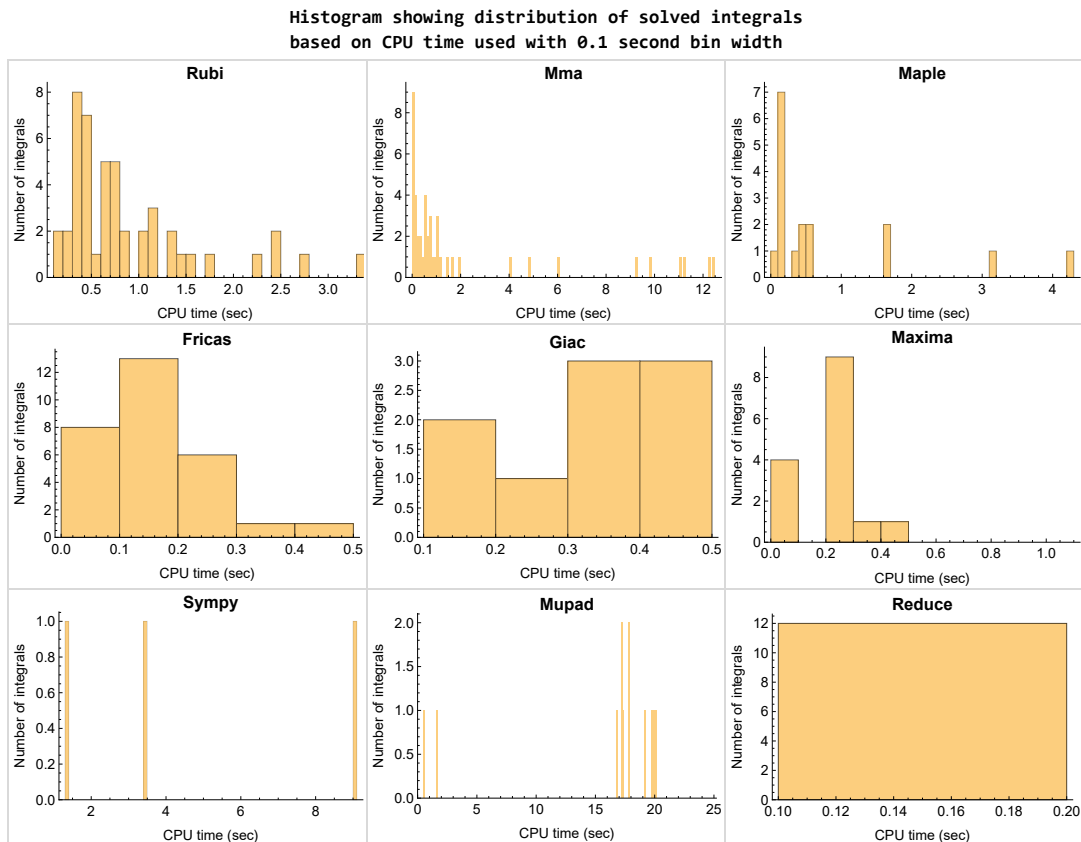


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fracas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

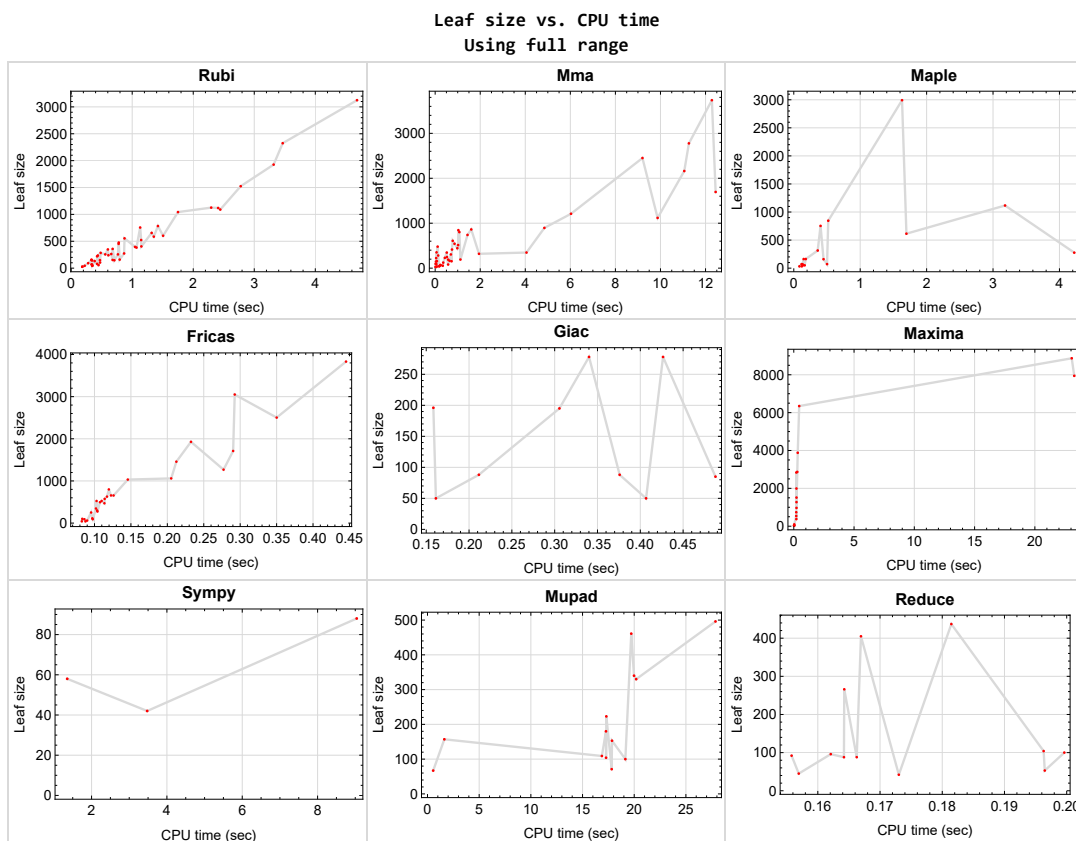


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 54, 55, 59, 60, 64, 65, 69, 70, 71}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {18, 25, 76, 79, 82}

Maple {72, 73, 75, 76, 78, 79, 81, 82}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for **Rubi** and **Mathematica**.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'  
'display2d : false'  
'domain : complex'  
'keepfloat : true'  
'load(to_poly_solve)'  
'load(simplify_sum)'  
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib  
maxima_lib.set('extra_definite_integration_methods', '[]')  
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:  
    # 1.7 is a fudge factor since it is low side from actual leaf count  
    leafCount = round(1.7*count_ops(anti))  
  
except Exception as ee:  
    leafCount = 1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')  
the_variable = evalin(symengine, 'x')  
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.15 Current tree layout of integration tests

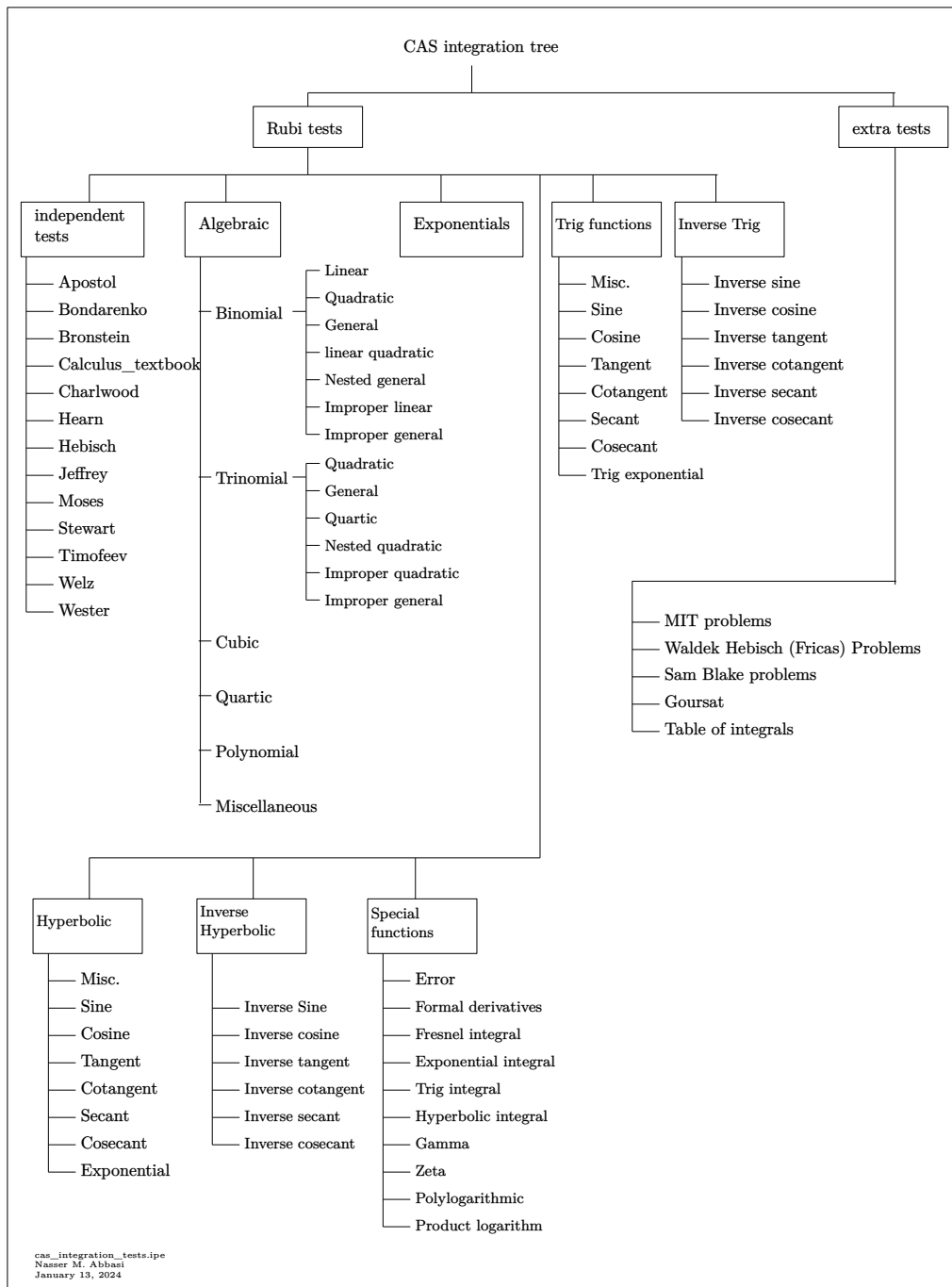
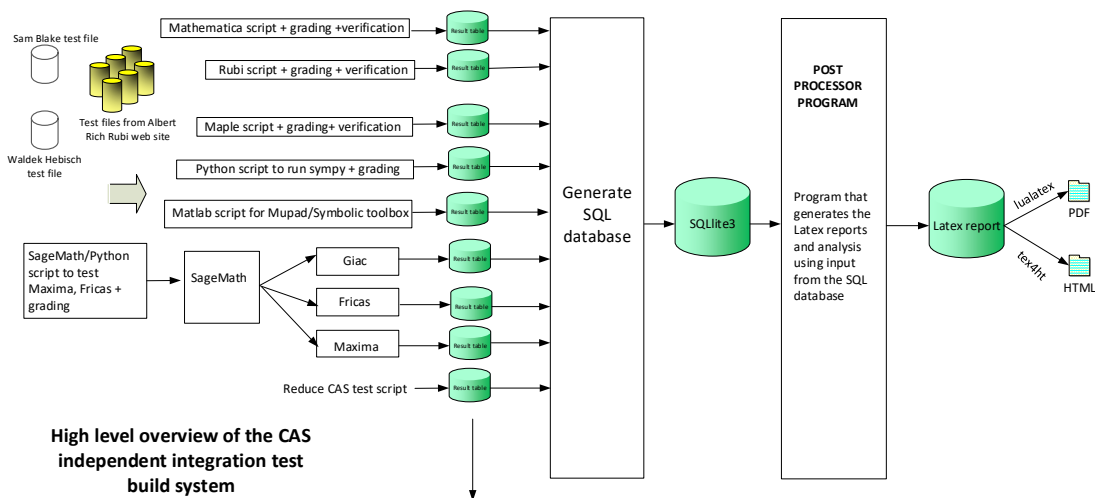


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "E"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in Rubi Table file

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Nasser M. Abbasi
January 13, 2024
Design note

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi	26
Mma	26
Maple	27
Fricas	27
Maxima	27
Giac	28
Mupad	28
Sympy	28
Reduce	29

Rubi

A grade { 1, 3, 5, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 56, 57, 58, 61, 62, 63, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 3, 5, 8, 10, 12, 15, 16, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 56, 57, 58, 61, 62, 63, 66, 67, 68, 72, 73, 75, 76, 78, 81 }

B grade { 18, 79, 82 }

C grade { }

F normal fail { 74, 77, 80, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 5, 12, 15, 20, 27, 53, 58, 63, 68 }

B grade { }

C grade { 72, 73, 75, 76, 78, 79, 81, 82 }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 74, 77, 80, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 5, 15, 20, 53, 63, 72, 75, 78, 81 }

B grade { 1, 3, 8, 10, 12, 16, 18, 23, 25, 27, 58, 68, 73, 74, 76, 77, 79, 80, 82, 83 }

C grade { }

F normal fail { 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 5, 53, 58 }

B grade { 12, 15, 20, 27, 31, 32, 33, 36, 37, 38, 51, 52, 56, 57 }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

F(-1) timedout fail { 60, 69, 70 }

F(-2) exception fail { 25, 41, 42, 43, 46, 47, 48, 61, 62, 63, 66, 67, 68 }

Giac

A grade { 15, 27, 68 }

B grade { 5, 12, 20, 53, 58, 63 }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 5, 12, 15, 20, 27, 53, 58, 63, 68, 72, 75, 78, 81 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 73, 74, 76, 77, 79, 80, 82, 83 }

F(-2) exception fail { }

Sympy

A grade { 53, 58 }

B grade { 5 }

C grade { }

F normal fail { 1, 3, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 63, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 5, 12, 15, 20, 53, 58, 63, 68, 72, 75, 78, 81 }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 66, 67, 73, 74, 76, 77, 79, 80, 82, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column **N.S.** means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	146	0	0	495	0	0	52	0
N.S.	1	1.00	1.02	0.00	0.00	3.46	0.00	0.00	0.36	0.00
time (sec)	N/A	0.344	0.036	0.000	0.000	0.108	0.000	0.000	0.160	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	115	21	15	18	52	20
N.S.	1	1.00	1.12	1.00	7.19	1.31	0.94	1.12	3.25	1.25
time (sec)	N/A	0.178	0.788	0.041	0.154	0.079	2.364	0.221	0.159	15.451

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	95	0	0	346	0	0	52	0
N.S.	1	1.00	1.03	0.00	0.00	3.76	0.00	0.00	0.57	0.00
time (sec)	N/A	0.278	0.019	0.000	0.000	0.102	0.000	0.000	0.157	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	115	21	15	18	52	20
N.S.	1	1.00	1.12	1.00	7.19	1.31	0.94	1.12	3.25	1.25
time (sec)	N/A	0.175	0.593	0.037	0.149	0.075	1.997	0.151	0.156	15.616

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	32	31	42	42	50	45	67
N.S.	1	1.00	1.00	1.23	1.19	1.62	1.62	1.92	1.73	2.58
time (sec)	N/A	0.182	0.012	0.121	0.034	0.083	3.477	0.161	0.157	0.551

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	108	18	14	18	36	20
N.S.	1	1.00	1.12	1.00	6.75	1.12	0.88	1.12	2.25	1.25
time (sec)	N/A	0.177	0.654	0.040	0.144	0.074	0.963	0.186	0.172	15.610

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	118	18	15	18	56	20
N.S.	1	1.00	1.12	1.00	7.38	1.12	0.94	1.12	3.50	1.25
time (sec)	N/A	0.180	0.601	0.039	0.138	0.071	0.623	0.262	0.185	15.361

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	240	229	0	0	799	0	0	250	0
N.S.	1	0.99	0.95	0.00	0.00	3.30	0.00	0.00	1.03	0.00
time (sec)	N/A	0.610	0.430	0.000	0.000	0.120	0.000	0.000	0.167	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	256	42	17	20	549	22
N.S.	1	1.00	1.11	1.00	14.22	2.33	0.94	1.11	30.50	1.22
time (sec)	N/A	0.184	9.218	0.079	0.289	0.072	3.266	0.671	0.170	15.240

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	131	123	0	0	525	0	0	203	0
N.S.	1	0.98	0.92	0.00	0.00	3.95	0.00	0.00	1.53	0.00
time (sec)	N/A	0.385	0.359	0.000	0.000	0.110	0.000	0.000	0.164	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	251	42	17	20	146	22
N.S.	1	1.00	1.11	1.00	13.94	2.33	0.94	1.11	8.11	1.22
time (sec)	N/A	0.181	6.686	0.073	0.284	0.077	3.231	0.648	0.165	15.561

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	43	41	50	96	91	0	88	96	100
N.S.	1	0.98	0.93	1.14	2.18	2.07	0.00	2.00	2.18	2.27
time (sec)	N/A	0.345	0.177	0.164	0.041	0.086	0.000	0.211	0.162	19.143

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	286	36	15	20	82	22
N.S.	1	1.00	1.11	1.00	15.89	2.00	0.83	1.11	4.56	1.22
time (sec)	N/A	0.181	22.991	0.076	0.275	0.072	3.140	0.232	0.167	18.843

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	289	36	17	20	106	22
N.S.	1	1.00	1.11	1.00	16.06	2.00	0.94	1.11	5.89	1.22
time (sec)	N/A	0.188	8.786	0.082	0.279	0.071	0.772	0.941	0.170	17.501

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	104	90	72	2838	100	0	85	266	496
N.S.	1	1.16	1.00	0.80	31.53	1.11	0.00	0.94	2.96	5.51
time (sec)	N/A	0.473	0.054	0.501	0.212	0.083	0.000	0.488	0.164	27.862

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	382	383	305	0	0	1457	0	0	165	0
N.S.	1	1.00	0.80	0.00	0.00	3.81	0.00	0.00	0.43	0.00
time (sec)	N/A	1.070	0.697	0.000	0.000	0.212	0.000	0.000	0.164	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	252	20	15	20	165	22
N.S.	1	1.00	1.11	1.00	14.00	1.11	0.83	1.11	9.17	1.22
time (sec)	N/A	0.188	1.196	0.062	0.237	0.078	0.557	0.360	0.161	16.548

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	256	845	0	0	1060	0	0	165	0
N.S.	1	0.98	3.24	0.00	0.00	4.06	0.00	0.00	0.63	0.00
time (sec)	N/A	0.767	1.030	0.000	0.000	0.205	0.000	0.000	0.162	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	252	20	15	20	165	22
N.S.	1	1.00	1.11	1.00	14.00	1.11	0.83	1.11	9.17	1.22
time (sec)	N/A	0.183	1.070	0.055	0.248	0.077	0.450	0.344	0.161	15.068

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	67	67	70	7945	251	0	278	92	157
N.S.	1	1.02	1.02	1.06	120.38	3.80	0.00	4.21	1.39	2.38
time (sec)	N/A	0.341	0.240	0.117	23.269	0.095	0.000	0.340	0.156	1.630

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	249	19	15	20	160	22
N.S.	1	1.00	1.11	1.00	13.83	1.06	0.83	1.11	8.89	1.22
time (sec)	N/A	0.189	1.267	0.054	0.219	0.070	0.881	0.187	0.162	15.887

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	118	18	15	18	56	20
N.S.	1	1.00	1.12	1.00	7.38	1.12	0.94	1.12	3.50	1.25
time (sec)	N/A	0.173	0.053	0.002	0.188	0.075	0.525	0.319	0.158	0.003

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1090	895	0	0	3050	0	0	38	0
N.S.	1	1.00	0.82	0.00	0.00	2.79	0.00	0.00	0.03	0.00
time (sec)	N/A	2.444	4.847	0.000	0.000	0.293	0.000	0.000	0.158	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1284	38	17	20	38	22
N.S.	1	1.00	1.11	1.00	71.33	2.11	0.94	1.11	2.11	1.22
time (sec)	N/A	0.174	5.000	0.077	0.711	0.081	1.451	0.405	0.158	15.898

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	596	586	1118	0	0	1928	0	0	38	0
N.S.	1	0.98	1.88	0.00	0.00	3.23	0.00	0.00	0.06	0.00
time (sec)	N/A	1.354	9.873	0.000	0.000	0.232	0.000	0.000	0.152	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1261	38	17	20	38	22
N.S.	1	1.00	1.11	1.00	70.06	2.11	0.94	1.11	2.11	1.22
time (sec)	N/A	0.179	4.814	0.076	0.661	0.078	1.188	0.406	0.160	16.631

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	146	153	162	8871	525	0	195	1053	340
N.S.	1	1.19	1.24	1.32	72.12	4.27	0.00	1.59	8.56	2.76
time (sec)	N/A	0.708	0.728	0.180	23.059	0.103	0.000	0.306	0.159	19.981

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	4629	38	17	20	1518	22
N.S.	1	1.00	1.11	1.00	257.17	2.11	0.94	1.11	84.33	1.22
time (sec)	N/A	0.183	7.408	0.078	5.280	0.080	1.533	0.727	0.181	16.999

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	4550	44	19	20	7251	22
N.S.	1	1.00	1.11	1.00	252.78	2.44	1.06	1.11	402.83	1.22
time (sec)	N/A	0.187	4.856	0.084	5.185	0.077	1.481	0.342	0.195	17.232

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3521	44	19	20	39791	22
N.S.	1	1.00	1.11	1.00	195.61	2.44	1.06	1.11	2210.61	1.22
time (sec)	N/A	0.189	6.522	0.079	5.205	0.081	2.101	1.004	0.451	17.973

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	479	0	1512	0	0	0	50	0
N.S.	1	1.00	1.01	0.00	3.18	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.780	0.110	0.000	0.225	0.000	0.000	0.000	0.163	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	351	0	966	0	0	0	50	0
N.S.	1	1.00	1.01	0.00	2.78	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.602	0.061	0.000	0.225	0.000	0.000	0.000	0.167	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	223	0	540	0	0	0	48	0
N.S.	1	1.00	1.01	0.00	2.45	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.425	0.045	0.000	0.203	0.000	0.000	0.000	0.160	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	106	18	15	18	36	20
N.S.	1	1.00	1.11	0.89	5.89	1.00	0.83	1.00	2.00	1.11
time (sec)	N/A	0.179	2.075	0.100	0.438	0.072	1.927	0.364	0.158	16.839

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	110	18	17	18	198	20
N.S.	1	1.00	1.11	0.89	6.11	1.00	0.94	1.00	11.00	1.11
time (sec)	N/A	0.180	11.370	0.105	0.469	0.075	1.538	0.371	0.163	16.316

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	749	756	739	0	6347	0	0	0	0	0
N.S.	1	1.01	0.99	0.00	8.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.130	1.436	0.000	0.450	0.000	0.000	0.000	0.253	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	551	554	543	0	3879	0	0	0	0	0
N.S.	1	1.01	0.99	0.00	7.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.874	0.860	0.000	0.327	0.000	0.000	0.000	0.246	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	355	358	347	0	1991	0	0	0	572	0
N.S.	1	1.01	0.98	0.00	5.61	0.00	0.00	0.00	1.61	0.00
time (sec)	N/A	0.680	0.516	0.000	0.220	0.000	0.000	0.000	0.187	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	319	36	17	20	82	22
N.S.	1	1.00	1.10	0.90	15.95	1.80	0.85	1.00	4.10	1.10
time (sec)	N/A	0.181	70.808	0.170	0.644	0.091	9.534	0.561	0.172	16.993

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	321	36	19	20	4800	22
N.S.	1	1.00	1.10	0.90	16.05	1.80	0.95	1.00	240.00	1.10
time (sec)	N/A	0.182	51.820	0.172	0.838	0.075	2.583	0.609	0.215	16.371

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1041	1043	802	0	0	0	0	0	159	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.751	1.073	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	781	783	608	0	0	0	0	0	159	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.422	0.779	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	521	523	414	0	0	0	0	0	155	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.147	0.747	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	241	19	17	20	155	22
N.S.	1	1.00	1.10	0.90	12.05	0.95	0.85	1.00	7.75	1.10
time (sec)	N/A	0.183	2.332	0.126	0.607	0.091	3.155	0.308	0.167	15.919

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	110	18	17	18	198	20
N.S.	1	1.00	1.11	0.89	6.11	1.00	0.94	1.00	11.00	1.11
time (sec)	N/A	0.179	0.058	0.000	0.486	0.075	1.521	0.505	0.173	0.002

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	3123	3124	3737	0	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.684	12.282	0.000	0.000	0.000	0.000	0.000	2.555	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	2323	2324	2777	0	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.466	11.262	0.000	0.000	0.000	0.000	0.000	1.564	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1523	1524	1695	0	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.778	12.448	0.000	0.000	0.000	0.000	0.000	0.433	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	4405	38	19	20	1453	22
N.S.	1	1.00	1.10	0.90	220.25	1.90	0.95	1.00	72.65	1.10
time (sec)	N/A	0.177	45.770	0.164	12.439	0.086	4.896	0.920	0.216	15.315

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	4406	44	20	20	1628	22
N.S.	1	1.00	1.10	0.90	220.30	2.20	1.00	1.00	81.40	1.10
time (sec)	N/A	0.176	33.138	0.162	18.273	0.082	9.908	1.339	0.218	15.420

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	281	0	738	0	0	0	54	0
N.S.	1	1.00	0.99	0.00	2.60	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.483	0.134	0.000	0.206	0.000	0.000	0.000	0.155	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	155	0	374	0	0	0	49	0
N.S.	1	1.00	0.98	0.00	2.37	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.333	0.077	0.000	0.211	0.000	0.000	0.000	0.168	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	32	31	41	58	50	42	71
N.S.	1	1.00	1.00	1.23	1.19	1.58	2.23	1.92	1.62	2.73
time (sec)	N/A	0.182	0.048	0.085	0.026	0.088	1.354	0.407	0.173	17.820

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	112	25	19	18	44	20
N.S.	1	1.00	1.10	0.80	5.60	1.25	0.95	0.90	2.20	1.00
time (sec)	N/A	0.177	18.578	0.103	0.506	0.076	1.055	0.470	0.176	15.907

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	113	25	19	18	49	20
N.S.	1	1.00	1.10	0.80	5.65	1.25	0.95	0.90	2.45	1.00
time (sec)	N/A	0.175	21.669	0.101	0.591	0.081	4.974	0.503	0.187	15.711

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	451	452	443	0	2869	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	6.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	0.984	0.000	0.291	0.000	0.000	0.000	0.265	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	256	247	0	1272	0	0	0	357	0
N.S.	1	1.00	0.97	0.00	4.99	0.00	0.00	0.00	1.40	0.00
time (sec)	N/A	0.559	0.525	0.000	0.225	0.000	0.000	0.000	0.179	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	51	50	91	88	88	88	109
N.S.	1	1.00	0.96	1.09	1.06	1.94	1.87	1.87	1.87	2.32
time (sec)	N/A	0.348	0.188	0.138	0.029	0.098	9.038	0.376	0.164	16.870

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	718	46	20	20	92	22
N.S.	1	1.00	1.09	0.82	32.64	2.09	0.91	0.91	4.18	1.00
time (sec)	N/A	0.183	50.647	0.180	0.973	0.086	2.167	0.351	0.186	15.178

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	46	20	20	97	22
N.S.	1	1.00	1.09	0.82	0.00	2.09	0.91	0.91	4.41	1.00
time (sec)	N/A	0.183	48.760	0.177	0.000	0.078	6.270	0.387	0.187	15.619

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	653	655	513	0	0	0	0	0	163	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.319	1.020	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	393	395	319	0	0	0	0	0	159	0
N.S.	1	1.01	0.81	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	1.047	1.946	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	69	70	0	274	0	278	88	153
N.S.	1	1.01	1.01	1.03	0.00	4.03	0.00	4.09	1.29	2.25
time (sec)	N/A	0.334	0.228	0.132	0.000	0.104	0.000	0.427	0.166	17.849

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	243	27	20	20	184	22
N.S.	1	1.00	1.09	0.82	11.05	1.23	0.91	0.91	8.36	1.00
time (sec)	N/A	0.187	3.335	0.121	0.755	0.075	2.621	0.301	0.182	15.998

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	244	27	20	20	211	22
N.S.	1	1.00	1.09	0.82	11.09	1.23	0.91	0.91	9.59	1.00
time (sec)	N/A	0.186	3.400	0.121	0.798	0.075	7.343	0.254	0.180	15.839

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1925	1926	2161	0	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.317	11.056	0.000	0.000	0.000	0.000	0.000	0.965	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1125	1126	1210	0	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.294	6.029	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	152	163	162	0	574	0	196	405	330
N.S.	1	1.20	1.28	1.28	0.00	4.52	0.00	1.54	3.19	2.60
time (sec)	N/A	0.680	0.639	0.149	0.000	0.114	0.000	0.158	0.167	20.191

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	48	22	20	38128	22
N.S.	1	1.00	1.09	0.82	0.00	2.18	1.00	0.91	1733.09	1.00
time (sec)	N/A	0.179	30.056	0.169	0.000	0.085	6.064	0.878	0.628	16.368

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	48	22	20	157108	22
N.S.	1	1.00	1.09	0.82	0.00	2.18	1.00	0.91	7141.27	1.00
time (sec)	N/A	0.181	31.618	0.173	0.000	0.084	34.136	1.647	1.982	15.632

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24	24
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20	1.20
time (sec)	N/A	0.245	2.861	0.198	1.456	0.085	47.798	0.619	0.213	15.705

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	159	0	60	0	0	53	104
N.S.	1	1.00	0.86	3.61	0.00	1.36	0.00	0.00	1.20	2.36
time (sec)	N/A	0.221	0.078	0.448	0.000	0.090	0.000	0.000	0.196	17.276

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	188	845	0	470	0	0	74	0
N.S.	1	1.00	1.26	5.67	0.00	3.15	0.00	0.00	0.50	0.00
time (sec)	N/A	0.332	0.579	0.519	0.000	0.114	0.000	0.000	0.204	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	0	0	0	655	0	0	74	0
N.S.	1	1.00	0.00	0.00	0.00	2.79	0.00	0.00	0.31	0.00
time (sec)	N/A	0.437	0.000	0.000	0.000	0.126	0.000	0.000	0.192	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	56	54	276	0	113	0	0	104	180
N.S.	1	0.71	0.68	3.49	0.00	1.43	0.00	0.00	1.32	2.28
time (sec)	N/A	0.449	0.336	4.223	0.000	0.097	0.000	0.000	0.196	17.268

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	221	148	347	1116	0	656	0	0	407	0
N.S.	1	0.67	1.57	5.05	0.00	2.97	0.00	0.00	1.84	0.00
time (sec)	N/A	0.472	4.053	3.181	0.000	0.123	0.000	0.000	0.203	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	263	0	0	0	1032	0	0	516	0
N.S.	1	0.67	0.00	0.00	0.00	2.65	0.00	0.00	1.32	0.00
time (sec)	N/A	0.657	0.000	0.000	0.000	0.146	0.000	0.000	0.236	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	87	80	80	314	0	300	0	0	100	223
N.S.	1	0.92	0.92	3.61	0.00	3.45	0.00	0.00	1.15	2.56
time (sec)	N/A	0.424	0.576	0.360	0.000	0.104	0.000	0.000	0.200	17.303

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	328	273	861	753	0	1268	0	0	191	0
N.S.	1	0.83	2.62	2.30	0.00	3.87	0.00	0.00	0.58	0.00
time (sec)	N/A	0.867	1.603	0.403	0.000	0.277	0.000	0.000	0.222	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	485	404	0	0	0	1711	0	0	191	0
N.S.	1	0.83	0.00	0.00	0.00	3.53	0.00	0.00	0.39	0.00
time (sec)	N/A	1.150	0.000	0.000	0.000	0.290	0.000	0.000	0.196	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	157	159	191	614	0	628	0	0	437	461
N.S.	1	1.01	1.22	3.91	0.00	4.00	0.00	0.00	2.78	2.94
time (sec)	N/A	0.795	1.121	1.696	0.000	0.117	0.000	0.000	0.181	19.714

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	757	603	2450	2992	0	2503	0	0	0	0
N.S.	1	0.80	3.24	3.95	0.00	3.31	0.00	0.00	0.00	0.00
time (sec)	N/A	1.506	9.200	1.630	0.000	0.350	0.000	0.000	0.394	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1384	1119	0	0	0	3831	0	0	0	0
N.S.	1	0.81	0.00	0.00	0.00	2.77	0.00	0.00	0.00	0.00
time (sec)	N/A	2.408	0.000	0.000	0.000	0.445	0.000	0.000	0.954	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [15] had the largest ratio of [.7500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	16	0.125
2	N/A	2	0	1.00	16	0.000
3	A	2	2	1.00	16	0.125
4	N/A	2	0	1.00	16	0.000
5	A	2	2	1.00	14	0.143
6	N/A	2	0	1.00	16	0.000
7	N/A	2	0	1.00	16	0.000
8	A	5	4	0.99	18	0.222
9	N/A	1	0	1.00	18	0.000
10	A	5	4	0.98	18	0.222
11	N/A	1	0	1.00	18	0.000
12	A	8	7	0.98	16	0.438
13	N/A	1	0	1.00	18	0.000
14	N/A	1	0	1.00	18	0.000
15	A	10	9	1.16	12	0.750
16	A	5	4	1.00	18	0.222
17	N/A	1	0	1.00	18	0.000
18	A	5	4	0.98	18	0.222
19	N/A	1	0	1.00	18	0.000
20	A	7	6	1.02	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	N/A	1	0	1.00	18	0.000
22	N/A	2	0	1.00	16	0.000
23	A	5	4	1.00	18	0.222
24	N/A	1	0	1.00	18	0.000
25	A	5	4	0.98	18	0.222
26	N/A	1	0	1.00	18	0.000
27	A	12	11	1.19	16	0.688
28	N/A	1	0	1.00	18	0.000
29	N/A	1	0	1.00	18	0.000
30	N/A	1	0	1.00	18	0.000
31	A	2	2	1.00	18	0.111
32	A	2	2	1.00	18	0.111
33	A	2	2	1.00	16	0.125
34	N/A	2	0	1.00	18	0.000
35	N/A	2	0	1.00	18	0.000
36	A	5	4	1.01	20	0.200
37	A	5	4	1.01	20	0.200
38	A	5	4	1.01	18	0.222
39	N/A	1	0	1.00	20	0.000
40	N/A	1	0	1.00	20	0.000
41	A	5	4	1.00	20	0.200
42	A	5	4	1.00	20	0.200
43	A	5	4	1.00	18	0.222
44	N/A	1	0	1.00	20	0.000
45	N/A	2	0	1.00	18	0.000
46	A	5	4	1.00	20	0.200
47	A	5	4	1.00	20	0.200
48	A	5	4	1.00	18	0.222
49	N/A	1	0	1.00	20	0.000
50	N/A	1	0	1.00	20	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
51	A	2	2	1.00	20	0.100
52	A	2	2	1.00	20	0.100
53	A	2	2	1.00	20	0.100
54	N/A	2	0	1.00	20	0.000
55	N/A	2	0	1.00	20	0.000
56	A	5	4	1.00	22	0.182
57	A	5	4	1.00	22	0.182
58	A	8	7	1.00	22	0.318
59	N/A	1	0	1.00	22	0.000
60	N/A	1	0	1.00	22	0.000
61	A	5	4	1.00	22	0.182
62	A	5	4	1.01	22	0.182
63	A	7	6	1.01	22	0.273
64	N/A	1	0	1.00	22	0.000
65	N/A	1	0	1.00	22	0.000
66	A	5	4	1.00	22	0.182
67	A	5	4	1.00	22	0.182
68	A	12	11	1.20	22	0.500
69	N/A	1	0	1.00	22	0.000
70	N/A	1	0	1.00	22	0.000
71	N/A	2	0	1.00	20	0.000
72	A	2	2	1.00	20	0.100
73	A	2	2	1.00	22	0.091
74	A	2	2	1.00	22	0.091
75	A	9	8	0.71	22	0.364
76	A	6	5	0.67	24	0.208
77	A	6	5	0.67	24	0.208
78	A	8	7	0.92	22	0.318
79	A	6	5	0.83	24	0.208
80	A	6	5	0.83	24	0.208
81	A	13	12	1.01	22	0.545

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	A	6	5	0.80	24	0.208
83	A	6	5	0.81	24	0.208

CHAPTER 3

LISTING OF INTEGRALS

3.1	$\int x^5(a + b \sec(c + dx^2)) dx$	58
3.2	$\int x^4(a + b \sec(c + dx^2)) dx$	64
3.3	$\int x^3(a + b \sec(c + dx^2)) dx$	69
3.4	$\int x^2(a + b \sec(c + dx^2)) dx$	74
3.5	$\int x(a + b \sec(c + dx^2)) dx$	79
3.6	$\int \frac{a+b \sec(c+dx^2)}{x} dx$	84
3.7	$\int \frac{a+b \sec(c+dx^2)}{x^2} dx$	89
3.8	$\int x^5(a + b \sec(c + dx^2))^2 dx$	94
3.9	$\int x^4(a + b \sec(c + dx^2))^2 dx$	101
3.10	$\int x^3(a + b \sec(c + dx^2))^2 dx$	106
3.11	$\int x^2(a + b \sec(c + dx^2))^2 dx$	113
3.12	$\int x(a + b \sec(c + dx^2))^2 dx$	118
3.13	$\int \frac{(a+b \sec(c+dx^2))^2}{x} dx$	125
3.14	$\int \frac{(a+b \sec(c+dx^2))^2}{x^2} dx$	130
3.15	$\int x \sec^7(a + bx^2) dx$	135
3.16	$\int \frac{x^5}{a+b \sec(c+dx^2)} dx$	143
3.17	$\int \frac{x^4}{a+b \sec(c+dx^2)} dx$	150
3.18	$\int \frac{x^3}{a+b \sec(c+dx^2)} dx$	155
3.19	$\int \frac{x^2}{a+b \sec(c+dx^2)} dx$	162
3.20	$\int \frac{x}{a+b \sec(c+dx^2)} dx$	167
3.21	$\int \frac{1}{x(a+b \sec(c+dx^2))} dx$	175
3.22	$\int \frac{a+b \sec(c+dx^2)}{x^2} dx$	180
3.23	$\int \frac{x^5}{(a+b \sec(c+dx^2))^2} dx$	185
3.24	$\int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx$	193

3.25	$\int \frac{x^3}{(a+b \sec(c+dx^2))^2} dx$	199
3.26	$\int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx$	207
3.27	$\int \frac{x}{(a+b \sec(c+dx^2))^2} dx$	213
3.28	$\int \frac{1}{x(a+b \sec(c+dx^2))^2} dx$	223
3.29	$\int \frac{1}{x^2(a+b \sec(c+dx^2))^2} dx$	229
3.30	$\int \frac{1}{x^3(a+b \sec(c+dx^2))^2} dx$	235
3.31	$\int x^3(a+b \sec(c+d\sqrt{x})) dx$	241
3.32	$\int x^2(a+b \sec(c+d\sqrt{x})) dx$	251
3.33	$\int x(a+b \sec(c+d\sqrt{x})) dx$	259
3.34	$\int \frac{a+b \sec(c+d\sqrt{x})}{x} dx$	265
3.35	$\int \frac{a+b \sec(c+d\sqrt{x})}{x^2} dx$	270
3.36	$\int x^3(a+b \sec(c+d\sqrt{x}))^2 dx$	275
3.37	$\int x^2(a+b \sec(c+d\sqrt{x}))^2 dx$	283
3.38	$\int x(a+b \sec(c+d\sqrt{x}))^2 dx$	292
3.39	$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x} dx$	300
3.40	$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^2} dx$	305
3.41	$\int \frac{x^3}{a+b \sec(c+d\sqrt{x})} dx$	311
3.42	$\int \frac{x^2}{a+b \sec(c+d\sqrt{x})} dx$	319
3.43	$\int \frac{x}{a+b \sec(c+d\sqrt{x})} dx$	327
3.44	$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx$	334
3.45	$\int \frac{a+b \sec(c+d\sqrt{x})}{x^2} dx$	339
3.46	$\int \frac{x^3}{(a+b \sec(c+d\sqrt{x}))^2} dx$	344
3.47	$\int \frac{x^2}{(a+b \sec(c+d\sqrt{x}))^2} dx$	352
3.48	$\int \frac{x}{(a+b \sec(c+d\sqrt{x}))^2} dx$	360
3.49	$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx$	368
3.50	$\int \frac{1}{x^2(a+b \sec(c+d\sqrt{x}))^2} dx$	374
3.51	$\int x^{3/2}(a+b \sec(c+d\sqrt{x})) dx$	380
3.52	$\int \sqrt{x}(a+b \sec(c+d\sqrt{x})) dx$	386
3.53	$\int \frac{a+b \sec(c+d\sqrt{x})}{\sqrt{x}} dx$	392
3.54	$\int \frac{a+b \sec(c+d\sqrt{x})}{x^{3/2}} dx$	397
3.55	$\int \frac{a+b \sec(c+d\sqrt{x})}{x^{5/2}} dx$	402
3.56	$\int x^{3/2}(a+b \sec(c+d\sqrt{x}))^2 dx$	407
3.57	$\int \sqrt{x}(a+b \sec(c+d\sqrt{x}))^2 dx$	415
3.58	$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{\sqrt{x}} dx$	422

3.59	$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{3/2}} dx$	429
3.60	$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{5/2}} dx$	435
3.61	$\int \frac{x^{3/2}}{a+b \sec(c+d\sqrt{x})} dx$	440
3.62	$\int \frac{\sqrt{x}}{a+b \sec(c+d\sqrt{x})} dx$	447
3.63	$\int \frac{1}{\sqrt{x}(a+b \sec(c+d\sqrt{x}))} dx$	454
3.64	$\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx$	461
3.65	$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx$	466
3.66	$\int \frac{x^{3/2}}{(a+b \sec(c+d\sqrt{x}))^2} dx$	471
3.67	$\int \frac{\sqrt{x}}{(a+b \sec(c+d\sqrt{x}))^2} dx$	479
3.68	$\int \frac{1}{\sqrt{x}(a+b \sec(c+d\sqrt{x}))^2} dx$	487
3.69	$\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))^2} dx$	496
3.70	$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))^2} dx$	501
3.71	$\int (ex)^m (a + b \sec(c + dx^n))^p dx$	506
3.72	$\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx$	511
3.73	$\int (ex)^{-1+2n} (a + b \sec(c + dx^n)) dx$	516
3.74	$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx$	522
3.75	$\int (ex)^{-1+n} (a + b \sec(c + dx^n))^2 dx$	528
3.76	$\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx$	535
3.77	$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx$	543
3.78	$\int \frac{(ex)^{-1+n}}{a+b \sec(c+dx^n)} dx$	550
3.79	$\int \frac{(ex)^{-1+2n}}{a+b \sec(c+dx^n)} dx$	557
3.80	$\int \frac{(ex)^{-1+3n}}{a+b \sec(c+dx^n)} dx$	565
3.81	$\int \frac{(ex)^{-1+n}}{(a+b \sec(c+dx^n))^2} dx$	572
3.82	$\int \frac{(ex)^{-1+2n}}{(a+b \sec(c+dx^n))^2} dx$	582
3.83	$\int \frac{(ex)^{-1+3n}}{(a+b \sec(c+dx^n))^2} dx$	593

3.1 $\int x^5(a + b \sec(c + dx^2)) dx$

Optimal result	58
Mathematica [A] (verified)	59
Rubi [A] (verified)	59
Maple [F]	60
Fricas [B] (verification not implemented)	61
Sympy [F]	61
Maxima [F]	62
Giac [F]	62
Mupad [F(-1)]	62
Reduce [F]	63

Optimal result

Integrand size = 16, antiderivative size = 143

$$\int x^5(a + b \sec(c + dx^2)) dx = \frac{ax^6}{6} - \frac{ibx^4 \arctan(e^{i(c+dx^2)})}{d} + \frac{ibx^2 \operatorname{PolyLog}(2, -ie^{i(c+dx^2)})}{d^2} - \frac{ibx^2 \operatorname{PolyLog}(2, ie^{i(c+dx^2)})}{d^2} - \frac{b \operatorname{PolyLog}(3, -ie^{i(c+dx^2)})}{d^3} + \frac{b \operatorname{PolyLog}(3, ie^{i(c+dx^2)})}{d^3}$$

output

```
1/6*a*x^6-I*b*x^4*arctan(exp(I*(d*x^2+c)))/d+I*b*x^2*polylog(2,-I*exp(I*(d*x^2+c)))/d^2-I*b*x^2*polylog(2,I*exp(I*(d*x^2+c)))/d^2-b*polylog(3,-I*exp(I*(d*x^2+c)))/d^3+b*polylog(3,I*exp(I*(d*x^2+c)))/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

$$\int x^5 (a + b \sec(c + dx^2)) dx = \frac{ax^6}{6} - \frac{ibx^4 \arctan(e^{ic+idx^2})}{d} + \frac{ibx^2 \operatorname{PolyLog}\left(2, -ie^{i(c+dx^2)}\right)}{d^2} - \frac{ibx^2 \operatorname{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{d^2} - \frac{b \operatorname{PolyLog}\left(3, -ie^{i(c+dx^2)}\right)}{d^3} + \frac{b \operatorname{PolyLog}\left(3, ie^{i(c+dx^2)}\right)}{d^3}$$

input

```
Integrate[x^5*(a + b*Sec[c + d*x^2]),x]
```

output

```
(a*x^6)/6 - (I*b*x^4*ArcTan[E^(I*c + I*d*x^2)])/d + (I*b*x^2*PolyLog[2, (-I)*E^(I*(c + d*x^2))])/d^2 - (I*b*x^2*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2 - (b*PolyLog[3, (-I)*E^(I*(c + d*x^2))])/d^3 + (b*PolyLog[3, I*E^(I*(c + d*x^2))])/d^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a + b \sec(c + dx^2)) dx$$

↓ 2010

$$\int (ax^5 + bx^5 \sec(c + dx^2)) dx$$

↓ 2009

$$\frac{ax^6}{6} - \frac{ibx^4 \arctan\left(e^{i(c+dx^2)}\right)}{d} - \frac{b \operatorname{PolyLog}\left(3, -ie^{i(dx^2+c)}\right)}{d^3} + \frac{b \operatorname{PolyLog}\left(3, ie^{i(dx^2+c)}\right)}{d^3} + \frac{ibx^2 \operatorname{PolyLog}\left(2, -ie^{i(dx^2+c)}\right)}{d^2} - \frac{ibx^2 \operatorname{PolyLog}\left(2, ie^{i(dx^2+c)}\right)}{d^2}$$

input `Int[x^5*(a + b*Sec[c + d*x^2]),x]`

output `(a*x^6)/6 - (I*b*x^4*ArcTan[E^(I*(c + d*x^2))])/d + (I*b*x^2*PolyLog[2, (-I)*E^(I*(c + d*x^2))])/d^2 - (I*b*x^2*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2 - (b*PolyLog[3, (-I)*E^(I*(c + d*x^2))])/d^3 + (b*PolyLog[3, I*E^(I*(c + d*x^2))])/d^3`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [F]

$$\int x^5 (a + b \sec(dx^2 + c)) dx$$

input `int(x^5*(a+b*sec(d*x^2+c)),x)`

output `int(x^5*(a+b*sec(d*x^2+c)),x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(115) = 230$.

Time = 0.11 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.46

$$\int x^5 (a + b \sec(c + dx^2)) dx$$

$$= \frac{2ad^3x^6 - 6ibdx^2\text{Li}_2(i \cos(dx^2 + c) + \sin(dx^2 + c)) - 6ibdx^2\text{Li}_2(i \cos(dx^2 + c) - \sin(dx^2 + c)) + 6ibdx^2\text{Li}_2(-i \cos(dx^2 + c) + \sin(dx^2 + c)) - 6ibdx^2\text{Li}_2(-i \cos(dx^2 + c) - \sin(dx^2 + c))}{d^3}$$

input `integrate(x^5*(a+b*sec(d*x^2+c)),x, algorithm="fricas")`

output

```
1/12*(2*a*d^3*x^6 - 6*I*b*d*x^2*dilog(I*cos(d*x^2 + c) + sin(d*x^2 + c)) -
6*I*b*d*x^2*dilog(I*cos(d*x^2 + c) - sin(d*x^2 + c)) + 6*I*b*d*x^2*dilog(
-I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*I*b*d*x^2*dilog(-I*cos(d*x^2 + c)
- sin(d*x^2 + c)) + 3*b*c^2*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + I) - 3
*b*c^2*log(cos(d*x^2 + c) - I*sin(d*x^2 + c) + I) + 3*b*c^2*log(-cos(d*x^2
+ c) + I*sin(d*x^2 + c) + I) - 3*b*c^2*log(-cos(d*x^2 + c) - I*sin(d*x^2
+ c) + I) + 3*(b*d^2*x^4 - b*c^2)*log(I*cos(d*x^2 + c) + sin(d*x^2 + c) +
1) - 3*(b*d^2*x^4 - b*c^2)*log(I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) + 3*
(b*d^2*x^4 - b*c^2)*log(-I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 3*(b*d^2
*x^4 - b*c^2)*log(-I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) - 6*b*polylog(3,
I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*b*polylog(3, I*cos(d*x^2 + c) - si
n(d*x^2 + c)) - 6*b*polylog(3, -I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*b*p
olylog(3, -I*cos(d*x^2 + c) - sin(d*x^2 + c))/d^3
```

Sympy [F]

$$\int x^5 (a + b \sec(c + dx^2)) dx = \int x^5 (a + b \sec(c + dx^2)) dx$$

input `integrate(x**5*(a+b*sec(d*x**2+c)),x)`

output `Integral(x**5*(a + b*sec(c + d*x**2)), x)`

Maxima [F]

$$\int x^5(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^5 dx$$

input `integrate(x^5*(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

output `1/6*a*x^6 + 2*b*integrate((x^5*cos(2*d*x^2 + 2*c))*cos(d*x^2 + c) + x^5*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^5*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2 + sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)`

Giac [F]

$$\int x^5(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^5 dx$$

input `integrate(x^5*(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)*x^5, x)`

Mupad [F(-1)]

Timed out.

$$\int x^5(a + b \sec(c + dx^2)) dx = \int x^5 \left(a + \frac{b}{\cos(dx^2 + c)} \right) dx$$

input `int(x^5*(a + b/cos(c + d*x^2)),x)`

output `int(x^5*(a + b/cos(c + d*x^2)), x)`

Reduce [F]

$$\int x^5(a + b \sec(c + dx^2)) dx = -2 \left(\int \frac{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 x^5}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 - 1} dx \right) b + \frac{ax^6}{6} + \frac{bx^6}{6}$$

input `int(x^5*(a+b*sec(d*x^2+c)),x)`

output `(- 12*int((tan((c + d*x**2)/2)**2*x**5)/(tan((c + d*x**2)/2)**2 - 1),x)*b + a*x**6 + b*x**6)/6`

3.2 $\int x^4(a + b \sec(c + dx^2)) dx$

Optimal result	64
Mathematica [N/A]	64
Rubi [N/A]	65
Maple [N/A]	66
Fricas [N/A]	66
Sympy [N/A]	66
Maxima [N/A]	67
Giac [N/A]	67
Mupad [N/A]	67
Reduce [N/A]	68

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^4(a + b \sec(c + dx^2)) dx = \frac{ax^5}{5} + b \operatorname{Int}(x^4 \sec(c + dx^2), x)$$

output `1/5*a*x^5+b*Defer(Int)(x^4*sec(d*x^2+c),x)`

Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4(a + b \sec(c + dx^2)) dx = \int x^4(a + b \sec(c + dx^2)) dx$$

input `Integrate[x^4*(a + b*Sec[c + d*x^2]),x]`

output `Integrate[x^4*(a + b*Sec[c + d*x^2]), x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a + b \sec(c + dx^2)) dx$$

$$\downarrow \text{2010}$$

$$\int (ax^4 + bx^4 \sec(c + dx^2)) dx$$

$$\downarrow \text{2009}$$

$$b \int x^4 \sec(dx^2 + c) dx + \frac{ax^5}{5}$$

input

```
Int[x^4*(a + b*Sec[c + d*x^2]),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^4(a + b \sec(dx^2 + c)) dx$$

input `int(x^4*(a+b*sec(d*x^2+c)),x)`output `int(x^4*(a+b*sec(d*x^2+c)),x)`**Fricas [N/A]**

Not integrable

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^4(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^4 dx$$

input `integrate(x^4*(a+b*sec(d*x^2+c)),x, algorithm="fricas")`output `integral(b*x^4*sec(d*x^2 + c) + a*x^4, x)`**Sympy [N/A]**

Not integrable

Time = 2.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^4(a + b \sec(c + dx^2)) dx = \int x^4(a + b \sec(c + dx^2)) dx$$

input `integrate(x**4*(a+b*sec(d*x**2+c)),x)`output `Integral(x**4*(a + b*sec(c + d*x**2)), x)`

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 7.19

$$\int x^4(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^4 dx$$

input `integrate(x^4*(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

output `1/5*a*x^5 + 2*b*integrate((x^4*cos(2*d*x^2 + 2*c))*cos(d*x^2 + c) + x^4*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^4*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2 + sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)`

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^4 dx$$

input `integrate(x^4*(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)*x^4, x)`

Mupad [N/A]

Not integrable

Time = 15.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^4(a + b \sec(c + dx^2)) dx = \int x^4 \left(a + \frac{b}{\cos(dx^2 + c)} \right) dx$$

input `int(x^4*(a + b/cos(c + d*x^2)),x)`

output `int(x^4*(a + b/cos(c + d*x^2)), x)`

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 3.25

$$\int x^4(a + b \sec(c + dx^2)) dx = -2 \left(\int \frac{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 x^4}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 - 1} dx \right) b + \frac{ax^5}{5} + \frac{bx^5}{5}$$

input `int(x^4*(a+b*sec(d*x^2+c)),x)`

output `(- 10*int((tan((c + d*x**2)/2)**2*x**4)/(tan((c + d*x**2)/2)**2 - 1),x)*b + a*x**5 + b*x**5)/5`

3.3 $\int x^3(a + b \sec(c + dx^2)) dx$

Optimal result	69
Mathematica [A] (verified)	69
Rubi [A] (verified)	70
Maple [F]	71
Fricas [B] (verification not implemented)	71
Sympy [F]	72
Maxima [F]	72
Giac [F]	73
Mupad [F(-1)]	73
Reduce [F]	73

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int x^3(a + b \sec(c + dx^2)) dx = \frac{ax^4}{4} - \frac{ibx^2 \arctan(e^{i(c+dx^2)})}{d} + \frac{ib \operatorname{PolyLog}(2, -ie^{i(c+dx^2)})}{2d^2} - \frac{ib \operatorname{PolyLog}(2, ie^{i(c+dx^2)})}{2d^2}$$

output

```
1/4*a*x^4-I*b*x^2*arctan(exp(I*(d*x^2+c)))/d+1/2*I*b*polylog(2,-I*exp(I*(d*x^2+c)))/d^2-1/2*I*b*polylog(2,I*exp(I*(d*x^2+c)))/d^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int x^3(a + b \sec(c + dx^2)) dx = \frac{ax^4}{4} - \frac{ibx^2 \arctan(e^{ic+idx^2})}{d} + \frac{ib \operatorname{PolyLog}(2, -ie^{i(c+dx^2)})}{2d^2} - \frac{ib \operatorname{PolyLog}(2, ie^{i(c+dx^2)})}{2d^2}$$

input `Integrate[x^3*(a + b*Sec[c + d*x^2]),x]`

output `(a*x^4)/4 - (I*b*x^2*ArcTan[E^(I*c + I*d*x^2)])/d + ((I/2)*b*PolyLog[2, (-I)*E^(I*(c + d*x^2))])/d^2 - ((I/2)*b*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2`

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a + b \sec(c + dx^2)) dx$$

$$\downarrow \text{2010}$$

$$\int (ax^3 + bx^3 \sec(c + dx^2)) dx$$

$$\downarrow \text{2009}$$

$$\frac{ax^4}{4} - \frac{ibx^2 \arctan\left(e^{i(c+dx^2)}\right)}{d} + \frac{ib \operatorname{PolyLog}\left(2, -ie^{i(dx^2+c)}\right)}{2d^2} - \frac{ib \operatorname{PolyLog}\left(2, ie^{i(dx^2+c)}\right)}{2d^2}$$

input `Int[x^3*(a + b*Sec[c + d*x^2]),x]`

output `(a*x^4)/4 - (I*b*x^2*ArcTan[E^(I*(c + d*x^2))])/d + ((I/2)*b*PolyLog[2, (-I)*E^(I*(c + d*x^2))])/d^2 - ((I/2)*b*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [F]

$$\int x^3(a + b \sec(dx^2 + c)) dx$$

input `int(x^3*(a+b*sec(d*x^2+c)),x)`

output `int(x^3*(a+b*sec(d*x^2+c)),x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(68) = 136$.

Time = 0.10 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.76

$$\int x^3(a + b \sec(c + dx^2)) dx$$

$$= \frac{ad^2x^4 - bc \log(\cos(dx^2 + c) + i \sin(dx^2 + c) + i) + bc \log(\cos(dx^2 + c) - i \sin(dx^2 + c) + i) - bc \log(\cos(dx^2 + c) + i \sin(dx^2 + c) - i) + bc \log(\cos(dx^2 + c) - i \sin(dx^2 + c) - i)}{d}$$

input `integrate(x^3*(a+b*sec(d*x^2+c)),x, algorithm="fricas")`

output

```
1/4*(a*d^2*x^4 - b*c*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + I) + b*c*log(
cos(d*x^2 + c) - I*sin(d*x^2 + c) + I) - b*c*log(-cos(d*x^2 + c) + I*sin(d
*x^2 + c) + I) + b*c*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) + I) - I*b*dil
og(I*cos(d*x^2 + c) + sin(d*x^2 + c)) - I*b*dilog(I*cos(d*x^2 + c) - sin(d
*x^2 + c)) + I*b*dilog(-I*cos(d*x^2 + c) + sin(d*x^2 + c)) + I*b*dilog(-I*
cos(d*x^2 + c) - sin(d*x^2 + c)) + (b*d*x^2 + b*c)*log(I*cos(d*x^2 + c) +
sin(d*x^2 + c) + 1) - (b*d*x^2 + b*c)*log(I*cos(d*x^2 + c) - sin(d*x^2 + c
) + 1) + (b*d*x^2 + b*c)*log(-I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - (b*
d*x^2 + b*c)*log(-I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1))/d^2
```

Sympy [F]

$$\int x^3(a + b \sec(c + dx^2)) dx = \int x^3(a + b \sec(c + dx^2)) dx$$

input

```
integrate(x**3*(a+b*sec(d*x**2+c)),x)
```

output

```
Integral(x**3*(a + b*sec(c + d*x**2)), x)
```

Maxima [F]

$$\int x^3(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^3 dx$$

input

```
integrate(x^3*(a+b*sec(d*x^2+c)),x, algorithm="maxima")
```

output

```
1/4*a*x^4 + 2*b*integrate((x^3*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + x^3*sin
(2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^3*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2
+ sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)
```

Giac [F]

$$\int x^3(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^3 dx$$

input `integrate(x^3*(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)*x^3, x)`

Mupad [F(-1)]

Timed out.

$$\int x^3(a + b \sec(c + dx^2)) dx = \int x^3 \left(a + \frac{b}{\cos(dx^2 + c)} \right) dx$$

input `int(x^3*(a + b/cos(c + d*x^2)),x)`

output `int(x^3*(a + b/cos(c + d*x^2)), x)`

Reduce [F]

$$\int x^3(a + b \sec(c + dx^2)) dx = -2 \left(\int \frac{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 x^3}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 - 1} dx \right) b + \frac{ax^4}{4} + \frac{bx^4}{4}$$

input `int(x^3*(a+b*sec(d*x^2+c)),x)`

output `(- 8*int((tan((c + d*x**2)/2)**2*x**3)/(tan((c + d*x**2)/2)**2 - 1),x)*b + a*x**4 + b*x**4)/4`

3.4 $\int x^2(a + b \sec(c + dx^2)) dx$

Optimal result	74
Mathematica [N/A]	74
Rubi [N/A]	75
Maple [N/A]	76
Fricas [N/A]	76
Sympy [N/A]	76
Maxima [N/A]	77
Giac [N/A]	77
Mupad [N/A]	77
Reduce [N/A]	78

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^2(a + b \sec(c + dx^2)) dx = \frac{ax^3}{3} + b \operatorname{Int}(x^2 \sec(c + dx^2), x)$$

output `1/3*a*x^3+b*Defer(Int)(x^2*sec(d*x^2+c),x)`

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2(a + b \sec(c + dx^2)) dx = \int x^2(a + b \sec(c + dx^2)) dx$$

input `Integrate[x^2*(a + b*Sec[c + d*x^2]),x]`

output `Integrate[x^2*(a + b*Sec[c + d*x^2]), x]`

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a + b \sec(c + dx^2)) dx$$

$$\downarrow \text{2010}$$

$$\int (ax^2 + bx^2 \sec(c + dx^2)) dx$$

$$\downarrow \text{2009}$$

$$b \int x^2 \sec(dx^2 + c) dx + \frac{ax^3}{3}$$

input

```
Int[x^2*(a + b*Sec[c + d*x^2]),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^2(a + b \sec(dx^2 + c)) dx$$

input `int(x^2*(a+b*sec(d*x^2+c)),x)`output `int(x^2*(a+b*sec(d*x^2+c)),x)`**Fricas [N/A]**

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^2(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sec(d*x^2+c)),x, algorithm="fricas")`output `integral(b*x^2*sec(d*x^2 + c) + a*x^2, x)`**Sympy [N/A]**

Not integrable

Time = 2.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2(a + b \sec(c + dx^2)) dx = \int x^2(a + b \sec(c + dx^2)) dx$$

input `integrate(x**2*(a+b*sec(d*x**2+c)),x)`output `Integral(x**2*(a + b*sec(c + d*x**2)), x)`

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 7.19

$$\int x^2(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

output `1/3*a*x^3 + 2*b*integrate((x^2*cos(2*d*x^2 + 2*c))*cos(d*x^2 + c) + x^2*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^2*cos(d*x^2 + c))/(cos(2*d*x^2 + 2*c)^2 + sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1), x)`

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2(a + b \sec(c + dx^2)) dx = \int (b \sec(dx^2 + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)*x^2, x)`

Mupad [N/A]

Not integrable

Time = 15.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^2(a + b \sec(c + dx^2)) dx = \int x^2 \left(a + \frac{b}{\cos(dx^2 + c)} \right) dx$$

input `int(x^2*(a + b/cos(c + d*x^2)),x)`

output `int(x^2*(a + b/cos(c + d*x^2)), x)`

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 3.25

$$\int x^2(a + b \sec(c + dx^2)) dx = -2 \left(\int \frac{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 x^2}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 - 1} dx \right) b + \frac{ax^3}{3} + \frac{bx^3}{3}$$

input `int(x^2*(a+b*sec(d*x^2+c)),x)`

output `(- 6*int((tan((c + d*x**2)/2)**2*x**2)/(tan((c + d*x**2)/2)**2 - 1),x)*b + a*x**3 + b*x**3)/3`

3.5 $\int x(a + b \sec(c + dx^2)) dx$

Optimal result	79
Mathematica [A] (verified)	79
Rubi [A] (verified)	80
Maple [A] (verified)	81
Fricas [A] (verification not implemented)	81
Sympy [B] (verification not implemented)	82
Maxima [A] (verification not implemented)	82
Giac [B] (verification not implemented)	82
Mupad [B] (verification not implemented)	83
Reduce [B] (verification not implemented)	83

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int x(a + b \sec(c + dx^2)) dx = \frac{ax^2}{2} + \frac{b \operatorname{arctanh}(\sin(c + dx^2))}{2d}$$

output `1/2*a*x^2+1/2*b*arctanh(sin(d*x^2+c))/d`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x(a + b \sec(c + dx^2)) dx = \frac{ax^2}{2} + \frac{b \operatorname{coth}^{-1}(\sin(c + dx^2))}{2d}$$

input `Integrate[x*(a + b*Sec[c + d*x^2]),x]`

output `(a*x^2)/2 + (b*ArcCoth[Sin[c + d*x^2]])/(2*d)`

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a + b \sec(c + dx^2)) dx$$

$$\downarrow \text{2010}$$

$$\int (ax + bx \sec(c + dx^2)) dx$$

$$\downarrow \text{2009}$$

$$\frac{ax^2}{2} + \frac{\text{barctanh}(\sin(c + dx^2))}{2d}$$

input `Int[x*(a + b*Sec[c + d*x^2]),x]`

output `(a*x^2)/2 + (b*ArcTanh[Sin[c + d*x^2]])/(2*d)`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

method	result	size
parts	$\frac{ax^2}{2} + \frac{b \ln(\sec(dx^2+c)+\tan(dx^2+c))}{2d}$	32
derivativedivides	$\frac{(dx^2+c)a+b \ln(\sec(dx^2+c)+\tan(dx^2+c))}{2d}$	36
default	$\frac{(dx^2+c)a+b \ln(\sec(dx^2+c)+\tan(dx^2+c))}{2d}$	36
parallelsch	$\frac{adx^2+b \left(-\ln\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+1\right) \right)}{2d}$	46
norman	$\frac{ax^2}{2} - \frac{b \ln\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)-1\right)}{2d} + \frac{b \ln\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+1\right)}{2d}$	48
risch	$\frac{ax^2}{2} + \frac{b \ln\left(e^{i(dx^2+c)}+i\right)}{2d} - \frac{b \ln\left(e^{i(dx^2+c)}-i\right)}{2d}$	50

input `int(x*(a+b*sec(d*x^2+c)),x,method=_RETURNVERBOSE)`

output `1/2*a*x^2+1/2*b/d*ln(sec(d*x^2+c)+tan(d*x^2+c))`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int x(a + b \sec(c + dx^2)) dx$$

$$= \frac{2adx^2 + b \log(\sin(dx^2 + c) + 1) - b \log(-\sin(dx^2 + c) + 1)}{4d}$$

input `integrate(x*(a+b*sec(d*x^2+c)),x, algorithm="fricas")`

output `1/4*(2*a*d*x^2 + b*log(sin(d*x^2 + c) + 1) - b*log(-sin(d*x^2 + c) + 1))/d`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 3.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int x(a + b \sec(c + dx^2)) dx = \begin{cases} \frac{a(c+dx^2)+b \log(\tan(c+dx^2)+\sec(c+dx^2))}{2d} & \text{for } d \neq 0 \\ \frac{x^2(a+b \sec(c))}{2} & \text{otherwise} \end{cases}$$

input `integrate(x*(a+b*sec(d*x**2+c)),x)`

output `Piecewise(((a*(c + d*x**2) + b*log(tan(c + d*x**2) + sec(c + d*x**2)))/(2*d), Ne(d, 0)), (x**2*(a + b*sec(c))/2, True))`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int x(a + b \sec(c + dx^2)) dx = \frac{1}{2} ax^2 + \frac{b \log(\sec(dx^2 + c) + \tan(dx^2 + c))}{2d}$$

input `integrate(x*(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

output `1/2*a*x^2 + 1/2*b*log(sec(d*x^2 + c) + tan(d*x^2 + c))/d`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\begin{aligned} & \int x(a + b \sec(c + dx^2)) dx \\ &= \frac{(dx^2 + c)a + b \log(|\tan(\frac{1}{2} dx^2 + \frac{1}{2} c) + 1|) - b \log(|\tan(\frac{1}{2} dx^2 + \frac{1}{2} c) - 1|)}{2d} \end{aligned}$$

input `integrate(x*(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output $\frac{1}{2} * ((d * x^2 + c) * a + b * \log(\text{abs}(\tan(1/2 * d * x^2 + 1/2 * c) + 1)) - b * \log(\text{abs}(\tan(1/2 * d * x^2 + 1/2 * c) - 1))) / d$

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

$$\int x(a + b \sec(c + dx^2)) dx = \frac{ax^2}{2} + \frac{b \ln(-bx^2i - 2bx e^{dx^2i} e^{ci})}{2d} - \frac{b \ln(bx^2i - 2bx e^{dx^2i} e^{ci})}{2d}$$

input `int(x*(a + b/cos(c + d*x^2)),x)`

output $(a * x^2) / 2 + (b * \log(-b * x^2 * i - 2 * b * x * \exp(d * x^2 * i) * \exp(c * i))) / (2 * d) - (b * \log(b * x^2 * i - 2 * b * x * \exp(d * x^2 * i) * \exp(c * i))) / (2 * d)$

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int x(a + b \sec(c + dx^2)) dx = \frac{-\log\left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right) - 1\right) b + \log\left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right) + 1\right) b + adx^2}{2d}$$

input `int(x*(a+b*sec(d*x^2+c)),x)`

output $(-\log(\tan((c + d * x^2) / 2) - 1) * b + \log(\tan((c + d * x^2) / 2) + 1) * b + a * d * x^2) / (2 * d)$

3.6 $\int \frac{a+b \sec(c+dx^2)}{x} dx$

Optimal result	84
Mathematica [N/A]	84
Rubi [N/A]	85
Maple [N/A]	86
Fricas [N/A]	86
Sympy [N/A]	86
Maxima [N/A]	87
Giac [N/A]	87
Mupad [N/A]	87
Reduce [N/A]	88

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a + b \sec(c + dx^2)}{x} dx = a \log(x) + b \operatorname{Int}\left(\frac{\sec(c + dx^2)}{x}, x\right)$$

output `a*ln(x)+b*Defer(Int)(sec(d*x^2+c)/x,x)`

Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \sec(c + dx^2)}{x} dx = \int \frac{a + b \sec(c + dx^2)}{x} dx$$

input `Integrate[(a + b*Sec[c + d*x^2])/x,x]`

output `Integrate[(a + b*Sec[c + d*x^2])/x, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec(c + dx^2)}{x} dx$$

↓ 2010

$$\int \left(\frac{a}{x} + \frac{b \sec(c + dx^2)}{x} \right) dx$$

↓ 2009

$$b \int \frac{\sec(dx^2 + c)}{x} dx + a \log(x)$$

input `Int[(a + b*Sec[c + d*x^2])/x,x]`

output `$Aborted`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(dx^2 + c)}{x} dx$$

input `int((a+b*sec(d*x^2+c))/x,x)`output `int((a+b*sec(d*x^2+c))/x,x)`**Fricas [N/A]**

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \sec(c + dx^2)}{x} dx = \int \frac{b \sec(dx^2 + c) + a}{x} dx$$

input `integrate((a+b*sec(d*x^2+c))/x,x, algorithm="fricas")`output `integral((b*sec(d*x^2 + c) + a)/x, x)`**Sympy [N/A]**

Not integrable

Time = 0.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a + b \sec(c + dx^2)}{x} dx = \int \frac{a + b \sec(c + dx^2)}{x} dx$$

input `integrate((a+b*sec(d*x**2+c))/x,x)`output `Integral((a + b*sec(c + d*x**2))/x, x)`

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 6.75

$$\int \frac{a + b \sec(c + dx^2)}{x} dx = \int \frac{b \sec(dx^2 + c) + a}{x} dx$$

input `integrate((a+b*sec(d*x^2+c))/x,x, algorithm="maxima")`

output `2*b*integrate((cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + cos(d*x^2 + c))/(x*cos(2*d*x^2 + 2*c)^2 + x*sin(2*d*x^2 + 2*c)^2 + 2*x*cos(2*d*x^2 + 2*c) + x), x) + a*log(x)`

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \sec(c + dx^2)}{x} dx = \int \frac{b \sec(dx^2 + c) + a}{x} dx$$

input `integrate((a+b*sec(d*x^2+c))/x,x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)/x, x)`

Mupad [N/A]

Not integrable

Time = 15.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b \sec(c + dx^2)}{x} dx = \int \frac{a + \frac{b}{\cos(dx^2+c)}}{x} dx$$

input `int((a + b/cos(c + d*x^2))/x,x)`

output `int((a + b/cos(c + d*x^2))/x, x)`

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{a + b \sec(c + dx^2)}{x} dx = -2 \left(\int \frac{1}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 x - x} dx \right) b + \log(x) a - \log(x) b$$

input `int((a+b*sec(d*x^2+c))/x,x)`

output `- 2*int(1/(tan((c + d*x**2)/2)**2*x - x),x)*b + log(x)*a - log(x)*b`

3.7 $\int \frac{a+b \sec(c+dx^2)}{x^2} dx$

Optimal result	89
Mathematica [N/A]	89
Rubi [N/A]	90
Maple [N/A]	91
Fricas [N/A]	91
Sympy [N/A]	91
Maxima [N/A]	92
Giac [N/A]	92
Mupad [N/A]	92
Reduce [N/A]	93

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = -\frac{a}{x} + b \operatorname{Int}\left(\frac{\sec(c + dx^2)}{x^2}, x\right)$$

output `-a/x+b*Defer(Int)(sec(d*x^2+c)/x^2,x)`

Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

input `Integrate[(a + b*Sec[c + d*x^2])/x^2,x]`

output `Integrate[(a + b*Sec[c + d*x^2])/x^2, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

↓ 2010

$$\int \left(\frac{a}{x^2} + \frac{b \sec(c + dx^2)}{x^2} \right) dx$$

↓ 2009

$$b \int \frac{\sec(dx^2 + c)}{x^2} dx - \frac{a}{x}$$

input `Int[(a + b*Sec[c + d*x^2])/x^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(dx^2 + c)}{x^2} dx$$

input `int((a+b*sec(d*x^2+c))/x^2,x)`output `int((a+b*sec(d*x^2+c))/x^2,x)`**Fricas [N/A]**

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{b \sec(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="fricas")`output `integral((b*sec(d*x^2 + c) + a)/x^2, x)`**Sympy [N/A]**

Not integrable

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

input `integrate((a+b*sec(d*x**2+c))/x**2,x)`output `Integral((a + b*sec(c + d*x**2))/x**2, x)`

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 7.38

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{b \sec(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="maxima")`

output `2*b*integrate((cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + cos(d*x^2 + c))/(x^2*cos(2*d*x^2 + 2*c)^2 + x^2*sin(2*d*x^2 + 2*c)^2 + 2*x^2*cos(2*d*x^2 + 2*c) + x^2), x) - a/x`

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{b \sec(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)/x^2, x)`

Mupad [N/A]

Not integrable

Time = 15.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{a + \frac{b}{\cos(dx^2+c)}}{x^2} dx$$

input `int((a + b/cos(c + d*x^2))/x^2,x)`

output `int((a + b/cos(c + d*x^2))/x^2, x)`

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.50

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \frac{-2 \left(\int \frac{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 x^2 - x^2} dx \right) bx - a - b}{x}$$

input `int((a+b*sec(d*x^2+c))/x^2,x)`

output `(- 2*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**2*x**2 - x**2),x)*b
*x - a - b)/x`

3.8 $\int x^5(a + b \sec(c + dx^2))^2 dx$

Optimal result	94
Mathematica [A] (verified)	95
Rubi [A] (verified)	95
Maple [F]	97
Fricas [B] (verification not implemented)	97
Sympy [F]	98
Maxima [F]	99
Giac [F]	99
Mupad [F(-1)]	99
Reduce [F]	100

Optimal result

Integrand size = 18, antiderivative size = 242

$$\begin{aligned}
 \int x^5(a + b \sec(c + dx^2))^2 dx = & -\frac{ib^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2iabx^4 \arctan(e^{i(c+dx^2)})}{d} \\
 & + \frac{b^2x^2 \log(1 + e^{2i(c+dx^2)})}{d^2} \\
 & + \frac{2iabx^2 \operatorname{PolyLog}(2, -ie^{i(c+dx^2)})}{d^2} \\
 & - \frac{2iabx^2 \operatorname{PolyLog}(2, ie^{i(c+dx^2)})}{d^2} \\
 & - \frac{ib^2 \operatorname{PolyLog}(2, -e^{2i(c+dx^2)})}{2d^3} \\
 & - \frac{2ab \operatorname{PolyLog}(3, -ie^{i(c+dx^2)})}{d^3} \\
 & + \frac{2ab \operatorname{PolyLog}(3, ie^{i(c+dx^2)})}{d^3} + \frac{b^2x^4 \tan(c + dx^2)}{2d}
 \end{aligned}$$

output

$$\begin{aligned} & -1/2*I*b^2*x^4/d+1/6*a^2*x^6-2*I*a*b*x^4*\arctan(\exp(I*(d*x^2+c)))/d+b^2*x^2* \\ & \ln(1+\exp(2*I*(d*x^2+c)))/d^2+2*I*a*b*x^2*\text{polylog}(2,-I*\exp(I*(d*x^2+c)))/ \\ & d^2-2*I*a*b*x^2*\text{polylog}(2,I*\exp(I*(d*x^2+c)))/d^2-1/2*I*b^2*\text{polylog}(2,-\exp \\ & (2*I*(d*x^2+c)))/d^3-2*a*b*\text{polylog}(3,-I*\exp(I*(d*x^2+c)))/d^3+2*a*b*\text{polylo} \\ & \text{g}(3,I*\exp(I*(d*x^2+c)))/d^3+1/2*b^2*x^4*\tan(d*x^2+c)/d \end{aligned}$$
Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int x^5 (a + b \sec(c + dx^2))^2 dx$$

$$= \frac{-3ib^2 d^2 x^4 + a^2 d^3 x^6 - 12iabd^2 x^4 \arctan(e^{i(c+dx^2)}) + 6b^2 dx^2 \log(1 + e^{2i(c+dx^2)}) + 12iabd x^2 \text{PolyLog}(2, -e^{i(c+dx^2)})}{d^3}$$

input

Integrate[x^5*(a + b*Sec[c + d*x^2])^2,x]

output

$$\begin{aligned} & ((-3*I)*b^2*d^2*x^4 + a^2*d^3*x^6 - (12*I)*a*b*d^2*x^4*\text{ArcTan}[E^{I*(c + d*} \\ & x^2)]) + 6*b^2*d*x^2*\text{Log}[1 + E^{((2*I)*(c + d*x^2))}] + (12*I)*a*b*d*x^2*\text{Pol} \\ & \text{yLog}[2, (-I)*E^{I*(c + d*x^2)}] - (12*I)*a*b*d*x^2*\text{PolyLog}[2, I*E^{I*(c +} \\ & d*x^2))] - (3*I)*b^2*\text{PolyLog}[2, -E^{((2*I)*(c + d*x^2))}] - 12*a*b*\text{PolyLog}[3 \\ & , (-I)*E^{I*(c + d*x^2)}] + 12*a*b*\text{PolyLog}[3, I*E^{I*(c + d*x^2)}] + 3*b^2 \\ & *d^2*x^4*\text{Tan}[c + d*x^2)]/(6*d^3) \end{aligned}$$
Rubi [A] (verified)Time = 0.61 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4692, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a + b \sec(c + dx^2))^2 dx$$

$$\begin{array}{c}
\downarrow 4692 \\
\frac{1}{2} \int x^4 (a + b \sec(dx^2 + c))^2 dx^2 \\
\downarrow 3042 \\
\frac{1}{2} \int x^4 \left(a + b \csc\left(dx^2 + c + \frac{\pi}{2}\right) \right)^2 dx^2 \\
\downarrow 4678 \\
\frac{1}{2} \int (a^2 x^4 + b^2 \sec^2(dx^2 + c) x^4 + 2ab \sec(dx^2 + c) x^4) dx^2 \\
\downarrow 2009 \\
\frac{1}{2} \left(\frac{a^2 x^6}{3} - \frac{4i ab x^4 \arctan\left(e^{i(c+dx^2)}\right)}{d} - \frac{4ab \operatorname{PolyLog}\left(3, -ie^{i(dx^2+c)}\right)}{d^3} + \frac{4ab \operatorname{PolyLog}\left(3, ie^{i(dx^2+c)}\right)}{d^3} + \frac{4i ab x^2 \operatorname{PolyLog}\left(3, -ie^{i(dx^2+c)}\right)}{d^3} + \frac{4i ab x^2 \operatorname{PolyLog}\left(3, ie^{i(dx^2+c)}\right)}{d^3} \right)
\end{array}$$

input `Int[x^5*(a + b*Sec[c + d*x^2])^2,x]`

output `(((-I)*b^2*x^4)/d + (a^2*x^6)/3 - ((4*I)*a*b*x^4*ArcTan[E^(I*(c + d*x^2))])/d + (2*b^2*x^2*Log[1 + E^((2*I)*(c + d*x^2))])/d^2 + ((4*I)*a*b*x^2*PolyLog[2, (-I)*E^(I*(c + d*x^2))])/d^2 - ((4*I)*a*b*x^2*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2 - (I*b^2*PolyLog[2, -E^((2*I)*(c + d*x^2))])/d^3 - (4*a*b*PolyLog[3, (-I)*E^(I*(c + d*x^2))])/d^3 + (4*a*b*PolyLog[3, I*E^(I*(c + d*x^2))])/d^3 + (b^2*x^4*Tan[c + d*x^2])/d)/2`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int x^5 (a + b \sec(dx^2 + c))^2 dx$$

input `int(x^5*(a+b*sec(d*x^2+c))^2,x)`

output `int(x^5*(a+b*sec(d*x^2+c))^2,x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(199) = 398$.

Time = 0.12 (sec) , antiderivative size = 799, normalized size of antiderivative = 3.30

$$\int x^5 (a + b \sec(c + dx^2))^2 dx = \text{Too large to display}$$

input `integrate(x^5*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output

```

1/6*(a^2*d^3*x^6*cos(d*x^2 + c) + 3*b^2*d^2*x^4*sin(d*x^2 + c) - 6*a*b*cos
(d*x^2 + c)*polylog(3, I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*a*b*cos(d*x^
2 + c)*polylog(3, I*cos(d*x^2 + c) - sin(d*x^2 + c)) - 6*a*b*cos(d*x^2 + c
)*polylog(3, -I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 6*a*b*cos(d*x^2 + c)*po
lylog(3, -I*cos(d*x^2 + c) - sin(d*x^2 + c)) - 3*(2*I*a*b*d*x^2 - I*b^2)*c
os(d*x^2 + c)*dilog(I*cos(d*x^2 + c) + sin(d*x^2 + c)) - 3*(2*I*a*b*d*x^2
+ I*b^2)*cos(d*x^2 + c)*dilog(I*cos(d*x^2 + c) - sin(d*x^2 + c)) - 3*(-2*I
*a*b*d*x^2 + I*b^2)*cos(d*x^2 + c)*dilog(-I*cos(d*x^2 + c) + sin(d*x^2 + c
)) - 3*(-2*I*a*b*d*x^2 - I*b^2)*cos(d*x^2 + c)*dilog(-I*cos(d*x^2 + c) - s
in(d*x^2 + c)) + 3*(a*b*c^2 - b^2*c)*cos(d*x^2 + c)*log(cos(d*x^2 + c) + I
*sin(d*x^2 + c) + I) - 3*(a*b*c^2 + b^2*c)*cos(d*x^2 + c)*log(cos(d*x^2 +
c) - I*sin(d*x^2 + c) + I) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)
*cos(d*x^2 + c)*log(I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 3*(a*b*d^2*x^
4 - b^2*d*x^2 - a*b*c^2 - b^2*c)*cos(d*x^2 + c)*log(I*cos(d*x^2 + c) - sin
(d*x^2 + c) + 1) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*cos(d*x^2
+ c)*log(-I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 3*(a*b*d^2*x^4 - b^2*d
*x^2 - a*b*c^2 - b^2*c)*cos(d*x^2 + c)*log(-I*cos(d*x^2 + c) - sin(d*x^2 +
c) + 1) + 3*(a*b*c^2 - b^2*c)*cos(d*x^2 + c)*log(-cos(d*x^2 + c) + I*sin(
d*x^2 + c) + I) - 3*(a*b*c^2 + b^2*c)*cos(d*x^2 + c)*log(-cos(d*x^2 + c) -
I*sin(d*x^2 + c) + I))/(d^3*cos(d*x^2 + c))

```

Sympy [F]

$$\int x^5 (a + b \sec(c + dx^2))^2 dx = \int x^5 (a + b \sec(c + dx^2))^2 dx$$

input

```
integrate(x**5*(a+b*sec(d*x**2+c))**2,x)
```

output

```
Integral(x**5*(a + b*sec(c + d*x**2))**2, x)
```

Maxima [F]

$$\int x^5 (a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^5 dx$$

input `integrate(x^5*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output `1/6*a^2*x^6 + (b^2*x^4*sin(2*d*x^2 + 2*c) + (d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)*integrate(4*(a*b*d*x^5*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + a*b*d*x^5*cos(d*x^2 + c) + (a*b*d*x^5*sin(d*x^2 + c) - b^2*x^3)*sin(2*d*x^2 + 2*c))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d), x))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)`

Giac [F]

$$\int x^5 (a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^5 dx$$

input `integrate(x^5*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)^2*x^5, x)`

Mupad [F(-1)]

Timed out.

$$\int x^5 (a + b \sec(c + dx^2))^2 dx = \int x^5 \left(a + \frac{b}{\cos(dx^2 + c)} \right)^2 dx$$

input `int(x^5*(a + b/cos(c + d*x^2))^2,x)`

output `int(x^5*(a + b/cos(c + d*x^2))^2, x)`

Reduce [F]

$$\int x^5 (a + b \sec(c + dx^2))^2 dx$$

$$= -24 \cos(dx^2 + c) \left(\int \frac{\tan\left(\frac{dx^2 + c}{2}\right)^2 x^5}{\tan\left(\frac{dx^2 + c}{2}\right)^2 - 1} dx \right) ab d^3 - 48 \cos(dx^2 + c) \left(\int \frac{\tan\left(\frac{dx^2 + c}{2}\right) x^3}{\tan\left(\frac{dx^2 + c}{2}\right)^4 - 2 \tan\left(\frac{dx^2 + c}{2}\right)^2 + 1} dx \right) b^2 c$$

input `int(x^5*(a+b*sec(d*x^2+c))^2,x)`

output `(- 24*cos(c + d*x**2)*int((tan((c + d*x**2)/2)**2*x**5)/(tan((c + d*x**2)/2)**2 - 1),x)*a*b*d**3 - 48*cos(c + d*x**2)*int((tan((c + d*x**2)/2)*x**3)/(tan((c + d*x**2)/2)**4 - 2*tan((c + d*x**2)/2)**2 + 1),x)*b**2*d**2 + 6*cos(c + d*x**2)*log(tan((c + d*x**2)/2) - 1)*b**2 - 6*cos(c + d*x**2)*log(tan((c + d*x**2)/2) + 1)*b**2 + cos(c + d*x**2)*a**2*d**3*x**6 + 2*cos(c + d*x**2)*a*b*d**3*x**6 + 3*sin(c + d*x**2)*b**2*d**2*x**4 + 6*b**2*d*x**2)/(6*cos(c + d*x**2)*d**3)`

3.9 $\int x^4(a + b \sec(c + dx^2))^2 dx$

Optimal result	101
Mathematica [N/A]	101
Rubi [N/A]	102
Maple [N/A]	102
Fricas [N/A]	103
Sympy [N/A]	103
Maxima [N/A]	104
Giac [N/A]	104
Mupad [N/A]	105
Reduce [N/A]	105

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^4(a + b \sec(c + dx^2))^2 dx = \text{Int}\left(x^4(a + b \sec(c + dx^2))^2, x\right)$$

output `Defer(Int)(x^4*(a+b*sec(d*x^2+c))^2,x)`

Mathematica [N/A]

Not integrable

Time = 9.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4(a + b \sec(c + dx^2))^2 dx = \int x^4(a + b \sec(c + dx^2))^2 dx$$

input `Integrate[x^4*(a + b*Sec[c + d*x^2])^2,x]`

output `Integrate[x^4*(a + b*Sec[c + d*x^2])^2, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a + b \sec(c + dx^2))^2 dx$$

↓ 4694

$$\int x^4 (a + b \sec(c + dx^2))^2 dx$$

input `Int[x^4*(a + b*Sec[c + d*x^2])^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
-> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4 (a + b \sec(dx^2 + c))^2 dx$$

input `int(x^4*(a+b*sec(d*x^2+c))^2,x)`

output `int(x^4*(a+b*sec(d*x^2+c))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^4(a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^4 dx$$

input `integrate(x^4*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `integral(b^2*x^4*sec(d*x^2 + c)^2 + 2*a*b*x^4*sec(d*x^2 + c) + a^2*x^4, x)`

Sympy [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^4(a + b \sec(c + dx^2))^2 dx = \int x^4(a + b \sec(c + dx^2))^2 dx$$

input `integrate(x**4*(a+b*sec(d*x**2+c))**2,x)`

output `Integral(x**4*(a + b*sec(c + d*x**2))**2, x)`

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 14.22

$$\int x^4 (a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^4 dx$$

input `integrate(x^4*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output `1/5*a^2*x^5 + (b^2*x^3*sin(2*d*x^2 + 2*c) + (d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)*integrate((4*a*b*d*x^4*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 4*a*b*d*x^4*cos(d*x^2 + c) + (4*a*b*d*x^4*sin(d*x^2 + c) - 3*b^2*x^2)*sin(2*d*x^2 + 2*c))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d), x))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)`

Giac [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4 (a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^4 dx$$

input `integrate(x^4*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)^2*x^4, x)`

Mupad [N/A]

Not integrable

Time = 15.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^4 (a + b \sec(c + dx^2))^2 dx = \int x^4 \left(a + \frac{b}{\cos(dx^2 + c)} \right)^2 dx$$

input `int(x^4*(a + b/cos(c + d*x^2))^2,x)`output `int(x^4*(a + b/cos(c + d*x^2))^2, x)`**Reduce [N/A]**

Not integrable

Time = 0.17 (sec) , antiderivative size = 549, normalized size of antiderivative = 30.50

$$\int x^4 (a + b \sec(c + dx^2))^2 dx = \text{Too large to display}$$

input `int(x^4*(a+b*sec(d*x^2+c))^2,x)`

output

```
(15*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4 - 2*tan((c + d*x**2)/2)**2 + 1),x)*tan((c + d*x**2)/2)**2*b**2 - 15*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4 - 2*tan((c + d*x**2)/2)**2 + 1),x)*b**2 - 40*int((tan((c + d*x**2)/2)**2*x**4)/(tan((c + d*x**2)/2)**2 - 1),x)*tan((c + d*x**2)/2)**2*a*b*d**2 + 40*int((tan((c + d*x**2)/2)**2*x**4)/(tan((c + d*x**2)/2)**2 - 1),x)*a*b*d**2 - 60*int((tan((c + d*x**2)/2)*x**2)/(tan((c + d*x**2)/2)**4 - 2*tan((c + d*x**2)/2)**2 + 1),x)*tan((c + d*x**2)/2)**2*b**2*d + 60*int((tan((c + d*x**2)/2)*x**2)/(tan((c + d*x**2)/2)**4 - 2*tan((c + d*x**2)/2)**2 + 1),x)*b**2*d - 15*int(1/(tan((c + d*x**2)/2)**4 - 2*tan((c + d*x**2)/2)**2 + 1),x)*tan((c + d*x**2)/2)**2*b**2 + 15*int(1/(tan((c + d*x**2)/2)**4 - 2*tan((c + d*x**2)/2)**2 + 1),x)*b**2 + 2*tan((c + d*x**2)/2)**2*a**2*d**2*x**5 + 4*tan((c + d*x**2)/2)**2*a*b*d**2*x**5 - 10*tan((c + d*x**2)/2)*b**2*d*x**3 - 2*a**2*d**2*x**5 - 4*a*b*d**2*x**5 - 15*b**2*x)/(10*d**2*(tan((c + d*x**2)/2)**2 - 1))
```

3.10 $\int x^3(a + b \sec(c + dx^2))^2 dx$

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Reduce [F]	111

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int x^3(a + b \sec(c + dx^2))^2 dx = \frac{a^2x^4}{4} - \frac{2iabx^2 \arctan(e^{i(c+dx^2)})}{d} + \frac{b^2 \log(\cos(c + dx^2))}{2d^2} + \frac{iab \operatorname{PolyLog}(2, -ie^{i(c+dx^2)})}{d^2} - \frac{iab \operatorname{PolyLog}(2, ie^{i(c+dx^2)})}{d^2} + \frac{b^2x^2 \tan(c + dx^2)}{2d}$$

output

```
1/4*a^2*x^4-2*I*a*b*x^2*arctan(exp(I*(d*x^2+c)))/d+1/2*b^2*ln(cos(d*x^2+c))
/d^2+I*a*b*polylog(2,-I*exp(I*(d*x^2+c)))/d^2-I*a*b*polylog(2,I*exp(I*(d*
x^2+c)))/d^2+1/2*b^2*x^2*tan(d*x^2+c)/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int x^3 (a + b \sec(c + dx^2))^2 dx$$

$$= \frac{a^2 d^2 x^4 - 8iabdx^2 \arctan\left(e^{i(c+dx^2)}\right) + 2b^2 \log(\cos(c + dx^2)) + 4iab \operatorname{PolyLog}\left(2, -ie^{i(c+dx^2)}\right) - 4iab \operatorname{PolyLog}\left(2, ie^{i(c+dx^2)}\right)}{4d^2}$$

input

```
Integrate[x^3*(a + b*Sec[c + d*x^2])^2,x]
```

output

```
(a^2*d^2*x^4 - (8*I)*a*b*d*x^2*ArcTan[E^(I*(c + d*x^2))] + 2*b^2*Log[Cos[c + d*x^2]] + (4*I)*a*b*PolyLog[2, (-I)*E^(I*(c + d*x^2))] - (4*I)*a*b*PolyLog[2, I*E^(I*(c + d*x^2))] + 2*b^2*d*x^2*Tan[c + d*x^2])/(4*d^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4692, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + b \sec(c + dx^2))^2 dx$$

$$\downarrow 4692$$

$$\frac{1}{2} \int x^2 (a + b \sec(dx^2 + c))^2 dx^2$$

$$\downarrow 3042$$

$$\frac{1}{2} \int x^2 \left(a + b \csc\left(dx^2 + c + \frac{\pi}{2}\right)\right)^2 dx^2$$

$$\downarrow 4678$$

$$\frac{1}{2} \int (a^2 x^2 + b^2 \sec^2(dx^2 + c) x^2 + 2ab \sec(dx^2 + c) x^2) dx^2$$

↓ 2009

$$\frac{1}{2} \left(\frac{a^2 x^4}{2} - \frac{4iabx^2 \arctan(e^{i(c+dx^2)})}{d} + \frac{2iab \operatorname{PolyLog}(2, -ie^{i(dx^2+c)})}{d^2} - \frac{2iab \operatorname{PolyLog}(2, ie^{i(dx^2+c)})}{d^2} + \frac{b^2 \log(\dots)}{d^2} \right)$$

input `Int[x^3*(a + b*Sec[c + d*x^2])^2,x]`

output `((a^2*x^4)/2 - ((4*I)*a*b*x^2*ArcTan[E^(I*(c + d*x^2))])/d + (b^2*Log[Cos[c + d*x^2]]/d^2 + ((2*I)*a*b*PolyLog[2, (-I)*E^(I*(c + d*x^2))])/d^2 - ((2*I)*a*b*PolyLog[2, I*E^(I*(c + d*x^2))])/d^2 + (b^2*x^2*Tan[c + d*x^2])/d)/2`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int x^3 (a + b \sec(dx^2 + c))^2 dx$$

input `int(x^3*(a+b*sec(d*x^2+c))^2,x)`

output `int(x^3*(a+b*sec(d*x^2+c))^2,x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(109) = 218$.

Time = 0.11 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.95

$$\int x^3 (a + b \sec(c + dx^2))^2 dx$$

$$= \frac{a^2 d^2 x^4 \cos(dx^2 + c) + 2 b^2 dx^2 \sin(dx^2 + c) - 2i ab \cos(dx^2 + c) \operatorname{Li}_2(i \cos(dx^2 + c) + \sin(dx^2 + c)) - 2i ab \cos(dx^2 + c) \operatorname{Li}_2(i \cos(dx^2 + c) - \sin(dx^2 + c))}{d^2 \cos(dx^2 + c)}$$

input `integrate(x^3*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `1/4*(a^2*d^2*x^4*cos(d*x^2 + c) + 2*b^2*d*x^2*sin(d*x^2 + c) - 2*I*a*b*cos(d*x^2 + c)*dilog(I*cos(d*x^2 + c) + sin(d*x^2 + c)) - 2*I*a*b*cos(d*x^2 + c)*dilog(I*cos(d*x^2 + c) - sin(d*x^2 + c)) + 2*I*a*b*cos(d*x^2 + c)*dilog(-I*cos(d*x^2 + c) + sin(d*x^2 + c)) + 2*I*a*b*cos(d*x^2 + c)*dilog(-I*cos(d*x^2 + c) - sin(d*x^2 + c)) - (2*a*b*c - b^2)*cos(d*x^2 + c)*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + I) + (2*a*b*c + b^2)*cos(d*x^2 + c)*log(cos(d*x^2 + c) - I*sin(d*x^2 + c) + I) + 2*(a*b*d*x^2 + a*b*c)*cos(d*x^2 + c)*log(I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 2*(a*b*d*x^2 + a*b*c)*cos(d*x^2 + c)*log(I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) + 2*(a*b*d*x^2 + a*b*c)*cos(d*x^2 + c)*log(-I*cos(d*x^2 + c) + sin(d*x^2 + c) + 1) - 2*(a*b*d*x^2 + a*b*c)*cos(d*x^2 + c)*log(-I*cos(d*x^2 + c) - sin(d*x^2 + c) + 1) - (2*a*b*c - b^2)*cos(d*x^2 + c)*log(-cos(d*x^2 + c) + I*sin(d*x^2 + c) + I) + (2*a*b*c + b^2)*cos(d*x^2 + c)*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) + I)/(d^2*cos(d*x^2 + c))`

Sympy [F]

$$\int x^3(a + b \sec(c + dx^2))^2 dx = \int x^3(a + b \sec(c + dx^2))^2 dx$$

input `integrate(x**3*(a+b*sec(d*x**2+c))**2,x)`

output `Integral(x**3*(a + b*sec(c + d*x**2))**2, x)`

Maxima [F]

$$\int x^3(a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^3 dx$$

input `integrate(x^3*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output `1/4*a^2*x^4 + 1/4*(4*b^2*d*x^2*sin(2*d*x^2 + 2*c) + 16*(a*b*d^3*cos(2*d*x^2 + 2*c)^2 + a*b*d^3*sin(2*d*x^2 + 2*c)^2 + 2*a*b*d^3*cos(2*d*x^2 + 2*c) + a*b*d^3)*integrate((x^3*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + x^3*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + x^3*cos(d*x^2 + c))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d), x) + (b^2*cos(2*d*x^2 + 2*c)^2 + b^2*sin(2*d*x^2 + 2*c)^2 + 2*b^2*cos(2*d*x^2 + 2*c) + b^2)*log(cos(2*d*x^2 + 2*c)^2 + sin(2*d*x^2 + 2*c)^2 + 2*cos(2*d*x^2 + 2*c) + 1)/(d^2*cos(2*d*x^2 + 2*c)^2 + d^2*sin(2*d*x^2 + 2*c)^2 + 2*d^2*cos(2*d*x^2 + 2*c) + d^2)`

Giac [F]

$$\int x^3 (a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^3 dx$$

input `integrate(x^3*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)^2*x^3, x)`

Mupad [F(-1)]

Timed out.

$$\int x^3 (a + b \sec(c + dx^2))^2 dx = \int x^3 \left(a + \frac{b}{\cos(dx^2 + c)} \right)^2 dx$$

input `int(x^3*(a + b/cos(c + d*x^2))^2,x)`

output `int(x^3*(a + b/cos(c + d*x^2))^2, x)`

Reduce [F]

$$\int x^3 (a + b \sec(c + dx^2))^2 dx$$

$$= \frac{-16 \cos(dx^2 + c) \left(\int \frac{\tan\left(\frac{dx^2 + c}{2}\right)^2 x^3}{\tan\left(\frac{dx^2 + c}{2}\right)^2 - 1} dx \right) ab d^2 - 2 \cos(dx^2 + c) \log\left(\tan\left(\frac{dx^2 + c}{2}\right)^2 + 1\right) b^2 + 2 \cos(dx^2 + c) a^2}{1}$$

input `int(x^3*(a+b*sec(d*x^2+c))^2,x)`

output

```
( - 16*cos(c + d*x**2)*int((tan((c + d*x**2)/2)**2*x**3)/(tan((c + d*x**2)/2)**2 - 1),x)*a*b*d**2 - 2*cos(c + d*x**2)*log(tan((c + d*x**2)/2)**2 + 1)*b**2 + 2*cos(c + d*x**2)*log(tan((c + d*x**2)/2) - 1)*b**2 + 2*cos(c + d*x**2)*log(tan((c + d*x**2)/2) + 1)*b**2 + cos(c + d*x**2)*a**2*d**2*x**4 + 2*cos(c + d*x**2)*a*b*d**2*x**4 + 2*sin(c + d*x**2)*b**2*d*x**2)/(4*cos(c + d*x**2)*d**2)
```

3.11 $\int x^2(a + b \sec(c + dx^2))^2 dx$

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Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^2(a + b \sec(c + dx^2))^2 dx = \text{Int}\left(x^2(a + b \sec(c + dx^2))^2, x\right)$$

output `Defer(Int)(x^2*(a+b*sec(d*x^2+c))^2,x)`

Mathematica [N/A]

Not integrable

Time = 6.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2(a + b \sec(c + dx^2))^2 dx = \int x^2(a + b \sec(c + dx^2))^2 dx$$

input `Integrate[x^2*(a + b*Sec[c + d*x^2])^2,x]`

output `Integrate[x^2*(a + b*Sec[c + d*x^2])^2, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + b \sec(c + dx^2))^2 dx$$

↓ 4694

$$\int x^2 (a + b \sec(c + dx^2))^2 dx$$

input `Int[x^2*(a + b*Sec[c + d*x^2])^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2 (a + b \sec(dx^2 + c))^2 dx$$

input `int(x^2*(a+b*sec(d*x^2+c))^2,x)`

output `int(x^2*(a+b*sec(d*x^2+c))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^2(a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `integral(b^2*x^2*sec(d*x^2 + c)^2 + 2*a*b*x^2*sec(d*x^2 + c) + a^2*x^2, x)`

Sympy [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^2(a + b \sec(c + dx^2))^2 dx = \int x^2(a + b \sec(c + dx^2))^2 dx$$

input `integrate(x**2*(a+b*sec(d*x**2+c))**2,x)`

output `Integral(x**2*(a + b*sec(c + d*x**2))**2, x)`

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 13.94

$$\int x^2(a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output `1/3*a^2*x^3 + (b^2*x*sin(2*d*x^2 + 2*c) + (d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)*integrate((4*a*b*d*x^2*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 4*a*b*d*x^2*cos(d*x^2 + c) + (4*a*b*d*x^2*sin(d*x^2 + c) - b^2)*sin(2*d*x^2 + 2*c))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d), x))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2 + 2*c) + d)`

Giac [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2(a + b \sec(c + dx^2))^2 dx = \int (b \sec(dx^2 + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)^2*x^2, x)`

Mupad [N/A]

Not integrable

Time = 15.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2 (a + b \sec(c + dx^2))^2 dx = \int x^2 \left(a + \frac{b}{\cos(dx^2 + c)} \right)^2 dx$$

input `int(x^2*(a + b/cos(c + d*x^2))^2,x)`output `int(x^2*(a + b/cos(c + d*x^2))^2, x)`**Reduce [N/A]**

Not integrable

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 8.11

$$\int x^2 (a + b \sec(c + dx^2))^2 dx$$

$$= \frac{-3 \cos(dx^2 + c) \left(\int \frac{\sin(dx^2+c)}{\cos(dx^2+c)} dx \right) b^2 - 24 \cos(dx^2 + c) \left(\int \frac{\tan\left(\frac{dx^2+c}{2}\right)^2 x^2}{\tan\left(\frac{dx^2+c}{2}\right)^2 - 1} dx \right) abd + 2 \cos(dx^2 + c) a^2 d}{6 \cos(dx^2 + c) d}$$

input `int(x^2*(a+b*sec(d*x^2+c))^2,x)`output `(- 3*cos(c + d*x**2)*int(sin(c + d*x**2)/cos(c + d*x**2),x)*b**2 - 24*cos(c + d*x**2)*int((tan((c + d*x**2)/2)**2*x**2)/(tan((c + d*x**2)/2)**2 - 1),x)*a*b*d + 2*cos(c + d*x**2)*a**2*d*x**3 + 4*cos(c + d*x**2)*a*b*d*x**3 + 3*sin(c + d*x**2)*b**2*x)/(6*cos(c + d*x**2)*d)`

3.12 $\int x(a + b \sec(c + dx^2))^2 dx$

Optimal result	118
Mathematica [A] (verified)	118
Rubi [A] (verified)	119
Maple [A] (verified)	121
Fricas [B] (verification not implemented)	121
Sympy [F]	122
Maxima [B] (verification not implemented)	122
Giac [B] (verification not implemented)	123
Mupad [B] (verification not implemented)	123
Reduce [B] (verification not implemented)	124

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x(a + b \sec(c + dx^2))^2 dx = \frac{a^2 x^2}{2} + \frac{ab \operatorname{arctanh}(\sin(c + dx^2))}{d} + \frac{b^2 \tan(c + dx^2)}{2d}$$

output

```
1/2*a^2*x^2+a*b*arctanh(sin(d*x^2+c))/d+1/2*b^2*tan(d*x^2+c)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int x(a + b \sec(c + dx^2))^2 dx = \frac{a^2 dx^2 + 2ab \operatorname{coth}^{-1}(\sin(c + dx^2)) + b^2 \tan(c + dx^2)}{2d}$$

input

```
Integrate[x*(a + b*Sec[c + d*x^2])^2,x]
```

output

```
(a^2*d*x^2 + 2*a*b*ArcCoth[Sin[c + d*x^2]] + b^2*Tan[c + d*x^2])/(2*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4692, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x(a + b \sec(c + dx^2))^2 dx \\
 & \quad \downarrow 4692 \\
 & \frac{1}{2} \int (a + b \sec(dx^2 + c))^2 dx^2 \\
 & \quad \downarrow 3042 \\
 & \frac{1}{2} \int \left(a + b \csc\left(dx^2 + c + \frac{\pi}{2}\right)\right)^2 dx^2 \\
 & \quad \downarrow 4260 \\
 & \frac{1}{2} \left(2ab \int \sec(dx^2 + c) dx^2 + b^2 \int \sec^2(dx^2 + c) dx^2 + a^2 x^2\right) \\
 & \quad \downarrow 3042 \\
 & \frac{1}{2} \left(2ab \int \csc\left(dx^2 + c + \frac{\pi}{2}\right) dx^2 + b^2 \int \csc\left(dx^2 + c + \frac{\pi}{2}\right)^2 dx^2 + a^2 x^2\right) \\
 & \quad \downarrow 4254 \\
 & \frac{1}{2} \left(2ab \int \csc\left(dx^2 + c + \frac{\pi}{2}\right) dx^2 - \frac{b^2 \int 1d(-\tan(dx^2 + c))}{d} + a^2 x^2\right) \\
 & \quad \downarrow 24 \\
 & \frac{1}{2} \left(2ab \int \csc\left(dx^2 + c + \frac{\pi}{2}\right) dx^2 + a^2 x^2 + \frac{b^2 \tan(c + dx^2)}{d}\right) \\
 & \quad \downarrow 4257 \\
 & \frac{1}{2} \left(a^2 x^2 + \frac{2ab \operatorname{arctanh}(\sin(c + dx^2))}{d} + \frac{b^2 \tan(c + dx^2)}{d}\right)
 \end{aligned}$$

input `Int[x*(a + b*Sec[c + d*x^2])^2,x]`

output `(a^2*x^2 + (2*a*b*ArcTanh[Sin[c + d*x^2]])/d + (b^2*Tan[c + d*x^2])/d)/2`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4254 `Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

rule 4257 `Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

rule 4260 `Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] := Simp[a^2*x, x] + (Simp[2*a*b Int[Csc[c + d*x], x], x] + Simp[b^2 Int[Csc[c + d*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

method	result
parts	$\frac{a^2 x^2}{2} + \frac{b^2 \tan(dx^2+c)}{2d} + \frac{ab \ln(\sec(dx^2+c)+\tan(dx^2+c))}{d}$
derivativedivides	$\frac{a^2(dx^2+c)+2ab \ln(\sec(dx^2+c)+\tan(dx^2+c))+b^2 \tan(dx^2+c)}{2d}$
default	$\frac{a^2(dx^2+c)+2ab \ln(\sec(dx^2+c)+\tan(dx^2+c))+b^2 \tan(dx^2+c)}{2d}$
risch	$\frac{a^2 x^2}{2} + \frac{ib^2}{d(1+e^{2i(dx^2+c)})} + \frac{ab \ln(e^{i(dx^2+c)}+i)}{d} - \frac{ab \ln(e^{i(dx^2+c)}-i)}{d}$
parallelrisch	$\frac{a^2 d x^2 \cos(dx^2+c) - 2 \cos(dx^2+c) ab \ln(\tan(\frac{dx^2}{2} + \frac{c}{2}) - 1) + 2 \cos(dx^2+c) ab \ln(\tan(\frac{dx^2}{2} + \frac{c}{2}) + 1) + b^2 \sin(dx^2+c)}{2 \cos(dx^2+c)d}$
norman	$\frac{-\frac{a^2 x^2}{2} + \frac{a^2 x^2 \tan(\frac{dx^2}{2} + \frac{c}{2})^2}{2} - \frac{b^2 \tan(\frac{dx^2}{2} + \frac{c}{2})}{d}}{\tan(\frac{dx^2}{2} + \frac{c}{2})^2 - 1} + \frac{ab \ln(\tan(\frac{dx^2}{2} + \frac{c}{2}) + 1)}{d} - \frac{ab \ln(\tan(\frac{dx^2}{2} + \frac{c}{2}) - 1)}{d}$

input `int(x*(a+b*sec(d*x^2+c))^2,x,method=_RETURNVERBOSE)`output `1/2*a^2*x^2+1/2*b^2*tan(d*x^2+c)/d+a*b/d*ln(sec(d*x^2+c)+tan(d*x^2+c))`**Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(40) = 80.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.07

$$\int x(a + b \sec(c + dx^2))^2 dx$$

$$= \frac{a^2 dx^2 \cos(dx^2 + c) + ab \cos(dx^2 + c) \log(\sin(dx^2 + c) + 1) - ab \cos(dx^2 + c) \log(-\sin(dx^2 + c) + 1)}{2d \cos(dx^2 + c)}$$

input `integrate(x*(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output

```
1/2*(a^2*d*x^2*cos(d*x^2 + c) + a*b*cos(d*x^2 + c)*log(sin(d*x^2 + c) + 1)
- a*b*cos(d*x^2 + c)*log(-sin(d*x^2 + c) + 1) + b^2*sin(d*x^2 + c))/(d*cos(d*x^2 + c))
```

Sympy [F]

$$\int x(a + b \sec(c + dx^2))^2 dx = \int x(a + b \sec(c + dx^2))^2 dx$$

input

```
integrate(x*(a+b*sec(d*x**2+c))**2,x)
```

output

```
Integral(x*(a + b*sec(c + d*x**2))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.18

$$\begin{aligned} & \int x(a + b \sec(c + dx^2))^2 dx \\ &= \frac{1}{2} a^2 x^2 + \frac{ab \log(\sec(dx^2 + c) + \tan(dx^2 + c))}{d} \\ &+ \frac{b^2 \sin(2dx^2 + 2c)}{d \cos(2dx^2 + 2c)^2 + d \sin(2dx^2 + 2c)^2 + 2d \cos(2dx^2 + 2c) + d} \end{aligned}$$

input

```
integrate(x*(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")
```

output

```
1/2*a^2*x^2 + a*b*log(sec(d*x^2 + c) + tan(d*x^2 + c))/d + b^2*sin(2*d*x^2
+ 2*c)/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 + 2*d*cos(2*d*x^2
+ 2*c) + d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(40) = 80$.

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int x(a + b \sec(c + dx^2))^2 dx$$

$$= \frac{(dx^2 + c)a^2 + 2ab \log\left(\left|\tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right) + 1\right|\right) - 2ab \log\left(\left|\tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right) - 1\right|\right) - \frac{2b^2 \tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)^2 - 1}}{2d}$$

input `integrate(x*(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`

output `1/2*((d*x^2 + c)*a^2 + 2*a*b*log(abs(tan(1/2*d*x^2 + 1/2*c) + 1)) - 2*a*b*log(abs(tan(1/2*d*x^2 + 1/2*c) - 1)) - 2*b^2*tan(1/2*d*x^2 + 1/2*c)/(tan(1/2*d*x^2 + 1/2*c)^2 - 1))/d`

Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.27

$$\int x(a + b \sec(c + dx^2))^2 dx = \frac{a^2 x^2}{2} + \frac{b^2 \operatorname{li}}{d (e^{2i dx^2 + c 2i} + 1)}$$

$$+ \frac{ab \ln\left(-abx^{4i} - 4abx e^{dx^2 \operatorname{li}} e^{c \operatorname{li}}\right)}{d}$$

$$- \frac{ab \ln\left(abx^{4i} - 4abx e^{dx^2 \operatorname{li}} e^{c \operatorname{li}}\right)}{d}$$

input `int(x*(a + b/cos(c + d*x^2))^2,x)`

output `(a^2*x^2)/2 + (b^2*li)/(d*(exp(c*2i + d*x^2*2i) + 1)) + (a*b*log(- a*b*x^4 i - 4*a*b*x*exp(d*x^2*1i)*exp(c*1i)))/d - (a*b*log(a*b*x^4 i - 4*a*b*x*exp(d*x^2*1i)*exp(c*1i)))/d`

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.18

$$\int x(a + b \sec(c + dx^2))^2 dx$$

$$= \frac{-2 \cos(dx^2 + c) \log\left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right) - 1\right) ab + 2 \cos(dx^2 + c) \log\left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right) + 1\right) ab + \cos(dx^2 + c) a^2 dx^2 + \sin(c + dx^2) b^2}{2 \cos(dx^2 + c) d}$$

input

```
int(x*(a+b*sec(d*x^2+c))^2,x)
```

output

```
( - 2*cos(c + d*x**2)*log(tan((c + d*x**2)/2) - 1)*a*b + 2*cos(c + d*x**2)
*log(tan((c + d*x**2)/2) + 1)*a*b + cos(c + d*x**2)*a**2*d*x**2 + sin(c +
d*x**2)*b**2)/(2*cos(c + d*x**2)*d)
```

$$3.13 \quad \int \frac{(a+b \sec(c+dx^2))^2}{x} dx$$

Optimal result	125
Mathematica [N/A]	125
Rubi [N/A]	126
Maple [N/A]	126
Fricas [N/A]	127
Sympy [N/A]	127
Maxima [N/A]	128
Giac [N/A]	128
Mupad [N/A]	129
Reduce [N/A]	129

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx = \text{Int}\left(\frac{(a + b \sec(c + dx^2))^2}{x}, x\right)$$

output `Defer(Int)((a+b*sec(d*x^2+c))^2/x,x)`

Mathematica [N/A]

Not integrable

Time = 22.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx = \int \frac{(a + b \sec(c + dx^2))^2}{x} dx$$

input `Integrate[(a + b*Sec[c + d*x^2])^2/x,x]`

output `Integrate[(a + b*Sec[c + d*x^2])^2/x, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx$$

↓ 4694

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx$$

input `Int[(a + b*Sec[c + d*x^2])^2/x,x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \sec(dx^2 + c))^2}{x} dx$$

input `int((a+b*sec(d*x^2+c))^2/x,x)`

output `int((a+b*sec(d*x^2+c))^2/x,x)`

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx = \int \frac{(b \sec(dx^2 + c) + a)^2}{x} dx$$

input `integrate((a+b*sec(d*x^2+c))^2/x,x, algorithm="fricas")`

output `integral((b^2*sec(d*x^2 + c)^2 + 2*a*b*sec(d*x^2 + c) + a^2)/x, x)`

Sympy [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx = \int \frac{(a + b \sec(c + dx^2))^2}{x} dx$$

input `integrate((a+b*sec(d*x**2+c))**2/x,x)`

output `Integral((a + b*sec(c + d*x**2))**2/x, x)`

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 15.89

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx = \int \frac{(b \sec(dx^2 + c) + a)^2}{x} dx$$

input `integrate((a+b*sec(d*x^2+c))^2/x,x, algorithm="maxima")`

output `a^2*log(x) + (b^2*sin(2*d*x^2 + 2*c) + (d*x^2*cos(2*d*x^2 + 2*c)^2 + d*x^2*sin(2*d*x^2 + 2*c)^2 + 2*d*x^2*cos(2*d*x^2 + 2*c) + d*x^2)*integrate(2*(2*a*b*d*x^2*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 2*a*b*d*x^2*cos(d*x^2 + c) + (2*a*b*d*x^2*sin(d*x^2 + c) + b^2)*sin(2*d*x^2 + 2*c))/(d*x^3*cos(2*d*x^2 + 2*c)^2 + d*x^3*sin(2*d*x^2 + 2*c)^2 + 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3), x))/(d*x^2*cos(2*d*x^2 + 2*c)^2 + d*x^2*sin(2*d*x^2 + 2*c)^2 + 2*d*x^2*cos(2*d*x^2 + 2*c) + d*x^2)`

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx = \int \frac{(b \sec(dx^2 + c) + a)^2}{x} dx$$

input `integrate((a+b*sec(d*x^2+c))^2/x,x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)^2/x, x)`

Mupad [N/A]

Not integrable

Time = 18.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx = \int \frac{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2}{x} dx$$

input `int((a + b/cos(c + d*x^2))^2/x,x)`output `int((a + b/cos(c + d*x^2))^2/x, x)`**Reduce [N/A]**

Not integrable

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.56

$$\int \frac{(a + b \sec(c + dx^2))^2}{x} dx = - \left(\int \frac{\sin(dx^2 + c)^2}{\sin(dx^2 + c)^2 x - x} dx \right) b^2$$

$$- 4 \left(\int \frac{1}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 x - x} dx \right) ab$$

$$+ \log(x) a^2 - 2 \log(x) ab + \log(x) b^2$$

input `int((a+b*sec(d*x^2+c))^2/x,x)`output `- int(sin(c + d*x**2)**2/(sin(c + d*x**2)**2*x - x),x)*b**2 - 4*int(1/(tan((c + d*x**2)/2)**2*x - x),x)*a*b + log(x)*a**2 - 2*log(x)*a*b + log(x)*b**2`

3.14 $\int \frac{(a+b \sec(c+dx^2))^2}{x^2} dx$

Optimal result	130
Mathematica [N/A]	130
Rubi [N/A]	131
Maple [N/A]	131
Fricas [N/A]	132
Sympy [N/A]	132
Maxima [N/A]	133
Giac [N/A]	133
Mupad [N/A]	134
Reduce [N/A]	134

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx = \text{Int}\left(\frac{(a + b \sec(c + dx^2))^2}{x^2}, x\right)$$

output `Defer(Int)((a+b*sec(d*x^2+c))^2/x^2,x)`

Mathematica [N/A]

Not integrable

Time = 8.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx = \int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx$$

input `Integrate[(a + b*Sec[c + d*x^2])^2/x^2,x]`

output `Integrate[(a + b*Sec[c + d*x^2])^2/x^2, x]`

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx$$

↓ 4694

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx$$

input `Int[(a + b*Sec[c + d*x^2])^2/x^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \sec(dx^2 + c))^2}{x^2} dx$$

input `int((a+b*sec(d*x^2+c))^2/x^2,x)`

output `int((a+b*sec(d*x^2+c))^2/x^2,x)`

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx = \int \frac{(b \sec(dx^2 + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sec(d*x^2+c))^2/x^2,x, algorithm="fricas")`

output `integral((b^2*sec(d*x^2 + c)^2 + 2*a*b*sec(d*x^2 + c) + a^2)/x^2, x)`

Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx = \int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx$$

input `integrate((a+b*sec(d*x**2+c))**2/x**2,x)`

output `Integral((a + b*sec(c + d*x**2))**2/x**2, x)`

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 289, normalized size of antiderivative = 16.06

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx = \int \frac{(b \sec(dx^2 + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sec(d*x^2+c))^2/x^2,x, algorithm="maxima")`

output

```
-a^2/x + (b^2*sin(2*d*x^2 + 2*c) + (d*x^3*cos(2*d*x^2 + 2*c))^2 + d*x^3*sin(2*d*x^2 + 2*c)^2 + 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3)*integrate((4*a*b*d*x^2*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 4*a*b*d*x^2*cos(d*x^2 + c) + (4*a*b*d*x^2*sin(d*x^2 + c) + 3*b^2)*sin(2*d*x^2 + 2*c))/(d*x^4*cos(2*d*x^2 + 2*c)^2 + d*x^4*sin(2*d*x^2 + 2*c)^2 + 2*d*x^4*cos(2*d*x^2 + 2*c) + d*x^4), x)/(d*x^3*cos(2*d*x^2 + 2*c)^2 + d*x^3*sin(2*d*x^2 + 2*c)^2 + 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3)
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx = \int \frac{(b \sec(dx^2 + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sec(d*x^2+c))^2/x^2,x, algorithm="giac")`

output

```
integrate((b*sec(d*x^2 + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 17.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx = \int \frac{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2}{x^2} dx$$

input `int((a + b/cos(c + d*x^2))^2/x^2,x)`output `int((a + b/cos(c + d*x^2))^2/x^2, x)`**Reduce [N/A]**

Not integrable

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 5.89

$$\int \frac{(a + b \sec(c + dx^2))^2}{x^2} dx$$

$$= \frac{-\left(\int \frac{\sin(dx^2+c)^2}{\sin(dx^2+c)^2 x^2 - x^2} dx\right) b^2 x - 4 \left(\int \frac{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 x^2 - x^2} dx\right) abx - a^2 - 2ab - b^2}{x}$$

input `int((a+b*sec(d*x^2+c))^2/x^2,x)`output `(- int(sin(c + d*x**2)**2/(sin(c + d*x**2)**2*x**2 - x**2),x)*b**2*x - 4*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**2*x**2 - x**2),x)*a*b*x - a**2 - 2*a*b - b**2)/x`

3.15 $\int x \sec^7(a + bx^2) dx$

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Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x \sec^7(a + bx^2) dx = \frac{5 \operatorname{arctanh}(\sin(a + bx^2))}{32b} + \frac{5 \sec(a + bx^2) \tan(a + bx^2)}{32b} + \frac{5 \sec^3(a + bx^2) \tan(a + bx^2)}{48b} + \frac{\sec^5(a + bx^2) \tan(a + bx^2)}{12b}$$

output 5/32*arctanh(sin(b*x^2+a))/b+5/32*sec(b*x^2+a)*tan(b*x^2+a)/b+5/48*sec(b*x^2+a)^3*tan(b*x^2+a)/b+1/12*sec(b*x^2+a)^5*tan(b*x^2+a)/b

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int x \sec^7(a + bx^2) dx = \frac{5 \operatorname{arctanh}(\sin(a + bx^2))}{32b} + \frac{5 \sec(a + bx^2) \tan(a + bx^2)}{32b} + \frac{5 \sec^3(a + bx^2) \tan(a + bx^2)}{48b} + \frac{\sec^5(a + bx^2) \tan(a + bx^2)}{12b}$$

input Integrate[x*Sec[a + b*x^2]^7,x]

output

```
(5*ArcTanh[Sin[a + b*x^2]])/(32*b) + (5*Sec[a + b*x^2]*Tan[a + b*x^2])/(32
*b) + (5*Sec[a + b*x^2]^3*Tan[a + b*x^2])/(48*b) + (Sec[a + b*x^2]^5*Tan[a
+ b*x^2])/(12*b)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4692, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x \sec^7(a + bx^2) dx \\
 & \quad \downarrow 4692 \\
 & \frac{1}{2} \int \sec^7(bx^2 + a) dx^2 \\
 & \quad \downarrow 3042 \\
 & \frac{1}{2} \int \csc\left(bx^2 + a + \frac{\pi}{2}\right)^7 dx^2 \\
 & \quad \downarrow 4255 \\
 & \frac{1}{2} \left(\frac{5}{6} \int \sec^5(bx^2 + a) dx^2 + \frac{\tan(a + bx^2) \sec^5(a + bx^2)}{6b} \right) \\
 & \quad \downarrow 3042 \\
 & \frac{1}{2} \left(\frac{5}{6} \int \csc\left(bx^2 + a + \frac{\pi}{2}\right)^5 dx^2 + \frac{\tan(a + bx^2) \sec^5(a + bx^2)}{6b} \right) \\
 & \quad \downarrow 4255 \\
 & \frac{1}{2} \left(\frac{5}{6} \left(\frac{3}{4} \int \sec^3(bx^2 + a) dx^2 + \frac{\tan(a + bx^2) \sec^3(a + bx^2)}{4b} \right) + \frac{\tan(a + bx^2) \sec^5(a + bx^2)}{6b} \right) \\
 & \quad \downarrow 3042
 \end{aligned}$$

$$\frac{1}{2} \left(\frac{5}{6} \left(\frac{3}{4} \int \csc \left(bx^2 + a + \frac{\pi}{2} \right)^3 dx^2 + \frac{\tan(a + bx^2) \sec^3(a + bx^2)}{4b} \right) + \frac{\tan(a + bx^2) \sec^5(a + bx^2)}{6b} \right)$$

↓ 4255

$$\frac{1}{2} \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \sec(bx^2 + a) dx^2 + \frac{\tan(a + bx^2) \sec(a + bx^2)}{2b} \right) + \frac{\tan(a + bx^2) \sec^3(a + bx^2)}{4b} \right) + \frac{\tan(a + bx^2) \sec^5(a + bx^2)}{6b} \right)$$

↓ 3042

$$\frac{1}{2} \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \csc \left(bx^2 + a + \frac{\pi}{2} \right) dx^2 + \frac{\tan(a + bx^2) \sec(a + bx^2)}{2b} \right) + \frac{\tan(a + bx^2) \sec^3(a + bx^2)}{4b} \right) + \frac{\tan(a + bx^2) \sec^5(a + bx^2)}{6b} \right)$$

↓ 4257

$$\frac{1}{2} \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\operatorname{arctanh}(\sin(a + bx^2))}{2b} + \frac{\tan(a + bx^2) \sec(a + bx^2)}{2b} \right) + \frac{\tan(a + bx^2) \sec^3(a + bx^2)}{4b} \right) + \frac{\tan(a + bx^2) \sec^5(a + bx^2)}{6b} \right)$$

input `Int[x*Sec[a + b*x^2]^7,x]`

output `((Sec[a + b*x^2]^5*Tan[a + b*x^2])/(6*b) + (5*((Sec[a + b*x^2]^3*Tan[a + b*x^2])/(4*b) + (3*(ArcTanh[Sin[a + b*x^2]]/(2*b) + (Sec[a + b*x^2]*Tan[a + b*x^2])/(2*b))))/4)/6)/2`

Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4255 `Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n-1)/(d*(n-1)), x] + Simp[b^2*(n-2)/(n-1)Int[(b*Csc[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

rule 4257 `Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\left(-\frac{\sec(bx^2+a)^5}{6} - \frac{5\sec(bx^2+a)^3}{24} - \frac{5\sec(bx^2+a)}{16}\right)\tan(bx^2+a) + \frac{5\ln(\sec(bx^2+a) + \tan(bx^2+a))}{16}}{2b}$
default	$-\frac{\left(-\frac{\sec(bx^2+a)^5}{6} - \frac{5\sec(bx^2+a)^3}{24} - \frac{5\sec(bx^2+a)}{16}\right)\tan(bx^2+a) + \frac{5\ln(\sec(bx^2+a) + \tan(bx^2+a))}{16}}{2b}$
risch	$-\frac{i\left(15e^{11i(bx^2+a)} + 85e^{9i(bx^2+a)} + 198e^{7i(bx^2+a)} - 198e^{5i(bx^2+a)} - 85e^{3i(bx^2+a)} - 15e^{i(bx^2+a)}\right)}{48b\left(e^{2i(bx^2+a)} + 1\right)^6} + \frac{5\ln\left(e^{i(bx^2+a)}\right)}{2b}$
parallelrisch	$\frac{(-225\cos(2bx^2+2a) - 90\cos(4bx^2+4a) - 15\cos(6bx^2+6a) - 150)\ln\left(\tan\left(\frac{a}{2} + \frac{bx^2}{2}\right) - 1\right) + (225\cos(2bx^2+2a) + 90\cos(4bx^2+4a) + 15\cos(6bx^2+6a) + 150)}{96b(10 + \cos(6bx^2+6a) + 6\cos(4bx^2+4a) + 2\cos(2bx^2+2a) + 1)}$

input `int(x*sec(b*x^2+a)^7,x,method=_RETURNVERBOSE)`

output `1/2/b*(-(-1/6*sec(b*x^2+a)^5-5/24*sec(b*x^2+a)^3-5/16*sec(b*x^2+a))*tan(b*x^2+a)+5/16*ln(sec(b*x^2+a)+tan(b*x^2+a)))`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x \sec^7(a + bx^2) dx = \frac{15 \cos(bx^2 + a)^6 \log(\sin(bx^2 + a) + 1) - 15 \cos(bx^2 + a)^6 \log(-\sin(bx^2 + a) + 1) + 2 \left(15 \cos(bx^2 + a)^4 + 10 \cos(bx^2 + a)^2 + 8\right) \sin(bx^2 + a)}{192 b \cos(bx^2 + a)^6}$$

input `integrate(x*sec(b*x^2+a)^7,x, algorithm="fricas")`

output `1/192*(15*cos(b*x^2 + a)^6*log(sin(b*x^2 + a) + 1) - 15*cos(b*x^2 + a)^6*log(-sin(b*x^2 + a) + 1) + 2*(15*cos(b*x^2 + a)^4 + 10*cos(b*x^2 + a)^2 + 8)*sin(b*x^2 + a))/(b*cos(b*x^2 + a)^6)`

Sympy [F]

$$\int x \sec^7(a + bx^2) dx = \int x \sec^7(a + bx^2) dx$$

input `integrate(x*sec(b*x**2+a)**7,x)`

output `Integral(x*sec(a + b*x**2)**7, x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2838 vs. 2(82) = 164.

Time = 0.21 (sec) , antiderivative size = 2838, normalized size of antiderivative = 31.53

$$\int x \sec^7(a + bx^2) dx = \text{Too large to display}$$

input `integrate(x*sec(b*x^2+a)^7,x, algorithm="maxima")`

output

```

1/192*(4*(15*sin(11*b*x^2 + 11*a) + 85*sin(9*b*x^2 + 9*a) + 198*sin(7*b*x^
2 + 7*a) - 198*sin(5*b*x^2 + 5*a) - 85*sin(3*b*x^2 + 3*a) - 15*sin(b*x^2 +
a))*cos(12*b*x^2 + 12*a) - 60*(6*sin(10*b*x^2 + 10*a) + 15*sin(8*b*x^2 +
8*a) + 20*sin(6*b*x^2 + 6*a) + 15*sin(4*b*x^2 + 4*a) + 6*sin(2*b*x^2 + 2*a
))*cos(11*b*x^2 + 11*a) + 24*(85*sin(9*b*x^2 + 9*a) + 198*sin(7*b*x^2 + 7*
a) - 198*sin(5*b*x^2 + 5*a) - 85*sin(3*b*x^2 + 3*a) - 15*sin(b*x^2 + a))*c
os(10*b*x^2 + 10*a) - 340*(15*sin(8*b*x^2 + 8*a) + 20*sin(6*b*x^2 + 6*a) +
15*sin(4*b*x^2 + 4*a) + 6*sin(2*b*x^2 + 2*a))*cos(9*b*x^2 + 9*a) + 60*(19
8*sin(7*b*x^2 + 7*a) - 198*sin(5*b*x^2 + 5*a) - 85*sin(3*b*x^2 + 3*a) - 15
*sin(b*x^2 + a))*cos(8*b*x^2 + 8*a) - 792*(20*sin(6*b*x^2 + 6*a) + 15*sin(
4*b*x^2 + 4*a) + 6*sin(2*b*x^2 + 2*a))*cos(7*b*x^2 + 7*a) - 80*(198*sin(5*
b*x^2 + 5*a) + 85*sin(3*b*x^2 + 3*a) + 15*sin(b*x^2 + a))*cos(6*b*x^2 + 6*
a) + 2376*(5*sin(4*b*x^2 + 4*a) + 2*sin(2*b*x^2 + 2*a))*cos(5*b*x^2 + 5*a)
- 300*(17*sin(3*b*x^2 + 3*a) + 3*sin(b*x^2 + a))*cos(4*b*x^2 + 4*a) - 15*
(2*(6*cos(10*b*x^2 + 10*a) + 15*cos(8*b*x^2 + 8*a) + 20*cos(6*b*x^2 + 6*a)
+ 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) + 1)*cos(12*b*x^2 + 12*a)
+ cos(12*b*x^2 + 12*a)^2 + 12*(15*cos(8*b*x^2 + 8*a) + 20*cos(6*b*x^2 + 6*
a) + 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) + 1)*cos(10*b*x^2 + 10*a
) + 36*cos(10*b*x^2 + 10*a)^2 + 30*(20*cos(6*b*x^2 + 6*a) + 15*cos(4*b*x^2
+ 4*a) + 6*cos(2*b*x^2 + 2*a) + 1)*cos(8*b*x^2 + 8*a) + 225*cos(8*b*x^...

```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int x \sec^7(a + bx^2) dx = \frac{2 \left(15 \sin(bx^2+a)^5 - 40 \sin(bx^2+a)^3 + 33 \sin(bx^2+a) \right)}{\left(\sin(bx^2+a)^2 - 1 \right)^3} - 15 \log(\sin(bx^2+a) + 1) + 15 \log(-\sin(bx^2+a) + 1)$$

192 b

input

```
integrate(x*sec(b*x^2+a)^7,x, algorithm="giac")
```

output

```

-1/192*(2*(15*sin(b*x^2 + a)^5 - 40*sin(b*x^2 + a)^3 + 33*sin(b*x^2 + a))/
(sin(b*x^2 + a)^2 - 1)^3 - 15*log(sin(b*x^2 + a) + 1) + 15*log(-sin(b*x^2
+ a) + 1))/b

```

Mupad [B] (verification not implemented)

Time = 27.86 (sec) , antiderivative size = 496, normalized size of antiderivative = 5.51

$$\int x \sec^7(a + bx^2) dx = \text{Too large to display}$$

input `int(x/cos(a + b*x^2)^7,x)`

output

$$\begin{aligned} & \frac{(5 \log(- (x^5 i)/8 - (5 x \exp(a i) \exp(b x^2 i))/8)) / (32 b) - (5 \log((x^5 i)/8 - (5 x \exp(a i) \exp(b x^2 i))/8)) / (32 b) + (\exp(a^3 i + b x^2 3 i) 8 i) / (3 b (5 \exp(a^2 i + b x^2 2 i) + 10 \exp(a^4 i + b x^2 4 i) + 10 \exp(a^6 i + b x^2 6 i) + 5 \exp(a^8 i + b x^2 8 i) + \exp(a^{10} i + b x^2 10 i) + 1)) - (\exp(a^1 i + b x^2 1 i) 1 i) / (6 b (3 \exp(a^2 i + b x^2 2 i) + 3 \exp(a^4 i + b x^2 4 i) + \exp(a^6 i + b x^2 6 i) + 1)) - (\exp(a^1 i + b x^2 1 i) 5 i) / (16 b (\exp(a^2 i + b x^2 2 i) + 1)) + (\exp(a^5 i + b x^2 5 i) 16 i) / (3 b (6 \exp(a^2 i + b x^2 2 i) + 15 \exp(a^4 i + b x^2 4 i) + 20 \exp(a^6 i + b x^2 6 i) + 15 \exp(a^8 i + b x^2 8 i) + 6 \exp(a^{10} i + b x^2 10 i) + \exp(a^{12} i + b x^2 12 i) + 1)) + (\exp(a^1 i + b x^2 1 i) 1 i) / (b (4 \exp(a^2 i + b x^2 2 i) + 6 \exp(a^4 i + b x^2 4 i) + 4 \exp(a^6 i + b x^2 6 i) + \exp(a^8 i + b x^2 8 i) + 1)) - (\exp(a^1 i + b x^2 1 i) 5 i) / (24 b (2 \exp(a^2 i + b x^2 2 i) + \exp(a^4 i + b x^2 4 i) + 1)) \end{aligned}$$
Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.96

$$\int x \sec^7(a + bx^2) dx$$

$$= \frac{-15 \log\left(\tan\left(\frac{bx^2}{2} + \frac{a}{2}\right) - 1\right) \sin(bx^2 + a)^6 + 45 \log\left(\tan\left(\frac{bx^2}{2} + \frac{a}{2}\right) - 1\right) \sin(bx^2 + a)^4 - 45 \log\left(\tan\left(\frac{bx^2}{2} + \frac{a}{2}\right) - 1\right) \sin(bx^2 + a)^2 + 15 \log\left(\tan\left(\frac{bx^2}{2} + \frac{a}{2}\right) - 1\right) \sin(bx^2 + a) + 15 \log\left(\tan\left(\frac{bx^2}{2} + \frac{a}{2}\right) - 1\right) \sin(bx^2 + a)^3 - 15 \log\left(\tan\left(\frac{bx^2}{2} + \frac{a}{2}\right) - 1\right) \sin(bx^2 + a)^5 + 15 \log\left(\tan\left(\frac{bx^2}{2} + \frac{a}{2}\right) - 1\right) \sin(bx^2 + a)^7}{24 b}$$

input `int(x*sec(b*x^2+a)^7,x)`

output

```
( - 15*log(tan((a + b*x**2)/2) - 1)*sin(a + b*x**2)**6 + 45*log(tan((a + b
*x**2)/2) - 1)*sin(a + b*x**2)**4 - 45*log(tan((a + b*x**2)/2) - 1)*sin(a
+ b*x**2)**2 + 15*log(tan((a + b*x**2)/2) - 1) + 15*log(tan((a + b*x**2)/2
) + 1)*sin(a + b*x**2)**6 - 45*log(tan((a + b*x**2)/2) + 1)*sin(a + b*x**2
)**4 + 45*log(tan((a + b*x**2)/2) + 1)*sin(a + b*x**2)**2 - 15*log(tan((a
+ b*x**2)/2) + 1) - 15*sin(a + b*x**2)**5 + 40*sin(a + b*x**2)**3 - 33*sin
(a + b*x**2))/(96*b*(sin(a + b*x**2)**6 - 3*sin(a + b*x**2)**4 + 3*sin(a +
b*x**2)**2 - 1))
```

3.16 $\int \frac{x^5}{a+b \sec(c+dx^2)} dx$

Optimal result	143
Mathematica [A] (verified)	144
Rubi [A] (verified)	144
Maple [F]	146
Fricas [B] (verification not implemented)	147
Sympy [F]	148
Maxima [F]	148
Giac [F]	148
Mupad [F(-1)]	149
Reduce [F]	149

Optimal result

Integrand size = 18, antiderivative size = 382

$$\int \frac{x^5}{a+b \sec(c+dx^2)} dx = \frac{x^6}{6a} + \frac{ibx^4 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^4 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}$$

$$+ \frac{bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$- \frac{bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$- \frac{ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

output

$$\frac{1}{6}x^6/a + \frac{1}{2}I*b*x^4*\ln(1+a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d - \frac{1}{2}I*b*x^4*\ln(1+a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d + b*x^2*\text{polylog}(2, -a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d^2 - b*x^2*\text{polylog}(2, -a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d^2 + I*b*\text{polylog}(3, -a*\exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d^3 - I*b*\text{polylog}(3, -a*\exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d^3$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{a + b \sec(c + dx^2)} dx$$

$$= \frac{\sqrt{-a^2 + b^2} d^3 x^6 + 3ibd^2 x^4 \log\left(1 - \frac{ae^{i(c+dx^2)}}{-b + \sqrt{-a^2 + b^2}}\right) - 3ibd^2 x^4 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}}\right) + 6bdx^2 \text{PolyLog}\left(2, \frac{ae^{i(c+dx^2)}}{-b + \sqrt{-a^2 + b^2}}\right) - 6bdx^2 \text{PolyLog}\left(2, \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}}\right) + 6bdx^2 \text{PolyLog}\left(3, \frac{ae^{i(c+dx^2)}}{-b + \sqrt{-a^2 + b^2}}\right) - 6bdx^2 \text{PolyLog}\left(3, \frac{ae^{i(c+dx^2)}}{b + \sqrt{-a^2 + b^2}}\right)}{6a\sqrt{-a^2 + b^2}}$$

input

Integrate[x^5/(a + b*Sec[c + d*x^2]),x]

output

$$\frac{(\text{Sqrt}[-a^2 + b^2]*d^3*x^6 + (3*I)*b*d^2*x^4*\text{Log}[1 - (a*E^{(I*(c + d*x^2))})/(-b + \text{Sqrt}[-a^2 + b^2])] - (3*I)*b*d^2*x^4*\text{Log}[1 + (a*E^{(I*(c + d*x^2))})/(b + \text{Sqrt}[-a^2 + b^2])] + 6*b*d*x^2*\text{PolyLog}[2, (a*E^{(I*(c + d*x^2))})/(-b + \text{Sqrt}[-a^2 + b^2])] - 6*b*d*x^2*\text{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b + \text{Sqrt}[-a^2 + b^2]))] + (6*I)*b*\text{PolyLog}[3, (a*E^{(I*(c + d*x^2))})/(-b + \text{Sqrt}[-a^2 + b^2])] - (6*I)*b*\text{PolyLog}[3, -((a*E^{(I*(c + d*x^2))})/(b + \text{Sqrt}[-a^2 + b^2]))])/(6*a*\text{Sqrt}[-a^2 + b^2]*d^3)$$

Rubi [A] (verified)Time = 1.07 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^5}{a + b \sec(c + dx^2)} dx \\
& \quad \downarrow 4692 \\
& \frac{1}{2} \int \frac{x^4}{a + b \sec(dx^2 + c)} dx^2 \\
& \quad \downarrow 3042 \\
& \frac{1}{2} \int \frac{x^4}{a + b \csc(dx^2 + c + \frac{\pi}{2})} dx^2 \\
& \quad \downarrow 4679 \\
& \frac{1}{2} \int \left(\frac{x^4}{a} - \frac{bx^4}{a(b + a \cos(dx^2 + c))} \right) dx^2 \\
& \quad \downarrow 2009 \\
& \frac{1}{2} \left(\frac{2ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{2ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{2bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{2bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} \right)
\end{aligned}$$

input `Int[x^5/(a + b*Sec[c + d*x^2]),x]`

output `(x^6/(3*a) + (I*b*x^4*Log[1 + (a*E^(I*(c + d*x^2))]/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^4*Log[1 + (a*E^(I*(c + d*x^2))]/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2))]/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) - (2*b*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2))]/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + ((2*I)*b*PolyLog[3, -((a*E^(I*(c + d*x^2))]/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - ((2*I)*b*PolyLog[3, -((a*E^(I*(c + d*x^2))]/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3))/2`

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x^5}{a + b \sec(dx^2 + c)} dx$$

input `int(x^5/(a+b*sec(d*x^2+c)),x)`

output `int(x^5/(a+b*sec(d*x^2+c)),x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(330) = 660$.

Time = 0.21 (sec) , antiderivative size = 1457, normalized size of antiderivative = 3.81

$$\int \frac{x^5}{a + b \sec(c + dx^2)} dx = \text{Too large to display}$$

input `integrate(x^5/(a+b*sec(d*x^2+c)),x, algorithm="fricas")`

output

```
1/12*(2*(a^2 - b^2)*d^3*x^6 - 6*a*b*d*x^2*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b
*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 +
c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 6*a*b*d*x^2*sqrt(-(a^2 - b^2)/a^
2)*dilog(-(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a
*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 6*a*b*d*x^2*sqrt(-(a
^2 - b^2)/a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^
2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 6*a*b*d*
x^2*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) -
(a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1
) + 3*I*a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 + c) + 2*I*a*sin(
d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 3*I*a*b*c^2*sqrt(-(a^2 -
b^2)/a^2)*log(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 -
b^2)/a^2) + 2*b) + 3*I*a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2
+ c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) - 3*I*a*b*
c^2*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c)
+ 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) - 6*I*a*b*sqrt(-(a^2 - b^2)/a^2)*polyl
og(3, -(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*si
n(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 6*I*a*b*sqrt(-(a^2 - b^2)/a^2)*
polylog(3, -(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I
*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 6*I*a*b*sqrt(-(a^2 - b^...
```

Sympy [F]

$$\int \frac{x^5}{a + b \sec(c + dx^2)} dx = \int \frac{x^5}{a + b \sec(c + dx^2)} dx$$

input `integrate(x**5/(a+b*sec(d*x**2+c)),x)`

output `Integral(x**5/(a + b*sec(c + d*x**2)), x)`

Maxima [F]

$$\int \frac{x^5}{a + b \sec(c + dx^2)} dx = \int \frac{x^5}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^5/(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

output `1/6*(x^6 - 12*a*b*integrate((a*x^5*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 2*b*x^5*cos(d*x^2 + c)^2 + a*x^5*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^5*sin(d*x^2 + c)^2 + a*x^5*cos(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*cos(d*x^2 + c)^2 + a^3*sin(2*d*x^2 + 2*c)^2 + 4*a^2*b*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c) + a^3 + 2*(2*a^2*b*cos(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a`

Giac [F]

$$\int \frac{x^5}{a + b \sec(c + dx^2)} dx = \int \frac{x^5}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^5/(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output `integrate(x^5/(b*sec(d*x^2 + c) + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a + b \sec(c + dx^2)} dx = \int \frac{x^5}{a + \frac{b}{\cos(dx^2+c)}} dx$$

input `int(x^5/(a + b/cos(c + d*x^2)),x)`output `int(x^5/(a + b/cos(c + d*x^2)), x)`**Reduce [F]**

$$\int \frac{x^5}{a + b \sec(c + dx^2)} dx$$

$$= \frac{12 \left(\int \frac{\tan\left(\frac{dx^2+c}{2}\right)^2 x^5}{\tan\left(\frac{dx^2+c}{2}\right)^2 a^2 - \tan\left(\frac{dx^2+c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) ab + 12 \left(\int \frac{\tan\left(\frac{dx^2+c}{2}\right)^2 x^5}{\tan\left(\frac{dx^2+c}{2}\right)^2 a^2 - \tan\left(\frac{dx^2+c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) b^2}{6a + 6b}$$

input `int(x^5/(a+b*sec(d*x^2+c)),x)`output `(12*int((tan((c + d*x**2)/2)**2*x**5)/(tan((c + d*x**2)/2)**2*a**2 - tan((c + d*x**2)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*a*b + 12*int((tan((c + d*x**2)/2)**2*x**5)/(tan((c + d*x**2)/2)**2*a**2 - tan((c + d*x**2)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*b**2 + x**6)/(6*(a + b))`

3.17 $\int \frac{x^4}{a+b \sec(c+dx^2)} dx$

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Rubi [N/A]	151
Maple [N/A]	151
Fricas [N/A]	152
Sympy [N/A]	152
Maxima [N/A]	153
Giac [N/A]	153
Mupad [N/A]	154
Reduce [N/A]	154

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx = \text{Int}\left(\frac{x^4}{a + b \sec(c + dx^2)}, x\right)$$

output `Defer(Int)(x^4/(a+b*sec(d*x^2+c)),x)`

Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx = \int \frac{x^4}{a + b \sec(c + dx^2)} dx$$

input `Integrate[x^4/(a + b*Sec[c + d*x^2]),x]`

output `Integrate[x^4/(a + b*Sec[c + d*x^2]), x]`

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx$$

↓ 4694

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx$$

input `Int[x^4/(a + b*Sec[c + d*x^2]),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a + b \sec(dx^2 + c)} dx$$

input `int(x^4/(a+b*sec(d*x^2+c)),x)`

output `int(x^4/(a+b*sec(d*x^2+c)),x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx = \int \frac{x^4}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^4/(a+b*sec(d*x^2+c)),x, algorithm="fricas")`

output `integral(x^4/(b*sec(d*x^2 + c) + a), x)`

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx = \int \frac{x^4}{a + b \sec(c + dx^2)} dx$$

input `integrate(x**4/(a+b*sec(d*x**2+c)),x)`

output `Integral(x**4/(a + b*sec(c + d*x**2)), x)`

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 14.00

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx = \int \frac{x^4}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^4/(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

output `1/5*(x^5 - 10*a*b*integrate((a*x^4*cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + 2*b*x^4*cos(d*x^2 + c)^2 + a*x^4*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^4*sin(d*x^2 + c)^2 + a*x^4*cos(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*cos(d*x^2 + c)^2 + a^3*sin(2*d*x^2 + 2*c)^2 + 4*a^2*b*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c) + a^3 + 2*(2*a^2*b*cos(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a`

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx = \int \frac{x^4}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^4/(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output `integrate(x^4/(b*sec(d*x^2 + c) + a), x)`

Mupad [N/A]

Not integrable

Time = 16.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx = \int \frac{x^4}{a + \frac{b}{\cos(dx^2+c)}} dx$$

input `int(x^4/(a + b/cos(c + d*x^2)),x)`output `int(x^4/(a + b/cos(c + d*x^2)), x)`**Reduce [N/A]**

Not integrable

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 9.17

$$\int \frac{x^4}{a + b \sec(c + dx^2)} dx$$

$$= \frac{10 \left(\int \frac{\tan\left(\frac{dx^2+c}{2}\right)^2 x^4}{\tan\left(\frac{dx^2+c}{2}\right)^2 a^2 - \tan\left(\frac{dx^2+c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) ab + 10 \left(\int \frac{\tan\left(\frac{dx^2+c}{2}\right)^2 x^4}{\tan\left(\frac{dx^2+c}{2}\right)^2 a^2 - \tan\left(\frac{dx^2+c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) b^2}{5a + 5b}$$

input `int(x^4/(a+b*sec(d*x^2+c)),x)`output `(10*int((tan((c + d*x**2)/2)**2*x**4)/(tan((c + d*x**2)/2)**2*a**2 - tan((c + d*x**2)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*a*b + 10*int((tan((c + d*x**2)/2)**2*x**4)/(tan((c + d*x**2)/2)**2*a**2 - tan((c + d*x**2)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*b**2 + x**5)/(5*(a + b))`

3.18 $\int \frac{x^3}{a+b \sec(c+dx^2)} dx$

Optimal result	155
Mathematica [B] (warning: unable to verify)	156
Rubi [A] (verified)	157
Maple [F]	158
Fricas [B] (verification not implemented)	159
Sympy [F]	160
Maxima [F]	160
Giac [F]	160
Mupad [F(-1)]	161
Reduce [F]	161

Optimal result

Integrand size = 18, antiderivative size = 261

$$\int \frac{x^3}{a+b \sec(c+dx^2)} dx = \frac{x^4}{4a} + \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2} - \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

output

```
1/4*x^4/a+1/2*I*b*x^2*ln(1+a*exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^(1/2))/a/(-a^2+b^2)^(1/2)/d-1/2*I*b*x^2*ln(1+a*exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^(1/2))/a/(-a^2+b^2)^(1/2)/d+1/2*b*polylog(2,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-1/2*b*polylog(2,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 845 vs. $2(261) = 522$.

Time = 1.03 (sec) , antiderivative size = 845, normalized size of antiderivative = 3.24

$$\int \frac{x^3}{a + b \sec(c + dx^2)} dx = \text{Too large to display}$$

input `Integrate[x^3/(a + b*Sec[c + d*x^2]),x]`

output

```
((b + a*Cos[c + d*x^2])*(x^4 - (2*b*(2*(c + d*x^2)*ArcTanh[((a + b)*Cot[(c + d*x^2)/2]])/Sqrt[a^2 - b^2]] - 2*(c + ArcCos[-(b/a)])*ArcTanh[((a - b)*Tan[(c + d*x^2)/2]])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] - (2*I)*ArcTanh[((a + b)*Cot[(c + d*x^2)/2]])/Sqrt[a^2 - b^2]] + (2*I)*ArcTanh[((a - b)*Tan[(c + d*x^2)/2]])/Sqrt[a^2 - b^2]))*Log[Sqrt[a^2 - b^2]/(Sqrt[2]*Sqrt[a]*E^((I/2)*(c + d*x^2))*Sqrt[b + a*Cos[c + d*x^2]])] + (ArcCos[-(b/a)] + (2*I)*ArcTanh[((a + b)*Cot[(c + d*x^2)/2]])/Sqrt[a^2 - b^2]] - ArcTanh[((a - b)*Tan[(c + d*x^2)/2]])/Sqrt[a^2 - b^2]))*Log[(Sqrt[a^2 - b^2]*E^((I/2)*(c + d*x^2)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Cos[c + d*x^2]])] - (ArcCos[-(b/a)] - (2*I)*ArcTanh[((a - b)*Tan[(c + d*x^2)/2]])/Sqrt[a^2 - b^2]))*Log[((a + b)*(a - b - I*Sqrt[a^2 - b^2])*(1 + I*Tan[(c + d*x^2)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))] - (ArcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Tan[(c + d*x^2)/2]])/Sqrt[a^2 - b^2]))*Log[((a + b)*((-I)*a + I*b + Sqrt[a^2 - b^2])*(I + Tan[(c + d*x^2)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2]))]))/(Sqrt[a^2 - b^2]*d^2)*Sec[c + d*x^2)/(4*a*(a + b*Sec[c + d*x^2]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3}{a + b \sec(c + dx^2)} dx \\
 & \quad \downarrow 4692 \\
 & \frac{1}{2} \int \frac{x^2}{a + b \sec(dx^2 + c)} dx^2 \\
 & \quad \downarrow 3042 \\
 & \frac{1}{2} \int \frac{x^2}{a + b \csc(dx^2 + c + \frac{\pi}{2})} dx^2 \\
 & \quad \downarrow 4679 \\
 & \frac{1}{2} \int \left(\frac{x^2}{a} - \frac{bx^2}{a(b + a \cos(dx^2 + c))} \right) dx^2 \\
 & \quad \downarrow 2009 \\
 & \frac{1}{2} \left(\frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}} - \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}} \right)
 \end{aligned}$$

input `Int[x^3/(a + b*Sec[c + d*x^2]),x]`

output `(x^4/(2*a) + (I*b*x^2*Log[1 + (a*E^(I*(c + d*x^2))]/(b - Sqrt[-a^2 + b^2]))/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^2*Log[1 + (a*E^(I*(c + d*x^2))]/(b + Sqrt[-a^2 + b^2]))/(a*Sqrt[-a^2 + b^2]*d) + (b*PolyLog[2, -((a*E^(I*(c + d*x^2))]/(b - Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^2) - (b*PolyLog[2, -((a*E^(I*(c + d*x^2))]/(b + Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^2)))/2`

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sinn[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x^3}{a + b \sec(dx^2 + c)} dx$$

input `int(x^3/(a+b*sec(d*x^2+c)),x)`

output `int(x^3/(a+b*sec(d*x^2+c)),x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(221) = 442$.

Time = 0.21 (sec) , antiderivative size = 1060, normalized size of antiderivative = 4.06

$$\int \frac{x^3}{a + b \sec(c + dx^2)} dx = \text{Too large to display}$$

input `integrate(x^3/(a+b*sec(d*x^2+c)),x, algorithm="fricas")`

output

```
1/4*((a^2 - b^2)*d^2*x^4 - I*a*b*c*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^
2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + I*a*b*
c*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2
*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - I*a*b*c*sqrt(-(a^2 - b^2)/a^2)*log(-2*a
*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b)
+ I*a*b*c*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^
2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) - a*b*sqrt(-(a^2 - b^2)/a^2)*di
log(-(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(
d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + a*b*sqrt(-(a^2 - b^2)/a^2
)*dilog(-(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a
sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - a*b*sqrt(-(a^2 - b^2)
/a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) -
I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + a*b*sqrt(-(a^2 -
b^2)/a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c)
- I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - (I*a*b*d*x^2
+ I*a*b*c)*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) + I*b*sin(d*x^2 +
c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a
) - (-I*a*b*d*x^2 - I*a*b*c)*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c)
+ I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2
- b^2)/a^2) + a)/a) - (-I*a*b*d*x^2 - I*a*b*c)*sqrt(-(a^2 - b^2)/a^2)*1...
```


Sympy [F]

$$\int \frac{x^3}{a + b \sec(c + dx^2)} dx = \int \frac{x^3}{a + b \sec(c + dx^2)} dx$$

input `integrate(x**3/(a+b*sec(d*x**2+c)),x)`

output `Integral(x**3/(a + b*sec(c + d*x**2)), x)`

Maxima [F]

$$\int \frac{x^3}{a + b \sec(c + dx^2)} dx = \int \frac{x^3}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^3/(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

output `1/4*(x^4 - 8*a*b*integrate((a*x^3*cos(2*d*x^2 + 2*c))*cos(d*x^2 + c) + 2*b*x^3*cos(d*x^2 + c)^2 + a*x^3*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^3*sin(d*x^2 + c)^2 + a*x^3*cos(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*cos(d*x^2 + c)^2 + a^3*sin(2*d*x^2 + 2*c)^2 + 4*a^2*b*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c) + a^3 + 2*(2*a^2*b*cos(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a`

Giac [F]

$$\int \frac{x^3}{a + b \sec(c + dx^2)} dx = \int \frac{x^3}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^3/(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output `integrate(x^3/(b*sec(d*x^2 + c) + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b \sec(c + dx^2)} dx = \int \frac{x^3}{a + \frac{b}{\cos(dx^2+c)}} dx$$

input `int(x^3/(a + b/cos(c + d*x^2)),x)`output `int(x^3/(a + b/cos(c + d*x^2)), x)`**Reduce [F]**

$$\int \frac{x^3}{a + b \sec(c + dx^2)} dx$$

$$= \frac{8 \left(\int \frac{\tan\left(\frac{dx^2+c}{2}\right)^2 x^3}{\tan\left(\frac{dx^2+c}{2}\right)^2 a^2 - \tan\left(\frac{dx^2+c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) ab + 8 \left(\int \frac{\tan\left(\frac{dx^2+c}{2}\right)^2 x^3}{\tan\left(\frac{dx^2+c}{2}\right)^2 a^2 - \tan\left(\frac{dx^2+c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) b^2 + \dots}{4a + 4b}$$

input `int(x^3/(a+b*sec(d*x^2+c)),x)`output `(8*int((tan((c + d*x**2)/2)**2*x**3)/(tan((c + d*x**2)/2)**2*a**2 - tan((c + d*x**2)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*a*b + 8*int((tan((c + d*x**2)/2)**2*x**3)/(tan((c + d*x**2)/2)**2*a**2 - tan((c + d*x**2)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*b**2 + x**4)/(4*(a + b))`

3.19 $\int \frac{x^2}{a+b \sec(c+dx^2)} dx$

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Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx = \text{Int}\left(\frac{x^2}{a + b \sec(c + dx^2)}, x\right)$$

output `Defer(Int)(x^2/(a+b*sec(d*x^2+c)),x)`

Mathematica [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx = \int \frac{x^2}{a + b \sec(c + dx^2)} dx$$

input `Integrate[x^2/(a + b*Sec[c + d*x^2]),x]`

output `Integrate[x^2/(a + b*Sec[c + d*x^2]), x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx$$

↓ 4694

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx$$

input `Int[x^2/(a + b*Sec[c + d*x^2]),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a + b \sec(dx^2 + c)} dx$$

input `int(x^2/(a+b*sec(d*x^2+c)),x)`

output `int(x^2/(a+b*sec(d*x^2+c)),x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx = \int \frac{x^2}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^2/(a+b*sec(d*x^2+c)),x, algorithm="fricas")`

output `integral(x^2/(b*sec(d*x^2 + c) + a), x)`

Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx = \int \frac{x^2}{a + b \sec(c + dx^2)} dx$$

input `integrate(x**2/(a+b*sec(d*x**2+c)),x)`

output `Integral(x**2/(a + b*sec(c + d*x**2)), x)`

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 14.00

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx = \int \frac{x^2}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^2/(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

output `1/3*(x^3 - 6*a*b*integrate((a*x^2*cos(2*d*x^2 + 2*c))*cos(d*x^2 + c) + 2*b*x^2*cos(d*x^2 + c)^2 + a*x^2*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^2*sin(d*x^2 + c)^2 + a*x^2*cos(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*cos(d*x^2 + c)^2 + a^3*sin(2*d*x^2 + 2*c)^2 + 4*a^2*b*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c) + a^3 + 2*(2*a^2*b*cos(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a`

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx = \int \frac{x^2}{b \sec(dx^2 + c) + a} dx$$

input `integrate(x^2/(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output `integrate(x^2/(b*sec(d*x^2 + c) + a), x)`

Mupad [N/A]

Not integrable

Time = 15.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx = \int \frac{x^2}{a + \frac{b}{\cos(dx^2+c)}} dx$$

input `int(x^2/(a + b/cos(c + d*x^2)),x)`output `int(x^2/(a + b/cos(c + d*x^2)), x)`**Reduce [N/A]**

Not integrable

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 9.17

$$\int \frac{x^2}{a + b \sec(c + dx^2)} dx$$

$$= \frac{6 \left(\int \frac{\tan\left(\frac{dx^2+c}{2}\right)^2 x^2}{\tan\left(\frac{dx^2+c}{2}\right)^2 a^2 - \tan\left(\frac{dx^2+c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) ab + 6 \left(\int \frac{\tan\left(\frac{dx^2+c}{2}\right)^2 x^2}{\tan\left(\frac{dx^2+c}{2}\right)^2 a^2 - \tan\left(\frac{dx^2+c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) b^2 + x^3}{3a + 3b}$$

input `int(x^2/(a+b*sec(d*x^2+c)),x)`output `(6*int((tan((c + d*x**2)/2)**2*x**2)/(tan((c + d*x**2)/2)**2*a**2 - tan((c + d*x**2)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*a*b + 6*int((tan((c + d*x**2)/2)**2*x**2)/(tan((c + d*x**2)/2)**2*a**2 - tan((c + d*x**2)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*b**2 + x**3)/(3*(a + b))`

3.20 $\int \frac{x}{a+b \sec(c+dx^2)} dx$

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Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{x}{a+b \sec(c+dx^2)} dx = \frac{x^2}{2a} - \frac{\operatorname{barctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+bd}}$$

output $\frac{1}{2}x^2/a-b*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*d*x^2+1/2*c)/(a+b)^{(1/2)})/a/(a-b)^{(1/2)}/(a+b)^{(1/2)}/d$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x}{a+b \sec(c+dx^2)} dx = \frac{\frac{c}{d} + x^2 + \frac{2b \operatorname{arctanh}\left(\frac{(-a+b) \tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}}{2a}$$

input `Integrate[x/(a + b*Sec[c + d*x^2]),x]`

output $(c/d + x^2 + (2*b*\operatorname{ArcTanh}[((-a + b)*\operatorname{Tan}[(c + d*x^2)/2])/ \operatorname{Sqrt}[a^2 - b^2]])/(\operatorname{Sqrt}[a^2 - b^2]*d)/(2*a)$

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4692, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{a + b \sec(c + dx^2)} dx \\
 & \quad \downarrow 4692 \\
 & \frac{1}{2} \int \frac{1}{a + b \sec(dx^2 + c)} dx^2 \\
 & \quad \downarrow 3042 \\
 & \frac{1}{2} \int \frac{1}{a + b \csc(dx^2 + c + \frac{\pi}{2})} dx^2 \\
 & \quad \downarrow 4270 \\
 & \frac{1}{2} \left(\frac{x^2}{a} - \frac{\int \frac{1}{\frac{a \cos(dx^2 + c)}{b} + 1} dx^2}{a} \right) \\
 & \quad \downarrow 3042 \\
 & \frac{1}{2} \left(\frac{x^2}{a} - \frac{\int \frac{1}{\frac{a \sin(dx^2 + c + \frac{\pi}{2})}{b} + 1} dx^2}{a} \right) \\
 & \quad \downarrow 3138 \\
 & \frac{1}{2} \left(\frac{x^2}{a} - \frac{2 \int \frac{1}{(1 - \frac{a}{b})x^4 + \frac{a+b}{b}} d \tan(\frac{1}{2}(dx^2 + c))}{ad} \right) \\
 & \quad \downarrow 221 \\
 & \frac{1}{2} \left(\frac{x^2}{a} - \frac{2b \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx^2))}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}} \right)
 \end{aligned}$$

input `Int[x/(a + b*Sec[c + d*x^2]),x]`

output `(x^2/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x^2)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/2`

Defintions of rubi rules used

rule 221 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3138 `Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 4270 `Int[(csc[(c_) + (d_)*(x_)]*(b_) + (a_))(-1), x_Symbol] := Simp[x/a, x] - Simp[1/a Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 4692 `Int[(x_)^(m_)*((a_) + (b_)*Sec[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{2 \arctan\left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)}{a} - \frac{2b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}$	70
default	$\frac{2 \arctan\left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)}{a} - \frac{2b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}$	70
risch	$\frac{x^2}{2a} + \frac{b \ln\left(\frac{e^{i(dx^2+c)} - ia^2 - ib^2 - b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}a}\right)}{2\sqrt{a^2-b^2}da} - \frac{b \ln\left(\frac{e^{i(dx^2+c)} + ia^2 - ib^2 + b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}a}\right)}{2\sqrt{a^2-b^2}da}$	160

```
input int(x/(a+b*sec(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
output 1/2/d*(2/a*arctan(tan(1/2*d*x^2+1/2*c))-2*b/a/((a-b)*(a+b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x^2+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 3.80

$$\int \frac{x}{a + b \sec(c + dx^2)} dx$$

$$= \frac{2(a^2 - b^2)dx^2 + \sqrt{a^2 - b^2}b \log\left(\frac{2ab \cos(dx^2+c) - (a^2 - 2b^2) \cos(dx^2+c)^2 - 2\sqrt{a^2-b^2}(b \cos(dx^2+c) + a) \sin(dx^2+c) + 2a^2 - b^2}{a^2 \cos(dx^2+c)^2 + 2ab \cos(dx^2+c) + b^2}\right)}{4(a^3 - ab^2)d}$$

```
input integrate(x/(a+b*sec(d*x^2+c)),x, algorithm="fricas")
```

output

```
[1/4*(2*(a^2 - b^2)*d*x^2 + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x^2 + c) -
(a^2 - 2*b^2)*cos(d*x^2 + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x^2 + c) + a)*
sin(d*x^2 + c) + 2*a^2 - b^2)/(a^2*cos(d*x^2 + c)^2 + 2*a*b*cos(d*x^2 + c)
+ b^2)))/((a^3 - a*b^2)*d), 1/2*((a^2 - b^2)*d*x^2 - sqrt(-a^2 + b^2)*b*a
rctan(-sqrt(-a^2 + b^2)*(b*cos(d*x^2 + c) + a)/((a^2 - b^2)*sin(d*x^2 + c)
)))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{x}{a + b \sec(c + dx^2)} dx = \int \frac{x}{a + b \sec(c + dx^2)} dx$$

input

```
integrate(x/(a+b*sec(d*x**2+c)),x)
```

output

```
Integral(x/(a + b*sec(c + d*x**2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7945 vs. 2(55) = 110.

Time = 23.27 (sec) , antiderivative size = 7945, normalized size of antiderivative = 120.38

$$\int \frac{x}{a + b \sec(c + dx^2)} dx = \text{Too large to display}$$

input

```
integrate(x/(a+b*sec(d*x^2+c)),x, algorithm="maxima")
```

output

```

1/2*(sqrt(-a^2 + b^2)*d*x^2 - b*arctan2(2*(4*(a^6 - a^4*b^2)*cos(d*x^2 + 2
*c)^4*cos(c)*sin(c) - 4*(a^6 - a^4*b^2)*cos(c)*sin(d*x^2 + 2*c)^4*sin(c) +
4*(3*(a^5*b - a^3*b^3)*cos(c)^2*sin(c) + (a^5*b - a^3*b^3)*sin(c)^3)*cos(
d*x^2 + 2*c)^3 - 4*((a^5*b - a^3*b^3)*cos(c)^3 + 3*(a^5*b - a^3*b^3)*cos(c)
)*sin(c)^2 + ((a^6 - a^4*b^2)*cos(c)^2 - (a^6 - a^4*b^2)*sin(c)^2)*cos(d*x
^2 + 2*c))*sin(d*x^2 + 2*c)^3 - 4*((a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)^3*
sin(c) + (a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2
+ 4*((a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)^3*sin(c) + (a^6 - 5*a^4*b^2 + 4
*a^2*b^4)*cos(c)*sin(c)^3 - 3*((a^5*b - a^3*b^3)*cos(c)^2*sin(c) - (a^5*b
- a^3*b^3)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 4*((a^5*b - 3*
a^3*b^3 + 2*a*b^5)*cos(c)^4*sin(c) + 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(c)
)^2*sin(c)^3 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*sin(c)^5)*cos(d*x^2 + 2*c) +
4*((a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(c)^5 + 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5
)*cos(c)^3*sin(c)^2 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(c)*sin(c)^4 - ((a^
6 - a^4*b^2)*cos(c)^2 - (a^6 - a^4*b^2)*sin(c)^2)*cos(d*x^2 + 2*c)^3 - 3*(
(a^5*b - a^3*b^3)*cos(c)^3 - (a^5*b - a^3*b^3)*cos(c)*sin(c)^2)*cos(d*x^2
+ 2*c)^2 + ((a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)^4 - (a^6 - 5*a^4*b^2 + 4*
a^2*b^4)*sin(c)^4)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c) + (a^5*cos(c)*sin(d*
x^2 + 2*c)^5 - a^5*cos(d*x^2 + 2*c)^5*sin(c) - 4*a^4*b*cos(d*x^2 + 2*c)^4*
cos(c)*sin(c) - (a^5*cos(d*x^2 + 2*c)*sin(c) - 4*a^4*b*cos(c)*sin(c))*s...

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(55) = 110.

Time = 0.34 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.21

$$\int \frac{x}{a + b \sec(c + dx^2)} dx$$

$$= \frac{(\sqrt{-a^2 + b^2}(a - 2b)d|-a + b| - \sqrt{-a^2 + b^2}|a||-a + b||d|) \left(\pi \left[\frac{dx^2+c}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\tan(\frac{1}{2} dx^2 + \frac{1}{2} c)}{\sqrt{-\frac{bd + \sqrt{b^2 d^2 + (ad+bd)(ad-bd)}}{ad-bd}}} \right)}{2((a^2 - 2ab + b^2)a^2 d^2 + (a^2 b - 2ab^2 + b^3)d|a||d|)} \right)}{2((a^2 - 2ab + b^2)a^2 d^2 + (a^2 b - 2ab^2 + b^3)d|a||d|)}$$

$$+ \frac{(ad - 2bd + |a||d|) \left(\pi \left[\frac{dx^2+c}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\tan(\frac{1}{2} dx^2 + \frac{1}{2} c)}{\sqrt{-\frac{bd - \sqrt{b^2 d^2 + (ad+bd)(ad-bd)}}{ad-bd}}} \right)}{2(a^2 d^2 - bd|a||d|)} \right)}{2(a^2 d^2 - bd|a||d|)}$$

input

```
integrate(x/(a+b*sec(d*x^2+c)),x, algorithm="giac")
```

output

```
1/2*(sqrt(-a^2 + b^2)*(a - 2*b)*d*abs(-a + b) - sqrt(-a^2 + b^2)*abs(a)*abs(-a + b)*abs(d))*(pi*floor(1/2*(d*x^2 + c)/pi + 1/2) + arctan(tan(1/2*d*x^2 + 1/2*c)/sqrt(-(b*d + sqrt(b^2*d^2 + (a*d + b*d)*(a*d - b*d)))/(a*d - b*d))))/((a^2 - 2*a*b + b^2)*a^2*d^2 + (a^2*b - 2*a*b^2 + b^3)*d*abs(a)*abs(d)) + 1/2*(a*d - 2*b*d + abs(a)*abs(d))*(pi*floor(1/2*(d*x^2 + c)/pi + 1/2) + arctan(tan(1/2*d*x^2 + 1/2*c)/sqrt(-(b*d - sqrt(b^2*d^2 + (a*d + b*d)*(a*d - b*d)))/(a*d - b*d))))/((a^2*d^2 - b*d*abs(a)*abs(d))
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.38

$$\int \frac{x}{a + b \sec(c + dx^2)} dx = \frac{x^2}{2a} + \frac{b \ln \left(2bx e^{dx^2} e^{ci} - \frac{bx(a + be^{dx^2} e^{ci})^{2i}}{\sqrt{a+b}\sqrt{a-b}} \right)}{2ad\sqrt{a+b}\sqrt{a-b}} - \frac{b \ln \left(2bx e^{dx^2} e^{ci} + \frac{bx(a + be^{dx^2} e^{ci})^{2i}}{\sqrt{a+b}\sqrt{a-b}} \right)}{2ad\sqrt{a+b}\sqrt{a-b}}$$

input

```
int(x/(a + b/cos(c + d*x^2)),x)
```

output

```
x^2/(2*a) + (b*log(2*b*x*exp(d*x^2*i)*exp(c*i) - (b*x*(a + b*exp(d*x^2*i)*exp(c*i))^2i)/((a + b)^(1/2)*(a - b)^(1/2))))/(2*a*d*(a + b)^(1/2)*(a - b)^(1/2)) - (b*log(2*b*x*exp(d*x^2*i)*exp(c*i) + (b*x*(a + b*exp(d*x^2*i)*exp(c*i))^2i)/((a + b)^(1/2)*(a - b)^(1/2))))/(2*a*d*(a + b)^(1/2)*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{x}{a + b \sec(c + dx^2)} dx = \frac{-2\sqrt{-a^2 + b^2} \operatorname{atan} \left(\frac{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)a - \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)b}{\sqrt{-a^2 + b^2}} \right) b + a^2 dx^2 - b^2 dx^2}{2ad(a^2 - b^2)}$$

input `int(x/(a+b*sec(d*x^2+c)),x)`

output `(- 2*sqrt(- a**2 + b**2)*atan((tan((c + d*x**2)/2)*a - tan((c + d*x**2)/2)*b)/sqrt(- a**2 + b**2))*b + a**2*d*x**2 - b**2*d*x**2)/(2*a*d*(a**2 - b**2))`

3.21 $\int \frac{1}{x(a+b \sec(c+dx^2))} dx$

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Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x(a+b \sec(c+dx^2))} dx = \text{Int}\left(\frac{1}{x(a+b \sec(c+dx^2))}, x\right)$$

output `Defer(Int)(1/x/(a+b*sec(d*x^2+c)), x)`

Mathematica [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+b \sec(c+dx^2))} dx = \int \frac{1}{x(a+b \sec(c+dx^2))} dx$$

input `Integrate[1/(x*(a + b*Sec[c + d*x^2])), x]`

output `Integrate[1/(x*(a + b*Sec[c + d*x^2])), x]`

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a + b \sec(c + dx^2))} dx$$

↓ 4694

$$\int \frac{1}{x(a + b \sec(c + dx^2))} dx$$

input `Int[1/(x*(a + b*Sec[c + d*x^2])),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a + b \sec(dx^2 + c))} dx$$

input `int(1/x/(a+b*sec(d*x^2+c)),x)`

output `int(1/x/(a+b*sec(d*x^2+c)),x)`

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x(a + b \sec(c + dx^2))} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)x} dx$$

input `integrate(1/x/(a+b*sec(d*x^2+c)),x, algorithm="fricas")`

output `integral(1/(b*x*sec(d*x^2 + c) + a*x), x)`

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a + b \sec(c + dx^2))} dx = \int \frac{1}{x(a + b \sec(c + dx^2))} dx$$

input `integrate(1/x/(a+b*sec(d*x**2+c)),x)`

output `Integral(1/(x*(a + b*sec(c + d*x**2))), x)`

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 13.83

$$\int \frac{1}{x(a + b \sec(c + dx^2))} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)x} dx$$

input `integrate(1/x/(a+b*sec(d*x^2+c)),x, algorithm="maxima")`

output `-(2*a*b*integrate((a*cos(2*d*x^2 + 2*c))*cos(d*x^2 + c) + 2*b*cos(d*x^2 + c)^2 + a*sin(2*d*x^2 + 2*c))*sin(d*x^2 + c) + 2*b*sin(d*x^2 + c)^2 + a*cos(d*x^2 + c))/(a^3*x*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*x*cos(d*x^2 + c)^2 + a^3*x*sin(2*d*x^2 + 2*c)^2 + 4*a^2*b*x*sin(2*d*x^2 + 2*c))*sin(d*x^2 + c) + 4*a*b^2*x*sin(d*x^2 + c)^2 + 4*a^2*b*x*cos(d*x^2 + c) + a^3*x + 2*(2*a^2*b*x*cos(d*x^2 + c) + a^3*x)*cos(2*d*x^2 + 2*c)), x) - log(x))/a`

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a + b \sec(c + dx^2))} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)x} dx$$

input `integrate(1/x/(a+b*sec(d*x^2+c)),x, algorithm="giac")`

output `integrate(1/((b*sec(d*x^2 + c) + a)*x), x)`

Mupad [N/A]

Not integrable

Time = 15.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(a + b \sec(c + dx^2))} dx = \int \frac{1}{x \left(a + \frac{b}{\cos(dx^2+c)} \right)} dx$$

input `int(1/(x*(a + b/cos(c + d*x^2))),x)`output `int(1/(x*(a + b/cos(c + d*x^2))), x)`**Reduce [N/A]**

Not integrable

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 8.89

$$\int \frac{1}{x(a + b \sec(c + dx^2))} dx$$

$$= \frac{2 \left(\int \frac{1}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 a^2 x - 2 \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 abx + \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 b^2 x - a^2 x + b^2 x} dx \right) ab - 2 \left(\int \frac{1}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 a^2 x - 2 \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 abx} dx \right)}{a - b}$$

input `int(1/x/(a+b*sec(d*x^2+c)),x)`output `(2*int(1/(tan((c + d*x**2)/2)**2*a**2*x - 2*tan((c + d*x**2)/2)**2*a*b*x + tan((c + d*x**2)/2)**2*b**2*x - a**2*x + b**2*x),x)*a*b - 2*int(1/(tan((c + d*x**2)/2)**2*a**2*x - 2*tan((c + d*x**2)/2)**2*a*b*x + tan((c + d*x**2)/2)**2*b**2*x - a**2*x + b**2*x),x)*b**2 + log(x))/(a - b)`

3.22 $\int \frac{a+b \sec(c+dx^2)}{x^2} dx$

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Reduce [N/A]	184

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = -\frac{a}{x} + b \operatorname{Int}\left(\frac{\sec(c + dx^2)}{x^2}, x\right)$$

output `-a/x+b*Defer(Int)(sec(d*x^2+c)/x^2,x)`

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

input `Integrate[(a + b*Sec[c + d*x^2])/x^2,x]`

output `Integrate[(a + b*Sec[c + d*x^2])/x^2, x]`

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

↓ 2010

$$\int \left(\frac{a}{x^2} + \frac{b \sec(c + dx^2)}{x^2} \right) dx$$

↓ 2009

$$b \int \frac{\sec(dx^2 + c)}{x^2} dx - \frac{a}{x}$$

input `Int[(a + b*Sec[c + d*x^2])/x^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(dx^2 + c)}{x^2} dx$$

input `int((a+b*sec(d*x^2+c))/x^2,x)`output `int((a+b*sec(d*x^2+c))/x^2,x)`**Fricas [N/A]**

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{b \sec(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="fricas")`output `integral((b*sec(d*x^2 + c) + a)/x^2, x)`**Sympy [N/A]**

Not integrable

Time = 0.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{a + b \sec(c + dx^2)}{x^2} dx$$

input `integrate((a+b*sec(d*x**2+c))/x**2,x)`output `Integral((a + b*sec(c + d*x**2))/x**2, x)`

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 7.38

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{b \sec(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="maxima")`

output `2*b*integrate((cos(2*d*x^2 + 2*c)*cos(d*x^2 + c) + sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + cos(d*x^2 + c))/(x^2*cos(2*d*x^2 + 2*c)^2 + x^2*sin(2*d*x^2 + 2*c)^2 + 2*x^2*cos(2*d*x^2 + 2*c) + x^2), x) - a/x`

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{b \sec(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sec(d*x^2+c))/x^2,x, algorithm="giac")`

output `integrate((b*sec(d*x^2 + c) + a)/x^2, x)`

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \int \frac{a + \frac{b}{\cos(dx^2+c)}}{x^2} dx$$

input `int((a + b/cos(c + d*x^2))/x^2,x)`

output `int((a + b/cos(c + d*x^2))/x^2, x)`

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.50

$$\int \frac{a + b \sec(c + dx^2)}{x^2} dx = \frac{-2 \left(\int \frac{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2}{\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 x^2 - x^2} dx \right) bx - a - b}{x}$$

input `int((a+b*sec(d*x^2+c))/x^2,x)`

output `(- 2*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**2*x**2 - x**2),x)*b
*x - a - b)/x`

$$3.23 \quad \int \frac{x^5}{(a+b \sec(c+dx^2))^2} dx$$

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Optimal result

Integrand size = 18, antiderivative size = 1092

$$\int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx = \text{Too large to display}$$

output

```
-I*b^2*polylog(2,-a*exp(I*(d*x^2+c))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/
d^3+1/6*x^6/a^2+b^2*x^2*ln(1+a*exp(I*(d*x^2+c)))/(b-I*(a^2-b^2)^(1/2))/a^2
/(a^2-b^2)/d^2+b^2*x^2*ln(1+a*exp(I*(d*x^2+c)))/(b+I*(a^2-b^2)^(1/2))/a^2/
(a^2-b^2)/d^2+1/2*I*b^3*x^4*ln(1+a*exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^(1/2))/
a^2/(-a^2+b^2)^(3/2)/d-I*b^3*polylog(3,-a*exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^(
1/2))/a^2/(-a^2+b^2)^(3/2)/d^3-I*b^2*polylog(2,-a*exp(I*(d*x^2+c)))/(b-I*(
a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^3-I*b*x^4*ln(1+a*exp(I*(d*x^2+c)))/(b+(-a^
2+b^2)^(1/2))/a^2/(-a^2+b^2)^(1/2)/d+I*b^3*polylog(3,-a*exp(I*(d*x^2+c)))/
(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^3-1/2*I*b^3*x^4*ln(1+a*exp(I*
(d*x^2+c)))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d-b^3*x^2*polylog(2,
-a*exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^2+2*b*x^2
*polylog(2,-a*exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(1/2)/
d^2+b^3*x^2*polylog(2,-a*exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+
b^2)^(3/2)/d^2-2*b*x^2*polylog(2,-a*exp(I*(d*x^2+c)))/(b+(-a^2+b^2)^(1/2))
/a^2/(-a^2+b^2)^(1/2)/d^2+I*b*x^4*ln(1+a*exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^(1
/2))/a^2/(-a^2+b^2)^(1/2)/d-2*I*b*polylog(3,-a*exp(I*(d*x^2+c)))/(b+(-a^2+
b^2)^(1/2))/a^2/(-a^2+b^2)^(1/2)/d^3-1/2*I*b^2*x^4/a^2/(a^2-b^2)/d+2*I*b*
polylog(3,-a*exp(I*(d*x^2+c)))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(1/2)/d
^3+1/2*b^2*x^4*sin(d*x^2+c)/a/(a^2-b^2)/d/(b+a*cos(d*x^2+c))
```

Mathematica [A] (verified)

Time = 4.85 (sec) , antiderivative size = 895, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx$$

$$= \frac{(b + a \cos(c + dx^2)) \sec^2(c + dx^2) \left(x^6(b + a \cos(c + dx^2)) - \frac{3b(b + a \cos(c + dx^2)) \left(2(1 + e^{2ic}) (ib\sqrt{-a^2 + b^2})e^{2ic} - 2a \right)}{\dots} \right)}{\dots}$$

input

```
Integrate[x^5/(a + b*Sec[c + d*x^2])^2,x]
```

output

```

((b + a*cos[c + d*x^2])*sec[c + d*x^2]^2*(x^6*(b + a*cos[c + d*x^2]) - (3*
b*(b + a*cos[c + d*x^2])*(2*(1 + E^((2*I)*c)))*(I*b*Sqrt[(-a^2 + b^2)*E^((2
*I)*c)] - 2*a^2*d*E^(I*c)*x^2 + b^2*d*E^(I*c)*x^2)*PolyLog[2, -((a*E^(I*(2
*c + d*x^2)))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))]) + 2*(1 + E^((
2*I)*c))*(I*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*x^2 - b^2*d
*E^(I*c)*x^2)*PolyLog[2, -((a*E^(I*(2*c + d*x^2)))/(b*E^(I*c) + Sqrt[(-a^2
+ b^2)*E^((2*I)*c)]))]) + I*(d*x^2*(2*b*d*E^((2*I)*c)*Sqrt[(-a^2 + b^2)*E^
((2*I)*c)]*x^2 + (1 + E^((2*I)*c))*((2*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]
- 2*a^2*d*E^(I*c)*x^2 + b^2*d*E^(I*c)*x^2)*Log[1 + (a*E^(I*(2*c + d*x^2))
)/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] + (1 + E^((2*I)*c))*((2*I)
*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*x^2 - b^2*d*E^(I*c)*x^
2)*Log[1 + (a*E^(I*(2*c + d*x^2)))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)
*c)])] - 2*(2*a^2 - b^2)*E^(I*c)*(1 + E^((2*I)*c))*PolyLog[3, -((a*E^(I*(
2*c + d*x^2)))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]) + 2*(2*a^2 -
b^2)*E^(I*c)*(1 + E^((2*I)*c))*PolyLog[3, -((a*E^(I*(2*c + d*x^2)))/(b*E^
(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])])])/(a^2 - b^2)*d^3*Sqrt[(-a^2 +
b^2)*E^((2*I)*c)]*(1 + E^((2*I)*c)) + (3*b^2*x^4*(-(b*sin[c]) + a*sin[d*
x^2]))/(a - b)*(a + b)*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])))/(
6*a^2*(a + b*Sec[c + d*x^2])^2)

```

Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 1090, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx \\
 & \quad \downarrow 4692 \\
 & \frac{1}{2} \int \frac{x^4}{(a + b \sec(dx^2 + c))^2} dx^2 \\
 & \quad \downarrow 3042 \\
 & \frac{1}{2} \int \frac{x^4}{(a + b \csc(dx^2 + c + \frac{\pi}{2}))^2} dx^2
 \end{aligned}$$

$$\frac{1}{2} \int \left(-\frac{2bx^4}{a^2(b+a\cos(dx^2+c))} + \frac{x^4}{a^2} + \frac{b^2x^4}{a^2(b+a\cos(dx^2+c))^2} \right) dx^2$$

$$\frac{1}{2} \left(\frac{x^6}{3a^2} + \frac{2ib \log\left(\frac{e^{i(dx^2+c)}a}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{a^2\sqrt{b^2-a^2}d} - \frac{ib^3 \log\left(\frac{e^{i(dx^2+c)}a}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{a^2(b^2-a^2)^{3/2}d} - \frac{2ib \log\left(\frac{e^{i(dx^2+c)}a}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{a^2\sqrt{b^2-a^2}d} + \frac{ib^3 \log\left(\frac{e^{i(dx^2+c)}a}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{a^2(b^2-a^2)^{3/2}d} \right)$$

input `Int[x^5/(a + b*Sec[c + d*x^2])^2,x]`

output

```
((-I)*b^2*x^4)/(a^2*(a^2 - b^2)*d) + x^6/(3*a^2) + (2*b^2*x^2*Log[1 + (a*
E^(I*(c + d*x^2)))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) + (2*b^
2*x^2*Log[1 + (a*E^(I*(c + d*x^2)))/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 -
b^2)*d^2) - (I*b^3*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]
)])/((a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^4*Log[1 + (a*E^(I*(c + d*x^2)
)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + (I*b^3*x^4*Log[1 + (
a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) -
((2*I)*b*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*
Sqrt[-a^2 + b^2]*d) - ((2*I)*b^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - I
*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - ((2*I)*b^2*PolyLog[2, -((a*E^
(I*(c + d*x^2)))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (2*b^3
*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a
^2 + b^2)^(3/2)*d^2) + (4*b*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b - Sq
rt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (2*b^3*x^2*PolyLog[2, -((a
*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2)
- (4*b*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2]))])/(
a^2*Sqrt[-a^2 + b^2]*d^2) - ((2*I)*b^3*PolyLog[3, -((a*E^(I*(c + d*x^2)))/
(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((4*I)*b*PolyLog[
3, -((a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]
*d^3) + ((2*I)*b^3*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + ...
```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x^5}{(a + b \sec(dx^2 + c))^2} dx$$

input `int(x^5/(a+b*sec(d*x^2+c))^2,x)`

output `int(x^5/(a+b*sec(d*x^2+c))^2,x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3050 vs. $2(958) = 1916$.

Time = 0.29 (sec) , antiderivative size = 3050, normalized size of antiderivative = 2.79

$$\int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx = \text{Too large to display}$$

input `integrate(x^5/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `Too large to include`

Sympy [F]

$$\int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx$$

input `integrate(x**5/(a+b*sec(d*x**2+c))**2,x)`

output `Integral(x**5/(a + b*sec(c + d*x**2))**2, x)`

Maxima [F]

$$\int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^5}{(b \sec(dx^2 + c) + a)^2} dx$$

input `integrate(x^5/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output

```

1/6*((a^4 - a^2*b^2)*d*x^6*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^6*
cos(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^6*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2
- b^4)*d*x^6*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*x^6*cos(d*x^2 + c) +
6*a*b^3*x^4*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6 + 2*(2*(a^3*b - a*b^3)*
d*x^6*cos(d*x^2 + c) - 3*a*b^3*x^4*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6)
*cos(2*d*x^2 + 2*c) - 6*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2
^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2
+ 4*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a
^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 -
a^4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)
*cos(2*d*x^2 + 2*c))*integrate(2*(2*(2*a^2*b^2 - b^4)*d*x^5*cos(d*x^2 + c)
^2 + 2*(2*a^2*b^2 - b^4)*d*x^5*sin(d*x^2 + c)^2 + (2*a^3*b - a*b^3)*d*x^5*
cos(d*x^2 + c) + 2*a*b^3*x^3*sin(d*x^2 + c) + ((2*a^3*b - a*b^3)*d*x^5*cos
(d*x^2 + c) - 2*a*b^3*x^3*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*a*b^3*x^
3*cos(d*x^2 + c) + (2*a^3*b - a*b^3)*d*x^5*sin(d*x^2 + c) + 2*a^2*b^2*x^3)
*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2
- a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4
*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*
b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^
4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)...

```

Giac [F]

$$\int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^5}{(b \sec(dx^2 + c) + a)^2} dx$$

input

```
integrate(x^5/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")
```

output

```
integrate(x^5/(b*sec(d*x^2 + c) + a)^2, x)
```


Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^5}{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

input `int(x^5/(a + b/cos(c + d*x^2))^2,x)`output `int(x^5/(a + b/cos(c + d*x^2))^2, x)`**Reduce [F]**

$$\int \frac{x^5}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^5}{\sec(dx^2 + c)^2 b^2 + 2 \sec(dx^2 + c) ab + a^2} dx$$

input `int(x^5/(a+b*sec(d*x^2+c))^2,x)`output `int(x**5/(sec(c + d*x**2)**2*b**2 + 2*sec(c + d*x**2)*a*b + a**2),x)`

$$3.24 \quad \int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx$$

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Mathematica [N/A]	193
Rubi [N/A]	194
Maple [N/A]	194
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Reduce [N/A]	197

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx = \text{Int}\left(\frac{x^4}{(a+b \sec(c+dx^2))^2}, x\right)$$

output `Defer(Int)(x^4/(a+b*sec(d*x^2+c))^2,x)`

Mathematica [N/A]

Not integrable

Time = 5.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx = \int \frac{x^4}{(a+b \sec(c+dx^2))^2} dx$$

input `Integrate[x^4/(a + b*Sec[c + d*x^2])^2,x]`

output `Integrate[x^4/(a + b*Sec[c + d*x^2])^2, x]`

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx$$

↓ 4694

$$\int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx$$

input `Int[x^4/(a + b*Sec[c + d*x^2])^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a + b \sec(dx^2 + c))^2} dx$$

input `int(x^4/(a+b*sec(d*x^2+c))^2,x)`

output `int(x^4/(a+b*sec(d*x^2+c))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^4}{(b \sec(dx^2 + c) + a)^2} dx$$

input `integrate(x^4/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `integral(x^4/(b^2*sec(d*x^2 + c)^2 + 2*a*b*sec(d*x^2 + c) + a^2), x)`

Sympy [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx$$

input `integrate(x**4/(a+b*sec(d*x**2+c))**2,x)`

output `Integral(x**4/(a + b*sec(c + d*x**2))**2, x)`

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 1284, normalized size of antiderivative = 71.33

$$\int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^4}{(b \sec(dx^2 + c) + a)^2} dx$$

input `integrate(x^4/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output

```
1/5*((a^4 - a^2*b^2)*d*x^5*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^5*
cos(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^5*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2
- b^4)*d*x^5*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*x^5*cos(d*x^2 + c) +
5*a*b^3*x^3*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^5 + (4*(a^3*b - a*b^3)*d*
x^5*cos(d*x^2 + c) - 5*a*b^3*x^3*sin(d*x^2 + c) + 2*(a^4 - a^2*b^2)*d*x^5)
*cos(2*d*x^2 + 2*c) - 5*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b
^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2
+ 4*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a
^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 -
a^4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)
*cos(2*d*x^2 + 2*c))*integrate((4*(2*a^2*b^2 - b^4)*d*x^4*cos(d*x^2 + c)^2
+ 4*(2*a^2*b^2 - b^4)*d*x^4*sin(d*x^2 + c)^2 + 2*(2*a^3*b - a*b^3)*d*x^4*
cos(d*x^2 + c) + 3*a*b^3*x^2*sin(d*x^2 + c) + (2*(2*a^3*b - a*b^3)*d*x^4*c
os(d*x^2 + c) - 3*a*b^3*x^2*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (3*a*b^3*
x^2*cos(d*x^2 + c) + 2*(2*a^3*b - a*b^3)*d*x^4*sin(d*x^2 + c) + 3*a^2*b^2*
x^2)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*
b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2
+ 4*(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 -
a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6
- a^4*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2...
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^4}{(b \sec(dx^2 + c) + a)^2} dx$$

input `integrate(x^4/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`output `integrate(x^4/(b*sec(d*x^2 + c) + a)^2, x)`**Mupad [N/A]**

Not integrable

Time = 15.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^4}{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

input `int(x^4/(a + b/cos(c + d*x^2))^2,x)`output `int(x^4/(a + b/cos(c + d*x^2))^2, x)`**Reduce [N/A]**

Not integrable

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^4}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^4}{\sec(dx^2 + c)^2 b^2 + 2 \sec(dx^2 + c) ab + a^2} dx$$

input `int(x^4/(a+b*sec(d*x^2+c))^2,x)`

output `int(x**4/(sec(c + d*x**2)**2*b**2 + 2*sec(c + d*x**2)*a*b + a**2),x)`

$$3.25 \quad \int \frac{x^3}{(a+b \sec(c+dx^2))^2} dx$$

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Mathematica [A] (warning: unable to verify)	201
Rubi [A] (verified)	201
Maple [F]	203
Fricas [B] (verification not implemented)	204
Sympy [F]	205
Maxima [F(-2)]	205
Giac [F]	205
Mupad [F(-1)]	206
Reduce [F]	206

Optimal result

Integrand size = 18, antiderivative size = 596

$$\begin{aligned}
 \int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx &= \frac{x^4}{4a^2} - \frac{ib^3 x^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
 &+ \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3 x^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
 &- \frac{ibx^2 \log\left(1 + \frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{b^2 \log(b + a \cos(c + dx^2))}{2a^2(a^2 - b^2)d^2} \\
 &- \frac{b^3 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} \\
 &+ \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
 &+ \frac{b^3 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} \\
 &- \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
 &+ \frac{b^2 x^2 \sin(c + dx^2)}{2a(a^2 - b^2)d(b + a \cos(c + dx^2))}
 \end{aligned}$$

output

```

1/4*x^4/a^2-1/2*I*b^3*x^2*ln(1+a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+I*b*x^2*ln(1+a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d+1/2*I*b^3*x^2*ln(1+a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-I*b*x^2*ln(1+a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d+1/2*b^2*ln(b+a*cos(d*x^2+c))/a^2/(a^2-b^2)/d^2-1/2*b^3*polylog(2,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+b*polylog(2,-a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2+1/2*b^3*polylog(2,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-b*polylog(2,-a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2+1/2*b^2*x^2*sin(d*x^2+c)/a/(a^2-b^2)/d/(b+a*cos(d*x^2+c))

```

Mathematica [A] (warning: unable to verify)

Time = 9.87 (sec) , antiderivative size = 1118, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx = \text{Too large to display}$$

input `Integrate[x^3/(a + b*Sec[c + d*x^2])^2,x]`

output

```
((-c + d*x^2)*(c + d*x^2)*(b + a*Cos[c + d*x^2])^2*Sec[c + d*x^2]^2)/(4*a^2*d^2*(a + b*Sec[c + d*x^2])^2) + ((b + a*Cos[c + d*x^2])*Sec[c + d*x^2]^2*(b^2*c*Sin[c + d*x^2] - b^2*(c + d*x^2)*Sin[c + d*x^2]))/(2*a*(-a + b)*(a + b)*d^2*(a + b*Sec[c + d*x^2])^2) + (b*Cos[(c + d*x^2)/2]^2*(b + a*Cos[c + d*x^2]))*(2*(2*a^2 - b^2)*c*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x^2)/2])/Sqrt[a + b]] - Sqrt[a - b]*b*Sqrt[a + b]*Log[Sec[(c + d*x^2)/2]^2] + Sqrt[a - b]*b*Sqrt[a + b]*Log[(b + a*Cos[c + d*x^2])*Sec[(c + d*x^2)/2]^2] + I*(2*a^2 - b^2)*(Log[1 - I*Tan[(c + d*x^2)/2]]*Log[(Sqrt[a + b] - Sqrt[a - b])*Tan[(c + d*x^2)/2]]/(I*Sqrt[a - b] + Sqrt[a + b])) + PolyLog[2, (Sqrt[a - b]*(1 - I*Tan[(c + d*x^2)/2]))/(Sqrt[a - b] - I*Sqrt[a + b])]) - I*(2*a^2 - b^2)*(Log[1 - I*Tan[(c + d*x^2)/2]]*Log[(I*(Sqrt[a + b] + Sqrt[a - b])*Tan[(c + d*x^2)/2]))/(Sqrt[a - b] + I*Sqrt[a + b])]) + PolyLog[2, (Sqrt[a - b]*(1 - I*Tan[(c + d*x^2)/2]))/(Sqrt[a - b] + I*Sqrt[a + b])]) + I*(2*a^2 - b^2)*(Log[1 + I*Tan[(c + d*x^2)/2]]*Log[(Sqrt[a + b] + Sqrt[a - b])*Tan[(c + d*x^2)/2]]/(I*Sqrt[a - b] + Sqrt[a + b])]) + PolyLog[2, (Sqrt[a - b]*(1 + I*Tan[(c + d*x^2)/2]))/(Sqrt[a - b] - I*Sqrt[a + b])]) - I*(2*a^2 - b^2)*(Log[1 + I*Tan[(c + d*x^2)/2]]*Log[(I*(Sqrt[a + b] - Sqrt[a - b])*Tan[(c + d*x^2)/2]))/(Sqrt[a - b] + I*Sqrt[a + b])]) + PolyLog[2, (Sqrt[a - b]*(1 + I*Tan[(c + d*x^2)/2]))/(Sqrt[a - b] + I*Sqrt[a + b])])]*Sec[c + d*x^2]^2*(2*a^2 - b^2)*d*x^2 + a*b*Sin[c + d*x^2]*(Sqrt[a + b] - Sqrt[a - b])*Ta...
```

Rubi [A] (verified)Time = 1.35 (sec) , antiderivative size = 586, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx \\
& \quad \downarrow 4692 \\
& \frac{1}{2} \int \frac{x^2}{(a + b \sec(dx^2 + c))^2} dx^2 \\
& \quad \downarrow 3042 \\
& \frac{1}{2} \int \frac{x^2}{(a + b \csc(dx^2 + c + \frac{\pi}{2}))^2} dx^2 \\
& \quad \downarrow 4679 \\
& \frac{1}{2} \int \left(-\frac{2bx^2}{a^2(b + a \cos(dx^2 + c))} + \frac{x^2}{a^2} + \frac{b^2x^2}{a^2(b + a \cos(dx^2 + c))^2} \right) dx^2 \\
& \quad \downarrow 2009 \\
& \frac{1}{2} \left(\frac{2b \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2 d^2 \sqrt{b^2-a^2}} - \frac{2b \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2 d^2 \sqrt{b^2-a^2}} + \frac{b^2 \log(a \cos(c + dx^2) + b)}{a^2 d^2 (a^2 - b^2)} + \frac{2ibx^2 \log\left(1 + \frac{b}{a^2 d \sqrt{b^2-a^2}}\right)}{a^2 d \sqrt{b^2-a^2}} \right)
\end{aligned}$$

input `Int[x^3/(a + b*Sec[c + d*x^2])^2,x]`

output
$$\begin{aligned}
& \left(\frac{x^4}{2a^2} - \frac{(I*b^3*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})]/(b - \operatorname{Sqrt}[-a^2 + b^2])]}{a^2*(-a^2 + b^2)^{(3/2)*d}} + \frac{((2*I)*b*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})]/(b - \operatorname{Sqrt}[-a^2 + b^2])]}{a^2*\operatorname{Sqrt}[-a^2 + b^2]*d} + \frac{(I*b^3*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})]/(b + \operatorname{Sqrt}[-a^2 + b^2])]}{a^2*(-a^2 + b^2)^{(3/2)*d}} \right. \\
& \left. - \frac{((2*I)*b*x^2*\operatorname{Log}[1 + (a*E^{(I*(c + d*x^2))})]/(b + \operatorname{Sqrt}[-a^2 + b^2])]}{a^2*\operatorname{Sqrt}[-a^2 + b^2]*d} + \frac{(b^2*\operatorname{Log}[b + a*\operatorname{Cos}[c + d*x^2]])}{a^2*(a^2 - b^2)*d^2} - \frac{(b^3*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b - \operatorname{Sqrt}[-a^2 + b^2])])]}{a^2*(-a^2 + b^2)^{(3/2)*d^2}} + \frac{(2*b*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b - \operatorname{Sqrt}[-a^2 + b^2])])]}{a^2*\operatorname{Sqrt}[-a^2 + b^2]*d^2} + \frac{(b^3*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b + \operatorname{Sqrt}[-a^2 + b^2])])]}{a^2*(-a^2 + b^2)^{(3/2)*d^2}} \right. \\
& \left. - \frac{(2*b*\operatorname{PolyLog}[2, -((a*E^{(I*(c + d*x^2))})/(b + \operatorname{Sqrt}[-a^2 + b^2])])]}{a^2*\operatorname{Sqrt}[-a^2 + b^2]*d^2} + \frac{(b^2*x^2*\operatorname{Sin}[c + d*x^2])}{a*(a^2 - b^2)*d*(b + a*\operatorname{Cos}[c + d*x^2])} \right) / 2
\end{aligned}$$

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x^3}{(a + b \sec(dx^2 + c))^2} dx$$

input `int(x^3/(a+b*sec(d*x^2+c))^2,x)`

output `int(x^3/(a+b*sec(d*x^2+c))^2,x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1928 vs. $2(522) = 1044$.

Time = 0.23 (sec) , antiderivative size = 1928, normalized size of antiderivative = 3.23

$$\int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx = \text{Too large to display}$$

input `integrate(x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output

```
1/4*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4*cos(d*x^2 + c) + (a^4*b - 2*a^2*b^3
+ b^5)*d^2*x^4 + 2*(a^3*b^2 - a*b^4)*d*x^2*sin(d*x^2 + c) - (2*a^3*b^2 -
a*b^4 + (2*a^4*b - a^2*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*dilog(-
(b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2
+ c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + (2*a^3*b^2 - a*b^4 + (2*a^4*b
- a^2*b^3)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cos(d*x^2 + c)
+ I*b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2
- b^2)/a^2) + a)/a + 1) - (2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cos(d*
x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2
+ c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)
/a + 1) + (2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cos(d*x^2 + c))*sqrt(-
(a^2 - b^2)/a^2)*dilog(-(b*cos(d*x^2 + c) - I*b*sin(d*x^2 + c) - (a*cos(d*x
^2 + c) - I*a*sin(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + (-I*(2*
a^3*b^2 - a*b^4)*d*x^2 - I*(2*a^3*b^2 - a*b^4)*c + (-I*(2*a^4*b - a^2*b^3)
*d*x^2 - I*(2*a^4*b - a^2*b^3)*c)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*l
og((b*cos(d*x^2 + c) + I*b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*
x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + (I*(2*a^3*b^2 - a*b^4)*d*x^2 +
I*(2*a^3*b^2 - a*b^4)*c + (I*(2*a^4*b - a^2*b^3)*d*x^2 + I*(2*a^4*b - a^2*
b^3)*c)*cos(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*log((b*cos(d*x^2 + c) + I*b
*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(-(a^2 - ...
```

Sympy [F]

$$\int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx$$

input `integrate(x**3/(a+b*sec(d*x**2+c))**2,x)`

output `Integral(x**3/(a + b*sec(c + d*x**2))**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f or more de`

Giac [F]

$$\int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^3}{(b \sec(dx^2 + c) + a)^2} dx$$

input `integrate(x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(x^3/(b*sec(d*x^2 + c) + a)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^3}{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

input `int(x^3/(a + b/cos(c + d*x^2))^2,x)`output `int(x^3/(a + b/cos(c + d*x^2))^2, x)`**Reduce [F]**

$$\int \frac{x^3}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^3}{\sec(dx^2 + c)^2 b^2 + 2 \sec(dx^2 + c) ab + a^2} dx$$

input `int(x^3/(a+b*sec(d*x^2+c))^2,x)`output `int(x**3/(sec(c + d*x**2)**2*b**2 + 2*sec(c + d*x**2)*a*b + a**2),x)`

$$3.26 \quad \int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx$$

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Maple [N/A]	208
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Giac [N/A]	211
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Reduce [N/A]	211

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx = \text{Int}\left(\frac{x^2}{(a+b \sec(c+dx^2))^2}, x\right)$$

output `Defer(Int)(x^2/(a+b*sec(d*x^2+c))^2,x)`

Mathematica [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx = \int \frac{x^2}{(a+b \sec(c+dx^2))^2} dx$$

input `Integrate[x^2/(a + b*Sec[c + d*x^2])^2,x]`

output `Integrate[x^2/(a + b*Sec[c + d*x^2])^2, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx$$

↓ 4694

$$\int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx$$

input `Int[x^2/(a + b*Sec[c + d*x^2])^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a + b \sec(dx^2 + c))^2} dx$$

input `int(x^2/(a+b*sec(d*x^2+c))^2,x)`

output `int(x^2/(a+b*sec(d*x^2+c))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^2}{(b \sec(dx^2 + c) + a)^2} dx$$

input `integrate(x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `integral(x^2/(b^2*sec(d*x^2 + c)^2 + 2*a*b*sec(d*x^2 + c) + a^2), x)`

Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx$$

input `integrate(x**2/(a+b*sec(d*x**2+c))**2,x)`

output `Integral(x**2/(a + b*sec(c + d*x**2))**2, x)`

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 1261, normalized size of antiderivative = 70.06

$$\int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^2}{(b \sec(dx^2 + c) + a)^2} dx$$

input `integrate(x^2/(a+b*sec(dx^2+c))^2,x, algorithm="maxima")`

output

```
1/3*((a^4 - a^2*b^2)*d*x^3*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^3*
cos(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^3*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2
- b^4)*d*x^3*sin(d*x^2 + c)^2 + 4*(a^3*b - a*b^3)*d*x^3*cos(d*x^2 + c) +
3*a*b^3*x*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^3 + (4*(a^3*b - a*b^3)*d*x^
3*cos(d*x^2 + c) - 3*a*b^3*x*sin(d*x^2 + c) + 2*(a^4 - a^2*b^2)*d*x^3)*cos
(2*d*x^2 + 2*c) - 3*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 -
a^2*b^4)*d*cos(d*x^2 + c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*
(a^5*b - a^3*b^3)*d*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b
^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4
*b^2)*d + 2*(2*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos
(2*d*x^2 + 2*c))*integrate((4*(2*a^2*b^2 - b^4)*d*x^2*cos(d*x^2 + c)^2 + 4
*(2*a^2*b^2 - b^4)*d*x^2*sin(d*x^2 + c)^2 + 2*(2*a^3*b - a*b^3)*d*x^2*cos(
d*x^2 + c) + a*b^3*sin(d*x^2 + c) + (2*(2*a^3*b - a*b^3)*d*x^2*cos(d*x^2 +
c) - a*b^3*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (a*b^3*cos(d*x^2 + c) + 2
*(2*a^3*b - a*b^3)*d*x^2*sin(d*x^2 + c) + a^2*b^2)*sin(2*d*x^2 + 2*c))/((a
^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 +
c)^2 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*sin
(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2
+ 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d + 2*(2*(a^5*b -
a^3*b^3)*d*cos(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c)), x)...
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^2}{(b \sec(dx^2 + c) + a)^2} dx$$

input `integrate(x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`output `integrate(x^2/(b*sec(d*x^2 + c) + a)^2, x)`**Mupad [N/A]**

Not integrable

Time = 16.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^2}{\left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

input `int(x^2/(a + b/cos(c + d*x^2))^2,x)`output `int(x^2/(a + b/cos(c + d*x^2))^2, x)`**Reduce [N/A]**

Not integrable

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x^2}{\sec(dx^2 + c)^2 b^2 + 2 \sec(dx^2 + c) ab + a^2} dx$$

input `int(x^2/(a+b*sec(d*x^2+c))^2,x)`

output `int(x**2/(sec(c + d*x**2)**2*b**2 + 2*sec(c + d*x**2)*a*b + a**2),x)`

3.27 $\int \frac{x}{(a+b \sec(c+dx^2))^2} dx$

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Giac [A] (verification not implemented)	220
Mupad [B] (verification not implemented)	220
Reduce [F]	221

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int \frac{x}{(a + b \sec(c + dx^2))^2} dx = \frac{x^2}{2a^2} - \frac{b(2a^2 - b^2) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2 \tan(c + dx^2)}{2a(a^2 - b^2)d(a + b \sec(c + dx^2))}$$

output

```
1/2*x^2/a^2-b*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x^2+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*b^2*tan(d*x^2+c)/a/(a^2-b^2)/d/(a+b*sec(d*x^2+c))
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.24

$$\int \frac{x}{(a + b \sec(c + dx^2))^2} dx = \frac{2b(-2a^2+b^2) \operatorname{arctanh}\left(\frac{(-a+b) \tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{a(a^2-b^2)(c+dx^2) \cos(c+dx^2)+b((a^2-b^2)(c+dx^2)+ab \sin(c+dx^2))}{b+a \cos(c+dx^2)}$$

$$= \frac{2a^2(a-b)(a+b)d}{2a^2(a-b)(a+b)d}$$

input `Integrate[x/(a + b*Sec[c + d*x^2])^2,x]`

output
$$\frac{((-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*x^2)*Cos[c + d*x^2] + b*((a^2 - b^2)*(c + d*x^2) + a*b*Sin[c + d*x^2]))/(b + a*Cos[c + d*x^2])}{(2*a^2*(a - b)*(a + b)*d)}$$

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {4692, 3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x}{(a + b \sec(c + dx^2))^2} dx \\ & \quad \downarrow 4692 \\ & \frac{1}{2} \int \frac{1}{(a + b \sec(dx^2 + c))^2} dx^2 \\ & \quad \downarrow 3042 \\ & \frac{1}{2} \int \frac{1}{(a + b \csc(dx^2 + c + \frac{\pi}{2}))^2} dx^2 \\ & \quad \downarrow 4272 \\ & \frac{1}{2} \left(\frac{b^2 \tan(c + dx^2)}{ad(a^2 - b^2)(a + b \sec(c + dx^2))} - \frac{\int -\frac{a^2 - b \sec(dx^2 + c)a - b^2}{a + b \sec(dx^2 + c)} dx^2}{a(a^2 - b^2)} \right) \\ & \quad \downarrow 25 \\ & \frac{1}{2} \left(\frac{\int \frac{a^2 - b \sec(dx^2 + c)a - b^2}{a + b \sec(dx^2 + c)} dx^2}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^2)}{ad(a^2 - b^2)(a + b \sec(c + dx^2))} \right) \\ & \quad \downarrow 3042 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left(\frac{\int \frac{a^2 - b \csc(dx^2 + c + \frac{\pi}{2}) a - b^2}{a + b \csc(dx^2 + c + \frac{\pi}{2})} dx^2}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^2)}{ad(a^2 - b^2)(a + b \sec(c + dx^2))} \right) \\
& \quad \downarrow 4407 \\
& \frac{1}{2} \left(\frac{\frac{x^2(a^2 - b^2)}{a} - \frac{b(2a^2 - b^2) \int \frac{\sec(dx^2 + c)}{a + b \sec(dx^2 + c)} dx^2}{a}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^2)}{ad(a^2 - b^2)(a + b \sec(c + dx^2))} \right) \\
& \quad \downarrow 3042 \\
& \frac{1}{2} \left(\frac{\frac{x^2(a^2 - b^2)}{a} - \frac{b(2a^2 - b^2) \int \frac{\csc(dx^2 + c + \frac{\pi}{2})}{a + b \csc(dx^2 + c + \frac{\pi}{2})} dx^2}{a}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^2)}{ad(a^2 - b^2)(a + b \sec(c + dx^2))} \right) \\
& \quad \downarrow 4318 \\
& \frac{1}{2} \left(\frac{\frac{x^2(a^2 - b^2)}{a} - \frac{(2a^2 - b^2) \int \frac{1}{\frac{a \cos(dx^2 + c)}{b} + 1} dx^2}{a}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^2)}{ad(a^2 - b^2)(a + b \sec(c + dx^2))} \right) \\
& \quad \downarrow 3042 \\
& \frac{1}{2} \left(\frac{\frac{x^2(a^2 - b^2)}{a} - \frac{(2a^2 - b^2) \int \frac{1}{\frac{a \sin(dx^2 + c + \frac{\pi}{2})}{b} + 1} dx^2}{a}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^2)}{ad(a^2 - b^2)(a + b \sec(c + dx^2))} \right) \\
& \quad \downarrow 3138 \\
& \frac{1}{2} \left(\frac{\frac{x^2(a^2 - b^2)}{a} - \frac{2(2a^2 - b^2) \int \frac{1}{(1 - \frac{a}{b})x^4 + \frac{a+b}{b}} d \tan(\frac{1}{2}(dx^2 + c))}{ad}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^2)}{ad(a^2 - b^2)(a + b \sec(c + dx^2))} \right) \\
& \quad \downarrow 221 \\
& \frac{1}{2} \left(\frac{\frac{x^2(a^2 - b^2)}{a} - \frac{2b(2a^2 - b^2) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c + dx^2))}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^2)}{ad(a^2 - b^2)(a + b \sec(c + dx^2))} \right)
\end{aligned}$$

input `Int[x/(a + b*Sec[c + d*x^2])^2,x]`

output `((((a^2 - b^2)*x^2)/a - (2*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x^2)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2)) + (b^2*Tan[c + d*x^2]/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x^2]))) / 2`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 221 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3138 `Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 4272 `Int[(csc[(c_) + (d_)*(x_)])*(b_) + (a_)^(n_), x_Symbol] := Simp[b^2*Cot[c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Simp[1/(a*(n + 1)*(a^2 - b^2)) Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]`

rule 4318 `Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)])*(b_) + (a_), x_Symbol] := Simp[1/b Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]`

rule 4407

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Simp[c*(x/a), x] - Simp[(b*c - a*d)/a Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4692

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{2 \arctan\left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)}{a^2} + \frac{2b \left(\frac{ab \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{(a^2-b^2) \left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)^2 a - \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 b - a - b} - \frac{(2a^2-b^2) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}} \right)}{2d a^2}$
default	$\frac{\frac{2 \arctan\left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)}{a^2} + \frac{2b \left(\frac{ab \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{(a^2-b^2) \left(\tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)^2 a - \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)^2 b - a - b} - \frac{(2a^2-b^2) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}} \right)}{2d a^2}$
risch	$\frac{x^2}{2a^2} + \frac{ib^2 \left(b e^{i(dx^2+c)} + a \right)}{a^2(a^2-b^2)d \left(a e^{2i(dx^2+c)} + 2b e^{i(dx^2+c)} + a \right)} + \frac{b \ln\left(e^{i(dx^2+c)} - \frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2} a} \right)}{\sqrt{a^2-b^2} (a+b)(a-b)d} - \frac{b^3 \ln\left(e^{i(dx^2+c)} \right)}{2\sqrt{a^2-b^2}}$

input

```
int(x/(a+b*sec(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/2/d*(2/a^2*arctan(tan(1/2*d*x^2+1/2*c))+2*b/a^2*(-a*b/(a^2-b^2)*tan(1/2*
d*x^2+1/2*c)/(tan(1/2*d*x^2+1/2*c)^2*a-tan(1/2*d*x^2+1/2*c)^2*b-a-b)-(2*a^
2-b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x^2+1/2*c)/
((a-b)*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(110) = 220$.

Time = 0.10 (sec) , antiderivative size = 525, normalized size of antiderivative = 4.27

$$\int \frac{x}{(a + b \sec(c + dx^2))^2} dx$$

$$= \frac{2(a^5 - 2a^3b^2 + ab^4)dx^2 \cos(dx^2 + c) + 2(a^4b - 2a^2b^3 + b^5)dx^2 + (2a^2b^2 - b^4 + (2a^3b - ab^3) \cos(dx^2 + c)) \sqrt{a^2 - b^2} \log((2ab \cos(dx^2 + c) - (a^2 - 2b^2) \cos(dx^2 + c))^2 - 2\sqrt{a^2 - b^2}(b \cos(dx^2 + c) + a) \sin(dx^2 + c) + 2a^2 - b^2) / (a^2 \cos(dx^2 + c)^2 + 2ab \cos(dx^2 + c) + b^2)) + 2(a^3b^2 - ab^4) \sin(dx^2 + c) / ((a^7 - 2a^5b^2 + a^3b^4) d \cos(dx^2 + c) + (a^6b - 2a^4b^3 + a^2b^5) d), 1/2((a^5 - 2a^3b^2 + ab^4) dx^2 \cos(dx^2 + c) + (a^4b - 2a^2b^3 + b^5) dx^2 - (2a^2b^2 - b^4 + (2a^3b - ab^3) \cos(dx^2 + c)) \sqrt{-a^2 + b^2} \arctan(-\sqrt{-a^2 + b^2}(b \cos(dx^2 + c) + a) / ((a^2 - b^2) \sin(dx^2 + c))) + (a^3b^2 - ab^4) \sin(dx^2 + c) / ((a^7 - 2a^5b^2 + a^3b^4) d \cos(dx^2 + c) + (a^6b - 2a^4b^3 + a^2b^5) d)}{4((a^7 - 2a^5b^2 + a^3b^4) d)}$$

input `integrate(x/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `[1/4*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*cos(d*x^2 + c) + 2*(a^4*b - 2*a^2*b^3 + b^5)*d*x^2 + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x^2 + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x^2 + c) - (a^2 - 2*b^2)*cos(d*x^2 + c))^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x^2 + c) + a)*sin(d*x^2 + c) + 2*a^2 - b^2)/(a^2*cos(d*x^2 + c)^2 + 2*a*b*cos(d*x^2 + c) + b^2)) + 2*(a^3*b^2 - a*b^4)*sin(d*x^2 + c)/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x^2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), 1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*cos(d*x^2 + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x^2 - (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x^2 + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x^2 + c) + a)/((a^2 - b^2)*sin(d*x^2 + c))) + (a^3*b^2 - a*b^4)*sin(d*x^2 + c)/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x^2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]`

Sympy [F]

$$\int \frac{x}{(a + b \sec(c + dx^2))^2} dx = \int \frac{x}{(a + b \sec(c + dx^2))^2} dx$$

input `integrate(x/(a+b*sec(d*x**2+c))**2,x)`

output `Integral(x/(a + b*sec(c + d*x**2))**2, x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8871 vs. $2(110) = 220$.

Time = 23.06 (sec) , antiderivative size = 8871, normalized size of antiderivative = 72.12

$$\int \frac{x}{(a + b \sec(c + dx^2))^2} dx = \text{Too large to display}$$

input `integrate(x/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output

```
1/2*((a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 -
2*a^2*b^4 + b^6)*d*x^2*cos(d*x^2 + c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x
^2*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*sin(d*x^2 +
c)^2 + 4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2 + c) + (a^6 - 2*a^4*b
^2 + a^2*b^4)*d*x^2 + (2*a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*cos(2*d*x^2
+ 2*c)^2 + 4*(2*a^2*b^3 - b^5)*cos(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*sin
(2*d*x^2 + 2*c)^2 + 4*(2*a^3*b^2 - a*b^4)*sin(2*d*x^2 + 2*c)*sin(d*x^2 + c
) + 4*(2*a^2*b^3 - b^5)*sin(d*x^2 + c)^2 + 2*(2*a^4*b - a^2*b^3 + 2*(2*a^3
*b^2 - a*b^4)*cos(d*x^2 + c))*cos(2*d*x^2 + 2*c) + 4*(2*a^3*b^2 - a*b^4)*c
os(d*x^2 + c))*sqrt(-a^2 + b^2)*arctan2(2*(4*(a^6 - a^4*b^2)*cos(d*x^2 + 2
*c)^4*cos(c)*sin(c) - 4*(a^6 - a^4*b^2)*cos(c)*sin(d*x^2 + 2*c)^4*sin(c) +
4*(3*(a^5*b - a^3*b^3)*cos(c)^2*sin(c) + (a^5*b - a^3*b^3)*sin(c)^3)*cos(
d*x^2 + 2*c)^3 - 4*((a^5*b - a^3*b^3)*cos(c)^3 + 3*(a^5*b - a^3*b^3)*cos(c
)*sin(c)^2 + ((a^6 - a^4*b^2)*cos(c)^2 - (a^6 - a^4*b^2)*sin(c)^2)*cos(d*x
^2 + 2*c))*sin(d*x^2 + 2*c)^3 - 4*((a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)^3*
sin(c) + (a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)*sin(c)^3)*cos(d*x^2 + 2*c)^2
+ 4*((a^6 - 5*a^4*b^2 + 4*a^2*b^4)*cos(c)^3*sin(c) + (a^6 - 5*a^4*b^2 + 4
*a^2*b^4)*cos(c)*sin(c)^3 - 3*((a^5*b - a^3*b^3)*cos(c)^2*sin(c) - (a^5*b
- a^3*b^3)*sin(c)^3)*cos(d*x^2 + 2*c))*sin(d*x^2 + 2*c)^2 - 4*((a^5*b - 3*
a^3*b^3 + 2*a*b^5)*cos(c)^4*sin(c) + 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*co...
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.59

$$\int \frac{x}{(a + b \sec(c + dx^2))^2} dx$$

$$= -\frac{b^2 \tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)}{(a^3 d - ab^2 d) \left(a \tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)^2 - b \tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)^2 - a - b\right)}$$

$$+ \frac{(2a^2 b - b^3) \left(\pi \left\lfloor \frac{dx^2 + c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2a - 2b) + \arctan\left(\frac{a \tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx^2 + \frac{1}{2} c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{(a^4 d - a^2 b^2 d) \sqrt{-a^2 + b^2}}$$

$$+ \frac{dx^2 + c}{2a^2 d}$$

input `integrate(x/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`output `-b^2*tan(1/2*d*x^2 + 1/2*c)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*x^2 + 1/2*c)^2 - b*tan(1/2*d*x^2 + 1/2*c)^2 - a - b)) + (2*a^2*b - b^3)*(pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x^2 + 1/2*c) - b*tan(1/2*d*x^2 + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*d - a^2*b^2*d)*sqrt(-a^2 + b^2)) + 1/2*(d*x^2 + c)/(a^2*d)`**Mupad [B] (verification not implemented)**

Time = 19.98 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.76

$$\int \frac{x}{(a + b \sec(c + dx^2))^2} dx$$

$$= \frac{\frac{b^2}{d(a b^2 \operatorname{li} - a^3 \operatorname{li})} + \frac{b^3 e^{\operatorname{li} dx^2 + c \operatorname{li}}}{a d (a b^2 \operatorname{li} - a^3 \operatorname{li})}}{a + a e^{2i dx^2 + c 2i} + 2 b e^{\operatorname{li} dx^2 + c \operatorname{li}}} + \frac{x^2}{2a^2}$$

$$+ \frac{b \ln\left(2 b x e^{\operatorname{li} dx^2 + c \operatorname{li}} (2 a^2 - b^2) - \frac{b x (a^2 - b^2) (2 a^2 - b^2) (a + b e^{\operatorname{li} dx^2 + c \operatorname{li}})^{2i}}{(a+b)^{3/2} (a-b)^{3/2}}\right) (2 a^2 - b^2)}{2 a^2 d (a + b)^{3/2} (a - b)^{3/2}}$$

$$- \frac{b \ln\left(2 b x e^{\operatorname{li} dx^2 + c \operatorname{li}} (2 a^2 - b^2) + \frac{b x (a^2 - b^2) (2 a^2 - b^2) (a + b e^{\operatorname{li} dx^2 + c \operatorname{li}})^{2i}}{(a+b)^{3/2} (a-b)^{3/2}}\right) (2 a^2 - b^2)}{2 a^2 d (a + b)^{3/2} (a - b)^{3/2}}$$

input `int(x/(a + b/cos(c + d*x^2))^2,x)`

output
$$\frac{(b^2/(d*(a*b^2*1i - a^3*1i)) + (b^3*\exp(c*1i + d*x^2*1i))/(a*d*(a*b^2*1i - a^3*1i)))/(a + a*\exp(c*2i + d*x^2*2i) + 2*b*\exp(c*1i + d*x^2*1i)) + x^2/(2*a^2) + (b*\log(2*b*x*\exp(c*1i + d*x^2*1i))*(2*a^2 - b^2) - (b*x*(a^2 - b^2))*(2*a^2 - b^2)*(a + b*\exp(c*1i + d*x^2*1i))*2i)/((a + b)^(3/2)*(a - b)^(3/2)))*(2*a^2 - b^2)/(2*a^2*d*(a + b)^(3/2)*(a - b)^(3/2)) - (b*\log(2*b*x*\exp(c*1i + d*x^2*1i))*(2*a^2 - b^2) + (b*x*(a^2 - b^2))*(2*a^2 - b^2)*(a + b*\exp(c*1i + d*x^2*1i))*2i)/((a + b)^(3/2)*(a - b)^(3/2)))*(2*a^2 - b^2)/(2*a^2*d*(a + b)^(3/2)*(a - b)^(3/2))$$

Reduce [F]

$$\int \frac{x}{(a + b \sec(c + dx^2))^2} dx = \text{Too large to display}$$

input `int(x/(a+b*sec(d*x^2+c))^2,x)`

output

```
( - 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x**2)/2)*a - tan((c + d*x**2)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x**2)*a**2 + 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x**2)/2)*a - tan((c + d*x**2)/2)*b)/sqrt( - a**2 + b**2))*cos(c + d*x**2)*a*b - 4*sqrt( - a**2 + b**2)*atan((tan((c + d*x**2)/2)*a - tan((c + d*x**2)/2)*b)/sqrt( - a**2 + b**2))*a*b + 2*sqrt( - a**2 + b**2)*atan((tan((c + d*x**2)/2)*a - tan((c + d*x**2)/2)*b)/sqrt( - a**2 + b**2))*b**2 + 8*cos(c + d*x**2)*int(x/(tan((c + d*x**2)/2)**4*a**2 - 2*tan((c + d*x**2)/2)**4*a*b + tan((c + d*x**2)/2)**4*b**2 - 2*tan((c + d*x**2)/2)**2*a**2 + 2*tan((c + d*x**2)/2)**2*b**2 + a**2 + 2*a*b + b**2),x)*a**5*d - 16*cos(c + d*x**2)*int(x/(tan((c + d*x**2)/2)**4*a**2 - 2*tan((c + d*x**2)/2)**4*a*b + tan((c + d*x**2)/2)**4*b**2 - 2*tan((c + d*x**2)/2)**2*a**2 + 2*tan((c + d*x**2)/2)**2*b**2 + a**2 + 2*a*b + b**2),x)*a**3*b**2*d + 8*cos(c + d*x**2)*int(x/(tan((c + d*x**2)/2)**4*a**2 - 2*tan((c + d*x**2)/2)**4*a*b + tan((c + d*x**2)/2)**4*b**2 - 2*tan((c + d*x**2)/2)**2*a**2 + 2*tan((c + d*x**2)/2)**2*b**2 + a**2 + 2*a*b + b**2),x)*a*b**4*d + 8*int(x/(tan((c + d*x**2)/2)**4*a**2 - 2*tan((c + d*x**2)/2)**4*a*b + tan((c + d*x**2)/2)**4*b**2 - 2*tan((c + d*x**2)/2)**2*a**2 + 2*tan((c + d*x**2)/2)**2*b**2 + a**2 + 2*a*b + b**2),x)*a**4*b*d - 16*int(x/(tan((c + d*x**2)/2)**4*a**2 - 2*tan((c + d*x**2)/2)**4*a*b + tan((c + d*x**2)/2)**4*b**2 - 2*tan((c + d*x**2)/2)**2*a**2 + 2*tan((c + d*x**2)/2)**2*b**2 + a**2 + 2*a*b...
```

3.28 $\int \frac{1}{x(a+b \sec(c+dx^2))^2} dx$

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Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x(a+b \sec(c+dx^2))^2} dx = \text{Int}\left(\frac{1}{x(a+b \sec(c+dx^2))^2}, x\right)$$

output

```
Defer(Int)(1/x/(a+b*sec(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 7.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+b \sec(c+dx^2))^2} dx = \int \frac{1}{x(a+b \sec(c+dx^2))^2} dx$$

input

```
Integrate[1/(x*(a + b*Sec[c + d*x^2]))^2,x]
```

output

```
Integrate[1/(x*(a + b*Sec[c + d*x^2]))^2, x]
```


Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a + b \sec(c + dx^2))^2} dx$$

↓ 4694

$$\int \frac{1}{x(a + b \sec(c + dx^2))^2} dx$$

input `Int[1/(x*(a + b*Sec[c + d*x^2]))^2],x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a + b \sec(dx^2 + c))^2} dx$$

input `int(1/x/(a+b*sec(d*x^2+c))^2,x)`

output `int(1/x/(a+b*sec(d*x^2+c))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x(a+b\sec(c+dx^2))^2} dx = \int \frac{1}{(b\sec(dx^2+c)+a)^2 x} dx$$

input `integrate(1/x/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `integral(1/(b^2*x*sec(d*x^2 + c)^2 + 2*a*b*x*sec(d*x^2 + c) + a^2*x), x)`

Sympy [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+b\sec(c+dx^2))^2} dx = \int \frac{1}{x(a+b\sec(c+dx^2))^2} dx$$

input `integrate(1/x/(a+b*sec(d*x**2+c))**2,x)`

output `Integral(1/(x*(a + b*sec(c + d*x**2))**2), x)`

Maxima [N/A]

Not integrable

Time = 5.28 (sec) , antiderivative size = 4629, normalized size of antiderivative = 257.17

$$\int \frac{1}{x(a + b \sec(c + dx^2))^2} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output

```
(a^6*d*x^2*cos(2*d*x^2 + 2*c)^2*log(x) + a^6*d*x^2*log(x)*sin(2*d*x^2 + 2*c)^2 + (a^2*b^4*cos(2*c)^2 + a^2*b^4*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2*log(x) + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(c)^2)*d*x^2*cos(d*x^2)^2*log(x) + (a^2*b^4*cos(2*c)^2 + a^2*b^4*sin(2*c)^2)*d*x^2*log(x)*sin(2*d*x^2)^2 + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(c)^2)*d*x^2*log(x)*sin(d*x^2)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2*log(x) - (a^2*b^4*sin(2*c) + 4*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*cos(d*x^2)*log(x) + 2*(a^4*b^2 - a^2*b^4)*d*x^2*cos(2*c)*log(x) + 4*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*log(x)*sin(d*x^2))*cos(2*d*x^2) - (2*a^4*b^2*d*x^2*cos(2*d*x^2)*cos(2*c)*log(x) - 2*a^4*b^2*d*x^2*log(x)*sin(2*d*x^2)*sin(2*c) - 4*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*cos(c)*log(x) + a^3*b^3*sin(d*x^2 + c) + 4*(a^5*b - a^3*b^3)*d*x^2*log(x)*sin(d*x^2)*sin(c) - 2*(a^6 - a^4*b^2)*d*x^2*log(x))*cos(2*d*x^2 + 2*c) - (a*b^5*cos(2*c)*sin(2*d*x^2) + a*b^5*cos(2*d*x^2)*sin(2*c) - 2*(a^2*b^4 - b^6)*cos(c)*sin(d*x^2) - 2*(a^2*b^4 - b^6)*cos(d*x^2)*sin(c))*cos(d*x^2 + c) + 2*(2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(c)*log(x) + (a^3*b^3 - a*b^5)*sin(c))*cos(d*x^2) + (a^8*d*x^2*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^2*sin(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 ...
```

Giac [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x (a + b \sec (c + dx^2))^2} dx = \int \frac{1}{(b \sec (dx^2 + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(1/((b*sec(d*x^2 + c) + a)^2*x), x)`

Mupad [N/A]

Not integrable

Time = 17.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x (a + b \sec (c + dx^2))^2} dx = \int \frac{1}{x \left(a + \frac{b}{\cos(dx^2+c)} \right)^2} dx$$

input `int(1/(x*(a + b/cos(c + d*x^2))^2),x)`

output `int(1/(x*(a + b/cos(c + d*x^2))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 1518, normalized size of antiderivative = 84.33

$$\int \frac{1}{x (a + b \sec (c + dx^2))^2} dx = \text{Too large to display}$$

input `int(1/x/(a+b*sec(d*x^2+c))^2,x)`

output

```
(4*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4*a**4*x - 4*tan((c +
d*x**2)/2)**4*a**3*b*x + 6*tan((c + d*x**2)/2)**4*a**2*b**2*x - 4*tan((c +
d*x**2)/2)**4*a*b**3*x + tan((c + d*x**2)/2)**4*b**4*x - 2*tan((c + d*x**
2)/2)**2*a**4*x + 4*tan((c + d*x**2)/2)**2*a**3*b*x - 4*tan((c + d*x**2)/
2)**2*a*b**3*x + 2*tan((c + d*x**2)/2)**2*b**4*x + a**4*x - 2*a**2*b**2*x +
b**4*x),x)*a**3*b - 12*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4
*a**4*x - 4*tan((c + d*x**2)/2)**4*a**3*b*x + 6*tan((c + d*x**2)/2)**4*a**
2*b**2*x - 4*tan((c + d*x**2)/2)**4*a*b**3*x + tan((c + d*x**2)/2)**4*b**4
*x - 2*tan((c + d*x**2)/2)**2*a**4*x + 4*tan((c + d*x**2)/2)**2*a**3*b*x -
4*tan((c + d*x**2)/2)**2*a*b**3*x + 2*tan((c + d*x**2)/2)**2*b**4*x + a**
4*x - 2*a**2*b**2*x + b**4*x),x)*a**2*b**2 + 12*int(tan((c + d*x**2)/2)**2
/(tan((c + d*x**2)/2)**4*a**4*x - 4*tan((c + d*x**2)/2)**4*a**3*b*x + 6*ta
n((c + d*x**2)/2)**4*a**2*b**2*x - 4*tan((c + d*x**2)/2)**4*a*b**3*x + tan
((c + d*x**2)/2)**4*b**4*x - 2*tan((c + d*x**2)/2)**2*a**4*x + 4*tan((c +
d*x**2)/2)**2*a**3*b*x - 4*tan((c + d*x**2)/2)**2*a*b**3*x + 2*tan((c + d*
x**2)/2)**2*b**4*x + a**4*x - 2*a**2*b**2*x + b**4*x),x)*a*b**3 - 4*int(ta
n((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4*a**4*x - 4*tan((c + d*x**2)/2)
)**4*a**3*b*x + 6*tan((c + d*x**2)/2)**4*a**2*b**2*x - 4*tan((c + d*x**2)/
2)**4*a*b**3*x + tan((c + d*x**2)/2)**4*b**4*x - 2*tan((c + d*x**2)/2)**2*
a**4*x + 4*tan((c + d*x**2)/2)**2*a**3*b*x - 4*tan((c + d*x**2)/2)**2*a...
```

$$3.29 \quad \int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx$$

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Rubi [N/A]	230
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Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx = \text{Int}\left(\frac{1}{x^2 (a + b \sec(c + dx^2))^2}, x\right)$$

output `Defer(Int)(1/x^2/(a+b*sec(d*x^2+c))^2,x)`

Mathematica [N/A]

Not integrable

Time = 4.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx$$

input `Integrate[1/(x^2*(a + b*Sec[c + d*x^2]))^2,x]`

output `Integrate[1/(x^2*(a + b*Sec[c + d*x^2]))^2, x]`

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx$$

↓ 4694

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx$$

input `Int[1/(x^2*(a + b*Sec[c + d*x^2])^2),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + b \sec(dx^2 + c))^2} dx$$

input `int(1/x^2/(a+b*sec(d*x^2+c))^2,x)`

output `int(1/x^2/(a+b*sec(d*x^2+c))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)^2 x^2} dx$$

input `integrate(1/x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `integral(1/(b^2*x^2*sec(d*x^2 + c)^2 + 2*a*b*x^2*sec(d*x^2 + c) + a^2*x^2), x)`

Sympy [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx$$

input `integrate(1/x**2/(a+b*sec(d*x**2+c))**2,x)`

output `Integral(1/(x**2*(a + b*sec(c + d*x**2))**2), x)`

Maxima [N/A]

Not integrable

Time = 5.18 (sec) , antiderivative size = 4550, normalized size of antiderivative = 252.78

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)^2 x^2} dx$$

input `integrate(1/x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output

```

-((a^6 - a^4*b^2)*d*x^2*cos(2*d*x^2 + 2*c)^2 + (a^6 - a^4*b^2)*d*x^2*sin(2
*d*x^2 + 2*c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2 + (a^2*b^4*sin(2*c) -
(a^4*b^2 - a^2*b^4)*d*x^2*cos(2*c))*cos(2*d*x^2) + (a^3*b^3*sin(d*x^2 + c)
- (a^4*b^2 - a^2*b^4)*d*x^2*cos(2*d*x^2)*cos(2*c) + 2*(a^5*b - 2*a^3*b^3
+ a*b^5)*d*x^2*cos(d*x^2)*cos(c) + (a^4*b^2 - a^2*b^4)*d*x^2*sin(2*d*x^2)*
sin(2*c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*sin(d*x^2)*sin(c) + 2*(a^5*
b - a^3*b^3)*d*x^2*cos(d*x^2 + c) + (2*a^6 - 3*a^4*b^2 + a^2*b^4)*d*x^2)*c
os(2*d*x^2 + 2*c) + (2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2 + (a*b^5*sin(2*c)
- 2*(a^3*b^3 - a*b^5)*d*x^2*cos(2*c))*cos(2*d*x^2) + 2*(2*(a^4*b^2 - 2*a^
2*b^4 + b^6)*d*x^2*cos(c) - (a^2*b^4 - b^6)*sin(c))*cos(d*x^2) + (a*b^5*co
s(2*c) + 2*(a^3*b^3 - a*b^5)*d*x^2*sin(2*c))*sin(2*d*x^2) - 2*(2*(a^4*b^2
- 2*a^2*b^4 + b^6)*d*x^2*sin(c) + (a^2*b^4 - b^6)*cos(c))*sin(d*x^2))*cos(
d*x^2 + c) + 2*((a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(c) - (a^3*b^3 - a*b^
5)*sin(c))*cos(d*x^2) - (a^8*d*x^3*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^3*sin(2*
d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*cos(2*d*x
^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4
+ a^2*b^6)*sin(c)^2)*d*x^3*cos(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)
*d*x^3*cos(d*x^2)*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3
*sin(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 -
2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^3*sin(d*x^2)^2 - 4*(a^7*b - 2*a^5*b...

```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)^2 x^2} dx$$

input `integrate(1/x^2/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(1/((b*sec(d*x^2 + c) + a)^2*x^2), x)`

Mupad [N/A]

Not integrable

Time = 17.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{x^2 \left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

input `int(1/(x^2*(a + b/cos(c + d*x^2))^2),x)`

output `int(1/(x^2*(a + b/cos(c + d*x^2))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 7251, normalized size of antiderivative = 402.83

$$\int \frac{1}{x^2 (a + b \sec(c + dx^2))^2} dx = \text{Too large to display}$$

input `int(1/x^2/(a+b*sec(d*x^2+c))^2,x)`

output

```
(4*cos(c + d*x**2)*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4*a**4
*x**2 - 4*tan((c + d*x**2)/2)**4*a**3*b*x**2 + 6*tan((c + d*x**2)/2)**4*a*
*2*b**2*x**2 - 4*tan((c + d*x**2)/2)**4*a*b**3*x**2 + tan((c + d*x**2)/2)*
*4*b**4*x**2 - 2*tan((c + d*x**2)/2)**2*a**4*x**2 + 4*tan((c + d*x**2)/2)*
*2*a**3*b*x**2 - 4*tan((c + d*x**2)/2)**2*a*b**3*x**2 + 2*tan((c + d*x**2)
/2)**2*b**4*x**2 + a**4*x**2 - 2*a**2*b**2*x**2 + b**4*x**2),x)*sec(c + d*
x**2)*a**5*b**2*x - 12*cos(c + d*x**2)*int(tan((c + d*x**2)/2)**2/(tan((c
+ d*x**2)/2)**4*a**4*x**2 - 4*tan((c + d*x**2)/2)**4*a**3*b*x**2 + 6*tan((c
+ d*x**2)/2)**4*a**2*b**2*x**2 - 4*tan((c + d*x**2)/2)**4*a*b**3*x**2 +
tan((c + d*x**2)/2)**4*b**4*x**2 - 2*tan((c + d*x**2)/2)**2*a**4*x**2 + 4*
tan((c + d*x**2)/2)**2*a**3*b*x**2 - 4*tan((c + d*x**2)/2)**2*a*b**3*x**2
+ 2*tan((c + d*x**2)/2)**2*b**4*x**2 + a**4*x**2 - 2*a**2*b**2*x**2 + b**4
*x**2),x)*sec(c + d*x**2)*a**4*b**3*x + 12*cos(c + d*x**2)*int(tan((c + d*
x**2)/2)**2/(tan((c + d*x**2)/2)**4*a**4*x**2 - 4*tan((c + d*x**2)/2)**4*a
**3*b*x**2 + 6*tan((c + d*x**2)/2)**4*a**2*b**2*x**2 - 4*tan((c + d*x**2)/
2)**4*a*b**3*x**2 + tan((c + d*x**2)/2)**4*b**4*x**2 - 2*tan((c + d*x**2)/
2)**2*a**4*x**2 + 4*tan((c + d*x**2)/2)**2*a**3*b*x**2 - 4*tan((c + d*x**2
)/2)**2*a*b**3*x**2 + 2*tan((c + d*x**2)/2)**2*b**4*x**2 + a**4*x**2 - 2*a
**2*b**2*x**2 + b**4*x**2),x)*sec(c + d*x**2)*a**3*b**4*x - 4*cos(c + d*x*
*2)*int(tan((c + d*x**2)/2)**2/(tan((c + d*x**2)/2)**4*a**4*x**2 - 4*ta...
```

3.30 $\int \frac{1}{x^3(a+b \sec(c+dx^2))^2} dx$

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Fricas [N/A]	237
Sympy [N/A]	237
Maxima [N/A]	238
Giac [N/A]	239
Mupad [N/A]	239
Reduce [N/A]	239

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3(a+b \sec(c+dx^2))^2} dx = \text{Int}\left(\frac{1}{x^3(a+b \sec(c+dx^2))^2}, x\right)$$

output

```
Defer(Int)(1/x^3/(a+b*sec(d*x^2+c))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 6.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3(a+b \sec(c+dx^2))^2} dx = \int \frac{1}{x^3(a+b \sec(c+dx^2))^2} dx$$

input

```
Integrate[1/(x^3*(a + b*Sec[c + d*x^2])^2),x]
```

output

```
Integrate[1/(x^3*(a + b*Sec[c + d*x^2])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx$$

↓ 4694

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx$$

input `Int[1/(x^3*(a + b*Sec[c + d*x^2])^2),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a + b \sec(dx^2 + c))^2} dx$$

input `int(1/x^3/(a+b*sec(d*x^2+c))^2,x)`

output `int(1/x^3/(a+b*sec(d*x^2+c))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)^2 x^3} dx$$

input `integrate(1/x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="fricas")`

output `integral(1/(b^2*x^3*sec(d*x^2 + c)^2 + 2*a*b*x^3*sec(d*x^2 + c) + a^2*x^3), x)`

Sympy [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx$$

input `integrate(1/x**3/(a+b*sec(d*x**2+c))**2,x)`

output `Integral(1/(x**3*(a + b*sec(c + d*x**2))**2), x)`

Maxima [N/A]

Not integrable

Time = 5.21 (sec) , antiderivative size = 3521, normalized size of antiderivative = 195.61

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)^2 x^3} dx$$

input `integrate(1/x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="maxima")`

output

```
-1/2*((a^4 - a^2*b^2)*d*x^2 + ((a^4 - a^2*b^2)*d*x^2*cos(2*c) - 2*a^2*b^2*
sin(2*c))*cos(2*d*x^2) + ((a^4 - a^2*b^2)*d*x^2*cos(2*d*x^2)*cos(2*c) + 2*
(a^3*b - a*b^3)*d*x^2*cos(d*x^2)*cos(c) - (a^4 - a^2*b^2)*d*x^2*sin(2*d*x^
2)*sin(2*c) - 2*(a^3*b - a*b^3)*d*x^2*sin(d*x^2)*sin(c) + (a^4 - a^2*b^2)*
d*x^2*cos(2*d*x^2 + 2*c) + 2*((a^3*b - a*b^3)*d*x^2 + ((a^3*b - a*b^3)*d*
x^2*cos(2*c) - a*b^3*sin(2*c))*cos(2*d*x^2) + 2*((a^2*b^2 - b^4)*d*x^2*cos
(c) - b^4*sin(c))*cos(d*x^2) - (a*b^3*cos(2*c) + (a^3*b - a*b^3)*d*x^2*sin
(2*c))*sin(2*d*x^2) - 2*(b^4*cos(c) + (a^2*b^2 - b^4)*d*x^2*sin(c))*sin(d*
x^2))*cos(d*x^2 + c) + 2*((a^3*b - a*b^3)*d*x^2*cos(c) - 2*a*b^3*sin(c))*c
os(d*x^2) - 2*(((a^6 - a^4*b^2)*cos(2*c)^2 + (a^6 - a^4*b^2)*sin(2*c)^2)*d
*x^4*cos(2*d*x^2)^2 + 4*((a^4*b^2 - a^2*b^4)*cos(c)^2 + (a^4*b^2 - a^2*b^4
)*sin(c)^2)*d*x^4*cos(d*x^2)^2 + 4*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^2)*cos(
c) + ((a^6 - a^4*b^2)*cos(2*c)^2 + (a^6 - a^4*b^2)*sin(2*c)^2)*d*x^4*sin(2
*d*x^2)^2 + 4*((a^4*b^2 - a^2*b^4)*cos(c)^2 + (a^4*b^2 - a^2*b^4)*sin(c)^2
)*d*x^4*sin(d*x^2)^2 - 4*(a^5*b - a^3*b^3)*d*x^4*sin(d*x^2)*sin(c) + (a^6
- a^4*b^2)*d*x^4 + 2*(2*((a^5*b - a^3*b^3)*cos(2*c)*cos(c) + (a^5*b - a^3*
b^3)*sin(2*c)*sin(c))*d*x^4*cos(d*x^2) + (a^6 - a^4*b^2)*d*x^4*cos(2*c) +
2*((a^5*b - a^3*b^3)*cos(c)*sin(2*c) - (a^5*b - a^3*b^3)*cos(2*c)*sin(c))*
d*x^4*sin(d*x^2))*cos(2*d*x^2) - 2*(2*((a^5*b - a^3*b^3)*cos(c)*sin(2*c) -
(a^5*b - a^3*b^3)*cos(2*c)*sin(c))*d*x^4*cos(d*x^2) - 2*((a^5*b - a^3*...
```

Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{(b \sec(dx^2 + c) + a)^2 x^3} dx$$

input `integrate(1/x^3/(a+b*sec(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(1/((b*sec(d*x^2 + c) + a)^2*x^3), x)`

Mupad [N/A]

Not integrable

Time = 17.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx = \int \frac{1}{x^3 \left(a + \frac{b}{\cos(dx^2+c)}\right)^2} dx$$

input `int(1/(x^3*(a + b/cos(c + d*x^2))^2),x)`

output `int(1/(x^3*(a + b/cos(c + d*x^2))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 39791, normalized size of antiderivative = 2210.61

$$\int \frac{1}{x^3 (a + b \sec(c + dx^2))^2} dx = \text{Too large to display}$$

input `int(1/x^3/(a+b*sec(d*x^2+c))^2,x)`

output

```
(16*int(tan((c + d*x**2)/2)**4/(tan((c + d*x**2)/2)**4*a**7*x**3 - 5*tan((c + d*x**2)/2)**4*a**6*b*x**3 + 9*tan((c + d*x**2)/2)**4*a**5*b**2*x**3 - 5*tan((c + d*x**2)/2)**4*a**4*b**3*x**3 - 5*tan((c + d*x**2)/2)**4*a**3*b**4*x**3 + 9*tan((c + d*x**2)/2)**4*a**2*b**5*x**3 - 5*tan((c + d*x**2)/2)**4*a*b**6*x**3 + tan((c + d*x**2)/2)**4*b**7*x**3 - 2*tan((c + d*x**2)/2)**2*a**7*x**3 + 6*tan((c + d*x**2)/2)**2*a**6*b*x**3 - 2*tan((c + d*x**2)/2)**2*a**5*b**2*x**3 - 10*tan((c + d*x**2)/2)**2*a**4*b**3*x**3 + 10*tan((c + d*x**2)/2)**2*a**3*b**4*x**3 + 2*tan((c + d*x**2)/2)**2*a**2*b**5*x**3 - 6*tan((c + d*x**2)/2)**2*a*b**6*x**3 + 2*tan((c + d*x**2)/2)**2*b**7*x**3 + a**7*x**3 - a**6*b*x**3 - 3*a**5*b**2*x**3 + 3*a**4*b**3*x**3 + 3*a**3*b**4*x**3 - 3*a**2*b**5*x**3 - a*b**6*x**3 + b**7*x**3),x)*tan((c + d*x**2)/2)**2*a**8*b*x**2 - 96*int(tan((c + d*x**2)/2)**4/(tan((c + d*x**2)/2)**4*a**7*x**3 - 5*tan((c + d*x**2)/2)**4*a**6*b*x**3 + 9*tan((c + d*x**2)/2)**4*a**5*b**2*x**3 - 5*tan((c + d*x**2)/2)**4*a**4*b**3*x**3 - 5*tan((c + d*x**2)/2)**4*a**3*b**4*x**3 + 9*tan((c + d*x**2)/2)**4*a**2*b**5*x**3 - 5*tan((c + d*x**2)/2)**4*a*b**6*x**3 + tan((c + d*x**2)/2)**4*b**7*x**3 - 2*tan((c + d*x**2)/2)**2*a**7*x**3 + 6*tan((c + d*x**2)/2)**2*a**6*b*x**3 - 2*tan((c + d*x**2)/2)**2*a**5*b**2*x**3 - 10*tan((c + d*x**2)/2)**2*a**4*b**3*x**3 + 10*tan((c + d*x**2)/2)**2*a**3*b**4*x**3 + 2*tan((c + d*x**2)/2)**2*a**2*b**5*x**3 - 6*tan((c + d*x**2)/2)**2*a*b**6*x**3 + 2*tan((c...
```

3.31 $\int x^3 (a + b \sec (c + d\sqrt{x})) dx$

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Optimal result

Integrand size = 18, antiderivative size = 476

$$\begin{aligned}
\int x^3(a + b \sec(c + d\sqrt{x})) dx = & \frac{ax^4}{4} - \frac{4ibx^{7/2} \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} \\
& + \frac{14ibx^3 \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
& - \frac{14ibx^3 \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
& - \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
& + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
& - \frac{420ibx^2 \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
& + \frac{420ibx^2 \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
& + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
& - \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
& + \frac{5040ibx \operatorname{PolyLog}\left(6, -ie^{i(c+d\sqrt{x})}\right)}{d^6} \\
& - \frac{5040ibx \operatorname{PolyLog}\left(6, ie^{i(c+d\sqrt{x})}\right)}{d^6} \\
& - \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, -ie^{i(c+d\sqrt{x})}\right)}{d^7} \\
& + \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, ie^{i(c+d\sqrt{x})}\right)}{d^7} \\
& - \frac{10080ib \operatorname{PolyLog}\left(8, -ie^{i(c+d\sqrt{x})}\right)}{d^8} \\
& + \frac{10080ib \operatorname{PolyLog}\left(8, ie^{i(c+d\sqrt{x})}\right)}{d^8}
\end{aligned}$$

output

$$\begin{aligned}
& 1/4*a*x^4-5040*I*b*x*polylog(6,I*exp(I*(c+d*x^(1/2))))/d^6+5040*I*b*x*poly \\
& log(6,-I*exp(I*(c+d*x^(1/2))))/d^6+10080*I*b*polylog(8,I*exp(I*(c+d*x^(1/2) \\
&)))/d^8-84*b*x^(5/2)*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+84*b*x^(5/2)* \\
& polylog(3,I*exp(I*(c+d*x^(1/2))))/d^3+420*I*b*x^2*polylog(4,I*exp(I*(c+d*x \\
& ^{(1/2)}))/d^4-4*I*b*x^(7/2)*arctan(exp(I*(c+d*x^(1/2))))/d+1680*b*x^(3/2)* \\
& polylog(5,-I*exp(I*(c+d*x^(1/2))))/d^5-1680*b*x^(3/2)*polylog(5,I*exp(I*(c \\
& +d*x^(1/2))))/d^5-420*I*b*x^2*polylog(4,-I*exp(I*(c+d*x^(1/2))))/d^4-14*I* \\
& b*x^3*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2-10080*b*x^(1/2)*polylog(7,-I*e \\
& xp(I*(c+d*x^(1/2))))/d^7+10080*b*x^(1/2)*polylog(7,I*exp(I*(c+d*x^(1/2)))) \\
& /d^7+14*I*b*x^3*polylog(2,-I*exp(I*(c+d*x^(1/2))))/d^2-10080*I*b*polylog(8 \\
& ,-I*exp(I*(c+d*x^(1/2))))/d^8
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.01

$$\begin{aligned}
\int x^3(a + b \sec(c + d\sqrt{x})) dx = & \frac{ax^4}{4} - \frac{4ibx^{7/2} \arctan(e^{ic+id\sqrt{x}})}{d} \\
& + \frac{14ibx^3 \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
& - \frac{14ibx^3 \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
& - \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
& + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
& - \frac{420ibx^2 \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
& + \frac{420ibx^2 \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
& + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
& - \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
& + \frac{5040ibx \operatorname{PolyLog}\left(6, -ie^{i(c+d\sqrt{x})}\right)}{d^6} \\
& - \frac{5040ibx \operatorname{PolyLog}\left(6, ie^{i(c+d\sqrt{x})}\right)}{d^6} \\
& - \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, -ie^{i(c+d\sqrt{x})}\right)}{d^7} \\
& + \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, ie^{i(c+d\sqrt{x})}\right)}{d^7} \\
& - \frac{10080ib \operatorname{PolyLog}\left(8, -ie^{i(c+d\sqrt{x})}\right)}{d^8} \\
& + \frac{10080ib \operatorname{PolyLog}\left(8, ie^{i(c+d\sqrt{x})}\right)}{d^8}
\end{aligned}$$

input `Integrate[x^3*(a + b*Sec[c + d*Sqrt[x]]),x]`

output `(a*x^4)/4 - ((4*I)*b*x^(7/2)*ArcTan[E^(I*(c + I*d*Sqrt[x]))]/d + ((14*I)*b*x^3*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))]/d^2 - ((14*I)*b*x^3*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))]/d^2 - (84*b*x^(5/2)*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))]/d^3 + (84*b*x^(5/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))]/d^3 - ((420*I)*b*x^2*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))]/d^4 + ((420*I)*b*x^2*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))]/d^4 + (1680*b*x^(3/2)*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))]/d^5 - (1680*b*x^(3/2)*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))]/d^5 + ((5040*I)*b*x*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))]/d^6 - ((5040*I)*b*x*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))]/d^6 - (10080*b*Sqrt[x]*PolyLog[7, (-I)*E^(I*(c + d*Sqrt[x]))]/d^7 + (10080*b*Sqrt[x]*PolyLog[7, I*E^(I*(c + d*Sqrt[x]))]/d^7 - ((10080*I)*b*PolyLog[8, (-I)*E^(I*(c + d*Sqrt[x]))]/d^8 + ((10080*I)*b*PolyLog[8, I*E^(I*(c + d*Sqrt[x]))]/d^8`

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + b \sec(c + d\sqrt{x})) dx$$

$$\downarrow \text{2010}$$

$$\int (ax^3 + bx^3 \sec(c + d\sqrt{x})) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{ax^4}{4} - \frac{4ibx^{7/2} \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{10080ib \operatorname{PolyLog}\left(8, -ie^{i(c+d\sqrt{x})}\right)}{d^8} + \\
& \frac{10080ib \operatorname{PolyLog}\left(8, ie^{i(c+d\sqrt{x})}\right)}{d^8} - \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, -ie^{i(c+d\sqrt{x})}\right)}{d^7} + \\
& \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, ie^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{5040ibx \operatorname{PolyLog}\left(6, -ie^{i(c+d\sqrt{x})}\right)}{d^6} - \\
& \frac{5040ibx \operatorname{PolyLog}\left(6, ie^{i(c+d\sqrt{x})}\right)}{d^6} + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} - \\
& \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{420ibx^2 \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} + \\
& \frac{420ibx^2 \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \\
& \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{14ibx^3 \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \\
& \frac{14ibx^3 \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2}
\end{aligned}$$

input `Int[x^3*(a + b*Sec[c + d*Sqrt[x]]),x]`

output `(a*x^4)/4 - ((4*I)*b*x^(7/2)*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((14*I)*b*x^3*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((14*I)*b*x^3*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - (84*b*x^(5/2)*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))])/d^3 + (84*b*x^(5/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((420*I)*b*x^2*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((420*I)*b*x^2*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (1680*b*x^(3/2)*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))])/d^5 - (1680*b*x^(3/2)*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])/d^5 + ((5040*I)*b*x*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((5040*I)*b*x*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))])/d^6 - (10080*b*Sqrt[x]*PolyLog[7, (-I)*E^(I*(c + d*Sqrt[x]))])/d^7 + (10080*b*Sqrt[x]*PolyLog[7, I*E^(I*(c + d*Sqrt[x]))])/d^7 - ((10080*I)*b*PolyLog[8, (-I)*E^(I*(c + d*Sqrt[x]))])/d^8 + ((10080*I)*b*PolyLog[8, I*E^(I*(c + d*Sqrt[x]))])/d^8`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

Maple [F]

$$\int x^3(a + b \sec(c + d\sqrt{x})) dx$$

input `int(x^3*(a+b*sec(c+d*x^(1/2))),x)`

output `int(x^3*(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int x^3(a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a)x^3 dx$$

input `integrate(x^3*(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(b*x^3*sec(d*sqrt(x) + c) + a*x^3, x)`

Sympy [F]

$$\int x^3(a + b \sec(c + d\sqrt{x})) dx = \int x^3(a + b \sec(c + d\sqrt{x})) dx$$

input `integrate(x**3*(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(x**3*(a + b*sec(c + d*sqrt(x))), x)`

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1512 vs. $2(352) = 704$.

Time = 0.22 (sec) , antiderivative size = 1512, normalized size of antiderivative = 3.18

$$\int x^3(a + b \sec(c + d\sqrt{x})) dx = \text{Too large to display}$$

input `integrate(x^3*(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output

```

1/4*((d*sqrt(x) + c)^8*a - 8*(d*sqrt(x) + c)^7*a*c + 28*(d*sqrt(x) + c)^6*
a*c^2 - 56*(d*sqrt(x) + c)^5*a*c^3 + 70*(d*sqrt(x) + c)^4*a*c^4 - 56*(d*sq
rt(x) + c)^3*a*c^5 + 28*(d*sqrt(x) + c)^2*a*c^6 - 8*(d*sqrt(x) + c)*a*c^7
- 8*b*c^7*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 8*(I*(d*sqrt(x) +
c)^7*b - 7*I*(d*sqrt(x) + c)^6*b*c + 21*I*(d*sqrt(x) + c)^5*b*c^2 - 35*I*
(d*sqrt(x) + c)^4*b*c^3 + 35*I*(d*sqrt(x) + c)^3*b*c^4 - 21*I*(d*sqrt(x) +
c)^2*b*c^5 + 7*I*(d*sqrt(x) + c)*b*c^6)*arctan2(cos(d*sqrt(x) + c), sin(d
*sqrt(x) + c) + 1) - 8*(I*(d*sqrt(x) + c)^7*b - 7*I*(d*sqrt(x) + c)^6*b*c
+ 21*I*(d*sqrt(x) + c)^5*b*c^2 - 35*I*(d*sqrt(x) + c)^4*b*c^3 + 35*I*(d*sq
rt(x) + c)^3*b*c^4 - 21*I*(d*sqrt(x) + c)^2*b*c^5 + 7*I*(d*sqrt(x) + c)*b*
c^6)*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 56*(I*(d*sqrt(
x) + c)^6*b - 6*I*(d*sqrt(x) + c)^5*b*c + 15*I*(d*sqrt(x) + c)^4*b*c^2 - 2
0*I*(d*sqrt(x) + c)^3*b*c^3 + 15*I*(d*sqrt(x) + c)^2*b*c^4 - 6*I*(d*sqrt(x
) + c)*b*c^5 + I*b*c^6)*dilog(I*e^(I*d*sqrt(x) + I*c)) - 56*(-I*(d*sqrt(x)
+ c)^6*b + 6*I*(d*sqrt(x) + c)^5*b*c - 15*I*(d*sqrt(x) + c)^4*b*c^2 + 20*
I*(d*sqrt(x) + c)^3*b*c^3 - 15*I*(d*sqrt(x) + c)^2*b*c^4 + 6*I*(d*sqrt(x)
+ c)*b*c^5 - I*b*c^6)*dilog(-I*e^(I*d*sqrt(x) + I*c)) + 4*((d*sqrt(x) + c)
^7*b - 7*(d*sqrt(x) + c)^6*b*c + 21*(d*sqrt(x) + c)^5*b*c^2 - 35*(d*sqrt(x
) + c)^4*b*c^3 + 35*(d*sqrt(x) + c)^3*b*c^4 - 21*(d*sqrt(x) + c)^2*b*c^5 +
7*(d*sqrt(x) + c)*b*c^6)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)...

```

Giac [F]

$$\int x^3(a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a)x^3 dx$$

input

```
integrate(x^3*(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
```

output

```
integrate((b*sec(d*sqrt(x) + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a + b \sec(c + d\sqrt{x})) dx = \int x^3 \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

input `int(x^3*(a + b/cos(c + d*x^(1/2))),x)`

output `int(x^3*(a + b/cos(c + d*x^(1/2))), x)`

Reduce [F]

$$\int x^3(a + b \sec(c + d\sqrt{x})) dx = -2 \left(\int \frac{\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2 x^3}{\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2 - 1} dx \right) b + \frac{ax^4}{4} + \frac{bx^4}{4}$$

input `int(x^3*(a+b*sec(c+d*x^(1/2))),x)`

output `(- 8*int((tan((sqrt(x)*d + c)/2)**2*x**3)/(tan((sqrt(x)*d + c)/2)**2 - 1),x)*b + a*x**4 + b*x**4)/4`

3.32 $\int x^2(a + b \sec(c + d\sqrt{x})) dx$

Optimal result	252
Mathematica [A] (verified)	253
Rubi [A] (verified)	254
Maple [F]	255
Fricas [F]	256
Sympy [F]	256
Maxima [B] (verification not implemented)	256
Giac [F]	257
Mupad [F(-1)]	258
Reduce [F]	258

Optimal result

Integrand size = 18, antiderivative size = 348

$$\begin{aligned}
 \int x^2(a + b \sec(c + d\sqrt{x})) dx = & \frac{ax^3}{3} - \frac{4ibx^{5/2} \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} \\
 & + \frac{10ibx^2 \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
 & - \frac{10ibx^2 \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
 & - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
 & + \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
 & - \frac{120ibx \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
 & + \frac{120ibx \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
 & + \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
 & - \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
 & + \frac{240ib \operatorname{PolyLog}\left(6, -ie^{i(c+d\sqrt{x})}\right)}{d^6} \\
 & - \frac{240ib \operatorname{PolyLog}\left(6, ie^{i(c+d\sqrt{x})}\right)}{d^6}
 \end{aligned}$$

output

```

1/3*a*x^3-4*I*b*x^(5/2)*arctan(exp(I*(c+d*x^(1/2))))/d+10*I*b*x^2*polylog(
2,-I*exp(I*(c+d*x^(1/2))))/d^2-10*I*b*x^2*polylog(2,I*exp(I*(c+d*x^(1/2))))
)/d^2-40*b*x^(3/2)*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+40*b*x^(3/2)*pol
ylog(3,I*exp(I*(c+d*x^(1/2))))/d^3-120*I*b*x*polylog(4,-I*exp(I*(c+d*x^(1/
2))))/d^4+120*I*b*x*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^4+240*b*x^(1/2)*po
lylog(5,-I*exp(I*(c+d*x^(1/2))))/d^5-240*b*x^(1/2)*polylog(5,I*exp(I*(c+d*
x^(1/2))))/d^5+240*I*b*polylog(6,-I*exp(I*(c+d*x^(1/2))))/d^6-240*I*b*pol
ylog(6,I*exp(I*(c+d*x^(1/2))))/d^6

```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.01

$$\begin{aligned}
\int x^2(a + b \sec(c + d\sqrt{x})) dx = & \frac{ax^3}{3} - \frac{4ibx^{5/2} \arctan(e^{ic+id\sqrt{x}})}{d} \\
& + \frac{10ibx^2 \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
& - \frac{10ibx^2 \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
& - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
& + \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
& - \frac{120ibx \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
& + \frac{120ibx \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
& + \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
& - \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
& + \frac{240ib \operatorname{PolyLog}\left(6, -ie^{i(c+d\sqrt{x})}\right)}{d^6} \\
& - \frac{240ib \operatorname{PolyLog}\left(6, ie^{i(c+d\sqrt{x})}\right)}{d^6}
\end{aligned}$$

input

```
Integrate[x^2*(a + b*Sec[c + d*Sqrt[x]]),x]
```

output

```
(a*x^3)/3 - ((4*I)*b*x^(5/2)*ArcTan[E^(I*c + I*d*Sqrt[x])])/d + ((10*I)*b*x^2*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((10*I)*b*x^2*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - (40*b*x^(3/2)*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))])/d^3 + (40*b*x^(3/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((120*I)*b*x*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((120*I)*b*x*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (240*b*Sqrt[x]*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))])/d^5 - (240*b*Sqrt[x]*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])/d^5 + ((240*I)*b*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((240*I)*b*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))])/d^6
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 (a + b \sec(c + d\sqrt{x})) \, dx \\
 & \quad \downarrow \text{2010} \\
 & \int (ax^2 + bx^2 \sec(c + d\sqrt{x})) \, dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{ax^3}{3} - \frac{4ibx^{5/2} \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{240ib \operatorname{PolyLog}\left(6, -ie^{i(c+d\sqrt{x})}\right)}{d^6} - \\
 & \frac{240ib \operatorname{PolyLog}\left(6, ie^{i(c+d\sqrt{x})}\right)}{d^6} + \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} - \\
 & \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{120ibx \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} + \\
 & \frac{120ibx \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \\
 & \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{10ibx^2 \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \\
 & \frac{10ibx^2 \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2}
 \end{aligned}$$

input `Int[x^2*(a + b*Sec[c + d*Sqrt[x]]),x]`

output `(a*x^3)/3 - ((4*I)*b*x^(5/2)*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((10*I)*b*x^2*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((10*I)*b*x^2*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - (40*b*x^(3/2)*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))])/d^3 + (40*b*x^(3/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((120*I)*b*x*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((120*I)*b*x*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (240*b*Sqrt[x]*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))])/d^5 - (240*b*Sqrt[x]*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])/d^5 + ((240*I)*b*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))])/d^6 - ((240*I)*b*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))])/d^6`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [F]

$$\int x^2(a + b \sec(c + d\sqrt{x})) dx$$

input `int(x^2*(a+b*sec(c+d*x^(1/2))),x)`

output `int(x^2*(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int x^2(a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(b*x^2*sec(d*sqrt(x) + c) + a*x^2, x)`

Sympy [F]

$$\int x^2(a + b \sec(c + d\sqrt{x})) dx = \int x^2(a + b \sec(c + d\sqrt{x})) dx$$

input `integrate(x**2*(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(x**2*(a + b*sec(c + d*sqrt(x))), x)`

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(256) = 512$.

Time = 0.22 (sec) , antiderivative size = 966, normalized size of antiderivative = 2.78

$$\int x^2(a + b \sec(c + d\sqrt{x})) dx = \text{Too large to display}$$

input `integrate(x^2*(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output

```

1/3*((d*sqrt(x) + c)^6*a - 6*(d*sqrt(x) + c)^5*a*c + 15*(d*sqrt(x) + c)^4*
a*c^2 - 20*(d*sqrt(x) + c)^3*a*c^3 + 15*(d*sqrt(x) + c)^2*a*c^4 - 6*(d*sqrt
(x) + c)*a*c^5 - 6*b*c^5*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 6
*(I*(d*sqrt(x) + c)^5*b - 5*I*(d*sqrt(x) + c)^4*b*c + 10*I*(d*sqrt(x) + c)
^3*b*c^2 - 10*I*(d*sqrt(x) + c)^2*b*c^3 + 5*I*(d*sqrt(x) + c)*b*c^4)*arctan2
(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 6*(I*(d*sqrt(x) + c)^5*b
- 5*I*(d*sqrt(x) + c)^4*b*c + 10*I*(d*sqrt(x) + c)^3*b*c^2 - 10*I*(d*sqrt(
x) + c)^2*b*c^3 + 5*I*(d*sqrt(x) + c)*b*c^4)*arctan2(cos(d*sqrt(x) + c), -
sin(d*sqrt(x) + c) + 1) - 30*(I*(d*sqrt(x) + c)^4*b - 4*I*(d*sqrt(x) + c)^
3*b*c + 6*I*(d*sqrt(x) + c)^2*b*c^2 - 4*I*(d*sqrt(x) + c)*b*c^3 + I*b*c^4)
*dilog(I*e^(I*d*sqrt(x) + I*c)) - 30*(-I*(d*sqrt(x) + c)^4*b + 4*I*(d*sqrt
(x) + c)^3*b*c - 6*I*(d*sqrt(x) + c)^2*b*c^2 + 4*I*(d*sqrt(x) + c)*b*c^3 -
I*b*c^4)*dilog(-I*e^(I*d*sqrt(x) + I*c)) + 3*((d*sqrt(x) + c)^5*b - 5*(d*
sqrt(x) + c)^4*b*c + 10*(d*sqrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c
^3 + 5*(d*sqrt(x) + c)*b*c^4)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)
)^2 + 2*sin(d*sqrt(x) + c) + 1) - 3*((d*sqrt(x) + c)^5*b - 5*(d*sqrt(x) +
c)^4*b*c + 10*(d*sqrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3 + 5*(d*
sqrt(x) + c)*b*c^4)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*si
n(d*sqrt(x) + c) + 1) - 720*I*b*polylog(6, I*e^(I*d*sqrt(x) + I*c)) + 720*
I*b*polylog(6, -I*e^(I*d*sqrt(x) + I*c)) - 720*((d*sqrt(x) + c)*b - b*c...

```

Giac [F]

$$\int x^2(a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a)x^2 dx$$

input

```
integrate(x^2*(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
```

output

```
integrate((b*sec(d*sqrt(x) + c) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a + b \sec(c + d\sqrt{x})) dx = \int x^2 \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

input `int(x^2*(a + b/cos(c + d*x^(1/2))),x)`output `int(x^2*(a + b/cos(c + d*x^(1/2))), x)`**Reduce [F]**

$$\int x^2(a + b \sec(c + d\sqrt{x})) dx = -2 \left(\int \frac{\tan\left(\frac{\sqrt{x}d + \frac{c}{2}}{2}\right)^2 x^2}{\tan\left(\frac{\sqrt{x}d + \frac{c}{2}}{2}\right)^2 - 1} dx \right) b + \frac{ax^3}{3} + \frac{bx^3}{3}$$

input `int(x^2*(a+b*sec(c+d*x^(1/2))),x)`output `(- 6*int((tan((sqrt(x)*d + c)/2)**2*x**2)/(tan((sqrt(x)*d + c)/2)**2 - 1),x)*b + a*x**3 + b*x**3)/3`

3.33 $\int x(a + b \sec(c + d\sqrt{x})) dx$

Optimal result	259
Mathematica [A] (verified)	260
Rubi [A] (verified)	261
Maple [F]	262
Fricas [F]	262
Sympy [F]	263
Maxima [B] (verification not implemented)	263
Giac [F]	264
Mupad [F(-1)]	264
Reduce [F]	264

Optimal result

Integrand size = 16, antiderivative size = 220

$$\begin{aligned}
 \int x(a + b \sec(c + d\sqrt{x})) dx = & \frac{ax^2}{2} - \frac{4ibx^{3/2} \arctan(e^{i(c+d\sqrt{x})})}{d} \\
 & + \frac{6ibx \operatorname{PolyLog}(2, -ie^{i(c+d\sqrt{x})})}{d^2} \\
 & - \frac{6ibx \operatorname{PolyLog}(2, ie^{i(c+d\sqrt{x})})}{d^2} \\
 & - \frac{12b\sqrt{x} \operatorname{PolyLog}(3, -ie^{i(c+d\sqrt{x})})}{d^3} \\
 & + \frac{12b\sqrt{x} \operatorname{PolyLog}(3, ie^{i(c+d\sqrt{x})})}{d^3} \\
 & - \frac{12ib \operatorname{PolyLog}(4, -ie^{i(c+d\sqrt{x})})}{d^4} \\
 & + \frac{12ib \operatorname{PolyLog}(4, ie^{i(c+d\sqrt{x})})}{d^4}
 \end{aligned}$$

output

```
1/2*a*x^2-4*I*b*x^(3/2)*arctan(exp(I*(c+d*x^(1/2))))/d+6*I*b*x*polylog(2,-
I*exp(I*(c+d*x^(1/2))))/d^2-6*I*b*x*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2-
12*b*x^(1/2)*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+12*b*x^(1/2)*polylog(3
,I*exp(I*(c+d*x^(1/2))))/d^3-12*I*b*polylog(4,-I*exp(I*(c+d*x^(1/2))))/d^4
+12*I*b*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.01

$$\int x(a + b \sec(c + d\sqrt{x})) dx = \frac{ax^2}{2} - \frac{4ibx^{3/2} \arctan(e^{ic+id\sqrt{x}})}{d} + \frac{6ibx \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x} \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x} \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{12ib \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{12ib \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4}$$

input

```
Integrate[x*(a + b*Sec[c + d*Sqrt[x]]),x]
```

output

```
(a*x^2)/2 - ((4*I)*b*x^(3/2)*ArcTan[E^(I*c + I*d*Sqrt[x])])/d + ((6*I)*b*x
*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))]/d^2 - ((6*I)*b*x*PolyLog[2, I*E^(
I*(c + d*Sqrt[x]))]/d^2 - (12*b*Sqrt[x]*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[
x]))])/d^3 + (12*b*Sqrt[x]*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((12
*I)*b*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((12*I)*b*PolyLog[4, I
*E^(I*(c + d*Sqrt[x]))])/d^4
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a + b \sec(c + d\sqrt{x})) dx$$

↓ 2010

$$\int (ax + bx \sec(c + d\sqrt{x})) dx$$

↓ 2009

$$\frac{ax^2}{2} - \frac{4ibx^{3/2} \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{12ib \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} +$$

$$\frac{12ib \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{12b\sqrt{x} \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} +$$

$$\frac{12b\sqrt{x} \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{6ibx \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} -$$

$$\frac{6ibx \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2}$$

input `Int[x*(a + b*Sec[c + d*Sqrt[x]]),x]`

output `(a*x^2)/2 - ((4*I)*b*x^(3/2)*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((6*I)*b*x*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((6*I)*b*x*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - (12*b*Sqrt[x]*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))])/d^3 + (12*b*Sqrt[x]*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((12*I)*b*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((12*I)*b*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))])/d^4`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [F]

$$\int x(a + b \sec(c + d\sqrt{x})) dx$$

input `int(x*(a+b*sec(c+d*x^(1/2))),x)`

output `int(x*(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int x(a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a)x dx$$

input `integrate(x*(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(b*x*sec(d*sqrt(x) + c) + a*x, x)`

Sympy [F]

$$\int x(a + b \sec(c + d\sqrt{x})) dx = \int x(a + b \sec(c + d\sqrt{x})) dx$$

input `integrate(x*(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(x*(a + b*sec(c + d*sqrt(x))), x)`

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(160) = 320$.

Time = 0.20 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.45

$$\int x(a + b \sec(c + d\sqrt{x})) dx = \text{Too large to display}$$

input `integrate(x*(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output `1/2*((d*sqrt(x) + c)^4*a - 4*(d*sqrt(x) + c)^3*a*c + 6*(d*sqrt(x) + c)^2*a*c^2 - 4*(d*sqrt(x) + c)*a*c^3 - 4*b*c^3*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 4*(I*(d*sqrt(x) + c)^3*b - 3*I*(d*sqrt(x) + c)^2*b*c + 3*I*(d*sqrt(x) + c)*b*c^2)*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 4*(I*(d*sqrt(x) + c)^3*b - 3*I*(d*sqrt(x) + c)^2*b*c + 3*I*(d*sqrt(x) + c)*b*c^2)*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 12*(I*(d*sqrt(x) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c + I*b*c^2)*dilog(I*e^(I*d*sqrt(x) + I*c)) - 12*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c - I*b*c^2)*dilog(-I*e^(I*d*sqrt(x) + I*c)) + 2*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)^2*b*c + 3*(d*sqrt(x) + c)*b*c^2)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) + 1) - 2*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)^2*b*c + 3*(d*sqrt(x) + c)*b*c^2)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c) + 1) + 24*I*b*polylog(4, I*e^(I*d*sqrt(x) + I*c)) - 24*I*b*polylog(4, -I*e^(I*d*sqrt(x) + I*c)) + 24*((d*sqrt(x) + c)*b - b*c)*polylog(3, I*e^(I*d*sqrt(x) + I*c)) - 24*((d*sqrt(x) + c)*b - b*c)*polylog(3, -I*e^(I*d*sqrt(x) + I*c)))/d^4`

Giac [F]

$$\int x(a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a)x dx$$

input `integrate(x*(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)*x, x)`

Mupad [F(-1)]

Timed out.

$$\int x(a + b \sec(c + d\sqrt{x})) dx = \int x \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

input `int(x*(a + b/cos(c + d*x^(1/2))),x)`

output `int(x*(a + b/cos(c + d*x^(1/2))), x)`

Reduce [F]

$$\int x(a + b \sec(c + d\sqrt{x})) dx = -2 \left(\int \frac{\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2 x}{\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2 - 1} dx \right) b + \frac{ax^2}{2} + \frac{bx^2}{2}$$

input `int(x*(a+b*sec(c+d*x^(1/2))),x)`

output `(- 4*int((tan((sqrt(x)*d + c)/2)**2*x)/(tan((sqrt(x)*d + c)/2)**2 - 1),x) *b + a*x**2 + b*x**2)/2`

3.34 $\int \frac{a+b \sec(c+d\sqrt{x})}{x} dx$

Optimal result	265
Mathematica [N/A]	265
Rubi [N/A]	266
Maple [N/A]	267
Fricas [N/A]	267
Sympy [N/A]	267
Maxima [N/A]	268
Giac [N/A]	268
Mupad [N/A]	269
Reduce [N/A]	269

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx = a \log(x) + b \operatorname{Int}\left(\frac{\sec(c + d\sqrt{x})}{x}, x\right)$$

output `a*ln(x)+b*Defer(Int)(sec(c+d*x^(1/2))/x,x)`

Mathematica [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x} dx$$

input `Integrate[(a + b*Sec[c + d*Sqrt[x]])/x,x]`

output `Integrate[(a + b*Sec[c + d*Sqrt[x]])/x, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx$$

↓ 2010

$$\int \left(\frac{a}{x} + \frac{b \sec(c + d\sqrt{x})}{x} \right) dx$$

↓ 2009

$$b \int \frac{\sec(c + d\sqrt{x})}{x} dx + a \log(x)$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])/x,x]`

output `$Aborted`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx$$

input `int((a+b*sec(c+d*x^(1/2)))/x,x)`output `int((a+b*sec(c+d*x^(1/2)))/x,x)`**Fricas [N/A]**

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x,x, algorithm="fricas")`output `integral((b*sec(d*sqrt(x) + c) + a)/x, x)`**Sympy [N/A]**

Not integrable

Time = 1.93 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x} dx$$

input `integrate((a+b*sec(c+d*x**(1/2)))/x,x)`

output `Integral((a + b*sec(c + d*sqrt(x)))/x, x)`

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 5.89

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x,x, algorithm="maxima")`

output `2*b*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c))/((cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x), x) + a*log(x)`

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x,x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)/x, x)`

Mupad [N/A]

Not integrable

Time = 16.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx = \int \frac{a + \frac{b}{\cos(c + d\sqrt{x})}}{x} dx$$

input `int((a + b/cos(c + d*x^(1/2)))/x,x)`output `int((a + b/cos(c + d*x^(1/2)))/x, x)`**Reduce [N/A]**

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x} dx = -2 \left(\int \frac{1}{\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2 x - x} dx \right) b - 2 \log(\sqrt{x}) b + \log(x) a$$

input `int((a+b*sec(c+d*x^(1/2)))/x,x)`output `- 2*int(1/(tan((sqrt(x)*d + c)/2)**2*x - x),x)*b - 2*log(sqrt(x))*b + log(x)*a`

3.35 $\int \frac{a+b \sec(c+d\sqrt{x})}{x^2} dx$

Optimal result	270
Mathematica [N/A]	270
Rubi [N/A]	271
Maple [N/A]	272
Fricas [N/A]	272
Sympy [N/A]	272
Maxima [N/A]	273
Giac [N/A]	273
Mupad [N/A]	274
Reduce [N/A]	274

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = -\frac{a}{x} + b \operatorname{Int}\left(\frac{\sec(c + d\sqrt{x})}{x^2}, x\right)$$

output `-a/x+b*Defer(Int)(sec(c+d*x^(1/2))/x^2,x)`

Mathematica [N/A]

Not integrable

Time = 11.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

input `Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^2,x]`

output `Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^2, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

↓ 2010

$$\int \left(\frac{a}{x^2} + \frac{b \sec(c + d\sqrt{x})}{x^2} \right) dx$$

↓ 2009

$$b \int \frac{\sec(c + d\sqrt{x})}{x^2} dx - \frac{a}{x}$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])/x^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

input `int((a+b*sec(c+d*x^(1/2)))/x^2,x)`output `int((a+b*sec(c+d*x^(1/2)))/x^2,x)`**Fricas [N/A]**

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="fricas")`output `integral((b*sec(d*sqrt(x) + c) + a)/x^2, x)`**Sympy [N/A]**

Not integrable

Time = 1.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

input `integrate((a+b*sec(c+d*x**(1/2)))/x**2,x)`

output `Integral((a + b*sec(c + d*sqrt(x)))/x**2, x)`

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.11

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="maxima")`

output `(2*b*x*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c))/((cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x^2), x) - a)/x`

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)/x^2, x)`

Mupad [N/A]

Not integrable

Time = 16.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{a + \frac{b}{\cos(c+d\sqrt{x})}}{x^2} dx$$

input `int((a + b/cos(c + d*x^(1/2)))/x^2,x)`output `int((a + b/cos(c + d*x^(1/2)))/x^2, x)`**Reduce [N/A]**

Not integrable

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 11.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

$$= \frac{-2 \cos(\sqrt{x}d + c) \left(\int \frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^4 x - 2 \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 x + x} dx \right) b d^2 x + 2 \cos(\sqrt{x}d + c) \left(\int \frac{\sqrt{x} \tan\left(\frac{\sqrt{x}d + c}{2}\right)}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^4 x^2 - 2 \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 x + x} dx \right)}{1}$$

input `int((a+b*sec(c+d*x^(1/2)))/x^2,x)`output `(- 2*cos(sqrt(x)*d + c)*int(tan((sqrt(x)*d + c)/2)**2/(tan((sqrt(x)*d + c)/2)**4*x - 2*tan((sqrt(x)*d + c)/2)**2*x + x),x)*b*d**2*x + 2*cos(sqrt(x)*d + c)*int((sqrt(x)*tan((sqrt(x)*d + c)/2))/(tan((sqrt(x)*d + c)/2)**4*x**2 - 2*tan((sqrt(x)*d + c)/2)**2*x**2 + x**2),x)*b*d*x - cos(sqrt(x)*d + c)*log(sqrt(x))*b*d**2*x - cos(sqrt(x)*d + c)*sec(sqrt(x)*d + c)*b - cos(sqrt(x)*d + c)*a + sqrt(x)*sin(sqrt(x)*d + c)*b*d)/(cos(sqrt(x)*d + c)*x)`

3.36 $\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx$

Optimal result	275
Mathematica [A] (verified)	276
Rubi [A] (verified)	276
Maple [F]	279
Fricas [F]	279
Sympy [F]	279
Maxima [B] (verification not implemented)	280
Giac [F]	281
Mupad [F(-1)]	281
Reduce [F]	281

Optimal result

Integrand size = 20, antiderivative size = 749

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx = \text{Too large to display}$$

output

```

315/2*b^2*polylog(7,-exp(2*I*(c+d*x^(1/2))))/d^8-315*b^2*x*polylog(5,-exp(
2*I*(c+d*x^(1/2))))/d^6+105*b^2*x^2*polylog(3,-exp(2*I*(c+d*x^(1/2))))/d^4
+14*b^2*x^3*ln(1+exp(2*I*(c+d*x^(1/2))))/d^2+2*b^2*x^(7/2)*tan(c+d*x^(1/2)
)/d-2*I*b^2*x^(7/2)/d-10080*I*a*b*x*polylog(6,I*exp(I*(c+d*x^(1/2))))/d^6-
840*I*a*b*x^2*polylog(4,-I*exp(I*(c+d*x^(1/2))))/d^4-28*I*a*b*x^3*polylog(
2,I*exp(I*(c+d*x^(1/2))))/d^2-8*I*a*b*x^(7/2)*arctan(exp(I*(c+d*x^(1/2))))
/d+10080*I*a*b*x*polylog(6,-I*exp(I*(c+d*x^(1/2))))/d^6+840*I*a*b*x^2*poly
log(4,I*exp(I*(c+d*x^(1/2))))/d^4+28*I*a*b*x^3*polylog(2,-I*exp(I*(c+d*x^(
1/2))))/d^2+20160*I*a*b*polylog(8,I*exp(I*(c+d*x^(1/2))))/d^8+210*I*b^2*x^(
3/2)*polylog(4,-exp(2*I*(c+d*x^(1/2))))/d^5+20160*a*b*x^(1/2)*polylog(7,I
*exp(I*(c+d*x^(1/2))))/d^7-20160*a*b*x^(1/2)*polylog(7,-I*exp(I*(c+d*x^(1/
2))))/d^7-3360*a*b*x^(3/2)*polylog(5,I*exp(I*(c+d*x^(1/2))))/d^5+3360*a*b*
x^(3/2)*polylog(5,-I*exp(I*(c+d*x^(1/2))))/d^5+168*a*b*x^(5/2)*polylog(3,I
*exp(I*(c+d*x^(1/2))))/d^3-168*a*b*x^(5/2)*polylog(3,-I*exp(I*(c+d*x^(1/2)
)))/d^3-42*I*b^2*x^(5/2)*polylog(2,-exp(2*I*(c+d*x^(1/2))))/d^3-315*I*b^2*
x^(1/2)*polylog(6,-exp(2*I*(c+d*x^(1/2))))/d^7-20160*I*a*b*polylog(8,-I*ex
p(I*(c+d*x^(1/2))))/d^8+1/4*a^2*x^4
    
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 739, normalized size of antiderivative = 0.99

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx = \text{Too large to display}$$

input `Integrate[x^3*(a + b*Sec[c + d*Sqrt[x]])^2,x]`

output

```
((-8*I)*b^2*d^7*x^(7/2) + a^2*d^8*x^4 - (32*I)*a*b*d^7*x^(7/2)*ArcTan[E^(I*(c + d*Sqrt[x]))] + 56*b^2*d^6*x^3*Log[1 + E^((2*I)*(c + d*Sqrt[x]))] + (112*I)*a*b*d^6*x^3*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (112*I)*a*b*d^6*x^3*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (168*I)*b^2*d^5*x^(5/2)*PolyLog[2, -E^((2*I)*(c + d*Sqrt[x]))] - 672*a*b*d^5*x^(5/2)*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))] + 672*a*b*d^5*x^(5/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] + 420*b^2*d^4*x^2*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))] - (3360*I)*a*b*d^4*x^2*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (3360*I)*a*b*d^4*x^2*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))] + (840*I)*b^2*d^3*x^(3/2)*PolyLog[4, -E^((2*I)*(c + d*Sqrt[x]))] + 13440*a*b*d^3*x^(3/2)*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))] - 13440*a*b*d^3*x^(3/2)*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))] - 1260*b^2*d^2*x*PolyLog[5, -E^((2*I)*(c + d*Sqrt[x]))] + (40320*I)*a*b*d^2*x*PolyLog[6, (-I)*E^(I*(c + d*Sqrt[x]))] - (40320*I)*a*b*d^2*x*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))] - (1260*I)*b^2*d*Sqrt[x]*PolyLog[6, -E^((2*I)*(c + d*Sqrt[x]))] - 80640*a*b*d*Sqrt[x]*PolyLog[7, (-I)*E^(I*(c + d*Sqrt[x]))] + 80640*a*b*d*Sqrt[x]*PolyLog[7, I*E^(I*(c + d*Sqrt[x]))] + 630*b^2*PolyLog[7, -E^((2*I)*(c + d*Sqrt[x]))] - (80640*I)*a*b*PolyLog[8, (-I)*E^(I*(c + d*Sqrt[x]))] + (80640*I)*a*b*PolyLog[8, I*E^(I*(c + d*Sqrt[x]))] + 8*b^2*d^7*x^(7/2)*Tan[c + d*Sqrt[x]]/(4*d^8)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4692, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx \\
& \quad \downarrow 4692 \\
& 2 \int x^{7/2} (a + b \sec(c + d\sqrt{x}))^2 d\sqrt{x} \\
& \quad \downarrow 3042 \\
& 2 \int x^{7/2} \left(a + b \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right) \right)^2 d\sqrt{x} \\
& \quad \downarrow 4678 \\
& 2 \int \left(a^2 x^{7/2} + b^2 \sec^2(c + d\sqrt{x}) x^{7/2} + 2ab \sec(c + d\sqrt{x}) x^{7/2} \right) d\sqrt{x} \\
& \quad \downarrow 2009 \\
& 2 \left(\frac{a^2 x^4}{8} - \frac{4iabx^{7/2} \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{10080iab \operatorname{PolyLog}\left(8, -ie^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{10080iab \operatorname{PolyLog}\left(8, ie^{i(c+d\sqrt{x})}\right)}{d^8} \right)
\end{aligned}$$

input

```
Int[x^3*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

$$\begin{aligned}
& 2*((-I)*b^2*x^{(7/2)}/d + (a^2*x^4)/8 - ((4*I)*a*b*x^{(7/2)}*ArcTan[E^{(I*(c + d*Sqrt[x])})])/d + (7*b^2*x^3*Log[1 + E^{((2*I)*(c + d*Sqrt[x])})])/d^2 + \\
& ((14*I)*a*b*x^3*PolyLog[2, (-I)*E^{(I*(c + d*Sqrt[x])})])/d^2 - ((14*I)*a*b*x^3*PolyLog[2, I*E^{(I*(c + d*Sqrt[x])})])/d^2 - ((21*I)*b^2*x^{(5/2)}*PolyLog[2, -E^{((2*I)*(c + d*Sqrt[x])})])/d^3 - \\
& (84*a*b*x^{(5/2)}*PolyLog[3, (-I)*E^{(I*(c + d*Sqrt[x])})])/d^3 + (84*a*b*x^{(5/2)}*PolyLog[3, I*E^{(I*(c + d*Sqrt[x])})])/d^3 + (105*b^2*x^2*PolyLog[3, -E^{((2*I)*(c + d*Sqrt[x])})})/(2*d^4) - \\
& ((420*I)*a*b*x^2*PolyLog[4, (-I)*E^{(I*(c + d*Sqrt[x])})])/d^4 + ((420*I)*a*b*x^2*PolyLog[4, I*E^{(I*(c + d*Sqrt[x])})])/d^4 + ((105*I)*b^2*x^{(3/2)}*PolyLog[4, -E^{((2*I)*(c + d*Sqrt[x])})])/d^5 + \\
& (1680*a*b*x^{(3/2)}*PolyLog[5, (-I)*E^{(I*(c + d*Sqrt[x])})])/d^5 - (1680*a*b*x^{(3/2)}*PolyLog[5, I*E^{(I*(c + d*Sqrt[x])})])/d^5 - (315*b^2*x*PolyLog[5, -E^{((2*I)*(c + d*Sqrt[x])})})/(2*d^6) + \\
& ((5040*I)*a*b*x*PolyLog[6, (-I)*E^{(I*(c + d*Sqrt[x])})])/d^6 - ((5040*I)*a*b*x*PolyLog[6, I*E^{(I*(c + d*Sqrt[x])})])/d^6 - (((315*I)/2)*b^2*Sqrt[x]*PolyLog[6, -E^{((2*I)*(c + d*Sqrt[x])})])/d^7 - \\
& (10080*a*b*Sqrt[x]*PolyLog[7, (-I)*E^{(I*(c + d*Sqrt[x])})])/d^7 + (10080*a*b*Sqrt[x]*PolyLog[7, I*E^{(I*(c + d*Sqrt[x])})])/d^7 + (315*b^2*PolyLog[7, -E^{((2*I)*(c + d*Sqrt[x])})})/(4*d^8) - \\
& ((10080*I)*a*b*PolyLog[8, (-I)*E^{(I*(c + d*Sqrt[x])})])/d^8 + ((10080*I)*a*b*PolyLog[8, I*E^{(I*(c + d*Sqrt[x])})])/d^8 + (b^2*x^{(7/2)}*Tan[c + d*Sqrt[x]])/d
\end{aligned}$$

Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Simp}[\text{IntSum}[u, x], x] \text{ /; } \text{SumQ}[u]$$

rule 3042

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ /; } \text{FunctionOfTrigOfLinearQ}[u, x]$$

rule 4678

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\text{Csc}[e + f*x])^n, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0]$$

rule 4692

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx$$

input

```
int(x^3*(a+b*sec(c+d*x^(1/2)))^2,x)
```

output

```
int(x^3*(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 x^3 dx$$

input

```
integrate(x^3*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

output

```
integral(b^2*x^3*sec(d*sqrt(x) + c)^2 + 2*a*b*x^3*sec(d*sqrt(x) + c) + a^2
*x^3, x)
```

Sympy [F]

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx = \int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx$$

input

```
integrate(x**3*(a+b*sec(c+d*x**(1/2)))**2,x)
```

output

```
Integral(x**3*(a + b*sec(c + d*sqrt(x)))**2, x)
```


Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6347 vs. $2(574) = 1148$.

Time = 0.45 (sec) , antiderivative size = 6347, normalized size of antiderivative = 8.47

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx = \text{Too large to display}$$

input `integrate(x^3*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output

```
1/4*((d*sqrt(x) + c)^8*a^2 - 8*(d*sqrt(x) + c)^7*a^2*c + 28*(d*sqrt(x) + c)^6*a^2*c^2 - 56*(d*sqrt(x) + c)^5*a^2*c^3 + 70*(d*sqrt(x) + c)^4*a^2*c^4 - 56*(d*sqrt(x) + c)^3*a^2*c^5 + 28*(d*sqrt(x) + c)^2*a^2*c^6 - 8*(d*sqrt(x) + c)*a^2*c^7 - 16*a*b*c^7*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 8*(60*b^2*c^7 + 60*((d*sqrt(x) + c)^7*a*b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d*sqrt(x) + c)^5*a*b*c^2 - 35*(d*sqrt(x) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)^3*a*b*c^4 - 21*(d*sqrt(x) + c)^2*a*b*c^5 + 7*(d*sqrt(x) + c)*a*b*c^6 + ((d*sqrt(x) + c)^7*a*b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d*sqrt(x) + c)^5*a*b*c^2 - 35*(d*sqrt(x) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)^3*a*b*c^4 - 21*(d*sqrt(x) + c)^2*a*b*c^5 + 7*(d*sqrt(x) + c)*a*b*c^6)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^7*a*b - 7*I*(d*sqrt(x) + c)^6*a*b*c + 21*I*(d*sqrt(x) + c)^5*a*b*c^2 - 35*I*(d*sqrt(x) + c)^4*a*b*c^3 + 35*I*(d*sqrt(x) + c)^3*a*b*c^4 - 21*I*(d*sqrt(x) + c)^2*a*b*c^5 + 7*I*(d*sqrt(x) + c)*a*b*c^6)*sin(2*d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) + 60*((d*sqrt(x) + c)^7*a*b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d*sqrt(x) + c)^5*a*b*c^2 - 35*(d*sqrt(x) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)^3*a*b*c^4 - 21*(d*sqrt(x) + c)^2*a*b*c^5 + 7*(d*sqrt(x) + c)*a*b*c^6 + ((d*sqrt(x) + c)^7*a*b - 7*(d*sqrt(x) + c)^6*a*b*c + 21*(d*sqrt(x) + c)^5*a*b*c^2 - 35*(d*sqrt(x) + c)^4*a*b*c^3 + 35*(d*sqrt(x) + c)^3*a*b*c^4 - 21*(d*sqrt(x) + c)^2*a*b*c^5 + 7*(d*sqrt(x) + c)*a*b*c^6)*cos(2*d*sqrt(x) + ...
```

Giac [F]

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 x^3 dx$$

input `integrate(x^3*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)^2*x^3, x)`

Mupad [F(-1)]

Timed out.

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx = \int x^3 \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

input `int(x^3*(a + b/cos(c + d*x^(1/2)))^2,x)`

output `int(x^3*(a + b/cos(c + d*x^(1/2)))^2, x)`

Reduce [F]

$$\int x^3 (a + b \sec(c + d\sqrt{x}))^2 dx = \text{too large to display}$$

input `int(x^3*(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(16*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a*b*d**7*x**3 + 6720
*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a*b*d**5*x**2 - 215040*
sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a*b*d**3*x - 645120*sqrt
(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a*b*d - 16*sqrt(x)*cos(sqrt(
x)*d + c)*tan((sqrt(x)*d + c)/2)*b**2*d**7*x**3 - 6720*sqrt(x)*cos(sqrt(x)
*d + c)*tan((sqrt(x)*d + c)/2)*b**2*d**5*x**2 + 215040*sqrt(x)*cos(sqrt(x)
*d + c)*tan((sqrt(x)*d + c)/2)*b**2*d**3*x + 645120*sqrt(x)*cos(sqrt(x)*d
+ c)*tan((sqrt(x)*d + c)/2)*b**2*d + 16*cos(sqrt(x)*d + c)*int(x**3/(tan((
sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*tan((sqrt(x)*d
+ c)/2)**2*a*b*d**8 - 16*cos(sqrt(x)*d + c)*int(x**3/(tan((sqrt(x)*d + c)/
2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*a*b*d**8 + 3360*cos(sqrt(x)*d
+ c)*int(x**2/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1
),x)*tan((sqrt(x)*d + c)/2)**2*a*b*d**6 - 3360*cos(sqrt(x)*d + c)*int(x**2
/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*tan((sqr
t(x)*d + c)/2)**2*b**2*d**6 - 3360*cos(sqrt(x)*d + c)*int(x**2/(tan((sqrt(
x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*a*b*d**6 + 3360*cos(
sqrt(x)*d + c)*int(x**2/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)
/2)**2 + 1),x)*b**2*d**6 + 2688*cos(sqrt(x)*d + c)*int((tan((sqrt(x)*d + c
)/2)**4*x**2)/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1
),x)*tan((sqrt(x)*d + c)/2)**2*a*b*d**6 - 2688*cos(sqrt(x)*d + c)*int((...
```

3.37 $\int x^2 (a + b \sec (c + d\sqrt{x}))^2 dx$

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Optimal result

Integrand size = 20, antiderivative size = 551

$$\begin{aligned}
\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx = & -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8iabx^{5/2} \arctan(e^{i(c+d\sqrt{x})})}{d} \\
& + \frac{10b^2x^2 \log(1 + e^{2i(c+d\sqrt{x})})}{d^2} \\
& + \frac{20iabx^2 \operatorname{PolyLog}(2, -ie^{i(c+d\sqrt{x})})}{d^2} \\
& - \frac{20iabx^2 \operatorname{PolyLog}(2, ie^{i(c+d\sqrt{x})})}{d^2} \\
& - \frac{20ib^2x^{3/2} \operatorname{PolyLog}(2, -e^{2i(c+d\sqrt{x})})}{d^3} \\
& - \frac{80abx^{3/2} \operatorname{PolyLog}(3, -ie^{i(c+d\sqrt{x})})}{d^3} \\
& + \frac{80abx^{3/2} \operatorname{PolyLog}(3, ie^{i(c+d\sqrt{x})})}{d^3} \\
& + \frac{30b^2x \operatorname{PolyLog}(3, -e^{2i(c+d\sqrt{x})})}{d^4} \\
& - \frac{240iabx \operatorname{PolyLog}(4, -ie^{i(c+d\sqrt{x})})}{d^4} \\
& + \frac{240iabx \operatorname{PolyLog}(4, ie^{i(c+d\sqrt{x})})}{d^4} \\
& + \frac{30ib^2\sqrt{x} \operatorname{PolyLog}(4, -e^{2i(c+d\sqrt{x})})}{d^5} \\
& + \frac{480ab\sqrt{x} \operatorname{PolyLog}(5, -ie^{i(c+d\sqrt{x})})}{d^5} \\
& - \frac{480ab\sqrt{x} \operatorname{PolyLog}(5, ie^{i(c+d\sqrt{x})})}{d^5} \\
& - \frac{15b^2 \operatorname{PolyLog}(5, -e^{2i(c+d\sqrt{x})})}{d^6} \\
& + \frac{480iab \operatorname{PolyLog}(6, -ie^{i(c+d\sqrt{x})})}{d^6} \\
& - \frac{480iab \operatorname{PolyLog}(6, ie^{i(c+d\sqrt{x})})}{d^6} \\
& + \frac{2b^2x^{5/2} \tan(c + d\sqrt{x})}{d}
\end{aligned}$$

output

```
-8*I*a*b*x^(5/2)*arctan(exp(I*(c+d*x^(1/2))))/d+1/3*a^2*x^3-20*I*a*b*x^2*
polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2+10*b^2*x^2*ln(1+exp(2*I*(c+d*x^(1/2)
)))/d^2+240*I*a*b*x*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^4-480*I*a*b*polylog
(6,I*exp(I*(c+d*x^(1/2))))/d^6-240*I*a*b*x*polylog(4,-I*exp(I*(c+d*x^(1/2)
)))/d^4-80*a*b*x^(3/2)*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+80*a*b*x^(3/
2)*polylog(3,I*exp(I*(c+d*x^(1/2))))/d^3+30*b^2*x*polylog(3,-exp(2*I*(c+d*
x^(1/2))))/d^4-20*I*b^2*x^(3/2)*polylog(2,-exp(2*I*(c+d*x^(1/2))))/d^3+30*
I*b^2*x^(1/2)*polylog(4,-exp(2*I*(c+d*x^(1/2))))/d^5+480*I*a*b*polylog(6,-
I*exp(I*(c+d*x^(1/2))))/d^6+480*a*b*x^(1/2)*polylog(5,-I*exp(I*(c+d*x^(1/2)
)))/d^5-480*a*b*x^(1/2)*polylog(5,I*exp(I*(c+d*x^(1/2))))/d^5-15*b^2*poly
log(5,-exp(2*I*(c+d*x^(1/2))))/d^6+20*I*a*b*x^2*polylog(2,-I*exp(I*(c+d*x^
(1/2))))/d^2-2*I*b^2*x^(5/2)/d+2*b^2*x^(5/2)*tan(c+d*x^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.99

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx$$

$$= \frac{-6ib^2d^5x^{5/2} + a^2d^6x^3 - 24iabd^5x^{5/2} \arctan\left(e^{i(c+d\sqrt{x})}\right) + 30b^2d^4x^2 \log\left(1 + e^{2i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(6, -e^{i(c+d\sqrt{x})}\right) + 60iabd^4x^2 \operatorname{PolyLog}\left(6, e^{i(c+d\sqrt{x})}\right)}{d^6}$$

input

```
Integrate[x^2*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

```
((-6*I)*b^2*d^5*x^(5/2) + a^2*d^6*x^3 - (24*I)*a*b*d^5*x^(5/2)*ArcTan[E^(I
*(c + d*Sqrt[x]))] + 30*b^2*d^4*x^2*Log[1 + E^((2*I)*(c + d*Sqrt[x]))] + (
60*I)*a*b*d^4*x^2*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (60*I)*a*b*d^4*
x^2*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (60*I)*b^2*d^3*x^(3/2)*PolyLog[2
, -E^((2*I)*(c + d*Sqrt[x]))] - 240*a*b*d^3*x^(3/2)*PolyLog[3, (-I)*E^(I*(
c + d*Sqrt[x]))] + 240*a*b*d^3*x^(3/2)*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))]
+ 90*b^2*d^2*x*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))] - (720*I)*a*b*d^2*x
*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (720*I)*a*b*d^2*x*PolyLog[4, I*E
^(I*(c + d*Sqrt[x]))] + (90*I)*b^2*d*Sqrt[x]*PolyLog[4, -E^((2*I)*(c + d*S
qrt[x]))] + 1440*a*b*d*Sqrt[x]*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))] - 14
40*a*b*d*Sqrt[x]*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))] - 45*b^2*PolyLog[5, -
E^((2*I)*(c + d*Sqrt[x]))] + (1440*I)*a*b*PolyLog[6, (-I)*E^(I*(c + d*Sqrt
[x]))] - (1440*I)*a*b*PolyLog[6, I*E^(I*(c + d*Sqrt[x]))] + 6*b^2*d^5*x^(5
/2)*Tan[c + d*Sqrt[x]]/(3*d^6)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4692, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx \\
 & \quad \downarrow 4692 \\
 & 2 \int x^{5/2} (a + b \sec(c + d\sqrt{x}))^2 d\sqrt{x} \\
 & \quad \downarrow 3042 \\
 & 2 \int x^{5/2} \left(a + b \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right) \right)^2 d\sqrt{x} \\
 & \quad \downarrow 4678 \\
 & 2 \int \left(a^2 x^{5/2} + b^2 \sec^2(c + d\sqrt{x}) x^{5/2} + 2ab \sec(c + d\sqrt{x}) x^{5/2} \right) d\sqrt{x} \\
 & \quad \downarrow 2009 \\
 & 2 \left(\frac{a^2 x^3}{6} - \frac{4iabx^{5/2} \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{240iab \operatorname{PolyLog}\left(6, -ie^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{240iab \operatorname{PolyLog}\left(6, ie^{i(c+d\sqrt{x})}\right)}{d^6} \right) +
 \end{aligned}$$

input `Int[x^2*(a + b*Sec[c + d*Sqrt[x]])^2,x]`

output

$$2 * (((-I) * b^2 * x^{(5/2)}) / d + (a^2 * x^3) / 6 - ((4 * I) * a * b * x^{(5/2)} * \text{ArcTan}[E^{(I * (c + d * \text{Sqrt}[x])}]]) / d + (5 * b^2 * x^2 * \text{Log}[1 + E^{((2 * I) * (c + d * \text{Sqrt}[x])]}]) / d^2 + ((10 * I) * a * b * x^2 * \text{PolyLog}[2, (-I) * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^2 - ((10 * I) * a * b * x^2 * \text{PolyLog}[2, I * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^2 - ((10 * I) * b^2 * x^{(3/2)} * \text{PolyLog}[2, -E^{((2 * I) * (c + d * \text{Sqrt}[x])}])]) / d^3 - (40 * a * b * x^{(3/2)} * \text{PolyLog}[3, (-I) * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^3 + (40 * a * b * x^{(3/2)} * \text{PolyLog}[3, I * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^3 + (15 * b^2 * x * \text{PolyLog}[3, -E^{((2 * I) * (c + d * \text{Sqrt}[x])}])]) / d^4 - ((120 * I) * a * b * x * \text{PolyLog}[4, (-I) * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^4 + ((120 * I) * a * b * x * \text{PolyLog}[4, I * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^4 + ((15 * I) * b^2 * \text{Sqrt}[x] * \text{PolyLog}[4, -E^{((2 * I) * (c + d * \text{Sqrt}[x])}])]) / d^5 + (240 * a * b * \text{Sqrt}[x] * \text{PolyLog}[5, (-I) * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^5 - (240 * a * b * \text{Sqrt}[x] * \text{PolyLog}[5, I * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^5 - (15 * b^2 * \text{PolyLog}[5, -E^{((2 * I) * (c + d * \text{Sqrt}[x])}])]) / (2 * d^6) + ((240 * I) * a * b * \text{PolyLog}[6, (-I) * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^6 - ((240 * I) * a * b * \text{PolyLog}[6, I * E^{(I * (c + d * \text{Sqrt}[x])}])]) / d^6 + (b^2 * x^{(5/2)} * \text{Tan}[c + d * \text{Sqrt}[x]]) / d$$

Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Simp}[\text{IntSum}[u, x], x] \text{ /; } \text{SumQ}[u]$$

rule 3042

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ /; } \text{FunctionOfTrigOfLinearQ}[u, x]$$

rule 4678

$$\text{Int}[(\text{csc}[(e_)] + (f_)*(x_)]*(b_)] + (a_)]^{(n_)}*((c_)] + (d_)]*(x_)]^{(m_)}, x_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\text{Csc}[e + f*x])^n, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0]$$

rule 4692

$$\text{Int}[(x_)]^{(m_)}*((a_)] + (b_)]*\text{Sec}[(c_)] + (d_)]*(x_)]^{(n_)]^{(p_)}, x_Symbol] \text{ :> } \text{Simp}[1/n \ \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)}*(a + b*\text{Sec}[c + d*x])^p, x], x, x^n], x] \text{ /; } \text{FreeQ}\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{IGtQ}[\text{Simplify}[(m + 1)/n], 0] \ \&\& \ \text{IntegerQ}[p]$$

Maple [F]

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx$$

input `int(x^2*(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(x^2*(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(b^2*x^2*sec(d*sqrt(x) + c)^2 + 2*a*b*x^2*sec(d*sqrt(x) + c) + a^2*x^2, x)`

Sympy [F]

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx = \int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx$$

input `integrate(x**2*(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(x**2*(a + b*sec(c + d*sqrt(x)))**2, x)`

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3879 vs. $2(422) = 844$.

Time = 0.33 (sec) , antiderivative size = 3879, normalized size of antiderivative = 7.04

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx = \text{Too large to display}$$

input `integrate(x^2*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output

```
1/3*((d*sqrt(x) + c)^6*a^2 - 6*(d*sqrt(x) + c)^5*a^2*c + 15*(d*sqrt(x) + c)^4*a^2*c^2 - 20*(d*sqrt(x) + c)^3*a^2*c^3 + 15*(d*sqrt(x) + c)^2*a^2*c^4 - 6*(d*sqrt(x) + c)*a^2*c^5 - 12*a*b*c^5*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 6*(12*b^2*c^5 + 12*((d*sqrt(x) + c)^5*a*b - 5*(d*sqrt(x) + c)^4*a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a*b*c^3 + 5*(d*sqrt(x) + c)*a*b*c^4 + ((d*sqrt(x) + c)^5*a*b - 5*(d*sqrt(x) + c)^4*a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a*b*c^3 + 5*(d*sqrt(x) + c)*a*b*c^4)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^5*a*b - 5*I*(d*sqrt(x) + c)^4*a*b*c + 10*I*(d*sqrt(x) + c)^3*a*b*c^2 - 10*I*(d*sqrt(x) + c)^2*a*b*c^3 + 5*I*(d*sqrt(x) + c)*a*b*c^4)*sin(2*d*sqrt(x) + 2*c)) *arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) + 12*((d*sqrt(x) + c)^5*a*b - 5*(d*sqrt(x) + c)^4*a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a*b*c^3 + 5*(d*sqrt(x) + c)*a*b*c^4 + ((d*sqrt(x) + c)^5*a*b - 5*(d*sqrt(x) + c)^4*a*b*c + 10*(d*sqrt(x) + c)^3*a*b*c^2 - 10*(d*sqrt(x) + c)^2*a*b*c^3 + 5*(d*sqrt(x) + c)*a*b*c^4)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^5*a*b - 5*I*(d*sqrt(x) + c)^4*a*b*c + 10*I*(d*sqrt(x) + c)^3*a*b*c^2 - 10*I*(d*sqrt(x) + c)^2*a*b*c^3 + 5*I*(d*sqrt(x) + c)*a*b*c^4)*sin(2*d*sqrt(x) + 2*c)) *arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 10*(6*(d*sqrt(x) + c)^4*b^2 - 16*(d*sqrt(x) + c)^3*b^2*c + 18*(d*sqrt(x) + c)^2*b^2*c^2 - 12*(d*sqrt(x) + c)*b^2*c^3 + 3*b^2*c^4 + (6*(d*s...
```

Giac [F]

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)^2*x^2, x)`

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx = \int x^2 \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

input `int(x^2*(a + b/cos(c + d*x^(1/2)))^2,x)`

output `int(x^2*(a + b/cos(c + d*x^(1/2)))^2, x)`

Reduce [F]

$$\int x^2 (a + b \sec(c + d\sqrt{x}))^2 dx = \text{too large to display}$$

input `int(x^2*(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(12*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a*b*d**5*x**2 - 1680
*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a*b*d**3*x - 5040*sqrt(
x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a*b*d - 12*sqrt(x)*cos(sqrt(x)
)*d + c)*tan((sqrt(x)*d + c)/2)*b**2*d**5*x**2 + 1680*sqrt(x)*cos(sqrt(x)*
d + c)*tan((sqrt(x)*d + c)/2)*b**2*d**3*x + 5040*sqrt(x)*cos(sqrt(x)*d + c
)*tan((sqrt(x)*d + c)/2)*b**2*d + 12*cos(sqrt(x)*d + c)*int(x**2/(tan((sqr
t(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*tan((sqrt(x)*d + c
)/2)**2*a*b*d**6 - 12*cos(sqrt(x)*d + c)*int(x**2/(tan((sqrt(x)*d + c)/2)*
**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*a*b*d**6 + 12*cos(sqrt(x)*d + c)*
int((tan((sqrt(x)*d + c)/2)**2*x**2)/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((s
qrt(x)*d + c)/2)**2 + 1),x)*tan((sqrt(x)*d + c)/2)**2*a*b*d**6 - 12*cos(sq
rt(x)*d + c)*int((tan((sqrt(x)*d + c)/2)**2*x**2)/(tan((sqrt(x)*d + c)/2)*
**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*tan((sqrt(x)*d + c)/2)**2*b**2*d*
**6 - 12*cos(sqrt(x)*d + c)*int((tan((sqrt(x)*d + c)/2)**2*x**2)/(tan((sqrt
(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*a*b*d**6 + 12*cos(s
qrt(x)*d + c)*int((tan((sqrt(x)*d + c)/2)**2*x**2)/(tan((sqrt(x)*d + c)/2)
**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*b**2*d**6 - 1920*cos(sqrt(x)*d +
c)*int((tan((sqrt(x)*d + c)/2)**2*x)/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((
sqrt(x)*d + c)/2)**2 + 1),x)*tan((sqrt(x)*d + c)/2)**2*a*b*d**4 + 1920*cos
(sqrt(x)*d + c)*int((tan((sqrt(x)*d + c)/2)**2*x)/(tan((sqrt(x)*d + c)/...
```

3.38 $\int x(a + b \sec(c + d\sqrt{x}))^2 dx$

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Optimal result

Integrand size = 18, antiderivative size = 355

$$\begin{aligned}
 \int x(a + b \sec(c + d\sqrt{x}))^2 dx = & -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8iabx^{3/2} \arctan(e^{i(c+d\sqrt{x})})}{d} \\
 & + \frac{6b^2x \log(1 + e^{2i(c+d\sqrt{x})})}{d^2} \\
 & + \frac{12iabx \operatorname{PolyLog}(2, -ie^{i(c+d\sqrt{x})})}{d^2} \\
 & - \frac{12iabx \operatorname{PolyLog}(2, ie^{i(c+d\sqrt{x})})}{d^2} \\
 & - \frac{6ib^2\sqrt{x} \operatorname{PolyLog}(2, -e^{2i(c+d\sqrt{x})})}{d^3} \\
 & - \frac{24ab\sqrt{x} \operatorname{PolyLog}(3, -ie^{i(c+d\sqrt{x})})}{d^3} \\
 & + \frac{24ab\sqrt{x} \operatorname{PolyLog}(3, ie^{i(c+d\sqrt{x})})}{d^3} \\
 & + \frac{3b^2 \operatorname{PolyLog}(3, -e^{2i(c+d\sqrt{x})})}{d^4} \\
 & - \frac{24iab \operatorname{PolyLog}(4, -ie^{i(c+d\sqrt{x})})}{d^4} \\
 & + \frac{24iab \operatorname{PolyLog}(4, ie^{i(c+d\sqrt{x})})}{d^4} \\
 & + \frac{2b^2x^{3/2} \tan(c + d\sqrt{x})}{d}
 \end{aligned}$$

output

```

-2*I*b^2*x^(3/2)/d+1/2*a^2*x^2-8*I*a*b*x^(3/2)*arctan(exp(I*(c+d*x^(1/2))))
)/d+6*b^2*x*ln(1+exp(2*I*(c+d*x^(1/2))))/d^2+12*I*a*b*x*polylog(2,-I*exp(I
*(c+d*x^(1/2))))/d^2-12*I*a*b*x*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2-6*I*
b^2*x^(1/2)*polylog(2,-exp(2*I*(c+d*x^(1/2))))/d^3-24*a*b*x^(1/2)*polylog(
3,-I*exp(I*(c+d*x^(1/2))))/d^3+24*a*b*x^(1/2)*polylog(3,I*exp(I*(c+d*x^(1/
2))))/d^3+3*b^2*polylog(3,-exp(2*I*(c+d*x^(1/2))))/d^4-24*I*a*b*polylog(4,
-I*exp(I*(c+d*x^(1/2))))/d^4+24*I*a*b*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^
4+2*b^2*x^(3/2)*tan(c+d*x^(1/2))/d

```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.98

$$\int x(a + b \sec(c + d\sqrt{x}))^2 dx$$

$$= \frac{-4ib^2d^3x^{3/2} + a^2d^4x^2 - 16iabd^3x^{3/2} \arctan\left(e^{i(c+d\sqrt{x})}\right) + 12b^2d^2x \log\left(1 + e^{2i(c+d\sqrt{x})}\right) + 24iabd^2x \operatorname{PolyLog}\left[2, -E^{i(c+d\sqrt{x})}\right] + 24iabd^2x \operatorname{PolyLog}\left[2, E^{i(c+d\sqrt{x})}\right] - (12I)*b^2*d*\sqrt{x}*\operatorname{PolyLog}\left[2, -E^{((2I)*(c+d*\sqrt{x}))}\right] - (24I)*a*b*d^2*x*\operatorname{PolyLog}\left[2, (-I)*E^{(I*(c+d*\sqrt{x}))}\right] - (24I)*a*b*d^2*x*\operatorname{PolyLog}\left[2, I*E^{(I*(c+d*\sqrt{x}))}\right] - (12I)*b^2*d*\sqrt{x}*\operatorname{PolyLog}\left[2, -E^{((2I)*(c+d*\sqrt{x}))}\right] - 48*a*b*d*\sqrt{x}*\operatorname{PolyLog}\left[3, (-I)*E^{(I*(c+d*\sqrt{x}))}\right] + 48*a*b*d*\sqrt{x}*\operatorname{PolyLog}\left[3, I*E^{(I*(c+d*\sqrt{x}))}\right] + 6*b^2*\operatorname{PolyLog}\left[3, -E^{((2I)*(c+d*\sqrt{x}))}\right] - (48I)*a*b*\operatorname{PolyLog}\left[4, (-I)*E^{(I*(c+d*\sqrt{x}))}\right] + (48I)*a*b*\operatorname{PolyLog}\left[4, I*E^{(I*(c+d*\sqrt{x}))}\right] + 4*b^2*d^3*x^{3/2}*\tan[c+d*\sqrt{x}]]/(2*d^4)$$

input

```
Integrate[x*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

```
((-4*I)*b^2*d^3*x^(3/2) + a^2*d^4*x^2 - (16*I)*a*b*d^3*x^(3/2)*ArcTan[E^(I*(c + d*Sqrt[x]))] + 12*b^2*d^2*x*Log[1 + E^((2*I)*(c + d*Sqrt[x]))] + (24*I)*a*b*d^2*x*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (24*I)*a*b*d^2*x*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (12*I)*b^2*d*Sqrt[x]*PolyLog[2, -E^((2*I)*(c + d*Sqrt[x]))] - 48*a*b*d*Sqrt[x]*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))] + 48*a*b*d*Sqrt[x]*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] + 6*b^2*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))] - (48*I)*a*b*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (48*I)*a*b*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))] + 4*b^2*d^3*x^(3/2)*Tan[c + d*Sqrt[x]])/(2*d^4)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4692, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a + b \sec(c + d\sqrt{x}))^2 dx$$

$$\downarrow 4692$$

$$2 \int x^{3/2}(a + b \sec(c + d\sqrt{x}))^2 d\sqrt{x}$$

$$\downarrow 3042$$

$$\begin{aligned}
& 2 \int x^{3/2} \left(a + b \csc \left(c + d\sqrt{x} + \frac{\pi}{2} \right) \right)^2 d\sqrt{x} \\
& \quad \downarrow 4678 \\
& 2 \int \left(x^{3/2} a^2 + 2bx^{3/2} \sec(c + d\sqrt{x}) a + b^2 x^{3/2} \sec^2(c + d\sqrt{x}) \right) d\sqrt{x} \\
& \quad \downarrow 2009 \\
& 2 \left(\frac{a^2 x^2}{4} - \frac{4iabx^{3/2} \arctan \left(e^{i(c+d\sqrt{x})} \right)}{d} - \frac{12iab \operatorname{PolyLog} \left(4, -ie^{i(c+d\sqrt{x})} \right)}{d^4} + \frac{12iab \operatorname{PolyLog} \left(4, ie^{i(c+d\sqrt{x})} \right)}{d^4} \right) - 1
\end{aligned}$$

input `Int[x*(a + b*Sec[c + d*Sqrt[x]])^2,x]`

output `2*(((-I)*b^2*x^(3/2))/d + (a^2*x^2)/4 - ((4*I)*a*b*x^(3/2)*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + (3*b^2*x*Log[1 + E^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((6*I)*a*b*x*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((6*I)*a*b*x*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - ((3*I)*b^2*Sqrt[x]*PolyLog[2, -E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (12*a*b*Sqrt[x]*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))])/d^3 + (12*a*b*Sqrt[x]*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 + (3*b^2*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))])/(2*d^4) - ((12*I)*a*b*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((12*I)*a*b*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (b^2*x^(3/2)*Tan[c + d*Sqrt[x]])/d`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 4692

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :-> Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int x(a + b \sec(c + d\sqrt{x}))^2 dx$$

```
input int(x*(a+b*sec(c+d*x^(1/2)))^2,x)
```

```
output int(x*(a+b*sec(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x(a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 x dx$$

```
input integrate(x*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

```
output integral(b^2*x*sec(d*sqrt(x) + c)^2 + 2*a*b*x*sec(d*sqrt(x) + c) + a^2*x,
x)
```

Sympy [F]

$$\int x(a + b \sec(c + d\sqrt{x}))^2 dx = \int x(a + b \sec(c + d\sqrt{x}))^2 dx$$

```
input integrate(x*(a+b*sec(c+d*x**(1/2)))**2,x)
```

```
output Integral(x*(a + b*sec(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1991 vs. $2(270) = 540$.

Time = 0.22 (sec) , antiderivative size = 1991, normalized size of antiderivative = 5.61

$$\int x(a + b \sec(c + d\sqrt{x}))^2 dx = \text{Too large to display}$$

input `integrate(x*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output

```
1/2*((d*sqrt(x) + c)^4*a^2 - 4*(d*sqrt(x) + c)^3*a^2*c + 6*(d*sqrt(x) + c)^2*a^2*c^2 - 4*(d*sqrt(x) + c)*a^2*c^3 - 8*a*b*c^3*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 4*(4*b^2*c^3 + 4*((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2 + ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^3*a*b - 3*I*(d*sqrt(x) + c)^2*a*b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) + 4*((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2 + ((d*sqrt(x) + c)^3*a*b - 3*(d*sqrt(x) + c)^2*a*b*c + 3*(d*sqrt(x) + c)*a*b*c^2)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^3*a*b - 3*I*(d*sqrt(x) + c)^2*a*b*c + 3*I*(d*sqrt(x) + c)*a*b*c^2)*sin(2*d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 6*((d*sqrt(x) + c)^2*b^2 - 2*(d*sqrt(x) + c)*b^2*c + b^2*c^2 + ((d*sqrt(x) + c)^2*b^2 - 2*(d*sqrt(x) + c)*b^2*c + b^2*c^2)*cos(2*d*sqrt(x) + 2*c) - (-I*(d*sqrt(x) + c)^2*b^2 + 2*I*(d*sqrt(x) + c)*b^2*c - I*b^2*c^2)*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(2*d*sqrt(x) + 2*c), cos(2*d*sqrt(x) + 2*c) + 1) + 4*((d*sqrt(x) + c)^3*b^2 - 3*(d*sqrt(x) + c)^2*b^2*c + 3*(d*sqrt(x) + c)*b^2*c^2)*cos(2*d*sqrt(x) + 2*c) + 6*((d*sqrt(x) + c)*b^2 - b^2*c + ((d*sqrt(x) + c)*b^2 - b^2*c)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)*b^2 - I*b^2*c)*sin(2*d*sqrt(x) + 2*c))*dilog(-e^(2*I*d*sqrt(x) + 2*I*c...
```

Giac [F]

$$\int x(a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 x dx$$

input `integrate(x*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)^2*x, x)`

Mupad [F(-1)]

Timed out.

$$\int x(a + b \sec(c + d\sqrt{x}))^2 dx = \int x \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

input `int(x*(a + b/cos(c + d*x^(1/2)))^2,x)`

output `int(x*(a + b/cos(c + d*x^(1/2)))^2, x)`

Reduce [F]

$$\int x(a + b \sec(c + d\sqrt{x}))^2 dx = \text{Too large to display}$$

input `int(x*(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(24*cos(sqrt(x)*d + c)*int((sqrt(x)*tan((sqrt(x)*d + c)/2))/(tan((sqrt(x)*
d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*a*b*d**3 - 24*cos(sqrt(
x)*d + c)*int((sqrt(x)*tan((sqrt(x)*d + c)/2))/(tan((sqrt(x)*d + c)/2)**4
- 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*b**2*d**3 + 8*cos(sqrt(x)*d + c)*int
(x/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*a*b*d*
*4 + 24*cos(sqrt(x)*d + c)*int(1/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(
x)*d + c)/2)**2 + 1),x)*a*b*d**2 - 24*cos(sqrt(x)*d + c)*int(1/(tan((sqrt(
x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*b**2*d**2 + 12*cos(s
qrt(x)*d + c)*log(tan((sqrt(x)*d + c)/2)**2 + 1)*a*b - 12*cos(sqrt(x)*d +
c)*log(tan((sqrt(x)*d + c)/2)**2 + 1)*b**2 - 12*cos(sqrt(x)*d + c)*log(tan
((sqrt(x)*d + c)/2) - 1)*a*b + 12*cos(sqrt(x)*d + c)*log(tan((sqrt(x)*d +
c)/2) - 1)*b**2 - 12*cos(sqrt(x)*d + c)*log(tan((sqrt(x)*d + c)/2) + 1)*a*
b + 12*cos(sqrt(x)*d + c)*log(tan((sqrt(x)*d + c)/2) + 1)*b**2 + cos(sqrt(
x)*d + c)*a**2*d**4*x**2 - cos(sqrt(x)*d + c)*a*b*d**4*x**2 - 6*cos(sqrt(x)
)*d + c)*a*b*d**2*x + 6*cos(sqrt(x)*d + c)*b**2*d**2*x - 4*sqrt(x)*sin(sqr
t(x)*d + c)*a*b*d**3*x - 12*sqrt(x)*sin(sqrt(x)*d + c)*a*b*d + 4*sqrt(x)*s
in(sqrt(x)*d + c)*b**2*d**3*x + 12*sqrt(x)*sin(sqrt(x)*d + c)*b**2*d - 12*
a*b*d**2*x + 12*b**2*d**2*x)/(2*cos(sqrt(x)*d + c)*d**4)
```

3.39 $\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x} dx$

Optimal result	300
Mathematica [N/A]	300
Rubi [N/A]	301
Maple [N/A]	301
Fricas [N/A]	302
Sympy [N/A]	302
Maxima [N/A]	303
Giac [N/A]	303
Mupad [N/A]	304
Reduce [N/A]	304

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx = \text{Int}\left(\frac{(a + b \sec(c + d\sqrt{x}))^2}{x}, x\right)$$

output

```
Defer(Int)((a+b*sec(c+d*x^(1/2)))^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 70.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx = \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx$$

input

```
Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x,x]
```

output

```
Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx$$

↓ 4694

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])^2/x,x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx$$

input `int((a+b*sec(c+d*x^(1/2)))^2/x,x)`

output `int((a+b*sec(c+d*x^(1/2)))^2/x,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x,x, algorithm="fricas")`

output `integral((b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2)/x, x)`

Sympy [N/A]

Not integrable

Time = 9.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx = \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx$$

input `integrate((a+b*sec(c+d*x**(1/2)))**2/x,x)`

output `Integral((a + b*sec(c + d*sqrt(x)))**2/x, x)`

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 319, normalized size of antiderivative = 15.95

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x,x, algorithm="maxima")`

output `(4*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) + (d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x*integrate(2*(b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) + 2*(a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c))*x)/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x) + (a^2*d*cos(2*d*sqrt(x) + 2*c)^2 + a^2*d*sin(2*d*sqrt(x) + 2*c)^2 + 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d)*x*log(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x)`

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x,x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)^2/x, x)`

Mupad [N/A]

Not integrable

Time = 16.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx = \int \frac{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2}{x} dx$$

input `int((a + b/cos(c + d*x^(1/2)))^2/x,x)`output `int((a + b/cos(c + d*x^(1/2)))^2/x, x)`**Reduce [N/A]**

Not integrable

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\begin{aligned} \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x} dx &= - \left(\int \frac{\sin(\sqrt{x}d + c)^2}{\sin(\sqrt{x}d + c)^2 x - x} dx \right) b^2 \\ &\quad - 4 \left(\int \frac{1}{\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2 x - x} dx \right) ab \\ &\quad - 4 \log(\sqrt{x}) ab + 2 \log(\sqrt{x}) b^2 + \log(x) a^2 \end{aligned}$$

input `int((a+b*sec(c+d*x^(1/2)))^2/x,x)`output `- int(sin(sqrt(x)*d + c)**2/(sin(sqrt(x)*d + c)**2*x - x),x)*b**2 - 4*int(1/(tan((sqrt(x)*d + c)/2)**2*x - x),x)*a*b - 4*log(sqrt(x))*a*b + 2*log(sqrt(x))*b**2 + log(x)*a**2`

3.40 $\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^2} dx$

Optimal result	305
Mathematica [N/A]	305
Rubi [N/A]	306
Maple [N/A]	306
Fricas [N/A]	307
Sympy [N/A]	307
Maxima [N/A]	308
Giac [N/A]	308
Mupad [N/A]	309
Reduce [N/A]	309

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx = \text{Int}\left(\frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2}, x\right)$$

output `Defer(Int)((a+b*sec(c+d*x^(1/2)))^2/x^2,x)`

Mathematica [N/A]

Not integrable

Time = 51.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx$$

input `Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^2,x]`

output `Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^2, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx$$

↓ 4694

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])^2/x^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx$$

input `int((a+b*sec(c+d*x^(1/2)))^2/x^2,x)`

output `int((a+b*sec(c+d*x^(1/2)))^2/x^2,x)`

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^2,x, algorithm="fricas")`

output `integral((b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2)/x^2, x)`

Sympy [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx$$

input `integrate((a+b*sec(c+d*x**(1/2)))**2/x**2,x)`

output `Integral((a + b*sec(c + d*sqrt(x)))**2/x**2, x)`

Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 321, normalized size of antiderivative = 16.05

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^2,x, algorithm="maxima")`

output `((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2*integrate(2*(3*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) + 2*(a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c)))*x)/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^3), x) + 4*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) - (a^2*d*cos(2*d*sqrt(x) + 2*c)^2 + a^2*d*sin(2*d*sqrt(x) + 2*c)^2 + 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d)*x)/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2)`

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^2,x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)^2/x^2, x)`

Mupad [N/A]

Not integrable

Time = 16.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2}{x^2} dx$$

input `int((a + b/cos(c + d*x^(1/2)))^2/x^2, x)`output `int((a + b/cos(c + d*x^(1/2)))^2/x^2, x)`**Reduce [N/A]**

Not integrable

Time = 0.21 (sec) , antiderivative size = 4800, normalized size of antiderivative = 240.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^2} dx = \text{Too large to display}$$

input `int((a+b*sec(c+d*x^(1/2)))^2/x^2, x)`

output

```
( - 10*cos(sqrt(x)*d + c)*sin(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**4*a*b
*d*x + 10*cos(sqrt(x)*d + c)*sin(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**4*
b**2*d*x + 20*cos(sqrt(x)*d + c)*sin(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)
**2*a*b*d*x - 20*cos(sqrt(x)*d + c)*sin(sqrt(x)*d + c)*tan((sqrt(x)*d + c)
/2)**2*b**2*d*x - 10*cos(sqrt(x)*d + c)*sin(sqrt(x)*d + c)*a*b*d*x + 10*co
s(sqrt(x)*d + c)*sin(sqrt(x)*d + c)*b**2*d*x - 30*sqrt(x)*int(tan((sqrt(x)
*d + c)/2)**4/(tan((sqrt(x)*d + c)/2)**6*x - 3*tan((sqrt(x)*d + c)/2)**4*x
+ 3*tan((sqrt(x)*d + c)/2)**2*x - x),x)*sin(sqrt(x)*d + c)**2*tan((sqrt(x)
)*d + c)/2)**4*a*b*d**2*x + 48*sqrt(x)*int(tan((sqrt(x)*d + c)/2)**4/(tan(
(sqrt(x)*d + c)/2)**6*x - 3*tan((sqrt(x)*d + c)/2)**4*x + 3*tan((sqrt(x)*d
+ c)/2)**2*x - x),x)*sin(sqrt(x)*d + c)**2*tan((sqrt(x)*d + c)/2)**4*b**2
*d**2*x + 60*sqrt(x)*int(tan((sqrt(x)*d + c)/2)**4/(tan((sqrt(x)*d + c)/2)
**6*x - 3*tan((sqrt(x)*d + c)/2)**4*x + 3*tan((sqrt(x)*d + c)/2)**2*x - x)
,x)*sin(sqrt(x)*d + c)**2*tan((sqrt(x)*d + c)/2)**2*a*b*d**2*x - 96*sqrt(x)
)*int(tan((sqrt(x)*d + c)/2)**4/(tan((sqrt(x)*d + c)/2)**6*x - 3*tan((sqrt
(x)*d + c)/2)**4*x + 3*tan((sqrt(x)*d + c)/2)**2*x - x),x)*sin(sqrt(x)*d +
c)**2*tan((sqrt(x)*d + c)/2)**2*b**2*d**2*x - 30*sqrt(x)*int(tan((sqrt(x)
*d + c)/2)**4/(tan((sqrt(x)*d + c)/2)**6*x - 3*tan((sqrt(x)*d + c)/2)**4*x
+ 3*tan((sqrt(x)*d + c)/2)**2*x - x),x)*sin(sqrt(x)*d + c)**2*a*b*d**2*x
+ 48*sqrt(x)*int(tan((sqrt(x)*d + c)/2)**4/(tan((sqrt(x)*d + c)/2)**6*x...
```

3.41 $\int \frac{x^3}{a+b \sec(c+d\sqrt{x})} dx$

Optimal result	311
Mathematica [A] (verified)	312
Rubi [A] (verified)	313
Maple [F]	315
Fricas [F]	316
Sympy [F]	316
Maxima [F(-2)]	316
Giac [F]	317
Mupad [F(-1)]	317
Reduce [F]	317

Optimal result

Integrand size = 20, antiderivative size = 1041

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx = \text{Too large to display}$$

output

```

1/4*x^4/a-84*I*b*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^3+1680*I*b*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^5+14*b*x^3*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-14*b*x^3*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-10080*I*b*x^(1/2)*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^7-2*I*b*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d-420*b*x^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4+420*b*x^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4+10080*I*b*x^(1/2)*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^7+2*I*b*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d+5040*b*x*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^6-5040*b*x*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^6-1680*I*b*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^5+84*I*b*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^3-10080*b*polylog(8,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^8+10080*b*polylog(8,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^8

```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 802, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx$$

$$= \frac{\sqrt{-a^2 + b^2} d^8 x^4 + 8 i b d^7 x^{7/2} \log\left(1 - \frac{a e^{i(c+d\sqrt{x})}}{-b + \sqrt{-a^2 + b^2}}\right) - 8 i b d^7 x^{7/2} \log\left(1 + \frac{a e^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}}\right) + 56 b d^6 x^3 \text{PolyLog}\left(\frac{1}{2}, \frac{a e^{i(c+d\sqrt{x})}}{-b + \sqrt{-a^2 + b^2}}\right) + 56 b d^6 x^3 \text{PolyLog}\left(\frac{1}{2}, \frac{a e^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}}$$

input

```
Integrate[x^3/(a + b*Sec[c + d*Sqrt[x]]),x]
```

output

```
(Sqrt[-a^2 + b^2]*d^8*x^4 + (8*I)*b*d^7*x^(7/2)*Log[1 - (a*E^(I*(c + d*Sqr
t[x])))/(-b + Sqrt[-a^2 + b^2])] - (8*I)*b*d^7*x^(7/2)*Log[1 + (a*E^(I*(c
+ d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])] + 56*b*d^6*x^3*PolyLog[2, (a*E^(I*(c
+ d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - 56*b*d^6*x^3*PolyLog[2, -((a*E
^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (336*I)*b*d^5*x^(5/2)*Pol
yLog[3, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (336*I)*b*d^5
*x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] -
1680*b*d^4*x^2*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2
])] + 1680*b*d^4*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
+ b^2]))] - (6720*I)*b*d^3*x^(3/2)*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/(-
b + Sqrt[-a^2 + b^2])] + (6720*I)*b*d^3*x^(3/2)*PolyLog[5, -((a*E^(I*(c +
d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + 20160*b*d^2*x*PolyLog[6, (a*E^(I*
(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - 20160*b*d^2*x*PolyLog[6, -((a
*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (40320*I)*b*d*Sqrt[x]*P
olyLog[7, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (40320*I)*b
*d*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]
- 40320*b*PolyLog[8, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] +
40320*b*PolyLog[8, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/
(4*a*Sqrt[-a^2 + b^2]*d^8)
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 1043, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx$$

$$\downarrow 4692$$

$$2 \int \frac{x^{7/2}}{a + b \sec(c + d\sqrt{x})} d\sqrt{x}$$

$$\downarrow 3042$$

$$2 \int \frac{x^{7/2}}{a + b \csc(c + d\sqrt{x} + \frac{\pi}{2})} d\sqrt{x}$$

$$\int \left(\frac{x^{7/2}}{a} - \frac{bx^{7/2}}{a(b + a \cos(c + d\sqrt{x}))} \right) d\sqrt{x}$$

$$2 \left(\frac{x^4}{8a} + \frac{ib \log \left(\frac{e^{i(c+d\sqrt{x})} a}{b - \sqrt{b^2 - a^2}} + 1 \right) x^{7/2}}{a\sqrt{b^2 - a^2}d} - \frac{ib \log \left(\frac{e^{i(c+d\sqrt{x})} a}{b + \sqrt{b^2 - a^2}} + 1 \right) x^{7/2}}{a\sqrt{b^2 - a^2}d} + \frac{7b \operatorname{PolyLog} \left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b - \sqrt{b^2 - a^2}} \right) x^3}{a\sqrt{b^2 - a^2}d^2} - \frac{7b \operatorname{PolyLog} \left(2, \frac{ae^{i(c+d\sqrt{x})}}{b + \sqrt{b^2 - a^2}} \right) x^3}{a\sqrt{b^2 - a^2}d^2} \right)$$

input

```
Int[x^3/(a + b*Sec[c + d*Sqrt[x]]),x]
```

output

```
2*(x^4/(8*a) + (I*b*x^(7/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]]))/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^(7/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]]))/(a*Sqrt[-a^2 + b^2]*d) + (7*b*x^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) - (7*b*x^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + ((42*I)*b*x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - ((42*I)*b*x^(5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (210*b*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (210*b*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) - ((840*I)*b*x^(3/2)*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) + ((840*I)*b*x^(3/2)*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) + (2520*b*x*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) - (2520*b*x*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) + ((5040*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^7) - ((5040*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^7) - ((5040*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^7) + ((5040*I)*b*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^7)
```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx$$

input `int(x^3/(a+b*sec(c+d*x^(1/2))),x)`

output `int(x^3/(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^3}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x^3/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(x^3/(b*sec(d*sqrt(x) + c) + a), x)`

Sympy [F]

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx$$

input `integrate(x**3/(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(x**3/(a + b*sec(c + d*sqrt(x))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x^3/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f or more de`

Giac [F]

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^3}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x^3/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(x^3/(b*sec(d*sqrt(x) + c) + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^3}{a + \frac{b}{\cos(c+d\sqrt{x})}} dx$$

input `int(x^3/(a + b/cos(c + d*x^(1/2))),x)`

output `int(x^3/(a + b/cos(c + d*x^(1/2))), x)`

Reduce [F]

$$\int \frac{x^3}{a + b \sec(c + d\sqrt{x})} dx$$

$$= \frac{8 \left(\int \frac{\tan\left(\frac{\sqrt{x}d + \frac{c}{2}}\right)^2 x^3}{\tan\left(\frac{\sqrt{x}d + \frac{c}{2}}\right)^2 a^2 - \tan\left(\frac{\sqrt{x}d + \frac{c}{2}}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) ab + 8 \left(\int \frac{\tan\left(\frac{\sqrt{x}d + \frac{c}{2}}\right)^2 x^3}{\tan\left(\frac{\sqrt{x}d + \frac{c}{2}}\right)^2 a^2 - \tan\left(\frac{\sqrt{x}d + \frac{c}{2}}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) b^2}{4a + 4b}$$

input `int(x^3/(a+b*sec(c+d*x^(1/2))),x)`

output

```
(8*int((tan((sqrt(x)*d + c)/2)**2*x**3)/(tan((sqrt(x)*d + c)/2)**2*a**2 -  
tan((sqrt(x)*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*a*b + 8*int((tan(  
(sqrt(x)*d + c)/2)**2*x**3)/(tan((sqrt(x)*d + c)/2)**2*a**2 - tan((sqrt(x)  
*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*b**2 + x**4)/(4*(a + b))
```

$$3.42 \quad \int \frac{x^2}{a+b \sec(c+d\sqrt{x})} dx$$

Optimal result	320
Mathematica [A] (verified)	321
Rubi [A] (verified)	322
Maple [F]	324
Fricas [F]	324
Sympy [F]	325
Maxima [F(-2)]	325
Giac [F]	325
Mupad [F(-1)]	326
Reduce [F]	326

Optimal result

Integrand size = 20, antiderivative size = 781

$$\begin{aligned}
 \int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx = & \frac{x^3}{3a} + \frac{2ibx^{5/2} \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
 & - \frac{2ibx^{5/2} \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
 & + \frac{10bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 & - \frac{10bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 & + \frac{40ibx^{3/2} \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
 & - \frac{40ibx^{3/2} \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
 & - \frac{120bx \operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
 & + \frac{120bx \operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
 & - \frac{240ib\sqrt{x} \operatorname{PolyLog}\left(5, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
 & + \frac{240ib\sqrt{x} \operatorname{PolyLog}\left(5, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
 & + \frac{240b \operatorname{PolyLog}\left(6, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} \\
 & - \frac{240b \operatorname{PolyLog}\left(6, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}
 \end{aligned}$$

output

```

1/3*x^3/a+2*I*b*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/
a/(-a^2+b^2)^(1/2)/d-2*I*b*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^
2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d+10*b*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))
/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2-10*b*x^2*polylog(2,-a*exp(I*
(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2+40*I*b*x^(3/2)
*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)
/d^3-40*I*b*x^(3/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)
))/a/(-a^2+b^2)^(1/2)/d^3-120*b*x*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a
^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4+120*b*x*polylog(4,-a*exp(I*(c+d*x^(
1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^4-240*I*b*x^(1/2)*polylo
g(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^5+2
40*I*b*x^(1/2)*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/(
-a^2+b^2)^(1/2)/d^5+240*b*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^
(1/2)))/a/(-a^2+b^2)^(1/2)/d^6-240*b*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(b+
(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^6

```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 608, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx$$

$$= \frac{\sqrt{-a^2 + b^2} d^6 x^3 + 6 i b d^5 x^{5/2} \log\left(1 - \frac{a e^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}}\right) - 6 i b d^5 x^{5/2} \log\left(1 + \frac{a e^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}}\right) + 30 b d^4 x^2 \text{PolyLog}\left(\frac{1}{2}, -\frac{a e^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}}\right) + 30 b d^4 x^2 \text{PolyLog}\left(\frac{1}{2}, \frac{a e^{i(c+d\sqrt{x})}}{b + \sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}}$$

input

```
Integrate[x^2/(a + b*Sec[c + d*Sqrt[x]]),x]
```

output

```
(Sqrt[-a^2 + b^2]*d^6*x^3 + (6*I)*b*d^5*x^(5/2)*Log[1 - (a*E^(I*(c + d*Sqr
t[x])))/(-b + Sqrt[-a^2 + b^2])] - (6*I)*b*d^5*x^(5/2)*Log[1 + (a*E^(I*(c
+ d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])] + 30*b*d^4*x^2*PolyLog[2, (a*E^(I*(c
+ d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - 30*b*d^4*x^2*PolyLog[2, -((a*E
^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (120*I)*b*d^3*x^(3/2)*Pol
yLog[3, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (120*I)*b*d^3
*x^(3/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] -
360*b*d^2*x*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])]
+ 360*b*d^2*x*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2
]))] - (720*I)*b*d*Sqrt[x]*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt
[-a^2 + b^2])] + (720*I)*b*d*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x]))
)/(b + Sqrt[-a^2 + b^2]))] + 720*b*PolyLog[6, (a*E^(I*(c + d*Sqrt[x])))/(-
b + Sqrt[-a^2 + b^2])] - 720*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(b +
Sqrt[-a^2 + b^2]))]/(3*a*Sqrt[-a^2 + b^2]*d^6)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 783, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx \\
 & \quad \downarrow 4692 \\
 & 2 \int \frac{x^{5/2}}{a + b \sec(c + d\sqrt{x})} d\sqrt{x} \\
 & \quad \downarrow 3042 \\
 & 2 \int \frac{x^{5/2}}{a + b \csc(c + d\sqrt{x} + \frac{\pi}{2})} d\sqrt{x} \\
 & \quad \downarrow 4679 \\
 & 2 \int \left(\frac{x^{5/2}}{a} - \frac{bx^{5/2}}{a(b + a \cos(c + d\sqrt{x}))} \right) d\sqrt{x}
 \end{aligned}$$

↓ 2009

$$2 \left(\frac{120b \operatorname{PolyLog} \left(6, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}} \right)}{ad^6\sqrt{b^2-a^2}} - \frac{120b \operatorname{PolyLog} \left(6, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}} \right)}{ad^6\sqrt{b^2-a^2}} - \frac{120ib\sqrt{x} \operatorname{PolyLog} \left(5, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}} \right)}{ad^5\sqrt{b^2-a^2}} + \frac{120i}{ad^5\sqrt{b^2-a^2}} \right)$$

input `Int[x^2/(a + b*Sec[c + d*Sqrt[x]]), x]`

output

```
2*(x^3/(6*a) + (I*b*x^(5/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x]))]/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^(5/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x]))]/(b + Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d) + (5*b*x^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x]))]/(b - Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^2) - (5*b*x^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x]))]/(b + Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^2) + ((20*I)*b*x^(3/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x]))]/(b - Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^3) - ((20*I)*b*x^(3/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x]))]/(b + Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^3) - (60*b*x*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x]))]/(b - Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^4) + (60*b*x*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x]))]/(b + Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^4) - ((120*I)*b*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x]))]/(b - Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^5) + ((120*I)*b*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x]))]/(b + Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^5) + (120*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x]))]/(b - Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^6) - (120*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x]))]/(b + Sqrt[-a^2 + b^2]))]/(a*Sqrt[-a^2 + b^2]*d^6))
```

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx$$

input `int(x^2/(a+b*sec(c+d*x^(1/2))),x)`

output `int(x^2/(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^2}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x^2/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(x^2/(b*sec(d*sqrt(x) + c) + a), x)`

Sympy [F]

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx$$

input `integrate(x**2/(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(x**2/(a + b*sec(c + d*sqrt(x))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x^2/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f or more de`

Giac [F]

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^2}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x^2/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(x^2/(b*sec(d*sqrt(x) + c) + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^2}{a + \frac{b}{\cos(c+d\sqrt{x})}} dx$$

input `int(x^2/(a + b/cos(c + d*x^(1/2))),x)`output `int(x^2/(a + b/cos(c + d*x^(1/2))), x)`**Reduce [F]**

$$\int \frac{x^2}{a + b \sec(c + d\sqrt{x})} dx$$

$$= \frac{6 \left(\int \frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 x^2}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 - \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) ab + 6 \left(\int \frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 x^2}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 - \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) b^2}{3a + 3b}$$

input `int(x^2/(a+b*sec(c+d*x^(1/2))),x)`output `(6*int((tan((sqrt(x)*d + c)/2)**2*x**2)/(tan((sqrt(x)*d + c)/2)**2*a**2 - tan((sqrt(x)*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*a*b + 6*int((tan((sqrt(x)*d + c)/2)**2*x**2)/(tan((sqrt(x)*d + c)/2)**2*a**2 - tan((sqrt(x)*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*b**2 + x**3)/(3*(a + b))`

3.43 $\int \frac{x}{a+b \sec(c+d\sqrt{x})} dx$

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Optimal result

Integrand size = 18, antiderivative size = 521

$$\begin{aligned}
 \int \frac{x}{a+b \sec(c+d\sqrt{x})} dx = & \frac{x^2}{2a} + \frac{2ibx^{3/2} \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
 & - \frac{2ibx^{3/2} \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
 & + \frac{6bx \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 & - \frac{6bx \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 & + \frac{12ib\sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
 & - \frac{12ib\sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
 & - \frac{12b \operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
 & + \frac{12b \operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}
 \end{aligned}$$

output

$$\begin{aligned} & \frac{1/2*x^2/a+2*I*b*x^{(3/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})}{a/(-a^2+b^2)^{(1/2)}/d-2*I*b*x^{(3/2)}*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})} \\ & /a/(-a^2+b^2)^{(1/2)}/d+6*b*x*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)}) \\ & /a/(-a^2+b^2)^{(1/2)}/d^2-6*b*x*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)}) \\ & /a/(-a^2+b^2)^{(1/2)}/d^2+12*I*b*x^{(1/2)}*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)}) \\ & /a/(-a^2+b^2)^{(1/2)}/d^3-12*I*b*x^{(1/2)}*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)}) \\ & /a/(-a^2+b^2)^{(1/2)}/d^3-12*b*polylog(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)}) \\ & /a/(-a^2+b^2)^{(1/2)}/d^4+12*b*polylog(4,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)}) \\ & /a/(-a^2+b^2)^{(1/2)}/d^4 \end{aligned}$$
Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.79

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx = \frac{\sqrt{-a^2 + b^2} d^4 x^2 + 4ibd^3 x^{3/2} \log\left(1 - \frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) - 4ibd^3 x^{3/2} \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right) + 12bd^2 x \text{PolyLog}\left(2, \frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) - 12bd^2 x \text{PolyLog}\left(2, \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{(2a^2 + b^2)d^4}$$

input

Integrate[x/(a + b*Sec[c + d*Sqrt[x]]),x]

output

$$\begin{aligned} & (\text{Sqrt}[-a^2 + b^2]*d^4*x^2 + (4*I)*b*d^3*x^{(3/2)}*\text{Log}[1 - (a*E^{(I*(c + d*\text{Sqrt}[x]))})/(-b + \text{Sqrt}[-a^2 + b^2])] - (4*I)*b*d^3*x^{(3/2)}*\text{Log}[1 + (a*E^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2])] + 12*b*d^2*x*\text{PolyLog}[2, (a*E^{(I*(c + d*\text{Sqrt}[x]))})/(-b + \text{Sqrt}[-a^2 + b^2])] - 12*b*d^2*x*\text{PolyLog}[2, -((a*E^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))] + (24*I)*b*d*\text{Sqrt}[x]*\text{PolyLog}[3, (a*E^{(I*(c + d*\text{Sqrt}[x]))})/(-b + \text{Sqrt}[-a^2 + b^2])] - (24*I)*b*d*\text{Sqrt}[x]*\text{PolyLog}[3, -((a*E^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))] - 24*b*\text{PolyLog}[4, (a*E^{(I*(c + d*\text{Sqrt}[x]))})/(-b + \text{Sqrt}[-a^2 + b^2])] + 24*b*\text{PolyLog}[4, -((a*E^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))])]/(2*a*\text{Sqrt}[-a^2 + b^2]*d^4) \end{aligned}$$

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{a + b \sec(c + d\sqrt{x})} dx \\
 & \quad \downarrow 4692 \\
 & 2 \int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} d\sqrt{x} \\
 & \quad \downarrow 3042 \\
 & 2 \int \frac{x^{3/2}}{a + b \csc(c + d\sqrt{x} + \frac{\pi}{2})} d\sqrt{x} \\
 & \quad \downarrow 4679 \\
 & 2 \int \left(\frac{x^{3/2}}{a} - \frac{bx^{3/2}}{a(b + a \cos(c + d\sqrt{x}))} \right) d\sqrt{x} \\
 & \quad \downarrow 2009 \\
 & 2 \left(-\frac{6b \operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{6b \operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{6ib\sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{6ib\sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} \right)
 \end{aligned}$$

input

```
Int[x/(a + b*Sec[c + d*Sqrt[x]]),x]
```

output

```

2*(x^2/(4*a) + (I*b*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x]))]/(b - Sqrt[-a
^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^(3/2)*Log[1 + (a*E^(I*(c + d*
Sqrt[x]))]/(b + Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d) + (3*b*x*PolyLo
g[2, -((a*E^(I*(c + d*Sqrt[x]))]/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 +
b^2]*d^2) - (3*b*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x]))]/(b + Sqrt[-a^2 +
b^2])))/(a*Sqrt[-a^2 + b^2]*d^2) + ((6*I)*b*Sqrt[x]*PolyLog[3, -((a*E^(I
*(c + d*Sqrt[x]))]/(b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^3) - ((
6*I)*b*Sqrt[x]*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x]))]/(b + Sqrt[-a^2 + b^2
])))/(a*Sqrt[-a^2 + b^2]*d^3) - (6*b*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])
))/b - Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d^4) + (6*b*PolyLog[4, -
(a*E^(I*(c + d*Sqrt[x]))]/(b + Sqrt[-a^2 + b^2])))/(a*Sqrt[-a^2 + b^2]*d
4))

```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 4692

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx$$

input `int(x/(a+b*sec(c+d*x^(1/2))),x)`

output `int(x/(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(x/(b*sec(d*sqrt(x) + c) + a), x)`

Sympy [F]

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x}{a + b \sec(c + d\sqrt{x})} dx$$

input `integrate(x/(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(x/(a + b*sec(c + d*sqrt(x))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f or more de

Giac [F]

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(x/(b*sec(d*sqrt(x) + c) + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x}{a + \frac{b}{\cos(c + d\sqrt{x})}} dx$$

input `int(x/(a + b/cos(c + d*x^(1/2))),x)`

output `int(x/(a + b/cos(c + d*x^(1/2))), x)`

Reduce [F]

$$\int \frac{x}{a + b \sec(c + d\sqrt{x})} dx$$

$$= \frac{4 \left(\int \frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 x}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 - \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) ab + 4 \left(\int \frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 x}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 - \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) b^2}{2a + 2b}$$

input `int(x/(a+b*sec(c+d*x^(1/2))),x)`

output `(4*int((tan((sqrt(x)*d + c)/2)**2*x)/(tan((sqrt(x)*d + c)/2)**2*a**2 - tan((sqrt(x)*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*a*b + 4*int((tan((sqrt(x)*d + c)/2)**2*x)/(tan((sqrt(x)*d + c)/2)**2*a**2 - tan((sqrt(x)*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*b**2 + x**2)/(2*(a + b))`

$$3.44 \quad \int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx$$

Optimal result	334
Mathematica [N/A]	334
Rubi [N/A]	335
Maple [N/A]	335
Fricas [N/A]	336
Sympy [N/A]	336
Maxima [N/A]	337
Giac [N/A]	337
Mupad [N/A]	338
Reduce [N/A]	338

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx = \text{Int}\left(\frac{1}{x(a+b \sec(c+d\sqrt{x}))}, x\right)$$

output `Defer(Int)(1/x/(a+b*sec(c+d*x^(1/2))), x)`

Mathematica [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx = \int \frac{1}{x(a+b \sec(c+d\sqrt{x}))} dx$$

input `Integrate[1/(x*(a + b*Sec[c + d*Sqrt[x]])), x]`

output `Integrate[1/(x*(a + b*Sec[c + d*Sqrt[x]])), x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx$$

↓ 4694

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx$$

input `Int[1/(x*(a + b*Sec[c + d*Sqrt[x]])),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx$$

input `int(1/x/(a+b*sec(c+d*x^(1/2))),x)`

output `int(1/x/(a+b*sec(c+d*x^(1/2))),x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)x} dx$$

input `integrate(1/x/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(1/(b*x*sec(d*sqrt(x) + c) + a*x), x)`

Sympy [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx$$

input `integrate(1/x/(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(1/(x*(a + b*sec(c + d*sqrt(x)))), x)`

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 241, normalized size of antiderivative = 12.05

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)x} dx$$

input `integrate(1/x/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output

```
-(2*a*b*integrate((a*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + 2*b*cos(d*sqrt(x) + c)^2 + a*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 2*b*sin(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) + c)))/((a^3*cos(2*d*sqrt(x) + 2*c)^2 + 4*a*b^2*cos(d*sqrt(x) + c)^2 + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a^2*b*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 4*a*b^2*sin(d*sqrt(x) + c)^2 + 4*a^2*b*cos(d*sqrt(x) + c) + a^3 + 2*(2*a^2*b*cos(d*sqrt(x) + c) + a^3)*cos(2*d*sqrt(x) + 2*c))*x), x) - log(x))/a
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)x} dx$$

input `integrate(1/x/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output

```
integrate(1/((b*sec(d*sqrt(x) + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 15.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{x \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right)} dx$$

input `int(1/(x*(a + b/cos(c + d*x^(1/2))))),x)`output `int(1/(x*(a + b/cos(c + d*x^(1/2))))), x)`**Reduce [N/A]**

Not integrable

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 7.75

$$\int \frac{1}{x(a + b \sec(c + d\sqrt{x}))} dx$$

$$= \frac{2 \left(\int \frac{1}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 x - 2 \tan\left(\frac{\sqrt{x}d + c}{2}\right) abx + \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 x - a^2 x + b^2 x} dx \right) ab - 2 \left(\int \frac{1}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 x - 2 \tan\left(\frac{\sqrt{x}d + c}{2}\right) abx + \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 x - a^2 x + b^2 x} dx \right)}{a - b}$$

input `int(1/x/(a+b*sec(c+d*x^(1/2))))),x)`output `(2*(int(1/(tan((sqrt(x)*d + c)/2)**2*a**2*x - 2*tan((sqrt(x)*d + c)/2)**2*a*b*x + tan((sqrt(x)*d + c)/2)**2*b**2*x - a**2*x + b**2*x),x)*a*b - int(1/(tan((sqrt(x)*d + c)/2)**2*a**2*x - 2*tan((sqrt(x)*d + c)/2)**2*a*b*x + tan((sqrt(x)*d + c)/2)**2*b**2*x - a**2*x + b**2*x),x)*b**2 + log(sqrt(x)))/(a - b)`

3.45 $\int \frac{a+b \sec(c+d\sqrt{x})}{x^2} dx$

Optimal result	339
Mathematica [N/A]	339
Rubi [N/A]	340
Maple [N/A]	341
Fricas [N/A]	341
Sympy [N/A]	341
Maxima [N/A]	342
Giac [N/A]	342
Mupad [N/A]	343
Reduce [N/A]	343

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = -\frac{a}{x} + b \operatorname{Int}\left(\frac{\sec(c + d\sqrt{x})}{x^2}, x\right)$$

output `-a/x+b*Defer(Int)(sec(c+d*x^(1/2))/x^2,x)`

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

input `Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^2,x]`

output `Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^2, x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

↓ 2010

$$\int \left(\frac{a}{x^2} + \frac{b \sec(c + d\sqrt{x})}{x^2} \right) dx$$

↓ 2009

$$b \int \frac{\sec(c + d\sqrt{x})}{x^2} dx - \frac{a}{x}$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])/x^2,x]`

output `$Aborted`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

input `int((a+b*sec(c+d*x^(1/2)))/x^2,x)`output `int((a+b*sec(c+d*x^(1/2)))/x^2,x)`**Fricas [N/A]**

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="fricas")`output `integral((b*sec(d*sqrt(x) + c) + a)/x^2, x)`**Sympy [N/A]**

Not integrable

Time = 1.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

input `integrate((a+b*sec(c+d*x**(1/2)))/x**2,x)`

output `Integral((a + b*sec(c + d*sqrt(x)))/x**2, x)`

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.11

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="maxima")`

output `(2*b*x*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c)))/((cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x^2), x) - a)/x`

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^2,x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)/x^2, x)`

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx = \int \frac{a + \frac{b}{\cos(c+d\sqrt{x})}}{x^2} dx$$

input `int((a + b/cos(c + d*x^(1/2)))/x^2,x)`output `int((a + b/cos(c + d*x^(1/2)))/x^2, x)`**Reduce [N/A]**

Not integrable

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 11.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^2} dx$$

$$= \frac{-2 \cos(\sqrt{x}d + c) \left(\int \frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^4 x - 2 \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 x + x} dx \right) b d^2 x + 2 \cos(\sqrt{x}d + c) \left(\int \frac{\sqrt{x} \tan\left(\frac{\sqrt{x}d + c}{2}\right)}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^4 x^2 - 2 \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 x + x} dx \right)}{1}$$

input `int((a+b*sec(c+d*x^(1/2)))/x^2,x)`output `(- 2*cos(sqrt(x)*d + c)*int(tan((sqrt(x)*d + c)/2)**2/(tan((sqrt(x)*d + c)/2)**4*x - 2*tan((sqrt(x)*d + c)/2)**2*x + x),x)*b*d**2*x + 2*cos(sqrt(x)*d + c)*int((sqrt(x)*tan((sqrt(x)*d + c)/2))/(tan((sqrt(x)*d + c)/2)**4*x**2 - 2*tan((sqrt(x)*d + c)/2)**2*x**2 + x**2),x)*b*d*x - cos(sqrt(x)*d + c)*log(sqrt(x))*b*d**2*x - cos(sqrt(x)*d + c)*sec(sqrt(x)*d + c)*b - cos(sqrt(x)*d + c)*a + sqrt(x)*sin(sqrt(x)*d + c)*b*d)/(cos(sqrt(x)*d + c)*x)`

$$3.46 \quad \int \frac{x^3}{(a+b \sec(c+d\sqrt{x}))^2} dx$$

Optimal result	344
Mathematica [A] (verified)	345
Rubi [A] (verified)	346
Maple [F]	348
Fricas [F]	349
Sympy [F]	349
Maxima [F(-2)]	349
Giac [F]	350
Mupad [F(-1)]	350
Reduce [F]	350

Optimal result

Integrand size = 20, antiderivative size = 3123

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

output

```

-3360*I*b*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/
a^2/(-a^2+b^2)^(1/2)/d^5-20160*I*b*x^(1/2)*polylog(7,-a*exp(I*(c+d*x^(1/2)
))/b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(1/2)/d^7-10080*I*b^3*x^(1/2)*poly
log(7,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d
^7-84*I*b^2*x^(5/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)
))/a^2/(a^2-b^2)/d^3-84*I*b^2*x^(5/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b
-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-1680*I*b^3*x^(3/2)*polylog(5,-a*exp
(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5+2*I*b^3*x
^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3
/2)/d+168*I*b*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)
))/a^2/(-a^2+b^2)^(1/2)/d^3+4*I*b*x^(7/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-
(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d+3360*I*b*x^(3/2)*polylog(5,-a*ex
p(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^5+20160*I*
b*x^(1/2)*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^
2+b^2)^(1/2)/d^7+10080*I*b^3*x^(1/2)*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(b+
(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^7+1680*I*b^2*x^(3/2)*polylog(4,-
a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^5+1680*I*b^3
*x^(3/2)*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(3/2)/d^5+84*I*b^3*x^(5/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^
2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+1680*I*b^2*x^(3/2)*polylog(4,-a...

```

Mathematica [A] (verified)

Time = 12.28 (sec) , antiderivative size = 3737, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Result too large to show}$$

input

```
Integrate[x^3/(a + b*Sec[c + d*sqrt[x]])^2,x]
```

output

```

((-4*I)*b^2*E^((2*I)*c)*x^(7/2)*(b + a*Cos[c + d*Sqrt[x]])^2*Sec[c + d*Sqr
t[x]]^2)/(a^2*(a^2 - b^2)*d*(1 + E^((2*I)*c))*(a + b*Sec[c + d*Sqrt[x]])^2
) + (x^4*(b + a*Cos[c + d*Sqrt[x]])^2*Sec[c + d*Sqrt[x]]^2)/(4*a^2*(a + b*
Sec[c + d*Sqrt[x]])^2) + (2*b*(b + a*Cos[c + d*Sqrt[x]])^2*(7*b*d^6*Sqrt[(-
a^2 + b^2)*E^((2*I)*c)]*x^3*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c
) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) + (2*I)*a^2*d^7*E^(I*c)*x^(7/2)*Log[1
+ (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)
]) - I*b^2*d^7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I
*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) + 7*b*d^6*Sqrt[(-a^2 + b^2)*E^((2*I
)*c)]*x^3*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c) + Sqrt[(-a^2 + b^
2)*E^((2*I)*c)]]) - (2*I)*a^2*d^7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c + d
*Sqrt[x])))]/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) + I*b^2*d^7*E^(I
*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c) + Sqrt[(-a^2 +
b^2)*E^((2*I)*c)]]) - 7*d^5*((6*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^
2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x^(5/2)*PolyLog[2, -((a*E^(I*
(2*c + d*Sqrt[x])))]/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 7*d^5
*((-6*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*
d*E^(I*c)*Sqrt[x])*x^(5/2)*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(
I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 210*b*d^4*Sqrt[(-a^2 + b^2)*E^
(2*I)*c)]*x^2*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c) - Sqr...

```

Rubi [A] (verified)

Time = 4.68 (sec) , antiderivative size = 3124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

$$\downarrow 4692$$

$$2 \int \frac{x^{7/2}}{(a + b \sec(c + d\sqrt{x}))^2} d\sqrt{x}$$

$$\downarrow 3042$$

$$\begin{aligned}
& 2 \int \frac{x^{7/2}}{(a + b \csc(c + d\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x} \\
& \quad \downarrow 4679 \\
& 2 \int \left(-\frac{2bx^{7/2}}{a^2(b + a \cos(c + d\sqrt{x}))} + \frac{x^{7/2}}{a^2} + \frac{b^2x^{7/2}}{a^2(b + a \cos(c + d\sqrt{x}))^2} \right) d\sqrt{x} \\
& \quad \downarrow 2009 \\
& 2 \left(\frac{x^4}{8a^2} + \frac{2ib \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a^2\sqrt{b^2-a^2}d} - \frac{ib^3 \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a^2(b^2-a^2)^{3/2}d} - \frac{2ib \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a^2\sqrt{b^2-a^2}d} + \frac{ib^3 \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a^2(b^2-a^2)^{3/2}d} \right)
\end{aligned}$$

input `Int[x^3/(a + b*Sec[c + d*Sqrt[x]])^2,x]`

output

```

2*(((-I)*b^2*x^(7/2))/(a^2*(a^2 - b^2)*d) + x^4/(8*a^2) + (7*b^2*x^3*Log[1
+ (a*E^(I*(c + d*Sqrt[x]))))/(b - I*Sqrt[a^2 - b^2]]))/(a^2*(a^2 - b^2)*d^
2) + (7*b^2*x^3*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]]
)/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x^(7/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x]))
)/(b - Sqrt[-a^2 + b^2]]))/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^(7/2)*Lo
g[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]]))/(a^2*Sqrt[-a^2 +
b^2]*d) + (I*b^3*x^(7/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
+ b^2]]))/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^(7/2)*Log[1 + (a*E^(I*(c
+ d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]]))/(a^2*Sqrt[-a^2 + b^2]*d) - ((42*I
)*b^2*x^(5/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2
]])))/(a^2*(a^2 - b^2)*d^3) - ((42*I)*b^2*x^(5/2)*PolyLog[2, -((a*E^(I*(c
+ d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]])))/(a^2*(a^2 - b^2)*d^3) - (7*b^3*x
^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])))/(a^2*(
-a^2 + b^2)^(3/2)*d^2) + (14*b*x^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/
(b - Sqrt[-a^2 + b^2])))/(a^2*Sqrt[-a^2 + b^2]*d^2) + (7*b^3*x^3*PolyLog[
2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])))/(a^2*(-a^2 + b^2)
^(3/2)*d^2) - (14*b*x^3*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-
a^2 + b^2])))/(a^2*Sqrt[-a^2 + b^2]*d^2) + (210*b^2*x^2*PolyLog[3, -((a*E
^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])))/(a^2*(a^2 - b^2)*d^4) + (
210*b^2*x^2*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]

```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `int(x^3/(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(x^3/(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^3}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^3/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(x^3/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2), x)`

Sympy [F]

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `integrate(x**3/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(x**3/(a + b*sec(c + d*sqrt(x)))**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^3/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f or more de`

Giac [F]

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^3}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^3/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(x^3/(b*sec(d*sqrt(x) + c) + a)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^3}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

input `int(x^3/(a + b/cos(c + d*x^(1/2)))^2,x)`

output `int(x^3/(a + b/cos(c + d*x^(1/2)))^2, x)`

Reduce [F]

$$\int \frac{x^3}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{too large to display}$$

input `int(x^3/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
( - 64*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**6*b*d**7*x**3
- 26880*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**6*b*d**5*x**2
+ 860160*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**6*b*d**3*x
+ 2580480*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**6*b*d - 160
*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**5*b**2*d**7*x**3 - 5
9136*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**5*b**2*d**5*x**2
- 430080*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**5*b**2*d**3
*x - 5160960*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**5*b**2*d
- 64*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**4*b**3*d**7*x**
3 - 10752*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**4*b**3*d**5
*x**2 - 1290240*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**4*b**
3*d**3*x + 128*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**3*b**4
*d**7*x**3 + 53760*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**3*
b**4*d**5*x**2 + 430080*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*
a**3*b**4*d**3*x + 5160960*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/
2)*a**3*b**4*d + 128*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**
2*b**5*d**7*x**3 + 37632*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)
*a**2*b**5*d**5*x**2 + 430080*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d +
c)/2)*a**2*b**5*d**3*x - 2580480*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d
+ c)/2)*a**2*b**5*d + 32*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c...
```


$$3.47 \quad \int \frac{x^2}{(a+b \sec(c+d\sqrt{x}))^2} dx$$

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Optimal result

Integrand size = 20, antiderivative size = 2323

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

output

```

-2*I*b^2*x^(5/2)/a^2/(a^2-b^2)/d+240*I*b^2*x^(1/2)*polylog(4,-a*exp(I*(c+d
*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^5+240*I*b^3*x^(1/2)*poly
log(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d
^5+480*I*b*x^(1/2)*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/
/a^2/(-a^2+b^2)^(1/2)/d^5+2*I*b^3*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-
a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+80*I*b*x^(3/2)*polylog(3,-a*exp(I
*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3+4*I*b*x^(5/
2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/
d+240*I*b^2*x^(1/2)*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)
))/a^2/(a^2-b^2)/d^5+2*b^2*x^(5/2)*sin(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cos
(c+d*x^(1/2)))-40*I*b^2*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+I*(a^
2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-40*I*b^2*x^(3/2)*polylog(2,-a*exp(I*(c+d*
x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-40*I*b^3*x^(3/2)*polylo
g(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3
-2*I*b^3*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a
^2+b^2)^(3/2)/d-80*I*b*x^(3/2)*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+
b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3-4*I*b*x^(5/2)*ln(1+a*exp(I*(c+d*x^(1
/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d-240*I*b^3*x^(1/2)*polyl
og(5,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^
5-20*b*x^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/...

```

Mathematica [A] (verified)

Time = 11.26 (sec) , antiderivative size = 2777, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Result too large to show}$$

input

```
Integrate[x^2/(a + b*Sec[c + d*sqrt[x]])^2,x]
```

output

```

((-4*I)*b^2*E^((2*I)*c)*x^(5/2)*(b + a*Cos[c + d*Sqrt[x]])^2*Sec[c + d*Sqr
t[x]]^2)/(a^2*(a^2 - b^2)*d*(1 + E^((2*I)*c))*(a + b*Sec[c + d*Sqrt[x]])^2
) + (x^3*(b + a*Cos[c + d*Sqrt[x]])^2*Sec[c + d*Sqrt[x]]^2)/(3*a^2*(a + b*
Sec[c + d*Sqrt[x]])^2) + (2*b*(b + a*Cos[c + d*Sqrt[x]])^2*(5*b*d^4*Sqrt[(-
a^2 + b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c
) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) + (2*I)*a^2*d^5*E^(I*c)*x^(5/2)*Log[1
+ (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)
]) - I*b^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I
*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) + 5*b*d^4*Sqrt[(-a^2 + b^2)*E^((2*I
)*c)]*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c) + Sqrt[(-a^2 + b^
2)*E^((2*I)*c)]]) - (2*I)*a^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d
*Sqrt[x])))]/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) + I*b^2*d^5*E^(I
*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c) + Sqrt[(-a^2 +
b^2)*E^((2*I)*c)]]) - 5*d^3*((4*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^
2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(I*
(2*c + d*Sqrt[x])))]/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 5*d^3
*((-4*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*
d*E^(I*c)*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(
I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 60*b*d^2*Sqrt[(-a^2 + b^2)*E^((
2*I)*c)]*x*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))]/(b*E^(I*c) - Sqrt[...

```

Rubi [A] (verified)

Time = 3.47 (sec) , antiderivative size = 2324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

$$\downarrow 4692$$

$$2 \int \frac{x^{5/2}}{(a + b \sec(c + d\sqrt{x}))^2} d\sqrt{x}$$

$$\downarrow 3042$$

$$\begin{aligned}
& 2 \int \frac{x^{5/2}}{(a + b \csc(c + d\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x} \\
& \quad \downarrow 4679 \\
& 2 \int \left(-\frac{2bx^{5/2}}{a^2(b + a \cos(c + d\sqrt{x}))} + \frac{x^{5/2}}{a^2} + \frac{b^2x^{5/2}}{a^2(b + a \cos(c + d\sqrt{x}))^2} \right) d\sqrt{x} \\
& \quad \downarrow 2009 \\
& 2 \left(-\frac{ix^{5/2} \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2(b^2-a^2)^{3/2}d} + \frac{ix^{5/2} \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2(b^2-a^2)^{3/2}d} - \frac{5x^2 \text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2(b^2-a^2)^{3/2}d^2} + \frac{5x^2 \text{PolyLog}\left(2, \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2(b^2-a^2)^{3/2}d^2} \right)
\end{aligned}$$

input `Int[x^2/(a + b*Sec[c + d*Sqrt[x]])^2,x]`

output

```

2*(((-I)*b^2*x^(5/2))/(a^2*(a^2 - b^2)*d) + x^3/(6*a^2) + (5*b^2*x^2*Log[1
+ (a*E^(I*(c + d*Sqrt[x]))))/(b - I*Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)*d^
2) + (5*b^2*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]])
)/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x^(5/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x]))
)/(b - Sqrt[-a^2 + b^2]])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^(5/2)*Lo
g[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 +
b^2]*d) + (I*b^3*x^(5/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
+ b^2]])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^(5/2)*Log[1 + (a*E^(I*(c
+ d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d) - ((20*I
)*b^2*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2
]))]/(a^2*(a^2 - b^2)*d^3) - ((20*I)*b^2*x^(3/2)*PolyLog[2, -((a*E^(I*(c
+ d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))]/(a^2*(a^2 - b^2)*d^3) - (5*b^3*x
^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(
-a^2 + b^2)^(3/2)*d^2) + (10*b*x^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/
(b - Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^2) + (5*b^3*x^2*PolyLog[
2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)
^(3/2)*d^2) - (10*b*x^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-
a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^2) + (60*b^2*x*PolyLog[3, -((a*E^(I
*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))]/(a^2*(a^2 - b^2)*d^4) + (60*
b^2*x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])...

```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `int(x^2/(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(x^2/(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^2}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(x^2/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2), x)`

Sympy [F]

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `integrate(x**2/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(x**2/(a + b*sec(c + d*sqrt(x)))**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f or more de`

Giac [F]

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^2}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(x^2/(b*sec(d*sqrt(x) + c) + a)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^2}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

input `int(x^2/(a + b/cos(c + d*x^(1/2)))^2,x)`

output `int(x^2/(a + b/cos(c + d*x^(1/2)))^2, x)`

Reduce [F]

$$\int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{too large to display}$$

input `int(x^2/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
( - 96*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**7*b*d**5*x**2
+ 13440*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**7*b*d**3*x +
40320*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**7*b*d - 192*sqrt
(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**6*b**2*d**5*x**2 + 14400
*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**6*b**2*d**3*x - 1728
0*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**6*b**2*d - 24*sqrt(
x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**5*b**3*d**5*x**2 - 7680*sqrt
(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**5*b**3*d**3*x - 57600*sqrt
(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**5*b**3*d + 168*sqrt(x)
*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**4*b**4*d**5*x**2 - 13440*sqrt
(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**4*b**4*d**3*x + 11520*sqrt
(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**4*b**4*d + 120*sqrt(x)*
cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**3*b**5*d**5*x**2 - 5760*sqrt(
x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**3*b**5*d**3*x + 17280*sqrt
(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**3*b**5*d + 24*sqrt(x)*cos
(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**2*b**6*d**5*x**2 - 960*sqrt(x)*c
os(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**2*b**6*d**3*x + 5760*sqrt(x)*c
os(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)*a**2*b**6*d - 192*cos(sqrt(x)*d +
c)*int(x**2/(2*tan((sqrt(x)*d + c)/2)**4*a**6 - 5*tan((sqrt(x)*d + c)/2)*
*4*a**5*b + tan((sqrt(x)*d + c)/2)**4*a**4*b**2 + 6*tan((sqrt(x)*d + c)...
```


3.48 $\int \frac{x}{(a+b \sec(c+d\sqrt{x}))^2} dx$

Optimal result	360
Mathematica [A] (verified)	361
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Optimal result

Integrand size = 18, antiderivative size = 1523

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

output

```

6*b^2*x*ln(1+a*exp(I*(c+d*x^(1/2))))/(b+I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d
^2+6*b^2*x*ln(1+a*exp(I*(c+d*x^(1/2))))/(b-I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2
)/d^2-12*b*x*polylog(2,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/(
-a^2+b^2)^(1/2)/d^2+12*b*x*polylog(2,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)
^(1/2))/a^2/(-a^2+b^2)^(1/2)/d^2+6*b^3*x*polylog(2,-a*exp(I*(c+d*x^(1/2)))
)/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^2-6*b^3*x*polylog(2,-a*exp(
I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^2-2*I*b^2*x^
(3/2)/a^2/(a^2-b^2)/d+1/2*x^2/a^2+12*b^2*polylog(3,-a*exp(I*(c+d*x^(1/2))))
/(b-I*(a^2-b^2)^(1/2))/a^2/(a^2-b^2)/d^4-12*b^3*polylog(4,-a*exp(I*(c+d*x
^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^4+12*b^3*polylog(4,-
a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^4+24*b
*polylog(4,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(1
/2)/d^4-24*b*polylog(4,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(
-a^2+b^2)^(1/2)/d^4+12*b^2*polylog(3,-a*exp(I*(c+d*x^(1/2))))/(b+I*(a^2-b^2
)^(1/2))/a^2/(a^2-b^2)/d^4+4*I*b*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2))))/(b-(
-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(1/2)/d+12*I*b^3*x^(1/2)*polylog(3,-a*exp
(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^3+24*I*b*x^
(1/2)*polylog(3,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^
2)^(1/2)/d^3+2*I*b^3*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1
/2))/a^2/(-a^2+b^2)^(3/2)/d+2*b^2*x^(3/2)*sin(c+d*x^(1/2))/a/(a^2-b^2)/...

```

Mathematica [A] (verified)

Time = 12.45 (sec) , antiderivative size = 1695, normalized size of antiderivative = 1.11

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

input

```
Integrate[x/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

```

((b + a*Cos[c + d*Sqrt[x]])*Sec[c + d*Sqrt[x]]^2*(x^2*(b + a*Cos[c + d*Sqr
t[x])) + (4*b*(b + a*Cos[c + d*Sqrt[x]])*(((2*I)*b*d^3*E^((2*I)*c)*x^(3/2
)))/(1 + E^((2*I)*c)) + (3*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x*Log[1 + (
a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] +
(2*I)*a^2*d^3*E^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I
*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - I*b^2*d^3*E^(I*c)*x^(3/2)*Log[1 +
(a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
+ 3*b*d^2*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x*Log[1 + (a*E^(I*(2*c + d*Sqrt[
x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (2*I)*a^2*d^3*E^(I*c
)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^
2)*E^((2*I)*c)])] + I*b^2*d^3*E^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqr
t[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 3*d*((2*I)*b*Sqrt[
(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x
])*Sqrt[x]*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^
2 + b^2)*E^((2*I)*c)])]) + 3*d*((-2*I)*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)] -
2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*Sqrt[x]*PolyLog[2, -((a*E
^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]) + 6
*b*Sqrt[(-a^2 + b^2)*E^((2*I)*c)]*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x]))
)/(b*E^(I*c) - Sqrt[(-a^2 + b^2)*E^((2*I)*c)])]) + (12*I)*a^2*d*E^(I*c)*Sqr
t[x]*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) - Sqrt[(-a^2 +...

```

Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 1524, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

$$\downarrow 4692$$

$$2 \int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} d\sqrt{x}$$

$$\downarrow 3042$$

$$\begin{aligned}
& 2 \int \frac{x^{3/2}}{(a + b \csc(c + d\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x} \\
& \quad \downarrow 4679 \\
& 2 \int \left(\frac{x^{3/2}b^2}{a^2 (b + a \cos(c + d\sqrt{x}))^2} - \frac{2x^{3/2}b}{a^2 (b + a \cos(c + d\sqrt{x}))} + \frac{x^{3/2}}{a^2} \right) d\sqrt{x} \\
& \quad \downarrow 2009 \\
& 2 \left(-\frac{ix^{3/2} \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{ix^{3/2} \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{3x \text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} + \frac{3x \text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} \right)
\end{aligned}$$

input `Int[x/(a + b*Sec[c + d*Sqrt[x]])^2,x]`

output

```

2*(((-I)*b^2*x^(3/2))/(a^2*(a^2 - b^2)*d) + x^2/(4*a^2) + (3*b^2*x*Log[1 +
(a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)*d^2)
+ (3*b^2*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]])/(a
^2*(a^2 - b^2)*d^2) - (I*b^3*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b
- Sqrt[-a^2 + b^2]])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^(3/2)*Log[1
+ (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]
*d) + (I*b^3*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^
2]])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^(3/2)*Log[1 + (a*E^(I*(c + d
*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d) - ((6*I)*b^2
*Sqrt[x]*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])
/(a^2*(a^2 - b^2)*d^3) - ((6*I)*b^2*Sqrt[x]*PolyLog[2, -((a*E^(I*(c + d*Sq
rt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (3*b^3*x*PolyL
og[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b
^2)^(3/2)*d^2) + (6*b*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-
a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (3*b^3*x*PolyLog[2, -((a*E^(I*
(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) -
(6*b*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(
a^2*Sqrt[-a^2 + b^2]*d^2) + (6*b^2*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/
(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (6*b^2*PolyLog[3, -((a*
E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4)...

```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sinn[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `int(x/(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(x/(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(x/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2), x)`

Sympy [F]

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `integrate(x/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(x/(a + b*sec(c + d*sqrt(x)))**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f or more de`

Giac [F]

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(x/(b*sec(d*sqrt(x) + c) + a)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

input `int(x/(a + b/cos(c + d*x^(1/2)))^2,x)`

output `int(x/(a + b/cos(c + d*x^(1/2)))^2, x)`

Reduce [F]

$$\int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{too large to display}$$

input `int(x/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
( - 384*cos(sqrt(x)*d + c)*int((sqrt(x)*tan((sqrt(x)*d + c)/2))/(4*tan((sqrt(x)*d + c)/2)**4*a**6 - 12*tan((sqrt(x)*d + c)/2)**4*a**5*b + 9*tan((sqrt(x)*d + c)/2)**4*a**4*b**2 + 4*tan((sqrt(x)*d + c)/2)**4*a**3*b**3 - 6*tan((sqrt(x)*d + c)/2)**4*a**2*b**4 + tan((sqrt(x)*d + c)/2)**4*b**6 - 8*tan((sqrt(x)*d + c)/2)**2*a**6 + 8*tan((sqrt(x)*d + c)/2)**2*a**5*b + 14*tan((sqrt(x)*d + c)/2)**2*a**4*b**2 - 12*tan((sqrt(x)*d + c)/2)**2*a**3*b**3 - 8*tan((sqrt(x)*d + c)/2)**2*a**2*b**4 + 4*tan((sqrt(x)*d + c)/2)**2*a*b**5 + 2*tan((sqrt(x)*d + c)/2)**2*b**6 + 4*a**6 + 4*a**5*b - 7*a**4*b**2 - 8*a**3*b**3 + 2*a**2*b**4 + 4*a*b**5 + b**6),x)*a**9*b*d**3 + 576*cos(sqrt(x)*d + c)*int((sqrt(x)*tan((sqrt(x)*d + c)/2))/(4*tan((sqrt(x)*d + c)/2)**4*a**6 - 12*tan((sqrt(x)*d + c)/2)**4*a**5*b + 9*tan((sqrt(x)*d + c)/2)**4*a**4*b**2 + 4*tan((sqrt(x)*d + c)/2)**4*a**3*b**3 - 6*tan((sqrt(x)*d + c)/2)**4*a**2*b**4 + tan((sqrt(x)*d + c)/2)**4*b**6 - 8*tan((sqrt(x)*d + c)/2)**2*a**6 + 8*tan((sqrt(x)*d + c)/2)**2*a**5*b + 14*tan((sqrt(x)*d + c)/2)**2*a**4*b**2 - 12*tan((sqrt(x)*d + c)/2)**2*a**3*b**3 - 8*tan((sqrt(x)*d + c)/2)**2*a**2*b**4 + 4*tan((sqrt(x)*d + c)/2)**2*a*b**5 + 2*tan((sqrt(x)*d + c)/2)**2*b**6 + 4*a**6 + 4*a**5*b - 7*a**4*b**2 - 8*a**3*b**3 + 2*a**2*b**4 + 4*a*b**5 + b**6),x)*a**8*b**2*d**3 + 288*cos(sqrt(x)*d + c)*int((sqrt(x)*tan((sqrt(x)*d + c)/2))/(4*tan((sqrt(x)*d + c)/2)**4*a**6 - 12*tan((sqrt(x)*d + c)/2)**4*a**5*b + 9*tan((sqrt(x)*d + c)/2)**4*a**4*b**2 ...
```


3.49 $\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx$

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Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx = \text{Int}\left(\frac{1}{x(a+b \sec(c+d\sqrt{x}))^2}, x\right)$$

output `Defer(Int)(1/x/(a+b*sec(c+d*x^(1/2)))^2,x)`

Mathematica [N/A]

Not integrable

Time = 45.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx = \int \frac{1}{x(a+b \sec(c+d\sqrt{x}))^2} dx$$

input `Integrate[1/(x*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output `Integrate[1/(x*(a + b*Sec[c + d*Sqrt[x]])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a + b \sec (c + d\sqrt{x}))^2} dx$$

↓ 4694

$$\int \frac{1}{x (a + b \sec (c + d\sqrt{x}))^2} dx$$

input `Int[1/(x*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x (a + b \sec (c + d\sqrt{x}))^2} dx$$

input `int(1/x/(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(1/x/(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{1}{x (a + b \sec (c + d \sqrt{x}))^2} dx = \int \frac{1}{(b \sec (d \sqrt{x} + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(1/(b^2*x*sec(d*sqrt(x) + c)^2 + 2*a*b*x*sec(d*sqrt(x) + c) + a^2*x), x)`

Sympy [N/A]

Not integrable

Time = 4.90 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x (a + b \sec (c + d \sqrt{x}))^2} dx = \int \frac{1}{x (a + b \sec (c + d \sqrt{x}))^2} dx$$

input `integrate(1/x/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(1/(x*(a + b*sec(c + d*sqrt(x)))**2), x)`

Maxima [N/A]

Not integrable

Time = 12.44 (sec) , antiderivative size = 4405, normalized size of antiderivative = 220.25

$$\int \frac{1}{x (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output

```
((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*cos(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*sin(d*sqrt(x))^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*sin(d*sqrt(x))*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*sin(d*sqrt(x))*sin(c) - (a^8 - a^6*b^2)*d*cos(2*d*sqrt(x) + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x*integr...
```

Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x (a + b \sec (c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \sec (d\sqrt{x} + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(1/((b*sec(d*sqrt(x) + c) + a)^2*x), x)`

Mupad [N/A]

Not integrable

Time = 15.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x (a + b \sec (c + d\sqrt{x}))^2} dx = \int \frac{1}{x \left(a + \frac{b}{\cos(c+d\sqrt{x})} \right)^2} dx$$

input `int(1/(x*(a + b/cos(c + d*x^(1/2)))^2),x)`

output `int(1/(x*(a + b/cos(c + d*x^(1/2)))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 1453, normalized size of antiderivative = 72.65

$$\int \frac{1}{x (a + b \sec (c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

input `int(1/x/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(2*(2*int(tan((sqrt(x)*d + c)/2)**2/(tan((sqrt(x)*d + c)/2)**4*a**4*x - 4*
tan((sqrt(x)*d + c)/2)**4*a**3*b*x + 6*tan((sqrt(x)*d + c)/2)**4*a**2*b**2
*x - 4*tan((sqrt(x)*d + c)/2)**4*a*b**3*x + tan((sqrt(x)*d + c)/2)**4*b**4
*x - 2*tan((sqrt(x)*d + c)/2)**2*a**4*x + 4*tan((sqrt(x)*d + c)/2)**2*a**3
*b*x - 4*tan((sqrt(x)*d + c)/2)**2*a*b**3*x + 2*tan((sqrt(x)*d + c)/2)**2*
b**4*x + a**4*x - 2*a**2*b**2*x + b**4*x),x)*a**3*b - 6*int(tan((sqrt(x)*d
+ c)/2)**2/(tan((sqrt(x)*d + c)/2)**4*a**4*x - 4*tan((sqrt(x)*d + c)/2)**
4*a**3*b*x + 6*tan((sqrt(x)*d + c)/2)**4*a**2*b**2*x - 4*tan((sqrt(x)*d +
c)/2)**4*a*b**3*x + tan((sqrt(x)*d + c)/2)**4*b**4*x - 2*tan((sqrt(x)*d +
c)/2)**2*a**4*x + 4*tan((sqrt(x)*d + c)/2)**2*a**3*b*x - 4*tan((sqrt(x)*d
+ c)/2)**2*a*b**3*x + 2*tan((sqrt(x)*d + c)/2)**2*b**4*x + a**4*x - 2*a**2
*b**2*x + b**4*x),x)*a**2*b**2 + 6*int(tan((sqrt(x)*d + c)/2)**2/(tan((sqr
t(x)*d + c)/2)**4*a**4*x - 4*tan((sqrt(x)*d + c)/2)**4*a**3*b*x + 6*tan((s
qrt(x)*d + c)/2)**4*a**2*b**2*x - 4*tan((sqrt(x)*d + c)/2)**4*a*b**3*x + t
an((sqrt(x)*d + c)/2)**4*b**4*x - 2*tan((sqrt(x)*d + c)/2)**2*a**4*x + 4*t
an((sqrt(x)*d + c)/2)**2*a**3*b*x - 4*tan((sqrt(x)*d + c)/2)**2*a*b**3*x +
2*tan((sqrt(x)*d + c)/2)**2*b**4*x + a**4*x - 2*a**2*b**2*x + b**4*x),x)*
a*b**3 - 2*int(tan((sqrt(x)*d + c)/2)**2/(tan((sqrt(x)*d + c)/2)**4*a**4*x
- 4*tan((sqrt(x)*d + c)/2)**4*a**3*b*x + 6*tan((sqrt(x)*d + c)/2)**4*a**2
*b**2*x - 4*tan((sqrt(x)*d + c)/2)**4*a*b**3*x + tan((sqrt(x)*d + c)/2)...
```

3.50 $\int \frac{1}{x^2 (a+b \sec(c+d\sqrt{x}))^2} dx$

Optimal result	374
Mathematica [N/A]	374
Rubi [N/A]	375
Maple [N/A]	375
Fricas [N/A]	376
Sympy [N/A]	376
Maxima [N/A]	377
Giac [N/A]	378
Mupad [N/A]	378
Reduce [N/A]	378

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x^2 (a+b \sec(c+d\sqrt{x}))^2} dx = \text{Int}\left(\frac{1}{x^2 (a+b \sec(c+d\sqrt{x}))^2}, x\right)$$

output `Defer(Int)(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x)`

Mathematica [N/A]

Not integrable

Time = 33.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 (a+b \sec(c+d\sqrt{x}))^2} dx = \int \frac{1}{x^2 (a+b \sec(c+d\sqrt{x}))^2} dx$$

input `Integrate[1/(x^2*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output `Integrate[1/(x^2*(a + b*Sec[c + d*Sqrt[x]])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx$$

↓ 4694

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `Int[1/(x^2*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `int(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^2} dx$$

input `integrate(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(1/(b^2*x^2*sec(d*sqrt(x) + c)^2 + 2*a*b*x^2*sec(d*sqrt(x) + c) + a^2*x^2), x)`

Sympy [N/A]

Not integrable

Time = 9.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `integrate(1/x**2/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(1/(x**2*(a + b*sec(c + d*sqrt(x)))**2), x)`

Maxima [N/A]

Not integrable

Time = 18.27 (sec) , antiderivative size = 4406, normalized size of antiderivative = 220.30

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^2} dx$$

```
input integrate(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

output

```
((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*cos(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*cos(c) + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*sin(d*sqrt(x))^2 - 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*sin(d*sqrt(x))*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*cos(2*c) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*sin(d*sqrt(x))*sin(c) - (a^8 - a^6*b^2)*d*cos(2*d*sqrt(x) + 2*c) + 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x^2*inte...
```

Giac [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^2} dx$$

input `integrate(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(1/((b*sec(d*sqrt(x) + c) + a)^2*x^2), x)`

Mupad [N/A]

Not integrable

Time = 15.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^2 \left(a + \frac{b}{\cos(c + d\sqrt{x})}\right)^2} dx$$

input `int(1/(x^2*(a + b/cos(c + d*x^(1/2)))^2),x)`

output `int(1/(x^2*(a + b/cos(c + d*x^(1/2)))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 1628, normalized size of antiderivative = 81.40

$$\int \frac{1}{x^2 (a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

input `int(1/x^2/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(4*int(tan((sqrt(x)*d + c)/2)**2/(tan((sqrt(x)*d + c)/2)**4*a**4*x**2 - 4*
tan((sqrt(x)*d + c)/2)**4*a**3*b*x**2 + 6*tan((sqrt(x)*d + c)/2)**4*a**2*b
**2*x**2 - 4*tan((sqrt(x)*d + c)/2)**4*a*b**3*x**2 + tan((sqrt(x)*d + c)/2
)**4*b**4*x**2 - 2*tan((sqrt(x)*d + c)/2)**2*a**4*x**2 + 4*tan((sqrt(x)*d
+ c)/2)**2*a**3*b*x**2 - 4*tan((sqrt(x)*d + c)/2)**2*a*b**3*x**2 + 2*tan((
sqrt(x)*d + c)/2)**2*b**4*x**2 + a**4*x**2 - 2*a**2*b**2*x**2 + b**4*x**2)
,x)*a**3*b*x - 12*int(tan((sqrt(x)*d + c)/2)**2/(tan((sqrt(x)*d + c)/2)**4
*a**4*x**2 - 4*tan((sqrt(x)*d + c)/2)**4*a**3*b*x**2 + 6*tan((sqrt(x)*d +
c)/2)**4*a**2*b**2*x**2 - 4*tan((sqrt(x)*d + c)/2)**4*a*b**3*x**2 + tan((s
qrt(x)*d + c)/2)**4*b**4*x**2 - 2*tan((sqrt(x)*d + c)/2)**2*a**4*x**2 + 4*
tan((sqrt(x)*d + c)/2)**2*a**3*b*x**2 - 4*tan((sqrt(x)*d + c)/2)**2*a*b**3
*x**2 + 2*tan((sqrt(x)*d + c)/2)**2*b**4*x**2 + a**4*x**2 - 2*a**2*b**2*x*
*2 + b**4*x**2),x)*a**2*b**2*x + 12*int(tan((sqrt(x)*d + c)/2)**2/(tan((sq
rt(x)*d + c)/2)**4*a**4*x**2 - 4*tan((sqrt(x)*d + c)/2)**4*a**3*b*x**2 + 6
*tan((sqrt(x)*d + c)/2)**4*a**2*b**2*x**2 - 4*tan((sqrt(x)*d + c)/2)**4*a*
b**3*x**2 + tan((sqrt(x)*d + c)/2)**4*b**4*x**2 - 2*tan((sqrt(x)*d + c)/2)
**2*a**4*x**2 + 4*tan((sqrt(x)*d + c)/2)**2*a**3*b*x**2 - 4*tan((sqrt(x)*d
+ c)/2)**2*a*b**3*x**2 + 2*tan((sqrt(x)*d + c)/2)**2*b**4*x**2 + a**4*x**
2 - 2*a**2*b**2*x**2 + b**4*x**2),x)*a*b**3*x - 4*int(tan((sqrt(x)*d + c)/
2)**2/(tan((sqrt(x)*d + c)/2)**4*a**4*x**2 - 4*tan((sqrt(x)*d + c)/2)**...
```

3.51 $\int x^{3/2} (a + b \sec (c + d\sqrt{x})) dx$

Optimal result	380
Mathematica [A] (verified)	381
Rubi [A] (verified)	381
Maple [F]	383
Fricas [F]	383
Sympy [F]	383
Maxima [B] (verification not implemented)	384
Giac [F]	385
Mupad [F(-1)]	385
Reduce [F]	385

Optimal result

Integrand size = 20, antiderivative size = 284

$$\int x^{3/2} (a + b \sec (c + d\sqrt{x})) dx = \frac{2}{5}ax^{5/2} - \frac{4ibx^2 \arctan \left(e^{i(c+d\sqrt{x})} \right)}{d} + \frac{8ibx^{3/2} \text{PolyLog} \left(2, -ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{8ibx^{3/2} \text{PolyLog} \left(2, ie^{i(c+d\sqrt{x})} \right)}{d^2} - \frac{24bx \text{PolyLog} \left(3, -ie^{i(c+d\sqrt{x})} \right)}{d^3} + \frac{24bx \text{PolyLog} \left(3, ie^{i(c+d\sqrt{x})} \right)}{d^3} - \frac{48ib\sqrt{x} \text{PolyLog} \left(4, -ie^{i(c+d\sqrt{x})} \right)}{d^4} + \frac{48ib\sqrt{x} \text{PolyLog} \left(4, ie^{i(c+d\sqrt{x})} \right)}{d^4} + \frac{48b \text{PolyLog} \left(5, -ie^{i(c+d\sqrt{x})} \right)}{d^5} - \frac{48b \text{PolyLog} \left(5, ie^{i(c+d\sqrt{x})} \right)}{d^5}$$

output

```
2/5*a*x^(5/2)-4*I*b*x^2*arctan(exp(I*(c+d*x^(1/2))))/d+8*I*b*x^(3/2)*polylog(2,-I*exp(I*(c+d*x^(1/2))))/d^2-8*I*b*x^(3/2)*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2-24*b*x*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+24*b*x*polylog(3,I*exp(I*(c+d*x^(1/2))))/d^3-48*I*b*x^(1/2)*polylog(4,-I*exp(I*(c+d*x^(1/2))))/d^4+48*I*b*x^(1/2)*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^4+48*b*polylog(5,-I*exp(I*(c+d*x^(1/2))))/d^5-48*b*polylog(5,I*exp(I*(c+d*x^(1/2))))/d^5
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.99

$$\int x^{3/2} (a + b \sec(c + d\sqrt{x})) dx = \frac{2 \left(ad^5 x^{5/2} - 10ibd^4 x^2 \arctan \left(e^{i(c+d\sqrt{x})} \right) + 20ibd^3 x^{3/2} \text{PolyLog} \left(2, -ie^{i(c+d\sqrt{x})} \right) - \right)}{5d^5}$$

input

```
Integrate[x^(3/2)*(a + b*Sec[c + d*Sqrt[x]]),x]
```

output

```
(2*(a*d^5*x^(5/2) - (10*I)*b*d^4*x^2*ArcTan[E^(I*(c + d*Sqrt[x]))]) + (20*I)*b*d^3*x^(3/2)*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (20*I)*b*d^3*x^(3/2)*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - 60*b*d^2*x*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))] + 60*b*d^2*x*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] - (120*I)*b*d*Sqrt[x]*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (120*I)*b*d*Sqrt[x]*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))] + 120*b*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))] - 120*b*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))]))/(5*d^5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} (a + b \sec(c + d\sqrt{x})) dx$$

$$\downarrow \text{2010}$$

$$\int (ax^{3/2} + bx^{3/2} \sec(c + d\sqrt{x})) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned} & \frac{2}{5}ax^{5/2} - \frac{4ibx^2 \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{48b \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} - \\ & \frac{48b \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{48ib\sqrt{x} \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} + \\ & \frac{48ib\sqrt{x} \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} - \frac{24bx \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \\ & \frac{24bx \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8ibx^{3/2} \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \\ & \frac{8ibx^{3/2} \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} \end{aligned}$$

input `Int[x^(3/2)*(a + b*Sec[c + d*Sqrt[x]]), x]`

output `(2*a*x^(5/2))/5 - ((4*I)*b*x^2*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((8*I)*b*x^(3/2)*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((8*I)*b*x^(3/2)*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - (24*b*x*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))])/d^3 + (24*b*x*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 - ((48*I)*b*Sqrt[x]*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))])/d^4 + ((48*I)*b*Sqrt[x]*PolyLog[4, I*E^(I*(c + d*Sqrt[x]))])/d^4 + (48*b*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))])/d^5 - (48*b*PolyLog[5, I*E^(I*(c + d*Sqrt[x]))])/d^5`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [F]

$$\int x^{\frac{3}{2}}(a + b \sec(c + d\sqrt{x})) dx$$

input `int(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x)`

output `int(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int x^{3/2}(a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a)x^{\frac{3}{2}} dx$$

input `integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(b*x^(3/2)*sec(d*sqrt(x) + c) + a*x^(3/2), x)`

Sympy [F]

$$\int x^{3/2}(a + b \sec(c + d\sqrt{x})) dx = \int x^{\frac{3}{2}}(a + b \sec(c + d\sqrt{x})) dx$$

input `integrate(x**(3/2)*(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(x**(3/2)*(a + b*sec(c + d*sqrt(x))), x)`

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(208) = 416$.

Time = 0.21 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.60

$$\int x^{3/2}(a + b \sec(c + d\sqrt{x})) dx = \text{Too large to display}$$

input `integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output

```
1/5*(2*(d*sqrt(x) + c)^5*a - 10*(d*sqrt(x) + c)^4*a*c + 20*(d*sqrt(x) + c)
^3*a*c^2 - 20*(d*sqrt(x) + c)^2*a*c^3 + 10*(d*sqrt(x) + c)*a*c^4 + 10*b*c^
4*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 10*(I*(d*sqrt(x) + c)^4*b
- 4*I*(d*sqrt(x) + c)^3*b*c + 6*I*(d*sqrt(x) + c)^2*b*c^2 - 4*I*(d*sqrt(x)
+ c)*b*c^3)*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 10*(I*
(d*sqrt(x) + c)^4*b - 4*I*(d*sqrt(x) + c)^3*b*c + 6*I*(d*sqrt(x) + c)^2*b*
c^2 - 4*I*(d*sqrt(x) + c)*b*c^3)*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x)
+ c) + 1) - 40*(I*(d*sqrt(x) + c)^3*b - 3*I*(d*sqrt(x) + c)^2*b*c + 3*I*
(d*sqrt(x) + c)*b*c^2 - I*b*c^3)*dilog(I*e^(I*d*sqrt(x) + I*c)) - 40*(-I*(
d*sqrt(x) + c)^3*b + 3*I*(d*sqrt(x) + c)^2*b*c - 3*I*(d*sqrt(x) + c)*b*c^2
+ I*b*c^3)*dilog(-I*e^(I*d*sqrt(x) + I*c)) + 5*((d*sqrt(x) + c)^4*b - 4*(
d*sqrt(x) + c)^3*b*c + 6*(d*sqrt(x) + c)^2*b*c^2 - 4*(d*sqrt(x) + c)*b*c^3
)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqrt(x) + c) +
1) - 5*((d*sqrt(x) + c)^4*b - 4*(d*sqrt(x) + c)^3*b*c + 6*(d*sqrt(x) + c)
^2*b*c^2 - 4*(d*sqrt(x) + c)*b*c^3)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x)
+ c)^2 - 2*sin(d*sqrt(x) + c) + 1) - 240*b*polylog(5, I*e^(I*d*sqrt(x)
+ I*c)) + 240*b*polylog(5, -I*e^(I*d*sqrt(x) + I*c)) - 240*(-I*(d*sqrt(x)
+ c)*b + I*b*c)*polylog(4, I*e^(I*d*sqrt(x) + I*c)) - 240*(I*(d*sqrt(x) +
c)*b - I*b*c)*polylog(4, -I*e^(I*d*sqrt(x) + I*c)) + 120*((d*sqrt(x) + c)^
2*b - 2*(d*sqrt(x) + c)*b*c + b*c^2)*polylog(3, I*e^(I*d*sqrt(x) + I*c))...
```

Giac [F]

$$\int x^{3/2}(a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a)x^{3/2} dx$$

input `integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)*x^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(a + b \sec(c + d\sqrt{x})) dx = \int x^{3/2} \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

input `int(x^(3/2)*(a + b/cos(c + d*x^(1/2))),x)`

output `int(x^(3/2)*(a + b/cos(c + d*x^(1/2))), x)`

Reduce [F]

$$\int x^{3/2}(a + b \sec(c + d\sqrt{x})) dx = \frac{2\sqrt{x} a x^2}{5} + \frac{2\sqrt{x} b x^2}{5} - 2 \left(\int \frac{\sqrt{x} \tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2 x}{\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2 - 1} dx \right) b$$

input `int(x^(3/2)*(a+b*sec(c+d*x^(1/2))),x)`

output `(2*(sqrt(x)*a*x**2 + sqrt(x)*b*x**2 - 5*int((sqrt(x)*tan((sqrt(x)*d + c)/2)**2*x)/(tan((sqrt(x)*d + c)/2)**2 - 1),x)*b))/5`

3.52 $\int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx$

Optimal result	386
Mathematica [A] (verified)	387
Rubi [A] (verified)	387
Maple [F]	388
Fricas [F]	388
Sympy [F]	389
Maxima [B] (verification not implemented)	389
Giac [F]	390
Mupad [F(-1)]	390
Reduce [F]	391

Optimal result

Integrand size = 20, antiderivative size = 158

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx = \frac{2}{3}ax^{3/2} - \frac{4ibx \arctan(e^{i(c+d\sqrt{x})})}{d} + \frac{4ib\sqrt{x} \operatorname{PolyLog}(2, -ie^{i(c+d\sqrt{x})})}{d^2} - \frac{4ib\sqrt{x} \operatorname{PolyLog}(2, ie^{i(c+d\sqrt{x})})}{d^2} - \frac{4b \operatorname{PolyLog}(3, -ie^{i(c+d\sqrt{x})})}{d^3} + \frac{4b \operatorname{PolyLog}(3, ie^{i(c+d\sqrt{x})})}{d^3}$$

output

```
2/3*a*x^(3/2)-4*I*b*x*arctan(exp(I*(c+d*x^(1/2))))/d+4*I*b*x^(1/2)*polylog(2,-I*exp(I*(c+d*x^(1/2))))/d^2-4*I*b*x^(1/2)*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2-4*b*polylog(3,-I*exp(I*(c+d*x^(1/2))))/d^3+4*b*polylog(3,I*exp(I*(c+d*x^(1/2))))/d^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx$$

$$= \frac{2(ad^3x^{3/2} - 6ibd^2x \arctan(e^{i(c+d\sqrt{x})}) + 6ibd\sqrt{x} \operatorname{PolyLog}(2, -ie^{i(c+d\sqrt{x})}) - 6ibd\sqrt{x} \operatorname{PolyLog}(2, ie^{i(c+d\sqrt{x})}))}{3d^3}$$

input

```
Integrate[Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]]),x]
```

output

```
(2*(a*d^3*x^(3/2) - (6*I)*b*d^2*x*ArcTan[E^(I*(c + d*Sqrt[x]))] + (6*I)*b*d*Sqrt[x]*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (6*I)*b*d*Sqrt[x]*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - 6*b*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))] + 6*b*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))]))/(3*d^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx$$

$$\downarrow \text{2010}$$

$$\int (a\sqrt{x} + b\sqrt{x} \sec(c + d\sqrt{x})) dx$$

$$\downarrow \text{2009}$$

$$\frac{2}{3}ax^{3/2} - \frac{4ibx \arctan(e^{i(c+d\sqrt{x})})}{d} - \frac{4b \operatorname{PolyLog}(3, -ie^{i(c+d\sqrt{x})})}{d^3} + \frac{4b \operatorname{PolyLog}(3, ie^{i(c+d\sqrt{x})})}{d^3} + \frac{4ib\sqrt{x} \operatorname{PolyLog}(2, -ie^{i(c+d\sqrt{x})})}{d^2} - \frac{4ib\sqrt{x} \operatorname{PolyLog}(2, ie^{i(c+d\sqrt{x})})}{d^2}$$

input `Int[Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]]),x]`

output `(2*a*x^(3/2))/3 - ((4*I)*b*x*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + ((4*I)*b*Sqrt[x]*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((4*I)*b*Sqrt[x]*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - (4*b*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))])/d^3 + (4*b*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

Maple [F]

$$\int \sqrt{x} (a + b \sec(c + d\sqrt{x})) dx$$

input `int(x^(1/2)*(a+b*sec(c+d*x^(1/2))),x)`

output `int(x^(1/2)*(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int \sqrt{x} (a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a) \sqrt{x} dx$$

input `integrate(x^(1/2)*(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(b*sqrt(x)*sec(d*sqrt(x) + c) + a*sqrt(x), x)`

Sympy [F]

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx = \int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx$$

input `integrate(x**(1/2)*(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(sqrt(x)*(a + b*sec(c + d*sqrt(x))), x)`

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(114) = 228$.

Time = 0.21 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.37

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx$$

$$= \frac{2(d\sqrt{x} + c)^3 a - 6(d\sqrt{x} + c)^2 ac + 6(d\sqrt{x} + c)ac^2 + 6bc^2 \log(\sec(d\sqrt{x} + c) + \tan(d\sqrt{x} + c)) - 6(i$$

input `integrate(x^(1/2)*(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output

```
1/3*(2*(d*sqrt(x) + c)^3*a - 6*(d*sqrt(x) + c)^2*a*c + 6*(d*sqrt(x) + c)*a
*c^2 + 6*b*c^2*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) - 6*(I*(d*sqrt
(x) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c)*arctan2(cos(d*sqrt(x) + c), sin(d*
sqrt(x) + c) + 1) - 6*(I*(d*sqrt(x) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c)*ar
ctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) - 12*(I*(d*sqrt(x) + c)
*b - I*b*c)*dilog(I*e^(I*d*sqrt(x) + I*c)) - 12*(-I*(d*sqrt(x) + c)*b + I*
b*c)*dilog(-I*e^(I*d*sqrt(x) + I*c)) + 3*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(
x) + c)*b*c)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*sin(d*sqr
t(x) + c) + 1) - 3*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x) + c)*b*c)*log(cos(d
*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*sin(d*sqrt(x) + c) + 1) + 12*b*
polylog(3, I*e^(I*d*sqrt(x) + I*c)) - 12*b*polylog(3, -I*e^(I*d*sqrt(x) +
I*c)))/d^3
```

Giac [F]

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx = \int (b \sec(d\sqrt{x} + c) + a)\sqrt{x} dx$$

input

```
integrate(x^(1/2)*(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")
```

output

```
integrate((b*sec(d*sqrt(x) + c) + a)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx = \int \sqrt{x} \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right) dx$$

input

```
int(x^(1/2)*(a + b/cos(c + d*x^(1/2))),x)
```

output

```
int(x^(1/2)*(a + b/cos(c + d*x^(1/2))), x)
```

Reduce [F]

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x})) dx = \frac{2\sqrt{x} ax}{3} + \frac{2\sqrt{x} bx}{3} - 2 \left(\int \frac{\sqrt{x} \tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2}{\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right)^2 - 1} dx \right) b$$

input `int(x^(1/2)*(a+b*sec(c+d*x^(1/2))),x)`

output `(2*(sqrt(x)*a*x + sqrt(x)*b*x - 3*int((sqrt(x)*tan((sqrt(x)*d + c)/2)**2)/
(tan((sqrt(x)*d + c)/2)**2 - 1),x)*b))/3`

3.53 $\int \frac{a+b \sec(c+d\sqrt{x})}{\sqrt{x}} dx$

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Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} + \frac{2b \operatorname{arctanh}(\sin(c + d\sqrt{x}))}{d}$$

output `2*a*x^(1/2)+2*b*arctanh(sin(c+d*x^(1/2)))/d`

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} + \frac{2b \operatorname{coth}^{-1}(\sin(c + d\sqrt{x}))}{d}$$

input `Integrate[(a + b*Sec[c + d*Sqrt[x]])/Sqrt[x],x]`

output `2*a*Sqrt[x] + (2*b*ArcCoth[Sin[c + d*Sqrt[x]]])/d`

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx$$

↓ 2010

$$\int \left(\frac{a}{\sqrt{x}} + \frac{b \sec(c + d\sqrt{x})}{\sqrt{x}} \right) dx$$

↓ 2009

$$2a\sqrt{x} + \frac{2b \operatorname{arctanh}(\sin(c + d\sqrt{x}))}{d}$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])/Sqrt[x], x]`

output `2*a*Sqrt[x] + (2*b*ArcTanh[Sin[c + d*Sqrt[x]]])/d`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_)*(x_)^(m_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$2a\sqrt{x} + \frac{2b \ln(\sec(c+d\sqrt{x})+\tan(c+d\sqrt{x}))}{d}$	32
default	$2a\sqrt{x} + \frac{2b \ln(\sec(c+d\sqrt{x})+\tan(c+d\sqrt{x}))}{d}$	32
parts	$2a\sqrt{x} + \frac{2b \ln(\sec(c+d\sqrt{x})+\tan(c+d\sqrt{x}))}{d}$	32

input `int((a+b*sec(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)`

output `2*a*x^(1/2)+2*b/d*ln(sec(c+d*x^(1/2))+tan(c+d*x^(1/2)))`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx$$

$$= \frac{2ad\sqrt{x} + b \log(\sin(d\sqrt{x} + c) + 1) - b \log(-\sin(d\sqrt{x} + c) + 1)}{d}$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")`

output `(2*a*d*sqrt(x) + b*log(sin(d*sqrt(x) + c) + 1) - b*log(-sin(d*sqrt(x) + c) + 1))/d`

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} + 2b \begin{cases} \frac{\sqrt{x}(\tan(c)\sec(c) + \sec^2(c))}{\tan(c) + \sec(c)} & \text{for } d = 0 \\ \frac{\log(\tan(c + d\sqrt{x}) + \sec(c + d\sqrt{x}))}{d} & \text{otherwise} \end{cases}$$

input `integrate((a+b*sec(c+d*x**(1/2)))/x**(1/2), x)`

output `2*a*sqrt(x) + 2*b*Piecewise((sqrt(x)*(tan(c)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), Eq(d, 0)), (log(tan(c + d*sqrt(x)) + sec(c + d*sqrt(x)))/d, True))`

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} + \frac{2b \log(\sec(d\sqrt{x} + c) + \tan(d\sqrt{x} + c))}{d}$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^(1/2), x, algorithm="maxima")`

output `2*a*sqrt(x) + 2*b*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c))/d`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx = \frac{2((d\sqrt{x} + c)a + b \log(|\tan(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c) + 1|) - b \log(|\tan(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c) - 1|))}{d}$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")`

output `2*((d*sqrt(x) + c)*a + b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c) - 1)))/d`

Mupad [B] (verification not implemented)

Time = 17.82 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} - \frac{2b \ln\left(\frac{b2i - 2be^{d\sqrt{x}}e^{c1i}}{\sqrt{x}}\right)}{d} + \frac{2b \ln\left(\frac{b2i + 2be^{d\sqrt{x}}e^{c1i}}{\sqrt{x}}\right)}{d}$$

input `int((a + b/cos(c + d*x^(1/2)))/x^(1/2),x)`

output `2*a*x^(1/2) - (2*b*log((b*2i - 2*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d + (2*b*log((b*2i + 2*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d`

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{a + b \sec(c + d\sqrt{x})}{\sqrt{x}} dx = \frac{2\sqrt{x}ad - 2 \log\left(\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right) - 1\right)b + 2 \log\left(\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right) + 1\right)b}{d}$$

input `int((a+b*sec(c+d*x^(1/2)))/x^(1/2),x)`

output `(2*(sqrt(x)*a*d - log(tan((sqrt(x)*d + c)/2) - 1)*b + log(tan((sqrt(x)*d + c)/2) + 1)*b)/d`

3.54 $\int \frac{a+b \sec(c+d\sqrt{x})}{x^{3/2}} dx$

Optimal result	397
Mathematica [N/A]	397
Rubi [N/A]	398
Maple [N/A]	399
Fricas [N/A]	399
Sympy [N/A]	399
Maxima [N/A]	400
Giac [N/A]	400
Mupad [N/A]	401
Reduce [N/A]	401

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx = -\frac{2a}{\sqrt{x}} + b \operatorname{Int}\left(\frac{\sec(c + d\sqrt{x})}{x^{3/2}}, x\right)$$

output

```
-2*a/x^(1/2)+b*Defer(Int)(sec(c+d*x^(1/2))/x^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 18.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx$$

input

```
Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^(3/2),x]
```

output

```
Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx$$

↓ 2010

$$\int \left(\frac{a}{x^{3/2}} + \frac{b \sec(c + d\sqrt{x})}{x^{3/2}} \right) dx$$

↓ 2009

$$b \int \frac{\sec(c + d\sqrt{x})}{x^{3/2}} dx - \frac{2a}{\sqrt{x}}$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])/x^(3/2), x]`

output `$Aborted`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{\frac{3}{2}}} dx$$

input `int((a+b*sec(c+d*x^(1/2)))/x^(3/2),x)`output `int((a+b*sec(c+d*x^(1/2)))/x^(3/2),x)`**Fricas [N/A]**

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^{\frac{3}{2}}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^(3/2),x, algorithm="fricas")`output `integral((b*sqrt(x)*sec(d*sqrt(x) + c) + a*sqrt(x))/x^2, x)`**Sympy [N/A]**

Not integrable

Time = 1.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x^{\frac{3}{2}}} dx$$

input `integrate((a+b*sec(c+d*x**(1/2)))/x**(3/2),x)`

output `Integral((a + b*sec(c + d*sqrt(x)))/x**(3/2), x)`

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 5.60

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^{3/2}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^(3/2),x, algorithm="maxima")`

output `2*(b*sqrt(x)*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c))/((cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x^(3/2)), x) - a)/sqrt(x)`

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^{3/2}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^(3/2),x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)/x^(3/2), x)`

Mupad [N/A]

Not integrable

Time = 15.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{a + \frac{b}{\cos(c+d\sqrt{x})}}{x^{3/2}} dx$$

input `int((a + b/cos(c + d*x^(1/2)))/x^(3/2), x)`output `int((a + b/cos(c + d*x^(1/2)))/x^(3/2), x)`**Reduce [N/A]**

Not integrable

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{3/2}} dx = \frac{\sqrt{x} \left(\int \frac{\sec(\sqrt{x}d+c) \tan(\sqrt{x}d+c)}{x} dx \right) bd - 2 \sec(\sqrt{x}d + c) b - 2a}{\sqrt{x}}$$

input `int((a+b*sec(c+d*x^(1/2)))/x^(3/2), x)`output `(sqrt(x)*int((sec(sqrt(x)*d + c)*tan(sqrt(x)*d + c))/x,x)*b*d - 2*sec(sqrt(x)*d + c)*b - 2*a)/sqrt(x)`

3.55 $\int \frac{a+b \sec(c+d\sqrt{x})}{x^{5/2}} dx$

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Mathematica [N/A]	402
Rubi [N/A]	403
Maple [N/A]	404
Fricas [N/A]	404
Sympy [N/A]	404
Maxima [N/A]	405
Giac [N/A]	405
Mupad [N/A]	406
Reduce [N/A]	406

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx = -\frac{2a}{3x^{3/2}} + b \operatorname{Int}\left(\frac{\sec(c + d\sqrt{x})}{x^{5/2}}, x\right)$$

output

```
-2/3*a/x^(3/2)+b*Defer(Int)(sec(c+d*x^(1/2))/x^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 21.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx$$

input

```
Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^(5/2),x]
```

output

```
Integrate[(a + b*Sec[c + d*Sqrt[x]])/x^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx$$

↓ 2010

$$\int \left(\frac{a}{x^{5/2}} + \frac{b \sec(c + d\sqrt{x})}{x^{5/2}} \right) dx$$

↓ 2009

$$b \int \frac{\sec(c + d\sqrt{x})}{x^{5/2}} dx - \frac{2a}{3x^{3/2}}$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])/x^(5/2), x]`

output `$Aborted`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{\frac{5}{2}}} dx$$

input `int((a+b*sec(c+d*x^(1/2)))/x^(5/2),x)`output `int((a+b*sec(c+d*x^(1/2)))/x^(5/2),x)`**Fricas [N/A]**

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^{\frac{5}{2}}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^(5/2),x, algorithm="fricas")`output `integral((b*sqrt(x)*sec(d*sqrt(x) + c) + a*sqrt(x))/x^3, x)`**Sympy [N/A]**

Not integrable

Time = 4.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{a + b \sec(c + d\sqrt{x})}{x^{\frac{5}{2}}} dx$$

input `integrate((a+b*sec(c+d*x**(1/2)))/x**(5/2),x)`

output `Integral((a + b*sec(c + d*sqrt(x)))/x**(5/2), x)`

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 113, normalized size of antiderivative = 5.65

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^{5/2}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^(5/2),x, algorithm="maxima")`

output `2/3*(3*b*x^(3/2)*integrate((cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + cos(d*sqrt(x) + c))/((cos(2*d*sqrt(x) + 2*c)^2 + sin(2*d*sqrt(x) + 2*c)^2 + 2*cos(2*d*sqrt(x) + 2*c) + 1)*x^(5/2)), x) - a)/x^(3/2)`

Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{b \sec(d\sqrt{x} + c) + a}{x^{5/2}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))/x^(5/2),x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)/x^(5/2), x)`

Mupad [N/A]

Not integrable

Time = 15.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{a + \frac{b}{\cos(c+d\sqrt{x})}}{x^{5/2}} dx$$

input `int((a + b/cos(c + d*x^(1/2)))/x^(5/2), x)`output `int((a + b/cos(c + d*x^(1/2)))/x^(5/2), x)`**Reduce [N/A]**

Not integrable

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

$$\int \frac{a + b \sec(c + d\sqrt{x})}{x^{5/2}} dx = \frac{\sqrt{x} \left(\int \frac{\sec(\sqrt{x}d+c) \tan(\sqrt{x}d+c)}{x^2} dx \right) b dx - 2 \sec(\sqrt{x}d + c) b - 2a}{3\sqrt{x} x}$$

input `int((a+b*sec(c+d*x^(1/2)))/x^(5/2), x)`output `(sqrt(x)*int((sec(sqrt(x)*d + c)*tan(sqrt(x)*d + c))/x**2,x)*b*d*x - 2*sec(sqrt(x)*d + c)*b - 2*a)/(3*sqrt(x)*x)`

3.56 $\int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx$

Optimal result	407
Mathematica [A] (verified)	408
Rubi [A] (verified)	409
Maple [F]	411
Fricas [F]	411
Sympy [F]	411
Maxima [B] (verification not implemented)	412
Giac [F]	413
Mupad [F(-1)]	413
Reduce [F]	413

Optimal result

Integrand size = 22, antiderivative size = 451

$$\begin{aligned}
 \int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx = & -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} \\
 & - \frac{8iabx^2 \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8b^2x^{3/2} \log\left(1 + e^{2i(c+d\sqrt{x})}\right)}{d^2} \\
 & + \frac{16iabx^{3/2} \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2} \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
 & - \frac{12ib^2x \operatorname{PolyLog}\left(2, -e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{48abx \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
 & + \frac{48abx \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b^2\sqrt{x} \operatorname{PolyLog}\left(3, -e^{2i(c+d\sqrt{x})}\right)}{d^4} \\
 & - \frac{96iab\sqrt{x} \operatorname{PolyLog}\left(4, -ie^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x} \operatorname{PolyLog}\left(4, ie^{i(c+d\sqrt{x})}\right)}{d^4} \\
 & + \frac{6ib^2 \operatorname{PolyLog}\left(4, -e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{96ab \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} \\
 & - \frac{96ab \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{2b^2x^2 \tan(c + d\sqrt{x})}{d}
 \end{aligned}$$

output

```
6*I*b^2*polylog(4,-exp(2*I*(c+d*x^(1/2))))/d^5+2/5*a^2*x^(5/2)-8*I*a*b*x^2
*arctan(exp(I*(c+d*x^(1/2))))/d+8*b^2*x^(3/2)*ln(1+exp(2*I*(c+d*x^(1/2))))
/d^2+96*I*a*b*x^(1/2)*polylog(4,I*exp(I*(c+d*x^(1/2))))/d^4+16*I*a*b*x^(3/
2)*polylog(2,-I*exp(I*(c+d*x^(1/2))))/d^2-2*I*b^2*x^2/d-48*a*b*x*polylog(3
,-I*exp(I*(c+d*x^(1/2))))/d^3+48*a*b*x*polylog(3,I*exp(I*(c+d*x^(1/2))))/d
^3+12*b^2*x^(1/2)*polylog(3,-exp(2*I*(c+d*x^(1/2))))/d^4-12*I*b^2*x*polylo
g(2,-exp(2*I*(c+d*x^(1/2))))/d^3-96*I*a*b*x^(1/2)*polylog(4,-I*exp(I*(c+d*
x^(1/2))))/d^4-16*I*a*b*x^(3/2)*polylog(2,I*exp(I*(c+d*x^(1/2))))/d^2+96*a
*b*polylog(5,-I*exp(I*(c+d*x^(1/2))))/d^5-96*a*b*polylog(5,I*exp(I*(c+d*x^
(1/2))))/d^5+2*b^2*x^2*tan(c+d*x^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.98

$$\int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx = \frac{2(-5ib^2d^4x^2 + a^2d^5x^{5/2} - 20iabd^4x^2 \arctan(e^{i(c+d\sqrt{x})}) + 20b^2d^3x^{3/2} \log(1 + e^{2i(c+d\sqrt{x})}))}{5d^5}$$

input

```
Integrate[x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

```
(2*((-5*I)*b^2*d^4*x^2 + a^2*d^5*x^(5/2) - (20*I)*a*b*d^4*x^2*ArcTan[E^(I*(
c + d*Sqrt[x]))] + 20*b^2*d^3*x^(3/2)*Log[1 + E^((2*I)*(c + d*Sqrt[x]))])
+ (40*I)*a*b*d^3*x^(3/2)*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (40*I)*a
*b*d^3*x^(3/2)*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (30*I)*b^2*d^2*x*Poly
Log[2, -E^((2*I)*(c + d*Sqrt[x]))] - 120*a*b*d^2*x*PolyLog[3, (-I)*E^(I*(c
+ d*Sqrt[x]))] + 120*a*b*d^2*x*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] + 30*b
^2*d*Sqrt[x]*PolyLog[3, -E^((2*I)*(c + d*Sqrt[x]))] - (240*I)*a*b*d*Sqrt[x
]*PolyLog[4, (-I)*E^(I*(c + d*Sqrt[x]))] + (240*I)*a*b*d*Sqrt[x]*PolyLog[4
, I*E^(I*(c + d*Sqrt[x]))] + (15*I)*b^2*PolyLog[4, -E^((2*I)*(c + d*Sqrt[x
]))] + 240*a*b*PolyLog[5, (-I)*E^(I*(c + d*Sqrt[x]))] - 240*a*b*PolyLog[5,
I*E^(I*(c + d*Sqrt[x]))] + 5*b^2*d^4*x^2*Tan[c + d*Sqrt[x]]))/(5*d^5)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4692, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx \\
 & \quad \downarrow 4692 \\
 & 2 \int x^2 (a + b \sec(c + d\sqrt{x}))^2 d\sqrt{x} \\
 & \quad \downarrow 3042 \\
 & 2 \int x^2 \left(a + b \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right) \right)^2 d\sqrt{x} \\
 & \quad \downarrow 4678 \\
 & 2 \int (a^2 x^2 + b^2 \sec^2(c + d\sqrt{x}) x^2 + 2ab \sec(c + d\sqrt{x}) x^2) d\sqrt{x} \\
 & \quad \downarrow 2009 \\
 & 2 \left(\frac{1}{5} a^2 x^{5/2} - \frac{4iabx^2 \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{48ab \operatorname{PolyLog}\left(5, -ie^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{48ab \operatorname{PolyLog}\left(5, ie^{i(c+d\sqrt{x})}\right)}{d^5} - \dots \right)
 \end{aligned}$$

input

```
Int[x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

$$2\left(\frac{(-1)b^2x^2}{d} + \frac{a^2x^{5/2}}{5} - \frac{(4I)abx^2\text{ArcTan}[E^{I(c+d\sqrt{x})}]}{d} + \frac{4b^2x^{3/2}\text{Log}[1+E^{(2I)(c+d\sqrt{x})}]}{d^2} + \frac{(8I)abx^{3/2}\text{PolyLog}[2, (-1)E^{I(c+d\sqrt{x})}]}{d^2} - \frac{(8I)abx^{3/2}\text{PolyLog}[2, I E^{I(c+d\sqrt{x})}]}{d^2} - \frac{(6I)b^2x\text{PolyLog}[2, -E^{(2I)(c+d\sqrt{x})}]}{d^3} - \frac{24abx\text{PolyLog}[3, (-1)E^{I(c+d\sqrt{x})}]}{d^3} + \frac{24abx\text{PolyLog}[3, I E^{I(c+d\sqrt{x})}]}{d^3} + \frac{6b^2\sqrt{x}\text{PolyLog}[3, -E^{(2I)(c+d\sqrt{x})}]}{d^4} - \frac{(48I)ab\sqrt{x}\text{PolyLog}[4, (-1)E^{I(c+d\sqrt{x})}]}{d^4} + \frac{(48I)ab\sqrt{x}\text{PolyLog}[4, I E^{I(c+d\sqrt{x})}]}{d^4} + \frac{(3I)b^2\text{PolyLog}[4, -E^{(2I)(c+d\sqrt{x})}]}{d^5} + \frac{48ab\text{PolyLog}[5, (-1)E^{I(c+d\sqrt{x})}]}{d^5} - \frac{48ab\text{PolyLog}[5, I E^{I(c+d\sqrt{x})}]}{d^5} + \frac{b^2x^2\text{Tan}[c+d\sqrt{x}]}{d}\right)$$

Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Simp}[\text{IntSum}[u, x], x] \text{ /; } \text{SumQ}[u]$$

rule 3042

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ /; } \text{FunctionOfTrigOfLinearQ}[u, x]$$

rule 4678

$$\text{Int}[(\text{csc}[e.] + (f.)(x.))(b.) + (a.)^{(n.)}((c.) + (d.)(x.))^{(m.)}, x_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[(c + dx)^m, (a + b\text{Csc}[e + fx])^n, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0]$$

rule 4692

$$\text{Int}[(x.)^{(m.)}((a.) + (b.)*\text{Sec}[(c.) + (d.)(x.)^{(n.)}])^{(p.)}, x_Symbol] \text{ :> } \text{Simp}[1/n \ \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)(a + b\text{Sec}[c + dx])^p}, x], x, x^n], x] \text{ /; } \text{FreeQ}\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{IGtQ}[\text{Simplify}[(m+1)/n], 0] \ \&\& \ \text{IntegerQ}[p]$$

Maple [F]

$$\int x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))^2 dx$$

input `int(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 x^{\frac{3}{2}} dx$$

input `integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(b^2*x^(3/2)*sec(d*sqrt(x) + c)^2 + 2*a*b*x^(3/2)*sec(d*sqrt(x) + c) + a^2*x^(3/2), x)`

Sympy [F]

$$\int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx = \int x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))^2 dx$$

input `integrate(x**(3/2)*(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(x**(3/2)*(a + b*sec(c + d*sqrt(x)))**2, x)`

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2869 vs. $2(346) = 692$.

Time = 0.29 (sec) , antiderivative size = 2869, normalized size of antiderivative = 6.36

$$\int x^{3/2}(a + b \sec(c + d\sqrt{x}))^2 dx = \text{Too large to display}$$

input `integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output

```
2/5*((d*sqrt(x) + c)^5*a^2 - 5*(d*sqrt(x) + c)^4*a^2*c + 10*(d*sqrt(x) + c)
)^3*a^2*c^2 - 10*(d*sqrt(x) + c)^2*a^2*c^3 + 5*(d*sqrt(x) + c)*a^2*c^4 + 1
0*a*b*c^4*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) + 5*(6*b^2*c^4 - 6*
((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a
*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3 + ((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x)
+ c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3)*c
os(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^4*a*b - 4*I*(d*sqrt(x) + c)^3*a
*b*c + 6*I*(d*sqrt(x) + c)^2*a*b*c^2 - 4*I*(d*sqrt(x) + c)*a*b*c^3)*sin(2*
d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 6*
((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x) + c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a
*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3 + ((d*sqrt(x) + c)^4*a*b - 4*(d*sqrt(x)
+ c)^3*a*b*c + 6*(d*sqrt(x) + c)^2*a*b*c^2 - 4*(d*sqrt(x) + c)*a*b*c^3)*c
os(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^4*a*b - 4*I*(d*sqrt(x) + c)^3*a
*b*c + 6*I*(d*sqrt(x) + c)^2*a*b*c^2 - 4*I*(d*sqrt(x) + c)*a*b*c^3)*sin(2*
d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x) + c) + 1) + 4
*(4*(d*sqrt(x) + c)^3*b^2 - 9*(d*sqrt(x) + c)^2*b^2*c + 9*(d*sqrt(x) + c)*
b^2*c^2 - 3*b^2*c^3 + (4*(d*sqrt(x) + c)^3*b^2 - 9*(d*sqrt(x) + c)^2*b^2*c
+ 9*(d*sqrt(x) + c)*b^2*c^2 - 3*b^2*c^3)*cos(2*d*sqrt(x) + 2*c) - (-4*I*(
d*sqrt(x) + c)^3*b^2 + 9*I*(d*sqrt(x) + c)^2*b^2*c - 9*I*(d*sqrt(x) + c)*b
^2*c^2 + 3*I*b^2*c^3)*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(2*d*sqrt(x) + ...
```

Giac [F]

$$\int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 x^{3/2} dx$$

input `integrate(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)^2*x^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx = \int x^{3/2} \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

input `int(x^(3/2)*(a + b/cos(c + d*x^(1/2)))^2,x)`

output `int(x^(3/2)*(a + b/cos(c + d*x^(1/2)))^2, x)`

Reduce [F]

$$\int x^{3/2} (a + b \sec(c + d\sqrt{x}))^2 dx = \text{too large to display}$$

input `int(x^(3/2)*(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```

(2*(sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**2*a**2*d**5*x**2 +
120*sqrt(x)*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**2*a*b*d - sqrt(x)*c
os(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**2*b**2*d**5*x**2 - 120*sqrt(x)*c
os(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**2*b**2*d - sqrt(x)*cos(sqrt(x)*d
+ c)*a**2*d**5*x**2 + 40*sqrt(x)*cos(sqrt(x)*d + c)*a*b*d**3*x + 120*sqrt
(x)*cos(sqrt(x)*d + c)*a*b*d + sqrt(x)*cos(sqrt(x)*d + c)*b**2*d**5*x**2 -
40*sqrt(x)*cos(sqrt(x)*d + c)*b**2*d**3*x - 120*sqrt(x)*cos(sqrt(x)*d + c
)*b**2*d + 120*cos(sqrt(x)*d + c)*int(sqrt(x)/(tan((sqrt(x)*d + c)/2)**4 -
2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*tan((sqrt(x)*d + c)/2)**2*a*b*d**3 -
120*cos(sqrt(x)*d + c)*int(sqrt(x)/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqr
t(x)*d + c)/2)**2 + 1),x)*tan((sqrt(x)*d + c)/2)**2*b**2*d**3 - 120*cos(sq
rt(x)*d + c)*int(sqrt(x)/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c
)/2)**2 + 1),x)*a*b*d**3 + 120*cos(sqrt(x)*d + c)*int(sqrt(x)/(tan((sqrt(x)
)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*b**2*d**3 + 240*cos(s
qrt(x)*d + c)*int(tan((sqrt(x)*d + c)/2)/(tan((sqrt(x)*d + c)/2)**4 - 2*ta
n((sqrt(x)*d + c)/2)**2 + 1),x)*tan((sqrt(x)*d + c)/2)**2*a*b*d**2 - 240*c
os(sqrt(x)*d + c)*int(tan((sqrt(x)*d + c)/2)/(tan((sqrt(x)*d + c)/2)**4 -
2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*tan((sqrt(x)*d + c)/2)**2*b**2*d**2 -
240*cos(sqrt(x)*d + c)*int(tan((sqrt(x)*d + c)/2)/(tan((sqrt(x)*d + c)/2)*
**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*a*b*d**2 + 240*cos(sqrt(x)*d + ...

```

3.57 $\int \sqrt{x} (a + b \sec(c + d\sqrt{x}))^2 dx$

Optimal result	415
Mathematica [A] (verified)	416
Rubi [A] (verified)	416
Maple [F]	418
Fricas [F]	418
Sympy [F]	419
Maxima [B] (verification not implemented)	419
Giac [F]	420
Mupad [F(-1)]	421
Reduce [F]	421

Optimal result

Integrand size = 22, antiderivative size = 255

$$\begin{aligned}
 \int \sqrt{x} (a + b \sec(c + d\sqrt{x}))^2 dx = & -\frac{2ib^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8iabx \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} \\
 & + \frac{4b^2\sqrt{x} \log\left(1 + e^{2i(c+d\sqrt{x})}\right)}{d^2} \\
 & + \frac{8iab\sqrt{x} \operatorname{PolyLog}\left(2, -ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
 & - \frac{8iab\sqrt{x} \operatorname{PolyLog}\left(2, ie^{i(c+d\sqrt{x})}\right)}{d^2} \\
 & - \frac{2ib^2 \operatorname{PolyLog}\left(2, -e^{2i(c+d\sqrt{x})}\right)}{d^3} \\
 & - \frac{8ab \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} \\
 & + \frac{8ab \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{2b^2x \tan(c + d\sqrt{x})}{d}
 \end{aligned}$$

output

```
-2*I*b^2*x/d+2/3*a^2*x^(3/2)-8*I*a*b*x*arctan(exp(I*(c+d*x^(1/2))))/d+4*b^
2*x^(1/2)*ln(1+exp(2*I*(c+d*x^(1/2))))/d^2+8*I*a*b*x^(1/2)*polylog(2,-I*ex
p(I*(c+d*x^(1/2))))/d^2-8*I*a*b*x^(1/2)*polylog(2,I*exp(I*(c+d*x^(1/2))))/
d^2-2*I*b^2*polylog(2,-exp(2*I*(c+d*x^(1/2))))/d^3-8*a*b*polylog(3,-I*exp(
I*(c+d*x^(1/2))))/d^3+8*a*b*polylog(3,I*exp(I*(c+d*x^(1/2))))/d^3+2*b^2*x*
tan(c+d*x^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.97

$$\int \sqrt{x} (a + b \sec(c + d\sqrt{x}))^2 dx$$

$$= \frac{2 \left(-3ib^2 d^2 x + a^2 d^3 x^{3/2} - 12iabd^2 x \arctan \left(e^{i(c+d\sqrt{x})} \right) + 6b^2 d \sqrt{x} \log \left(1 + e^{2i(c+d\sqrt{x})} \right) + 12iabd \sqrt{x} \operatorname{PolyLog} \right)}{\dots}$$

input

```
Integrate[Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

```
(2*((-3*I)*b^2*d^2*x + a^2*d^3*x^(3/2) - (12*I)*a*b*d^2*x*ArcTan[E^(I*(c +
d*Sqrt[x]))] + 6*b^2*d*Sqrt[x]*Log[1 + E^((2*I)*(c + d*Sqrt[x]))] + (12*I
)*a*b*d*Sqrt[x]*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))] - (12*I)*a*b*d*Sqrt
[x]*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))] - (3*I)*b^2*PolyLog[2, -E^((2*I)*(
c + d*Sqrt[x]))] - 12*a*b*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))] + 12*a*b*
PolyLog[3, I*E^(I*(c + d*Sqrt[x]))] + 3*b^2*d^2*x*Tan[c + d*Sqrt[x]]))/(3*
d^3)
```

Rubi [A] (verified)Time = 0.56 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4692, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \sqrt{x}(a + b \sec(c + d\sqrt{x}))^2 dx \\
& \quad \downarrow 4692 \\
& 2 \int x(a + b \sec(c + d\sqrt{x}))^2 d\sqrt{x} \\
& \quad \downarrow 3042 \\
& 2 \int x\left(a + b \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right)\right)^2 d\sqrt{x} \\
& \quad \downarrow 4678 \\
& 2 \int (xa^2 + 2bx \sec(c + d\sqrt{x})a + b^2x \sec^2(c + d\sqrt{x})) d\sqrt{x} \\
& \quad \downarrow 2009 \\
& 2 \left(\frac{1}{3}a^2x^{3/2} - \frac{4iabx \arctan\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{4ab \operatorname{PolyLog}\left(3, -ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{4ab \operatorname{PolyLog}\left(3, ie^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{4iab}{d} \right)
\end{aligned}$$

input

```
Int[Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

```
2*((( -I)*b^2*x)/d + (a^2*x^(3/2))/3 - ((4*I)*a*b*x*ArcTan[E^(I*(c + d*Sqrt[x]))])/d + (2*b^2*Sqrt[x]*Log[1 + E^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((4*I)*a*b*Sqrt[x]*PolyLog[2, (-I)*E^(I*(c + d*Sqrt[x]))])/d^2 - ((4*I)*a*b*Sqrt[x]*PolyLog[2, I*E^(I*(c + d*Sqrt[x]))])/d^2 - (I*b^2*PolyLog[2, -E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (4*a*b*PolyLog[3, (-I)*E^(I*(c + d*Sqrt[x]))])/d^3 + (4*a*b*PolyLog[3, I*E^(I*(c + d*Sqrt[x]))])/d^3 + (b^2*x*Tan[c + d*Sqrt[x]])/d)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \sqrt{x} (a + b \sec(c + d\sqrt{x}))^2 dx$$

input `int(x^(1/2)*(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(x^(1/2)*(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int \sqrt{x} (a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 \sqrt{x} dx$$

input `integrate(x^(1/2)*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(b^2*sqrt(x)*sec(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sec(d*sqrt(x) + c) + a^2*sqrt(x), x)`

Sympy [F]

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x}))^2 dx = \int \sqrt{x}(a + b \sec(c + d\sqrt{x}))^2 dx$$

input `integrate(x**(1/2)*(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(sqrt(x)*(a + b*sec(c + d*sqrt(x)))**2, x)`

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1272 vs. $2(194) = 388$.

Time = 0.23 (sec) , antiderivative size = 1272, normalized size of antiderivative = 4.99

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x}))^2 dx = \text{Too large to display}$$

input `integrate(x^(1/2)*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output

```

2/3*((d*sqrt(x) + c)^3*a^2 - 3*(d*sqrt(x) + c)^2*a^2*c + 3*(d*sqrt(x) + c)
*a^2*c^2 + 6*a*b*c^2*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c)) + 3*(2*b
^2*c^2 - 2*((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c + ((d*sqrt(x)
+ c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x)
+ c)^2*a*b - 2*I*(d*sqrt(x) + c)*a*b*c)*sin(2*d*sqrt(x) + 2*c))*arctan2
(cos(d*sqrt(x) + c), sin(d*sqrt(x) + c) + 1) - 2*((d*sqrt(x) + c)^2*a*b -
2*(d*sqrt(x) + c)*a*b*c + ((d*sqrt(x) + c)^2*a*b - 2*(d*sqrt(x) + c)*a*b*c)
*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)^2*a*b - 2*I*(d*sqrt(x) + c)*
a*b*c)*sin(2*d*sqrt(x) + 2*c))*arctan2(cos(d*sqrt(x) + c), -sin(d*sqrt(x)
+ c) + 1) + 2*((d*sqrt(x) + c)*b^2 - b^2*c + ((d*sqrt(x) + c)*b^2 - b^2*c)
*cos(2*d*sqrt(x) + 2*c) - (-I*(d*sqrt(x) + c)*b^2 + I*b^2*c)*sin(2*d*sqrt(x)
+ 2*c))*arctan2(sin(2*d*sqrt(x) + 2*c), cos(2*d*sqrt(x) + 2*c) + 1) - 2
*((d*sqrt(x) + c)^2*b^2 - 2*(d*sqrt(x) + c)*b^2*c)*cos(2*d*sqrt(x) + 2*c)
- (b^2*cos(2*d*sqrt(x) + 2*c) + I*b^2*sin(2*d*sqrt(x) + 2*c) + b^2)*dilog(
-e^(2*I*d*sqrt(x) + 2*I*c)) - 4*((d*sqrt(x) + c)*a*b - a*b*c + ((d*sqrt(x)
+ c)*a*b - a*b*c)*cos(2*d*sqrt(x) + 2*c) + (I*(d*sqrt(x) + c)*a*b - I*a*b
*c)*sin(2*d*sqrt(x) + 2*c))*dilog(I*e^(I*d*sqrt(x) + I*c)) + 4*((d*sqrt(x)
+ c)*a*b - a*b*c + ((d*sqrt(x) + c)*a*b - a*b*c)*cos(2*d*sqrt(x) + 2*c) -
(-I*(d*sqrt(x) + c)*a*b + I*a*b*c)*sin(2*d*sqrt(x) + 2*c))*dilog(-I*e^(I*
d*sqrt(x) + I*c)) + (-I*(d*sqrt(x) + c)*b^2 + I*b^2*c + (-I*(d*sqrt(x) ...

```

Giac [F]

$$\int \sqrt{x}(a + b \sec(c + d\sqrt{x}))^2 dx = \int (b \sec(d\sqrt{x} + c) + a)^2 \sqrt{x} dx$$

input

```
integrate(x^(1/2)*(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")
```

output

```
integrate((b*sec(d*sqrt(x) + c) + a)^2*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x} (a + b \sec(c + d\sqrt{x}))^2 dx = \int \sqrt{x} \left(a + \frac{b}{\cos(c + d\sqrt{x})} \right)^2 dx$$

input `int(x^(1/2)*(a + b/cos(c + d*x^(1/2)))^2,x)`output `int(x^(1/2)*(a + b/cos(c + d*x^(1/2)))^2, x)`**Reduce [F]**

$$\int \sqrt{x} (a + b \sec(c + d\sqrt{x}))^2 dx$$

$$= \frac{2\sqrt{x} \cos(\sqrt{x}d+c)a^2d^3x}{3} - \frac{2\sqrt{x} \cos(\sqrt{x}d+c)abd^3x}{3} + 4 \cos(\sqrt{x}d+c) \left(\int \frac{\sqrt{x}}{\tan\left(\frac{\sqrt{x}d+c}{2}\right)^4 - 2\tan\left(\frac{\sqrt{x}d+c}{2}\right)^2 + 1} dx \right) ab d^3 +$$

input `int(x^(1/2)*(a+b*sec(c+d*x^(1/2)))^2,x)`output `(2*(sqrt(x)*cos(sqrt(x)*d + c)*a**2*d**3*x - sqrt(x)*cos(sqrt(x)*d + c)*a*b*d**3*x + 6*cos(sqrt(x)*d + c)*int(sqrt(x)/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*a*b*d**3 + 12*cos(sqrt(x)*d + c)*int(tan((sqrt(x)*d + c)/2)/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*a*b*d**2 - 12*cos(sqrt(x)*d + c)*int(tan((sqrt(x)*d + c)/2)/(tan((sqrt(x)*d + c)/2)**4 - 2*tan((sqrt(x)*d + c)/2)**2 + 1),x)*b**2*d**2 - 6*cos(sqrt(x)*d + c)*log(tan((sqrt(x)*d + c)/2) - 1)*a*b + 6*cos(sqrt(x)*d + c)*log(tan((sqrt(x)*d + c)/2) - 1)*b**2 + 6*cos(sqrt(x)*d + c)*log(tan((sqrt(x)*d + c)/2) + 1)*a*b - 6*cos(sqrt(x)*d + c)*log(tan((sqrt(x)*d + c)/2) + 1)*b**2 - 6*sqrt(x)*a*b*d + 6*sqrt(x)*b**2*d - 3*sin(sqrt(x)*d + c)*a*b*d**2*x + 3*sin(sqrt(x)*d + c)*b**2*d**2*x))/(3*cos(sqrt(x)*d + c)*d**3)`

3.58 $\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{\sqrt{x}} dx$

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Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx = 2a^2\sqrt{x} + \frac{4ab \operatorname{arctanh}(\sin(c + d\sqrt{x}))}{d} + \frac{2b^2 \tan(c + d\sqrt{x})}{d}$$

output `2*a^2*x^(1/2)+4*a*b*arctanh(sin(c+d*x^(1/2)))/d+2*b^2*tan(c+d*x^(1/2))/d`

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx = \frac{2(a^2 d \sqrt{x} + 2ab \operatorname{coth}^{-1}(\sin(c + d\sqrt{x})) + b^2 \tan(c + d\sqrt{x}))}{d}$$

input `Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/Sqrt[x],x]`

output `(2*(a^2*d*Sqrt[x] + 2*a*b*ArcCoth[Sin[c + d*Sqrt[x]]] + b^2*Tan[c + d*Sqrt[x]]))/d`

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4692, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx \\
 & \quad \downarrow \text{4692} \\
 & 2 \int (a + b \sec(c + d\sqrt{x}))^2 d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int \left(a + b \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right)\right)^2 d\sqrt{x} \\
 & \quad \downarrow \text{4260} \\
 & 2 \left(2ab \int \sec(c + d\sqrt{x}) d\sqrt{x} + b^2 \int \sec^2(c + d\sqrt{x}) d\sqrt{x} + a^2 \sqrt{x} \right) \\
 & \quad \downarrow \text{3042} \\
 & 2 \left(2ab \int \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right) d\sqrt{x} + b^2 \int \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right)^2 d\sqrt{x} + a^2 \sqrt{x} \right) \\
 & \quad \downarrow \text{4254} \\
 & 2 \left(2ab \int \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right) d\sqrt{x} - \frac{b^2 \int 1 d(-\tan(c + d\sqrt{x}))}{d} + a^2 \sqrt{x} \right) \\
 & \quad \downarrow \text{24} \\
 & 2 \left(2ab \int \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right) d\sqrt{x} + a^2 \sqrt{x} + \frac{b^2 \tan(c + d\sqrt{x})}{d} \right) \\
 & \quad \downarrow \text{4257} \\
 & 2 \left(a^2 \sqrt{x} + \frac{2ab \operatorname{arctanh}(\sin(c + d\sqrt{x}))}{d} + \frac{b^2 \tan(c + d\sqrt{x})}{d} \right)
 \end{aligned}$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])^2/Sqrt[x], x]`

output `2*(a^2*Sqrt[x] + (2*a*b*ArcTanh[Sin[c + d*Sqrt[x]]])/d + (b^2*Tan[c + d*Sqrt[x]])/d)`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4254 `Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

rule 4257 `Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

rule 4260 `Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(2, x_Symbol] := Simp[a^2*x, x] + (Simp[2*a*b Int[Csc[c + d*x], x], x] + Simp[b^2 Int[Csc[c + d*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

method	result	size
parts	$2a^2\sqrt{x} + \frac{2b^2 \tan(c+d\sqrt{x})}{d} + \frac{4ab \ln(\sec(c+d\sqrt{x})+\tan(c+d\sqrt{x}))}{d}$	51
derivativedivides	$\frac{2a^2(c+d\sqrt{x})+4ab \ln(\sec(c+d\sqrt{x})+\tan(c+d\sqrt{x}))+2b^2 \tan(c+d\sqrt{x})}{d}$	52
default	$\frac{2a^2(c+d\sqrt{x})+4ab \ln(\sec(c+d\sqrt{x})+\tan(c+d\sqrt{x}))+2b^2 \tan(c+d\sqrt{x})}{d}$	52

input `int((a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)`

output `2*a^2*x^(1/2)+2*b^2*tan(c+d*x^(1/2))/d+4*a*b/d*ln(sec(c+d*x^(1/2))+tan(c+d*x^(1/2)))`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(41) = 82$.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx$$

$$= \frac{2(a^2 d \sqrt{x} \cos(d\sqrt{x} + c) + ab \cos(d\sqrt{x} + c) \log(\sin(d\sqrt{x} + c) + 1) - ab \cos(d\sqrt{x} + c) \log(-\sin(d\sqrt{x} + c)))}{d \cos(d\sqrt{x} + c)}$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")`

output `2*(a^2*d*sqrt(x)*cos(d*sqrt(x) + c) + a*b*cos(d*sqrt(x) + c)*log(sin(d*sqrt(x) + c) + 1) - a*b*cos(d*sqrt(x) + c)*log(-sin(d*sqrt(x) + c) + 1) + b^2*sin(d*sqrt(x) + c))/(d*cos(d*sqrt(x) + c))`

Sympy [A] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx$$

$$= \begin{cases} \frac{2a^2(c+d\sqrt{x})+4ab \log(\tan(c+d\sqrt{x})+\sec(c+d\sqrt{x}))+2b^2 \tan(c+d\sqrt{x})}{d} & \text{for } d \neq 0 \\ -\sqrt{x}(-2a^2 - 4ab \sec(c) - 2b^2 \sec^2(c)) & \text{otherwise} \end{cases}$$

input `integrate((a+b*sec(c+d*x**(1/2)))**2/x**(1/2),x)`output `Piecewise(((2*a**2*(c + d*sqrt(x)) + 4*a*b*log(tan(c + d*sqrt(x)) + sec(c + d*sqrt(x))) + 2*b**2*tan(c + d*sqrt(x)))/d, Ne(d, 0)), (-sqrt(x)*(-2*a**2 - 4*a*b*sec(c) - 2*b**2*sec(c)**2), True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx = 2a^2\sqrt{x} + \frac{4ab \log(\sec(d\sqrt{x} + c) + \tan(d\sqrt{x} + c))}{d} + \frac{2b^2 \tan(d\sqrt{x} + c)}{d}$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")`output `2*a^2*sqrt(x) + 4*a*b*log(sec(d*sqrt(x) + c) + tan(d*sqrt(x) + c))/d + 2*b^2*tan(d*sqrt(x) + c)/d`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(41) = 82$.

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx$$

$$= \frac{2 \left((d\sqrt{x} + c)a^2 + 2ab \log \left(\left| \tan \left(\frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right) + 1 \right| \right) - 2ab \log \left(\left| \tan \left(\frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2b^2 \tan \left(\frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right)}{\tan \left(\frac{1}{2} d\sqrt{x} + \frac{1}{2} c \right) + 1} \right)}{d}$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")`

output `2*((d*sqrt(x) + c)*a^2 + 2*a*b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c) + 1)) - 2*a*b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c) - 1)) - 2*b^2*tan(1/2*d*sqrt(x) + 1/2*c)/(tan(1/2*d*sqrt(x) + 1/2*c)^2 - 1))/d`

Mupad [B] (verification not implemented)

Time = 16.87 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.32

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx = 2a^2 \sqrt{x} + \frac{b^2 4i}{d (e^{c2i+d\sqrt{x}2i} + 1)}$$

$$+ \frac{4ab \ln \left(-\frac{ab4i}{\sqrt{x}} - \frac{4abe^d \sqrt{x}^{1i} e^{c1i}}{\sqrt{x}} \right)}{d}$$

$$- \frac{4ab \ln \left(\frac{ab4i}{\sqrt{x}} - \frac{4abe^d \sqrt{x}^{1i} e^{c1i}}{\sqrt{x}} \right)}{d}$$

input `int((a + b/cos(c + d*x^(1/2)))^2/x^(1/2),x)`

output `2*a^2*x^(1/2) + (b^2*4i)/(d*(exp(c*2i + d*x^(1/2)*2i) + 1)) + (4*a*b*log(-(a*b*4i)/x^(1/2) - (4*a*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d - (4*a*b*log((a*b*4i)/x^(1/2) - (4*a*b*exp(d*x^(1/2)*1i)*exp(c*1i))/x^(1/2)))/d`

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{\sqrt{x}} dx$$

$$= \frac{2\sqrt{x} \cos(\sqrt{x}d + c) a^2 d - 4 \cos(\sqrt{x}d + c) \log\left(\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right) - 1\right) ab + 4 \cos(\sqrt{x}d + c) \log\left(\tan\left(\frac{\sqrt{x}d}{2} + \frac{c}{2}\right) + 1\right) ab + \sin(\sqrt{x}d + c) b^2}{\cos(\sqrt{x}d + c) d}$$

input `int((a+b*sec(c+d*x^(1/2)))^2/x^(1/2),x)`output `(2*(sqrt(x)*cos(sqrt(x)*d + c)*a**2*d - 2*cos(sqrt(x)*d + c)*log(tan((sqrt(x)*d + c)/2) - 1)*a*b + 2*cos(sqrt(x)*d + c)*log(tan((sqrt(x)*d + c)/2) + 1)*a*b + sin(sqrt(x)*d + c)*b**2)/(cos(sqrt(x)*d + c)*d)`

$$3.59 \quad \int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

Optimal result	429
Mathematica [N/A]	429
Rubi [N/A]	430
Maple [N/A]	430
Fricas [N/A]	431
Sympy [N/A]	431
Maxima [N/A]	432
Giac [N/A]	433
Mupad [N/A]	433
Reduce [N/A]	433

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx = \text{Int}\left(\frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}}, x\right)$$

output `Defer(Int)((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x)`

Mathematica [N/A]

Not integrable

Time = 50.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx$$

input `Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^(3/2),x]`

output `Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^(3/2), x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx$$

↓ 4694

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])^2/x^(3/2), x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx$$

input `int((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x)`

output `int((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^{3/2}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="fricas")`

output `integral((b^2*sqrt(x)*sec(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sec(d*sqrt(x) + c) + a^2*sqrt(x))/x^2, x)`

Sympy [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx$$

input `integrate((a+b*sec(c+d*x**(1/2)))**2/x**(3/2),x)`

output `Integral((a + b*sec(c + d*sqrt(x)))**2/x**(3/2), x)`

Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 718, normalized size of antiderivative = 32.64

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^{3/2}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="maxima")`

output

```
(4*b^2*sin(2*d*sqrt(x) + 2*c) + (d*cos(2*d*sqrt(x) + 2*c)^2*integrate(4*(b^2*sin(2*d*sqrt(x) + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c))*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x) + d*integrate(4*(b^2*sin(2*d*sqrt(x) + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c))*sqrt(x)))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x)*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c)*integrate(4*(b^2*sin(2*d*sqrt(x) + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c))*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x) + d*integrate(4*(b^2*sin(2*d*sqrt(x) + 2*c) + (a*b*d*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + a*b*d*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + a*b*d*cos(d*sqrt(x) + c))*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2), x))*x - 2*(a^2*d*cos(2*d*sqrt(x) + 2*c)^2 + a^2*d*sin(2*d*sqrt(x) + 2*c)^2 + 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d)*sqrt(x))/((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 + 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^{3/2}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)^2/x^(3/2), x)`

Mupad [N/A]

Not integrable

Time = 15.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2}{x^{3/2}} dx$$

input `int((a + b/cos(c + d*x^(1/2)))^2/x^(3/2),x)`

output `int((a + b/cos(c + d*x^(1/2)))^2/x^(3/2), x)`

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.18

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{3/2}} dx = \frac{2\sqrt{x} \left(\int \frac{\sec(\sqrt{x}d+c)^2 \tan(\sqrt{x}d+c)}{x} dx \right) b^2 d + 2\sqrt{x} \left(\int \frac{\sec(\sqrt{x}d+c) \tan(\sqrt{x}d+c)}{x} dx \right) a b}{\sqrt{x}}$$

input `int((a+b*sec(c+d*x^(1/2)))^2/x^(3/2),x)`

output `(2*(sqrt(x)*int((sec(sqrt(x)*d + c)**2*tan(sqrt(x)*d + c))/x,x)*b**2*d + sqrt(x)*int((sec(sqrt(x)*d + c)*tan(sqrt(x)*d + c))/x,x)*a*b*d - sec(sqrt(x)*d + c)**2*b**2 - 2*sec(sqrt(x)*d + c)*a*b - a**2))/sqrt(x)`

3.60
$$\int \frac{(a+b \sec(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

Optimal result	435
Mathematica [N/A]	435
Rubi [N/A]	436
Maple [N/A]	436
Fricas [N/A]	437
Sympy [N/A]	437
Maxima [F(-1)]	438
Giac [N/A]	438
Mupad [N/A]	438
Reduce [N/A]	439

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx = \text{Int}\left(\frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}}, x\right)$$

output `Defer(Int)((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x)`

Mathematica [N/A]

Not integrable

Time = 48.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

input `Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^(5/2),x]`

output `Integrate[(a + b*Sec[c + d*Sqrt[x]])^2/x^(5/2), x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

↓ 4694

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

input `Int[(a + b*Sec[c + d*Sqrt[x]])^2/x^(5/2), x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

input `int((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x)`

output `int((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^{5/2}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="fricas")`

output `integral((b^2*sqrt(x)*sec(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sec(d*sqrt(x) + c) + a^2*sqrt(x))/x^3, x)`

Sympy [N/A]

Not integrable

Time = 6.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

input `integrate((a+b*sec(c+d*x**(1/2)))**2/x**(5/2),x)`

output `Integral((a + b*sec(c + d*sqrt(x)))**2/x**(5/2), x)`

Maxima [F(-1)]

Timed out.

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx = \text{Timed out}$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="maxima")`

output `Timed out`

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(b \sec(d\sqrt{x} + c) + a)^2}{x^{5/2}} dx$$

input `integrate((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="giac")`

output `integrate((b*sec(d*sqrt(x) + c) + a)^2/x^(5/2), x)`

Mupad [N/A]

Not integrable

Time = 15.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2}{x^{5/2}} dx$$

input `int((a + b/cos(c + d*x^(1/2)))^2/x^(5/2),x)`

output `int((a + b/cos(c + d*x^(1/2)))^2/x^(5/2), x)`

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.41

$$\int \frac{(a + b \sec(c + d\sqrt{x}))^2}{x^{5/2}} dx = \frac{2\sqrt{x} \left(\int \frac{\sec(\sqrt{x}d+c)^2 \tan(\sqrt{x}d+c)}{x^2} dx \right) b^2 dx}{3} + \frac{2\sqrt{x} \left(\int \frac{\sec(\sqrt{x}d+c) \tan(\sqrt{x}d+c)}{x^2} dx \right) abd x}{3} - \frac{2 \sec(\sqrt{x}d+c)}{3\sqrt{x}}$$

input

```
int((a+b*sec(c+d*x^(1/2)))^2/x^(5/2),x)
```

output

```
(2*(sqrt(x)*int((sec(sqrt(x)*d + c)**2*tan(sqrt(x)*d + c))/x**2,x)*b**2*d*x + sqrt(x)*int((sec(sqrt(x)*d + c)*tan(sqrt(x)*d + c))/x**2,x)*a*b*d*x - sec(sqrt(x)*d + c)**2*b**2 - 2*sec(sqrt(x)*d + c)*a*b - a**2))/(3*sqrt(x)*x)
```


3.61 $\int \frac{x^{3/2}}{a+b \sec(c+d\sqrt{x})} dx$

Optimal result	440
Mathematica [A] (verified)	441
Rubi [A] (verified)	442
Maple [F]	444
Fricas [F]	444
Sympy [F]	444
Maxima [F(-2)]	445
Giac [F]	445
Mupad [F(-1)]	445
Reduce [F]	446

Optimal result

Integrand size = 22, antiderivative size = 653

$$\begin{aligned} \int \frac{x^{3/2}}{a+b \sec(c+d\sqrt{x})} dx &= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\ &- \frac{2ibx^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{8bx^{3/2} \text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\ &- \frac{8bx^{3/2} \text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\ &+ \frac{24ibx \text{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24ibx \text{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\ &- \frac{48b\sqrt{x} \text{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b\sqrt{x} \text{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\ &- \frac{48ib \text{PolyLog}\left(5, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{48ib \text{PolyLog}\left(5, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \end{aligned}$$

output

```

2/5*x^(5/2)/a+2*I*b*x^2*ln(1+a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/
a/(-a^2+b^2)^(1/2)/d-2*I*b*x^2*ln(1+a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(
1/2))/a/(-a^2+b^2)^(1/2)/d+8*b*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2))))/
(b-(-a^2+b^2)^(1/2))/a/(-a^2+b^2)^(1/2)/d^2-8*b*x^(3/2)*polylog(2,-a*exp(
I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a/(-a^2+b^2)^(1/2)/d^2+24*I*b*x*pol
ylog(3,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a/(-a^2+b^2)^(1/2)/d^
3-24*I*b*x*polylog(3,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a/(-a^2
+b^2)^(1/2)/d^3-48*b*x^(1/2)*polylog(4,-a*exp(I*(c+d*x^(1/2))))/(b-(-a^2+b^
2)^(1/2))/a/(-a^2+b^2)^(1/2)/d^4+48*b*x^(1/2)*polylog(4,-a*exp(I*(c+d*x^(
1/2))))/(b+(-a^2+b^2)^(1/2))/a/(-a^2+b^2)^(1/2)/d^4-48*I*b*polylog(5,-a*ex
p(I*(c+d*x^(1/2))))/(b-(-a^2+b^2)^(1/2))/a/(-a^2+b^2)^(1/2)/d^5+48*I*b*pol
ylog(5,-a*exp(I*(c+d*x^(1/2))))/(b+(-a^2+b^2)^(1/2))/a/(-a^2+b^2)^(1/2)/d^
5

```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} dx = \frac{2\left(\sqrt{-a^2 + b^2} d^5 x^{5/2} + 5ibd^4 x^2 \log\left(1 - \frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) - 5ibd^4 x^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)\right)}{5a\sqrt{-a^2 + b^2} d^5}$$

input

```
Integrate[x^(3/2)/(a + b*Sec[c + d*Sqrt[x]]),x]
```

output

```

(2*(Sqrt[-a^2 + b^2]*d^5*x^(5/2) + (5*I)*b*d^4*x^2*Log[1 - (a*E^(I*(c + d*
Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (5*I)*b*d^4*x^2*Log[1 + (a*E^(I*(c +
d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(b + Sqrt[-a^2 + b^2]) + 20*b*d^3*x^(3/2)*PolyLog[2, (a*E^(
I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - 20*b*d^3*x^(3/2)*PolyLog[2,
-((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] + (60*I)*b*d^2*x*Pol
yLog[3, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] - (60*I)*b*d^2*
x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))] - 120*b*
d*Sqrt[x]*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^2])] +
120*b*d*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^
2]))] - (120*I)*b*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/(-b + Sqrt[-a^2 + b^
2])] + (120*I)*b*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 +
b^2]))])]/(5*a*Sqrt[-a^2 + b^2]*d^5)

```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} dx \\
 & \quad \downarrow 4692 \\
 & 2 \int \frac{x^2}{a + b \sec(c + d\sqrt{x})} d\sqrt{x} \\
 & \quad \downarrow 3042 \\
 & 2 \int \frac{x^2}{a + b \csc(c + d\sqrt{x} + \frac{\pi}{2})} d\sqrt{x} \\
 & \quad \downarrow 4679 \\
 & 2 \int \left(\frac{x^2}{a} - \frac{bx^2}{a(b + a \cos(c + d\sqrt{x}))} \right) d\sqrt{x} \\
 & \quad \downarrow 2009 \\
 & 2 \left(-\frac{24ib \operatorname{PolyLog}\left(5, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} + \frac{24ib \operatorname{PolyLog}\left(5, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{24b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{24b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} \right)
 \end{aligned}$$

input

```
Int[x^(3/2)/(a + b*Sec[c + d*sqrt[x]]),x]
```

output

```

2*(x^(5/2)/(5*a) + (I*b*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (4*b*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) - (4*b*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + ((12*I)*b*x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - ((12*I)*b*x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (24*b*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (24*b*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) - ((24*I)*b*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) + ((24*I)*b*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5)
)

```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 4679

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```

rule 4692

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{a + b \sec(c + d\sqrt{x})} dx$$

input `int(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)`

output `int(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^{\frac{3}{2}}}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(x^(3/2)/(b*sec(d*sqrt(x) + c) + a), x)`

Sympy [F]

$$\int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^{\frac{3}{2}}}{a + b \sec(c + d\sqrt{x})} dx$$

input `integrate(x**(3/2)/(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(x**(3/2)/(a + b*sec(c + d*sqrt(x))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Giac [F]

$$\int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^{3/2}}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(x^(3/2)/(b*sec(d*sqrt(x) + c) + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{x^{3/2}}{a + \frac{b}{\cos(c+d\sqrt{x})}} dx$$

input `int(x^(3/2)/(a + b/cos(c + d*x^(1/2))),x)`

output `int(x^(3/2)/(a + b/cos(c + d*x^(1/2))), x)`

Reduce [F]

$$\int \frac{x^{3/2}}{a + b \sec(c + d\sqrt{x})} dx = \frac{2\sqrt{x}x^2 + 10 \left(\int \frac{\sqrt{x} \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 x}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 - \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) ab + 10 \left(\int \frac{1}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)} dx \right)}{5a + 5b}$$

input

```
int(x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)
```

output

```
(2*(sqrt(x)*x**2 + 5*int((sqrt(x)*tan((sqrt(x)*d + c)/2)**2*x)/(tan((sqrt(x)*d + c)/2)**2*a**2 - tan((sqrt(x)*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*a*b + 5*int((sqrt(x)*tan((sqrt(x)*d + c)/2)**2*x)/(tan((sqrt(x)*d + c)/2)**2*a**2 - tan((sqrt(x)*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*b**2))/(5*(a + b))
```

3.62 $\int \frac{\sqrt{x}}{a+b \sec(c+d\sqrt{x})} dx$

Optimal result	447
Mathematica [A] (verified)	448
Rubi [A] (verified)	449
Maple [F]	451
Fricas [F]	451
Sympy [F]	451
Maxima [F(-2)]	452
Giac [F]	452
Mupad [F(-1)]	452
Reduce [F]	453

Optimal result

Integrand size = 22, antiderivative size = 393

$$\begin{aligned}
 \int \frac{\sqrt{x}}{a+b \sec(c+d\sqrt{x})} dx = & \frac{2x^{3/2}}{3a} + \frac{2ibx \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
 & - \frac{2ibx \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
 & + \frac{4b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 & - \frac{4b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 & + \frac{4ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
 & - \frac{4ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}
 \end{aligned}$$

output

$$\frac{2/3*x^{(3/2)}/a+2*I*b*x*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d-2*I*b*x*\ln(1+a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d+4*b*x^{(1/2)}*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d^2-4*b*x^{(1/2)}*polylog(2,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d^2+4*I*b*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b-(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d^3-4*I*b*polylog(3,-a*\exp(I*(c+d*x^{(1/2)})))/(b+(-a^2+b^2)^{(1/2)})/a/(-a^2+b^2)^{(1/2)}/d^3}{d^3}$$
Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx$$

$$= \frac{2\left(\sqrt{-a^2 + b^2} d^3 x^{3/2} + 3ibd^2 x \log\left(1 - \frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) - 3ibd^2 x \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right) + 6bd\sqrt{x} \text{PolyLog}\left(2, \frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) + 6bd\sqrt{x} \text{PolyLog}\left(2, \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right) + 6bd\sqrt{x} \text{PolyLog}\left(3, \frac{ae^{i(c+d\sqrt{x})}}{-b+\sqrt{-a^2+b^2}}\right) + 6bd\sqrt{x} \text{PolyLog}\left(3, \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)\right)}{3a^2 d^3}$$

input

`Integrate[Sqrt[x]/(a + b*Sec[c + d*Sqrt[x]]),x]`

output

$$\frac{(2*(\text{Sqrt}[-a^2 + b^2]*d^3*x^{(3/2)} + (3*I)*b*d^2*x*\text{Log}[1 - (a*\text{E}^{(I*(c + d*\text{Sqrt}[x]))})/(-b + \text{Sqrt}[-a^2 + b^2])]) - (3*I)*b*d^2*x*\text{Log}[1 + (a*\text{E}^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2])]) + 6*b*d*\text{Sqrt}[x]*\text{PolyLog}[2, (a*\text{E}^{(I*(c + d*\text{Sqrt}[x]))})/(-b + \text{Sqrt}[-a^2 + b^2])]) - 6*b*d*\text{Sqrt}[x]*\text{PolyLog}[2, -((a*\text{E}^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))]) + (6*I)*b*\text{PolyLog}[3, (a*\text{E}^{(I*(c + d*\text{Sqrt}[x]))})/(-b + \text{Sqrt}[-a^2 + b^2])]) - (6*I)*b*\text{PolyLog}[3, -((a*\text{E}^{(I*(c + d*\text{Sqrt}[x]))})/(b + \text{Sqrt}[-a^2 + b^2]))])]/(3*a*\text{Sqrt}[-a^2 + b^2]*d^3)$$

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx \\
 & \quad \downarrow 4692 \\
 & 2 \int \frac{x}{a + b \sec(c + d\sqrt{x})} d\sqrt{x} \\
 & \quad \downarrow 3042 \\
 & 2 \int \frac{x}{a + b \csc(c + d\sqrt{x} + \frac{\pi}{2})} d\sqrt{x} \\
 & \quad \downarrow 4679 \\
 & 2 \int \left(\frac{x}{a} - \frac{bx}{a(b + a \cos(c + d\sqrt{x}))} \right) d\sqrt{x} \\
 & \quad \downarrow 2009 \\
 & 2 \left(\frac{2ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{2ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{2b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{2b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} \right)
 \end{aligned}$$

input `Int[Sqrt[x]/(a + b*Sec[c + d*Sqrt[x]]),x]`

output

$$2*(x^{3/2}/(3*a) + (I*b*x*\text{Log}[1 + (a*E^{I*(c + d*\text{Sqrt}[x])})]/(b - \text{Sqrt}[-a^2 + b^2])])/ (a*\text{Sqrt}[-a^2 + b^2]*d) - (I*b*x*\text{Log}[1 + (a*E^{I*(c + d*\text{Sqrt}[x])})]/(b + \text{Sqrt}[-a^2 + b^2])])/ (a*\text{Sqrt}[-a^2 + b^2]*d) + (2*b*\text{Sqrt}[x]*\text{PolyLog}[2, -((a*E^{I*(c + d*\text{Sqrt}[x])})/(b - \text{Sqrt}[-a^2 + b^2]))])/ (a*\text{Sqrt}[-a^2 + b^2]*d^2) - (2*b*\text{Sqrt}[x]*\text{PolyLog}[2, -((a*E^{I*(c + d*\text{Sqrt}[x])})/(b + \text{Sqrt}[-a^2 + b^2]))])/ (a*\text{Sqrt}[-a^2 + b^2]*d^2) + ((2*I)*b*\text{PolyLog}[3, -((a*E^{I*(c + d*\text{Sqrt}[x])})/(b - \text{Sqrt}[-a^2 + b^2]))])/ (a*\text{Sqrt}[-a^2 + b^2]*d^3) - ((2*I)*b*\text{PolyLog}[3, -((a*E^{I*(c + d*\text{Sqrt}[x])})/(b + \text{Sqrt}[-a^2 + b^2]))])/ (a*\text{Sqrt}[-a^2 + b^2]*d^3)$$

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Ssin[e + f*x]^n)], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx$$

input `int(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x)`

output `int(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x)`

Fricas [F]

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{\sqrt{x}}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(sqrt(x)/(b*sec(d*sqrt(x) + c) + a), x)`

Sympy [F]

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx$$

input `integrate(x**(1/2)/(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(sqrt(x)/(a + b*sec(c + d*sqrt(x))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more de

Giac [F]

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{\sqrt{x}}{b \sec(d\sqrt{x} + c) + a} dx$$

input `integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(sqrt(x)/(b*sec(d*sqrt(x) + c) + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx = \int \frac{\sqrt{x}}{a + \frac{b}{\cos(c+d\sqrt{x})}} dx$$

input `int(x^(1/2)/(a + b/cos(c + d*x^(1/2))),x)`

output `int(x^(1/2)/(a + b/cos(c + d*x^(1/2))), x)`

Reduce [F]

$$\int \frac{\sqrt{x}}{a + b \sec(c + d\sqrt{x})} dx$$

$$= \frac{2\sqrt{x}x + 6 \left(\int \frac{\sqrt{x} \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 - \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right) ab + 6 \left(\int \frac{\sqrt{x} \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2}{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 - \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 - a^2 - 2ab - b^2} dx \right)}{3a + 3b}$$

input `int(x^(1/2)/(a+b*sec(c+d*x^(1/2))),x)`

output `(2*(sqrt(x)*x + 3*int((sqrt(x)*tan((sqrt(x)*d + c)/2)**2)/(tan((sqrt(x)*d + c)/2)**2*a**2 - tan((sqrt(x)*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x) *a*b + 3*int((sqrt(x)*tan((sqrt(x)*d + c)/2)**2)/(tan((sqrt(x)*d + c)/2)**2*a**2 - tan((sqrt(x)*d + c)/2)**2*b**2 - a**2 - 2*a*b - b**2),x)*b**2))/(3*(a + b))`

3.63 $\int \frac{1}{\sqrt{x}(a+b \sec(c+d\sqrt{x}))} dx$

Optimal result	454
Mathematica [A] (verified)	454
Rubi [A] (verified)	455
Maple [A] (verified)	457
Fricas [A] (verification not implemented)	457
Sympy [F]	458
Maxima [F(-2)]	458
Giac [B] (verification not implemented)	459
Mupad [B] (verification not implemented)	459
Reduce [B] (verification not implemented)	460

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{1}{\sqrt{x}(a+b \sec(c+d\sqrt{x}))} dx = \frac{2\sqrt{x}}{a} - \frac{4b \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+bd}}$$

output 2*x^(1/2)/a-4*b*arctanh((a-b)^(1/2)*tan(1/2*c+1/2*d*x^(1/2))/(a+b)^(1/2))/a/(a-b)^(1/2)/(a+b)^(1/2)/d

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{x}(a+b \sec(c+d\sqrt{x}))} dx = \frac{2\left(\frac{c}{d} + \sqrt{x} + \frac{2b \operatorname{arctanh}\left(\frac{(-a+b) \tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}\right)}{a}$$

input Integrate[1/(Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])),x]

output

$$(2*(c/d + \text{Sqrt}[x] + (2*b*\text{ArcTanh}[((-a + b)*\text{Tan}[(c + d*\text{Sqrt}[x])/2])/ \text{Sqrt}[a^2 - b^2]])/(\text{Sqrt}[a^2 - b^2]*d)))/a$$
Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4692, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))} dx$$

$$\downarrow 4692$$

$$2 \int \frac{1}{a + b \sec(c + d\sqrt{x})} d\sqrt{x}$$

$$\downarrow 3042$$

$$2 \int \frac{1}{a + b \csc(c + d\sqrt{x} + \frac{\pi}{2})} d\sqrt{x}$$

$$\downarrow 4270$$

$$2 \left(\frac{\sqrt{x}}{a} - \frac{\int \frac{1}{\frac{a \cos(c + d\sqrt{x})}{b} + 1} d\sqrt{x}}{a} \right)$$

$$\downarrow 3042$$

$$2 \left(\frac{\sqrt{x}}{a} - \frac{\int \frac{1}{\frac{a \sin(c + d\sqrt{x} + \frac{\pi}{2})}{b} + 1} d\sqrt{x}}{a} \right)$$

$$\downarrow 3138$$

$$2 \left(\frac{\sqrt{x}}{a} - \frac{2 \int \frac{1}{\frac{a+b}{b} + (1-\frac{a}{b})x} d \tan(\frac{1}{2}(c + d\sqrt{x}))}{ad} \right)$$

$$\downarrow 221$$

$$2 \left(\frac{\sqrt{x}}{a} - \frac{2b \operatorname{arctanh} \left(\frac{\sqrt{a-b} \tan \left(\frac{1}{2} (c + d\sqrt{x}) \right)}{\sqrt{a+b}} \right)}{ad\sqrt{a-b}\sqrt{a+b}} \right)$$

input `Int[1/(Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])),x]`

output `2*(Sqrt[x]/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*Sqrt[x])/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))`

Defintions of rubi rules used

rule 221 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3138 `Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 4270 `Int[(csc[(c_) + (d_)*(x_)])*(b_) + (a_)^(-1), x_Symbol] := Simp[x/a, x] - Simp[1/a Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 4692 `Int[(x_)^(m_)*((a_) + (b_)*Sec[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{4b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right) + \frac{4 \operatorname{arctan}\left(\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)}{a}}{a\sqrt{(a-b)(a+b)}} \cdot d$	70
default	$\frac{4b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right) + \frac{4 \operatorname{arctan}\left(\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)}{a}}{a\sqrt{(a-b)(a+b)}} \cdot d$	70

input `int(1/x^(1/2)/(a+b*sec(c+d*x^(1/2))),x,method=_RETURNVERBOSE)`

output `2/d*(-2*b/a/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*c+1/2*d*x^(1/2))/((a-b)*(a+b))^(1/2))+2/a*arctan(tan(1/2*c+1/2*d*x^(1/2))))`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.03

$$\int \frac{1}{\sqrt{x}(a + b \sec(c + d\sqrt{x}))} dx$$

$$= \left[\frac{2(a^2 - b^2)d\sqrt{x} + \sqrt{a^2 - b^2}b \log\left(\frac{2ab \cos(d\sqrt{x}+c) - (a^2 - 2b^2) \cos(d\sqrt{x}+c)^2 + 2a^2 - b^2 - 2(\sqrt{a^2 - b^2}b \cos(d\sqrt{x}+c) + \sqrt{a^2 - b^2}a)}{a^2 \cos(d\sqrt{x}+c)^2 + 2ab \cos(d\sqrt{x}+c) + b^2}\right)}{(a^3 - ab^2)d} \right]$$

input `integrate(1/x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output

```
[(2*(a^2 - b^2)*d*sqrt(x) + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*sqrt(x) + c) - (a^2 - 2*b^2)*cos(d*sqrt(x) + c)^2 + 2*a^2 - b^2 - 2*(sqrt(a^2 - b^2)*b*cos(d*sqrt(x) + c) + sqrt(a^2 - b^2)*a)*sin(d*sqrt(x) + c))/(a^2*cos(d*sqrt(x) + c)^2 + 2*a*b*cos(d*sqrt(x) + c) + b^2)))/((a^3 - a*b^2)*d), 2*((a^2 - b^2)*d*sqrt(x) - sqrt(-a^2 + b^2)*b*arctan(-(sqrt(-a^2 + b^2)*b*cos(d*sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*sin(d*sqrt(x) + c))))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))} dx$$

input

```
integrate(1/x**(1/2)/(a+b*sec(c+d*x**(1/2))),x)
```

output

```
Integral(1/(sqrt(x)*(a + b*sec(c + d*sqrt(x)))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))} dx = \text{Exception raised: ValueError}$$

input

```
integrate(1/x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(55) = 110$.

Time = 0.43 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.09

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))} dx$$

$$= \frac{2(\sqrt{-a^2 + b^2}(a - 2b)d|-a + b| - \sqrt{-a^2 + b^2}|a||-a + b||d|) \left(\pi \left[\frac{d\sqrt{x} + c}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\tan(\frac{1}{2} d\sqrt{x} + \frac{1}{2} c)}{\sqrt{-\frac{bd + \sqrt{b^2 d^2 + (ad + bd)(ad - bd)}}{ad - bd}}} \right)}{(a^2 - 2ab + b^2)a^2 d^2 + (a^2 b - 2ab^2 + b^3)d|a||d|} \right)}{2(ad - 2bd + |a||d|) \left(\pi \left[\frac{d\sqrt{x} + c}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\tan(\frac{1}{2} d\sqrt{x} + \frac{1}{2} c)}{\sqrt{-\frac{bd - \sqrt{b^2 d^2 + (ad + bd)(ad - bd)}}{ad - bd}}} \right)} \right)} + \frac{2(ad - 2bd + |a||d|) \left(\pi \left[\frac{d\sqrt{x} + c}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\tan(\frac{1}{2} d\sqrt{x} + \frac{1}{2} c)}{\sqrt{-\frac{bd - \sqrt{b^2 d^2 + (ad + bd)(ad - bd)}}{ad - bd}}} \right)} \right)}{a^2 d^2 - bd|a||d|}$$

input `integrate(1/x^(1/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output

```
2*(sqrt(-a^2 + b^2)*(a - 2*b)*d*abs(-a + b) - sqrt(-a^2 + b^2)*abs(a)*abs(-a + b)*abs(d))*(pi*floor(1/2*(d*sqrt(x) + c)/pi + 1/2) + arctan(tan(1/2*d*sqrt(x) + 1/2*c)/sqrt(-(b*d + sqrt(b^2*d^2 + (a*d + b*d)*(a*d - b*d)))/(a*d - b*d))))/((a^2 - 2*a*b + b^2)*a^2*d^2 + (a^2*b - 2*a*b^2 + b^3)*d*abs(a)*abs(d)) + 2*(a*d - 2*b*d + abs(a)*abs(d))*(pi*floor(1/2*(d*sqrt(x) + c)/pi + 1/2) + arctan(tan(1/2*d*sqrt(x) + 1/2*c)/sqrt(-(b*d - sqrt(b^2*d^2 + (a*d + b*d)*(a*d - b*d)))/(a*d - b*d))))/((a^2*d^2 - b*d*abs(a)*abs(d))
```

Mupad [B] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))} dx = \frac{2\sqrt{x}}{a} + \frac{2b \ln \left(2be^{d\sqrt{x}} e^{c} - \frac{b(a + be^{d\sqrt{x}} e^c)^{2i}}{\sqrt{a+b}\sqrt{a-b}} \right)}{ad\sqrt{a+b}\sqrt{a-b}} - \frac{2b \ln \left(2be^{d\sqrt{x}} e^c + \frac{b(a + be^{d\sqrt{x}} e^c)^{2i}}{\sqrt{a+b}\sqrt{a-b}} \right)}{ad\sqrt{a+b}\sqrt{a-b}}$$

input `int(1/(x^(1/2)*(a + b/cos(c + d*x^(1/2))))),x)`

output `(2*x^(1/2))/a + (2*b*log(2*b*exp(d*x^(1/2)*1i)*exp(c*1i) - (b*(a + b*exp(d*x^(1/2)*1i)*exp(c*1i))*2i)/((a + b)^(1/2)*(a - b)^(1/2)))/(a*d*(a + b)^(1/2)*(a - b)^(1/2)) - (2*b*log(2*b*exp(d*x^(1/2)*1i)*exp(c*1i) + (b*(a + b*exp(d*x^(1/2)*1i)*exp(c*1i))*2i)/((a + b)^(1/2)*(a - b)^(1/2)))/(a*d*(a + b)^(1/2)*(a - b)^(1/2))`

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{x}(a + b \sec(c + d\sqrt{x}))} dx$$

$$= \frac{-4\sqrt{-a^2 + b^2} \operatorname{atan}\left(\frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)a - \tan\left(\frac{\sqrt{x}d + c}{2}\right)b}{\sqrt{-a^2 + b^2}}\right) b + 2\sqrt{x} a^2 d - 2\sqrt{x} b^2 d}{ad(a^2 - b^2)}$$

input `int(1/x^(1/2)/(a+b*sec(c+d*x^(1/2))),x)`

output `(2*(- 2*sqrt(- a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt(- a**2 + b**2))*b + sqrt(x)*a**2*d - sqrt(x)*b**2*d)/(a*d*(a**2 - b**2))`

$$3.64 \quad \int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Optimal result	461
Mathematica [N/A]	461
Rubi [N/A]	462
Maple [N/A]	462
Fricas [N/A]	463
Sympy [N/A]	463
Maxima [N/A]	464
Giac [N/A]	464
Mupad [N/A]	465
Reduce [N/A]	465

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx = \text{Int}\left(\frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))}, x\right)$$

output `Defer(Int)(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)`

Mathematica [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx = \int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))} dx$$

input `Integrate[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])),x]`

output `Integrate[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])), x]`

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))} dx$$

↓ 4694

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))} dx$$

input `Int[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))} dx$$

input `int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)`

output `int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x)`

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a) x^{\frac{3}{2}}} dx$$

input `integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(sqrt(x)/(b*x^2*sec(d*sqrt(x) + c) + a*x^2), x)`

Sympy [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))} dx$$

input `integrate(1/x**(3/2)/(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(1/(x**(3/2)*(a + b*sec(c + d*sqrt(x))))), x)`

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 243, normalized size of antiderivative = 11.05

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a) x^{3/2}} dx$$

input `integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output

```
-2*(a*b*sqrt(x)*integrate((a*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + 2
*b*cos(d*sqrt(x) + c)^2 + a*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 2*
b*sin(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) + c)))/((a^3*cos(2*d*sqrt(x) + 2*c)
)^2 + 4*a*b^2*cos(d*sqrt(x) + c)^2 + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a^2*
b*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 4*a*b^2*sin(d*sqrt(x) + c)^2
+ 4*a^2*b*cos(d*sqrt(x) + c) + a^3 + 2*(2*a^2*b*cos(d*sqrt(x) + c) + a^3)
*cos(2*d*sqrt(x) + 2*c))*x^(3/2)), x) + 1)/(a*sqrt(x))
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a) x^{3/2}} dx$$

input `integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output

```
integrate(1/((b*sec(d*sqrt(x) + c) + a)*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 16.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{x^{3/2} \left(a + \frac{b}{\cos(c+d\sqrt{x})} \right)} dx$$

input `int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2)))) , x)`

output `int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2)))) , x)`

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 8.36

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))} dx = \frac{2\sqrt{x} \left(\int \frac{\tan\left(\frac{\sqrt{x}d + \frac{c}{2}}{2}\right)^2}{\sqrt{x} \tan\left(\frac{\sqrt{x}d + \frac{c}{2}}{2}\right)^2 a^2 x - \sqrt{x} \tan\left(\frac{\sqrt{x}d + \frac{c}{2}}{2}\right)^2 b^2 x - \sqrt{x} a^2 x - 2\sqrt{x} a b x - \sqrt{x} b^2 x} dx \right)}{1}$$

input `int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2))), x)`

output `(2*(sqrt(x)*int(tan((sqrt(x)*d + c)/2)**2/(sqrt(x)*tan((sqrt(x)*d + c)/2)*
*2*a**2*x - sqrt(x)*tan((sqrt(x)*d + c)/2)**2*b**2*x - sqrt(x)*a**2*x - 2*
sqrt(x)*a*b*x - sqrt(x)*b**2*x), x)*a*b + sqrt(x)*int(tan((sqrt(x)*d + c)/2
)**2/(sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**2*x - sqrt(x)*tan((sqrt(x)*d +
c)/2)**2*b**2*x - sqrt(x)*a**2*x - 2*sqrt(x)*a*b*x - sqrt(x)*b**2*x), x)*b*
2 - 1))/(sqrt(x)(a + b))`

3.65
$$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx$$

Optimal result	466
Mathematica [N/A]	466
Rubi [N/A]	467
Maple [N/A]	467
Fricas [N/A]	468
Sympy [N/A]	468
Maxima [N/A]	469
Giac [N/A]	469
Mupad [N/A]	470
Reduce [N/A]	470

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx = \text{Int}\left(\frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))}, x\right)$$

output `Defer(Int)(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))), x)`

Mathematica [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx = \int \frac{1}{x^{5/2}(a+b \sec(c+d\sqrt{x}))} dx$$

input `Integrate[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])), x]`

output `Integrate[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])), x]`

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))} dx$$

↓ 4694

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))} dx$$

input `Int[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))} dx$$

input `int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x)`

output `int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a) x^{\frac{5}{2}}} dx$$

input `integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(sqrt(x)/(b*x^3*sec(d*sqrt(x) + c) + a*x^3), x)`

Sympy [N/A]

Not integrable

Time = 7.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{x^{\frac{5}{2}} (a + b \sec(c + d\sqrt{x}))} dx$$

input `integrate(1/x**(5/2)/(a+b*sec(c+d*x**(1/2))),x)`

output `Integral(1/(x**(5/2)*(a + b*sec(c + d*sqrt(x))))), x)`

Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 244, normalized size of antiderivative = 11.09

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a) x^{5/2}} dx$$

input `integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="maxima")`

output `-2/3*(3*a*b*x^(3/2)*integrate((a*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) + 2*b*cos(d*sqrt(x) + c)^2 + a*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 2*b*sin(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) + c)))/((a^3*cos(2*d*sqrt(x) + 2*c)^2 + 4*a*b^2*cos(d*sqrt(x) + c)^2 + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a^2*b*sin(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 4*a*b^2*sin(d*sqrt(x) + c)^2 + 4*a^2*b*cos(d*sqrt(x) + c) + a^3 + 2*(2*a^2*b*cos(d*sqrt(x) + c) + a^3)*cos(2*d*sqrt(x) + 2*c))*x^(5/2)), x) + 1)/(a*x^(3/2))`

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a) x^{5/2}} dx$$

input `integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(1/((b*sec(d*sqrt(x) + c) + a)*x^(5/2)), x)`

Mupad [N/A]

Not integrable

Time = 15.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))} dx = \int \frac{1}{x^{5/2} \left(a + \frac{b}{\cos(c+d\sqrt{x})} \right)} dx$$

input `int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2)))) , x)`

output `int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2)))) , x)`

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 9.59

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))} dx = \frac{2\sqrt{x} \left(\int \frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)^2}{\sqrt{x} \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 a^2 x^2 - \sqrt{x} \tan\left(\frac{\sqrt{x}d + c}{2}\right)^2 b^2 x^2 - \sqrt{x} a^2 x^2 - 2\sqrt{x} ab x^2 - \sqrt{x} b^2 x^2} \right)}{1}$$

input `int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2))), x)`

output `(2*(3*sqrt(x)*int(tan((sqrt(x)*d + c)/2)**2/(sqrt(x)*tan((sqrt(x)*d + c)/2))**2*a**2*x**2 - sqrt(x)*tan((sqrt(x)*d + c)/2)**2*b**2*x**2 - sqrt(x)*a**2*x**2 - 2*sqrt(x)*a*b*x**2 - sqrt(x)*b**2*x**2), x)*a*b*x + 3*sqrt(x)*int(tan((sqrt(x)*d + c)/2)**2/(sqrt(x)*tan((sqrt(x)*d + c)/2))**2*a**2*x**2 - sqrt(x)*tan((sqrt(x)*d + c)/2)**2*b**2*x**2 - sqrt(x)*a**2*x**2 - 2*sqrt(x)*a*b*x**2 - sqrt(x)*b**2*x**2), x)*b**2*x - 1)/(3*sqrt(x)*x*(a + b))`

$$3.66 \quad \int \frac{x^{3/2}}{(a+b \sec(c+d\sqrt{x}))^2} dx$$

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Rubi [A] (verified)	473
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Maxima [F(-2)]	476
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Reduce [F]	477

Optimal result

Integrand size = 22, antiderivative size = 1925

$$\int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

output

```

-2*I*b^2*x^2/a^2/(a^2-b^2)/d+48*I*b^3*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b
-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5+48*I*b^2*polylog(4,-a*exp(I*(
c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^5+96*I*b*polylog(5,-a
*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^5-96*I*
b*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(
1/2)/d^5+8*b^2*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/
a^2/(a^2-b^2)/d^2+8*b^2*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)
^(1/2)))/a^2/(a^2-b^2)/d^2+8*b^3*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2))
)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-8*b^3*x^(3/2)*polylog(2,-a
*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-16*b*
x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+
b^2)^(1/2)/d^2+16*b*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2
)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2+48*b^2*x^(1/2)*polylog(3,-a*exp(I*(c+d*
x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+48*b^2*x^(1/2)*polylog(
3,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4-48*b^3*
x^(1/2)*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+
b^2)^(3/2)/d^4+48*b^3*x^(1/2)*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b
^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4+96*b*x^(1/2)*polylog(4,-a*exp(I*(c+d*
x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^4-96*b*x^(1/2)*poly
log(4,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)...

```

Mathematica [A] (verified)

Time = 11.06 (sec) , antiderivative size = 2161, normalized size of antiderivative = 1.12

$$\int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Result too large to show}$$

input

```
Integrate[x^(3/2)/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

```
(2*(b + a*cos[c + d*sqrt(x)])*sec[c + d*sqrt(x)]^2*(x^(5/2)*(b + a*cos[c +
d*sqrt(x)]) + (5*b*(b + a*cos[c + d*sqrt(x)])*((-2*I)*b*d^4*E^((2*I)*c)*
x^2)/(1 + E^((2*I)*c)) + (4*b*d^3*sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x^(3/2)*L
og[1 + (a*E^(I*(2*c + d*sqrt(x))))]/(b*E^(I*c) - sqrt[(-a^2 + b^2)*E^((2*I)
*c)]]) + (2*I)*a^2*d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*sqrt(x))))]/(b*
E^(I*c) - sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - I*b^2*d^4*E^(I*c)*x^2*Log[1 +
(a*E^(I*(2*c + d*sqrt(x))))]/(b*E^(I*c) - sqrt[(-a^2 + b^2)*E^((2*I)*c)])]
+ 4*b*d^3*sqrt[(-a^2 + b^2)*E^((2*I)*c)]*x^(3/2)*Log[1 + (a*E^(I*(2*c + d
*sqrt(x))))]/(b*E^(I*c) + sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - (2*I)*a^2*d^4*
E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*sqrt(x))))]/(b*E^(I*c) + sqrt[(-a^2 +
b^2)*E^((2*I)*c)])] + I*b^2*d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*sqrt[
x])))]/(b*E^(I*c) + sqrt[(-a^2 + b^2)*E^((2*I)*c)])] - 4*d^2*((3*I)*b*sqrt[
(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*sqrt[x] + b^2*d*E^(I*c)*sqrt[x
])*x*PolyLog[2, -((a*E^(I*(2*c + d*sqrt(x))))/(b*E^(I*c) - sqrt[(-a^2 + b^
2)*E^((2*I)*c)]))] + 4*d^2*((-3*I)*b*sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^
2*d*E^(I*c)*sqrt[x] + b^2*d*E^(I*c)*sqrt[x])*x*PolyLog[2, -((a*E^(I*(2*c +
d*sqrt(x))))/(b*E^(I*c) + sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + 24*b*d*sqrt
[(-a^2 + b^2)*E^((2*I)*c)]*sqrt[x]*PolyLog[3, -((a*E^(I*(2*c + d*sqrt(x)))
)/(b*E^(I*c) - sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] + (24*I)*a^2*d^2*E^(I*c)*
x*PolyLog[3, -((a*E^(I*(2*c + d*sqrt(x))))/(b*E^(I*c) - sqrt[(-a^2 + b^...
```

Rubi [A] (verified)

Time = 3.32 (sec) , antiderivative size = 1926, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

$$\downarrow 4692$$

$$2 \int \frac{x^2}{(a + b \sec(c + d\sqrt{x}))^2} d\sqrt{x}$$

$$\downarrow 3042$$

$$\begin{aligned}
 & 2 \int \frac{x^2}{(a + b \csc(c + d\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x} \\
 & \quad \downarrow \text{4679} \\
 & 2 \int \left(-\frac{2bx^2}{a^2 (b + a \cos(c + d\sqrt{x}))} + \frac{x^2}{a^2} + \frac{b^2 x^2}{a^2 (b + a \cos(c + d\sqrt{x}))^2} \right) d\sqrt{x} \\
 & \quad \downarrow \text{2009} \\
 & 2 \left(-\frac{ix^2 \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{ix^2 \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{4x^{3/2} \text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} + \frac{4x^{3/2} \text{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} \right)
 \end{aligned}$$

input `Int[x^(3/2)/(a + b*Sec[c + d*Sqrt[x]])^2,x]`

output

```

2*(((-I)*b^2*x^2)/(a^2*(a^2 - b^2)*d) + x^(5/2)/(5*a^2) + (4*b^2*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) + (4*b^2*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + (I*b^3*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((12*I)*b^2*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - ((12*I)*b^2*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (4*b^3*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (8*b*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (4*b^3*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (8*b*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (24*b^2*Sqrt[x]*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (24*b^2*Sqrt[x]*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4)

```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `int(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^{\frac{3}{2}}}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(x^(3/2)/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2), x)`

Sympy [F]

$$\int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^{\frac{3}{2}}}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `integrate(x**(3/2)/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(x**(3/2)/(a + b*sec(c + d*sqrt(x)))**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f or more de`

Giac [F]

$$\int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^{3/2}}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(x^(3/2)/(b*sec(d*sqrt(x) + c) + a)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{x^{3/2}}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

input `int(x^(3/2)/(a + b/cos(c + d*x^(1/2)))^2,x)`

output `int(x^(3/2)/(a + b/cos(c + d*x^(1/2)))^2, x)`

Reduce [F]

$$\int \frac{x^{3/2}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{too large to display}$$

input `int(x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(2*( - 960*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**2*a**4*b + 2880*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**2*a**3*b**2 - 2880*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**2*a**2*b**3 + 960*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*tan((sqrt(x)*d + c)/2)**2*a*b**4 + 960*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*a**4*b - 960*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*a**3*b**2 - 960*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*a**2*b**3 + 960*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*a*b**4 - 960*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*tan((sqrt(x)*d + c)/2)**2*a**3*b**2 + 2880*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*tan((sqrt(x)*d + c)/2)**2*a**...
```

3.67 $\int \frac{\sqrt{x}}{(a+b \sec(c+d\sqrt{x}))^2} dx$

Optimal result	479
Mathematica [A] (verified)	480
Rubi [A] (verified)	481
Maple [F]	483
Fricas [F]	484
Sympy [F]	484
Maxima [F(-2)]	484
Giac [F]	485
Mupad [F(-1)]	485
Reduce [F]	485

Optimal result

Integrand size = 22, antiderivative size = 1125

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

output

```

-8*I*b*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b
^2)^(1/2)/d^3+2/3*x^(3/2)/a^2+4*b^2*x^(1/2)*ln(1+a*exp(I*(c+d*x^(1/2)))/(b
-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+4*b^2*x^(1/2)*ln(1+a*exp(I*(c+d*x^(
1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+4*I*b*x*ln(1+a*exp(I*(c+d*
x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d-4*I*b^2*polylog(2,-
a*exp(I*(c+d*x^(1/2)))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3+4*I*b^3*po
lylog(3,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)
/d^3-2*I*b^3*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(3/2)/d-4*I*b*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^
2/(-a^2+b^2)^(1/2)/d-4*I*b^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b-I*(a^2-b
^2)^(1/2)))/a^2/(a^2-b^2)/d^3-4*b^3*x^(1/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)
)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+8*b*x^(1/2)*polylog(2,-
a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2+4*b^
3*x^(1/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^
2+b^2)^(3/2)/d^2-8*b*x^(1/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^
2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^2-4*I*b^3*polylog(3,-a*exp(I*(c+d*x^(1/2)
)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-2*I*b^2*x/a^2/(a^2-b^2)
/d+8*I*b*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(1/2)/d^3+2*I*b^3*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2))
)/a^2/(-a^2+b^2)^(3/2)/d+2*b^2*x*sin(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*co...

```

Mathematica [A] (verified)

Time = 6.03 (sec) , antiderivative size = 1210, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

input

```
Integrate[Sqrt[x]/(a + b*Sec[c + d*Sqrt[x]])^2,x]
```

output

```
(2*(b + a*cos[c + d*sqrt(x)])*sec[c + d*sqrt(x)]^2*(x^(3/2)*(b + a*cos[c +
d*sqrt(x)]) + (3*b*(b + a*cos[c + d*sqrt(x)])*((-2*I)*b*d^2*E^((2*I)*c)*
x)/(1 + E^((2*I)*c)) + (2*b*d*sqrt[(-a^2 + b^2)*E^((2*I)*c)]*sqrt[x]*Log[1
+ (a*E^(I*(2*c + d*sqrt(x)))]/(b*E^(I*c) - sqrt[(-a^2 + b^2)*E^((2*I)*c)]
)] + (2*I)*a^2*d^2*E^(I*c)*x*Log[1 + (a*E^(I*(2*c + d*sqrt(x)))]/(b*E^(I*c)
) - sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) - I*b^2*d^2*E^(I*c)*x*Log[1 + (a*E^(I
*(2*c + d*sqrt(x)))]/(b*E^(I*c) - sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) + 2*b*d
*sqrt[(-a^2 + b^2)*E^((2*I)*c)]*sqrt[x]*Log[1 + (a*E^(I*(2*c + d*sqrt(x)))]
)/(b*E^(I*c) + sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) - (2*I)*a^2*d^2*E^(I*c)*x*
Log[1 + (a*E^(I*(2*c + d*sqrt(x)))]/(b*E^(I*c) + sqrt[(-a^2 + b^2)*E^((2*I
)*c)]]) + I*b^2*d^2*E^(I*c)*x*Log[1 + (a*E^(I*(2*c + d*sqrt(x)))]/(b*E^(I*
c) + sqrt[(-a^2 + b^2)*E^((2*I)*c)]]) + 2*((-I)*b*sqrt[(-a^2 + b^2)*E^((2*
I)*c)] + 2*a^2*d*E^(I*c)*sqrt[x] - b^2*d*E^(I*c)*sqrt[x])*PolyLog[2, -((a*
E^(I*(2*c + d*sqrt(x)))]/(b*E^(I*c) - sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] +
2*((-I)*b*sqrt[(-a^2 + b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*sqrt[x] + b^2*d
*E^(I*c)*sqrt[x])*PolyLog[2, -((a*E^(I*(2*c + d*sqrt(x)))]/(b*E^(I*c) + sq
rt[(-a^2 + b^2)*E^((2*I)*c)]))] + (4*I)*a^2*E^(I*c)*PolyLog[3, -((a*E^(I*(
2*c + d*sqrt(x)))]/(b*E^(I*c) - sqrt[(-a^2 + b^2)*E^((2*I)*c)]))] - (2*I)*
b^2*E^(I*c)*PolyLog[3, -((a*E^(I*(2*c + d*sqrt(x)))]/(b*E^(I*c) - sqrt[(-a
^2 + b^2)*E^((2*I)*c)]))] - (4*I)*a^2*E^(I*c)*PolyLog[3, -((a*E^(I*(2*c...
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 1126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

$$\downarrow 4692$$

$$2 \int \frac{x}{(a + b \sec(c + d\sqrt{x}))^2} d\sqrt{x}$$

$$\downarrow 3042$$

$$\begin{aligned}
& 2 \int \frac{x}{\left(a + b \csc\left(c + d\sqrt{x} + \frac{\pi}{2}\right)\right)^2} d\sqrt{x} \\
& \quad \downarrow 4679 \\
& 2 \int \left(\frac{xb^2}{a^2 (b + a \cos(c + d\sqrt{x}))^2} - \frac{2xb}{a^2 (b + a \cos(c + d\sqrt{x}))} + \frac{x}{a^2} \right) d\sqrt{x} \\
& \quad \downarrow 2009 \\
& 2 \left(-\frac{ix \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{ix \log\left(\frac{e^{i(c+d\sqrt{x})}a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{2\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} + \frac{2\sqrt{x} \operatorname{PolyLog}\left(2, \frac{ae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} \right)
\end{aligned}$$

input `Int[Sqrt[x]/(a + b*Sec[c + d*Sqrt[x]])^2,x]`

output

```

2*((( -I)*b^2*x)/(a^2*(a^2 - b^2)*d) + x^(3/2)/(3*a^2) + (2*b^2*Sqrt[x]*Log
[1 + (a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*
d^2) + (2*b^2*Sqrt[x]*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 -
b^2])])/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x*Log[1 + (a*E^(I*(c + d*Sqrt[x]))
)/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x*Log[1 +
(a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d
) + (I*b^3*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a
^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(b
+ Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((2*I)*b^2*PolyLog[2, -(
(a*E^(I*(c + d*Sqrt[x])))/(b - I*Sqrt[a^2 - b^2])])]/(a^2*(a^2 - b^2)*d^3)
- ((2*I)*b^2*PolyLog[2, -(a*E^(I*(c + d*Sqrt[x])))/(b + I*Sqrt[a^2 - b^2
])])]/(a^2*(a^2 - b^2)*d^3) - (2*b^3*Sqrt[x]*PolyLog[2, -(a*E^(I*(c + d*S
qrt[x])))/(b - Sqrt[-a^2 + b^2])])]/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (4*b*Sq
rt[x]*PolyLog[2, -(a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])]/(a^
2*Sqrt[-a^2 + b^2]*d^2) + (2*b^3*Sqrt[x]*PolyLog[2, -(a*E^(I*(c + d*Sqrt[
x])))/(b + Sqrt[-a^2 + b^2])])]/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (4*b*Sqrt[x
]*PolyLog[2, -(a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])]/(a^2*Sq
rt[-a^2 + b^2]*d^2) - ((2*I)*b^3*PolyLog[3, -(a*E^(I*(c + d*Sqrt[x])))/(b
- Sqrt[-a^2 + b^2])])]/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((4*I)*b*PolyLog[3,
-(a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])]/(a^2*Sqrt[-a^2 + ...

```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sinn[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

Maple [F]

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `int(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{\sqrt{x}}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(sqrt(x)/(b^2*sec(d*sqrt(x) + c)^2 + 2*a*b*sec(d*sqrt(x) + c) + a^2), x)`

Sympy [F]

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx$$

input `integrate(x**(1/2)/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(sqrt(x)/(a + b*sec(c + d*sqrt(x)))**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f or more de`

Giac [F]

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{\sqrt{x}}{(b \sec(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(sqrt(x)/(b*sec(d*sqrt(x) + c) + a)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{\sqrt{x}}{\left(a + \frac{b}{\cos(c+d\sqrt{x})}\right)^2} dx$$

input `int(x^(1/2)/(a + b/cos(c + d*x^(1/2)))^2,x)`

output `int(x^(1/2)/(a + b/cos(c + d*x^(1/2)))^2, x)`

Reduce [F]

$$\int \frac{\sqrt{x}}{(a + b \sec(c + d\sqrt{x}))^2} dx = \text{too large to display}$$

input `int(x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(2*( - 24*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*a**2*b + 24*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*cos(sqrt(x)*d + c)*a*b**2 - 24*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*a*b**2 + 24*sqrt( - a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt( - a**2 + b**2))*b**3 + 2*sqrt(x)*cos(sqrt(x)*d + c)*a**4*d**3*x + sqrt(x)*cos(sqrt(x)*d + c)*a**3*b*d**3*x - sqrt(x)*cos(sqrt(x)*d + c)*a**2*b**2*d**3*x - 24*cos(sqrt(x)*d + c)*int(sqrt(x)/(2*tan((sqrt(x)*d + c)/2)**4*a**5 - 7*tan((sqrt(x)*d + c)/2)**4*a**4*b + 8*tan((sqrt(x)*d + c)/2)**4*a**3*b**2 - 2*tan((sqrt(x)*d + c)/2)**4*a**2*b**3 - 2*tan((sqrt(x)*d + c)/2)**4*a*b**4 + tan((sqrt(x)*d + c)/2)**4*b**5 - 4*tan((sqrt(x)*d + c)/2)**2*a**5 + 6*tan((sqrt(x)*d + c)/2)**2*a**4*b + 4*tan((sqrt(x)*d + c)/2)**2*a**3*b**2 - 8*tan((sqrt(x)*d + c)/2)**2*a**2*b**3 + 2*tan((sqrt(x)*d + c)/2)**2*b**5 + 2*a**5 + a**4*b - 4*a**3*b**2 - 2*a**2*b**3 + 2*a*b**4 + b**5),x)*a**8*b*d**3 + 12*cos(sqrt(x)*d + c)*int(sqrt(x)/(2*tan((sqrt(x)*d + c)/2)**4*a**5 - 7*tan((sqrt(x)*d + c)/2)**4*a**4*b + 8*tan((sqrt(x)*d + c)/2)**4*a**3*b**2 - 2*tan((sqrt(x)*d + c)/2)**4*a**2*b**3 - 2*tan((sqrt(x)*d + c)/2)**4*a*b**4 + tan((sqrt(x)*d + c)/2)**4*b**5 - 4*tan((sqrt(x)*d + c)/2)**2*a**5 + 6*tan((sqrt(x)*d + c)/2)**2*a...
```

3.68 $\int \frac{1}{\sqrt{x}(a+b \sec(c+d\sqrt{x}))^2} dx$

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Mathematica [A] (verified)	487
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Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{1}{\sqrt{x}(a+b \sec(c+d\sqrt{x}))^2} dx = \frac{2\sqrt{x}}{a^2} - \frac{4b(2a^2 - b^2) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{2b^2 \tan(c+d\sqrt{x})}{a(a^2 - b^2)d(a+b \sec(c+d\sqrt{x}))}$$

output

```
2*x^(1/2)/a^2-4*b*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*c+1/2*d*x^(1/2))
/(a+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+2*b^2*tan(c+d*x^(1/2))/a/(a^2-
b^2)/d/(a+b*sec(c+d*x^(1/2)))
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{x}(a+b \sec(c+d\sqrt{x}))^2} dx = \frac{2b(-2a^2+b^2) \operatorname{arctanh}\left(\frac{(-a+b) \tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2-b^2}}\right) + \frac{a(a^2-b^2)(c+d\sqrt{x}) \cos(c+d\sqrt{x}) + b((a^2-b^2)(c+d\sqrt{x}) + ab \sin(c+d\sqrt{x}))}{b+a \cos(c+d\sqrt{x})}}{a^2(a-b)(a+b)d}$$

input `Integrate[1/(Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output $(2*((-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*Sqrt[x])/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*Sqrt[x])*Cos[c + d*Sqrt[x]] + b*((a^2 - b^2)*(c + d*Sqrt[x]) + a*b*Sin[c + d*Sqrt[x]]))/(b + a*Cos[c + d*Sqrt[x]])))/(a^2*(a - b)*(a + b)*d)$

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4692, 3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx \\
 & \quad \downarrow 4692 \\
 & 2 \int \frac{1}{(a + b \sec(c + d\sqrt{x}))^2} d\sqrt{x} \\
 & \quad \downarrow 3042 \\
 & 2 \int \frac{1}{(a + b \csc(c + d\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x} \\
 & \quad \downarrow 4272 \\
 & 2 \left(\frac{b^2 \tan(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \sec(c + d\sqrt{x}))} - \frac{\int -\frac{a^2 - b \sec(c + d\sqrt{x})a - b^2}{a + b \sec(c + d\sqrt{x})} d\sqrt{x}}{a(a^2 - b^2)} \right) \\
 & \quad \downarrow 25 \\
 & 2 \left(\frac{\int \frac{a^2 - b \sec(c + d\sqrt{x})a - b^2}{a + b \sec(c + d\sqrt{x})} d\sqrt{x}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \sec(c + d\sqrt{x}))} \right) \\
 & \quad \downarrow 3042
 \end{aligned}$$

$$\begin{aligned}
& 2 \left(\frac{\int \frac{a^2 - b \csc(c + d\sqrt{x} + \frac{\pi}{2}) a - b^2}{a + b \csc(c + d\sqrt{x} + \frac{\pi}{2})} d\sqrt{x}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \sec(c + d\sqrt{x}))} \right) \\
& \quad \downarrow \text{4407} \\
& 2 \left(\frac{\frac{\sqrt{x}(a^2 - b^2)}{a} - \frac{b(2a^2 - b^2) \int \frac{\sec(c + d\sqrt{x})}{a + b \sec(c + d\sqrt{x})} d\sqrt{x}}{a(a^2 - b^2)}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \sec(c + d\sqrt{x}))} \right) \\
& \quad \downarrow \text{3042} \\
& 2 \left(\frac{\frac{\sqrt{x}(a^2 - b^2)}{a} - \frac{b(2a^2 - b^2) \int \frac{\csc(c + d\sqrt{x} + \frac{\pi}{2})}{a + b \csc(c + d\sqrt{x} + \frac{\pi}{2})} d\sqrt{x}}{a(a^2 - b^2)}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \sec(c + d\sqrt{x}))} \right) \\
& \quad \downarrow \text{4318} \\
& 2 \left(\frac{\frac{\sqrt{x}(a^2 - b^2)}{a} - \frac{(2a^2 - b^2) \int \frac{1}{\frac{a \cos(c + d\sqrt{x})}{b} + 1} d\sqrt{x}}{a(a^2 - b^2)}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \sec(c + d\sqrt{x}))} \right) \\
& \quad \downarrow \text{3042} \\
& 2 \left(\frac{\frac{\sqrt{x}(a^2 - b^2)}{a} - \frac{(2a^2 - b^2) \int \frac{1}{\frac{a \sin(c + d\sqrt{x} + \frac{\pi}{2})}{b} + 1} d\sqrt{x}}{a(a^2 - b^2)}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \sec(c + d\sqrt{x}))} \right) \\
& \quad \downarrow \text{3138} \\
& 2 \left(\frac{\frac{\sqrt{x}(a^2 - b^2)}{a} - \frac{2(2a^2 - b^2) \int \frac{1}{\frac{a+b}{b} + (1-\frac{a}{b})x} d \tan(\frac{1}{2}(c + d\sqrt{x}))}{ad}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \sec(c + d\sqrt{x}))} \right) \\
& \quad \downarrow \text{221} \\
& 2 \left(\frac{\frac{\sqrt{x}(a^2 - b^2)}{a} - \frac{2b(2a^2 - b^2) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c + d\sqrt{x}))}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \sec(c + d\sqrt{x}))} \right)
\end{aligned}$$

input `Int[1/(Sqrt[x]*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output `2*(((a^2 - b^2)*Sqrt[x])/a - (2*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*Sqrt[x])/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2)) + (b^2*Tan[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*Sqrt[x]]))`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 221 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3138 `Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 4272 `Int[(csc[(c_) + (d_)*(x_)])*(b_) + (a_)^(n_), x_Symbol] := Simp[b^2*Cot[c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Simp[1/(a*(n + 1)*(a^2 - b^2)) Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]`

rule 4318 `Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)])*(b_) + (a_), x_Symbol] := Simp[1/b Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]`

```
rule 4407 Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.), x_Symbol] := Simp[c*(x/a), x] - Simp[(b*c - a*d)/a Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

```
rule 4692 Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.28

method	result
derivativedivides	$4b \frac{\left(\frac{ab \tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{(a^2-b^2) \left(\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)^2 a - \tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)^2 b - a - b} \right) - \frac{(2a^2-b^2) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{a^2} \right) + \frac{4 \operatorname{arctan}\left(\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)}{a^2}}{d}$
default	$4b \frac{\left(\frac{ab \tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{(a^2-b^2) \left(\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)^2 a - \tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)^2 b - a - b} \right) - \frac{(2a^2-b^2) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{a^2} \right) + \frac{4 \operatorname{arctan}\left(\tan\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)}{a^2}}{d}$

```
input int(1/x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

```
output 2/d*(2*b/a^2*(-a*b/(a^2-b^2)*tan(1/2*c+1/2*d*x^(1/2))/(tan(1/2*c+1/2*d*x^(
1/2))^2*a-tan(1/2*c+1/2*d*x^(1/2))^2*b-a-b)-(2*a^2-b^2)/(a-b)/(a+b)/((a-b)
*(a+b))^(1/2)*operatorname{arctanh}((a-b)*tan(1/2*c+1/2*d*x^(1/2))/((a-b)*(a+b))^(1/2)))
+2/a^2*operatorname{arctan}(tan(1/2*c+1/2*d*x^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(110) = 220$.

Time = 0.11 (sec) , antiderivative size = 574, normalized size of antiderivative = 4.52

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx$$

$$= \left[\frac{2(a^5 - 2a^3b^2 + ab^4)d\sqrt{x} \cos(d\sqrt{x} + c) + 2(a^4b - 2a^2b^3 + b^5)d\sqrt{x} + ((2a^3b - ab^3)\sqrt{a^2 - b^2} \cos(d\sqrt{x} + c) + (a^7 - 2a^5b^2 + a^3b^4)d \cos(d\sqrt{x} + c) + (a^6b - 2a^4b^3 + a^2b^5)d \sin(d\sqrt{x} + c) + ((2a^3b - ab^3)\sqrt{-a^2 + b^2}) \arctan\left(\frac{\sqrt{-a^2 + b^2} \cos(d\sqrt{x} + c) + \sqrt{-a^2 + b^2} a}{(a^2 - b^2) \sin(d\sqrt{x} + c)}\right) + (a^3b^2 - a^2b^4) \sin(d\sqrt{x} + c)}{(a^7 - 2a^5b^2 + a^3b^4)d \cos(d\sqrt{x} + c) + (a^6b - 2a^4b^3 + a^2b^5)d \sin(d\sqrt{x} + c)} \right]$$

input `integrate(1/x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `[(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cos(d*sqrt(x) + c) + 2*(a^4*b - 2*a^2*b^3 + b^5)*d*sqrt(x) + ((2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*cos(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(a^2 - b^2))*log((2*a*b*cos(d*sqrt(x) + c) - (a^2 - 2*b^2)*cos(d*sqrt(x) + c)^2 + 2*a^2 - b^2 - 2*(sqrt(a^2 - b^2)*b*cos(d*sqrt(x) + c) + sqrt(a^2 - b^2)*a)*sin(d*sqrt(x) + c)))/(a^2*cos(d*sqrt(x) + c)^2 + 2*a*b*cos(d*sqrt(x) + c) + b^2)) + 2*(a^3*b^2 - a*b^4)*sin(d*sqrt(x) + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*sqrt(x) + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), 2*((a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cos(d*sqrt(x) + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*sqrt(x) - ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*cos(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2))*arctan(-(sqrt(-a^2 + b^2)*b*cos(d*sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*sin(d*sqrt(x) + c)))) + (a^3*b^2 - a*b^4)*sin(d*sqrt(x) + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*sqrt(x) + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]`

Sympy [F]

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `integrate(1/x**(1/2)/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(1/(sqrt(x)*(a + b*sec(c + d*sqrt(x)))**2), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(1/x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` or more de`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.54

$$\begin{aligned} & \int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx \\ &= -\frac{4b^2 \tan\left(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c\right)}{(a^3d - ab^2d) \left(a \tan\left(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c\right)^2 - b \tan\left(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c\right) - a - b\right)} \\ &+ \frac{4(2a^2b - b^3) \left(\pi \left[\frac{d\sqrt{x}+c}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(2a - 2b) + \arctan\left(\frac{a \tan\left(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c\right) - b \tan\left(\frac{1}{2}d\sqrt{x} + \frac{1}{2}c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{(a^4d - a^2b^2d)\sqrt{-a^2 + b^2}} \\ &+ \frac{2(d\sqrt{x} + c)}{a^2d} \end{aligned}$$

input `integrate(1/x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output

```
-4*b^2*tan(1/2*d*sqrt(x) + 1/2*c)/((a^3*d - a*b^2*d)*(a*tan(1/2*d*sqrt(x)
+ 1/2*c)^2 - b*tan(1/2*d*sqrt(x) + 1/2*c)^2 - a - b)) + 4*(2*a^2*b - b^3)*
(pi*floor(1/2*(d*sqrt(x) + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2
*d*sqrt(x) + 1/2*c) - b*tan(1/2*d*sqrt(x) + 1/2*c))/sqrt(-a^2 + b^2)))/((a
^4*d - a^2*b^2*d)*sqrt(-a^2 + b^2)) + 2*(d*sqrt(x) + c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 20.19 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.60

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx$$

$$= \frac{\frac{b^2 \operatorname{Li}_4}{a d (a^2 - b^2)} + \frac{b^3 e^{c \operatorname{Li} + d \sqrt{x} \operatorname{Li}} \operatorname{Li}_4}{a^2 d (a^2 - b^2)}}{a + a e^{c \operatorname{Li} + d \sqrt{x} \operatorname{Li}} + 2 b e^{c \operatorname{Li} + d \sqrt{x} \operatorname{Li}}} + \frac{2 \sqrt{x}}{a^2}$$

$$+ \frac{\ln \left(e^{c \operatorname{Li} + d \sqrt{x} \operatorname{Li}} (4 a^2 b - 2 b^3) - \frac{(4 a^2 b - 2 b^3) (a^2 - b^2) (a + b e^{c \operatorname{Li} + d \sqrt{x} \operatorname{Li}}) \operatorname{Li}}{(a + b)^{3/2} (a - b)^{3/2}} \right) (4 a^2 b - 2 b^3)}{a^2 d (a + b)^{3/2} (a - b)^{3/2}}$$

$$- \frac{2 b \ln \left(e^{c \operatorname{Li} + d \sqrt{x} \operatorname{Li}} (4 a^2 b - 2 b^3) + \frac{b (a^2 - b^2) (2 a^2 - b^2) (a + b e^{c \operatorname{Li} + d \sqrt{x} \operatorname{Li}}) \operatorname{Li}}{(a + b)^{3/2} (a - b)^{3/2}} \right) (2 a^2 - b^2)}{a^2 d (a + b)^{3/2} (a - b)^{3/2}}$$

input

```
int(1/(x^(1/2)*(a + b/cos(c + d*x^(1/2))))^2,x)
```

output

```
((b^2*4i)/(a*d*(a^2 - b^2)) + (b^3*exp(c*1i + d*x^(1/2)*1i)*4i)/(a^2*d*(a^
2 - b^2)))/(a + a*exp(c*2i + d*x^(1/2)*2i) + 2*b*exp(c*1i + d*x^(1/2)*1i))
+ (2*x^(1/2))/a^2 + (log(exp(c*1i + d*x^(1/2)*1i)*(4*a^2*b - 2*b^3) - ((4
*a^2*b - 2*b^3)*(a^2 - b^2)*(a + b*exp(c*1i + d*x^(1/2)*1i))*1i)/((a + b)
^(3/2)*(a - b)^(3/2)))*(4*a^2*b - 2*b^3))/(a^2*d*(a + b)^(3/2)*(a - b)^(3/2
)) - (2*b*log(exp(c*1i + d*x^(1/2)*1i)*(4*a^2*b - 2*b^3) + (b*(a^2 - b^2)*
(2*a^2 - b^2)*(a + b*exp(c*1i + d*x^(1/2)*1i))*2i)/((a + b)^(3/2)*(a - b)
^(3/2)))*(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{x} (a + b \sec(c + d\sqrt{x}))^2} dx$$

$$= \frac{-8\sqrt{-a^2 + b^2} \operatorname{atan}\left(\frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)a - \tan\left(\frac{\sqrt{x}d + c}{2}\right)b}{\sqrt{-a^2 + b^2}}\right) \cos(\sqrt{x}d + c) a^3 b + 4\sqrt{-a^2 + b^2} \operatorname{atan}\left(\frac{\tan\left(\frac{\sqrt{x}d + c}{2}\right)a - \tan\left(\frac{\sqrt{x}d + c}{2}\right)b}{\sqrt{-a^2 + b^2}}\right)}{\dots}$$

input `int(1/x^(1/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(2*(-4*sqrt(-a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt(-a**2 + b**2))*cos(sqrt(x)*d + c)*a**3*b + 2*sqrt(-a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt(-a**2 + b**2))*cos(sqrt(x)*d + c)*a*b**3 - 4*sqrt(-a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt(-a**2 + b**2))*a**2*b**2 + 2*sqrt(-a**2 + b**2)*atan((tan((sqrt(x)*d + c)/2)*a - tan((sqrt(x)*d + c)/2)*b)/sqrt(-a**2 + b**2))*b**4 + sqrt(x)*cos(sqrt(x)*d + c)*a**5*d - 2*sqrt(x)*cos(sqrt(x)*d + c)*a**3*b**2*d + sqrt(x)*cos(sqrt(x)*d + c)*a*b**4*d + sqrt(x)*a**4*b*d - 2*sqrt(x)*a**2*b**3*d + sqrt(x)*b**5*d + sin(sqrt(x)*d + c)*a**3*b**2 - sin(sqrt(x)*d + c)*a*b**4)/(a**2*d*(cos(sqrt(x)*d + c)*a**5 - 2*cos(sqrt(x)*d + c)*a**3*b**2 + cos(sqrt(x)*d + c)*a*b**4 + a**4*b - 2*a**2*b**3 + b**5))
```


3.69 $\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))^2} dx$

Optimal result	496
Mathematica [N/A]	496
Rubi [N/A]	497
Maple [N/A]	497
Fricas [N/A]	498
Sympy [N/A]	498
Maxima [F(-1)]	499
Giac [N/A]	499
Mupad [N/A]	499
Reduce [N/A]	500

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))^2} dx = \text{Int}\left(\frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))^2}, x\right)$$

output `Defer(Int)(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

Mathematica [N/A]

Not integrable

Time = 30.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))^2} dx = \int \frac{1}{x^{3/2}(a+b \sec(c+d\sqrt{x}))^2} dx$$

input `Integrate[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output `Integrate[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

↓ 4694

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `Int[1/(x^(3/2)*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^{\frac{3}{2}}} dx$$

input `integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(sqrt(x)/(b^2*x^2*sec(d*sqrt(x) + c)^2 + 2*a*b*x^2*sec(d*sqrt(x) + c) + a^2*x^2), x)`

Sympy [N/A]

Not integrable

Time = 6.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{\frac{3}{2}} (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `integrate(1/x**(3/2)/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(1/(x**(3/2)*(a + b*sec(c + d*sqrt(x)))**2), x)`

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \text{Timed out}$$

input `integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Timed out`

Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^{\frac{3}{2}}} dx$$

input `integrate(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(1/((b*sec(d*sqrt(x) + c) + a)^2*x^(3/2)), x)`

Mupad [N/A]

Not integrable

Time = 16.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{3/2} \left(a + \frac{b}{\cos(c + d\sqrt{x})}\right)^2} dx$$

input `int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2))))^2,x)`

output `int(1/(x^(3/2)*(a + b/cos(c + d*x^(1/2))))^2, x)`

Reduce [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 38128, normalized size of antiderivative = 1733.09

$$\int \frac{1}{x^{3/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

input `int(1/x^(3/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(2*(4*sqrt(x)*int(tan((sqrt(x)*d + c)/2)**4/(sqrt(x)*tan((sqrt(x)*d + c)/2)
)**4*a**7*x - 5*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**6*b*x + 9*sqrt(x)*tan
((sqrt(x)*d + c)/2)**4*a**5*b**2*x - 5*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a
**4*b**3*x - 5*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**3*b**4*x + 9*sqrt(x)*t
an((sqrt(x)*d + c)/2)**4*a**2*b**5*x - 5*sqrt(x)*tan((sqrt(x)*d + c)/2)**4
*a*b**6*x + sqrt(x)*tan((sqrt(x)*d + c)/2)**4*b**7*x - 2*sqrt(x)*tan((sqrt
(x)*d + c)/2)**2*a**7*x + 6*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**6*b*x - 2
*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**5*b**2*x - 10*sqrt(x)*tan((sqrt(x)*d
+ c)/2)**2*a**4*b**3*x + 10*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**3*b**4*x
+ 2*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**2*b**5*x - 6*sqrt(x)*tan((sqrt(x)
)*d + c)/2)**2*a*b**6*x + 2*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*b**7*x + sqr
t(x)*a**7*x - sqrt(x)*a**6*b*x - 3*sqrt(x)*a**5*b**2*x + 3*sqrt(x)*a**4*b*
**3*x + 3*sqrt(x)*a**3*b**4*x - 3*sqrt(x)*a**2*b**5*x - sqrt(x)*a*b**6*x +
sqrt(x)*b**7*x),x)*tan((sqrt(x)*d + c)/2)**2*a**8*b - 24*sqrt(x)*int(tan((
sqrt(x)*d + c)/2)**4/(sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**7*x - 5*sqrt(x)
*tan((sqrt(x)*d + c)/2)**4*a**6*b*x + 9*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*
a**5*b**2*x - 5*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**4*b**3*x - 5*sqrt(x)*
tan((sqrt(x)*d + c)/2)**4*a**3*b**4*x + 9*sqrt(x)*tan((sqrt(x)*d + c)/2)**
4*a**2*b**5*x - 5*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a*b**6*x + sqrt(x)*tan
((sqrt(x)*d + c)/2)**4*b**7*x - 2*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**...
```

$$3.70 \quad \int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

Optimal result	501
Mathematica [N/A]	501
Rubi [N/A]	502
Maple [N/A]	502
Fricas [N/A]	503
Sympy [N/A]	503
Maxima [F(-1)]	504
Giac [N/A]	504
Mupad [N/A]	504
Reduce [N/A]	505

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \text{Int} \left(\frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2}, x \right)$$

output `Defer(Int)(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

Mathematica [N/A]

Not integrable

Time = 31.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `Integrate[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output `Integrate[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

↓ 4694

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `Int[1/(x^(5/2)*(a + b*Sec[c + d*Sqrt[x]])^2),x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}} (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

output `int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^{\frac{5}{2}}} dx$$

input `integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(sqrt(x)/(b^2*x^3*sec(d*sqrt(x) + c)^2 + 2*a*b*x^3*sec(d*sqrt(x) + c) + a^2*x^3), x)`

Sympy [N/A]

Not integrable

Time = 34.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{\frac{5}{2}} (a + b \sec(c + d\sqrt{x}))^2} dx$$

input `integrate(1/x**(5/2)/(a+b*sec(c+d*x**(1/2)))**2,x)`

output `Integral(1/(x**(5/2)*(a + b*sec(c + d*sqrt(x)))**2), x)`

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \text{Timed out}$$

input `integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Timed out`

Giac [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \sec(d\sqrt{x} + c) + a)^2 x^{5/2}} dx$$

input `integrate(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(1/((b*sec(d*sqrt(x) + c) + a)^2*x^(5/2)), x)`

Mupad [N/A]

Not integrable

Time = 15.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{5/2} \left(a + \frac{b}{\cos(c + d\sqrt{x})}\right)^2} dx$$

input `int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2))))^2),x)`

output `int(1/(x^(5/2)*(a + b/cos(c + d*x^(1/2))))^2), x)`

Reduce [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 157108, normalized size of antiderivative = 7141.27

$$\int \frac{1}{x^{5/2} (a + b \sec(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

input `int(1/x^(5/2)/(a+b*sec(c+d*x^(1/2)))^2,x)`

output

```
(2*(12*sqrt(x)*int(tan((sqrt(x)*d + c)/2)**4/(sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**9*x**2 - 7*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**8*b*x**2 + 20*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**7*b**2*x**2 - 28*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**6*b**3*x**2 + 14*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**5*b**4*x**2 + 14*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**4*b**5*x**2 - 28*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**3*b**6*x**2 + 20*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**2*b**7*x**2 - 7*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a*b**8*x**2 + sqrt(x)*tan((sqrt(x)*d + c)/2)**4*b**9*x**2 - 2*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**9*x**2 + 10*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**8*b*x**2 - 16*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**7*b**2*x**2 + 28*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**5*b**4*x**2 - 28*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**4*b**5*x**2 + 16*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a**2*b**7*x**2 - 10*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*a*b**8*x**2 + 2*sqrt(x)*tan((sqrt(x)*d + c)/2)**2*b**9*x**2 + sqrt(x)*a**9*x**2 - 3*sqrt(x)*a**8*b*x**2 + 8*sqrt(x)*a**6*b**3*x**2 - 6*sqrt(x)*a**5*b**4*x**2 - 6*sqrt(x)*a**4*b**5*x**2 + 8*sqrt(x)*a**3*b**6*x**2 - 3*sqrt(x)*a*b**8*x**2 + sqrt(x)*b**9*x**2),x)*tan((sqrt(x)*d + c)/2)**2*a**12*b*x - 96*sqrt(x)*int(tan((sqrt(x)*d + c)/2)**4/(sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**9*x**2 - 7*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**8*b*x**2 + 20*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**7*b**2*x**2 - 28*sqrt(x)*tan((sqrt(x)*d + c)/2)**4*a**6*b**3*x**2 + 14*sqrt(x)*tan((sqr...
```

3.71 $\int (ex)^m (a + b \sec(c + dx^n))^p dx$

Optimal result	506
Mathematica [N/A]	506
Rubi [N/A]	507
Maple [N/A]	508
Fricas [N/A]	508
Sympy [N/A]	508
Maxima [N/A]	509
Giac [N/A]	509
Mupad [N/A]	509
Reduce [N/A]	510

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx = x^{-m} (ex)^m \text{Int}(x^m (a + b \sec(c + dx^n))^p, x)$$

output `(e*x)^m*Defer(Int)(x^m*(a+b*sec(c+d*x^n))^p,x)/(x^m)`

Mathematica [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx = \int (ex)^m (a + b \sec(c + dx^n))^p dx$$

input `Integrate[(e*x)^m*(a + b*Sec[c + d*x^n])^p,x]`

output `Integrate[(e*x)^m*(a + b*Sec[c + d*x^n])^p, x]`

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4696, 4694}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx$$

$$\downarrow 4696$$

$$x^{-m}(ex)^m \int x^m (a + b \sec(dx^n + c))^p dx$$

$$\downarrow 4694$$

$$x^{-m}(ex)^m \int x^m (a + b \sec(dx^n + c))^p dx$$

input `Int[(e*x)^m*(a + b*Sec[c + d*x^n])^p,x]`

output `$Aborted`

Defintions of rubi rules used

rule 4694 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sec[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

rule 4696 `Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx$$

input `int((e*x)^m*(a+b*sec(c+d*x^n))^p,x)`output `int((e*x)^m*(a+b*sec(c+d*x^n))^p,x)`**Fricas [N/A]**

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx = \int (ex)^m (b \sec(dx^n + c) + a)^p dx$$

input `integrate((e*x)^m*(a+b*sec(c+d*x^n))^p,x, algorithm="fricas")`output `integral((e*x)^m*(b*sec(d*x^n + c) + a)^p, x)`**Sympy [N/A]**

Not integrable

Time = 47.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx = \int (ex)^m (a + b \sec(c + dx^n))^p dx$$

input `integrate((e*x)**m*(a+b*sec(c+d*x**n))**p,x)`output `Integral((e*x)**m*(a + b*sec(c + d*x**n))**p, x)`

Maxima [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx = \int (ex)^m (b \sec(dx^n + c) + a)^p dx$$

input `integrate((e*x)^m*(a+b*sec(c+d*x^n))^p,x, algorithm="maxima")`

output `integrate((e*x)^m*(b*sec(d*x^n + c) + a)^p, x)`

Giac [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx = \int (ex)^m (b \sec(dx^n + c) + a)^p dx$$

input `integrate((e*x)^m*(a+b*sec(c+d*x^n))^p,x, algorithm="giac")`

output `integrate((e*x)^m*(b*sec(d*x^n + c) + a)^p, x)`

Mupad [N/A]

Not integrable

Time = 15.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx = \int \left(a + \frac{b}{\cos(c + dx^n)} \right)^p (ex)^m dx$$

input `int((a + b/cos(c + d*x^n))^p*(e*x)^m,x)`

output `int((a + b/cos(c + d*x^n))^p*(e*x)^m, x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m (a + b \sec(c + dx^n))^p dx = e^m \left(\int x^m (\sec(x^n d + c) b + a)^p dx \right)$$

input `int((e*x)^m*(a+b*sec(c+d*x^n))^p,x)`

output `e**m*int(x**m*(sec(x**n*d + c)*b + a)**p,x)`

3.72 $\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx$

Optimal result	511
Mathematica [A] (verified)	511
Rubi [A] (verified)	512
Maple [C] (warning: unable to verify)	513
Fricas [A] (verification not implemented)	513
Sympy [F]	514
Maxima [F]	514
Giac [F]	514
Mupad [B] (verification not implemented)	515
Reduce [B] (verification not implemented)	515

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx = \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \operatorname{arctanh}(\sin(c + dx^n))}{den}$$

output `a*(e*x)^n/e/n+b*(e*x)^n*arctanh(sin(c+d*x^n))/d/e/n/(x^n)`

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx = \frac{x^{-n}(ex)^n (adx^n + b \coth^{-1}(\sin(c + dx^n)))}{den}$$

input `Integrate[(e*x)^(-1 + n)*(a + b*Sec[c + d*x^n]),x]`

output `((e*x)^n*(a*d*x^n + b*ArcCoth[Sin[c + d*x^n]]))/(d*e*n*x^n)`

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} (a + b \sec(c + dx^n)) dx$$

↓ 2010

$$\int (a(ex)^{n-1} + b(ex)^{n-1} \sec(c + dx^n)) dx$$

↓ 2009

$$\frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \arctanh(\sin(c + dx^n))}{den}$$

input `Int[(e*x)^(-1 + n)*(a + b*Sec[c + d*x^n]),x]`

output `(a*(e*x)^n)/(e*n) + (b*(e*x)^n*ArcTanh[Sin[c + d*x^n]])/(d*e*n*x^n)`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.61

method	result
risch	$\frac{(-1+n)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie)^2 \pi - i \operatorname{csgn}(ie)^3 \pi + 2 \ln(x) + 2 \ln(e))}{2} \frac{ax e}{n} - 2i \arctan(e^{i(\dots)})$

input `int((e*x)^(-1+n)*(a+b*sec(c+d*x^n)),x,method=_RETURNVERBOSE)`

output `a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(x)+2*ln(e))-2*I*arctan(exp(I*(c+d*x^n)))/d/e*e^n/n*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx$$

$$= \frac{2ade^{n-1}x^n + be^{n-1} \log(\sin(dx^n + c) + 1) - be^{n-1} \log(-\sin(dx^n + c) + 1)}{2dn}$$

input `integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n)),x, algorithm="fricas")`

output `1/2*(2*a*d*e^(n-1)*x^n + b*e^(n-1)*log(sin(d*x^n + c) + 1) - b*e^(n-1)*log(-sin(d*x^n + c) + 1))/(d*n)`

Sympy [F]

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx = \int (ex)^{n-1} (a + b \sec(c + dx^n)) dx$$

input `integrate((e*x)**(-1+n)*(a+b*sec(c+d*x**n)),x)`

output `Integral((e*x)**(n - 1)*(a + b*sec(c + d*x**n)), x)`

Maxima [F]

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx = \int (b \sec(dx^n + c) + a)(ex)^{n-1} dx$$

input `integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n)),x, algorithm="maxima")`

output `2*b*e^n*integrate((x^n*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + x^n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + x^n*cos(d*x^n + c))/(e*x*cos(2*d*x^n + 2*c)^2 + e*x*sin(2*d*x^n + 2*c)^2 + 2*e*x*cos(2*d*x^n + 2*c) + e*x), x) + (e*x)^n*a/(e*n)`

Giac [F]

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx = \int (b \sec(dx^n + c) + a)(ex)^{n-1} dx$$

input `integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n)),x, algorithm="giac")`

output `integrate((b*sec(d*x^n + c) + a)*(e*x)^(n - 1), x)`

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx$$

$$= \frac{(ex)^n (b \ln(-b(ex)^{n-1} 2i - 2be^{c1i} e^{dx^n 1i} (ex)^{n-1}) - b \ln(b(ex)^{n-1} 2i - 2be^{c1i} e^{dx^n 1i} (ex)^{n-1}) + a dx^n)}{den x^n}$$

input `int((a + b/cos(c + d*x^n))*(e*x)^(n - 1),x)`output `((e*x)^n*(b*log(- b*(e*x)^(n - 1)*2i - 2*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1)) - b*log(b*(e*x)^(n - 1)*2i - 2*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1)) + a*d*x^n)/(d*e*n*x^n)`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n)) dx$$

$$= \frac{e^n (x^n a d - \log(\tan(\frac{x^n d}{2} + \frac{c}{2}) - 1) b + \log(\tan(\frac{x^n d}{2} + \frac{c}{2}) + 1) b)}{den}$$

input `int((e*x)^(-1+n)*(a+b*sec(c+d*x^n)),x)`output `(e**n*(x**n*a*d - log(tan((x**n*d + c)/2) - 1)*b + log(tan((x**n*d + c)/2 + 1)*b))/(d*e*n)`

3.73 $\int (ex)^{-1+2n} (a + b \sec (c + dx^n)) dx$

Optimal result	516
Mathematica [A] (verified)	516
Rubi [A] (verified)	517
Maple [C] (warning: unable to verify)	518
Fricas [B] (verification not implemented)	519
Sympy [F]	520
Maxima [F]	520
Giac [F]	521
Mupad [F(-1)]	521
Reduce [F]	521

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int (ex)^{-1+2n} (a + b \sec (c + dx^n)) dx = \frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n} \arctan (e^{i(c+dx^n)})}{den} + \frac{ibx^{-2n}(ex)^{2n} \text{PolyLog} (2, -ie^{i(c+dx^n)})}{d^2en} - \frac{ibx^{-2n}(ex)^{2n} \text{PolyLog} (2, ie^{i(c+dx^n)})}{d^2en}$$

output

```
1/2*a*(e*x)^(2*n)/e/n-2*I*b*(e*x)^(2*n)*arctan(exp(I*(c+d*x^n)))/d/e/n/(x^n)+I*b*(e*x)^(2*n)*polylog(2,-I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-I*b*(e*x)^(2*n)*polylog(2,I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.26

$$\int (ex)^{-1+2n} (a + b \sec (c + dx^n)) dx = \frac{(ex)^{2n} \cos (c + dx^n) \left(a + \frac{bx^{-2n} \left((-2c+\pi-2dx^n) \left(\log (1-ie^{-i(c+dx^n)}) - \log (1+ie^{-i(c+dx^n)}) \right) - (-2c+\pi) \log \left(\cot \left(\frac{1}{4} (2c+\pi+2dx^n) \right) \right) \right)}{d^2} \right)}{2en (b + a \cos (c + dx^n))}$$

input `Integrate[(e*x)^(-1 + 2*n)*(a + b*Sec[c + d*x^n]),x]`

output
$$\frac{((e*x)^{(2*n)}*\text{Cos}[c + d*x^n]*(a + (b*((-2*c + \text{Pi} - 2*d*x^n))*(\text{Log}[1 - I/E^{(I*(c + d*x^n))}] - \text{Log}[1 + I/E^{(I*(c + d*x^n))}]) - (-2*c + \text{Pi})*\text{Log}[\text{Cot}[(2*c + \text{Pi} + 2*d*x^n)/4]]) + (2*I)*(PolyLog[2, (-I)/E^{(I*(c + d*x^n))}] - PolyLog[2, I/E^{(I*(c + d*x^n))}])))/(d^2*x^{(2*n)}))*(a + b*Sec[c + d*x^n])/(2*e*n*(b + a*\text{Cos}[c + d*x^n]))}$$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} (a + b \sec(c + dx^n)) dx$$

↓ 2010

$$\int (a(ex)^{2n-1} + b(ex)^{2n-1} \sec(c + dx^n)) dx$$

↓ 2009

$$\frac{a(ex)^{2n}}{2en} - \frac{2ibx^{-n}(ex)^{2n} \arctan(e^{i(c+dx^n)})}{\frac{den}{ibx^{-2n}(ex)^{2n} \text{PolyLog}(2, ie^{i(dx^n+c)})}} + \frac{ibx^{-2n}(ex)^{2n} \text{PolyLog}(2, -ie^{i(dx^n+c)})}{\frac{d^2en}{ibx^{-2n}(ex)^{2n} \text{PolyLog}(2, ie^{i(dx^n+c)})}}$$

input `Int[(e*x)^(-1 + 2*n)*(a + b*Sec[c + d*x^n]),x]`

output
$$\frac{(a*(e*x)^{(2*n)})/(2*e*n) - ((2*I)*b*(e*x)^{(2*n)}*ArcTan[E^{(I*(c + d*x^n))})]/(d*e*n*x^n) + (I*b*(e*x)^{(2*n)}*PolyLog[2, (-I)*E^{(I*(c + d*x^n))}])/(d^2*e*n*x^{(2*n)}) - (I*b*(e*x)^{(2*n)}*PolyLog[2, I*E^{(I*(c + d*x^n))}])/(d^2*e*n*x^{(2*n)})}$$

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 845, normalized size of antiderivative = 5.67

method	result	size
risch	Expression too large to display	845

input `int((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n)),x,method=_RETURNVERBOSE)`

output

```

I/e/n/d*b*(-exp(2*I*c))^(1/2)*ln(1+exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*(-1)^(
(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(1
/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(e^n)^2*x^n*exp(-1/2*I*(2*Pi*n*csgn(I*
e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*
n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+2*c))-I/e/n/d*b*(-exp(2
*I*c))^(1/2)*ln(1-exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*(-1)^(-1/2*csgn(I*e)*c
sgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e)*csgn
(I*x)*csgn(I*e*x))*(e^n)^2*x^n*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csg
n(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(
I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+2*c))+1/e/n/d^2*b*(-exp(2*I*c))^(1/2)*(-
1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)
^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*dilog(1+exp(I*x^n*d)*(-exp(2*I*c))^(
1/2))*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*
x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)
-Pi*csgn(I*e*x)^3+2*c))-1/e/n/d^2*b*(-exp(2*I*c))^(1/2)*(-1)^(-1/2*csgn(I*
e)*csgn(I*e*x)^2)*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e)*
csgn(I*x)*csgn(I*e*x))*dilog(1-exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*(e^n)^2*
exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn
(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^
3+2*c))+1/2*a/n*x*exp(1/2*(-1+2*n))*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*...

```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(133) = 266$.

Time = 0.11 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.15

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n)) dx$$

$$= \frac{ad^2 e^{2n-1} x^{2n} - bce^{2n-1} \log(\cos(dx^n + c) + i \sin(dx^n + c) + i) + bce^{2n-1} \log(\cos(dx^n + c) - i \sin(dx^n + c))}{2}$$

input

```
integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n)),x, algorithm="fricas")
```


output

```
1/2*(a*d^2*e^(2*n - 1)*x^(2*n) - b*c*e^(2*n - 1)*log(cos(d*x^n + c) + I*sin(d*x^n + c) + I) + b*c*e^(2*n - 1)*log(cos(d*x^n + c) - I*sin(d*x^n + c) + I) - b*c*e^(2*n - 1)*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + I) + b*c*e^(2*n - 1)*log(-cos(d*x^n + c) - I*sin(d*x^n + c) + I) - I*b*e^(2*n - 1)*dilog(I*cos(d*x^n + c) + sin(d*x^n + c)) - I*b*e^(2*n - 1)*dilog(I*cos(d*x^n + c) - sin(d*x^n + c)) + I*b*e^(2*n - 1)*dilog(-I*cos(d*x^n + c) + sin(d*x^n + c)) + I*b*e^(2*n - 1)*dilog(-I*cos(d*x^n + c) - sin(d*x^n + c)) + (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(I*cos(d*x^n + c) - sin(d*x^n + c) + 1) + (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(-I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(-I*cos(d*x^n + c) - sin(d*x^n + c) + 1))/(d^2*n)
```

Sympy [F]

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n)) dx = \int (ex)^{2n-1} (a + b \sec(c + dx^n)) dx$$

input

```
integrate((e*x)**(-1+2*n)*(a+b*sec(c+d*x**n)),x)
```

output

```
Integral((e*x)**(2*n - 1)*(a + b*sec(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n)) dx = \int (b \sec(dx^n + c) + a)(ex)^{2n-1} dx$$

input

```
integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n)),x, algorithm="maxima")
```

output

```
2*b*e^(2*n)*integrate((x^(2*n)*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + x^(2*n)*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + x^(2*n)*cos(d*x^n + c))/(e*x*cos(2*d*x^n + 2*c)^2 + e*x*sin(2*d*x^n + 2*c)^2 + 2*e*x*cos(2*d*x^n + 2*c) + e*x), x) + 1/2*(e*x)^(2*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n)) dx = \int (b \sec(dx^n + c) + a)(ex)^{2n-1} dx$$

input `integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n)),x, algorithm="giac")`

output `integrate((b*sec(d*x^n + c) + a)*(e*x)^(2*n - 1), x)`

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n)) dx = \int \left(a + \frac{b}{\cos(c + dx^n)} \right) (ex)^{2n-1} dx$$

input `int((a + b/cos(c + d*x^n))*(e*x)^(2*n - 1),x)`

output `int((a + b/cos(c + d*x^n))*(e*x)^(2*n - 1), x)`

Reduce [F]

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n)) dx = \frac{e^{2n} \left(x^{2n} a + x^{2n} b - 4 \left(\int \frac{x^{2n} \tan\left(\frac{x^n d + c}{2}\right)^2}{\tan\left(\frac{x^n d + c}{2}\right)^2 x-x} dx \right) b n \right)}{2en}$$

input `int((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n)),x)`

output `(e**(2*n)*(x**(2*n)*a + x**(2*n)*b - 4*int((x**(2*n))*tan((x**n*d + c)/2)**2)/(tan((x**n*d + c)/2)**2*x - x),x)*b*n)/(2*e*n)`

3.74 $\int (ex)^{-1+3n} (a + b \sec (c + dx^n)) dx$

Optimal result	522
Mathematica [F]	523
Rubi [A] (verified)	523
Maple [F]	524
Fricas [B] (verification not implemented)	524
Sympy [F]	525
Maxima [F]	525
Giac [F]	526
Mupad [F(-1)]	526
Reduce [F]	527

Optimal result

Integrand size = 22, antiderivative size = 235

$$\int (ex)^{-1+3n} (a + b \sec (c + dx^n)) dx = \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n} \arctan (e^{i(c+dx^n)})}{den} + \frac{2ibx^{-2n}(ex)^{3n} \text{PolyLog} (2, -ie^{i(c+dx^n)})}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n} \text{PolyLog} (2, ie^{i(c+dx^n)})}{d^2en} - \frac{2bx^{-3n}(ex)^{3n} \text{PolyLog} (3, -ie^{i(c+dx^n)})}{d^3en} + \frac{2bx^{-3n}(ex)^{3n} \text{PolyLog} (3, ie^{i(c+dx^n)})}{d^3en}$$

output

```
1/3*a*(e*x)^(3*n)/e/n-2*I*b*(e*x)^(3*n)*arctan(exp(I*(c+d*x^n)))/d/e/n/(x^n)+2*I*b*(e*x)^(3*n)*polylog(2,-I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*I*b*(e*x)^(3*n)*polylog(2,I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*b*(e*x)^(3*n)*polylog(3,-I*exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))+2*b*(e*x)^(3*n)*polylog(3,I*exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))
```

Mathematica [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx = \int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx$$

input `Integrate[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n]),x]`

output `Integrate[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n]), x]`

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (ex)^{3n-1} (a + b \sec(c + dx^n)) dx \\ & \quad \downarrow \text{2010} \\ & \int (a(ex)^{3n-1} + b(ex)^{3n-1} \sec(c + dx^n)) dx \\ & \quad \downarrow \text{2009} \\ & \frac{a(ex)^{3n}}{3en} - \frac{2ibx^{-n}(ex)^{3n} \arctan(e^{i(c+dx^n)})}{2bx^{-3n}(ex)^{3n} \text{PolyLog}(3, ie^{i(dx^n+c)})} - \frac{2bx^{-3n}(ex)^{3n} \text{PolyLog}(3, -ie^{i(dx^n+c)})}{d^3en} + \\ & \frac{d^3en}{2ibx^{-2n}(ex)^{3n} \text{PolyLog}(2, ie^{i(dx^n+c)})} + \frac{d^2en}{2ibx^{-2n}(ex)^{3n} \text{PolyLog}(2, -ie^{i(dx^n+c)})} - \frac{d^2en}{d^2en} \end{aligned}$$

input `Int[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n]),x]`

output

```
(a*(e*x)^(3*n))/(3*e*n) - ((2*I)*b*(e*x)^(3*n)*ArcTan[E^(I*(c + d*x^n))])/
(d*e*n*x^n) + ((2*I)*b*(e*x)^(3*n)*PolyLog[2, (-I)*E^(I*(c + d*x^n))]/(d^
2*e*n*x^(2*n)) - ((2*I)*b*(e*x)^(3*n)*PolyLog[2, I*E^(I*(c + d*x^n))]/(d^
2*e*n*x^(2*n)) - (2*b*(e*x)^(3*n)*PolyLog[3, (-I)*E^(I*(c + d*x^n))]/(d^3
*e*n*x^(3*n)) + (2*b*(e*x)^(3*n)*PolyLog[3, I*E^(I*(c + d*x^n))]/(d^3*e*n
*x^(3*n)))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Maple [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx$$

input

```
int((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n)),x)
```

output

```
int((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(215) = 430$.

Time = 0.13 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.79

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx = \text{Too large to display}$$

input

```
integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n)),x, algorithm="fricas")
```

output

```

1/6*(2*a*d^3*e^(3*n - 1)*x^(3*n) - 6*I*b*d*e^(3*n - 1)*x^n*dilog(I*cos(d*x^n + c) + sin(d*x^n + c)) - 6*I*b*d*e^(3*n - 1)*x^n*dilog(I*cos(d*x^n + c) - sin(d*x^n + c)) + 6*I*b*d*e^(3*n - 1)*x^n*dilog(-I*cos(d*x^n + c) + sin(d*x^n + c)) + 6*I*b*d*e^(3*n - 1)*x^n*dilog(-I*cos(d*x^n + c) - sin(d*x^n + c)) + 3*b*c^2*e^(3*n - 1)*log(cos(d*x^n + c) + I*sin(d*x^n + c) + I) - 3*b*c^2*e^(3*n - 1)*log(cos(d*x^n + c) - I*sin(d*x^n + c) + I) + 3*b*c^2*e^(3*n - 1)*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + I) - 3*b*c^2*e^(3*n - 1)*log(-cos(d*x^n + c) - I*sin(d*x^n + c) + I) - 6*b*e^(3*n - 1)*polylog(3, I*cos(d*x^n + c) + sin(d*x^n + c)) + 6*b*e^(3*n - 1)*polylog(3, I*cos(d*x^n + c) - sin(d*x^n + c)) - 6*b*e^(3*n - 1)*polylog(3, -I*cos(d*x^n + c) + sin(d*x^n + c)) + 6*b*e^(3*n - 1)*polylog(3, -I*cos(d*x^n + c) - sin(d*x^n + c)) + 3*(b*d^2*e^(3*n - 1)*x^(2*n) - b*c^2*e^(3*n - 1))*log(I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - 3*(b*d^2*e^(3*n - 1)*x^(2*n) - b*c^2*e^(3*n - 1))*log(I*cos(d*x^n + c) - sin(d*x^n + c) + 1) + 3*(b*d^2*e^(3*n - 1)*x^(2*n) - b*c^2*e^(3*n - 1))*log(-I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - 3*(b*d^2*e^(3*n - 1)*x^(2*n) - b*c^2*e^(3*n - 1))*log(-I*cos(d*x^n + c) - sin(d*x^n + c) + 1))/(d^3*n)

```

Sympy [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx = \int (ex)^{3n-1} (a + b \sec(c + dx^n)) dx$$

input

```
integrate((e*x)**(-1+3*n)*(a+b*sec(c+d*x**n)),x)
```

output

```
Integral((e*x)**(3*n - 1)*(a + b*sec(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx = \int (b \sec(dx^n + c) + a)(ex)^{3n-1} dx$$

input

```
integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n)),x, algorithm="maxima")
```

output

```
2*b*e^(3*n)*integrate((x^(3*n)*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + x^(3*n)
*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + x^(3*n)*cos(d*x^n + c))/(e*x*cos(2*d*
x^n + 2*c)^2 + e*x*sin(2*d*x^n + 2*c)^2 + 2*e*x*cos(2*d*x^n + 2*c) + e*x),
x) + 1/3*(e*x)^(3*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx = \int (b \sec(dx^n + c) + a)(ex)^{3n-1} dx$$

input

```
integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n)),x, algorithm="giac")
```

output

```
integrate((b*sec(d*x^n + c) + a)*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx = \int \left(a + \frac{b}{\cos(c + dx^n)} \right) (ex)^{3n-1} dx$$

input

```
int((a + b/cos(c + d*x^n))*(e*x)^(3*n - 1),x)
```

output

```
int((a + b/cos(c + d*x^n))*(e*x)^(3*n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n)) dx = \frac{e^{3n} \left(x^{3n} a + x^{3n} b - 6 \left(\int \frac{x^{3n} \tan\left(\frac{x^n d + c}{2}\right)^2}{\tan\left(\frac{x^n d + c}{2}\right)^2 x - x} dx \right) b n \right)}{3en}$$

input `int((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n)),x)`

output `(e**(3*n)*(x**(3*n)*a + x**(3*n)*b - 6*int((x**(3*n)*tan((x**n*d + c)/2)**2)/(tan((x**n*d + c)/2)**2*x - x),x)*b*n)/(3*e*n)`

3.75 $\int (ex)^{-1+n} (a + b \sec (c + dx^n))^2 dx$

Optimal result	528
Mathematica [A] (verified)	528
Rubi [A] (verified)	529
Maple [C] (warning: unable to verify)	531
Fricas [A] (verification not implemented)	532
Sympy [F]	532
Maxima [F]	532
Giac [F]	533
Mupad [B] (verification not implemented)	533
Reduce [B] (verification not implemented)	534

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int (ex)^{-1+n} (a + b \sec (c + dx^n))^2 dx = \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \arctanh(\sin (c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tan (c + dx^n)}{den}$$

output `a^2*(e*x)^n/e/n+2*a*b*(e*x)^n*arctanh(sin(c+d*x^n))/d/e/n/(x^n)+b^2*(e*x)^n*tan(c+d*x^n)/d/e/n/(x^n)`

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int (ex)^{-1+n} (a + b \sec (c + dx^n))^2 dx = \frac{x^{-n}(ex)^n (a^2 dx^n + 2ab \coth^{-1}(\sin (c + dx^n)) + b^2 \tan (c + dx^n))}{den}$$

input `Integrate[(e*x)^(-1 + n)*(a + b*Sec[c + d*x^n])^2,x]`

output $((e*x)^n*(a^2*d*x^n + 2*a*b*ArcCoth[Sin[c + d*x^n]] + b^2*Tan[c + d*x^n]))/(d*e*n*x^n)$

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4696, 4692, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} (a + b \sec(c + dx^n))^2 dx$$

$$\downarrow 4696$$

$$\frac{x^{-n}(ex)^n \int x^{n-1}(a + b \sec(dx^n + c))^2 dx}{e}$$

$$\downarrow 4692$$

$$\frac{x^{-n}(ex)^n \int (a + b \sec(dx^n + c))^2 dx^n}{en}$$

$$\downarrow 3042$$

$$\frac{x^{-n}(ex)^n \int (a + b \csc(dx^n + c + \frac{\pi}{2}))^2 dx^n}{en}$$

$$\downarrow 4260$$

$$\frac{x^{-n}(ex)^n (2ab \int \sec(dx^n + c) dx^n + b^2 \int \sec^2(dx^n + c) dx^n + a^2 x^n)}{en}$$

$$\downarrow 3042$$

$$\frac{x^{-n}(ex)^n (2ab \int \csc(dx^n + c + \frac{\pi}{2}) dx^n + b^2 \int \csc(dx^n + c + \frac{\pi}{2})^2 dx^n + a^2 x^n)}{en}$$

$$\downarrow 4254$$

$$\frac{x^{-n}(ex)^n (2ab \int \csc(dx^n + c + \frac{\pi}{2}) dx^n - \frac{b^2 \int 1d(-\tan(dx^n+c))}{d} + a^2 x^n)}{en}$$

$$\downarrow 24$$

$$\frac{x^{-n}(ex)^n \left(2ab \int \csc(dx^n + c + \frac{\pi}{2}) dx^n + a^2x^n + \frac{b^2 \tan(c+dx^n)}{d} \right)}{en}$$

↓ 4257

$$\frac{x^{-n}(ex)^n \left(a^2x^n + \frac{2ab \operatorname{arctanh}(\sin(c+dx^n))}{d} + \frac{b^2 \tan(c+dx^n)}{d} \right)}{en}$$

input `Int[(e*x)^(-1 + n)*(a + b*Sec[c + d*x^n])^2,x]`

output `((e*x)^n*(a^2*x^n + (2*a*b*ArcTanh[Sin[c + d*x^n]])/d + (b^2*Tan[c + d*x^n])/d))/(e*n*x^n)`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4254 `Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

rule 4257 `Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

rule 4260 `Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Simp[a^2*x, x] + (Simp[2*a*b Int[Csc[c + d*x], x], x] + Simp[b^2 Int[Csc[c + d*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
-> Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

rule 4696 `Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
-> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n, p}, x]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.22 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.49

method	result
risch	$\frac{a^2 x e^{\frac{(-1+n)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(x) + 2 \ln(e))}{2}}}{n} + \frac{2 i x b^2 e^{(-1+n)}}{n}$

input `int((e*x)^(-1+n)*(a+b*sec(c+d*x^n))^2,x,method=_RETURNVERBOSE)`

output `a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(x)+2*ln(e)))+2*I*x*b^2*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(x)+2*ln(e)))/d/n/(x^n)/(1+exp(2*I*(c+d*x^n)))-4*I*arctan(exp(I*(c+d*x^n)))/d/e*e^n/n*b*a*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))`

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.43

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n))^2 dx$$

$$= \frac{a^2 de^{n-1} x^n \cos(dx^n + c) + abe^{n-1} \cos(dx^n + c) \log(\sin(dx^n + c) + 1) - abe^{n-1} \cos(dx^n + c) \log(-\sin(dx^n + c) + 1) + b^2 e^{n-1} \sin(dx^n + c)}{dn \cos(dx^n + c)}$$

input `integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")`

output `(a^2*d*e^(n - 1)*x^n*cos(d*x^n + c) + a*b*e^(n - 1)*cos(d*x^n + c)*log(sin(d*x^n + c) + 1) - a*b*e^(n - 1)*cos(d*x^n + c)*log(-sin(d*x^n + c) + 1) + b^2*e^(n - 1)*sin(d*x^n + c))/(d*n*cos(d*x^n + c))`

Sympy [F]

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n))^2 dx = \int (ex)^{n-1} (a + b \sec(c + dx^n))^2 dx$$

input `integrate((e*x)**(-1+n)*(a+b*sec(c+d*x**n))**2,x)`

output `Integral((e*x)**(n - 1)*(a + b*sec(c + d*x**n))**2, x)`

Maxima [F]

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n))^2 dx = \int (b \sec(dx^n + c) + a)^2 (ex)^{n-1} dx$$

input `integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")`

output

```
(e*x)^n*a^2/(e*n) + 2*(b^2*e^n*sin(2*d*x^n + 2*c) + 2*(a*b*d*e^(n + 1)*n*cos(2*d*x^n + 2*c)^2 + a*b*d*e^(n + 1)*n*sin(2*d*x^n + 2*c)^2 + 2*a*b*d*e^(n + 1)*n*cos(2*d*x^n + 2*c) + a*b*d*e^(n + 1)*n)*integrate((x^n*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + x^n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + x^n*cos(d*x^n + c))/(e*x*cos(2*d*x^n + 2*c)^2 + e*x*sin(2*d*x^n + 2*c)^2 + 2*e*x*cos(2*d*x^n + 2*c) + e*x), x))/(d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)
```

Giac [F]

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n))^2 dx = \int (b \sec(dx^n + c) + a)^2 (ex)^{n-1} dx$$

input

```
integrate((e*x)^(-1+n)*(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
```

output

```
integrate((b*sec(d*x^n + c) + a)^2*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 17.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.28

$$\begin{aligned} & \int (ex)^{-1+n} (a + b \sec(c + dx^n))^2 dx \\ &= \frac{a^2 x (ex)^{n-1}}{n} + \frac{b^2 x (ex)^{n-1} 2i}{dn x^n (e^{c 2i + dx^n 2i} + 1)} \\ &+ \frac{2abx \ln(-ab(ex)^{n-1} 4i - 4ab e^{c 1i} e^{dx^n 1i} (ex)^{n-1}) (ex)^{n-1}}{dn x^n} \\ &- \frac{2abx \ln(ab(ex)^{n-1} 4i - 4ab e^{c 1i} e^{dx^n 1i} (ex)^{n-1}) (ex)^{n-1}}{dn x^n} \end{aligned}$$

input

```
int((a + b/cos(c + d*x^n))^2*(e*x)^(n - 1), x)
```

output

```
(a^2*x*(e*x)^(n - 1))/n + (b^2*x*(e*x)^(n - 1)*2i)/(d*n*x^n*(exp(c*2i + d*x^n*2i) + 1)) + (2*a*b*x*log(- a*b*(e*x)^(n - 1)*4i - 4*a*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1)))/(d*n*x^n) - (2*a*b*x*log(a*b*(e*x)^(n - 1)*4i - 4*a*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1)))/(d*n*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int (ex)^{-1+n} (a + b \sec(c + dx^n))^2 dx$$

$$= \frac{e^n (x^n \cos(x^n d + c) a^2 d - 2 \cos(x^n d + c) \log(\tan(\frac{x^n d}{2} + \frac{c}{2}) - 1) ab + 2 \cos(x^n d + c) \log(\tan(\frac{x^n d}{2} + \frac{c}{2}) - 1) ab + 2 \cos(x^n d + c) \log(\tan(\frac{x^n d}{2} + \frac{c}{2}) + 1) ab + \sin(x^n d + c) b^2)}{\cos(x^n d + c) den}$$

input

```
int((e*x)^(-1+n)*(a+b*sec(c+d*x^n))^2,x)
```

output

```
(e**n*(x**n*cos(x**n*d + c)*a**2*d - 2*cos(x**n*d + c)*log(tan((x**n*d + c)/2) - 1)*a*b + 2*cos(x**n*d + c)*log(tan((x**n*d + c)/2) + 1)*a*b + sin(x**n*d + c)*b**2))/(cos(x**n*d + c)*d*e**n)
```

3.76 $\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx$

Optimal result	535
Mathematica [A] (warning: unable to verify)	536
Rubi [A] (verified)	536
Maple [C] (warning: unable to verify)	538
Fricas [B] (verification not implemented)	539
Sympy [F]	540
Maxima [F]	540
Giac [F]	541
Mupad [F(-1)]	541
Reduce [F]	542

Optimal result

Integrand size = 24, antiderivative size = 221

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx = \frac{a^2(ex)^{2n}}{2en} - \frac{4iabx^{-n}(ex)^{2n} \arctan(e^{i(c+dx^n)})}{den} + \frac{b^2x^{-2n}(ex)^{2n} \log(\cos(c + dx^n))}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n} \text{PolyLog}(2, -ie^{i(c+dx^n)})}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n} \text{PolyLog}(2, ie^{i(c+dx^n)})}{d^2en} + \frac{b^2x^{-n}(ex)^{2n} \tan(c + dx^n)}{den}$$

output

```
1/2*a^2*(e*x)^(2*n)/e/n-4*I*a*b*(e*x)^(2*n)*arctan(exp(I*(c+d*x^n)))/d/e/n
/(x^n)+b^2*(e*x)^(2*n)*ln(cos(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*a*b*(e*x)^(2
*n)*polylog(2,-I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*I*a*b*(e*x)^(2*n)*p
olylog(2,I*exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))+b^2*(e*x)^(2*n)*tan(c+d*x^n
)/d/e/n/(x^n)
```


Mathematica [A] (warning: unable to verify)

Time = 4.05 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.57

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx$$

$$= \frac{x^{-2n} (ex)^{2n} \left(8ab \arctan(\cot(c)) \operatorname{arctanh}(\sin(c) + \cos(c) \tan(\frac{dx^n}{2})) - \frac{4ab \csc(c) ((dx^n - \arctan(\cot(c))) (\log(1 - e^{i(dx^n - \arctan(\cot(c))))} \right)}{2} \right)}{e^{2n}}$$

input `Integrate[(e*x)^(-1 + 2*n)*(a + b*Sec[c + d*x^n])^2,x]`

output `((e*x)^(2*n)*(8*a*b*ArcTan[Cot[c]]*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x^n)/2]] - (4*a*b*Csc[c]*((d*x^n - ArcTan[Cot[c]])*(Log[1 - E^(I*(d*x^n - ArcTan[Cot[c]])]) - Log[1 + E^(I*(d*x^n - ArcTan[Cot[c]])]) + I*PolyLog[2, -E^(I*(d*x^n - ArcTan[Cot[c]])]) - I*PolyLog[2, E^(I*(d*x^n - ArcTan[Cot[c]])])]) / Sqrt[Csc[c]^2 + (2*b^2*d*x^n*Sin[(d*x^n)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x^n)/2] - Sin[(c + d*x^n)/2])) + (2*b^2*d*x^n*Sin[(d*x^n)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x^n)/2] + Sin[(c + d*x^n)/2])) - 2*b^2*d*x^n*Tan[c] + d*x^n*(a^2*d*x^n + 2*b^2*Tan[c]) + 2*b^2*(Log[Cos[c + d*x^n]] + d*x^n*Tan[c])))/(2*d^2*e^n*x^(2*n))`

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4696, 4692, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} (a + b \sec(c + dx^n))^2 dx$$

$$\downarrow 4696$$

$$\frac{x^{-2n} (ex)^{2n} \int x^{2n-1} (a + b \sec(dx^n + c))^2 dx}{e}$$

$$\downarrow 4692$$

$$\begin{array}{c}
 \frac{x^{-2n}(ex)^{2n} \int x^n (a + b \sec(dx^n + c))^2 dx^n}{en} \\
 \downarrow 3042 \\
 \frac{x^{-2n}(ex)^{2n} \int x^n (a + b \csc(dx^n + c + \frac{\pi}{2}))^2 dx^n}{en} \\
 \downarrow 4678 \\
 \frac{x^{-2n}(ex)^{2n} \int (a^2 x^n + b^2 \sec^2(dx^n + c) x^n + 2ab \sec(dx^n + c) x^n) dx^n}{en} \\
 \downarrow 2009 \\
 \frac{x^{-2n}(ex)^{2n} \left(\frac{1}{2} a^2 x^{2n} - \frac{4iabx^n \arctan\left(e^{i(c+dx^n)}\right)}{d} + \frac{2iab \operatorname{PolyLog}\left(2, -ie^{i(dx^n+c)}\right)}{d^2} - \frac{2iab \operatorname{PolyLog}\left(2, ie^{i(dx^n+c)}\right)}{d^2} + \frac{b^2 \log(\cos(c+dx^n))}{d^2} \right)}{en}
 \end{array}$$

input `Int[(e*x)^(-1 + 2*n)*(a + b*Sec[c + d*x^n])^2,x]`

output `((e*x)^(2*n)*((a^2*x^(2*n))/2 - ((4*I)*a*b*x^n*ArcTan[E^(I*(c + d*x^n))])/d + (b^2*Log[Cos[c + d*x^n]])/d^2 + ((2*I)*a*b*PolyLog[2, (-I)*E^(I*(c + d*x^n))])/d^2 - ((2*I)*a*b*PolyLog[2, I*E^(I*(c + d*x^n))])/d^2 + (b^2*x^n*Tan[c + d*x^n])/d))/(e*n*x^(2*n))`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e._) + (f._)*(x_)]*(b._) + (a._))^(n._)*((c._) + (d._)*(x_))^(m._), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

rule 4696 `Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.18 (sec) , antiderivative size = 1116, normalized size of antiderivative = 5.05

method	result	size
risch	Expression too large to display	1116

input `int((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n))^2,x,method=_RETURNVERBOSE)`

output

```

1/2*a^2/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn
(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln
(x)+2*ln(e))+2*I*x*b^2*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e
*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I
e*x)^3*Pi+2*ln(x)+2*ln(e)))/d/n/(x^n)/(1+exp(2*I*(c+d*x^n)))+2*I*b/d/n*(e^
n)^2/e*a*ln(1+exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*(-1)^(-1/2*csgn(I*e)*csgn(
I*e*x)^2)*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e)*csgn(I*x
)*csgn(I*e*x))*(-exp(2*I*c))^(1/2)*x^n*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*
Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I
e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e*x)^3+2*c))-2*I*b/d/n*(e^n)^2/e*a*ln(1
-exp(I*x^n*d)*(-exp(2*I*c))^(1/2))*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1
)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x)
)*(-exp(2*I*c))^(1/2)*x^n*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I
e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*
csgn(I*e*x)-Pi*csgn(I*e*x)^3+2*c))+2*b/d^2/n*(e^n)^2/e*a*dilog(1+exp(I*x^n
*d)*(-exp(2*I*c))^(1/2))*(-1)^(-1/2*csgn(I*e)*csgn(I*e*x)^2)*(-1)^(-1/2*cs
gn(I*x)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(-exp(2*
I*c))^(1/2)*exp(-1/2*I*(2*Pi*n*csgn(I*e*x)^3-2*Pi*n*csgn(I*e)*csgn(I*e*x)^
2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi
*csgn(I*e*x)^3+2*c))-2*b/d^2/n*(e^n)^2/e*a*dilog(1-exp(I*x^n*d)*(-exp(2...

```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(207) = 414$.

Time = 0.12 (sec) , antiderivative size = 656, normalized size of antiderivative = 2.97

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx = \text{Too large to display}$$

input

```
integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")
```

output

```

1/2*(a^2*d^2*e^(2*n - 1)*x^(2*n)*cos(d*x^n + c) + 2*b^2*d*e^(2*n - 1)*x^n*
sin(d*x^n + c) - 2*I*a*b*e^(2*n - 1)*cos(d*x^n + c)*dilog(I*cos(d*x^n + c)
+ sin(d*x^n + c)) - 2*I*a*b*e^(2*n - 1)*cos(d*x^n + c)*dilog(I*cos(d*x^n
+ c) - sin(d*x^n + c)) + 2*I*a*b*e^(2*n - 1)*cos(d*x^n + c)*dilog(-I*cos(d
*x^n + c) + sin(d*x^n + c)) + 2*I*a*b*e^(2*n - 1)*cos(d*x^n + c)*dilog(-I*
cos(d*x^n + c) - sin(d*x^n + c)) - (2*a*b*c - b^2)*e^(2*n - 1)*cos(d*x^n +
c)*log(cos(d*x^n + c) + I*sin(d*x^n + c) + I) + (2*a*b*c + b^2)*e^(2*n -
1)*cos(d*x^n + c)*log(cos(d*x^n + c) - I*sin(d*x^n + c) + I) - (2*a*b*c -
b^2)*e^(2*n - 1)*cos(d*x^n + c)*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + I
) + (2*a*b*c + b^2)*e^(2*n - 1)*cos(d*x^n + c)*log(-cos(d*x^n + c) - I*sin
(d*x^n + c) + I) + 2*(a*b*d*e^(2*n - 1)*x^n + a*b*c*e^(2*n - 1))*cos(d*x^n
+ c)*log(I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - 2*(a*b*d*e^(2*n - 1)*x^n
+ a*b*c*e^(2*n - 1))*cos(d*x^n + c)*log(I*cos(d*x^n + c) - sin(d*x^n + c
) + 1) + 2*(a*b*d*e^(2*n - 1)*x^n + a*b*c*e^(2*n - 1))*cos(d*x^n + c)*log(
-I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - 2*(a*b*d*e^(2*n - 1)*x^n + a*b*c
*e^(2*n - 1))*cos(d*x^n + c)*log(-I*cos(d*x^n + c) - sin(d*x^n + c) + 1))/
(d^2*n*cos(d*x^n + c))

```

Sympy [F]

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx = \int (ex)^{2n-1} (a + b \sec(c + dx^n))^2 dx$$

input

```
integrate((e*x)**(-1+2*n)*(a+b*sec(c+d*x**n))**2,x)
```

output

```
Integral((e*x)**(2*n - 1)*(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx = \int (b \sec(dx^n + c) + a)^2 (ex)^{2n-1} dx$$

input

```
integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")
```

output

```
1/2*(e*x)^(2*n)*a^2/(e*n) + (2*b^2*e^(2*n)*x^n*sin(2*d*x^n + 2*c) + (d*e*n
*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*cos(2*d*x^n +
2*c) + d*e*n)*integrate(2*(2*a*b*d*e^(2*n)*x^(2*n)*cos(2*d*x^n + 2*c)*cos
(d*x^n + c) + 2*a*b*d*e^(2*n)*x^(2*n)*cos(d*x^n + c) + (2*a*b*d*e^(2*n)*x^
(2*n)*sin(d*x^n + c) - b^2*e^(2*n)*x^n*sin(2*d*x^n + 2*c))/(d*e*x*cos(2*d
*x^n + 2*c)^2 + d*e*x*sin(2*d*x^n + 2*c)^2 + 2*d*e*x*cos(2*d*x^n + 2*c) +
d*e*x), x))/(d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d
*e*n*cos(2*d*x^n + 2*c) + d*e*n)
```

Giac [F]

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx = \int (b \sec(dx^n + c) + a)^2 (ex)^{2n-1} dx$$

input

```
integrate((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
```

output

```
integrate((b*sec(d*x^n + c) + a)^2*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx = \int \left(a + \frac{b}{\cos(c + dx^n)} \right)^2 (ex)^{2n-1} dx$$

input

```
int((a + b/cos(c + d*x^n))^2*(e*x)^(2*n - 1),x)
```

output

```
int((a + b/cos(c + d*x^n))^2*(e*x)^(2*n - 1), x)
```

Reduce [F]

$$\int (ex)^{-1+2n} (a + b \sec(c + dx^n))^2 dx$$

$$= \frac{e^{2n} \left(x^{2n} \cos(x^n d + c) a^2 d^2 + 3x^{2n} \cos(x^n d + c) ab d^2 - 8 \cos(x^n d + c) \left(\int \frac{x^{2n} \tan\left(\frac{x^n d + c}{2}\right)^4}{\tan\left(\frac{x^n d + c}{2}\right)^4 x - 2 \tan\left(\frac{x^n d + c}{2}\right)^2 x + x} \right) \right)}{2}$$

input `int((e*x)^(-1+2*n)*(a+b*sec(c+d*x^n))^2,x)`

output

```
(e**(2*n)*(x**(2*n)*cos(x**n*d + c)*a**2*d**2 + 3*x**(2*n)*cos(x**n*d + c)
*a*b*d**2 - 8*cos(x**n*d + c)*int((x**(2*n)*tan((x**n*d + c)/2)**4)/(tan((
x**n*d + c)/2)**4*x - 2*tan((x**n*d + c)/2)**2*x + x),x)*a*b*d**2*n + 16*c
os(x**n*d + c)*int((x**(2*n)*tan((x**n*d + c)/2)**2)/(tan((x**n*d + c)/2)*
**4*x - 2*tan((x**n*d + c)/2)**2*x + x),x)*a*b*d**2*n + 2*cos(x**n*d + c)*l
og(tan((x**n*d + c)/2)**2 + 1)*a*b - 2*cos(x**n*d + c)*log(tan((x**n*d + c
)/2)**2 + 1)*b**2 - 2*cos(x**n*d + c)*log(tan((x**n*d + c)/2) - 1)*a*b + 2
*cos(x**n*d + c)*log(tan((x**n*d + c)/2) - 1)*b**2 - 2*cos(x**n*d + c)*log
(tan((x**n*d + c)/2) + 1)*a*b + 2*cos(x**n*d + c)*log(tan((x**n*d + c)/2)
+ 1)*b**2 - 2*x**n*sin(x**n*d + c)*a*b*d + 2*x**n*sin(x**n*d + c)*b**2*d)
/(2*cos(x**n*d + c)*d**2*e**n)
```

3.77 $\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx$

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Optimal result

Integrand size = 24, antiderivative size = 390

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx = \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} - \frac{4iabx^{-n}(ex)^{3n} \arctan(e^{i(c+dx^n)})}{den} + \frac{2b^2x^{-2n}(ex)^{3n} \log(1 + e^{2i(c+dx^n)})}{d^2en} + \frac{4iabx^{-2n}(ex)^{3n} \text{PolyLog}(2, -ie^{i(c+dx^n)})}{d^2en} - \frac{4iabx^{-2n}(ex)^{3n} \text{PolyLog}(2, ie^{i(c+dx^n)})}{d^2en} - \frac{ib^2x^{-3n}(ex)^{3n} \text{PolyLog}(2, -e^{2i(c+dx^n)})}{d^3en} - \frac{4abx^{-3n}(ex)^{3n} \text{PolyLog}(3, -ie^{i(c+dx^n)})}{d^3en} + \frac{4abx^{-3n}(ex)^{3n} \text{PolyLog}(3, ie^{i(c+dx^n)})}{d^3en} + \frac{b^2x^{-n}(ex)^{3n} \tan(c + dx^n)}{den}$$

output

```

1/3*a^2*(e*x)^(3*n)/e/n-I*b^2*(e*x)^(3*n)/d/e/n/(x^n)-4*I*a*b*(e*x)^(3*n)*
arctan(exp(I*(c+d*x^n)))/d/e/n/(x^n)+2*b^2*(e*x)^(3*n)*ln(1+exp(2*I*(c+d*x
^n)))/d^2/e/n/(x^(2*n))+4*I*a*b*(e*x)^(3*n)*polylog(2,-I*exp(I*(c+d*x^n)))
/d^2/e/n/(x^(2*n))-4*I*a*b*(e*x)^(3*n)*polylog(2,I*exp(I*(c+d*x^n)))/d^2/e
/n/(x^(2*n))-I*b^2*(e*x)^(3*n)*polylog(2,-exp(2*I*(c+d*x^n)))/d^3/e/n/(x^(
3*n))-4*a*b*(e*x)^(3*n)*polylog(3,-I*exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))+4
*a*b*(e*x)^(3*n)*polylog(3,I*exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))+b^2*(e*x)
^(3*n)*tan(c+d*x^n)/d/e/n/(x^n)

```

Mathematica [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx = \int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx$$

input

```
Integrate[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n])^2,x]
```

output

```
Integrate[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n])^2, x]
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4696, 4692, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int (ex)^{3n-1} (a + b \sec(c + dx^n))^2 dx \\
 \downarrow 4696 \\
 \frac{x^{-3n} (ex)^{3n} \int x^{3n-1} (a + b \sec(dx^n + c))^2 dx}{e} \\
 \downarrow 4692
 \end{array}$$

$$\begin{array}{c}
 \frac{x^{-3n}(ex)^{3n} \int x^{2n}(a + b \sec(dx^n + c))^2 dx^n}{en} \\
 \downarrow 3042 \\
 \frac{x^{-3n}(ex)^{3n} \int x^{2n}(a + b \csc(dx^n + c + \frac{\pi}{2}))^2 dx^n}{en} \\
 \downarrow 4678 \\
 \frac{x^{-3n}(ex)^{3n} \int (a^2x^{2n} + b^2 \sec^2(dx^n + c)x^{2n} + 2ab \sec(dx^n + c)x^{2n}) dx^n}{en} \\
 \downarrow 2009 \\
 x^{-3n}(ex)^{3n} \left(\frac{1}{3}a^2x^{3n} - \frac{4iabx^{2n} \arctan(e^{i(c+dx^n)})}{d} - \frac{4ab \operatorname{PolyLog}(3, -ie^{i(dx^n+c)})}{d^3} + \frac{4ab \operatorname{PolyLog}(3, ie^{i(dx^n+c)})}{d^3} + \frac{4iabx^n \operatorname{PolyLog}(\dots)}{d^3} \right)
 \end{array}$$

input `Int[(e*x)^(-1 + 3*n)*(a + b*Sec[c + d*x^n])^2,x]`

output `((e*x)^(3*n)*((-1)*b^2*x^(2*n))/d + (a^2*x^(3*n))/3 - ((4*I)*a*b*x^(2*n)*ArcTan[E^(I*(c + d*x^n))])/d + (2*b^2*x^n*Log[1 + E^((2*I)*(c + d*x^n))])/d^2 + ((4*I)*a*b*x^n*PolyLog[2, (-I)*E^(I*(c + d*x^n))])/d^2 - ((4*I)*a*b*x^n*PolyLog[2, I*E^(I*(c + d*x^n))])/d^2 - (I*b^2*PolyLog[2, -E^((2*I)*(c + d*x^n))])/d^3 - (4*a*b*PolyLog[3, (-I)*E^(I*(c + d*x^n))])/d^3 + (4*a*b*PolyLog[3, I*E^(I*(c + d*x^n))])/d^3 + (b^2*x^(2*n)*Tan[c + d*x^n])/d)/(e*n*x^(3*n))`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

rule 4696 `Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol] := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

Maple [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx$$

input `int((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x)`

output `int((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1032 vs. $2(365) = 730$.

Time = 0.15 (sec) , antiderivative size = 1032, normalized size of antiderivative = 2.65

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx = \text{Too large to display}$$

input `integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")`

output

```

1/3*(a^2*d^3*e^(3*n - 1)*x^(3*n)*cos(d*x^n + c) + 3*b^2*d^2*e^(3*n - 1)*x^(
(2*n))*sin(d*x^n + c) - 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*polylog(3, I*cos(d
*x^n + c) + sin(d*x^n + c)) + 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*polylog(3,
I*cos(d*x^n + c) - sin(d*x^n + c)) - 6*a*b*e^(3*n - 1)*cos(d*x^n + c)*poly
log(3, -I*cos(d*x^n + c) + sin(d*x^n + c)) + 6*a*b*e^(3*n - 1)*cos(d*x^n +
c)*polylog(3, -I*cos(d*x^n + c) - sin(d*x^n + c)) + 3*(a*b*c^2 - b^2*c)*e
^(3*n - 1)*cos(d*x^n + c)*log(cos(d*x^n + c) + I*sin(d*x^n + c) + I) - 3*(
a*b*c^2 + b^2*c)*e^(3*n - 1)*cos(d*x^n + c)*log(cos(d*x^n + c) - I*sin(d*x
^n + c) + I) + 3*(a*b*c^2 - b^2*c)*e^(3*n - 1)*cos(d*x^n + c)*log(-cos(d*x
^n + c) + I*sin(d*x^n + c) + I) - 3*(a*b*c^2 + b^2*c)*e^(3*n - 1)*cos(d*x
^n + c)*log(-cos(d*x^n + c) - I*sin(d*x^n + c) + I) - 3*(2*I*a*b*d*e^(3*n -
1)*x^n - I*b^2*e^(3*n - 1))*cos(d*x^n + c)*dilog(I*cos(d*x^n + c) + sin(d
*x^n + c)) - 3*(2*I*a*b*d*e^(3*n - 1)*x^n + I*b^2*e^(3*n - 1))*cos(d*x^n +
c)*dilog(I*cos(d*x^n + c) - sin(d*x^n + c)) - 3*(-2*I*a*b*d*e^(3*n - 1)*x
^n + I*b^2*e^(3*n - 1))*cos(d*x^n + c)*dilog(-I*cos(d*x^n + c) + sin(d*x^n
+ c)) - 3*(-2*I*a*b*d*e^(3*n - 1)*x^n - I*b^2*e^(3*n - 1))*cos(d*x^n + c)
*dilog(-I*cos(d*x^n + c) - sin(d*x^n + c)) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n
) + b^2*d*e^(3*n - 1)*x^n - (a*b*c^2 - b^2*c)*e^(3*n - 1))*cos(d*x^n + c)*
log(I*cos(d*x^n + c) + sin(d*x^n + c) + 1) - 3*(a*b*d^2*e^(3*n - 1)*x^(2*n
) - b^2*d*e^(3*n - 1)*x^n - (a*b*c^2 + b^2*c)*e^(3*n - 1))*cos(d*x^n + ...

```

Sympy [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx = \int (ex)^{3n-1} (a + b \sec(c + dx^n))^2 dx$$

input

```
integrate((e*x)**(-1+3*n)*(a+b*sec(c+d*x**n))**2,x)
```

output

```
Integral((e*x)**(3*n - 1)*(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx = \int (b \sec(dx^n + c) + a)^2 (ex)^{3n-1} dx$$

input `integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")`

output `1/3*(e*x)^(3*n)*a^2/(e*n) + (2*b^2*e^(3*n)*x^(2*n)*sin(2*d*x^n + 2*c) + (d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)*integrate(4*(a*b*d*e^(3*n)*x^(3*n)*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + a*b*d*e^(3*n)*x^(3*n)*cos(d*x^n + c) + (a*b*d*e^(3*n)*x^(3*n)*sin(d*x^n + c) - b^2*e^(3*n)*x^(2*n)*sin(2*d*x^n + 2*c))/(d*e*x*cos(2*d*x^n + 2*c)^2 + d*e*x*sin(2*d*x^n + 2*c)^2 + 2*d*e*x*cos(2*d*x^n + 2*c) + d*e*x), x)/(d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 + 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)`

Giac [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx = \int (b \sec(dx^n + c) + a)^2 (ex)^{3n-1} dx$$

input `integrate((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x, algorithm="giac")`

output `integrate((b*sec(d*x^n + c) + a)^2*(e*x)^(3*n - 1), x)`

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx = \int \left(a + \frac{b}{\cos(c + dx^n)} \right)^2 (ex)^{3n-1} dx$$

input `int((a + b/cos(c + d*x^n))^2*(e*x)^(3*n - 1), x)`

output `int((a + b/cos(c + d*x^n))^2*(e*x)^(3*n - 1), x)`

Reduce [F]

$$\int (ex)^{-1+3n} (a + b \sec(c + dx^n))^2 dx$$

$$= e^{3n} \left(x^{3n} \cos(x^n d + c) a^2 d^3 + 3x^{3n} \cos(x^n d + c) a b d^3 - 12 \cos(x^n d + c) \left(\int \frac{x^{3n} \tan\left(\frac{x^n d + c}{2}\right)^4}{\tan\left(\frac{x^n d + c}{2}\right)^4 x - 2 \tan\left(\frac{x^n d + c}{2}\right)^2 x + x} dx \right) \right)$$

input `int((e*x)^(-1+3*n)*(a+b*sec(c+d*x^n))^2,x)`

output `(e**(3*n)*(x**(3*n)*cos(x**n*d + c)*a**2*d**3 + 3*x**(3*n)*cos(x**n*d + c)*a*b*d**3 - 12*cos(x**n*d + c)*int((x**(3*n)*tan((x**n*d + c)/2)**4)/(tan((x**n*d + c)/2)**4*x - 2*tan((x**n*d + c)/2)**2*x + x),x)*a*b*d**3*n + 24*cos(x**n*d + c)*int((x**(3*n)*tan((x**n*d + c)/2)**2)/(tan((x**n*d + c)/2)**4*x - 2*tan((x**n*d + c)/2)**2*x + x),x)*a*b*d**3*n + 24*cos(x**n*d + c)*int((x**(2*n)*tan((x**n*d + c)/2))/(tan((x**n*d + c)/2)**4*x - 2*tan((x**n*d + c)/2)**2*x + x),x)*a*b*d**2*n - 24*cos(x**n*d + c)*int((x**(2*n)*tan((x**n*d + c)/2))/(tan((x**n*d + c)/2)**4*x - 2*tan((x**n*d + c)/2)**2*x + x),x)*b**2*d**2*n - 6*cos(x**n*d + c)*log(tan((x**n*d + c)/2) - 1)*a*b + 6*cos(x**n*d + c)*log(tan((x**n*d + c)/2) - 1)*b**2 + 6*cos(x**n*d + c)*log(tan((x**n*d + c)/2) + 1)*a*b - 6*cos(x**n*d + c)*log(tan((x**n*d + c)/2) + 1)*b**2 - 3*x**(2*n)*sin(x**n*d + c)*a*b*d**2 + 3*x**(2*n)*sin(x**n*d + c)*b**2*d**2 - 6*x**n*a*b*d + 6*x**n*b**2*d))/(3*cos(x**n*d + c)*d**3*e**n)`

3.78 $\int \frac{(ex)^{-1+n}}{a+b \sec(c+dx^n)} dx$

Optimal result	550
Mathematica [A] (verified)	550
Rubi [A] (verified)	551
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Reduce [B] (verification not implemented)	556

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{(ex)^{-1+n}}{a+b \sec(c+dx^n)} dx = \frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b} - b\sqrt{a+b}den}$$

output

```
(e*x)^n/a/e/n-2*b*(e*x)^n*arctanh((a-b)^(1/2)*tan(1/2*c+1/2*d*x^n)/(a+b)^(1/2))/a/(a-b)^(1/2)/(a+b)^(1/2)/d/e/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^{-1+n}}{a+b \sec(c+dx^n)} dx = \frac{(ex)^n \left(d + cx^{-n} + \frac{2bx^{-n} \operatorname{arctanh}\left(\frac{(-a+b) \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \right)}{aden}$$

input

```
Integrate[(e*x)^(-1 + n)/(a + b*Sec[c + d*x^n]),x]
```

output $((e*x)^n*(d + c/x^n + (2*b*ArcTanh[(-a + b)*Tan[(c + d*x^n)/2]]/Sqrt[a^2 - b^2]))/(Sqrt[a^2 - b^2]*x^n))/(a*d*e^n)$

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4696, 4692, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{n-1}}{a + b \sec(c + dx^n)} dx$$

$$\downarrow 4696$$

$$\frac{x^{-n}(ex)^n \int \frac{x^{n-1}}{a+b \sec(dx^n+c)} dx}{e}$$

$$\downarrow 4692$$

$$\frac{x^{-n}(ex)^n \int \frac{1}{a+b \sec(dx^n+c)} dx^n}{en}$$

$$\downarrow 3042$$

$$\frac{x^{-n}(ex)^n \int \frac{1}{a+b \csc(dx^n+c+\frac{\pi}{2})} dx^n}{en}$$

$$\downarrow 4270$$

$$\frac{x^{-n}(ex)^n \left(\frac{x^n}{a} - \frac{\int \frac{1}{a \cos(dx^n+c)+1} dx^n}{b} \right)}{en}$$

$$\downarrow 3042$$

$$\frac{x^{-n}(ex)^n \left(\frac{x^n}{a} - \frac{\int \frac{1}{a \sin(dx^n+c+\frac{\pi}{2})+1} dx^n}{b} \right)}{en}$$

$$\downarrow 3138$$

$$\frac{x^{-n}(ex)^n \left(\frac{x^n}{a} - \frac{2 \int \frac{1}{(1-\frac{a}{b})x^{2n} + \frac{a+b}{b}} d \tan\left(\frac{1}{2}(dx^n+c)\right)}{ad} \right)}{en}$$

↓ 221

$$\frac{x^{-n}(ex)^n \left(\frac{x^n}{a} - \frac{2b \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}} \right)}{en}$$

input `Int[(e*x)^(-1 + n)/(a + b*Sec[c + d*x^n]),x]`

output `((e*x)^n*(x^n/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x^n)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(e*n*x^n)`

Defintions of rubi rules used

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3138 `Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 4270 `Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^-1, x_Symbol] := Simp[x/a, x] - Simp[1/a Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

```
rule 4692 Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

```
rule 4696 Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.61

method	result
risch	$\frac{x e^{\frac{(-1+n)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(x) + 2 \ln(e))}{2}}}{an} + \frac{2i \arctan\left(\frac{2ae}{2}\right)}{2}$

```
input int((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
output 1/a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(x)+2*ln(e))+2*I*arctan(1/2*(2*a*exp(I*(d*x^n+2*c))+2*exp(I*c)*b)/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/d/e*e^n/n/a*b*exp(1/2*I*(-Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I*e)*csgn(I*e*x)^2+Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x)*csgn(I*e*x)^2+Pi*csgn(I*e*x)^3+2*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.45

$$\int \frac{(ex)^{-1+n}}{a + b \sec(c + dx^n)} dx$$

$$= \frac{2(a^2 - b^2)de^{n-1}x^n + \sqrt{a^2 - b^2}be^{n-1} \log\left(\frac{2ab \cos(dx^n+c) - (a^2 - 2b^2) \cos(dx^n+c)^2 + 2a^2 - b^2 - 2(\sqrt{a^2 - b^2}b \cos(dx^n+c) + \sqrt{a^2 - b^2})}{a^2 \cos(dx^n+c)^2 + 2ab \cos(dx^n+c) + b^2}\right)}{2(a^3 - ab^2)dn}$$

input `integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x, algorithm="fricas")`

output `[1/2*(2*(a^2 - b^2)*d*e^(n - 1)*x^n + sqrt(a^2 - b^2)*b*e^(n - 1)*log((2*a*b*cos(d*x^n + c) - (a^2 - 2*b^2)*cos(d*x^n + c)^2 + 2*a^2 - b^2 - 2*(sqrt(a^2 - b^2)*b*cos(d*x^n + c) + sqrt(a^2 - b^2)*a)*sin(d*x^n + c))/(a^2*cos(d*x^n + c)^2 + 2*a*b*cos(d*x^n + c) + b^2)))/((a^3 - a*b^2)*d*n), ((a^2 - b^2)*d*e^(n - 1)*x^n - sqrt(-a^2 + b^2)*b*e^(n - 1)*arctan(-(sqrt(-a^2 + b^2)*b*cos(d*x^n + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*sin(d*x^n + c))))/((a^3 - a*b^2)*d*n)]`

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{n-1}}{a + b \sec(c + dx^n)} dx$$

input `integrate((e*x)**(-1+n)/(a+b*sec(c+d*x**n)),x)`

output `Integral((e*x)**(n - 1)/(a + b*sec(c + d*x**n)), x)`

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{n-1}}{b \sec(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x, algorithm="maxima")`

output `-(2*a*b*e^(n + 1)*n*integrate((a*x^n*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + 2*b*x^n*cos(d*x^n + c)^2 + a*x^n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 2*b*x^n*sin(d*x^n + c)^2 + a*x^n*cos(d*x^n + c)))/(a^3*e*x*cos(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*cos(d*x^n + c)^2 + a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a^2*b*e*x*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*a*b^2*e*x*sin(d*x^n + c)^2 + 4*a^2*b*e*x*cos(d*x^n + c) + a^3*e*x + 2*(2*a^2*b*e*x*cos(d*x^n + c) + a^3*e*x)*cos(2*d*x^n + 2*c)), x) - e^n*x^n)/(a*e^n)`

Giac [F]

$$\int \frac{(ex)^{-1+n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{n-1}}{b \sec(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x, algorithm="giac")`

output `integrate((e*x)^(n - 1)/(b*sec(d*x^n + c) + a), x)`

Mupad [B] (verification not implemented)

Time = 17.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.56

$$\int \frac{(ex)^{-1+n}}{a + b \sec(c + dx^n)} dx$$

$$= \frac{x(ex)^{n-1}}{an} + \frac{bx \ln \left(2bx e^{c \operatorname{li} e^{dx^n \operatorname{li} (ex)^{n-1}} - \frac{bx(a+be^{c \operatorname{li} e^{dx^n \operatorname{li} (ex)^{n-1} 2i})}{\sqrt{a+b}\sqrt{a-b}} \right) (ex)^{n-1}}{adnx^n \sqrt{a+b}\sqrt{a-b}}$$

$$- \frac{bx \ln \left(2bx e^{c \operatorname{li} e^{dx^n \operatorname{li} (ex)^{n-1}} + \frac{bx(a+be^{c \operatorname{li} e^{dx^n \operatorname{li} (ex)^{n-1} 2i})}{\sqrt{a+b}\sqrt{a-b}} \right) (ex)^{n-1}}{adnx^n \sqrt{a+b}\sqrt{a-b}}$$

input `int((e*x)^(n - 1)/(a + b/cos(c + d*x^n)),x)`output `(x*(e*x)^(n - 1))/(a*n) + (b*x*log(2*b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1) - (b*x*(a + b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n - 1)*2i)/((a + b)^(1/2)*(a - b)^(1/2)))*(e*x)^(n - 1))/(a*d*n*x^n*(a + b)^(1/2)*(a - b)^(1/2)) - (b*x*log(2*b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1) + (b*x*(a + b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n - 1)*2i)/((a + b)^(1/2)*(a - b)^(1/2)))*(e*x)^(n - 1))/(a*d*n*x^n*(a + b)^(1/2)*(a - b)^(1/2))`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{(ex)^{-1+n}}{a + b \sec(c + dx^n)} dx$$

$$= \frac{e^n \left(-2\sqrt{-a^2 + b^2} \operatorname{atan} \left(\frac{\tan\left(\frac{x^n d + c}{2}\right)a - \tan\left(\frac{x^n d + c}{2}\right)b}{\sqrt{-a^2 + b^2}} \right) b + x^n a^2 d - x^n b^2 d \right)}{aden(a^2 - b^2)}$$

input `int((e*x)^(-1+n)/(a+b*sec(c+d*x^n)),x)`output `(e**n*(-2*sqrt(-a**2 + b**2)*atan((tan((x**n*d + c)/2)*a - tan((x**n*d + c)/2)*b)/sqrt(-a**2 + b**2))*b + x**n*a**2*d - x**n*b**2*d)/(a*d*e*n*(a**2 - b**2))`

3.79 $\int \frac{(ex)^{-1+2n}}{a+b \sec(c+dx^n)} dx$

Optimal result	557
Mathematica [B] (warning: unable to verify)	558
Rubi [A] (verified)	559
Maple [C] (warning: unable to verify)	561
Fricas [B] (verification not implemented)	561
Sympy [F]	562
Maxima [F]	563
Giac [F]	563
Mupad [F(-1)]	563
Reduce [F]	564

Optimal result

Integrand size = 24, antiderivative size = 328

$$\int \frac{(ex)^{-1+2n}}{a+b \sec(c+dx^n)} dx = \frac{(ex)^{2n}}{2aen} + \frac{ibx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{ibx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-2n}(ex)^{2n} \text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{bx^{-2n}(ex)^{2n} \text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$

output

```
1/2*(e*x)^(2*n)/a/e/n+I*b*(e*x)^(2*n)*ln(1+a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d/e/n/(x^n)-I*b*(e*x)^(2*n)*ln(1+a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d/e/n/(x^n)+b*(e*x)^(2*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))-b*(e*x)^(2*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a/(-a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 861 vs. $2(328) = 656$.

Time = 1.60 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.62

$$\int \frac{(ex)^{-1+2n}}{a + b \sec(c + dx^n)} dx = \text{Too large to display}$$

input `Integrate[(e*x)^(-1 + 2*n)/(a + b*Sec[c + d*x^n]),x]`

output

```
((e*x)^(2*n)*(b + a*Cos[c + d*x^n])*(1 - (2*b*(2*(c + d*x^n)*ArcTanh[((a +
b)*Cot[(c + d*x^n)/2]])/Sqrt[a^2 - b^2]] - 2*(c + ArcCos[-(b/a)])*ArcTanh[
((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] - (2*I)*Ar
cTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + (2*I)*ArcTanh[((a -
b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[Sqrt[a^2 - b^2]/(Sqrt[2]*Sqrt
[a]*E^((I/2)*(c + d*x^n))*Sqrt[b + a*Cos[c + d*x^n]])] + (ArcCos[-(b/a)] +
(2*I)*(ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTanh[((
a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]))*Log[(Sqrt[a^2 - b^2]*E^((I/2
)*(c + d*x^n)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Cos[c + d*x^n]])] - (ArcCos[-(
b/a)] - (2*I)*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[(
(a + b)*(a - b - I*Sqrt[a^2 - b^2])*(1 + I*Tan[(c + d*x^n)/2]))/(a*(a + b
+ Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))] - (ArcCos[-(b/a)] + (2*I)*ArcTanh[
((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[((a + b)*((-I)*a + I*b
+ Sqrt[a^2 - b^2])*(I + Tan[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*T
an[(c + d*x^n)/2]))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqr
t[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x
^n)/2]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*T
an[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))])))/(
Sqrt[a^2 - b^2]*d^2*x^(2*n))*Sec[c + d*x^n]/(2*a*e^n*(a + b*Sec[c + d*x^
n]))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4696, 4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ex)^{2n-1}}{a + b \sec(c + dx^n)} dx \\
 & \quad \downarrow \text{4696} \\
 & \frac{x^{-2n}(ex)^{2n} \int \frac{x^{2n-1}}{a+b \sec(dx^n+c)} dx}{e} \\
 & \quad \downarrow \text{4692} \\
 & \frac{x^{-2n}(ex)^{2n} \int \frac{x^n}{a+b \sec(dx^n+c)} dx^n}{en} \\
 & \quad \downarrow \text{3042} \\
 & \frac{x^{-2n}(ex)^{2n} \int \frac{x^n}{a+b \csc(dx^n+c+\frac{\pi}{2})} dx^n}{en} \\
 & \quad \downarrow \text{4679} \\
 & \frac{x^{-2n}(ex)^{2n} \int \left(\frac{x^n}{a} - \frac{bx^n}{a(b+a \cos(dx^n+c))} \right) dx^n}{en} \\
 & \quad \downarrow \text{2009} \\
 & \frac{x^{-2n}(ex)^{2n} \left(\frac{b \operatorname{PolyLog}\left(2, -\frac{ae^i(dx^n+c)}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^i(dx^n+c)}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{ibx^n \log\left(1+\frac{ae^i(c+dx^n)}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}} - \frac{ibx^n \log\left(1+\frac{ae^i(c+dx^n)}{\sqrt{b^2-a^2}+b}\right)}{ad\sqrt{b^2-a^2}} \right)}{en}
 \end{aligned}$$

input `Int[(e*x)^(-1 + 2*n)/(a + b*Sec[c + d*x^n]),x]`

output

$$\frac{((e*x)^{(2*n)}*(x^{(2*n)})/(2*a) + (I*b*x^n*\text{Log}[1 + (a*E^{(I*(c + d*x^n))})]/(b - \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d) - (I*b*x^n*\text{Log}[1 + (a*E^{(I*(c + d*x^n))})]/(b + \text{Sqrt}[-a^2 + b^2]))/(a*\text{Sqrt}[-a^2 + b^2]*d) + (b*\text{PolyLog}[2, -((a*E^{(I*(c + d*x^n))})/(b - \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2) - (b*\text{PolyLog}[2, -((a*E^{(I*(c + d*x^n))})/(b + \text{Sqrt}[-a^2 + b^2]))]/(a*\text{Sqrt}[-a^2 + b^2]*d^2)))/(e*n*x^{(2*n)})$$

Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 3042

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$$

rule 4679

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, 1/(\text{Sin}[e + f*x]^n/(b + a*\text{Sin}[e + f*x]^n)), x], x] \text{ ; FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 0]$$

rule 4692

$$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*\text{Sec}[(c_.) + (d_.)*(x_)^{(n_.)}])^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/n \ \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*\text{Sec}[c + d*x])^p, x], x, x^n], x] \text{ ; FreeQ}\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{IGtQ}[\text{Simplify}[(m + 1)/n], 0] \ \&\& \ \text{IntegerQ}[p]$$

rule 4696

$$\text{Int}[(e*(x_))^{(m_.)}*((a_.) + (b_.)*\text{Sec}[(c_.) + (d_.)*(x_)^{(n_.)}])^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[e^{\text{IntPart}[m]}*(e*x)^{\text{FracPart}[m]}/x^{\text{FracPart}[m]} \ \text{Int}[x^m*(a + b*\text{Sec}[c + d*x^n])^p, x], x] \text{ ; FreeQ}\{a, b, c, d, e, m, n, p\}, x]$$

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.30

method	result
risch	$\frac{x e^{\frac{(-1+2n)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(x) + 2 \ln(e))}{2}}}{2an} - \left(ix^n d \ln \left(\frac{ae}{\dots} \right) \right)$

```
input int((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
output 1/2/a/n*x*exp(1/2*(-1+2*n))*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I
*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(x
)+2*ln(e))-1/(a^2-b^2)*(I*x^n*d*ln((a*exp(I*(d*x^n+2*c))+exp(I*c)*b-(exp
(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))/(exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I
*c))^(1/2)))-I*x^n*d*ln((a*exp(I*(d*x^n+2*c))+exp(I*c)*b+(exp(2*I*c)*b^2-a
^2*exp(2*I*c))^(1/2))/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)))+
dilog(a/(exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*(d*x^n+2*
c))+1/(exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I*c)*b-1/(exp
(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*(exp(2*I*c)*b^2-a^2*exp(2*I
*c))^(1/2))-dilog(a/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp
(I*(d*x^n+2*c))+1/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*exp(I
*c)*b+1/(exp(I*c)*b+(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2))*(exp(2*I*c)*b^2
-a^2*exp(2*I*c))^(1/2))*(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)/d^2/n/e*(e^
n)^2*b/a*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*
e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3-Pi*c
sgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn
(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1268 vs. 2(300) = 600.

Time = 0.28 (sec) , antiderivative size = 1268, normalized size of antiderivative = 3.87

$$\int \frac{(ex)^{-1+2n}}{a + b \sec(c + dx^n)} dx = \text{Too large to display}$$

input `integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n)),x, algorithm="fricas")`

output

```

1/2*(-I*a*b*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) +
2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + I*a*b*c*e^(2*n
- 1)*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c)
+ 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - I*a*b*c*e^(2*n - 1)*sqrt(-(a^2 - b^2
)/a^2)*log(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b
^2)/a^2) - 2*b) + I*a*b*c*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos(
d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) + (a
^2 - b^2)*d^2*e^(2*n - 1)*x^(2*n) - a*b*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)
*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (I*a*sqrt(-(a^2 -
b^2)/a^2) + I*b)*sin(d*x^n + c) + a)/a + 1) - a*b*e^(2*n - 1)*sqrt(-(a^2
- b^2)/a^2)*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (-I*a*
sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) + a)/a + 1) + a*b*e^(2*n - 1)
*sqrt(-(a^2 - b^2)/a^2)*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n +
c) + (I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) - a)/a + 1) + a*b*e
^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*co
s(d*x^n + c) + (-I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a +
1) + (I*a*b*d*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) + I*a*b*c*e^(2*n - 1)
)*sqrt(-(a^2 - b^2)/a^2))*log(((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n +
c) - (I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) + a)/a) + (-I*a*b*d
*e^(2*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2) - I*a*b*c*e^(2*n - 1)*sqrt(-(a^...

```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{2n-1}}{a + b \sec(c + dx^n)} dx$$

input `integrate((e*x)**(-1+2*n)/(a+b*sec(c+d*x**n)),x)`

output `Integral((e*x)**(2*n - 1)/(a + b*sec(c + d*x**n)), x)`

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{2n-1}}{b \sec(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n)),x, algorithm="maxima")`

output `-1/2*(4*a*b*e^(2*n + 1)*n*integrate((a*x^(2*n)*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + 2*b*x^(2*n)*cos(d*x^n + c)^2 + a*x^(2*n)*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 2*b*x^(2*n)*sin(d*x^n + c)^2 + a*x^(2*n)*cos(d*x^n + c))/(a^3*e*x*cos(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*cos(d*x^n + c)^2 + a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a^2*b*e*x*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*a*b^2*e*x*sin(d*x^n + c)^2 + 4*a^2*b*e*x*cos(d*x^n + c) + a^3*e*x + 2*(2*a^2*b*e*x*cos(d*x^n + c) + a^3*e*x)*cos(2*d*x^n + 2*c)), x) - e^(2*n)*x^(2*n)/(a*e*n)`

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{2n-1}}{b \sec(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n)),x, algorithm="giac")`

output `integrate((e*x)^(2*n - 1)/(b*sec(d*x^n + c) + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{2n-1}}{a + \frac{b}{\cos(c+dx^n)}} dx$$

input `int((e*x)^(2*n - 1)/(a + b/cos(c + d*x^n)),x)`

output `int((e*x)^(2*n - 1)/(a + b/cos(c + d*x^n)), x)`

Reduce [F]

$$\int \frac{(ex)^{-1+2n}}{a + b \sec(c + dx^n)} dx$$

$$= \frac{e^{2n} \left(x^{2n} + 4 \left(\int \frac{x^{2n} \tan\left(\frac{x^n d + c}{2}\right)^2}{\tan\left(\frac{x^n d + c}{2}\right)^2 a^2 x - \tan\left(\frac{x^n d + c}{2}\right)^2 b^2 x - a^2 x - 2abx - b^2 x} dx \right) abn + 4 \left(\int \frac{x^{2n} \tan\left(\frac{x^n d + c}{2}\right)^2}{\tan\left(\frac{x^n d + c}{2}\right)^2 a^2 x - \tan\left(\frac{x^n d + c}{2}\right)^2 b^2 x} dx \right) \right)}{2en(a + b)}$$

input `int((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n)), x)`

output `(e**(2*n)*(x**(2*n) + 4*int((x**(2*n)*tan((x**n*d + c)/2)**2)/(tan((x**n*d + c)/2)**2*a**2*x - tan((x**n*d + c)/2)**2*b**2*x - a**2*x - 2*a*b*x - b**2*x), x)*a*b*n + 4*int((x**(2*n)*tan((x**n*d + c)/2)**2)/(tan((x**n*d + c)/2)**2*a**2*x - tan((x**n*d + c)/2)**2*b**2*x - a**2*x - 2*a*b*x - b**2*x), x)*b**2*n)/(2*e*n*(a + b))`

3.80 $\int \frac{(ex)^{-1+3n}}{a+b \sec(c+dx^n)} dx$

Optimal result	565
Mathematica [F]	566
Rubi [A] (verified)	566
Maple [F]	568
Fricas [B] (verification not implemented)	569
Sympy [F]	570
Maxima [F]	570
Giac [F]	570
Mupad [F(-1)]	571
Reduce [F]	571

Optimal result

Integrand size = 24, antiderivative size = 485

$$\begin{aligned}
 \int \frac{(ex)^{-1+3n}}{a+b \sec(c+dx^n)} dx = & \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\
 & - \frac{ibx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\
 & + \frac{2bx^{-2n}(ex)^{3n} \text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} \\
 & - \frac{2bx^{-2n}(ex)^{3n} \text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} \\
 & + \frac{2ibx^{-3n}(ex)^{3n} \text{PolyLog}\left(3, -\frac{ae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en} \\
 & - \frac{2ibx^{-3n}(ex)^{3n} \text{PolyLog}\left(3, -\frac{ae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}
 \end{aligned}$$

output

$$\frac{1}{3} \frac{(e^x)^{3n}}{a e^n + I b (e^x)^{3n} \ln(1 + a \exp(I(c + d x^n))) / (b - (-a^2 + b^2)^{1/2})} / \frac{a}{(-a^2 + b^2)^{1/2}} \frac{d e^n}{(x^n)^2} - I b (e^x)^{3n} \ln(1 + a \exp(I(c + d x^n))) / (b + (-a^2 + b^2)^{1/2})} / \frac{a}{(-a^2 + b^2)^{1/2}} \frac{d e^n}{(x^n)^2} + 2 b (e^x)^{3n} \operatorname{polylog}(2, -a \exp(I(c + d x^n))) / (b - (-a^2 + b^2)^{1/2})} / \frac{a}{(-a^2 + b^2)^{1/2}} \frac{d^2 e^n}{(x^{2n})^2} - 2 b (e^x)^{3n} \operatorname{polylog}(2, -a \exp(I(c + d x^n))) / (b + (-a^2 + b^2)^{1/2})} / \frac{a}{(-a^2 + b^2)^{1/2}} \frac{d^2 e^n}{(x^{2n})^2} + 2 I b (e^x)^{3n} \operatorname{polylog}(3, -a \exp(I(c + d x^n))) / (b - (-a^2 + b^2)^{1/2})} / \frac{a}{(-a^2 + b^2)^{1/2}} \frac{d^3 e^n}{(x^{3n})^3} - 2 I b (e^x)^{3n} \operatorname{polylog}(3, -a \exp(I(c + d x^n))) / (b + (-a^2 + b^2)^{1/2})} / \frac{a}{(-a^2 + b^2)^{1/2}} \frac{d^3 e^n}{(x^{3n})^3}$$
Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx$$

input

`Integrate[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n]), x]`

output

`Integrate[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n]), x]`
Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4696, 4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3n-1}}{a + b \sec(c + dx^n)} dx$$

$$\downarrow 4696$$

$$\frac{x^{-3n} (ex)^{3n} \int \frac{x^{3n-1}}{a + b \sec(dx^n + c)} dx}{e}$$

$$\begin{array}{c}
 \downarrow 4692 \\
 \frac{x^{-3n}(ex)^{3n} \int \frac{x^{2n}}{a+b \sec(dx^n+c)} dx^n}{en} \\
 \downarrow 3042 \\
 \frac{x^{-3n}(ex)^{3n} \int \frac{x^{2n}}{a+b \csc(dx^n+c+\frac{\pi}{2})} dx^n}{en} \\
 \downarrow 4679 \\
 \frac{x^{-3n}(ex)^{3n} \int \left(\frac{x^{2n}}{a} - \frac{bx^{2n}}{a(b+a \cos(dx^n+c))} \right) dx^n}{en} \\
 \downarrow 2009 \\
 \frac{x^{-3n}(ex)^{3n} \left(\frac{2ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{2ib \operatorname{PolyLog}\left(3, -\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{2bx^n \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{2bx^n \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} \right)}{en}
 \end{array}$$

input `Int[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n]),x]`

output `((e*x)^(3*n)*(x^(3*n)/(3*a) + (I*b*x^(2*n)*Log[1 + (a*E^(I*(c + d*x^n))]/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (I*b*x^(2*n)*Log[1 + (a*E^(I*(c + d*x^n))]/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x^n*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) - (2*b*x^n*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((2*I)*b*PolyLog[3, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - ((2*I)*b*PolyLog[3, -((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3)))/(e*n*x^(3*n))`

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

rule 4696 `Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx$$

input `int((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n)),x)`

output `int((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n)),x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1711 vs. $2(445) = 890$.

Time = 0.29 (sec) , antiderivative size = 1711, normalized size of antiderivative = 3.53

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx = \text{Too large to display}$$

input `integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n)),x, algorithm="fricas")`

output

```
-1/6*(6*a*b*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2)*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) + a)/a + 1) + 6*a*b*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2)*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (-I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) + a)/a + 1) - 6*a*b*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2)*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) - a)/a + 1) - 6*a*b*d*e^(3*n - 1)*x^n*sqrt(-(a^2 - b^2)/a^2)*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (-I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a + 1) - 3*I*a*b*c^2*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + 3*I*a*b*c^2*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 3*I*a*b*c^2*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) + 3*I*a*b*c^2*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*log(-2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) - 2*b) - 2*(a^2 - b^2)*d^3*e^(3*n - 1)*x^(3*n) + 6*I*a*b*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) + (I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c))/a) - 6*I*a*b*e^(3*n - 1)*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) + (I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c))/a)
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{3n-1}}{a + b \sec(c + dx^n)} dx$$

input `integrate((e*x)**(-1+3*n)/(a+b*sec(c+d*x**n)),x)`

output `Integral((e*x)**(3*n - 1)/(a + b*sec(c + d*x**n)), x)`

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{3n-1}}{b \sec(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n)),x, algorithm="maxima")`

output `-1/3*(6*a*b*e^(3*n + 1)*n*integrate((a*x^(3*n)*cos(2*d*x^n + 2*c)*cos(d*x^n + c) + 2*b*x^(3*n)*cos(d*x^n + c)^2 + a*x^(3*n)*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 2*b*x^(3*n)*sin(d*x^n + c)^2 + a*x^(3*n)*cos(d*x^n + c))/(a^3*e*x*cos(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*cos(d*x^n + c)^2 + a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a^2*b*e*x*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*a*b^2*e*x*sin(d*x^n + c)^2 + 4*a^2*b*e*x*cos(d*x^n + c) + a^3*e*x + 2*(2*a^2*b*e*x*cos(d*x^n + c) + a^3*e*x)*cos(2*d*x^n + 2*c)), x) - e^(3*n)*x^(3*n)/(a*e*n)`

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{3n-1}}{b \sec(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n)),x, algorithm="giac")`

output `integrate((e*x)^(3*n - 1)/(b*sec(d*x^n + c) + a), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx = \int \frac{(ex)^{3n-1}}{a + \frac{b}{\cos(c+dx^n)}} dx$$

input `int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n)), x)`

output `int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n)), x)`

Reduce [F]

$$\int \frac{(ex)^{-1+3n}}{a + b \sec(c + dx^n)} dx$$

$$= \frac{e^{3n} \left(x^{3n} + 6 \left(\int \frac{x^{3n} \tan\left(\frac{x^n d + c}{2}\right)^2}{\tan\left(\frac{x^n d + c}{2}\right)^2 a^2 x - \tan\left(\frac{x^n d + c}{2}\right)^2 b^2 x - a^2 x - 2abx - b^2 x} dx \right) abn + 6 \left(\int \frac{x^{3n} \tan\left(\frac{x^n d + c}{2}\right)^2}{\tan\left(\frac{x^n d + c}{2}\right)^2 a^2 x - \tan\left(\frac{x^n d + c}{2}\right)^2 b^2 x} dx \right) \right)}{3en(a + b)}$$

input `int((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n)), x)`

output `(e**(3*n)*(x**(3*n) + 6*int((x**(3*n)*tan((x**n*d + c)/2)**2)/(tan((x**n*d + c)/2)**2*a**2*x - tan((x**n*d + c)/2)**2*b**2*x - a**2*x - 2*a*b*x - b**2*x), x)*a*b*n + 6*int((x**(3*n)*tan((x**n*d + c)/2)**2)/(tan((x**n*d + c)/2)**2*a**2*x - tan((x**n*d + c)/2)**2*b**2*x - a**2*x - 2*a*b*x - b**2*x), x)*b**2*n)/(3*e**n*(a + b))`

3.81 $\int \frac{(ex)^{-1+n}}{(a+b \sec(c+dx^n))^2} dx$

Optimal result	572
Mathematica [A] (verified)	572
Rubi [A] (verified)	573
Maple [C] (warning: unable to verify)	576
Fricas [A] (verification not implemented)	577
Sympy [F]	578
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Giac [F]	579
Mupad [B] (verification not implemented)	580
Reduce [B] (verification not implemented)	580

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(ex)^{-1+n}}{(a+b \sec(c+dx^n))^2} dx = \frac{(ex)^n}{a^2 e n} - \frac{2b(2a^2 - b^2) x^{-n} (ex)^n \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2} den} + \frac{b^2 x^{-n} (ex)^n \tan(c+dx^n)}{a(a^2 - b^2) den (a+b \sec(c+dx^n))}$$

output

```
(e*x)^n/a^2/e/n-2*b*(2*a^2-b^2)*(e*x)^n*arctanh((a-b)^(1/2)*tan(1/2*c+1/2*d*x^n)/(a+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d/e/n/(x^n)+b^2*(e*x)^n*tan(c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(a+b*sec(c+d*x^n))
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22

$$\int \frac{(ex)^{-1+n}}{(a+b \sec(c+dx^n))^2} dx = \frac{x^{-n} (ex)^n \left(-2b(-2a^2 + b^2) \operatorname{arctanh}\left(\frac{(-a+b) \tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right) (b + a \cos(c+dx^n)) + \sqrt{a^2 - b^2} (a(a^2 - b^2) (c + dx^n) + a^2(a-b)(a+b)\sqrt{a^2 - b^2} den (b + a \cos(c + dx^n))) \right)}{a^2(a-b)(a+b)\sqrt{a^2 - b^2} den (b + a \cos(c + dx^n))}$$

input `Integrate[(e*x)^(-1 + n)/(a + b*Sec[c + d*x^n])^2,x]`

output $((e*x)^n*(-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x^n]) + Sqrt[a^2 - b^2]*(a*(a^2 - b^2)*(c + d*x^n)*Cos[c + d*x^n] + b*((a^2 - b^2)*(c + d*x^n) + a*b*Sin[c + d*x^n]))) / (a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d*e*n*x^n*(b + a*Cos[c + d*x^n]))$

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4696, 4692, 3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{n-1}}{(a + b \sec(c + dx^n))^2} dx$$

↓ 4696

$$\frac{x^{-n}(ex)^n \int \frac{x^{n-1}}{(a+b \sec(dx^n+c))^2} dx}{e}$$

↓ 4692

$$\frac{x^{-n}(ex)^n \int \frac{1}{(a+b \sec(dx^n+c))^2} dx^n}{en}$$

↓ 3042

$$\frac{x^{-n}(ex)^n \int \frac{1}{(a+b \csc(dx^n+c+\frac{\pi}{2}))^2} dx^n}{en}$$

↓ 4272

$$\frac{x^{-n}(ex)^n \left(\frac{b^2 \tan(c+dx^n)}{ad(a^2-b^2)(a+b \sec(c+dx^n))} - \frac{\int -\frac{a^2-b \sec(dx^n+c)a-b^2}{a+b \sec(dx^n+c)} dx^n}{a(a^2-b^2)} \right)}{en}$$

↓ 25

$$x^{-n}(ex)^n \left(\frac{\int \frac{a^2 - b \sec(dx^n + c) a - b^2}{a + b \sec(dx^n + c)} dx^n}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^n)}{ad(a^2 - b^2)(a + b \sec(c + dx^n))} \right)$$

en
↓ 3042

$$x^{-n}(ex)^n \left(\frac{\int \frac{a^2 - b \csc(dx^n + c + \frac{\pi}{2}) a - b^2}{a + b \csc(dx^n + c + \frac{\pi}{2})} dx^n}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^n)}{ad(a^2 - b^2)(a + b \sec(c + dx^n))} \right)$$

en
↓ 4407

$$x^{-n}(ex)^n \left(\frac{\frac{(a^2 - b^2)x^n}{a} - \frac{b(2a^2 - b^2) \int \frac{\sec(dx^n + c)}{a + b \sec(dx^n + c)} dx^n}{a}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^n)}{ad(a^2 - b^2)(a + b \sec(c + dx^n))} \right)$$

en
↓ 3042

$$x^{-n}(ex)^n \left(\frac{\frac{(a^2 - b^2)x^n}{a} - \frac{b(2a^2 - b^2) \int \frac{\csc(dx^n + c + \frac{\pi}{2})}{a + b \csc(dx^n + c + \frac{\pi}{2})} dx^n}{a}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^n)}{ad(a^2 - b^2)(a + b \sec(c + dx^n))} \right)$$

en
↓ 4318

$$x^{-n}(ex)^n \left(\frac{\frac{(a^2 - b^2)x^n}{a} - \frac{(2a^2 - b^2) \int \frac{1}{a \cos(dx^n + c) + 1} dx^n}{a}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^n)}{ad(a^2 - b^2)(a + b \sec(c + dx^n))} \right)$$

en
↓ 3042

$$x^{-n}(ex)^n \left(\frac{\frac{(a^2 - b^2)x^n}{a} - \frac{(2a^2 - b^2) \int \frac{1}{a \sin(dx^n + c + \frac{\pi}{2}) + 1} dx^n}{a}}{a(a^2 - b^2)} + \frac{b^2 \tan(c + dx^n)}{ad(a^2 - b^2)(a + b \sec(c + dx^n))} \right)$$

en
↓ 3138

$$x^{-n}(ex)^n \left(\frac{\frac{(a^2 - b^2)x^n}{a} - \frac{2(2a^2 - b^2) \int \frac{1}{(1 - \frac{a}{b})x^{2n} + \frac{a+b}{b}} d \tan(\frac{1}{2}(dx^n + c))}{a(a^2 - b^2)}}{ad} + \frac{b^2 \tan(c + dx^n)}{ad(a^2 - b^2)(a + b \sec(c + dx^n))} \right)$$

en

$$x^{-n}(ex)^n \left(\frac{\frac{(a^2-b^2)x^n}{a} - \frac{2b(2a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)} + \frac{b^2 \tan(c+dx^n)}{ad(a^2-b^2)(a+b\sec(c+dx^n))} \right)$$

en

input `Int[(e*x)^(-1 + n)/(a + b*Sec[c + d*x^n])^2,x]`

output `((e*x)^n*(((a^2 - b^2)*x^n)/a - (2*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x^n)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2)) + (b^2*Tan[c + d*x^n])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x^n])))/(e*n*x^n)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3138 `Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`


```
rule 4272 Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2)) Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

```
rule 4318 Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] := Simp[1/b Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

```
rule 4407 Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] := Simp[c*(x/a), x] - Simp[(b*c - a*d)/a Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

```
rule 4692 Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

```
rule 4696 Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(
a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.70 (sec) , antiderivative size = 614, normalized size of antiderivative = 3.91

method	result
risch	$\frac{x e^{(-1+n)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(x) + 2 \ln(e))}}{a^{2n}} + \frac{2ib^2 e^n (-1)^{-cn}}{\dots}$

input `int((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x,method=_RETURNVERBOSE)`

output

$$\frac{1/a^2/n*x*\exp(1/2*(-1+n)*(-I*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)*\operatorname{Pi}+I*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*e*x)^2*\operatorname{Pi}+I*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)^2*\operatorname{Pi}-I*\operatorname{csgn}(I*e*x)^3*\operatorname{Pi}+2*\ln(x)+2*\ln(e)))+2*I*b^2/a^2/(a^2-b^2)/d/n/(a*\exp(2*I*(c+d*x^n))+2*b*\exp(I*(c+d*x^n))+a)*e^n*(-1)^{-1/2*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)^2}*(-1)^{-1/2*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*e*x)^2}*(-1)^{1/2*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)}*(b*\exp(1/2*I*(-\operatorname{Pi}*n*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)+\operatorname{Pi}*n*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*e*x)^2+\operatorname{Pi}*n*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)^2-\operatorname{Pi}*n*\operatorname{csgn}(I*e*x)^3+\operatorname{Pi}*n*\operatorname{csgn}(I*e*x)^3+2*d*x^n+2*c)))+\exp(1/2*I*\operatorname{Pi}*n*\operatorname{csgn}(I*e*x)*(-\operatorname{csgn}(I*e)*\operatorname{csgn}(I*x)*n+\operatorname{csgn}(I*e)*\operatorname{csgn}(I*e*x)*n+n*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)-n*\operatorname{csgn}(I*e*x)^2+\operatorname{csgn}(I*e*x)^2))*a)/e+2*I*\arctan(1/2*(2*a*\exp(I*(d*x^n+2*c))+2*\exp(I*c)*b)/(a^2*\exp(2*I*c)-\exp(2*I*c)*b^2)^{1/2})/(a^2*\exp(2*I*c)-\exp(2*I*c)*b^2)^{1/2}/d/e*e^n/n/(-a^2+b^2)*(-2*a^2+b^2)/a^2*b*\exp(1/2*I*(-\operatorname{Pi}*n*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)+\operatorname{Pi}*n*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*e*x)^2+\operatorname{Pi}*n*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)^2-\operatorname{Pi}*n*\operatorname{csgn}(I*e*x)^3+\operatorname{Pi}*n*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)-\operatorname{Pi}*n*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*e*x)^2-\operatorname{Pi}*n*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)^2+\operatorname{Pi}*n*\operatorname{csgn}(I*e*x)^3+2*c))}$$

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 628, normalized size of antiderivative = 4.00

$$\int \frac{(ex)^{-1+n}}{(a+b\sec(c+dx^n))^2} dx$$

$$= \frac{2(a^5 - 2a^3b^2 + ab^4)de^{n-1}x^n \cos(dx^n + c) + 2(a^4b - 2a^2b^3 + b^5)de^{n-1}x^n + 2(a^3b^2 - ab^4)e^{n-1} \sin(dx^n)}{2((a^2 - b^2)^2 \sec^2(c + dx^n))^{3/2}}$$

input `integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")`

output

```
[1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*e^(n - 1)*x^n*cos(d*x^n + c) + 2*(a^4*b - 2*a^2*b^3 + b^5)*d*e^(n - 1)*x^n + 2*(a^3*b^2 - a*b^4)*e^(n - 1)*sin(d*x^n + c) + ((2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*e^(n - 1)*cos(d*x^n + c) + (2*a^2*b^2 - b^4)*sqrt(a^2 - b^2)*e^(n - 1))*log((2*a*b*cos(d*x^n + c) - (a^2 - 2*b^2)*cos(d*x^n + c)^2 + 2*a^2 - b^2 - 2*(sqrt(a^2 - b^2)*b*cos(d*x^n + c) + sqrt(a^2 - b^2)*a)*sin(d*x^n + c))/(a^2*cos(d*x^n + c)^2 + 2*a*b*cos(d*x^n + c) + b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cos(d*x^n + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n), ((a^5 - 2*a^3*b^2 + a*b^4)*d*e^(n - 1)*x^n*cos(d*x^n + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*e^(n - 1)*x^n + (a^3*b^2 - a*b^4)*e^(n - 1)*sin(d*x^n + c) - ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*e^(n - 1)*cos(d*x^n + c) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*e^(n - 1))*arctan(-(sqrt(-a^2 + b^2)*b*cos(d*x^n + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*sin(d*x^n + c)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cos(d*x^n + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n)]
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{n-1}}{(a + b \sec(c + dx^n))^2} dx$$

input

```
integrate((e*x)**(-1+n)/(a+b*sec(c+d*x**n))**2,x)
```

output

```
Integral((e*x)**(n - 1)/(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{n-1}}{(b \sec(dx^n + c) + a)^2} dx$$

input

```
integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")
```

output

```

((a^4 - a^2*b^2)*d*e^n*x^n*cos(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*e^n*
x^n*cos(d*x^n + c)^2 + (a^4 - a^2*b^2)*d*e^n*x^n*sin(2*d*x^n + 2*c)^2 + 2*
a*b^3*e^n*sin(d*x^n + c) + 4*(a^2*b^2 - b^4)*d*e^n*x^n*sin(d*x^n + c)^2 +
4*(a^3*b - a*b^3)*d*e^n*x^n*cos(d*x^n + c) + (a^4 - a^2*b^2)*d*e^n*x^n - 2
*(a*b^3*e^n*sin(d*x^n + c) - 2*(a^3*b - a*b^3)*d*e^n*x^n*cos(d*x^n + c) -
(a^4 - a^2*b^2)*d*e^n*x^n)*cos(2*d*x^n + 2*c) - 2*((2*a^8*b - 3*a^6*b^3 +
a^4*b^5)*d*e^(n + 1)*n*cos(2*d*x^n + 2*c)^2*cos(c) + 4*(2*a^6*b^3 - 3*a^4*
b^5 + a^2*b^7)*d*e^(n + 1)*n*cos(d*x^n + c)^2*cos(c) + (2*a^8*b - 3*a^6*b^
3 + a^4*b^5)*d*e^(n + 1)*n*cos(c)*sin(2*d*x^n + 2*c)^2 + 4*(2*a^7*b^2 - 3*
a^5*b^4 + a^3*b^6)*d*e^(n + 1)*n*cos(c)*sin(2*d*x^n + 2*c)*sin(d*x^n + c)
+ 4*(2*a^6*b^3 - 3*a^4*b^5 + a^2*b^7)*d*e^(n + 1)*n*cos(c)*sin(d*x^n + c)^
2 + 4*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b^6)*d*e^(n + 1)*n*cos(d*x^n + c)*cos(c)
+ (2*a^8*b - 3*a^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*cos(c) + 2*(2*(2*a^7*b^2
- 3*a^5*b^4 + a^3*b^6)*d*e^(n + 1)*n*cos(d*x^n + c)*cos(c) + (2*a^8*b - 3
*a^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*cos(c))*cos(2*d*x^n + 2*c))*integrate((a
^3*x^n*cos(2*d*x^n + 2*c)*cos(d*x^n) + a^3*x^n*sin(2*d*x^n + 2*c)*sin(d*x^
n) + 2*(a^2*b - b^3)*x^n*cos(d*x^n)^2*cos(c) + 2*(a^2*b - b^3)*x^n*cos(c)*
sin(d*x^n)^2 + (a^3 - a*b^2)*x^n*cos(d*x^n) - (a*b^2*x^n*cos(d*x^n)*cos(2*
c) + a*b^2*x^n*sin(d*x^n)*sin(2*c))*cos(2*d*x^n) - (a*b^2*x^n*cos(2*c)*sin
(d*x^n) - a*b^2*x^n*cos(d*x^n)*sin(2*c))*sin(2*d*x^n))/(a^8*e*x*cos(2*d...

```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{n-1}}{(b \sec(dx^n + c) + a)^2} dx$$

input

```
integrate((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
```

output

```
integrate((e*x)^(n - 1)/(b*sec(d*x^n + c) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.94

$$\int \frac{(ex)^{-1+n}}{(a + b \sec(c + dx^n))^2} dx = \frac{\frac{b^2 x (ex)^{n-1} 2i}{a^n x^n (a^2 - b^2)} + \frac{b^3 x e^{c 1i + dx^n 1i} (ex)^{n-1} 2i}{a^2 dx^n (a^2 - b^2)}}{a + a e^{c 2i + dx^n 2i} + 2 b e^{c 1i + dx^n 1i}} + \frac{x (ex)^{n-1}}{a^2 n}$$

$$+ \frac{bx \ln \left(-2 e^{c 1i} e^{dx^n 1i} (b^3 x (ex)^{n-1} - 2 a^2 b x (ex)^{n-1}) - \frac{bx (a^4 - a^2 b^2) (a + b e^{c 1i} e^{dx^n 1i}) (ex)^{n-1} (2 a^2 - b^2) 2i}{a^2 (a+b)^{3/2} (a-b)^{3/2}} \right)}{a^2 dx^n (a+b)^{3/2} (a-b)^{3/2}}$$

$$- \frac{bx \ln \left(-2 e^{c 1i} e^{dx^n 1i} (b^3 x (ex)^{n-1} - 2 a^2 b x (ex)^{n-1}) + \frac{bx (a^4 - a^2 b^2) (a + b e^{c 1i} e^{dx^n 1i}) (ex)^{n-1} (2 a^2 - b^2) 2i}{a^2 (a+b)^{3/2} (a-b)^{3/2}} \right)}{a^2 dx^n (a+b)^{3/2} (a-b)^{3/2}}$$

input

```
int((e*x)^(n - 1)/(a + b/cos(c + d*x^n))^2,x)
```

output

```
((b^2*x*(e*x)^(n - 1)*2i)/(a*d*n*x^n*(a^2 - b^2)) + (b^3*x*exp(c*1i + d*x^n*1i)*(e*x)^(n - 1)*2i)/(a^2*d*n*x^n*(a^2 - b^2)))/(a + a*exp(c*2i + d*x^n*2i) + 2*b*exp(c*1i + d*x^n*1i)) + (x*(e*x)^(n - 1))/(a^2*n) + (b*x*log(-2*exp(c*1i)*exp(d*x^n*1i)*(b^3*x*(e*x)^(n - 1) - 2*a^2*b*x*(e*x)^(n - 1)) - (b*x*(a^4 - a^2*b^2)*(a + b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n - 1)*(2*a^2 - b^2)*2i)/(a^2*(a + b)^(3/2)*(a - b)^(3/2)))*(e*x)^(n - 1)*(2*a^2 - b^2))/((a^2*d*n*x^n*(a + b)^(3/2)*(a - b)^(3/2)) - (b*x*log((b*x*(a^4 - a^2*b^2)*(a + b*exp(c*1i)*exp(d*x^n*1i))*(e*x)^(n - 1)*(2*a^2 - b^2)*2i)/(a^2*(a + b)^(3/2)*(a - b)^(3/2)) - 2*exp(c*1i)*exp(d*x^n*1i)*(b^3*x*(e*x)^(n - 1) - 2*a^2*b*x*(e*x)^(n - 1)))*(e*x)^(n - 1)*(2*a^2 - b^2)))/(a^2*d*n*x^n*(a + b)^(3/2)*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.78

$$\int \frac{(ex)^{-1+n}}{(a + b \sec(c + dx^n))^2} dx$$

$$= \frac{e^n \left(-4\sqrt{-a^2 + b^2} \operatorname{atan} \left(\frac{\tan \left(\frac{x^n d + c}{2} \right) a - \tan \left(\frac{x^n d + c}{2} \right) b}{\sqrt{-a^2 + b^2}} \right) \cos(x^n d + c) a^3 b + 2\sqrt{-a^2 + b^2} \operatorname{atan} \left(\frac{\tan \left(\frac{x^n d + c}{2} \right) a - \tan \left(\frac{x^n d + c}{2} \right) b}{\sqrt{-a^2 + b^2}} \right) \right)}{(a + b \sec(c + dx^n))^2}$$

input `int((e*x)^(-1+n)/(a+b*sec(c+d*x^n))^2,x)`

output `(e**n*(- 4*sqrt(- a**2 + b**2)*atan((tan((x**n*d + c)/2)*a - tan((x**n*d + c)/2)*b)/sqrt(- a**2 + b**2))*cos(x**n*d + c)*a**3*b + 2*sqrt(- a**2 + b**2)*atan((tan((x**n*d + c)/2)*a - tan((x**n*d + c)/2)*b)/sqrt(- a**2 + b**2))*cos(x**n*d + c)*a*b**3 - 4*sqrt(- a**2 + b**2)*atan((tan((x**n*d + c)/2)*a - tan((x**n*d + c)/2)*b)/sqrt(- a**2 + b**2))*a**2*b**2 + 2*sqrt(- a**2 + b**2)*atan((tan((x**n*d + c)/2)*a - tan((x**n*d + c)/2)*b)/sqrt(- a**2 + b**2))*b**4 + x**n*cos(x**n*d + c)*a**5*d - 2*x**n*cos(x**n*d + c)*a**3*b**2*d + x**n*cos(x**n*d + c)*a*b**4*d + x**n*a**4*b*d - 2*x**n*a**2*b**3*d + x**n*b**5*d + sin(x**n*d + c)*a**3*b**2 - sin(x**n*d + c)*a*b**4)/(a**2*d*e*n*(cos(x**n*d + c)*a**5 - 2*cos(x**n*d + c)*a**3*b**2 + cos(x**n*d + c)*a*b**4 + a**4*b - 2*a**2*b**3 + b**5))`

$$3.82 \quad \int \frac{(ex)^{-1+2n}}{(a+b \sec(cx+dx^n))^2} dx$$

Optimal result	583
Mathematica [B] (warning: unable to verify)	584
Rubi [A] (verified)	585
Maple [C] (warning: unable to verify)	587
Fricas [B] (verification not implemented)	588
Sympy [F]	589
Maxima [F]	590
Giac [F]	590
Mupad [F(-1)]	591
Reduce [F]	591

Optimal result

Integrand size = 24, antiderivative size = 757

$$\begin{aligned}
\int \frac{(ex)^{-1+2n}}{(a + b \sec(c + dx^n))^2} dx = & \frac{(ex)^{2n}}{2a^2 en} - \frac{ib^3 x^{-n} (ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} den} \\
& + \frac{2ibx^{-n} (ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} den} \\
& + \frac{ib^3 x^{-n} (ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} den} \\
& - \frac{2ibx^{-n} (ex)^{2n} \log\left(1 + \frac{ae^{i(c+dx^n)}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} den} \\
& + \frac{b^2 x^{-2n} (ex)^{2n} \log(b + a \cos(c + dx^n))}{a^2 (a^2 - b^2) d^2 en} \\
& - \frac{b^3 x^{-2n} (ex)^{2n} \text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2 en} \\
& + \frac{2bx^{-2n} (ex)^{2n} \text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^2 en} \\
& + \frac{b^3 x^{-2n} (ex)^{2n} \text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2 en} \\
& - \frac{2bx^{-2n} (ex)^{2n} \text{PolyLog}\left(2, -\frac{ae^{i(c+dx^n)}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^2 en} \\
& + \frac{b^2 x^{-n} (ex)^{2n} \sin(c + dx^n)}{a (a^2 - b^2) den (b + a \cos(c + dx^n))}
\end{aligned}$$

output

```

1/2*(e*x)^(2*n)/a^2/e/n-I*b^3*(e*x)^(2*n)*ln(1+a*exp(I*(c+d*x^n))/(b-(-a^2
+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)+2*I*b*(e*x)^(2*n)*ln(1+a*ex
p(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d/e/n/(x^n)+I*b^
3*(e*x)^(2*n)*ln(1+a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)
^(3/2)/d/e/n/(x^n)-2*I*b*(e*x)^(2*n)*ln(1+a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)
^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d/e/n/(x^n)+b^2*(e*x)^(2*n)*ln(b+a*cos(c+d*x
^n))/a^2/(a^2-b^2)/d^2/e/n/(x^(2*n))-b^3*(e*x)^(2*n)*polylog(2,-a*exp(I*(c
+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))+2*b*
(e*x)^(2*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+
b^2)^(1/2)/d^2/e/n/(x^(2*n))+b^3*(e*x)^(2*n)*polylog(2,-a*exp(I*(c+d*x^n)
)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))-2*b*(e*x)^(2
*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/
2)/d^2/e/n/(x^(2*n))+b^2*(e*x)^(2*n)*sin(c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/
(b+a*cos(c+d*x^n))

```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2450 vs. $2(757) = 1514$.

Time = 9.20 (sec) , antiderivative size = 2450, normalized size of antiderivative = 3.24

$$\int \frac{(ex)^{-1+2n}}{(a + b \sec(c + dx^n))^2} dx = \text{Result too large to show}$$

input

```
Integrate[(e*x)^(-1 + 2*n)/(a + b*Sec[c + d*x^n])^2,x]
```

output

```
(-2*b*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*(b + a*cos[c + d*x^n])^2*(2*(c + d*x^n)
*ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - 2*(c + ArcCos[-(b
/a)])*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/
a)] - (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTa
nh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[Sqrt[a^2 - b^2]/(Sq
rt[2]*Sqrt[a]*E^((I/2)*(c + d*x^n))*Sqrt[b + a*cos[c + d*x^n]])] + (ArcCos
[-(b/a)] + (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] -
ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^
2])*E^((I/2)*(c + d*x^n))]/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*cos[c + d*x^n]])] -
(ArcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^
2]])*Log[1 - ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*
x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))] + (-ArcCos[-(b
/a)] + (2*I)*ArcTanh[((a - b)*Tan[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[1
- ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(
a*(a + b + Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))] + I*(PolyLog[2, ((b - I*S
qrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(a*(a + b +
Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2]
)*(a + b - Sqrt[a^2 - b^2]*Tan[(c + d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2
]*Tan[(c + d*x^n)/2]))])*Sec[c + d*x^n]^2/((a^2 - b^2)^(3/2)*d^2*n*(a +
b*Sec[c + d*x^n])^2) + (b^3*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*(b + a*cos[c + ...
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 603, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4696, 4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{2n-1}}{(a + b \sec(c + dx^n))^2} dx$$

↓ 4696

$$\frac{x^{-2n}(ex)^{2n} \int \frac{x^{2n-1}}{(a + b \sec(dx^n + c))^2} dx}{e}$$

↓ 4692

$$\begin{array}{c}
\frac{x^{-2n}(ex)^{2n} \int \frac{x^n}{(a+b \sec(dx^n+c))^2} dx^n}{en} \\
\downarrow 3042 \\
\frac{x^{-2n}(ex)^{2n} \int \frac{x^n}{(a+b \csc(dx^n+c+\frac{\pi}{2}))^2} dx^n}{en} \\
\downarrow 4679 \\
\frac{x^{-2n}(ex)^{2n} \int \left(-\frac{2bx^n}{a^2(b+a \cos(dx^n+c))} + \frac{x^n}{a^2} + \frac{b^2x^n}{a^2(b+a \cos(dx^n+c))^2} \right) dx^n}{en} \\
\downarrow 2009 \\
x^{-2n}(ex)^{2n} \left(\frac{2b \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2 d^2 \sqrt{b^2-a^2}} - \frac{2b \operatorname{PolyLog}\left(2, -\frac{ae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2 d^2 \sqrt{b^2-a^2}} + \frac{b^2 \log(a \cos(c+dx^n)+b)}{a^2 d^2 (a^2-b^2)} + \frac{2ibx^n \log\left(1+\frac{ae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{a^2 d \sqrt{b^2-a^2}} \right)
\end{array}$$

input `Int[(e*x)^(-1 + 2*n)/(a + b*Sec[c + d*x^n])^2,x]`

output `((e*x)^(2*n)*(x^(2*n)/(2*a^2) - (I*b^3*x^n*Log[1 + (a*E^(I*(c + d*x^n))]/(b - Sqrt[-a^2 + b^2])))/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^n*Log[1 + (a*E^(I*(c + d*x^n))]/(b - Sqrt[-a^2 + b^2]))/(a^2*Sqrt[-a^2 + b^2]*d) + (I*b^3*x^n*Log[1 + (a*E^(I*(c + d*x^n))]/(b + Sqrt[-a^2 + b^2]))/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^n*Log[1 + (a*E^(I*(c + d*x^n))]/(b + Sqrt[-a^2 + b^2]))/(a^2*Sqrt[-a^2 + b^2]*d) + (b^2*Log[b + a*Cos[c + d*x^n]])/(a^2*(a^2 - b^2)*d^2) - (b^3*PolyLog[2, -((a*E^(I*(c + d*x^n))]/(b - Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (2*b*PolyLog[2, -((a*E^(I*(c + d*x^n))]/(b - Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^2) + (b^3*PolyLog[2, -((a*E^(I*(c + d*x^n))]/(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (2*b*PolyLog[2, -((a*E^(I*(c + d*x^n))]/(b + Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^2) + (b^2*x^n*Sin[c + d*x^n])/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x^n])))/(e*n*x^(2*n))`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] :=> Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] :=> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :=> Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

rule 4696 `Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :=> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.63 (sec) , antiderivative size = 2992, normalized size of antiderivative = 3.95

method	result	size
risch	Expression too large to display	2992

input `int((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n))^2,x,method=_RETURNVERBOSE)`

output

```

1/2/a^2/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn
(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*I
n(x)+2*I*ln(e))+2*I*b^2/a^2/(a^2-b^2)/d/n*x^n/(a*exp(2*I*(c+d*x^n))+2*b*exp
(I*(c+d*x^n))+a)*(e^n)^2*(-1)^(-1/2*csgn(I*x)*csgn(I*e*x)^2)*(-1)^(-1/2*cs
gn(I*e)*csgn(I*e*x)^2)*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(b*exp(1
/2*I*(-2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+2*Pi*n*csgn(I*e)*csgn(I*e*x)
^2+2*Pi*n*csgn(I*x)*csgn(I*e*x)^2-2*Pi*n*csgn(I*e*x)^3+Pi*csgn(I*e*x)^3+2*
d*x^n+2*c))+exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e)*csgn(I*x)*n+2*csgn(I*e)
*csgn(I*e*x)*n+2*n*csgn(I*x)*csgn(I*e*x)-2*n*csgn(I*e*x)^2+csgn(I*e*x)^2))
*a)/e-2*I*b/(a^2-b^2)^2/d*(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)/n/e*(e^n)^
2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*e)*csgn
(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)
*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)
^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((a*exp(I*(d*x^n+2*c))+exp(I*c)*b-(exp(2*I
*c)*b^2-a^2*exp(2*I*c))^(1/2))/(exp(I*c)*b-(exp(2*I*c)*b^2-a^2*exp(2*I*c))
^(1/2)))+I*b^3/a^2/(a^2-b^2)^2/d*(exp(2*I*c)*b^2-a^2*exp(2*I*c))^(1/2)/n/e
*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*csgn(I
e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3-Pi*cs
gn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn
(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((a*exp(I*(d*x^n+2*c))+exp(I*c)*...

```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2503 vs. $2(705) = 1410$.

Time = 0.35 (sec) , antiderivative size = 2503, normalized size of antiderivative = 3.31

$$\int \frac{(ex)^{-1+2n}}{(a + b \sec(c + dx^n))^2} dx = \text{Too large to display}$$

input

```
integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")
```

output

```

1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*e^(2*n - 1)*x^(2*n)*cos(d*x^n + c) + (a
^4*b - 2*a^2*b^3 + b^5)*d^2*e^(2*n - 1)*x^(2*n) + 2*(a^3*b^2 - a*b^4)*d*e^
(2*n - 1)*x^n*sin(d*x^n + c) - ((2*a^4*b - a^2*b^3)*e^(2*n - 1)*sqrt(-(a^2
- b^2)/a^2)*cos(d*x^n + c) + (2*a^3*b^2 - a*b^4)*e^(2*n - 1)*sqrt(-(a^2 -
b^2)/a^2))*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cos(d*x^n + c) - (I*a*s
qrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) + a)/a + 1) - ((2*a^4*b - a^2*
b^3)*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*cos(d*x^n + c) + (2*a^3*b^2 - a*b^
4)*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) +
b)*cos(d*x^n + c) - (-I*a*sqrt(-(a^2 - b^2)/a^2) - I*b)*sin(d*x^n + c) +
a)/a + 1) + ((2*a^4*b - a^2*b^3)*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2)*cos(d*
x^n + c) + (2*a^3*b^2 - a*b^4)*e^(2*n - 1)*sqrt(-(a^2 - b^2)/a^2))*dilog((
(a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) + (I*a*sqrt(-(a^2 - b^2)/a^2
) - I*b)*sin(d*x^n + c) - a)/a + 1) + ((2*a^4*b - a^2*b^3)*e^(2*n - 1)*sqr
t(-(a^2 - b^2)/a^2)*cos(d*x^n + c) + (2*a^3*b^2 - a*b^4)*e^(2*n - 1)*sqrt(
-(a^2 - b^2)/a^2))*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cos(d*x^n + c) +
(-I*a*sqrt(-(a^2 - b^2)/a^2) + I*b)*sin(d*x^n + c) - a)/a + 1) + ((a^3*b^2
- a*b^4 - I*(2*a^4*b - a^2*b^3)*c*sqrt(-(a^2 - b^2)/a^2))*e^(2*n - 1)*cos
(d*x^n + c) + (a^2*b^3 - b^5 - I*(2*a^3*b^2 - a*b^4)*c*sqrt(-(a^2 - b^2)/a
^2))*e^(2*n - 1))*log(2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt
(-(a^2 - b^2)/a^2) + 2*b) + ((a^3*b^2 - a*b^4 + I*(2*a^4*b - a^2*b^3)*c...

```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{2n-1}}{(a + b \sec(c + dx^n))^2} dx$$

input

```
integrate((e*x)**(-1+2*n)/(a+b*sec(c+d*x**n))**2,x)
```

output

```
Integral((e*x)**(2*n - 1)/(a + b*sec(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{2n-1}}{(b \sec(dx^n + c) + a)^2} dx$$

input `integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")`

output

```
1/2*(4*a*b^3*e^(2*n)*x^n*sin(d*x^n + c) + (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n)
)*cos(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*e^(2*n)*x^(2*n)*cos(d*x^n + c
)^2 + (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n)*sin(2*d*x^n + 2*c)^2 + 4*(a^2*b^2
- b^4)*d*e^(2*n)*x^(2*n)*sin(d*x^n + c)^2 + 4*(a^3*b - a*b^3)*d*e^(2*n)*x^
(2*n)*cos(d*x^n + c) + (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n) - 2*(2*a*b^3*e^(2
*n)*x^n*sin(d*x^n + c) - 2*(a^3*b - a*b^3)*d*e^(2*n)*x^(2*n)*cos(d*x^n + c
) - (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n))*cos(2*d*x^n + 2*c) + 2*((a^6 - a^4*
b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n +
c)^2 + (a^6 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*
e*n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*
x^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*
e*n + 2*(2*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)
*cos(2*d*x^n + 2*c))*integrate(2*(a^2*b^4*e^(2*n)*x^n*cos(2*c)*sin(2*d*x^n
) + a^2*b^4*e^(2*n)*x^n*cos(2*d*x^n)*sin(2*c) - 2*(a^3*b^3 - a*b^5)*e^(2*n
)*x^n*cos(c)*sin(d*x^n) - 2*(a^3*b^3 - a*b^5)*e^(2*n)*x^n*cos(d*x^n)*sin(c
) + (a^3*b^3*e^(2*n)*x^n*sin(d*x^n + c) - (2*a^5*b - a^3*b^3)*d*e^(2*n)*x^
(2*n)*cos(d*x^n + c))*cos(2*d*x^n + 2*c) - ((2*a^5*b - 3*a^3*b^3 + a*b^5)*
d*e^(2*n)*x^(2*n) - (a*b^5*e^(2*n)*x^n*sin(2*c) + (2*a^3*b^3 - a*b^5)*d*e^
(2*n)*x^(2*n)*cos(2*c))*cos(2*d*x^n) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*
e^(2*n)*x^(2*n)*cos(c) + (a^2*b^4 - b^6)*e^(2*n)*x^n*sin(c))*cos(d*x^n)...
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{2n-1}}{(b \sec(dx^n + c) + a)^2} dx$$

input `integrate((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="giac")`

output `integrate((e*x)^(2*n - 1)/(b*sec(d*x^n + c) + a)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{2n-1}}{\left(a + \frac{b}{\cos(c+dx^n)}\right)^2} dx$$

input `int((e*x)^(2*n - 1)/(a + b/cos(c + d*x^n))^2,x)`

output `int((e*x)^(2*n - 1)/(a + b/cos(c + d*x^n))^2, x)`

Reduce [F]

$$\int \frac{(ex)^{-1+2n}}{(a + b \sec(c + dx^n))^2} dx = \text{too large to display}$$

input `int((e*x)^(-1+2*n)/(a+b*sec(c+d*x^n))^2,x)`

output

```
(e**(2*n)*(4*x**(2*n)*cos(x**n*d + c)*tan((x**n*d + c)/2)**2*a**6*d**2 - 4
*x**(2*n)*cos(x**n*d + c)*tan((x**n*d + c)/2)**2*a**5*b*d**2 - x**(2*n)*co
s(x**n*d + c)*tan((x**n*d + c)/2)**2*a**4*b**2*d**2 + 2*x**(2*n)*cos(x**n*
d + c)*tan((x**n*d + c)/2)**2*a**3*b**3*d**2 - x**(2*n)*cos(x**n*d + c)*ta
n((x**n*d + c)/2)**2*a**2*b**4*d**2 - 4*x**(2*n)*cos(x**n*d + c)*a**6*d**2
- 4*x**(2*n)*cos(x**n*d + c)*a**5*b*d**2 + x**(2*n)*cos(x**n*d + c)*a**4*
b**2*d**2 - x**(2*n)*cos(x**n*d + c)*a**2*b**4*d**2 - 16*x**n*cos(x**n*d +
c)*tan((x**n*d + c)/2)*a**5*b*d - 16*x**n*cos(x**n*d + c)*tan((x**n*d + c
)/2)*a**4*b**2*d + 8*x**n*cos(x**n*d + c)*tan((x**n*d + c)/2)*a**3*b**3*d
+ 8*x**n*cos(x**n*d + c)*tan((x**n*d + c)/2)*a**2*b**4*d - 32*cos(x**n*d +
c)*int(x**(2*n)/(4*cos(x**n*d + c)*a**4*b*x - 6*cos(x**n*d + c)*a**3*b**2
*x + 2*cos(x**n*d + c)*a*b**4*x - 2*sin(x**n*d + c)**2*a**5*x + 3*sin(x**n
*d + c)**2*a**4*b*x - sin(x**n*d + c)**2*a**2*b**3*x + 2*a**5*x - 3*a**4*b
*x + 2*a**3*b**2*x - 2*a**2*b**3*x + b**5*x),x)*tan((x**n*d + c)/2)**2*a**
10*b*d**2*n + 32*cos(x**n*d + c)*int(x**(2*n)/(4*cos(x**n*d + c)*a**4*b*x
- 6*cos(x**n*d + c)*a**3*b**2*x + 2*cos(x**n*d + c)*a*b**4*x - 2*sin(x**n*
d + c)**2*a**5*x + 3*sin(x**n*d + c)**2*a**4*b*x - sin(x**n*d + c)**2*a**2
*b**3*x + 2*a**5*x - 3*a**4*b*x + 2*a**3*b**2*x - 2*a**2*b**3*x + b**5*x),
x)*tan((x**n*d + c)/2)**2*a**9*b**2*d**2*n + 72*cos(x**n*d + c)*int(x**(2*
n)/(4*cos(x**n*d + c)*a**4*b*x - 6*cos(x**n*d + c)*a**3*b**2*x + 2*cos(...
```

$$3.83 \quad \int \frac{(ex)^{-1+3n}}{(a+b \sec(c+dx^n))^2} dx$$

Optimal result	593
Mathematica [F]	594
Rubi [A] (verified)	595
Maple [F]	597
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Sympy [F]	598
Maxima [F]	598
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Reduce [F]	600

Optimal result

Integrand size = 24, antiderivative size = 1384

$$\int \frac{(ex)^{-1+3n}}{(a+b \sec(c+dx^n))^2} dx = \text{Too large to display}$$

output

```

1/3*(e*x)^(3*n)/a^2/e/n+I*b^3*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n))/(b+(-a^2
+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)+2*b^2*(e*x)^(3*n)*ln(1+a*ex
p(I*(c+d*x^n))/(b-I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2/e/n/(x^(2*n))+2*b^
2*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)
/d^2/e/n/(x^(2*n))+4*I*b*(e*x)^(3*n)*polylog(3,-a*exp(I*(c+d*x^n))/(b+(-a^
2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3/e/n/(x^(3*n))-2*I*b^2*(e*x)^(3*n)*
polylog(2,-a*exp(I*(c+d*x^n))/(b+I*(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3/e/n
/(x^(3*n))-2*I*b^2*(e*x)^(3*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b-I*(a^2-b^2)
^(1/2)))/a^2/(a^2-b^2)/d^3/e/n/(x^(3*n))-4*I*b*(e*x)^(3*n)*polylog(3,-a*ex
p(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(1/2)/d^3/e/n/(x^(3*n
))+2*I*b*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a
^2+b^2)^(1/2)/d/e/n/(x^n)-I*b^3*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n))/(b+(-a
^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)-2*b^3*(e*x)^(3*n)*polylog
(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/
(x^(2*n))+4*b*(e*x)^(3*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)
))/a^2/(-a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))+2*b^3*(e*x)^(3*n)*polylog(2,-a*
exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*
n))-4*b*(e*x)^(3*n)*polylog(2,-a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^
2/(-a^2+b^2)^(1/2)/d^2/e/n/(x^(2*n))-2*I*b^3*(e*x)^(3*n)*polylog(3,-a*exp(
I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n)...

```

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2} dx = \int \frac{(ex)^{-1+3n}}{(a+b\sec(c+dx^n))^2} dx$$

input

```
Integrate[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n])^2,x]
```

output

```
Integrate[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n])^2, x]
```

Rubi [A] (verified)

Time = 2.41 (sec) , antiderivative size = 1119, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4696, 4692, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ex)^{3n-1}}{(a + b \sec(c + dx^n))^2} dx \\
 & \quad \downarrow \text{4696} \\
 & \frac{x^{-3n}(ex)^{3n} \int \frac{x^{3n-1}}{(a+b \sec(dx^n+c))^2} dx}{e} \\
 & \quad \downarrow \text{4692} \\
 & \frac{x^{-3n}(ex)^{3n} \int \frac{x^{2n}}{(a+b \sec(dx^n+c))^2} dx^n}{en} \\
 & \quad \downarrow \text{3042} \\
 & \frac{x^{-3n}(ex)^{3n} \int \frac{x^{2n}}{(a+b \csc(dx^n+c+\frac{\pi}{2}))^2} dx^n}{en} \\
 & \quad \downarrow \text{4679} \\
 & \frac{x^{-3n}(ex)^{3n} \int \left(-\frac{2bx^{2n}}{a^2(b+a \cos(dx^n+c))} + \frac{x^{2n}}{a^2} + \frac{b^2x^{2n}}{a^2(b+a \cos(dx^n+c))^2} \right) dx^n}{en} \\
 & \quad \downarrow \text{2009} \\
 & x^{-3n}(ex)^{3n} \left(\frac{2b^2 \log\left(\frac{e^{i(dx^n+c)} \frac{a}{b-i\sqrt{a^2-b^2}} + 1}{a^2(a^2-b^2)d^2}\right) x^n}{a^2(a^2-b^2)d^2} + \frac{2b^2 \log\left(\frac{e^{i(dx^n+c)} \frac{a}{b+i\sqrt{a^2-b^2}} + 1}{a^2(a^2-b^2)d^2}\right) x^n}{a^2(a^2-b^2)d^2} + \frac{4b \text{PolyLog}\left(2, -\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right) x^n}{a^2\sqrt{b^2-a^2}d^2} - \frac{2b^3 \text{PolyLog}\left(2, -\frac{ae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right) x^n}{a^2(b^2-a^2)^{3/2}d^2} \right)
 \end{aligned}$$

input

```
Int[(e*x)^(-1 + 3*n)/(a + b*Sec[c + d*x^n])^2,x]
```

output

```

((e*x)^(3*n)*((-1)*b^2*x^(2*n))/(a^2*(a^2 - b^2)*d) + x^(3*n)/(3*a^2) + (
2*b^2*x^n*Log[1 + (a*E^(I*(c + d*x^n)))/(b - I*Sqrt[a^2 - b^2])])/(a^2*(a^
2 - b^2)*d^2) + (2*b^2*x^n*Log[1 + (a*E^(I*(c + d*x^n)))/(b + I*Sqrt[a^2 -
b^2])])/(a^2*(a^2 - b^2)*d^2) - (I*b^3*x^(2*n)*Log[1 + (a*E^(I*(c + d*x^n
)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((2*I)*b*x^(2*n)
*Log[1 + (a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b
^2]*d) + (I*b^3*x^(2*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2
])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((2*I)*b*x^(2*n)*Log[1 + (a*E^(I*(c + d*
x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((2*I)*b^2*Poly
Log[2, -((a*E^(I*(c + d*x^n)))/(b - I*Sqrt[a^2 - b^2])])]/(a^2*(a^2 - b^2)
*d^3) - ((2*I)*b^2*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b + I*Sqrt[a^2 - b^
2])])]/(a^2*(a^2 - b^2)*d^3) - (2*b^3*x^n*PolyLog[2, -((a*E^(I*(c + d*x^n)
)))/(b - Sqrt[-a^2 + b^2])])]/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (4*b*x^n*PolyL
og[2, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])]/(a^2*Sqrt[-a^2 + b
^2]*d^2) + (2*b^3*x^n*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 +
b^2])])]/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (4*b*x^n*PolyLog[2, -((a*E^(I*(c +
d*x^n)))/(b + Sqrt[-a^2 + b^2])])]/(a^2*Sqrt[-a^2 + b^2]*d^2) - ((2*I)*b^
3*PolyLog[3, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])]/(a^2*(-a^2
+ b^2)^(3/2)*d^3) + ((4*I)*b*PolyLog[3, -((a*E^(I*(c + d*x^n)))/(b - Sqrt[
-a^2 + b^2])])]/(a^2*Sqrt[-a^2 + b^2]*d^3) + ((2*I)*b^3*PolyLog[3, -((a...

```

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 4692 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
-> Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

rule 4696 `Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
-> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sec[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n, p}, x]`

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{(a + b \sec(c + dx^n))^2} dx$$

input `int((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n))^2,x)`

output `int((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n))^2,x)`

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3831 vs. $2(1282) = 2564$.

Time = 0.45 (sec) , antiderivative size = 3831, normalized size of antiderivative = 2.77

$$\int \frac{(ex)^{-1+3n}}{(a + b \sec(c + dx^n))^2} dx = \text{Too large to display}$$

input `integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="fricas")`

output `Too large to include`

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{3n-1}}{(a + b \sec(c + dx^n))^2} dx$$

input `integrate((e*x)**(-1+3*n)/(a+b*sec(c+d*x**n))**2,x)`

output `Integral((e*x)**(3*n - 1)/(a + b*sec(c + d*x**n))**2, x)`

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{3n-1}}{(b \sec(dx^n + c) + a)^2} dx$$

input `integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="maxima")`

output

```

1/3*(6*a*b^3*e^(3*n)*x^(2*n)*sin(d*x^n + c) + (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n)*cos(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*e^(3*n)*x^(3*n)*cos(d*x^n + c)^2 + (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n)*sin(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*e^(3*n)*x^(3*n)*sin(d*x^n + c)^2 + 4*(a^3*b - a*b^3)*d*e^(3*n)*x^(3*n)*cos(d*x^n + c) + (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n) - 2*(3*a*b^3*e^(3*n)*x^(2*n)*sin(d*x^n + c) - 2*(a^3*b - a*b^3)*d*e^(3*n)*x^(3*n)*cos(d*x^n + c) - (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n))*cos(2*d*x^n + 2*c) + 3*((a^6 - a^4*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n + c)^2 + (a^6 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*sin(2*d*x^n + 2*c)*sin(d*x^n + c) + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n + 2*(2*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n*cos(2*d*x^n + 2*c))*integrate(2*(2*a^2*b^4*e^(3*n)*x^(2*n)*cos(2*c)*sin(2*d*x^n) + 2*a^2*b^4*e^(3*n)*x^(2*n)*cos(2*d*x^n)*sin(2*c) - 4*(a^3*b^3 - a*b^5)*e^(3*n)*x^(2*n)*cos(c)*sin(d*x^n) - 4*(a^3*b^3 - a*b^5)*e^(3*n)*x^(2*n)*cos(d*x^n)*sin(c) + (2*a^3*b^3*e^(3*n)*x^(2*n)*sin(d*x^n + c) - (2*a^5*b - a^3*b^3)*d*e^(3*n)*x^(3*n)*cos(d*x^n + c))*cos(2*d*x^n + 2*c) - ((2*a^5*b - 3*a^3*b^3 + a*b^5)*d*e^(3*n)*x^(3*n) - (2*a*b^5*e^(3*n)*x^(2*n)*sin(2*c) + (2*a^3*b^3 - a*b^5)*d*e^(3*n)*x^(3*n)*cos(2*c))*cos(2*d*x^n) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(3*n)*x^(3*n)*cos(c) + 2*(a^2...

```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{3n-1}}{(b \sec(dx^n + c) + a)^2} dx$$

input

```
integrate((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n))^2,x, algorithm="giac")
```

output

```
integrate((e*x)^(3*n - 1)/(b*sec(d*x^n + c) + a)^2, x)
```


Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a + b \sec(c + dx^n))^2} dx = \int \frac{(ex)^{3n-1}}{\left(a + \frac{b}{\cos(c+dx^n)}\right)^2} dx$$

input `int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n))^2,x)`output `int((e*x)^(3*n - 1)/(a + b/cos(c + d*x^n))^2, x)`**Reduce [F]**

$$\int \frac{(ex)^{-1+3n}}{(a + b \sec(c + dx^n))^2} dx = \text{too large to display}$$

input `int((e*x)^(-1+3*n)/(a+b*sec(c+d*x^n))^2,x)`

output

```
(e**(3*n)*(- 24*sqrt(- a**2 + b**2)*atan((tan((x**n*d + c)/2)*a - tan((x
**n*d + c)/2)*b)/sqrt(- a**2 + b**2))*cos(x**n*d + c)*a**2*b - 24*sqrt(-
a**2 + b**2)*atan((tan((x**n*d + c)/2)*a - tan((x**n*d + c)/2)*b)/sqrt(-
a**2 + b**2))*cos(x**n*d + c)*a*b**2 - 24*sqrt(- a**2 + b**2)*atan((tan(
(x**n*d + c)/2)*a - tan((x**n*d + c)/2)*b)/sqrt(- a**2 + b**2))*a*b**2 -
24*sqrt(- a**2 + b**2)*atan((tan((x**n*d + c)/2)*a - tan((x**n*d + c)/2)*
b)/sqrt(- a**2 + b**2))*b**3 + 2*x**(3*n)*cos(x**n*d + c)*a**4*d**3 - 3*x
**(3*n)*cos(x**n*d + c)*a**3*b*d**3 - 3*x**(3*n)*cos(x**n*d + c)*a**2*b**2
*d**3 + 96*cos(x**n*d + c)*int((x**(3*n)*tan((x**n*d + c)/2)**4)/(4*tan((x
**n*d + c)/2)**4*a**8*x - 4*tan((x**n*d + c)/2)**4*a**7*b*x - 11*tan((x**n
*d + c)/2)**4*a**6*b**2*x + 10*tan((x**n*d + c)/2)**4*a**5*b**3*x + 11*tan
((x**n*d + c)/2)**4*a**4*b**4*x - 8*tan((x**n*d + c)/2)**4*a**3*b**5*x - 5
*tan((x**n*d + c)/2)**4*a**2*b**6*x + 2*tan((x**n*d + c)/2)**4*a*b**7*x +
tan((x**n*d + c)/2)**4*b**8*x - 8*tan((x**n*d + c)/2)**2*a**8*x - 8*tan((x
**n*d + c)/2)**2*a**7*b*x + 22*tan((x**n*d + c)/2)**2*a**6*b**2*x + 24*tan
((x**n*d + c)/2)**2*a**5*b**3*x - 18*tan((x**n*d + c)/2)**2*a**4*b**4*x -
24*tan((x**n*d + c)/2)**2*a**3*b**5*x + 2*tan((x**n*d + c)/2)**2*a**2*b**6
*x + 8*tan((x**n*d + c)/2)**2*a*b**7*x + 2*tan((x**n*d + c)/2)**2*b**8*x +
4*a**8*x + 12*a**7*b*x + 5*a**6*b**2*x - 18*a**5*b**3*x - 21*a**4*b**4*x
+ 11*a**3*b**5*x + 6*a*b**7*x + b**8*x),x)*a**11*b*d**3*n - 96*cos(x**n...
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 602
4.2 Links to plain text integration problems used in this report for each CAS . 620

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                               Small rewrite of logic in main function to make it*)
(*                               match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCountOptimal},
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A",""}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
          ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
    ,(*ELSE*)(*result does not contains complex*)
    If[leafCountResult<=2*leafCountOptimal,
      finalresult={"A",""}
      ,(*ELSE*)
      finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
      ]
    ]
  ,(*ELSE*)(*expnResult>expnOptimal*)
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    finalresult={"C","Result contains higher order function than in optimal. Order "
    ,

```

```

        finalresult={"F","Contains unresolved integral."}
    ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],

```

```
Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
If[AppellFunctionQ[Head[expn]],
Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
If[Head[expn] === RootSum,
Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
If[Head[expn] === Integrate || Head[expn] === Int,
Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
MemberQ[{
Exp, Log,
Sin, Cos, Tan, Cot, Sec, Csc,
ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
Sinh, Cosh, Tanh, Coth, Sech, Csch,
ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
}, func]
```

```
SpecialFunctionQ[func_] :=
MemberQ[{
Erf, Erfc, Erfi,
FresnelS, FresnelC,
ExpIntegralE, ExpIntegralEi, LogIntegral,
SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
Gamma, LogGamma, PolyGamma,
Zeta, PolyLog, ProductLog,
EllipticF, EllipticE, EllipticPi
}, func]
```

```
HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
```

```
AppellFunctionQ[func_] :=
MemberQ[{AppellF1}, func]
```

Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B","result has leaf size over 500,000. Avoiding possible recursion issue";
      fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result := ExpnType(result);
      ExpnType_optimal := ExpnType(optimal);

      if debug then
          print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal

```

```

#   antiderivative
#   "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (" ,
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf

            end if
        else #result contains complex but optimal is not
            if debug then
                print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
        # this assumes optimal do not as well. No check is needed here.
        if debug then
            print("result do not contain complex, this assumes optimal do not as well
        fi;

```



```

        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of
                            convert(leaf_count_result,string)," $ vs. $2(",
                            convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;
end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function

```

```

# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+'') or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9

```

```

    end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,
    GAMMA, lnGAMMA, Psi, Zeta, polylog, dilog, LambertW,
    EllipticF, EllipticE, EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.

```

```
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc;
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]
```

```

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')

```

```

    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""

```

```

else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = ""
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType

#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

```

```

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arcsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:

```



```

    if m:
        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral',
        'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
        'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
    if m:
        print ("func ", func , " is special_function")
    else:
        print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False

```

```

if debug:
    print ("Enter is_atom, expn=",expn)

if not hasattr(expn, 'parent'):
    return False

#thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print ("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)

```

```

    return expnType(expn.operands()[0]) #expnType(expn.args[0])
elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
    if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

if debug:
    print ("Enter grade_antiderivative for sagemath")
    print ("Enter grade_antiderivative, result=",result)
    print ("Enter grade_antiderivative, optimal=",optimal)
    print ("type(anti)=", type(result))
    print ("type(optimal)=", type(optimal))

```

```

leaf_count_result = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

expnType_result = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result - 2*leaf_count_optimal)
else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_result - expnType_optimal)

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file